MANNING RETARDANCE COEFFICIENTS FOR UPRIGHT STEMS OF VEGETATION IN CHANNELS WITH MODERATE FLOW VELOCITIES

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

For bare earth waterways such as those for carrying water on farm lands, spillways of small flood control reservoirs, terrace outlet channels, drainage ditches of highways and airports, and similar conservation usage, a lining of grass is found to be advantageous. The effect of grass is to stabilize the body of the channel, form a mat of vegetation, and resist the scouring force of flows which occur infrequently but are rapid and of great rates because of the particular characteristics of those waterways.

The investigation of the hydraulic characteristics and performance of grass lining in such channels has been developed for the past several decades, and some useful discoveries have been made to contribute to the practical design and field application. The research and observations have been undertaken mostly by the Agricultural Research Service, U.S.D.A., through numerous experiments.

Yet, analytical approach to explore the inter-dependent relationship among the possible factors affecting the hydraulic characteristics of flow in this specific type of channel has seldom been carried out. However, after a broad study of the related literature in contemporary hydraulics, it could be concluded that no satisfactory theory is available for purely theoretical and exact solution of flow problems in ordinary alluvial channels. In grassed channels, the problem is much more complex, and the existing theories which are based on simplified conditions are unlikely applicable to practical engineering use in such channels.

While investigations are still being undertaken by many workers to find a new universal equation that involves every possible factor based on modern concept of mechanics of turbulent flow, to replace the rather empirically natured formulas in open channel problems, there is likely no choice other than to employ the widely used formula such as Manning's.

The Manning formula is written as

$$V = \frac{1.486}{n} R^{2/3} S^{1/2}, \qquad (1)$$

where V is the mean velocity in fps, R the hydraulic radius in ft., S the energy slope and n is the mean roughness coefficient and considered to involve all factors affecting the resistance to the flow. In other words, it is a general measure of flow resistance in the channel, including the resistance due to the boundary and that due to obstacles in the flow.

In usual open channel flow problems, the value of n is most difficult to determine. The accuracy of selected n values is rather dependent on the exercise of sound judgement based on experience, and sometimes it even inclines to be a guesswork. From the theoretical point of view, the value of n could be determined analytically by the theoretical velocity distribution in the channel cross section. However, as it was pointed out, the purely analytical solution of flow resistance by the application of the existing theories explaining such law would not be achieved.

Actually, the value of n is highly variable and depends on a number of factors. However, in a particular problem, the factors considered to be involved in the n value can be carefully investigated. If there is a certain physical relationship between n and those factors, it is also possible to reduce those factors to some functional relationship that is expressible in a generalized form.

A rational approach to the determination of n value is proposed in this thesis by dimensional consideration to investigate the physical quantities that are regarded to be involved in the problem. A hypothesis of approximate functional relationship of the variables was set up and tested by appropriate experimental data.

The study was limited to the type of vegetation that had up-right stems which were not wholly submerged in the flow of moderate velocity. Further rigorous research is necessary for the extension of this study.

CHAPTER II

OBJECTIVES

The general objective of this thesis was to determine the hydraulic effect of grasses that have up-right stems on the retardance of flow in earth channels. The following particular studies were made:

1. By dimensional consideration, to establish a possible functional relationship among the physical variables involved in the problem.
2. To analyze and determine quantitatively the relationship of the Manning mean coefficient of roughness with the physical characteristics of vegetation and the variation of earth bottom.

CHAPTER III

REVIEW OF THE LITERATURE

Historical Development of Open Channel Formula

The research of stability of erodible channels has absorbed much effort and keen concern in any field of engineering related with the design of open channels. Lining with vegetation to check erosion in subsidiary or secondary water conveyance systems could be regarded as belonging to the method of permissible velocity. The permissible velocity is the maximum value of non-erodible mean velocity that will not cause scour of the channel body. Apparently it is rather a relative concept, since this velocity is very uncertain and indefinite, and often based on experience and judgement.

The concept of permissible velocity could be considered to be derived from the so-called regime theory, which was developed among the British engineers engaged in the reclamation of Northern India during the late 19th and early years of this century. The Kennedy formula

$$v_o = Cy^x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

and the Lacey formula

$$\frac{v^2}{R} = 1.325\, f_{VR} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

are the representatives of their concept, in which

v_o = the non-silting and non-eroding mean velocity in fps,

y = the depth of flow in ft.,

$x = 0.64$, an exponent which varies only slightly,

$C = 6.84$, depending primarily on the firmness of the bed material,

v = the average velocity in the flow,

R = the hydraulic radius of the channel,

and f_{VR} = the silt factor.

The formula has been extensively used in India and Pakistan for the design of irrigation canals.

The author introduces these formulas here as reference tracing the historical development of the concept and theory of treating the scouring problems on open channels.

Not specifically for the artificial canals, but for general alluvial channels, investigation of erosion problems has been extensively undertaken more recently appearing as sedimentation hydraulics in the form of Einstein's bed-load function and Maddock-Leopold's principle of channel geometry (2,3).

However, apparently those methods of approach cannot directly be applicable to the design of channels in the conservation field, for the flow in those channels is clear or with slight sediment load which is of minor importance, and the boundary geometry could generally be considered rigid.

In practical hydraulic literature, the equations of estimating mean velocity or discharge is for clear flow in rigid conduits, either pipes or open channels. For the latter case, the mean velocity of a uniform flow is usually expressed approximately by a so-called uniform-flow formula, generally in the form of

$$v = CR^xS^y \qquad (4)$$

where v = the mean velocity in fps,

R = the hydraulic radius in ft.,

S = the energy slope,

C = the coefficient depending on the boundary roughness.

The well-known Chezy's formula and Manning's formula have been most widely used, especially the latter one in the United States, for open channel flow computations.

The Manning formula was presumably derived from the classical hydraulic point of view, the Manning n is evaluated only regarding the effect of boundary roughness. The fluid properties, of which viscosity is of primary importance, are excluded from consideration.

In the light of the theory of the mechanics of turbulent flow along boundaries, it has been revealed that the mean velocity of a turbulent flow depends on the velocity distribution. The problem of the velocity distribution of turbulent flow in open channels can be stated as essentially one of a turbulent boundary layer. A brief review of the literature would be helpful for comprehension of evaluating mean velocity in open channels.

Concept of Velocity Distribution

Hydraulic engineers frequently use the logarithmic law proposed for velocity distribution in turbulent flow, which is obtainable from Prandtl's or Karman's hypothesis. The law can be written as

$$\frac{u}{v*} = \frac{1}{k} \ln \left(\frac{y}{y'}\right) \qquad (5)$$

in which u is the local mean velocity along the flow direction at a distance y from the boundary, v* is the shear velocity $\sqrt{\frac{\tau_o}{\rho}}$ in which τ_o is the local boundary shear and ρ is the mass density of fluid, k is the so-called Karman universal constant and the value of y' is dependent on a length parameter representing the hydraulic condition of the boundary

Keulegan (8) developed the formulas of mean velocity for turbulent flow in smooth and rough pipes, and also applied Nikuradse's results for turbulent flow in pipes to open channels. It was shown that when the hydraulic radius was used as the characteristic length, the Nikuradse formula for pipe flow can be applied to open channel flow.

However, Powell (12) has pointed out that such an extension of Nikuradse's results to open channels cannot be satisfactory since there is a free surface in the open channel flow.

A number of investigators working with open channel flow have proposed the use of Chezy's formula

$$V = C \sqrt{RS} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which C is the resistance coefficient. Some workers (12,13) have proposed for rough channels a relationship of the form

$$C = a + b \log_{10}(R/k) \ldots\ldots\ldots (7)$$

The constants a and b must be determined experimentally. This equation is of the analogous form as the Karman-Prandtl equation for rough pipes. Values of these constants have been determined for various types of artificial roughness. Robinson and Albertson (13) conducted an investigation of artificial roughness in open channels and obtained the resistance formula as

$$C = 26.65 \log_{10} (1.891\ d/a) \ldots (8)$$

where d is the mean depth of flow and a is the height of artificial roughness, which is similar to the general form of Eq. (7). Strictly empirical equations cannot be justified for their use merely on the basis of their simplicity. It would be questionable for using supposedly simple flow equations when supposedly constant retardance coefficients are found to be not constant.

The foregoing concepts and hypotheses trying to explain the distribution law and thereby solve the turbulent flow equations are classified as phenomenological theories (6), because they are on the basis of observation that in turbulent shear flow there is a lateral transport of fluid between zones of high and low velocity brought about by the turbulent mixing process and conclusion that the mixing process and the velocity gradient are interrelated. However, there has been considerable criticism of those theories, mainly concerning inferences as to the details of the structure of the turbulence. It can be pointed out that the logarithmic law is not applicable to the flow near the center of the pipe, as it is evident from the assumptions made in its derivation. Furthermore, it has been found that without employing the Prandtl-Karman hypotheses the velocity distribution would satisfy linear logarithmic relations in the overlapping zone of turbulent boundary layer (6).

While theories concerning the turbulent flow mechanism are still being developed (6), at the present stage there is no satisfactorily rigorous theory of turbulent flow that can give the complete solution of velocity distribution in turbulent flow.

The following remark may be made concerning boundary roughness, that for a rough boundary the effect of viscosity on the velocity distributions is of negligible importance. Therefore a formula like Manning's which does not take into account the concept of viscosity would be limited to the case of turbulent flow along rough boundaries (5,14).

Vegetated Channel Hydraulic Experiments

The investigation of hydraulics of vegetated channels, on the other hand, has been carried out by wholly empirical observations. M. B. Cox (16) made some experiments during 1939 to 1944, and the results have been reported. Experiments made by W. O. Ree and V. J. Palmer (18,19,20,21) both at Spartanburg, South Carolina, and Stillwater, Oklahoma, have contributed practical and useful information to the solution of grassed channels and the experiments have been continued. The Manning formula is used to compute flows in grassed channels, and Manning's coefficient of roughness is specifically known as the retardance coefficient. According to the investigation by W. O. Ree and the Agricultural Research Service (20, 21), it was found that Manning's n for just one kind of grass varied over wide range depending on the depth of flow and the slope of the channel. It seemed the selection of a design value for n would be nearly impossible. However, it was discovered that the retardance coefficient n has a certain relationship with the product of the mean velocity of flow, V, and the hydraulic radius, R. This relationship is characteristic of the vegetation and practically independent of channel slope and shape. Thus a number of experimental curves for n against VR were developed for five different degrees of retardance: very high, high, moderate, low and very low. The results can be summarized as follows:

1. The hydraulic behavior of grasses is similar if their physical characteristics are similar.
2. If R is considered as a linear characteristic of the channel, and since kinematic viscosity of water is practically constant throughout the experiment, VR value can be regarded as a measure of the

Reynolds number.

3. The retardance of the grass could be divided into two considerably different stages: when grasses are not submerged and when grasses are submerged. The transition between the two stages is abrupt.

The practical application of the above results to the design of grassed channels is given in the Handbook of Channel Design for Soil and Water Conservation, U. S. Soil Conservation Service (22). The charts are produced by extensive experiments, and the solution of problems is given by direct application of the test data. The functional relationship among the physical quantities or variables in the flow has not yet been established.

Theory of Dimensional Analysis

An experiment that involves considerable number of variables can be processed systematically and given proper control by predicting their functional behavior employing dimensional analysis, if it is properly handled.

In modern mechanics of fluids, dimensional analysis appears to be a powerful measure in solving many problems and has been extensively applied. Individually, the theory of dimensional analysis has been remarkably developed as a branch of applied mathematics in physical problems since Supre, Lord Rayleigh and Buckingham. Since this method will be used in analyzing the problem of resistance of grasses in vegetated channels, a brief review of the theory will help grasp the concept.

The application of dimensional analysis to a practical problem is based on the hypothesis that the solution of the problem is expressible by means of a dimensionally homogeneous equation in terms of specified variables. An equation is said to be dimensionally homogeneous if the form of the equation is independent of the fundamental units chosen for measurement. Evidently, any equation that relates dimensionless products is dimensionally homogeneous; i.e., the form of the equation does not depend on the fundamental units of measurement. If formally stated, a sufficient and necessary condition that an equation be dimensionally homogeneous is that it be reducible to an equation among dimensionless products. The entire theory of dimensional analysis can be summarized by Buckingham's theorem, which states as follows:

> If an equation is dimensionally homogeneous, it can be reduced to a relationship among a complete set of dimensionless products.

A non-dimensional product of n dimensional quantities can be represented generally in the form

$$\pi = A_1^{x_1} A_2^{x_2} A_3^{x_3} \ldots\ldots A_n^{x_n} \ldots\ldots (9)$$

in which the exponents are pure numbers of such magnitudes that the net power of each of the m dimensional categories involved is reduced to zero. The condition for which the exponents of dimensions, say L, T, and M will be zero is represented by the independent simultaneous equations

$$a_1x_1 + a_2x_2 + \ldots\ldots + a_nx_n = 0$$

$$b_1x_1 + b_2x_2 + \ldots\ldots + b_nx_n = 0$$

$$c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n = 0$$

If in general, the coefficients of any set of linear equations form a matrix in which the number of any set of column n is greater than the number of rows m, then the rank r of the matrix is the maximum order of the non-vanishing determinants that can be formed from the m rows and n columns. It can be proved by the theory of linear algebra that there is a unique solution of the equations for any r terms for which the determinant of the coefficients is not zero, whatever choice is made for the remaining terms other than all zeros. The theory further states that there are only n - r linearly independent solutions; that is, any other choice of the remaining terms will simply yield combinations of those already obtained. In the general case, therefore, the number of linearly independent solutions of the linear equations for the powers of the dimensional quantities also gives the number of non-dimensional products in a complete set. Then, the coefficients a_i, b_i, etc., are the rows in the dimensional matrix.

The well-known and useful principle π-theorem can be stated as follows:

> The number of dimensionless products in a complete set is equal to the total number of variables minus the rank of their dimensional matrix.

Thus, if

$$f\ (A_1, A_2, \ldots A_n) = 0$$

The π-theorem states these n dimensional variables can be combined into an equally valid expression involving r fewer terms,

$$F\ (\pi_1, \pi_2, \ldots \pi_{n-r}) = 0$$

each of these non-dimensional variables being composed of $r + 1$ of the original quantities.

Buckingham has pointed out that we obtain the maximum amount of experimental control over the dimensionless variables if the original variables that can be regulated each occur in only one dimensionless product. In general, in choosing the m variables that are to occur in each of the dimensionless groups, the following conditions are to be satisfied: first, these repeating variables must together include all m dimensional categories; second, they must not be sufficient in themselves to form a dimensionless group. Selection of the quantities satisfying these requirements should be dependent on the results desired. Generally speaking, the selection of a length, a velocity, and a density will be expedient as repeating variables in usual fluid problems. Langhaar (25) presents the following rule for the arrangement of variables in dimensional matrix:

> In the dimensional matrix, let the first variable be the dependent variable. Let the second variable be that which is easiest to regulate experimentally. Let the third variable be that which is next easiest to regulate experimentally, and so on.

The dimensionless groups that are obtained from the preceding procedure are rather arbitrary. Buckingham (27) has suggested that some sets of products are more useful in practice than others to achieve greater experimental control of variables. Occasionally transformations are desirable. When it is found that a certain variable that was introduced in the dimensional matrix has a negligible influence on the phenomenon and that that variable occurs in more than one dimensionless product, the transformation will be taken place to obtain another complete set of dimensionless products.

CHAPTER IV

EXPERIMENTAL FACILITIES AND PROCEDURE

Test Channels

The data used in this thesis were the results of a series of experiments run in the Stillwater Outdoor Hydraulic Laboratory which is a facility of the Agricultural Research Service and is located 10 miles west of Stillwater, Oklahoma, adjoining to the Blackwell Lake.

Eight flat-bottomed, earth channels, especially called unit channels, were used for the experiment. The channel dimension was 3 ft. wide and 96 ft. long approximately (Fig. 1), with considerably steep average slope of 5%. The sides were fixed by concrete with the top about 3 in. above the bottom (Fig. 2). When the flow depth increased during tests, smooth wooden side walls were erected on the top of the concrete, preventive measures being taken against leakage. The effect of wall friction on flow was very small. Thus, the individual channel could be considered as an elementary portion of a very wide, flat channel in the field. The name unit channel was given for this reason.

The soil of the bottom was silt loam, the average density of which is given in Table I. In constructing the bottom, the top ground was first removed about 8 to 10 in., and the silt loam was placed and leveled off by a heavy board. The soil was then packed by a lawn roller moving forward and back once.

The vegetation was planted by the personnel of the Department of

Agronomy using a fertilizer spreader. It was so seeded that the plant population was supposed to be one plant in every square inch of the bottom. But the actual result might not be strictly as planned. The kinds of vegetation in the various channels with their age when the tests were run are given in Table I.

TABLE I

PHYSICAL CONDITION OF THE TEST CHANNELS

Channel Number	Average Soil Density (lbs/ft^3)	Vegetation	Age of Vegetation (days)	Vegetation Population ($Stems/ft^2$)
5	82.0	Sudan	17	47
6	82.0	Sudan	17	57
7	82.0	Sudan	17	42
8	82.0	Sudan	17	52

The population of the plants in each channel was estimated by the personnel in the Department of Agronomy.

Test water was conveyed by 12 in. pipe from the adjoining lake to the test channel upper ends, where a stilling apparatus leads the inflow water into the channels (Fig. 3). The flow rate was measured by an orifice meter.

In the following analysis the data from an experiment performed in 1946 were adopted in order to test the validity of the hypothesis that the relationship derived in the theory based on dimensional analysis is applicable to the kind of plant that has similar physical condition of up-right plant. The 1946 experiment was run for mixed grasses of sudan and crab grass. The data are shown in Table III.

Test Procedure

The retardance effect of vegetation has a certain relationship with the age of the plants, because the physical conditions of vegetation are variable with its age. When the vegetation grew up to a certain stage, the flow tests were run. The procedure of a single test was as the following:

1. The channel cross-sections were taken at the nine selected stations.
2. A pre-assigned amount of flow was led into the channel, and was kept steadily during the test interval, which was arbitrarily chosen to be forty minutes.
3. When equilibrium was reached, the water-surface elevation was measured at each of fifteen substations across the channel.

The details of measurements will be described in the following section.

In this experiment, the following assumptions were made:

1. In some shallow flows, meandering of the path of flow was observed. However, uniform flow was assumed, and the Manning formula was employed in the hydraulic computations.
2. Infiltration loss might be considerable, especially for low flows. But actually clay content in the soil of the channel bottom was high, and this kind of loss was therefore ignorable. Infiltration rate was estimated to be maximum 0.2 in. per hour generally.
3. Erosion was taking place uniformly with respect to time under consideration.

Measurements and Observations

Measurement of Bottom Elevations

Along the channel, nine stations were established. The locations of the stations along the longitudinal section of a channel are shown in Figures 1 and 2. At each station there was a 3 x 3 x $\frac{1}{4}$-in. steel cross-angle carefully fixed level by two columns. The cross-angles were about 3.0 ft. high from the datum bolts, which were set at each station on the concrete walls. The datum bolts were checked for elevations by leveling precisely by Engineer's level before the experiment. At each station, the cross-angle was marked that 15 vertical readings at 0.2 ft. intervals except at the end points where the distance from the edge of the concrete sides was 0.1 ft., could be taken. Bottom elevation observations were taken both before and after each test. Before the first measurement would be taken, the bottom of the channel was wetted moderately. The measuring instrument was a vertical gage with blunt point rod named point gage, mounted on the cross-angle. The degree of roughness of the steel cross-angle surface was approximately 0.005 ft., and the error due to this roughness in measuring would be balanced if the readings were always taken on the same points of observation.

Each three stations in a group located at the upper, middle, and lower reaches were observed by a single observer, and the observation was made simultaneously at the three reaches.

The contact of the point of the point gage on the ground surface was usually visible, and care was always taken not to cause error by penetrating the rod into the top soil.

The point gage zero was defined as the value obtained by subtracting the reading of the gage on the datum bolt from the known value of elevation of the datum bolt. The elevation of a particular point on the bottom was obtained by adding the point gage reading on the point to the point gage zero reading. The sample is given in the Appendix A.

Measurement of Channel Width at the Stations

The channels were so constructed as to have nominal width of 3 ft., but the accurate value at each station was measured by a measuring rod just on the ground surface, at the point 1 in. above, and at the point 2 in. above the ground surface.

Measurement of Test Flow

The flow rate in each test was pre-determined arbitrarily, and it was kept nearly equal for the same number of test in all channels. The discharge was measured by an orifice meter and the amount was precisely determined by taking 10 readings successively on the manometer. Water temperature for each test was noted.

Measurement of Elevation of Water Surface

The general procedure was the same as the bottom measurement. When the water was turned in and when it was considered to reach steady condition, the observation was made simultaneously at the upper, middle and lower stations, so that the error due to the time lag in the observation by a single man from the upper reach through to the lower reach could be eliminated, since during the time lag there might be erosion which would cause some experimental error. The error due to the surface roughness of the cross-angle in measuring the slope of the channel water surface was supposed to be 0.02 ft. at the maximum. The first observation was taken 15 minutes after water was led in, and another set of observations was made just before the flow was shut-off. Instead of a blunt point rod, a sharp point rod was used in the point gage to measure water surface elevations. The sample notes are given in the Appendix B.

Measurement of Plant Population and Stem Diameters

The measurement of plant population, the number of stems per square feet of bottom area, was carried out by the personnel of the Department of Agronomy. Thirteen random observations were taken for each channel. The data are shown in the Appendix C with the standard deviation of the population distribution of the plants calculated. It is seen that the plant distribution was not uniform. The mean population of each channel was given in Table I.

The plant stem diameter was measured for selected typical plants on the enlarged photograph on which a scale is on the same vertical plane of the plants (Fig. 4). The data are given in Table III with the record of the 1946 experiment of sudan and crab-grass.

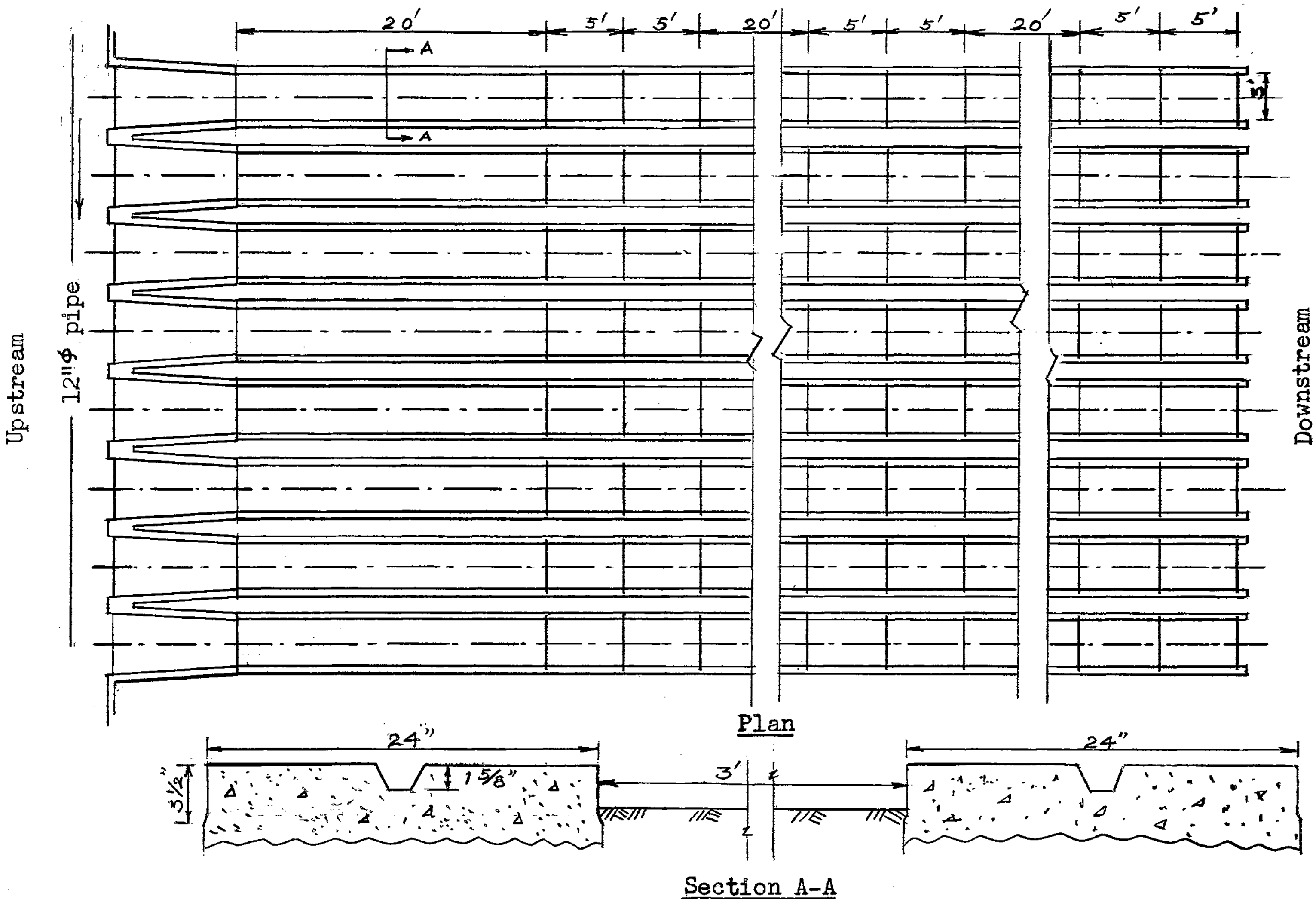

FIG. 1 Dimensions of Test Channels

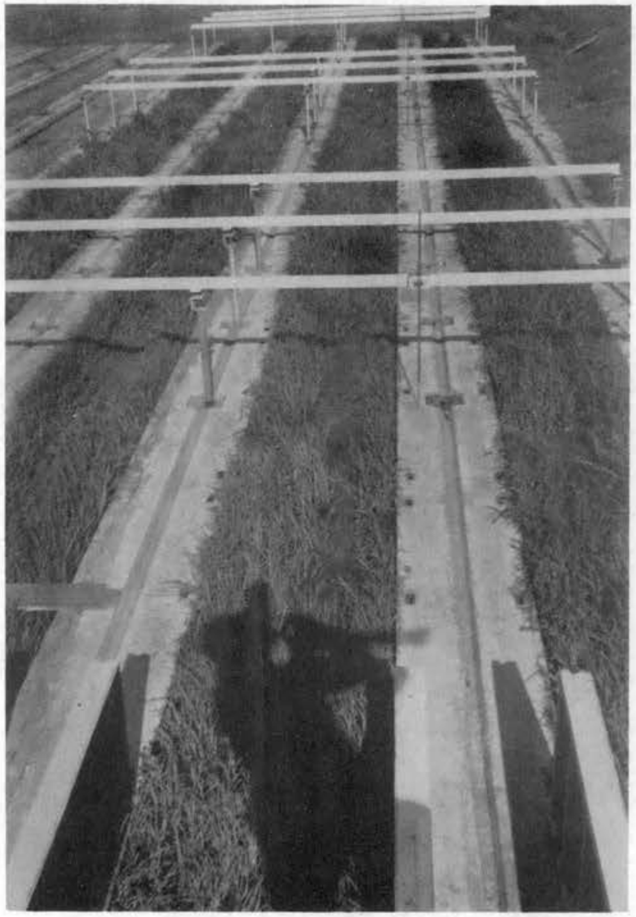

Figure 2.
General View of the Test Channels Looking Downstream.

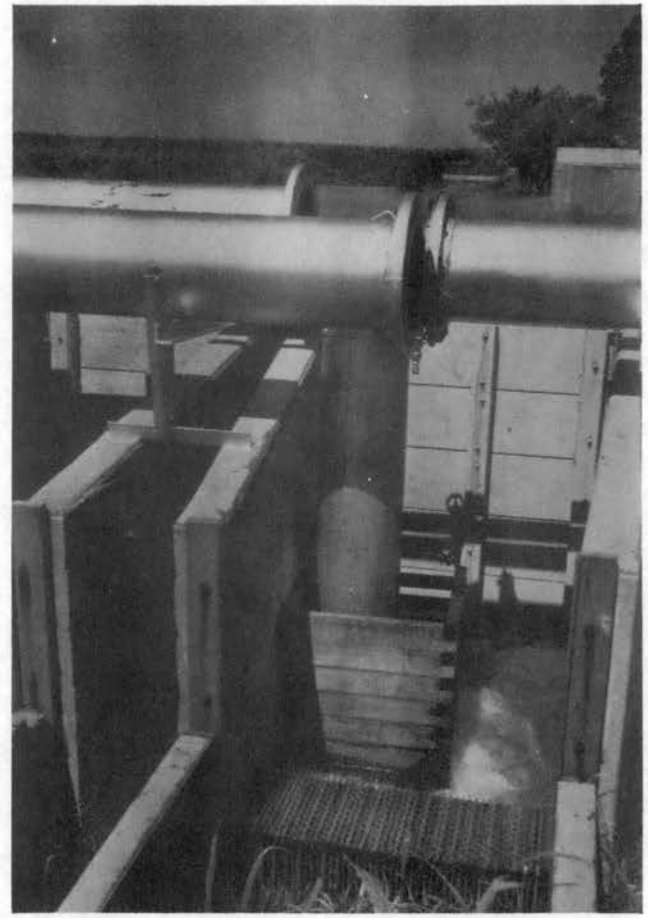

Figure 3.
Apparatus for Stilling Flow at Upstream End of Channels.

Figure 4.
Typical Sudan Grass Plants
Used in Tests.

Figure 5.
A View of a Unit Channel after a Test.

CHAPTER V

ANALYSIS OF DATA

The Dimensional Analysis

In the following analysis, the length-time-mass system will be employed. The three primary quantities are expressed symbolically by L, T and M, respectively.

The variables in this particular problem of flow are selected with their dimensions as follows:

R is the hydraulic radius, the variable expressing the characteristic of the geometry of the boundary. There has been controversy about the use of hydraulic radius to denote a linear dimension of channel geometry, for it is a hypothetical length derived as the quotient of channel cross-sectional area by the wetted perimeter. However, in this experiment, R is actually equal to the mean depth of flow, as will be seen in the later discussion. The dimension is L.

V is the mean velocity of flow through the cross-section of the of the channel. The dimensions are LT^{-1}.

N is the population of plants in the flow per square feet of the channel bottom, the dimension of which is L^{-2}.

D is the mean diameter of plant stems in the flow, and is expressed as L.

μ is the dynamic viscosity of water, having the dimensions of $ML^{-1}T^{-1}$. The concept of viscous effect on fluid motion is much emphasized in the modern mechanics of fluid (5), the dynamic viscosity as a parameter denoting viscous influence must be taken into account.

σ_b is the standard deviation of the bottom variation computed from the bottom readings of the point gage. This can be considered as expressing the bottom roughness and has the dimension of L.

ρ is the mass density of water, a characteristic of the fluid. The dimensions are ML^{-3}.

g is the gravitational acceleration, having the dimensions of LT^{-2}. Since there exists free water surface, and the entire flow is subjected to the gravitational force, g is considered to be included in the variables.

n is the Manning resistance coefficient, and here it is the dependent variable. The dimension of n is rather controversial. From the Manning formula, it seems that the n dimension would take $TL^{-1/3}$. Since it is unreasonable to suppose that the roughness coefficient would contain the dimension T, Keulegan (8) assumed that the numerator contains $\sqrt{g}$, thus yielding the dimensions of $L^{1/6}$ for n. Besides, it can be shown that n = $[\phi(R/K)]\,K^{1/6}$ (7), where K is a linear measure of roughness and ϕ (R/K) is a function of R/K. Since R/K is dimensionless, n will have the dimensions of $K^{1/6}$, namely, $L^{1/6}$. From the above dimensional reasoning, n is here considered to have the dimensions of $L^{1/6}$.

R, N, D and σ_b can be combined to yield the dimensionless products

$$\Pi_1 = \frac{\sigma_b}{R}$$

$$\Pi_2 = \frac{D}{R} \qquad (11)$$

and

$$\Pi_3 = NR^2$$

hence N, D and σ_b can be excluded from the dimensional matrix. The rest of the variables yield a dimensional matrix as follows:

	n	V	R	ρ	μ	g
L	1/6	1	1	-3	-1	1
T	0	-1	0	0	-1	-2
M	0	0	0	1	1	0

The rank of the matrix is determined by the determinant picking up arbitrarily from the right-hand corner of the matrix, which is written as

$$\Delta = \begin{vmatrix} -3 & -1 & 1 \\ 0 & -1 & -2 \\ 1 & 1 & 0 \end{vmatrix}, \qquad (12)$$

which is not an identity of zero. The non-vanishing determinant, Δ, is of third order. The rank of the matrix is thus seen to be 3.

The number of variables arranged in the matrix is 6, from which subtracting the rank of the matrix, 3, we must have three dimensionless products constituted by those variables, which are independent.

The most probable way of combining the 6 variables into dimensionless products will, by inspection, be

$$\Pi_4 = \frac{R^{1/6}}{n}$$

$$\Pi_5 = \frac{\rho VR}{\mu}$$
$$\Pi_6 = \frac{V^2}{Rg} \qquad (13)$$

The above π -terms can also be produced by taking ρ, V and R as the repeating variables, and the rest of the variables that each occurs in only one Π -term, and solving the power coefficient equations of demensions, the same results as those in (11) and (13) will be obtained.

If R is set equal to the mean depth of the flow in this experiment, the term Π_5 is just the Reynolds number, and if we take the square root of Π_6, as it is still dimensionless, it is just the Froude number. The general equation then will be the form

$$\frac{R^{1/6}}{n} = f\left(\frac{\sigma_b}{R}, \frac{D}{R}, NR^2, R_e, F\right) \quad , \ldots(14)$$

in which R_e represents the Reynolds number, and F the Froude number.

For the convenience, a transformation will yield

$$\Pi_7 = \Pi_1 \cdot \Pi_2 \cdot \Pi_3 = \sigma_b DN \quad \ldots\ldots\ldots\ldots(15)$$

Due to the scope of the experimental data available, it was decided to exclude the terms $\frac{\sigma_b}{R}$ and D/R from the correlation analysis. The study was restricted to the two Π- terms below which were the ones of primary interest.

By the above transformation in (15), we may employ the new dimensionless term, Π_7, which can be given a meaning of expressing the gross resistance elements of the boundary and the obstacles in the flow, in the place of Π_3. The equation will become

$$\frac{R^{1/6}}{n} = g\left(\sigma_b DN, R_e, F\right) \quad \ldots\ldots\ldots\ldots(16)$$

In case that the flow is such that the turbulance is significantly developed, the effect of viscosity is very small and the Reynolds number may be discarded. As R is equal to the mean depth of flow, the unit value of $\sqrt{gR}$ is identified as the celerity of the small gravity waves that occur in shallow water in channels due to any momentary change in the local depth of the water. In this experiment, though actually there is free water surface in the channel, and there might be waves existing on the water surface, but the influence and hence the significance is so small that it could be ignored, the Froude number can be dropped.

The above discussion finally leads to the possible functional relationship

$$\frac{R^{1/6}}{n} = \phi\,(\sigma_b DN), \ldots\ldots\ldots\ldots\ldots(17a)$$

or

$$n = \psi\,(R^{1/6}, \sigma_b DN), \ldots\ldots(17b)$$

where $\phi(\sigma_b DN)$ denotes the function of $\sigma_b DN$, and $\psi(R^{1/6}, \sigma_b DN)$ the function in terms of $R^{1/6}$ and $\sigma_b DN$.

There might be an argument that the flexibility of the vegetation stems has considerable effect on the resistance to flow. Here we may limit for the time being our consideration to the stage of the depth of flow in which the vegetation is not submerged to the extent as to cause serious bending of the vegetation. Within this scope of discussion, the complex effect of leaves can also be excluded.

Hydraulic Computations

A transformation of the Manning formula yields

$$n = \frac{1.486}{V} R^{2/3} S^{1/2} \ldots\ldots\ldots\ldots\ldots(18)$$

If all of the elements in the right-hand side of Eq. (18) are known or can be calculated from the experiment, n - value can be determined.

As will be seen in the Appendix D, the channel characteristics such as the cross-sectional area, A, the wetted perimeter, w.p., and hence the hydraulic radius were determined at each station along the channel, and the retardance coefficients were computed for reaches in each channel. Six reaches were taken from the combinations of stations as in Table II.

TABLE II

THE CLASSIFICATION OF CHANNEL REACHES

Reach	Upper End	Lower End
A	Sta. 20	Sta. 50
B	25	55
C	30	60
D	50	80
E	55	85
F	60	90

As discharge, Q, was preassigned and measured by an orifice meter, the mean velocity, V, can be calculated from the determined cross-sectional area. The cross-sectional area at each station along the channel was taken as the actual area occupied by the flowing water, and was determined by computation or graphical solution from the data of the width, bottom and water surface observations. For determination of the n-value in a reach, those data of the channel characteristics for the reach were obtained by simply taking the mean value of those at the upper and lower ends. The mean velocity, assumed as steady - uniform in a reach, was then computed from the mean cross-sectional area and the discharge.

In calculating the slope of energy line along the channel, the mean velocity through each station was first determined and the value of the velocity head was obtained, which was added to the water surface elevation at each station. The slope of the energy line of a reach was thus determined from the difference of the energy heads at the upper and lower ends and the precisely measured slope distance between the ends.

As every element was determined, substitution of those data into Equation (18) yielded the n-value for the reach. The sample table of calculation is seen in the Appendix D.

Some assumptions were made for the computation:

(a) The uniform variation in both dimensions, i.e., along the longitudinal direction and the transverse section of a reach and hence, of the channel, was assumed. Consequently, the actual variation between the two ends of the reach was neglected.

(b) The area occupied by the plants in the flow was not considered. This caused the calculated area being somewhat greater than the actual, and consequently to obtain smaller velocity.

(c) By hypothesis, the wall effect on the flow was neglected. However, the effect would actually be small, the wetted perimeter was determined by only taking the width of the cross-section. This is also the reason that the hydraulic radius, R, is actually equal to the mean depth of flow in this experiment.

(d) The influence of suspension load being carried in the flow caused by scouring was ignored.

Bottom Variation Computations

The frictional resistance of the channel bottom is dependent on its roughness. The channel bottom was originally constructed as smooth and uniform as possible. Thereafter the roughness was increased as the bottom was eroded by the flow (Fig. 5). The bottom elevation measurements show the feature of scouring at the particular points on the bottom and also indicate the variation of the bottom surface at the particular station. In general, the greater the relative variation of the bottom, the higher the degree of roughness is. To express the variation of the bottom surface quantitatively, the standard deviation of the bottom measurements by the point gage was computed for each station. The mean values of the standard deviation of the bottom readings were calculated for every reach and entire channels taking arithemetical averages. The same assumption of the first of those made in the hydraulic computation was applied.

The standard deviation thus computed is not necessarily a measure of erosion of the bottom, as the variation of bottom might be either due to erosion or due to sedimentation.

Presentation of the Results

In Table III, the numerical values of the variables involved in Eq. (17) are shown.

For convenience, $R^{1/6}/n$ is taken to vary along the ordinate, and $\frac{1}{\sigma_b ND}$, instead of $\sigma_b ND$, is taken on the abscissa. The two numerical values are given in Table IV, and the points for these two dimensionless groups are plotted in Fig. 6. By inspection, it is seen that the curve would very likely be linear. Consequently, by the principle of linear regression, statistical computation of the regression coefficient is shown in Table V. Assuming the bivariate normal model, the correlation coefficient is also calculated. The results are as follows:

The regression coefficient of $\frac{R^{1/6}}{n}$ on $\frac{1}{\sigma_b ND}$

$$b_{2.1} = 0.102,$$

The regression coefficient of $\frac{1}{\sigma_b ND}$ on $\frac{R^{1/6}}{n}$

$$b_{1.2} = -8.83,$$

The correlation coefficient

$$r = 0.95$$

Hence the regression equation of $\frac{R^{1/6}}{n}$ on $\frac{1}{\sigma_b ND}$ is

$$\frac{R^{1/6}}{n} = 19.7 - \frac{0.1}{\sigma_b ND}, \qquad (19)$$

and that of $\frac{1}{\sigma_b ND}$ on $\frac{R^{1/6}}{n}$

$$\frac{1}{\sigma_b ND} = 182.9 - 8.8\,\frac{R^{1/6}}{n} \;. \qquad (20)$$

Eqs. (19) and (20) are the most probable formulas of the curves in Fig. 6.

TABLE 111

RESULTS OF TESTS AND MEASUREMENTS

Expt.	Channel	Type of Grass	Test No.	Avg. V (ft/sec)	R (ft)	n (ft$^{1/6}$)	σ_b (ft)	D (ft)	N (Per ft^2)
OK-R-3 (1946)	U10-1	Sudan Crab	5	.4290	.0545	.110	.01291	.00865	63
			6	.5480	.101	.129	.01291	.00865	63
			7	.6972	.184	.151	.01291	.00865	63
			8	.7919	.271	.171	.01317	.00865	63
			9	.9475	.394	.182	.01317	.00865	63
			10	1.092	.510	.189	.01317	.00865	63
NEV-4 (1959)	U-5	Sudan	1	.600	.0258	.0489	.0131	.0171	47
			2	.840	.0493	.0546	.0137	.0171	47
			3	1.112	.1130	.0712	.0162	.0171	47
			4	1.788	.1744	.0593	.0170	.0171	47
	U-6	Sudan	1	.674	.0251	.0428	.0131	.0171	57
			2	.850	.0476	.0523	.0120	.0171	57
			3	1.154	.1130	.0677	.0134	.0171	57
			4	1.906	.1640	.0532	.0120	.0171	57
	U-7	Sudan	1	.700	.0258	.0430	.0147	.0171	42
			2	.984	.0413	.0410	.0177	.0171	42
			3	1.391	.0903	.0498	.0261	.0171	42
			4	2.224	.1378	.0423	.0347	.0171	42
	U-8	Sudan	1	.726	.0253	.0397	.0171	.0171	52
			2	.945	.0432	.0442	.0187	.0171	52
			3	1.323	.0938	.0528	.0239	.0171	52
			4	1.959	.1558	.0466	.0320	.0171	52

TABLE IV

CALCULATED VALUES OF THE DIMENSIONLESS TERMS, $1/\sigma_b ND$ AND $R^{1/6}/n$

Channel	Grass	Test No.	$1/\sigma_b ND (= X_1)$	$R^{1/6}/n (= X_2)$
U 10-1	Sudan Crab	5	142	5.60
		6	142	5.30
		7	142	4.99
		8	139	4.69
		9	139	4.70
		10	139	4.73
U-5	Sudan	1	94.9	11.12
		2	90.7	11.09
		3	76.9	9.77
		4	73.2	12.61
U-6	Sudan	1	78.4	12.64
		2	85.5	11.51
		3	76.6	10.27
		4	50.4	13.91
U-7	Sudan	1	94.7	12.64
		2	78.6	14.34
		3	53.4	13.45
		4	37.5	16.99
U-8	Sudan	1	65.7	13.65
		2	60.1	13.40
		3	47.0	12.77
		4	35.1	15.74
Total			1941.7	235.91
Mean			88.26	10.72

TABLE V

COMPUTATION OF THE REGRESSION COEFFICIENT

$X_1 = 1941.7$		$X_2 = 235.91$
$\bar{x}_1 = 88.26$		$\bar{x}_2 = 10.72$
$X_1^2 = 199{,}597.25$	$X_1X_2 = 17951.699$	$X_2^2 = 2854.80$
$\frac{(X_1)^2}{n} = 171{,}372.68$	$\frac{X_1\ X_2}{n} = 20821.202$	$\frac{(X_2)^2}{n} = 2529.71$
$x_1^2 = 28{,}224.57$	$x_1x_2 = -2869.503$	$x_2^2 = 325.09$

$$b_{1.2} = \frac{x_1x_2}{x_2} = -8.82$$

$$b_{2.1} = \frac{x_1x_2}{x_1^2} = -0.102$$

$$r = \sqrt{b_{1.2}b_{2.1}} = 0.95$$

The high value of the correlation coefficient, r, shows the close relationship of the two curves; in other words, the tendencies of the two quantities are closely associated with each other. However, in this experiment, it is not appropriate to assume a bivariate model for these quantities, for a value is looked upon as a consequence of the other factors; namely, $R^{1/6}/n$ is measured on $\frac{1}{\sigma ND}$. Therefore, no statistical significance could be made on the correlation coefficient, r, other than it is only an indication of the distribution of the points treated in this problem.

Writing out an analysis of variance, the result is as shown in Table VI.

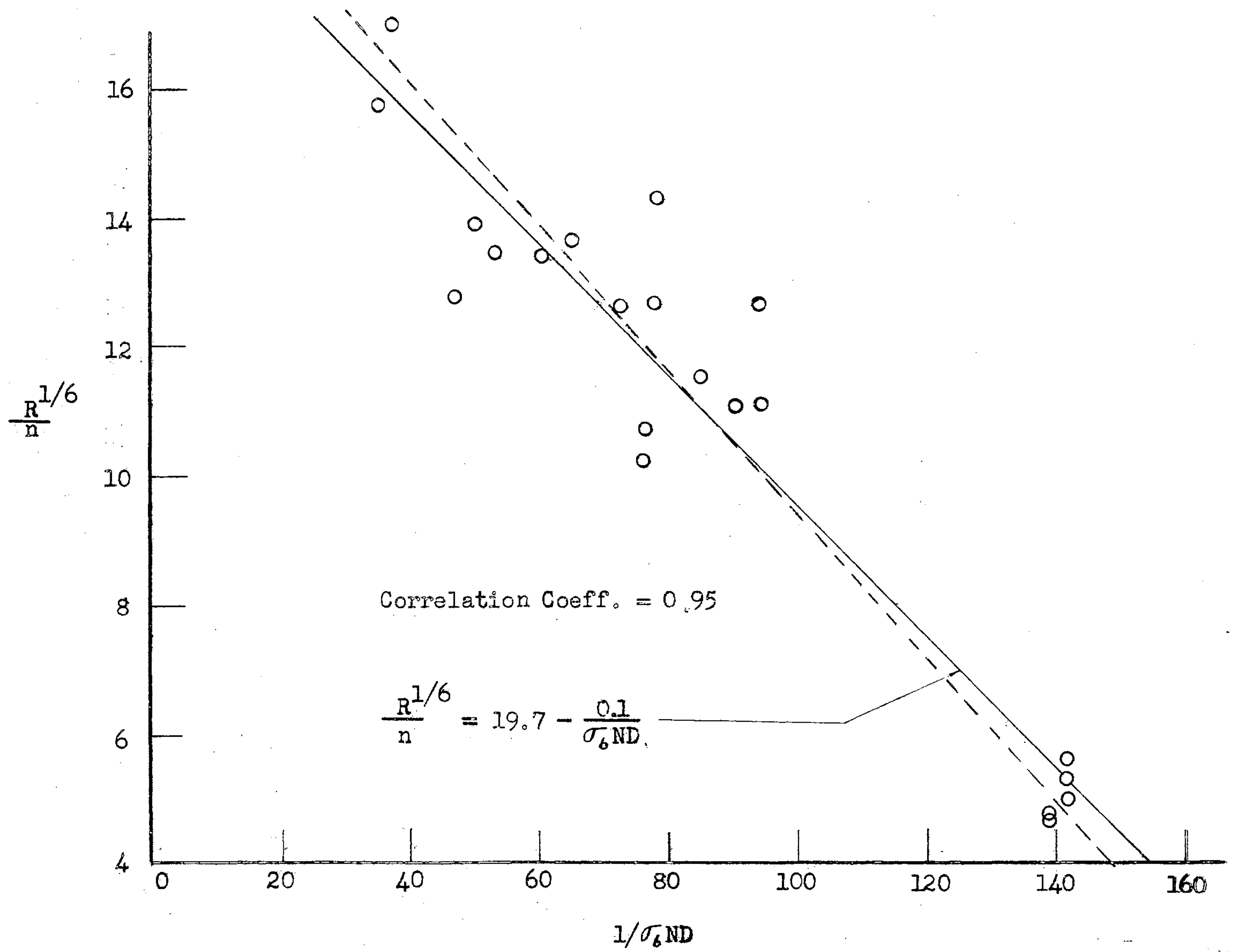

Figure 6. Relationship between the Manning n Relative to the Hydraulic Radius and the Product of Resistance Elements in Vegetated Channels.

TABLE VI

AN ANALYSIS OF VARIANCE OF THE REGRESSION COEFFIEIENT

Source	d.f.	Sum of Squares	Mean Squares
Total	22	2,854.7981	
Mean	1	2,529.7058	
Regression	1	29.17	
Error	20	295.92	14.796

Hence the experimental error is measured by 14.796.

An estimation of the variance of the regression coefficient is given by

$$s_b = 0.00723,$$

which yields the limits of the regression coefficient, $\beta_{2.1}$, at 95% confidence level as

$$-0.117 \leq \beta_{2.1} \leq -0.087.$$

It is seen that it is highly improbable that $\beta_{2.1}$ would be equal to zero.

The source of the experimental error could be thought as follows:

1. The error due to the deviation from the assumption that the flow was uniform. Actually, the flow could not be strictly uniform, for the population distribution of plant was not homogeneous, and local erosion caused considerable meandering during shallow flow. Some deviation from the assumptions previously made in

the measurements and the aalysis of data could also be considered.

2. The error possibly caused both in measurement and by personal behavior. Every measurement was made only once; the error might not be compensating.

3. The error due to the fluctuation of discharge. Although the discharge was supposed to be kept constant and measured by an orifice meter, some extent of fluctuation in discharge was seen. However, the error would be very small.

4. During the experiment, all the variables were co-existant; in other words, the experiment might have to be run under more strict control on appropriate factors.

5. The error due to the influence of plant leaves. The assumption was simplified so that the effect of leaves was not considered. However, actually there might be considerable complex influence of leaves on the resistance to flow.

Equation (19) yields

$$\frac{1}{n} = \frac{1}{R^{1/6}}\left(19.7 - \frac{1}{\sigma_b ND}\right), \qquad \ldots\ldots\ldots\ldots (21)$$

or

$$n = \frac{R^{1/6}}{19.7 - \dfrac{0.1}{\sigma_b ND}} . \qquad \ldots\ldots\ldots\ldots (22)$$

Equation (21) shows that N and D cannot be equal to zero; either of the two quantities is equal to zero will imply the other vanishes also, since if there is no plant, both N and D vanish simultaneously. Therefore the particular relationship derived in Equation (19) would only be applicable to the vegetated channels, and for the vegetation that has up-right, considerably clean stems. For the other open channel problems, some other analysis must be made.

Equation (19) also indicates the large influence of plant population. σ_b and D are usually of small value and the range of their variance would also be small. The swift change of n value would be greatly attributed to the denser plant population.

From Figure 6, the fact can be seen that a fivefold increase in n corresponds roughly to a tenfold increase in the value of resistance elements for a constant depth of flow. In practice, the bottom variation parameter, σ_b, will be most difficult to estimate, for the determination of the numerical value must pass through a process of computation. Moreover, the variation of bottom is not only interrelated with the population and the diameter of plant stems, but also highly dependent upon the characteristics of soil and possibly on the pre-existing condition of bottom. These factors affecting the variance of bottom yet remain to be investigated. The bottom variation is actually not independent of time, nor constant with respect to time, for during flow passing, the bottom would be scoured and there would be change in bottom roughness from time to time. However, within the limit of "permissible degree of erosion", which corresponds to the definition of permissible velocity, the change in bottom roughness could be considered limited and the time factor could be ignored.

CHAPTER VI

CONCLUSIONS

By dimensional consideration, a new approach to the determination of the Manning mean coefficient of roughness, specifically named as the retardance coefficient in vegetated channels, was made by considering its functional relationship with other physical factors involved in the problem. The possible formulation of the relationship was presented in Equation (14),

$$\frac{R^{1/6}}{n} = f\left(\frac{\sigma_b}{R}, \frac{D}{R}, NR^2, R_e, F\right) .$$

Pending extensive experiment to provide sufficient data, the numerical correlation was not found.

A simple form representing approximate relationship of the retardance coefficient with the physical characteristics of vegetation -- the mean diameter of stems and the population of plants -- and the standard deviation of measurement of bottom change as a parameter indicative of bottom surface variation, was then produced by fitting the data obtained by experiment to previously predicted form based on simplified assumptions. An algebraic function was found as in Equation (19),

$$\frac{R^{1/6}}{n} = 19.7 - \frac{0.1}{\sigma_b ND} .$$

It denotes that n in a depth of flow under consideration varies linearly with the product of the three elements. Its most serious limitation is that the expression would only be applicable to unsubmerged vegetation that has considerably clean, upright stems in the flow of moderate velocity.

It was found that for a certain depth of flow a fivefold increase in n corresponded roughly to a tenfold increase in the value of the product of the resistance elements.

From Equation (19), it could be predicted that the general form of the relation would be

$$\frac{R^{1/6}}{n} = A + \frac{B}{\sigma_b N D} ,$$

in which A and B are some numerical constants which must be determined by experiment for particular conditions.

CHAPTER VII

RECOMMENDATION FOR FURTHER STUDY

1. More study of velocity distribution in vegetated channels is desirable such that more precise estimation of the mean velocity in the channel can be made.
2. A parameter expressing shape of plant stem would be required in the consideration of such flow resistance problem. This kind of shape effect must be clarified.
3. A basic proposal was made in the form of Equation (14) that the Manning n could be rationally estimated closely by establishing the functional relationship among the variables. Accurate empirical correlation has yet to be made by setting proper variables involved under proper control in extensive experiment.
4. From the view of momentum principle, it is desirable to investigate the effect of elastic characteristic of plant stems, since the bending of plant was seen unavoidable.
5. The role of mechanical aspects of soil should be explored, since bottom variation is greatly dependent on such characteristics.

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A P P E N D I X

APPENDIX A

Stillwater Outdoor Hydraulic Laboratory

Channel U6

BOTTOM READINGS - Unit Channels

Expt. NEV-4

Profile Station 55

Datum bolt reading 1.000
Elevation datum bolt 11.081
Gage zero elevation 10.081

Date	6-9-59	6-9-59	6-9-59	6-9-59	6-9-54
Time	9:16 A	12:17 P	1:36 P	2:46 P	4:31 P
	BT-1	AT-1	AT-2	AT-3	AT-4
By	CAL/WCC	CAL/WCC	CAL/WCC	CAL/WCC	CAL/WCC
	Condition of Bottom				
Section Station	Moist Firm	Wet	Wet	Wet	Wet
0.1	.762	.761	.761	.762	.760
.3	.758	.753	.755	.755	.754
.5	.748	.744	.741	.746	.735
.7	.733	.728	.726	.727	.647
.9	.733	.729	.731	.728	.707
1.1	.734	.735	.734	.732	.698
1.3	.737	.739	.736	.735	.685
1.5	.743	.743	.740	.741	.695
1.7	.731	.735	.734	.731	.669
1.9	.730	.728	.727	.725	.674
2.1	.730	.738	.730	.729	.720
2.3	.730	.729	.729	.729	.729
2.5	.734	.734	.733	.733	.733
2.7	.732	.732	.732	.732	.730
2.9	.747	.742	.742	.739	.735
Average	.739	.708	.737	.736	.711
Gage Zero	10.081	10.081	10.081	10.081	10.081
Mean Elev.	10.820	10.819	10.818	10.817	10.792

Note: BT = Before Test
AT = After Test

APPENDIX B

Channel U6

Stillwater Outdoor Hydraulic Laboratory

Expt. Nev-4

WATER SURFACE READINGS
Unit Channel Tests

Checked ______

	Test No. 3		
	Station		
	50	55	60
Date	6-9-59		
Time	1:52 p.m./ 1/56 p.m.	1:50 p.m./ 2:01 p.m.	2:03 p.m./ 2:06 p.m.
By	CAL/WCC	CAL/WCC	CAL/WCC
Section Station			
.1	.859	.841	.819
.3	.863	.847	.811
.5	.860	.849	.817
.7	.874	.841	.823
.9	.863	.828	.824
1.1	.850	.815	.833
1.3	.849	.839	.827
1.5	.851	.845	.825
1.7	.850	.840	.840
1.9	.848	.847	.823
2.1	.863	.851	.829
2.3	.845	.849	.810
2.5	.847	.842	.825
2.7	.856	.853	.822
2.9	.859	.841	.824
Total	12.837	12.628	12.352
Average	.856	.842	.823
Datum bolt elev.	11.327	11.081	10.834
Datum bolt read.	1.000	1.000	1.000
Gage zero	10.327	10.081	9.834
Mean W.S. Elevation	11.183	10.923	10.657

APPENDIX C

PLANT POPULATION

NUMBER OF PLANTS PER SQUARE FEET

No. of Observation	Channel 5	Channel 6	Channel 7	Channel 8
1	43	39	29	50
2	40	29	31	61
3	51	81	35	29
4	39	75	42	35
5	27	67	44	34
6	31	74	45	60
7	45	53	60	61
8	40	52	30	70
9	57	60	57	46
10	80	42	29	52
11	61	41	39	67
12	52	60	71	71
13	45	71	54	42
Total	611	744	544	678
Mean	47	57	42	52
Standard Deviation	13.75	15.52	12.9	14.2

APPENDIX D

CALCULATION OF RETARDANCE COEFFICIENTS

Calc. BPF Date Nov. 1959
Check_____ Date ______
Channel U6 Expt. NEV-4

Reach Lengths:

A. 30.00 B. 29.97 C. 30.00 D. 30.00 E. 29.98 F. 29.98

Test	Sta. and Reach	Q	P	A	W.S Elev.	$\frac{V^2}{2g}$	E.L. Elev.	Fall	Slope	V	R	n
4	20	.9042	2.910	.3812	12.682	.0873	12.7693					
	A		2.915	.4826				1.4721	.0491	1.874	.1655	.0535
	50	.9042	2.920	.5840	11.260	.0372	11.2972					
	25	.9042	2.910	.3463	12.384	.1059	12.4899					
	B		2.915	.4024				1.4555	.0486	2.247	.1380	.0389
	55	.9042	2.920	.4584	10.974	.0604	11.0344					
	30	.9042	2.920	.4059	12.193	.0770	12.2700					
	C		2.925	.4711				1.5139	.0505	1.919	.1610	.0515
	60	.9042	2.930	.5362	10.712	.0441	10.7561					
	50	.9042	2.920	.5840	11.260	.0372	11.2972					
	D		2.910	.5493				1.5273	.0509	1.645	.1887	.0668
	80	.9042	2.900	.5146	9.722	.0479	9.7699					
	55	.9042	2.920	.4584	10.974	.0604	11.0244					
	E		2.910	.4539				1.5336	.0512	1.992	.1559	.0488
	85	.9042	2.900	.4493	9.438	.0628	9.5008					
	60	.9042	2.930	.5362	10.712	.0441	10.7561					
	F		2.940	.5145				1.5159	.0506	1.758	.1750	.0594
	90	.9042	2.950	.4927	9.188	.0522	9.2402					

APPENDIX E

STANDARD DEVIATION OF BOTTOM READINGS

	Reach	U-5	U-6	U-7	U-8
BT-1	A	.01357	.0132	.0153	.0119
	B	.00979	.0110	.0173	.0155
	C	.01190	.0275	.0105	.0116
	D	.0149	.0119	.0146	.0163
	E	.0119	.0117	.0236	.0309
	F	.0212	.0196	.0091	.0151
Total		.08326	.0949	.0904	.1003
Mean		.01221	.0158	.0151	.0167
AT-1	A	.01349	.0131	.0149	.0123
	B	.00981	.0099	.0199	.0140
	C	.01190	.0142	.0152	.0116
	D	.0135	.0113	.0169	.0137
	E	.0129	.0110	.0284	.0318
	F	.0247	.0180	.0151	.0258
Total		.08730	.0775	.1104	.1092
Mean		.01438	.0129	.0184	.0182
AT-2	A	.01446	.0135	.0287	.0124
	B	.01149	.0116	.0256	.0141
	C	.01287	.0142	.0214	.0147
	D.	.0165	.0114	.0215	.0242
	E	.0163	.0102	.0385	.0341
	F	.01640	.0206	.0237	.0364
Total		.08802	.0815	.1591	.1367
Mean		.01467	.01358	.0265	.0228
AT-3	A	.01770	.0161	.0316	.0165
	B	.01168	.0275	.0517	.0233
	C	.01572	.0192	.0286	.0240
	D	.0176	.0134	.0240	.0286
	E	.0175	.0147	.0532	.0438
	F	.0263	.0269	.0329	.0521
Total		.10650	.1178	.2220	.1883
Mean		.01742	.0196	.0370	.0314

VITA

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Candidate for the Degree of

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