ANALYSIS OF CONTINUOUS TRUSSES ON ELASTIC SUPPORTS

By

JAMES D. COBB

Bachelor of Science

Oklahoma State University

Stillwater, Oklahoma

1959

Submitted to the faculty of the Graduate School
of the Oklahoma State University of
Agriculture and Applied Science
in partial fulfillment of
the requirements for
the degree of
MASTER OF SCIENCE
1960

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ON ELASTIC SUPPORTS

Thesis Approved:

Thesis Advisor

Dean of the Graduate School

PREFACE

The author wishes to express indebtness and appreciation to the following persons:

To Professor Jan J. Tuma, Head of the School of Civil Engineering, for his sincere guidance in preparation of this thesis, and for his encouragement, instruction, and assistance given to the author throughout his graduate and undergraduate study. The author considers it a privilege to have studied under a person such as Professor Tuma.

To the Faculty in the School of Civil Engineering for appointing the author Graduate Assistant, and for their many contributions, directly or indirectly, to his education.

To James W. Gillespie and Charles O. Heller for their constructive criticism and careful reading of the manuscript.

To Mrs. Willie Bernardi, Secretary in the School of Civil Engineering, for her careful typing of the manuscript.

Finally, to his beloved wife, LaVerna, and parents, Mr. and Mrs. Fred Cobb, for their encouragement, understanding, and also making it possible for the author to complete his graduate study.

J. D. C.

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NOMENCLATURE

d_m, d_n . . . . Length of bars

h, i, j, k . . . . Letters designating intermediate supports

m, n . . . . Letters designating bars or panels

q_{ji}, q_{jk} . . . . Reactions due to unit moment

A_m, A_n . . . . Areas of bar m or n

BR_j . . . . Sum of end shears

BN_m, BN_n . . . . Axial forces in bar m or n of the basic structure due to loads

BV_{ji}, BV_{jk} . . . . End shears of a simple truss

C_j . . . . Spring constant

E . . . . Modulus of elasticity

F'_{ji}, F'_{jk} . . . . Angular flexibilities due to rotation

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G''_{hj}, G''_{lj} . . . . Angular carry-over values (far end) due to displacement of supports

L_j . . . . Length of span

M_j . . . . Final moment

P_m, P_n . . . . Concentrated loads

Q_{ji}, Q_{jk} . . . . Displacement of supports

R_j . . . . Reaction at the support

V_{ji}, V_{jk} . . . . End shears

α_m, α_n . . . . Axial forces in bar m or n due to unit moment at left end

β_m, β_n . . . . Axial forces in bar m or n due to unit moment at right end

Δ . . . . Displacement due to loads

$\Delta^{(1)}$ . . . . Displacement due to unit moment

λ_m, λ_n . . . . Axial flexibilities

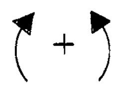
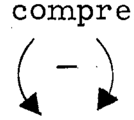
λ_m', λ_n' . . . . Relative axial flexibilities

τ_{ji}', τ_{jk}' . . . . Angular load functions due to rotation

τ_{ji}'', τ_{jk}'' . . . . Angular load functions due to displacement of supports

$\tau_{ji}^{(\Delta)}$, $\tau_{jk}^{(\Delta)}$ . . . . Angular displacement slopes

SIGN CONVENTION

Axial forces	+ tension	- compression
Moment		

PART I

INTRODUCTION

A general procedure for the analysis of continuous trusses on elastic supports is presented. The depth of the truss may be constant or variable and deformations may be caused by transverse loads, applied couples, displacement of the elastic supports or inaccuracies in fabrication and erection. The presentation is an extension of the study of continuous beams on elastic supports (1) which deals with continuous beams whose supports cannot be assumed rigid and the elasticity of which must be considered.

The recommended procedure is the direct solution of simultaneous equations or successive approximations which may be carried to any desired degree of accuracy. The general five-moment equation is derived for a continuous truss and can be used as a mathematical model to develop the algebraic carry-over procedure (1, 2, 3).

The author was introduced to the subject in a graduate seminar, C. E. 620, taught by Professor Tuma in the spring semester of 1959 at the Oklahoma State University. The possibility of its extension to continuous trusses was suggested in an "Analysis of Guyed Towers" course also taught by Professor Tuma for the Flint Steel Corporation, Tulsa, Oklahoma.

The method presented can easily be applied to guyed towers. The elasticity of the guys would be considered to be the spring at the supports. The method recommended to analyze the guys is presented in (4, 5, 6).

This study is restricted to coplanar trusses, and the customary assumptions of truss analysis are introduced. The sign convention of the three-moment equation is adopted.

The following discussion is divided into six parts. The first contains the derivation of the five-moment equation; the second is the physical interpretation of the angular functions due to rotation; the third is the physical interpretation of the angular functions due to displacement of supports. The fourth is the recommended procedure, while the fifth demonstrates this procedure through an illustrative example. Finally, the results are summarized and a conclusion is drawn.

In the physical interpretation of the angular functions due to rotation, the basic structure $\overline{hijkl}$ is assumed elastic and the supports rigid. Fictitious hinges are assumed placed at the supports, and the vertical bars are assumed split at the supports to allow rotation. In the physical interpretation of the angular functions due to displacement of supports, the basic structure $\overline{hijkl}$ is assumed rigid and the supports elastic. Fictitious hinges are again assumed placed at the supports and vertical bars are split at the supports to allow displacement. Once these functions are calculated the continuity is established by determining the required moment.

PART II

DERIVATION OF THE FIVE-MOMENT EQUATION

1. Statement of the Problem

A continuous truss, resting on elastic supports, subjected to a general system of loads is considered. The supports are denoted by 0, 1, 2, 3, ... h, i, j, k, l, ... and their elastic constants by C_0, C_1, C_2, C_3, ... C_h, C_i, C_j, C_k, C_l, ... The exterior ends are simply supported and the bending moments at the supports, $M_0, M_1, M_2, M_3, \ldots M_h, M_i, M_j, M_k, M_l, \ldots$ are selected as unknowns. A portion of this continuous truss is shown in Fig. (2-1).

2. Three Moment Equation

The general three-moment equation written for the intermediate joint j (any joint) in terms of the angular flexibilities (F'), angular carry-over values (G'), angular load functions (τ') due to rotation; angular displacements slopes ($\tau^{(\Delta)}$) due to displacement of supports; and redundant moments is (1):

$$G'_{ij}M_i + (F'_{ji} + F'_{jk})M_j + G'_{kj}M_k = -\tau'_{ji} - \tau'_{jk} - \tau^{(\Delta)}_{ji} - \tau^{(\Delta)}_{jk} . \qquad (2\text{-}1)$$

The meaning and physical interpretation of the angular functions due to rotation (F', G', τ') are explained in Part III.

3. Angular Displacement Slope

The angular displacement slope $\tau^{(\Delta)}_{ji}$ (or $\tau^{(\Delta)}_{jk}$) is defined as

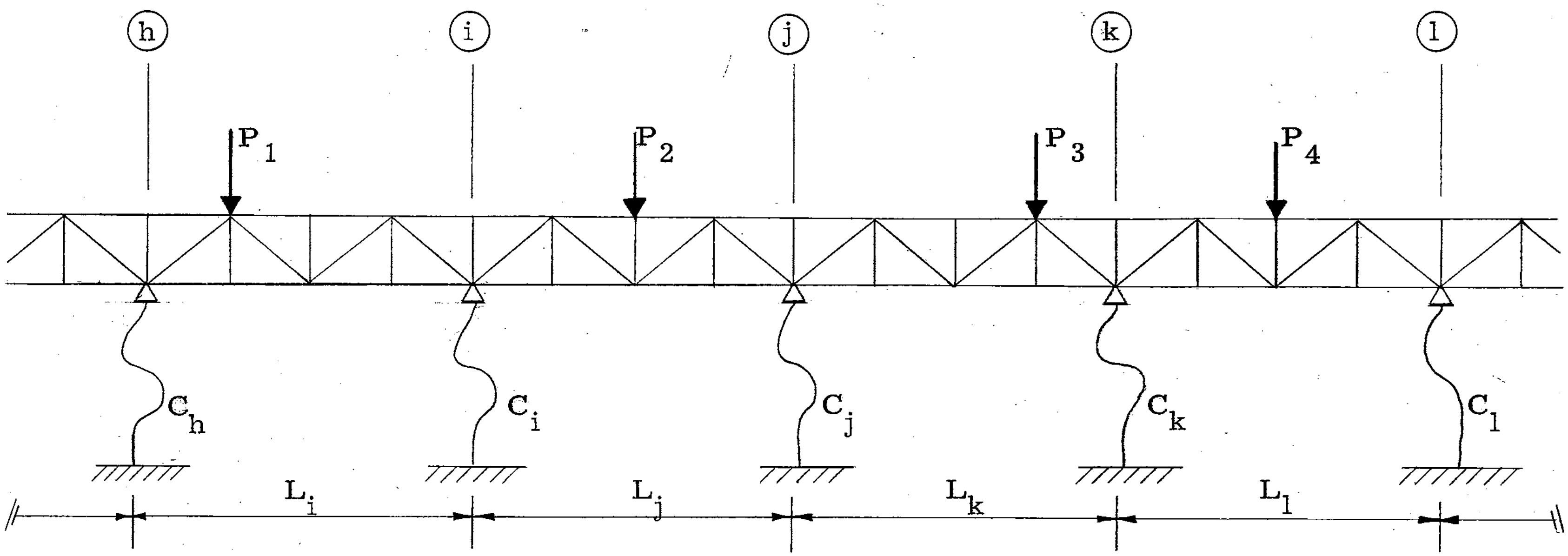


Fig. 2-1

Typical Portion of a Continuous Truss

the end slope at the elastic support (j). Denoting the displacements by Δ_i, Δ_j, and Δ_k (Fig. 2-2), the end slopes become:

$$\tau_{ji}^{(\Delta)} = \frac{\Delta_i - \Delta_j}{L_j} \qquad \Big| \qquad \tau_{jk}^{(\Delta)} = - \frac{\Delta_j - \Delta_k}{L_k} \; . \tag{2-2}$$

Introducing

$$\Sigma\tau_j^{(\Delta)} = \tau_{ji}^{(\Delta)} + \tau_{jk}^{(\Delta)} \; ,$$

Eqs. (2-2) become:

$$\Sigma\tau_j^{(\Delta)} = \frac{\Delta_i}{L_j} - \Delta_j \left[\frac{1}{L_j} + \frac{1}{L_k}\right] + \frac{\Delta_k}{L_k} \; . \tag{2-3}$$

The Eq. (2-3) is the change in the angular displacement slope at j expressed as a function of displacements Δ_i, Δ_j, and Δ_k. These displacements can be expressed as functions of the elastic support constants (Spring Constants) and the corresponding reactions. From Hooke's Law they are:

$$\Delta_i = \frac{R_i}{C_i} \qquad \Big| \qquad \Delta_j = \frac{R_j}{C_j} \qquad \Big| \qquad \Delta_k = \frac{R_k}{C_k} \; . \tag{2-4}$$

With these new equivalents (Eqs. 2-4), Eq. (2-3) becomes:

$$\Sigma\tau_j^{(\Delta)} = \frac{1}{L_j C_i} R_i - R_j \left[\frac{1}{L_j C_j} + \frac{1}{L_k C_j}\right] + \frac{1}{L_k C_k} R_k \; . \tag{2-5}$$

In order to express the reactions as functions of the loads and redundant moments, the basic structure $\overline{hijkl}$ is isolated into four free bodies by splitting the vertical bars h, i, j, k, and l (Fig. 2-3).

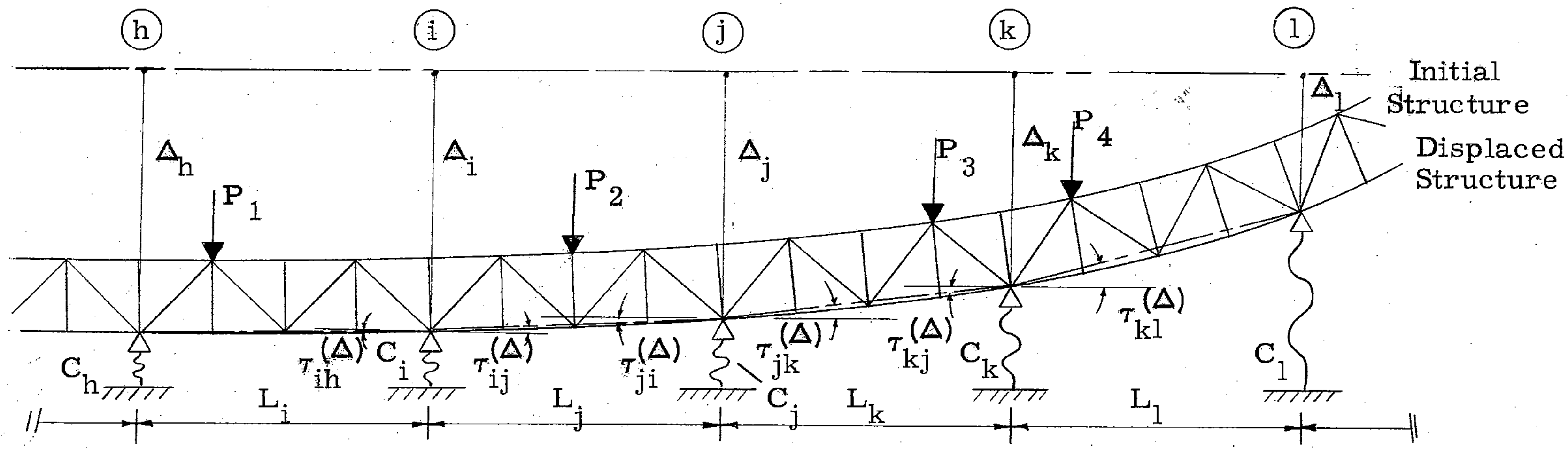


Fig. 2-2

Angular Displacement Slopes

Elastic Structure - Elastic Supports

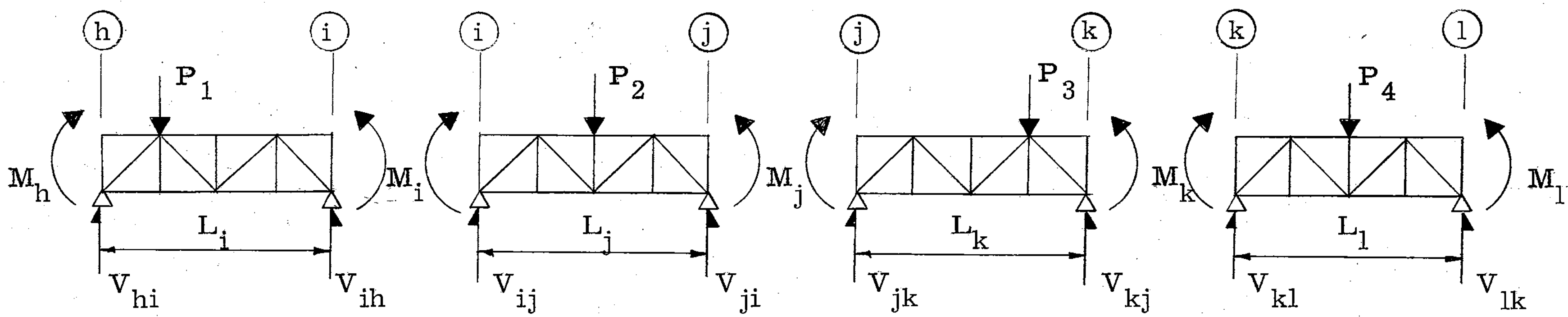


Fig. 2-3

Free Bodies $\overline{hi}$, $\overline{ij}$, $\overline{jk}$, and $\overline{kl}$

The end shears as found from static equilibrium are:

$$\left.\begin{aligned} V_{ji} &= BV_{ji} + \frac{M_i}{L_j} - \frac{M_j}{L_j} \\ V_{jk} &= BV_{jk} - \frac{M_j}{L_k} + \frac{M_k}{L_k} \end{aligned}\right\} \quad (2\text{-}6)$$

where

BV_{ji}, (BV_{jk}) = the end shear of the simple truss $\overline{ij}$ (or $\overline{jk}$).

The reaction at the support j of the basic structure $\overline{ijk}$ is the sum of the end shears. Thus,

$$R_j = V_{ji} + V_{jk} \, . \quad (2\text{-}7)$$

The substitution of Eqs. (2-6) into Eq. (2-7) results in the following equation for the reaction at j:

$$R_j = BR_j + \frac{M_i}{L_j} - M_j \left[\frac{1}{L_j} + \frac{1}{L_k}\right] + \frac{M_k}{L_k} \, , \quad (2\text{-}8)$$

where

$$BR_j = BV_{ji} + BV_{jk} \, .$$

Similarly, the reactions R_i and R_k are developed (Eqs. 2-6, 7). Thus,

$$\left.\begin{aligned} R_i &= BR_i + \frac{M_h}{L_i} - M_i \left[\frac{1}{L_i} + \frac{1}{L_j}\right] + \frac{M_j}{L_j} \\ R_k &= BR_k + \frac{M_j}{L_k} - M_k \left[\frac{1}{L_k} + \frac{1}{L_l}\right] + \frac{M_l}{L_l} \, . \end{aligned}\right\} \quad (2\text{-}9)$$

Finally, substituting equivalent R's (Eqs. 2-8, 9) into the Eq. (2-5) results in the following equation for the change in the angular displacement slope:

$$\Sigma\tau_j^{(\Delta)} = \underbrace{\frac{1}{L_j C_i}}_{Q_{ij}} \left[BR_i + \underbrace{\frac{1}{L_i}}_{q_{ih}} M_h - \underbrace{\left(\frac{1}{L_i} + \frac{1}{L_j}\right)}_{\Sigma q_i} M_i + \underbrace{\frac{1}{L_j}}_{q_{ij}} M_j \right]$$

$$- \underbrace{\left[\frac{1}{L_j C_j} + \frac{1}{L_k C_j}\right]}_{\Sigma Q_j} \left[BR_j + \underbrace{\frac{1}{L_j}}_{q_{ji}} M_i - \underbrace{\left(\frac{1}{L_j} + \frac{1}{L_k}\right)}_{\Sigma q_j} M_j + \underbrace{\frac{1}{L_k}}_{q_{jk}} M_k \right] \qquad (2\text{-}10)$$

$$+ \underbrace{\frac{1}{L_k C_k}}_{Q_{kj}} \left[BR_k + \underbrace{\frac{1}{L_k}}_{q_{kj}} M_j - \underbrace{\left(\frac{1}{L_k} + \frac{1}{L_l}\right)}_{\Sigma q_k} M_k + \underbrace{\frac{1}{L_l}}_{q_{kl}} M_l \right].$$

4. Development of the Five-Moment Equation

Rearranging and substituting equivalents of Eq. (2-10) into the three moment equation (Eq. 2-1) results in the five-moment equation for a general system of loads:

$$\underbrace{Q_{ij} q_{ih}}_{G''_{hj}} M_h + \underbrace{\left[- Q_{ij}\Sigma q_i - \Sigma Q_j q_{ji}\right]}_{G''_{ij}} M_i$$

$$+ \underbrace{\left[Q_{ij} q_{ij} + \Sigma Q_j \Sigma q_j + Q_{kj} q_{kj}\right]}_{\Sigma F''_j} M_j$$

$$+ \underbrace{\left[-\Sigma Q_j q_{jk} - Q_{kj}\Sigma q_k\right]}_{G''_{kj}} M_k + \underbrace{Q_{kj} q_{kl}}_{G''_{lj}} M_l \qquad (2\text{-}11)$$

$$+ G'_{ij} M_i + \underbrace{(F'_{ji} + F'_{jk})}_{\Sigma F'_j} M_j + G'_{kj} M_k$$

$$+ \underbrace{Q_{ij} BR_i - \Sigma Q_j BR_j + Q_{kj} BR_k}_{\Sigma\tau''_j} + \underbrace{\tau'_{ji} + \tau'_{jk}}_{\Sigma\tau'_j} = 0.$$

Combining the functions due to rotation (G'_{ij}, $\Sigma F'_j$, G'_{kj}, $\Sigma\tau'_j$) with the functions due to displacement of supports (G''_{hj}, G''_{ij}, $\Sigma F''_j$, G''_{kj}, G''_{lj}, $\Sigma\tau''_j$), and introducing the expressions:

$$\Sigma F_j = \Sigma F'_j + \Sigma F''_j \quad , \tag{2-12}$$

$$G_{ij} = G'_{ij} + G''_{ij} \quad , \tag{2-13}$$

$$G_{kj} = G'_{kj} + G''_{kj} \quad , \tag{2-14}$$

and

$$\Sigma\tau_j = \Sigma\tau'_j + \Sigma\tau''_j \quad , \tag{2-15}$$

as new notations, Eq. (2-11) results in the five-moment equation in a new form:

$$\begin{aligned} & G''_{hj} M_h + G_{ij} M_i + \Sigma F_j M_j + G_{kj} M_k \\ & + G''_{lj} M_l + \Sigma\tau_j = 0. \end{aligned} \tag{2-16}$$

The physical interpretation of the functions due to displacement of supports will be explained in Part IV.

PART III

ANGULAR FUNCTIONS DUE TO ROTATION

In the physical interpretation of the angular functions due to rotation, the basic structure $\overline{hijkl}$ is assumed elastic and on rigid supports. The following angular functions have been derived from the principle of minimum energy (2).

1. Angular Flexibility

The angular flexibility F'_{ji} (or F'_{jk}) is the end slope of the basic structure $\overline{hijkl}$ at j due to a unit moment applied at j (Fig. 3-1).

$$F'_{ji} = \sum_{i}^{j} \beta_m^{\,2} \lambda_m \qquad\qquad F'_{jk} = \sum_{j}^{k} \alpha_n^{\,2} \lambda_n , \qquad (3\text{-}1)$$

where

α_n = the axial force in bar n due to unit moment at j,

β_m = the axial force in bar m due to unit moment at j,

and

$$\lambda_m = \frac{d_m}{A_m E} \qquad\qquad \lambda_n = \frac{d_n}{A_n E} .$$

The notations used in the axial flexibilities (λ_m, λ_n) are:

d_m, d_n = the length of the bar,

A_m, A_n = the cross-sectional area of the bar,

and

E = modulus of elasticity of the bar.

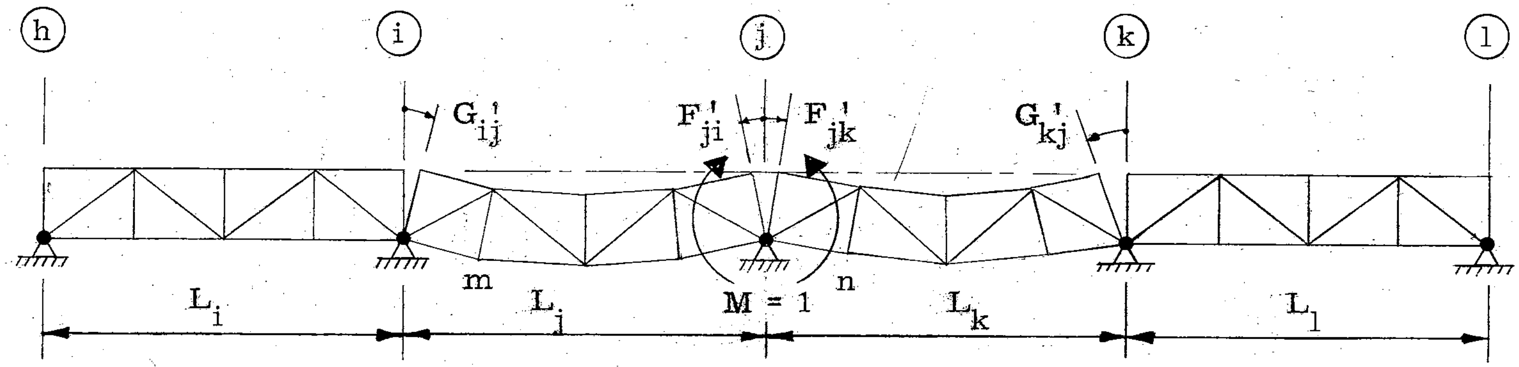


Fig. 3-1

Angular Flexibilities and Carry-Over Values

Elastic Structure - Rigid Supports

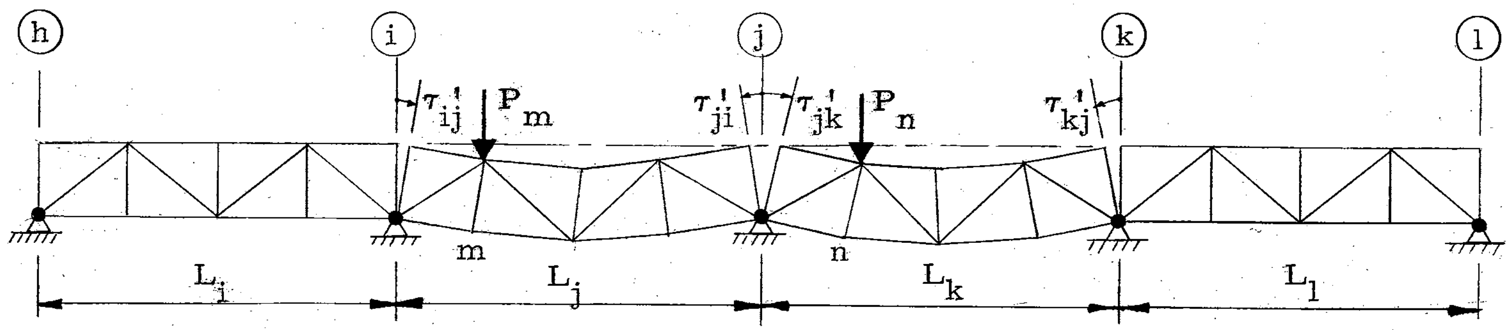


Fig. 3-2

Angular Load Functions

Elastic Structure - Rigid Supports

2. Angular Carry-over Value

The angular carry-over value G'_{ij} (or G'_{kj}) is the end slope of the basic structure $\overline{hijkl}$ at i (or k) due to a unit moment applied at j (Fig. 3-1).

$$G'_{ij} = \sum_{i}^{j} \alpha_m \beta_m \lambda_m \qquad\qquad G'_{kj} = \sum_{j}^{k} \alpha_n \beta_n \lambda_n . \tag{3-2}$$

3. Angular Load Function

The angular load function τ'_{ji} (or τ'_{jk}) is the end slope of the basic structure $\overline{hijkl}$ at j due to loads (Fig. 3-2).

$$\tau'_{ji} = \sum_{i}^{j} BN_m \beta_m \lambda_m \qquad\qquad \tau'_{jk} = \sum_{j}^{k} BN_n \alpha_n \lambda_n . \tag{3-3}$$

where

BN_m = the axial force in bar m of the basic structure $\overline{ij}$ due to loads,

and

BN_n = the axial force in bar n of the basic structure $\overline{jk}$ due to loads.

4. Axial Force

The total axial force in the bar m (or n) of $\overline{ij}$ (or $\overline{jk}$) is (Fig. 3-3):

$$\left.\begin{aligned} N_m &= BN_m + M_i \alpha_m + M_j \beta_m \\ N_n &= BN_n + M_j \alpha_n + M_k \beta_n . \end{aligned}\right\} \tag{3-4}$$

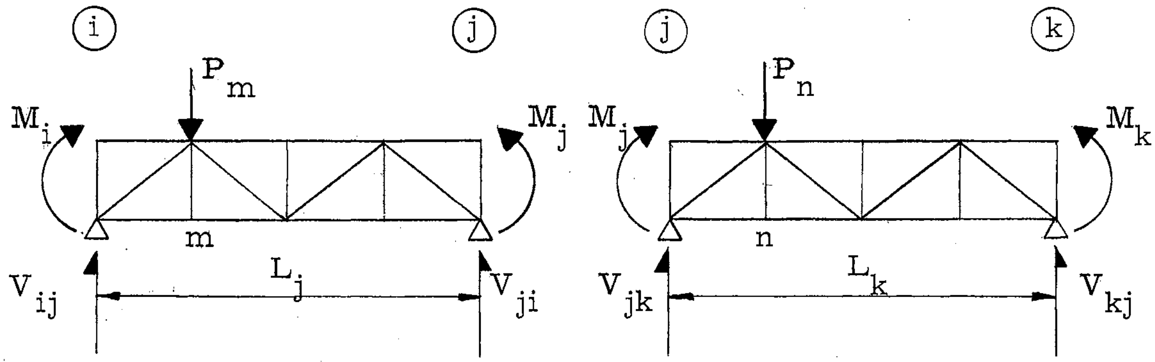


Fig. 3-3

Free Bodies $\overline{ij}$ and $\overline{jk}$

PART IV

ANGULAR FUNCTIONS DUE TO DISPLACEMENT OF SUPPORTS

For the purpose of explaining the physical interpretation of the analytical expressions and angular functions due to displacement of supports, the basic structures are assumed rigid and on elastic supports.

1. Reaction Due to Unit Moment

For a unit moment applied at the left or right end of the basic structure $\overline{ij}$ (or $\overline{jk}$), the reaction at the elastic support i, j, or k is (Fig. 4-1a):

$$q_{ij} = q_{ji} = \frac{1}{L_j} \qquad \bigg| \qquad q_{jk} = q_{kj} = \frac{1}{L_k}, \tag{4-1}$$

and

$$\Sigma q_j = q_{ji} + q_{jk} . \tag{4-2}$$

2. Displacement Due to Unit Moment

For a unit moment applied at the left or right end of the basic structure $\overline{ij}$ (or $\overline{jk}$), the displacement at the elastic support i, j, or k is (Fig. 4-1a):

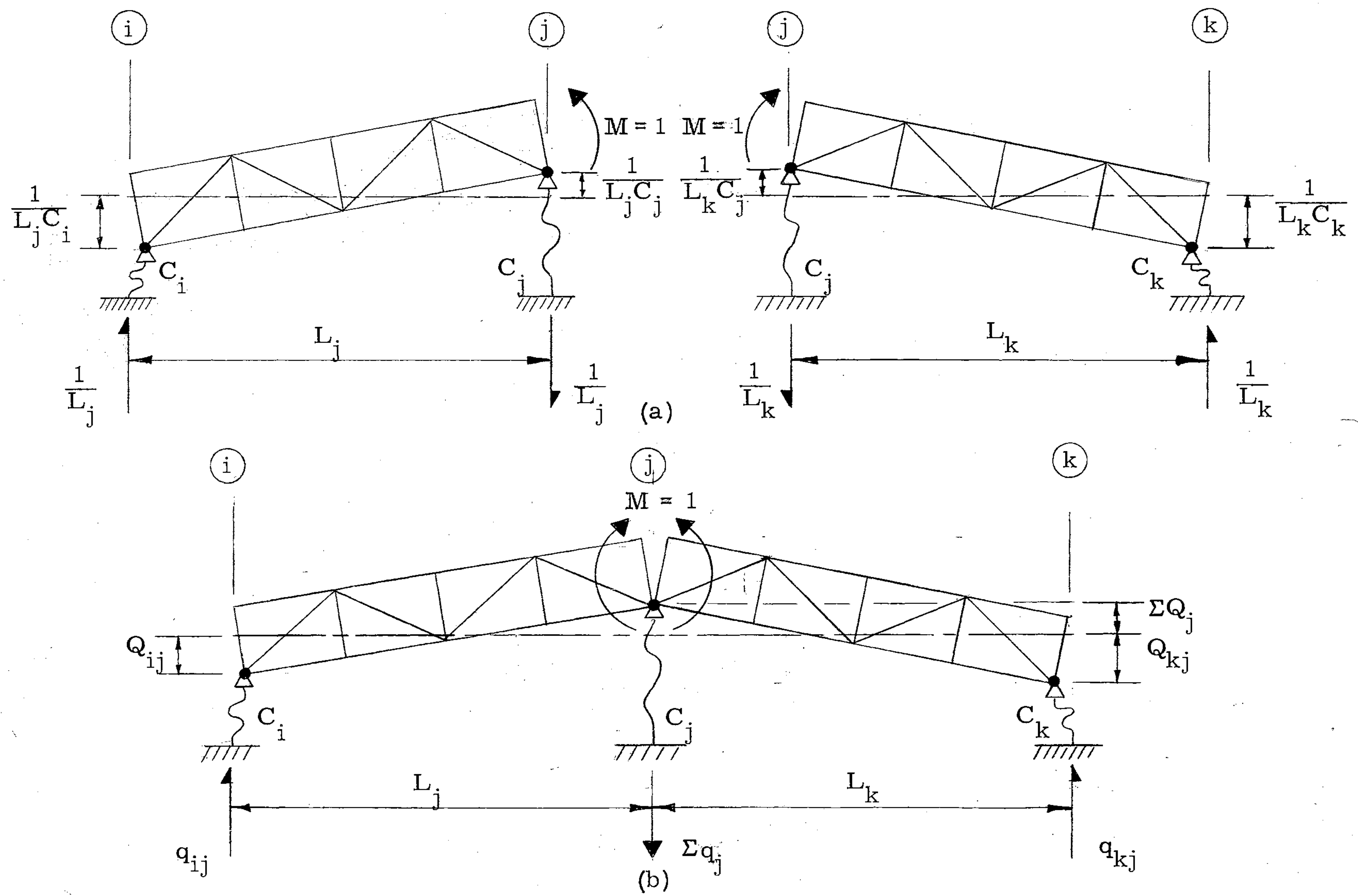


Fig. 4-1

Reactions and Displacements Due to Unit Moment
- Rigid Structure Elastic Supports -

From Eqs. (2-4),

$$Q_{ij} = \frac{1}{L_j C_i} \qquad Q_{ji} = \frac{1}{L_j C_j}$$
$$Q_{jk} = \frac{1}{L_k C_j} \qquad Q_{kj} = \frac{1}{L_k C_k} \qquad (4-3)$$

and

$$\Sigma Q_j = Q_{ji} + Q_{jk} . \qquad (4-4)$$

By combining the two basic structures $\overline{ij}$ and $\overline{jk}$ and applying a unit moment at j, the terms Σq_j (Eq. 4-2) and ΣQ_j (Eq. 4-4) become the reaction and the displacement of the elastic support j, respectively (Fig. 4-1b).

3. Angular Flexibility

The angular flexibility F''_{ji} (or F''_{jk}) is the end slope of the basic structure $\overline{hijkl}$ at j due to a unit moment applied at j (Fig. 4-2).

$$F''_{ji} = \frac{\Delta_i^{(1)} + \Delta_j^{(1)}}{L_j} = q_{ji} \left[Q_{ij} + \Sigma Q_j \right]$$
$$F''_{jk} = \frac{\Delta_k^{(1)} + \Delta_j^{(1)}}{L_k} = q_{jk} \left[Q_{kj} + \Sigma Q_j \right] . \qquad (4-5)$$

4. Angular Carry-Over Value (Near End)

The angular carry-over value G''_{ij} (or G''_{kj}) of the near end i (or k) is the change in the end slope of the basic structure $\overline{hijkl}$ due to a unit moment applied at j (Fig. 4-2).

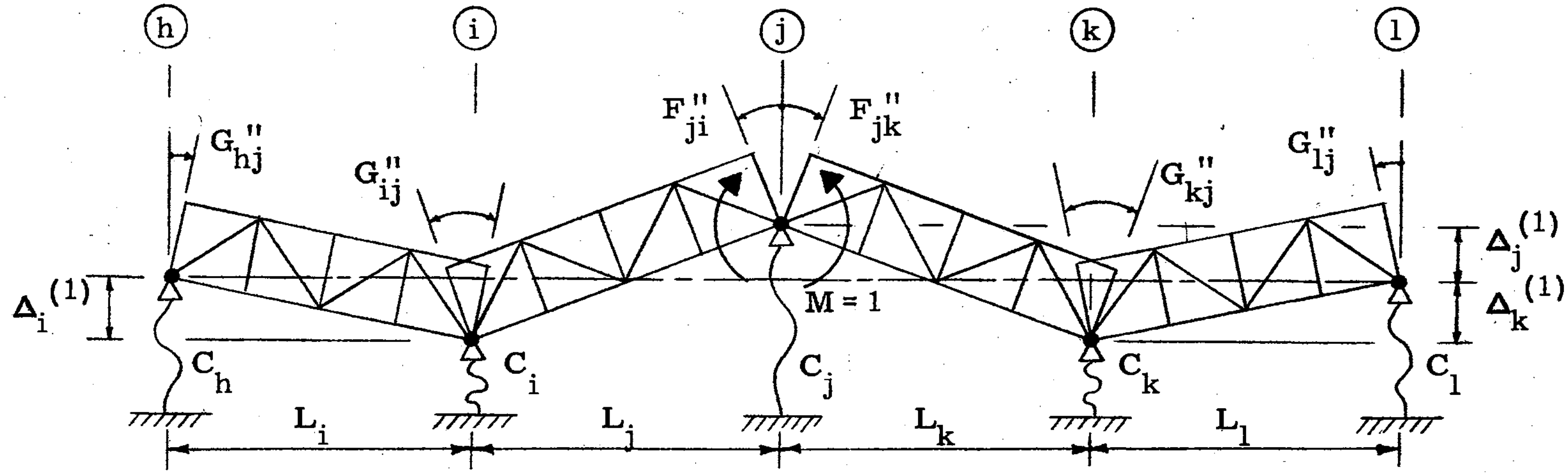


Fig. 4-2

Angular Flexibilities and Carry-Over Values

Rigid Structure - Elastic Supports

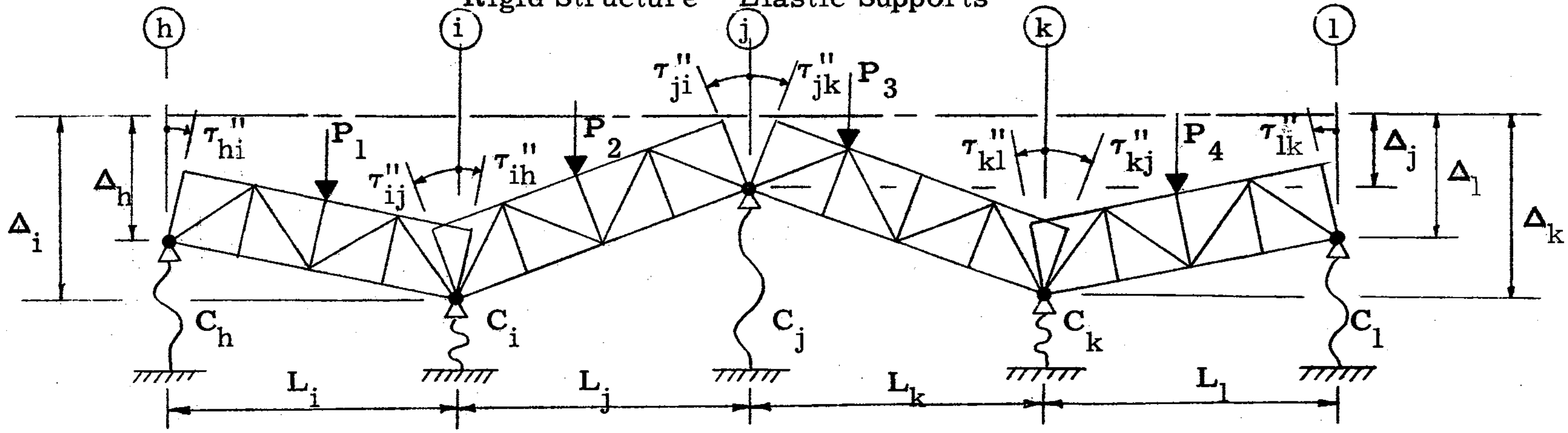


Fig. 4-3

Angular Load Functions

Rigid Structure - Elastic Supports

$$\left.\begin{aligned} G''_{ij} &= \Delta_i^{(1)} \left[\frac{1}{L_j} + \frac{1}{L_i} \right] + \frac{\Delta_j^{(1)}}{L_j} = \\ &= Q_{ij}\Sigma q_i + \Sigma Q_j q_{ji} \\ G''_{kj} &= \Delta_k^{(1)} \left[\frac{1}{L_k} + \frac{1}{L_l} \right] + \frac{\Delta_j^{(1)}}{L_k} = \\ &= \Sigma Q_j q_{jk} + Q_{kj}\Sigma q_k \quad . \end{aligned}\right\} \quad (4\text{-}6)$$

5. Angular Carry-Over Value (Far End)

The angular carry-over value G''_{hj} (or G''_{lj}) at the far end h (or l) is the end slope of the basic structure $\overline{hijkl}$ at that end due to a unit moment applied at j (Fig. 4-2).

$$\left.\begin{aligned} G''_{hj} &= \frac{\Delta_i^{(1)}}{L_i} = Q_{ij}q_{hi} \\ G''_{lj} &= \frac{\Delta_k^{(1)}}{L_k} = Q_{kj}q_{lk} \quad . \end{aligned}\right\} \quad (4\text{-}7)$$

6. Angular Load Function

The angular load function τ''_{ji} (or τ''_{jk}) is the end slope of the basic structure $\overline{hijkl}$ at j due to loads (Fig. 4-3).

$$\left.\begin{aligned} \tau''_{ji} &= Q_{ij}BR_i - Q_{ji}BR_j \\ \tau''_{jk} &= Q_{kj}BR_k - Q_{jk}BR_j \quad , \end{aligned}\right\} \quad (4\text{-}8)$$

where Q_{ij} (or Q_{kj}) is as defined in Eqs. (4-3) and BR_i (or BR_j, BR_k) as defined in Eqs. (2-6, 7).

PART V

PROCEDURE

The analysis of statically indeterminate trusses cannot be accomplished unless the cross-sectional areas of all members are predetermined. How to "produce" these areas of the truss members is the common dilemma of indeterminate truss analysis. "The design of such structures is essentially a "cut and try" process; a structure must be assumed, the redundants determined, the stresses calculated, and the parts proportioned; if the section values differ substantially from those orginally assumed, the process is repeated - one or more times, as necessary" (7). An alternate approach is to calculate the redundants by using the relative values of the elastic constants, i. e. using a ratio of the spring constants to the areas of the truss members (assumed equal for all members) and modulus of elasticity (1).

Whichever approach is used, the following steps are recommended for the analysis of continuous trusses on elastic supports.

1. Angular Truss Constants Due to Rotation
   - (a) Angular Flexibilities (Eqs. 3-1)
   - (b) Angular Carry-Over Values (Eqs. 3-2)
   - (c) Angular Load Functions (Eqs. 3-3)
2. Angular Truss Constants Due to Displacement of Supports
   - (a) Reactions of the Basic Structure $\overline{ij}$ (or $\overline{jk}$) Due to Unit Moment (Eqs. 4-1, 2)

(b) Displacement of Supports of the Basic Structure $\overline{ij}$ (or $\overline{jk}$) Due to Unit Moment (Eqs. 4-3, 4)

(c) Reactions of the Basic Structures $\overline{hi}$, $\overline{ij}$, $\overline{jk}$, and $\overline{kl}$ Due to Loads as Defined in Eqs. (2-6, 8)

(d) Angular Flexibilities (Eqs. 4-5)

(e) Angular Carry-Over Values (Near End) (Eqs. 4-6)

(f) Angular Carry-Over Values (Far End) (Eqs. 4-7)

(g) Angular Load Functions (Eqs. 4-8)

3. Angular Truss Constants Due to Rotation and Displacement of Supports

    (a) Angular Flexibilities (Eq. 2-12)

    (b) Angular Carry-Over Values (Near End) (Eqs. 2-13, 14)

    (c) Angular Carry-Over Values (Far End) (Eqs. 4-7)

    (d) Angular Load Functions (Eq. 2-15)

4. Write Five-Moment Equations

5. Solve for Redundant Moment in Five-Moment Equations

6. Numerical Control - Redundant Moments Must Satisfy Five-Moment Equations

7. Axial Forces

PART VI

ILLUSTRATIVE EXAMPLE

A numerical example is presented to illustrate the application of the five-moment equations to the analysis of continuous trusses on elastic supports. To simplify the numerical labor the areas of all truss members are assumed to be equal to 1 in.^{2}, the modulus of elasticity,

$$E = 30 \times 10^3 \text{ k/in.}^2$$

and the spring constants,

$$C_1 = C_2 = C_3 = 10 \text{ k/ft.}$$

All other values are given in feet, kips, or kip-feet.

EXAMPLE: A four span continuous truss of constant depth, resting on elastic supports and loaded as shown (Fig. 6-1) is analyzed. The real structure $\overline{04}$ is resolved into three basic structures $\overline{12}$, $\overline{23}$, and $\overline{34}$. The computation of the relative angular truss constants due to rotation is demonstrated in Table (6-1) and Fig. (6-2). Because the computed values are relative, they must be divided by AE (1) (30 x 10^3) in order for the real angular constants to be obtained.

1. Angular Truss Constants Due to Rotation

The angular truss constants due to rotation are calculated by means of Eqs. (3-1, 2, and 3), as shown in Table (6-1) and Fig. (6-2).

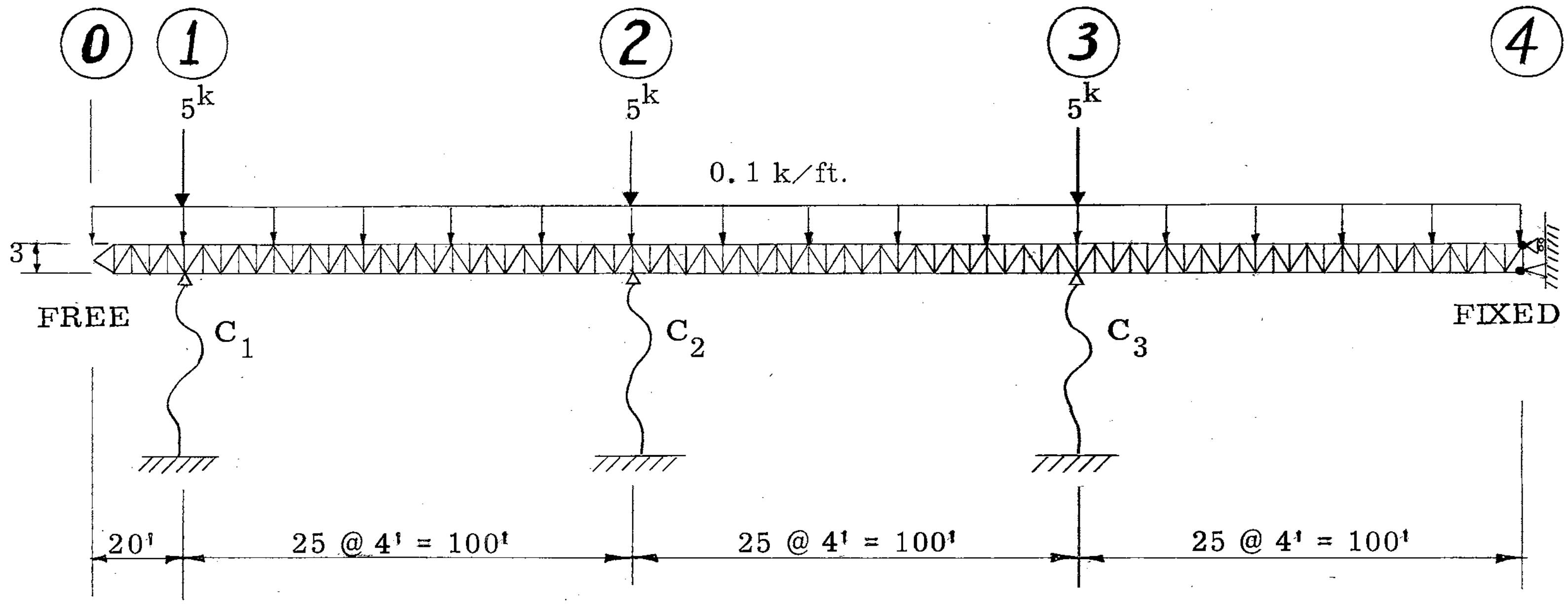


Fig. 6-1

Four Span Continuous Truss

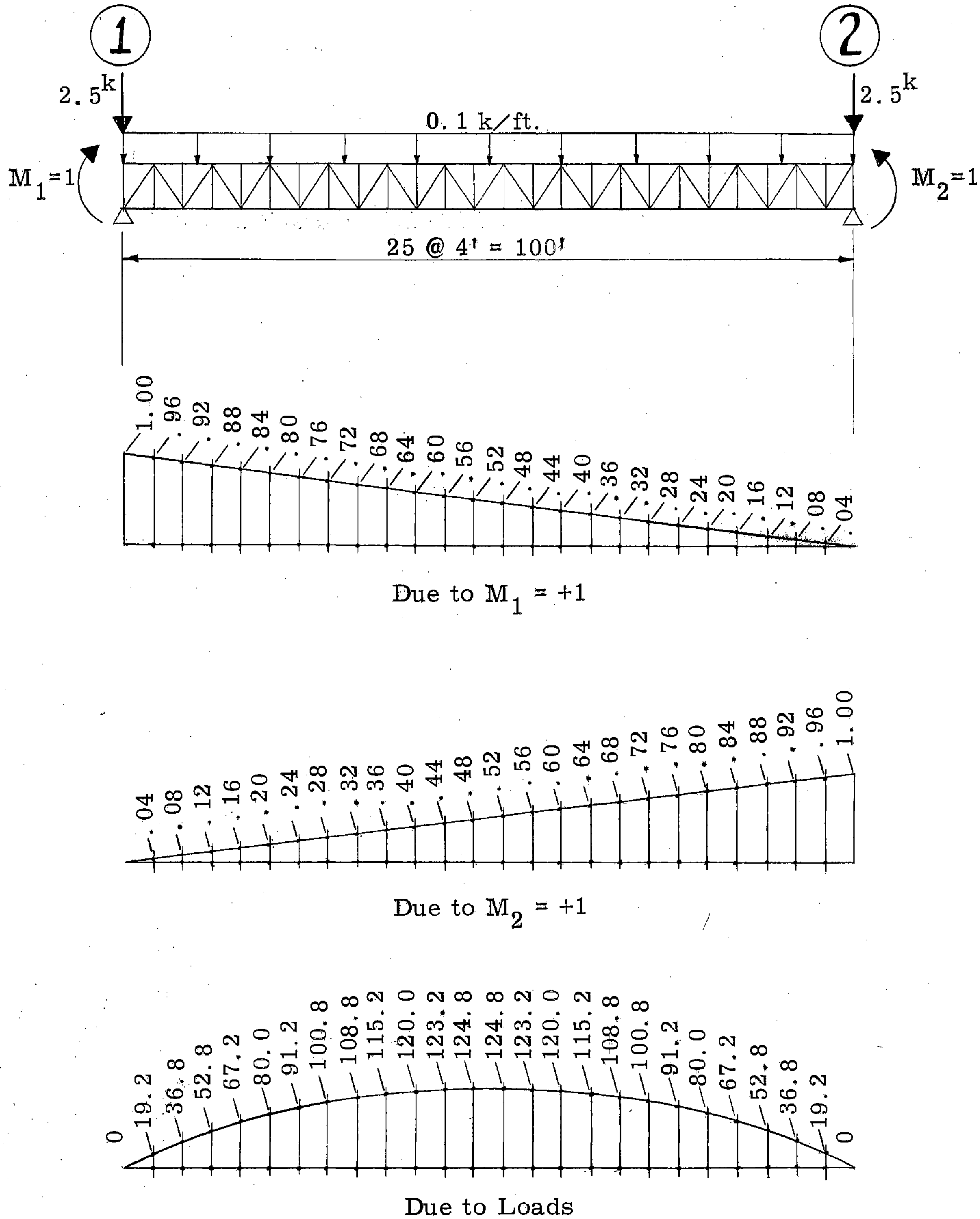


Fig. 6-2

Bending Moment Diagrams
of Basic Structure $\overline{12}$

TABLE I RELATIVE ANGULAR CONSTANTS-DUE TO ROTATION

	m	λ'_m	R'_m	R''_m	α_m	β_m	$\alpha_m^2\lambda_m$	$\beta_m^2\lambda_m$	$\alpha_m\beta_m\lambda_m$	BN_m	$BN_m\alpha_m\lambda_m$	$BN_m\beta_m\lambda_m$
BOTTOM	1	4	3		+.320	+.013	+.410	+.001	+.016	+ 6.40	+ 8.19	+ .33
	2	4	3		+.320	+.013	+.410	+.001	+.016	+ 6.40	+ 8.19	+ .33
	3	4	3		+.293	+.040	+.343	+.006	+.048	+17.60	+20.63	+ 2.82
	4	4	3		+.293	+.040	+.343	+.006	+.048	+17.60	+20.63	+ 2.82
	5	4	3		+.267	+.067	+.285	+.018	+.072	+26.67	+28.48	+ 7.15
	6	4	3		+.267	+.067	+.285	+.018	+.072	+26.67	+28.48	+ 7.15
	7	4	3		+.240	+.093	+.230	+.035	+.088	+33.60	+32.26	+12.50
	8	4	3		+.240	+.093	+.230	+.035	+.088	+33.60	+32.26	+12.50
	9	4	3		+.213	+.120	+.181	+.058	+.104	+38.40	+32.72	+18.43
	10	4	3		+.213	+.120	+.181	+.058	+.104	+38.40	+32.72	+18.43
	11	4	3		+.187	+.147	+.140	+.086	+.108	+41.07	+30.72	+24.15
	12	4	3		+.187	+.147	+.140	+.086	+.108	+41.07	+30.72	+24.15
	13	4	3		+.160	+.173	+.102	+.120	+.112	+41.60	+26.62	+28.79
	14	4	3		+.160	+.173	+.102	+.120	+.112	+41.60	+26.62	+28.79
	15	4	3		+.133	+.200	+.071	+.160	+.108	+40.00	+21.28	+32.00
	16	4	3		+.133	+.200	+.071	+.160	+.108	+40.00	+21.28	+32.00
	17	4	3		+.107	+.227	+.046	+.206	+.096	+36.27	+15.52	+32.93
	18	4	3		+.107	+.227	+.046	+.206	+.096	+36.27	+15.52	+32.93
	19	4	3		+.080	+.253	+.026	+.256	+.080	+30.40	+ 9.73	+30.76
	20	4	3		+.080	+.253	+.026	+.256	+.080	+30.40	+ 9.73	+30.76
	21	4	3		+.053	+.280	+.011	+.314	+.060	+22.40	+ 4.75	+25.09
	22	4	3		+.053	+.280	+.011	+.314	+.060	+22.40	+ 4.75	+25.09
	23	4	3		+.027	+.307	+.003	+.377	+.032	+12.27	+ 1.32	+15.06
	24	4	3		+.027	+.307	+.003	+.377	+.032	+12.27	+ 1.32	+15.06
	25	4	3		0	+.333	0	+.444	0	0	0	0
TOP	1	4		3	-.333	0	+.444	0	0	0	0	0
	2	4		3	-.307	-.027	+.377	+.003	+.033	-12.27	+15.06	+ 1.32
	3	4		3	-.307	-.027	+.377	+.003	+.033	-12.27	+15.06	+ 1.32
	4	4		3	-.280	-.053	+.314	+.011	+.059	-22.40	+25.09	+ 4.75
	5	4		3	-.280	-.053	+.314	+.011	+.059	-22.40	+25.09	+ 4.75
	6	4		3	-.253	-.080	+.256	+.026	+.081	-30.40	+30.76	+ 9.73
	7	4		3	-.253	-.080	+.256	+.026	+.081	-30.40	+30.76	+ 9.73
	8	4		3	-.227	-.107	+.206	+.046	+.097	-36.27	+32.93	+15.52
	9	4		3	-.227	-.107	+.206	+.046	+.097	-36.27	+32.93	+15.52
	10	4		3	-.200	-.133	+.160	+.071	+.106	-40.00	+32.00	+21.28
	11	4		3	-.200	-.133	+.160	+.071	+.106	-40.00	+32.00	+21.28
	12	4		3	-.173	-.160	+.120	+.102	+.111	-41.60	+28.79	+26.62
	13	4		3	-.173	-.160	+.120	+.102	+.111	-41.60	+28.79	+26.62
	14	4		3	-.147	-.187	+.086	+.140	+.110	-41.07	+24.15	+30.72
	15	4		3	-.147	-.187	+.086	+.140	+.110	-41.07	+24.15	+30.72
	16	4		3	-.120	-.213	+.058	+.181	+.102	-38.40	+18.43	+32.72
	17	4		3	-.120	-.213	+.058	+.181	+.102	-38.40	+18.43	+32.72
	18	4		3	-.093	-.240	+.035	+.230	+.089	-33.60	+12.50	+32.26
	19	4		3	-.093	-.240	+.035	+.230	+.089	-33.60	+12.50	+32.26
	20	4		3	-.067	-.267	+.018	+.285	+.072	-26.67	+ 7.15	+28.48
	21	4		3	-.067	-.267	+.018	+.285	+.072	-26.67	+ 7.15	+28.48
	22	4		3	-.040	-.293	+.006	+.343	+.047	-17.60	+ 2.82	+20.63
	23	4		3	-.040	-.293	+.006	+.343	+.047	-17.60	+ 2.82	+20.63
	24	4		3	-.013	-.320	+.001	+.410	+.017	- 6.40	+ .33	+ 8.19
	25	4		3	-.013	-.320	+.001	+.410	+.017	- 6.40	+ .33	+ 8.19
DIAGONAL	1	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	- 8.00	- .68	+ .68
	2	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	+ 7.33	- .62	+ .62
	3	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	- 6.67	- .57	+ .57
	4	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	+ 6.00	- .51	+ .51
	5	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	- 5.33	- .45	+ .45
	6	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	+ 4.67	- .40	+ .40
	7	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	- 4.00	- .34	+ .34
	8	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	+ 3.33	- .28	+ .28
	9	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	- 2.67	- .23	+ .23
	10	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	+ 2.00	- .17	+ .17
	11	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	- 1.33	- .11	+ .11
	12	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	+ .67	- .06	+ .06
	13	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	0	0	0
	14	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	- .67	+ .06	- .06
	15	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	+ 1.33	+ .11	- .11
	16	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	- 2.00	+ .17	- .17
	17	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	+ 2.67	+ .23	- .23
	18	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	- 3.33	+ .28	- .28
	19	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	+ 4.00	+ .34	- .34
	20	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	- 4.67	+ .40	- .40
	21	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	+ 5.33	+ .45	- .45
	22	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	- 6.00	+ .51	- .51
	23	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	+ 6.67	+ .57	- .57
	24	5	2.4	2.4	-.017	+.017	+.001	+.001	-.001	- 7.33	+ .62	- .62
	25	5	2.4	2.4	+.017	-.017	+.001	+.001	-.001	+ 8.00	+ .68	- .68
VERTICAL	1	3			0	0	0	0	0	- 2.70	0	0
	2	3			0	0	0	0	0	0	0	0
	3	3			0	0	0	0	0	- .40	0	0
	4	3			0	0	0	0	0	0	0	0
	5	3			0	0	0	0	0	- .40	0	0
	6	3			0	0	0	0	0	0	0	0
	7	3			0	0	0	0	0	- .40	0	0
	8	3			0	0	0	0	0	0	0	0
	9	3			0	0	0	0	0	- .40	0	0
	10	3			0	0	0	0	0	0	0	0
	11	3			0	0	0	0	0	- .40	0	0
	12	3			0	0	0	0	0	0	0	0
	13	3			0	0	0	0	0	- .40	0	0
	14	3			0	0	0	0	0	0	0	0
	15	3			0	0	0	0	0	- .40	0	0
	16	3			0	0	0	0	0	0	0	0
	17	3			0	0	0	0	0	- .40	0	0
	18	3			0	0	0	0	0	0	0	0
	19	3			0	0	0	0	0	- .40	0	0
	20	3			0	0	0	0	0	0	0	0
	21	3			0	0	0	0	0	- .40	0	0
	22	3			0	0	0	0	0	0	0	0
	23	3			0	0	0	0	0	- .40	0	0
	24	3			0	0	0	0	0	0	0	0
	25	3			-.010	+.010	—	—	—	- 7.50	—	—
Σ							+7.414	+7.414	+3.671		+924.46	+924.46

(a) Angular Flexibilities (Eqs. 3-1)

$$F'_{23} = F'_{34} = \Sigma\alpha_m^{\;2}\lambda_m = \frac{7.414}{(1)(30 \times 10^3)} = 247.1 \times 10^{-6}$$

$$F'_{21} = F'_{32} = F'_{43} = \Sigma\beta_m^{\;2}\lambda_m = \frac{7.414}{(1)(30 \times 10^3)} = 247.1 \times 10^{-6}$$

$$\Sigma F'_2 = 494.2 \times 10^{-6} \quad \Big| \quad \Sigma F'_3 = 494.2 \times 10^{-6} \quad \Big| \quad \Sigma F'_4 = 247.1 \times 10^{-6}$$

(b) Angular Carry-Over Values (Eqs. 3-2)

$$G'_{12} = G'_{23} = G'_{34} = \Sigma\alpha_m\beta_m\lambda_m = \frac{3.671}{(1)(30 \times 10^3)} = 122.4 \times 10^{-6}$$

$$G'_{21} = G'_{32} = G'_{43} = \Sigma\beta_m\alpha_m\lambda_m = \frac{3.671}{(1)(30 \times 10^3)} = 122.4 \times 10^{-6}$$

(c) Angular Load Functions (Eqs. 3-3)

$$\tau'_{12} = \tau'_{23} = \tau'_{34} = \Sigma BN_m\alpha_m\lambda_m = \frac{924.46}{(1)(30 \times 10^3)} = 30.82 \times 10^{-3}$$

$$\tau'_{21} = \tau'_{32} = \tau'_{43} = \Sigma BN_m\beta_m\lambda_m = \frac{924.46}{(1)(30 \times 10^3)} = 30.82 \times 10^{-3}$$

$$\Sigma\tau'_2 = 61.64 \times 10^{-3} \quad \Big| \quad \Sigma\tau'_3 = 61.64 \times 10^{-3} \quad \Big| \quad \Sigma\tau'_4 = 30.82 \times 10^{-3}$$

2. Angular Truss Constants Due to Displacement of Supports

The angular truss constants due to displacement of supports are calculated by means of Eqs. (4-1, 2, 3, 4, 5, 6, 7, 2-6, 8).

(a) Reactions Due to Unit Moment (Eqs. 4-1, 2)

$$q_{12} = q_{23} = q_{34} = \frac{1}{L} = \frac{1}{100} = 10 \times 10^{-3}$$

$$q_{21} = q_{32} = q_{43} = \frac{1}{L} = \frac{1}{100} = 10 \times 10^{-3}$$

$$\Sigma q_2 = 20 \times 10^{-3} \quad \Big| \quad \Sigma q_3 = 20 \times 10^{-3} \quad \Big| \quad \Sigma q_4 = 10 \times 10^{-3}$$

(b) Displacement of Supports Due to Unit Moment (Eq's. 4-3, 4)

$$Q_{12} = Q_{23} = Q_{34} = \frac{1}{L\ C} = \frac{1}{(100)(10)} = 1 \times 10^{-3}$$

$$Q_{21} = Q_{32} = \frac{1}{L\ C} = \frac{1}{(100)(10)} = 1 \times 10^{-3}$$

$$Q_{43} = 0$$

$$\Sigma Q_2 = 2 \times 10^{-3} \quad \Big| \quad \Sigma Q_3 = 2 \times 10^{-3} \quad \Big| \quad \Sigma Q_4 = 0$$

(c) Reactions of the Basic Structures Due to Loads (Eqs. 2-6, 8)

$$BR_1 = 2 + 5 + 5 = 12 \quad \Big| \quad BR_2 = 5 + 5 + 5 = 15$$

$$BR_3 = 5 + 5 + 5 = 15 \quad \Big| \quad BR_4 = 5$$

(d) Angular Flexibilities (Eq's. 4-5)

$$F''_{23} = q_{23} \left[Q_{32} + \Sigma Q_2 \right] = 10 \left[1 + 2 \right] 10^{-6} = 30 \times 10^{-6}$$

$$F''_{34} = q_{34} \left[Q_{43} + \Sigma Q_3 \right] = 10 \left[0 + 2 \right] 10^{-6} = 20 \times 10^{-6}$$

$$F''_{21} = F''_{32} = q_{21} \left[Q_{12} + \Sigma Q_2 \right] = q_{32} \left[Q_{23} + \Sigma Q_3 \right] =$$

$$= 10 \left[1 + 2 \right] 10^{-6} = 30 \times 10^{-6}$$

$$F''_{43} = q_{43} \left[Q_{34} + \Sigma Q_4 \right] = 10 \left[1 + 0 \right] 10^{-6} = 10 \times 10^{-6}$$

$\Sigma F_2'' = 60 \times 10^{-6}$ | $\Sigma F_3'' = 50 \times 10^{-6}$ | $\Sigma F_4'' = 10 \times 10^{-6}$

(e) Angular Carry-Over Values (Near End)(Eqs. 4-6)

$$G_{12}'' = G_{21}'' = - Q_{12}\Sigma q_1 - \Sigma Q_2 q_{21} = - \Sigma Q_1 q_{12} - Q_{21}\Sigma q_2 =$$

$$= - \left[(1)(10) + (2)(10)\right] 10^{-6} = - 30 \times 10^{-6}$$

$$G_{23}'' = G_{32}'' = - Q_{23}\Sigma q_2 - \Sigma Q_3 q_{32} = - \Sigma Q_2 q_{23} - Q_{32}\Sigma q_3 =$$

$$= - \left[(1)(20) + (2)(10)\right] 10^{-6} = - 40 \times 10^{-6}$$

$$G_{34}'' = G_{43}'' = - Q_{34}\Sigma q_3 - \Sigma Q_4 q_{43} = - \Sigma Q_3 q_{34} - Q_{43}\Sigma q_4 =$$

$$= - \left[(1)(20) + (0)(10)\right] 10^{-6} = - 20 \times 10^{-6}$$

(f) Angular Carry-Over Values (Far End) (Eqs. 4-7)

$$G_{13}'' = G_{24}'' = Q_{23} q_{12} = Q_{34} q_{23} = (1)(10) \times 10^{-6}$$

$$G_{31}'' = G_{42}'' = Q_{21} q_{32} = Q_{32} q_{43} = (1)(10) \times 10^{-6}$$

(g) Angular Load Functions (Eqs. 4-8)

$$\tau_{12}'' = - \tau_{21}'' = - Q_{12} BR_1 + Q_{21} BR_2 = \left[- (1)(12) + (1)(15)\right] 10^{-3} = 3 \times 10^{-3}$$

$$\tau_{23}'' = - \tau_{32}'' = - Q_{23} BR_2 + Q_{32} BR_3 = \left[- (1)(15) + (1)(15)\right] 10^{-3} = 0$$

$$\tau_{34}'' = - \tau_{43}'' = - Q_{34} BR_3 + Q_{43} BR_4 = \left[- (1)(15) + (0)(5)\right] 10^{-3} = - 15 \times 10^{-3}$$

$\Sigma \tau_2'' = - 3 \times 10^{-3}$ | $\Sigma \tau_3'' = - 15 \times 10^{-3}$ | $\Sigma \tau_4'' = + 15 \times 10^{-3}$

3. Angular Truss Constants Due to Rotation and Displacement of Supports

The results of steps (1) and (2) combined according to Eqs. (2-12, 13, 14, 15) give the final values of the angular flexibilities, carry-over values (near end), and load functions. The angular carry-over values (far end) are functions of the displacements only (Eq. 4-7).

(a) Angular Flexibilities (Eq. 2-12)

$$\Sigma F_2 = \Sigma F_2' + \Sigma F_2'' = 554.2 \times 10^{-6}$$

$$\Sigma F_3 = \Sigma F_3' + \Sigma F_3'' = 554.2 \times 10^{-6}$$

$$\Sigma F_4 = \Sigma F_4' + \Sigma F_4'' = 257.1 \times 10^{-6}$$

(b) Angular Carry-Over Values (Near End) (Eqs. 2-13, 14)

$$G_{12} = G_{12}' + G_{12}'' = 92.4 \times 10^{-6}$$

$$G_{23} = G_{32} = G_{23}' + G_{23}'' = G_{32}' + G_{32}'' = 82.4 \times 10^{-6}$$

$$G_{34} = G_{43} = G_{34}' + G_{34}'' = G_{43}' + G_{43}'' = 102.4 \times 10^{-6}$$

(c) Angular Carry-Over Values (Far End) (Eq. 4-7)

$$G_{13}'' = G_{24}'' = 10 \times 10^{-6} \qquad G_{31}'' = G_{42}'' = 10 \times 10^{-6}$$

(d) Angular Load Functions (Eq. 2-15)

$$\Sigma \tau_2 = \Sigma \tau_2' + \Sigma \tau_2'' = 58.64 \times 10^{-3}$$

$$\Sigma \tau_3 = \Sigma \tau_3' + \Sigma \tau_3'' = 46.64 \times 10^{-3}$$

$$\Sigma \tau_4 = \Sigma \tau_4' + \Sigma \tau_4'' = 45.82 \times 10^{-3}$$

4. Write Five-Moment Equations (Eq. 2-16)

Since there are three redundant moments, M_2, M_3, and M_4. three five-moment equations must be stated:

$$G_{12}M_1 + \Sigma F_2 M_2 + G_{32}M_3 + G_{42}'' M_4 + \Sigma\tau_2 = 0$$

$$G_{13}'' M_1 + G_{23}M_2 + \Sigma F_3 M_3 + G_{43} M_4 + \Sigma\tau_3 = 0$$

$$G_{24}'' M_2 + G_{34}M_3 + \Sigma F_4 M_4 + \Sigma\tau_4 = 0$$

Computing M_1 as the cantilever moment (-20 k-ft.) in span $\overline{01}$ and with results of step (3) these equations simplify to:

$$556.6\, M_2 + 84.4\, M_3 + 10.0\, M_4 + 56,552 = 0$$

$$84.4\, M_2 + 546.6\, M_3 + 104.4\, M_4 + 46,440 = 0$$

$$10.0\, M_2 + 104.4\, M_3 + 258.3\, M_4 + 45,820 = 0$$

5. Solve for Redundant Moments

From the five-moment equations the solution for redundant moments is:

$M_2 = -92.87$ k-ft.	$M_3 = -40.55$ k-ft.	$M_4 = -157.23$ k-ft.

6. Numerical Control

The results of step (5) must satisfy the five-moment equations of step (4).

$$(556.6)(-92.87) + (84.4)(-40.55) + (10.0)(157.23) + 56,552 \doteq 0$$

$$84.4\,(-92.87) + (546.6)(-40.55) + (104.4)(-157.23) + 46,440 \doteq 0$$

$$(10.0)(-92.87) + (104.4)(-40.55) + (258.3)(-157.23) + 45,820 \doteq 0$$

7. Axial Forces

Once the redundant moments are known the final values of the axial forces for all truss members are calculated by means of Eqs. (3-4).

For example, in span $\overline{12}$:

$$N_{B3} = BN_{B3} + \alpha_{B3}M_1 + \beta_{B3}M_2$$

$$= 17.60 + (.293)(-20.00) + (.040)(-92.87)$$

$$= 8.03 \text{ k}$$

PART VII

SUMMARY AND CONCLUSIONS

In this study the general procedure for the analysis for continuous trusses on elastic supports is presented. The general five-moment equation is derived and the physical interpretation of its terms is explained.

The presented method is adequate for application in engineering practice. The derived five-moment equation is applicable to continuous trusses for any number of spans. However, the numerical labor increases with the number of spans and the method used for solution of the system of simultaneous equations becomes impractical when the number of unknown moments reaches five or more. In such a case, it is suggested a carry-over procedure (1, 2, 3) be adapted.

However, since most continuous trusses on elastic supports are limited to less than five spans, the use of the method outlined in this study will prove to be desirable.

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VITA

James D. Cobb

Candidate for the Degree of

Master of Science

Thesis: ANALYSIS OF CONTINUOUS TRUSSES ON ELASTIC SUPPORTS

Major Field: Civil Engineering

Biographical:

Personal Data: Born in Norman, Oklahoma, on November 5, 1931, the son of Fred and Kate Cobb.

Education: Graduated from Norman High School in May, 1949. Attended Oklahoma University from September, 1949 to May, 1951. Entered Oklahoma State University in September 1955 and spent one intervening summer at Oklahoma City University. Completed the requirements for the degree of Bachelor of Science in Civil Engineering at the Oklahoma State University in June, 1959. Completed the requirements for the degree of Master of Science at the Oklahoma State University in January, 1960.

Professional Experience: Entered the United States Marine Corps in May, 1952 and received honorable discharge in May, 1954. Served in the Korean War from December, 1952 to June, 1953. Employed by Cobb Engineering Company, Oklahoma City, Oklahoma, as a draftsman and inspector from May, 1954 to September, 1955, as resident engineer from June, 1956 to September, 1956, and as draftsman from June, 1957 to September, 1957. Employed by the School of Civil Engineering at the Oklahoma State University as a Student Assistant from September, 1957 to June, 1958, from September, 1958 to June, 1959, and as Graduate Assistant (Research and Teaching) from June, 1959 until January, 1960. Author of one article in the <u>Oklahoma State Engineer</u>. Presently, an Associate Member of the American Society of Civil Engineers and Engineer-in-Training as a Professional Engineer in the State of Oklahoma.