APPLICATION OF BOOLEAN ALGEBRA TO HYDRAULIC CIRCUIT SIMPLIFICATION

By

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APPLICATION OF BOOLEAN ALGEBRA TO HYDRAULIC

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CHAPTER I

INTRODUCTION

In the past few years the trend in industry has been toward automation. More and more machines with automatic controls have appeared. Finding a suitable control for a machine has often been an expensive and laborious task for the engineer. When a control procedure is so complicated as to need a memory device, the electronic control is probably the most commonly used. The reason is that more is known of design procedures for electrical components and also because more electronic control components are available on the market.

Most large machines already have a hydraulic power source available if any of the machine has hydraulic actuators. The control system and the machine might be much cheaper to build and simpler to service if electronic components were eliminated. This, of course, is one of the factors to be determined and considered in the design of a particular machine. It is the opinion of the author that more hydraulic control systems would be used if more were known about their design.

A more or less trial and error procedure is now in general use for the design of hydraulic controls. Since more progress has been made in the electronic controls field, an investigation of the methods of design used in this field was undertaken to determine if a similar design method could be used for hydraulic controls.

Boolean algebra is one mathematical device used for simplification of two-state devices (on or off) used in electronic design. Since most hydraulic power components are of the type to be either at one state or another (cylinder extended or retracted), the algebra of George Boole will apply as long as time sequence is considered.

The problem under consideration here is to select a sequence of operations for two double-acting cylinders such that one operation does not always follow another operation in the sequence of operation. This, in the opinion of the author, determines the need for a "memory device" in the control system. Since the same end result in the memory part of the control system can be accomplished in different ways by a different arrangement and number of components, the determination of all possible variations in arrangement is unimportant because one arrangement will probably be of more value (perhaps determined by economics) than the other possibilities. A method by which all other possible arrangements may be determined from one arrangement would then be of extreme value. Boolean algebra representation of the situation is one method of accomplishing this. After a function of the state of the output of the system (cylinders) is determined, the simplification methods of Boolean algebra may be applied to the function to determine all possible forms of the function and thus all arrangements of the system can then be determined. This investigation follows this procedure. The problem is not difficult to solve by trial and error methods but the problem illustrates the point that the author wishes to convey as well as would a more complicated problem or situation. The usefulness of the methods will be stated as opinions of the author further in this investigation but it should be realized that the material is presented with the idea that those with more experience in the field could probably draw a better conclusion from the material presented as it applies to their particular problem. For this reason, the author will state the conclusions of the investigation more as a means of stimulating others to consider how the methods presented may be applied to their particular problems.

CHAPTER II

BOOLEAN ALGEBRA

Years ago George Boole deduced an algebra by which a reasoning process could be carried out. It was possible to represent an animal or thing by a symbol, use algebraic mechanics on simple equations containing the symbols, and obtain new equations representing a meaning that the mind usually derives in its process of thought. Of course, the original statement had to be written in equation form before the algebraic manipulations could be exercised. After the form of the equation was changed, the translation of the meaning back into words had to be performed. Some of the meaning of the matter could be lost in this translation process or perhaps the original situation was too involved to be represented by a single equation of usable size.

In the electrical engineering field, complicated switching requirements of computers can be stated by a Boolean function that can be changed to different forms by the same type processes as were used in Boolean algebra. Certain types of electrical circuits represent a Boolean term (shown later). Thus, the requirements of the output of a circuit for certain inputs can be determined if the circuit is represented in Boolean form.

Boolean terms have the characteristic of being two-valued--

either one or zero. This is analogous to the "on" state or "off" state of a switch, and thus fits the requirements of switching circuits in this respect. For notation purposes, assume that the inputs of a circuit are A and B. The output of the circuit can then be represented by a function of A and B.

$$\text{output} = f(A, B)$$

Let A represent a 1 on "on" value of input A and B represent a 1 or "on" value of input B. Similarly let $\overline{A}$ (read not-A) represent the off or zero state of input A. $\overline{B}$ represents no signal at input B. Then the output of a switching circuit with inputs A and B could be represented by

$$\text{output} = f(A, \overline{A}) + f(B, \overline{B}) + f(A, B, \overline{A}, \overline{B})$$

From the above equation the output function can be seen to consist of terms containing any of the variables A, $\overline{A}$, B, and $\overline{B}$. With the theorems and postulates of Boolean algebra or other methods developed later, the form of the equation (output) can be changed without changing the value of the output.

The postulates of interest here are:

1. If an element A is a member of a class, and B is also an element of the same class, then (A+B) is also a part of the given class.
2. Element A is a part of a given class as is element B. Then A•B is also a member of the given class.
3. An element 0 exists such that element A + element 0 = A for every element A in a given class.

4. There is an element 1 such that in a given class there exists $A \cdot 1 = A$.

5. When two elements A and B are in the same class, $(A+B) = (B+A)$.

6. When two elements A and B are in the same class, then $(A \cdot B) = (B \cdot A)$.

7. When elements A and B are in the same class then $A+(B \cdot C) = (A+B) \cdot (A+C)$.

8. When elements A and B are in the same class, $A \cdot (B+C) = (A \cdot B) + (A \cdot C)$.

9. Elements 0 and 1 mentioned in postulates 3 and 4 must be unique and then for every element A in a given class, there is an element $\overline{A}$ such that $A \cdot \overline{A} = 0$, and also $A+\overline{A} = 1$.

10. At least two elements, A and B, exist in a given class such that $A \neq B$.

From the above postulates a set of formal theorems has been deduced. The theorems are stated below in equation form when convenient with no proof.

1. The 0 element in postulate 3 is unique.

2. The element 1 in postulate 4 is unique.

3. $A + A = A$.

4. $A \cdot A = A$.

5. $A + 1 = 1$.

6. $A \cdot 0 = 0$.

7. $A + A \cdot B = A$.

8. $A(A + B) = A$.

9. $\overline{A}$ can be uniquely determined.

10. $\overline{(\overline{A})} = A$.

11. $\overline{(A + B)} = \overline{A}\,\overline{B}$.

12. $A + (\overline{A} + C) = 1$.

13. $A \cdot (\overline{A}C) = 0$.

14. $\overline{AB} = \overline{A} + \overline{B}$.

15. $(A + B) + C = A + (B + C)$.

16. $(A \cdot B) \cdot C = A \cdot (B \cdot C)$.

17. $A + \overline{A}B = A + B$.

18. $A \cdot (\overline{A} + B) = AB$.

19. $(A + B)\ (\overline{A} + C) = AC + \overline{A}B$.

20. $\overline{(AB + B\overline{C})} = \overline{A}C + \overline{B}\ \overline{C}$.

21. $\overline{(A + C)(B + \overline{C})} = (\overline{A} + C)(\overline{B} + \overline{C})$.

The above theorems are useful when the form of a Boolean expression is to be changed. The simplification procedure is a change in the form of a Boolean function so that it is the simplest expression possible for given specifications. In other words, there is no simplest form if no qualifications are to be met.

A certain function is represented by the equation

$$f = A \cdot B + A \cdot C.$$

By factoring the A, the function is then

$$f = A \cdot (B + C).$$

This illustrates that a function can have its form changed with no change in its value. The terms A, B, and C can have either the value zero or one. Take the case of $A = 1$, $B = 1$, and $C = 0$.

$$f = (1)\cdot(1) + (1)\cdot(0)$$
$$= 1 + 0$$
$$= 1$$

or

$$f = 1\cdot(1 + 0)$$
$$= 1\cdot(1)$$
$$= 1$$

Then either form of the function obtains the same end result although the form of each is different.

The function can be expressed in many different forms by using the theorems stated earlier in this chapter.

A visual simplification or form change will be introduced to help explain the simplification processes of Boolean algebra.

This particular visual method uses a diagram called a Veitch diagram. If the variables are A, $\bar{A}$, B, and $\bar{B}$, then the Veitch diagram would be as follows. B is represented by the area of the top two squares and A is represented by the area of the two left squares.

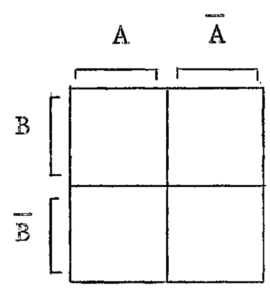

Figure 1. Two-Variable Veitch Diagram

Considering the function $f = A\cdot B + A\cdot C$, the diagram would contain three sets of variables A, B, and C. The Veitch diagram would then be:

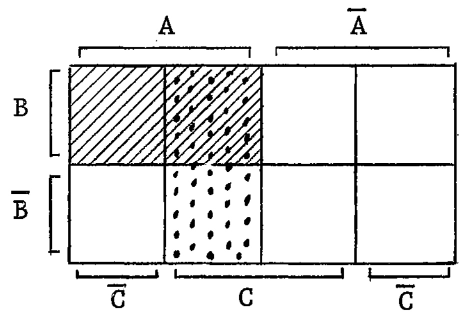

Figure 2. Three-Variable Veitch Diagram

The cross-hatched area represents that part of the function which is A·B. It is the cross-hatched area common to both A and B. The dotted area represents that area common to A and C. Notice that the areas are common to A and C and that the areas overlap. The total cross-hatched and dotted area represents the function. A function can have an area duplicated in its terms but this has no particular meaning. The duplication is similar to two buttons to the same doorbell being pushed at the same time. The bell does not ring twice as loud.

By inspecting the Veitch diagram, one could ascertain that the area could also be represented by the following terms:

$$f = A \cdot C + A \cdot B \cdot \overline{C}$$

$$f = A \cdot C \cdot \overline{B} + A \cdot B \cdot C + A \cdot B \cdot \overline{C} \ .$$

Each form is equivalent with the original equation. The original equation is probably the simplest form depending on the conditions.

It should be noted that the function $f = A \cdot C \cdot \overline{B} + A \cdot B \cdot C + A \cdot B \cdot \overline{C}$ contains a form of each of the three variables in each term. The name minterm has been given to such terms. They represent a minimum area (one square) on the Veitch diagram.

The term A•C can be read that the signal A "and" signal C must be present to have a signal out. From this comes the name of a circuit called an "and" circuit which has the characteristics wanted. In electronics the circuit could be represented by diodes as follows:

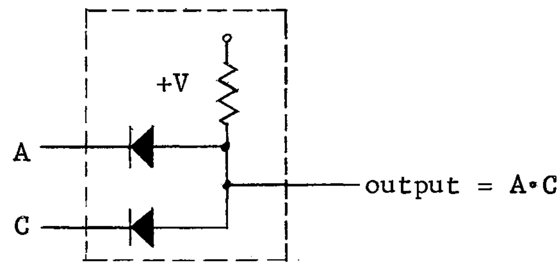

Figure 3. "And" Circuit

In hydraulic components the same effect could be obtained by the following:

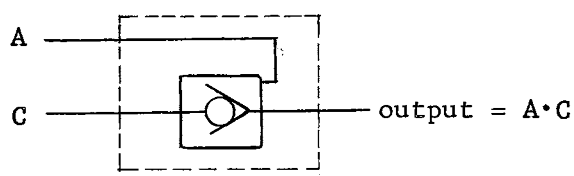

Figure 4. Pilot-Check Valve "And" Circuit

The term (B + C) could be represented in electronic components by the circuit below.

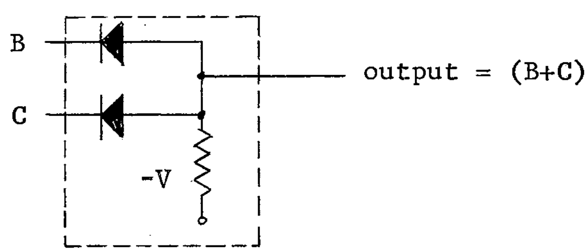

Figure 5. "Or" Circuit

In hydraulic components the circuit would be:

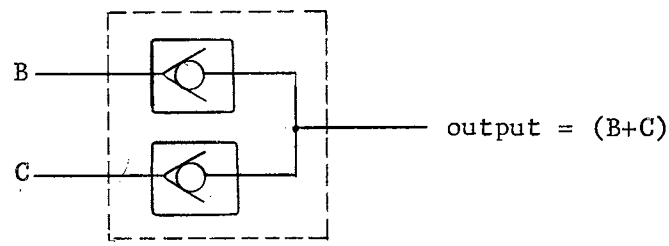

Figure 6. Hydraulic "Or" Circuit

The name given to this circuit is the "or" circuit because either signal at B "or" C would place a signal at the output.

Another circuit of interest is the flip-flop circuit. This circuit has two outputs, and when one signal is an "on" or one, the other is an "off" or zero. The toggle flip-flop circuit remains at one state until a signal is received and then the outputs change to opposite states. The next signal received again reverses the state of the outputs. A typical circuit is as follows:

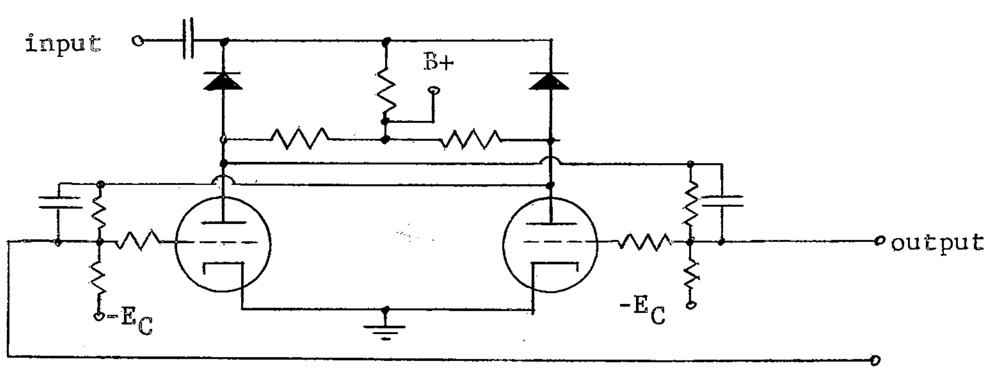

Figure 7. Toggle Flip-Flop Circuit

It is believed that a single hydraulic component which serves the same purpose as the toggle flip-flop does not exist.

Another bi-stable state flip-flop circuit exists which has two inputs and two outputs. A signal at one input switches one of the outputs to either a 1 or 0 state depending on the particular circuit. If the state of the flip-flop is already correct, then nothing will happen to the state of the circuit. A signal at the other input would, of course, reverse the zero or one state of each output. A typical circuit follows.

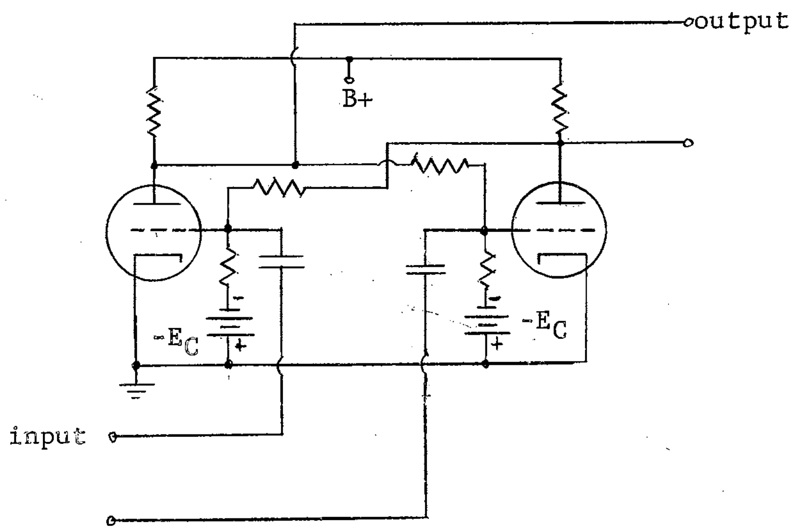

Figure 8. Flip-Flop Circuit

CHAPTER III

THE STATEMENT AND SOLUTION OF A PROBLEM

For illustrative purposes a problem will be stated and then Boolean algebra will be applied to the problem to illustrate the use of Boolean algebra.

The problem concerns the operation of two hydraulic actuators or cylinders. The cylinders are double-acting or have the property of applying force and movement in two directions. The two cylinders have the identification numbers (1) and (2).

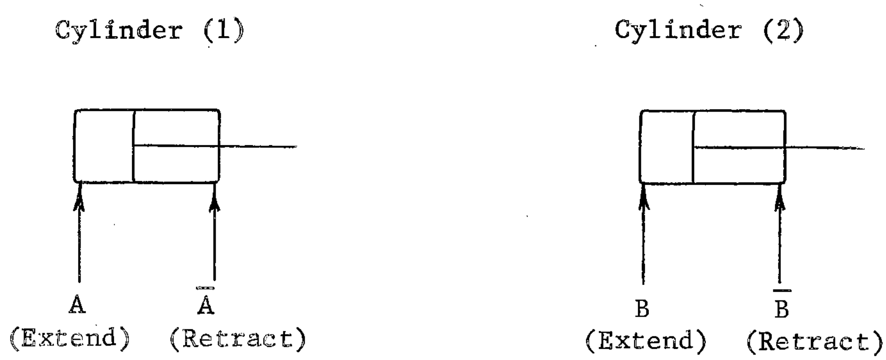

Figure 9. Cylinders

The sequence of operation is:

Cylinder (1) extends.

Cylinder (1) retracts.

Cylinder (2) extends.

Cylinder (1) extends.

Cylinder (2) retracts.

Cylinder (1) retracts.

The terms A, $\overline{A}$, B, and $\overline{B}$ refer to the signals which cause the cylinders to either extend or retract. The functions A and $\overline{A}$ are respectively the signal for cylinder (1) to extend and the signal for cylinder (1) to retract. The term B is the signal for cylinder (2) to extend and $\overline{B}$ is the signal for cylinder (2) to retract.

In symbol form the sequence of operation will be used as follows:

(1)E

(1)R

(2)E

(1)E

(2)R

(1)R.

Where (1)E represents the extend operation of cylinder (1), and (1)R represents the retract operation of cylinder (1). The rest of the sequence of operation symbols are self-explanatory.

In Boolean terms the state of the input signals for each operation in the sequence are as follows.

(1)E	A $\overline{B}$
(1)R	$\overline{A}$ $\overline{B}$
(2)E	$\overline{A}$ B
(1)E	A B

(2)R $\quad A\ \overline{B}$

(1)R $\quad \overline{A}\ \overline{B}$.

A simple Boolean function f(A) in terms of the variables cannot be written because of the time factor in the sequence. It is possible for the sequence of operations to be either synchronous or asynchronous. Either type of operation is suitable although asynchronous operation is more prevalent in hydraulic circuits.

The memory element must be employed in this problem since one operation in the sequence does not always follow the same previous operation. This necessitates the use of the electrical flip-flop circuit or the two-position valve in hydraulic components.

For the sequence of operations stated, the following will be assumed given.

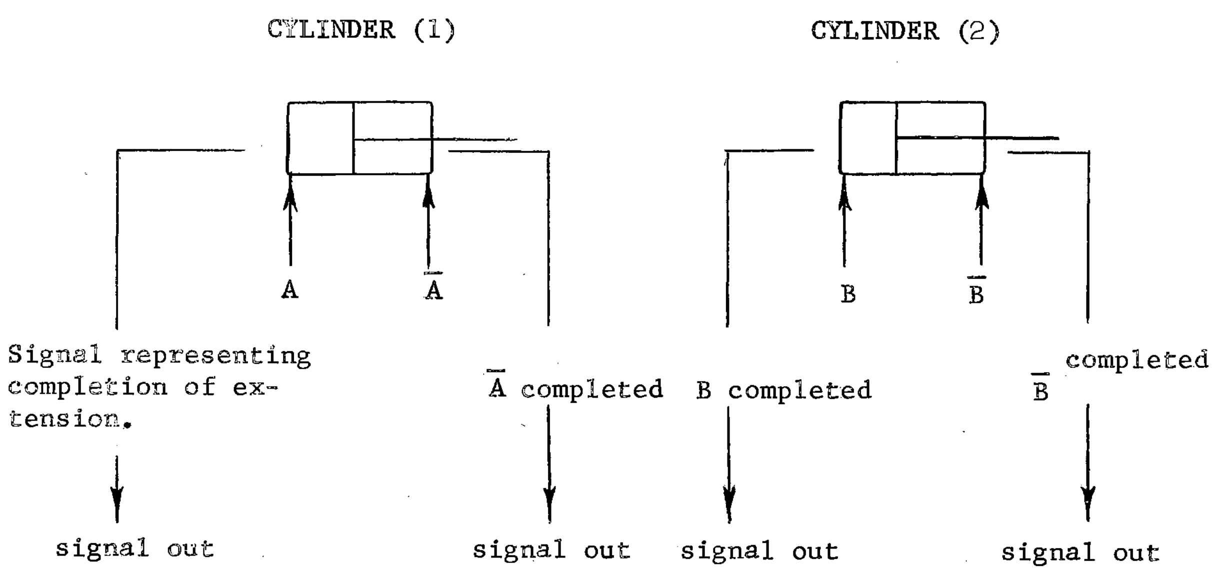

Figure 10. Data

The above assumptions reduce the problem to the sequence-time operation of the two cylinders. A signal out will be assumed to exist at the completion of each operation.

The figure which follows represents one form of a hydraulic circuit that will perform the sequence of operations stated in the problem.

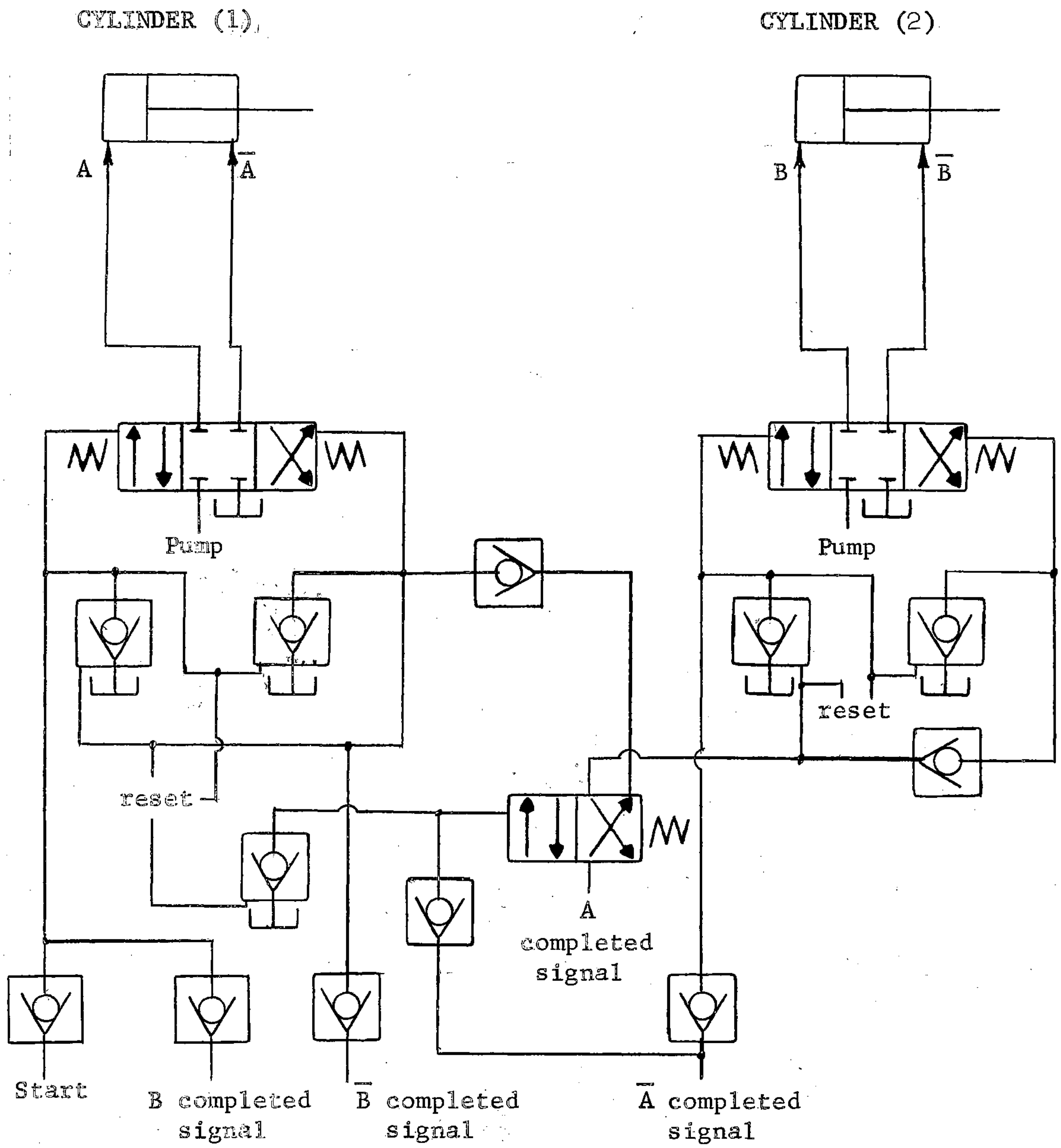

Figure 11. General Hydraulic Circuit for Problem.

The figures below show the brain circuitry of Figure 11 and an electrical analogy.

All but essential components have been deleted.

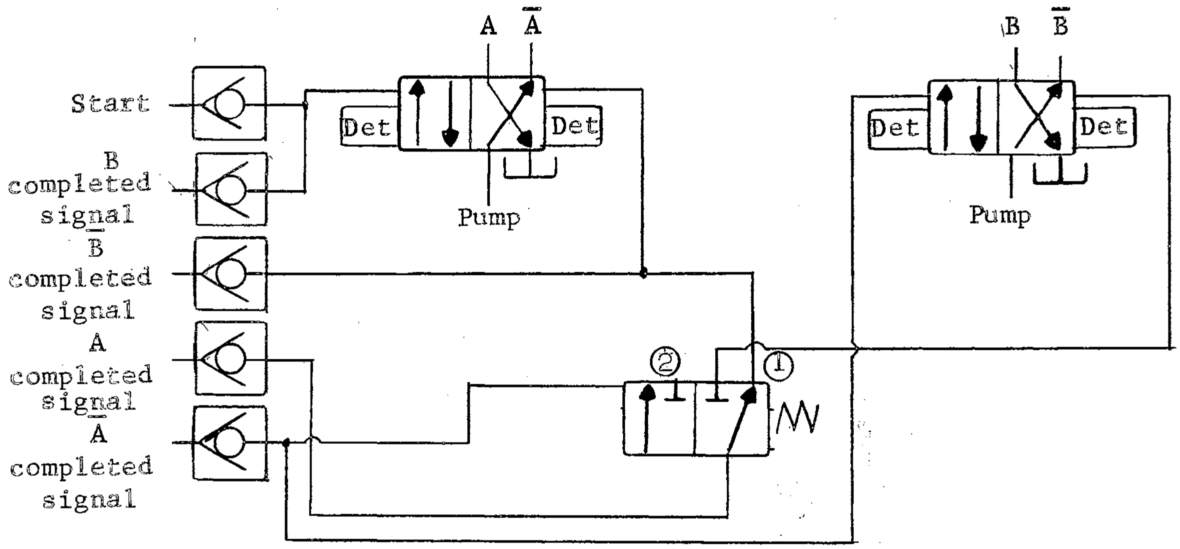

Figure 12. Partial Control Circuit

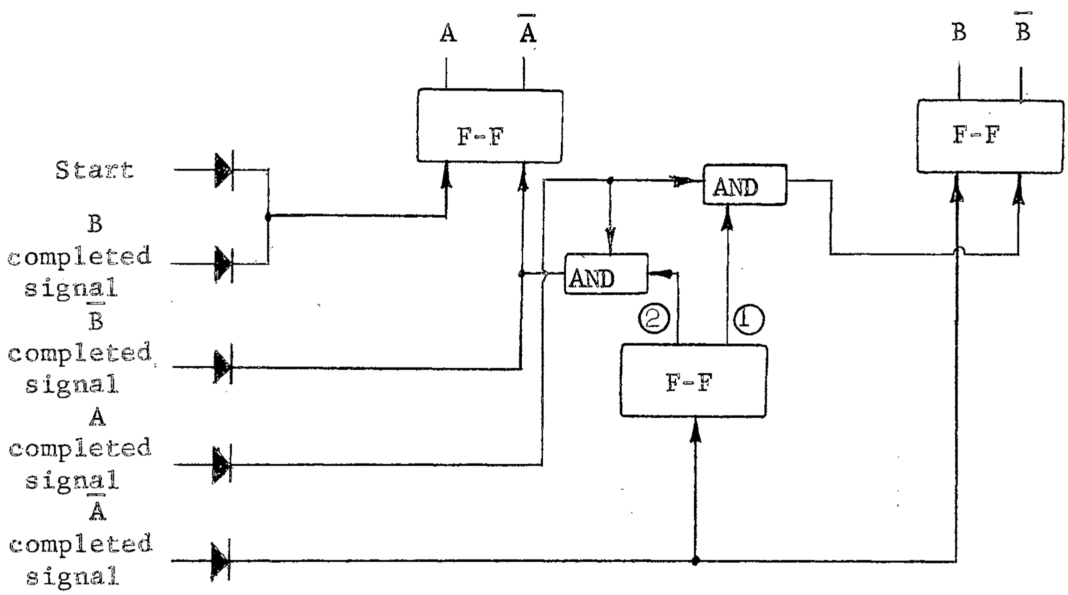

Figure 13. Electrical Analogy of Partial Control Circuit

The next figure shows another possible form of the brain circuitry. The circuit was determined by using hydraulic analogies for the electrical components of Figure 13.

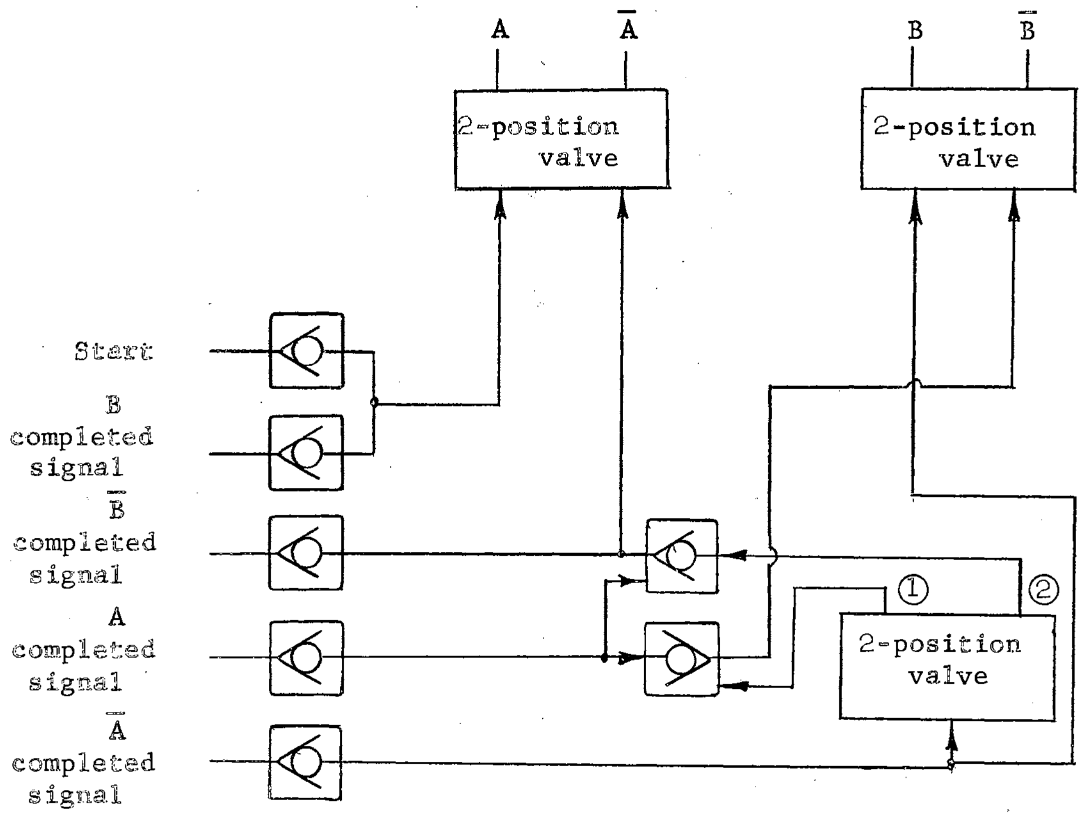

Figure 14. Modified Brain Circuit

Any of the functions f(A), f($\overline{A}$), f(B) and f($\overline{B}$) could have their form changed by Boolean algebra methods but they are in as simple form as is needed for this problem.

For illustration purposes the form of f($\overline{B}$) and f($\overline{A}$) will be changed.

$$f(\overline{B}) = A_c \cdot ① = A_c(\overline{A}_c + ①)$$

$$f(\overline{A}) = \overline{B}_c + A_c \cdot ② = \overline{B}_c + A_c(\overline{A}_c + ②)$$

The circuit for the changed forms of f($\overline{B}$) and f($\overline{A}$) follows, leaving the reader to make the comparison between Figure 14 and Figure 15.

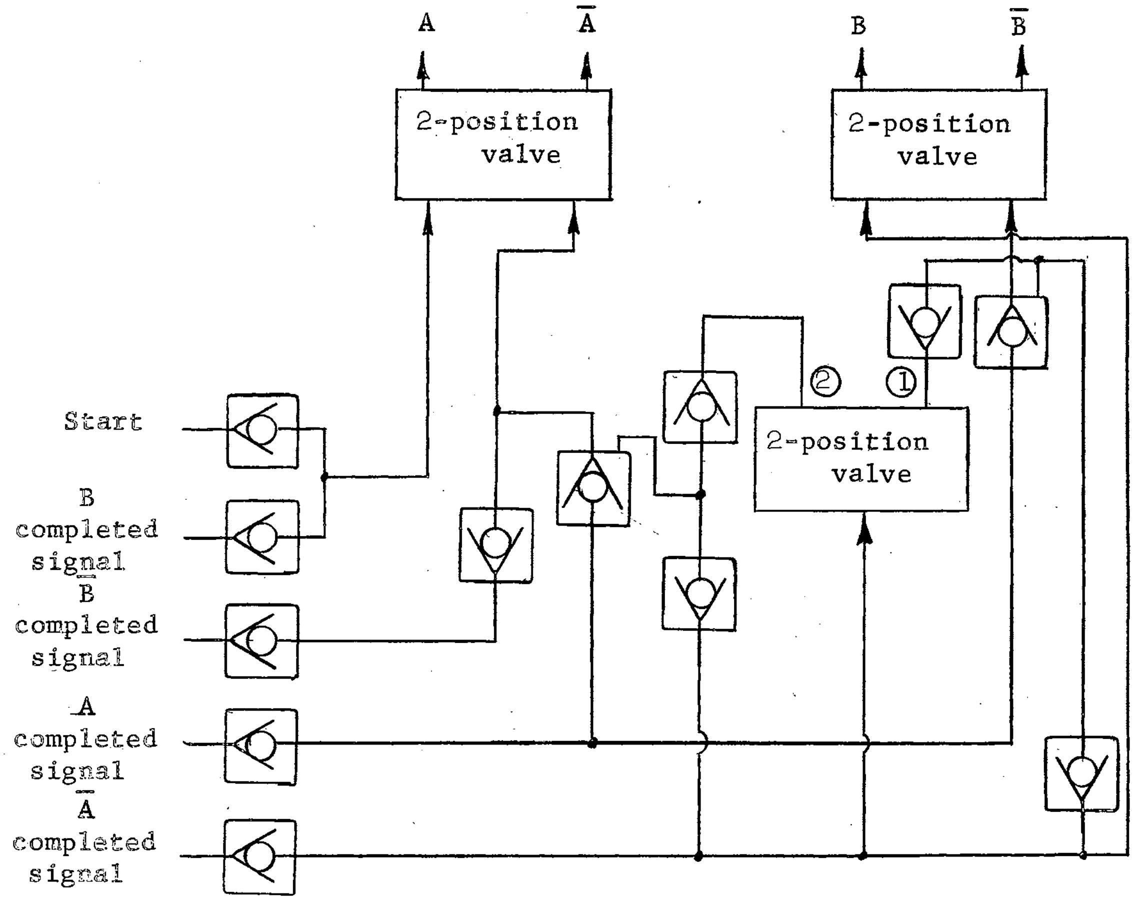

Figure 15. Different Form of Brain Circuit

It is evident that this circuit is more complicated than Figure 14. The form has been changed but the output sequence is the same.

CHAPTER IV

CONCLUSIONS

A system that uses a small element to control a larger power element usually has three distinct divisions. One division is a central power source. Another division is a control system that controls the last division. The last division is composed of components that change the power from the source into the desired end result. A system using an electrical power source has an analogous system using fluid under pressure as a power source. It would be a valid conclusion that the general system could be designed on the basis of using one type of power source alone. Then if any part or perhaps the whole system need be changed to another type power source, a direct analogy translation could be made. On this basis one could use the known design methods of each or all systems using a particular type power source. Which design method one uses would depend on the proficiency of the designer. The type of central power source used in the actual system would depend on such factors as initial cost, operating cost, maintenance cost, size, speed of operation, safety, etc.

In this investigation a hydraulically controlled and operated system was determined which would produce a given end result. The system was then converted to an analogous electrical system. The electrical system was expressed as a Boolean function and was then operated on so as to change its form without changing the end result. The new form of the electrical system was converted back into an analogous hydraulic system to provide an evaluation between the original hydraulic design and the changed form of the hydraulic system. The conclusions reached are that the form of the hydraulic circuit can be changed by using the Boolean algebra techniques of simplification. After the form of the system has been changed, an evaluation of which form is better can be made, depending on the particular circumstances under which the system is to be used.

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