VIRTUAL MASS OF A CYLINDER EXECUTING HARMONIC MOTION

By

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VIRTUAL MASS OF A CYLINDER EXECUTING HARMONIC MOTION

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Of the people who helped with the investigation, two deserve special recognition. The counsel and guidance afforded the writer by Professor L. J. Fila is very much appreciated. Furthermore, the writer is grateful for the aid his wife, Jean, provided in taking the experimental data.

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CHAPTER I

INTRODUCTION

The object of this thesis is to determine experimentally the virtual mass of a cylinder executing harmonic motion in water. The virtual mass concept permits a different approach to the motion analysis of accelerating bodies which encounter fluid resistance. Evaluation of the fluid resistance of a body moving at a particular velocity through an incompressible fluid is easily accomplished with an equation that is parabolic in form with respect to velocity, provided the velocity does not exceed that of sound. However, the overall effect of this type of fluid resistance on the complicated harmonic motion of a freely vibrating body is difficult to determine analytically.

One noticeable effect is the lowering of the natural frequency of vibration of the body. A differential equation containing a term with the velocity raised to a power of three or greater will define the motion of the body but most of the forms encountered are difficult to solve. An alternate approach to the problem utilizes the virtual mass concept and gives a differential equation which does not contain a velocity term. The second differential equation gives the same natural frequency of vibration as the first by increasing the mass of the vibrating body by multiplying by an appropriate constant. As an added advantage the second differential equation has a standard form.

The apparent increase in the mass of a body accelerating through a fluid as compared with a vacuum is called the virtual mass of the body. It is regarded as a constant for a particular geometric body and is expressed in terms of the mass of the fluid displaced by the body in the works of other investigations. The experimental results obtained compared favorably with those of previous investigators but an expected relationship introducing the frequency of vibration as a parameter of a vibrating body's virtual mass was not definitely established.

CHAPTER II

PREVIOUS INVESTIGATIONS

A sufficient number of investigations involving the virtual mass of accelerating bodies in fluids have been conducted to establish certain facts. The important features of these investigations that pertain to the virtual mass of a cylinder will be discussed in this chapter.

Dryden, Munaghan and Bateman (1), in summarizing the previous works involving virtual mass, noted that either rigid finite boundaries or a compressible fluid will increase the virtual mass of an object. They indicated the common method of experimental analysis of virtual mass involved a sphere mounted on a pendulum. The change in the natural frequency of oscillation of the pendulum from air to water determined the virtual mass and the results agreed with analytical works that considered viscosity effects. The authors cited further analytical work which gave a value of 1.00 times the displaced mass of the fluid for the virtual mass of a cylinder accelerated transversely to its axis.

A different experimental method was used by T. E. Stelson and F. T. Mavis (2) to determine the virtual mass of accelerating objects. They determined the virtual mass of an object by the change in the natural frequency of vibration of a vibrating beam which drove the cylinder in water and then in air. The frequency of vibration did not exceed 208 cycles per second. Results of their tests established the virtual mass of a cylinder as 1.00 times the displaced mass of the fluid for motion transverse to the cylinder's axis if the ratio of the length to diameter was less than infinity but greater than 1.2.

The energy approach to analytical development of the virtual mass of an accelerating cylinder is presented by H. R. Vallentine (3) in his Applied Hydraulics for rotation flow in a fluid with infinite boundaries, and the results gave a value of 1.00 times the displaced mass of the fluid as the virtual mass for a cylinder moving transverse to its axis. Vallentine noted that since pure irrotational flow is difficult to achieve and fluid boundaries in practical applications are finite, the value of the virtual mass obtained from the energy method would be expected to differ from actual measured values. Also a time lag between the fluid's motion and the cylinder's motion should be expected when the fluid has finite boundaries.

Each investigation pointed out that the effects of virtual mass in low density fluids, particularly air, could be neglected. Further evidence of this fact is their use of the natural frequency of vibration in air in place of the natural frequency of vibration in a vacuum when they experimentally determined the virtual mass of an object accelerating in a fluid.

CHAPTER III

TEST APPARATUS AND EQUIPMENT

An apparatus was constructed to determine experimentally the virtual mass of a cylinder executing harmonic motion in water. Since the funds that were available were limited, the test apparatus was designed to be as simple as possible and to utilize existing equipment in the Mechanical Engineering laboratory.

A special three-horsepower varidrive with a speed range of 600 to 4200 revolutions per minute was modified to serve as the power source by extending the drive shaft. A mechanism was designed to drive the model with a harmonic motion having an amplitude of one-half inch and to measure the driving force. This mechanism used a bushing, a yoke, an eccentric, a guide for the yoke, and a torque arm. The oilite bushing attached to the varidrive shaft supported and positioned the yoke guide which was held in place by the eccentric fastened to the shaft. The yoke guide permitted the shaft and bushing to rotate freely as the eccentric drove the yoke and the attached model. Rotation of the yoke guide due to the force required to drive the reciprocating mass and the model was prevented by a torque arm attached to the yoke guide.

Numbered weights made of hex stock were used to prevent the yoke guide from rotating due to the force required to drive the model. The weights were fastened to the yoke guide's torque arm by a weight rod.

The weight required to prevent rotation of the yoke guide was later used to determine the force driving the model.

Lubrication of the yoke guide's bearing and the supporting oilite bushing was provided by an oil cup which fed 30 weight oil to the rotating bushing through a felt wick. The lubrication of the other moving parts was accomplished with grease.

A separate torque arm support which was used to support the torque arm when the machine was not running also served as a reference point for the torque arm during the actual runs. The torque arm support also held the covered water tank. Details of the torque arm support and the covered water tank are shown in Figures 10, 11, and 12.

The cylindrical model was made of two inch outside diameter aluminum tubing. Aluminum end plugs pressed into the tubing gave the cylinder an overall length of 9.25 inches. A one inch diameter hole was drilled in the middle of the cylinder normal to its axis, and then an aluminum bushing 3.75 inches long with an outside diameter of one inch and an inside diameter of .516 inches was pressed into the hole and welded. Two holes were tapped for set screws in the bushing so the model could be attached to the yoke's drive rod.

Other equipment used consisted of a chronotach, two sets of precision weights, a balance, and a special set of calibration weights. The chronotach was used to measure the rotative speed of the varidrive, and it contained a tachometer, a revolutions counter and a timing clock. The two sets of precision weights ranged from .0625 ounces to two pounds, were used with the balance to weight the weights required to balance the torque arm and the special calibration weights which were later attached to the yoke's drive rod. The special calibration weights were made of two inch steel shafting with a .516 axial hole to permit them to be placed on the yoke's drive rod. A set screw was used to attach the weights to the yoke's rod. A strobe light was used to help check the relationship of the driving force to the displacement of the model.

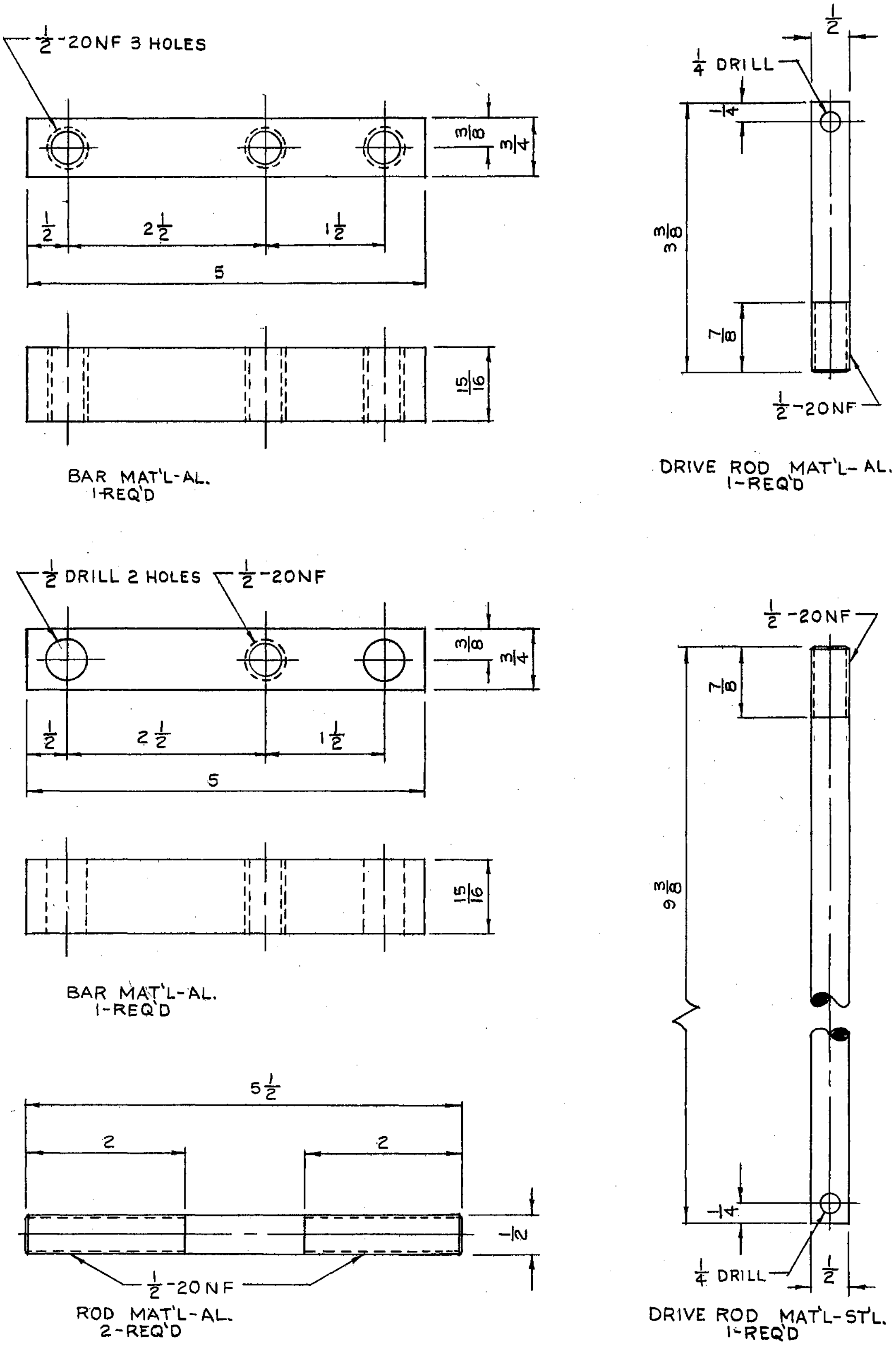


Figure 1. Yoke and Drive Rod Details

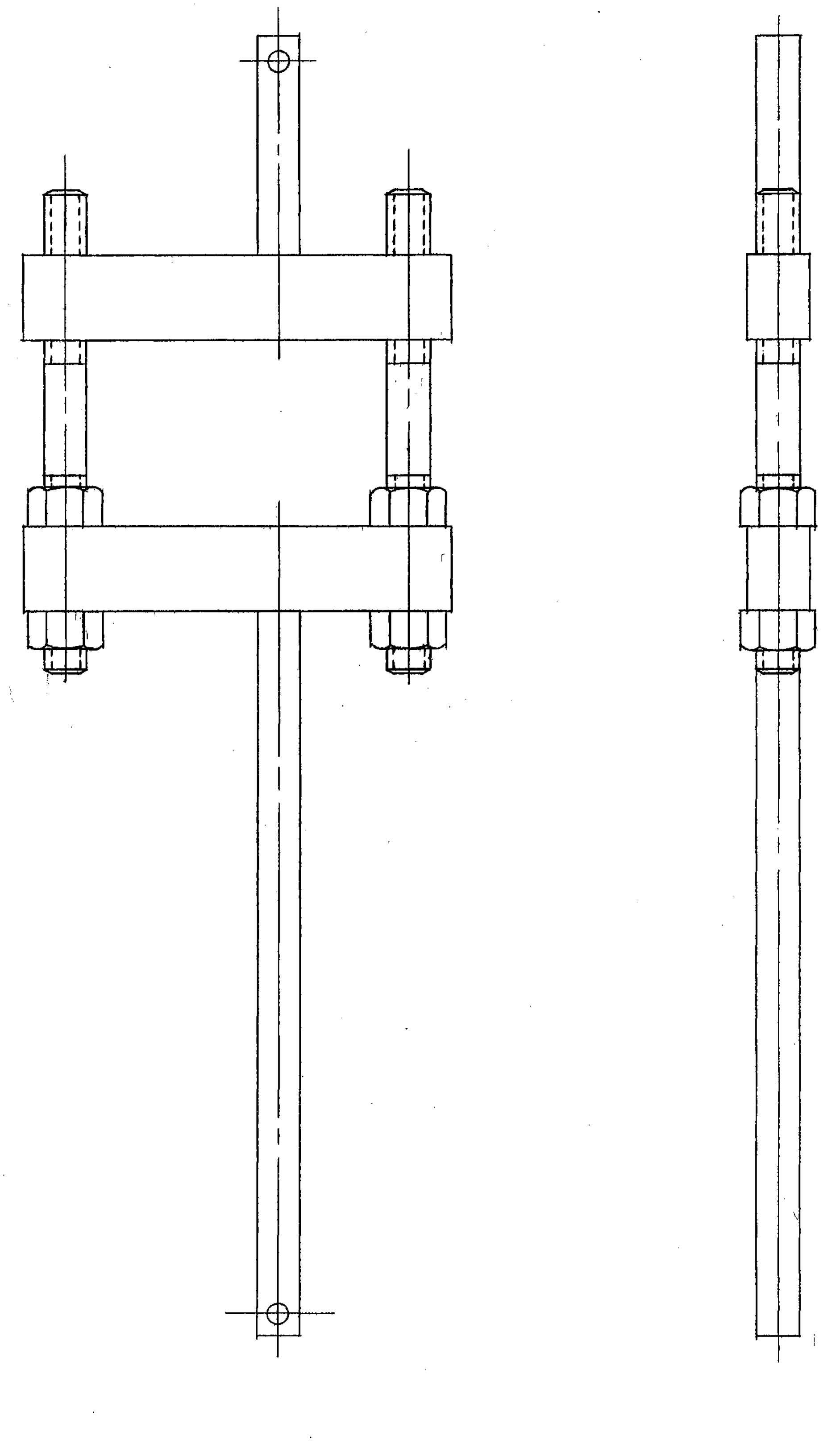


Figure 2. Yoke and Drive Rod Assembly

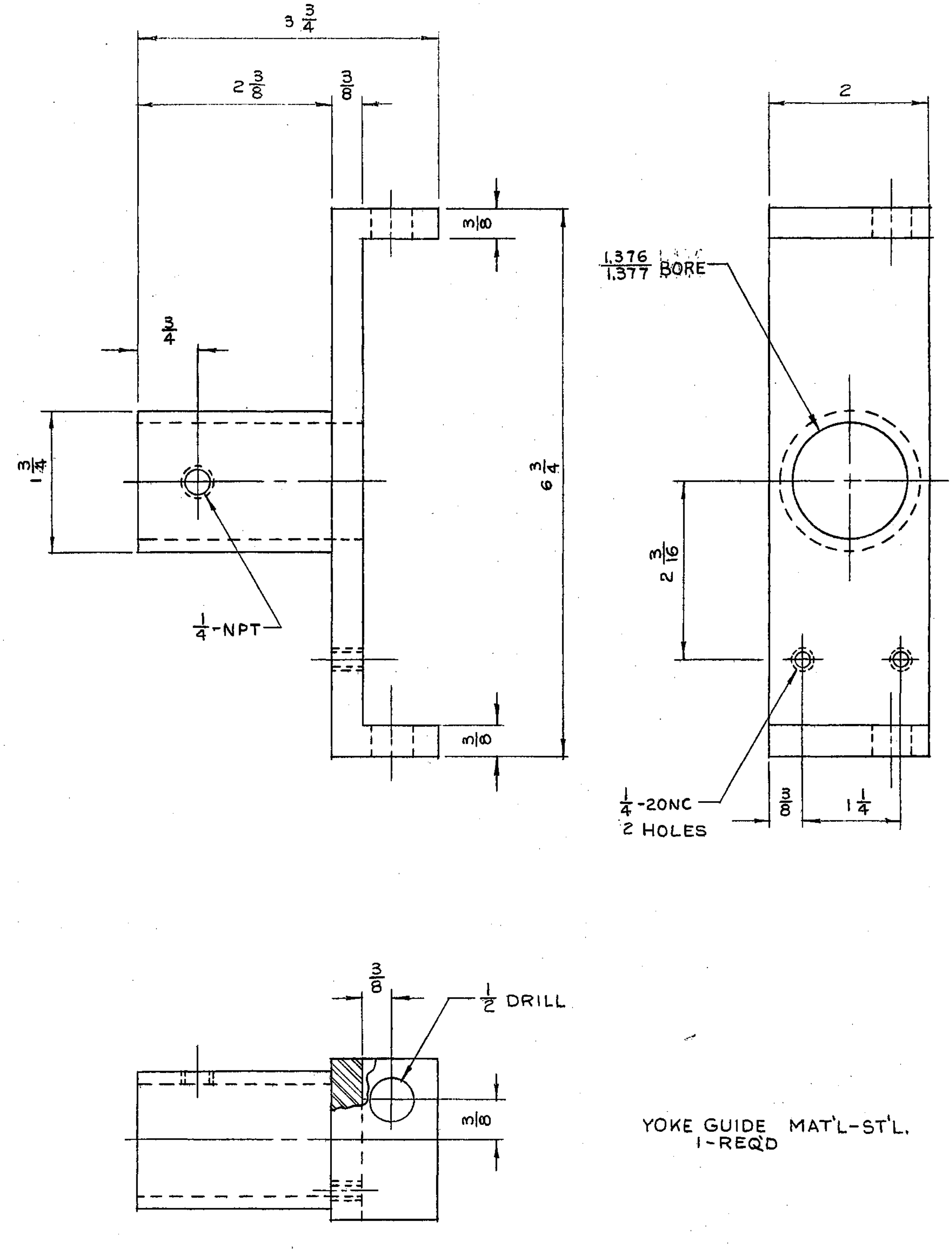
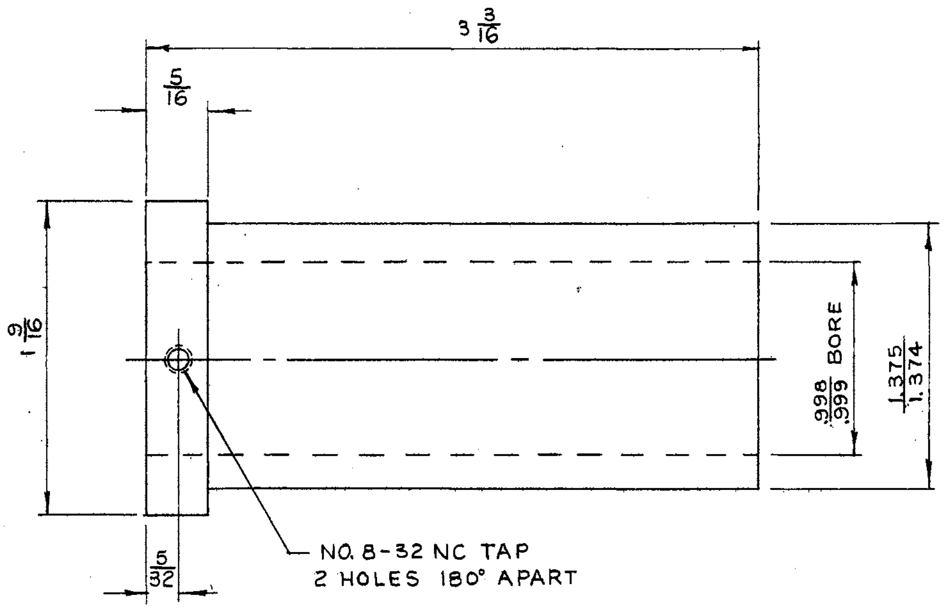
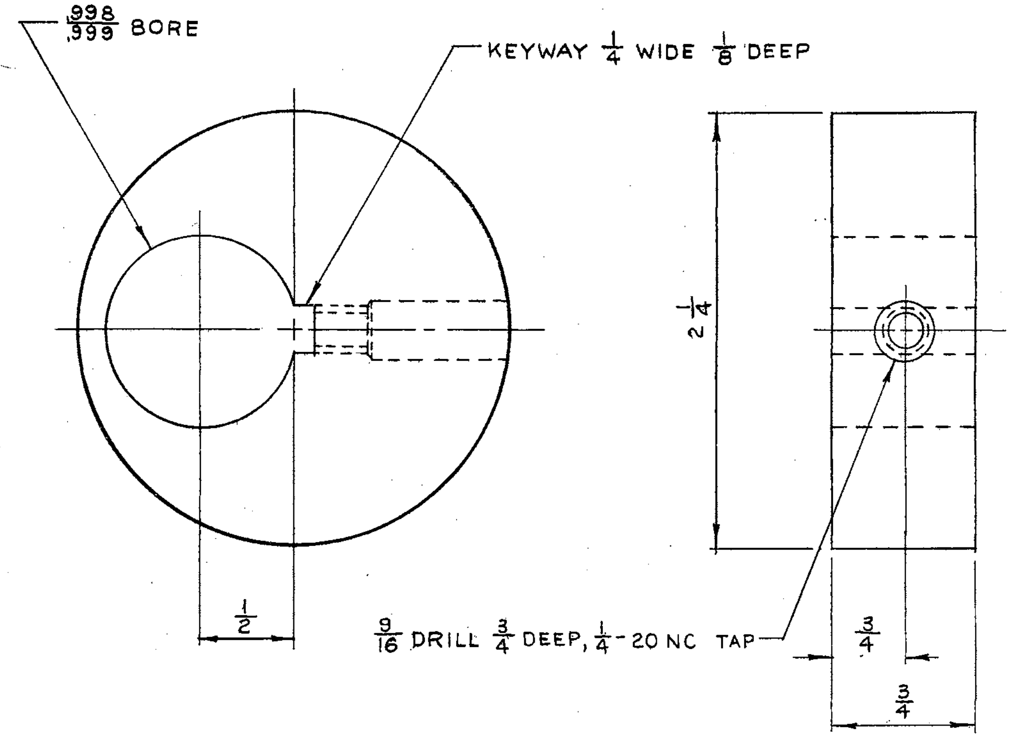


Figure 3. Yoke Guide Details



BUSHING MAT'L-OILITE
1-REQ'D

Figure 4. Bushing Details



ECCENTRIC MAT'L-ST'L.
1-REQ'D

Figure 5. Eccentric Details

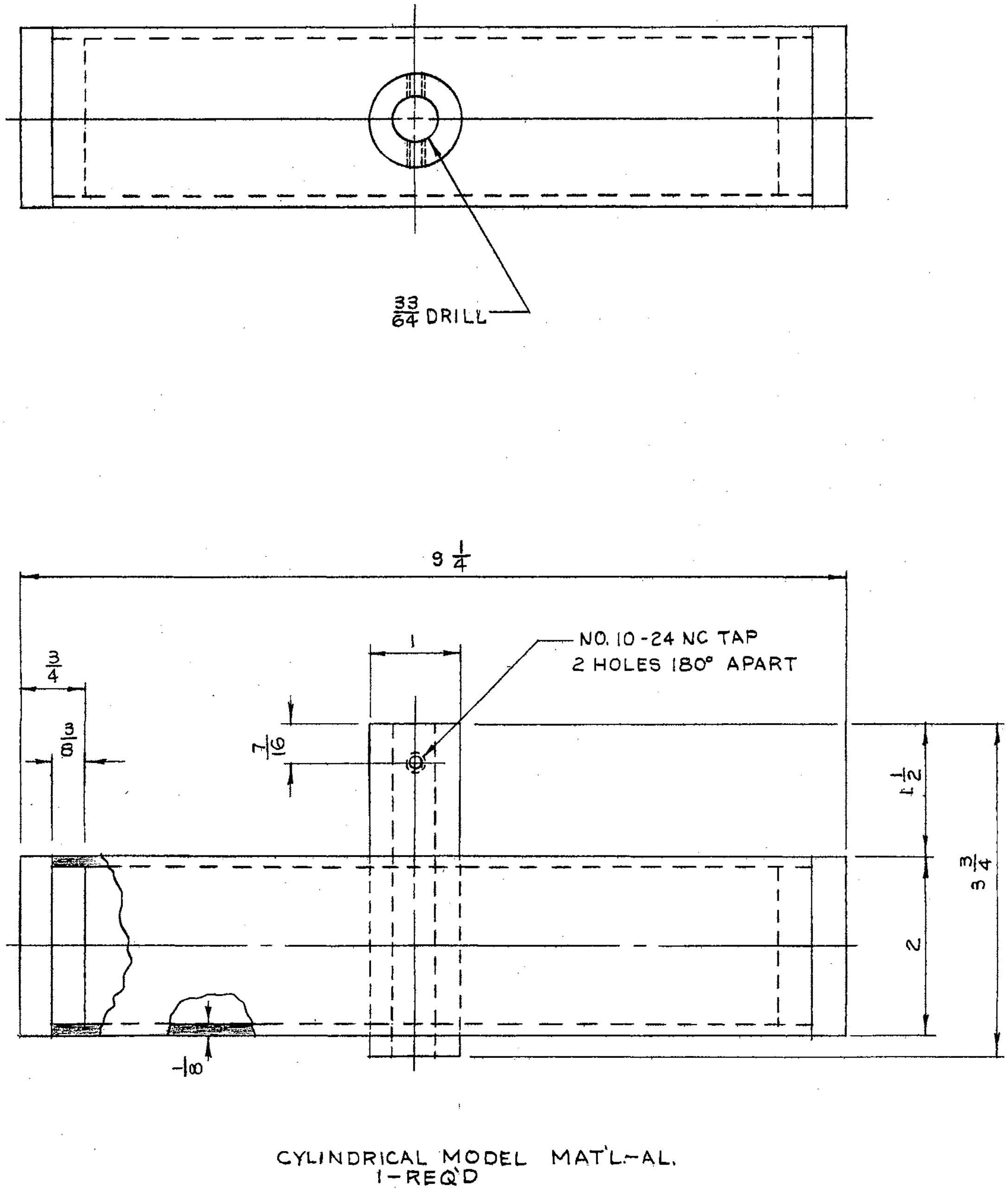


Figure 6. Cylindrical Model Details

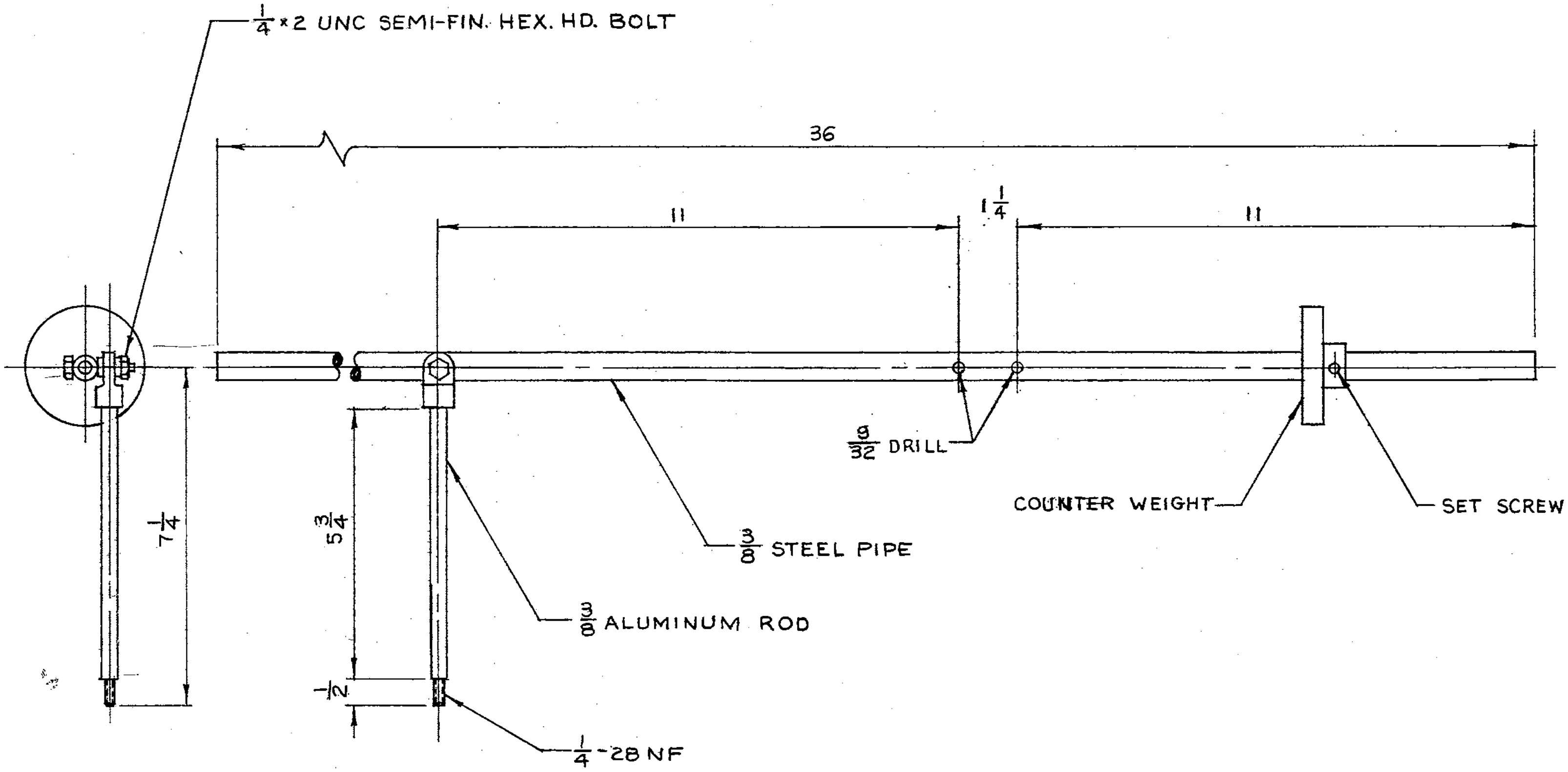


Figure 7. Torque Arm Details

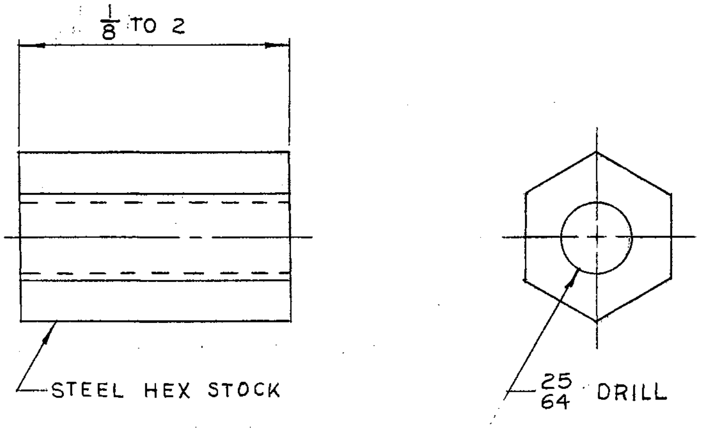


Figure 8. Torque Arm Balance Weight Details

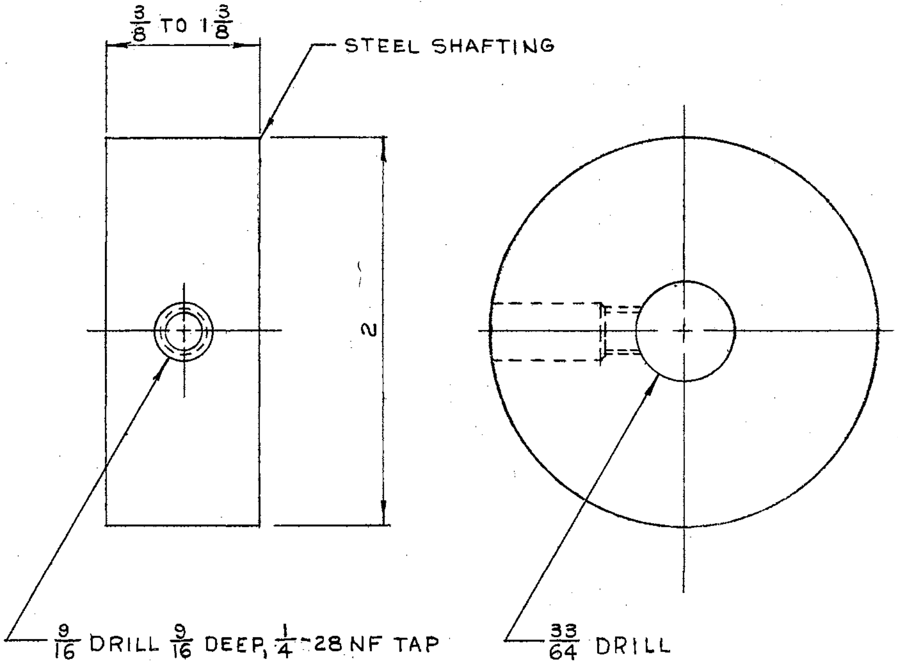


Figure 9. Calibration Weight Details

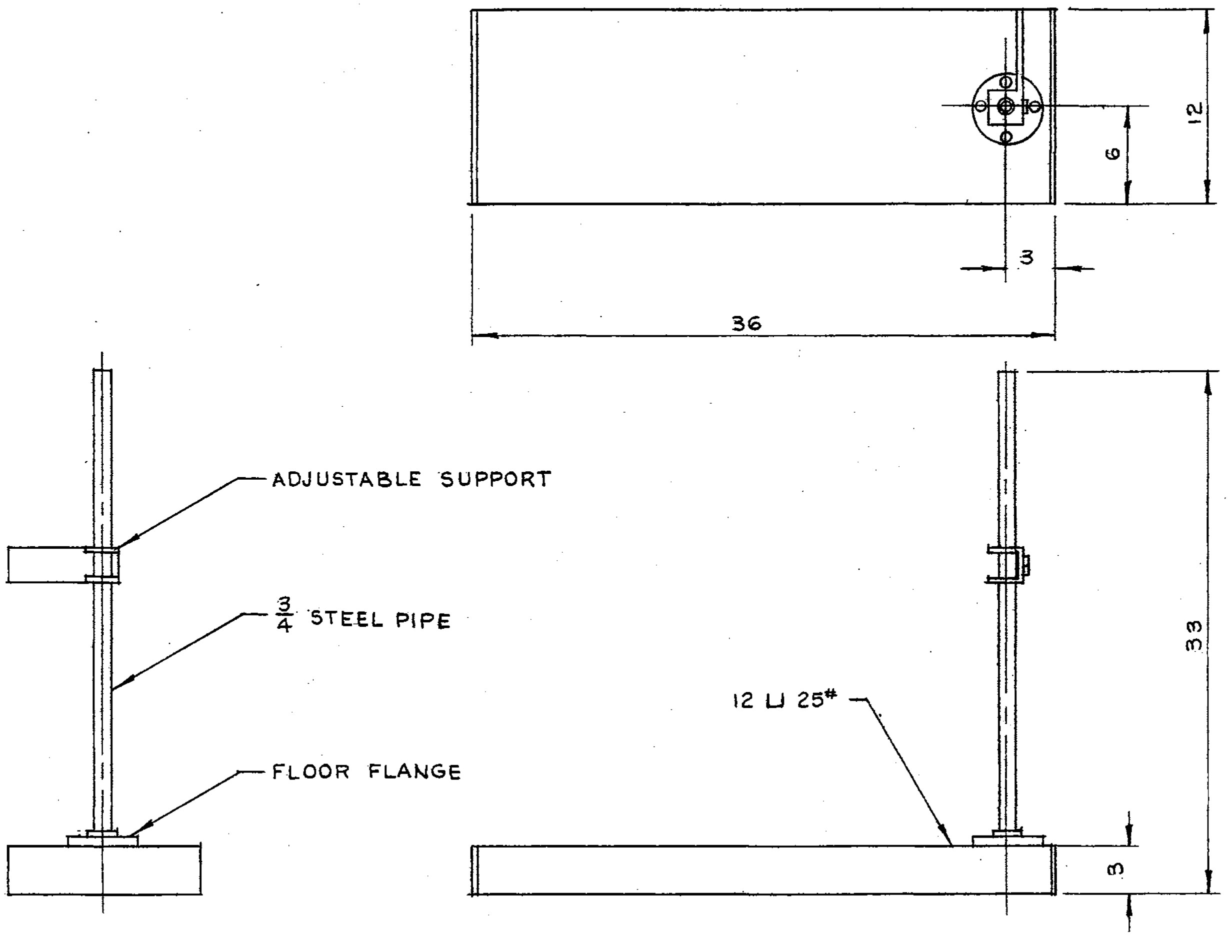


Figure 10. Torque Arm Support Details

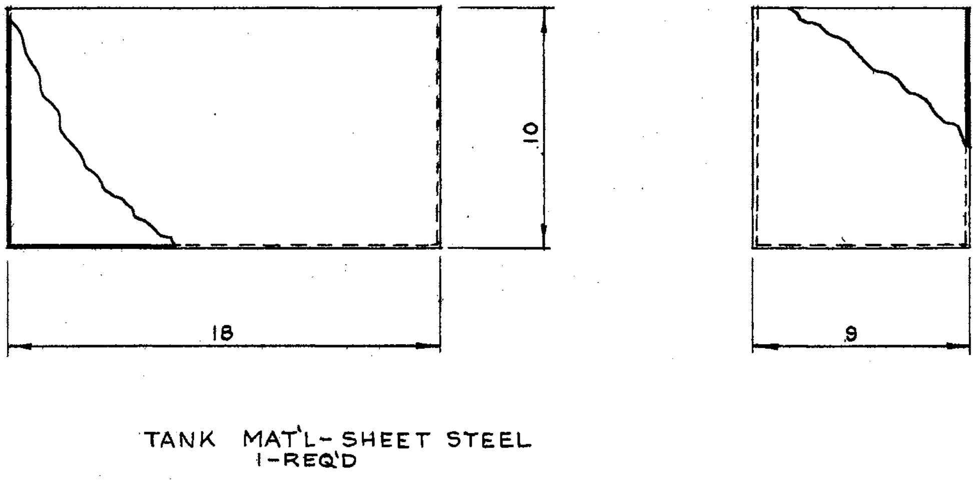


Figure 11. Tank Details

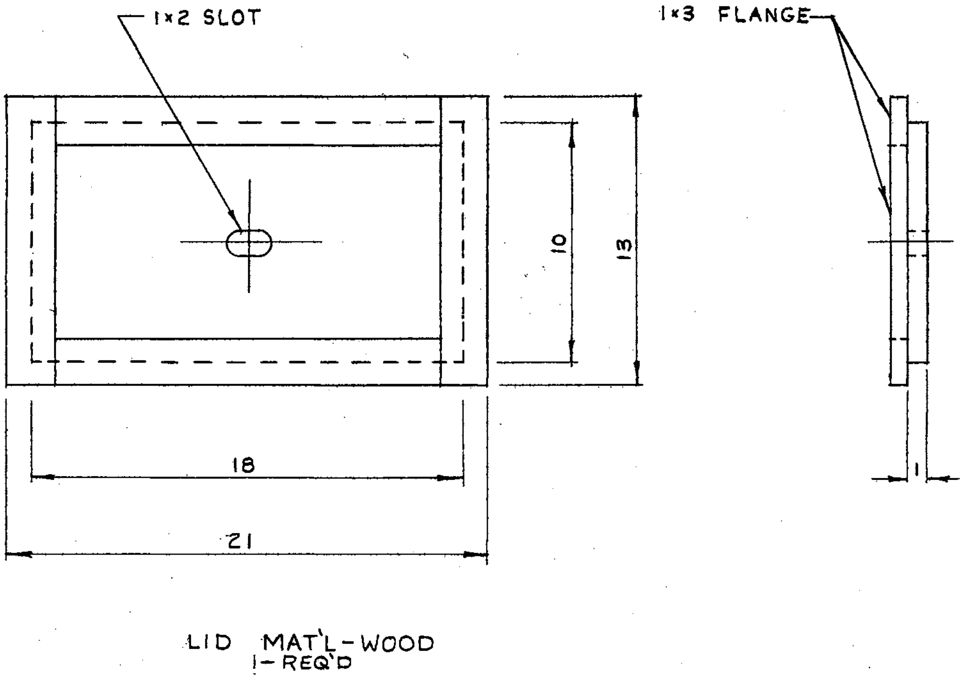


Figure 12. Lid Details

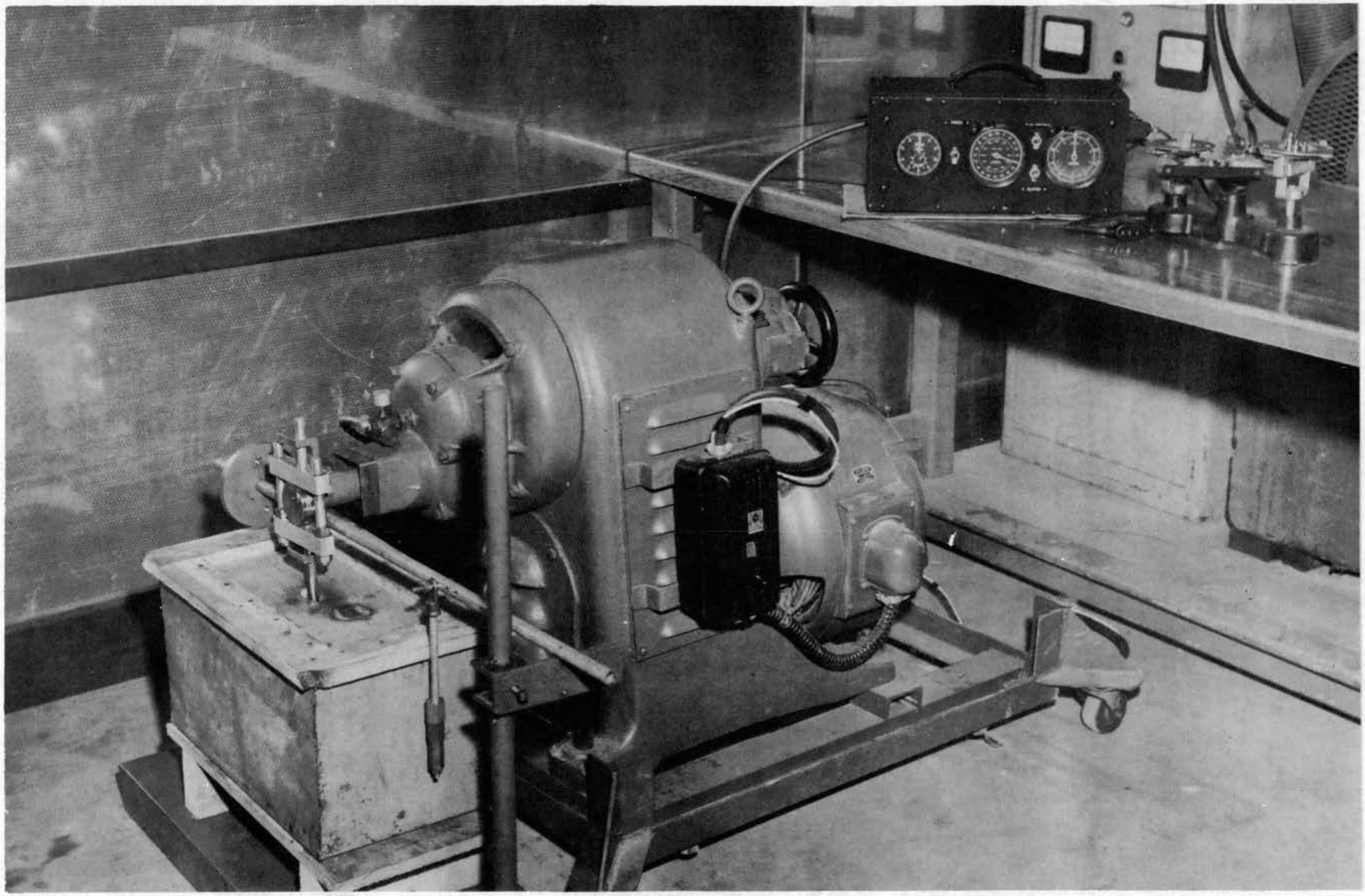


Figure 13. Test Set-Up

CHAPTER IV

EXPERIMENTAL PROCEDURE

The procedure used to determine the frequency-virtual mass relationship of a cylinder executing harmonic motion utilized the apparatus and equipment described in the previous chapter. Tests were run to obtain the torque arm weight required to maintain the arm's equilibrium position when the model was driven at various frequencies in air and then in water. Other tests were run for the determination of the equivalent weight on the yoke's drive rod in terms of torque arm weight and the driving force-displacement relationship of the driven model. The test procedures are described in detail in this chapter.

A sufficient number of runs with the varidrive speed ranging from 600 to 1600 revolutions per minute in 100 revolutions per minute increments with the cylindrical model were made to determine the torque arm equilibrium weights and to establish the reproducibility of the results. Then the model was driven in water at varidrive speeds from 600 to 1300 revolutions per minute in 100 revolutions per minute increments and again the torque arm weights required for equilibrium were recorded and the results checked to establish their reproducibility. The reference point that was used in establishing the torque arm equilibrium weight was set at the bottom of the arm's swing by the torque arm support.

These runs were followed by special calibration runs made in air with the model replaced by special calibration weights of various sizes. The runs were made at varidrive speeds ranging from 600 to 1200 revolutions per minute in 100 revolutions per minute increments and the equilibrium torque arm weights were recorded. Then the torque arm weight required for equilibrium when the model was driven in water had an identical torque arm weight corresponding to an equivalent calibration weight. This equivalent calibration weight equalled the weight of the model plus the weight of its virtual mass while operating in water since they required identical torque arm weights.

The balance and the precision weights were used to determine the value of the torque arm balance weights, the model and the calibration weight to the nearest .031 ounce. The torque arm balance weights required to maintain equilibrium were recorded by groups in terms of their respective identifying numbers and then each group was weighed on the balance. The model and the various combinations of calibration weights were also weighed in the groups in which they were used. The recorded weights of the model and the different combinations of the calibration weights included the weight of the yoke and its driving rods.

The frequency of the model's harmonic motion was determined by using the chronotach to measure the number of revolutions the varidrive turned during a selected time interval of the timing clock. The time interval was 0.1 of a minute and the total number of revolutions were recorded to the nearest revolution.

A strobe light operating at twice the rotative speed of the varidrive was set to stop the reciprocating motion of the yoke and the oscillating movement of the torque arm and determine the driving force-displacement relationship of the device by fixing the displacement point where the driving force was a maximum.

CHAPTER V

EXPERIMENTAL OBSERVATIONS

Results of experimental work are always questionable to a certain degree. Sometimes explanations of obviously incorrect or questionable points can be found by careful observation of the behavior of the equipment and apparatus and of the test procedure that was used. Therefore the following observations and comments are presented.

Analysis of the forces required for equilibrium shows that the device depends on friction to transmit the driving force to the torque arm. Since friction is variable and unpredictable some difficulty could be expected in obtaining reproducible results.

Any change in the coefficient of friction produced erratic results. Two changes in the coefficient of friction were observed. One change resulted from the breakdown of the grease film on the eccentric when large calibration weights were attached to the yoke's drive rod. After a considerable number of calibration runs using large calibration weights, wear between the eccentric and the yoke produced another change in the coefficient of friction. However, the coefficient of friction was returned to its original value by adjusting the yoke's clearance with respect to the eccentric until the torque arm weights required for equilibrium with the model coincided with the initial runs of the model in air.

Considerable vibration occurred at high varidrive speeds and two distinctly different vibrations were notice. The varidrive and the device it was driving experienced severe vibration at speeds above 1200 revolutions per minute. Combinations of varidrive speeds in the vicinity of 1300 revolutions per minute and of large drive rod loads, particularly in the form of fluid drag, produced a vibratory whip of the yoke's drive rod. Any appreciable amount of vibration could effect the weight needed on the torque arm for equilibrium by imposing additional vibratory forces on the device.

Some oscillatory motion of the torque arm was expected since the driving force was not constant; however, the amplitude of the torque arm's motion was small for most of the varidrive speeds that were used. When the varidrive and the device experienced vibration, an increase in the amplitude of the torque arm's motion usually occurred.

The harmonic movement of the model in the tank produced violent motion of the water for all but the lower varidrive speeds. In order to prevent excessive splashing of the water, a lid was used to cover the tank.

CHAPTER VI

TEST RESULTS

Results of the test performed to determine the virtual mass of the cylinder are presented in the tables and figures of this chapter. They showed the value of the virtual mass of a cylinder executing harmonic motion was dependent on the frequency. Selected runs of the model in air, the model in water and the calibration runs with the special calibration weights in air were used to determine the values of the virtual mass.

Interpolation of the calibration curves determined the equivalent weight required on the yoke's drive rod to produce the same effect as when the model was running in water at a particular varidrive speed. After determining the equivalent weight for the model moving in water at a particular speed, the weight of the model was subtracted from the equivalent weight. The remaining weight was the weight of the virtual mass contributed by the water.

In order to compare the values of the virtual mass of the cylinder with those of previous investigators the weight of the virtual mass was divided by the weight of the water displaced by the cylinder and plotted in Figure 19. The virtual mass values determined from the tests were 1.10 to 1.51 times greater than the values reported by other investigators.

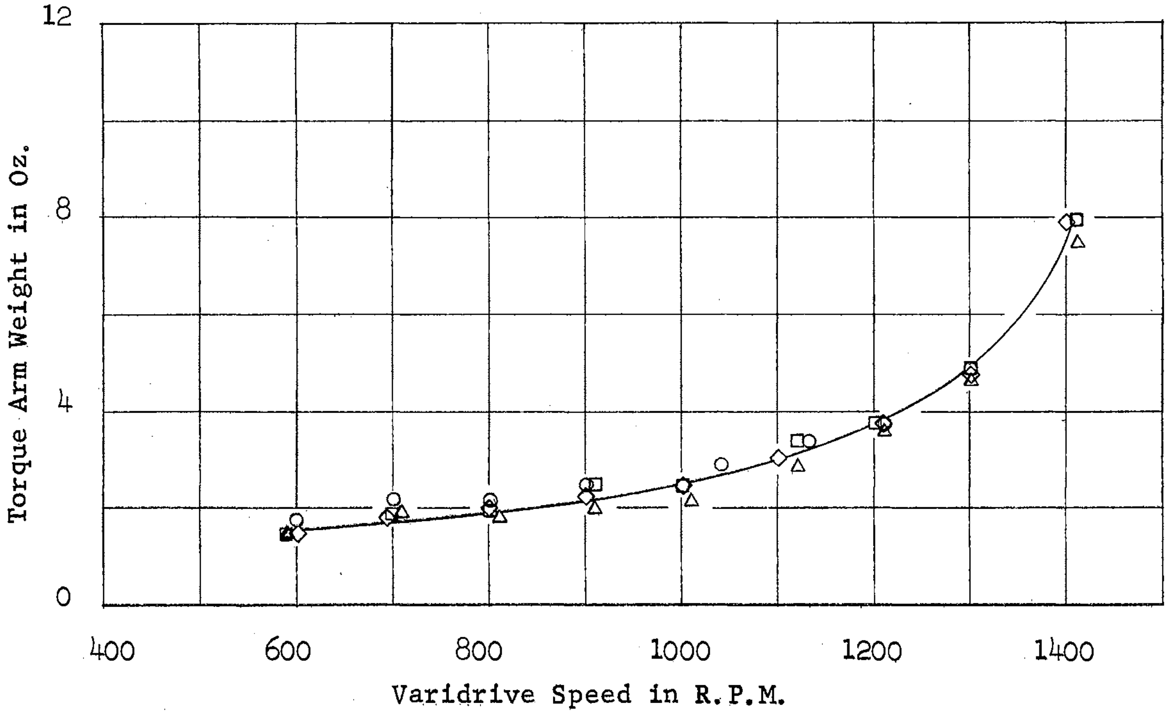


Figure 14. Plot of Torque Arm Weight for Model in Air

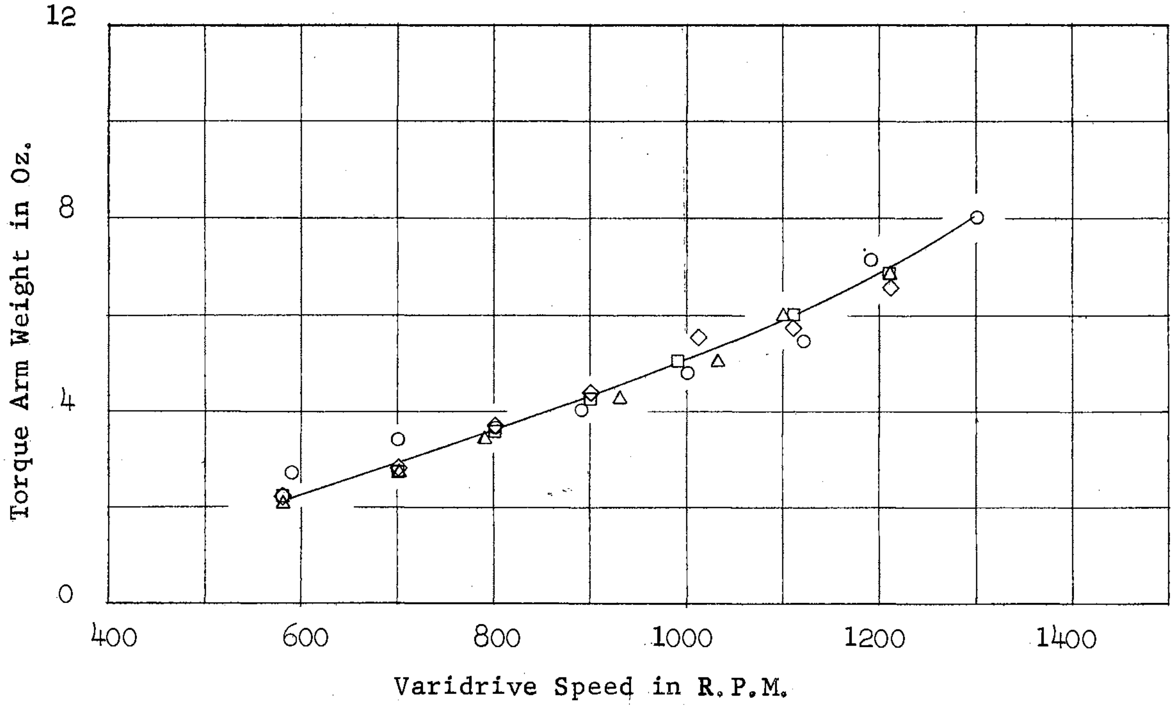


Figure 15. Plot of Torque Arm Weight for Model in Water

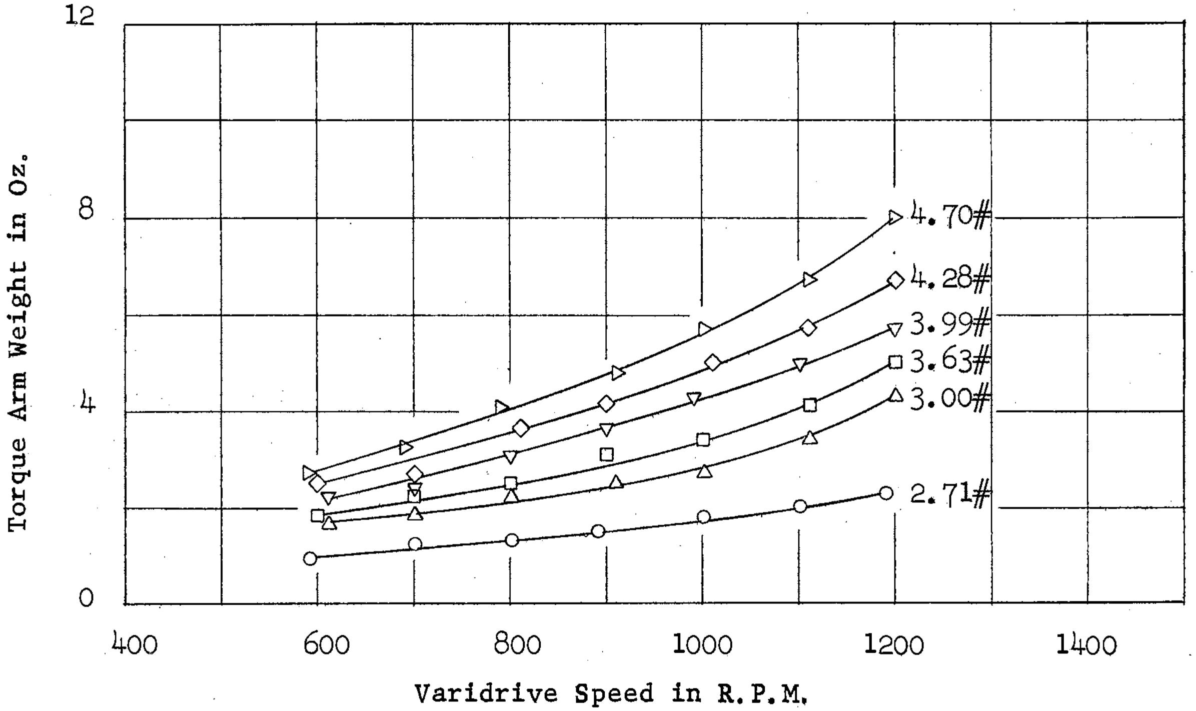


Figure 16. Plot of Torque Arm Weight for the Calibration Runs

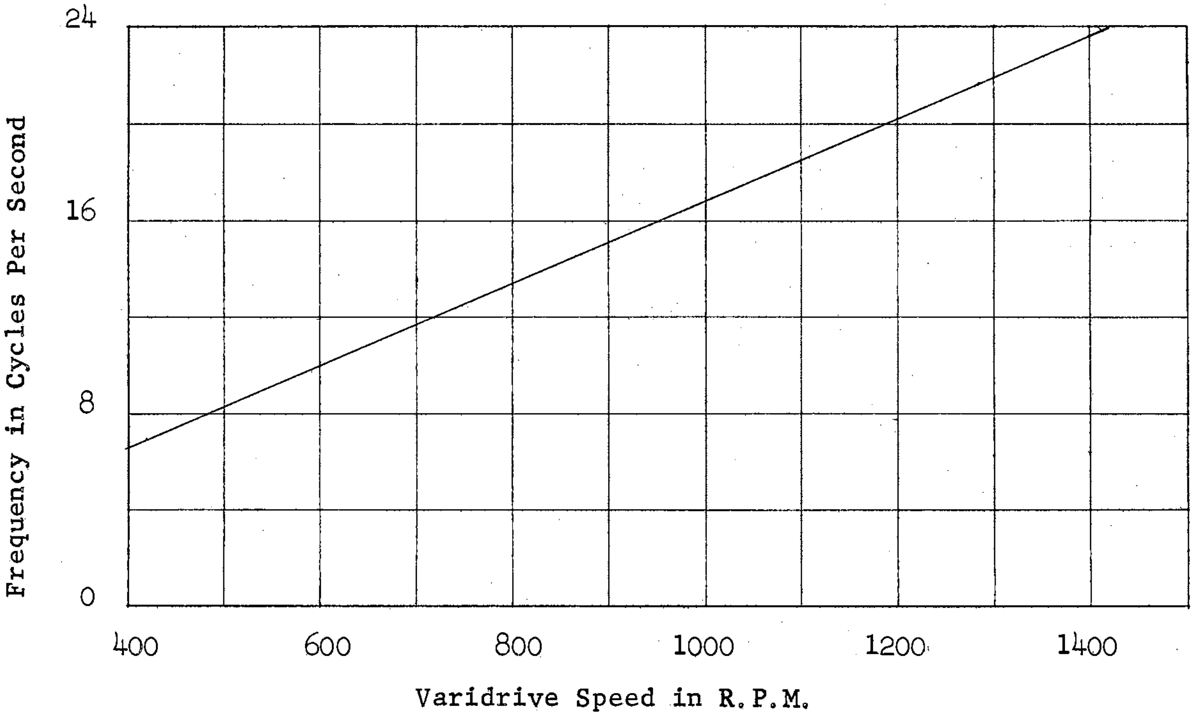


Figure 17. Plot of Frequency vs. Varidrive Speed

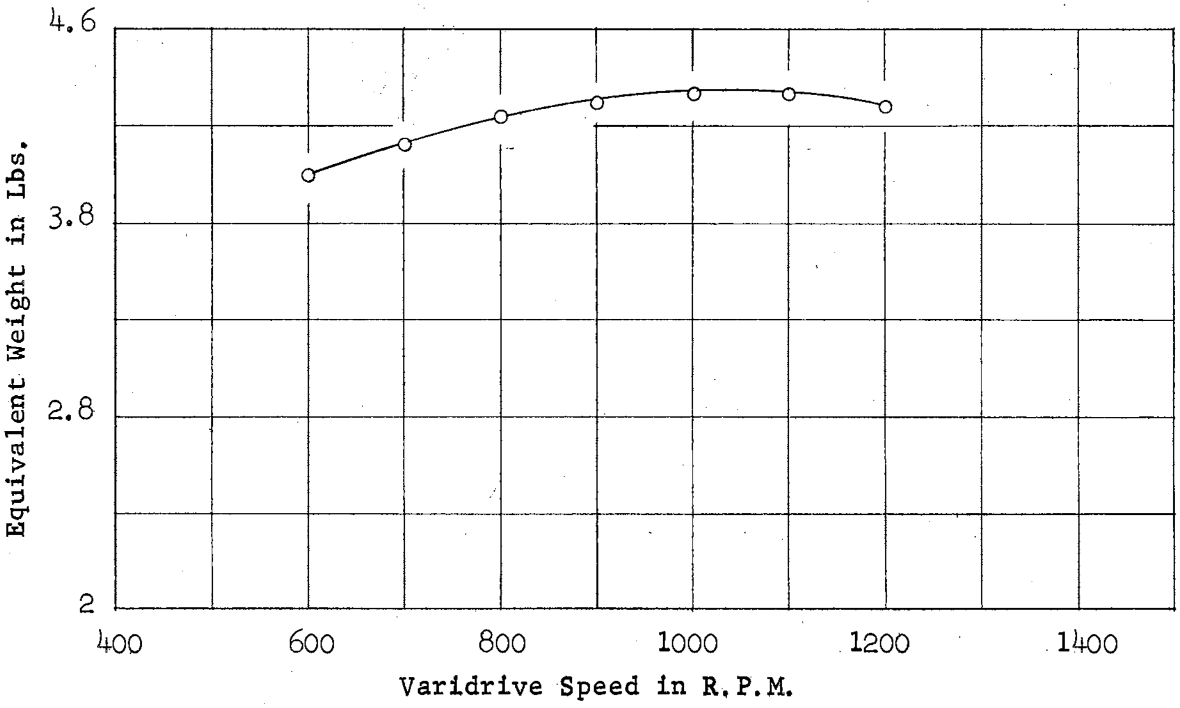


Figure 18. Plot of Equivalent Weight

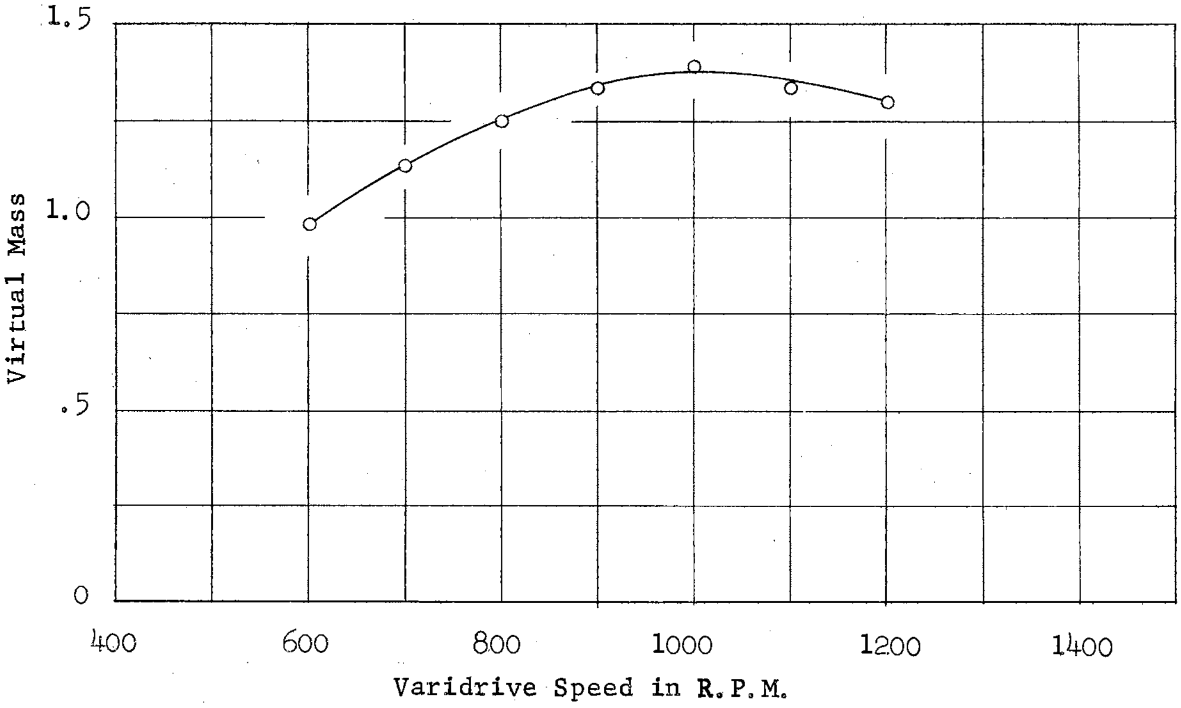


Figure 19. Plot of Virtual Mass

TABLE I

SELECTED TEST DATA

Tachometer Reading	Revolutions	Time in Minutes	Speed in R. P. M.	Torque Arm Weight In Ounces
(Model in Air 2.84 Lb. Total Moving Weight)				
600	60	.100	600	1.78
700	70	.100	700	2.19
800	81	.101	800	2.19
900	89	.099	900	2.47
1000	104	.100	1040	2.88
1100	112	.099	1130	3.38
1200	121	.100	1210	3.75
1300	131	.101	1300	4.88
1400	140	.099	1410	7.94
1500	152	.100	1520	8.13
1600	164	.101	1620	8.13
600	60	.101	590	1.47
700	69	.099	700	1.88
800	80	.100	800	1.97
900	91	.100	910	2.56
1000	100	.100	1000	2.47
1100	112	.100	1120	3.38
1200	120	.100	1200	3.75
1300	129	.099	1300	4.88
1400	141	.100	1410	7.94
1500	152	.099	1540	7.84
1600	159	.099	1600	8.13
600	60	.101	590	1.47
700	70	.099	710	1.88
800	81	.100	810	1.78
900	90	.099	910	1.97
1000	101	.100	1010	2.16
1100	113	.101	1120	2.88
1200	120	.099	1210	3.56
1300	130	.100	1300	4.59
1400	142	.101	1410	7.47
1500	150	.099	1520	6.75
1600	158	.098	1610	7.94
600	59	.099	600	1.47
700	69	.100	690	1.78
800	79	.099	800	1.97
900	90	.100	900	2.25
1000	101	.101	1000	2.44
1100	110	.100	1100	3.03
1200	121	.100	1210	3.75
1300	129	.099	1300	4.75
1400	140	.100	1400	7.88
1500	150	.100	1500	8.06
1600	161	.101	1600	8.25
(Model in Water 2.84 lb. Total Moving Weight)				
600	59	.100	590	2.66
700	70	.100	700	3.38
800	81	.101	800	3.63
900	88	.099	890	4.03
1000	102	.102	1000	4.75
1100	111	.099	1120	5.44
1200	118	.099	1190	7.13
1300	131	.101	1300	8.00
600	58	.100	580	2.25
700	70	.100	700	2.66
800	80	.100	800	3.56
900	90	.100	900	4.25
1000	99	.100	990	5.03
1100	112	.101	1110	5.94
1200	120	.099	1210	6.84
600	59	.100	590	2.06
700	70	.100	700	2.66
800	80	.101	790	3.38
900	94	.101	930	4.25
1000	103	.100	1030	5.03
1100	111	.101	1100q	5.94
1200	121	.100	1210	6.84
600	59	.100	590	2.25
700	70	.100	700	2.84
800	80	.100	800	3.75
900	90	.100	900	4.38
1000	101	.100	1010	5.47
1100	111	.100	1110	5.69
1200	122	.101	1210	6.56

Tachometer Reading	Revolutions	Time in Minutes	Speed in R. P. M.	Torque Arm Weight In Ounces
(Calibration Run in Air 2.71 Lb. Moving Weight				
600	59	.100	590	.94
700	70		700	1.25
800	80		800	1.31
900	89		890	1.50
1000	100		1000	1.81
1100	110		1100	2.00
1200	119		1190	2.31
(Calibration Run in Air 3.00 Lb. Total Moving Weight)				
600	61	.100	610	1.63
700	70		700	1.81
800	80		800	2.19
900	91		910	2.50
1000	100		1000	2.69
1100	111		1110	3.41
1200	120		1200	4.31
(Calibration Run in Air 3.63 Lb. Total Moving Weight)				
600	60	.100	600	1.81
700	70		700	2.22
800	80		800	2.50
900	90		900	3.09
1000	100		1000	3.41
1100	111		1110	4.13
1200	121		1210	5.00
(Calibration Run in Air 3.99 Lb. Total Moving Weight)				
600	61	.100	610	2.22
700	70		700	2.41
800	80		800	3.09
900	90		900	3.63
1000	99		990	4.31
1100	110		1100	4.97
1200	120		1200	5.69
(Calibration Run in Air 4.28 Lb. Total Moving Weight)				
600	60	.100	600	2.50
700	70		700	2.69
800	81		810	3.63
900	90		900	4.13
1000	101		1010	4.97
1100	111		1110	5.69
1200	120		1200	6.66
(Calibration Run in Air 4.70 Lb. Total Moving Weight)				
600	59	.100	590	2.69
700	69		690	3.22
800	79		790	4.09
900	91		910	4.78
1000	100		1000	5.69
1100	111		1110	6.66
1200	120		1200	8.00

TABLE II

TEST RESULTS

Speed in R. P. M.	Torque Arm Weight In Ounces	Equivalent Weight In Pounds	Weight Of Virtual Mass In Pounds	Virtual Mass*
600	2.20	3.99	1.15	1.10
700	2.85	4.16	1.32	1.26
800	3.60	4.28	1.44	1.37
900	4.30	4.37	1.53	1.46
1000	5.10	4.42	1.58	1.51
1100	5.85	4.37	1.53	1.46
1200	6.85	4.33	1.49	1.42

Model Volume 29 in.^{3}

Weight of Displaced Water = 1.05 Lb.

Total Moving Weight = 2.84 Lb.

$$^{*}\text{Virtual Mass} = \frac{\text{Equivalent Weight - Moving Weight}}{\text{Weight of Displaced Water}}$$

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

The frequency of the harmonic motion of the model affected the virtual mass of the cylinder. As the frequency increased the virtual mass increased from 1.10 to 1.51 times the mass of the water displaced by the model. Even though the results were reproducible from run to run with slight variation in the required torque arm weights, the writer believes some of the test results may be questionable, particularly those which gave virtual mass values of about 1.5 times greater than what other investigators had found. The use of the small covered tank and the presence of vibration at some of the high varidrive speeds could have caused this rather large increase in the virtual mass.

Although the test results indicated the frequency of the model's harmonic motion affected the virtual mass of the cylinder the writer does not feel justified in stating the relationship presented by the data to be conclusive in view of the previously mentioned questionable test results. However, the dependency of the virtual mass of a cylinder on the frequency of its harmonic motion has been shown and is worthy of further investigation.

Certain modifications in the apparatus that was used to determine the virtual mass should be considered before further work using this method is attempted. More accurate results could be achieved by reducing the possibility of vibration occurring and increasing the magnitude of the driving force that is to be measured. This could be done by using slower varidrive speeds to reduce vibration, by increasing the amplitude of the model to maintain the same maximum acceleration, and by increasing the model size. A larger tank should also be used. Replacement of the sliding friction with rolling friction would be another aid to accuracy. A device with sufficient sensitivity to record the driving rod force-displacement relationship would be a valuable aid.

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