ANALYSIS OF VIERENDEEL TRUSSES WITH PARALLEL CHORDS BY CARRY-OVER JOINT MOMENTS

By

SAMUEL THEOPHILUS

Bachelor of Science

Government College of Engineering

Bangalore, India

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JOINT MOMENTS

Thesis Adviser

James V. Parcher

Dean of the Graduate School

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S. T.

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NOMENCLATURE

L_j . . . . Length of Panel ij.

h. . . . . Height of Truss.

θ_j . . . . Slope of Truss Member at Joint j.

Δ_j . . . . Relative Displacement Between Joints i and j.

ψ_j . . . . $\dfrac{\Delta_j}{L_j}$

BV_{jk} . . . . . . End Shear of the Simple Beam jk at j.

V_{jk} . . . . End Shear of Member jk at j.

V_j . . . . Truss Shear at j.

M_{jk} . . . . End Moment of Member jk at j.

C_{jk} . . . . Carry-Over Factor of Member jk from j to k.

K_{jk} . . . . Stiffness Factor of Member jk.

K''_{jk} . . . . . . $K_{jk}\,(1 - C_{jk})$

K'''_{jk} . . . . . . $K_{jk}\,(1 + C_{jk})$

K^*_{jk} . . . . . . Modified Stiffness Factor of Member jk.

CK_{jk} . . . . . . $C_{jk}\,K_{jk}$ = Carry-Over Stiffness Factor of Member jk.

CK^*_{jk} . . . . . Modified Carry-Over Stiffness Factor of Member jk.

D^*_{jk} . . . . . . $\dfrac{K^*_{jk}}{\Sigma K^*_j}$ = New Distribution Factor

CD^*_{jk} . . . . . . New Carry-Over Distribution Factor.

γ^*_{jk} . . . . Joint Moment Carry-Over Factor from j to k.

FM_{jk} . . . . . . Fixed End Moment of Member jk at j.

FM^*_{jk} . . . . . Modified Fixed End Moment of Member jk.

m^*_j . . . . Starting Joint Moment at j.

JM^*_j . . . . . . Joint Moment at j.

D'_{jk} . . . . Displacement Distribution Factor.

w. . . . . Intensity of Load.

S_{jk}. . . . . Displacement Stiffness Factor for Member jk.

CHAPTER I

INTRODUCTION

A Vierendeel Truss is a frame composed of a series of rectangular or trapezoidal panels without diagonal members (Fig. 1-1 a, b, c). The end connections of all members are rigid and are designed to resist bending. Vierendeel Trusses are used for highway bridges, railway bridges, steel frame towers, and steel building frames.

The Vierendeel Truss was invented by Professor Arthur Vierendeel of Belgium, in 1893. He used this type of truss for the steel steeple of a church in Flanders, Belgium. (2). Recently, the Vierendeel Truss has been used more frequently in the United States. In 1936, eight bridges of the Vierendeel Truss type were constructed in Los Angeles alone. (2).

The analysis of a Vierendeel Truss by the method of slope deflection or moment distribution becomes quite tedious and laborious when the number of unknowns becomes large. In such cases, the method of carry-over joint moments might have some advantages. The general carry-over philosophy is the outgrowth of extensive research being done at Oklahoma State University under the direction of Professor Jan J. Tuma (4, 5, 6). Professor Tuma has applied the carry-over method to the analysis of continuous beams. (4). The method of carry-over joint moments has been applied to the analysis of continuous rigid frames with joint translation prevented by Gregory (5). Sturm has extended the carry-over joint moment method of analysis to multi-story rigid frames. (6).

The carry-over joint moment method of analyzing Vierendeel Trusses is developed and applied in this thesis. Advantages and disadvantages of this method of analysis are noted.

The sign convention of the slope deflection method is adopted. All deformations are assumed to be small and elastic. Deformations due to normal and shearing forces are neglected.

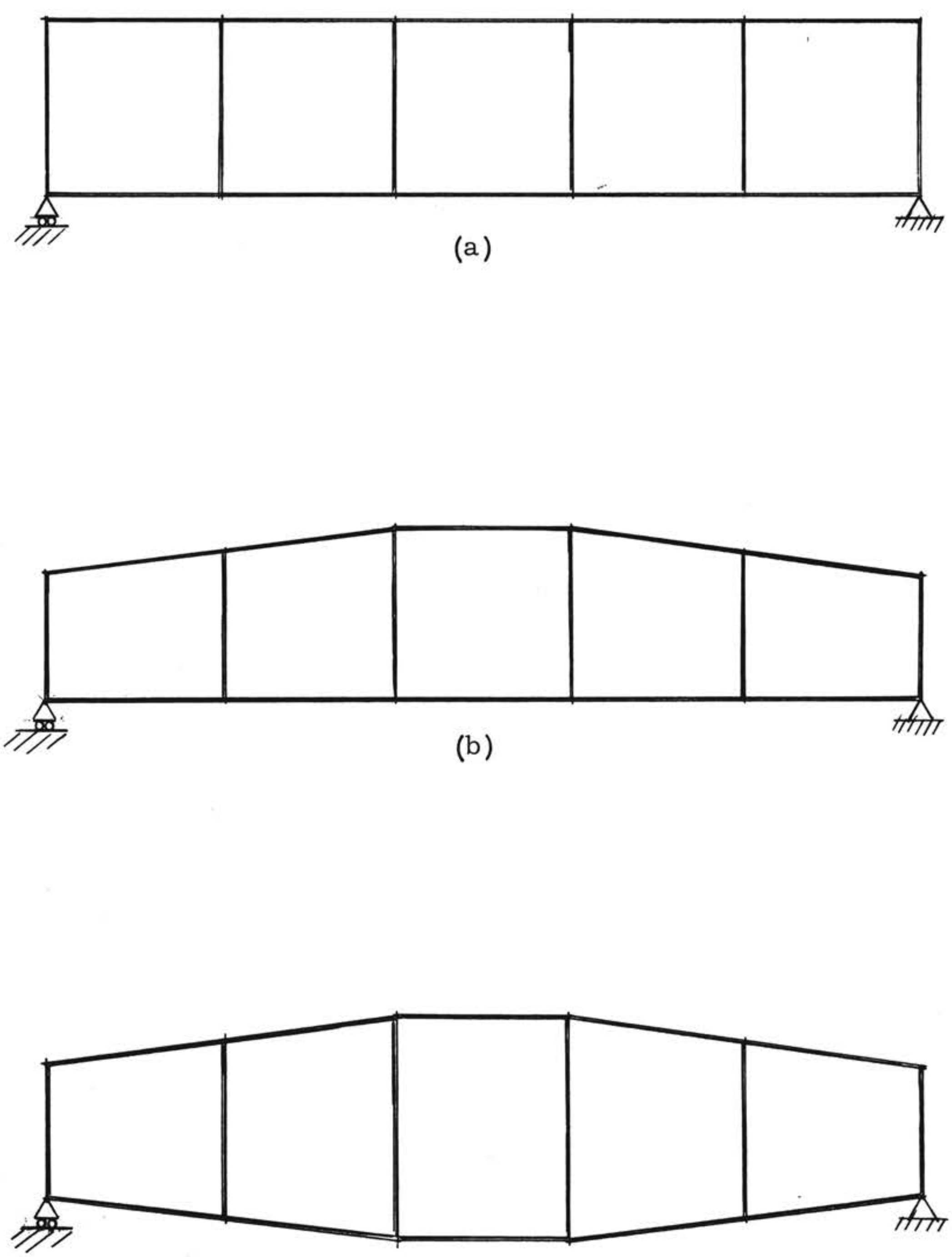


Fig. 1-1

Typical Vierendeel Trusses

CHAPTER II

STATEMENT OF THE PROBLEM

A Vierendeel Truss with parallel top and bottom chords is considered (Fig. 2-1). The truss is loaded by a general system of loads on the top and bottom chords. Relative horizontal displacement of the top and bottom chords is prevented.

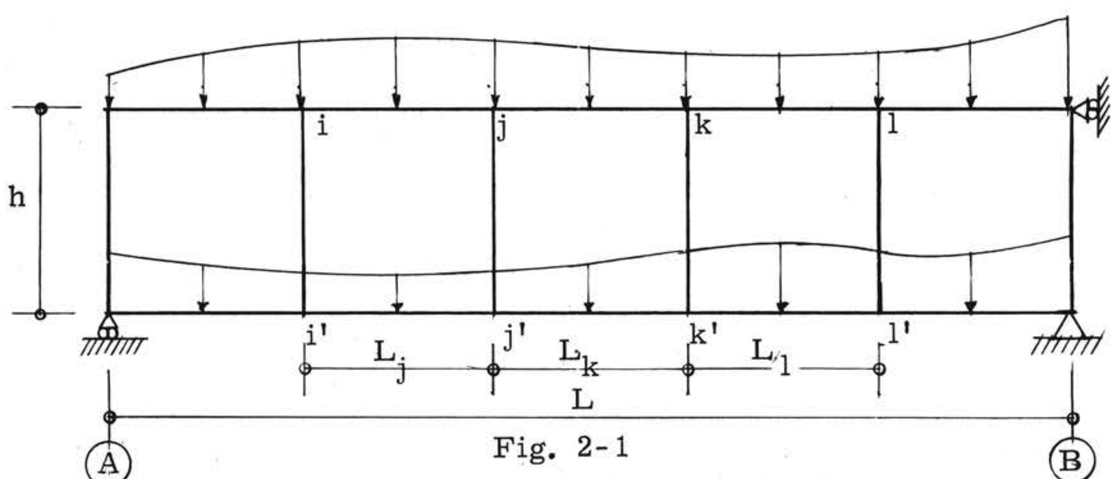


Fig. 2-1
Vierendeel Truss Loaded With A General System of Loads

A general procedure for the analysis of Vierendeel Trusses with parallel chords is derived. A typical intermediate portion of a truss is considered. A six slope equation is derived from the general slope deflection equations for end moments by eliminating the vertical displacements using the conditions of joint equilibrium and shear equilibrium.

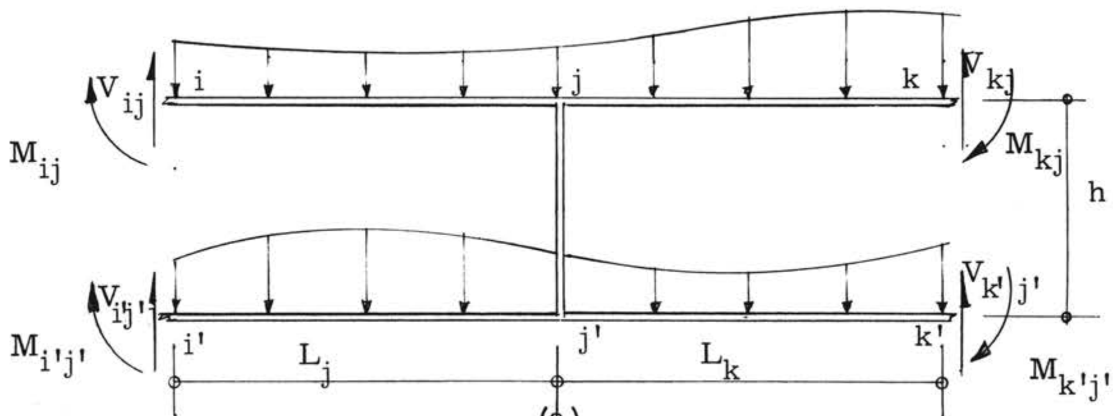
The six slope equation is converted into a six joint moment equation, and the final end moments are expressed in terms of joint moments.

These new joint moment equations involve the use of new constants; the new constants are joint moments, new stiffness factors, new carry-over factors, and starting joint moments. These new constants are defined and interpretated physically.

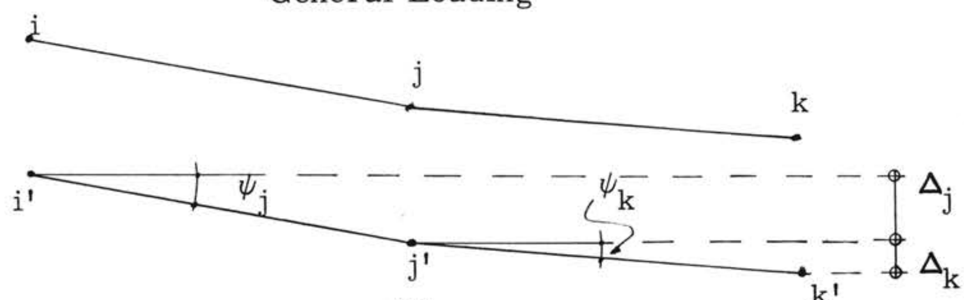
CHAPTER III

DERIVATION OF THE SIX SLOPE EQUATION

A typical segment of a Vierendeel Truss subjected to a general system of vertical loads is shown (Fig. 3-1).



(a)
General Loading



(b)
Relative Displacement of Joints

Fig. 3-1
Typical Segment of a Vierendeel Truss

Since all deformations are assumed to be small,

$$\psi_j = \frac{\Delta_j}{L_j} \qquad\qquad \psi_k = \frac{\Delta_k}{L_k}$$

The general slope deflection equations for end moments at joints j and j' are

Joint j:

$$M_{ji} = K_{ji}\,\theta_j + CK_{ij}\,\theta_i - S_{ji}\,\psi_j + FM_{ji}$$

$$M_{jk} = K_{jk}\,\theta_j + CK_{kj}\,\theta_k - S_{jk}\,\psi_k + FM_{jk} \qquad (3\text{-}1)$$

$$M_{jj'} = K_{jj'}\theta_j + CK_{j'j}\,\theta_j$$

Joint j':

$$M_{j'i'} = K_{j'i'}\;\theta_{j'} + CK_{i'j'}\;\theta_{i'} - S_{j'i'}\;\psi_{j'} + FM_{j'i'}$$

$$M_{j'k'} = K_{j'k'}\;\theta_{j'} + CK_{k'j'}\;\theta_{k'} - S_{j'k'}\,\psi_{k'} + FM_{j'k'} \qquad (3\text{-}2)$$

$$M_{j'j} = K_{j'j}\;\;\theta_{j'} + CK_{jj'}\;\;\theta_j$$

From the condition of equilibrium of moments at each joint, two equations are written:

$$\Sigma M_j = 0, \qquad M_{ji} + M_{jk} + M_{jj'} = 0$$

Substituting the values from Eq's (3-1) and simplifying,

$$\Sigma K_j\;\theta_j + CK_{ij}\;\theta_i + CK_{kj}\;\theta_k + CK_{j'j}\;\theta_{j'} - S_{ji}\;\psi_j$$

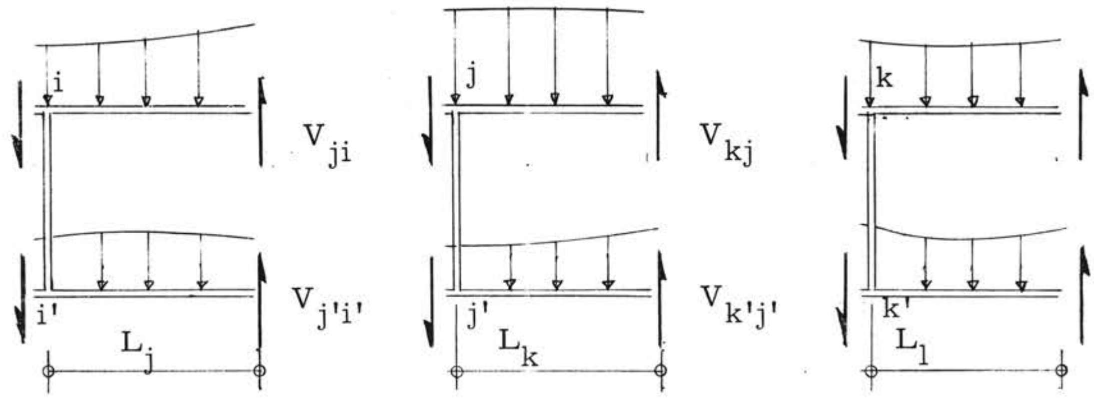
$$- S_{jk}\,\psi_k + FM_{jk} + FM_{ji} = 0 \qquad (3\text{-}3)$$

$$\Sigma M_{j'} = 0, \quad M_{j'i'} + M_{j'k'} + M_{j'j} = 0$$

Substituting the values from Eq's (3-2) and simplifying,

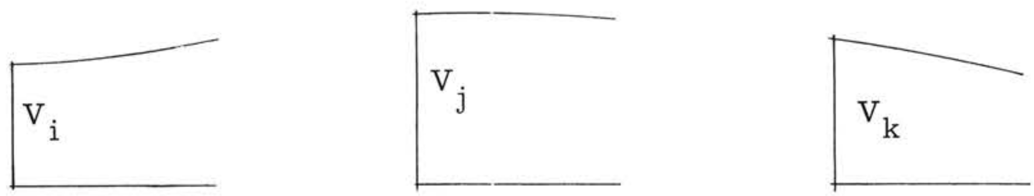
$$\Sigma K_{j'}\;\theta_{j'} + CK_{i'j'}\;\theta_{i'} + CK_{k'j'}\;\theta_{k'} + CK_{jj'}\;\theta_j - S_{j'i'}\,\psi_{j'}$$

$$- S_{j'k'}\,\psi_{k'} + FM_{j'i'} + FM_{j'k'} = 0 \qquad (3\text{-}4)$$



(a)

Cross Sectional Shears



(b)

Truss Shear

Fig. 3-2

Vertical Shears

Applying the condition of equilibrium of vertical forces at joints j and j' (Fig. 3-2), the following shear equation can be written:

$$V_{ji} + V_{j'i'} + V_j = 0 ,$$

but

$$V_{ji} = BV_{ji} + \frac{M_{ji} + M_{ij}}{L_j}$$

and

$$V_{j'i'} = BV_{j'i'} + \frac{M_{j'i'} + M_{i'j'}}{L_j}$$

so,

$$\frac{M_{ji} + M_{ij}}{L_j} + \frac{M_{j'i'} + M_{i'j'}}{L_j} + BV_{j'i'} + BV_{ji} + V_j = 0$$

or

$$M_{ji} + M_{ij} + M_{j'i'} + M_{i'j'} + L_j (BV_{j'i'} + BV_{ji} + V_j) = 0$$

Substituting the values of M_{ji}, M_{ij}, $M_{j'i'}$, $M_{i'j'}$ from the slope deflection Eq's (3-1, 3-2), simplifying, and solving for ψ_j, $\psi_{j'}$:

$$\theta_j (K_{ji} + CK_{ji}) + \theta_i (K_{ij} + CK_{ij}) + \theta_{j'} (K_{j'i'} + CK_{j'i'}) + \theta_{i'} (K_{i'j'} + CK_{i'j'})$$
$$+ FM_{ji} + FM_{ij} + FM_{j'i'} + FM_{i'j'} + L_j (BV_{j'i'} + BV_{ji} + V_j)$$
$$= \psi_j (S_{ji} + S_{ij} + S_{j'i'} + S_{i'j'})$$

$$\psi_j = \frac{1}{(S_{ji} + S_{ij} + S_{j'i'} + S_{i'j'})} \Big[\theta_j K'''_{ji} + \theta_i K'''_{ij} + \theta_{j'} K'''_{j'i'} +$$
$$+ \theta_{i'} K'''_{i'j'} + FM_{ji} + FM_{ij} + FM_{i'j'} + FM_{j'i'} + L_j (BV_{ji} + BV_{j'i'} + V_j) \Big]$$
$$= \psi_{j'}$$

The equilibrium of vertical forces at section k yield the equation

$$V_{kj} + V_{k'j'} + V_k = 0$$

but,

$$V_{kj} = BV_{kj} + \frac{M_{kj} + M_{jk}}{L_k}$$

and

$$V_{k'j'} = BV_{k'j'} + \frac{M_{k'j'} + M_{j'k'}}{L_k}$$

so,

$$\frac{M_{kj} + M_{jk}}{L_k} + \frac{M_{k'j'} + M_{j'k'}}{L_k} + BV_{kj} + BV_{k'j'} + V_k = 0$$

or

$$M_{jk} + M_{kj} + M_{j'k'} + M_{k'j'} + L_k (BV_{kj} + BV_{k'j'} + V_k) = 0$$

Substituting the values of M_{kj}, M_{jk}, $M_{k'j'}$ and $M_{j'k'}$ from the slope deflection Eq's (3-1, 3-2), simplyfying, and solving for ψ_k, $\psi_{k'}$:

$$\theta_k (K_{kj} + CK_{kj}) + \theta_j (K_{jk} + CK_{jk}) + \theta_{k'} (K_{k'j'} + CK_{k'j'})$$

$$+ \theta_{j'} (K_{j'k'} + CK_{j'k'}) + FM_{kj} + FM_{jk} + FM_{k'j'} + FM_{j'k'} +$$

$$L_k (BV_{kj} + BV_{k'j'} + V_k) \quad = \psi_k (S_{kj} + S_{jk} + S_{k'j'} + S_{j'k'})$$

So,

$$\psi_k = \frac{1}{(S_{kj} + S_{jk} + S_{k'j'} + S_{j'k'})} \Big[\theta_k K'''_{kj} + \theta_j K'''_{jk} + \theta_{k'} K'''_{k'j'}$$

$$+ \theta_{j'} k'''_{j'k'} + FM_{kj} + FM_{jk} + FM_{k'j'} + FM_{j'k'} + L_k (BV_{kj} + BV_{k'j'} + V_k) \Big]$$

$$= \psi_{k'}$$

Let

$$\frac{S_{ji}}{(S_{ji} + S_{ij} + S_{j'i'} + S_{i'j'})} = D'_{ji}$$

And

$$\frac{S_{kj}}{(S_{kj} + S_{jk} + S_{k'j'} + S_{j'k'})} = D'_{kj}$$

where D'_{ji} and D'_{kj} are designated as displacement distribution factors.

Substituting the values of ψ_j, $\psi_{j'}$, ψ_k, $\psi_{k'}$, into the Eq's (3-3, 3-4):

From Eq's (3-3)

$$\Sigma K_j \theta_j + CK_{ij} \theta_i + CK_{kj} \theta_k + CK_{j'j} \theta_{j'} - D'_{ji} \Big[\theta_j K'''_{ji} + \theta_i K'''_{ij}$$

$$+ \theta_{j'} K'''_{j'i'} + \theta_{i'} K'''_{i'j'} + FM_{ji} + FM_{ij} + FM_{j'i'} + FM_{i'j'} + L_j (BV_{ji}$$

$$+ BV_{j'i'} + V_j) \Big] - D'_{jk} \Big[\theta_k K'''_{kj} + \theta_j K'''_{jk} + \theta_{k'} K'''_{k'j'} + \theta_{j'} K'''_{j'k'}$$

$$+ FM_{kj} + FM_{jk} + FM_{k'j'} + FM_{j'k'} + L_k (BV_{kj} + BV_{k'j'} + V_k) \Big] + \Sigma FM_j = 0$$

Rearranging:

$$\theta_j (\Sigma K_j - D'_{ji} K'''_{ji} - D'_{jk} K'''_{jk}) + \theta_i (CK_{ij} - D'_{ji} K'''_{ij}) + \theta_k (CK_{kj}$$

$$- D'_{jk} K'''_{kj}) + \theta_{j'} (CK_{j'j} - D'_{ji} K'''_{j'i'} - D'_{jk} K'''_{j'k'}) + \theta_{i'} (-D'_{ji} K'''_{i'j'})$$

$$+ \theta_{k'} (-D'_{jk} K'''_{k'j'}) - D'_{ji} \Big[FM_{ji} + FM_{ij} + FM_{j'i'} + FM_{i'j'}$$

$$+ L_j (BV_{ji} + BV_{j'i'} + V_j) \Big] - D'_{jk} \Big[FM_{kj} + FM_{jk} + FM_{k'j'} + FM_{j'k'}$$

$$+ L_k (BV_{kj} + BV_{k'j'} + V_k) \Big] + \Sigma FM_j = 0 \qquad (3\text{-}5)$$

From Eq's (3-4)

$$\Sigma K_{j'} \theta_{j'} + CK_{i'j'} \theta_{i'} + CK_{k'j'} \theta_{k'} + CK_{jj'} \theta_j - D'_{j'i'} \Big[\theta_j K'''_{ji} +$$

$$\theta_i K'''_{ij} + \theta_{j'} K'''_{j'i'} + \theta_{i'} K'''_{i'j'} + FM_{ji} + FM_{ij} + FM_{j'i'} + FM_{i'j'}$$

$$+ L_j (BV_{ji} + BV_{j'i'} + V_j) \Big] - D'_{j'k'} \Big[\theta_k K'''_{kj} + \theta_j K'''_{jk} + \theta_{k'} K'''_{k'j'}$$

$$+ \theta_{j'} K'''_{j'k'} + FM_{kj} + FM_{jk} + FM_{k'j'} + FM_{j'k'} + L_k (BV_{kj} + BV_{k'j'} + V_k) \Big]$$

$$+ \Sigma FM_{j'} = 0$$

Rearranging:

$$\theta_{j'} (\Sigma K_{j'} - D'_{j'i'} K'''_{j'i'} - D'_{j'k'} K'''_{j'k'}) + \theta_{i'} (CK_{i'j'} - D'_{j'i'} K'''_{ij'})$$

$$+ \theta_{k'} (CK_{k'j'} - D'_{j'k'} K'''_{k'j'}) + \theta_j (CK_{jj'} - D'_{j'i'} K'''_{jk})$$

$$+ \theta_i (-D'_{j'i'} K'''_{ij}) + \theta_k (-D'_{j'k'} K'''_{kj}) - D'_{j'i'} \Big[FM_{ij} + FM_{ji}$$

$$+ FM_{j'i'} + FM_{i'j'} + L_j (BV_{ji} + BV_{j'i'} + V_j) \Big] - D'_{j'k'} \Big[FM_{jk} \qquad (3\text{-}6)$$

$$+ FM_{kj} + FM_{k'j'} + FM_{j'k'} + L_k (BV_{kj} + BV_{k'j'} + V_k) \Big] + \Sigma FM_{j'} = 0$$

Rewriting these equations in terms of modified stiffness factors and modified carry-over stiffness factors, Eq. (3-5) becomes

$$\Sigma K^*_j \theta_j + CK^*_{ij} \theta_i + CK^*_{kj} \theta_k + CK^*_{j'j} \theta_{j'} + CK^*_{i'j} \theta_{i'}$$

$$+ CK^*_{k'j} \theta_{k'} + \Sigma SM^*_j = 0 \qquad (3\text{-}7)$$

Eq. (3-6) becomes

$$\Sigma K^*_{j'} \theta_{j'} + CK^*_{i'j'} \theta_{i'} + CK^*_{k'j'} \theta_{k'} + CK^*_{jj'} \theta_j + CK^*_{ij'} \theta_i$$

$$+ CK^*_{kj'} \theta_k + \Sigma SM^*_{j'} = 0 \qquad (3\text{-}8)$$

Eq's (3-7 and 3-8) are called six slope equations which contain redundant slopes θ's. ΣK^*_j and $\Sigma K^*_{j'}$ are called modified joint stiffness factors and CK^*_{ij}, CK^*_{kj}------are called modified carry-over stiffness factors.

CHAPTER IV

DERIVATION OF THE SIX JOINT MOMENT EQUATION

The following new terms are introduced and defined by the accompanying equation.

a.) Joint Moment $JM^*_j = \Sigma K^*_j \, \theta_j$

b.) Joint Moment Carry-Over Factor $\gamma^*_{ij} = - \dfrac{CK^*_{ij}}{\Sigma K^*_i}$

c.) Starting Moment $m^*_j = - \Sigma SM^*_j$

Substituting the new equivalents into Eq's (3-7, 3-8) and solving for JM_j and $JM_{j'}$, the following joint moment equations are obtained.

$$JM^*_j = \gamma^*_{ij} JM^*_i + \gamma^*_{kj} JM^*_k + \gamma^*_{j'j} JM^*_{j'} + \gamma^*_{i'j} JM^*_{i'} + \gamma^*_{k'j} JM^*_{k'} + m^*_j \qquad (4\text{-}1)$$

$$JM^*_{j'} = \gamma^*_{i'j'} JM^*_{i'} + \gamma^*_{k'j'} JM^*_{k'} + \gamma^*_{jj'} JM^*_j + \gamma^*_{ij'} JM^*_i + \gamma^*_{kj'} JM^*_k + m^*_{j'} \qquad (4\text{-}2)$$

Where,

Joint Moments

$\Sigma K^*_j \, \theta_j = JM^*_j$ $\qquad$ $\Sigma K^*_{j'} \, \theta_{j'} = JM^*_{j'}$

Joint Moment Carry-Over Factors

$\gamma^*_{ij} = - \dfrac{CK^*_{ij}}{\Sigma K^*_i}$ $\qquad$ $\gamma^*_{kj} = - \dfrac{CK^*_{kj}}{\Sigma K^*_k}$ $\qquad$ $\gamma^*_{j'j} = - \dfrac{CK^*_{j'j}}{\Sigma K^*_{j'}}$

$$\gamma^*_{i'j} = - \frac{CK^*_{i'j}}{\Sigma K^*_{i'}} \qquad \gamma^*_{k'j} = - \frac{CK^*_{k'j}}{\Sigma K^*_{k'}}$$

Starting Moments

$$m^*_j = - \Sigma SM^*_j \qquad\qquad m^*_{j'} = - \Sigma SM^*_{j'}$$

The physical interpretation and definitions of the joint moment carry-over factor and the starting moment follows.

Starting Moment - m^*_j -

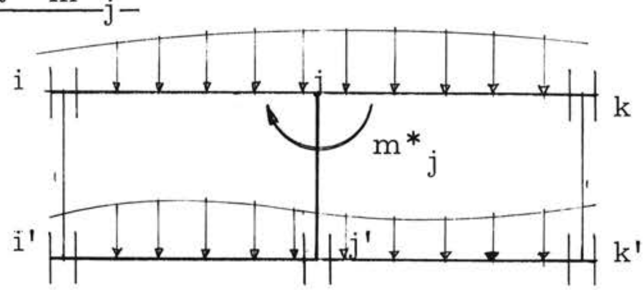


Fig. 4-1

Physical Interpretation of Starting Moment m^*_j

In Eq. (4-1), if the joints i, k, i', j', and k' are fixed against rotation but free to translate, the equation reduces to $JM^*_j = m^*_j$. Hence, the starting moment m^*_j may be explained physically as the joint moment at j due to loads if all the adjacent joints are fixed against rotation but free to translate.

Joint Moment Carry-Over Factor

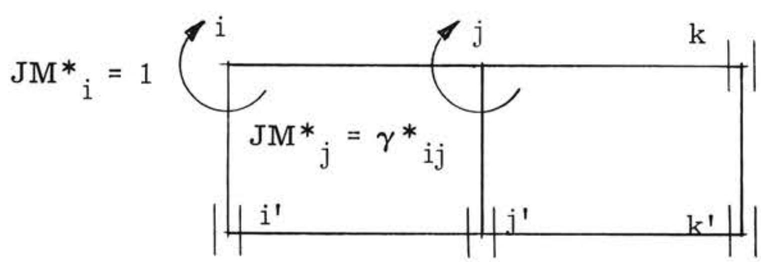


Fig. 4-2

Physical Interpretation of Joint Moment Carry-Over Factor γ^*_{ij}

In Eq. (4-1), if the joints i', j', k', k are fixed against rotation but free to translate, and if no loads exist on the structure, the equation reduces to $JM^*_j = \gamma^*_{ij}\, JM^*_i$.

If $JM^*_i = 1$, $JM^*_j = \gamma^*_{ij}$.

Hence, the joint moment carry-over factor γ^*_{ij} may be explained physically as the joint moment developed at j due to application of unit joint moment at i, if the joints i', j', k', and k are fixed against rotation but free to translate.

The carry-over pattern is illustrated for typical joints j and j' (Fig. 4-3). The joint moment at any joint is influenced by the joint moments at the five adjacent joints as evidenced from Eq's (4-1, 4-2) and Fig. 4-3.

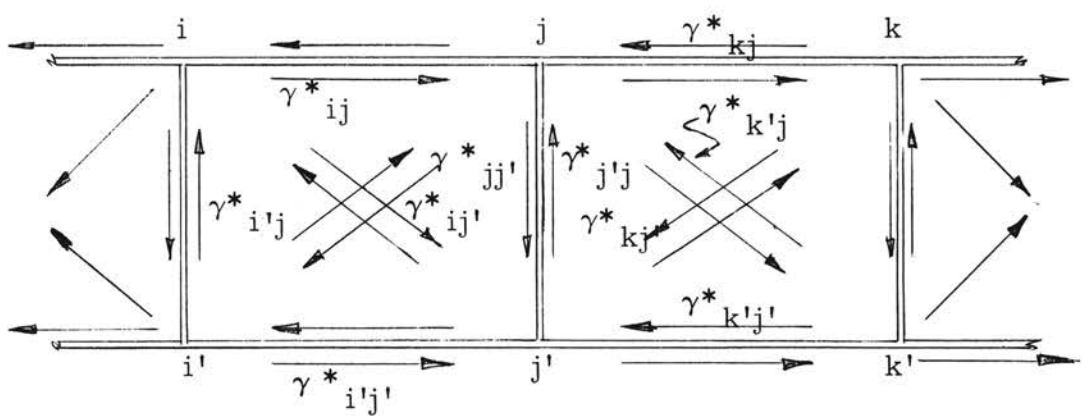


Fig. 4-3
Carry-Over Pattern

End moment equations are derived in terms of the new equivalents. The slopes θ's and ψ's are eliminated from the general slope deflection equations for end moments (Eq's 3-1, 3-2). The final end moment equations in terms of joint moments, new distribution factors, new carry-over distribution factors, and modified fixed end moments are expressed as Eq's (4-6, 4-7). The elimination of the slopes is shown as intermediate steps (Eq's 4-3, 4-4, 4-5).

Final End Moment Equations:

$$
\begin{aligned}
M_{ji} &= K_{ji}\,\theta_j + CK_{ij}\,\theta_i - S_{ji}\,\psi_j + FM_{ji} \\
&= K_{ji}\,\theta_j + CK_{ij}\,\theta_i - D'_{ji}\Big[\theta_j K'''_{ji} + \theta_i K'''_{ij} + \theta_{j'} K'''_{j'i'} + \theta_{i'} K'''_{i'j'} \\
&\qquad + FM_{ji} + FM_{ij} + FM_{j'i'} + FM_{i'j'} + L_j\,(BV_{ji} + BV_{j'i'} + V_j)\Big] + FM_{ji} \\
&= \theta_j\,(K_{ji} - D'_{ji} K'''_{ji}) + \theta_i\,(CK_{ij} - D'_{ji} K'''_{ij}) + \theta_{j'}\,(-D'_{ji} K'''_{j'i'}) \\
&\quad + \theta_{i'}\,(-D'_{ji} K'''_{i'j'}) - D'_{ji}\Big[FM_{ji} + FM_{ij} + FM_{j'i'} + FM_{i'j'} \\
&\qquad + L_j\,(BV_{ji} + BV_{j'i'} + V_j)\Big] + FM_{ji} \\
&= \frac{JM^*_j}{\Sigma K^*_j}\,(K_{ji} - D'_{ji} K'''_{ji}) + \frac{JM^*_i}{\Sigma K^*_i}\,(CK_{ij} - D'_{ji} K'''_{ij}) \\
&\quad + \frac{JM^*_{j'}}{\Sigma K^*_{j'}}\,(-D'_{ji} K'''_{j'i'}) + \frac{JM^*_{i'}}{\Sigma K^*_{i'}}\,(-D'_{ji} K'''_{i'j'}) \\
&\quad - D'_{ji}\Big[FM_{ji} + FM_{ij} + FM_{j'i'} + FM_{i'j'} + L_j\,(BV_{ji} + BV_{j'i'} + V_j)\Big] \\
&\qquad + FM_{ji} \qquad\qquad (4\text{-}3)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
M_{jk} &= \frac{JM^*_j}{\Sigma K^*_j}\,(K_{jk} - D'_{jk} K'''_{jk}) + \frac{JM^*_k}{\Sigma K^*_k}\,(CK_{kj} - D'_{jk} K'''_{kj}) \\
&\quad + \frac{JM^*_{j'}}{\Sigma K^*_{j'}}\,(-D'_{jk} K'''_{j'k'}) + \frac{JM^*_{k'}}{\Sigma K^*_{k'}}\,(-D'_{jk} K'''_{k'j'}) \\
&\quad - D'_{jk}\Big[FM_{jk} + FM_{kj} + FM_{j'k'} + FM_{k'j'} + L_k\,(BV_{kj} + BV_{k'j'} + V_k)\Big] \\
&\qquad + FM_{jk} \qquad\qquad (4\text{-}4)
\end{aligned}
$$

$$M_{jj'} = \frac{JM^*_j}{\Sigma K^*_j} K_{jj'} + \frac{JM^*_{j'}}{\Sigma K^*_{j'}} CK_{j'j} \tag{4-5}$$

The new final end moment equations are

Joint j

$$M_{ji} = D^*_{ji} JM^*_j + CD^*_{ij} JM^*_i + CD^{*(ji)}_{j'j} JM^*_{j'} + CD^*_{i'j} JM^*_{i'} + FM^*_{ji}$$

$$M_{jk} = D^*_{jk} JM^*_j + CD^*_{kj} JM^*_k + CD^{*(jk)}_{j'j} JM^*_{j'} + CD^*_{k'j} JM^*_{k'} + FM^*_{jk} \tag{4-6}$$

$$M_{jj'} = D^*_{jj'} JM^*_j + CD_{j'j} JM^*_{j'}$$

Joint j'

$$M_{j'i'} = D^*_{j'i'} JM^*_{j'} + CD^*_{i'j'} JM^*_{i'} + CD^*_{ij'} JM^*_i + CD^{*(j'i')}_{jj'} JM^*_j + FM^*_{j'i'}$$

$$M_{j'k'} = D^*_{j'k'} JM^*_{j'} + CD^*_{k'j'} JM^*_{k'} + CD^*_{kj'} JM^*_k + CD^{*(j'k')}_{jj'} JM^*_j + FM^*_{j'k'} \tag{4-7}$$

$$M_{j'j} = D^*_{j'j} JM^*_{j'} + CD_{jj'} JM^*_j$$

The new symbols in the equations are:

<u>New Distribution Factor</u>

$$D^*_{ji} = \frac{K_{ji} - D'_{ji} K'''_{ji}}{(\Sigma K_j - D'_{ji} K'''_{ji} - D'_{jk} K'''_{jk})} = \frac{K^*_{ji}}{\Sigma K^*_j}$$

<u>New Carry-Over Distribution Factors</u>

$$CD^*_{ij} = \frac{CK_{ij} - D'_{ji} K'''_{ij}}{\Sigma K^*_i}$$

$$CD^*_{j'j}{}^{(ji)} = \frac{-D'_{ji}\ K'''_{j'i'}}{\Sigma K^*_{j'}}$$

$$CD^*_{j'j}{}^{(jk)} = \frac{-D'_{jk}\ K'''_{j'k'}}{\Sigma K^*_{j'}}$$

$$CD_{j'j} = \frac{CK_{j'j}}{\Sigma K^*_{j'}}$$

$$CD^*_{i'j} = \frac{-D'_{ji}\ K'''_{i'j'}}{\Sigma K^*_{i'}}$$

Modified Fixed End Moment

$$FM^*_{ji} = -D'_{ji}\Big[FM_{ji} + FM_{ij} + FM_{j'i'} + FM_{i'j'} + L_j\,(BV_{ji} + BV_{j'i'} + V_j)\Big] + FM_{ji}$$

CHAPTER V

ILLUSTRATIVE EXAMPLES

The application of the carry-over joint moment procedure to the analysis of Vierendeel Frames is illustrated by two numerical examples.

Example No. 1: A symmetrical five panel Vierendeel Truss with parallel top and bottom chords loaded as shown is analyzed (Fig. 5-1). EI is constant for all members.

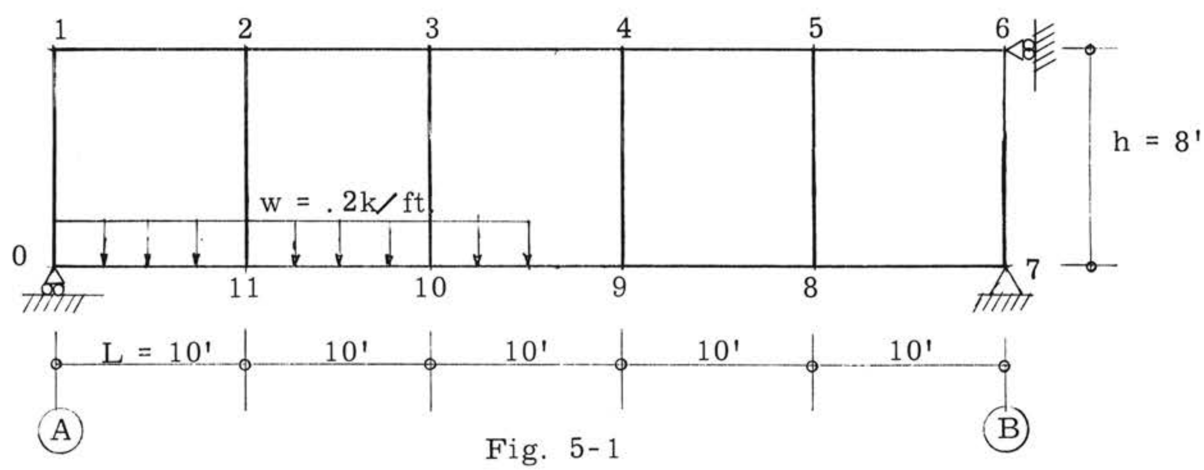


Fig. 5-1
A Five Panel Vierendeel Truss

Since the structure is symmetrical, it will be advantageous to consider two cases in the analysis:

Case I - Symmetrical Loading (Fig. 5-2).

Case II - Antisymmetrical Loading (Fig. 5-5).

The results from the analysis of Cases I and II will be superimposed to obtain the final results.

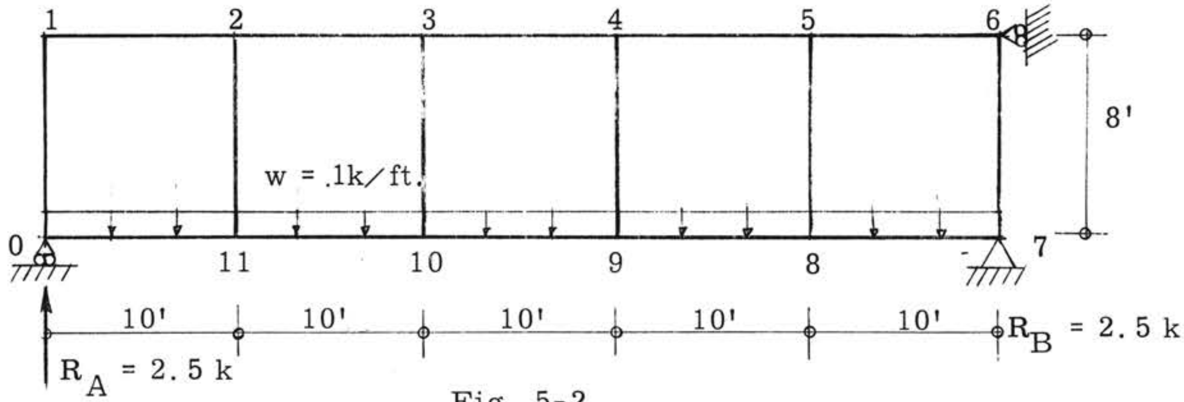


Fig. 5-2

Loaded Truss - Case I

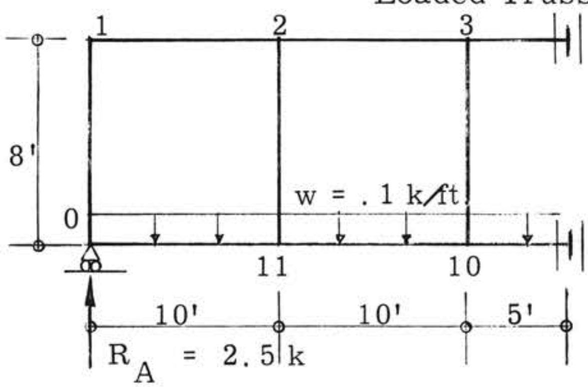


Fig. 5-3

Basic Structure - Case I

The original structure is loaded by a uniformly distributed load (Fig. 5-2). The structure and loading are symmetrical; thus, the resulting deformation is symmetrical with respect to the center line. Since the deformations on the left are symmetrical with the corresponding deformations on the right, only the left portion will be considered. This is done by imagining the structure to be cut through the center, and these cut ends to be placed in guides permitting vertical translation, but not rotation (Fig. 5-3).

This involves change in the modification of stiffness factors of members 34 and (10)9 from K_{34}, $K_{(10)9}$ to K''_{34}, $K''_{(10)9}$.

1.) Deformations

$$\theta_1 = -\theta_6 \qquad \theta_0 = -\theta_7$$

$$\theta_2 = -\theta_5 \qquad \theta_{(11)} = -\theta_8$$

$$\theta_3 = -\theta_4 \qquad \theta_{(10)} = -\theta_9$$

$$\Delta_1 = \Delta_0 = \Delta_6 = \Delta_7 = 0$$

$$\Delta_2 = \Delta_{(11)} = \Delta_5 = \Delta_8$$

$$\Delta_3 = \Delta_{(10)} = \Delta_4 = \Delta_9$$

2.) Stiffness Factors (EI constant)

$$K_{12} = K_{21} = K_{23} = K_{32} = K_{34} = K_{0(11)} = K_{(11)0} = K_{(11)(10)}$$

$$= K_{(10)(11)} = K_{(10)9} = \frac{4EI}{L} = \frac{4EI}{10} = .4EI$$

$$K_{01} = K_{10} = K_{2(11)} = K_{(11)2} = K_{3(10)} = K_{(10)3}$$

$$= \frac{4EI}{h} = \frac{4EI}{8}$$

$$= .5EI$$

$$K''_{34} = K''_{(10)9} = K_{34}(1 - C_{34})$$

$$= \frac{4EI}{L}(1 - .5) = \frac{1}{2}\,\frac{4EI}{10}$$

$$= .2EI$$

3.) Carry-Over Factors

All carry-over factors = + .5, since EI = constant.

4.) Fixed End Moments

$$FM_{0(11)} = FM_{(11)(10)} = FM_{(10)9}$$

$$= -\frac{wL^2}{12} = -\frac{(.1)(10)^2}{12} = -.833 \text{ k-ft.}$$

$$FM_{(11)0} = FM_{(10)(11)} = FM_{(9)(10)} = +\frac{wL^2}{12}$$

$$= \frac{(.1)(10)^2}{12} = +.833 \text{ k-ft.}$$

5.) Displacement Distribution Factors

$$D'_{12} = \frac{S_{12}}{S_{12} + S_{21} + S_{09} + S_{90}} = .25 \quad \text{Since } S = \frac{6EI}{L}$$

Since EI and L are constants

$$D'_{21} = D'_{23} = D'_{32} = D'_{0(11)} = D'_{(11)0} = D'_{(11)(10)} = D'_{(10)(11)}$$

$$= .25$$

6.) Modified Stiffness Factors

$$\Sigma K^*_1 = \Sigma K_1 - D'_{12} K'''_{12} = ((.4 + .5) - .25\,(.6))\,EI$$

$$= (.9 - .15)\,EI = .75\,EI$$

$$\Sigma K^*_0 = \Sigma K_0 - D'_{0(11)} K'''_{(11)0} = ((.4 + .5) - .25\,(.6))\,EI$$

$$= .75\,EI$$

$$\Sigma K^*_2 = \Sigma K_2 - D'_{21} K'''_{21} - D'_{23} K'''_{23}$$

$$= ((.4 + .4 + .5) - .25\,(.6) - .25\,(.6))\,EI$$

$$= (1.3 - .15 - .15)\,EI = 1\,EI$$

$$\Sigma K^*_{(11)} = \Sigma K_{11} - D'_{(11)0} K'''_{11(0)} - D'_{(11)(10)} K'''_{(11)(10)}$$

$$= ((.4 + .4 + .5) - .25\,(.6) - .25\,(.6))\,EI$$

$$= 1\,EI$$

$$\Sigma K^*_3 = \Sigma K_3 - D'_{32} K'''_{32}$$

$$= ((.4 + .2 + .5) - .25\,(.6))\,EI$$

$$= (1.1 - .15)\,EI = .95\,EI$$

$$\Sigma K^*_{(10)} = \Sigma K_{(10)} - D'_{(10)(11)} K'''_{(10)(11)}$$

$$= ((.4 + .2 + .5) - .25\,(.6))\,EI$$

$$= .95\,EI$$

7.) Modified Carry-Over Stiffness Factors

$CK^*_{12} = CK_{12} - D'_{21} K'''_{12} = ((.5)(.4) - (.25)(.6))\ EI$

$= (.2 - .15)\ EI = .05\ EI$

$CK^*_{21} = CK_{21} - D'_{12} K'''_{21} = .05\ EI$

Similarly, since EI and L are constants,

$CK^*_{23} = CK^*_{32} = CK^*_{0(11)} = CK^*_{(11)0} = CK^*_{(11)(10)}$

$= CK^*_{(10)(11)} = .05\ EI$

$CK^*_{10} = CK_{10} - D'_{0(11)} K'''_{12} = ((.5)(.5) - .25\ (.6))\ EI$

$= (.25 - .15)\ EI = .1\ EI$

$CK^*_{01} = .1\ EI$

$CK^*_{2(11)} = CK_{2(11)} - D'_{(11)(10)} K'''_{23} - D'_{(11)0} K'''_{21} = (.5\ (.5)$

$- .25\ (.6) - .25\ (.6))\ EI$

$= -.05\ EI$

$CK^*_{(11)2} = CK_{11(2)} - D'_{21} K'''_{(11)0} - D'_{23} K'''_{(11)(10)}$

$= (.5\ (.5) - .25\ (.6) - .25\ (.6))\ EI$

$= -.05\ EI$

$CK^*_{3(10)} = CK_{3(10)} - D'_{\cancel{(10)}9} K'''_{34} - D'_{(10)(11)} K'''_{32}$

$= (.5\ (.5) - .25\ (.6))\ EI$

$= .1\ EI$

$CK^*_{(10)3} = .1\ EI$

$CK^*_{1(11)} = - D'_{(11)0} K'''_{12} = -.25\ (.6)\ EI = -.15\ EI$

Similarly,

$CK^*_{02} = CK^*_{20} = CK^*_{(11)1} = CK^*_{2(10)} = CK^*_{(10)2}$

$= CK^*_{(11)3} = CK^*_{3(11)} = -.15\ EI$

8.) Joint Moment Carry-Over Factors

$$\gamma^*_{12} = - \frac{CK^*_{12}}{\Sigma K^*_1} = - \frac{.05\ EI}{.75\ EI} = - .0667$$

$$\gamma^*_{10} = - \frac{CK^*_{10}}{\Sigma K^*_1} = - \frac{.1\ EI}{.75\ EI} = - .134$$

$$\gamma^*_{1(11)} = - \frac{CK^*_{1(11)}}{\Sigma K^*_1} = - \frac{(-.15)\ EI}{.75\ \ EI} = + .200$$

$$\gamma^*_{21} = - \frac{CK^*_{21}}{\Sigma K^*_2} = - \frac{.05\ EI}{1\ EI} = - .050$$

$$\gamma^*_{23} = - \frac{CK^*_{23}}{\Sigma K^*_2} = - \frac{.05\ EI}{1\ EI} = - .050$$

$$\gamma^*_{2(10)} = - \frac{CK^*_{2(10)}}{\Sigma K^*_2} = - \frac{(-.15)}{1\ EI}\ EI = + .150$$

$$\gamma^*_{2(11)} = - \frac{CK^*_{2(11)}}{\Sigma K^*_2} = - \frac{(-.05)\ EI}{1\ EI} = + .050$$

$$\gamma^*_{20} = - \frac{CK^*_{20}}{\Sigma K^*_2} = - \frac{(-.15)}{1\ EI}\ EI = + .15$$

$$\gamma^*_{01} = - \frac{CK^*_{01}}{\Sigma K^*_0} = - \frac{.1\ EI}{.75\ EI} = - .134$$

$$\gamma^*_{02} = - \frac{CK^*_{02}}{\Sigma K^*_0} = - \frac{(-.15)\ EI}{.75\ \ EI} = + .200$$

$$\gamma^*_{0(11)} = - \frac{CK^*_{0(11)}}{\Sigma K^*_0} = - \frac{.05\ EI}{.75\ EI} = - .0667$$

$$\gamma^*_{(11)0} = - \frac{CK^*_{(11)0}}{\Sigma K^*_{(11)}} = - \frac{.05\ EI}{1\ EI} = - .050$$

$$\gamma^*_{(11)1} = - \frac{CK^*_{(11)1}}{\Sigma K^*_{(11)}} = - \frac{(-.15\ EI)}{1\ EI} = + .150$$

$$\gamma^*_{(11)2} = -\frac{CK^*_{(11)2}}{\Sigma K^*_{(11)}} = -\frac{(-.05)\ EI}{1\ EI} = +.050$$

$$\gamma^*_{(11)3} = -\frac{CK^*_{(11)3}}{\Sigma K^*_{(11)}} = -\frac{(-.15)\ EI}{1\ EI} = +.150$$

$$\gamma^*_{(11)(10)} = -\frac{CK^*_{(11)(10)}}{\Sigma K^*_{(11)}} = -\frac{.05\ EI}{1\ EI} = -.050$$

$$\gamma^*_{32} = -\frac{CK^*_{32}}{\Sigma K^*_{3}} = -\frac{.05\ EI}{.95\ EI} = -.053$$

$$\gamma^*_{3(11)} = -\frac{CK^*_{3(11)}}{\Sigma K^*_{3}} = -\frac{(-.15)\ EI}{.95\ EI} = +.158$$

$$\gamma^*_{3(10)} = -\frac{CK^*_{3(10)}}{\Sigma K^*_{3}} = -\frac{.1\ EI}{.95\ EI} = -.105$$

Similarly,

$$\gamma^*_{(10)3} = -.105, \quad \gamma^*_{(10)2} = +.158, \quad \gamma^*_{(10)(11)} = -.053$$

9.) <u>New Distribution Factors</u>

$$D^*_{12} = \frac{K_{12} - D'_{12}\,K'''_{12}}{\Sigma K^*_{1}} = \frac{(.4 - .25\,(.6))\ EI}{.75\ EI} = .334$$

$$D^*_{10} = \frac{K_{10}}{\Sigma K^*_{1}} = \frac{.5\ EI}{.75\ EI} = .667$$

$$D^*_{21} = \frac{K_{21} - D'_{21}\,K'''_{21}}{\Sigma K^*_{2}} = \frac{(.4 - .25\,(.6))\ EI}{1\ EI} = .25$$

$$D^*_{23} = \frac{K_{23} - D'_{23}\,K'''_{23}}{\Sigma K^*_{2}} = \frac{(.4 - .25\,(.6))\ EI}{1\ EI} = .25$$

$$D^*_{2(11)} = \frac{K_{2(11)}}{\Sigma K^*_{2}} = \frac{.5\ EI}{1\ EI} = .5$$

$$D^*_{32} = \frac{K_{32} - D'_{32}\,K'''_{32}}{\Sigma K^*_3} = \frac{(.4 - .25\,(.6))\ EI}{.95\ EI} = .264$$

$$D^*_{34} = \frac{K''_{32}}{\Sigma K^*_3} = \frac{.2\ EI}{.95\ EI} = .211 = D^*_{(10)9}$$

$$D^*_{3(10)} = \frac{K_{3(10)}}{\Sigma K^*_3} = \frac{.5\ EI}{.95\ EI} = .526$$

$$D^*_{01} = \frac{K_{01}}{\Sigma K^*_0} = \frac{.5\ EI}{.75\ EI} = .667$$

$$D^*_{0(11)} = \frac{K_{0(11)} - D'_{0(11)}\,K'''_{0(11)}}{\Sigma K^*_0} = \frac{(.4 - .15)\ EI}{.75} = .334$$

$$D^*_{(11)0} = \frac{K_{(11)0} - D'_{(11)0}\,K'''_{(11)0}}{\Sigma K^*_{(11)}} = \frac{(.4 - .15)\ EI}{1EI} = .25$$

$$D^*_{(11)2} = \frac{K_{(11)2}}{\Sigma K^*_{(11)}} = \frac{.5\ EI}{1\ EI} = .5$$

$$D^*_{(11)(10)} = \frac{K_{(11)(10)} - D'_{(11)(10)}\,K'''_{(11)(10)}}{\Sigma K^*_{(11)}} = \frac{(.4 - .15)EI}{1EI} = .25$$

$$D^*_{(10)(11)} = \frac{K_{(10)(11)} - D'_{(10)(11)}\,K'''_{(10)(11)}}{\Sigma K^*_{(10)}} = \frac{(.4 - .15)EI}{.95\ EI} = .264$$

$$D^*_{(10)3} = \frac{K_{(10)3}}{\Sigma K^*_{(10)}} = \frac{.5\ EI}{.95\ EI} = .526$$

10.) New Carry-Over Distribution Factors

$$CD^*_{12} = \frac{CK_{12} - D'_{21}\,K'''_{12}}{\Sigma K^*_1} = \frac{(.2 - .15)\ EI}{.75\ EI} = \frac{.05}{.75} = .066$$

$$CD^*_{10} = -\frac{D'_{0(11)}K''_{12}}{\Sigma K^*_1} = -\frac{.25\,(.6)\ EI}{.75\ EI} = -.2$$

$$CD_{10} = \frac{CK_{10}}{\Sigma K^*_1} = \frac{(.5)(.5)\ EI}{.75\ EI} = \frac{.25}{.75} = .333$$

$$CD^*_{1(11)} = -\frac{D'_{(11)0}\,K'''_{12}}{\Sigma K^*_1} = -\frac{.25\,(.6)\,EI}{.75\,EI} = -.2$$

$$CD^*_{01} = -\frac{D'_{12}\,K'''_{0(11)}}{\Sigma K^*_0} = -\frac{.25\,(.6)\,EI}{.75\,EI} = -.2$$

$$CD_{01} = +\frac{CK_{01}}{\Sigma K^*_0} = \frac{.25\,EI}{.75\,EI} = .333$$

$$CD^*_{02} = -\frac{D'_{21}\,K'''_{0(11)}}{\Sigma K^*_0} = -\frac{.15\,EI}{.75\,EI} = -.2$$

$$CD^*_{0(11)} = \frac{CK_{0(11)} - D'_{(11)0}\,K'''_{0(11)}}{\Sigma K^*_0} = \frac{.05\,EI}{.75\,EI} = .066$$

$$CD^*_{21} = \frac{CK_{21} - D'_{12}\,K'''_{21}}{\Sigma K^*_2} = \frac{(.2 - .15)\,EI}{1\,EI} = .05$$

$$CD^*_{20} = -\frac{D'_{0(11)}\,K'''_{21}}{\Sigma K^*_2} = -\frac{.15\,EI}{1\,EI} = -.15$$

$$CD^*_{2(11)} = -\frac{D'_{(11)0}\,K'''_{21}}{\Sigma K^*_2} = -\frac{D'_{(11)(10)}\,K'''_{23}}{\Sigma K^*_2} = -\frac{.15\,EI}{1\,EI}$$

$$= -.15$$

$$CD_{2(11)} = +\frac{CK_{2(11)}}{\Sigma K^*_2} = \frac{.5\,(.5)\,EI}{1\,EI} = .25$$

$$CD^*_{2(10)} = -\frac{D_{(10)(11)}\,K'''_{23}}{\Sigma K^*_2} = -\frac{.15\,EI}{1\,EI} = -.15$$

$$CD^*_{23} = \frac{CK_{23} - D'_{32}\,K'''_{23}}{\Sigma K^*_2} = \frac{(.2 - .15)\,EI}{1\,EI} = .05$$

$$CD^*_{32} = \frac{CK_{32} - D'_{23}\,K'''_{32}}{\Sigma K^*_3} = \frac{(.2 - .15)\,EI}{.95\,EI} = .053$$

$$CD^*_{3(11)} = -\frac{D'_{(11)(10)}\,K'''_{32}}{\Sigma K^*_3} = -\frac{.15\,EI}{.95\,EI} = -.158$$

$$CD^*_{3(10)} = - \frac{D'_{(11)(10)} K'''_{32}}{\Sigma K^*_3} = - \frac{.15\ EI}{.95\ EI} = - .158$$

$$CD_{3(10)} = \frac{CK_{3(10)}}{\Sigma K^*_3} = \frac{.5\ (.5)\ EI}{.95\ \ EI} = .264$$

Similarly,

$$CD^*_{(11)0} = .05,\ CD^*_{(11)1} = -.15,\ CD^*_{(11)2} = -.15$$

$$CD_{(11)2} = .25,\ CD^*_{(11)3} = -.15,\ CD^*_{(11)(10)} = .05$$

$$CD^*_{(10)(11)} = .053,\ CD^*_{(10)2} = -.158,\ CD^*_{(10)3} = -.158$$

$$CD_{(10)3} = .264$$

11.) <u>Starting Moments</u>

$$m^*_1 = - \Sigma SM^*_1$$

$$= D'_{12} \Big[\cancel{FM}_{12} + \cancel{FM}_{21} + FM_{0(11)} + FM_{(11)0} + L\ (\cancel{BV}_{21} + BV_{(11)0} + V_2) \Big] - \cancel{\Sigma FM}_1$$

$$= .25 \left[- .833 + .833 + 10(.5 + 1.5) \right]$$

$$= 5$$

$$m^*_2 = D'_{21} \Big[\cancel{FM}_{21} + \cancel{FM}_{12} + FM_{(11)0} + FM_{0(11)} + L\ (\cancel{BV}_{21} + BV_{(11)0} + V_2) \Big]$$

$$+ D'_{23} \Big[\cancel{FM}_{23} + \cancel{FM}_{32} + FM_{(11)(10)} + FM_{(10)(11)} + L\ (\cancel{BV}_{32} + BV_{(10)(11)} + V_3) \Big] - \cancel{\Sigma FM}_2$$

$$= .25 \left[- .833 + .833 + 10(2) \right] + .25 \left[- .833 + .833 + 10\ (.5 + .5) \right]$$

$$= 5 + 2.5 = 7.5$$

$$m^*_3 = + D'_{32}\left[\cancel{FM_{32}} + \cancel{FM_{23}} + FM_{(10)(11)} + FM_{(11)(10)} + L(\cancel{BV_{32}} + BV_{(10)(11)} + V_3\right] - \Sigma\cancel{FM_3}$$

$$= .25\left[- .833 + .833 + 10(.5 + .5)\right]$$

$$= 2.5$$

Similarly,

$$m^*_0 = 5 + .833 = 5.833,\quad m^*_{(11)} = 7.5,\quad m^*_{(10)} = 2.5$$

12.) <u>Modified Fixed End Moments</u>

$$FM^*_{12} = - D'_{12}\left[\cancel{FM_{12}} + \cancel{FM_{21}}\ FM_{0(11)} + FM_{(11)0} + L(\cancel{BV_{21}} + BV_{(11)0} + V_2)\right] + \cancel{FM_{12}}$$

$$= - .25(- .833 + .833 + 10(.5 + 1.5))$$

$$= - 5$$

$$FM^*_{21} = - D'_{21}\left[\cancel{FM_{21}} + \cancel{FM_{12}} + FM_{0(11)} + FM_{(11)0} + L(\cancel{BV_{21}} + BV_{(11)0} + V_2)\right] + \cancel{FM_{12}}$$

$$= - .25(- .833 + .833 + 10(.5 + 1.5))$$

$$= - 5$$

$$FM^*_{23} = - D'_{23}\left[\cancel{FM_{23}} + \cancel{FM_{32}} + FM_{(11)(10)} + FM_{(10)(11)} + L(\cancel{BV_{32}} + BV_{(10)(11)} + V_3)\right] + \cancel{FM_{23}}$$

$$= - .25(- .833 + .833 + 10(.5 + .5))$$

$$= - 2.5$$

Similarly $FM^*_{32} = -2.5$

$$FM^*_{0(11)} = - D'_{0(11)}\left[\cancel{FM_{12}} + \cancel{FM_{21}} + FM_{0(11)} + FM_{(11)0} + L(\cancel{BV_{21}} + BV_{(11)0} + V_2)\right] + FM_{0(11)}$$

$$= -.25\,(-.833 + .833 + 10\,(.5 + 1.5)) + (-.833)$$

$$= -5 - .833 = -5.833$$

$$FM^*_{(11)0} = -5 + .833 = -4.167$$

$$FM^*_{(11)(10)} = -D'_{(11)(10)}\left[\cancel{FM_{23}} + \cancel{FM_{32}} + FM_{(11)(10)} + FM_{(10)(11)} + L\,(\cancel{BV_{32}} + BV_{(10)(11)} + V_3)\right] + FM_{(11)(10)}$$

$$= -.25\,(-.833 + .833 + 10\,(.5 + .5)) - .833$$

$$= -2.5 - .833 = -3.333$$

$$FM^*_{(10)(11)} = -2.5 + .833 = -1.667$$

$$FM^*_{34} = 0$$

$$FM^*_{(10)9} = FM_{(10)9} = -.833$$

13. Carry-Over Procedure

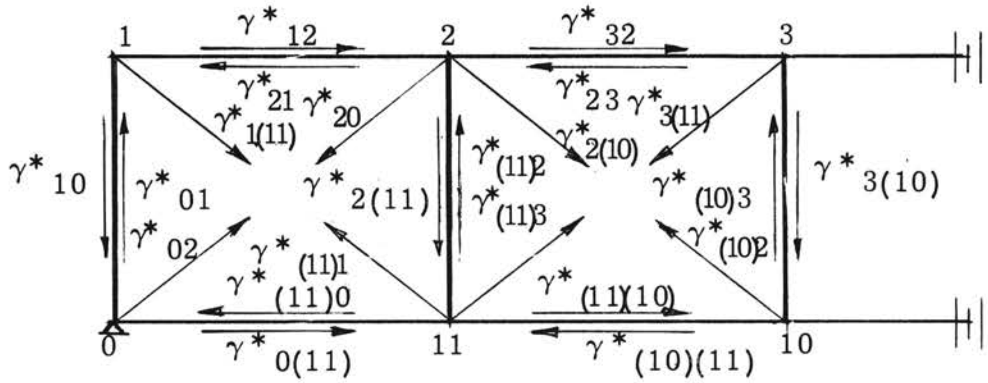


Fig. 5-4
Carry-Over Procedure (Example 1, Case 1)

	1	2	3	10	11	0
γ* 's	- .0667→ - .1340↓ + .2000↘	- .050→ + .150↘ + .050↓ + .150↙ - .050←	- .053← + .158↙ - .105↓	- .105↑ + .158↖ + .053→	- .050← + .050↑ - .150↗ - .050→ + .150↖	- .134↑ + .200↗ - .066→
m* 's	5	7.5	2.5	2.5	7.5	5.833
	- .3750 +1.1250 - .7820	- .3335 - .1325 + .3950 + .3750 +1.6660	- .3750 - .2630 +1.1250	+1.1250 - .2630 - .3750	+1.0000 + .3750 + .3960 - .1325 - .2750	- .6700 +1.1250 - .3840
	- .0320	+1.4700	0.4870	+ .4870	+1.364	+ .0710
	- .0735 + .2046 - .0095	+ .0021 - .0258 + .0769 + .0682 + .0140	- .0735 - .0511 + .2046	+ .2205 - .0511 - .0682	+ .0032 + .0735 + .0769 - .0258 - .0046	- .0042 + .2205 - .0682
	+ .1216	+ .2054	+ .080	.1012	.1230	.1565
JM*'s	5.0896	9.1754	+3.0670	3.0882	8.9870	6.0605

TABLE 5-1

Carry-Over Table - Case I (Example 1)

The carry-over procedure is performed from Joints 1, 2, 3, 10, 11, 0 in sequence. The direction of arrows in the table should be followed in the sketch. For example, the carry-over factor $-.667 \rightarrow$ in the table under Joint (1), indicates the carry-over factor from 1 to 2 as followed from the sketch. Hence, for all the carry-over factors in the table, the directions shown in the table have to be referred to the sketch.

14.) Check of The Carry-Over Procedure

$$\begin{aligned}
JM^*_1 &= 5.089 = ? \\
&= m^*_1 + \gamma^*_{01}\, JM^*_0 + \gamma^*_{(11)1}\, JM^*_{(11)} + \gamma^*_{21}\, JM^*_2 \\
&= 5 + (-.134)(6.06) + (.15)\, 8.98 + (-.05)\, 9.175 \\
&= 5 - .812 + 1.348 - .4587 \\
&= 5.08
\end{aligned}$$

$$\begin{aligned}
JM^*_2 &= 9.175 = ? \\
&= m^*_2 + \gamma^*_{12}\, JM^*_1 + \gamma^*_{02}\, JM^*_0 + \gamma^*_{(11)2}\, JM^*_{(11)} \\
&\quad + \gamma^*_{(10)2}\, JM^*_{10} + \gamma^*_{32}\, JM^*_3 \\
&= 7.5 + (-.0667)(5.089) + .2\,(6.06) + .05\,(8.996) \\
&\quad + .158\,(3.088) + (-.053)(3.067) \\
&= 7.5 - .339 + 1.212 + .449 + .487 - .163 \\
&= 9.14
\end{aligned}$$

$$\begin{aligned}
JM^*_3 &= 3.067 = ? \\
&= m^*_3 + \gamma^*_{23}\, JM^*_2 + \gamma^*_{(11)3}\, JM^*_{(11)} + \gamma^*_{(10)3}\, JM^*_{(10)} \\
&= 2.5 + (-.05)(9.175) + (.15)(8.996) + (-.105)(3.088) \\
&= 2.5 - .458 + 1.349 - .324 \\
&= 3.067
\end{aligned}$$

$$JM^*_0 = 6.06 = ?$$

$$= m^*_0 + \gamma^*_{(1)0} JM^*_1 + \gamma^*_{20} JM^*_2 + \gamma^*_{(11)0} \ JM^*_{(11)}$$

$$= 5.883 + (-.134)(5.089) + .15(9.175) + (-.05)(8.996)$$

$$= 5.883 - .681 + 1.376 - .449$$

$$= 6.12$$

$$JM^*_{(11)} = 8.98 = ?$$

$$= m^*_{(11)} + \gamma^*_{0(11)} JM^*_0 + \gamma^*_{1(11)} JM^*_1$$

$$+ \gamma^*_{2(11)} JM^*_2 + \gamma^*_{3(11)} JM^*_3 + \gamma^*_{(10)(11)} JM^*_{(10)}$$

$$= 7.5 + (-.066)(6.06) + .2(5.089) + .05(9.175)$$

$$+ (.158)(3.067) + (-.053)(3.088)$$

$$= 7.5 - .399 + 1.017 + .458 + .4845 - .163$$

$$= 8.91$$

$$JM^*_{(10)} = 3.088 = ?$$

$$= m^*_{10} + \gamma^*_{(11)(10)} JM^*_{(11)} + \gamma^*_{2(10)} JM^*_2$$

$$+ \gamma^*_{3(10)} JM^*_3$$

$$= 2.5 + (-.05)(8.996) + .15(9.175) + (-.105)(3.067)$$

$$= 2.5 - .449 + 1.376 - .322$$

$$= 3.100$$

15.) Final End Moments

$$M_{12} = D^*_{12} JM^*_1 + CD^*_{21} JM^*_2 + CD^*_{(11)1} JM^*_{(11)}$$

$$+ CD^*_{01} JM^*_0 + FM^*_{12}$$

$$= .334(5.089) + .05(9.175) - .15(8.996) - .2(6.06) - 5$$

$$= 1.699 + .458 - 1.35 - 1.212 - 5 = -5.407$$

$$M_{10} = D^*_{10}\, JM^*_1 + CD_{01}\, JM^*_0$$
$$= .667\,(5.089) + .333\,(6.06)$$
$$= 3.394 + 2.017$$
$$= +5.41$$

$$M_{21} = D^*_{21}\, JM^*_2 + CD^*_{12}\, JM^*_1 + CD^*_{02}\, JM^*_0$$
$$+ CD^*_{(11)2}\, JM^*_{(11)} + FM^*_{21}$$
$$= .25\,(9.175) + .066\,(5.089) - .2\,(6.06) - .15\,(8.996) - 5$$
$$= 2.29 + .335 - 1.212 - 1.35 - 5$$
$$= -4.93$$

$$M_{23} = D^*_{23}\, JM^*_2 + CD^*_{32}\, JM^*_3 + CD^*_{(10)2}\, JM^*_{(10)}$$
$$+ CD^*_{(11)2}\, JM^*_{(11)} + FM^*_{23}$$
$$= .25\,(9.175) + .053\,(3.067) + (-.158)\,(3.088)$$
$$- .15\,(8.996) - 2.5$$
$$= 2.29 + .162 - .487 - 1.350 - 2.5$$
$$= -1.885$$

$$M_{2(11)} = D^*_{2(11)}\, JM^*_2 + CD_{(11)2}\, JM^*_{(11)}$$
$$= .5\,(9.175) + .25\,(8.996)$$
$$= 4.588 + 2.249$$
$$= 6.837$$

$$M_{32} = D^*_{32}\, JM^*_3 + CD^*_{23}\, JM^*_2 + CD^*_{(11)3}\, JM^*_{(11)}$$
$$+ CD^*_{(10)3}\, JM^*_{(10)} + FM^*_{32}$$
$$= .264\,(3.067) + .05\,(9.175) - .15\,(8.996) - .158\,(3.088)$$
$$- 2.5$$
$$= .809 + .458 - 1.35 - .487 - 2.5$$
$$= -3.07$$

$$M_{34} = D^*{}_{34}\, JM^*{}_3 + \cancel{CD^*{}_{(10)3}}\, JM^*{}_{(10)} + \cancel{FM^*{}_{34}}$$

$$= .211\ (3.067)$$

$$= .647$$

$$M_{3(10)} = D^*{}_{3(10)}\, JM^*{}_3 + CD_{(10)3}\, JM^*{}_{10}$$

$$= .526\ (3.067) + .264\ (3.088)$$

$$= 1.613 + .815$$

$$= 2.428$$

$$M_{01} = D^*{}_{01}\, JM^*{}_0 + CD_{10}\, JM^*{}_1$$

$$= .667\ (6.06) + .333\ (5.089)$$

$$= 4.04 + 1.694$$

$$= 5.734$$

$$M_{0(11)} = D^*{}_{0(11)}\, JM^*{}_0 + CD^*{}_{(11)0}\, JM^*{}_{(11)} + CD^*{}_{20}\, JM^*{}_2$$

$$+ CD^*{}_{10}\, JM^*{}_1 + FM^*{}_{0(11)}$$

$$= .334\ (6.06) + .05\ (8.996) + (-.15)\ (9.175)$$

$$+ (-.2)\ (5.089) - 5.833$$

$$= 2.024 + .449 - 1.376 - 1.017 - 5.833$$

$$= -5.753$$

$$M_{(11)0} = D^*{}_{(11)0}\, JM^*{}_{(11)} + CD^*{}_{0(11)}\, JM^*{}_0 + CD^*{}_{1(11)}\, JM^*{}_1$$

$$+ CD^*{}_{2(11)}\, JM^*{}_2 + FM^*{}_{(11)0}$$

$$= (.25)\ (8.996) + (.066)\ (6.06) + (-.2)\ (5.089)$$

$$+ (-.15)\ (9.175) - 4.167$$

$$= 2.249 + .399 - 1.017 - 1.376 - 4.167$$

$$= -3.91$$

$$M_{(11)(10)} = D^*{}_{(11)(10)}\, JM^*{}_{(11)} + CD^*{}_{(10)(11)}\, JM^*{}_{(10)}$$

$$+ CD^*{}_{3(11)}\, JM^*{}_3 + CD^*{}_{2(11)}\, JM^*{}_2 + FM^*{}_{(11)(10)}$$

$$= .25\,(8.996) + (.053)\,(3.088) + (-\,.158)\,(3.067)$$

$$+ (-\,.15)\,(9.175) - 3.333$$

$$= 2.249 + .1636 - .484 - 1.376 - 3.333$$

$$= -\,2.78$$

$$M_{(11)2} = D^*{}_{(11)2}\,JM^*{}_{(11)} + CD_{2(11)}\,JM^*{}_2$$

$$= (.5)\,(8.996) + (.25)\,(9.175)$$

$$= 4.498 + 2.29$$

$$= 6.78$$

$$M_{(10)(11)} = D^*{}_{(10)(11)}\,JM^*{}_{(10)} + CD^*{}_{(11)(10)}\,JM^*{}_{(11)}$$

$$+ CD^*{}_{2(10)}\,JM^*{}_2 + CD^*{}_{3(10)}\,JM^*{}_3 + FM^*{}_{(10)(11)}$$

$$= (.264)\,(3.088) + .05\,(8.996) + (-\,.15)\,(9.175)$$

$$+ (-\,.158)\,(3.067) - 1.667$$

$$= .815 + .45 - 1.376 - .484 - 1.667$$

$$= -\,2.26$$

$$M_{10(9)} = D^*{}_{(10)9}\,JM^*{}_{(10)} + CD^*{}_{3(10)}\,JM^*{}_3 + FM^*{}_{(10)9}$$

$$= (.211)\,(3.088) - .833$$

$$= .651 - .833$$

$$= -\,.182$$

$$M_{(10)3} = D^*{}_{(10)3}\,JM^*{}_{(10)} + CD_{3(10)}\,JM^*{}_3$$

$$= (.526)\,(3.088) + .264\,(3.067)$$

$$= 1.62 + .809$$

$$= 2.429$$

The Final End Moments for Case I are:

$M_{12} = -M_{65} = -5.407$ $\quad$ $M_{34} = -M_{43} = .647$

$M_{10} = -M_{67} = +5.41$ $\quad$ $M_{3(10)} = -M_{49} = 2.428$

$M_{21} = -M_{56} = -4.93$ $\quad$ $M_{01} = -M_{76} = -5.734$

$M_{23} = -M_{54} = -1.885$ $\quad$ $M_{0(11)} = -M_{78} = 5.753$

$M_{2(11)} = -M_{58} = 6.837$ $\quad$ $M_{(11)0} = -M_{87} = -3.91$

$M_{32} = -M_{45} = -3.07$ $\quad$ $M_{(11)(10)} = -M_{89} = -2.78$

$$M_{(11)2} = -M_{85} = 6.78$$

$$M_{(10)(11)} = -M_{98} = -2.26$$

$$M_{(10)9} = -M_{9(10)} = -.182$$

$$M_{(10)3} = -M_{94} = 2.429$$

CASE II - ANTISYMMETRICAL

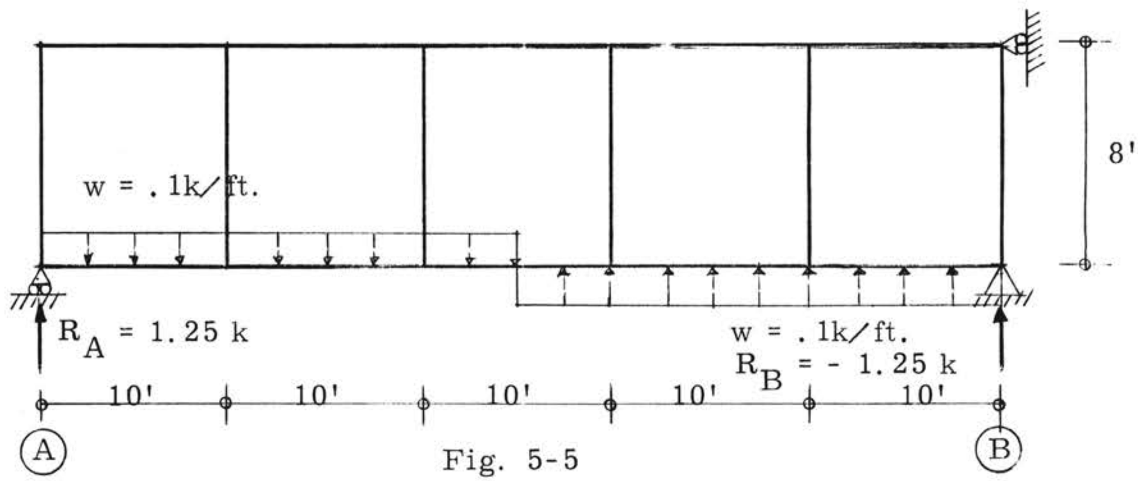


Fig. 5-5

Loaded Truss - Case II

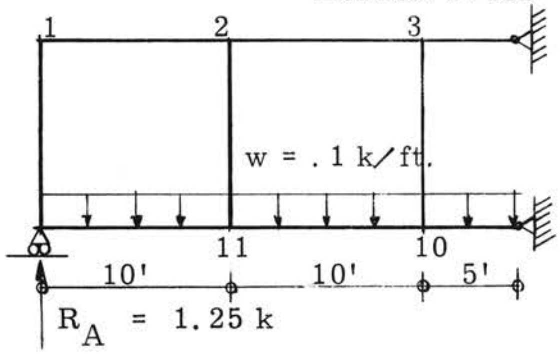


Fig. 5-6

Basic Structure - Case II

The original structure is loaded by an antisymmetrical distributed load of uniform intensity (Fig. 5-5). The structure is symmetrical; thus, the resulting deformation is antisymmetrical with respect to the center line. Since the deformations on the left are antisymmetrical with the corresponding deformations on the right, only the left portion will be considered. This is done by imagining the structure to be cut through the center and these cut ends to be hinged permitting rotation but no translation (Fig. 5-6).

This involves change in the modification of stiffness factors of members 34 and (10)9 from K_{34}, $K_{(10)9}$, to $K'''_{3\,4}$, $K'''_{(10)9}$.

1.) Deformations

$$\theta_1 = \theta_6 \qquad \theta_0 = \theta_7$$

$$\theta_2 = \theta_5 \qquad \theta_{(11)} = \theta_8$$

$$\theta_3 = \theta_4 \qquad \theta_{(10)} = \theta_9$$

$$\Delta_1 = \Delta_0 = \Delta_6 = \Delta_7 = 0$$

$$\Delta_2 = \Delta_{(11)} = -\Delta_5 = -\Delta_8$$

$$\Delta_3 = \Delta_{(10)} = -\Delta_4 = -\Delta_9$$

2.) Stiffness Factors EI (Constant)

$$K_{12} = K_{21} = K_{23} = K_{32} = K_{34} = K_{0(11)}$$

$$= K_{(11)0} = K_{(11)(10)} = K_{(10)(11)} = K_{(10)9}$$

$$= \frac{4\ EI}{L} = \frac{4\ EI}{10}$$

$$= .4\ EI$$

$$K_{01} = K_{10} = K_{2(11)} = K_{(11)2} = K_{3(10)} = K_{(10)3}$$

$$= \frac{4\ EI}{L} = \frac{4\ EI}{8}$$

$$= .5\ EI$$

$$K'''_{34} = K'''_{(10)9} = K_{34}\ (1 + C_{34})$$

$$= \frac{4\ EI}{10}\ (1 + .5)$$

$$= .6\ EI$$

3.) Carry-Over Factors

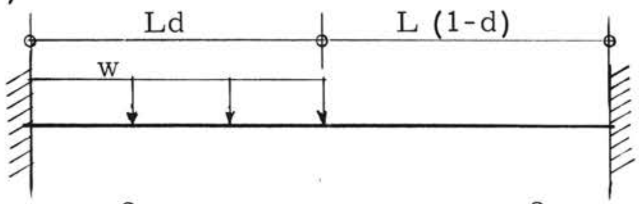
All carry-over factors = + .5.

4.) Fixed End Moments

$$FM_{0(11)} = FM_{(11)(10)} = -\ .833 \text{ k-ft.}$$

$$FM_{(11)0} = FM_{(10)(11)} = +\ .833 \text{ k-ft.}$$

$FM_{(10)9}$



$-\frac{d^2(6 - 8d + 3d^2)}{12}$ $\frac{d^3(4 - 3d)}{12}$

$FM = wL^2 \times \text{coefficient}$

Since, in our case, the loading is antisymmetrical, we have

$$FM_{(10)9} = -wL^2 \frac{d^2(6 - 8d + 3d^2)}{12} + wL^2 \frac{d^3(4-3d)}{12}$$

$$= -.574 + .26 \qquad d = .5, \ w = .1$$

$$= -.314 \text{ k-ft.}$$

5.) Displacement Distribution Factors

$$D'_{12} = .25$$
$$= D'_{21} = D'_{23} = D'_{32} = D'_{0(11)} = D'_{(11)0} = D'_{(11)(10)}$$
$$= D'_{(10)(11)}$$

6.) Modified Joint Stiffness Factors

$$\Sigma K^*_1 = .75 \text{ EI}$$

$$\Sigma K^*_0 = .75 \text{ EI}$$

$$\Sigma K^*_2 = 1.00 \text{ EI}$$

$$\Sigma K^*_{(11)} = 1.00 \text{ EI}$$

$$\Sigma K^*_3 = \Sigma K_3 - D'_{32} K'''_{32}$$
$$= ((.4 + .6 + .5) - .25(.6)) \text{ EI}$$
$$= 1.35 \text{ EI}$$

$$\Sigma K^*_{(10)} = \Sigma K_{(10)} - D'_{(10)(11)} K'''_{(10)(11)}$$

$$= ((.4 + .6 + .5) - .25\,(.6))\ EI$$

$$= 1.35\ EI$$

7.) <u>Modified Carry-Over Stiffness Factors</u>

$$CK^*_{12} = CK^*_{21} = CK^*_{23} = CK^*_{32} = CK^*_{0(11)} = CK^*_{(11)0}$$

$$= CK^*_{(11)(10)} = CK^*_{(10)(11)}$$

$$= .05\ EI$$

$$CK^*_{10} = CK^*_{01} = .1\ EI$$

$$CK^*_{2(11)} = CK^*_{(11)2} = -.05\ EI$$

$$CK^*_{3(10)} = CK^*_{(10)3} = +.1\ EI$$

$$CK^*_{1(11)} = CK^*_{(11)1} = CK^*_{02} = CK^*_{20} = CK^*_{2(10)} = CK^*_{(10)2}$$

$$= CK^*_{(11)3} = CK^*_{3(11)} = -.15\ EI$$

8.) <u>Joint Moment Carry-Over Factors</u>

$$\gamma^*_{12} = \gamma^*_{0(11)} = -.0667$$

$$\gamma^*_{10} = \gamma^*_{01} = -.134$$

$$\gamma^*_{21} = \gamma^*_{23} = \gamma^*_{(11)0} = \gamma^*_{(11)(10)} = -.05$$

$$\gamma^*_{2(11)} = \gamma^*_{(11)2} = +.05$$

$$\gamma^*_{1(11)} = \gamma^*_{02} = +.2$$

$$\gamma^*_{20} = \gamma^*_{(11)1} = \gamma^*_{2(10)} = \gamma^*_{(11)3} = +.15$$

$$\gamma^*_{32} = \gamma^*_{(10)(11)} = -\frac{CK^*_{32}}{\Sigma K^*_3} = -\frac{.05\ EI}{1.35\ EI} = -.037$$

$$\gamma^*_{3(10)} = \gamma^*_{(10)3} = -\frac{CK^*_{3(10)}}{\Sigma K^*_3} = -\frac{.1\ EI}{1.35\ EI} = -074$$

$$\gamma^*_{3(11)} = \gamma^*_{(10)2} = - \frac{CK^*_{3(11)}}{\Sigma K^*_3} = - \frac{(-.15\ EI)}{1.35\ EI} = +.111$$

9.) New Distribution Factors

$$D^*_{12} = D^*_{0(11)} = .334$$

$$D^*_{10} = D^*_{01} = .667$$

$$D^*_{21} = D^*_{23} = D^*_{(11)0} = D^*_{(11)(10)} = .25$$

$$D^*_{2(11)} = D^*_{(11)2} = .5$$

$$D^*_{32} = D^*_{(10)(11)} = \frac{K_{32} - D'_{32}\, K'''_{32}}{\Sigma K^*_3} = \frac{(.4 - .25(.6))EI}{1.35\ EI} = .18$$

$$D^*_{34} = D^*_{(10)9} = \frac{K'''_{34} - D'_{34}\, K'''_{34}}{\Sigma K^*_3} = \frac{.6\ EI}{1.35\ EI} = .444$$

$$D^*_{3(10)} = D^*_{(10)3} = \frac{K_{(3)10}}{\Sigma K^*_3} = \frac{.5\ EI}{1.35\ EI} = .37$$

10.) New Carry-Over Distribution Factors

$$CD^*_{12} = CD^*_{0(11)} = .066$$

$$CD^*_{21} = CD^*_{23} = CD^*_{(11)0} = CD^*_{(11)(10)} = .05$$

$$CD^*_{32} = CD^*_{(10)(11)} = \frac{CK_{32} - D'_{23}\, K'''_{32}}{\Sigma K^*_3} = \frac{.05\ EI}{1.35\ EI} = .037$$

$$CD^*_{01} = CD^*_{10} = -.2$$

$$CD_{01} = CD_{10} = .333$$

$$CD^*_{2(11)} = CD^*_{(11)2} = -.15$$

$$CD_{2(11)} = CD_{(11)2} = .25$$

$$CD^*_{3(10)} = CD^*_{(10)3} = - \frac{D'_{(10)(11)}\, K'''_{32}}{\Sigma K^*_3} = - \frac{.15\ EI}{1.35\ EI} = -.111$$

$$CD_{3(10)} = CD_{(10)3} = \frac{CK_{3(10)}}{\Sigma K^*_3} = \frac{.5\,(.5)\,EI}{1.35\,EI} = .185$$

$$CD^*_{1(11)} = CD^*_{(11)1} = CD^*_{02} = CD^*_{20} = CD^*_{2(10)}$$

$$= CD^*_{(11)3} = -.2$$

$$CD^*_{(10)2} = CD^*_{3(11)} = -\frac{D'_{(11)(10)}\,K'''_{3(10)}}{\Sigma K^*_3} = -\frac{.15\,EI}{1.35\,EI} = -.111$$

11.) <u>Starting Moments</u>

$$m^*_1 = D'_{12}\Big[\cancel{FM}_{12} + \cancel{FM}_{21} + FM_{0(11)} + FM_{(11)0}$$

$$+ L\ (\cancel{BV}_{21} + BV_{(11)0} + V_2\Big] - \Sigma\cancel{FM}_1$$

$$= .25\,(-.833 + .833 + 10\,(.5 + .25))$$

$$= 1.875$$

$$m^*_2 = D'_{21}\Big[\cancel{FM}_{21} + \cancel{FM}_{12} + FM_{(11)0} + FM_{0(11)}$$

$$+ L\,(\cancel{BV}_{21} + BV_{(11)0} + V_2)\Big]$$

$$+ D'_{23}\Big[\cancel{FM}_{23} + \cancel{FM}_{32} + FM_{11(10)} + FM_{10(11)}$$

$$L\,(\cancel{BV}_{32} + BV_{10(11)} + V_3)\Big] - \Sigma\cancel{FM}_2$$

$$= .25\,(-.833 + .833 + 10\,(.5 + .25))$$

$$+ .25\,(-.833 + .833 + 10\,(.5 + (-.75)))$$

$$= 1.875 - .625 = 1.25$$

$$m^*_3 = D'_{32}\Big[\cancel{FM}_{32} + \cancel{FM}_{23} + FM_{10(11)} + FM_{11(10)}$$

$$+ L\,(\cancel{BV}_{32} + BV_{10(11)} + V_3)\Big] - \Sigma\cancel{FM}_3$$

$$= .25\,(-.833 + .833 + 10\,(.5 - .75))$$

$$= -.625$$

$$m^*_0 = 1.875 + .833 = 2.708$$

$$m^*_{(11)} = 1.25$$

$$m^*_{(10)} = -.625 - (.833 - .314)$$

$$= -1.144$$

12.) Modified Fixed End Moments

$$FM^*_{12} = -D'_{12}\left[\cancel{FM_{12}} + \cancel{FM_{21}} + FM_{0(11)} + FM_{(11)0} + L\left(\cancel{BV_{21}} + BV_{(11)0} + V_2\right)\right] + \cancel{FM_{12}}$$

$$= -.25(-.833 + .833 + 10(.5 + .25))$$

$$= -1.875$$

$$FM^*_{21} = -D'_{21}\left[\cancel{FM_{21}} + \cancel{FM_{12}} + FM_{0(11)} + FM_{(11)0} + L\left(\cancel{BV_{21}} + BV_{(11)0} + V_2\right)\right] + \cancel{FM_{21}}$$

$$= -.25(-.833 + .833 + 10(.5 + .25))$$

$$= -1.875$$

$$FM^*_{23} = -D'_{23}\left[\cancel{FM_{23}} + \cancel{FM_{32}} + FM_{(11)(10)} + FM_{(10)(11)} + L\left(\cancel{BV_{32}} + BV_{(10)(11)} + V_3\right)\right] + \cancel{FM_{23}}$$

$$= -.25(-.833 + .833 + 10(.5 + (-.75)))$$

$$= +.625$$

$$FM^*_{32} = +.625$$

$$FM^*_{0(11)} = -D'_{0(11)}\left[\cancel{FM_{12}} + \cancel{FM_{21}} + FM_{0(11)} + FM_{(11)0} + L\left(\cancel{BV_{21}} + BV_{(11)0} + V_2\right)\right] + FM_{0(11)}$$

$$= -.25(-.833 + .833 + 10(.5 + (.25)) + (-.833)$$

$$= -1.875 - .833 = -2.708$$

$$FM^*_{(11)0} = -1.875 + .833 = -1.042$$

$$FM^*_{(11)(10)} = -D'_{(11)(10)}\left[\cancel{FM_{23}} + \cancel{FM_{32}} + FM_{(11)(10)} + FM_{(10)(11)} + L\left(\cancel{BV_{32}} + BV_{(10)(11)} + V_3\right)\right] + FM_{(11)(10)}$$

$$= -.25(-.833 + .833 + 10(.5 + (-.75))) - .833$$

$$= +.625 - .833 = -.202$$

$$FM^*_{(10)(11)} = +.625 + .833 = 1.458$$

$$FM^*_{34} = 0$$

$$FM^*_{(10)9} = FM_{(10)9} = -.314$$

13. Carry-Over Procedure

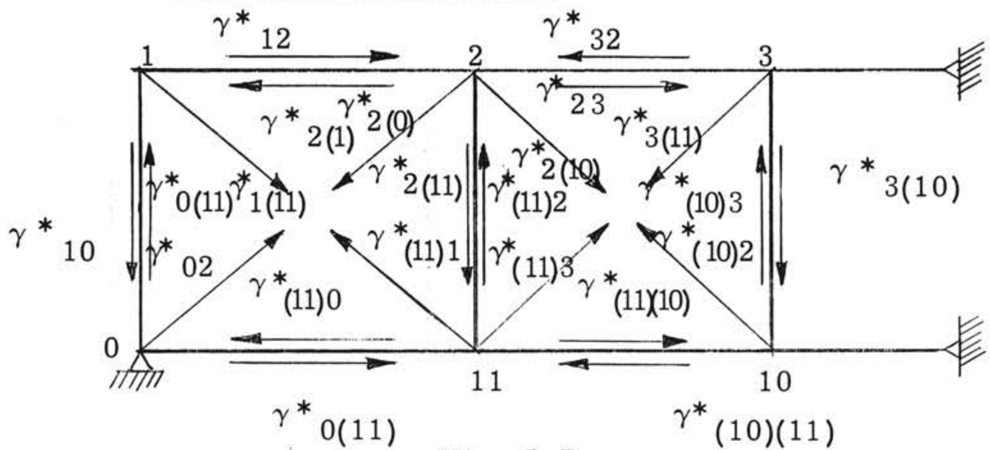


Fig. 5-7

Carry-Over Pattern - Case II

	1	2	3	10	11	0
γ* 's	- .0667 → - .1340 ↓ + .200 ↘	- .050 → + .150 ↘ + .050 ↓ + .150 ↙ - .050 ←	- .037 ← + .111 ↙ - .074 ↓	- .074 ↑ + .111 ↖ - .037 ←	- .050 ← + .050 ↑ + .150 ↗ - .050 → + .150 ↖	- .134 ↑ + .200 ↗ - .066 →
m* 's	1.875	1.250	- .625	-1.144	1.25	2.708
	- .0625 + .1875 - .3628	- .1250 + .0231 - .1269 + .0625 + .5410	- .0625 + .0846 + .1875	+ .1875 + .0462 - .0625	+ .3750 + .0625 - .0693 + .0423 - .1787	- .2512 + .1875 - .0625
	- .2378	+ .3747	+ .2096	+ .1712	+ .2318	- .1262
	- .0187 + .0347 + .0169	+ .0158 - .0077 + .0190 + .0115 - .0252	- .01873 - .01266 + .03477	+ .0562 - .0155 - .0115	- .0475 + .0187 + .0232 - .0063 + .0083	+ .0318 + .0562 - .0115
	+ .0329	+ .0134	+ .0034	+ .0292	- .0036	+ .0765
JM* 's	1.670	+1.638	- .412	- .943	+1.478	2.658

TABLE 5-2

Carry-Over Table - Case II (Example 1)

Check

$$JM^*_1 = 1.670 = ?$$

$$= m^*_1 + \gamma^*_{01} JM^*_0 + \gamma^*_{(11)1} JM^*_{(11)} + \gamma^*_{21} JM^*_2$$

$$= 1.875 + (-.134)(2.658) + (.15)(1.478) + (-.05)(1.638)$$

$$= 1.875 - .356 + .221 - .081$$

$$= 1.65$$

$$JM^*_2 = 1.638 = ?$$

$$= m^*_2 + \gamma^*_{12} JM^*_1 + \gamma^*_{02} JM^*_0 + \gamma^*_{(11)2} JM^*_{(11)}$$

$$+ \gamma^*_{(10)2} JM^*_{10} + \gamma^*_{32} JM^*_3$$

$$= 1.25 + (-.0667)(1.67) + (.2)(2.658) + (.05)(1.478)$$

$$+ (.111)(-.943) + (-.037)(-.412)$$

$$= 1.25 - .1113 + .5316 + .0739 - .104 + .0152$$

$$= 1.65$$

$$JM^*_3 = -.412 = ?$$

$$= m_3 + \gamma^*_{23} JM^*_2 + \gamma^*_{(11)3} JM^*_{(11)} + \gamma^*_{(10)3} JM^*_{(10)}$$

$$= -.625 + (-.05)\ 1.638 + (.15)(1.478) + (-.074)(-.943)$$

$$= -.625 - .080 + .221 + .069$$

$$= -.415$$

$$JM^*_0 = 2.658 = ?$$

$$= m^*_0 + \gamma^*_{10} JM^*_1 + \gamma^*_{20} JM^*_2 + \gamma^*_{(11)0} JM^*_{(11)}$$

$$= 2.708 + (-.134)(1.67) + (.15)(1.638) + (-.05)(1.478)$$

$$= 2.708 - .223 + .245 - .0739$$

$$= 2.657$$

$$JM^*_{(11)} = 1.478 = ?$$

$$= m^*_{(11)} + \gamma^*_{0(11)} JM^*_0 + \gamma^*_{1(11)} JM^*_1 + \gamma^*_{2(11)} JM^*_2$$

$$+ \gamma^*_{3(11)} JM^*_3 + \gamma^*_{(10)(11)} JM^*_{(10)}$$

$= 1.478 + (-.066)(2.658) + .2\,(1.67) + .05\,(1.638)$

$+ (.111)(-.412) + (-.037)(-.943)$

$= 1.25 - .1754 + .334 + .0819 - .0457 + .0348$

$= 1.479$

$JM^*_{(10)} = -.943 = ?$

$= m^*_{(10)} + \gamma^*_{(11)(10)} JM^*_{(11)} + \gamma^*_{2(10)} JM^*_2$

$+ \gamma^*_{3(10)} JM^*_3$

$= -1.144 + (-.05)\,1.478 + (.15)(1.638) + (-.074)(-.412)$

$= -1.144 - .073 + .245 + .0304$

$= -.942$

14.) Final End Moments

$M_{12} = D^*_{12} JM^*_1 + CD^*_{21} JM^*_2 + CD^*_{(11)1} JM^*_{(11)}$

$+ CD^*_{01} JM^*_0 + FM^*_{12}$

$= .557 + .0819 - .295 - .531 - 1.875$

$= -2.06$

$M_{10} = D^*_{10} JM^*_1 + CD_{01} JM^*_0$

$= .667\,(1.67) + .333\,(2.658)$

$= 1.113 + .895$

$= 2.00$

$M_{21} = D^*_{21} JM^*_2 + CD^*_{12} JM^*_1 + CD^*_{02} JM^*_0$

$+ CD^*_{(11)2} JM^*_{(11)} + FM^*_{21}$

$= .25\,(1.638) + (.066)(1.67) + (-.2)(2.658) - (.15)(1.478)$

$+ (-1.875)$

$= .409 + .1102 - .531 - .2217 - 1.875$

$= -2.10$

$$M_{23} = D^*_{23}\ JM^*_2 + CD^*_{32}\ JM^*_3 + CD^*_{(10)2}\ JM^*_{(10)} + CD^*_{(11)2}\ JM^*_{(11)} + FM^*_{23}$$

$$= .25\ (1.638) + (.037)\ (-\ .412) + (-\ .111)\ (-\ .943) - (.15)\ (1.478) + .625$$

$$= .409 - .0152 + .1046 - .2217 + .625$$

$$= +\ .901$$

$$M_{2(11)} = D^*_{2(11)}\ JM^*_2 + CD_{(11)2}\ JM^*_{(11)}$$

$$= .5\ (1.638) + .25\ (1.478)$$

$$= .819 + .3695$$

$$= 1.190$$

$$M_{32} = D^*_{32}\ JM^*_0 + CD^*_{23}\ JM^*_2 + CD^*_{(11)3}\ JM^*_{(11)} + CD^*_{(10)3}\ JM^*_{(10)} + FM^*_{32}$$

$$= (.185)\ (-\ .412) + .05\ (1.638) + (-\ .2)\ 1.478 + (-\ .111)(-.943) + (.625)$$

$$= -\ .0762 + .0819 - .2956 + .1046 + .625$$

$$= +\ .439$$

$$M_{34} = D^*_{34}\ JM^*_3 + CD^*_{(10)3}\ JM^*_{(10)} + FM^*_{34}$$

$$= .444\ (-\ .412) + (-\ .111)\ (-\ .943) + 0$$

$$= -\ .1829 + .104 = -\ .078$$

$$M_{3(10)} = D^*_{3(10)}\ JM^*_3 + CD_{(10)3}\ JM^*_{(10)}$$

$$= .37\ (-\ .412) + (.185)\ (-\ .943)$$

$$= -\ .152 - .1744 = -\ .326$$

$$M_{01} = D^*_{01}\ JM^*_0 + CD_{10}\ JM^*_1$$

$$= .667\ (2.658) + (.333)\ (1.67)$$

$$= 1.772 + .556$$

$$= 2.328$$

$$M_{0(11)} = D^*{}_{0(11)} JM^*{}_0 + CD^*{}_{(11)0} JM^*{}_{(11)} + CD^*{}_{20} JM^*{}_2$$

$$+ CD^*{}_{10} JM^*{}_1 + FM^*{}_{0(11)}$$

$$= (.334)(2.658) + (.05)(1.478) + (-.2)(1.638) + (-.2)(1.67)$$

$$- 2.708$$

$$= .887 + .0739 - .327 - .334 - 2.708$$

$$= -2.39$$

$$M_{(11)0} = D^*{}_{(11)0} JM^*{}_{(11)} + CD^*{}_{0(11)} JM^*{}_0 + CD^*{}_{1(11)} JM^*{}_1$$

$$+ CD^*{}_{2(11)} JM^*{}_2 + FM^*{}_{(11)0}$$

$$= (.25)(1.478) + (.066)(2.658) + (-.2)\,167 - (.15)(1.638)$$

$$- 1.042$$

$$= .3695 + .1754 - .334 - .245 - 1.042$$

$$= -1.076$$

$$M_{(11)(10)} = D^*{}_{(11)(10)} JM^*{}_{(11)} + CD^*{}_{(10)(11)} JM^*{}_{(10)} + CD^*{}_{3(11)} JM^*{}_3$$

$$+ CD^*{}_{2(11)} JM^*{}_2\ FM^*{}_{(11)(10)}$$

$$= (.25)(1.478) + (.037)(-.943) + (-.111)(-.412)$$

$$-(.15)(1.638) + (-.202)$$

$$= .3695 - .0348 + .0457 - .245 - .202$$

$$= -.066$$

$$M_{(11)2} = D^*{}_{(11)2} JM^*{}_{(11)} + CD_{2(11)} JM^*{}_2$$

$$= (.5)(1.478) + (.25)(1.638)$$

$$= .739 + .409$$

$$= 1.148$$

$$M_{(10)(11)} = D^*_{(10)(11)} JM^*{}_{(10)} + CD^*_{(11)(10)} JM^*{}_{(11)} + CD^*{}_{2(10)} JM^*{}_2$$

$$+ CD^*{}_{3(10)} JM^*{}_3 + FM^*{}_{(10)(11)}$$

$$= (.185)(-.943) + (.05)(1.478) + (-.2)(1.638) + (-.111)(-.412) + 1.461$$

$$= -.1744 + .074 - .327 + .046 + 1.461$$

$$= +1.081$$

$$M_{(10)9} = D^*_{(10)9}\, JM^*_{(10)} + CD^*_{3(10)}\, JM^*_3 + FM^*_{(10)9}$$

$$= .444(-.943) + (-.111)(-.412) - (.314)$$

$$= -.418 + .046 - .314$$

$$= -.686$$

$$M_{(10)3} = D^*_{(10)3}\, JM^*_{(10)} + CD_{3(10)}\, JM^*_3$$

$$= (.37)(-.943) + (.185)(-.412)$$

$$= -.348 - .076 = -.424$$

<u>Final End Moments for Case II</u>

$M_{12} = M_{65} = -2.06$

$M_{10} = M_{67} = 2.00$

$M_{21} = M_{56} = -2.10$

$M_{23} = M_{54} = .901$

$M_{2(11)} = M_{58} = 1.19$

$M_{32} = M_{45} = .439$

$M_{34} = M_{43} = -.078$

$M_{3(10)} = M_{49} = -.326$

$M_{01} = M_{76} = 2.328$

$M_{0(11)} = M_{78} = -2.39$

$M_{(11)0} = M_{87} = -1.076$

$M_{(11)(10)} = M_{89} = -.066$

$M_{(11)2} = M_{85} = 1.148$

$M_{(10)(11)} = M_{98} = 1.081$

$M_{(10)9} = M_{9(10)} = -.686$

$M_{(10)3} = M_{94} = -.424$

The superposition of Cases I, II yields the final end moments for the original loading (Table 5-3).

Final End Moments (Case I and Case II)

	Case I	Case II	Final Moments		Case I	Case II	Final Moments
M_{10}	+5.41	+2.00	+7.410	M_{78}	-5.753	-2.39	-8.143
M_{12}	-5.407	-2.06	-7.464	M_{76}	5.734	2.328	8.062
M_{21}	-4.93	-2.10	-7.03	M_{87}	+3.91	-1.076	+2.834
$M_{2(11)}$	6.837	1.19	+8.027	M_{85}	-6.78	1.148	-5.632
M_{23}	-1.885	.901	- .984	M_{89}	+2.78	- .066	+2.714
M_{32}	-3.07	.439	-2.63	M_{98}	+2.26	1.081	+3.341
$M_{3(10)}$	2.428	- .326	+2.102	M_{94}	-2.429	- .424	-2.853
M_{34}	.647	- .078	+0.569	$M_{9(10)}$	+ .182	- .686	- .504
M_{43}	- .647	- .078	- .725	$M_{(10)9}$	- .182	- .686	- .868
M_{49}	-2.428	- .326	-2.754	$M_{(10)3}$	2.429	- .424	2.005
M_{45}	+3.07	+ .439	+3.509	$M_{(10)(11)}$	-2.26	+1.081	-1.179
M_{54}	+1.885	+ .901	+2.794	$M_{(11)(10)}$	-2.78	- .066	-2.846
M_{58}	-6.837	+1.19	-5.647	$M_{(11)2}$	6.78	1.148	7.928
M_{56}	+4.93	-2.10	+2.820	$M_{(11)0}$	-3.91	-1.076	-4.986
M_{65}	+5.407	-2.06	+3.347	$M_{0(11)}$	5.753	-2.39	3.363
M_{67}	-5.41	+2.00	-3.410	M_{01}	-5.734	2.328	-3.406

TABLE 5-3

Superposition of Cases I, II

EXAMPLE NO. 2

A Vierendeel Truss with concentrated loads acting at its joints is shown (Fig. 5-8). The truss has four panels with parallel top and bottom chords. EI is constant.

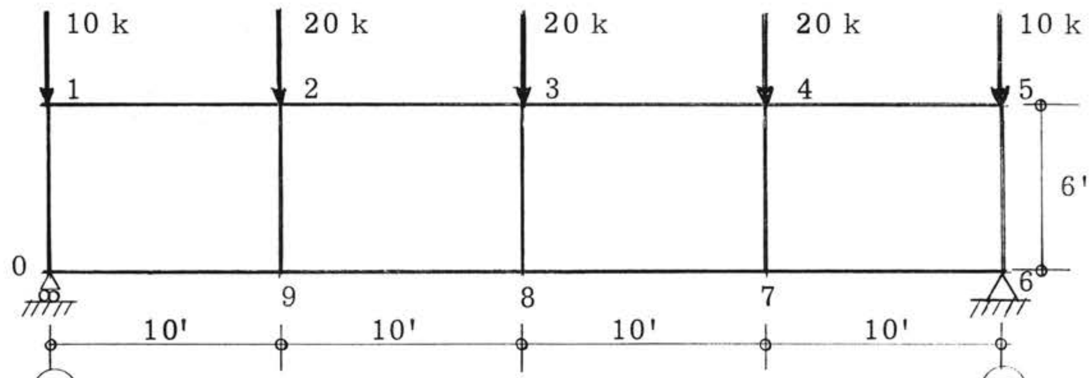


Fig. 5-8

A Four Panel Vierendeel Truss

Since the structure is symmetrical and symmetrically loaded, only half the structure will be considered as was done in Case I of Example No. 1.

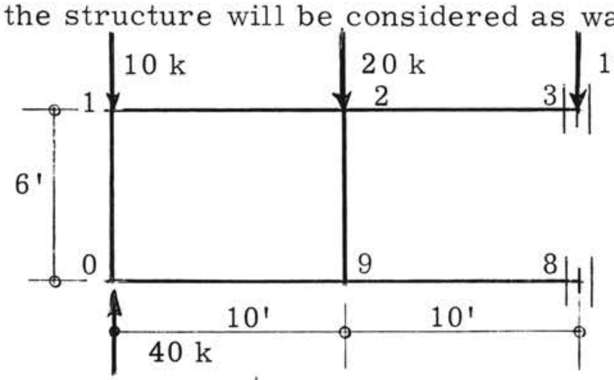


Fig. 5-9

Basic Structure

1.) Deformations

$$\theta_1 = \theta_0 = -\theta_5 = -\theta_6$$

$$\theta_2 = \theta_9 = -\theta_4 = -\theta_7$$

$$\theta_3 = \theta_8 = 0$$

$$\Delta_1 = \Delta_0 = \Delta_5 = \Delta_6 = 0$$

$$\Delta_2 = \Delta_9 = \Delta_4 = \Delta_7$$

$$\Delta_3 = \Delta_8$$

2.) Stiffness Factors

Since EI = constant

$$K_{01} = K_{10} = K_{29} = \frac{4\ EI}{6} = .666\ EI$$

$$K_{12} = K_{21} = K_{23} = K_{32} = K_{09} = K_{90} = K_{98} = K_{89} = \frac{4\ EI}{10}$$

$$= .4\ EI$$

3.) Carry-Over Factors

All carry-over factors = + .5

4.) Fixed End Moments

All FM's due to loads = 0

5.) Displacement Distribution Factors

$$D'_{12} = \frac{S_{12}}{S_{12} + S_{21} + S_{09} + S_{90}} = .25 \qquad \text{Since } S = \frac{6\ EI}{L}$$

Similarly,

$$D'_{21} = D'_{23} = D'_{32} = D'_{09} = D'_{90} = D'_{98} = D'_{89} = .25$$

6.) Modified Stiffness Factors

$$\Sigma K^*_1 = \Sigma K_1 - D'_{12}\, K'''_{12} = ((.666 + .4) - .25\,(.6))\ EI$$

$$= .916\ EI$$

$$\Sigma K^*_0 = \Sigma K_0 - D'_{09}\, K'''_{09} = ((.666 + .4) - .25\,(.6))\ EI$$

$$= .916\ EI$$

$$\Sigma K^*_2 = \Sigma K_2 - D'_{21}\, K'''_{21} - D'_{23}\, K'''_{23} = ((.666 + .4 + .4)$$

$$- .15 - .15)\ EI$$

$$= 1.166\ EI$$

$$\Sigma K^*_9 = \Sigma K_9 - D'_{90}\, K'''_{90} - D'_{98}\, K'''_{98} = ((.666 + .4 + .4)$$

$$- .15 - .15)\ EI$$

$$= 1.166\ EI$$

7.) Modified Carry-Over Stiffness Factors

$$CK^*_{12} = CK_{12} - D'_{21} K'''_{12} = (.5(.4) - .25(.6))\, EI$$
$$= .05\, EI$$

$$CK^*_{21} = CK_{21} - D'_{12} K'''_{21} = (.5(.4) - .25(.6))\, EI$$
$$= .05\, EI$$

Similarly,

$$CK^*_{23} = CK^*_{09} = CK^*_{90} = CK^*_{98} = .05\, EI$$

$$CK^*_{10} = CK_{10} - D'_{90} K'''_{12} = (.5(.666) - .25(.600))\, EI$$
$$= .183\, EI$$

$$CK^*_{01} = CK_{01} - D'_{12} K'''_{01} = (.5(.666) - .25(.600))\, EI$$
$$= .183\, EI$$

$$CK^*_{29} = CK_{29} - D'_{90} K'''_{21} - D'_{98} K'''_{23} = (.333 - .30)\, EI$$
$$= +.033\, EI = CK^*_{92}$$

$$CK^*_{19} = - D'_{90} K'''_{12} = -.25(.6)EI = -.15\, EI$$

$$CK^*_{91} = - D'_{12} K'''_{90} = -.25(.6)EI = -.15\, EI$$

Similarly,

$$CK^*_{02} = CK^*_{20} = CK^*_{28} = CK^*_{93} = -.15\, EI$$

8.) Joint Moment Carry-Over Factors

$$\gamma^*_{12} = -\frac{CK^*_{12}}{\Sigma K^*_1} = -\frac{.05\, EI}{.916\, EI} = -.0545$$

$$\gamma^*_{19} = -\frac{CK^*_{19}}{\Sigma K^*_1} = -\frac{(-.15)\, EI}{.916\, EI} = +.164$$

$$\gamma^*_{10} = -\frac{CK^*_{10}}{\Sigma K^*_1} = -\frac{.183\, EI}{.916\, EI} = -.200$$

$$\gamma^*_{01} = -\frac{CK^*_{01}}{\Sigma K^*_0} = -\frac{.183\ EI}{.916\ EI} = -.200$$

$$\gamma^*_{02} = -\frac{CK^*_{02}}{\Sigma K^*_0} = -\frac{(-.15)\ EI}{.916\ EI} = +.164$$

$$\gamma^*_{09} = -\frac{CK^*_{09}}{\Sigma K^*_0} = -\frac{(.05)\ EI}{.916EI} = = -.0545$$

$$\gamma^*_{21} = -\frac{CK^*_{21}}{\Sigma K^*_2} = -\frac{.05\ EI}{1.166\ EI} = -.043$$

$$\gamma^*_{20} = -\frac{CK^*_{20}}{\Sigma K^*_2} = -\frac{(-.15\ EI)}{1.166EI} = +.128$$

$$\gamma^*_{29} = -\frac{CK_{29}}{\Sigma K_2} = -\frac{.033\ EI}{1.166\ EI} = -.028$$

$$\gamma^*_{23} = -\frac{CK^*_{23}}{\Sigma K^*_2} = -\frac{.05\ EI}{1.166EI} = -.043$$

$$\gamma^*_{28} = -\frac{CK^*_{28}}{\Sigma K^*_2} = -\frac{(-.15\ EI)}{1.166EI} = +.128$$

Similarly,

$$\gamma^*_{90} = -.043,\ \gamma^*_{91} = +.128,\ \gamma^*_{92} = -.028$$

$$\gamma^*_{93} = +.128,\ \gamma^*_{98} = -.043$$

9.) New Distribution Factors

$$D^*_{12} = \frac{K_{12} - D'_{12}\ K'''_{12}}{\Sigma K^*_1} = \frac{(.4 - .25\ (.6))\ EI}{.916\ EI}$$

$$= \frac{.25}{.916} = .273$$

$$D^*_{10} = \frac{K_{10}}{\Sigma K^*_1} = \frac{.666\ EI}{.916\ EI} = .726$$

$$D^*_{01} = \frac{K_{01}}{\Sigma K^*_0} = \frac{.666\ EI}{.916\ EI} = .726$$

$$D^*_{09} = \frac{K_{09} - D'_{09}\, K'''_{09}}{\Sigma K^*_0} = \frac{(.4 - .25\,(.6))\ EI}{.916\ EI}$$

$$= .273$$

$$D^*_{21} = \frac{K_{21} - D'_{21}\, K'''_{21}}{\Sigma K^*_2} = \frac{.25}{1.166} = .215$$

$$D^*_{23} = \frac{K_{23} - D'_{23}\, K'''_{23}}{\Sigma K^*_2} = \frac{.25}{1.166} = .215$$

$$D^*_{29} = \frac{K_{29}}{\Sigma K^*_2} = \frac{.666}{1.166} = .572$$

$$D^*_{92} = \frac{K_{92}}{\Sigma K^*_9} = \frac{.666}{1.166} = .572$$

$$D^*_{90} = \frac{K_{90} - D'_{90}\, K'''_{90}}{\Sigma K^*_9} = \frac{.25}{1.166} = .215$$

$$D^*_{98} = \frac{K_{98} - D'_{98}\, K'''_{98}}{\Sigma K^*_9} = \frac{.25}{1.166} = .215$$

10.) <u>New Carry-Over Distribution Factors</u>

$$CD^*_{12} = \frac{CK_{12} - D'_{21}\, K'''_{12}}{\Sigma K^*_1} = \frac{(.5\,(.4) - .25\,(.6))EI}{.916\ EI}$$

$$= \frac{.05}{.916} = .0545$$

$$CD^*_{10} = -\frac{D'_{09}\, K'''_{12}}{\Sigma K^*_1} = -\frac{.25\,(.600)\ EI}{.916\ EI}$$

$$= -.164$$

$$CD_{10} = +\frac{CK_{10}}{\Sigma K^*_1} = +\frac{(.5)\,(.666)\ EI}{.916\ EI} = .363$$

$$CD^*_{19} = -\frac{D'_{90}\,K'''_{12}}{\Sigma K^*_1} = -\frac{.25\,(.6)}{.916} = -.164$$

Similarly,

$$CD^*_{01} = -.164, \quad CD_{01} = .363$$

$$CD^*_{02} = -.164, \quad CD^*_{09} = .0545$$

$$CD^*_{21} = \frac{CK_{21} - D'_{12}\,K'''_{21}}{\Sigma K^*_2} = \frac{.05\ EI}{1.166\ EI} = .043$$

$$CD^*_{20} = -\frac{D'_{09}\,K'''_{21}}{\Sigma K^*_2} = -\frac{.25\,(.6)\ EI}{1.166\ EI} = -.13$$

$$CD^*_{29} = -\frac{D'_{90}\,K'''_{21}}{\Sigma K^*_2} = -\frac{D'_{98}\,K'''_{23}}{\Sigma K^*_2} = -.13$$

$$CD_{29} = \frac{CK_{29}}{\Sigma K^*_2} = +\frac{.5\,(.666)}{1.166} = +.286$$

Similarly,

$$CD^*_{23} = .043$$

$$CD^*_{28} = -.13$$

$$CD^*_{92} = -.13$$

$$CD_{92} = +.286$$

$$CD^*_{93} = -.13$$

$$CD^*_{98} = .043$$

$$CD^*_{91} = -.130$$

$$CD^*_{90} = .043$$

11.) Starting Moments

$$m^*_1 = -\Sigma SM^*_1$$

$$= +D'_{12}\left[\cancel{FM_{12}} + \cancel{FM_{21}} + \cancel{FM_{09}} + \cancel{FM_{90}}\right.$$

$$\left. + L\,(\cancel{BV_{21}} + \cancel{BV_{90}} + V_2)\right] + \cancel{FM_{12}}$$

$$= D'_{12}\,(L)\,V_2 = .25\ (10)30$$

$$= 75$$

$$m^*_0 = D'_{09}\,LV_2 = 75$$

$$m^*_2 = +D'_{21}\left[\cancel{FM_{21}} + \cancel{FM_{12}} + \cancel{FM_{90}} + \cancel{FM_{09}}\right.$$

$$\left. + L\,(\cancel{BV_{21}} + \cancel{BV_{90}} + V_2)\right] + D'_{23}\left[\cancel{FM_{23}} + \cancel{FM_{32}} + \cancel{FM_{98}}\right.$$

$$\left. + \cancel{FM_{89}} + L\,(\cancel{BV_{32}} + \cancel{BV_{98}} + V_3)\right]$$

$$+ \cancel{FM_{21}} + \cancel{FM_{23}}$$

$$= D'_{21}\,(L)\,V_2 + D'_{23}\,LV_3$$

$$= .25\ (10)\ 30 + .25\ (10)(10) = 75 + 25$$

$$= 100$$

Similarly,

$$m^*_9 = 100$$

12.) Modified Fixed End Moments

$$FM^*_{12} = -D'_{12}\left[\cancel{FM_{12}} + \cancel{FM_{21}} + \cancel{FM_{09}} + FM_{90}\right.$$

$$\left. + L\,(\cancel{BV_{21}} + \cancel{BV_{90}} + V_2)\right] + \cancel{FM_{12}}$$

$$= -D'_{12}\,LV_2 = -.25\ (10)\ (30) = -75$$

Similarly,

$$FM^*_{09} = -75$$

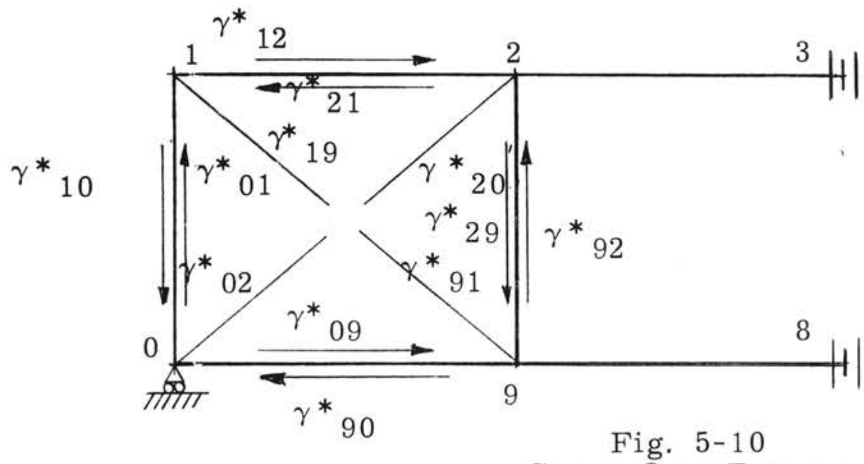
$$FM^*_{21} = -75 = FM^*_{90}$$

$$FM^*_{23} = -D'_{23}\,LV_3 = -.25\,(10)\,(10) = -25$$

$$FM^*_{32} = -25 = FM^*_{98} = FM^*_{89}$$

13. Carry-Over Procedure

The carry-over procedure is performed considering joints 1, 2, 9, 0 in that order. The remaining procedure is the same as shown for Case I, Example 1. In this Example, since JM^*'s are zero at joints 3 and 8, there is no carry-over to or from joints 3 and 8.



Since JM's are zero at 3 and 8, there is no carry-over to or from 3 and 8.

Fig. 5-10
Carry-Over Pattern

	1	2	9	0
γ*'s	- .0545 → + .164 ↘ - .200 ↓	- .043 ← + .128 ↙ - .028 ↓	- .043 ← + .128 ↖ - .028 ↑	- .200 ↑ + .164 ↗ - .0545 →
m*'s	75	100	100	75
	-4.300 +12.800 -15.000	-4.100 -2.800 +12.300	+12.300 -2.800 -4.100	+15.00 +12.80 -4.30
m*'s	-6.500	+5.400	+5.400	-6.500
	- .233 + .692 +1.300	+ .354 - .151 -1.065	-1.065 + .151 + .354	+1.300 + .692 - .233
m*'s	+1.759	- .862	- .862	+1.759
	+ .037 - .110 - .358	- .0976 + .0240 + .2940	+ .294 + .024 - .097	- .358 - .110 + .037
m*'s	- .431	+ .221	+ .221	- .431
	- .0095 + .0283 + .0862	+ .0236 - .0062 - .0710	- .0710 - .0062 + .0236	+ .0862 + .0283 - .0095
	+ .105	- .0536	- .0536	+ .105
JM*'s	69.933	104.706	104.706	69.933

TABLE 5-4

Carry-Over Table (Example 2)

14.) Check of The Carry-Over Procedure

$$JM^*_1 = 69.933 = ?$$

$$= m^* + \gamma^*_{21} JM_2 + \gamma^*_{91} JM^*_9 + \gamma^*_{01} JM^*_0$$

$$= 75 - .043 (104.706) + .128 (104.7) - .200 (69.933)$$

$$= 75 - 4.5 + 13.41 - 13.98 = 69.93$$

$$JM^*_2 = 104.706 = ? = m^*_2 + \gamma^*_{12} JM^*_1 + \gamma^*_{02} JM^*_0$$

$$+ \gamma^*_{92} JM^*_9$$

$$= 100 - .0545 (69.933) + .164 (69.933) - .028 (104.706)$$

$$= 100 - 3.81 + 11.44 - 2.93 = 104.70$$

$$JM^*_9 = m^*_9 + \gamma^*_{09} JM^*_0 + \gamma^*_{19} JM^*_1 + \gamma^*_{29} JM^*_2$$

$$= 100 - (.0545) 69.933 + .164 (69.933) - .028 (104.706)$$

$$= 104.70$$

$$JM^*_0 = 69.933 = ?$$

$$= m^*_0 + \gamma^*_{10} JM^*_1 + \gamma^*_{20} JM^*_2 + \gamma^*_{90} JM^*_9$$

$$= 75 - .200 (69.933) + .128 (104.706) + (-.043) (104.7)$$

$$= 69.93$$

15.) Final End Moments

$$M_{12} = D^*_{12} JM^*_1 + CD^*_{21} JM^*_2 + CD^*_{91} JM^*_9 + CD^*_{01} JM^*_0$$

$$+ FM^*_{12}$$

$$= .273 (69.933) + .043 (104.706) - .13 (104.706) - .164 (69.933)$$

$$- 75$$

$$= 19.1 + 4.51 - 13.6 - 11.46 - 75$$

$$= - 76.45$$

$$M_{10} = D^*_{10} JM^*_1 + CD_{01} JM^*_0$$

$$= .727 (69.933) + .363 (69.933)$$

$$= 50.90 + 25.45$$

$$= + 76.35$$

$$M_{21} = D^*_{21}\, JM^*_2 + CD^*_{12}\, JM^*_1 + CD^*_{02}\, JM^*_0 + CD^*_{92}\, JM^*_9 + FM^*_{21}$$

$$= .215\,(104.706) + .0545\,(69.933) - .164\,(69.933) - (.13)\,(104.706) - 75$$

$$= 22.5 + 3.81 - 11.46 - 13.6 - 75$$

$$= - 73.75$$

$$M_{29} = D^*_{29}\, JM^*_2 + CD_{92}\, JM^*_9$$

$$= .572\,(104.706) + .286\,(104.706)$$

$$= 60 + 30 = 90$$

$$M_{23} = D^*_{23}\, JM^*_2 + CD^*_{92}\, JM^*_9 + FM^*_{23}$$

$$= .215\,(104.7) - .13\,(104.70) - 25$$

$$= 22.5 - 13.6 - 25 = - 16.5$$

$$M_{32} = D^*_{32}\, \overset{0}{JM^*_3} + CD^*_{23}\, JM^*_2 + CD^*_{93}\, JM^*_9 + CD^*_{83} JM^*_8 + FM^*_{32}$$

$$= .043\,(104.706) - .13\,(104.706) - 25$$

$$= 4.5 - 13.6 - 25 = - 34.1$$

$$M_{38} = D^*_{38}\, \overset{0}{JM^*_3} + CD_{83}\, \overset{0}{JM^*_8} = 0$$

$$M_{83} = D^*_{83}\, \overset{0}{JM^*_8} + CD^*_{38}\, \overset{0}{JM^*_3} = 0$$

$$M_{98} = D^*_{89}\, \overset{0}{JM^*_8} + CD^*_{98}\, JM^*_9 + CD^*_{28}\, JM^*_2 + CD^*_{38}\, \overset{0}{JM^*_3} + FM^*_{89}$$

$= .215\,(104.76) - .13\,(104.706)$

$= 22.5 - 13.6 - 25 = -16.1$

$M_{92} = D^*_{92}\,JM^*_9 + CD_{29}\,JM^*_2$

$= .572\,(104.706) + (.286)\,(104.706)$

$= 60 + 30 = 90$

$M_{90} = D^*_{90}\,JM^*_9 + CD^*_{09}\,JM^*_0 + CD^*_{19}\,JM^*_1 + CD^*_{29}JM^*_2$

$+ FM^*_{90}$

$= .215\,(104.706) + .0545\,(69.933) + (-.164)\,(69.933)$

$+ (-.13)\,(104.706) - 75$

$= 22.5 + 3.81 - 11.46 - 13.6 - 75$

$= -73.68$

$M_{01} = D^*_{01}\,JM^*_0 + CD_{10}\,JM^*_1$

$= .726\,(69.933) + (.363)\,69.933$

$= 50.90 + 25.45$

$= +76.35$

$M_{09} = D^*_{09}\,JM^*_0 + CD^*_{90}\,JM^*_9 + CD^*_{20}\,JM^*_2$

$+ CD^*_{10}\,JM^*_1 + FM^*_{09}$

$= .273\,(69.933) + .043\,(104.706) + (-.13)\,(104.706)$

$- .164\,(69.933) - 75$

$= 19.1 + 4.5 - 13.60 - 11.46 - 75$

$= -76.32$

Hence,

$$M_{10} = -M_{56} = 76.20$$

$$M_{12} = -M_{54} = -76.32$$

$$M_{21} = -M_{45} = -73.68$$

$$M_{23} = -M_{43} = -16.5$$

$$M_{32} = -M_{34} = -34.1$$

$$M_{29} = -M_{47} = 90$$

$$M_{38} = 0$$

$$M_{09} = -M_{67} = -76.32$$

$$M_{90} = -M_{76} = -73.68$$

$$M_{98} = -M_{78} = -16.1$$

$$M_{89} = -M_{87} = -34.1$$

$$M_{01} = -M_{65} = +76.35$$

$$M_{92} = -M_{74} = +90$$

$$M_{83} = 0$$

CHAPTER 6

SUMMARY AND CONCLUSIONS

The illustrative examples show the application of the carry-over joint moment method to the analysis of Vierendeel Trusses. Joint moments are obtained at the finish of the carry-over procedure. These joint moments must then be distributed to the members at that joint in proper proportions to obtain final end moments. Since the displacements were eliminated in the derivation of the carry-over joint moment method, only one carry-over procedure is necessary. Some additional constants must be evaluated for this method which are unnecessary for the method of slope deflection or moment distribution. These new constants are simply modifications of the original slope deflection constants.

The method of slope deflection is impractical to use in the general analysis of a Vierendeel Truss because of the quantity of unknown deformations. The slope deflection method might be desirable if an electronic computer is readily available to solve the many simultaneous equations.

The primary disadvantages of the moment distribution method are:

1.) Separate distribution for each independent displacement

2.) Simultaneous solution of a number of shear equations equal to the number of independent displacements.

The carry-over joint moment method of analysis is not convenient for trusses with few panels. As the number of panels becomes greater, the advantages of the carry-over joint moment method becomes more apparent.

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VITA

Samuel Theophilus

Candidate for the Degree of

Master of Science

Thesis: ANALYSIS OF VIERENDEEL TRUSSES WITH PARALLEL CHORDS BY CARRY-OVER JOINT MOMENTS

Major Field: Civil Engineering

Biographical:

Personal Data: Born in Bangalore, India, on October 22, 1927, the son of Henry and Manonmani Theophilus.

Education: Graduated from St. Aloysious High School, Bangalore, India, in May 1944. Entered the intermediate course in Mysore University in 1944 and passed the intermediate examination in May 1946. Joined the Government College of Engineering, Bangalore, in 1946 and completed the requirements for the Degree of Bachelor of Engineering in Civil Engineering in June 1950. Entered the Oklahoma State University in September 1959, and completed the requirements for the Degree of Master of Science in Civil Engineering with major in Structures in May, 1961.

Professional experience: Joined the Mysore State Public Works Department, India, in July 1950 and served as engineer in buildings, bridges, highways and irrigation works until June 1956. In July 1956, was appointed as instructor in Civil Engineering in the C. P. C. Polytechnic Institute, Mysore, India, and served in this capacity until July 1959. Employed as Graduate Assistant in the Oklahoma State University from September 1959 to May 1960.