INVENTORY CONTROL IN A CONCRETE BATCH OPERATION

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PREFACE

Management of inventories is recognized today as one of the key responsibilities in achieving continuous and economical plant operation. Inventory dollars are no longer regarded as a drain on working capital; they are a factor to be used and administered with skill and intelligence.

The purpose of this thesis is to outline methods for establishing an inventory control program. It is my hope that this thesis will make managers cognizant of the importance of control and the advantages which may be obtained from installing an inventory control program.

I am deeply indebted to the Ideal Cement Company for granting me the research fellowship which has made this study possible.

I am grateful to Professor Wilson J. Bentley, Dr. Paul E. Torgersen and Professor Scott T. Poage for their invaluable guidance and assistance in the preparation of this thesis.

With regard to the actual investigation, the writer wishes to acknowledge the helpful information made available by the following officials of the Dolese Company: Mr. Shelton, Vice President; Mr. Keith Harris, Controller; and Mr. Valdie M. Carr, Sales Manager.

I take this means of expressing my gratitude to Aurelio S. Lagman, Jr. for his cooperation in making this study. I appreciate having had the opportunity to work with such a capable and energetic person.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Management has many functions in an organization. Its primary duty is to maintain the organization so that a profit can be made from its operation. In order to maximize profits, entrepreneurs have been applying more scientific means of running an organization. The use of mathematics, especially statistics, has been recognized as a basic tool in aiding the entrepreneur to make decisions. In the past, decisions concerning the running of an organization were often made on an intuitive or a guess method. Many organizations were highly successful using this approach, but, as times changed, the entrepreneurs were often faced with more complex decisions brought on by technological advances as well as rising costs and increased competition. It became necessary to plan operations in greater detail and emphasis was placed on cost cutting in an effort to produce products that could be priced within reach of the consumer and also meet competition.

To be able to predict and adopt policies for running an organization is not enough; management must apply controls so that the policies will be carried out and the organization can accomplish its objective of profit and providing a customer service. Standard cost accounting systems, inventory control, production planning and scheduling are a few of the methods being used by managers as means for regulating and controlling the organization.

The idea of "Management Control", although it is not a new concept, has been gaining wider acceptance. Smaller businessmen have realized that in order to maximize the profits in an organization, it is necessary to reduce expenses. The effectiveness of any type of control program initiated by management must be under constant supervision and review. No matter what policies are adopted, they will not be effective unless they are carried out.

Statement of the Problem

Inventory control refers to the process whereby the investment in materials and parts carried in stock is regulated within predetermined limits set in accordance with the inventory policies established by management.[1] It is the objective of this thesis to present to the management of a concrete batch plant operation a method for establishing a means of controlling stone aggregate inventories and encourage the installation of a sound inventory program.

The first area discussed in the context of this thesis is forecasting; without it a sound inventory program could

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[1]Carson, Gordon B. Production Handbook (New York, 1958), p. 4.1.

not be initiated. A company must know what to plan for and what revenue it can expect. From forecasting, are discussed the actual levels of inventories to be carried and a reorder rule is introduced as a means of determining inventory levels. Finally, techniques of management control are discussed and methods explained for the actual control of inventories. The application of quality control charts to inventory is shown as a simple means of providing control.

The techniques discussed in this thesis are similar to the ones recommended to the Dolese Company of Oklahoma City, Oklahoma for establishing an inventory control program. Some of the data used in this thesis was furnished by the Dolese Company and figures have been changed to protect confidential sales information.

CHAPTER II

BUSINESS FORECASTING

Business forecasting is a prerequisite to a sound inventory control program. This forecasting may be accomplished either formally, i.e., with mathematical models, or informally, i.e. using intuitive judgments. The forecast provides the manager with an estimate of the future demand for his product or service and then permits the manager to provide for that time when the anticipated demand becomes a reality.

The demand forecast is a link between evaluation of external factors in the economy which influence the business and the management of the company's internal affairs.[2] Actually, no forecast of demand is a guarantee of certainty. It should be kept in mind that a demand figure is only an estimate and would be interpreted in the following manner: "If conditions continue as they have in the past and in view of the current situation, we can expect, e. g., to have a 5000 cubic yard demand next month."

Although a demand cannot be predicted exactly, fore-

[2]Magee, John F. Production Planning and Inventory Control (New York, 1958), p. 105.

casting can be a very useful tool. Demand estimates can be reasonably close to actual demands; these estimated demands can be helpful in several areas. To mention a few of these areas:

1. Forecasts aid in controlling inventory levels - a knowledge of what demand to expect even though it may not be exact allows a company to be able to set optimum inventory levels which are required to give satisfactory customer service in an uncertain future. These optimum levels are established against the criteria of minimizing the expected cost in a probabilistic sense of shortages and overages.
2. Forecasts aid in sales analysis - useful information can be obtained as to what specific types of a product to carry or in what areas to concentrate sales effort. In the concrete industry, this could include city growth and contracts to be awarded.
3. Forecasting aids in determining the need for possible future expansions of facilities - in some cases it may be necessary to expand plant capacity or build new plants in order to meet increased customer demands. Such decisions have to be made in advance of the actual demand.
4. Forecasting can aid a company in determining its financial policies. What revenue can a

firm expect? During winter months, sales may be very low and policies may have to be adopted to carry the company through this period of time.

5. Forecasting aids in determining personnel policies - the scheduling or addition of employees is sometimes troublesome, but with a reasonable estimate of the demand, it is possible to plan ahead. Stabilizing the work force is a good means of improving employee morale and increasing organizational loyalty.

Thus, it can be seen that forecasting can aid many departments in the company. No organization can exist for any length of time without an estimate of what demand it must meet in order to operate successfully for many profitable years.

There are several principle steps in making a forecast. First, the company must have records available of past sales so that trends may be studied. Secondly, a company should estimate its share of the market and should have an estimate of the potential of the total market. This percentage can be applied to estimated sales in the company's locality and a demand figure can be arrived at very easily. The percentage of market sales should be considered as only one of the factors in setting demand requirements. Thirdly, and probably one of the most useful factors, is to correlate the company's sales to some external or national statistic. These indicators are very useful in the prediction of trends. Finally, the demand figures obtained from a forecast should be interpreted in light of the current business conditions. The forecast must be a reasonable one and not divorced from reality.

The techniques discussed in the remaining part of this chapter are by no means the only methods used in forecasting. The purpose is to familiarize the manager with some methods which are easy to apply and will aid in developing some system of forecasting.

Techniques of Forecasting

The objective of a business forecast is to aid the plant manager in estimating a reasonable demand for a given future time period. The important mathematical tool used in analyzing trend data is statistics. Along with statistical analyses, three questions should be asked when analyzing time series. First, what factors caused business to develop as it did in the past? Second, what are the important factors now? Third, how are the present factors related to the future? By the use of statistical methods, it is possible to isolate several factors which can aid us in planning for the future.

In order to study a time series, it is first necessary to graph the sales data available as shown in Figure 1. The reliability of the forecast is increased if there are from ten to twenty years data available, although, in

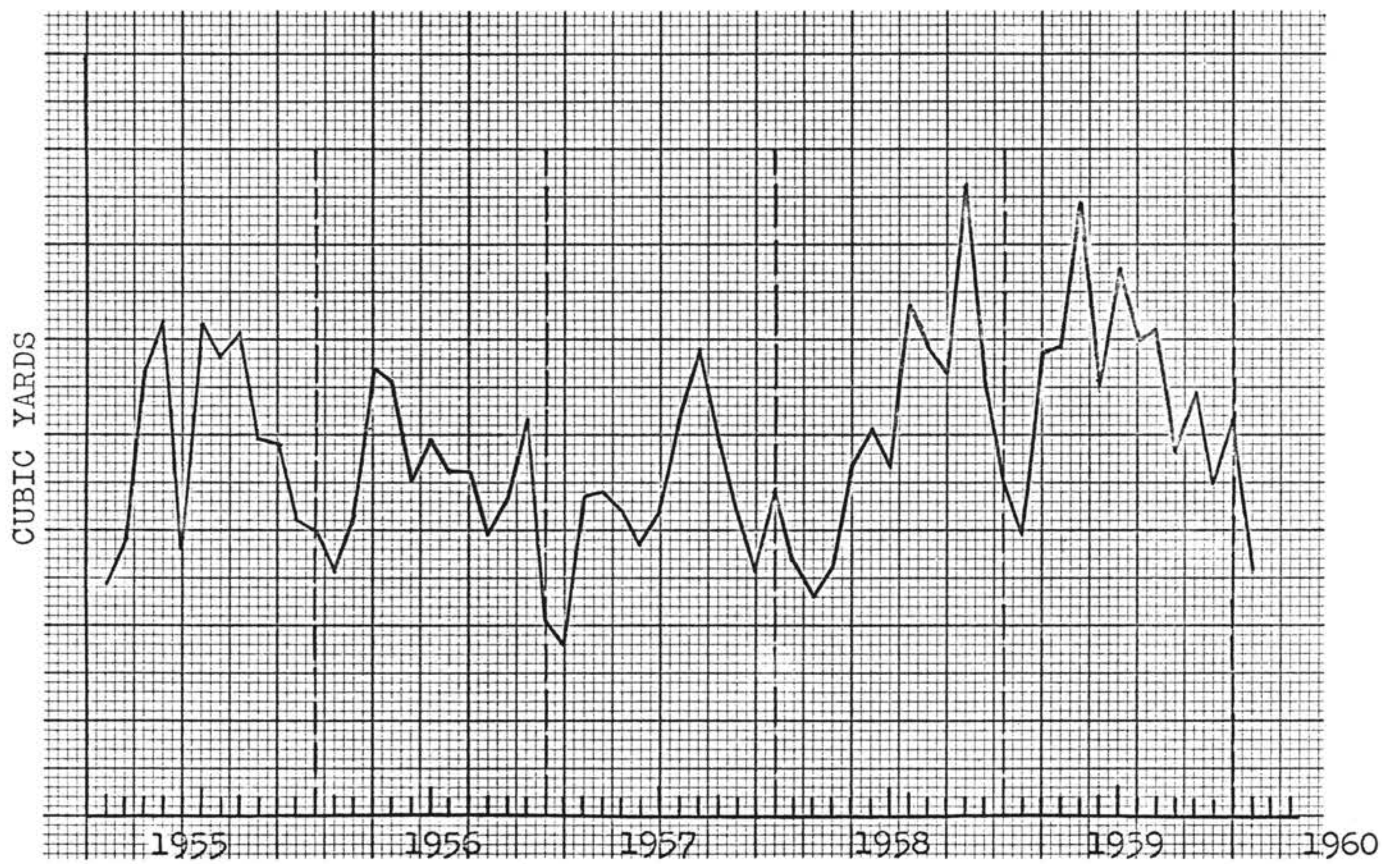

Figure 1. Monthly Concrete Sales

most cases, companies do not keep more than five years data.

The time series shows that three well defined movements may be distinguished in the data. The fourth, which is erratic or random variation, is more difficult to detect. The movements, or factors distinguishable, are:

1. Basic trend
2. Seasonal variation
3. Business cycle

These factors are important to our analysis because they aid us to explain the past and play an important role in predicting the future.

The Basic Trend

Even though the data displayed in Figure 1 fluctuates very badly, a closer examination would show a definite increasing trend which represents company growth. It must be remembered when attempting to fit a trend that the data could represent a random process, i. e., a definite movement up or down with no systematic factors operating on the data. From the statistical viewpoint, random data is from the same universe or population while non-random data is assumed to come from different populations. If the above is true, then we have two alternatives:

1. A trend should be fitted if the series shows the presence of systematic factors.
2. No trend should be fitted if the time se-

ries is a random process.

Tests made on the sales data included the "Multiple Run Test", the "Non-Parametric Test" and "Run Test". It was found that the sales data was from a non-random process and the factors present were those mentioned above. When systematic factors are present in the data, it is possible to isolate a trend.

Seasonal Variation

The data also reveals fluctuations due to seasonal effects. Winter and Spring months usually show the effects of weather conditions on concrete pours. Knowledge of the seasonal pattern can be very informative and helpful when considering the demands on the plant facilities. Summer and Fall months' usage rates are much higher relative to the other seasons of the year.

Business Cycles

Time series usually show tendencies for business to move in never ending sequences. For example, business cycles have a tendency to prosper, decline, stagnate, recover and then prosper again. The business cycle is distinguishable from seasonal variation in that it does not occur at regular time intervals during any year.

Erratic (Random) Movements

Probably, one of the hardest factors to isolate is the erratic movements. These factors are usually the result of such unpredictable occurrences as droughts, floods, wars, etc. In some cases, they are not distinguishable from business cycles.

Once these factors are isolated, it is possible to eliminate them from the data.

The method used in this analysis is the removal of trend and seasonal variation. Working with unadjusted, or raw data, is very difficult if not impossible in this case.

Elimination of Trend

The method used in this thesis for time series analysis is the linear approximation. Higher degree curves require many hours of calculation and the evaluation of five, fourth order determinates for the third degree case. The use of the polynomial equations of higher degree than two or three is not recommended because of the difficulty; and time required is out of proportion to the expected return. The end result of extrapolating the trend equation is still only an estimate of the demand. For only five years data, these higher degree polynomials would not be as accurate as if ten to twenty years of sales data had been used.

The linear trend approximation shown in Figure 2 is not only a measure of company growth, but it can also be used to forecast demands. This linear function is found by the method of least squares and its common notation is:

$$\hat{Y} = ax + b$$

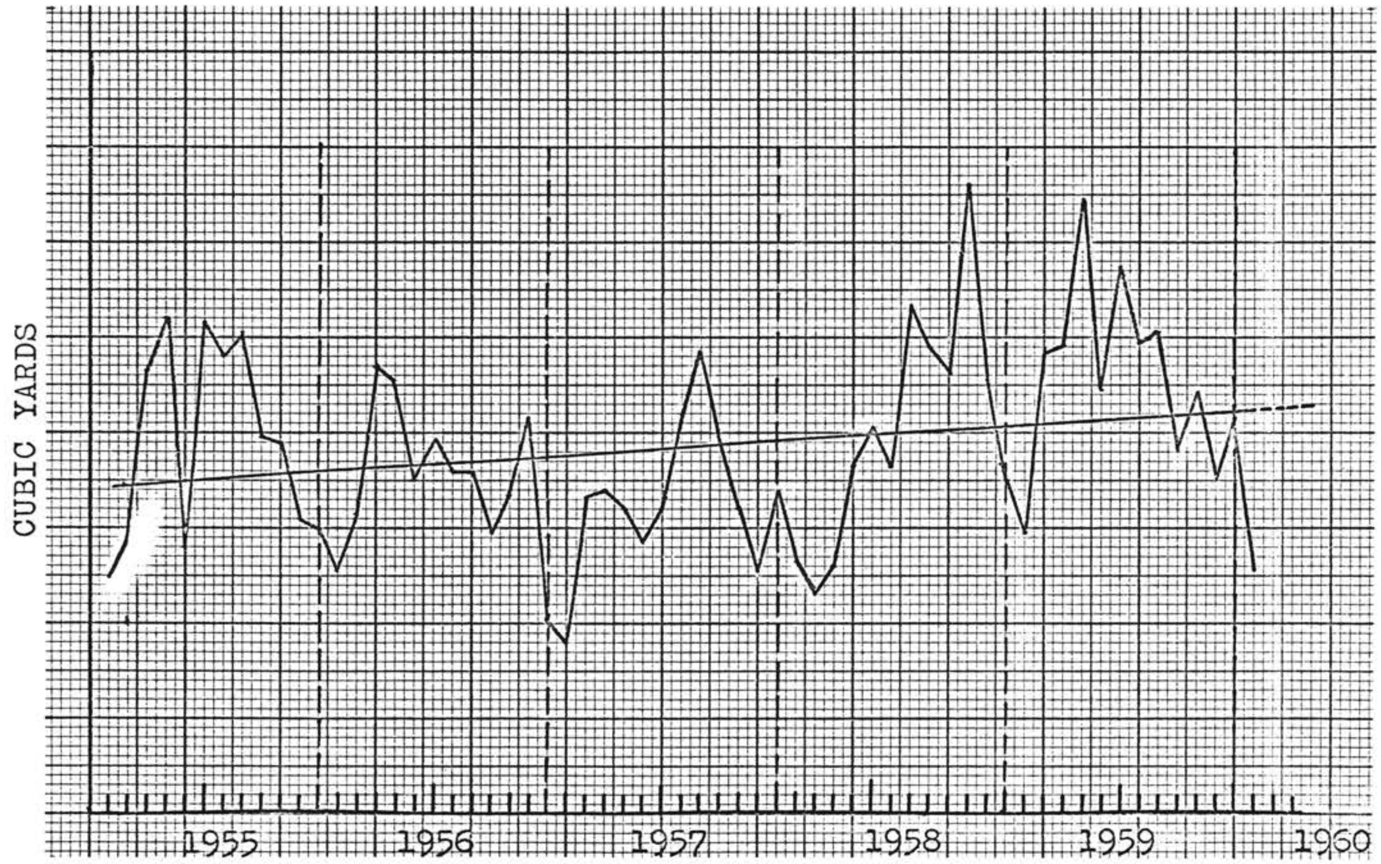

Figure 2. Monthly Concrete Sales and the Trend

where:

$\hat{Y}$ is the calculated or trend value

b is the slope of the regression line

c is the intercept of the Y ordinate at $x = 0$

x is the independent variable which can be days, months, years, quarters, etc.

The application of this equation is mechanical and the method of calculating the trend line is demonstrated in Table I and the graph of Figure 3.

Once the trend has been established in a time series, it can be removed in two ways. The first method of adjusting for trend influence is to express the actual sales data as per cent of corresponding trend. That is, divide the actual sales figure of a given date by corresponding trend value.

The second method of eliminating the trend effect is to subtract the trend value from the corresponding monthly sales data. The result of this operation leaves the data in cubic yards of concrete which will deviate about a zero base line.

Elimination of Seasonal Variation

Another factor which can be removed from the sales data is the seasonal variation. The seasonal effect along with the trend value may be expressed in "normal" years as shown in Figure 4. "Normal" is the trend with usual seasonal variation. The deviations from the "normal" are

TABLE I

EXAMPLE OF TREND LINE CALCULATION
(IN THOUSANDS OF CUBIC YARDS)

Yearly	Sales	X	X^2	XY
1955	200	-2	4	-400
1956	210	-1	1	-210
1957	208	0	0	0
1958	218	1	1	218
1959	225	2	4	450
	1061		10	58

$$b = \frac{\Sigma Y}{n} = \frac{1061}{5} = 212.2 \text{ cu. yds.}$$

$$a = \frac{\Sigma x Y}{\Sigma x^2} = \frac{58}{10} = 5.8$$

$$\hat{Y} = ax + b = 5.8x + 212.2$$

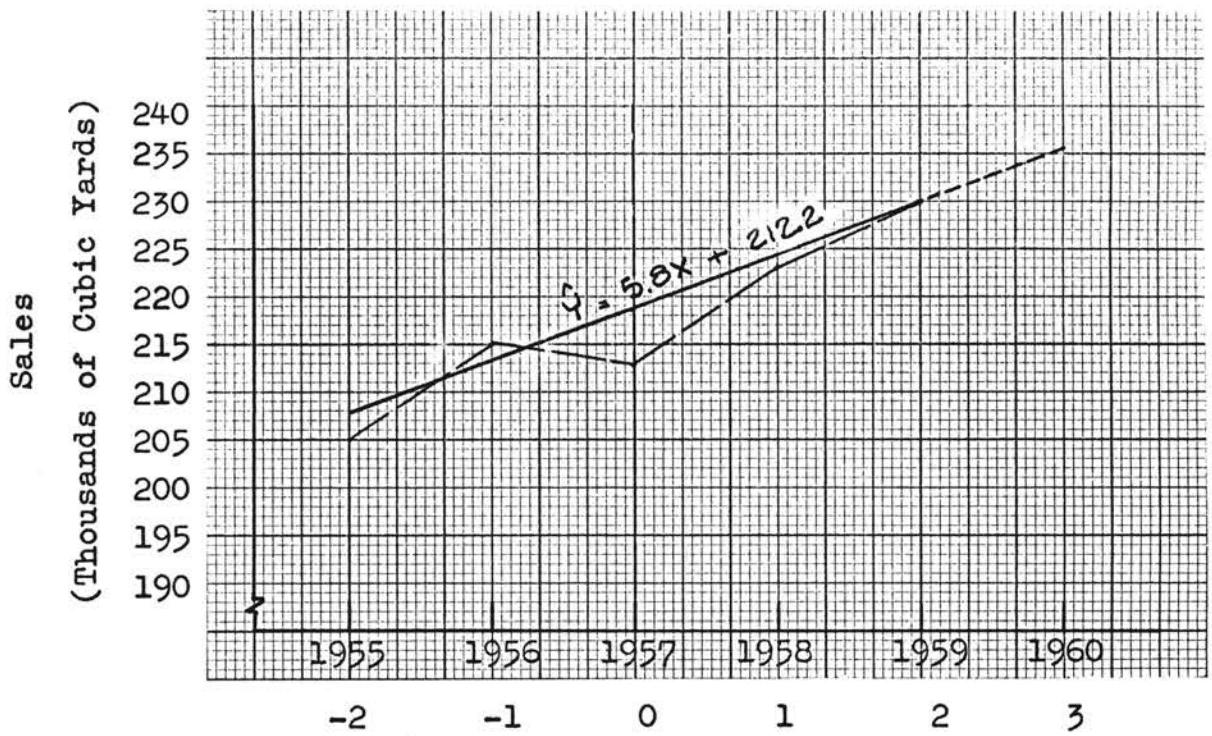

Figure 3. Graphic Representation of Trend Line

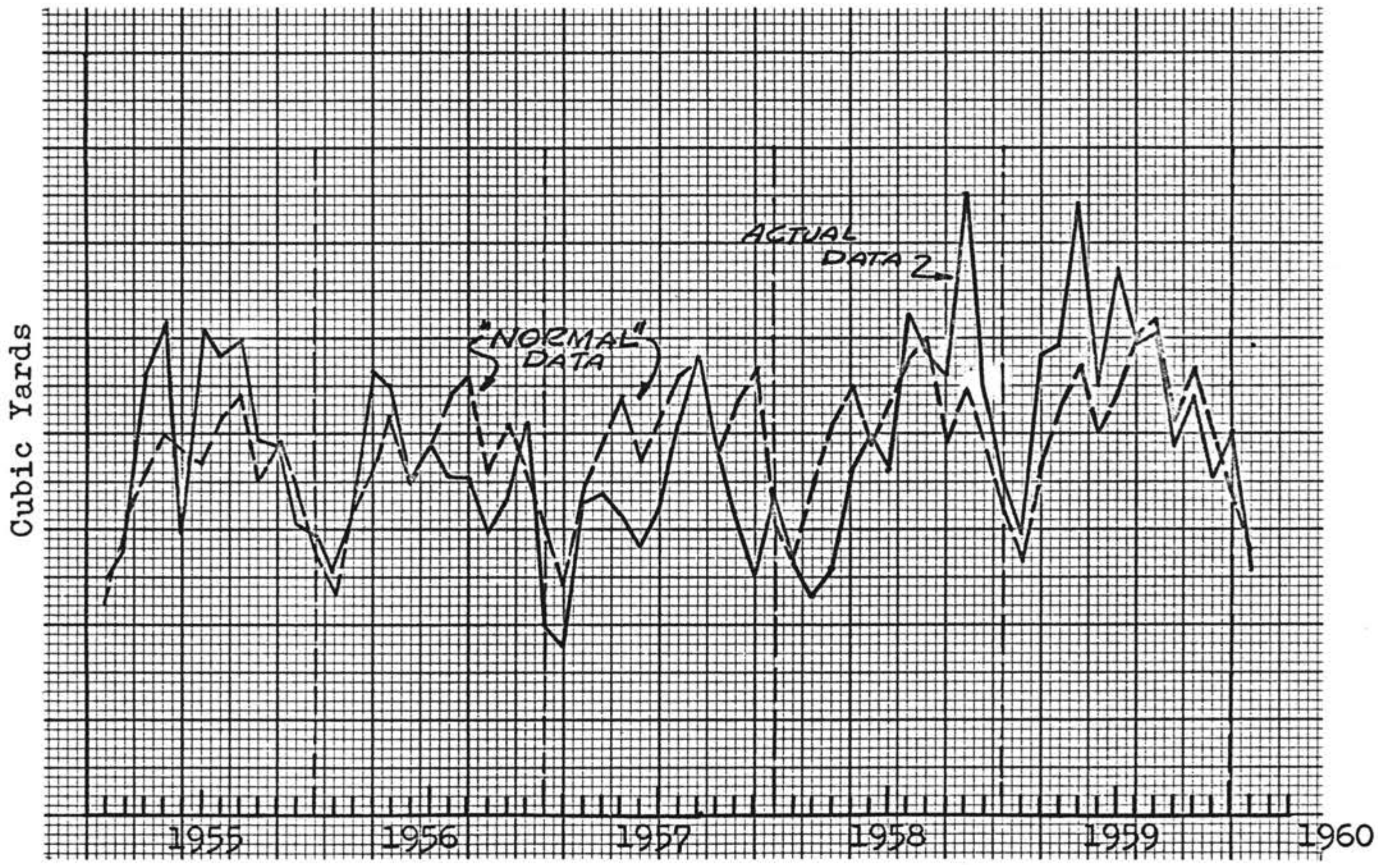

Figure 4. Concrete Sales - Actual Data With "Normal"

business cycles and random movements. These normal values may be arrived at by finding the product of the proper seasonal and trend value.

In cases where it is desired to seasonally adjust data, not considering trend, the actual sales data is divided by the seasonal index. The result of this operation is a smoothing effect which eliminates the dips and high points in a time series. The seasonal indices for the sales figures in Figure 1 (page 8) are:

Month	Seasonal Index
January	65.0
February	88.6
March	102.9
April	114.0
May	96.6
June	106.6
July	119.9
August	123.8
September	98.1
October	110.4
November	95.2
December	78.8

The seasonal index can be computed in a number of ways. The indices above were calculated on a twelve month moving average which is probably one of the most accurate ways of calculating the seasonal index.

One point which should be emphasized is that it is possible to remove the trend and seasonal in any order. Table II shows the method for removing the trend and seasonal variation. The residues are the business cycle and random movement.

Cyclic and Random Movements

The graph of Figure 5 shows the deviations from "normal" which are the result of cyclic and random movements. The author assumes that the random movements will eliminate themselves over a long period of years. The cyclic movement could be very useful in forecasting. In order to establish a reliable business cycle, at least twenty years data is required. This sample of five years is not large enough to draw a valid inference as to the period or length of the cycle. It should be noted that the use of business cycles is a means of prediction, but limited in most cases because of the limited data which is available.

The Business Forecast

The batch plant manager should realize when he is starting a business forecast that the objective is not to determine the curve of best fit which might tell you what will happen in the next few months, but it is to make analyses using statistical methods, which will enable an executive to take advantage of future conditions better than he

TABLE II

ELIMINATION OF TREND AND SEASONAL VARIATION (ADJUSTED FIGURES)

Year and Month		Actual Sales A	Trend Value B	Seasonal Index (Per cent) C	"Normal" (B x C) D	Deviation From "Normal" (A - C) E	Deviation Squared F	Sigma Distances ($E \div \sigma$) G
1955	Jan.	14,735	24,346	65.0	12,325	2,410	5,808,100	+0.31
	Feb.	18,898	24,487	88.6	20,555	- 1,657	2,745,649	-0.21
	Mar.	36,694	24,629	102.9	25,633	11,061	122,345,721	+1.44
	Apr.	41,916	24,770	114.0	29,638	12,278	150,749,284	+1.59
	May	17,897	24,912	96.6	23,725	- 5,828	33,965,584	-0.75
	June	41,688	25,054	106.6	27,368	14,322	205,119,684	+1.86
	July	37,774	25,195	119.9	32,199	5,575	31,080,625	+0.72
	Aug.	40,487	25,337	123.8	33,747	6,740	45,427,600	+0.88
	Sept.	29,516	25,478	98.1	24,804	4,712	22,202,944	+0.61
	Oct.	29,227	25,620	110.4	29,324	- 97	9,409	-0.01
	Nov.	20,886	25,761	95.2	24,044	- 3,158	9,972,964	-0.41
	Dec.	19,997	25,903	78.8	18,292	1,705	2,907,025	+0.22
1956	Jan.	15,572	26,044	65.0	13,429	2,143	4,592,449	+0.28

Where: $\sigma = \sqrt{\dfrac{\Sigma d^2}{N - K - 1}} = \sqrt{\dfrac{4.2193 \times 10^9}{61 - 2}} =$ 7697 cubic yards

Note: Data is shown for one year only.

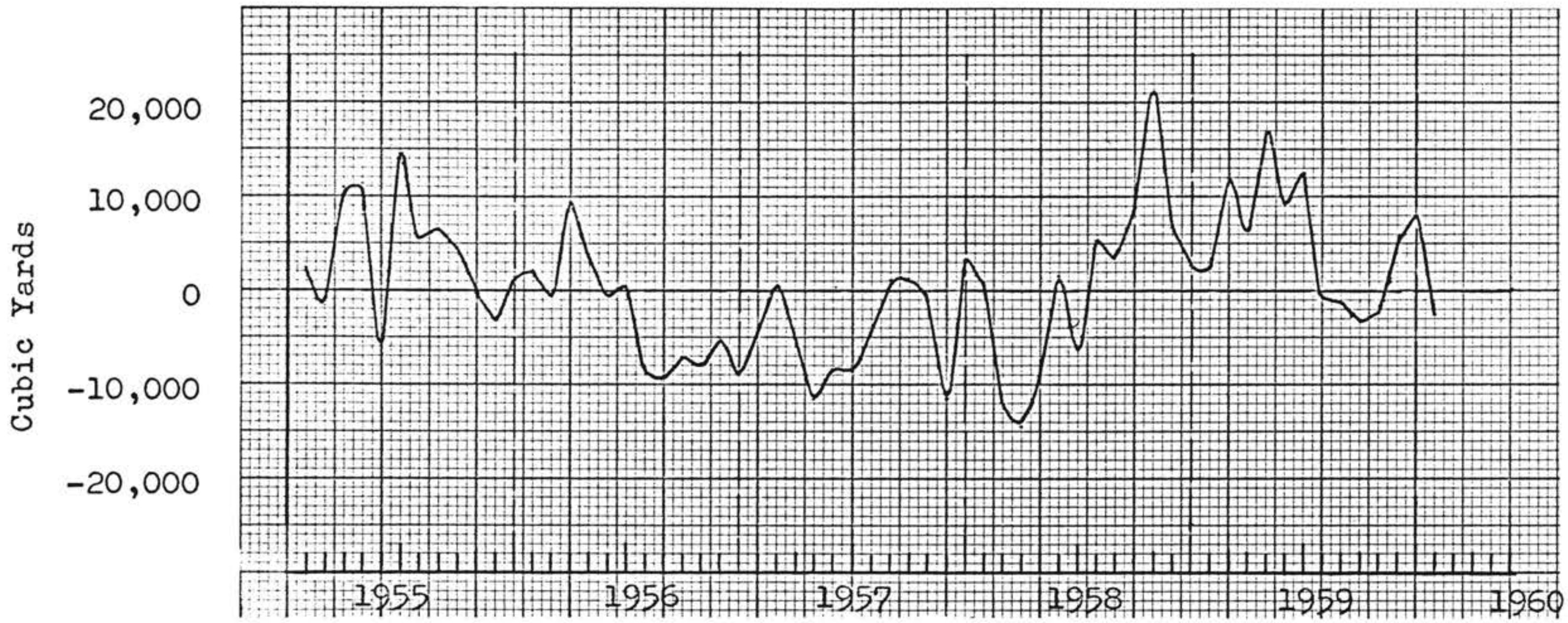

Figure 5. Variation of Actual Sales From "Normal" Sales
(Trend and Seasonal Variation Eliminated)

could do without the aid of a forecast.[3] Besides the factors mentioned previously, others must be considered and evaluated before the final demand figure is reached. Consideration should be given to economic conditions, business activity, weather, and building activity.

Weather is a factor which must be given serious consideration before forecasts are accepted. In some instances, rain and continued cold weather may cause concrete usage to fall far below that which was anticipated. As soon as weather conditions improve, concrete sales increase. The fact that weather may cause customers to delay concrete pours on jobs, places an extra burden upon the manager. This requires that the inventories must be ready to absorb the fluctuation due to increased sales.

Weather forecasts can usually be obtained on an extended basis predicting for a month in advance. If weather is not considered when making a business forecast, the predicted demand figure may not be a true representation for the desired time period causing loss of time and money due to the fact that this factor was not taken into consideration.

Trend Extrapolation

From the trend equation, it is possible to extend the regression line to the next time period. The dotted line

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[3]Frisbee, J. R. and I. N. Riggleman. Business Statistics (New York, 1938), p. 360.

in Figure 3 (page 15), shows the trend line extended using a yearly forecast. The trend value is calculated in the following manner:

$$\hat{Y} = a\,x + b = 5.8\ (3) + 212.2 = 229.6 \text{ cubic yards}$$

The yearly value may be broken down into monthly values using the following method:

$$\text{Monthly average} = \frac{\text{Yearly Forecast}}{12} = \frac{229.6}{12} = 19.13$$

cubic yards.

Expected monthly = Monthly average x Seasonal index usage.

Table III is an example of how this method is used.

Extrapolation done on the monthly basis is accomplished in the same manner. First, finding the trend value and then multiplying by the appropriate index will give the demand forecast for the next succeeding month.

The tendency for predicting by extrapolation is to project too far into the future. Values calculated must be interpreted along with other factors before a final figure is reached.

Forecasting in Terms of Standard Deviations

Table II (page 19) shows in Column G the standard deviations from "normal". It is possible to correlate the company's sales data with some other time series. For example,

TABLE III

ESTIMATED MONTHLY DEMANDS
USING EXTRAPOLATED
YEARLY TREND

Month	Seasonal Index	Thousands of Cubic Yards	
		Estimated Mean Usage	Predicted Demand (B x C)
A	B	C	D
January	65.0	19.13	12.43
February	88.6	19.13	16.95
March	102.9	19.13	19.68
April	114.0	19.13	21.81
May	96.6	19.13	18.48
June	106.6	19.13	20.39
July	119.9	19.13	22.94
August	123.8	19.13	23.68
September	98.1	19.13	18.77
October	110.4	19.13	21.12
November	95.2	19.13	18.21
December	78.8	19.13	15.07

$$\frac{229.6}{12} = 19.13 \ \frac{\text{thousand cubic yards}}{\text{month}}$$

it is possible that there is a high correlation between construction activity and concrete sales. The basis for comparing the two time series is to eliminate the assignable factors from each series and express both in terms of their standard deviation from "normal" so that a fixed ratio can be determined between the two series. Then, if an increase of six per cent is expected in construction activity and one standard deviation of concrete sales was equal to twelve per cent of construction activity, an increase of one-half a standard deviation in concrete sales would be expected. It should be emphasized once more that a correlation of around 0.90 should exist between the time series before such a forecasting method should be used.

Multiple Regression

Another means of forecasting is the use of economic indicators which also require that a relationship between the indicators and company sales exist. Some companies have found that they have been able to correlate their sales with some economic indicator such as Gross National Product, National Cement Shipment, New Construction activity or Housing starts. The economic indicators that were tried with the Dolese sales data could not be used in sales forecasting. The reason for this was due to the fact that concrete batch sales respond more to local conditions and relationship should be checked on a local level. The highest correlation that was obtained was 0.78 with cement shipments into Oklahoma.

To be effective in sales prediction, it is advantageous to choose an indicator from which the future sales can be predicted. That is, find the indicators with which the company sales are highly correlated, but where there is a sufficient lag to forecast the future. These indicators are called leading indicators.

CHAPTER III

INVENTORY LEVELS

Arriving at the optimum level between too much and too little stock in inventory is a difficult task. In cases where stocks are an important part of a production process, there is a tendency to let stocks expand beyond economic limits. On the other hand, a shortage of inventories may result in costs brought about by customer dissatisfaction and/or some expensive turnover, and handling. This chapter deals with techniques which may be used to determine the optimum inventories to maintain when "uncertainty of demand" is a significant factor.

Stone inventories play a vital role in the operation of a concrete batch plant. They serve many purposes in the organization, some of which are often overlooked. To mention a few:

1. Stone inventories kept in stock act as a cushion against unexpected and seasonal sales.
2. Inventories may be used to help take advantage of price changes so that when discounts or notice of price increases are are given, the level may be adjusted ac-

cordingly.

3. Inventories reduce the risk of run-out which is costly to the organization.
4. Ordering larger quantities may result in quantity discounts and reduce costs associated with placing orders.

Due to the uncertainty involved in establishing proper inventory levels and from the above discussion, one may jump to the conclusion that inventories should be kept high enough so that no possibility of run-out exists. There are several disadvantages to carrying excessive inventories, some of which are as follows:

1. The overages (amount greater than is needed to protect against run-out) represents invested capital which could possibly be used to greater advantage in some other areas of the organization. The optimum level may be determined and maintained by a well defined inventory control program.
2. Stone aggregates require large storage areas which are expensive in themselves.
3. Tax assessments on large inventories may be significant and it may be desirable to carry smaller amounts of inventory during the assessment period.

It is therefore necessary to exercise close control over the stone aggregate so that the associated costs are held to a minimum and the inventories held at a level which will guarantee satisfactory customer service.

In the above discussion on the purpose of inventories, it was noted that the level of stock is important due to the amount of capital which is invested in it. The plant manager should try to strike a balance between increased cost and declining return from increased inventories. When conditions exist such that the replenishment order arrives just as the stock on hand is depleted, it may be stated that inventories are at an ideal level. In most cases, the condition of being able to accurately predict consumption is impossible and safety allowances are required to protect the system against the unexpected, thus requiring a departure from this ideal system. The uncertainty of demand requires that we design a system which will be flexible and absorb the fluctuations of demand from time period to time period.

Inventory stock levels will vary from period to period because the demands on the system are not constant. In actual practice, management may feel that it is necessary to keep inventories as high as possible regardless of cost. However, the methods discussed in this thesis allow inventory level to fluctuate with the customer demand forecast.

The total inventory in a system is the sum of the inventory on hand, plus on order. Setting inventory levels may be a very difficult task and in instances where records are not available, the tendency may be to overstock.

Inventory levels may be arrived at by selecting some type of a reorder rule which depends on three conditions, namely:

1. A demand forecast
2. The lead time
3. The distribution of demand-forecast errors

Included in the requirements are conditions which are management controlled such as the length of the review period and the allowable risk of run-out.

There are two basic types of reorder systems which will handle uncertainty conditions: The two bin system where the order size is fixed, but the order interval is dependent on consumer demand and the fixed cycle, in which the inventory is reviewed at certain fixed time intervals and orders placed to return the inventory to a predetermined level.

<u>The Distribution of Demand-Forecast Error</u>

Throughout the remaining part of this chapter, use will be made of $\hat{\sigma}$ which is the standard deviation of the distribution of demand-forecast errors. The standard deviation of this distribution is calculated from the difference between the forecast and actual demand over a given time period. These deviations were assumed to come from a normally distributed universe, but the actual sample distribution, using Dolese sales data, was slightly skewed to the right. The author made the hypothesis that there was no difference between the obtained distribution and a normal distribution. To check this hypothesis, a "Test of Normality" was run checking the distribution for the non-normal characteristics of skewness and kurtosis. The results of the test did not reject the hypothesis that the variance of the demand-forecast errors was normally distributed.

Safety Stock

The common factor to any reorder rule chosen is safety stock which is required to protect the system against unexpected fluctuations in demand. In order to determine the amount of safety stock to carry, it is necessary to know the variance of the distribution of daily demand-forecast errors. Once this variance is known, the next step is to determine the risk of run-out. This factor depends on the level of protection desired. For example, if the risk of run-out is allowed to one chance in one hundred, or a one per cent chance of run-out, then the factor associated with this level of risk is 2.33. The notation for the risk factor is usually designated by "K" and is found using the normal distribution tables. This is the distance from the mean expressed in sigma distances and approximately ninety-nine per cent of the area under a normal curve lies below the point 2.33. For the author's purposes, this means that in the long run, it may be expected that customer orders can be filled ninety-nine per cent of the time. Another way to make the preceding statement is to say that the probability of a daily demand exceeding the average by 2.33 times the standard deviation of the daily demand-forecast error is one per cent.

The safety stock is the product of the standard deviation of the distribution of daily demand-forecast error, the risk factor and the length of the review period plus lead time;

$$\text{safety stock} = \frac{K\,\hat{\sigma}_D}{\sqrt{t+\gamma}} \cdot (t+\gamma) = K\,\hat{\sigma}_D\sqrt{t+\gamma}$$

where:

t = length of the review period

γ = length of the lead time

In cases where it is impossible to determine $\hat{\sigma}_D$, it is necessary to take a different approach in setting safety stock levels. By using this method, the risk of run-out is very, very small. The procedure is as follows:

1. Determine the standard deviation of the actual daily demand for each month. The standard deviation may be found using the following formula:

$$\sigma_D = \sqrt{\frac{\Sigma\,(X_i - \bar{X})^2}{n - 1}}$$

where:

σ_D = standard deviation of actual orders

X_i = the actual daily orders

$\overline{X}$ = the mean of the actual daily orders

n = the number of working days in the month

Table IV shows the calculation of σ_D for a given month.

2. Next, determine the risk factor. If it is decided that 97.72 per cent of the time an order should be able to be filled, then from the normal table, K would equal 2.00. Knowing the length of the review period plus lead time, the level of safety stock may be found. The mathematical notation for this condition is:

$$\text{Safety Stock} = K\sigma_D\ (t + \gamma)$$

The following example demonstrates the use of the above equation. It is assumed that σ_D is given in cubic yards of concrete and a conversion factor of 0.91 was used to change cubic yards of concrete to tons of aggregate. This factor was determined by assuming that on the average, 1820 lbs. of aggregate are used per cubic yard of concrete poured. In the example, let:

σ_D = 200 cubic yards of concrete

$(t + \gamma)$ = 9 days

K = 2.00

Safety stock = $K\sigma_D\ (t + \gamma).91$ = 3276 tons

TABLE IV

ILLUSTRATION SHOWING CALCULATION OF STANDARD DEVIATION OF DAILY CONCRETE USAGE

Month and Day	Actual Orders (Cubic Yds.) X	X^2
Oct. 1	10.25	105.06
2	75.0	5,625.00
5	173.5	30,102.25
6	153.5	23,562.25
7	246.75	60,885.56
8	446.0	198,916.00
9	285.75	81,653.00
12	1,182.75	1,398,897.56
13	657.0	431,649.00
14	1,302.0	1,695,204.00
15	816.75	667,080.56
16	999.75	999,500.06
19	789.25	622,915.56
20	900.0	810,000.00
21	637.5	406,406.25
22	275.25	75,762.56
23	397.0	157,609.00
26	155.0	24,025.00
27	229.25	52,555.56
28	207.5	43,056.25
29	213.0	45,369.00
	$\Sigma x = 10,152.75$	$\Sigma x^2 = 7,830,879.54$

$$\sigma = \sqrt{\frac{\Sigma x^2 - (\Sigma x)^2/n}{n - 1}}$$

$$\sigma = \sqrt{\frac{2,922,387.52}{20}} = 382.26 \text{ cubic yards}$$

The probability of run-out under this condition:

$$P = (1 - 0.9772)^9 = (0.0228)^9 = 1.66 \times 10^{-15}$$

When selecting the standard deviation of actual sales to use in determining the safety allowance, it should be remembered that this method is only an approximation which became necessary because the daily demand-forecast distribution was not known. This method should suffice until management can determine the actual distribution of demand-forecast error. The highest value of σ_D may be used because it is the best known protection calculated from recent data to protect against the worst conditions which might occur in the future. However, due to the uncertainty involved in such a problem, the safety stocks will be higher. Thomas M. Whittin, in his book on "The Theory of Inventory Management", points out that safety allowances should be larger (1) the higher the costs of depletion; (2) the lower the carrying charges; (3) the greater the unpredictable variations in demand; and (4) the longer the lead time.

Table V shows the effect of the risk factor on the safety stock and the probability of run-out associated with each level.

Reorder Rules

Fixed Order System

The fixed order system is one where a fixed quantity is ordered when the inventory level reaches a predetermined

TABLE V

LEVELS OF SAFETY STOCK FOR GIVEN K VALUE

Probability of Run-out per day	K	Safety Stock	Probability Run-out Over Lead Time and Review Period
0.01	2.33	4660	1×10^{-18}
0.0228	2.00	4000	1.66×10^{-15}
0.05	1.65	3300	1.95×10^{-12}
0.10	1.28	2560	1×10^{-9}

minimum quantity. An example of this type of system is commonly referred to as the (S,s) rule which operates in the following manner:[4]

1. Two inventory levels are chosen where $\underline{S} > \underline{s}$.
2. When inventories are reviewed and the actual inventory is between S and s, no order is placed.
3. When the inventory level falls below s, a standard order quantity is placed restoring the level of inventories to S.

The reorder quantity which is placed when the inventory levels fall below the lower inventory level s is usually found by the optimum order equation:

$$q = \sqrt{\frac{2As}{i}}$$

where:

q = average order quantity

A = cost of placing an order

S = usage rate for a given time period

i = cost of holding one ton of aggregate over the same time period

The mathematical model for the (S,s) rule is:

$$\underline{S} = \sqrt{\frac{2As}{i}} + \underline{s} - \frac{d}{2} V$$

where:

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[4] Arrow, K., T. Harris, and J. Marschak. "Optimal Inventory Policy", Econometrica, Vol. 19, July, 1951, pp. 250-272.

$\underline{S}$ = maximum inventory level

$\underline{s}$ = reorder level

V = length of the review period

$\frac{dV}{2}$ = expected usage during the review period

The fixed order system serves well where inventory records are not kept and continuous monitoring of inventory is possible. The tendency is to loosen control over inventories. This method also serves best where low valued stock may be purchased in large quantities and the stock plays a minor role in the plant production output.

The Fixed Cycle System

The fixed cycle system is one where the review period is fixed and orders placed at regular intervals to cover the usage for the next time period. This system works well where tighter, more frequent control is required over the inventories and a number of different types of aggregates may be ordered at one time. Fixed cycle is better where the cost of carrying inventories is very low.

Fixed Cycle - Fixed Order Point System

Many inventory control programs use combinations of the two basic systems. When applying a reorder rule to a given problem, it is necessary that it be tailored to fit the given conditions. The ordering rule which is developed in this thesis is a combination of the two basic systems in which the review period and the reorder level have been fixed. It operates in the following manner: At the time the inventory is reviewed, an order quantity is placed to cover the expected demand for the next review period plus the difference between the actual and reorder inventory levels. The assumption was made that the lead time is fixed.

The mathematical model which describes the reorder rule may be stated in the following way:

$$P_i = U_{(i + 1)} + p_i$$

$$Q_i = U_{(i + 1)} \pm \Delta_i$$

where:

$$U_{(i + 1)} = \bar{\bar{X}}_{(i + 1)} t$$

$$\Delta = \begin{cases} -, \text{ for } r - p > 0 \\ 0, \text{ for } r = p \\ +, \text{ for } r - p < 0 \end{cases}$$

P_i = the target inventory which includes the inventory on hand plus on order

p_i = the reorder level which is equal to the expected or average demand over the lead time plus safety stock.

U_{i+1} = the estimated usage for the next time period

Δ_i = the difference between the actual and reorder inventory levels at time of review

r = the actual inventory on hand at the time of review

Q_i = the replenishment order placed to satisfy customer demand for the next time period

$\bar{\bar{x}}$ = the average demand per day

The objective of the reorder rule is to have reasonable control over the aggregate inventories which are so vital to the batch plants' concrete output. The following example along with Table VI and the graph in Figure 6 will help to explain the use of the model.

The following assumptions are made: (1) The lead time is four days; (2) the review period is five days; (3) the standard deviation of actual daily sales is 200 tons of aggregate; (4) the number of working days is twenty days for each example month; (5) the expected demand for the first month is 6000 tons; (6) for the second month, the expected demand is 4000 tons; (7) the risk factor is equal to 1.28 at a probability of run-out of 0.10; (8) $\hat{\sigma}_M$ is equal to 76 tons.

$$\bar{\bar{X}}_1 = \frac{6000}{20} = 300 \text{ tons/day}$$

$$p_1 = \bar{\bar{X}}_1 \gamma + K\sigma_D(t + \gamma) + \frac{K\hat{\sigma}_M}{20}(t + \gamma) = 3548 \text{ tons}$$

where:

$$K\sigma_D (t + \gamma) + \frac{K\hat{\sigma}_M}{20} (t + \gamma) \text{ is (equal) the safety stock}$$

The first term is that portion required to offset

TABLE VI

INVENTORY RECORD

	Month and Period* A	P_i Target Inventory B	$\overline{X}$ Estimated Mean Usage C	Actual Daily Usage D	Received E	r Actual Inventory F	P Reorder Point G	Δ H	Q Reorder Quantity I	Inventory On Hand Plus on Order J
MONTH 1	1	5048	300	308		3548	3548	0	1500	5048
			300	384		3164				4664
			300	466		2698				4198
			300	200		2498				3998
	*		300	391	1500	3607				3607
	2	5048	300	400		3207	3548	341	1841	5048
			300	332		2875				4716
			300	440		2435				4276
			300	50		2385				4226
	*		300	246	1841	3980				3980
	3	5048	300	290		3690	3548	142	1358	5048
			300	400		3290				4648
			300	560		2730				4088
			300	800		1930				3288
	*		300	775	1358	2513				2513
	4	4548	300	237		2276	3548	1272	2272	4548
			300	456		1820				4092
			300	257		1563				3835
			300	50		1513				3785
	*		300	250	2272	3535				3535
MONTH 2	5	4148	200	540		2995	3148	153	1153	4148
			200	397		2598				3751
			200	145		2453				3606
			200	84		2369				3522
	*		200	299	1153	3223				3223
	6	4148	200	123		3100	3148	48	1048	4148
			200	247		2853				3901
			200	446		2407				3455
			200	305		2102				3150
	*		200	269	1048	2881				2881
	7	4148	200	68		2831	3148	317	1317	4148

*One period is equivalent to five days.

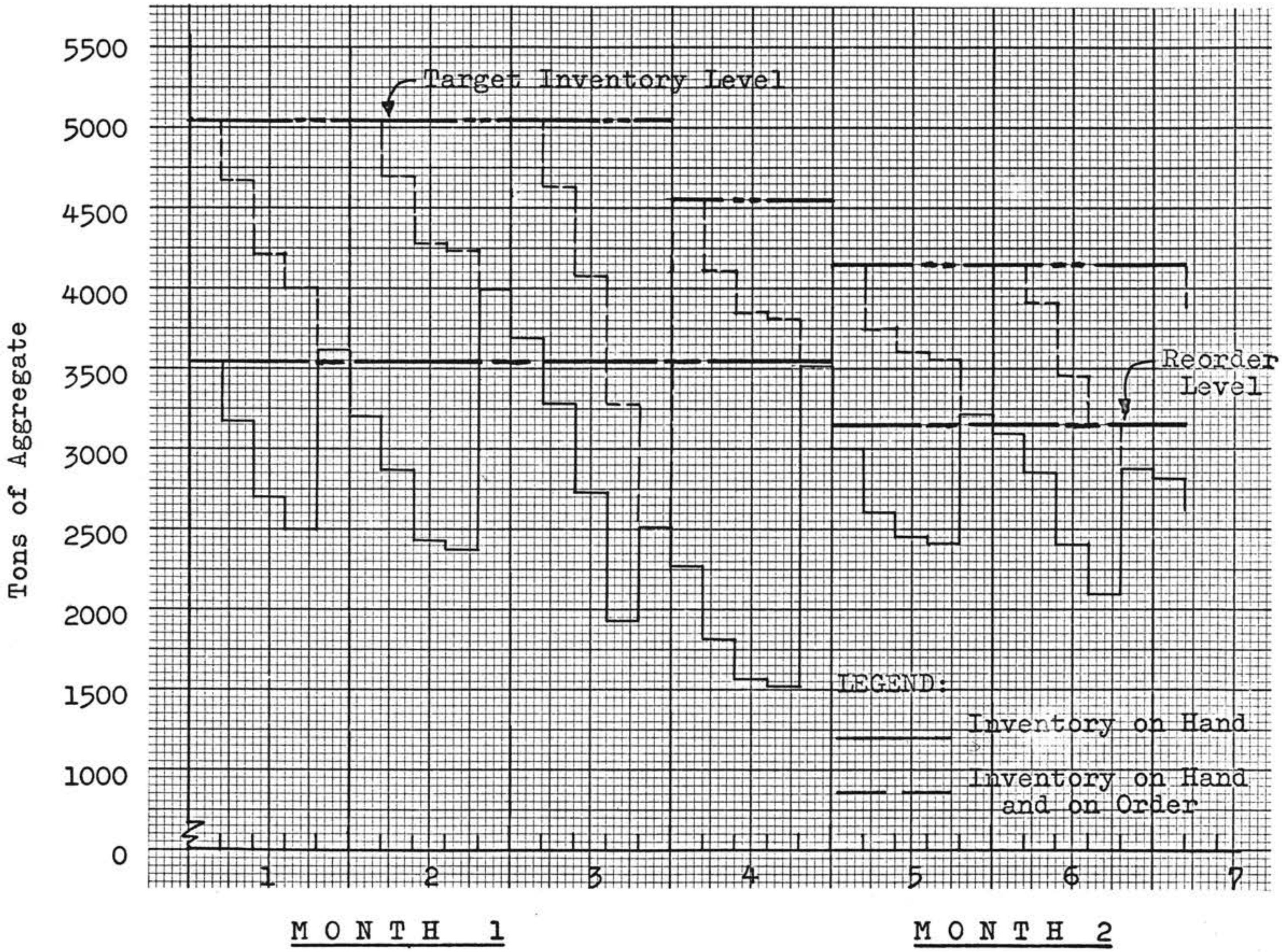

Figure 6. Typical Daily Fluctuations in Inventories

variations due to daily sales and the second term is the portion of the safety stock required to absorb the variations due to inability to accurately predict the daily mean usage.

$$P_1 = \bar{\bar{X}}_1 t + p_1 = 5048 \text{ tons}$$

$$Q_1 = \bar{\bar{X}}_2{}^t \pm \Delta_1 = 1500 \pm \Delta \text{ tons}$$

The calculation for the second month is made in the same manner and the results are shown in Table VI, (page 40), which operates in the following way:

1. The target inventory is calculated and entered at the beginning of each review period.
2. The average daily demand is calculated using the total number of working days during the month and entered in column (C).
3. Column (D) represents the actual daily usage which is entered at the close of each day, giving management a chance to keep a running account of their inventories.
4. Column (E) represents inventories received.
5. Column (F) is used in conjunction with the actual daily usage and represents the actual level of the inventories on hand.
6. Column (G) shows the reorder levels which are subtracted from Column (F) at the time of review to get Δ.

7. Column (H) is Δ.

8. Column (I) is the replenishment order quantity for the next review period and arrives four days after an order is placed. Delivery days are starred.

9. Column (J) represents the actual inventory on hand plus on order in the system.

The graph of Figure 6 (page 41) shows the inventory fluctuations and it should be emphasized once again that these variations are due to the uncertainty inherent in the problem and the inability to "pin point" demands.

The Lead Time and Review Period

Up to this point, no mention has been given to the determination of the lead time and review period. The lead time is usually determined at the aggregate source and depends on the facilities (railroad) the plant manager has at his disposal. In cases where the lead time varies, it is necessary to determine the standard deviation of the distribution of lead times so that a safety allowance can be made for the varying delivery times.

The length of the review period is usually determined by the plant manager and is dependent on the storage capacity available at the batch plant site. As the length of the review period becomes longer, the amount of safety stock increases which leaves the remaining portion to act as working stock and increases the amount of inventory on hand plus on order. The length of the review period is actually an economic decision which involves the balancing of certain costs due to carrying inventories, placing of replenishment orders and costs associated with the risk of run-out.

Total Variable Operating Cost

Once a reorder rule has been established, it is possible to find the expected cost per time period. The total variable cost involves:

1. Replenishment cost - Cost incurred when an order is placed. This value will be designated by C_o.
2. Cost of carrying inventory - The cost of carrying the inventories and is designated by C_I.
3. Cost of depletion - Cost that occurs when customer orders cannot be filled due to run-out. This value is designated by C_D and often times a value that can only be arrived at through a management decision involving the risk of run-out that they are willing to accept.

The expression for the total variable cost is:

$$T.V.C. = C_o + C_I \left(\frac{Q}{2} + S.S.\right) + C_D \cdot P$$

where:

P = probability of run-out

$\frac{Q}{2}$ = average order quantity

S.S. = safety stock

The T.V.C. per period is not constant because the order quantity and safety stock may vary from period to period. It should be pointed out that the cost of the stone is not included in the equation because under any reorder rule which may be chosen, the object is to select the rule that will cost less to use in the long run. The cost of the aggregates are a fixed cost and will not vary with the reorder rule adopted.

CHAPTER IV

CONTROL OF FORECASTS AND INVENTORY LEVELS

An inventory control program is a responsibility of management. It requires that the objectives of the program be defined so that policies may be adopted which will provide maximum control at minimum cost. The objectives of any organization may include the following:

1. To maintain optimum inventory levels which will satisfy customer demands.
2. To reveal any excessive accumulation or shortages of inventories.
3. To maintain records so that periodic review is possible.

Once these objectives are formulated, a control system may be designed to establish inventory levels that will meet forecasted demand and methods adopted to adjust operating levels by replenishment orders to take account of the actual demand as it materializes. The actual control may be accomplished by maintaining adequate records and selecting a reordering system which will assist management in making decisions pertaining to inventory policies.

Development of Controls and Procedures

Management controls are necessary in guiding an organization towards its objectives. Several measures must be taken to insure a successful control program. Records must be kept which will log materials as they are received and signal any over and under stocked condition which may arise in relation to the present or projected demands. Records must reflect information that will insure that the investment in inventories is at a minimum level consistent with the sales demands and the operating requirements of the batch plant facilities. Frequent reviews of stores are necessary to test the validity of the records and periodic checks made on the forecasting technique to be certain satisfactory results are being obtained from the forecast.

The procedures which management develop should be aimed at reaching the objectives. The process includes setting up procedures for all inventory, issuing requisitions for all orders, coordinating sales and production needs. Once the policies and procedures are established, management must be sure the inventory control program is operating according to plan.

Controlling the Forecast

The importance of the demand forecast cannot be over emphasized. The method of forecasting used in the second chapter may also be used as a means of control. By expressing the deviations from the "normal" in sigma distances (a measure of statistical dispersion), we may adopt the use of the quality control chart. The application of a control chart provides the manager with a method of "watching" the forecasting process in operation. Interest is based primarily on the stability of the forecast so that any significant difference between actual and forecasted demand will be quickly noticed and remedial action taken to adjust the forecast.

The control limits are used in the following fashion and data which fall either above or below "normal" are treated as follows. If seven consecutive months fall above "normal" an assignable cause is probably operating. If one point falls above the two sigma limit, no action is taken, but if two consecutive months fall above the two sigma limits, an inquiry should be made to find an assignable cause. When one month falls above the three sigma limit, an assignable cause is operating and a thorough investigation should be made in the forecasting method so that any irregularities may be corrected. A more complete discussion of runs and run test is presented in most Quality Control texts.

The control chart also aids management in detecting a change of trend. When points form a definite pattern up or down and continue towards one of the outer control limits, there may have been a change in trend. Under such circumstances, it might be advisable to recalculate the trend equation from the existing data to see whether or not it agrees with the original equation. If there is a significant difference, the new trend should be adopted and a new control chart constructed for the revised forecasting procedure.

The control chart in Figure 7 shows signs of being "out of control", that is runs of as many as seven months above or below "normal". The runs are the result of assignable causes operating in the system due to the business cycle and random variations. These factors could not be removed from the data because there was not enough information available to isolate them.

The process being "in control" or "out of control" is a condition which is defined by management. In the above situation where the business cycle seems to have an affect on the system, runs above and below normal will tend to fall into a pattern resembling the apparent business cycle.

Controlling Inventories

A method of control is built into the reorder system which was developed in the third chapter. The Table VI and graph of Figure 6 aid management to establish proper inventory levels and decisions concerning them are based on fact. The levels may be carefully noted and adjusted for any predicted change in demand.

A point which must be remembered is that an accumulation or lack of inventories may be costly and levels never

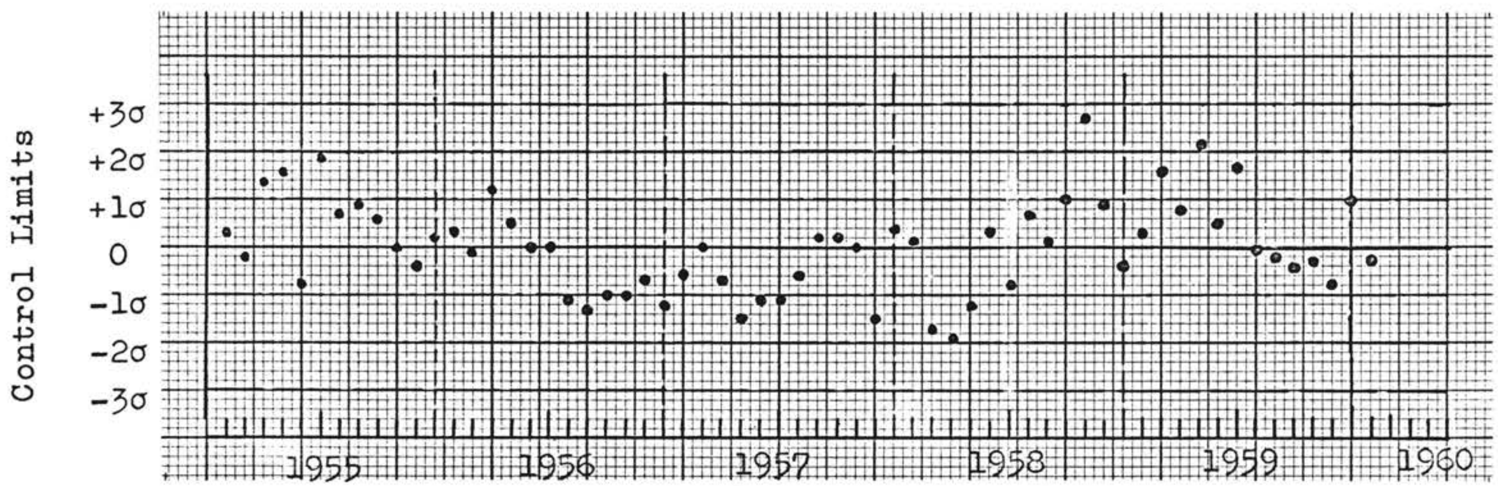

Figure 7. Control Chart

become fixed or automatic. Records are only an aid, a manager must keep analyzing and looking for danger signals when making periodic review.

Organizing for Control of Inventories

Inventories usually represent a large asset in any business and a part of the organization should be given the responsibility of supervising the inventory control program. Authority should be vested in personnel who are capable of making sound decisions, understand the forecasting and production process, and are cognizant of the organization objectives. Each person should understand the decisions he is expected to make and just what implications are concerned. He should know by analyzing the records and demand figures what level of inventories is required and the size of the replenishment orders to be placed. Above all, he must be able to detect changes in demand regardless of the uncertainty involved so that the plant facilities will be in the best position possible to satisfy customer demands.

CHAPTER V

SUMMARY AND CONCLUSIONS

In this thesis, an attempt has been made to establish methods by which the management of a concrete batch operation can make use of in establishing a reliable system of controlling its inventories. A sound inventory program depends on a reasonable forecast of demand, the ability to establish optimum inventory levels and control procedures to restore inventories to desired levels.

Forecasting for concrete sales can be a very difficult task because it is a seasonal product and sales are influenced by local business conditions. Methods are discussed and examples shown for arriving at a demand forecast. Demands are calculated on a monthly basis because it was possible to eliminate certain factors. Forecasting is done in terms of "normal" years, that is the trend with the usual seasonal variation. A dependable forecasting plan aids management in making decisions, however, there is no substitute for good judgment.

Statistical methods were used in determining the proper inventory levels to carry in the "face of uncertainty". Safety allowances were discussed and a temporary method shown for determining safety allowances when limited data is available. The routine will suffice until enough data is accumulated to more accurately determine the safety stocks. A reorder rule was developed to give management a systematic plan for establishing inventory levels and is applicable under uncertainty conditions.

The final step of an inventory control program is establishing proper restrictions which are necessary in aiding management to reach its objectives. Control is necessary to correct errors in the forecast as they occur and to insure the inventory control program is operating according to plan.

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