

MINIMUM STAGE CALCULATIONS FOR COMPLEX  
FRACTIONATORS.

By  
RICHARD SIBLEY JOYNER  
" "  
Bachelor of Science  
Oklahoma State University  
Stillwater, Oklahoma  
1961

Submitted to the faculty of the Graduate School of  
the Oklahoma State University in partial  
fulfillment of the requirements  
for the degree of  
MASTER OF SCIENCE  
May, 1961

OCT 10 1961

MINIMUM STAGE CALCULATIONS FOR COMPLEX  
FRACTIONATORS

Thesis Approved:

*R. N. Maddox*  
\_\_\_\_\_  
Thesis Adviser

*W. E. Edmister*  
\_\_\_\_\_  
Thesis Adviser

*Allen Maudsler*  
\_\_\_\_\_  
Dean of the Graduate School

## PREFACE

A method has been developed for estimating the minimum number of theoretical stages in multifeed, multiproduct distillation columns. In addition to the minimum number of stages the method predicts product compositions, product flow rates and feed and product entry or withdrawal points. The method has been programmed for the IBM 650 Computer although it is well suited to hand calculations. The method was tested by comparison with a simulation of a complex column operating at total reflux. The results of the comparison indicate that the method will give reliable estimates of the performance of a complex column at total reflux.

The author wishes to thank Dr. R. N. Maddox, whose advice and encouragement made this project possible; the staff of the Oklahoma State University Computing Center for their cooperation and assistance; and Continental Oil Company for its fellowship which in part supported this work.

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## CHAPTER I

### INTRODUCTION

The determination of the minimum number of theoretical stages required to achieve a given separation is a useful tool in the design of multicomponent fractional distillation columns. The calculation of the minimum number of theoretical stages provides a fast method of estimating the performance of a proposed column. In a short time, several alternate designs of the column could be evaluated for feasibility before applying more rigorous design techniques such as relaxation methods or plate-to-plate calculations.

Several methods have been suggested for calculating the minimum number of theoretical stages in single feed, two-product columns. The most suitable and most widely accepted short-cut methods are those which were proposed by Fenske<sup>(3)</sup> and Winn<sup>(10)</sup>.

The Fenske<sup>(3)</sup> method relates the minimum number of stages, separation and relative volatility, assuming constant relative volatility throughout the column. The Winn<sup>(10)</sup> method relates minimum number of stages, separation and two characteristic constants which are functions of the equilibrium distribution ratios (K-values) of the arbitrary key components.

A method for use on multifeed, multiproduct distillation columns has been developed and was tested in this work using an

IBM 650 Computer. A modification of the method has made possible the calculation of the feed entry point.

Although an IBM 650 was used for evaluation of the method in this work, the method is readily adapted to slide rule or desk calculator computation.

## CHAPTER II

### SURVEY OF LITERATURE

Several methods have been suggested for calculating the performance of a fractionator at total reflux. These may be separated into two general categories; those methods which utilize tray-by-tray calculations and those which employ semi-empirical equations.

The first of these types is by far the most difficult and time consuming. Several procedures have been employed in making the tray-by-tray calculations. Amundson and Pontinen<sup>(1)</sup> perform the tray calculations by solving the heat and material balance equations using matrix techniques. This method requires the solution and inversion of large matrices, thereby rendering the method practically useless for hand calculations. Lyster, et al<sup>(5)</sup> make tray calculations utilizing the Theile-Geddes<sup>(8)</sup> technique. Their method requires the use of a large computer, although it is well known that the Theile-Geddes<sup>(8)</sup> method is easily adapted to hand calculations.

Edmister<sup>(2)</sup> performs the tray calculations using a method based on absorbing and stripping factors.

For any of these tray-by-tray techniques an estimate of the product compositions, total stream flow rates and the number of stages would be extremely helpful.



The second of the general types, semi-empirical equations, has been well received by the Chemical Engineering profession, largely because of the ease by which calculations are made and the reduced time requirements. Since this type of calculation is the basis of this work, it will be treated in greater detail in the Theory Chapter than the first method.

## CHAPTER III

### THEORY

The operation of a fractionator at total reflux may best be visualized by referring to the classical McCabe-Theile<sup>(6)</sup> diagram, Figure I. At total reflux, all of the overhead product is returned to the column and no bottom product is withdrawn. This condition is of theoretical interest only because a column operating at total reflux produces no product and performs no useful function.

Another concept of total reflux is that of considering the column to be of infinite cross-section with finite feed and product streams. Under these circumstances the column is making the desired products from the given feed composition.

From a design standpoint a column operating at total reflux indicates the minimum number of stages required to make a specified separation. Since no overhead product is withdrawn from the column, or the reflux is very much larger than the distillate product, the slope of the operating line,  $\frac{L}{L+D}$ , is unity and coincides with the diagonal  $y = x$  line. With the slope of the operating line equal to one, the step from the operating line to the equilibrium line is a maximum, hence, the smallest number of steps for a given separation.

Referring again to Figure I, it may be seen that the number of stages at total reflux is independent of the composition at which the feed is introduced as well as the condition of the feed

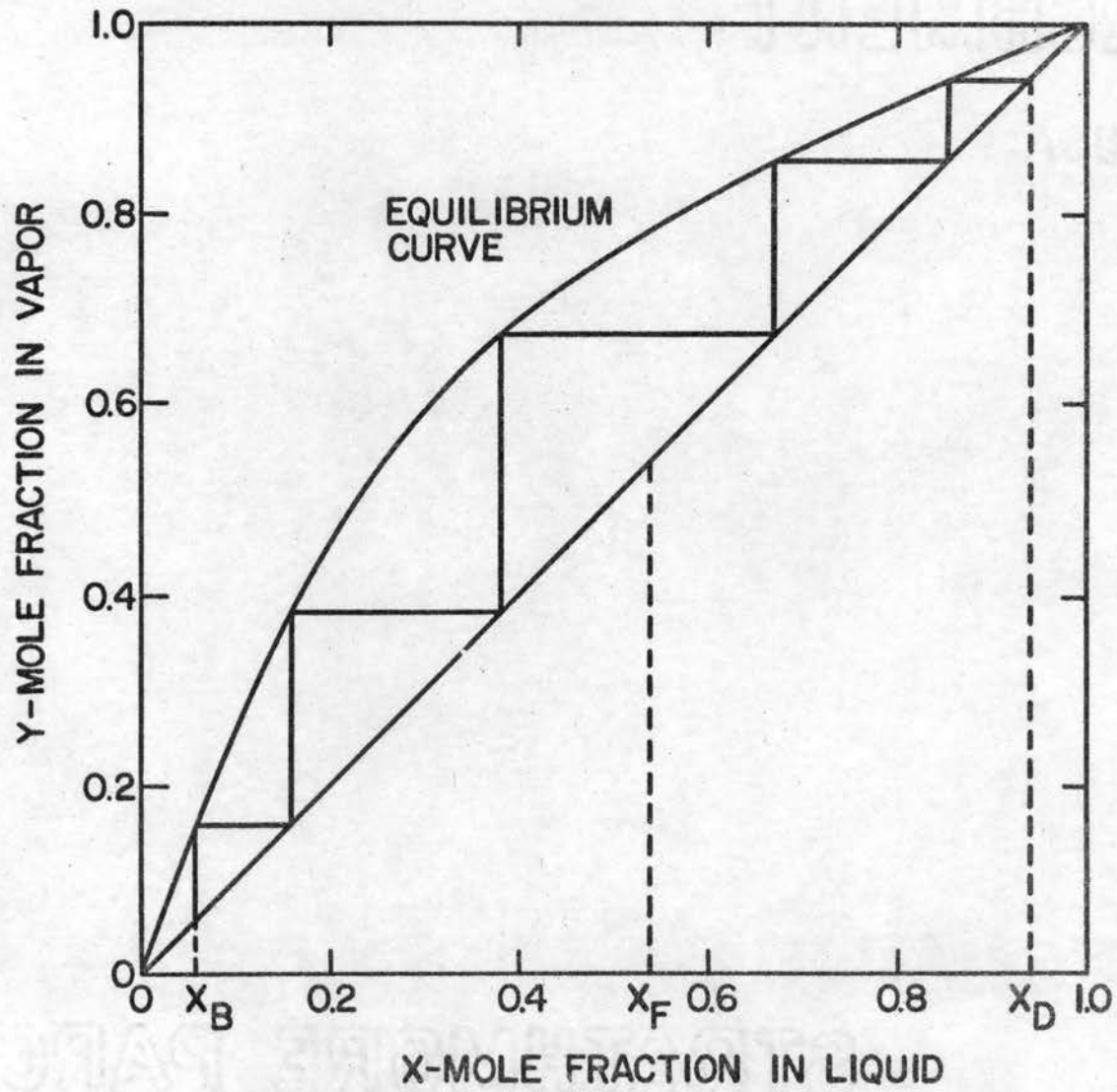


FIGURE 1  
McCABE-THIELE DIAGRAM

(whether it is a liquid, vapor or a mixture of liquid and vapor). Obviously, the movement of the feed entry point must be confined to any point between the distillate and bottoms product compositions.

Some authors have based their derivations of equations describing column operation at total reflux on constant molal overflow from plate-to-plate. The difference between passing streams on any plate above the feed plate is the distillate and difference below the feed plate is the bottoms product.

Or

$$V = L + D$$

$$L = V + B$$

The total reflux condition implies  $V$  and  $L \gg D$  and  $B$  so that  $V = L$ . Thus, the constant molal overflow assumption is unnecessary.

Fenske<sup>(3)</sup> derived relations to calculate the minimum number of stages in single feed, two product fractionators. Constant relative volatility and constant molal overflow were the basic assumptions. The Fenske equation is derived as follows:

$$y_{LK} = K_{LK} x_{LK}$$

$$y_{HK} = K_{HK} x_{HK}$$

dividing:

$$\frac{y_{LK}}{y_{HK}} = \alpha_{LK-HK} \frac{x_{LK}}{x_{HK}} \quad (1)$$

Equation (1) may be converted to molar ratios

$$\left(\frac{v_{LK}}{v}\right) \left(\frac{v}{v_{HK}}\right) \alpha = \text{LK-HK} \left(\frac{l_{LK}}{L}\right) \left(\frac{L}{l_{HK}}\right)$$

hence:

$$\frac{v_{LK}}{v_{HK}} = \alpha_{LK-HK} \frac{l_{LK}}{l_{HK}} \quad (2)$$

Material balance around the column above the feed gives

$$v_{nLK} = l_{(n+1)LK} + d_{LK} \quad (3)$$

$$v_{nHK} = l_{(n+1)HK} + d_{HK} \quad (4)$$

At total reflux  $d_{LK}$  and  $d_{HK}$  are very small when compared to column internal stream flows.

Dividing equation (3) by equation (4), gives:

$$\frac{v_{nLK}}{v_{nHK}} = \frac{l_{(n+1)LK}}{l_{(n+1)HK}}$$

Substituting in equation (2) gives

$$\frac{l_{(n+1)LK}}{l_{(n+1)HK}} = \alpha_{LK-HK} \frac{l_{nLK}}{l_{nHK}} \quad (5)$$

Equation (5) relates the ratio of the mols of liquid of the light key and the heavy key components in the liquid on any tray to their ratio on the plate above. If these ratios were obtained from plate 1 through n the result would be

$$\left( \frac{1_{nLK}}{1_{nHK}} \right) = \alpha_{LK-HK}^n \left( \frac{1_{DLK}}{1_{DHK}} \right) \quad (6)$$

Thus, the exponent of  $\alpha$  is the number of perfect theoretical trays required to make the desired separation. Equation (6) may be rearranged to:

$$\alpha_{LK-HK}^n = \left( \frac{d}{b} \right)_{LK} \left( \frac{b}{d} \right)_{HK} \quad (7)$$

Which is the usual form of Fenske's equation.

Winn's relation for calculating the minimum number of stages at total reflux is similar to Fenske's equation. Winn found that if the K-values of two components were plotted on log-log coordinates at various temperatures, an essentially straight line resulted. With this fact and the fact that a straight line on log-log coordinates is expressed analytically by

$$K = \beta (K')^\theta \quad (8)$$

Where  $\beta$  and  $\theta$  are constants, Winn proceeded in a manner similar to Fenske. The resulting equation is

$$\beta_{LK-HK}^n = \left( \frac{x_D}{x_B} \right)_{LK} \left( \frac{x_B}{x_D} \right)_{HK}^\theta$$

This equation may be rearranged to give

$$\beta_{LK-HK}^n = \left( \frac{d}{b} \right)_{LK} \left( \frac{b}{d} \right)_{HK}^\theta \left( \frac{B}{D} \right)^{1-\theta} \quad (9)$$

which is similar to equation (7).  $\beta$  and  $\Theta$  are determined by writing two equations of the form of (8), one for the temperature at the top of the column and one for the bottom of the column.

$$K_{LK} = \beta (K_{HK})^{\Theta} \quad \text{at } T_D$$

$$K_{LK} = \beta (K_{HK})^{\Theta} \quad \text{at } T_B$$

Solving these equations simultaneously yields  $\Theta$ .  $\beta$  is determined by back substitution.

The Winn equation does not suffer from the assumption of constant relative volatility. Rather, it is limited only by the reliability of the K-data available, or if equation (8) does not adequately represent the K-value data.

Underwood<sup>(9)</sup>, derived an expression which is similar to that of Fenske and employed the same assumptions.

The Winn equation as it was originally derived was intended for single feed, two product fractionators. If, however, one considers a complex fractionator, Figure II, as being composed of several "sections", one section between each product stream, an expression similar to the Winn equation may be written. The section concept has been used successfully by Edmister<sup>(2)</sup>, in absorber calculations and in distillation calculations by absorption factor methods. In the case of an overhead product, a side product and a bottoms product, there would be two sections. The calculations are made from the distillate composition to the side draw composition, that is, calculating over section 1. If we assume equations of the type of (8) to be valid then the equation would be written

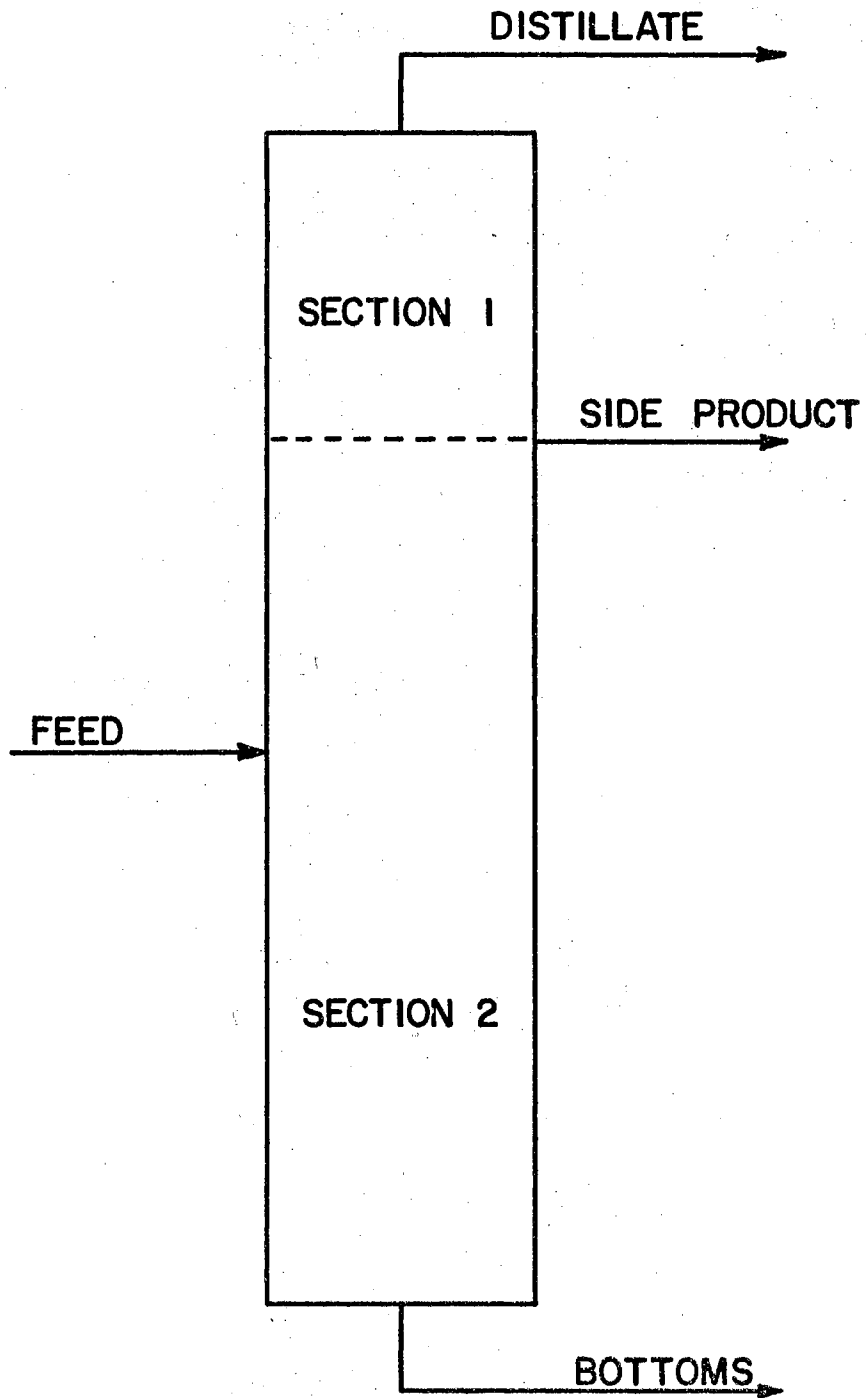


FIGURE 2  
COMPLEX COLUMN



$$\beta^{S_{M_1}} = \left(\frac{d}{p}\right)_{LK} \left(\frac{p}{b}\right)_{HK}^{\theta_1} \left(\frac{P}{D}\right)^{1-\theta_1} \quad (10)$$

for the section between the distillate and the side product and

$$\beta^{S_{M_2}} = \left(\frac{p}{b}\right)_{LK} \left(\frac{b}{p}\right)_{HK}^{\theta_2} \left(\frac{B}{P}\right)^{1-\theta_2} \quad (11)$$

for the section between the side product and the bottoms product.

These equations neglect the location of the feed plate in relation to the side product. As pointed out above, the location of the feed plate and feed condition have no effect on the total number of stages. Therefore, by analogy, the feed plate location should have no effect on the number of stages in a complex fractionator.

To extend the method to more than one side product and/or more than one feed, it is necessary only to write an additional equation of the form of (10) for each additional "section" of the column. Since the location of the feed plate has no effect on the number of stages, it follows that any number of feeds would be treated in the same way. In fact, the feeds may be summed and treated as one feed for calculation purposes. The material balance

$$d + p_1 + p_2 + \dots + p_n + b = f_1 + f_2 + \dots + f_n = \sum f_i = f_T \quad (12)$$

assumes that each feed will be introduced at the proper point in the column. The actual location of the feed entries will be considered later.

The total number of plates required at total reflux is the sum of the number of stages in each section. In addition to the total number of stages, the component distributions in each stream may also be calculated. To calculate the product distributions, equation (10)

may be written

$$\beta_i^{S_{M_1}} = \left(\frac{d}{p}\right)_i \left(\frac{p}{b}\right)_{HK}^{\theta_i} \left(\frac{P}{D}\right)^{1-\theta_i} \quad (13)$$

with the subscript,  $i$ , referring to any component, using the heavy key component as a base for the calculation of  $\beta_i$  and  $\theta_i$ .

A material balance around the column gives:

$$f_T = d + b + p_1 + p_2 + \dots + p_n \quad (12)$$

for each component.

Dividing both sides by  $p_1$  (if there are no side products divide by  $d$ ) gives:

$$\frac{f_T}{p_1} = 1 + \frac{d}{p_1} + \frac{b}{p_1} + \frac{p_2}{p_1} + \frac{p_3}{p_1} + \dots + \frac{p_n}{p_1} \quad (14)$$

The component distribution ratios as calculated by equation (13) will be

$$\frac{d}{p_1}, \frac{p_1}{p_2}, \frac{p_2}{p_3}, \dots, \frac{p_{n-1}}{p_n}, \frac{p_n}{b}$$

The ratios may be converted for use in (12) by noting that:

$$\frac{p_2}{p_1} = \frac{1}{\cancel{p_1}} \frac{p_1}{p_2}, \quad \frac{p_3}{p_1} = \frac{p_2}{p_1} \cdot \frac{p_3}{p_2}$$

rearranging (13) so that

$$p_1 = \frac{f_T}{1 + \frac{d}{p_1} + \frac{p_2}{p_1} + \frac{p_3}{p_1} + \dots + \frac{p_n}{p_1} + \frac{b}{p_1}} \quad (15)$$

or

$$P_1 = \frac{f_T}{1 + \frac{d}{P_1} + \frac{P_2}{P_1} + \left(\frac{P_2}{P_1}\right) \left(\frac{P_3}{P_2}\right) + \dots + \left(\frac{P_n}{P_{n-1}}\right) \left(\frac{b}{P_n}\right)} \quad (16)$$

$$\text{and } d = \left(\frac{d}{P_1}\right) P_1, \quad P_2 = \left(\frac{P_2}{P_1}\right) P_1, \quad P_3 = \left(\frac{P_3}{P_1}\right) P_1 \quad \text{etc.}$$

For a two product column, (15) reduces to

$$d = \frac{f_T}{1 + \frac{b}{d}}$$

To test the utility of the above equations a program for the IBM 650 Computer was written. The equations calculate the minimum number of stages for each section of the column thereby locating the position of the side product streams. Since the feed(s) are summed, it is desirable to determine the position of the feed(s) relative to the product streams.

Robinson and Gilliland<sup>(7)</sup> define an optimum intersection ratio,  $\phi$ , which relates the ratio of the compositions of the key components at the feed plate and the plate above. The ratio,  $\phi$ , is defined such that the optimum feed plate location is given by

$$\left(\frac{x_{LK}}{x_{HK}}\right)_f \leq \phi \leq \left(\frac{x_{LK}}{x_{HK}}\right)_{f+1} \quad (17)$$

where  $f+1$  is the plate above the feed plate. Since the calculations performed to find the minimum number of stages gives product distributions only, the feed plate location must be calculated on the basis of

stream compositions rather than stream flow rates. If the ratio to be compared with the feed ratio occurs (n) plates from the feed plate, equation (17) must comply with this stipulation. The Fenske equation indicates a convenient relationship which may be utilized.

Rewriting the Fenske equation for the section above the feed for a simple column gives

$$\alpha_{\text{LK-HK}}^{S_{\text{ME}}} = \left(\frac{d}{f}\right)_{\text{LK}} \left(\frac{f}{d}\right)_{\text{HK}}$$

where:  $\alpha_{\text{LK-HK}}^{S_{\text{ME}}}$  = minimum number of stages in the enriching section

This may be arranged so that:

$$\frac{d_{\text{LK}}}{d_{\text{HK}}} = \alpha_{\text{LK-HK}}^{S_{\text{ME}}} \frac{f_{\text{LK}}}{f_{\text{HK}}} \quad (18a)$$

or

$$\left(\frac{x_{\text{LK}}}{x_{\text{HK}}}\right)_{\text{D}} = \alpha_{\text{LK-HK}}^{S_{\text{ME}}} \left(\frac{x_{\text{LK}}}{x_{\text{HK}}}\right)_{\text{F}} \quad (18b)$$

Equation (18a) is similar to (17). Since the Winn  $\beta$  is similar to  $\alpha$ , a better relation would be:

$$\left(\frac{x_{\text{LK}}}{x_{\text{HK}}}\right)_{\text{D}} = \beta_{\text{E}}^{S_{\text{ME}}} \left(\frac{x_{\text{LK}}}{x_{\text{HK}}}\right)_{\text{F}}^{\Theta_{\text{E}}} \quad (19)$$

The above indicates that the feed should be introduced at a point in the column where the ratio of the key components is equal to their ratio in the feed. Equation (19) provides a method of calculating the number of stages below the top of the column that the feed should be introduced.

For a complex fractionator, the ratio of the key components in each feed may be checked against their ratio in each product stream. For example, consider a single feed, three-product column. The key component ratio in the feed must be less than the ratio in the distillate, and greater than the ratio in the side product, if the feed is to be introduced above the side product.

Symbolically:

$$\left( \frac{P_{LK}}{P_{HK}} \right) < \left( \frac{f_{LK}}{f_{HK}} \right) < \left( \frac{d_{LK}}{d_{HK}} \right) \quad \begin{array}{l} \text{feed between the side product} \\ \text{and distillate (Case 1)} \end{array}$$

$$\left( \frac{f_{LK}}{f_{HK}} \right) < \left( \frac{P_{LK}}{P_{HK}} \right) \quad \begin{array}{l} \text{feed below side product} \\ \text{(Case 2)} \end{array}$$

Obviously, the key component ratios in the feed cannot be greater than the ratio in the distillate or less than the ratio in the bottoms. The location of the feed entry in Case 1 would be found by

$$\beta_1^{S_{FE}} = \left( \frac{d_{LK}}{d_{HK}} \right) \left( \frac{f_{HK}}{f_{LK}} \right)^{\theta_1} \left( \frac{F}{D} \right)^{1-\theta_1}$$

and in Case 2 by

$S_{m_1} + S_{FP}$  where  $S_{FP}$  is computed from

$$\beta \frac{S_{FP}}{2} = \left( \frac{P_{LK}}{P_{HK}} \right) \left( \frac{f_{LK}}{f_{HK}} \right)^{\theta_2} \left( \frac{F}{P} \right)^{1-\theta_2} \quad (21)$$

Similar expressions and procedures apply to more than one feed and more than one side product.

## CHAPTER IV

### RESULTS

To determine the effect of feed plate location on product composition at total reflux would be extremely difficult using an actual column. However, total reflux may be simulated. The simulation may be accomplished by using a digital computer for which a plate-by-plate calculation program has been written.

It was found using the above simulation, with an internal vapor rate of 10,000 mols/hr, that there was a negligible effect of feed plate location on the distillate composition. This may be seen in the following table:

TABLE I

Effect of Feed Plate Location on the Distillate  
Composition at Total Reflux

<u>Feed Entry Point Plate No., Top Down</u>	<u>Mol Frn Light Key in Distillate</u>	<u>Mol Frn Heavy Key in Distillate</u>
3	0.7559	0.2437
4	0.7552	0.2442
5	0.7552	0.2446
7	0.7554	0.2443

The total number of theoretical stages was eleven.

The results of this study indicate that the assumption that the location of the feed had little or no effect on the composition

of the products is valid. Consequently, one may assume that the same negligible effect will occur if multiple feeds are summed and treated as one feed when dealing with complex fractionators.

As pointed out in the previous discussion of feed plate location, the feed should be introduced at a point in the column at which the ratio of the compositions of the key components in the feed and at the feed plate are equal.

This assumption was checked using the above simulation procedure with the same system and vapor rate. The results are listed in the following table.

TABLE II  
Optimum Feed Plate Location

Feed Entry Point (Plate No., Top Down)	Feed Plate $\frac{f_{LK}}{f_{HK}}$	Distillate $\frac{d_{LK}}{d_{HK}}$	Feed $\frac{f_{LK}}{f_{HK}}$
3	1.696	3.102	1
4	1.255	3.093	1
5	0.932	3.087	1
7	0.522	3.092	1

As may be seen in Table II, the point at which the ratio of the keys in the feed are equal to the ratio at the feed plate occurs between plates 4 and 5. The feed plate location calculated from equation (19) was 4.35. By analogy, one may assume that the location of more than one feed may also be calculated from equation (19).



Tables III and IV show the comparison between two complex fractionators calculated by the method of this work and a plate-by-plate calculation procedure. In both cases the total reflux condition was simulated in the plate-by-plate calculations by using a reflux ratio of  $(L_0/D) = 99.0$ .

The fractionator compared in Table III is a single feed, 3-product column. The column in Table IV is almost the same as the column in Table III except the feed was altered slightly and split into two streams. The side product in both cases was withdrawn as saturated liquid. The feeds in both cases were also saturated liquids. In both examples plates are numbered from the top plate in the column to the reboiler. That is, the top plate is 1 and the reboiler is 14.

These results show that the method of this work may be used to good advantage in the preliminary design of a complex fractionator. The method of calculating the location of feed plates has been shown to be reliable. In both of the cases investigated the new method correctly indicated the trays between which the feeds should be introduced. Until now no total reflux method has been available for evaluating alternate designs of complex fractionators. The new method is fast, easy to use and well suited to hand or desk calculator computations.

TABLE III  
 Comparison Between Proposed Method and Plate-by-Plate  
 Calculation for a Complex Column

Comp.	Feed	This Work			Plate-by-Plate		
		Distillate	Side-Draw	Bottoms	Distillate	Side-Draw	Bottoms
C <sub>2</sub>	1.38	1.12311	0.25689	0.00000	1.12309	0.25686	0.00003
C <sub>3</sub>	4.25	0.22631	3.89744	0.12625	0.22684	3.89661	0.12655
iC <sub>4</sub>	1.48	0.00035	0.13139	1.34826	0.00036	0.13139	1.34825
nC <sub>4</sub>	2.10	0.00002	0.02532	2.07466	0.00002	0.02769	2.07229
iC <sub>5</sub>	1.38	0.00000	0.00006	1.37994	0.00000	0.00016	1.37984
nC <sub>5</sub>	0.75	0.00000	0.00001	0.74999	0.00000	0.00003	0.74997
C <sub>6</sub>	2.25	0.00000	0.00000	2.25000	0.00000	0.00000	2.25000
Totals	13.59	1.34979	4.31111	7.92910	1.35031	4.31274	7.92693

(Cont.)

TABLE III (Cont.)

	This Work	Plate-By-Plate
$T_D$	518.098	518.195
$T_P$	591.913	591.901
$T_B$	756.757	756.854
$Sm_1$	3.90041	4-5*
$Sm_2$	9.53652	9-10*
$Sm_T$	13.43693	14 <sup>+</sup>
$S_{FP}$	8.28286	8-9*

\* Indicates Stream Withdrawn or Feed Between Trays

+ Includes Reboiler, Excludes Total Condenser

TABLE IV  
 Comparison Between Proposed Method and Plate-by-Plate  
 Calculation for a Complex Column

Comp.	Feed 1	Feed 2	This Work			Plate-By-Plate		
			Distillate	Side-Draw	Bottoms	Distillate	Side-Draw	Bottoms
C <sub>2</sub>	0.92	0.46	1.12312	0.25688	0.0000	1.12310	0.25686	0.00004
C <sub>3</sub>	2.95	1.30	0.22683	3.89603	0.12714	0.22683	3.89603	0.12714
iC <sub>4</sub>	0.48	1.00	0.00034	0.12662	1.35304	0.00035	0.12663	1.35302
nC <sub>4</sub>	1.40	0.70	0.00002	0.02405	2.07593	0.00003	0.03245	2.06752
iC <sub>5</sub>	0.82	0.46	0.00000	0.00005	1.27995	0.00000	0.00062	1.27938
nC <sub>5</sub>	0.50	0.25	0.00000	0.00001	0.74999	0.00000	0.00018	0.74982
C <sub>6</sub>	1.5	0.75	0.00000	0.00000	2.25000	0.00000	0.00000	2.25000
Totals	8.57	4.92	1.35031	4.30364	7.83605	1.35031	4.31277	7.82691

(Cont.)

TABLE IV (Cont.)

	This Work	Plate-By-Plate
$T_D$	518.144	518.192
$T_P$	591.848	591.927
$T_B$	756.715	756.627
$Sm_1$	3.89782	4-5*
$Sm_2$	9.58008	9-10*
$Sm_T$	13.47790	14 <sup>+</sup>
$S_{FP1}$	6.89218	6-7*
$S_{FP2}$	9.43945	9-10*

\* Indicates Stream Withdrawn or Fed Between Trays

+ Includes Reboiler, Excludes Total Condenser

## CHAPTER V

## CONCLUSIONS

The new equation for complex fractionators will provide the design engineer with a short, reliable method of estimating the performance of complex columns operating at total reflux. The method will give estimates of the component distributions in the various product streams and the rates of those streams as well as the relative locations of the product and feed streams.

The assumption that the feed streams may be summed and treated as one feed is valid because it was shown that the location of the feed at total reflux had a negligible effect on the composition of the product streams.

The Winn method for representing equilibrium data is probably better than the assumption of constant relative volatility for a section of the column. For either case the proposed method is a preliminary estimate only. For final designs a more rigorous technique such as plate-by-plate calculation must be used.

## LIST OF NOMENCLATURE

- B - total mols of bottom product stream
- D - total mols of distillate product stream
- F - total mols of feed stream
- K - equilibrium constant,  $\frac{Y}{x}$
- L - total mols of liquid stream
- N - number of actual theoretical stages
- P - total mols of side stream
- S - minimum number of theoretical stages
- V - total mols of vapor stream
- 
- b - mols of a component in bottom product stream
- d - mols of a component in distillate product stream
- f - mols of a component in a feed stream
- l - mols of a component in a liquid stream
- p - mols of a component in a side stream
- v - mols of a component in a vapor stream
- x - mol fraction of a component in liquid
- y - mol fraction of a component in vapor

Greek

$\alpha$  - relative volatility,  $\frac{K_i}{K_R}$

$\beta$  - relative operability in Gilliland equation or a characteristic constant in the Winn equation

$\theta$  - a characteristic constant in the Winn equation or roots in the Underwood equation

$\phi$  - roots in the Underwood equation

Subscripts

B - refers to bottom plate in column or bottoms product

D - refers to distillate

E - enriching section

F - refers to feed streams or feed plate

LK - light key component

HK - heavy key component

M - minimum

T - refers to top plate in column

b - component in the bottoms product

d - component in the distillate product

f - component in the feed stream or feed plate

i - any component

m - refers to plate in stripping section

n - refers to plate in the enriching section

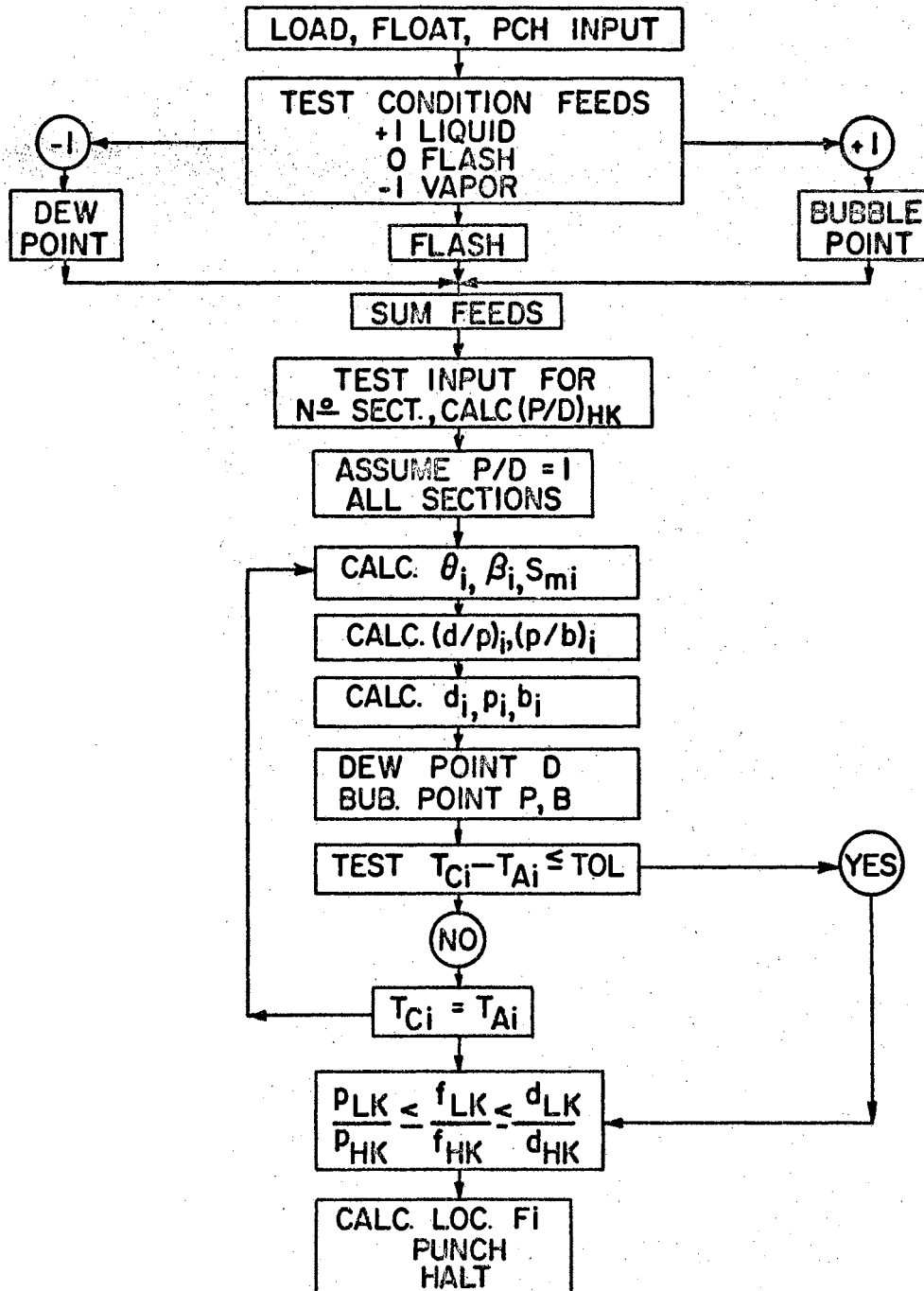


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APPENDIX A

## BLOCK DIAGRAM—MAIN PROGRAM



## SUBROUTINES

SR-1, Punch (PCH)

LDD n-1

STD n-1

RAL P-Pch

LDD EXIT  $\rightarrow$  Pch

SR-2, Block transfer (BT)

SET 9040

LDI 2000

SET 9040

STI 4000

SR-3, K-Evaluation (K-eval)

SET

LDI

RAB n-1

STU T

(A-1) RAU ( $d_i$  of K) B

FMP T

FAD ( $C_i$  of K) B

FMP T

FAD ( $b_i$  of K) B

FMP T

## SUBROUTINES (Cont.)

FAD  $(A_i \text{ of } K) B$   
 NZB  $\rightarrow$  EXIT  
 SXB-1 Go A-1  
  
 SR-4, Mol Fraction (MF)  
 SET  
 LDI  
 RAB n-1  
 LDD 0  
 STD  $\sum 1_i$   
 STD  $\sum \mathcal{V}_i$   
 (-) A-2  
 (+) A-3  
 (A-1) BMC  
 (A-2) RAU  $(\mathcal{V}_i) B$   
 FAD  $\sum \mathcal{V}_i$   
 STU  $\sum \mathcal{V}_i \rightarrow$  A-4  
 (A-3) RAU  $(1_i) B$   
 FAD  $\sum 1_i$   
 STU  $\sum 1_i \rightarrow$  (A-4)  
 (A-4) NZB  $\rightarrow$  (A-5)  
 SXB-1  
 (A-5) RAB n-1

## SUBROUTINES (Cont.)

	(-)	(A-7)
(A-6) BMC	(+)	(A-8)
(A-7) RAU	( $\mathcal{V}$ )	B
FDV	$\sum \mathcal{V}_i$	
STU	( $y_i$ )	B $\longrightarrow$ (A-9)
(A-8) RAU	( $l_i$ )	B
FDV	$\sum l_i$	
STU	( $x_i$ )	B $\longrightarrow$ (A-9)
NZE	EXIT	
SXB-1	$\longrightarrow$	(A-6)
SR-5	Bubble Pt. - Dew Pt.	(BP-DP)
SET		
LDI		
LDD	EXIT	
STD	EXIT	
LDD	0	
STD	$\sum K_i x_i$	or $\sum \frac{y_i}{K_i}$
(A-10) STU	T	
RAC	$\longrightarrow$	SR-3 (K-eval)
RAB	n-1	
(A-1) RAU	EXIT	
	(-)	(A-2)
BMI	(+)	(A-3)

## SUBROUTINES (Cont.)

(A-2)	RAU	$(y_i)$	B
	FDV	$(K_i)$	B
	STU	$(\frac{y_i}{K_i})$	B
	FAD	$\sum \frac{y_i}{K_i}$	
	STU	$\sum \frac{y_i}{K_i}$	$\rightarrow$ (A-4)
(A-3)	RAU	$(x_i)$	B
	FMP	$(K_i)$	B
	STU	$(K_i x_i)$	B
	FAD	$\sum K_i x_i$	
	STU	$\sum K_i x_i$	
(A-4)	NZA		$\rightarrow$ (A-5)
	SXA-1		$\rightarrow$ (A-1)
(A-5)	RAU	1	
	FSB	$\sum K_i x_i$ or $\sum \frac{y_i}{K_i}$	
	STU	$1 - \sum K_i x_i$ or $(\Delta)$	
	RAU	Tolerance	
	FSM	$\Delta$	
		(-) (A-6)	
	BMI	(+) EXIT	

## SUBROUTINES (Cont.)

(A-6) RAU EXIT

BMI (-) (A-7)  
 (+) (A-8)

(A-7) RSU  $1 - \sum \frac{y}{K} \rightarrow (A-9)$ (A-8) RAU  $1 - \sum K_x$ 

(A-9) FDV 7.5

FAD 1

FMP T  $\rightarrow (A-10)$ SR-6,  $\theta_i$ 

RAB n-1

STD EXIT

(A-1) RAU  $(K_{HK})_T^C$ FDV  $(K_{HK})_B^C$ LDD  $\rightarrow \ln X$ STU  $\ln \frac{(K_{HK})_T}{(K_{HK})_B}$ RAU  $(K_i)_T^B$ FDV  $(K_i)_B^B$ LDD  $\rightarrow \ln X$ FDV  $\ln \frac{(K_{HK})_T}{(K_{HK})_B}$



## SUBROUTINES (Cont.)

STU  $(\theta_i)$  BNZB  $\rightarrow$  EXITSXB-1  $\rightarrow$  (A-1)SR-7,  $\beta_i$ 

RAB n-1

STD EXIT

(A-1) RAU  $(K_{HK})$  CLDD  $\rightarrow$   $\ln X$ FMP  $(\theta_i)$  BLDD  $\rightarrow$   $e^x$ STU  $(K_{HK})^{\theta_i}$ RAU  $(K_i)$  BFDV  $(K_{HK})^{\theta_i}$ STU  $(\beta_i)$  B

NZB EXIT

SXB-1  $\rightarrow$  (A-1)SR-8,  $S_m$ 

STD EXIT

RAU  $d_{LK}$ FDV  $P_{LK}$

## SUBROUTINES (Cont.)

STU  $\left(\frac{d}{p}\right)_{LK}$   
 RAU  $\left(\frac{p}{d}\right)_{HK} B$   
 LDD  $\longrightarrow \ln X$   
 FMP  $(\theta) C$   
 LDD  $\longrightarrow e^x$   
 STU  $\left(\frac{p}{d}\right)_{HK}^{\theta}$   
 RAU 1  
 FSB  $(\theta) C$   
 STU  $1-\theta$   
 RAU  $\frac{P}{D}$   
 LDD  $\longrightarrow \ln X$   
 FMP  $1 - \theta$   
 LDD  $\longrightarrow e^x$   
 FMP  $\left(\frac{p}{d}\right)_{HK}^{\theta}$   
 FMP  $\left(\frac{d}{p}\right)_{LK}$   
 LDD  $\longrightarrow \ln X$   
 STU  $\ln X$   
 RAU  $\beta$   
 LDD  $\longrightarrow \ln X$   
 STU  $\ln \beta$

## SUBROUTINES (Cont.)

RAU  $\ln X$   
 FDV  $\ln \beta$   
 STU  $S_m \rightarrow$  EXIT

SR-9,  $\left(\frac{d}{p}\right)_i$

RAB  $n-1$   
 STU EXIT  
 (A-1) RAU  $\left(\frac{P}{d}\right)_{HK}$   
 LDD  $\ln X$   
 FMP  $(\theta) B$   
 LDD  $\rightarrow e^x$   
 STU  $\left(\frac{P}{d}\right)_{HK}^{\theta}$   
 RAU  $\beta$   
 LDD  $\rightarrow \ln X$   
 FMP  $S_m$   
 LDD  $\leftarrow e^x$   
 STU  $\beta^{S_m}$   
 RAU 1  
 FSB  $(\theta) B$   
 STU  $1 - \theta$   
 RAU  $\frac{P}{D}$   
 LDD  $\ln X$

## SUBROUTINES (Cont.)

FMP  $1 - \theta$   
 LDD  $\longrightarrow e^x$   
 STU  $\left(\frac{P}{D}\right)^{1 - \theta}$   
 RAU  $\beta^{S_m}$   
 FDV  $\left(\frac{P}{d}\right)_{HK}^{\theta}$   
 FDV  $\left(\frac{P}{D}\right)^{1 - \theta}$   
 STU  $\left(\frac{d}{P}\right)_i B$   
 NZB EXIT  
 SXB-1  $\longrightarrow (A-1)$

SR-10, S (Feed Plate Loc.)

STD EXIT  
 RAU  $\beta$   
 LDD  $\longrightarrow \ln X$   
 STU  $\ln \beta$   
 RAU 1  
 FSB  $\theta$   
 STU  $1 - \theta$   
 RAU F  
 FDV P  
 LDD  $\longrightarrow \ln X$   
 FMP  $1 - \theta$

## SUBROUTINES (Cont.)

LDD  $\longrightarrow e^x$   
 STU  $\left(\frac{F}{P}\right)^{1-\theta}$   
 RAU 1  
 FDV Rf  
 LDD  $\longrightarrow \ln X$   
 FMP  $\theta$   
 LDD  $\longrightarrow e^x$   
 FMP  $R_p$   
 FMP  $\left(\frac{F}{P}\right)^{1-\theta}$   
 LDD  $\longrightarrow \ln X$   
 FDV  $\ln \beta \longrightarrow \text{EXIT}$

## MAIN PROGRAM

RAL n  
 SRT 1  
 SLO .1  
 STL  $(n-1)_1$   
 SRT 4  
 STL  $(n-1)_2$   
 RAL  $(n-1)_1$   
 RAA 10  
 (A-1) LDD  
 SDA  
 NZA (A-2)  
 SXA-1 (A-1)  
 (A-2) RAL No. of Sections  
 SRT 1  
 SLO .1  
 STL  $N_{s-1}$   
 RAL No. of Feeds  
 SRT 1  
 SLO .1  
 STL  $N_{F-1}$   
 SET

## MAIN PROGRAM (Cont.)

```

        LDI   Float Loop
        RAC   250
        RAA   n - 1
        SET
(A-5)  LDI   K's & F's
        RAB   (A-3) → Float
(A-3)  SET
        STI   Floated Data
        NZC   → (A-4)
        SXC-SO → (A-5)
(A-4)  RAA   No. of Key Nos (NKi)
(A-7)  RAL   (NKi) A
        SRT   5
        STL   (NKi) A
        NZA   (A-6)
        SXA-1 (A-7)
(A-6)  RAA   Input
        SET
        LDI   Input Data
        RAB   → Float
        SET
        STI   Floated Input
        LDD   P-Pch
        STD   P-Pch
        RAC   → Pch

```

## MAIN PROGRAM (Cont.)

	LDD	P-Pch	
	STD	P-Pch	
	RAC	→ Pch	
	RAU	$(p_i)_{HK}$	
	NZU	(A-8)	
	FDV	$(d)_{HK}$	
	STU	→ (A-9)	
(A-8)	RAU	$(b)_{HK}$	
	FDV	$(d)_{HK}$	
	STU	$(\frac{b}{d})_{HK}$ → READ	
(A-9)	RAU	$(p_2)_{HK}$	
	NZU	→ (A-10)	
	FDV	$(p_1)_{HK}$	
	STU	$(\frac{p_2}{p_1})_{HK}$ → (A-11)	
(A-10)	RAU	$(b)_{HK}$	
	FDV	$(p_1)_{HK}$	
	STU	$(\frac{b}{p_1})_{HK}$ → READ	
(A-11)	RAU	$(p_3)_{HK}$	
	NZU	→ (A-12)	



## MAIN PROGRAM (Cont.)

	FDV	$(p_2)_{HK}$	
	STU	$\left(\frac{p_3}{p_2}\right)_{HK}$	$\longrightarrow (A-13)$
(A-12)	RAU	$(b)_{HK}$	
	FDV	$(p_2)_{HK}$	
	STU	$\left(\frac{b}{p_2}\right)_{HK}$	$\longrightarrow \text{READ}$
(A-13)	RAU	$(b)_{HK}$	
	FDV	$(p_3)_{HK}$	
	STU	$\left(\frac{b}{p_3}\right)_{HK}$	$\longrightarrow \text{READ}$
	RAA	19	
(A-14)	RAU	$(f_i)_1^A$	
	FAD	$(f_i)_2^A$	
	FAD	$(f_i)_3^A$	
	STU	$(\sum f_i) A$	
	NZA		$\longrightarrow (A-15)$
	SKA-1		$\longrightarrow (A-14)$
(A-15)	RAU	$T_b$	
	RAC		$\longrightarrow \text{K-Eval}$
	RAA	Loc $K_b$	

## MAIN PROGRAM (Cont.)

RAB  $\text{Loc}_2 K_b$   
 RAC  $\longrightarrow$  BT  
 RAU  $T_{p3}$   
 NZU (A-16)  
 RAC  $\longrightarrow$  K-Eval  
 RAA  $\text{Loc } K_{p3}$   
 RAB  $\text{Loc}_2 K_{p3}$   
 RAC  $\longrightarrow$  BT  
 LDD  $HK_4$   
 RAC  $HK_4$   
 LDD  $\longrightarrow \theta$   
 SET  
 STI  $\theta_4$   
 RAA  $\text{loc } K_T$   
 RAB  $\text{Loc}_2 K_T$   
 RAC  $\longrightarrow$  BT  
 RAU  $T_{p2}$   
 RAC  $\longrightarrow$  K-Eval  
 LDD  $HK_3$   
 RAC  $HK_3$   
 LDD  $\longrightarrow \theta$   
 SET

## MAIN PROGRAM (Cont.)

```

STI   $\theta_3$ 

RAA  Loc K

RAB  Loc2K

RAC   $\longrightarrow$  BT

RAU  TP1

RAC   $\longrightarrow$  K-Eval

LDD  HK2

RAC  HK2

LDD   $\longrightarrow$   $\theta$ 

SET

STI   $\theta_2$ 

RAA  LocK

RAB  Loc2K

RAC   $\longrightarrow$  BT

RAU  Td

RAC   $\longrightarrow$  K-Eval

LDD  HK1

RAC  HK1

LDD   $\longrightarrow$   $\theta$ 

SET

STI   $\theta_1$ 

RAU  TP1

RAC   $\longrightarrow$  K-Eval

```

## MAIN PROGRAM (Cont.)

```
SET
LDI   $\theta_1$ 
LDD  HK1
RAC  HK1
LDD   $\longrightarrow \beta$ 
SET
STI   $\beta_1$ 
RAV  Tp2
RAC   $\longrightarrow$  K-Eval
SET
LDI   $\theta_2$ 
LDD  HK2
RAC  HK2
LDD   $\longrightarrow \beta$ 
SET
STI   $\beta_2$ 
RAU  Tp3
RAC   $\longrightarrow$  K-Eval
SET
LDI   $\theta_3$ 
LDD  HK3
RAC  HK3
LDD   $\longrightarrow \beta$ 
```

## MAIN PROGRAM (Cont.)

```

SET
STI     $\beta_3$ 
RAU     $T_b$ 
RAC     $\longleftarrow$  K-Eval
SET
LDI     $\theta_4$ 
LDD     $HK_4$ 
RAC     $HK_4$ 
LDD     $\longrightarrow$   $\beta$ 
SET
STI     $\longrightarrow$   $\beta_4$ 
LDD    1
STD    P
STD    D
LDD     $d_{LK}$ 
STD     $d_{LK}$ 
RAU     $(\bar{p}_1)_{LK}$ 
NZU     $\longrightarrow$  (A-16)
STU     $(p_1)_{LK}$ 
(A-20) RAA  loc  $\theta_1$ 
RAB    loc2  $\theta_1$ 
RAC     $\longrightarrow$  BT

```

## MAIN PROGRAM (Cont.)

RAA loc  $\beta_1$   
 RAB loc<sub>2</sub>  $\beta_1$   
 RAC  $\longrightarrow$  BT  
 LDD Section No.  
 RAB Section No.  
 LDD HK<sub>1</sub>  
 RAC HK<sub>1</sub>  
 LDD  $\longrightarrow$  S<sub>m</sub>  
 STU Sm<sub>1</sub>  
 RAU ( $p_1$ )LK  
 NZU  $\longrightarrow$  (A-19)  
 STU ( $p_1$ )LK  
 RAU ( $p_2$ )LK  
 NZU  $\longrightarrow$  (A-17)  
 (A-21) STU ( $p_2$ )LK  
 RAA loc  $\theta_2$   
 RAB loc<sub>2</sub>  $\theta_2$   
 RAC  $\longrightarrow$  BT  
 RAA loc  $\beta_2$   
 RAB loc<sub>2</sub>  $\beta_2$   
 RAC  $\longrightarrow$  BT  
 LDD Section No.  
 RAB Section No.

## MAIN PORGRAM (Cont.)

LDD  $HK_2$   
 RAC  $HK_2$   
 LDD  $\longrightarrow S_m$   
 STU  $S_{m2}$   
 RAU  $(p_2)_{LK}$   
 NZU  $\longrightarrow (A-19)$   
 STU  $(p_2)_{LK}$   
 RAU  $(p_3)_{LK}$   
 NZU  $\longrightarrow (A-18)$   
 (A-22) STU  $(p_3)_{LK}$   
 RAA  $loc \theta_3$   
 RAB  $loc_2 \theta_3$   
 RAC  $\longrightarrow BT$   
 RAA  $loc \beta_3$   
 RAB  $loc_2 \beta_3$   
 RAC  $\longrightarrow BT$   
 LDD Section No.  
 RAB Section No.  
 LDD  $HK_3$   
 RAC  $HK_3$   
 LDD  $\longrightarrow S_m$   
 STU  $S_{m3}$

## MAIN PROGRAM (Cont.)

RAU  $(p_3)_{LK}$   
 NZU  $\longrightarrow (A-19)$   
 STU  $(p_3)_{LK}$   
 LDD  $(b)_{LK}$   
 STU  $(b)_{LK}$   
 RAA  $loc \theta_4$   
 RAB  $loc_2 \theta_4$   
 RAC  $\longleftarrow BT$   
 RAA  $loc \beta_4$   
 RAB  $loc_2 \beta_4$   
 RAC  $\longrightarrow BT$   
 LDD Section No.  
 RAB Section No.  
 LDD  $HK_4$   
 RAC  $HK_4$   
 LDD  $\longrightarrow S_m$   
 STU  $S_{m4} \longrightarrow (A-2)$   
 (A-16) RAU  $(b)_{LK}$   
 STU  $(b)_{LK} \longrightarrow (A-20)$   
 (A-17) RAU  $(b)_{LK}$   
 STU  $(b)_{LK} \longrightarrow (A-21)$



## MAIN PROGRAM (Cont.)

(A-18) RAU  $(b)_{LK}$   
 STU  $(b)_{LK} \longrightarrow$  (A-22)

(A-19) RAA  $loc \theta_1$   
 RAB  $loc_2 \theta_1$   
 RAC  $\longrightarrow$  BT  
 RAA  $loc \beta_1$   
 RAB  $loc_2 \beta_1$   
 RAC  $\longrightarrow$  BT  
 LDD Section No.  
 RAC Section No.  
 LDD  $S_{m1}$   
 STD  $S_{m1}$   
 LDD  $\longrightarrow \left(\frac{d}{p}\right)_i$   
 RAA  $loc \left(\frac{d}{p_1}\right)_i$   
 RAB  $loc_2 \left(\frac{d}{p_1}\right)_i$   
 RAC  $\longrightarrow$  BT  
 RAU  $S_{m2}$   
 NZU  $\longrightarrow$  (A-23)

(A-27) STU  $S_{m2}$   
 RAA  $loc \theta_2$

## MAIN PROGRAM (Cont.)

RAB  $\text{loc}_2 \theta_2$   
 RAC  $\longleftrightarrow$  BT  
 RAA  $\text{loc } \beta_2$   
 RAB  $\text{loc}_2 \beta_2$   
 RAC  $\longrightarrow$  BT  
 LDD Section No.  
 RAC Section No.  
 LDD  $\longrightarrow \left(\frac{d}{p}\right)_i$   
 RAA  $\text{loc } \left(\frac{p_1}{p_2}\right)_i$   
 RAB  $\text{loc}_2 \left(\frac{p_1}{p_2}\right)_i$   
 RAC  $\longrightarrow$  BT  
 RAU  $S_{m3}$   
 NZU  $\longrightarrow$  (A-24)  
 (A-28) STU  $S_{m3}$   
 RAA  $\text{loc } \theta_3$   
 RAB  $\text{loc}_2 \theta_3$   
 RAC  $\longrightarrow$  BT  
 RAA  $\text{loc } \beta_3$   
 RAB  $\text{loc}_2 \beta_3$   
 RAC  $\longrightarrow$  BT

## MAIN PROGRAM (Cont.)

LDD Section No.  
 RAC Section No.  
 LDD  $\rightarrow \left(\frac{d}{p}\right)_i$   
  
 RAA  $\text{loc} \left(\frac{p_2}{p_3}\right)_i$   
  
 RAB  $\text{loc}_2 \left(\frac{p_2}{p_3}\right)_i$   
  
 RAC  $\rightarrow$  BT  
  
 RAU  $S_{m4}$   
  
 NZU  $\rightarrow$  (A-25)  
  
 (A-29) STU  $S_{m4}$   
  
 RAA  $\text{loc} \theta_4$   
  
 RAB  $\text{loc}_2 \theta_4$   
  
 RAC  $\rightarrow$  BT  
  
 RAA  $\text{loc} \beta_4$   
  
 RAB  $\text{loc}_2 \beta_4$   
  
 RAC  $\rightarrow$  BT  
  
 LDD Section No.  
 RAC Section No.  
 LDD  $\rightarrow \left(\frac{d}{p}\right)_i$   
  
 RAA  $\text{loc} \left(\frac{p_3}{b}\right)_i$

## MAIN PROGRAM (Cont.)

RAB  $\text{loc}_2 \left(\frac{p_3}{b}\right)_i$   
 RAC  $\longrightarrow$  BT  $\longrightarrow$  (A-26)  
 (A-23) LDD  $\frac{p_2}{p_1}$   
 STD  $\longleftarrow$  (A-27)  
 (A-24) LDD  $\frac{p_3}{p_2}$   
 STD  $\longrightarrow$  (A-28)  
 (A-25) LDD  $\frac{B}{p_3}$   
 STD  $\longleftarrow$  (A-29)  
 RAB  $n - 1$   
 (A-26) RAU  $1$   
 FAD  $\frac{d}{p_1}$   
 STU  $1 + \frac{d}{p_1}$   
 RAU  $S_{m2}$   
 NZU  $\longrightarrow$  (A-30)  
 RAU  $1$   
 FDV  $\left(\frac{p_1}{p_2}\right)_i B$   
 STU  $\left(\frac{p_2}{p_1}\right)_i$   
 RAU  $S_{m3}$

## MAIN PROGRAM (Cont.)

NZU  $\longrightarrow$  (A-31)

RAU 1

FDV  $\left(\frac{p_2}{p_3}\right)_i B$

STU  $\left(\frac{p_3}{p_2}\right)_i$

RAU  $S_{m4}$

NZU  $\longrightarrow$  (A-32)

RAU  $\left(\frac{p_2}{p_1}\right)_i$

FMP  $\left(\frac{p_3}{p_2}\right)_i$

STU  $\left(\frac{p_3}{p_1}\right)_i$

FDV  $\left(\frac{p_3}{b}\right)_i B$

STU  $\left(\frac{b}{p_1}\right)_i$

FAD  $\left(\frac{p_3}{p_1}\right)_i$

FAD  $\left(\frac{p_2}{p_1}\right)_i$

FAD  $1 + \frac{d}{p_1}$

STU  $\Sigma$

RAU  $(f_i) B$

## MAIN PROGRAM (Cont.)

	FDV	$\Sigma$	
	STU	$(p_1)_i \rightarrow$	(A-33)
(A-30)	RAU	$(f_i) B$	
	FDV	$1 + \frac{d}{p_1}$	
	STU	$(b_i) B$	
	RSU	$(b_i) B$	
	FAD	$(f_i) B$	
	STU	$d_i \rightarrow$	(A-33)
(A-31)	RAU	$1 + \frac{d}{p_1}$	
	FAD	$\frac{b}{p_1}$	
	STU	$\Sigma$	
	RAU	$(f_i) B$	
	FDV	$\Sigma$	
	STU	$(p_1)_i B$	
	RAU	$(\frac{b}{p_1})$	
	FMP	$(p_1)_i B$	
	STU	$(b_i) B$	
	RAU	$(\frac{d}{p_1})_i B$	
	FMP	$(p_1)_i B$	

## MAIN PROGRAM (Cont.)

STU  $(d_i) B \rightarrow (A-33)$   
 (A-32) RAU  $\left(\frac{p_2}{p_1}\right)$   
 FMP  $\left(\frac{b}{p_2}\right)$   
 STU  $\left(\frac{b}{p_1}\right)$   
 FAD  $\left(\frac{p_2}{p_1}\right)$   
 FAD  $1 + \frac{d}{p_1}$   
 STU  $\sum$   
 RAU  $(f_i) B$   
 FDV  $\sum$   
 STU  $(p_1)_i B$   
 RAU  $\left(\frac{b}{p_1}\right)$   
 FMP  $(p_1)_i B$   
 STU  $(b_i) B$   
 RAU  $\left(\frac{p_2}{p_1}\right)$   
 FMP  $(p_1)_i B$   
 STU  $(p_2)_i B$   
 RAU  $\left(\frac{d}{p_1}\right) B$   
 FMP  $(p_1)_i$   
 STU  $(d_i) B$

## MAIN PROGRAM (Cont.)

(A-33) NZB (A-34)  
 SXB-1  $\longrightarrow$  (A-26)

(A-34) RAA loc d  
 RAB loc<sub>2</sub> d  
 RAC  $\longrightarrow$  BT  
 RSC  $\longrightarrow$  MF  
 RAU T<sub>d</sub>  
 RSC  $\longrightarrow$  DP  
 LDD T<sub>d</sub>  
 STD T<sub>d</sub>  
 RAU P<sub>1</sub>  
 NZU  $\longrightarrow$  (A-35)  
 RAA loc p<sub>1</sub>  
 RAB loc<sub>2</sub>p<sub>1</sub>  
 RAC  $\longrightarrow$  BT  
 RAC  $\longrightarrow$  MF  
 RAU T<sub>P<sub>1</sub></sub>  
 RAC  $\longrightarrow$  BP  
 LDD T<sub>P<sub>1</sub></sub>  
 STD T<sub>P<sub>1</sub></sub>  
 RAU P<sub>2</sub>  
 NZU  $\longrightarrow$  (A-35)



## MAIN PROGRAM (Cont.)

RAA	loc $p_2$
RAB	loc <sub>2</sub> $p_2$
RAC	→ BT
RAC	→ MF
RAU	T <sub><math>p_2</math></sub>
RAC	→ BP
LDD	T <sub><math>p_2</math></sub>
STD	T <sub><math>p_2</math></sub>
RAU	$p_3$
NZU	→ (A-35)
RAA	loc $p_3$
RAB	loc <sub>2</sub> $p_3$
RAC	→ BT
RAC	→ MF
RAU	T <sub><math>p_3</math></sub>
RAC	→ BP
LDD	T <sub><math>p_3</math></sub>
STD	T <sub><math>p_3</math></sub>
(A-35) RAA	loc b
RAB	loc <sub>2</sub> b
RAC	→ BT

## MAIN PROGRAM (Cont.)

RAC  $\longrightarrow$  MF  
 RAU  $T_b$   
 RAC  $\longrightarrow$  BP  
 LDD  $T_b$   
 STD  $T_b$   
 RAA 4  
 (A-38) RAU  $(T_{d_1}) A$   
 FSB  $(T_{d_2}) A$   
 STU  $\Delta T$   
 RAU tolerance  
 FSM  $\Delta T$   
 BMI  $\longrightarrow$  (A-36)  
 NZA (A-37)  
 SXA-1 (A-38)  
 (A-37) LDD D  
 STD D  
 LDD  $P_1$   
 STD  $P_1$   
 LDD  $P_2$   
 STD  $P_2$   
 LDD  $P_3$   
 STD  $P_3$

## MAIN PROGRAM (Cont.)

```
LDD  B
STU  B
LDD  P-Pch
STD  P-Pch
RAC   $\longleftrightarrow$  Pch
LDD  0
STD  0
RAL  P-Pch
LDD   $\longleftrightarrow$  Pch
LDD  Td
STD  Td
LDD  TP1
STD  TP1
LDD  TP2
STD  TP2
LDD  TP3
STD  TP3
LDD  Tb
STD  Tb
LDD  0
STD  0
```

## MAIN PROGRAM (Cont.)

RAL P-Pch  
 LDD  $\longrightarrow$  Pch  $\longrightarrow$  load  
 LDD LK<sub>1</sub>  
 RAC LK<sub>1</sub>  
 LDD HK<sub>1</sub>  
 RAA HK<sub>1</sub>  
 RAU (d<sub>i</sub>) C  
 FDV (d<sub>i</sub>) A  
 STU  $\left(\frac{d_{LK}}{d_{HK}}\right)$   
 RAU (f<sub>i</sub>)<sub>1</sub> C  
 FDV (f<sub>i</sub>)<sub>1</sub> A  
 STU  $\left(\frac{f_{LK}}{f_{HK\ 1}}\right)$   
 RAU (f<sub>i</sub>)<sub>2</sub> C  
 NZU  $\longrightarrow$  (A-39)  
 FDV (f<sub>i</sub>)<sub>2</sub> A  
 STU  $\left(\frac{f_{LK}}{f_{HK\ 2}}\right)$   
 RAU (f<sub>i</sub>)<sub>3</sub> C  
 NZU  $\longrightarrow$  (A-39)  
 FDV (f<sub>i</sub>)<sub>3</sub> A

## MAIN PROGRAM (Cont.)

STU	$\left(\frac{f_{LK}}{f_{HK}}\right)_3$
(A-39) RAU	$LK_2$
NZU	$\longrightarrow$ (A-40)
RAC	$LK_2$
LDD	$HK_2$
RAA	$HK_2$
RAU	$(p_i)_1^C$
FDV	$(p_i)_1^A$
STU	$\left(\frac{P_{LK}}{P_{HK}}\right)_1$
RAU	$(f_i)_1^C$
FDV	$(f_i)_1^A$
STU	$\left(\frac{f_{LK}}{f_{HK}}\right)_i$
RAU	$(f_i)_2^C$
NZU	$\longrightarrow$ (A-41)
FDV	$(f_i)_2^A$
STU	$\left(\frac{f_{LK}}{f_{HK}}\right)_2$
RAU	$(f_i)_3^C$
NZU	$\longrightarrow$ (A-41)
FDV	$(f_i)_3^A$

## MAIN PROGRAM (Cont.)

STU	$\left(\frac{f_{LK}}{f_{HK}}\right)_3$
(A-41) RAU	$LK_3$
NZU	$\longrightarrow$ (A-40)
RAC	$LK_3$
LDD	$HK_3$
RAA	$HK_3$
RAU	$(p_i)_2^C$
FDV	$(p_i)_2^A$
STU	$\left(\frac{P_{LK}}{P_{HK}}\right)_2$
RAU	$(f_i)_1^C$
FDV	$(f_i)_1^A$
STU	$\left(\frac{f_{LK}}{f_{HK}}\right)_1$
RAU	$(f_i)_2^C$
NZU	$\longleftrightarrow$ (A-42)
FDV	$(f_i)_2^A$
STU	$\left(\frac{f_{LK}}{f_{HK}}\right)_2$
RAU	$(f_i)_3^C$
NZU	$\longrightarrow$ (A-42)
FDV	$(f_i)_3^A$

## MAIN PROGRAM (Cont.)

$$\text{STU} \quad \left( \frac{f_{LK}}{f_{HK}} \right)_3$$

$$(A-42) \text{ RAU} \quad LK_4$$

$$\text{NZU} \quad \longrightarrow (A-40)$$

$$\text{RAC} \quad LK_4$$

$$\text{LDD} \quad HK_4$$

$$\text{RAA} \quad HK_4$$

$$\text{RAU} \quad (p_i)_3^C$$

$$\text{FDV} \quad (p_i)_3^A$$

$$\text{STU} \quad \left( \frac{P_{LK}}{P_{HK}} \right)_3$$

$$\text{RAU} \quad (f_i)_1^C$$

$$\text{FDV} \quad (f_i)_1^A$$

$$\text{STU} \quad \left( \frac{f_{LK}}{f_{HK}} \right)_1$$

$$\text{RAU} \quad (f_i)_2^C$$

$$\text{NZU} \quad \longrightarrow (A-40)$$

$$\text{FDV} \quad (f_i)_2^A$$

$$\text{STU} \quad \left( \frac{f_{LK}}{f_{HK}} \right)_2$$

$$\text{RAU} \quad (f_i)_3^C$$

$$\text{NZU} \quad \longrightarrow (A-40)$$

$$\text{FDV} \quad (f_i)_3^A$$

## MAIN PROGRAM (Cont.)

STU  $\left(\frac{f_{LK}}{f_{HK}}\right)_3$   
 (A-40) RAU  $(Rf_1)_2$   
 FSB  $Rp_1$   
 BMI  $\longrightarrow$  (A-43)  
 RAU  $(Rf_1)_3$   
 FSB  $Rp_2$   
 BMI  $\longrightarrow$  (A-44)  
 RAU  $(Rf_1)_4$   
 FSB  $Rp_3$   
 BMI  $\longrightarrow$  (A-45) or (A-46)  
 (A-43) LDD  $LK_1$   
 RAC  $LK_1$   
 LDD  $(\beta_1)_C$   
 STD  $(\beta_1)_C$   
 LDD  $(\theta_1)_C$   
 STD  $(\theta_1)_C$   
 LDD  $F_1$   
 STD  $F_1$   
 LDD  $D$   
 STD  $D$



## MAIN PROGRAM (Cont.)

```

LDD  $R_d$ 
STD  $R_d$ 
LDD  $(Rf_1)_1$ 
STD  $(Rf_1)_1$ 
LDD  $\longrightarrow S$ 
STU  $S_{11} \longrightarrow (A-47)$ 
(A-44) LDD  $LK_2$ 
RAC  $LK_2$ 
LDD  $(\beta_2)C$ 
STD  $(\beta_2)C$ 
LDD  $(\theta_2)C$ 
STD  $(\theta_2)C$ 
LDD  $F_1$ 
STD  $F_1$ 
LDD  $p_1$ 
STD  $p_1$ 
LDD  $Rp_1$ 
STD  $Rp_1$ 
LDD  $(Rf_1)_2$ 
STD  $(Rf_1)_2$ 

```

## MAIN PROGRAM (Cont.)

```

LDD  → S
STU  S21 → (A-47)
(A-45) LDD  LK3
      RAC  LK3
      LDD  (β3)C
      STD  (β3)C
      LDD  (θ3)C
      STD  (θ3)C
      LDD  F1
      STD  F1
      LDD  P2
      STD  P2
      LDD  Rp2
      STD  Rp2
      LDD  (Rf1)3
      STD  (Rf1)3
      LDD  → S
      STU  S31 → (A-47)
(A-46) LDD  LK4
      RAC  LK4
      LDD  (β4)C

```

## MAIN PROGRAM (Cont.)

STD  $(\beta_4)C$   
 LDD  $(\theta_4)C$   
 STD  $(\theta_4)C$   
 LDD  $F_1$   
 STD  $F_1$   
 LDD  $P_3$   
 STD  $P_3$   
 LDD  $Rp_3$   
 STD  $Rp_3$   
 LDD  $(Rf_1)_4$   
 STD  $(Rf_1)_4$   
 LDD  $\longrightarrow S$   
 STU  $S_{41}$   
 (A-47) RAU  $(Rf_2)_2$   
 NZU  $\longrightarrow (A-57)$   
 FSB  $Rp_1$   
 BMI  $\longrightarrow (A-48)$   
 RAU  $(Rf_2)_3$   
 FSB  $Rp_2$   
 BMI  $\longrightarrow (A-49)$   
 RAU  $(Rf_2)_4$

## MAIN PROGRAM (Cont.)

```

          FSB   Rp3
          BMI   → (A-50) or (A-51)
(A-48) LDD   LK1
          RAC   LK1
          LDD   (β1)C
          STD   (β1)C
          LDD   (θ4)C
          STD   (θ4)C
          LDD   F2
          STD   F2
          LDD   D
          STD   D
          LDD   Rd
          STD   Rd
          LDD   (Rf2)1
          STD   (Rf2)1
          LDD   → S
          STU   S12 → (A-52)
(A-49) LDD   LK2
          RAC   LK2
          LDD   (β2)C

```

## MAIN PROGRAM (Cont.)

```

          STD  ( $\beta_2$ )C
          LDD  ( $\theta_2$ )C
          STD  ( $\theta_2$ )C
          LDD  F2
          STD  F2
          LDD  P1
          STD  P1
          LDD  Rp1
          STD  Rp1
          LDD  (Rf2)2
          STD  (Rf2)2
          LDD  → S
          STU  S22 → (A-52)
(A-50) LDD  LK3
          RAC  LK3
          LDD  ( $\beta_3$ )C
          STD  ( $\beta_3$ )C
          LDD  ( $\theta_3$ )C
          STD  ( $\theta_3$ )C
          LDD  F2
          STD  F2
          LDD  P2

```

## MAIN PROGRAM (Cont.)

```

          STD   P2
          LDD   Rp2
          STD   Rp2
          LDD   (Rf2)3
          STD   (Rf2)3
          LDD   → S
          STU   S32 → (A-52)
(A-51) LDD   LK4
          RAC   LK4
          LDD   (β4)C
          STD   (β4)C
          LDD   (θ4)C
          STD   (θ4)C
          LDD   F2
          STD   F2
          LDD   P3
          STD   P3
          LDD   Rp3
          STD   Rp3
          LDD   (Rf2)4
          STD   (Rf2)4

```

## MAIN PROGRAM (Cont.)

LDD  $\longrightarrow$  S  
 STU  $S_{42}$   
 (A-52) RAU  $(Rf_3)_2$   
 NZU (A-57)  
 FSB  $Rp_1$   
 BMI  $\longrightarrow$  (A-53)  
 RAU  $(Rf_3)_3$   
 FSB  $Rp_2$   
 BMI  $\longrightarrow$  (A-54)  
 RAU  $(Rf_3)_4$   
 FSB  $Rp_3$   
 BMI  $\longrightarrow$  (A-55) or (A-56)  
 (A-53) LDD  $LK_1$   
 RAC  $LK_1$   
 LDD  $(\beta_1)C$   
 STD  $(\beta_1)C$   
 LDD  $(\theta_1)C$   
 STD  $(\theta_1)C$   
 LDD  $F_3$   
 STD  $F_3$   
 LDD D

## MAIN PROGRAM (Cont.)

```

          STD   D
          LDD   Rd
          STD   Rd
          LDD   (Rf3)1
          STD   (Rf3)1
          LDD   → S
          STU   S13 → (A-57)
(A-54) LDD   LK2
          RAC   LK2
          LDD   (β2)C
          STD   (β2)C
          LDD   (θ2)C
          STD   (θ2)C
          LDD   F3
          STD   F3
          LDD   P1
          STD   P1
          LDD   Rp1
          STD   Rp1
          LDD   (Rf3)2
          STD   (Rf2)2

```



## MAIN PROGRAM (Cont.)

```

          LDD      S
          STU      S23      (A-57)
(A-55) LDD      LK3
          RAC      LK3
          LDD      (β3)C
          STD      (β3)C
          LDD      (θ3)C
          STD      (θ3)C
          LDD      F3
          STD      F3
          LDD      P2
          STD      P2
          LDD      Rp2
          STD      Rp2
          LDD      (Rf3)3
          STD      (Rf3)3
          LDD      → S
          STD      S33 → (A-57)
(A-56) LDD      LK4
          RAC      LK4
          LDD      (β4)C
          STD      (β4)C

```

## MAIN PROGRAM (Cont.)

```
LDD  ( $\theta_4$ )C
STD  ( $\theta_4$ )C
LDD  F3
STD  F3
LDD  P3
STD  P3
LDD  Rp3
STD  Rp3
LDD  (Rf3)4
STD  (Rf3)4
LDD  → S
STU  S43
(A-57) LDD  0
      STD  0
      RAU  P-Pch
      LDD  → Pch
      LDD  0
      STD  0
      RAL  P-Pch
      LDD  → Pch
      HALT
```

VITA

Richard Sibley Joyner

Candidate for the Degree of

Master of Science

Thesis: MINIMUM STAGE CALCULATIONS FOR COMPLEX FRACTIONATORS

Major Field: Chemical Engineering

Biographical:

Personal Data: Born in Guthrie, Oklahoma, December 7, 1932, the son of Walthall R. and Marjorie P. Joyner.

Education: Attended elementary, secondary and high school in Guthrie, Oklahoma, graduated from Guthrie High School; received the Bachelor of Science degree from Oklahoma State University in May, 1957; completed requirements for Master of Science degree in May, 1961. Membership in scholarly or professional societies includes Sigma Tau and the American Institute of Chemical Engineers.

Professional experience: Process Engineer for Texas Eastman Company, Longview, Texas from June, 1957 to August, 1959; Process Engineer for Phillips Petroleum Company from June, 1960 to September, 1960.