THE ANALYSIS OF RIGID TRUSS-FRAMES BY THE STRING POLYGON METHOD

By

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PREFACE

The material presented in this thesis is the outgrowth of the regular course CIVEN 5A4 lectures delivered by Professor Jan J. Tuma in the Fall, 1960- 1961. The literature survey and the general theory recorded in the Introduction were prepared by Professor Tuma (1).

Upon finishing the last requirements of his present program for the degree of Master of Science in Civil Engineering, the writer wishes to acknowledge his indebtedness and gratitude to the following individuals:

To Miss Mary M. Graves, Head, Physical Sciences and Engineering Library, Oklahoma State University, for her noble and magnificient help in obtaining for him a student assistantship in the Library which made this graduate study possible.

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Y. İ. Gönülşen

July 13, 1961
Stillwater, Oklahoma

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d	Length of Finite Segment ij.
dm, dn	Lengths of Bars m, n, respectively.
e	Depth of Truss.
f	Free Height of Column.
h	Height of Column.
w	Intensity of Load.
x, y	Coordinates of Cross-Sections.
A_m, A_n	Areas of Cross-Section of Bars m, n, Respectively.
BM	Bending Moment Due to Loads.
BN_m, BN_n	Axial Forces in the Bars m, n, Due to Loads, Respectively.
BV	Shearing Force Due to Loads.
E	Modulus of Elasticity.
E_{ij}	Angular-Linear (Linear-Angular) Carry-Over Value.
F_{ij}, F_{ji}	Angular Flexibilities.
G_{ij}, G_{ji}	Angular Carry-Over Values.
H	End Thrust.
J_c	Moment of Inertia of Column.
L	Length of Span.
M_i, M_j, M_k	Bending Moments at points i, j, k, Respectively.

$\overline{M}$ . . . . Moment Elastic Weight.

$\overline{P}$ . . . . Force Elastic Weight.

V_i, V_j . . . . Shears at Points i, j, Respectively.

$\overline{W}_j$, $\overline{W}_k$ . . . . Total Elastic Weights of Segments ij, and jk, Respectively.

α_m . . . . Axial Force in Bar m Due to Unit Moment Applied at Left Hand.

β_m . . . . Axial Force in Bar m Due to Unit Moment Applied at Right Hand.

γ_m . . . . Axial Force in Bar m Due to Horizontal Thrust.

λ_m . . . . Axial Flexibility of Bar m.

Δ_i, Δ_j . . . . Displacements of Points i, j, Respectively.

Δ_i^C . . . . Displacement of Column at Point i.

Δ_i^T . . . . Displacement of Truss at Point i.

Δ_{ij} . . . . Linear Displacement of Member ij.

ϵ_{ij} . . . . Linear Displacement Load Function.

$_{ij}$ . . . . Linear Flexibility.

θ_i . . . . Slope of Point i.

θ_i^C . . . . Slope of Column at Point i.

θ_i^T . . . . Slope of Truss at Point i.

ϕ_{ji} . . . . Angle Between String Line ij and Tangent to Elastic Curve at j.

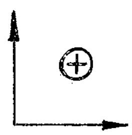
ϕ_j . . . . Change in Angle of String Line at j.

ω . . . . Angle Between Horizontal and String Line.

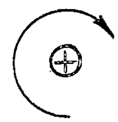
τ_{ij}, τ_{ji} . . . . Angular Displacement Load Functions.

SIGN CONVENTION

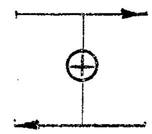
For forces and moments:



Forces



Moments



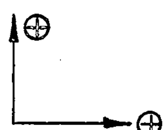
Couples

For axial forces:

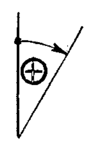
Tension

Compression

For deformations:

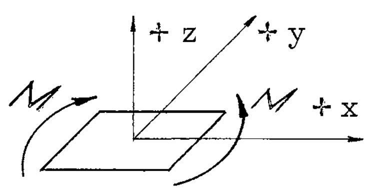


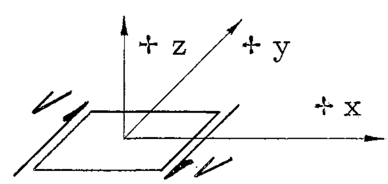
Displacements



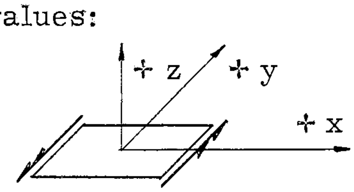
Rotations

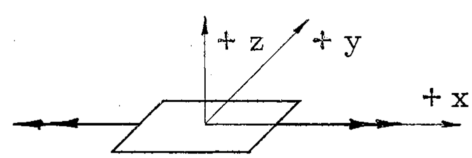
For bending moments and shears:

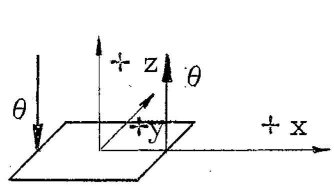


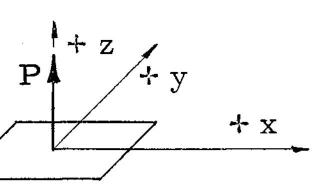


For elastic values:









CHAPTER I

INTRODUCTION

The application of the String Polygon Method to the analysis of straight members was first introduced by Tuma (1) and fully explored by Chu (2) and Harvey (3).

In the Spring Semester, 1960, Tuma (4) presented the extension of the String Polygon Theory to bent members, fixed-end frames, frames with hinged-ends, and to the derivation of slope deflection equations.

Following this seminar several investigations have been made. Boecker (5) applied the String Polygon to frames with hinged-ends, Oden (6) investigated fixed-end frames, and Houser (7) developed the slope deflection equations for bent members.

The investigation of members of a variable cross section was carried out by Exline (8) and Yu (9).

The general theory of the String Polygon Method as applied to rigid frames was proved by Tuma and Oden (10) and the extension to the plastic analysis was made by Gauger (11).

In this thesis the analysis of truss-frames by the String Polygon Method is developed and the numerical procedure is demonstrated by two examples.

The theories and applications developed are valid for elasto-static cases only and their extension to plasto-static cases will require additional study.

The historical background of the String Polygon Method was given by Tuma and Oden (10) and is not repeated here. The following presentation is divided into seven parts.

The nomenclature and the sign convention are shown in the first part of the thesis and are fully explained.

The review of the literature and the principle of conjugation are explained in the First Two Chapters.

The String Polygon expressions for straight solid members and truss members are given in Chapters Three and Four. Also the definition of the conjugate structure and the application of elastic weights are shown in the same Chapter.

The real contribution presented in this thesis is the development of compatability equations which add two new principles to the theory of the String Polygon Method.

Finally the application of the theory and the final summary and conclusions are shown in the last part of this thesis.

CHAPTER II

CONJUGATE FRAME

2-1. Real Frame

A rigid frame of variable cross section acted upon by a general system of loads is considered (Fig. 2-1).

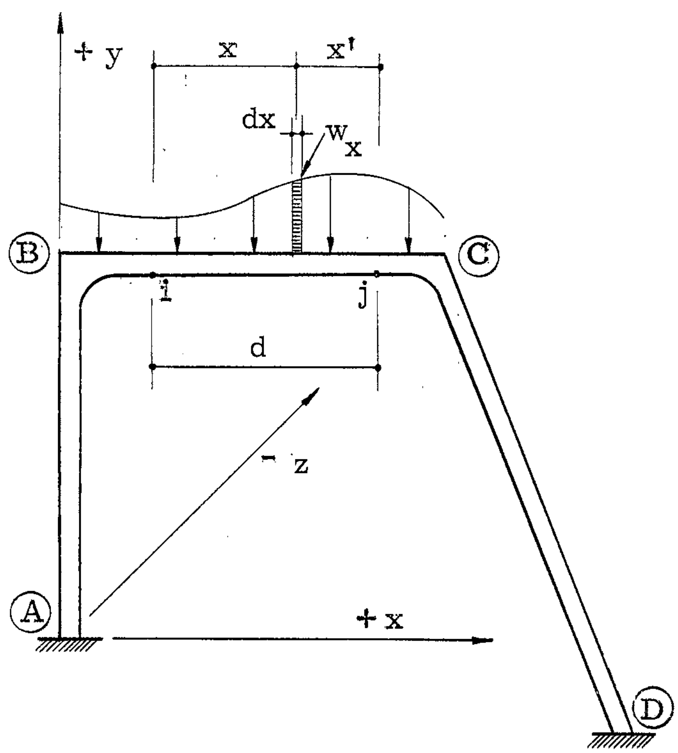


Figure 2-1

Real Frame

A finite segment ij of this frame is isolated (Fig. 2-2) and the end shears and moments are calculated.

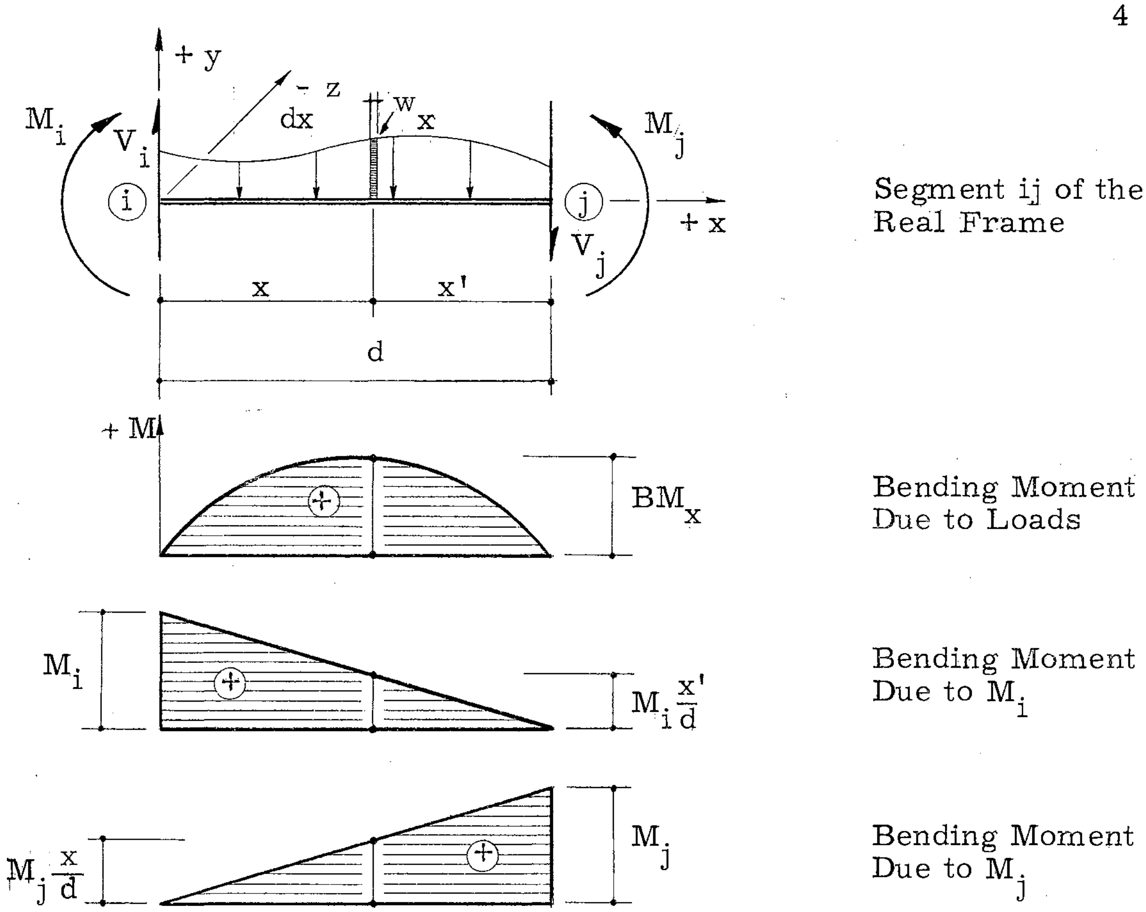


Figure 2-2

Bending Moment Diagrams of Segment ij

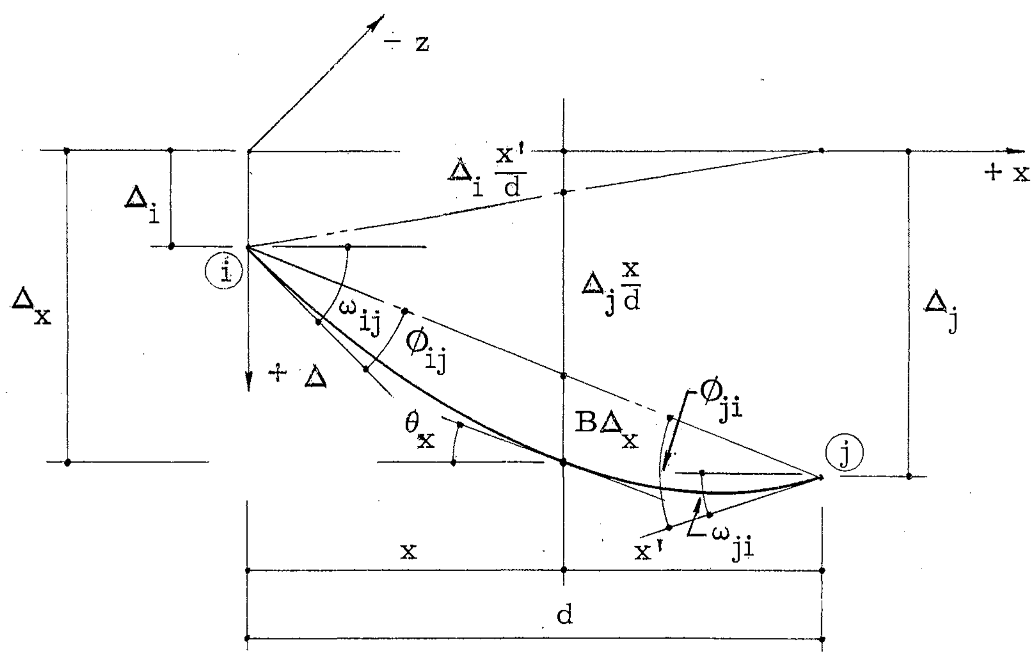


Figure 2-3

Deflection of the Elastic Curve

With notation;

$$w_x = \text{Intensity of load at } x$$

The end shears of the segment ij are:

$$\left.\begin{aligned} V_i &= BV_i - \frac{M_i}{d} + \frac{M_j}{d} \\ V_j &= BV_j - \frac{M_i}{d} + \frac{M_j}{d} \end{aligned}\right\} \qquad (2\text{-}1)$$

The new symbols BV_i and BV_j represent the end shears of segment ij due to loads acting on that segment only and it may be represented as the end shear of the simple beam ij acted on by the corresponding loads. The analytical expressions for these beam end shears are

$$\left.\begin{aligned} BV_i &= \int_i^j w_x \frac{x'}{d}\, dx \\ BV_j &= -\int_i^j w_x \frac{x}{d}\, dx \end{aligned}\right\} \qquad (2\text{-}2)$$

The shear at a given point of the segment ij is:

$$V_x = BV_x - \frac{M_i}{d} + \frac{M_j}{d} \qquad (2\text{-}3)$$

Where the expression BV_x is the shear of the simple beam ij loaded by the given system of loads:

$$BV_x = BV_i - \int_o^x w_x\, dx \qquad (2\text{-}4)$$

The bending moment at the same point:

$$M_x = BM_x + M_i \frac{x'}{d} + M_j \frac{x}{d} \qquad (2\text{-}5)$$

Similarly as in the previous cases BM_x is the bending moment of the simple beam loaded by the corresponding loads (Fig 2-2):

$$BM_x = BV_i(x) - \int_o^x w_x x dx \tag{2-6}$$

Thus it was shown that the end shears and the bending moments at the points can be calculated as functions of the bending moment at i and j and of the simple beam ij.

If now the deformed segment ij is shown in a larger scale (Fig. 2-3), a definite similarity is observed between the moment diagram (Fig. 2-2) and the deformation diagram (Fig. 2-3).

From Fig. 2-3, the slope at x:

$$\theta_x = B\theta_x + \frac{\Delta_i}{d} + \frac{\Delta_j}{d} \tag{2-7}$$

Where $B\theta_x$ is the slope of the simple beam ij at x:

Similarly the deflection of x:

$$\Delta_x = B\Delta_x + \Delta_i \frac{x'}{d} + \Delta_j \frac{x}{d} \tag{2-8}$$

Where $B\Delta_x$ is the deflection of the simple beam ij at x. The similarity of Equations (2-3, 4) with Equations (2-7, 8) is well apparent and leads to the conjugation principle.

2-2. Conjugate Frame

The segment ij of the real frame (Fig. 2-2) is taken. The change in slope of an element dx of this finite segment ij due to the bending moment M_x may be considered as a force vector and denoted as an elemental elastic weight:

$$d\phi_x = \overline{W}_x = \frac{M_x dx}{EI_x} \qquad (2\text{-}9)$$

With the application of the sum of these elastic weights for the segment ij, the conjugate segment ij is introduced (Fig. 2-4).

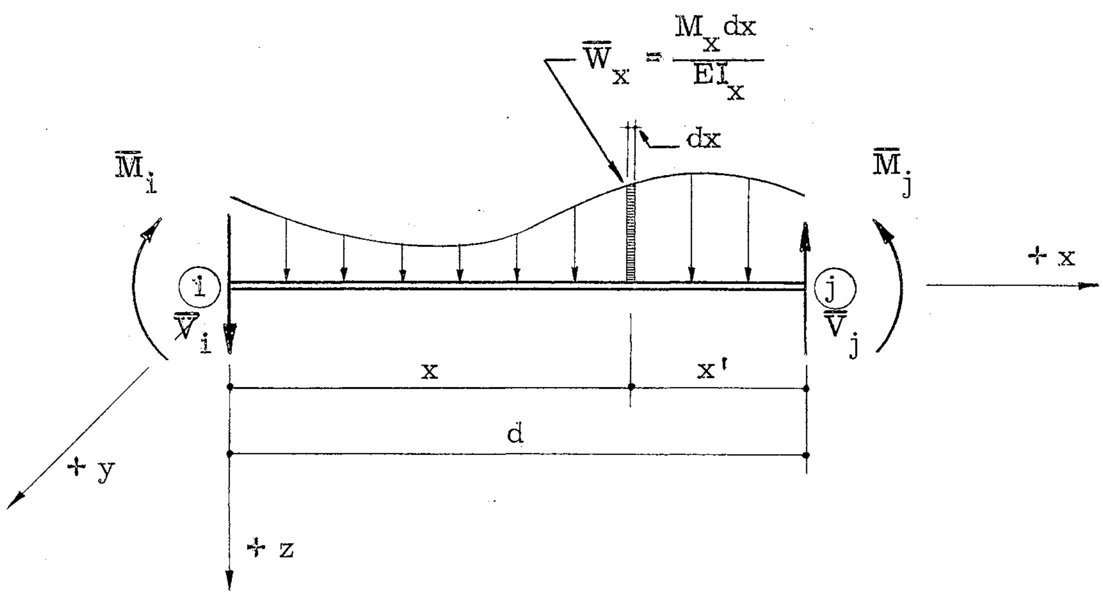


Figure 2-4

Conjugate Segment ij

The end shears of the segment ij are:

$$\left.\begin{aligned} \theta_i &= \overline{V}_i = \overline{BV}_i - \frac{\overline{M}_i}{d} + \frac{\overline{M}_j}{d} \\ \theta_j &= \overline{V}_j = \overline{BV}_j - \frac{\overline{M}_i}{d} + \frac{\overline{M}_j}{d} \end{aligned}\right\} \qquad (2\text{-}10)$$

Where $\overline{BV}_i$ and $\overline{BV}_j$ represent the end shears of the segment ij due to $\frac{M}{EI}$ diagram acting on that segment only and it may be represented as the end shear of the conjugate segment ij acted on by the corresponding $\frac{M}{EI}$ diagram. The analytical expressions for these conjugate beam end shears are:

$$\left.\begin{aligned} B\theta_i &= \overline{BV}_i = \int_i^j \overline{W}_x \frac{x'}{d}\,dx \\ B\theta_j &= \overline{BV}_j = \int_i^j \overline{W}_x \frac{x}{d}\,dx \end{aligned}\right\} \qquad (2\text{-}11)$$

The shear at a given point of segment ij;

$$\theta_x = \overline{V}_x = \overline{BV}_x + \frac{\overline{M}_i}{d} + \frac{\overline{M}_j}{d} \qquad (2\text{-}12)$$

Where the expression $\overline{BV}_x$ is the shear of the conjugate segment ij loaded by the given $\frac{M}{EI}$ diagram

$$B\theta_x = \overline{BV}_x - \int_o^x \overline{W}_x dx \qquad (2\text{-}13)$$

The bending moment at the same point:

$$\Delta_x = \overline{M}_x = \overline{BM}_x + \overline{M}_i \frac{x'}{d} + \overline{M}_j \frac{x}{d} \qquad (2\text{-}14)$$

Similarly as in the previous cases $\overline{BM}_x$ is the moment of the conjugate segment loaded by the corresponding $\frac{M}{EI}$ diagram:

$$B\Delta_x = \overline{BM}_x = \overline{BV}_i(x) - \int_o^x \overline{W}_x x dx \qquad (2\text{-}15)$$

Thus it was shown that the shears (slopes) and the bending moments (deflections) at a point of the segment are functions of the bending moments (deflections) at i and j and of the functions of simple segment ij.

From the similarity of Equations (2-3, 4) with Equations (2-7, 8) following analogies may be stated:

(a) The deformations of the real segment ij are defined by the beam functions of the conjugate segment ij (Fig. 2-5).

(b) The conjugate segment ij is loaded by a series of elemental elastic loads (one shown only) acting in the plane zx and causing bending about y (EQ. 2-9).

(c) The shear of the conjugate segment (EQ. 2-12) is the slope of the real segment (EQ. 2-7).

(d) The bending moment of the conjugate segment (EQ. 2-14) is the deflection of the real segment (EQ. 2-8).

These relationships are shown graphically in Fig. 2-5.

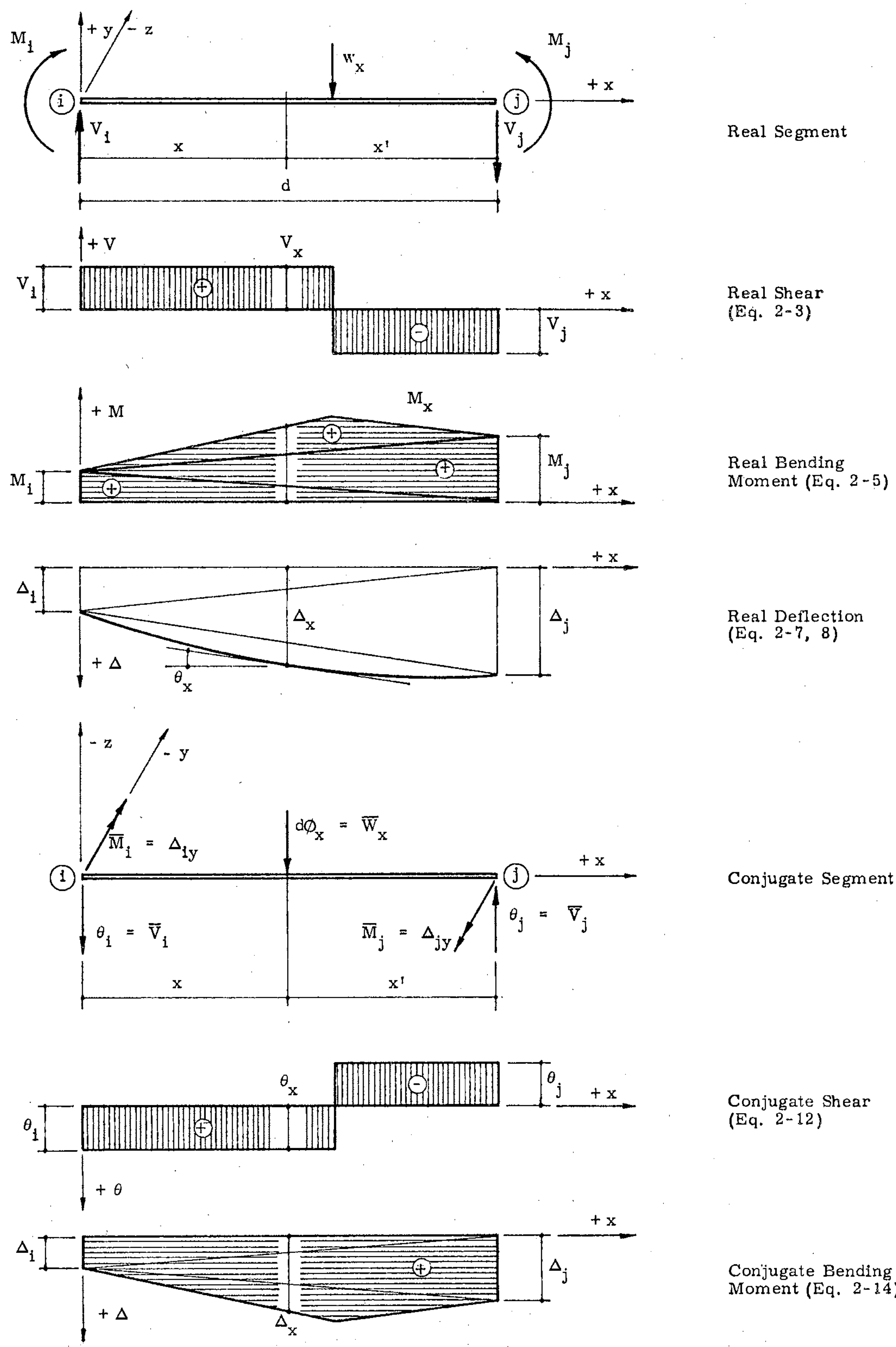


Figure 2-5

Relationships Between Real and Conjugate Segments ij

CHAPTER III

THE STRING POLYGON - STRAIGHT MEMBERS

3-1. Conjugate Reactions of a segment

From the relationships between the real and the conjugate frame as it was discussed in Chapter II (Arts. 2-1, 2-2) the end slopes at i and j of a simple segment ij loaded by a general systems of loads (Fig. 3-1) may be calculated from the reactions of the equivalent conjugate segment ij.

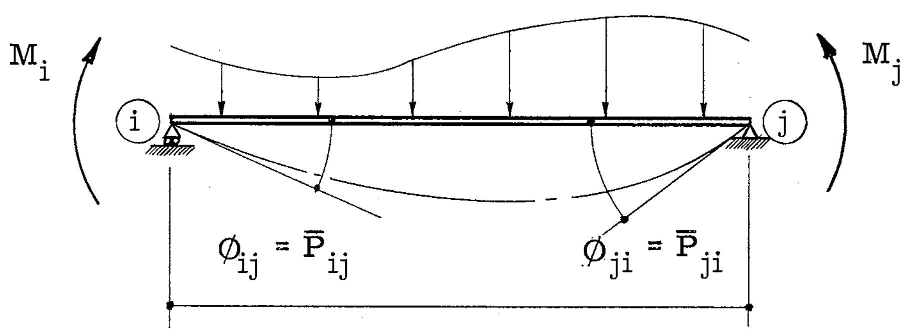


Figure 3-1

Simple Segment ij

The algebraic expressions for these end slopes are;

$$
\begin{aligned}
\phi_{ij} &= M_i F_{ij} + M_j G_{ji} + \tau_{ij} = \overline{P}_{ij} \\
\phi_{ji} &= M_j F_{ji} + M_i G_{ij} + \tau_{ji} = \overline{P}_{ji}
\end{aligned}
\qquad (3\text{-}1)
$$

The notations of these equations follow:

M_i (or M_j) is the bending moment of the simple beam ij at i (or j).

F_{ij} (or F_{ji}) is the angular flexibility of the equivalent simple beam ij at i (or j).

G_{ij} (or G_{ji}) is the angular carry-over value of the equivalent simple beam ij at i (or j).

τ_{ij} (or τ_{ji}) is the angular load function of the equivalent simple beam ij at i (or j).

The full account of these expressions and definitions of algebraic formulas are given elsewhere (11). For completeness the most important functions are restated in this chapter.

As shown in Fig. 3-2, the expressions for the total elastic weight of segments ij and jk are:

$$\overline{W}_j = \sum_i^j \overline{W}_x \qquad \overline{W}_k = \sum_j^k \overline{W}_x \tag{3-2}$$

The respective reactions of the separate beams are:

$$\begin{aligned} \overline{P}_{ij} &= \sum_i^j \overline{W}_x \frac{x'}{d} & \overline{P}_{jk} &= \sum_j^k \overline{W}_x \frac{x'}{d} \\ \overline{P}_{ji} &= \sum_i^j \overline{W}_x \frac{x}{d} & \overline{P}_{kj} &= \sum_j^k \overline{W}_x \frac{x}{d} \end{aligned} \tag{3-3}$$

where they represent the end slopes of the respective simple beams. (Fig. 3-2).

3-2. Classification of Elastic Weights

Once the relationship between the real and conjugate frames is established, the question arises as to how the elastic weight should be represented. It was shown by Tuma and Oden (10) that there are three types of elastic weights.

a- Elemental Elastic Weights

b- Segmental Elastic Weights

c- Joint Elastic Weights

The application of the elemental elastic weights to the analysis of a closed ring is well known under the name of column analogy developed by Cross (12) and the application of segmental elastic weights under the name of conjugate method was developed by Kinney (13) and Lee (14). The segmental elastic weights may be also represented by reactions of each segment. If these reactions are applied on the conjugate frame, the joint elastic weights are developed.

The joint elastic weight $\overline{P}_j$ is expressed by the general formula:

$$\overline{P}_j = \overline{P}_{ji} + \overline{P}_{jk} \tag{3-4}$$

and may be defined as the change of change in slope of the polygonal line ijk at j.

These elastic weights may be used for calculation of bending moments at joints and calculation of joint displacements. This approach is called the String Polygon Method.

The above mentioned three types of elastic weights are illustrated in Fig. 3-2.

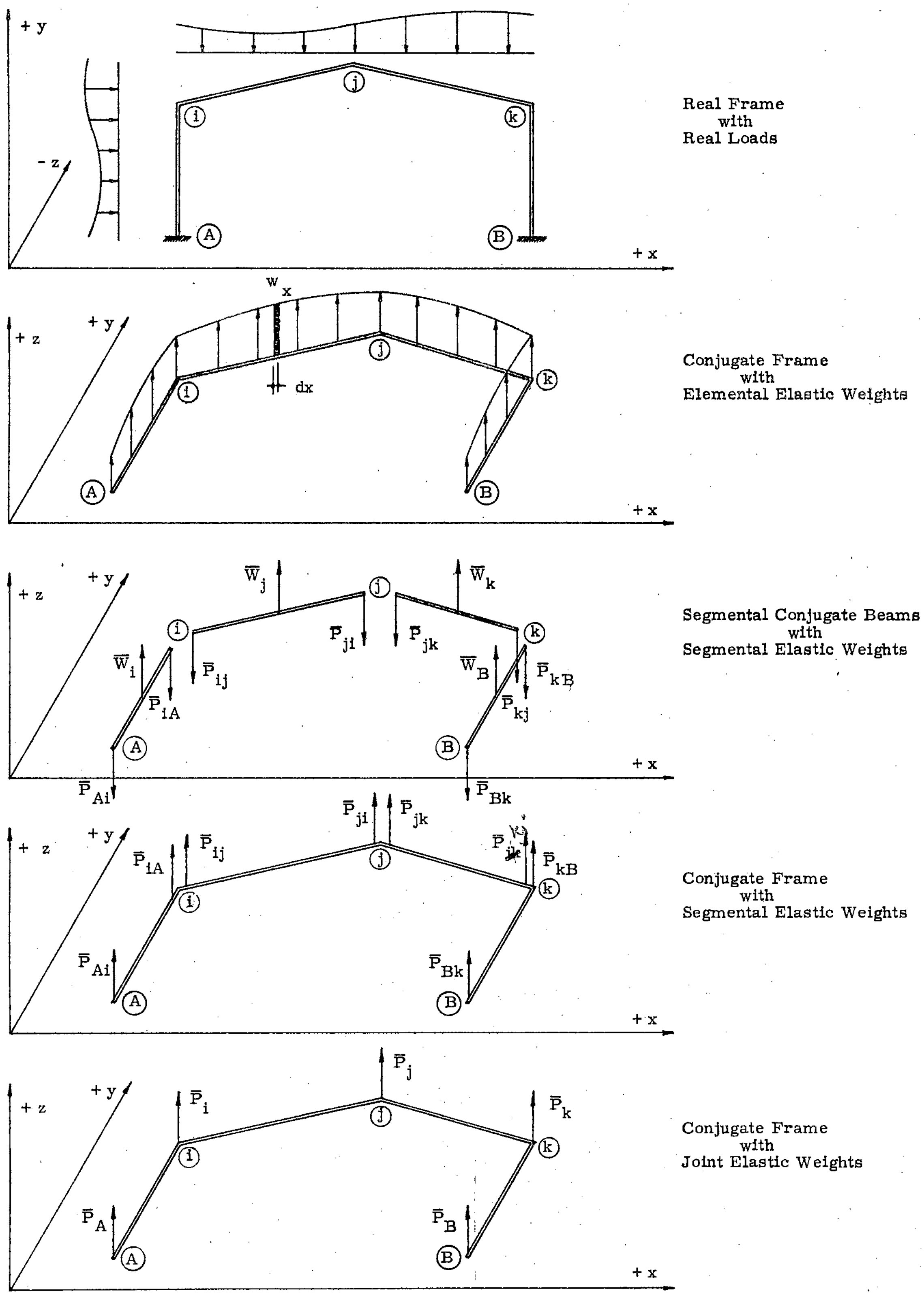


Figure 3-2

Elastic Weights

3-3. Elasto-static Equations

The joint elastic weights represent a new set of force-vectors in a state of static equilibrium and equivalent to the initial set of elemental elastic weights.

Thus:

$$\sum \overline{P}_j = 0 \qquad (3\text{-}5)$$

$$\sum \overline{P}_j x = 0 \qquad (3\text{-}6)$$

$$\sum \overline{P}_j y = 0 \qquad (3\text{-}7)$$

In addition to this any part of the conjugate frame may be isolated and end shears and moments of this isolated part are the deformations of the real structure at the end respectively. These statements are illustrated by Fig. 3-3.

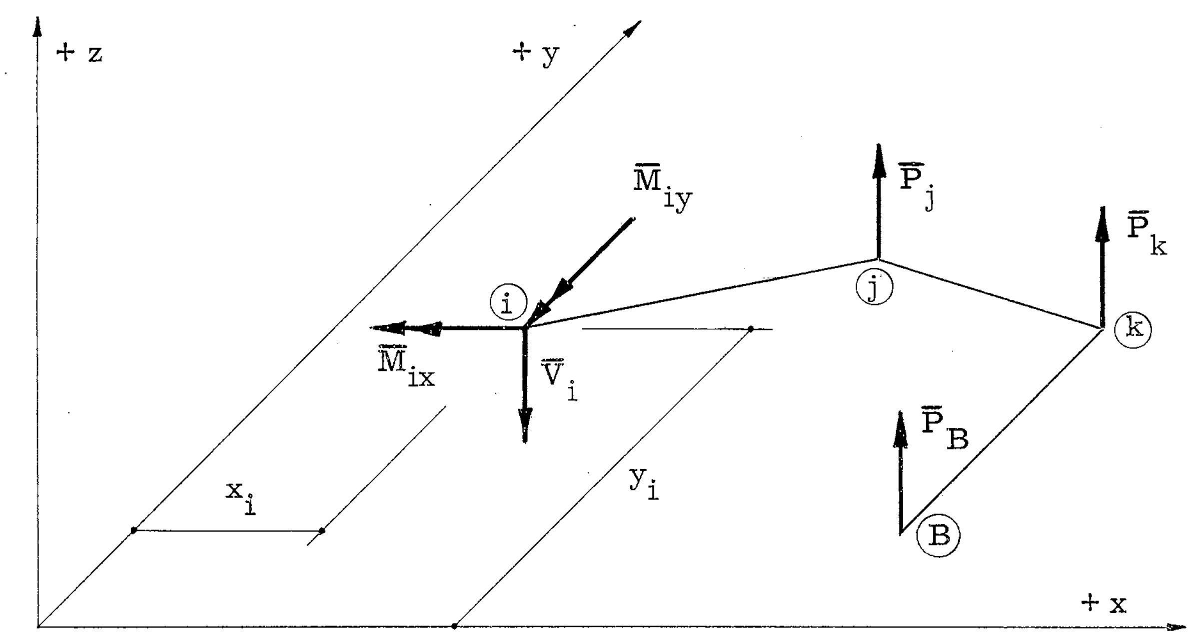


Figure 3-3

Isolated Part of Conjugate Frame

$$\overline{V}_i = \sum_i^B \overline{P}_j = \theta_i \tag{3-7a}$$

$$\overline{M}_{ix} = \sum_i^B \overline{P}_j \, (y - y_i) = \Delta_{ix} \tag{3-7b}$$

$$\overline{M}_{iy} = \sum_i^B \overline{P}_j \, (x - x_i) = \Delta_{iy} \tag{3-7c}$$

The extension of the String Polygon Method to the analysis of truss members is explained in the following chapter.

CHAPTER IV

THE STRING POLYGON - TRUSS MEMBERS

4-1. Conjugate reactions of a segment

If a curved truss segment of variable depth with a general system of transverse loads is considered as shown in Fig. 4-1, the end slopes and horizontal displacement of supports may be again represented as reactions of a conjugate structure.

It is well known from the theory of structures that the angle changes of each truss panel may be represented as the elastic weights. A typical truss elastic weight $\overline{P}_m$ applied on the conjugate structure is shown in Fig. 4-2.

The elastic weight $\overline{P}_m$ is calculated by means of virtual work and the position coordinates of this elastic weight are x_m, y_m respectively. Such elastic weight can be calculated for each joint of the truss. The sum of all elastic weights is the total elastic weight acting on the conjugate structure.

$$\overline{W} = \sum \overline{P}_m \tag{4-1}$$

The end slope of the real structure at point i;

$$\overline{P}_{ij} = \sum \overline{P}_m \frac{x'_m}{L} \tag{4-2}$$

at point j;

$$\overline{P}_{ji} = \sum \overline{P}_m \frac{x_m}{L} \tag{4-3}$$

The end displacement of the real structure at point i;

$$\overline{M}_{ijx} = \sum \overline{P}_m y_m \tag{4-4}$$

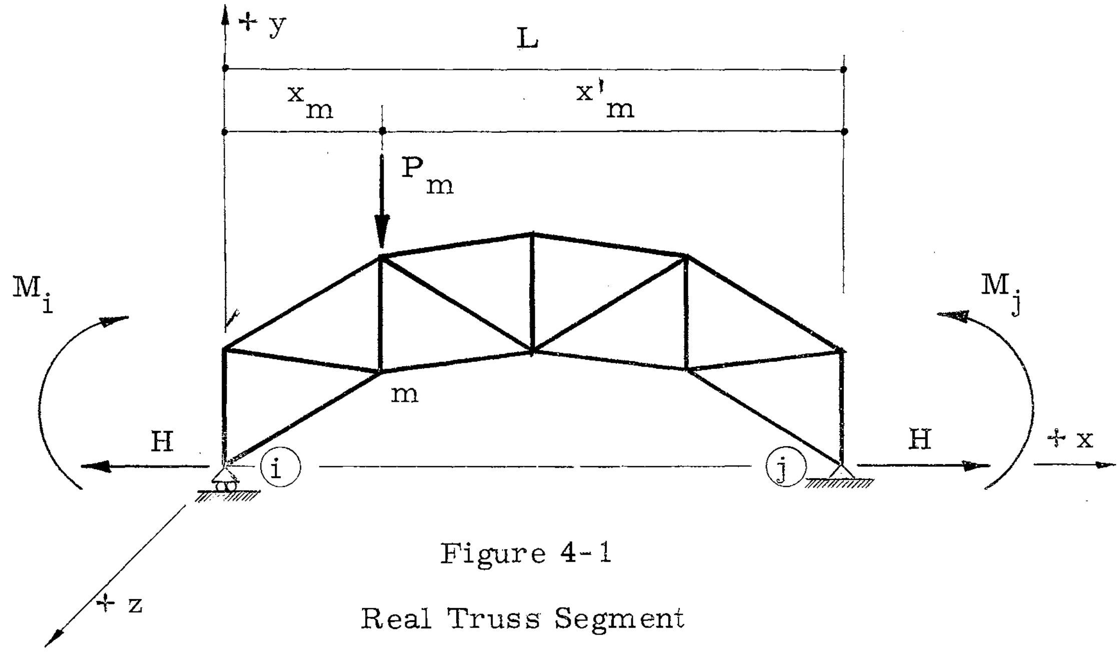


Figure 4-1

Real Truss Segment

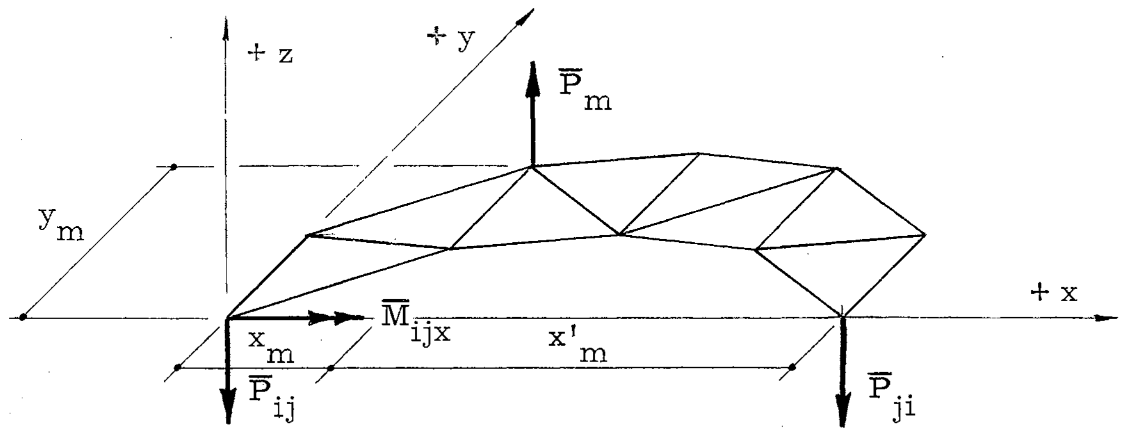


Figure 4-2

Conjugate Truss Segment

If $\overline{P}_m$ is evaluated in terms of moments M_i and M_j, horizontal thrust H and loads, and if new symbols are introduced, the final form for these expressions (Eqs. 4-2, 3, 4,) is obtained.

$$\phi_{ij} = M_i F_{ij} + M_j G_{ji} + HE_{ij} + \tau_{ij} = \overline{P}_{ij} \quad (4\text{-}5)$$

$$\phi_{ji} = M_j F_{ji} + M_i G_{ij} + HE_{ji} + \tau_{ji} = \overline{P}_{ji} \quad (4\text{-}6)$$

$$\Delta_{ijx} = M_i E_{ij} + M_j E_{ji} + H\Omega_{ij} + \epsilon_{ij} = \overline{M}_{ijx} \quad (4\text{-}7)$$

The algebraic expressions for the constants are recorded in the Table 4-1.

All expressions in these tables are in terms of the following nomenclature;

BN_m = The axial force due to loads

α_m = The axial force due to $M_i = + 1$

β_m = The axial force due to $M_j = + 1$

γ_m = The axial force due to horizontal trust $H = + 1$

d_m = The length of the bar

A_m = The area of the cross-section of the bar

E = Modulus of elasticity of the bar

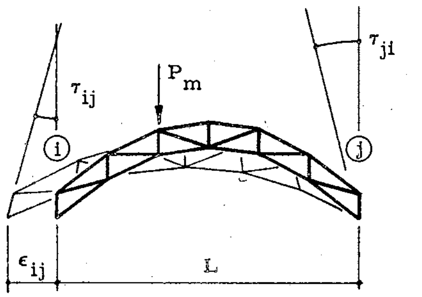
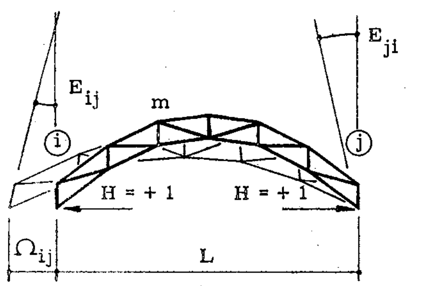
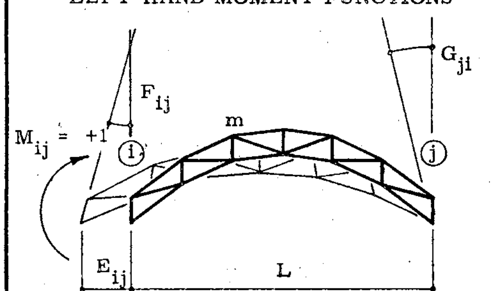
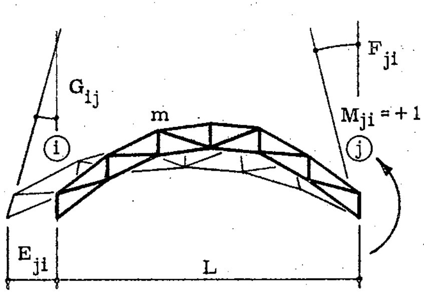
λ_m = The axial flexibility

where:

$$\lambda_m = \frac{d_m}{A_m E} \qquad (4-8)$$

The derivation of these constants is a part of the regular CIVEN 5A4 and reference is made to the lecture notes (15).

CURVED TRUSS FUNCTIONS — Table 4-1

LOAD FUNCTIONS 	$\tau_{ij} = \sum_i^j BN_m \alpha_m \lambda_m$	The angular displacement load function τ_{ij} is the end slope of the simple curved truss ij, at i, due to loads.
	$\tau_{ji} = \sum_i^j BN_m \beta_m \lambda_m$	The angular displacement load function τ_{ij} is the end slope of the simple curved truss ij, at j, due to loads.
	$\epsilon_{ij} = \sum_i^j BN_m \gamma_m \lambda_m$	The linear displacement load function ϵ_{ij} is the relative horizontal displacement of the ends of the simple curved truss ij due to loads.
THRUST FUNCTIONS 	$E_{ij} = \sum_i^j \alpha_m \gamma_m \lambda_m$	The angular-linear (linear-angular) carry-over value E_{ij} is the end slope (relative horizontal displacement)of the simple curved truss ij, at i, due to H=1 (M_{ij}=+1).
	$E_{ji} = \sum_i^j \beta_m \gamma_m \lambda_m$	The angular-linear (linear-angular) carry-over value E_{ij} is the end slope (relative horizontal displacement) of the simple curved truss ij, at j, due to H=+1 (M_{ji}=+1).
	$\Omega_{ij} = \sum_i^j \gamma_m^2 \lambda_m$	The linear flexibility ij is the relative horizontal displacement of the simple curved truss ij due to H=+1.
LEFT-HAND MOMENT FUNCTIONS 	$F_{ij} = \sum_i^j \alpha_m^2 \lambda_m$	The angular flexibility F_{ij} is the end slope of the simple curved truss ij, at i, due to M_{ij}=+1.
	$G_{ji} = \sum_i^j \alpha_m \beta_m \lambda_m$	The angular carry-over value G_{ij} is the end slope of the simple curved truss ij, at j, due to M_{ij}=+1.
	$E_{ij} = \sum_i^j \alpha_m \gamma_m \lambda_m$	The linear-angular(angular-linear) carry-over value E_{ij} is the end slope (relative horizontal displacement) of the simple curved truss ij, at i, due to M_{ij}=+1 (H=+1).
RIGHT-HAND MOMENT FUNCTIONS 	$F_{ji} = \sum_i^j \beta_m^2 \lambda_m$	The angular flexibility F_{ij} is the end slope of the simple curved truss ij, at i, due to M_{ji}=+1.
	$G_{ij} = \sum_i^j \alpha_m \beta_m \lambda_m$	The angular carry-over value G_{ij} is the end slope of the simple curved truss ij, at i, due to M_{ji} = +1.
	$E_{ji} = \sum_i^j \beta_m \gamma_m \lambda_m$	The linear angular(angular-linear) carry-over value E_{ji} is the end slope(relative horizontal displacement) of the simple curved truss ij at j, due to M_{ji}=+1 (H=+1).

CHAPTER V

COMPATABILITY EQUATIONS

5-1. Compatability of Slopes

It becomes obvious from the deformation sketch of the truss frame that the angular deformation of the truss at the connection of the truss to the column must be equal to the angular displacement of the string line.

This can be expressed in terms of conjugate structure as the equality of the column conjugate structure shear with the truss conjugate structure shear.

$$\overline{V}_j^{(\text{column})} = \overline{V}_{ji}^{(\text{truss})} \qquad \overline{V}_j^{(\text{column})} = \overline{V}_{jk}^{(\text{truss})} \tag{5-1}$$

The conjugate shear of the column may be easily calculated from the conjugate beam. The shear of the conjugate column corresponding to the rotation of truss;

$$\overline{V}_j^{(\text{column})} = \overline{P}_{oj} + \overline{P}_{jB} \tag{5-2}$$

The end slope of truss is given by a similar equation (4-2).

A similar procedure is applicable for the adjacent panel and the reversed sign approach has to be observed.

The compatability of slopes statement is illustrated in Fig. 5-1. Where $\overline{P}_{jB}$ is the column joint elastic weight at j.

5-2. Compatability of Displacements

In a similar way to the compatability of slopes (Art. 5-1), it is true that the linear deformation of the truss at the connection of the truss to the column must be equal to the linear displacement of the string line on the column.

This can be expressed in terms of the conjugate structure as the equality of the column conjugate structure bending moment (displacement) with the truss conjugate structure bending moment (displacement).

$$\Delta_j^{(column)} = \Delta_{ji}^{(truss)} \qquad \Delta_j^{(column)} = \Delta_{jk}^{(truss)} \qquad (5\text{-}3)$$

The conjugate bending moment (displacement) of the column may be easily calculated from the conjugate beam. The bending moment (displacement) of the conjugate column corresponding to the displacement of the truss:

$$\Delta_j^{(column)} = \overline{\overline{P}}_{oj}\, f \qquad (5\text{-}4)$$

The end displacement of the truss is given by the similar Equation (4-4).

A similar procedure is applicable for the adjacent panel and the reversed sign approach has to be observed.

The compatability of displacements statement together with the compatability of slopes statement is illustrated in Fig. 5-1.

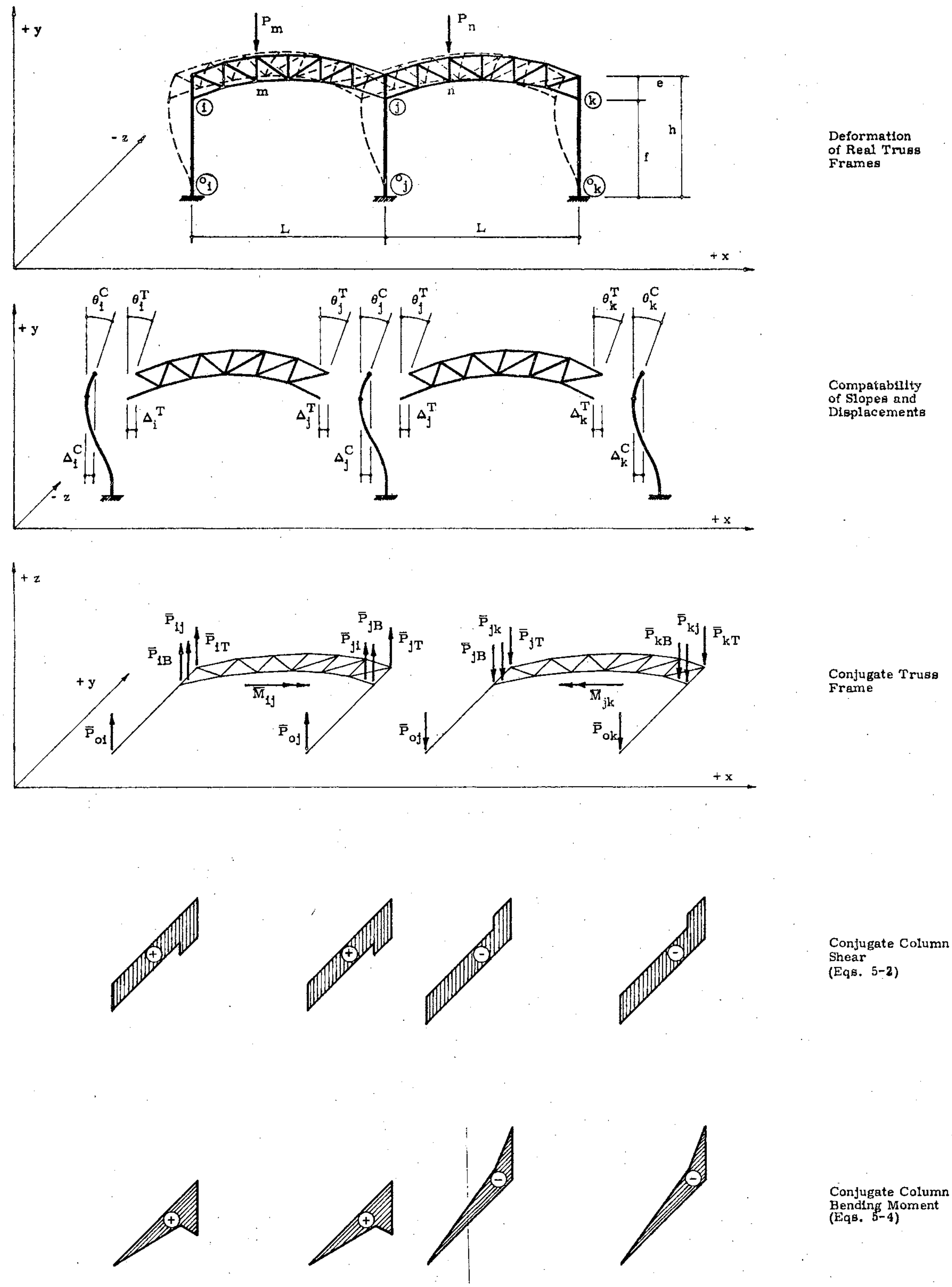


Figure 5-1

Compatability of Slopes and Displacements

CHAPTER VI

NUMERICAL PROCEDURE & APPLICATION

6-1. Procedure

Two examples illustrate the numerical procedure of analysis by the string polygon method applied to truss-frames.

The first illustrates the application of the string polygon method to the solution of a single panel truss-frame (Fig. 6-1), while the second deals with the solution of a two-panel continuous truss-frame (Fig. 6-2).

First, the basic structure of the truss frame is drawn and the end bending moments and end thrusts are selected as unknowns. Then the moment and force elastic weights in terms of angular and linear functions are developed and applied on the conjugate structure. The moment and force matrix is written in terms of elasto-static equations and the solution of the moment and force matrix yields the values of the redundants.

In the solution of problems all values are in kips, inches, or kip-inches.

References are made in each example to the equations, and tables used.

6-2. Single Span Frame (Example No. 1)

The unsymmetrical single panel truss-frame is analyzed for the given conditions (Fig. 1-a). The end bending moments and thrusts are taken as the unknowns (Fig. 1-b).

a.) Angular and Linear Functions (1)

Column Members

$$F_{AC} = F_{CA} = F_{BD} = F_{DB} = \frac{200}{(3)(300)E} = \frac{.222}{E}$$

$$F_{CE} = F_{EC} = F_{DF} = F_{FD} = \frac{60}{(3)(300)E} = \frac{.066}{E}$$

$$G_{AC} = G_{CA} = G_{BD} = G_{DB} = \frac{200}{(6)(300)E} = \frac{.111}{E}$$

$$G_{CE} = G_{EC} = G_{DF} = G_{FD} = \frac{60}{(6)(300)E} = \frac{.033}{E}$$

Truss Members (Table 6-1)

$$F_{CD} = \frac{.189350}{E} \qquad F_{DC} = \frac{.189400}{E}$$

$$G_{CD} = G_{DC} = \frac{-.05055}{E}$$

$$E_{CD} = \frac{5.001}{E} \qquad E_{DC} = \frac{3.334}{E}$$

$$\Omega_{CE} = \frac{500}{E}$$

$$\tau_{CD} = \frac{.0250}{E} \qquad \tau_{DC} = \frac{16.645}{E}$$

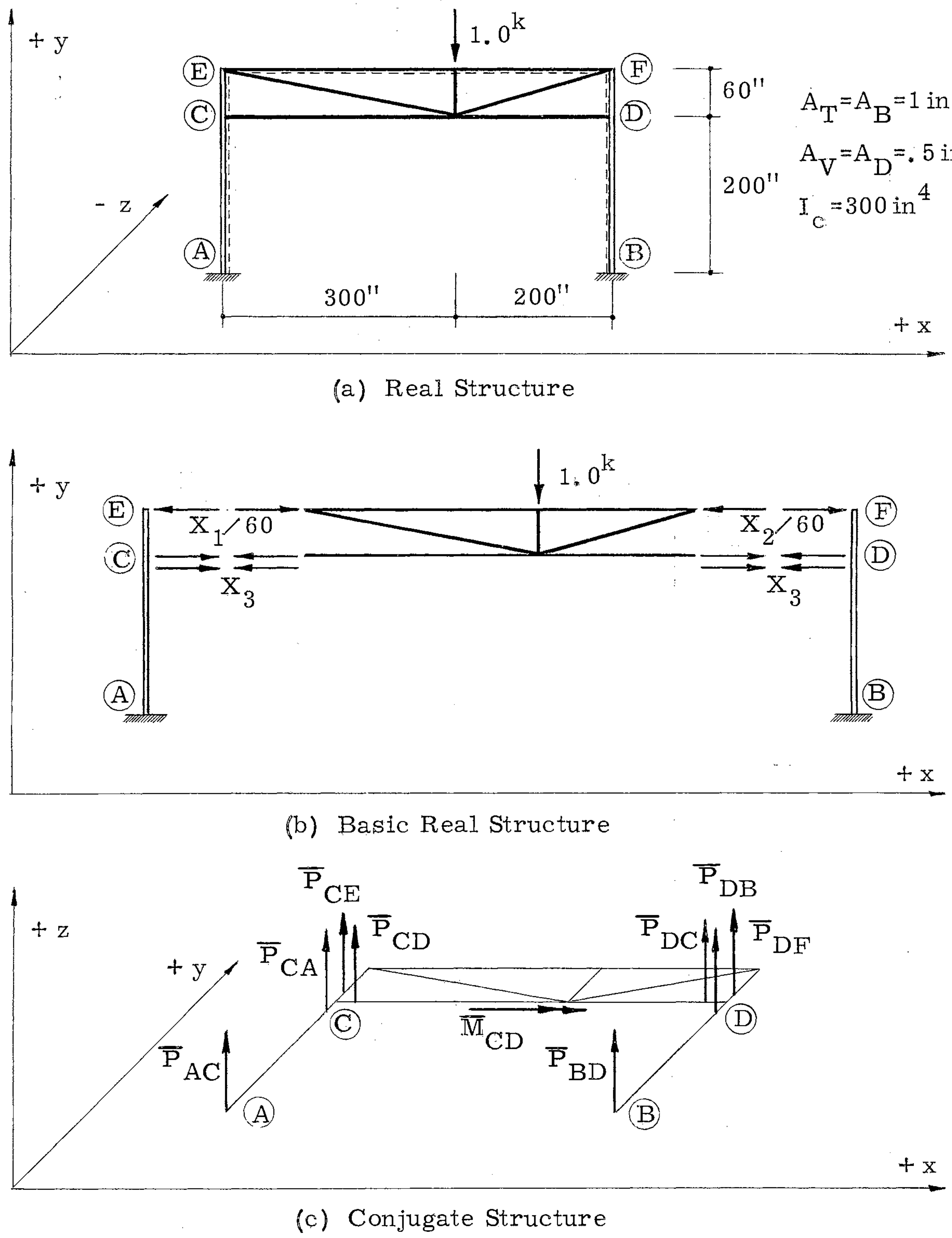
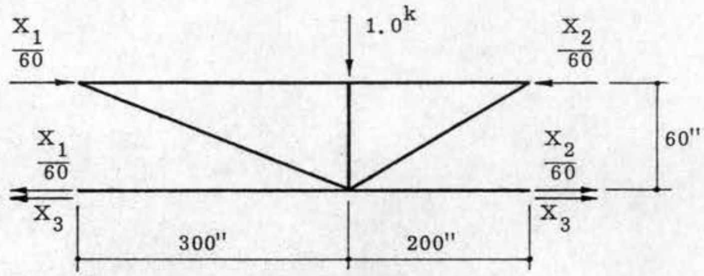
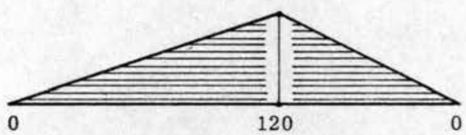


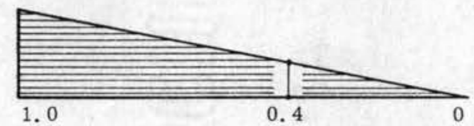
Figure 6-1
Single Panel Truss-Frame



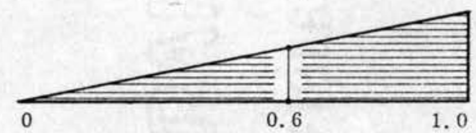


Bending Moment
Due to Loads

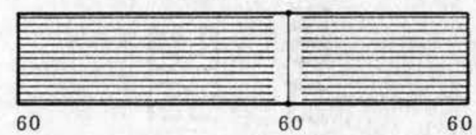
Bending Moment
Due to X_1 = 1.0



Bending Moment
Due to X_2 = 1.0



Bending Moment
Due to X_3 = 1.0



m		λ_m	α_m	β_m	γ_m	BN_m	$\alpha_m^2\lambda_m$	$\beta_m^2\lambda_m$	$\alpha_m\beta_m\lambda_m$	$\alpha_m\gamma_m\lambda_m$	$\beta_m\gamma_m\lambda_m$	$\gamma_m^2\lambda_m$	$BN_m\alpha_m\lambda_m$	$BN_m\beta_m\lambda_m$
T	1	300	-.0066	-.0100	0	-2.000	.013350	.030000	+.020010	0	0	0	+4.002	+6.000
	2	200	-.0066	-.0100	0	-2.000	.008900	.020000	+.013340	0	0	0	+2.668	+4.000
B	1	300	+.0166	.0	+1.000	0	.083200	0	0	+5.001	0	300	0	0
	2	200	0	+.0166	+1.000	0	0	.055500	0	0	+3.334	200	0	0
D	1	612	-.0102	+.0102	0	+2.040	.063600	.063600	-.063600	0	0	0	-12.734	+12.734
	2	418	+.0070	-.0070	0	+2.090	.020300	.020300	-.020300	0	0	0	+ 6.089	- 6.089
V	1	120	0	0	0	-1.000	0	0	0	0	0	0	0	0
							.189350	.189400	-.050550	5.001	3.334	500	.025	16.645

b.) Elastic Weights (Eqs. 3-1, 4-5, 6, 7)

$$E\overline{P}_{AC} = M_{AC} F_{AC} + M_{CA} G_{CA}$$
$$= .333 X_1 - 44.444 X_3$$

$$E\overline{P}_{CA} = M_{CA} F_{CA} + M_{AC} G_{AC}$$
$$= .333 X_1 - 22.222 X_3$$

$$E\overline{P}_{BD} = M_{BD} F_{BD} + M_{DB} G_{DB}$$
$$= .333 X_2 - 44.444 X_3$$

$$E\overline{P}_{CE} = M_{CE} F_{CE}$$
$$= .066 X_1$$

$$E\overline{P}_{DF} = M_{DF} F_{DF}$$
$$= .033 X_2$$

$$E\overline{P}_{CD} = M_{CD} F_{CD} + M_{DC} G_{DC} + H_{CD} E_{CD} + \tau_{CD}$$
$$= .189350 X_1 - .05055 X_2 + 5.001 X_3 + .0250$$

$$E\overline{P}_{DC} = M_{DC} F_{DC} + M_{CD} G_{CD} + H_{DC} E_{DC} + \tau_{DC}$$
$$= .189350 X_2 - .05055 X_1 + 3.334 X_3 + 16.645$$

$$E\overline{M}_{CD} = M_{CD} E_{CD} + M_{DC} E_{DC} + H_{CD} \Omega_{CD}$$
$$= 5.001 X_1 + 3.334 X_2 + 500 X_3$$

c.) Elasto-Static Equations (Eqs. 3-4, 5, 6, 7)

Elastic weights are applied on the conjugate structure as shown in Fig. 1-c.

Static Moment about $\overline{BD}$

$$(\overline{P}_{AC} + \overline{P}_{CA} + \overline{P}_{CE} + \overline{P}_{CD})\ (500) = 0$$

$$461.145\,X_1 - 25.315\,X_2 - 31630.000\,X_3$$

Static Moment about $\overline{AC}$

$$(\overline{P}_{BD} + \overline{P}_{DB} + \overline{P}_{DF} + \overline{P}_{DC})\ (500) = 0$$

$$462.000\,X_2 - 25.315\,X_1 - 3163.000\,X_3 = 8320.000 = 0$$

Static Moment about $\overline{CD}$

$$(\overline{P}_{AC} + \overline{P}_{BD})\ (200) - \overline{M}_{CD} = 0$$

$$61.666 + 63.333 - 18277.667 = 0$$

d.) Moment and Force Matrix

$$\begin{bmatrix} (461.145) & (-25.315) & (-30832.500) \\ (-25.315) & (462.000) & (-31630.000) \\ (61.666) & (63.333) & (-18277.667) \end{bmatrix} \begin{bmatrix} (X_1) \\ (X_2) \\ (X_3) \end{bmatrix} = \begin{bmatrix} (-12.320) \\ (-8320.000) \\ (0) \end{bmatrix}$$

e.) Final Moments and Forces

The solution of the moment and force matrix yields the following values:

$$M_{CD} = -10.183 \text{ kip-in.}$$

$$M_{DC} = -27.425 \text{ kip-in.}$$

$$H_{CD} = -0.129 \text{ kip}$$

6-3. Continuous Frame (Example No. 2)

The symmetrical two-panel truss-frame is analyzed for the loading shown (Fig. 6-2a). The end bending moments and thrusts are selected as the unknowns (Fig. 6-2b).

a.) Angular and Linear Functions

The values of angular and linear truss functions are computed by means of virtual work (Table 6-2) in terms of the new symbols listed in Table 4-1.

The values of the angular and linear functions for the complete structure are shown in Table 6-3.

Example No. 2		Angular and Linear Functions					Table 6-3
Member		F	G	E	Ω	τ	ϵ
Truss	45 = 54	$\frac{.190622}{E}$	$\frac{.092094}{E}$	$\frac{20.3763}{E}$	$\frac{3623.16}{E}$	$\frac{962.396}{E}$	$\frac{167817.4}{E}$
	56 = 65	$\frac{.190622}{E}$	$\frac{.092094}{E}$	$\frac{20.3763}{E}$	$\frac{3623.16}{E}$	$\frac{962.396}{E}$	$\frac{167817.4}{E}$
Column	14 = 41	$\frac{.222}{E}$	$\frac{.111}{E}$	----	----	----	----
	25 = 52	$\frac{.222}{E}$	$\frac{.111}{E}$	----	----	----	----
	36 = 63	$\frac{.222}{E}$	$\frac{.111}{E}$	----	----	----	----
	47 = 74	$\frac{.066}{E}$	$\frac{.033}{E}$	----	----	----	----
	58 = 85	$\frac{.066}{E}$	$\frac{.033}{E}$	----	----	----	----
	69 = 96	$\frac{.066}{E}$	$\frac{.033}{E}$	----	----	----	----

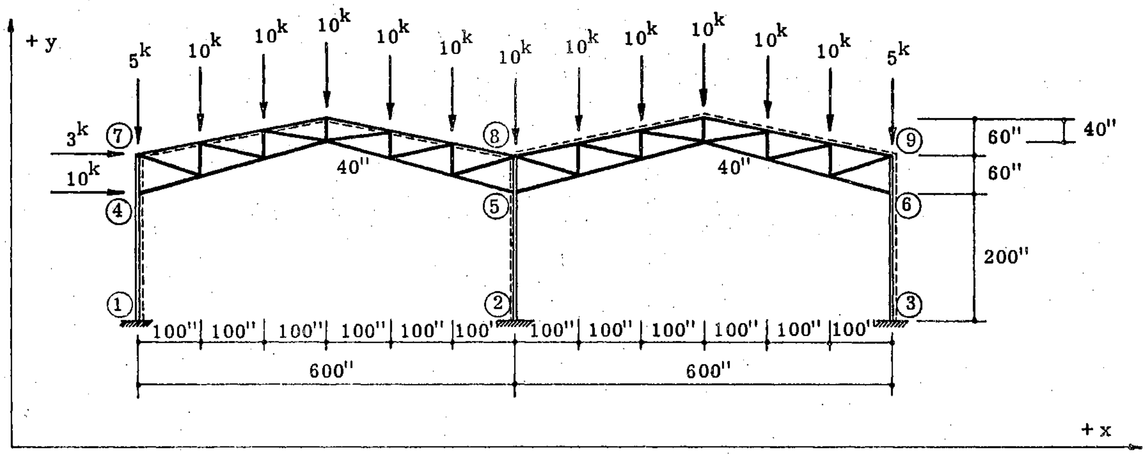


Figure 6-2a

Real Structure

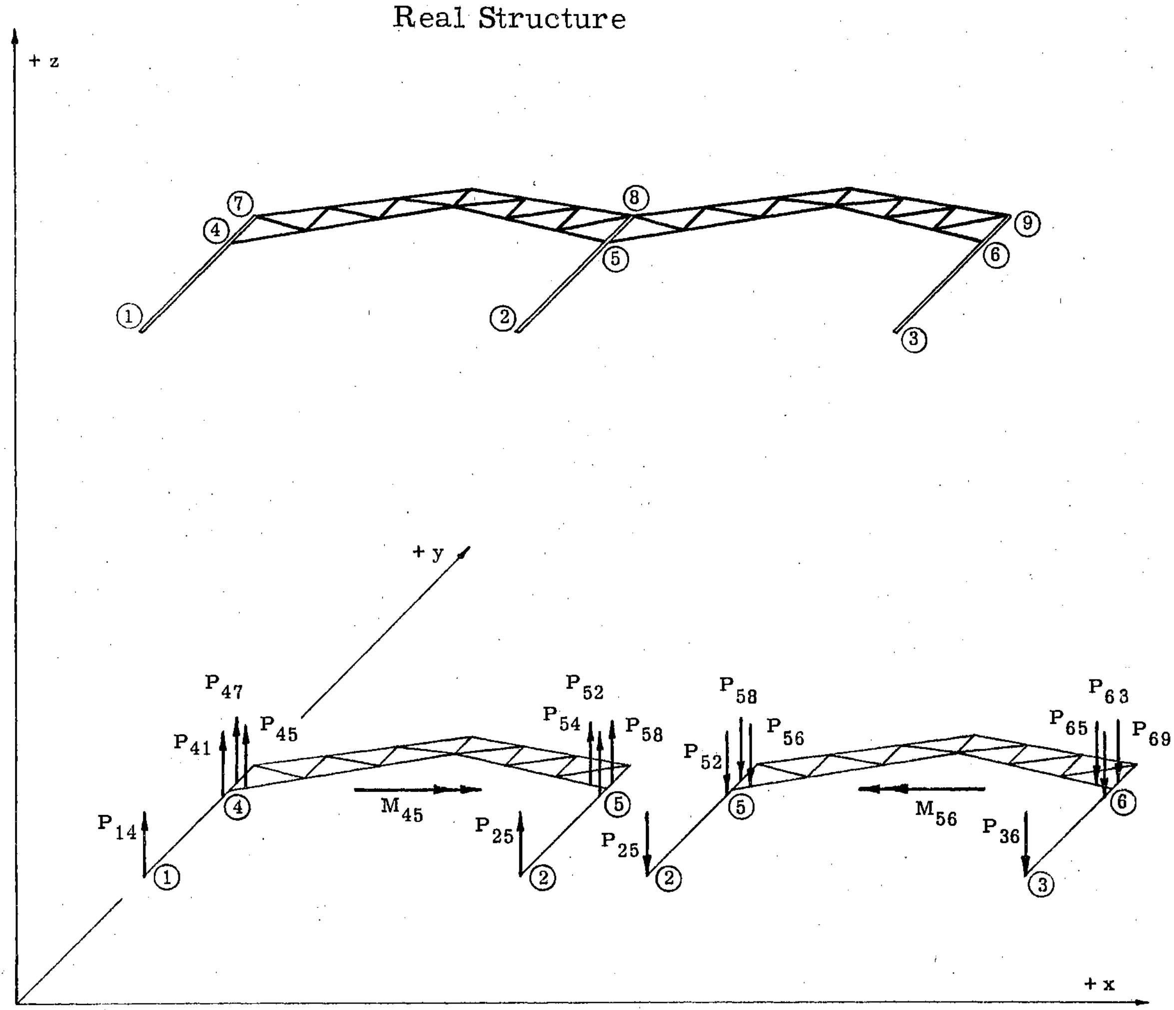
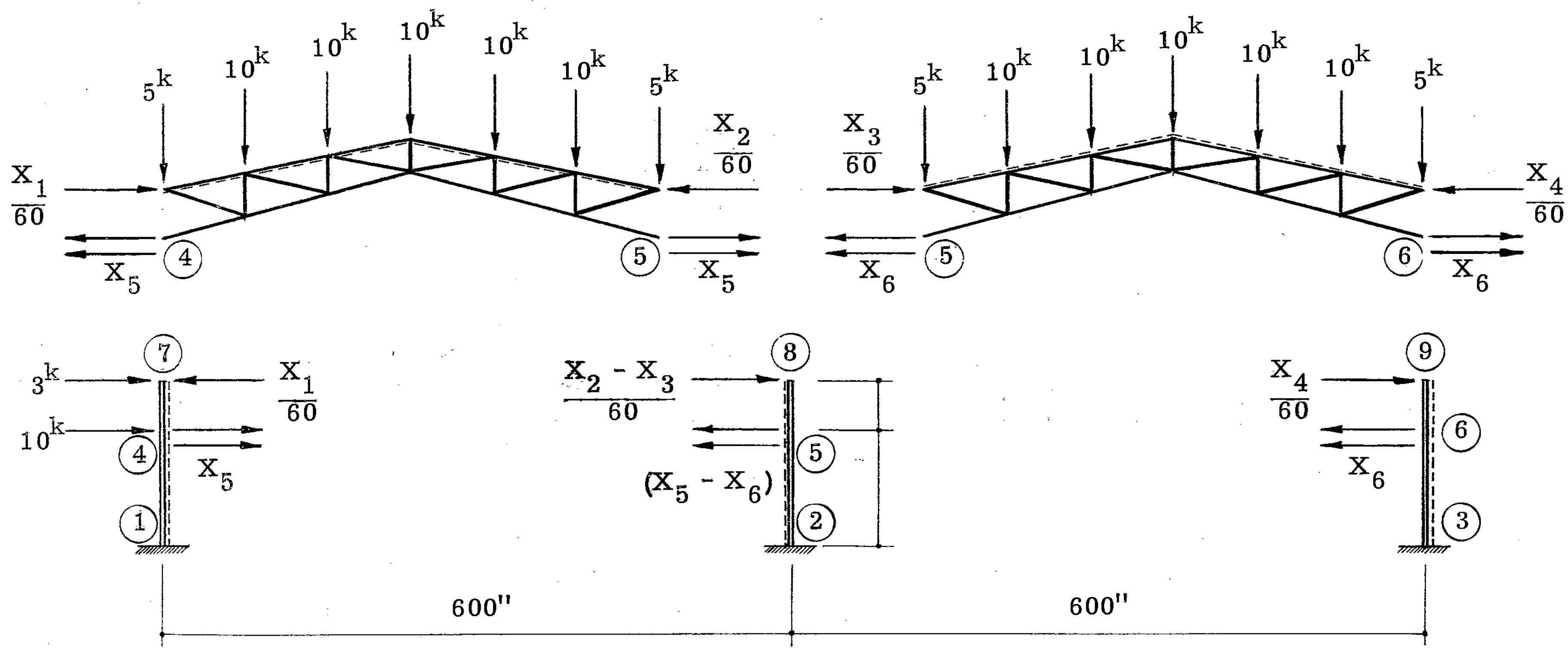


Figure 6-2c

Conjugate Structure



$X_1 = M_{45}$

$X_2 = M_{54}$

$X_5 = H_{45} = H_{54}$

$A_T = A_B = 1 \text{ in}^2$

$A_V = A_D = .5 \text{ in}^2$

$I_c = 300 \text{ in}^4$

$X_3 = M_{56}$

$X_5 = M_{65}$

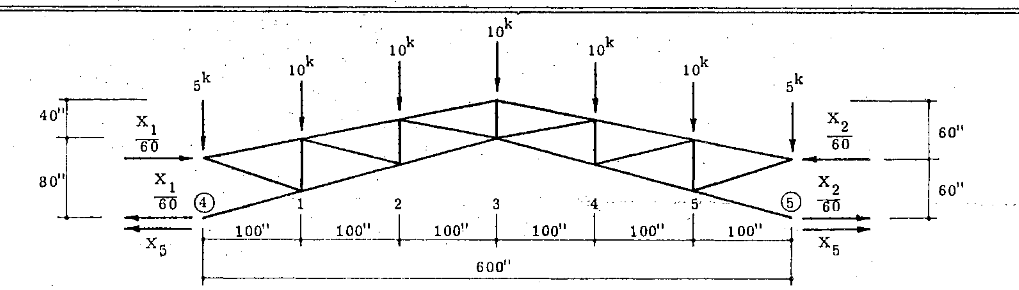
$X_6 = H_{56} = H_{65}$

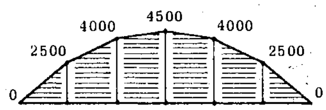
Figure 6-2b

Basic Structure

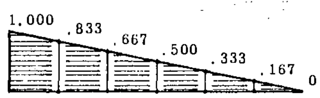
Two-Panel Continuous Truss-Frame

Example No. 2 | Basic Structure $\overline{\overline{45}}$ | Table 6-2

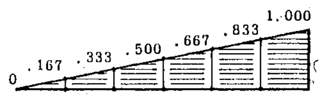




Bending Moment Due to Loads

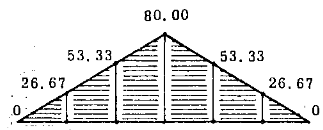


Due to X_1 = 1.0

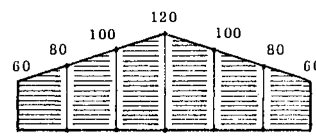


Due to X_2 = 1.0

Due to X_5 = 1.0 Top Chord



Due to X_5 = 1.0 Bottom Chord



	m	d_m	λ_m	α_m	β_m	γ_m	BN_m	$\alpha_m^2\lambda_m$	$\alpha_m\beta_m\lambda_m$	$\alpha_m\gamma_m\lambda_m$	$\gamma_m^2\lambda_m$	$BN_m\alpha_m\lambda_m$	$BN_m\gamma_m\lambda_m$
TOP	1	102.00	102.00	-.01593	-.00319	- .51000	- 47.820	.025884	+.005183	+ .828700	26.530		+ 2,487.60
	2	102.00	102.00	-.01458	-.00728	-1.16500	- 87.430	.021683	+.010826	+1.732500	138.680	+130.022	+10,398.20
	3	102.00	102.00	-.01275	-.01275	-2.04000	-114.730	.016581	+.016581	+2.653500	424.448	+149.206	+23,248.80
	4	102.00	102.00	-.01275	-.01275	-2.04000	-114.730	.016581	+.016581	+2.653500	424.448	+149.206	+23,248.80
	5	102.00	102.00	-.00728	-.01458	-1.16500	- 87.430	.005406	+.010826	+ .865100	138.680	+ 64.922	+10,398.20
	6	102.00	102.00	-.00319	-.01593	- .51000	- 47.820	.001038	+.005183	+ .165900	26.530	+ 15.560	+ 2,487.60
BOTTOM	1	103.50	103.50	+.01725	0	+1.03500	0	.030798	0	+1.847800	110.870	0	0
	2	103.50	103.50	+.01617	+.00324	+1.55200	+ 48.520	.027062	+.005422	+2.597400	249.300	+ 81.203	+ 7,773.90
	3	103.50	103.50	+.01479	+.00739	+2.21800	+ 88.710	.022640	+.011312	+3.395200	509.170	+135.794	+20,364.50
	4	103.50	103.50	+.00739	+.01479	+2.21800	+ 88.710	.005652	+.011312	+1.696500	509.170	+ 67.851	+20,364.50
	5	103.50	103.50	+.00324	+.01617	+1.55200	+ 48.520	.001087	+.005422	+ .520400	249.300	+ 16.271	+ 7,773.90
	6	103.50	103.50	0	+.01725	+1.03500	0	0	0	0	110.870	0	0
DIAGONAL	1	105.35	210.70	-.00110	+.00330	+ .52700	+ 49.24	.000255	-.000765	- .122100	58.520	- 11.412	+ 5,467.60
	2	103.50	207.00	-.00138	+.00415	+ .66600	+ 40.19	.000394	-.001185	- .190000	91.540	- 11.480	+ 5,532.30
	3	102.00	204.00	-.00183	+.00547	+ .87500	+ 27.30	.000683	-.002042	- .326700	156.180	- 10.192	+ 5,893.10
	4	102.00	204.00	+.00547	-.00183	+ .87500	+ 27.30	.006014	-.002042	- .976400	156.180	+ 30.463	+ 5,893.10
	5	103.50	207.00	+.00415	-.00138	+ .66600	+ 40.19	.003565	-.001185	- .571300	91.540	+ 34.525	+ 5,532.30
	6	105.35	210.70	+.00330	-.00110	+ .52700	+ 49.24	.002295	-.000765	- .366400	58.520	+ 34.237	+ 5,467.60
VERTICAL	0												
	1	53.33	106.66	+.00063	-.00188	- .30200	- 28.08	.000042	-.000126	- .020900	9.730	- 1.887	+ 932.60
	2	46.67	93.34	+.00071	-.00240	- .34300	- 21.40	.000047	-.000159	- .022700	10.980	- 1.418	+ 685.10
	3	40.00	80.00	+.00500	+.00500	+ .80000	+ 35.00	.002000	+.002000	+ .320000	51.200	+ 1.400	+ 2,240.00
	4	46.67	93.34	-.00240	+.00071	- .34300	- 21.40	.000638	-.000159	- .068500	10.980	+ 4.794	+ 685.10
	5	53.33	106.66	-.00188	+.00063	- .30200	- 28.08	.000377	-.000126	- .062400	9.730	+ 5.631	+ 932.60
	6												
								.190622	+.092094	+20.376300	3,623.160	962.396	167,817.40

b.) <u>Elastic Weights</u> (Eqs. 3-1, 4-5, 6, 7)

$$E\overline{P}_{45} = M_{45} F_{45} + M_{54} G_{54} + H_{45} E_{45} + \tau_{45}$$

$$= .190622 X_1 + 092094 X_2 + 20.3763 X_5 + 962.396$$

$$E\overline{P}_{54} = M_{54} F_{54} + M_{45} G_{45} + H_{54} E_{54} + \tau_{54}$$

$$= .190622 X_2 + .092094 X_1 + 20.3763 X_5 + 962.396$$

$$E\overline{M}_{45} = M_{45} E_{45} + M_{54} E_{54} + H_{45} \Omega_{45} + \epsilon_{45}$$

$$= 20.3763 X_1 + 20.3763 X_2 + 3623.16 X_5 + 167817.40$$

$$E\overline{P}_{56} = M_{56} F_{56} + M_{65} G_{65} + H_{56} E_{56} + \tau_{56}$$

$$= -.190622 X_3 - 092094 X_4 - 20.3763 X_6 - 962.396$$

$$E\overline{P}_{65} = M_{65} F_{65} + H_{56} G_{56} + H_{65} E_{65} + \tau_{65}$$

$$= -.190622 X_4 - .092094 - 20.3763 X_6 - 962.396$$

$$E\overline{M}_{56} = M_{56} E_{56} + M_{65} E_{56} + H_{65} \Omega_{56} + \epsilon_{56}$$

$$= -20.3763 X_3 - 20.3763 X_4 - 3623.16 X_6 - 167817.40$$

$$E\overline{P}_{14} = M_{14} F_{14} + M_{41} G_{41} = .333 X_1 - 44.444 X_5 - 637.140$$

$$E\overline{P}_{41} = M_{41} F_{41} + M_{14} G_{14} = .333 X_1 - 22.222 X_5 - 348.540$$

$$E\overline{P}_{47} = M_{47} F_{47} = .066 X_1 - 12.00$$

$$*E\overline{P}_{25} = M_{25} F_{25} + M_{52} G_{52} = .333 X_2 - .333 X_3 - 44.444 X_5 + 44.44 X_6$$

$$*E\overline{P}_{52} = M_{52} F_{52} + M_{25} G_{25} = .333 X_2 - .333 X_3 - 22.222 X_5 - 22.222 X_6$$

$$*E\overline{P}_{58} = M_{58} F_{58} = .066 X_2 - .066 X_3$$

$$E\overline{P}_{36} = M_{36} F_{36} + M_{63} G_{63} = 44.444 X_6 - .333 X_4$$

$$E\overline{P}_{63} = M_{63} F_{63} + M_{36} G_{36} = 22.222 X_6 - .333 X_6$$

$$E\overline{P}_{69} = M_{69} F_{69} = -.033 X_4$$

* Positive for left-hand panel,
Negative for right-hand panel.

c) <u>Elasto-Static Equations</u> (Eq. 3-4, 5, 6, 7)

Elastic weights are applied on the conjugate structure as shown in Fig. 6-2c.

<u>Panel 1</u>

<u>Static Moment about $\overline{25}$</u>

$$(\overline{P}_{14} + \overline{P}_{41} + \overline{P}_{47} + \overline{P}_{45})\ (600) = 0$$

1. $553.992\, X_1 + 55.254\, X_2 - 27773.820\, X_5 - 21174 = 0$

<u>Static Moment about $\overline{14}$</u>

$$(\overline{P}_{25} + \overline{P}_{52} + \overline{P}_{58} + \overline{P}_{54})\ (600) = 0$$

2. $55.254\, X_1 + 553.992\, X_2 - 439.620\, X_3 - 27773.820\, X_5 + 39999.600\, X_6 + 577437 = 0$

<u>Static Moment about $\overline{45}$</u>

$$(\overline{P}_{14} + \overline{P}_{25})\ (200) - (\overline{M}_{45}) = 0$$

3. $46.290\, X_1 + 46.290\, X_2 - 66.666\, X_3 - 21400.760\, X_5 + 8888.800\, X_6 - 295245 = 0$

Panel 2

Static Moment about $\overline{36}$

$$(\overline{P}_{25} + \overline{P}_{52} + \overline{P}_{58} + \overline{P}_{56})\ (600) = 0$$

4. $-439620\,X_2 + 553.992\,X_3 + 55254\,X_4 + 39999.600\,X_5 - 27773.820\,X_6 + 577437 = 0$

Static Moment about $\overline{25}$

$$(\overline{P}_{36} + \overline{P}_{63} + \overline{P}_{69} + \overline{P}_{65})\ (600) = 0$$

5. $55.254\,X_3 + 553.992\,X_4 - 27773.820 + 577437 = 0$

Static Moment about $\overline{56}$

$$(\overline{P}_{25} + \overline{P}_{36})\ (200) - (\overline{M}_{56}) = 0$$

6. $66.666\,X_2 - 46.290\,X_3 - 46.290\,X_4 - 8888.800\,X_5 + 21400.760\,X_6 + 167817 = 0$

d.) Moment and Force Matrix

$$
\begin{bmatrix}
(553.992) & (55.254) & (0) & (0) & (-27773.820) & (0) \\
(55.254) & (553.992) & (-439.620) & (0) & (-27773.820) & (39994.60) \\
(46.290) & (46.290) & (-66.666) & (0) & (-21400.760) & (8888.80) \\
(0) & (-439.620) & (553.992) & (55.254) & (39999.600) & (-27773.82) \\
(0) & (0) & (55.254) & (553.992) & (0) & (-27773.82) \\
(0) & (66.666) & (-46.290) & (-46.290) & (-8888.800) & (21400.76)
\end{bmatrix}
\begin{bmatrix}
(X_1) \\ (X_2) \\ (X_3) \\ (X_4) \\ (X_5) \\ (X_6)
\end{bmatrix}
=
\begin{bmatrix}
(21174.) \\ (-577437.) \\ (295245.) \\ (-577437.) \\ (-577437.) \\ (-167817.)
\end{bmatrix}
$$

e.) Final Moments and Forces

The moment and force matrix is solved by high speed computer and the results are compared in Table 6-4 to those found by the virtual work method.

Example No. 2	COMPARISON OF RESULTS	Table 6-4
Redundant	Virtual Work	String Polygon
$X_1 = M_{34}$	- 758.50 kip-in.	- 759.77 kip-in.
$X_2 = M_{43}$	-2588.77 kip-in.	-2587.85 kip-in.
$X_3 = M_{45}$	-2262.97 kip-in.	-2262.42 kip-in.
$X_4 = M_{54}$	-1670.12 kip-in.	-1670.82 kip-in.
$X_5 = H_{34}$	- 21.07 kips	- 21.07 kips
$X_6 = H_{56}$	- 17.04 kips	- 17.04 kips

CHAPTER VII

SUMMARY AND CONCLUSION

The extension of the String Polygon Method to the analysis of rigid truss-frames having straight, bent, or curved members is presented in this thesis.

A means of establishing compatability of slopes and displacements at the connection of the truss to the column is established.

The theory presented in this thesis is illustrated by two numerical examples. Truss-frames are analyzed by the String Polygon Method and by the Virtual Work Method, and the results are compared.

The String Polygon Method offers the following advantages:

a. The application of the differential elastic weights is simplified by concentrating their effect at the joints in the form of joint elastic weights.

b. The conjugate structure offers a physical model for the analyst.

c. Elasto-static equations compatable with the deformation of the real structure are obtained from the conjugate structure.

The elasto-static equations combined with the equations of static equilibrium offer the necessary conditions for analyzing a rigid truss-frame.

The application of the String Polygon Method offers numerical controls and yields theoretically exact results. The necessary computations are more direct and easier organized than for most conventional methods.

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