STRING POLYGON CONSTANTS FOR MEMBERS WITH SUDDEN CHANGE IN SECTION

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CONSTANTS FOR MEMBERS
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SECTION

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NOMENCLATURE

b........................Width of Beam.

L_i, L_j, L_k..............Length of Spans i, j, and k.

f........................Angular Flexibility Coefficient.

g........................Angular Carry-Over Coefficient.

q........................Specific Weight of Beam.

t........................End Slope Coefficient.

w........................Intensity of Load.

BM.......................Bending Moment Due to Loads.

F_{ij}, F_{ji}.................Angular Flexibilities.

G_{ij}, G_{ji}.................Carry-Over Values.

I_o.......................Minimum Moment of Inertia.

$L\beta$.......................Length of Haunch.

M_i, M_j, M_k..............Bending Moments at Points i, j, and k.

τ_{ji}, τ_{ij}.................Angular Load Functions.

$\emptyset_j$.......................Change in Angle of String Polygon at j.

μ........................Ratio of Minimum Moment of Inertia to the Moment of Inertia of the Haunch.

n........................Moving Load Position Coefficient.

A, B, C, D, E, F.........Letters Designating Cross Sections of Beam.

(DL)....................Due to Dead Load.

(HL)....................Due to Haunch Load.

(LL)....................Due to Live Load.

(UL)....................Due to Uniform Load.

E.......................Modulus of Elasticity.

L.......................Length of Span.

P.......................Intensity of Moving Load.

INTRODUCTION

The purpose of this thesis is the derivation of the general algebraic expressions for the angular functions of beams with sudden changes in section. The string polygon is used for the derivation of these formulas. An IBM 650 computer program was compiled for the solution of these expressions. Using the program, tables of numerical coefficients for angular functions were compiled.

The material in this thesis is an outgrowth of a series of seminar lectures given by Professor J. J. Tuma (1) in the Spring of 1960. Chu (2) showed how the string polygon could be used to solve for beam constants. Tuma, Lassley, and French (17) developed tables of angular functions for beams with parabolic haunches. Boecker (5) compiled tables of angular functions for beams with linear haunches.

CHAPTER I

THE THEORY OF THE STRING POLYGON

1-1. Definition

The development of the string polygon equation is based upon elementary structural relationships and makes use of the theory of simple bending. A simple beam of variable cross section loaded by a general system of loads is considered (Fig. 1-1).

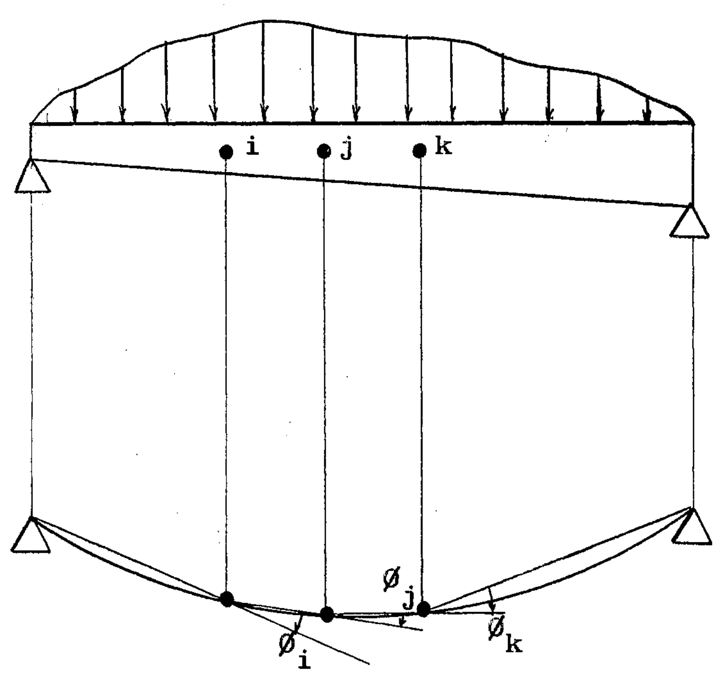


Fig. 1-1

Simple Beam $\overline{AB}$

The elastic curve of the simple beam $\overline{AB}$ is shown in exaggerated form. The string polygon is a straight line representation of the elastic curve.

Three arbitrarily selected points on the elastic curve are denoted as i, j, and k. The change in slope between line ij and jk is designated by ϕ_j (the change in slope of the string polygon). The algebraic expression for this change in slope is given by:

$$\phi_j = G_{ij}M_i + M_j \sum F_j + G_{kj}M_k + \sum \tau_j \qquad (1\text{-}1)$$

where:

M_i = the bending moment at i.

M_j = the bending moment at j.

M_k = the bending moment at k.

F_{ji} = the angular flexibility of the simple beam ij at j.

F_{jk} = the angular flexibility of the simple beam jk at j.

G_{ij} = the angular carry-over value of the simple beam ij at i.

G_{kj} = the angular carry-over value of the simple beam kj at k.

τ_{ji} = the angular load function of the simple beam ij at j.

τ_{jk} = the angular load function of the simple beam jk at j.

This form of the change in slope was derived by Tuma (1).

The physical interpretation of the constants G_{ij}, G_{ji}, F_{ji}, F_{ij}, τ_{ij}, and τ_{ji} can easily be shown. These constants are called angular functions.

1-2. Angular Functions

a.) The angular flexibility F_{ji} is the end slope of the simple beam $\overline{ij}$ at j due to a unit moment applied at that end (Fig. 1-2).

$$F_{ji} = \int_{o}^{L_i} \frac{x^2 dx}{L_i^2 \, EI_x} \qquad (1\text{-}2)$$

b.) The carry-over value G_{ij} is the end slope of the simple beam $\overline{ij}$ at i due to a unit moment applied at the far end j (Fig. 1-2).

$$G_{ij} = \int_{o}^{L_i} \frac{xx' dx}{L_i^2 \, EI_x} \qquad (1\text{-}3)$$

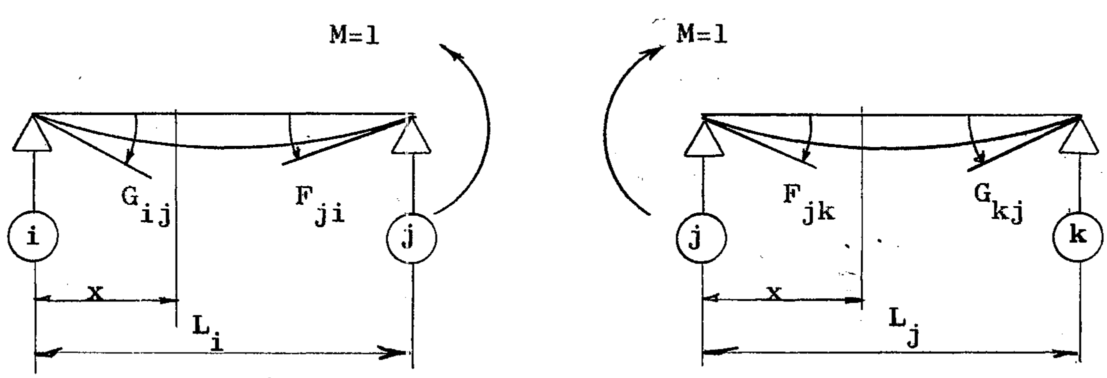


Fig. 1-2

Angular Flexibilities and Carry-Over Values c.) The angular load function $\mathcal{T}_{ji}$ is the end slope of the simple beam $\overline{ij}$ at j due to loads (Fig. 1-3).

$$\mathcal{T}_{ij} = \int_{o}^{L_i} \frac{BM_x \; x \; dx}{L \; EI_x} \qquad (1\text{-}4)$$

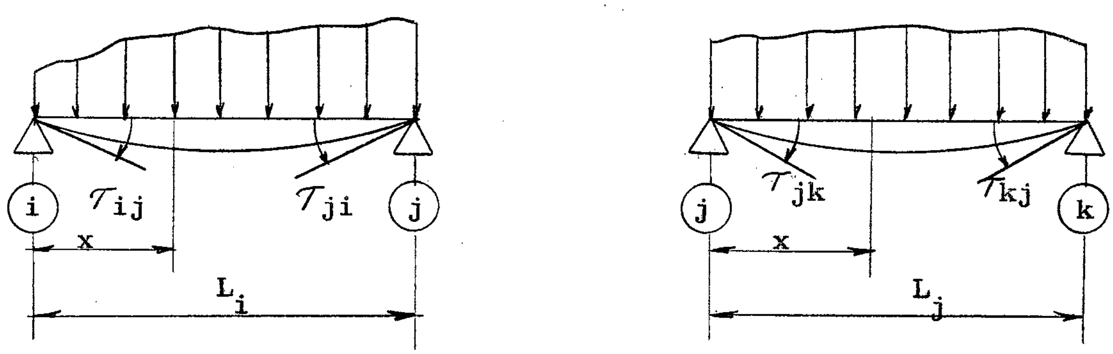


Fig. 1-3

Angular Load Functions

It has been shown that when the changes of slope of the string polygon ϕ_i, ϕ_j, ϕ_k are applied to the conjugate beam as elastic loads at points i, j, k the resulting shear and moment diagrams represent the slope and deflection of these points on the real beam (1,5,6).

CHAPTER II

PROPERTIES OF BEAMS
WITH SUDDEN CHANGES IN CROSS SECTION

2-1. Definition of Problem

The general expressions for the angular flexibilities, carry-over values, and load functions for two typical beams with sudden changes in cross section will be derived in a later chapter. The geometry of these beams and some typical functions are investigated first. It will be shown that the final angular functions are always a combination of the angular functions of the straight portions of the beam and haunch.

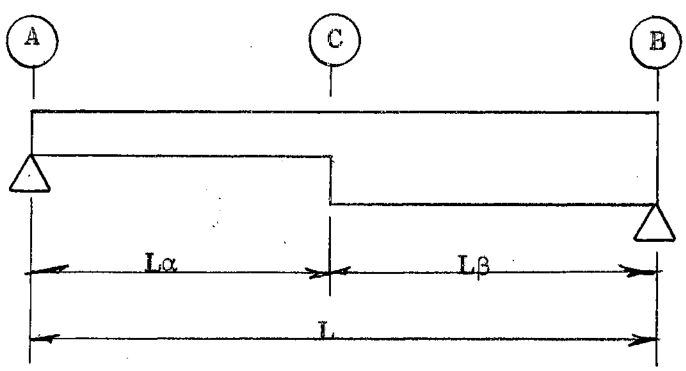


Fig. 2-1

Unsymmetrical Beam
With a Sudden Change in Cross Section

Figure 2-1 shows an unsymmetrical beam with a sudden change in cross section where:

β = the ratio of the haunch length to the total length.

α = $1-\beta$.

I_A = the moment of inertia of section $\overline{AC}$.

I_B = the moment of inertia of section $\overline{BC}$.

I_o = I_A.

μ = I_A/I_B.

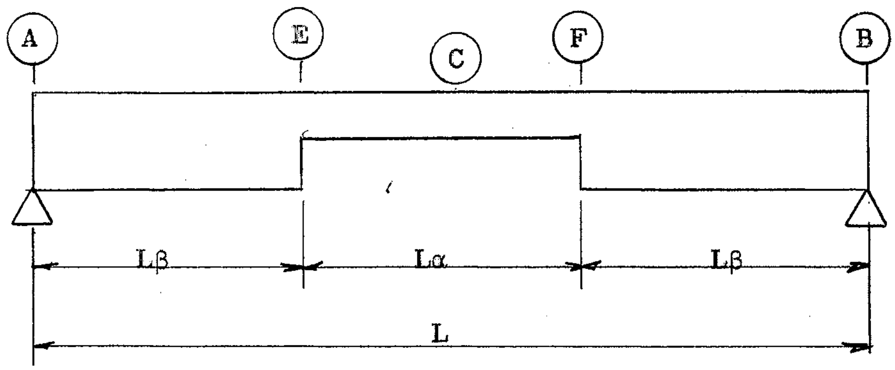


Fig. 2-2

Symmetrical Beams
With Sudden Changes in Cross Section

Figure 2-2 shows a symmetrical beam with sudden changes in cross section where:

β = the ratio of the haunch length to the total length.

α = $1-2\beta$.

I_A = I_B = the moment of inertia of the haunch.

$I_C = I_o$ = the moment of inertia of section $\overline{EF}$.

$\mu = I_o/I_A$.

2-2. Geometry of Bar

A bar of constant cross section shown in Fig. 2-3 is considered.

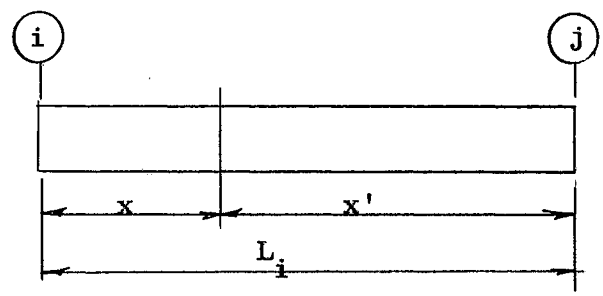


Fig. 2-3

Bar of Constant Cross Section

a.) Angular Flexibilities and Carry-Over Values

The angular flexibilities for a bar of constant cross section are (Eq. 1-2):

$$F_{ij} = \frac{L_i}{3EI_o}$$

$$F_{ji} = \frac{L_i}{3EI_o} \qquad (2\text{-}1)$$

The carry-over values are (Eq. 1-3):

$$G_{ij} = \frac{L_i}{6EI_o}$$

$$G_{ji} = \frac{L_i}{6EI_o} \tag{2-2}$$

b.) Angular Load Functions

The angular load functions are found using equation 1-4.

The angular load function for a uniform load is:

$$\mathcal{T}_{ij} = \mathcal{T}_{ji} = \frac{L_i^3}{24\ EI_o}$$

CHAPTER III

BEAM CONSTANTS BY THE STRING POLYGON

3-1. General

This chapter is devoted to developing beam constants for symmetrical and unsymmetrical beams with sudden changes in cross section by the method of the string polygon.

3-2. Unsymmetrical Beam

A beam of length L with one haunch of length $L\beta$ is considered (Table 3-1). The angular functions for a straight bar of constant cross section in portion AC and another bar of constant cross section in portion CB are considered. The moment of inertia of any cross section in AC is I_o. Similarly, the moment of inertia of any cross section in BC is I_o/μ.

The angular constants of this beam are derived for five separate cases. First, a unit couple is applied at end B and the end slopes of the elastic curve are calculated by the string polygon method. Similarly, the end slopes of the elastic curve are then found when the beam is loaded by: a unit couple at A, a uniform load, a uniform haunch load, and a concentrated load.

In all cases the following procedure is used:

(a) The bending moment diagram due to the applied load is drawn.

(b) The given beam is divided into a necessary number of segments and the string polygon elastic weights are calculated for each segment.

(c) The elastic weights are applied to the corresponding conjugate beam and the reactions of the conjugate beam are the required functions.

For simplicity the calculations are recorded in the following self-explanatory tables.

TABLE 3-1a Unit Moment at B--- Sketch

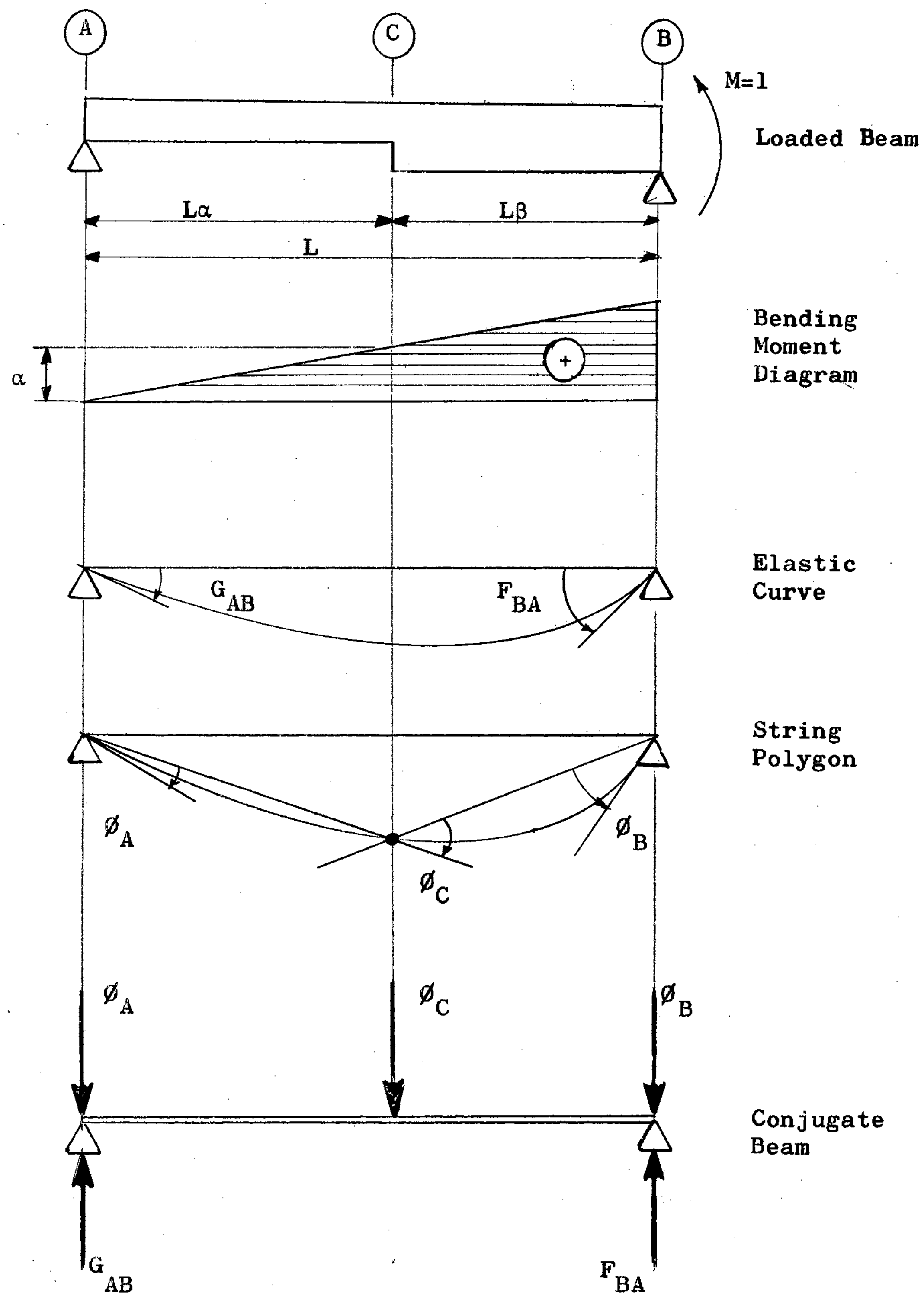


TABLE 3-1b Unit Moment at B--- Calculations

Elastic Weights:

$$\phi_A = \alpha G_{CA}$$

$$\phi_C = \alpha(F_{CA} + F_{CB}) + G_{BC}$$

$$\phi_B = \alpha G_{BC} + F_{BC}$$

Reactions:

$$F_{BA} = \frac{1}{3}(\alpha^3 + \mu - \mu\alpha^3) \frac{L}{EI_o}$$

$$G_{AB} = \frac{1}{2}(\alpha^2 + \mu - \mu\alpha^2) \frac{L}{EI_o} - F_{BA}$$

TABLE 3-2a Unit Moment at A--- Sketch

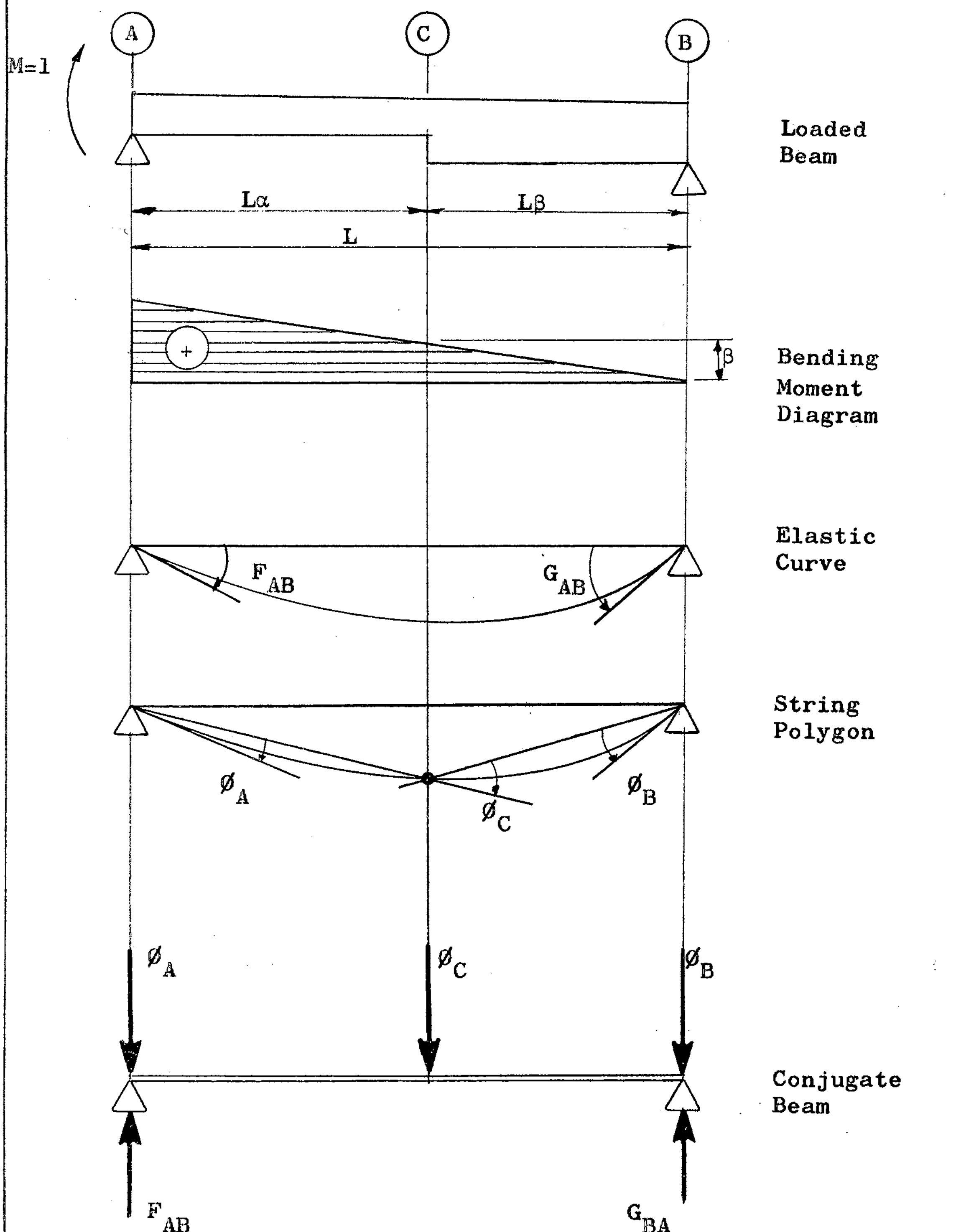


TABLE 3-2b Unit Moment at A--- Calculations

Elastic Weights:

$$\phi_A = F_{AC} + \beta G_{CA}$$

$$\phi_C = G_{AC} + \beta(F_{CA} + F_{CB})$$

$$\phi_B = \beta G_{CB}$$

Reactions:

$$F_{AB} = \left[\alpha + \mu - \mu\alpha\right]\frac{L}{EI_o} - 2G_{AB} - F_{BA}$$

$$G_{BA} = G_{AB}$$

TABLE 3-3a Uniform Load on $\overline{AB}$--- Sketch

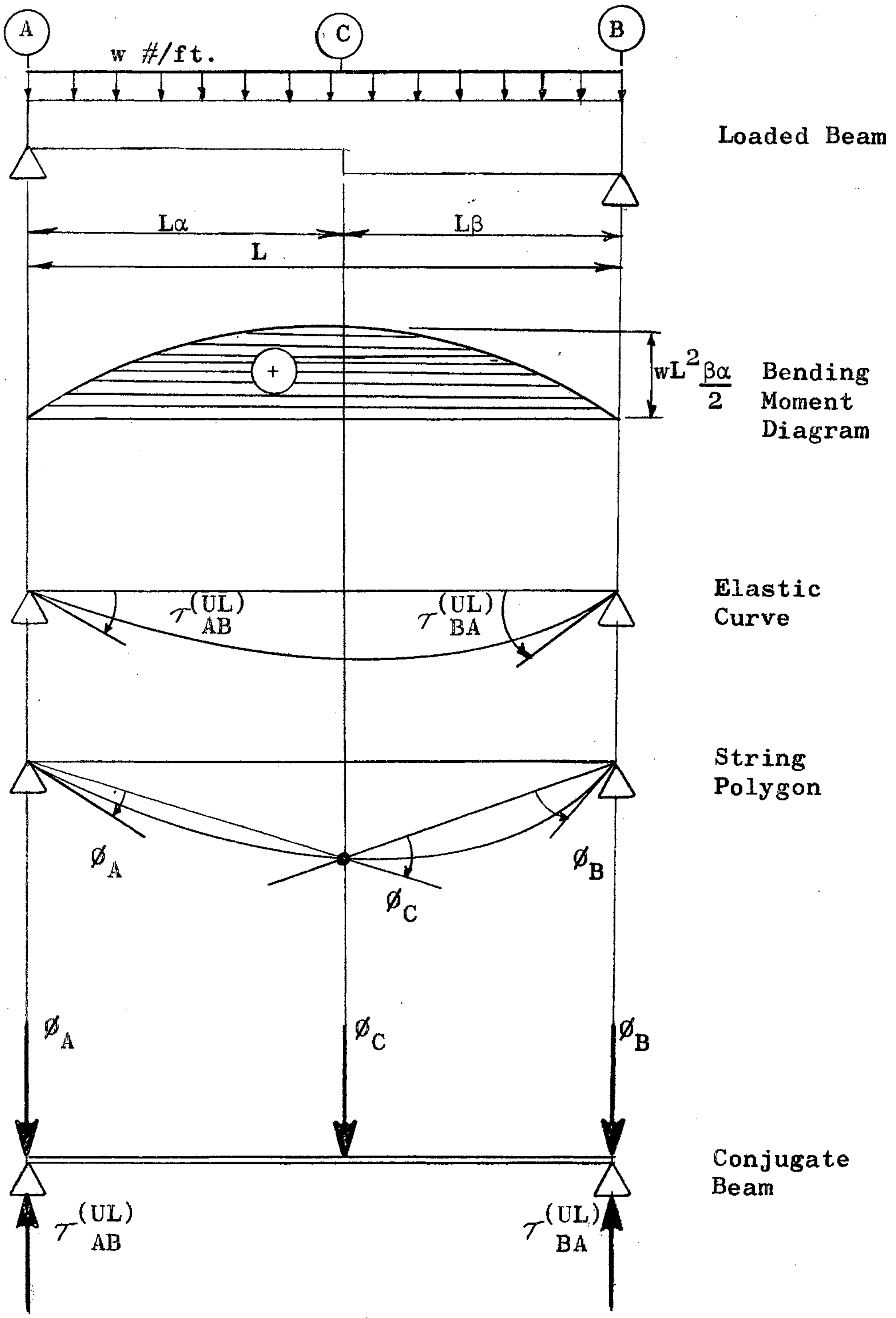


TABLE 3-3b Uniform Load on $\overline{AB}$--- Calculations

Elastic Weights:

$$\phi_A = wL^2 \frac{\beta\alpha}{2} G_{CA} + \tau_{CA}^{(UL)}$$

$$\phi_C = wL^2 \frac{\beta\alpha}{2}(F_{AC} + F_{CB}) + \sum \tau_C^{(UL)}$$

$$\phi_B = wL^2 \frac{\beta\alpha}{2} G_{BC} + \tau_{BC}^{(UL)}$$

Reactions:

$$\tau_{BA}^{(UL)} = \frac{wL^2}{2} F_{BA} - \frac{1}{8} (\alpha^4 + \mu - \mu\alpha^4) \frac{wL^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = \frac{wL^2}{2} G_{AB} - \tau_{BA}^{(UL)}$$

TABLE 3-4a Uniform Load on $\overline{CB}$--- Sketch

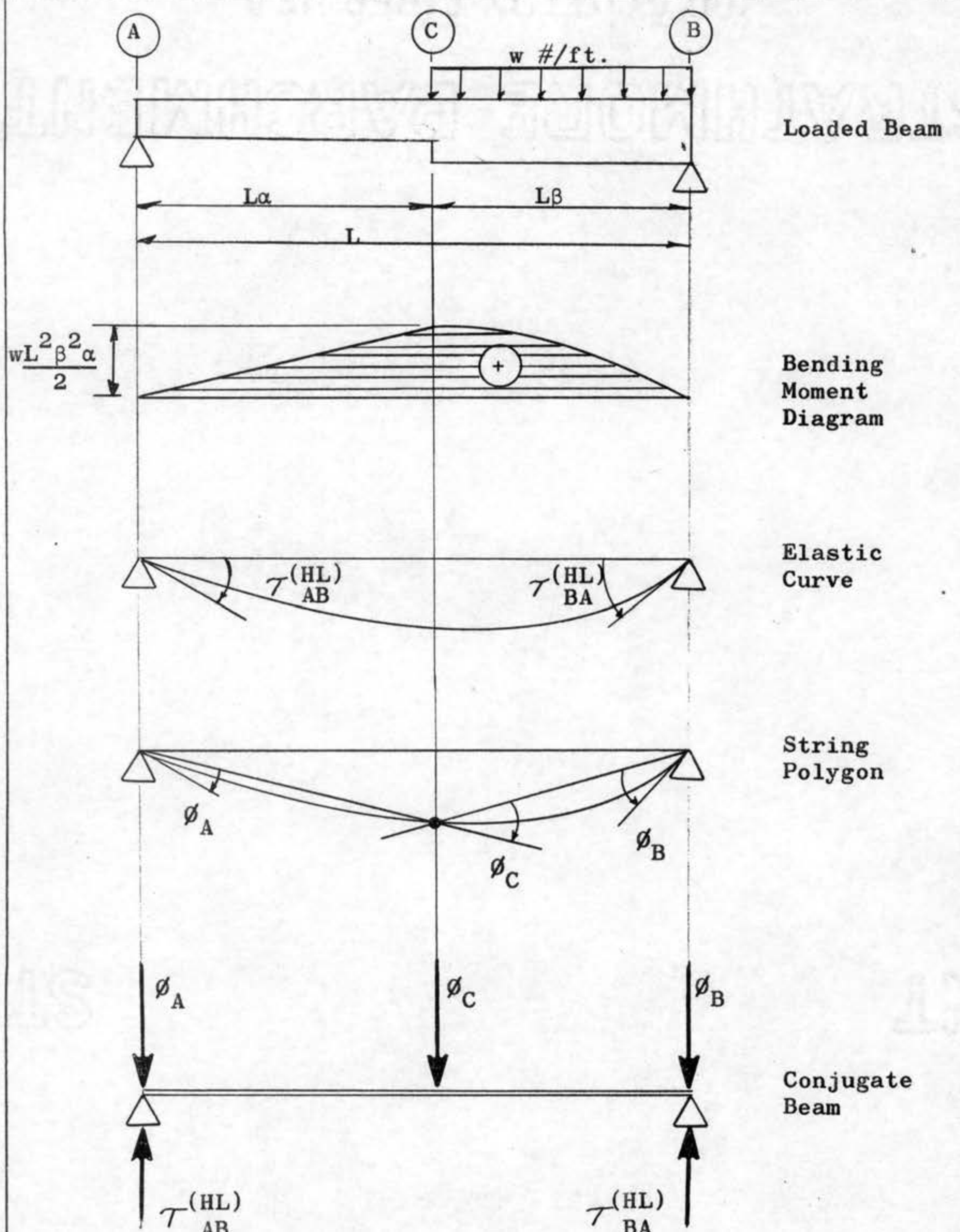


TABLE 3-4b Uniform Load on $\overline{CB}$--- Calculations

Elastic Weights:

$$\phi_A = \frac{wL^2}{2} \beta\alpha G_{AC}$$

$$\phi_C = \frac{wL^2}{2} \beta^2\alpha(F_{CA} + F_{CB}) + \tau_{CB}^{(UL)}$$

$$\phi_B = \frac{wL^2}{2} \beta^2\alpha G_{BC} + \tau_{BC}^{(UL)}$$

Reactions:

$$\tau_{BA}^{(HL)} = \frac{\beta^2}{2} \left[wL^2 F_{BA} - \frac{1}{6} \left(\mu - \alpha\mu + \frac{\mu}{2} - \frac{\mu\alpha^2}{2}\right) \frac{wL^3}{EI_o} \right]$$

$$\tau_{AB}^{(HL)} = \frac{\beta^2}{2} \left(\frac{\alpha^2}{2} + \frac{\mu}{2} - \frac{\mu\alpha^2}{2} - \frac{\mu}{3} - \frac{\mu\alpha}{3}\right) \frac{wL^3}{EI_o} - \tau_{BA}^{(HL)}$$

TABLE 3-5a Unit Load on $\overline{AC}$--- Sketch

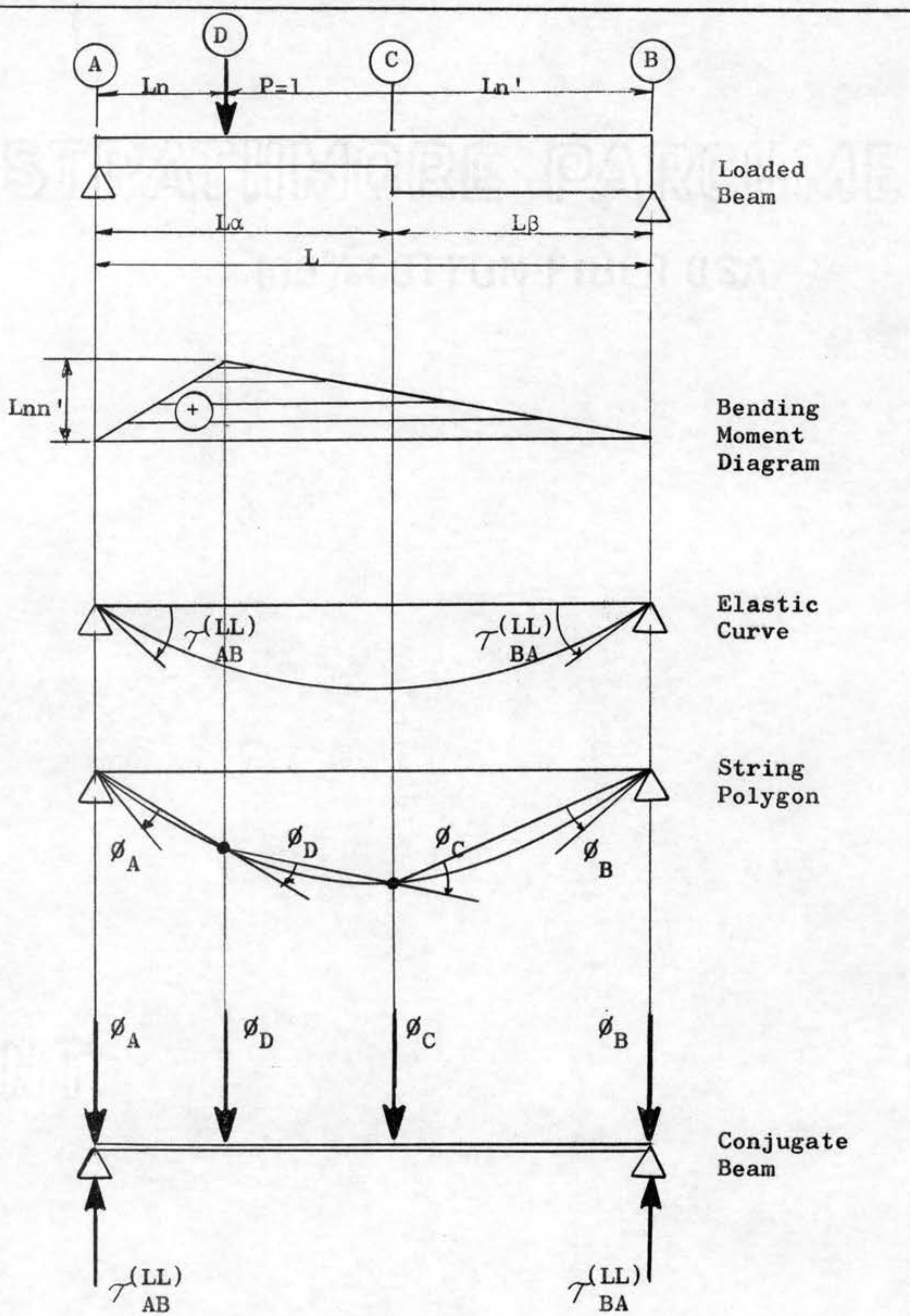


TABLE 3-5b Unit Load on $\overline{AC}$--- Calculations

Elastic Weights:

$$\phi_A = Lnn'G_{DA}$$

$$\phi_D = Lnn'(F_{DA} + F_{DC}) + Ln\beta G_{CD}$$

$$\phi_C = Lnn'G_{DC} + L\beta n(F_{CD} + F_{CB})$$

$$\phi_B = Ln\beta G_{CB}$$

Reactions:

$$\tau_{BA}^{(LL)} = LnG_{BA} - \left[\frac{n^3}{6}\right]\frac{L^2}{EI_o}$$

$$\tau_{AB}^{(LL)} = Ln(G_{BA} + F_{AB}) - \left[\frac{n^2}{2}\right]\frac{L^2}{EI_o} - \tau_{BA}^{(LL)}$$

TABLE 3-6a Unit Load on $\overline{CB}$--- Sketch

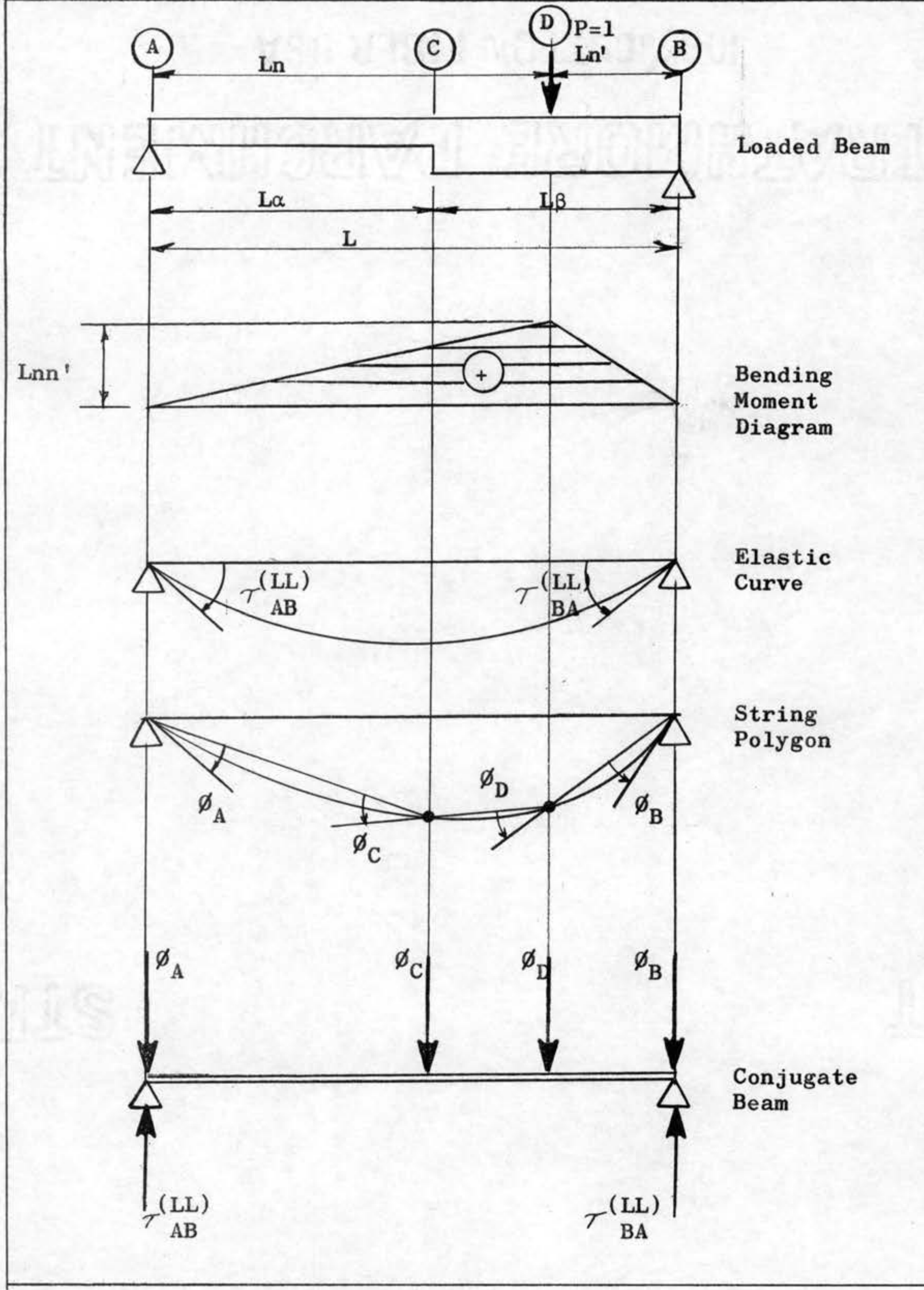


TABLE 3-6b Unit Load on $\overline{CB}$--- Calculations

Elastic Weights:

$$\phi_A = L\alpha n' G_{CA}$$

$$\phi_C = L\alpha n'(F_{CA} + F_{CA}) + Ln'nG_{DC}$$

$$\phi_D = L\alpha n' G_{DC} + Lnn'(F_{DC} + F_{DB})$$

$$\phi_B = Lnn' G_{DB}$$

Reactions:

$$\tau_{AB}^{(LL)} = Ln' G_{BA} - \left[(n')^3 \frac{\mu}{6}\right] \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(LL)} = Ln'(G_{BA} + F_{AB}) - \left[\frac{\mu}{2} (n')^2\right] \frac{L^2}{EI_o} - \tau_{AB}^{(LL)}$$

3-3. Symmetrical Beam

A beam of length L with two symmetrical haunches of length $L\beta$ is considered (Table 3-7). The angular functions of a bar of constant cross section in portion AE, a bar of constant cross section in portion EF, and another bar of constant cross section in portion FB are considered. The moment of inertia of any cross section in EF is I_o. Similarly, the moment of inertia of any cross section in AE or FB is I_o/μ.

The angular functions of this beam are derived for four separate cases. First, a unit couple is applied at A and the end slopes of the elastic curve are calculated by the string polygon method. Similarly, the end slopes of the elastic curve are then found when the beam is loaded by: a uniform load, a uniform haunch load, and a concentrated load. In all cases the procedure used is the same as that used for unsymmetrical beams.

TABLE 3-7a Unit Moment at A--- Sketch

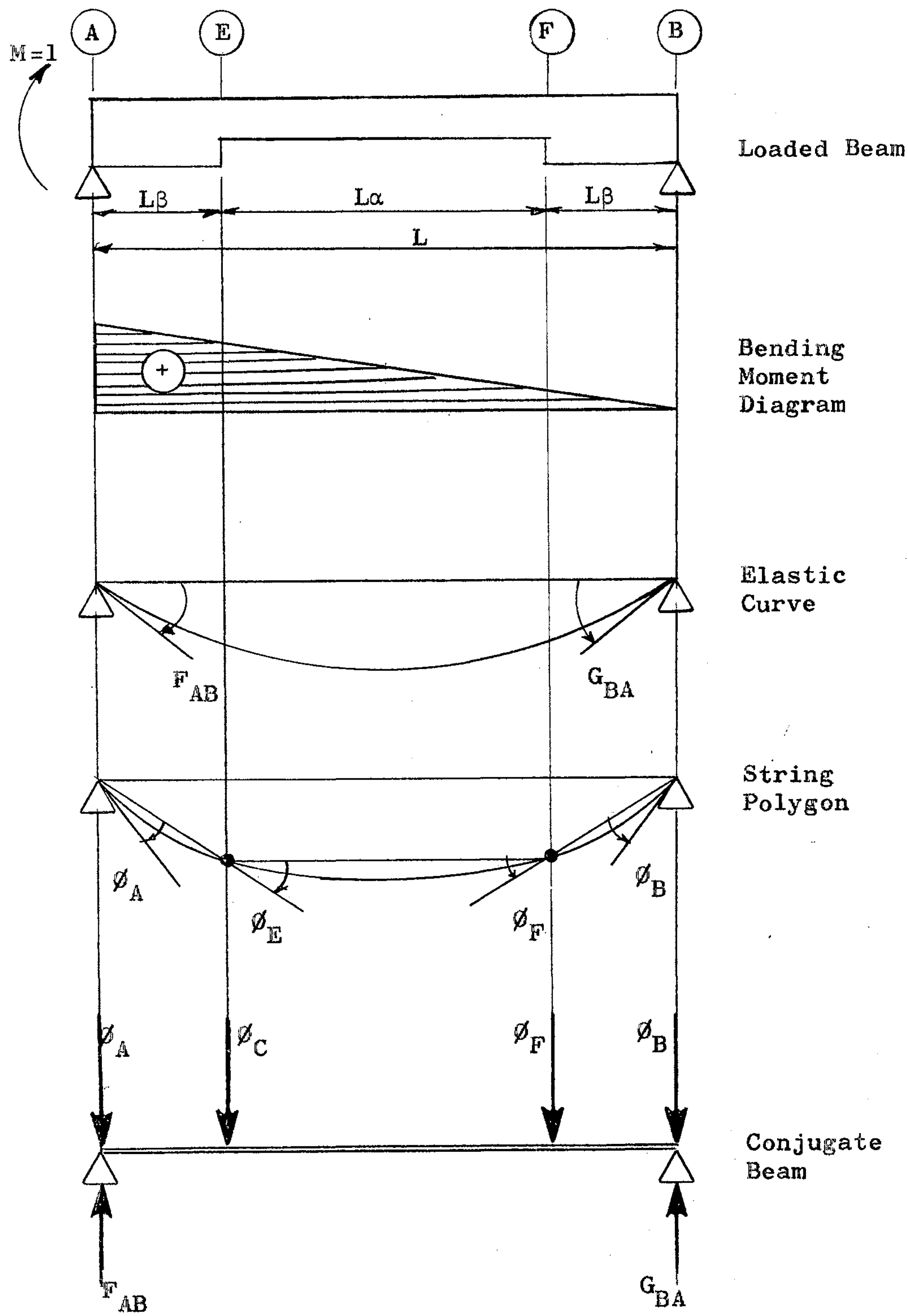


TABLE 3-7b Unit Moment at A--- Calculations

Elastic Weights:

$$\phi_A = F_{AE} + (1-\beta)G_{EA}$$

$$\phi_E = G_{AE} + (1-\beta)(F_{EA} + F_{EF}) + \beta G_{EF}$$

$$\phi_F = (1-\beta)G_{EF} + \beta(F_{EF} + F_{FB})$$

$$\phi_B = \beta G_{FB}$$

Reactions:

$$F_{AB} = \left(\frac{\alpha}{4} + \frac{\alpha^3}{12} + \frac{\mu}{3} - \frac{\mu\alpha}{4} - \frac{\mu\alpha^3}{12}\right)\frac{L}{EI_o}$$

$$G_{BA} = G_{AB} = \left(\frac{\alpha}{4} - \frac{\alpha^3}{12} - \frac{\mu}{6} - \frac{\mu\alpha}{4} + \frac{\mu\alpha^3}{12}\right)\frac{L}{EI_o}$$

By symmetry:

$$F_{AB} = F_{BA}$$

$$G_{AB} = G_{BA}$$

TABLE 3-8a Uniform Load on $\overline{AB}$--- Sketch

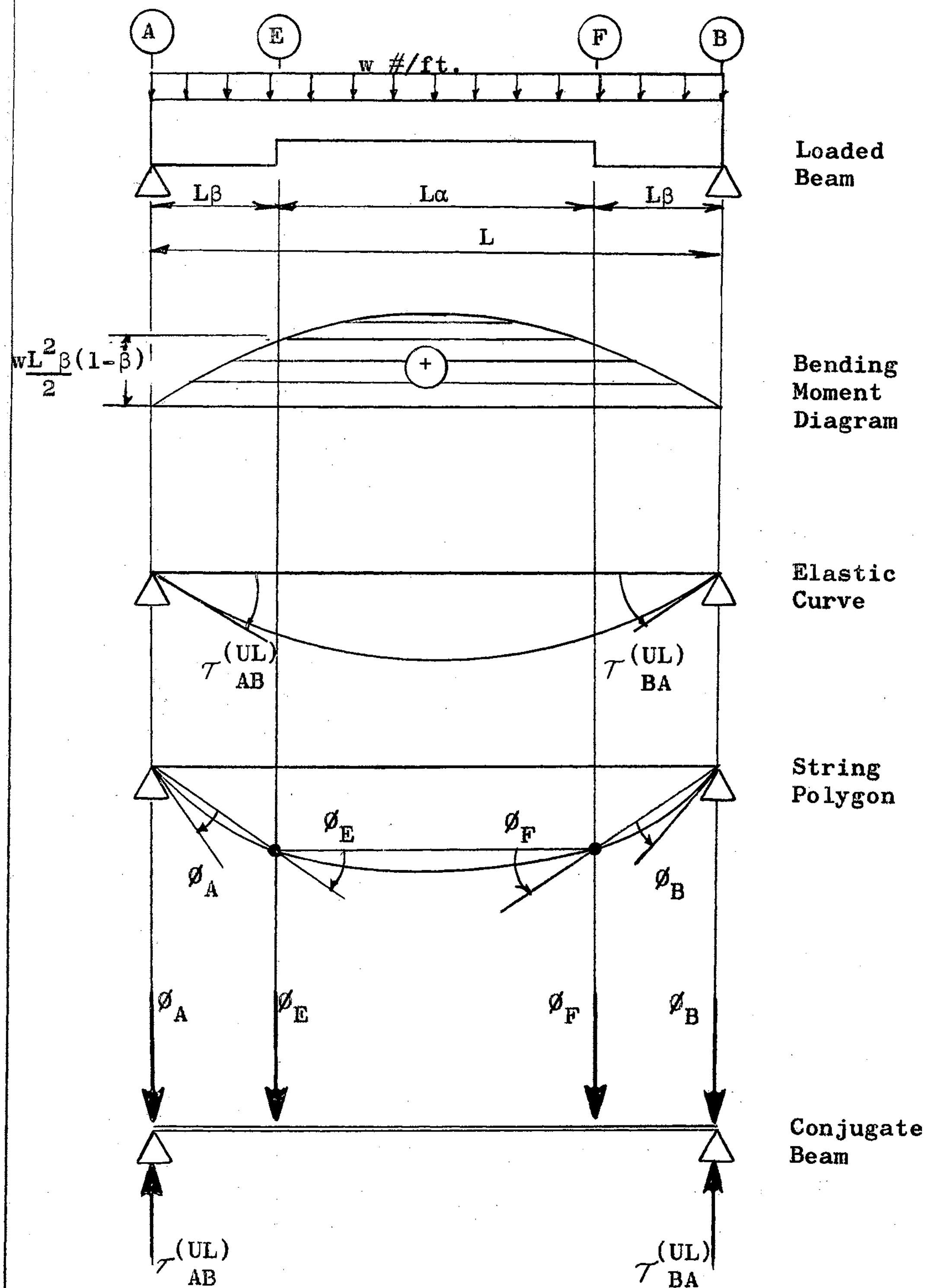


TABLE 3-8b Uniform Load on $\overline{AB}$--- Calculations

Elastic Weights:

$$\phi_A = \frac{wL^2}{2} \beta(1-\beta)G_{EA} + \tau_{AE}^{(UL)}$$

$$\phi_E = \frac{wL^2}{2} \beta(1-\beta)(F_{EA} + F_{EF} + G_{EF}) + \sum \tau_E$$

$$\phi_F = \phi_E$$

$$\phi_B = \phi_A$$

Reaction:

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = \frac{G_{BA}}{4} wL^2$$

TABLE 3-9a Uniform Load on Haunches--- Sketch

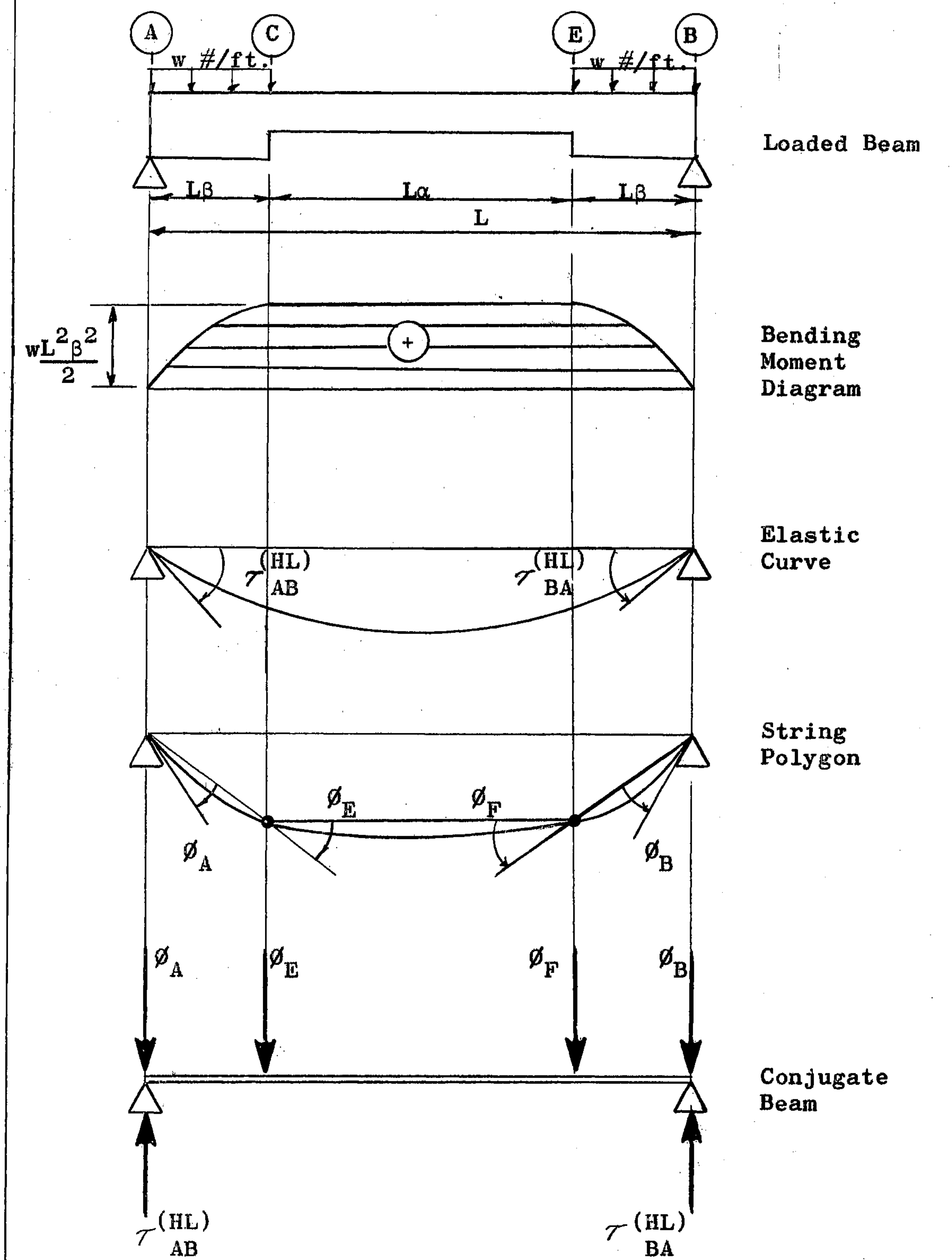


TABLE 3-9b Uniform Load on the Haunches--- Calculations

Elastic Weights:

$$\phi_A = \frac{wL^2}{4} \beta^2 G_{EA} + \tau_{AE}^{(UL)}$$

$$\phi_E = \frac{wL^2}{2} \beta^2 (F_{EA} + F_{EF} + G_{EF}) + \tau_{EA}^{(UL)}$$

$$\phi_F = \phi_E$$

$$\phi_B = \phi_A$$

Reactions:

$$\tau_{AB}^{(HL)} = \tau_{BA}^{(HL)} = \frac{\beta^2}{2} \left[\frac{\mu}{3} - \frac{\alpha\mu}{3} + \frac{\alpha}{2} \right] \frac{L^3}{EI_o}$$

TABLE 3-10a Unit Load on $\overline{AE}$--- Sketch

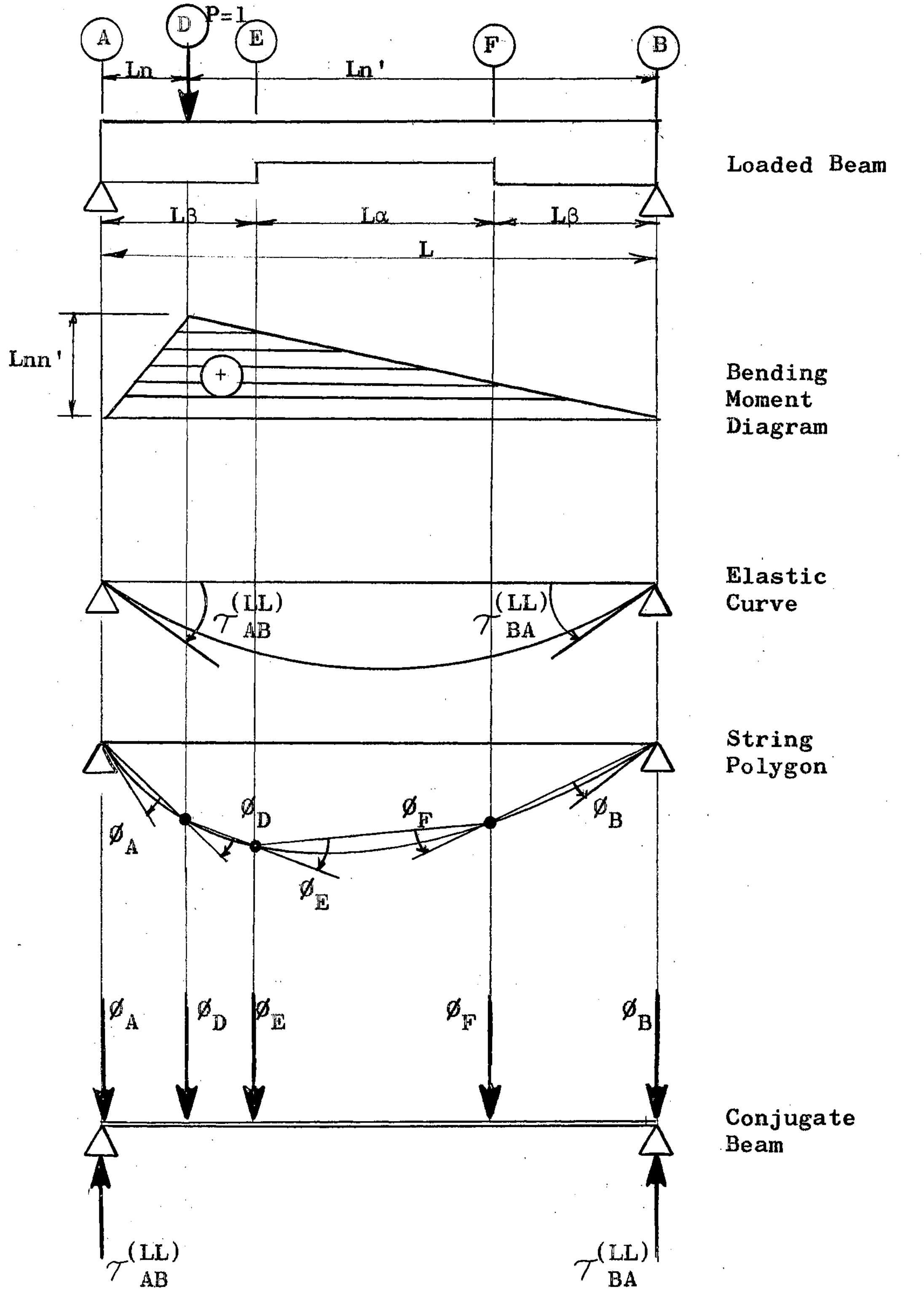


TABLE 3-10b Unit Load on $\overline{AE}$--- Calculations

Elastic Weights:

$$\phi_A = Lnn'G_{DA}$$

$$\phi_D = Lnn'(F_{DA} + F_{DE}) + L(1-\beta)nG_{DE}$$

$$\phi_E = Lnn'(G_{DE} + L(1-\beta)n(F_{ED} + F_{EF}) + L\beta nG_{FE}$$

$$\phi_F = L(1-\beta)nG_{EF} + L\beta n(F_{EF} + F_{FB})$$

$$\phi_B = L\beta nG_{FB}$$

Reactions:

$$\tau_{BA}^{(LL)} = nLG_{BA} - \left[\frac{(\mu n^3)}{6}\right]\frac{L^2}{EI_o}$$

$$\tau_{AB}^{(LL)} = nL(F_{BA} + G_{BA}) - \left[\frac{(\mu n^2)}{2}\right]\frac{L^2}{EI_o} - \tau_{BA}^{(LL)}$$

TABLE 3-11a Unit Load on $\overline{EF}$--- Sketch

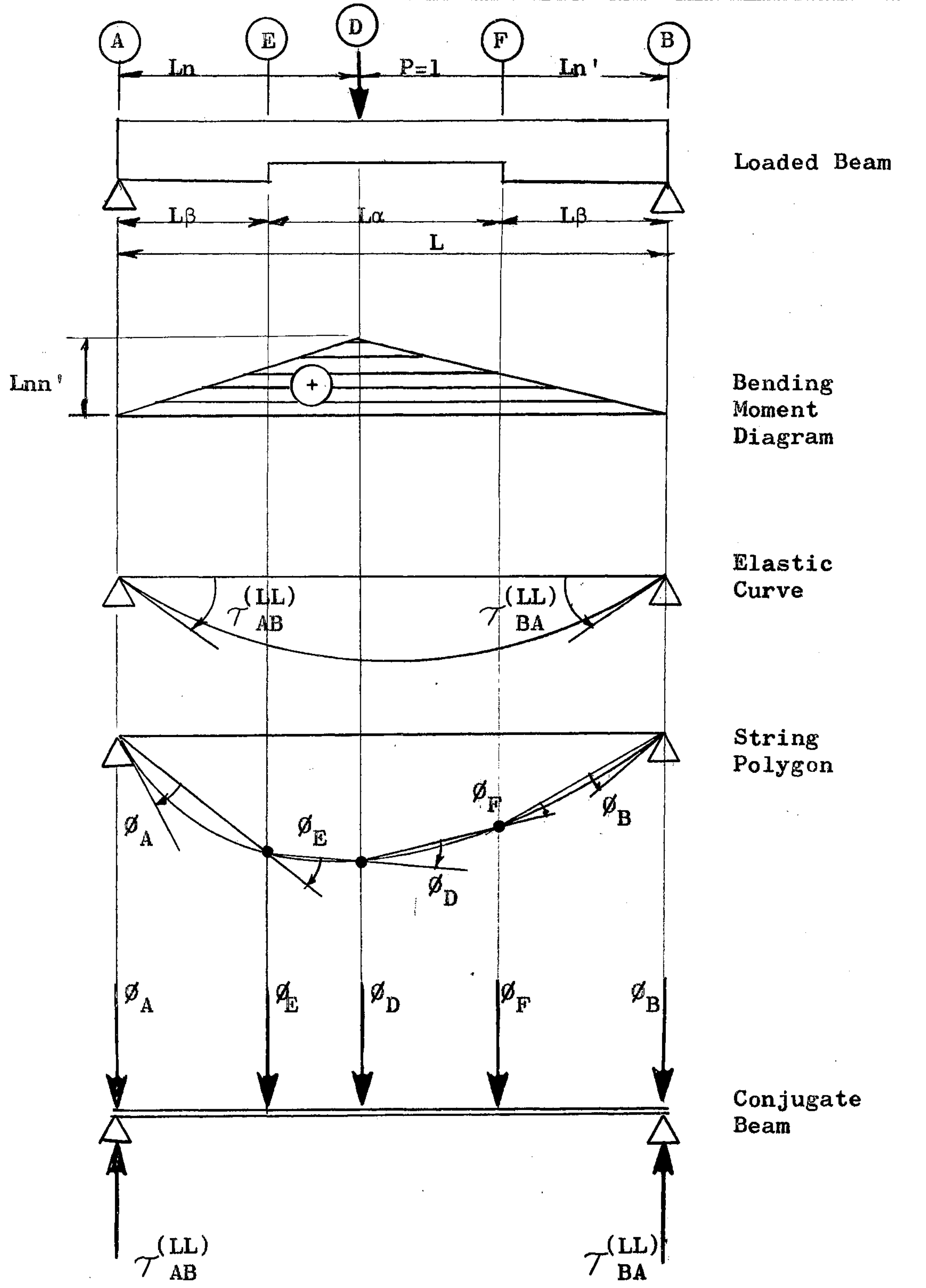


TABLE 3-11a Unit Load on $\overline{EF}$--- Calculations

Elastic Weights:

$$\phi_A = Ln'\beta G_{EA}$$

$$\phi_E = Ln'\beta(F_{EA} + F_{ED}) + Lnn'G_{DE}$$

$$\phi_D = Ln'\beta G_{ED} + Lnn'(F_{DE} + F_{DF}) + Ln\beta G_{FD}$$

$$\phi_F = Lnn'\beta G_{DF} + Lnn'(F_{FD} + F_{FB})$$

$$\phi_B = Ln\beta G_{FB}$$

Reactions:

$$\tau_{BA}^{(LL)} = \frac{Ln}{2}G_{BA} + \left[\frac{n'}{4}\left(\frac{\alpha^3}{3} + \frac{\mu}{3} - \frac{\mu\alpha^3}{3} - \frac{\alpha^2}{2} - \frac{\mu}{2} + \frac{\mu}{2}\alpha^2\right) - \frac{1}{24} + \frac{n}{8} - \frac{n^3}{6}\right]\frac{L^2}{EI_o}$$

$$\tau_{AB}^{(LL)} = \left[\frac{1}{4}\left(\alpha + \mu - \mu\alpha - \frac{\alpha^2}{2} - \frac{\mu}{2} + \mu\frac{\alpha^2}{2}\right) - \frac{1}{8} + \frac{n}{2} - \frac{n^2}{2}\right]\frac{L^2}{EI_o} - \tau_{BA}^{(LL)}$$

CHAPTER IV

PROGRAM FOR THE IBM 650 ELECTRONIC COMPUTER

4-1. General

Programming for the solution of any problem on a digital computer is accomplished in two steps. First, a drawing showing each phase of the problem and the sequence of operations is made. Second, from the drawing, flow chart, a series of instructions for the computer is established.

The program in this chapter was prepared in floating decimal arithmetic for the IBM 650 Electronic Computer at Oklahoma State University's computing center. The coding form used is that of I.B.M.'s Formula Translator hereafter referred to as Fortran. The Fortran system may also be used with the I.B.M. 704.

4-2. Input Card Format

The description of the beam for which constants are desired is introduced to the computer with ten words on two cards. The dimension coefficients are μ, the ratio of the moments of inertias, β, the length factor, and N, the live load factor. The symobls $\Delta\mu$, $\Delta\beta$, and ΔN are the increments by which the dimension coefficients are to be increased. The symbols μmax, βmax, and Nmax are the maximum values which the dimension coefficients may attain. The beam type number is one for unsymmetrical beams and two for symmetrical beams. Floating decimal arithmetic is used for all words (Fig. 4-1).

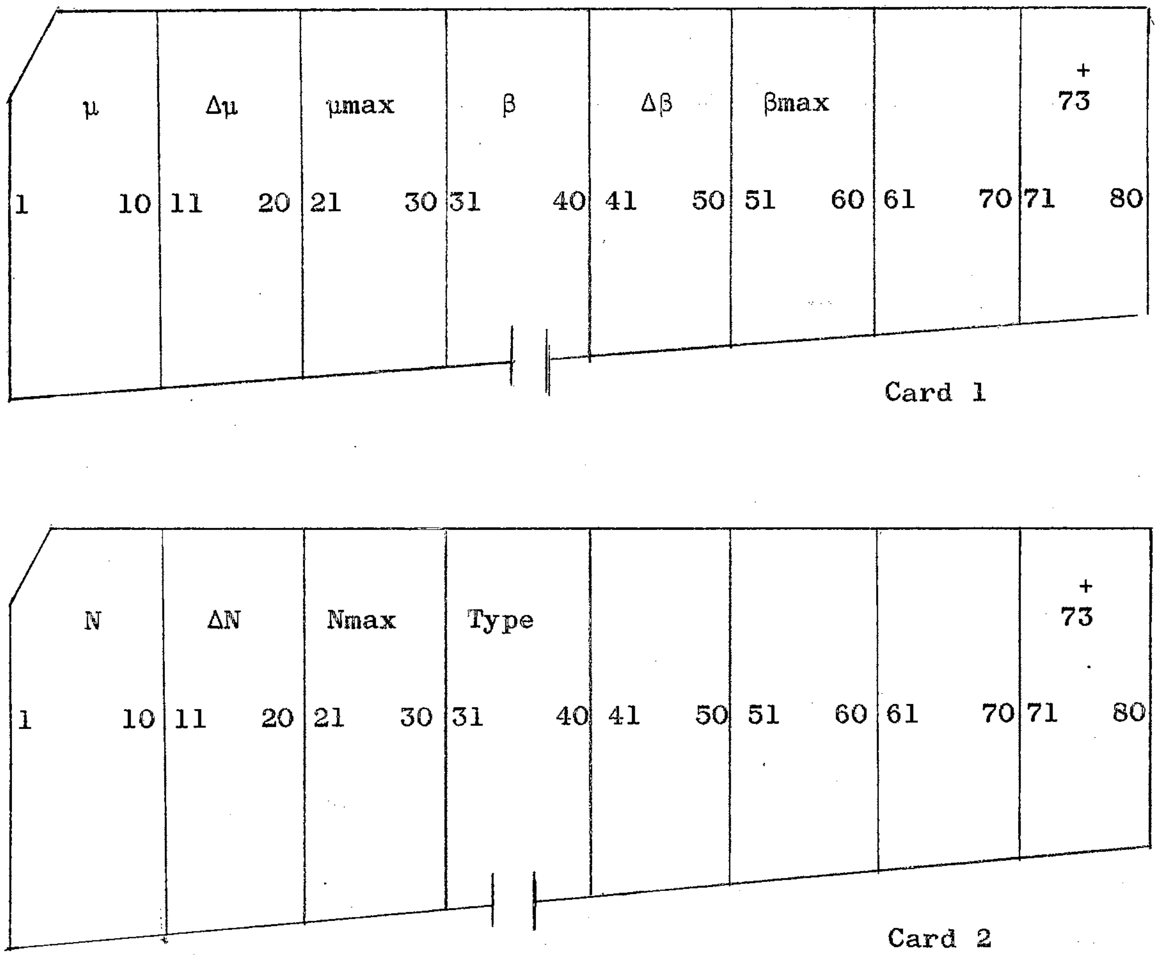


Fig. 4-1
Input Data Cards

The general data cards for unsymmetrical beams will be (Fig. 4-2):

$\mu = 0 \longrightarrow .95$ in increments of 0.05.

$\beta = 0.1 \longrightarrow 1.0$ in increments of 0.1.

$N = 0.1 \longrightarrow 0.9$ in increments of 0.1.

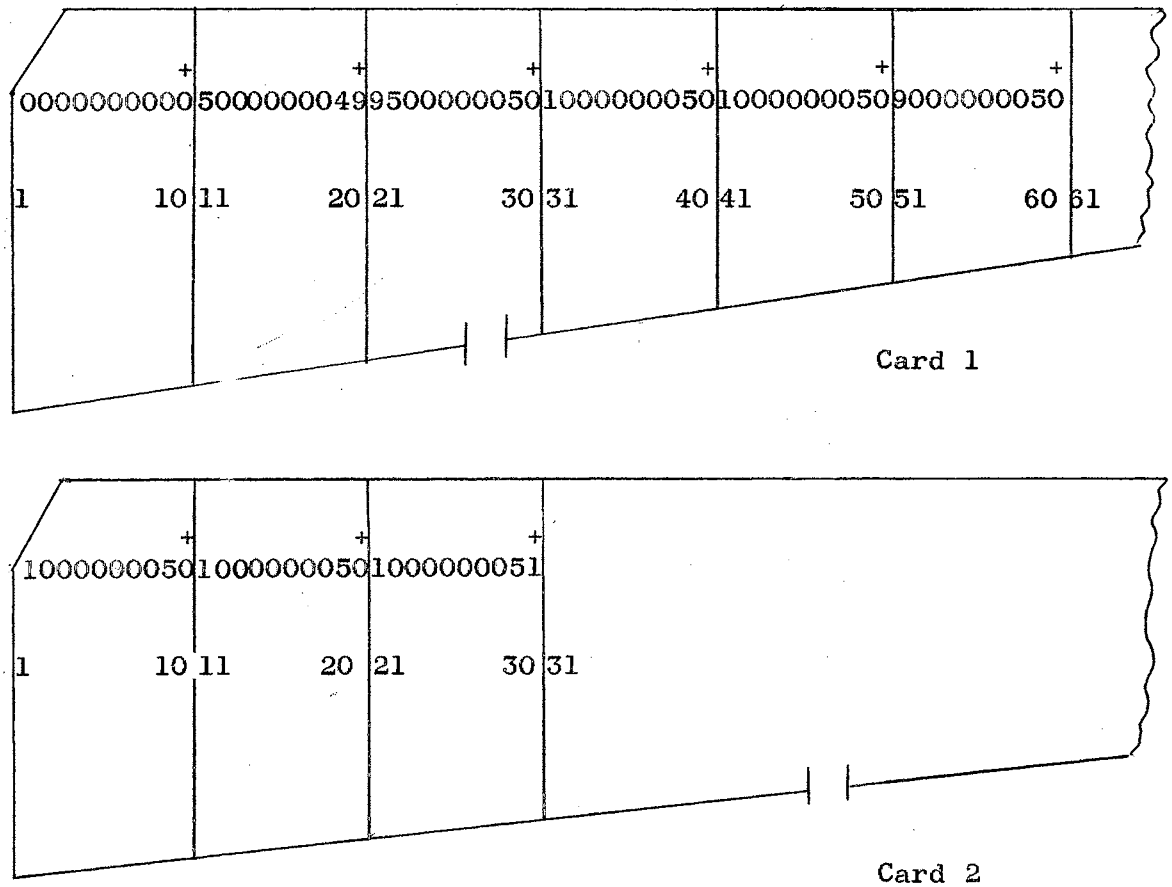


Fig. 4-2
General Input Data Cards for Unsymmetrical Beams

4-3. Output Card Format

The angular function coefficients will be in floating decimal form on a number of cards depending upon the input data.

The beam identification numbers

$$\overset{+}{5000000050} \text{ and } \overset{+}{3000000050}$$

will appear on the first output card for which

$$\mu = 0.5$$

$$\beta = 0.3$$

and the number

$$\overset{+}{4000000050}$$

will appear card containing influence coefficients for n = 0.4.

The first output cards will be arranged as follows:

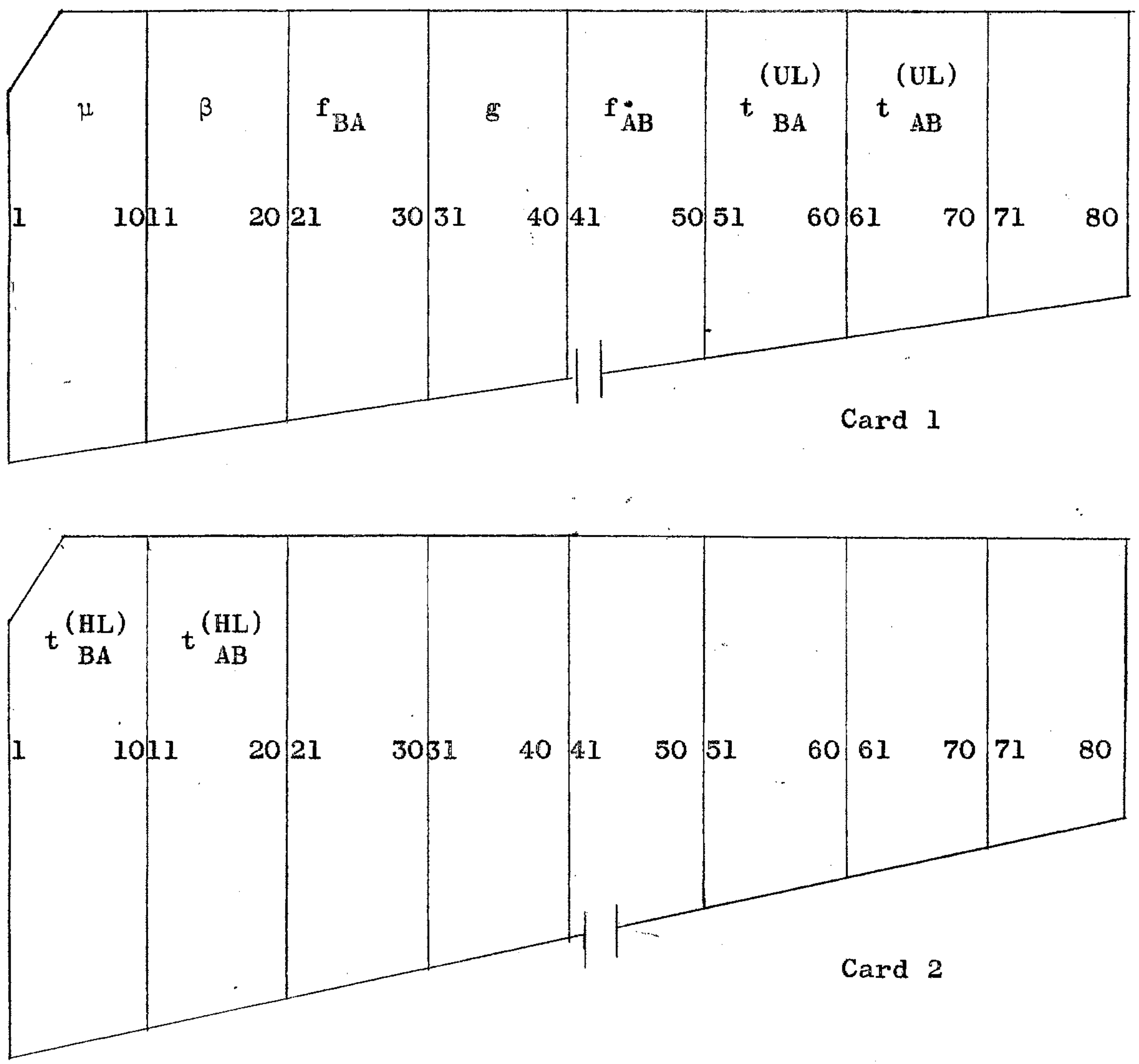


Fig. 4-3
First Output Cards

The angular live load coefficients will appear as follows:

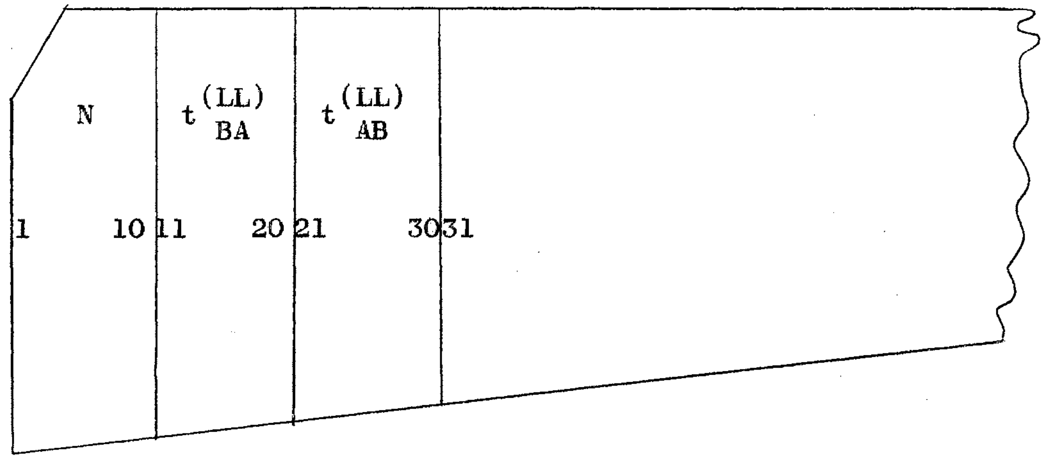


Fig. 4-4
Output Card for Live Load Coefficients

4-4. Flow Chart

The flow chart (Fig. 4-5) was prepared as an aid to setting up the sequence of instructions to the computer.

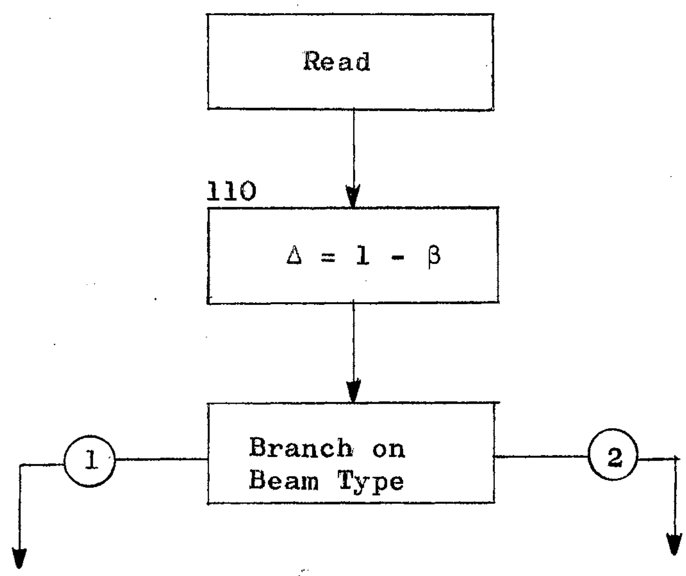


Fig. 4-5
Flow Chart

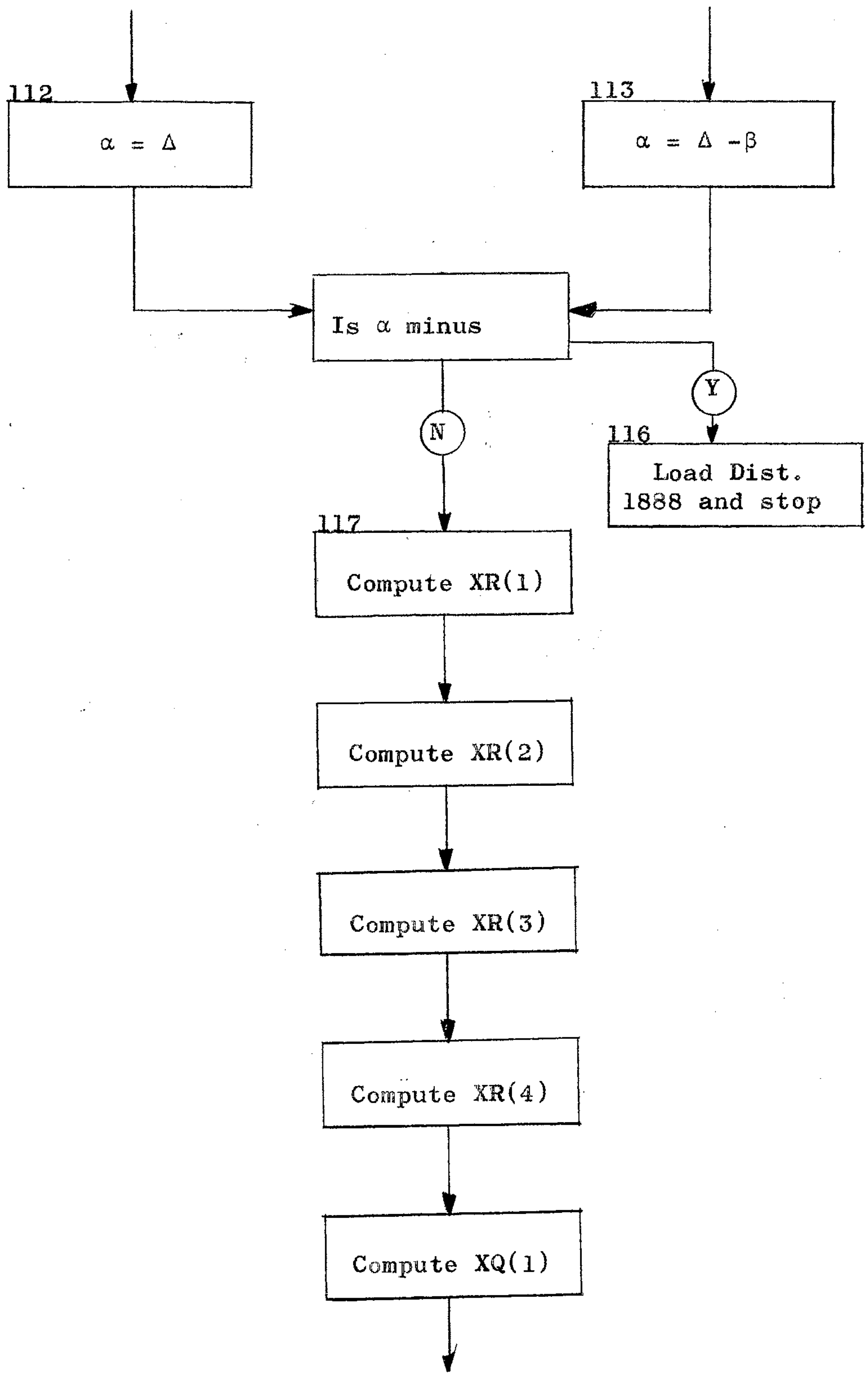


Fig. 4-5 (Con't.)
Flow Chart

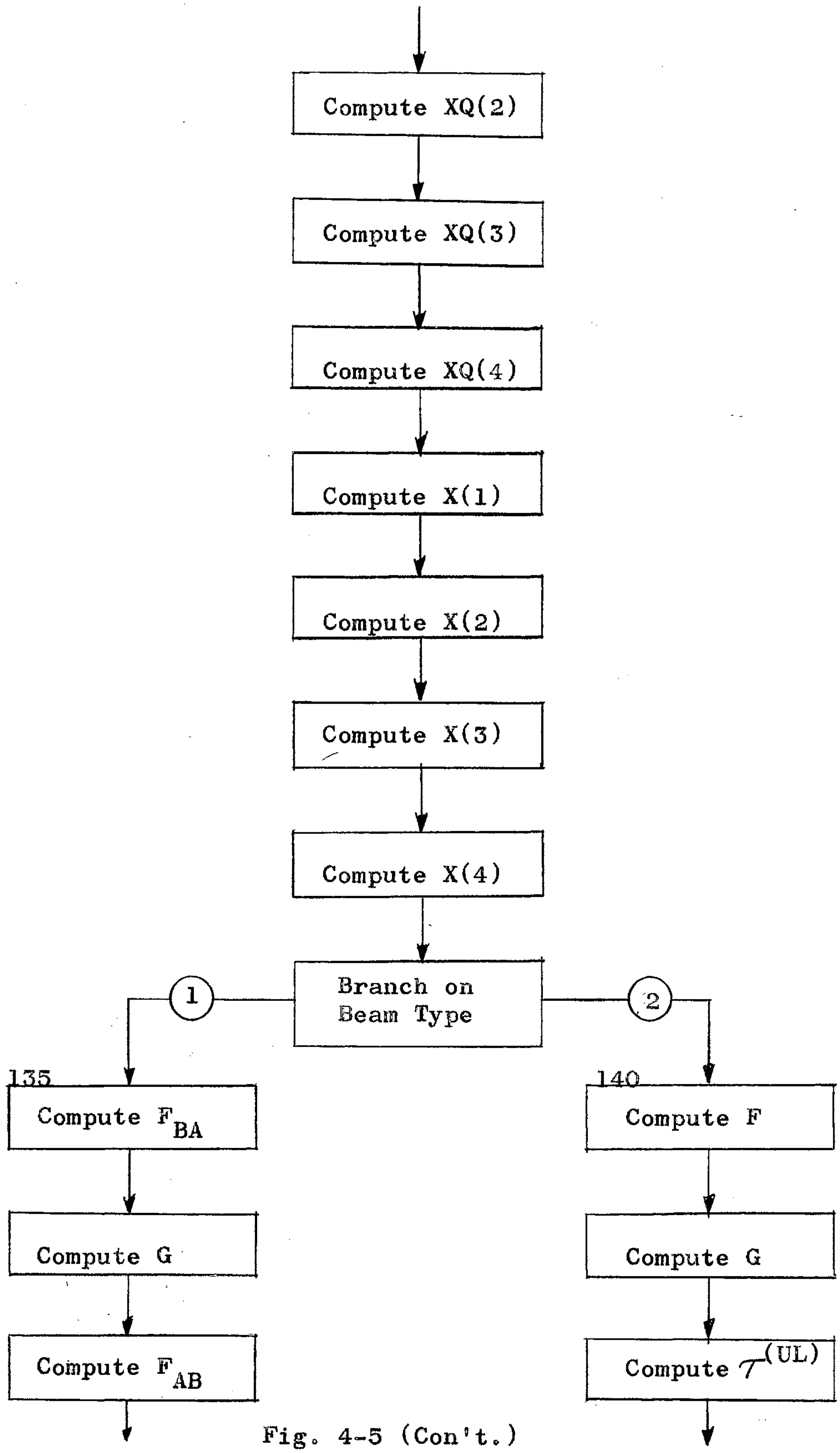


Fig. 4-5 (Con't.)
Flow Chart

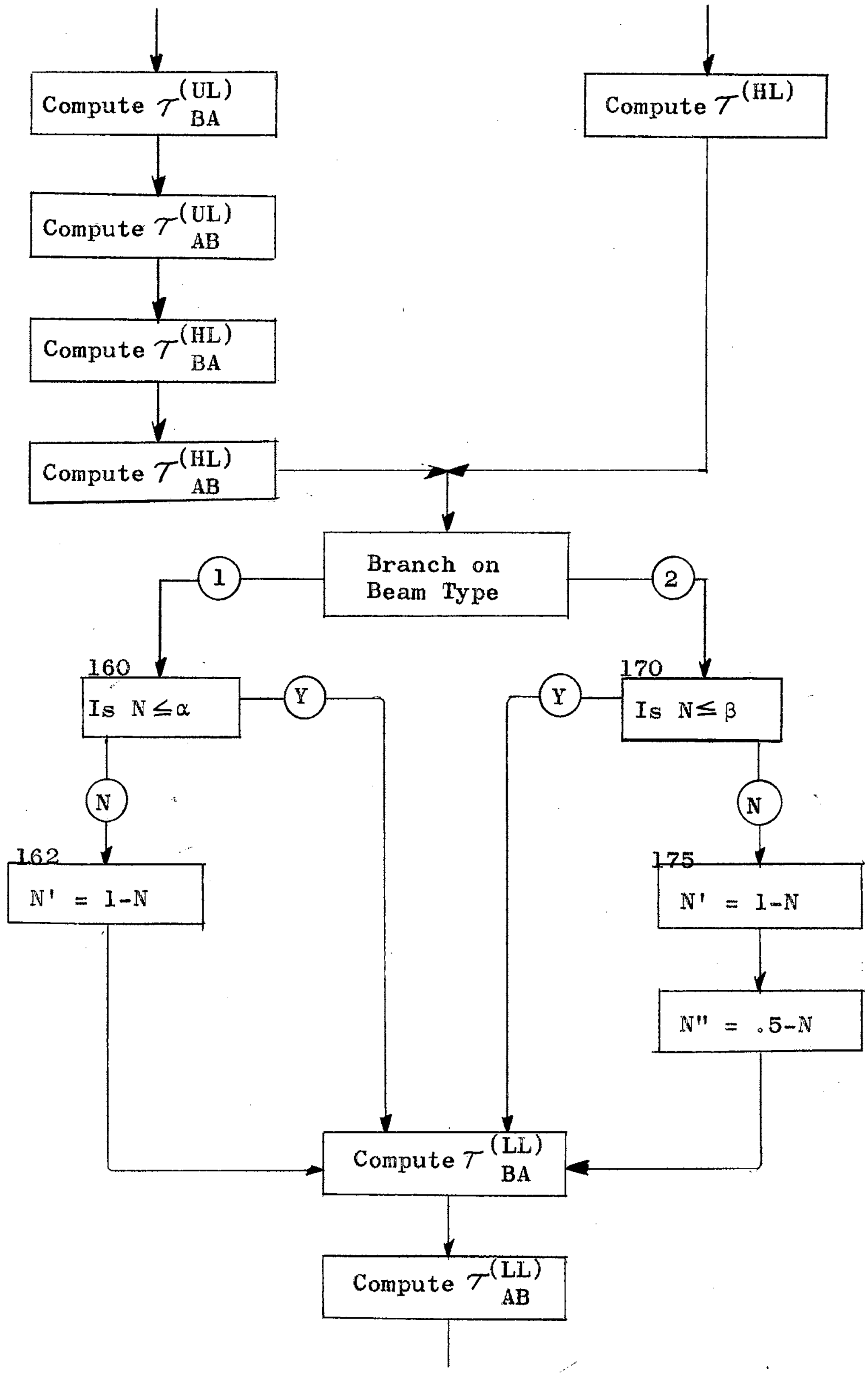


Fig. 4-5 (Con't.)
Flow Chart

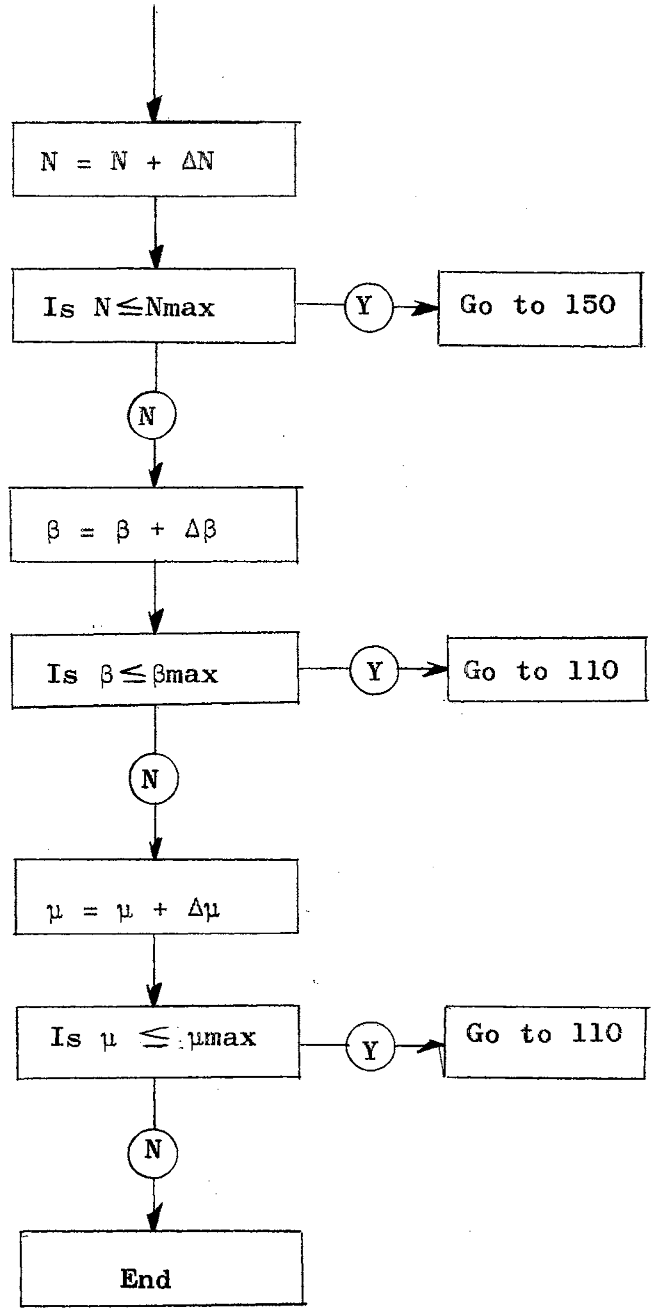


Fig. 4-5 (Con't)
Flow Chart

4-5. Statement of Program

The following program (Fig. 4-6), assembled in I.B.M. 650 Fortran, will calculate beam constants including 10-point influence coefficients for beams of either type for which μ is expressed as a multiple of 5 hundredths. Provision is made for beams of constant moment of inertia and the program will give correct results for β equal to zero.

The program will give misleading results for β equal to 1 for the unsymmetrical beam and β equal to .5 for the symmetrical beam as the resulting coefficient will be multiplied by μ, which, of course, is incorrect.

The computer will stop if α becomes minus as would be the case if β for a symmetrical beam would be entered as 0.6. No other stops are incorporated.

```
C00000  EXLINE BEAM CONSTANTS
000010  DIMENSION ALPHA (4), XR(4),
000011  XQ(4),X(4)
000020  READ, AMU,DAMU,AMUM,BETA,DBETA
000021  ,AN,DAN,ANM,TYPE
000030  BETA3=BETA
000040  AN3=AN
000050  AMU3=AMU
001100  DELTA=1.-BETA
000070  IF(TYPE-1.)111,112,113
```

Fig. 4-6
I.B.M. 650 Fortran Program

```
001110  PAUSE 1111
001120  ALPHA(1)=DELTA
000100  GO TO 115
001130  ALPHA(1)=DELTA-BETA
001150  IF(ALPHA)116,117,117
001160  STOP 1888
001170  DO 118 I=1,3
001180  ALPHA(I+1)=ALPHA(I)*ALPHA(1)
000140  DO 120 I=1,4
000150  AI=I
000160  XR(I)=ALPHA(I)/AI
000170  XQ(I)=((1.-ALPHA(I))*AMU)/AI
001200  X(I)=XQ(I)+XR(I)
000190  IF(TYPE-1.)130,135,140
001300  PAUSE 1112
001350  F2=X(3)
000220  G=X(2)-F2
000230  F1=X(1)-G*2.-F2
000240  T6=(F2-X(4))/2.
000250  T5=G/2.-T6
000260  THL=F2-(XQ(1)+XQ(2))/6.
000270  T8=((BETA*BETA)*THL)/2.
000280  THLA=X(2)-XQ(1)/3.
000290  T7=((BETA*BETA)*THLA)/2.-T8
```

Fig. 4-6 (Con't.)
I.B.M. 650 Fortran Program

```
000300  PUNCH,AMU,BETA,F2,G,F1,T6,T5
000301  T8,T7
000310  GO TO 150
001400  F=(X(1)+X(3))/4.
000330  G=(X(1)-X(3))/4.
000340  T4=G/4.
000350  T5=(BETA*BETA)*(X(1)/4.-XQ(1)
000351  /12.)
000360  PUNCH,AMU,BETA,F,G,T4,T5
001500  CONTINUE
001510  IF(TYPE-1.)152,160,170
001520  PAUSE 1113
001600  IF(AN-ALPHA(1))161,161,162
001610  T2=AN*(G-(AN*AN)/6.)
000420  T1=AN*(G+F1-AN/2.)-T2
000430  GO TO 180
001620  AN1=1.-AN
000450  T1=AN1*(G-(AN1*AN1)*AMU/6.)
000460  T2=AN1*(G+F2-AN1*AMU/2.)-T1
000470  GO TO 180
001700  IF(AN-BETA)171,171,175
001710  T2=AN*(G-(AN*AN)*AMU/6.)
000500  T1=AN*(G+F-(AN*AMU)/2.)-T2
000510  GO TO 180
```

Fig. 4-6 (Con't.)
I.B.M. 650 Fortran Program

```
001750   AN1=1.-AN
000520   AN2=0.5-AN
000530   T2=G/2.+(AN1*(X(3)-X(2)))/4.
000531   -((AN2*AN2)*(AN+1))/6.
000540   T1=(X(1)-X(2))/4.-(AN2*AN2)/2.
000541   -T2
001700   PUNCH,AN,T2,T1
000560   AN=AN+DAN
000570   IF(ANM-AN)185,150,150
001850   AN=AN3
000590   BETA=BETA+DBETA
000600   IF(BETAM-BETA)190,110,110
001900   BETA=BETA3
000620   AMU=AMU+DAMU
000630   IF(AMUM-AMU)200,110,110
002000   STOP 1888
000650   END
```

Fig. 4-6 (Con't.)
I.B.M. 650 Fortran Program

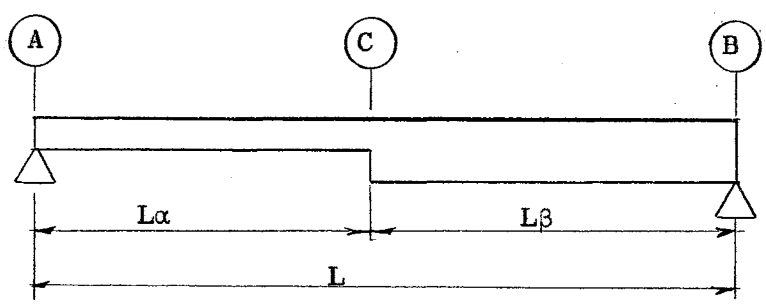
CHAPTER V

TABLES OF BEAM CONSTANTS

5-1. General

Tables of coefficients for the calculation of angular functions, for parabolic haunch beams, have been compiled by Tuma, Lessley, and French (17). Tables of coefficients for the calculation of angular functions, for straight haunched beams, have been compiled by Boecker (5).

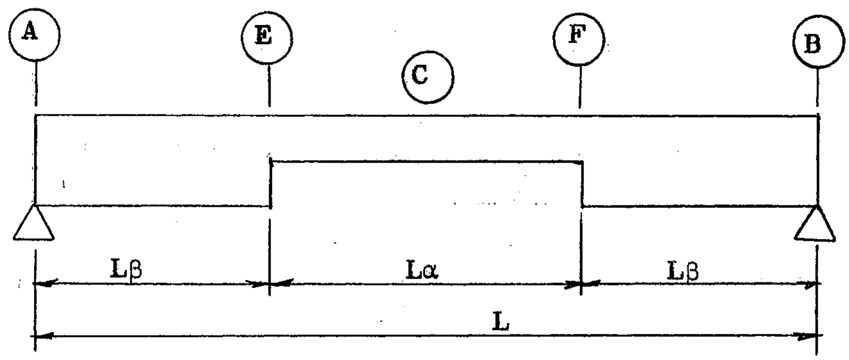
The tables compiled in this chapter are for beams with a sudden change in cross section. A specific beam is located by the ratio of the haunch length to the total length and by the ratio of the minimum and maximum moment of inertia (Fig. 5-1, 5-2).



$I_A = I_o$ $\qquad$ $I_B = I_o/\mu$

Fig. 5-1

Unsymmetrical Beam



$I_C = I_o$ $\qquad$ $I_A = I_B = I_o/\mu$

Fig. 5-2

Symmetrical Beam

The following combinations of ratios are used:

$\beta = 0, .1, .2, .3 \ldots .8, .9, 1.0$

$\mu = 0, .05, .1, .15, \ldots .85, .90, .95$

5-2. Table A-0

The beam constants for a prismatic beam of constant cross section are recorded in table A-0. The geometry of the beam is given by the sketch found at the top of the table. Constants listed in the table are:

a.) Angular Flexibilities

$F_{AB} = F_{BA} = F = L/3EI_o$

b.) Angular Carry-Over Values

$G_{AB} = G_{BA} = G = L/GEI_o$

c.) Angular Load Functions

1.) Due to a Unit Load

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o} \qquad \tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

t_1 = left end slope coefficient due to unit load at L_n.

t_2 = right end slope coefficient due to unit load at L_n.

Influence values of these coefficients for 100 positions of unit load are shown in the table.

2.) Due to Uniformly Distributed Load

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = \frac{wL^3}{24EI_o}$$

w = intensity of the load.

3.) Due to a Dead Load

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = \frac{bh_o qL^3}{24EI_o}$$

b = width of beam.

h_o = depth of beam.

q = specific weight of beam.

5-3. <u>A-1, 2, - 17, 18</u>

The beam constants for an unsymmetrical beam with a sudden change in cross section are recorded in tables A-1, 2, ... 17, 18. The geometry of the beam is defined by the sketch and the parameters.

$L\beta$ = length of haunch.

I_o = minimum moment of inertia.

I_h = moment of inertia of the haunch.

μ = I_o/I_h.

w_o = minimum weight.

w_h = weight of haunch.

The beam constants listed in each table are:

a.) Angular Flexibilities

$$F_{AB} = f_1 \frac{L}{EI_o} \qquad F_{BA} = f_2 \frac{L}{EI_o}$$

f = angular flexibility coefficient.

b.) Carry-Over Values

$$G_{AB} = G_{BA} = g \frac{L}{EI_o}$$

g = angular carry-over value coefficient.

c.) Angular Load Functions

1.) Due to a Unit Load

$$\tau^{(LL)}_{AB} = t_1 \frac{L^2}{EI_o} \qquad \tau^{(LL)}_{BA} = t_2 \frac{L^2}{EI_o}$$

t_1 = left end slope coefficient due to a unit load at L_n.

t_2 = right end slope coefficient due to a unit load at L_n.

2.) Due to a Uniformly Distributed Load

$$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o} \qquad \tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

t_5 = left end slope coefficient due to a uniformly distributed load.

t_6 = right end slope coefficient due to a uniformly distributed load.

3.) Due to a Uniformly Distributed Haunch Load

$$\tau_{AB}^{(HL)} = t_7 \frac{wL^3}{EI_o} \qquad \tau_{BA}^{(HL)} = t_8 \frac{wL^3}{EI_o}$$

t_7 = left end slope coefficient due to a uniformly distributed load along the haunch.

t_8 = right end slope coefficient due to a uniformly distributed load along the haunch.

5-4. Tables B-1, 2, 3, 4

Beam constants for a beam with two symmetrical haunches are recorded in Tables B-1, 2, 3, 4. The geometry is defined by the sketch and the parameters.

$L\beta$ = length of the haunch.

I_o = minimum moment of inertia.

I_h = moment of inertia of the haunch.

μ = I_o/I_h.

w_o = minimum weight.

w_h = weight of the haunch.

The beam constants listed in each table are:

a.) Angular Flexibilities

$$F_{AB} = F_{BA} = f \frac{L}{EI_o}$$

f = angular flexibility coefficient.

b.) Carry-Over Values

$$G_{AB} = G_{BA} = g \frac{L}{EI_o}$$

g = angular carry-over value coefficient.

c.) Angular Load Functions

1.) Due to a Unit Load

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o} \qquad \tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

t_1 = left end slope coefficient due to a unit load at L_n.

t_2 = right end slope coefficient due to a unit load at L_n.

2.) Due to a Uniformly Distributed Load

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_o}$$

t_4 = end coefficient due to a uniformly distributed load.

3.) Due to a Uniformly Distributed Haunch Load

$$\tau_{AB}^{(HL)} = \tau_{BA}^{(HL)} = t_5 \frac{wL^3}{EI_o}$$

t_5 = end slope coefficient due to a uniformly distributed load along the haunch.

5-5. Dead Load Functions

Using simple methods of superposition the angular load function due to the dead load may be found easily using values from the table.

For unsymmetrical beams they are:

$$\tau_{AB}^{(DL)} = \left[t_5 \cdot w_o + t_7 \,(w_h - w_o)\right] \frac{L^3}{EI_o}$$

$$\tau_{BA}^{(DL)} = \left[t_6 \cdot w_o + t_8 \,(w_h - w_o)\right] \frac{L^3}{EI_o}$$

For symmetrical beams they are:

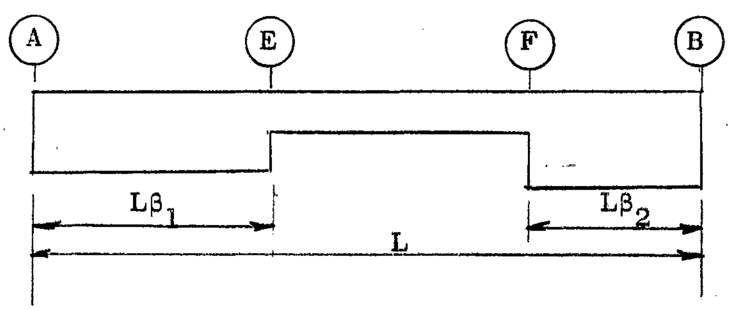
$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = \left[t_4 \cdot w_o + t_5 \,(w_h - w_o)\right] \frac{L^3}{EI_o}$$

5-6. <u>Members with Unsymmetrical Haunches</u>

Often the structural engineer finds it necessary to analyze members with unsymmetrical haunches. The beam constants tabulated in this chapter may be used for such a problem.

Leontovich (18) and Gulden (19) are among those who have shown that beam constants for unsymmetrically haunched beams may be found by superimposing the constants of component parts of the member. The procedure is described using the following example.

Consider a beam with unsymmetrical haunches (Fig. 5-3).



$$I_{EF} = I_o \qquad I_A = \frac{I_o}{\mu_1} \qquad I_B = \frac{I_o}{\mu_2}$$

Fig. 5-3

Unsymmetrically Haunched Beam

To find, for instance, the angular flexibilities F_{BA}, F_{AB}, and the angular carry-over values G_{AB}, G_{BA} of the beam shown, obtain from the table constants for beam number one and two (Fig. 5-4) and for a beam of constant cross section and with an equal moment of inertia, I_o. The desired constants are found by simply adding those of beams number one and two, and subtracting from this sum the appropriate constants of the beam of constant cross section, as shown in Fig. 5-4.

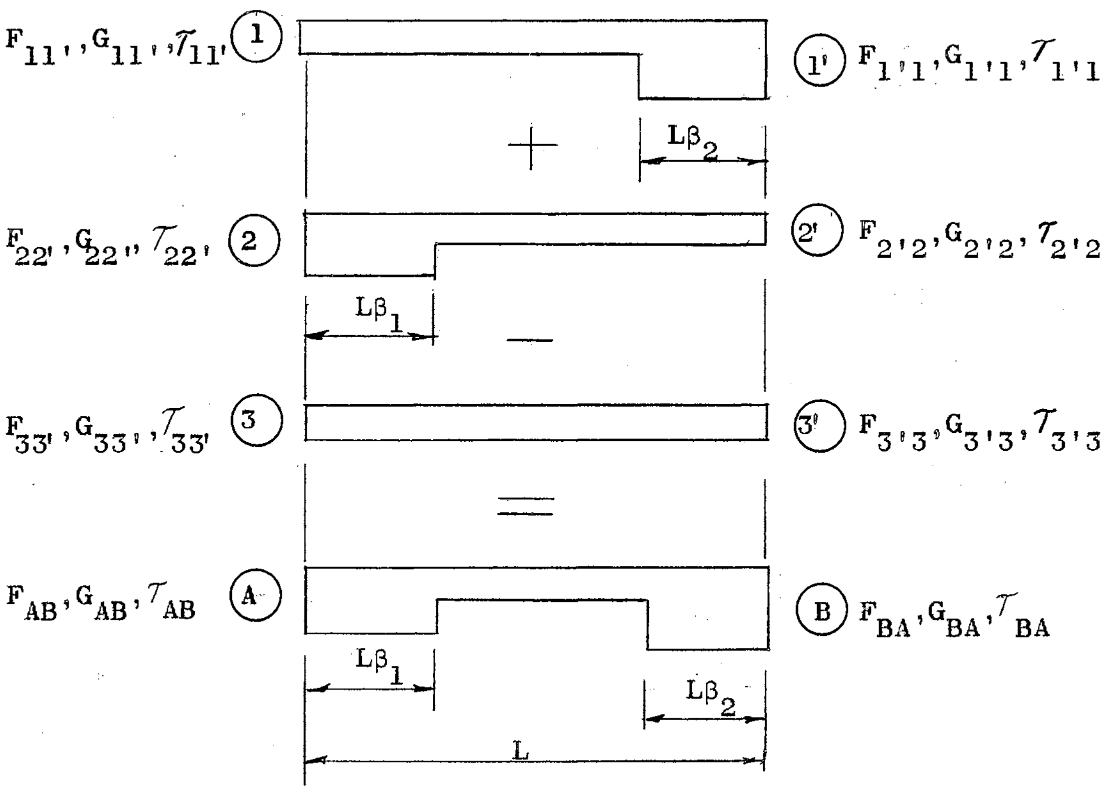


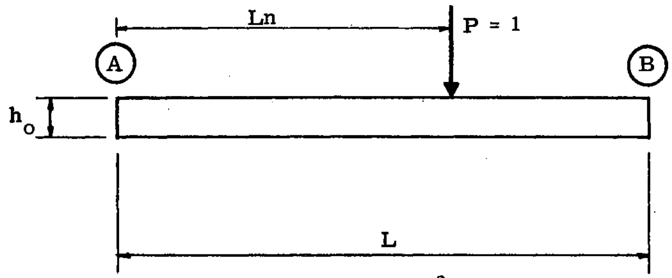
Fig. 5-4

Superposition of Beam Constants

5-7. Tables

TABLE A-0

$\beta = 0.0$



$$I_o = \frac{bh_o^3}{12}$$

$$F_{AB} = F_{BA} = \frac{L}{3EI_o}$$

$$G_{AB} = G_{BA} = \frac{L}{6EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = \frac{bh_o q L^3}{24 EI_o}$$

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = \frac{w L^3}{24 EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

Influence Coefficients t_1

n	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0033	.0065	.0096	.0125	.0154	.0182	.0209	.0236	.0261
0.1	.0285	.0309	.0331	.0353	.0373	.0393	.0412	.0430	.0448	.0464
0.2	.0480	.0495	.0509	.0522	.0535	.0547	.0558	.0568	.0578	.0587
0.3	.0595	.0603	.0609	.0615	.0621	.0626	.0630	.0633	.0636	.0638
0.4	.0640	.0641	.0642	.0641	.0641	.0639	.0638	.0635	.0632	.0629
0.5	.0625	.0621	.0616	.0610	.0604	.0598	.0591	.0584	.0577	.0569
0.6	.0560	.0551	.0542	.0532	.0522	.0512	.0501	.0490	.0479	.0467
0.7	.0455	.0443	.0430	.0417	.0404	.0391	.0377	.0363	.0349	.0335
0.8	.0320	.0305	.0290	.0275	.0260	.0244	.0229	.0213	.0197	.0181
0.9	.0165	.0149	.0133	.0116	.0100	.0083	.0067	.0050	.0033	.0017

Influence Coefficients t_2

n	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0017	.0033	.0050	.0067	.0083	.0100	.0116	.0133	.0149
0.1	.0165	.0181	.0197	.0213	.0229	.0244	.0260	.0275	.0290	.0305
0.2	.0320	.0335	.0349	.0363	.0377	.0391	.0404	.0417	.0430	.0443
0.3	.0455	.0467	.0479	.0490	.0501	.0512	.0522	.0532	.0542	.0551
0.4	.0560	.0569	.0577	.0584	.0591	.0598	.0604	.0610	.0616	.0621
0.5	.0625	.0629	.0632	.0635	.0638	.0639	.0641	.0641	.0642	.0641
0.6	.0640	.0638	.0636	.0633	.0630	.0626	.0621	.0615	.0609	.0603
0.7	.0595	.0587	.0578	.0568	.0558	.0547	.0535	.0522	.0509	.0495
0.8	.0480	.0464	.0448	.0430	.0412	.0393	.0373	.0353	.0331	.0309
0.9	.0285	.0261	.0236	.0209	.0182	.0154	.0125	.0096	.0065	.0033

TABLE A-1
β=0.1

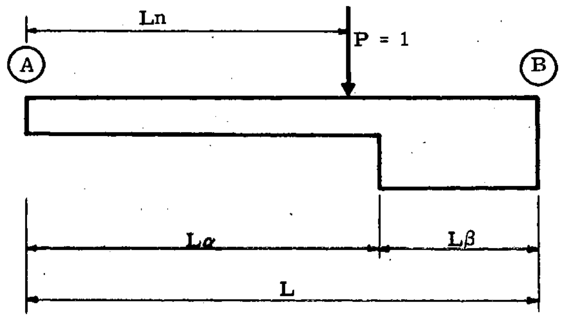
$$F_{AB} = f_1 \frac{L}{EI_o}$$

$$G_{AB} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o}$$

$$\tau_{AB}^{(HL)} = t_7 \frac{wL^3}{EI_o}$$

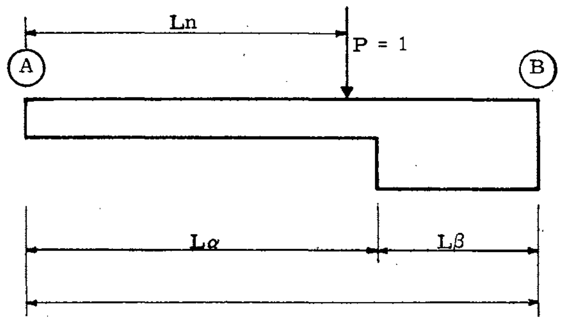


$I_A = I_o$ $\mu = \frac{I_A}{I_B}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_1	g	t_5	t_7
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0285	.0479	.0594	.0639	.0623	.0558	.0452	.0317	.0162	.3330	.1620	.0415	.0008
0.05	.0285	.0479	.0594	.0639	.0623	.0558	.0453	.0317	.0162	.3330	.1622	.0415	.0008
0.10	.0285	.0479	.0594	.0639	.0623	.0558	.0453	.0318	.0162	.3330	.1625	.0415	.0008
0.15	.0285	.0479	.0594	.0639	.0624	.0558	.0453	.0318	.0162	.3330	.1627	.0415	.0008
0.20	.0285	.0479	.0594	.0639	.0624	.0558	.0453	.0318	.0163	.3331	.1629	.0415	.0008
0.25	.0285	.0479	.0594	.0639	.0624	.0558	.0453	.0318	.0163	.3331	.1632	.0416	.0008
0.30	.0285	.0480	.0594	.0639	.0624	.0559	.0453	.0318	.0163	.3331	.1634	.0416	.0008
0.35	.0285	.0480	.0594	.0639	.0624	.0559	.0453	.0318	.0163	.3331	.1636	.0416	.0008
0.40	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0318	.0163	.3331	.1639	.0416	.0008
0.45	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0319	.0163	.3331	.1641	.0416	.0008
0.50	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0319	.0164	.3332	.1643	.0416	.0008
0.55	.0285	.0480	.0595	.0639	.0624	.0559	.0454	.0319	.0164	.3332	.1646	.0416	.0008
0.60	.0285	.0480	.0595	.0639	.0624	.0559	.0454	.0319	.0164	.3332	.1648	.0416	.0008
0.65	.0285	.0480	.0595	.0640	.0624	.0559	.0454	.0319	.0164	.3332	.1650	.0416	.0008
0.70	.0285	.0480	.0595	.0640	.0624	.0559	.0454	.0319	.0164	.3332	.1653	.0416	.0008
0.75	.0285	.0480	.0595	.0640	.0625	.0559	.0454	.0319	.0164	.3332	.1655	.0416	.0008
0.80	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0164	.3333	.1657	.0416	.0008
0.85	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1660	.0416	.0008
0.90	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1662	.0417	.0008
0.95	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1664	.0417	.0008

TABLE A-2
β = 0.1



$I_A = I_o$ $\mu = \frac{I_A}{I_B}$

$F_{BA} = f_2 \frac{L}{EI_o}$

$G_{BA} = g \frac{L}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$

$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$

$\tau_{BA}^{(HL)} = t_8 \frac{wL^3}{EI_o}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_2	g	t_6	t_8
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0160	.0311	.0441	.0541	.0602	.0612	.0562	.0443	.0243	.2430	.1620	.0395	.0012
0.05	.0161	.0311	.0442	.0542	.0603	.0613	.0564	.0445	.0245	.2475	.1622	.0396	.0012
0.10	.0161	.0312	.0442	.0543	.0604	.0615	.0566	.0446	.0247	.2520	.1625	.0397	.0012
0.15	.0161	.0312	.0443	.0544	.0605	.0616	.0567	.0448	.0249	.2565	.1627	.0398	.0013
0.20	.0161	.0313	.0444	.0545	.0606	.0618	.0569	.0450	.0251	.2611	.1629	.0399	.0013
0.25	.0161	.0313	.0445	.0546	.0607	.0619	.0570	.0452	.0254	.2656	.1632	.0400	.0013
0.30	.0162	.0313	.0445	.0547	.0609	.0620	.0572	.0454	.0256	.2701	.1634	.0401	.0013
0.35	.0162	.0314	.0446	.0548	.0610	.0622	.0574	.0456	.0258	.2746	.1636	.0403	.0013
0.40	.0162	.0314	.0447	.0549	.0611	.0623	.0575	.0458	.0260	.2791	.1639	.0404	.0013
0.45	.0162	.0315	.0447	.0550	.0612	.0625	.0577	.0459	.0262	.2836	.1641	.0405	.0013
0.50	.0163	.0315	.0448	.0551	.0613	.0626	.0579	.0461	.0264	.2882	.1643	.0406	.0014
0.55	.0163	.0316	.0449	.0552	.0614	.0627	.0580	.0463	.0266	.2927	.1646	.0407	.0014
0.60	.0163	.0316	.0449	.0553	.0616	.0629	.0582	.0465	.0268	.2972	.1648	.0408	.0014
0.65	.0163	.0317	.0450	.0553	.0617	.0630	.0584	.0467	.0270	.3017	.1650	.0409	.0014
0.70	.0164	.0317	.0451	.0554	.0618	.0632	.0585	.0469	.0272	.3062	.1653	.0410	.0014
0.75	.0164	.0318	.0451	.0555	.0619	.0633	.0587	.0471	.0274	.3107	.1655	.0411	.0014
0.80	.0164	.0318	.0452	.0556	.0620	.0634	.0588	.0473	.0277	.3153	.1657	.0412	.0014
0.85	.0164	.0319	.0453	.0557	.0621	.0636	.0590	.0474	.0279	.3198	.1660	.0413	.0015
0.90	.0165	.0319	.0454	.0558	.0623	.0637	.0592	.0476	.0281	.3243	.1662	.0415	.0015
0.95	.0165	.0320	.0454	.0559	.0624	.0639	.0593	.0478	.0283	.3288	.1664	.0416	.0015

TABLE A-3
β = 0.2

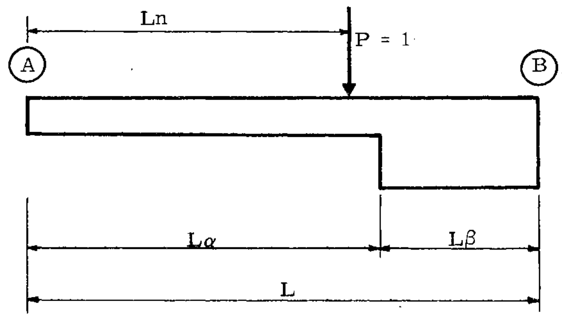
$$F_{AB} = f_1 \frac{L}{EI_o}$$

$$G_{AB} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o}$$

$$\tau_{AB}^{(HL)} = t_7 \frac{wL^3}{EI_o}$$

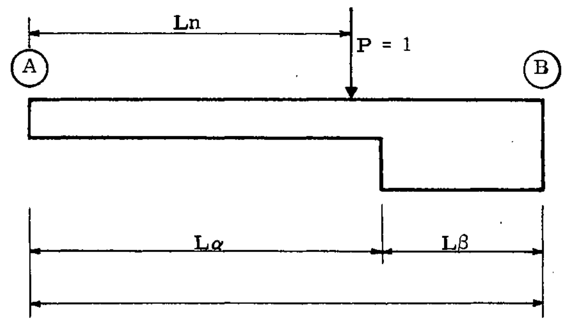


$I_A = I_o$ $\mu = \frac{I_A}{I_B}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_1	g	t_5	t_7
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0282	.0475	.0587	.0629	.0612	.0544	.0436	.0299	.0149	.3307	.1493	.0405	.0029
0.05	.0282	.0475	.0587	.0630	.0612	.0545	.0437	.0300	.0150	.3308	.1502	.0406	.0030
0.10	.0283	.0475	.0588	.0630	.0613	.0546	.0438	.0301	.0151	.3309	.1511	.0406	.0030
0.15	.0283	.0475	.0588	.0631	.0614	.0546	.0439	.0302	.0152	.3311	.1519	.0407	.0030
0.20	.0283	.0476	.0589	.0631	.0614	.0547	.0440	.0303	.0152	.3312	.1528	.0408	.0030
0.25	.0283	.0476	.0589	.0632	.0615	.0548	.0441	.0304	.0513	.3313	.1537	.0408	.0031
0.30	.0283	.0476	.0589	.0633	.0616	.0549	.0442	.0305	.0154	.3315	.1545	.0409	.0031
0.35	.0283	.0477	.0590	.0633	.0616	.0550	.0443	.0306	.0155	.3316	.1554	.0409	.0031
0.40	.0283	.0477	.0590	.0634	.0617	.0550	.0444	.0307	.0156	.3317	.1563	.0410	.0031
0.45	.0284	.0477	.0591	.0634	.0618	.0551	.0445	.0308	.0156	.3319	.1571	.0410	.0031
0.50	.0284	.0477	.0591	.0635	.0618	.0552	.0446	.0309	.0157	.3320	.1580	.0411	.0031
0.55	.0284	.0478	.0591	.0635	.0619	.0553	.0447	.0310	.0158	.3321	.1589	.0412	.0031
0.60	.0284	.0478	.0592	.0636	.0620	.0554	.0448	.0311	.0159	.3323	.1597	.0412	.0032
0.65	.0284	.0478	.0592	.0636	.0620	.0554	.0448	.0313	.0160	.3324	.1606	.0413	.0032
0.70	.0284	.0478	.0593	.0637	.0621	.0555	.0449	.0314	.0160	.3325	.1615	.0413	.0032
0.75	.0284	.0479	.0593	.0637	.0622	.0556	.0450	.0315	.0161	.3327	.1623	.0414	.0032
0.80	.0284	.0479	.0593	.0638	.0622	.0557	.0451	.0316	.0162	.3328	.1632	.0414	.0032
0.85	.0285	.0479	.0594	.0638	.0623	.0558	.0452	.0317	.0163	.3329	.1641	.0415	.0032
0.90	.0285	.0479	.0594	.0639	.0624	.0558	.0453	.0318	.0163	.3331	.1649	.0416	.0032
0.95	.0285	.0480	.0595	.0639	.0624	.0559	.0454	.0319	.0164	.3332	.1658	.0416	.0033

TABLE A-4
β = 0.2



$I_A = I_o$ $\qquad \mu = \frac{I_A}{I_B}$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

$$\tau_{BA}^{(HL)} = t_8 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_2	g	t_6	t_8
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0148	.0285	.0403	.0491	.0538	.0536	.0474	.0341	.0171	.1707	.1493	.0341	.0034
0.05	.0149	.0287	.0406	.0494	.0543	.0541	.0480	.0348	.0176	.0788	.1502	.0345	.0035
0.10	.0149	.0289	.0408	.0498	.0547	.0546	.0486	.0355	.0182	.1869	.1511	.0349	.0036
0.15	.0150	.0291	.0411	.0501	.0551	.0552	.0492	.0362	.0188	.1951	.1519	.0353	.0037
0.20	.0151	.0292	.0413	.0505	.0556	.0557	.0498	.0369	.0194	.2032	.1528	.0356	.0038
0.25	.0152	.0294	.0416	.0508	.0560	.0562	.0504	.0376	.0199	.2113	.1537	.0360	.0039
0.30	.0153	.0296	.0419	.0511	.0564	.0567	.0510	.0383	.0205	.2195	.1545	.0364	.0040
0.35	.0154	.0297	.0421	.0515	.0569	.0572	.0516	.0390	.0211	.2276	.1554	.0368	.0041
0.40	.0155	.0299	.0424	.0518	.0573	.0578	.0522	.0397	.0216	.2357	.1563	.0371	.0042
0.45	.0155	.0301	.0426	.0522	.0577	.0583	.0528	.0404	.0222	.2439	.1571	.0375	.0043
0.50	.0156	.0303	.0429	.0525	.0582	.0588	.0534	.0411	.0228	.2520	.1580	.0379	.0044
0.55	.0157	.0304	.0432	.0529	.0586	.0593	.0540	.0418	.0234	.2601	.1589	.0383	.0045
0.60	.0158	.0306	.0434	.0532	.0590	.0598	.0546	.0425	.0239	.2683	.1597	.0387	.0046
0.65	.0159	.0308	.0437	.0536	.0595	.0604	.0553	.0431	.0245	.2764	.1606	.0390	.0047
0.70	.0160	.0310	.0439	.0539	.0599	.0609	.0559	.0438	.0251	.2845	.1615	.0394	.0048
0.75	.0161	.0311	.0442	.0543	.0603	.0614	.0565	.0445	.0256	.2927	.1623	.0398	.0049
0.80	.0162	.0313	.0445	.0546	.0608	.0619	.0571	.0452	.0262	.3008	.1632	.0402	.0050
0.85	.0162	.0315	.0447	.0550	.0612	.0624	.0577	.0459	.0268	.3089	.1641	.0405	.0051
0.90	.0163	.0317	.0450	.0553	.0616	.0630	.0583	.0466	.0274	.3171	.1649	.0409	.0052
0.95	.0164	.0318	.0452	.0557	.0621	.0635	.0589	.0473	.0279	.3252	.1658	.0413	.0053

TABLE A-5
β = 0.3

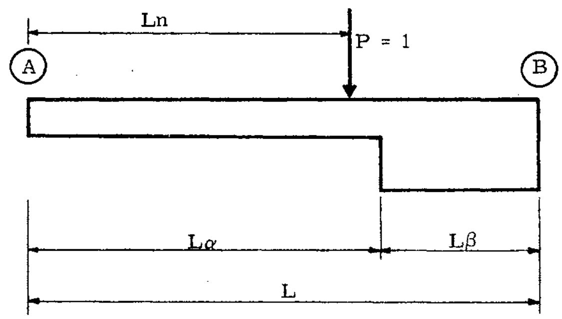
$F_{AB} = f_1 \frac{L}{EI_o}$

$G_{AB} = g \frac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$

$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o}$

$\tau_{AB}^{(HL)} = t_7 \frac{wL^3}{EI_o}$



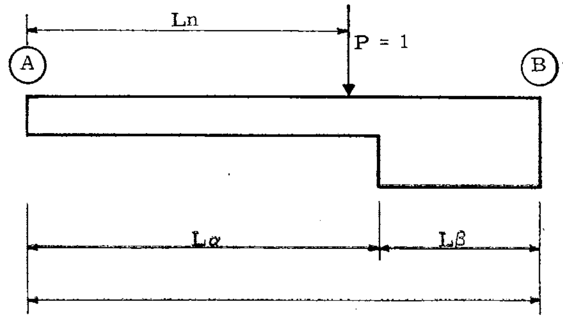
$I_A = I_o$

$\mu = \frac{I_A}{I_B}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_1	g	t_5	t_7
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0276	.0462	.0568	.0604	.0580	.0506	.0392	.0261	.0131	.3243	.1307	.0382	.0059
0.05	.0276	.0463	.0569	.0606	.0582	.0509	.0395	.0264	.0132	.3247	.1325	.0383	.0059
0.10	.0277	.0464	.0571	.0608	.0584	.0511	.0398	.0267	.0134	.3252	.1343	.0385	.0060
0.15	.0277	.0465	.0572	.0609	.0587	.0514	.0401	.0270	.0136	.3257	.1361	.0387	.0061
0.20	.0278	.0466	.0573	.0611	.0589	.0517	.0405	.0273	.0138	.3261	.1379	.0389	.0061
0.25	.0278	.0467	.0575	.0613	.0591	.0519	.0408	.0276	.0139	.3266	.1397	.0391	.0062
0.30	.0279	.0467	.0576	.0615	.0593	.0522	.0411	.0279	.0141	.3270	.1415	.0392	.0063
0.35	.0279	.0468	.0577	.0617	.0596	.0525	.0414	.0282	.0143	.3275	.1433	.0394	.0063
0.40	.0280	.0469	.0579	.0618	.0598	.0528	.0417	.0285	.0144	.3279	.1451	.0396	.0064
0.45	.0280	.0470	.0580	.0620	.0600	.0530	.0420	.0288	.0146	.3284	.1469	.0397	.0065
0.50	.0280	.0471	.0581	.0622	.0603	.0533	.0423	.0291	.0148	.3288	.1487	.0399	.0065
0.55	.0281	.0472	.0583	.0624	.0605	.0536	.0427	.0294	.0150	.3293	.1505	.0401	.0066
0.60	.0281	.0473	.0584	.0626	.0607	.0538	.0430	.0297	.0151	.3297	.1523	.0403	.0066
0.65	.0282	.0474	.0586	.0627	.0609	.0541	.0433	.0299	.0153	.3302	.1541	.0404	.0067
0.70	.0282	.0475	.0587	.0629	.0612	.0544	.0436	.0302	.0155	.3306	.1559	.0406	.0068
0.75	.0283	.0476	.0588	.0631	.0614	.0546	.0439	.0305	.0156	.3311	.1577	.0408	.0068
0.80	.0283	.0476	.0590	.0633	.0616	.0549	.0442	.0308	.0158	.3315	.1595	.0410	.0069
0.85	.0284	.0477	.0591	.0635	.0618	.0552	.0446	.0311	.0160	.3320	.1613	.0411	.0070
0.90	.0284	.0478	.0592	.0636	.0620	.0555	.0449	.0314	.0162	.3324	.1631	.0413	.0070
0.95	.0285	.0479	.0594	.0638	.0623	.0557	.0452	.0317	.0163	.3329	.1649	.0415	.0071

TABLE A-6
β = 0.3



$F_{BA} = f_2 \dfrac{L}{EI_o}$

$G_{BA} = g \dfrac{L}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \dfrac{L^2}{EI_o}$

$\tau_{BA}^{(UL)} = t_6 \dfrac{wL^3}{EI_o}$

$\tau_{BA}^{(HL)} = t_8 \dfrac{wL^3}{EI_o}$

$I_A = I_o$ $\mu = \dfrac{I_A}{I_B}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_2	g	t_6	t_8
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0129	.0248	.0347	.0416	.0445	.0424	.0343	.0229	.0114	.1143	.1307	.0272	.0051
0.05	.0131	.0252	.0352	.0423	.0454	.0435	.0356	.0241	.0123	.1253	.0325	.0279	.0054
0.10	.0133	.0255	.0358	.0430	.0463	.0446	.0368	.0254	.0131	.1362	.0343	.0286	.0557
0.15	.0134	.0259	.0363	.0438	.0472	.0456	.0381	.0266	.0140	.1472	.0361	.0293	.0060
0.20	.0136	.0262	.0369	.0445	.0481	.0467	.0393	.0279	.0148	.1581	.0379	.0301	.0063
0.25	.0138	.0266	.0374	.0452	.0490	.0478	.0406	.0292	.0157	.1691	.0397	.0308	.0066
0.30	.0140	.0270	.0379	.0459	.0499	.0489	.0419	.0304	.0166	.1800	.1415	.0315	.0069
0.35	.0142	.0273	.0385	.0466	.0508	.0500	.0431	.0317	.0174	.1910	.1433	.0322	.0071
0.40	.0143	.0277	.0390	.0474	.0517	.0510	.0444	.0329	.0183	.2019	.1451	.0330	.0074
0.45	.0145	.0280	.0396	.0481	.0526	.0521	.0456	.0342	.0191	.2129	.1469	.0337	.0077
0.50	.0147	.0284	.0401	.0488	.0535	.0532	.0469	.0354	.0200	.2238	.1487	.0344	.0080
0.55	.0149	.0288	.0406	.0495	.0544	.0543	.0482	.0367	.0208	.2348	.1505	.0351	.0083
0.60	.0151	.0291	.0412	.0502	.0553	.0554	.0494	.0379	.0217	.2457	.1523	.0359	.0086
0.65	.0152	.0295	.0417	.0510	.0562	.0564	.0507	.0392	.0225	.2567	.1541	.0366	.0088
0.70	.0154	.0298	.0423	.0517	.0571	.0575	.0519	.0405	.0234	.2676	.1559	.0373	.0091
0.75	.0156	.0302	.0428	.0524	.0580	.0586	.0532	.0417	.0242	.2786	.1577	.0380	.0094
0.80	.0158	.0306	.0433	.0531	.0589	.0597	.0545	.0430	.0251	.2895	.1595	.0388	.0097
0.85	.0160	.0309	.0439	.0538	.0598	.0608	.0557	.0442	.0259	.3005	.1613	.0395	.0100
0.90	.0161	.0313	.0444	.0546	.0607	.0618	.0570	.0455	.0268	.3114	.1631	.0402	.0103
0.95	.0163	.0316	.0450	.0553	.0616	.0629	.0582	.0467	.0276	.3224	.1649	.0409	.0106

TABLE A-7
β = 0.4

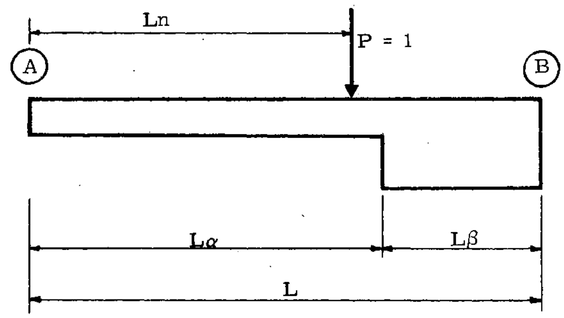
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$

$$\tau_{AB}^{(HL)} = t_7 \frac{w L^3}{E I_o}$$

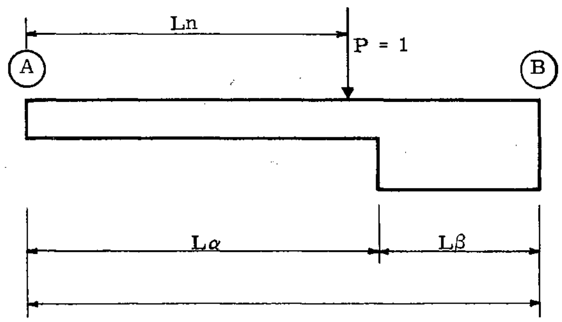


$I_A = I_o$ $\qquad \mu = \frac{I_A}{I_B}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_1	g	t_5	t_7
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0264	.0437	.0531	.0555	.0518	.0432	.0324	.0216	.0108	.3120	.1080	.0342	.0086
0.05	.0265	.0439	.0534	.0559	.0524	.0438	.0331	.0221	.0111	.3131	.1109	.0346	.0088
0.10	.0266	.0442	.0537	.0563	.0529	.0445	.0337	.0226	.0114	.3141	.1139	.0349	.0090
0.15	.0267	.0444	.0541	.0567	.0534	.0451	.0344	.0232	.0117	.3152	.1168	.0353	.0092
0.20	.0268	.0446	.0544	.0572	.0540	.0458	.0350	.0237	.0119	.3163	.1197	.0357	.0094
0.25	.0269	.0448	.0547	.0576	.0545	.0464	.0357	.0242	.0122	.3173	.1227	.0361	.0095
0.30	.0270	.0450	.0550	.0580	.0550	.0470	.0363	.0247	.0125	.3184	.1256	.0364	.0097
0.35	.0271	.0452	.0553	.0584	.0556	.0477	.0370	.0252	.0128	.3195	.1285	.0368	.0099
0.40	.0272	.0454	.0557	.0589	.0561	.0483	.0376	.0258	.0131	.3205	.1315	.0372	.0101
0.45	.0273	.0457	.0560	.0593	.0566	.0490	.0383	.0263	.0134	.3216	.1344	.0376	.0103
0.50	.0274	.0459	.0563	.0597	.0572	.0496	.0389	.0268	.0136	.3227	.1373	.0379	.0105
0.55	.0275	.0461	.0566	.0602	.0577	.0502	.0396	.0273	.0139	.3237	.1403	.0383	.0106
0.60	.0276	.0463	.0569	.0606	.0582	.0509	.0403	.0278	.0142	.3248	.1432	.0387	.0108
0.65	.0278	.0465	.0573	.0610	.0588	.0515	.0409	.0284	.0145	.3259	.1461	.0391	.0110
0.70	.0279	.0467	.0576	.0614	.0593	.0522	.0416	.0289	.0148	.3269	.1491	.0394	.0112
0.75	.0280	.0469	.0579	.0619	.0598	.0528	.0422	.0294	.0151	.3280	.1520	.0398	.0114
0.80	.0281	.0471	.0582	.0623	.0604	.0534	.0429	.0299	.0154	.3291	.1549	.0402	.0115
0.85	.0282	.0474	.0585	.0627	.0609	.0541	.0435	.0304	.0156	.3301	.1579	.0405	.0117
0.90	.0283	.0476	.0589	.0631	.0614	.0547	.0442	.0310	.0159	.3312	.1608	.0409	.0119
0.95	.0284	.0478	.0592	.0636	.0620	.0554	.0448	.0315	.0162	.3323	.1637	.0413	.0121

TABLE A-8
β = 0.4



$I_A = I_o$ $\mu = \frac{I_A}{I_B}$

$F_{BA} = f_2 \frac{L}{EI_o}$

$G_{BA} = g \frac{L}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$

$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$

$\tau_{BA}^{(HL)} = t_8 \frac{wL^3}{EI_o}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_2	g	t_6	t_8
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0106	.0203	.0279	.0325	.0332	.0288	.0216	.0144	.0072	.0720	.1080	.0198	.0058
0.05	.0109	.0209	.0288	.0337	.0346	.0306	.0235	.0161	.0083	.0851	.1109	.0209	.0063
0.10	.0112	.0214	.0297	.0349	.0361	.0323	.0254	.0178	.0093	.0981	.1139	.0220	.0069
0.15	.0115	.0220	.0305	.0361	.0376	.0341	.0273	.0194	.0104	.1112	.1168	.0231	.0075
0.20	.0118	.0226	.0314	.0372	.0390	.0358	.0292	.0211	.0115	.1243	.1197	.0242	.0080
0.25	.0121	.0232	.0323	.0384	.0405	.0376	.0311	.0228	.0125	.1373	.1227	.0253	.0086
0.30	.0124	.0238	.0332	.0396	.0420	.0394	.0330	.0245	.0136	.1504	.1256	.0264	.0092
0.35	.0127	.0244	.0341	.0407	.0434	.0411	.0349	.0262	.0147	.1635	.1285	.0275	.0097
0.40	.0130	.0250	.0349	.0419	.0449	.0429	.0368	.0278	.0157	.1765	.1315	.0285	.0103
0.45	.0133	.0255	.0358	.0431	.0464	.0446	.0387	.0295	.0168	.1896	.1344	.0296	.0108
0.50	.0136	.0261	.0367	.0443	.0478	.0464	.0406	.0312	.0178	.2027	.1373	.0307	.0114
0.55	.0139	.0267	.0376	.0454	.0493	.0482	.0424	.0329	.0189	.2157	.1403	.0318	.0120
0.60	.0142	.0273	.0385	.0466	.0508	.0499	.0443	.0346	.0200	.2288	.1432	.0329	.0125
0.65	.0144	.0279	.0393	.0478	.0522	.0517	.0462	.0362	.0210	.2419	.1461	.0340	.0131
0.70	.0147	.0285	.0402	.0490	.0537	.0534	.0481	.0379	.0221	.2549	.1491	.0351	.0137
0.75	.0150	.0291	.0411	.0501	.0552	.0552	.0500	.0396	.0232	.2680	.1520	.0362	.0142
0.80	.0153	.0297	.0420	.0513	.0566	.0570	.0519	.0413	.0242	.2811	.1549	.0373	.0148
0.85	.0156	.0302	.0429	.0525	.0581	.0587	.0538	.0430	.0253	.2941	.1579	.0384	.0154
0.90	.0159	.0308	.0437	.0537	.0596	.0605	.0557	.0446	.0264	.3072	.1608	.0395	.0159
0.95	.0162	.0314	.0446	.0548	.0610	.0622	.0576	.0463	.0274	.3203	.1637	.0406	.0165

TABLE A-9
$\beta = 0.5$

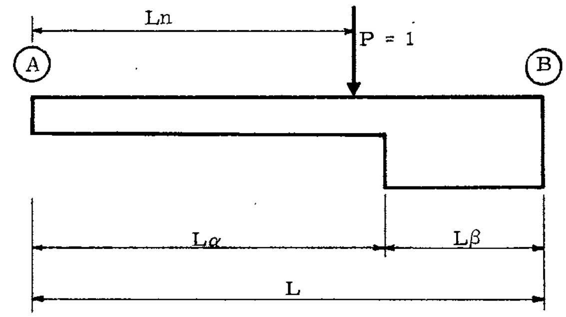
$$F_{AB} = f_1 \frac{L}{EI_o}$$

$$G_{AB} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o}$$

$$\tau_{AB}^{(HL)} = t_7 \frac{wL^3}{EI_o}$$



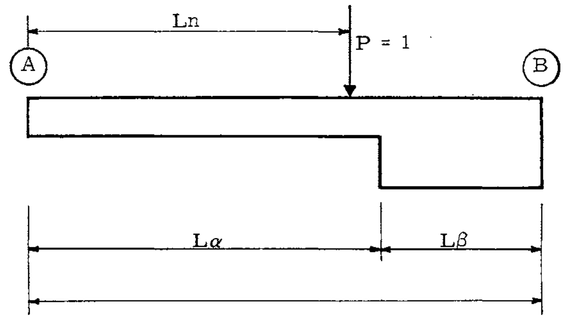
$I_A = I_o$

$\mu = \frac{I_A}{I_B}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_1	g	t_5	t_7
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0243	.0397	.0470	.0473	.0417	.0333	.0250	.0167	.0083	.2917	.0833	.0286	.0104
0.05	.0245	.0401	.0476	.0482	.0427	.0345	.0260	.0174	.0087	.2937	.0875	.0293	.0108
0.10	.0247	.0405	.0482	.0490	.0437	.0356	.0270	.0182	.0091	.2958	.0917	.0299	.0112
0.15	.0250	.0409	.0489	.0498	.0448	.0367	.0281	.0190	.0096	.2979	.0958	.0306	.0116
0.20	.0252	.0413	.0495	.0507	.0458	.0379	.0291	.0197	.0100	.3000	.1000	.0312	.0120
0.25	.0254	.0417	.0501	.0515	.0469	.0390	.0301	.0205	.0104	.3021	.1042	.0319	.0124
0.30	.0256	.0422	.0508	.0523	.0479	.0401	.0311	.0213	.0108	.3042	.1083	.0326	.0128
0.35	.0258	.0426	.0514	.0532	.0490	.0413	.0322	.0220	.0112	.3062	.1125	.0332	.0132
0.40	.0260	.0430	.0520	.0540	.0500	.0424	.0332	.0228	.0116	.3083	.1167	.0339	.0135
0.45	.0262	.0434	.0526	.0548	.0510	.0435	.0342	.0236	.0120	.3104	.1208	.0345	.0139
0.50	.0264	.0438	.0532	.0557	.0521	.0447	.0352	.0243	.0124	.3125	.1250	.0352	.0143
0.55	.0266	.0443	.0539	.0565	.0531	.0458	.0363	.0251	.0128	.3146	.1292	.0358	.0147
0.60	.0268	.0447	.0545	.0573	.0542	.0469	.0373	.0259	.0132	.3167	.1333	.0365	.0151
0.65	.0270	.0451	.0551	.0582	.0552	.0481	.0383	.0266	.0136	.3187	.1375	.0371	.0155
0.70	.0273	.0455	.0558	.0590	.0563	.0492	.0393	.0274	.0140	.3208	.1417	.0378	.0159
0.75	.0275	.0459	.0564	.0598	.0573	.0503	.0404	.0282	.0145	.3229	.1458	.0384	.0163
0.80	.0277	.0463	.0570	.0607	.0583	.0515	.0414	.0289	.0149	.3250	.1500	.0391	.0167
0.85	.0279	.0468	.0576	.0615	.0594	.0526	.0424	.0297	.0153	.3271	.1542	.0397	.0171
0.90	.0281	.0472	.0582	.0623	.0604	.0537	.0434	.0305	.0157	.3292	.1583	.0404	.0174
0.95	.0283	.0476	.0589	.0632	.0615	.0549	.0445	.0312	.0161	.3313	.1625	.0410	.0178

TABLE A-10
β = 0.5



$I_A = I_o$

$\mu = \frac{I_A}{I_B}$

$F_{BA} = f_2 \frac{L}{EI_o}$

$G_{BA} = g \frac{L}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$

$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$

$\tau_{BA}^{(HL)} = t_8 \frac{wL^3}{EI_o}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_2	g	t_6	t_8
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0082	.0153	.0205	.0227	.0208	.0167	.0125	.0083	.0042	.0417	.0833	.0130	.0052
0.05	.0086	.0162	.0217	.0243	.0229	.0190	.0148	.0103	.0054	.0562	.0875	.0145	.0061
0.10	.0090	.0170	.0230	.0260	.0250	.0214	.0172	.0123	.0066	.0708	.0917	.0159	.0070
0.15	.0094	.0178	.0242	.0277	.0271	.0238	.0196	.0143	.0078	.0854	.0958	.0173	.0079
0.20	.0098	.0187	.0255	.0293	.0292	.0261	.0219	.0163	.0090	.1000	.1000	.0187	.0088
0.25	.0102	.0195	.0268	.0310	.0312	.0285	.0242	.0182	.0102	.1146	.1042	.0202	.0098
0.30	.0107	.0203	.0280	.0327	.0333	.0309	.0266	.0202	.0115	.1292	.1083	.0216	.0107
0.35	.0111	.0212	.0292	.0343	.0354	.0332	.0289	.0222	.0127	.1437	.1125	.0230	.0116
0.40	.0115	.0220	.0305	.0360	.0375	.0356	.0313	.0242	.0139	.1583	.1167	.0245	.0125
0.45	.0119	.0228	.0317	.0377	.0396	.0380	.0337	.0262	.0151	.1729	.1208	.0259	.0134
0.50	.0123	.0237	.0330	.0393	.0417	.0403	.0360	.0282	.0163	.1875	.1250	.0273	.0143
0.55	.0127	.0245	.0342	.0410	.0437	.0427	.0384	.0302	.0176	.2021	.1292	.0288	.0152
0.60	.0132	.0253	.0355	.0427	.0458	.0451	.0407	.0321	.0188	.2167	.1333	.0302	.0161
0.65	.0136	.0262	.0367	.0443	.0479	.0474	.0430	.0341	.0200	.2312	.1375	.0316	.0171
0.70	.0140	.0270	.0380	.0460	.0500	.0498	.0454	.0361	.0212	.2458	.1417	.0331	.0180
0.75	.0144	.0278	.0392	.0477	.0521	.0522	.0478	.0381	.0224	.2604	.1458	.0345	.0189
0.80	.0148	.0287	.0405	.0493	.0542	.0545	.0501	.0401	.0236	.2750	.1500	.0359	.0198
0.85	.0152	.0295	.0417	.0510	.0562	.0569	.0525	.0421	.0249	.2896	.1542	.0374	.0207
0.90	.0157	.0303	.0430	.0527	.0583	.0593	.0548	.0440	.0261	.3042	.1583	.0388	.0216
0.95	.0161	.0312	.0442	.0543	.0604	.0616	.0571	.0460	.0273	.3187	.1625	.0402	.0225

TABLE A-11
β = 0.6

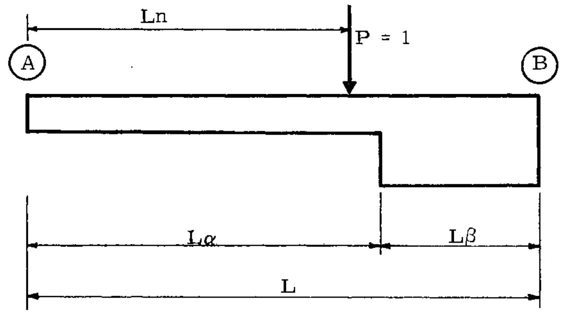
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$

$$\tau_{AB}^{(HL)} = t_7 \frac{w L^3}{E I_o}$$



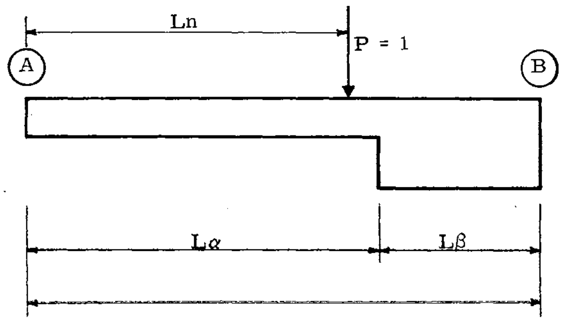
$I_A = I_o$

$\mu = \frac{I_A}{I_B}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_1	g	t_5	t_7
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0213	.0336	.0379	.0352	.0293	.0235	.0176	.0117	.0059	.2613	.0587	.0219	.0106
0.05	.0217	.0343	.0290	.0366	.0310	.0251	.0190	.0127	.0064	.2649	.0641	.0229	.0113
0.10	.0220	.0350	.0401	.0381	.0327	.0267	.0204	.0138	.0069	.2685	.0695	.0238	.0120
0.15	.0224	.0358	.0411	.0395	.0343	.0283	.0218	.0148	.0075	.2721	.0749	.0248	.0127
0.20	.0227	.0365	.0422	.0410	.0360	.0300	.0232	.0158	.0080	.2757	.0803	.0258	.0134
0.25	.0231	.0372	.0433	.0424	.0376	.0316	.0246	.0168	.0085	.2793	.0857	.0268	.0141
0.30	.0235	.0379	.0444	.0438	.0393	.0332	.0260	.0178	.0091	.2829	.0911	.0278	.0148
0.35	.0238	.0386	.0455	.0453	.0409	.0349	.0274	.0188	.0096	.2865	.0965	.0288	.0155
0.40	.0242	.0394	.0465	.0467	.0426	.0365	.0288	.0198	.0101	.2901	.1009	.0298	.0162
0.45	.0245	.0401	.0476	.0482	.0443	.0381	.0302	.0209	.0107	.2937	.1073	.0308	.0169
0.50	.0249	.0408	.0487	.0496	.0459	.0397	.0316	.0219	.0112	.2973	.1127	.0318	.0176
0.55	.0253	.0415	.0498	.0510	.0476	.0414	.0329	.0229	.0117	.3009	.1181	.0328	.0183
0.60	.0256	.0422	.0509	.0525	.0492	.0430	.0343	.0239	.0122	.3045	.1235	.0337	.0190
0.65	.0260	.0430	.0519	.0539	.0509	.0446	.0357	.0249	.0128	.3081	.1289	.0347	.0197
0.70	.0263	.0437	.0530	.0554	.0526	.0462	.0371	.0259	.0133	.3117	.1343	.0357	.0204
0.75	.0267	.0444	.0541	.0568	.0542	.0479	.0385	.0269	.0138	.3153	.1397	.0367	.0211
0.80	.0271	.0451	.0552	.0582	.0559	.0495	.0399	.0279	.0144	.3189	.1451	.0377	.0218
0.85	.0274	.0458	.0563	.0597	.0575	.0511	.0413	.0290	.0149	.3225	.1505	.0387	.0225
0.90	.0278	.0466	.0573	.0611	.0592	.0527	.0427	.0300	.0154	.3261	.1559	.0397	.0232
0.95	.0281	.0473	.0584	.0626	.0608	.0544	.0441	.0310	.0160	.3297	.1613	.0407	.0239

TABLE A-12
β = 0.6



$I_A = I_o$

$\mu = \dfrac{I_A}{I_B}$

$F_{BA} = f_2 \dfrac{L}{EI_o}$

$G_{BA} = g \dfrac{L}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \dfrac{L^2}{EI_o}$

$\tau_{BA}^{(UL)} = t_6 \dfrac{wL^3}{EI_o}$

$\tau_{BA}^{(HL)} = t_8 \dfrac{wL^3}{EI_o}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_2	g	t_6	t_8
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0057	.0104	.0131	.0128	.0107	.0085	.0064	.0043	.0021	.0213	.0587	.0075	.0038
0.05	.0062	.0115	.0147	.0150	.0133	.0113	.0090	.0065	.0035	.0369	.0641	.0092	.0051
0.10	.0068	.0126	.0163	.0171	.0158	.0141	.0117	.0086	.0048	.0525	.0695	.0109	.0064
0.15	.0073	.0136	.0180	.0193	.0184	.0169	.0144	.0108	.0061	.0681	.0749	.0126	.0077
0.20	.0079	.0147	.0196	.0214	.0210	.0196	.0170	.0130	.0074	.0837	.0803	.0143	.0090
0.25	.0084	.0158	.0212	.0236	.0236	.0224	.0197	.0152	.0087	.0993	.0857	.0160	.0102
0.30	.0089	.0169	.0228	.0258	.0262	.0252	.0223	.0174	.0100	.1149	.0911	.0177	.0115
0.35	.0095	.0180	.0244	.0279	.0288	.0279	.0250	.0196	.0114	.1305	.0965	.0194	.0128
0.40	.0100	.0190	.0261	.0301	.0314	.0307	.0276	.0218	.0127	.1461	.1019	.0211	.0141
0.45	.0106	.0201	.0277	.0322	.0340	.0335	.0303	.0239	.0140	.1617	.1073	.0229	.0153
0.50	.0111	.0212	.0293	.0344	.0366	.0363	.0329	.0261	.0153	.1773	.1127	.0246	.0166
0.55	.0116	.0223	.0309	.0366	.0392	.0390	.0356	.0283	.0166	.1929	.1181	.0263	.0179
0.60	.0122	.0234	.0325	.0387	.0418	.0418	.0383	.0305	.0180	.2085	.1235	.0280	.0192
0.65	.0127	.0244	.0342	.0409	.0444	.0446	.0409	.0327	.0193	.2241	.1289	.0297	.0205
0.70	.0133	.0255	.0358	.0430	.0469	.0474	.0436	.0349	.0206	.2397	.1343	.0314	.0217
0.75	.0138	.0266	.0374	.0452	.0495	.0501	.0462	.0371	.0219	.2553	.1397	.0331	.0230
0.80	.0143	.0277	.0390	.0474	.0521	.0529	.0489	.0393	.0232	.2709	.1451	.0348	.0243
0.85	.0149	.0288	.0406	.0495	.0547	.0557	.0515	.0414	.0245	.2865	.1505	.0365	.0256
0.90	.0154	.0298	.0423	.0517	.0573	.0585	.0542	.0436	.0259	.3021	.1559	.0382	.0268
0.95	.0160	.0309	.0439	.0538	.0599	.0612	.0568	.0458	.0272	.3177	.1613	.0400	.0281

TABLE A-13
β = 0.7

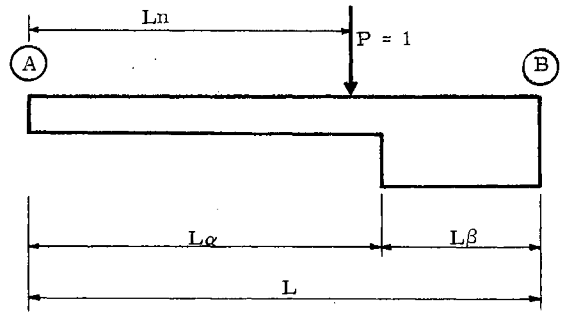
$F_{AB} = f_1 \dfrac{L}{EI_o}$

$G_{AB} = g \dfrac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \dfrac{L^2}{EI_o}$

$\tau_{AB}^{(UL)} = t_5 \dfrac{wL^3}{EI_o}$

$\tau_{AB}^{(HL)} = t_7 \dfrac{wL^3}{EI_o}$



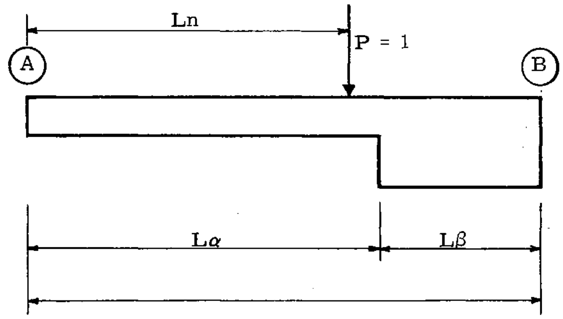
$I_A = I_o$

$\mu = \dfrac{I_A}{I_B}$

Coefficients for Angular Functions

μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9	f_1	g	t_5	t_7
	Influence Coefficient t_2												
0.00	.0171	.0251	.0252	.0216	.0180	.0144	.0108	.0072	.0036	.2190	.0360	.0145	.0088
0.05	.0176	.0263	.0269	.0237	.0202	.0165	.0125	.0084	.0042	.2247	.0425	.0159	.0099
0.10	.0182	.0274	.0286	.0258	.0224	.0186	.0143	.0097	.0049	.2304	.0491	.0172	.0110
0.15	.0188	.0286	.0303	.0280	.0247	.0206	.0160	.0109	.0055	.2361	.0556	.0186	.0121
0.20	.0194	.0297	.0321	.0301	.0269	.0227	.0177	.0122	.0062	.2419	.0621	.0199	.0132
0.25	.0199	.0308	.0338	.0322	.0291	.0248	.0195	.0134	.0068	.2476	.0687	.0213	.0143
0.30	.0205	.0320	.0355	.0343	.0313	.0269	.0212	.0146	.0075	.2533	.0752	.0227	.0154
0.35	.0211	.0331	.0372	.0364	.0336	.0290	.0229	.0159	.0081	.2590	.0817	.0240	.0165
0.40	.0216	.0343	.0389	.0386	.0358	.0310	.0247	.0171	.0088	.2647	.0883	.0254	.0176
0.45	.0222	.0354	.0406	.0407	.0380	.0331	.0264	.0184	.0094	.2704	.0948	.0267	.0187
0.50	.0228	.0366	.0424	.0428	.0402	.0352	.0281	.0196	.0100	.2762	.1013	.0281	.0198
0.55	.0234	.0377	.0441	.0449	.0425	.0373	.0299	.0208	.0107	.2819	.1079	.0294	.0209
0.60	.0239	.0389	.0458	.0470	.0447	.0394	.0316	.0221	.0113	.2876	.1144	.0308	.0220
0.65	.0245	.0400	.0475	.0492	.0469	.0414	.0334	.0233	.0120	.2933	.1209	.0322	.0231
0.70	.0251	.0411	.0492	.0513	.0491	.0435	.0351	.0246	.0126	.2990	.1275	.0335	.0242
0.75	.0256	.0423	.0509	.0534	.0514	.0456	.0368	.0258	.0133	.3047	.1340	.0349	.0253
0.80	.0262	.0434	.0526	.0555	.0536	.0477	.0386	.0270	.0139	.3105	.1405	.0362	.0264
0.85	.0268	.0446	.0543	.0576	.0558	.0498	.0403	.0283	.0146	.3162	.1471	.0376	.0275
0.90	.0274	.0457	.0561	.0598	.0581	.0518	.0420	.0295	.0152	.3219	.1536	.0390	.0286
0.95	.0279	.0469	.0578	.0619	.0603	.0539	.0438	.0308	.0159	.3276	.1601	.0403	.0297

TABLE A-14
β = 0.7



$I_A = I_o$ $\qquad \mu = \frac{I_A}{I_B}$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

$$\tau_{BA}^{(HL)} = t_8 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_2	g	t_6	t_8
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0034	.0059	.0063	.0054	.0045	.0036	.0027	.0018	.0009	.0090	.0360	.0035	.0022
0.05	.0041	.0072	.0083	.0079	.0074	.0066	.0055	.0041	.0023	.0252	.0425	.0054	.0038
0.10	.0047	.0085	.0102	.0105	.0103	.0096	.0084	.0064	.0037	.0414	.0491	.0073	.0054
0.15	.0054	.0098	.0122	.0130	.0132	.0127	.0112	.0087	.0050	.0576	.0556	.0092	.0070
0.20	.0060	.0111	.0141	.0155	.0161	.0157	.0141	.0110	.0064	.0739	.0621	.0111	.0087
0.25	.0067	.0124	.0161	.0180	.0190	.0187	.0169	.0133	.0078	.0901	.0687	.0130	.0103
0.30	.0074	.0137	.0181	.0206	.0219	.0217	.0197	.0157	.0092	.1063	.0752	.0149	.0119
0.35	.0080	.0150	.0200	.0231	.0248	.0247	.0226	.0180	.0106	.1225	.0817	.0169	.0135
0.40	.0087	.0163	.0220	.0256	.0277	.0278	.0254	.0203	.0119	.1387	.0883	.0188	.0151
0.45	.0093	.0176	.0239	.0282	.0306	.0308	.0283	.0226	.0133	.1549	.0948	.0207	.0167
0.50	.0100	.0189	.0259	.0307	.0335	.0338	.0311	.0249	.0147	.1712	.1013	.0226	.0184
0.55	.0106	.0202	.0279	.0332	.0364	.0368	.0339	.0272	.0161	.1874	.1079	.0245	.0200
0.60	.0113	.0215	.0298	.0358	.0393	.0398	.0368	.0295	.0175	.2036	.1144	.0264	.0216
0.65	.0119	.0229	.0318	.0383	.0422	.0429	.0397	.0318	.0188	.2198	.1209	.0283	.0232
0.70	.0126	.0242	.0337	.0408	.0451	.0459	.0425	.0341	.0202	.2360	.1275	.0302	.0248
0.75	.0132	.0255	.0357	.0433	.0480	.0489	.0453	.0364	.0216	.2522	.1340	.0321	.0264
0.80	.0139	.0268	.0377	.0459	.0509	.0519	.0481	.0388	.0230	.2685	.1405	.0340	.0280
0.85	.0145	.0281	.0396	.0484	.0538	.0549	.0510	.0411	.0244	.2847	.1471	.0359	.0297
0.90	.0152	.0294	.0416	.0509	.0567	.0580	.0538	.0434	.0257	.3009	.1536	.0378	.0313
0.95	.0158	.0307	.0435	.0535	.0596	.0610	.0567	.0457	.0271	.3171	.1601	.0398	.0329

TABLE A-15
β = 0.8

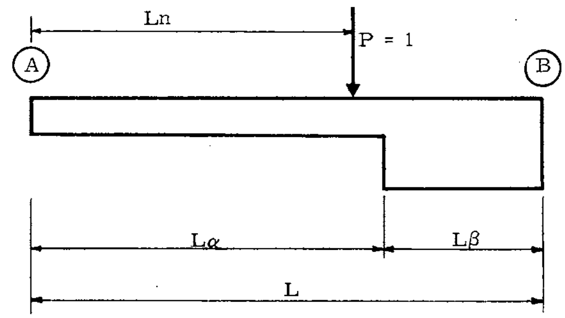
$F_{AB} = f_1 \dfrac{L}{E I_o}$

$G_{AB} = g \dfrac{L}{E I_o}$

$\tau_{AB}^{(LL)} = t_1 \dfrac{L^2}{E I_o}$

$\tau_{AB}^{(UL)} = t_5 \dfrac{wL^3}{E I_o}$

$\tau_{AB}^{(HL)} = t_7 \dfrac{wL^3}{E I_o}$

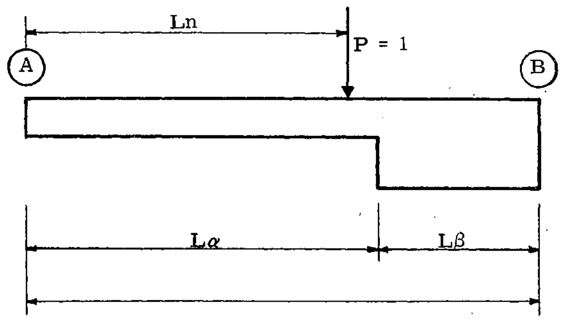


$I_A = I_o$ $\mu = \dfrac{I_A}{I_B}$

Coefficients for Angular Functions

μ \ n	Influence Coefficient t_2 .1	.2	.3	.4	.5	.6	.7	.8	.9	f_1	g	t_5	t_7
0.00	.0114	.0139	.0121	.0104	.0087	.0069	.0052	.0035	.0017	.1627	.0173	.0075	.0055
0.05	.0123	.0156	.0145	.0131	.0114	.0094	.0072	.0049	.0025	.1712	.0248	.0092	.0071
0.10	.0131	.0173	.0169	.0158	.0140	.0118	.0092	.0063	.0032	.1797	.0323	.0109	.0086
0.15	.0140	.0190	.0192	.0184	.0167	.0143	.0112	.0077	.0039	.1883	.0397	.0127	.0102
0.20	.0148	.0207	.0216	.0211	.0194	.0167	.0133	.0092	.0047	.1968	.0472	.0144	.0117
0.25	.0157	.0224	.0240	.0238	.0221	.0192	.0153	.0106	.0054	.2053	.0547	.0161	.0132
0.30	.0166	.0241	.0263	.0265	.0248	.0217	.0173	.0120	.0062	.2139	.0621	.0178	.0148
0.35	.0174	.0258	.0287	.0292	.0275	.0241	.0193	.0135	.0069	.2224	.0696	.0195	.0163
0.40	.0183	.0275	.0311	.0318	.0302	.0266	.0213	.0149	.0076	.2309	.0771	.0212	.0178
0.45	.0191	.0292	.0334	.0345	.0329	.0290	.0233	.0163	.0084	.2395	.0845	.0229	.0194
0.50	.0200	.0309	.0358	.0372	.0356	.0315	.0253	.0177	.0092	.2480	.0920	.0246	.0209
0.55	.0208	.0326	.0382	.0399	.0383	.0340	.0274	.0192	.0099	.2565	.0995	.0263	.0224
0.60	.0217	.0343	.0406	.0426	.0410	.0364	.0294	.0206	.0106	.2651	.1069	.0280	.0240
0.65	.0225	.0361	.0429	.0452	.0437	.0388	.0314	.0220	.0113	.2736	.1144	.0297	.0255
0.70	.0234	.0378	.0453	.0479	.0463	.0413	.0334	.0234	.0121	.2821	.1219	.0314	.0270
0.75	.0242	.0395	.0477	.0506	.0490	.0437	.0354	.0249	.0128	.2906	.1293	.0331	.0286
0.80	.0251	.0412	.0500	.0533	.0517	.0462	.0374	.0263	.0135	.2992	.1368	.0348	.0301
0.85	.0259	.0429	.0524	.0560	.0544	.0486	.0395	.0277	.0143	.3077	.1443	.0365	.0317
0.90	.0268	.0446	.0548	.0586	.0571	.0511	.0415	.0291	.0150	.3163	.1517	.0383	.0332
0.95	.0276	.0463	.0571	.0613	.0598	.0535	.0435	.0306	.0158	.3248	.1592	.0400	.0347

TABLE A-16
$\beta = 0.8$



$I_A = I_o$

$\mu = \frac{I_A}{I_B}$

$F_{BA} = f_2 \frac{L}{EI_o}$

$G_{BA} = g \frac{L}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$

$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$

$\tau_{BA}^{(HL)} = t_8 \frac{wL^3}{EI_o}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_2	g	t_6	t_8
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0016	.0021	.0019	.0016	.0013	.0011	.0008	.0005	.0003	.0027	.0173	.0011	.0009
0.05	.0023	.0036	.0040	.0043	.0044	.0042	.0037	.0029	.0017	.0192	.0248	.0032	.0027
0.10	.0031	.0051	.0062	.0070	.0074	.0074	.0067	.0053	.0031	.0357	.0323	.0052	.0046
0.15	.0038	.0066	.0084	.0098	.0105	.0105	.0096	.0077	.0045	.0523	.0397	.0072	.0065
0.20	.0046	.0081	.0106	.0125	.0136	.0137	.0125	.0100	.0059	.0688	.0472	.0092	.0084
0.25	.0053	.0096	.0128	.0152	.0166	.0168	.0155	.0124	.0073	.0853	.0547	.0113	.0102
0.30	.0060	.0111	.0150	.0179	.0197	.0199	.0184	.0148	.0087	.1019	.0621	.0133	.0121
0.35	.0068	.0126	.0171	.0206	.0227	.0231	.0213	.0171	.0101	.1184	.0696	.0153	.0140
0.40	.0075	.0141	.0193	.0234	.0258	.0262	.0243	.0195	.0116	.1349	.0771	.0173	.0159
0.45	.0083	.0156	.0215	.0261	.0289	.0294	.0272	.0219	.0130	.1515	.0845	.0194	.0177
0.50	.0090	.0171	.0237	.0288	.0319	.0325	.0301	.0243	.0144	.1680	.0920	.0214	.0196
0.55	.0098	.0186	.0259	.0315	.0350	.0357	.0331	.0266	.0158	.1845	.0995	.0234	.0215
0.60	.0105	.0201	.0280	.0342	.0380	.0388	.0360	.0290	.0172	.2011	.1069	.0255	.0234
0.65	.0113	.0215	.0302	.0370	.0411	.0420	.0390	.0314	.0186	.2176	.1144	.0275	.0253
0.70	.0120	.0230	.0324	.0397	.0442	.0451	.0419	.0338	.0200	.2341	.1219	.0295	.0271
0.75	.0128	.0245	.0346	.0424	.0472	.0483	.0448	.0361	.0214	.2507	.1293	.0315	.0290
0.80	.0135	.0260	.0368	.0451	.0503	.0514	.0478	.0385	.0229	.2672	.1368	.0336	.0309
0.85	.0143	.0275	.0390	.0478	.0533	.0546	.0507	.0409	.0243	.2837	.1443	.0356	.0328
0.90	.0150	.0290	.0411	.0506	.0564	.0577	.0536	.0433	.0257	.3003	.1517	.0376	.0346
0.95	.0158	.0305	.0433	.0533	.0594	.0609	.0566	.0456	.0271	.3168	.1592	.0396	.0365

TABLE A-17
β = 0.9

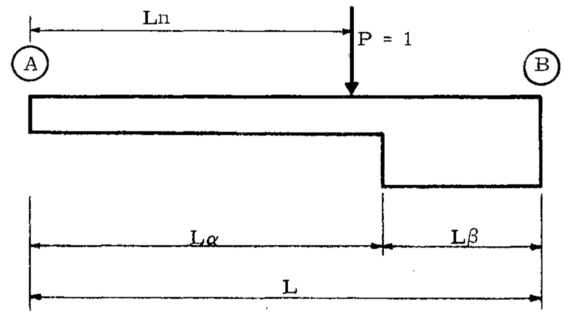
$$F_{AB} = f_1 \frac{L}{EI_o}$$

$$G_{AB} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o}$$

$$\tau_{AB}^{(HL)} = t_7 \frac{wL^3}{EI_o}$$

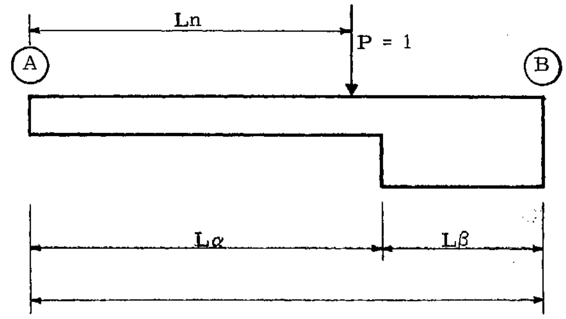


$I_A = I_o$ $\mu = \frac{I_A}{I_B}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_1	g	t_5	t_7
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0042	.0037	.0033	.0028	.0023	.0019	.0014	.0009	.0005	.0903	.0047	.0022	.0019
0.05	.0054	.0059	.0061	.0059	.0053	.0046	.0036	.0025	.0013	.1025	.0128	.0042	.0038
0.10	.0066	.0082	.0089	.0089	.0084	.0073	.0058	.0040	.0021	.1146	.0209	.0061	.0057
0.15	.0078	.0104	.0117	.0120	.0114	.0100	.0080	.0056	.0029	.1268	.0290	.0081	.0076
0.20	.0091	.0126	.0145	.0150	.0144	.0127	.0102	.0071	.0037	.1389	.0371	.0101	.0095
0.25	.0103	.0148	.0173	.0181	.0174	.0154	.0124	.0087	.0045	.1511	.0452	.0121	.0115
0.30	.0115	.0170	.0201	.0212	.0204	.0181	.0146	.0103	.0053	.1632	.0533	.0140	.0134
0.35	.0127	.0192	.0229	.0242	.0234	.0208	.0168	.0118	.0061	.1754	.0614	.0160	.0153
0.40	.0139	.0214	.0258	.0273	.0264	.0235	.0190	.0134	.0069	.1875	.0695	.0180	.0172
0.45	.0151	.0237	.0286	.0303	.0294	.0262	.0212	.0149	.0077	.1997	.0776	.0199	.0191
0.50	.0163	.0259	.0314	.0334	.0324	.0289	.0235	.0165	.0085	.2118	.0857	.0219	.0210
0.55	.0176	.0281	.0342	.0365	.0354	.0316	.0257	.0180	.0093	.2240	.0938	.0239	.0229
0.60	.0188	.0303	.0370	.0395	.0384	.0343	.0279	.0196	.0101	.2361	.1019	.0259	.0248
0.65	.0200	.0325	.0398	.0426	.0414	.0371	.0301	.0211	.0109	.2483	.1100	.0278	.0268
0.70	.0212	.0347	.0426	.0456	.0444	.0398	.0323	.0227	.0117	.2604	.1181	.0298	.0287
0.75	.0224	.0369	.0454	.0487	.0475	.0425	.0345	.0242	.0125	.2726	.1262	.0318	.0306
0.80	.0236	.0391	.0483	.0518	.0505	.0452	.0367	.0258	.0133	.2847	.1343	.0338	.0325
0.85	.0249	.0414	.0511	.0548	.0535	.0479	.0389	.0273	.0141	.2969	.1424	.0357	.0344
0.90	.0261	.0436	.0539	.0579	.0565	.0506	.0411	.0289	.0149	.3090	.1505	.0377	.0363
0.95	.0273	.0458	.0567	.0609	.0595	.0533	.0433	.0304	.0157	.3212	.1586	.0397	.0382

TABLE A-18
β = 0.9



$I_A = I_o$

$\mu = \frac{I_A}{I_B}$

$F_{BA} = f_2 \frac{L}{EI_o}$

$G_{BA} = g \frac{L}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$

$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$

$\tau_{BA}^{(HL)} = t_8 \frac{wL^3}{EI_o}$

Coefficients for Angular Functions

	Influence Coefficient t_2									f_2	g	t_6	t_8
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0003	.0003	.0002	.0002	.0002	.0001	.0001	.0001	.0000	.0003	.0047	.0002	.0001
0.05	.0011	.0019	.0025	.0030	.0033	.0033	.0031	.0025	.0015	.0170	.0128	.0022	.0022
0.10	.0019	.0034	.0048	.0058	.0064	.0065	.0060	.0049	.0029	.0336	.0209	.0043	.0042
0.15	.0027	.0050	.0070	.0086	.0095	.0097	.0090	.0073	.0043	.0503	.0290	.0064	.0062
0.20	.0035	.0066	.0093	.0114	.0126	.0129	.0120	.0097	.0057	.0669	.0371	.0085	.0083
0.25	.0043	.0082	.0115	.0141	.0157	.0161	.0149	.0120	.0072	.0836	.0452	.0105	.0103
0.30	.0052	.0098	.0138	.0169	.0189	.0193	.0179	.0144	.0086	.1002	.0533	.0126	.0123
0.35	.0060	.0114	.0161	.0197	.0220	.0225	.0209	.0168	.0100	.1168	.0614	.0147	.0144
0.40	.0068	.0130	.0183	.0225	.0251	.0257	.0239	.0192	.0114	.1335	.0695	.0168	.0164
0.45	.0076	.0145	.0206	.0253	.0282	.0289	.0268	.0216	.0128	.1502	.0776	.0188	.0185
0.50	.0084	.0161	.0229	.0281	.0313	.0321	.0298	.0240	.0143	.1668	.0857	.0209	.0205
0.55	.0092	.0177	.0251	.0309	.0344	.0353	.0328	.0264	.0157	.1835	.0938	.0230	.0225
0.60	.0100	.0193	.0274	.0337	.0376	.0385	.0357	.0288	.0171	.2001	.1019	.0251	.0246
0.65	.0108	.0209	.0297	.0365	.0407	.0416	.0387	.0312	.0185	.2168	.1100	.0271	.0266
0.70	.0116	.0225	.0319	.0393	.0438	.0448	.0417	.0336	.0200	.2334	.1181	.0292	.0286
0.75	.0124	.0241	.0342	.0421	.0469	.0480	.0446	.0360	.0214	.2501	.1262	.0313	.0306
0.80	.0133	.0257	.0364	.0448	.0500	.0512	.0476	.0384	.0228	.2667	.1343	.0334	.0327
0.85	.0141	.0272	.0387	.0476	.0531	.0544	.0506	.0408	.0242	.2834	.1424	.0354	.0347
0.90	.0149	.0288	.0410	.0504	.0563	.0576	.0536	.0432	.0257	.3000	.1505	.0375	.0368
0.95	.0157	.0304	.0432	.0532	.0594	.0608	.0565	.0456	.0271	.3167	.1586	.0396	.0388

TABLE B-1
β = 0.1

$F_{AB} = F_{BA} = f \dfrac{L}{EI_o}$

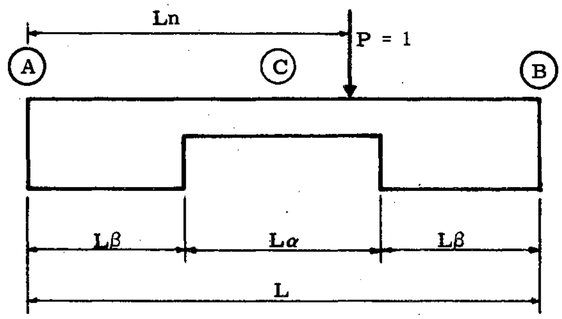
$G_{AB} = G_{BA} = g \dfrac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \dfrac{L^2}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \dfrac{L^2}{EI_o}$

$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \dfrac{wL^3}{EI_o}$

$\tau_{AB}^{(HL)} = \tau_{BA}^{(HL)} = t_5 \dfrac{wL^3}{EI_o}$



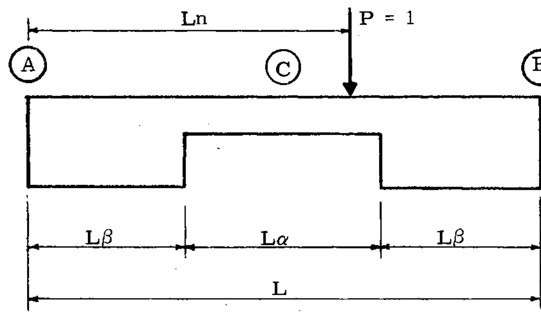
$I_C = I_o$

$\mu = \dfrac{I_o}{I_A} = \dfrac{I_o}{I_B}$

Coefficients for Angular Functions

	Influence Coefficient t_1									f	g	t_4	t_5
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0243	.0442	.0561	.0611	.0600	.0539	.0439	.0308	.0157	.2427	.1573	.0393	.0020
0.05	.0245	.0444	.0563	.0612	.0601	.0540	.0439	.0309	.0158	.2472	.1578	.0394	.0020
0.10	.0247	.0446	.0565	.0614	.0602	.0541	.0440	.0309	.0158	.2517	.1583	.0396	.0020
0.15	.0249	.0448	.0566	.0615	.0604	.0542	.0441	.0310	.0158	.2563	.1587	.0397	.0020
0.20	.0251	.0450	.0568	.0617	.0605	.0543	.0442	.0310	.0159	.2608	.1592	.0398	.0021
0.25	.0253	.0451	.0570	.0618	.0606	.0545	.0443	.0311	.0159	.2653	.1597	.0399	.0021
0.30	.0255	.0453	.0571	.0619	.0607	.0546	.0444	.0312	.0160	.2699	.1601	.0400	.0021
0.35	.0257	.0455	.0573	.0621	.0609	.0547	.0444	.0312	.0160	.2744	.1606	.0401	.0021
0.40	.0260	.0457	.0575	.0622	.0610	.0548	.0445	.0313	.0160	.2789	.1611	.0403	.0021
0.45	.0262	.0459	.0576	.0624	.0611	.0549	.0446	.0313	.0161	.2835	.1615	.0404	.0021
0.50	.0264	.0461	.0578	.0625	.0612	.0550	.0447	.0314	.0161	.2880	.1620	.0405	.0022
0.55	.0266	.0463	.0580	.0627	.0614	.0551	.0448	.0315	.0162	.2925	.1625	.0406	.0022
0.60	.0268	.0465	.0582	.0628	.0615	.0552	.0448	.0315	.0162	.2971	.1629	.0407	.0022
0.65	.0270	.0467	.0583	.0630	.0616	.0553	.0449	.0316	.0162	.3016	.1634	.0408	.0022
0.70	.0272	.0469	.0585	.0631	.0617	.0554	.0450	.0316	.0163	.3061	.1639	.0410	.0022
0.75	.0274	.0470	.0587	.0633	.0619	.0555	.0451	.0317	.0163	.3107	.1643	.0411	.0022
0.80	.0277	.0472	.0588	.0634	.0620	.0556	.0452	.0318	.0163	.3152	.1648	.0412	.0023
0.85	.0279	.0474	.0590	.0636	.0621	.0557	.0453	.0318	.0164	.3197	.1653	.0413	.0023
0.90	.0281	.0476	.0592	.0637	.0622	.0558	.0453	.0319	.0164	.3243	.1657	.0414	.0023
0.95	.0283	.0478	.0593	.0639	.0624	.0559	.0454	.0319	.0165	.3288	.1662	.0415	.0023
μ / n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_4	t_5
	Influence Coefficients t_2												

TABLE B-2
$\beta = 0.2$



$I_C = I_o$

$\mu = \frac{I_o}{I_A} = \frac{I_o}{I_B}$

$F_{AB} = F_{BA} = f \frac{L}{EI_o}$

$G_{AB} = G_{BA} = g \frac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$

$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_o}$

$\tau_{AB}^{(HL)} = \tau_{BA}^{(HL)} = t_5 \frac{wL^3}{EI_o}$

Coefficients for Angular Functions

	Influence Coefficient t_1									f	g	t_4	t_5
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0168	.0336	.0466	.0525	.0525	.0475	.0384	.0264	.0132	.1680	.1320	.0330	.0060
0.05	.0174	.0343	.0472	.0531	.0530	.0479	.0388	.0267	.0134	.1763	.1337	.0334	.0061
0.10	.0180	.0350	.0479	.0537	.0535	.0483	.0391	.0270	.0135	.1845	.1355	0339	.0062
0.15	.0186	.0358	.0485	.0542	.0540	.0487	.0395	.0272	.0137	.1928	.1372	.0343	.0064
0.20	.0191	.0365	.0492	.0548	.0545	.0492	.0398	.0275	.0139	.2011	.1389	.0347	.0065
0.25	.0197	.0372	.0498	.0554	.0550	.0496	.0402	.0278	.0140	.2093	.1407	.0352	.0067
0.30	.0203	.0379	.0504	.0560	.0555	.0500	.0406	.0281	.0142	.2176	.1424	.0356	.0068
0.35	.0209	.0386	.0511	.0565	.0560	.0505	.0409	.0284	.0144	.2259	.1441	.0360	.0069
0.40	.0215	.0394	.0517	.0571	.0565	.0509	.0413	.0286	.0145	.2341	.1459	.0365	.0071
0.45	.0221	.0401	.0524	.0577	.0570	.0513	.0416	.0289	.0147	.2424	.1476	.0369	.0072
0.50	.0227	.0408	.0530	.0583	.0575	.0517	.0420	.0292	.0149	.2507	.1493	.0373	.0073
0.55	.0232	.0415	.0537	.0588	.0580	.0522	.0423	.0295	.0150	.2589	.1511	.0378	.0075
0.60	.0238	.0422	.0543	.0594	.0585	.0526	.0427	.0298	.0152	.2672	.1528	.0382	.0076
0.65	.0244	.0430	.0550	.0600	.0590	.0530	.0430	.0300	.0153	.2755	.1545	.0386	.0077
0.70	.0250	.0437	.0556	.0606	.0595	.0534	.0434	.0303	.0155	.2837	.1563	.0390	.0078
0.75	.0256	.0444	.0563	.0611	.0600	.0539	.0437	.0306	.0157	.2920	.1580	.0395	.0080
0.80	.0262	.0451	.0569	.0617	.0605	.0543	.0441	.0309	.0158	.3003	.1597	.0399	.0081
0.85	.0267	.0458	.0576	.0623	.0610	.0547	.0444	.0312	.0160	.3085	.1615	.0404	.0082
0.90	.0273	.0466	.0582	.0629	.0615	.0551	.0448	.0314	.0162	.3168	.1632	.0408	.0084
0.95	.0279	.0473	.0588	.0634	.0620	.0556	.0451	.0317	.0163	.3251	.1649	.0412	.0085
μ / n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_4	t_5
	Influence Coefficients t_2												

TABLE B-3
β = 0.3

$F_{AB} = F_{BA} = f \dfrac{L}{EI_o}$

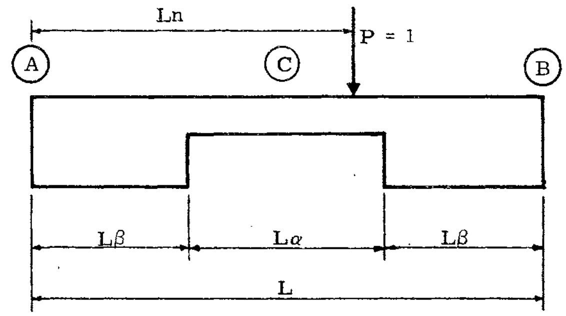
$G_{AB} = G_{BA} = g \dfrac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \dfrac{L^2}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \dfrac{L^2}{EI_o}$

$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \dfrac{wL^3}{EI_o}$

$\tau_{AB}^{(HL)} = \tau_{BA}^{(HL)} = t_5 \dfrac{wL^3}{EI_o}$

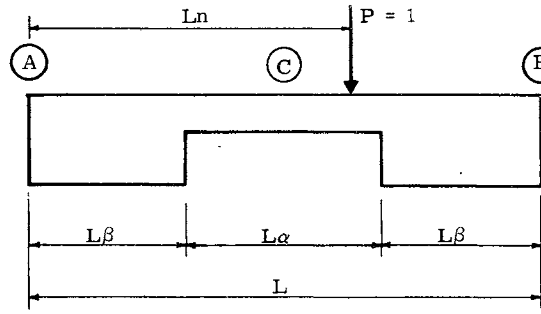


$I_C = I_o$ $\mu = \dfrac{I_o}{I_A} = \dfrac{I_o}{I_B}$

Coefficients for Angular Functions

	Influence Coefficient t_1									f	g	t_4	t_5
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0105	.0211	.0316	.0388	0400	.0362	.0284	.0189	.0095	.1053	.0947	10237	.0090
0.05	.0114	.0224	.0330	.0401	.0411	.0372	.0293	.0196	.0098	.1167	.0983	.0246	.0094
0.10	.0123	.0238	.0344	.0413	.0422	.0382	.0301	.0202	.0102	.1281	.1019	.0255	.0099
0.15	.0132	.0251	.0358	.0426	.0434	.0392	.0310	.0209	.0105	.1395	.1055	.0264	.0103
0.20	.0141	.0265	.0372	.0438	.0445	.0402	.0318	.0215	.0109	.1509	.1091	.0273	.0108
0.25	.0150	.0278	.0386	.0451	.0456	.0411	.0327	.0222	.0112	.1623	.1127	.0282	.0112
0.30	.0159	.0291	.0400	.0464	.0467	.0421	.0335	.0228	.0116	.1737	.1163	.0291	.0117
0.35	.0168	.0305	.0414	.0476	.0479	.0431	0344	.0235	.0119	.1851	.1199	.0300	.0122
0.40	.0177	.0318	.0428	.0489	.0490	.0441	.0352	.0242	.0123	.1965	.1235	.0309	.0126
0.45	.0186	.0332	.0442	.0501	.0501	.0451	.0361	.0248	.0126	.2079	.1271	.0318	.0130
0.50	.0195	.0345	.0455	.0514	.0513	.0461	.0370	.0255	.0130	.2193	.1307	.0327	.0135
0.55	.0204	.0359	.0469	.0527	.0524	.0471	.0378	.0261	.0133	.2307	.1343	.0336	.0139
0.60	.0213	.0372	.0483	.0539	.0535	.0481	.0387	.0268	.0137	.2421	.1379	.0345	.0144
0.65	.0222	.0386	.0497	.0552	.0546	.0491	.0395	.0274	.0140	.2535	.1415	.0354	.0148
0.70	.0231	.0399	.0511	.0564	.0558	.0501	.0404	.0281	.0144	.2649	.1451	.0363	.0153
0.75	.0240	.0413	.0525	.0577	.0569	.0511	.0412	.0287	.0147	.2763	.1487	.0372	.0157
0.80	.0249	.0426	.0539	.0590	.0580	.0520	.0421	.0294	.0151	.2877	.1523	.0381	.0162
0.85	.0258	.0440	.0553	.0602	.0591	.0530	.0429	.0300	.0154	.2991	.1559	.0390	.0166
0.90	.0267	.0453	.0567	.0615	.0603	.0540	.0438	.0307	.0158	.3105	.1595	.0399	.0171
0.95	.0276	.0467	.0581	.0627	.0614	.0550	.0446	.0313	.0161	.3219	.1631	.0408	.0175
μ \ n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_4	t_5
	Influence Coefficients t_2												

TABLE B-4
β = 0.4



$I_C = I_o$

$\mu = \frac{I_o}{I_A} = \frac{I_o}{I_B}$

$F_{AB} = F_{BA} = f \frac{L}{EI_o}$

$G_{AB} = G_{BA} = g \frac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$

$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_o}$

$\tau_{AB}^{(HL)} = \tau_{BA}^{(HL)} = t_5 \frac{wL^3}{EI_o}$

Coefficients for Angular Functions

	Influence Coefficient t_1									f	g	t_4	t_5
μ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
0.00	.0051	.0101	.0152	.0203	.0225	.0197	.0148	.0099	.0049	.0507	.0493	.0123	.0080
0.05	.0062	.0120	.0174	.0224	.0245	.0215	.0163	.0110	.0055	.0648	.0552	.0138	.0091
0.10	.0074	.0139	.0196	.0246	.0265	.0234	.0179	.0121	.0061	.0789	.0611	.0153	.0101
0.15	.0086	.0158	.0218	.0268	.0285	.0252	.0194	.0132	.0067	.0931	.0669	.0167	.0112
0.20	.0098	.0177	.0241	.0290	.0305	.0270	.0209	.0143	.0072	.1072	.0728	.0182	.0123
0.25	.0109	.0196	.0263	.0312	.0325	.0288	.0225	.0154	.0078	.1213	.0787	.0197	.0133
0.30	.0121	.0215	.0285	.0334	.0345	.0306	.0240	.0165	.0084	.1355	.0845	.0211	.0144
0.35	.0133	.0234	.0307	.0356	.0365	.0324	.0255	.0176	.0090	.1496	.0904	.0226	.0155
0.40	.0144	.0253	.0329	.0378	.0385	.0342	.0271	.0187	.0096	.1637	.0963	.0241	.0165
0.45	.0156	.0272	.0351	.0399	.0405	.0360	.0286	.0198	.0101	.1779	.1021	.0255	.0176
0.50	.0168	.0291	.0373	.0421	.0425	.0379	.0301	.0209	.0107	.1920	.1080	.0270	.0187
0.55	.0180	.0310	.0396	.0443	.0445	.0397	.0317	.0220	.0113	.2061	.1139	.0285	.0197
0.60	.0191	.0329	.0418	.0465	.0465	.0415	.0332	.0231	.0119	.2203	.1197	.0299	.0208
0.65	.0203	.0347	.0440	.0487	.0485	.0433	.0348	.0243	.0125	.2344	.1256	.0314	.0219
0.70	.0215	.0366	.0462	.0509	.0505	.0451	.0363	.0254	.0130	.2485	.1315	.0329	.0229
0.75	.0226	.0385	.0484	.0531	.0525	.0469	.0378	.0265	.0136	.2627	.1373	.0343	.0240
0.80	.0238	.0404	.0506	.0553	.0545	.0487	.0394	.0276	.0142	.2768	.1432	.0358	.0251
0.85	.0250	.0423	.0529	.0574	.0565	.0506	.0409	.0287	.0148	.2909	.1491	.0373	.0261
0.90	.0262	.0442	.0551	.0596	.0585	.0524	.0424	.0298	.0153	.3051	.1549	.0387	.0272
0.95	.0273	.0461	.0573	.0618	.0605	.0542	.0440	.0309	.0159	.3192	.1608	.0402	.0283
μ / n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_4	t_5
	Influence Coefficients t_2												

The tables on the preceding pages may be used to evaluate constants for the analysis of pinned-end frames, continuous beams, and continuous frames of variable cross section. The use of these beam constants is not restricted solely to analyses by the String Polygon Method as there exists a series of simple relationships between the angular functions of a structural system and the so-called moment functions.

The moment functions are:

(a) Stiffness Factor

(b) Carry-Over Stiffness Factor

(c) Fixed-End Moments.

The relationship between these functions and the angular functions is given by:

(a) Stiffness Factor - The stiffness factor K_{ij} is defined as the moment required to produce unit rotation at i, end j being fixed.

$$K_{ij} = \frac{F_{ji}}{N}$$

$$K_{ji} = \frac{F_{ij}}{N}$$

$$N = F_{ij} F_{ji} - G_{ij}^2$$

(b) Carry-Over Stiffness Factor - The carry-over stiffness factor C_{ij} is defined as the ratio of the moment produced at the fixed end j to the moment applied at i.

$$C_{ij} = \frac{G_{ij}}{F_{ji}}$$

$$C_{ji} = \frac{G_{ji}}{F_{ij}}$$

(c) Fixed-End Moments - The fixed-end moment FM_{ij} is defined as the moment developed at i of the member ij if ends i and j are held fixed.

$$FM_{ij} = \frac{G_{ij}\tau_{ji} - F_{ji}\tau_{ij}}{N}$$

$$FM_{ji} = \frac{F_{ij}\tau_{ji} - G_{ij}\tau_{ij}}{N}$$

CHAPTER VI

SUMMARY AND CONCLUSIONS

The analytical expressions for the end slopes of a straight prismatic beam with sudden changes in cross section were calculated by means of the string polygon method of analysis. It was shown that resulting algebraic equations were simple and can be readily used in engineering practice.

Angular functions included in this thesis are:

(a) Angular flexibilities and carry-over values.

(b) Load functions due to uniform and haunch loads.

(c) Load functions due to a unit moving load.

All these angular functions have been evaluated on a electronic computer IBM 650 and are tabulated in this thesis.

These angular functions can be used for three purposes:

(a) Analysis of straight plate girders with sudden changes in cross section.

(b) Calculation of end slopes and deflections of straight plate girders.

(c) Analysis of rigid frames with straight members with sudden changes in cross section.

A SELECTED BIBLIOGRAPHY

1. Tuma, Jan J., "Carry-Over Procedures Applied to Civil Engineering Problems," Lecture Notes, C.E. 620 - Seminar, Oklahoma State University, Stillwater, Spring 1959, 1960.

2. Chu, Shih L., "Beam Constants by the String Polygon Method," M.S. Thesis, Oklahoma State University, Stillwater, Summer 1959.

3. Maydayag, Angel F., "Deflection of Airplane Wings by the String Polygon Method," Seminar Report, Oklahoma State University, Stillwater, Spring 1960.

4. Harvey, John W., "Column-Beams by the String Polygon and Carry-Over Method," M.S. Thesis, Oklahoma State University, Stillwater, Summer 1960.

5. Boecker, Henry C., "Analysis of Pinned-End Frames by the String Polygon Method," M.S. Thesis, Oklahoma State University, Stillwater, Summer 1960.

6. Oden, John T., "Analysis of Fixed End Frames by the String Polygon Method," M.S. Thesis, Oklahoma State University, Stillwater, Summer 1960.

7. Wu, Chien M., "The General String Polygon," M.S. Thesis, Oklahoma State University, Stillwater, Summer 1960.

8. Mohr, O., "Behandlung der Elastischen Als Seillinie," Zeitschr D. Architekt. u. Ing. Vereins Zu, Hannover, 1868.

9. Müller-Breslau, H. F. B., "Bietrag Zur Theorie Des Fachwerks," Zeitschr D. Architekt.u. Ing, Hannover, 1885, pp. 31, 418.

10. Müller-Breslau, H. F. B., Die Graphische Statik Der Baukonstruktionen, Vol. II, Part 2, 2nd Ed., Leipzig, 1925, pp. 337 - 365.

11. Wanke, J., Zur Berechnung Der Formänderungen Vollwandiger Tragwerke, Der Stahlbau, 1939, No. 23, 24.

12. Chmelka, F., "Näherung Formeln," Der Stahlbau, 1940, No. 23, 24.

13. Biezeno, C. B. and R. Grammel, "Engineering Dynamics, Vol. II," Elastic Problems of Single Machine Elements, tr. M. L. Meyer, Blackie and Son, Limited, Glasgow, 1956, pp. 2 - 8.

14. Kaufmann, W., Statik Der Tragwerke, 4th Ed., Berlin, 1957, pp. 144 - 153.

15. Cross, Hardy and N. D. Morgan, Continuous Frames of Reinforced Concrete, New York, 1945, pp. 26 - 76.

16. Michalos, J., Theory of Structural Analysis and Design, New York, 1958, pp. 20 - 37.

17. Tuma, Jan J., T. Lassley, and S. French, "Analysis of Continuous Beam Bridges, Vol. I, Carry-Over Procedure," School of Civil Engineering Research Publication, Oklahoma State University, Stillwater, No. 3, 1959.

18. Leontovich, V., Frames and Arches, McGraw-Hill Book Co., Inc., New York, N.Y., 1959, pp. 222, 241 - 243.

19. Guldan, R., Die Cross-Methode, Springer-Verlag, Wien, 1955, pp. 131 - 138.

20. Lassley, T. I., "Beam Constants by High Speed Computers," M.S. Thesis, Oklahoma State University, Stillwater, 1959.

21. 650 Data Processing System Bulletin, G-24-5003-0, International Business Machines Corporation, June 1959.

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Master of Science

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Professional Experience: Served in the United States Army from September, 1953, to September, 1955. Computer, Western Geophysical Company of America, Galveston, Texas, Summer, 1956. Engineer, Exline Engineering, Tulsa, Oklahoma, Summer, 1958, 1960. Jr. Engineer, W. N. Holway and Associates, Tulsa, Oklahoma, Summer 1959.