A COMPUTER PROGRAM FOR SOLVING TWO-DIMENSIONAL UNSTEADY-STATE FLOW PROBLEMS BY THE ALTERNATING-DIRECTION IMPLICIT METHOD

By

Dean M. DeMoss

Bachelor of Science

Oklahoma State University of Agriculture and Science

Stillwater, Oklahoma

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Thesis Approved:

David Cornell
Thesis Adviser

J. H. Boggs

Robert MacVicar
Dean of the Graduate School

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NOMENCLATURE

U	variable in the basic differential equation
x	distance in horizontal direction, centimeters and feet
y	distance in vertical direction, centimeters and feet
C	coefficients in the basic differential equation
t	time, seconds and hours
W	numerical approximation to U
Δx	increment of x
Δy	increment of y
Δt	increment of time, t
A,B,C,D	coefficients of simultaneous equations
G,Z	intermediate values in solution of simultaneous equations
T	temperature, $^{\circ}R$
c_p	heat capacity, B.t.u./(lb.) ($^{\circ}F$)
ρ	density, lb./(ft.^{3})
K	thermal conductivity, B.t.u./(hr.) (ft.) ($^{\circ}F$)
α	parameter introduced in working difference equations
Q	flow rate for interior network point, $^{\circ}R$, pounds per square inch and (pounds per square inch)^2
q	flow rate, B.t.u./(hr.) (ft.), barrels per day and MCF/day
P	pressure, atmospheres and pounds per square inch
$\bar{\mu}$	average viscosity, centipoises
ϕ	fractional porosity
c	compressibility factor, $\frac{1}{\text{atmospheres}}$ and $\frac{1}{\text{pounds per square inch}}$
Δt_D	incremental dimensionless time

$\bar{z}$	average compressibility factor
h	formation thickness, feet
k	permeability, darcys and millidarcys
$\bar{P}$	average reservoir pressure, pounds per square inch
$\bar{Q}$	flow rate for corner point, (pounds per square inch)^2
Ei	exponential integral
r_i	distance from producing well to drawdown point
P_o	initial pressure, pounds per square inch

FORTRAN PROGRAM SYMBOLS

W	14 x 14 point variable array
KCON	14 x 14 point control array
J	subscript denoting positive x-direction
I	subscript denoting negative y-direction
Q	8 x 1 flow rate array
MAX	maximum number of points in either the I or J direction
ALPHA	symbol for the alpha value
POR	porosity
VIS	viscosity
DELX	incremental distance
COE	conversion factor
COM	compressibility factor
PERM	permeability
TTIME	total desired time
TIME	cumulative time

TMAX sub-time total

KDIR symbol denoting the direction in which the calculations are being made

DELT time increment

CHAPTER I

INTRODUCTION

It is important in the prediction of the behavior of oil and gas reservoirs to be able to calculate potential and flow distributions in the reservoir. Since only a few limited analytical solutions have been found to solve the second order partial differential equations which describe these distributions in finite cases, it becomes necessary to obtain approximate solutions by finite difference methods.

Since the development of the electronic digital computer, many such methods have been devised. Because of the large amount of work involved in solving distribution problems and the high cost of computing time, it becomes very important to obtain a method which requires a minimum of work and time. Much attention has been given to this problem of minimizing the work and time. The alternating-direction implicit method recently introduced by Peaceman and Rachford (7) represents one of the best known schemes for solving potential and flow distribution in two-dimensional systems. The method is advantageous because it has been proven by Douglas (2) that the method is stable for any size time step.

Although some applications of the method have been published, a computer program to solve other problems of this type was not available. The object of this study was to program the method on the

IBM 650 Computer for use in solving linearized unsteady-state gas, compressible fluid and heat flow problems. After such a program had been written, it was to be checked by solving a problem which had a known solution. When this had been accomplished, solutions to other problems could be obtained.

CHAPTER II

PREVIOUS INVESTIGATIONS

Recently, much attention has been given to the solution of unsteady-state problems by approximate finite difference methods. Since this study is concerned with applications of the alternating direction implicit method, only those articles which deal with this method or those directly related to it will be discussed here.

Bruce, et al, (1) developed a stable numerical procedure for solving the equation for production of gas at constant rates from linear and radial systems. A digital computer was used to perform this integration using an implicit form of an approximating difference equation. The solutions were compared with a laboratory study of gas depletion in a linear system.

Peaceman and Rachford (7) introduced the alternating-direction implicit method for solving parabolic and elliptic differential equations. The method was applied to the simplest type of problem, that of unsteady-state heat flow in a square. Also, the solution of Laplace's equation in a square was solved as an example of a steady-state problem. An analysis was presented that showed the alternating-direction implicit method to require less work than the best previously known iterative solution for solving Laplace's equation.

Douglas, Peaceman and Rachford (4) applied the alternating-direction implicit method to the problem of unsteady-state gas flow through porous media for a two dimensional square reservoir. The square region, containing a perfect gas, was depleted and solutions at various stages of the depletion were presented in graphical form in terms of dimensionless parameters.

Douglas and Peaceman (3) solved the heat flow equation in two dimensions by use of the alternating direction implicit method. Although all the examples worked were for steady-state condition, equations were developed for unsteady-state problems and a thorough discussion of the method was included. Examples were presented for heat flow around a corner, a problem involving a radiation boundary condition, and point heat sources and sinks in an elliptical region.

Douglas and Rachford (5) described a method similar to the alternating-direction implicit method for solving problems in three dimensions. The procedure was applicable in predicting flow patterns and potentials for both the steady- and unsteady-state flow of a single phase fluid.

CHAPTER III

THE MATHEMATICAL METHOD

The alternating-direction implicit method solves differential equations of the type

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = C \frac{\partial U}{\partial t} \qquad (3\text{-}1)$$

The method consists of replacing the continuous derivatives in Equation (3-1) by ratios of finite differences and solving the resulting difference equations.

To form the required difference equations, an integration net with mesh widths Δx and Δy, is placed over the two-dimensional region in which the integration is to be carried out. For convenience, Δx and Δy are set equal to each other.

One of the second derivatives, for instance, $\frac{\partial^2 U}{\partial x^2}$, is replaced by a second difference in terms of unknown values of U at the time level, $t + \Delta t$. The other second derivative, $\frac{\partial^2 U}{\partial y^2}$, is replaced by a second difference in terms of known values of U at the time level, t. When this has been done, Equation (3-1) becomes

$$\frac{W(x-\Delta x,y,t+\Delta t) - 2W(x,y,t+\Delta t) + W(x+\Delta x,y,t+\Delta t)}{(\Delta x)^2} +$$

$$\frac{W(x,y-\Delta y,t) - 2W(x,y,t) + W(x,y+\Delta y,t)}{(\Delta y)^2} = \frac{C\,(W(x,y,t+\Delta t) - W(x,y,t))}{\Delta t} \qquad (3\text{-}2)$$

where W is the numerical approximation for U.

Equation (3-2) is used for evaluating the unknown values of W at the time level, $t + \Delta t$, and is said to be implicit in the x-direction. Unknown values of W at the time level, $t + 2\Delta t$, are found by forming an equation, similar to Equation (3-2), but implicit in the y-direction. Equation (3-3) gives the necessary equation for the y-direction.

$$\frac{W(x-\Delta x,y,t+\Delta t) - 2W(x,y,t+\Delta t) + W(x+\Delta x,y,t+\Delta t)}{(\Delta x)^2} +$$

$$\frac{W(x,y-\Delta y,t+2\Delta t) - 2W(x,y,t+2\Delta t) + W(x,y+\Delta y,t+2\Delta t)}{(\Delta y)^2} = \frac{C\,(W(x,y,t+2\Delta t) - W(x,y,t+\Delta t))}{\Delta t} \qquad (3\text{-}3)$$

It should be noted that the unknown values of W in Equation (3-2) become the known values of W in Equation (3-3). Also, according to Douglas (2), Δt in the y-direction must be equal to the Δt used in the x-direction for any one double time step in order for the method to be stable.

If $\Delta x = \Delta y$ and $\alpha = C\frac{(\Delta x)^2}{\Delta t}$, Equations (3-2) and (3-3) can be rearranged in the following form, respectively:

In the x-direction

$$- W(x-\Delta x,y,t+\Delta t) + (2+\alpha)\, W(x,y,t+\Delta t) - W(x+\Delta x,y,t+\Delta t) = W(x,y-\Delta y,t) + (\alpha-2)\, W(x,y,t) + W(x,y+\Delta y,t) \qquad (3\text{-}4)$$

In the y-direction

$$- W(x,y-\Delta y,t+2\Delta t) + (2+\alpha)\, W(x,y,t+2\Delta t) - W(x,y+\Delta y,t+2\Delta t) = W(x-\Delta x,y,t+\Delta t) + (\alpha-2)\, W(x,y,t+\Delta t) + W(x+\Delta x,y,t+\Delta t) \qquad (3\text{-}5)$$

To start the problem, Equation (3-4) is written for each point in the x-direction preceding from the left of the integration network going toward the right. This results in the formation of small sets of simultaneous equations. There will be as many sets of equations as there are lines in the network in the x-direction. Each set of equations will have as many unknowns as there are points on the line. After boundary conditions have been accounted for, these sets of simultaneous equations can always be arranged as follows:

$$B_1W_1 + C_1W_2 = D_1$$

$$A_iW_{i-1} + B_iW_i + C_iW_{i+1} = D_i \qquad 2 \leq i \leq n-1 \qquad (3\text{-}6)$$

$$A_nW_{n-1} + B_nW_n = D_n$$

where n is the number of points per line and A, B, C and D are constant coefficients.

L. H. Thomas (3) solved these equations by the following non-iterative technique.

Let

$$Z = B_1$$

$$Z_i = B_i - \frac{A_i C_{i-1}}{Z_{i-1}} \qquad 2 \leqslant i \leqslant n \qquad (3\text{-}7)$$

and

$$G_1 = \frac{D_1}{Z_1}$$

$$G_i = \frac{D_i - A_i G_{i-1}}{Z_i} \qquad 2 \leqslant i \leqslant n \qquad (3\text{-}8)$$

The solution is

$$W_n = G_n$$

$$W_i = G_i - \frac{C_i W_{i+1}}{Z_i} \qquad 1 \leqslant i \leqslant n-1 \qquad (3\text{-}9)$$

G and Z are computed in order of increasing i, and W is computed in order of decreasing i.

After new values of W at time, $t + \Delta t$ have been calculated for the entire network in the x-direction, the procedure outlined above is repeated in the y-direction at time, $t + 2\ \Delta t$. This constitutes one double time step increasing t by $2\ \Delta t$. A new Δt can now be selected for use in the next double time step. The process is repeated over and over until a solution for the desired time is obtained.

CHAPTER IV

APPLICATIONS OF THE METHOD

The differential equations describing linearized unsteady-state gas, heat and single-phase fluid flow through porous media are very similar. With the proper modifications, Equation (3-1), can be used for each type of flow in two dimensions. Also it is possible to have a point source or sink at each point in the integration network by making a simple addition to Equation (3-1). For adaptation to reservoir work, the source would correspond to an injection well and the sink to a production well.

For heat flow, Equation (3-1) may be modified by making the substitutions $C = \frac{c_p \rho}{K}$ and $U = T$. This gives

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{c_p \rho}{K} \frac{\partial T}{\partial t} \tag{4-1}$$

After substituting finite differences for the derivatives in Equation (4-1) as was described in Chapter III, working equations similar to Equations (3-4) and (3-5) are obtained for each mesh point in the x- and y-direction.

For the x-direction

$$\begin{aligned} &- T(x-\Delta x,y,t+\Delta t) + (2+\alpha)\, T(x,y,t+\Delta t) \\ &- T(x+\Delta x,y,t+\Delta t) = T(x,y-\Delta y,t) + (\alpha-2) \\ &\quad T(x,y,t) + T(x,y+\Delta y,t) \end{aligned} \tag{4-2}$$

For the y-direction

$$- T(x,y-\Delta y,t+2\Delta t) + (2+\alpha)\, T(x,y,t+2\Delta t)$$
$$- T(x,y+\Delta y,t+2\Delta t) = T(x-\Delta x,y,t+\Delta t) + (\alpha -2) \qquad (4\text{-}3)$$
$$T(x,y,t+\Delta t) + T(x+\Delta x,y,t+\Delta t)$$

where $\alpha = \frac{c p \rho}{K} \frac{(\Delta x)^2}{\Delta t}$

It remains to form difference equations for the source or sink points. This can be achieved by assuming linear flow into a small finite block of unit thickness such as the one shown in Figure 1.

The flow of heat across the right hand face is approximated by the equation

$$q_1 = K(\Delta y \cdot 1) \frac{(T_o - T_1)}{\Delta x} \qquad (4\text{-}4)$$

Similar equations hold for the other sides of the square. For $\Delta x = \Delta y$, the total flow into the square would be

$$q = K(4T_0 - T_1 - T_2 - T_3 - T_4) \qquad (4\text{-}5)$$

By letting $Q = \frac{q}{K}$, the equation for a heat source at point x,y, which satisfies steady state conditions, would be

$$- T(x-\Delta x,y) - T(x+\Delta x,y) - T(x,y-\Delta y)$$
$$- T(x,y+\Delta y) + 4T(x,y) = Q \qquad (4\text{-}6)$$

Working Equations (4-2) and (4-3) are modified in order to converge to this solution. After modification, they are

For the x-direction

$$- T(x-\Delta x,y,t+\Delta t) + (2+\alpha)\, T(x,y,t+\Delta t)$$
$$- T(x+\Delta x,y,t+\Delta t) = Q + T(x,y-\Delta y,t) + (\alpha -2) \qquad (4\text{-}7)$$
$$T(x,y,t) + T(x,y+\Delta y,t)$$

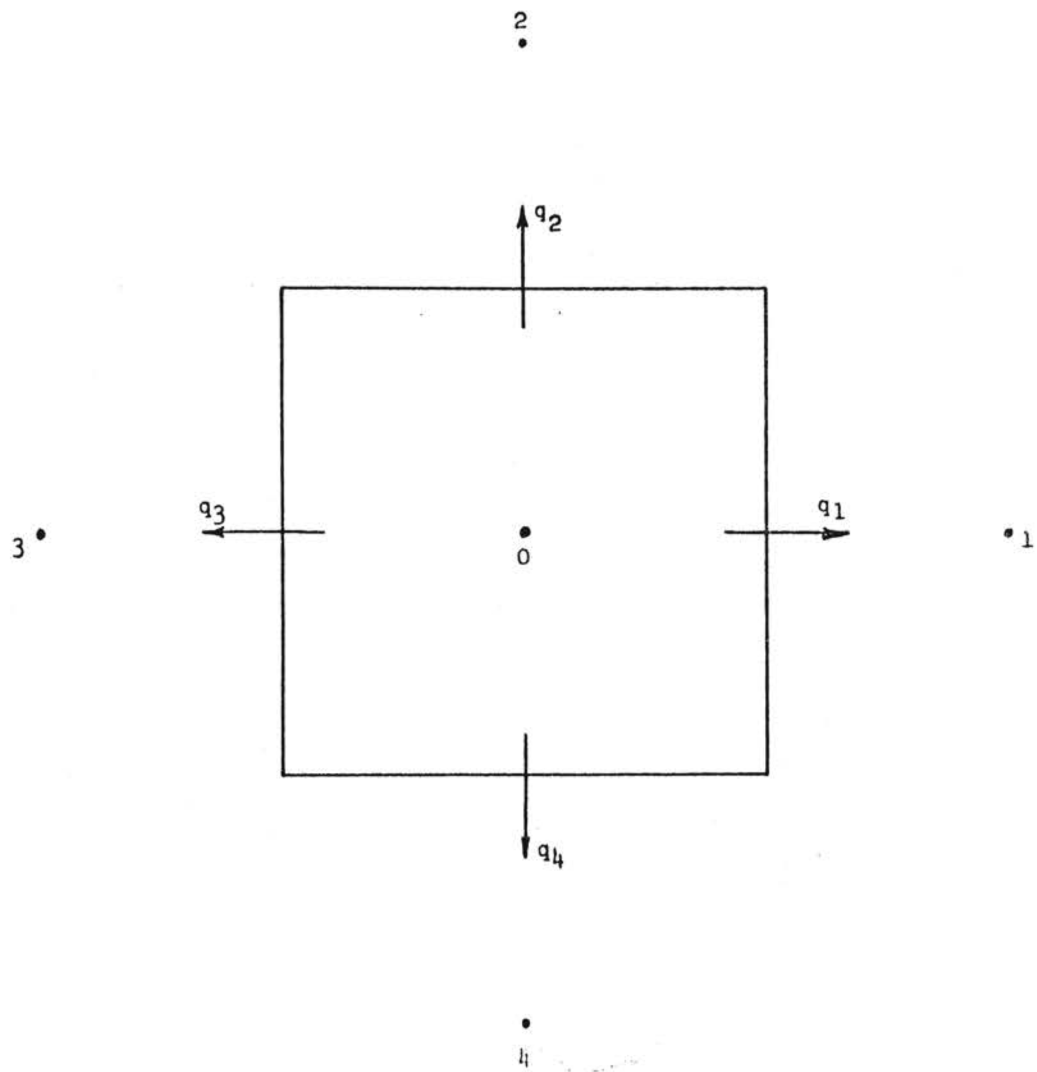


FIG. 1 SOURCE AT A POINT

For the y-direction

$$- T(x,y-\Delta y,t+2\Delta t) + (2+\alpha)\,T(x,y,t+2\Delta t) - T(x,y+\Delta y,t+2\Delta t) = Q + T(x-\Delta x,y,t+\Delta t) + (\alpha-2)\,T(x,y,t+\Delta t) + T(x+\Delta x,y,t+\Delta t) \qquad (4\text{-}8)$$

These equations were presented by Douglas and Peaceman (3) and may be used for both steady- and unsteady-state heat flow problems.

Equations for finding pressure distributions and flow patterns for compressible fluid flow may be formed by modifying Equation (3-1). If P is substituted for U and $\frac{\bar{\mu}\,\phi\,c}{k}$ for C, Equation (3-1) describes fluid flow in two dimensions. These substitutions give

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = \frac{\bar{\mu}\phi c}{k}\,\frac{\partial P}{\partial t} \qquad (4\text{-}9)$$

where

P = Pressure, atmospheres

x and y = Distance, centimeters

$\bar{\mu}$ = Average viscosity, centipoises

ϕ = Fractional porosity

c = Compressibility factor, $\frac{1}{\text{atmospheres}}$

k = Permeability, darcys

t = Time, seconds

Working equations are obtained for each mesh point in the manner described previously. The classical units in Equation (4-9) are converted to practical field units.

For the x-direction

$$- P(x-\Delta x,y,t+\Delta t) + (2+\alpha)\,P(x,y,t+\Delta t) - P(x+\Delta x,y,t+\Delta t) = P(x,y-\Delta y) + (\alpha-2)\,P(x,y,t) + P(x,y+\Delta y,t) \qquad (4\text{-}10)$$

For the y-direction

$$- P(x,y-\Delta y,t+2\Delta t) + (2+\alpha) P(x,y,t+2\Delta t) - P(x,y+\Delta y,t+2\Delta t) = P(x-\Delta x,y,t+\Delta t) + (\alpha-2) P(x,y,t+\Delta t) + P(x+\Delta x,y,t+\Delta t) \qquad (4\text{-}11)$$

where $\alpha = \frac{3793.5\,\bar{\mu}\phi c\,(\Delta x)^2}{k\ \Delta t}$, or in terms of dimensionless time, $\frac{1}{\Delta t_D}$.

The units are

P = Pressure, pounds per square inch

Δx = Distance increment, feet

Δt = Time increment, hours

c = Compressibility factor, $\frac{1}{\text{pounds per square inch}}$

ϕ = Fractional porosity

$\bar{\mu}$ = Viscosity, centipoises

k = Permeability, millidarcys

For a source point at the point x,y, equations similar to Equations (4-7) and (4-8) can be developed for fluid flow. These are

For the x-direction

$$- P(x+\Delta x,y,t+\Delta t) + (2+\alpha) P(x,y,t+\Delta t) - P(x+\Delta x,y,t+\Delta t) = Q + P(x,y-\Delta y,t) + (\alpha-2) P(x,y,t) + P(x,y+\Delta y,t) \qquad (4\text{-}12)$$

For the y-direction

$$- P(x,y-\Delta y,t+2\Delta t) + (2+\alpha) P(x,y,t+2\Delta t) - P(x,y+\Delta y,t+2\Delta t) = Q + P(x-\Delta x,y,t+\Delta t) + (\alpha-2) P(x,y,t+\Delta t) + P(x+\Delta x,y,t+\Delta t) \qquad (4\text{-}13)$$

The units are the same as in Equations (4-10) and (4-11) with Q equal to $\frac{158.5\,\bar{\mu}q}{hk}$.

Units for Q are

Q = Flow rate, pounds per square inch

$\bar{\mu}$ = Viscosity, centipoises

q = Flow rate, barrels per day

h = Formation thickness, feet

k = Permeability, millidarcys

Equation (3-1) can also be adapted to gas flow problems by letting $U = P^2$ and $C = \frac{\phi \bar{\mu}}{k\bar{P}}$. The equation then becomes

$$\frac{\partial^2 P^2}{\partial x^2} + \frac{\partial^2 P^2}{\partial y^2} = \frac{\phi \bar{\mu}}{k \bar{P}} \frac{\partial P^2}{\partial t} \tag{4-14}$$

which has the same classical units as Equation (4-9). The term, $\bar{P}$, is taken as the average reservoir pressure in atmospheres. This assumption makes the differential equation linear, thus simplifying the problem. Dr. Rachford (8) has suggested that such an assumption will cause considerable error in material balance calculations. He suggests that unsteady-state gas flow be handled in the manner set forth by Douglas, Peaceman and Rachford (4).

It was found, however, that the method which assumes an average reservoir pressure would check with existing analytical solutions in which the same assumption was made. This would not necessarily mean that the method was correct, but it would provide a means of determining the computer program's validity.

Difference equations are again formulated for each mesh point and the units are converted to practical field units. This gives

For the x-direction

$$- P^2(x-\Delta x,y,t+\Delta t) + (\alpha +2)\, P^2(x,y,t+\Delta t)$$

$$- P^2(x+\Delta x,y,t+\Delta t) = P^2(x,y-\Delta y,t) + (\alpha-2)\,P^2(x,y,t) + P^2(x,y+\Delta y,t) \tag{4-15}$$

For the y-direction

$$- P^2(x,y-\Delta y,t+2\Delta t) + (\alpha+2)\,P^2(x,y,t+2\Delta t) - P^2(x,y+\Delta y,t+2\Delta t) = P^2(x-\Delta x,y,t+\Delta t) + (\alpha-2)\,P^2(x,y,t+\Delta t) + P^2(x+\Delta x,y,t+\Delta t) \tag{4-16}$$

where $\alpha = \dfrac{3793.5\,\bar{\mu}\phi\,(\Delta x)^2}{k\,\bar{\bar{P}}\,(\Delta t)}$ which is the reciprocal of dimensionless time, Δt_D.

The units are

P^2 = Pressure^2, (pounds per square inch)^2

μ = Viscosity, centipoises

ϕ = Fractional porosity

Δx = Distance increment, feet

k = Permeability, millidarcys

$\bar{\bar{P}}$ = Average pressure, pounds per square inch

Δt = Time increment, hours

For a source at point x,y in gas flow problems, difference equations can be developed. They are

For the x-direction

$$- P^2(x-\Delta x,y,t+\Delta t) + (\alpha+2)\,P^2(x,y,t+\Delta t) - P^2(x+\Delta x,y,t+\Delta t) = Q + P^2(x,y-\Delta y,t) + (\alpha-2)\,P^2(x,y,t) + P^2(x,y+\Delta y,t) \tag{4-17}$$

For the y-direction

$$- P^2(x,y-\Delta y,t+2\Delta t) + (\alpha+2)\,P^2(x,y,t+2\Delta t) - P^2(x,y+\Delta y,t+2\Delta t) = Q + P^2(x-\Delta x,y,t+\Delta t) + (\alpha-2)\,P^2(x,y,t+\Delta t) + P^2(x+\Delta x,y,t+\Delta t) \tag{4-18}$$

The flow rate, Q equals $\frac{8{,}930\,\bar{\mu}\,\bar{z}\,T\,q}{hk}$

where

$\bar{\mu}$ = Average viscosity, centipoises

$\bar{z}$ = Average compressibility factor

T = Reservoir temperature, ^{o}R

q = Flow rate, MCF/day

h = Formation thickness, feet

k = Permeability, millidarcys

Other terms are defined after Equation (4-16).

At this point it should be noted that the Q developed in the foregoing discussion was for an interior network point. For a corner source point, only one-fourth of Q would be used for the flow rate in the difference equations, since it is bounded by only one-fourth of the region in question. Similarily for a side boundary point, one-half of Q would be used in the difference equations.

CHAPTER V

THE COMPUTER PROGRAM

The alternating-direction implicit method described in Chapter III was programed for the IBM 650 computer using 650 Fortran language. The program solves the general two dimensional unsteady-state equation in a rectilinear region having uniform properties. With the proper data selection, it will solve each of the applications described in Chapter IV.

Two boundary conditions were incorporated in the program. The first was, $U = U_o$, where U_o is the initial value of U. The other condition was $\frac{\partial U}{\partial L} = 0$, where L is a symbol for distance. The second condition imposes the restriction of no flow across a boundary line. The condition, $U = U_o$, may also be met at any point in the region by choosing the proper data.

A point source may be located at any point in the region if the particular point is not beign held constant. By data arrangement, flow rates from the source points may be changed at preselected times. This provides the program with the ability to handle multiple transient flow problems in irregular shaped finite regions. As many as eight different flow rates may be used in any one problem. A given flow rate can be used at any desired number of points. A positive rate is used for an injection point and a negative sign is used for a production point.

A 14 x 14 point grid, denoted by the doubly subscripted variable, W, comprises the region in which any figure, composed of straight lines in the vertical and horizontal direction, may be imposed. The boundary of the figure must be approximated by the grid lines. The conventional positive x-direction is denoted in the program by an increasing J value. The conventional negative y-direction is indicated by an increasing I value. This is shown in Figure 3.

Another 14 x 14 array, denoted by KCON, serves as a control for each of the points in W. KCON values are subscripted by I and J, as were the W values.

The general plan of the program is to begin with W values at an initial time and calculate values of W after some increment of time. This calculation is made in the J direction by following the mathematical method described in Chapter III. After this calculation has been completed, the time increment is increased and the calculations are made again in the I direction. The procedure is repeated until the desired time has been reached.

A generalized block diagram showing all important steps in the program is given in Figure 2. Many details were omitted for brevity. The complete 650 Fortran program is listed in Table I.

The DIMENSION statement (600) reserves 527 locations in memory for the variable array, W, the fixed control array, KCON, and an eight place array for flow rate (Q) values. Space is also reserved for all other subscripted variables used in the program.

Statement (601) reads the maximum number of calculation points in the I direction (M) and the J direction (N). The working variable array,

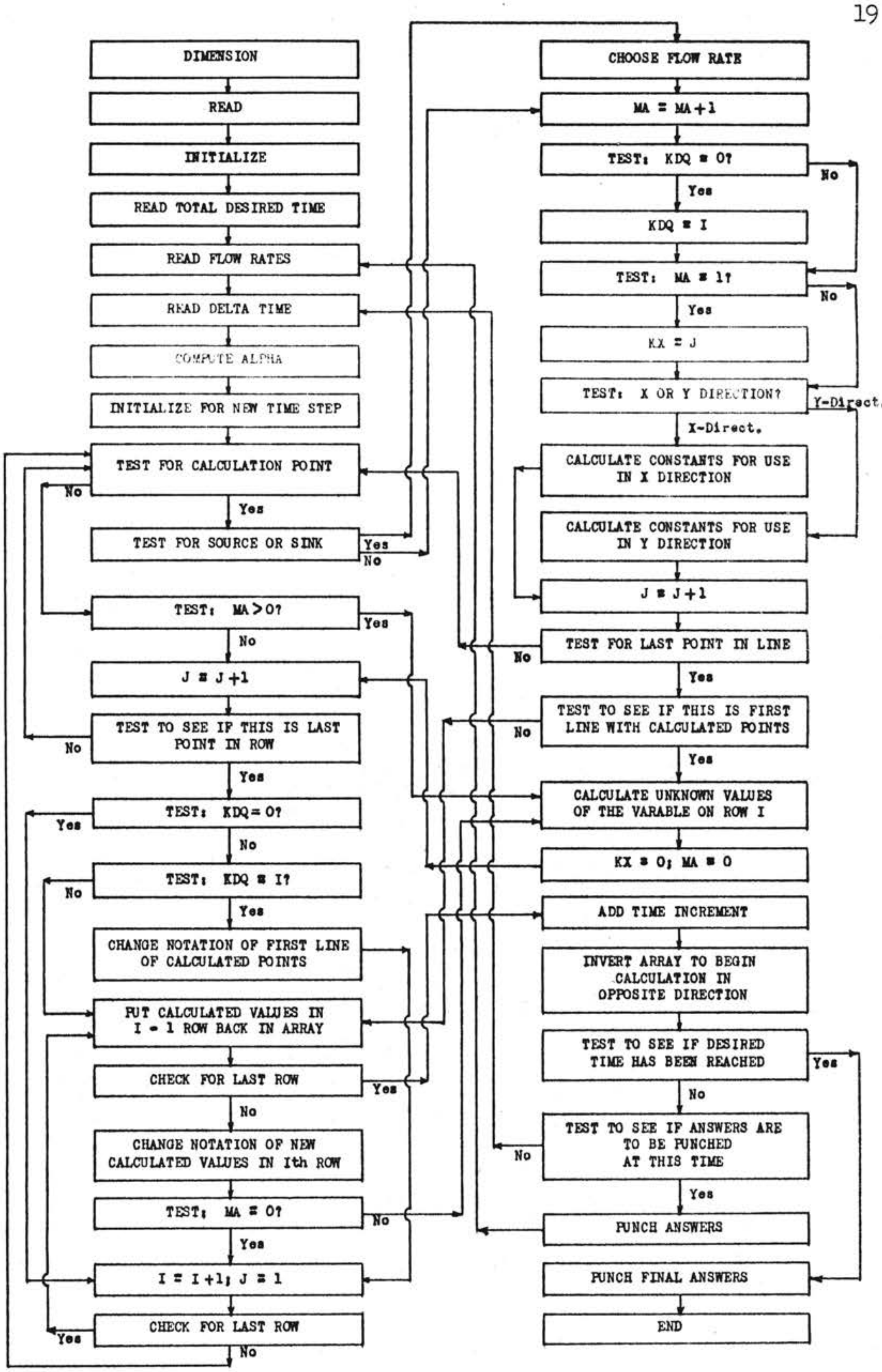


FIG. 2 BLOCK DIAGRAM FOR COMPUTER PROGRAM

TABLE I

THE 650 FORTRAN PROGRAM

```
600 0 DIMENSIONW(14,14),KCON(14,14),
600 1 A(14),B(14),D(15),C(14),Z(14),
600 2 G(14),R(14),KP(14),Q(8),R2(14)
601 0 READ,M,N
602 0 READ,W
603 0 READ,KCON
607 0 READ,MAX
650 0 READ,POR,VIS,DELX,COE,COM,PERM
655 0 READ,TTIME
654 0 TIME=0.0
200 0 KDIR=1
  3 0 KTHEA=1
651 0 READ,TMAX
996 0 TTIMX=0.0
604 0 READ,Q
652 0 READ,DELT
653 0 ALPHA=(COE*POR*COM*VIS*DELX*DE
653 1 LX)/(DELT*PERM)
  4 0 I=1
  5 0 J=1
202 0 KDQ=0
  6 0 MA=0
  7 0 D(J)=0.0
  8 0 IF(KCON(I,J)-100)29,210,23
210 0 IF(MA)9,9,95
  9 0 J=J+1
 10 0 IF(J-(N+1))7,212,212
212 0 IF(KDQ)11,11,213
213 0 IF(KDQ-I)218,214,214
214 0 DO217K=1,N
215 0 IF(KCON(I,K)-100)216,217,216
216 0 R2(K)=R(K)
217 0 CONTINUE
230 0 GOTO11
218 0 DO221K=1,N
219 0 IF(KCON(I-1,K)-100)220,221,220
220 0 W(I-1,K)=R2(K)
221 0 CONTINUE
222 0 IF(I-(M+1))223,115,115
223 0 DO226K=1,N
224 0 IF(KCON(I,K)-100)225,226,225
225 0 R2(K)=R(K)
226 0 CONTINUE
227 0 IF(MA)11,11,95
 11 0 I=I+1
 12 0 J=1
 13 0 IF(I-(M+1))7,218,218
 23 0 DO28K=1,8
 24 0 IF(KCON(I,J)-(11-K)*100)28,28,
 24 1 701
701 0 MA=MA+1
702 0 D(MA)=0.0
 25 0 D(MA)=D(MA)+Q(K)
 26 0 ISUB=KCON(I,J)-(11-K)*100
 27 0 GOTO500
 28 0 CONTINUE
 29 0 ISUB=KCON(I,J)
 30 0 MA=MA+1
499 0 D(MA)=0.0
500 0 IF(KDQ)420,501,420
501 0 KDQ=1
420 0 IF(MA-1)421,421,31
421 0 KX=J
 31 0 GOTO(32,33),KDIR
 32 0 GOTO(56,34,37,45,48,63,40,66,5
 32 1 1,70,56,70,73,56,73,70,73,63,6
 32 2 3,66,66,83,85,76,80),ISUB
 33 0 GOTO(56,34,45,37,48,40,63,51,6
 33 1 6,56,70,70,56,73,73,73,70,76,8
 33 2 0,83,85,66,66,63,63),ISUB
 34 0 C(MA)=-2.0
 36 0 GOTO43
 37 0 A(MA)=-2.0
 39 0 GOTO43
 40 0 A(MA)=-1.0
 41 0 C(MA)=-1.0
 43 0 D(MA)=D(MA)+2.0*W(I+1,J)
 44 0 GOTO90
 45 0 C(MA)=-2.0
 47 0 GOTO54
 48 0 A(MA)=-2.0
 50 0 GOTO54
 51 0 C(MA)=-1.0
 52 0 A(MA)=-1.0
 54 0 D(MA)=D(MA)+2.0*W(I-1,J)
 55 0 GOTO90
 56 0 A(MA)=-1.0
 57 0 C(MA)=-1.0
 59 0 GOTO68
 63 0 C(MA)=-2.0
 65 0 GOTO68
 66 0 A(MA)=-2.0
 68 0 D(MA)=D(MA)+W(I+1,J)+W(I-1,J)
 69 0 GOTO90
 70 0 A(MA)=-1.0
 71 0 D(MA)=D(MA)+W(I,J+1)
 72 0 GOTO68
 73 0 C(MA)=-1.0
 74 0 D(MA)=D(MA)+W(I,J-1)
 75 0 GOTO68
 76 0 A(MA)=-1.0
 77 0 D(MA)=D(MA)+W(I,J+1)
 78 0 GOTO43
 80 0 D(MA)=D(MA)+W(I,J-1)
 81 0 GOTO41
 83 0 D(MA)=D(MA)+W(I,J+1)
 84 0 GOTO52
 85 0 C(MA)=-1.0
 86 0 D(MA)=D(MA)+W(I,J-1)
 87 0 GOTO54
 90 0 B(MA)=ALPHA+2.0
 91 0 D(MA)=D(MA)+(ALPHA-2.0)*W(I,J)
 92 0 J=J+1
 93 0 IF(J-(N+1))7,440,440
440 0 IF(KDQ-(I-1))218,218,95
 95 0 Z(1)=B(1)
 96 0 DO98K=2,MA
 97 0 KM=K-1
 98 0 Z(K)=B(K)-(A(K)*C(KM))/Z(KM)
 99 0 G(1)=D(1)/Z(1)
100 0 DO102K=2,MA
101 0 KM=K-1
102 0 G(K)=(D(K)-A(K)*G(KM))/Z(K)
430 0 NA=KX+MA-1
103 0 R(NA)=G(MA)
104 0 MB=MA-1
105 0 DO107K=1,MB
106 0 JQ=MA-K
431 0 NQ=KX+JQ-1
107 0 R(NQ)=G(JQ)-(C(JQ)*R(NQ+1))/Z(
107 1 JQ)
402 0 KX=0
108 0 MA=0
109 0 GOTO9
115 0 KTHEA=KTHEA+1
799 0 TIME=TIME+DELT
995 0 TTIMX=TTIMX+DELT
800 0 MZ=N
801 0 N=M
802 0 M=MZ
116 0 GOTO(117,119),KDIR
117 0 KDIR=2
118 0 GOTO120
119 0 KDIR=1
120 0 DO129KI=1,MAX
122 0 DO128K=KI,MAX
123 0 R(K)=W(KI,K)
124 0 KP(K)=KCON(KI,K)
125 0 W(KI,K)=W(K,KI)
126 0 KCON(KI,K)=KCON(K,KI)
127 0 W(K,KI)=R(K)
128 0 KCON(K,KI)=KP(K)
129 0 CONTINUE
449 0 IF(TIME-TTIME)550,452,452
550 0 GOTO(450,4),KDIR
450 0 IF(TTIMX-TMAX)652,998,998
998 0 PUNCH,W
990 0 PUNCH,ALPHA,TTIME,TTIMX,TMAX,D
990 1 ELT,KTHEA
997 0 GOTO651
452 0 GOTO(132,116),KDIR
132 0 PUNCH,W
133 0 PUNCH,KCON
135 0 PUNCH,KTHEA,TTIME
138 0 GOTO601
136 0 END
```

W, is read in memory by statement (602). The control array, KCON, is read into memory by statement (603). Selection of KCON control values will be discussed later. Statement (607) reads the maximum number of points (MAX) in either the I or J direction. This number equals the larger of the two numbers M and N. Statement (650) reads the different variables involved in calculating the ALPHA value discussed in Chapter IV. For oil and gas field applications, POR is the symbol for porosity, VIS for viscosity, DELX for incremental distance, COE for conversion factor, COM for compressibility factor and PERM for permeability. Units and values for the various properties and factors are discussed in Chapter IV.

The total time (TTIME) desired is read by statement (655). Statements (654) through (3) initialize the following variables to begin the first time step: TIME is a variable which represents cumulative time in the program. KDIR denotes the direction in which the calculations are proceding. For KDIR equal to one, the calculations are in the J direction. For KDIR equal to two, the calculations are in the I direction. KTHEA represents the number of times the calculations have been made across the array plus one.

The total time, TTIME, may be sub-divided into parts. Each of these parts are denoted by TMAX in the program. Statement (651) reads the value of TMAX into memory. At the end of each of these sub-times, the flow rates at each of the source points may be changed. Statement (996) initializes the cumulative sub-time variable TTIMX to begin a new calculation. Flow rates for existing point sources are read by statement (604).

DELT, the time increment, is read by statement (652). This time increment is used for one double time step in the calculations, once in the J direction and once in the I direction. For the program to work properly, DELT values must be arranged so that twice the summation of the time increments in any sub-time increment equals the sub-time total read for TMAX.

Statement (653) calculates ALPHA. Statements (4) through (7) initialize the variables to start the calculations. The calculations are started in the upper left corner of the W array. Each point in the first row of the array is inspected to determine if it is a valid calculation point by statement (8). Also determined is whether the point is a source point. If the point is not a calculation point, the J value of the point is increased by statement (9) and the next point is inspected. If the point is a valid calculation point but not a source point, the calculations are routed through statements (29) to (92). These statements decide what type of point has been encountered and calculates the proper coefficients for the point. These coefficients are described in Chapter III. The J value of the point is then increased and the next point in the row is investigated. If the point had been a source point, the calculations would have gone through statements (23) to (92) which makes the additions suggested by Equations (4-7), (4-12) and (4-17).

When coefficients for the existing calculation points are determined, the unknown values at each of the points are solved for by statements (95) to (107). This is done in the manner outlined in Chapter III.

When all points in the first row have been examined and all unknowns are found, the I value is increased and the above procedure is repeated for the next row. After the entire array has been covered, the time variables are appropriately increased, and the second part of the double time step can begin.

In order to use the same equations for the next time step in the I direction as were used in the J direction, both the W and KCON arrays were inverted about the dotted diagonal shown in Figure 3. This was accomplished by statements (120) to (129).

Statement (449) checks to see if the desired time (TTIME) has been reached. If it has, W and KCON are punched by statements (132) and (133). Statement (135) punches KTHEA and TTIME. Control is then referred back to statement (601) to begin an entirely new problem.

If the desired time has not been reached, calculations for the second part of the double time step begin in the I direction.

When the complete double time step has been finished, statement (449) again checks to see if the desired time has been reached. If it has, the answers described previously are punched. If the desired time has not been reached, statement (450) checks to see if the desired value for sub-time, TMAX, has been reached. If it has not, a new time increment, DELT, is read by statement (652) and calculations begin on the next double time step.

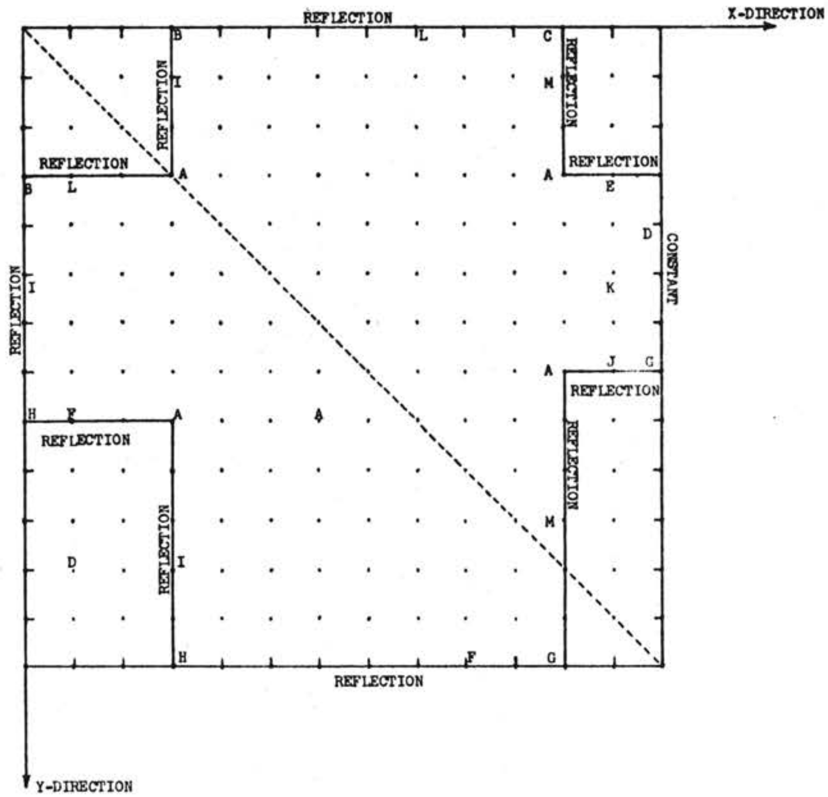
If TMAX has been reached, statement (998) punches the W array and statement (990) punches ALPHA, TTIME, TTIMX, TMAX, DELT and KTHEA. TTIMX should be equal to TMAX at this time.

Statement (997) sends the program to statement (651) which reads

TABLE II

POINT CONTROLS

KCON Value	POINT CONDITION
1	Point is a common point in both X- and Y-direction.
2	Point is an outer upper left side corner point and is a reflection point in both X- and Y-direction.
3	Point is an outer upper right side corner point and is a reflection point in both X- and Y-direction.
4	Point is an outer lower left side corner point and is a reflection point in both X- and Y-direction.
5	Point is an outer lower right side corner point and is a reflection point in both X- and Y-direction.
6	Point is a reflection point on the left side in the X-direction and a reflection point on the top line in the Y-direction.
7	Point is a reflection point on the top line in the X-direction and a reflection point on the left side in the Y-direction.
8	Point is a reflection point on the right side in the X-direction and bottom reflection point in the Y-direction.
9	Point is a reflection point on the bottom in the X-direction and right side reflection point in the Y-direction.
10	Point is preceded by a constant in the X-direction and not in Y-direction.
11	Point is preceded by a constant in the Y-direction and not in the X-direction.
12	Point is preceded by a constant in both directions.
13	Point is followed by a constant in the X-direction and not in the Y-direction.
14	Point is followed by a constant in the Y-direction and not in the X-direction.
15	Point is followed by a constant in both the X- and Y-direction.
16	Point is preceded by a constant in the X-direction and followed by a constant in the Y-direction.
17	Point is preceded by a constant in the Y-direction and followed by a constant in the X-direction.
18	Point is a reflection point on the left side in the X-direction and a reflection point on the top line in the Y-direction. The point is preceded by a constant in the Y-direction and not in the X-direction.
19	Point is a reflection point on the left side in the X-direction and a reflection point on the top line in the Y-direction. The point is followed by a constant in the Y-direction and not in the X-direction.
20	Point is a reflection point on the right side in the X-direction and bottom reflection point in the Y-direction. The point is preceded by a constant in the Y-direction and not the X-direction.
21	Point is a reflection point on the right side in the X-direction and bottom reflection point in the Y-direction. Point is followed by a constant in the Y-direction and not in the X-direction.
22	Point is a reflection point on the bottom in the X-direction and right side reflection point in the Y-direction. Point is preceded by a constant in the X-direction and not in the Y-direction.
23	Point is a reflection point on the bottom in the X-direction and right side reflection point in the Y-direction. The point is followed by a constant in the X-direction and not in the Y-direction.
24	Point is a reflection point on the top line in the X-direction and a reflection point on the left side in the Y-direction. Point is preceded by a constant in the X-direction and not in the Y-direction.
25	Point is a reflection point on the top line in the X-direction and a reflection point on the left side in the Y-direction. The point is followed by a constant in the X-direction and not in the Y-direction.
100	Point is a constant or does not enter into the calculations.



Point	KCON Value*
A	1
B	2
C	3
D	100
E	24
F	9
G	5
H	4
I	6
J	22
K	10
L	7
M	8

* Values obtained from TABLE II

FIG. 3 ILLUSTRATION OF POINT CONDITIONS

a new value of TMAX. Statement (604) reads new values of flow rates, and statement (652) reads a new value of DELT to begin calculations on a new double time step.

It is advantageous when handling a configuration which doesn't require the full 14 x 14 array to place the configuration in the upper left corner of the array. The maximum number of points used in either the J or I direction should be determined and tabulated as MAX. This will speed the calculations since the entire 14 x 14 array will not have to be scanned.

Data input for the program is made up by first determining the maximum number of points used in any column in the I direction. This number is tabulated as M. Then the maximum number of points used in any row in the J direction is tabulated as N. M and N are punched on the same data card in fixed point form.

Initial values for the variable array, W, are punched into data cards in floating point form. The first two data cards contain W values for the extreme left column of the array. The second two data cards contain W values for the second column of the array, etc.

KCON values are determined by looking at each point in the W array and assigning a KCON number to it. These numbers along with the corresponding point conditions are tabulated in Table II.

A KCON number of 100 was assigned to constant points and other points that do not enter into the calculations. The boundary condition, $\frac{\partial U}{\partial L} = 0$, leads to the situation involving fictitious reflection points along the boundary in question. This and other conditions are illustrated in Figure 3.

For source points, KCON numbers are modified by adding a constant to the number determined from Table II. If the first value in the Q array is to be used as a flow rate, 1,000 is added to the KCON number. This is decreased by 100 for each succeeding Q value. For example, to use the eighth Q value, 300 would be added to the KCON value.

Values for KCON are read in memory in fixed point form in the manner previously described for W values.

On the next data card, the value for MAX is punched in fixed point form. This is followed by a card punched with floating point values for the properties and factors: POR, VIS, DELX, COE, COM, and PERM.

Next, the total time (TTIME) desired is punched in floating point form in the next data card. This is followed by a card punched with the value for the first sub-time total, TMAX. Two cards with Q values follow. The first card contains seven flow rates while the second contains one.

Following this, comes cards with time increments, DELT. Twice the sum of the time increments should equal the value read in for TMAX. This is followed by cycles of TMAX, Q and DELT until the total desired time is reached.

When each sub-total time, TMAX, is reached, values for W at that cumulative time level will be punched. These W values will have the same arrangement as the original data input. Values for ALPHA, TTIME, TTIMX, TMAX, DELTA, and KTHEA are also punched.

When the total desired time TTIME has been reached, the W array will be punched followed by KCON array, KTHEA and TTIME.

CHAPTER VI

APPLICATION OF THE COMPUTER PROGRAM

The following unsteady-state gas flow problem will illustrate the use and also check the validity of the computer program.

A one-mile square portion of a natural gas reservoir exists at an initial uniform pressure of 480 psia. The reservoir is characterized by the following physical properties:

Permeability, k	=	20 millidarcys
Porosity, ϕ	=	0.10
Viscosity, $\bar{\mu}$	=	0.012 centipoises
Average pressure, $\bar{P}$	=	400 psia
Formation thickness, h	=	60 feet
Compressibility factor, $\bar{z}$	=	0.95
Temperature, T	=	550 ^{o}R

Producing wells have been drilled at each of the corners of the square. Each well is produced at the rate of 400 MCF per day for 20,000 hours.

It is desired to calculate a pressure drawdown curve for a point at the center of the square reservoir during this flow period.

For the computer solution, the problem was reduced by symmetry to that of only one well at the upper left corner of the square producing at four times the rate previously described for the four wells, or 16,000 MCF per day.

The upper left quarter of the square is covered by a 14 x 14 net corresponding to the W array described in Chapter V. The lower right corner of the array becomes the point for which the drawdown is to be found.

KCON values are selected for each point in the W array from Table II.

The flow rate, $\bar{Q}$, at the upper left point of the array is determined in the manner described in Chapter IV. The point in question is a corner point; therefore,

$$\bar{Q} = \frac{Q}{4} = \frac{8{,}930\,\bar{\mu}\,\bar{z}\,T\,q}{4\,h\,k} \tag{6-1}$$

$$\bar{Q} = \frac{8{,}930\,(0.012)\,(0.95)\,(550)\,(400)}{4\,(60)\,(20)} = 18{,}664 \text{ psia}^2$$

The other variables involved in the calculations are determined and are punched in data cards. Data format is shown in Table III.

A time increment of 10 hours was used to start the problem. This was increased by a factor of approximately 1.2 for each succeeding double time step.

Answers for the problem were punched at pre-determined times. One set of answers is given in Table IV for 190 hours of flow.

To check the computer results, Horner's (6) point source solution for an infinite reservoir was used to calculate a similar drawdown at the point in question.

TABLE III

DATA FORMAT

```
M            N
        14+          14+
W ARRAY      INITIAL      VALUES
2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+
2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+
2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+
2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+
2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+
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2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+ 2304000056+

  KCON       CONTROL      ARRAY
      1002+          6+          6+          6+          6+          6+          6+
        6+          6+          6+          6+          6+          6+          4+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        7+          1+          1+          1+          1+          1+          1+
        1+          1+          1+          1+          1+          1+          9+
        3+          8+          8+          8+          8+          8+          8+
        8+          8+          8+          8+          8+          8+          5+
    MAX
       14+
POR          VIS          DELX         COE          COM          PERM
1000000050+ 1200000049+ 2030800053+ 3793500054+ 2500000048+ 2000000052+
TTIME
2000000055+
TMAX
1980000053+
Q    FLOW    RATES
1866400055+         51+         51+         51+         51+         51+         51+
        51+
DELT
5000000051+
6000000051+
7000000051+
8000000051+
1000000052+
1200000052+
1400000052+
1700000052+
2000000052+
```

TABLE IV

ANSWER FORMAT

W ARRAY

2212297856+	2258600656+	2278506656+	2288981656+	2295102756+	2298808756+	2301047856+
2302372356+	2303131956+	2303551856+	2303775056+	2303887756+	2303939156+	2303953856+
2258600556+	2270935756+	2282433156+	2290479456+	2295759456+	2299121756+	2301202956+
2302449856+	2303170256+	2303570656+	2303784056+	2303892056+	2303941456+	2303955456+
2278506756+	2282433256+	2288279256+	2293425356+	2297265656+	2299900456+	2301605856+
2302656456+	2303274556+	2303622156+	2303809056+	2303904256+	2303947856+	2303960356+
2288981656+	2290479456+	2293425256+	2296472056+	2299000456+	2300862556+	2302126856+
2302931656+	2303416256+	2303693256+	2303843956+	2303921356+	2303957056+	2303967256+
2295102556+	2295759556+	2297265556+	2299000556+	2300561056+	2301780856+	2302645956+
2303214656+	2303565256+	2303769156+	2303881656+	2303939956+	2303966956+	2303974756+
2298808656+	2299121856+	2299900256+	2300862556+	2301780856+	2302533556+	2303087556+
2303462256+	2303698556+	2303838356+	2303916456+	2303957356+	2303976456+	2303981956+
2301047856+	2301202956+	2301605556+	2302126756+	2302646056+	2303087456+	2303422556+
2303654856+	2303804156+	2303893956+	2303944656+	2303971556+	2303984256+	2303987856+
2302372356+	2302449956+	2302656356+	2302931756+	2303214756+	2303462356+	2303654956+
2303791156+	2303880456+	2303934756+	2303965756+	2303982256+	2303990156+	2303992456+
2303131756+	2303170356+	2303274356+	2303416156+	2303565256+	2303698456+	2303804156+
2303880256+	2303930856+	2303961956+	2303979856+	2303989556+	2303994156+	2303995456+
2303551756+	2303570656+	2303622056+	2303693156+	2303769256+	2303838356+	2303894056+
2303934656+	2303961956+	2303979056+	2303988856+	2303994256+	2303996756+	2303997556+
2303774956+	2303784056+	2303808956+	2303843756+	2303881556+	2303916456+	2303944756+
2303965656+	2303979956+	2303988856+	2303994056+	2303996856+	2303998256+	2303998656+
2303887856+	2303892256+	2303904256+	2303921256+	2303940056+	2303957356+	2303971656+
2303982356+	2303989656+	2303994156+	2303996856+	2303998356+	2303999056+	2303999256+
2303939056+	2303941456+	2303947856+	2303956856+	2303966956+	2303976356+	2303984156+
2303990056+	2303994156+	2303996756+	2303998256+	2303999056+	2303999556+	2303999656+
2303953956+	2303955756+	2303960356+	2303967256+	2303974856+	2303981956+	2303987956+
2303992456+	2303995556+	2303997456+	2303998656+	2303999356+	2303999656+	2303999756+

ALPHA	TTIME	TTIMX	TMAX	DELT	KTHEA
1173371851+	2000000055+	1980000053+	1980000053+	2000000052+	19+

The equation used was

$$P^2 = P_o^2 + \sum_{i=1}^{\infty} \frac{1424 \bar{\mu} \bar{z} T q}{2hk} Ei\left(\frac{-\bar{\mu} \phi r_i^2}{4(2.634 \times 10^{-4}) k \bar{P}}\right) \quad (6\text{-}2)$$

where

P = Drawdown pressure at the point in question, pounds per square inch

P_o = Initial pressure, pounds per square inch

$\bar{\mu}$ = Viscosity, centipoises

$\bar{z}$ = Compressibility factor

T = Temperature, °R

q = Flow rate, MCF/Day

h = Formation thickness, feet

k = Permeability, millidarcy's

ϕ = Fractional porosity

r = Distance from the producing well to the point in question, feet

$\bar{P}$ = Average reservoir pressure, pounds per square inch

Ei = Symbol for exponential integral

To obtain a pressure drawdown comparable to the one calculated by the numerical solution for a finite reservoir, Equation (6-2) would have to be evaluated for an infinite number of wells spaced around the desired point in the manner used for the finite case. This would result in an infinite array of producing wells spaced at one-mile intervals.

For an approximation, thirty-six producing wells were included in

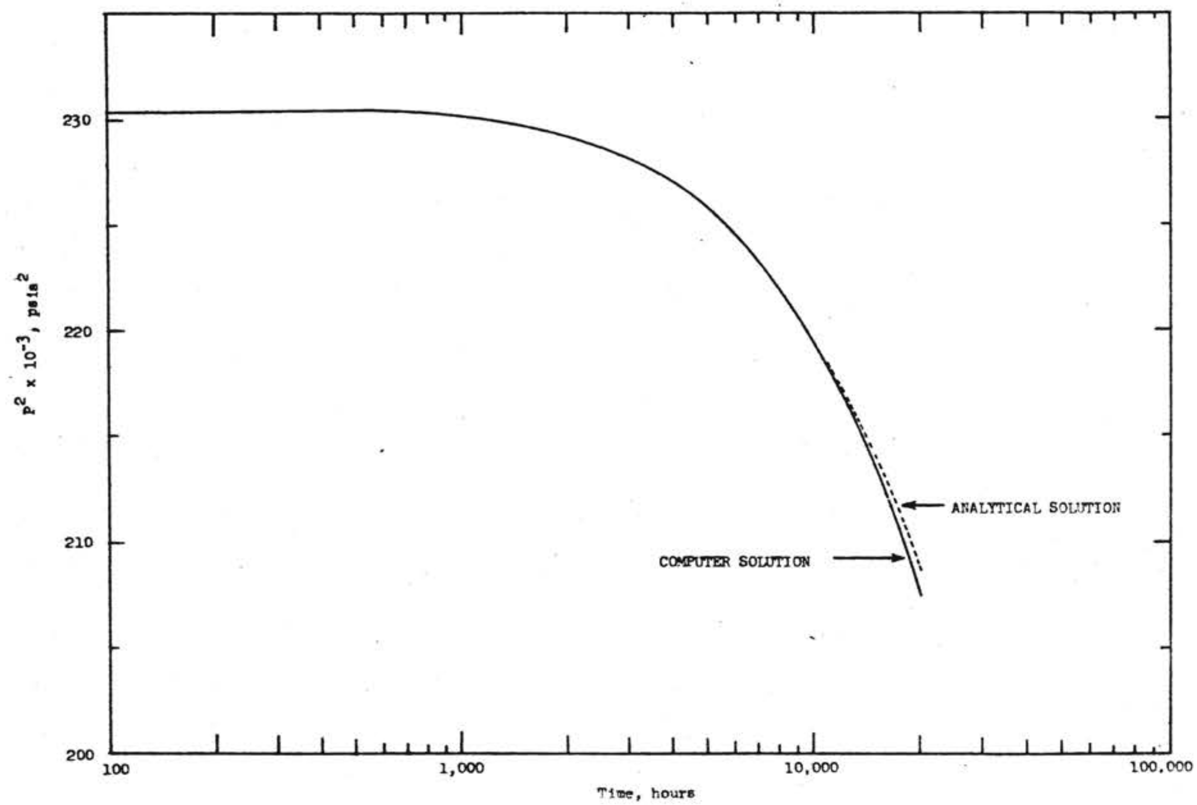

FIG. 4 RESULTS OF CALCULATIONS

the summation indicated in Equation (6-2). Figure 4 shows the comparison between the two methods of calculation. Until approximately 10,000 hours of production was reached, the two methods produced the same drawdown at the central point. After 10,000 hours, some difference appeared in the two methods with the analytical method giving less drawdown than the numerical solution. This was caused by including only a finite number of wells when using Equation (6-2). It was felt that by including a sufficient number of wells in Equation (6-2), the two solutions would be identical for all practical purposes.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The purpose of this study was to provide a working IBM 650 computer program that would calculate potential and flow distributions for unsteady-state heat, compressible fluid and linearized gas flow problems in two dimensions.

By using the alternating-direction implicit numerical method, a program for solving these types of unsteady problems was obtained. The program was checked by comparing the numerical solution with a known analytical solution for a pressure drawdown at the center point of a square gas reservoir having uniform properties. The two methods compared favorable as was shown in Figure 4.

Two boundary conditions were incorporated in the program; that of a constant boundary value and that of no flow across the boundary. Point sources or sinks may be located at any point in the region if the point is not being held constant. The program will handle multiple transient flow problems in irregular shaped finite regions.

The unsteady-state gas flow problem was solved by making the differential equation describing the flow, linear. This was accomplished by assuming an average pressure for the entire reservoir. This assumption would perhaps cause appreciable error in the calculations. It is felt, however, that the solution that was presented is as accurate as existing analytical solutions in which the same average pressure assumption is made. This was substantiated by the calculations presented in Chapter VI.

CHAPTER VIII

RECOMMENDATIONS FOR FUTURE STUDY

The computer program presented in this study could be used to investigate many interesting problems encountered in compressible fluid and gas flow in uniform reservoirs.

One such problem would be that of determining if existing spacing of producing wells is adequate to sufficiently drain the reservoir in a reasonable length of time. The effects of infill drilling could also be determined.

Potential and flow distributions due to producing at unequal rates at a number of wells in a finite reservoir could be obtained.

The effect that a neighboring producing well would have on a well's build-up curve could be studied.

It might be possible to trace an entire field's production history in order to determine desired unknown reservoir parameters.

Dr. Rachford (8) has suggested an improvement in the programing technique presented in this study. He proposes that a basic region be defined and at every point in the region, values for permeability in both the x- and the y-direction be read as variables. This would make it possible to treat cases in which non-uniformity occurs, such as, variable permeability in the x- and y-directions.

For reflection boundaries, he suggests that the permeability be set equal to zero instead of resorting to fictitious reflection points as was done in this study. Although this would increase the computer time required, since calculations would have to be made over the entire basic region, it is said to eliminate certain unstable conditions sometimes encountered when reflection points are used.

The program for non-uniform conditions could be used to study the many effects of non-uniform conditions which certainly exist in the actual reservoir. In particular, potential and flow distributions might be obtained for horizontally fractured systmes.

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VITA

Dean Monroe DeMoss

Candidate for the Degree of

Master of Science

Thesis: A COMPUTER PROGRAM FOR SOLVING TWO-DIMENSIONAL UNSTEADY-STATE FLOW PROBLEMS BY THE ALTERNATING-DIRECTION IMPLICIT METHOD

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born at Enid, Oklahoma, June 16, 1937, the son of Monroe and Ida DeMoss.

Education: Attended grade school in Southard, Oklahoma; graduated from Southard High School, Southard, Oklahoma in 1955; received the Bachelor of Science Degree from Oklahoma State University in January, 1960; completed the requirements for the Master of Science Degree in August, 1961.

Experience: Employed by the U. S. Gypsum Company in Southard, Oklahoma in the summer of 1955 as a laborer; employed by Cessna Aircraft Company at Wichita, Kansas for the summer of 1956 as a laborer; employed by Mobil Oil Company in Healdton, Oklahoma during the summers of 1957, 1958 and 1959 as a roustabout; employed by Pan American Petroleum Corporation at Liberal, Kansas in the summer of 1960 as a Jr. Petroleum Engineer; employed by Mobil Oil Company in Dallas, Texas as a laboratory technician.