DESIGN OF A NARROW-BAND FREQUENCY MODULATION DISCRIMINATOR

By

DAVID ALLEN CORYELL

Bachelor of Science

Oklahoma State University

Stillwater, Oklahoma

1954

Submitted to the faculty of the Graduate School
of the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
August, 1961

DESIGN OF A NARROW-BAND FREQUENCY
MODULATION DISCRIMINATOR

Thesis Approved:

H. D. Crawford
Thesis Adviser

C. M. Summers

Calvin Marlin
Dean of the Graduate School

PREFACE

A version of the circuit described was suggested to the author's thesis adviser by Mr. R. T. Bruss of Minneapolis, Minnesota. Mr. Bruss and Dr. Crawford subsequently built a discriminator using only crude design procedures to determine its efficacy in solving their problems. They found that, even though the circuit operated, a much more detailed treatment was indicated. It is the purpose of this thesis to carry out this analysis in greater detail, and to describe a discriminator which fulfills the requirements of the original application.

The author wishes to thank Drs. Harry D. Crawford and William L. Hughes for their advice and encouragement in the preparation of the thesis study. Indebtedness is also acknowledged to the Douglas Aircraft Company, Incorporated, Santa Monica, California, through whose Employee Scholarship Plan the author was able to undertake graduate study.

Thanks are also due my wife, Pat, and the children, who have put up with the author through the throes of thesis writing. I am humbly grateful for their continued support and encouragement.

The typing of this thesis by Miss Velda D. Davis is also recognized.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Modulation is defined in the 1938 Standards Report of the Institute of Radio Engineers as "the process of producing a wave some characteristic of which varies as a function of the instantaneous value of another wave, called the modulating wave." (1). Three of the characteristics of a sinusoidal wave which may be so varied are the amplitude, the frequency, and the phase; each of which lends its name to a type of modulation.

Demodulation, or detection, is the process employed to recover the original information from a modulated wave. The complexity of demodulator circuits varies with the type of modulation and with the accuracy desired in the reproduction of the modulating wave. In general, amplitude modulation detectors are relatively simple, followed in order of complexity by frequency and phase modulation detectors.

Detection of frequency modulated signals is accomplished by one of two basic methods. In the first, the signal is applied to a network the purpose of which is to convert the variations in frequency into variations in amplitude. The resulting amplitude modulated wave is then demodulated, usually by a diode detector. (2).

In the second method, variations in the period of the modulated wave are compared cycle-by-cyle with a reference, often the mean frequency of the input. The result of this comparison is an output voltage proportional to the period of the input, and thus proportional to the frequency of the input, which is equivalent to the modulating wave.

The detector to be described is of the first type. The distorting network used simulates the response of a pair of RLC tuned circuits by means of feedback amplifiers with twin-T networks in the feedback path. Included in the investigation is an analysis of the linearity of such demodulators with respect to the input frequency and the design of a linear demodulator for use with a carrier frequency in the audio range.

CHAPTER II

HISTORICAL BACKGROUND

The use of frequency modulation was suggested early in the history of radio communication. It was hoped that the use of frequency modulation would permit the transmission of signals over channels of narrower bandwidth than that required for the use of amplitude modulation. It was reasoned that modulation consisting of very slight shifts in the frequency of the transmitted carrier could be received and demodulated by a receiver of very narrow bandwidth. The narrow bandwidth of such a receiver would provide an improvement in the signal-to-noise ratio of the receiver.

The fallacy of this reasoning was shown by J. R. Carson, (3), in 1922, in a classic article. Carson was the first to derive the mathematical expression for a frequency modulated signal, in a form which indicated an infinite series of signal components, or sidebands, spaced symmetrically about the carrier frequency, rather than the single pair involved in an amplitude modulated signal.

As a result, Carson concluded, "the transmission of the signal by frequency modulation requires the transmission of a band of frequencies at least [twice the highest signal frequency] in width." From a further (approximate) analysis, Carson concluded, "Consequently this type of modulation inherently distorts without any compensating advantages whatsoever." The video tape recorder has proved the first of these conclusions wrong, and commercial frequency modulation broadcasting, the second.

Carson's conclusions seemingly discouraged work on frequency modulation, and little progress was made until 1936. "In 1929 Roder confirmed the results of Carson... In 1930 van der Pol treated the subject. He drew no conclusions regarding the utility of the method." (4).

Armstrong's article (4)* showed the advantage of frequency modulation over amplitude modulation in the reduction of noise and the virtual elimination of interfering signals.

> The basis of the method consists in introducing into the transmitted wave a characteristic which cannot be reproduced in disturbances of natural origin and utilizing a receiving means which is substantially not responsive to the current resulting from the ordinary types of disturbances and fully responsive only to the type of wave which has the special characteristics.

The article includes information on a complete system for producing and detecting frequency modulated signals.

Frequency Modulation Detectors

One of the earliest systems of detection of frequency

---

*Also see Lawrence B. Arguimbau, Vacuum-Tube Circuits and Transistors, (New York, 1956) p. 515. The discussion on spectra and "Background Notes," pp. 504-516, are particularly good.

modulated signals involved applying the signal to a resonant circuit which was detuned so as to place the carrier and significant sidebands on the slope of the selectivity curve, hence the name, slope detector. The frequency modulation was converted to amplitude modulation, which was in turn detected by a square-law detector. (5)

This system had the advantage of simplicity, requiring only the same components used in conventional amplitude modulation detectors, but the distortion due to curvature of the response curve of the tuned circuit was undesirable. Another disadvantage was that the signal could be detected at two settings of the resonant frequency of the tuned circuit, above and below the signal frequency.

These two disadvantages of the slope detector were practically eliminated by combining two slope detectors, tuned above and below the signal frequency, and combining their output signals in opposition. (4).

The hardware required for the dual slope detector was combined into a single special transformer in the design of the Foster-Seeley (6) discriminator. This detector was first used in automatic frequency control of broadcast receivers. This discriminator became the most widely used detector of frequency modulated signals and has only recently been given some strong competition by circuits which have the advantage of requiring fewer tubes or components. The discriminator can be shown to be equivalent in its action to the dual slope detector. (7).

Both the discriminator and the ratio detector are based upon the action of a special transformer with two outputs, the relative amplitude of which depends on the frequency of the input signal. The two outputs are connected to diode detectors and the output signal is derived as the difference in output of the two detectors. The discriminator is sensitive not only to variations in the frequency of the input, but also to amplitude variations, and thus must be preceded by limiting amplifiers which provide a constant amplitude input to the discriminator. The ratio detector has reduced sensitivity to amplitude variations, and is capable of faster recovery when overloaded by noise impulses, but provides an output signal voltage one-half that of a discriminator operating with similar input. (2).

A second general method of demodulation of frequency modulated signals involves the concept of generalized frequency as the rate of alternation (passing through zero) of the signal. (3).

> The function sin $[\Omega(t)]$ alternates when the function $\Omega(t)$ changes by π, i. e., the interval between zeros corresponds to the time intervals during which $\Omega(t)$ changes by the amount π ... The rate of alternation of the function sin $[\Omega(t)]$ is approximately the same at any time, t, as that of the function sin wt, where $w = \Omega'(t)$...Generalized frequency, as defined above, corresponds with the "instantaneous frequency" [of a siren] which the ear apperceives.

The method operates on the principle of comparing the period (and thus the rate of alternation) of the incoming signal with that of a reference signal. The reference signal may be from an oscillator locked to the incoming signal, or may consist of the voltage developed across a tuned circuit when the input is applied.

In this detector, a pulse of plate current is initiated as the reference (quadrature) signal approaches its peak value. This occurs approximately at the zero-crossing of the input signal, which terminates the plate-current pulse. Variation of the phase of the quadrature signal maxima with respect to the zero-crossings of the input signal causes variation in the area of the pulses of plate current. The output signal is obtained by filtering the carrier-frequency components out of the plate voltage waveform.

The first commercial application of this principle was the Bradley detector, which utilized a specially constructed pentagrid tube in a locked-oscillator circuit. (8). Difficulties were encountered with stray coupling between the input circuit and the quadrature (reference) signal circuit, which led to poor rejection of amplitude modulation. Giacolletto (9) achieved improved operation using a beam-deflection tube in a similar circuit. This tube permitted use of a push-pull input circuit and single-ended reference circuit, enhancing isolation of the two circuits and reducing sensitivity to amplitude modulation of the input.

The gated beam tube described by Adler (10) has become widely used in commercial television receivers. Its internal construction permits nearly complete isolation of the input and quadrature circuits. Applied as a detector, this tube replaces three tubes required with a discriminator or ratio detector circuit. In addition, the tube is useful in other applications, as a limiter or sync separator, for example.*

Another detector of this type uses a conventional twin-triode tube. It may be operated either as a simple detector, or with one of the triodes functioning as a locked-in oscillator if especially high amplitude modulation rejection is required. (11).

A third "type" of demodulator operates in a similar manner to the second, but uses a trigger circuit to generate a pulse of constant volt-second area at each alternation of the input. The direct component of the pulses thus generated is applied to the output circuitry. These "frequency counters" are used primarily in monitoring low frequency shifts (drift), using carrier frequencies which are also low, so as to permit the generation of pulses of reasonable area during the interval between alternations of the input. (12).

---

*This tube is available commercially as type 6BN6.

CHAPTER III

THEORY OF FREQUENCY MODULATION AND DEMODULATION

A frequency modulated signal is formed by varying the frequency of the carrier $e_c = E_c \sin[w(t) + \theta]$ in accordance with a signal frequency $e_s = E_s \cos w_s t$ in such a manner that the instantaneous carrier frequency w(t) is shifted by an amount proportional to the magnitude of the signal frequency, and at a rate proportional to the frequency of the signal. Of course, in the practical case, the modulating frequency will usually be much more complex than the form given. If, however, the signal is periodic and analyzable into Fourier components, the spectrum of the modulated wave will contain the same components as would arise if each of the Fourier components of the modulating wave acted independently. (3).

Let the carrier frequency wave be represented by

$$e_c = E_c \sin \phi(t), \qquad (1)$$

where $\phi(t)$ is the total phase angle excursion of the carrier since some (arbitrary) time t = 0. Further, following Carson, let the instantaneous frequency of the carrier be defined as the rate of change of phase,

$$w(t) = d\phi(t)/dt. \tag{2}$$

It follows that

$$\phi(t) = \int_0^t w(t)\, dt. \tag{3}$$

If w(t) is a constant, this leads to the familiar expression

$$e_c = E_c \sin wt \tag{4}$$

for the carrier. If, however, as in the present case, w(t) is a variable function of time, such as

$$w(t) = w_c + k_f E_s \cos w_s t, \tag{5}$$

then

$$\phi(t) = \int_0^t w_c\, dt + \int_0^t (k_f E_s \cos w_s t)\, dt \tag{6}$$

$$= w_c t + (k_f E_s / w_s) \sin w_s t \tag{7}$$

within a constant of integration which corresponds to fixing the zero of phase reference.

Finally, substitution of (7) into (1) leads to the expression of the frequency modulated wave:

$$e_c = E_c \sin [w_c t + (k_f E_s / w_s) \sin w_s t]. \tag{8}$$

The equation of a phase modulated wave is similar:

$$e_c = E_c \sin [w_c t + (k_p E_s) \sin w_s t]. \tag{9}$$

The difference in the two lies in the form of the deviation ratio; $k_f E_x / w_s$ in frequency modulation, and $k_p E_s$ in phase modulation. (13). The dependence of this ratio on the signal frequency results in a more economical utilization of the spectrum within the bandwidth required for one channel. In phase modulation, the number of significant sidebands is constant as the frequency of modulation decreases, but their spacing decreases, so that the utilized bandwidth decreases rapidly with decreasing modulating frequency. In frequency modulation, however, the decreased spacing of the sidebands as the modulating frequency decreases is partially offset by the increased number of sidebands required by the increasing deviation ratio, so that the bandwidth utilized is nearly constant. The use of pre-emphasis to increase the amplitude of the higher-frequency components thus can be used to advantage in frequency modulation, without significantly increasing the total required bandwidth.

It is convenient to define the frequency-deviation ratio

$$\beta = k_f E_s / w_s. \qquad (10)$$

It is the ratio of the peak frequency deviation to the audio rate at which it occurs. (7). Equation (8) then becomes

$$e_c = E_c \sin[w_c t + (k_f E_s / w_s) \sin w_s t] \qquad (8)$$

$$e_c = E_c \sin [w_c t + \beta \sin w_s t]$$

$$= E_c [\sin w_c t \cos (\beta \sin w_s t) +$$

$$\cos w_c t \sin (\beta \sin w_s t)]. \qquad (11)$$

In order to interpret the terms of the form $\cos(u \sin x)$ and $\sin(u \sin x)$, it is necessary to make use of the identities (14):

$$\cos(u \sin x) = J_0(u) + 2\sum_1^\infty J_{2n}(u) \cos 2nx, \qquad (12)$$

and

$$\sin(u \sin x) = 2\sum_1^\infty J_{2n-1}(u) \sin(2n-1)x. \qquad (13)$$

Thus

$$e_c = E_c[J_0(\beta)\sin w_c t + 2\sum_1^\infty J_{2n-1}(\beta)\sin(2n-1)w_s t \cos w_c t$$

$$+ 2\sum_1^\infty J_{2n}(\beta)\sin w_c t \cos 2nw_s t] \qquad (14)$$

$$= E_c[J_0(\beta)\sin w_c t$$

$$+ 2\sum_1^\infty J_{2n-1}(\beta)\tfrac{1}{2}\left\{\sin[(2n-1)w_s + w_c]t\right.$$

$$\left. + \sin [(2n-1)w_s - w_c]t\right\}$$

$$+ 2\sum_1^\infty J_{2n}(\beta)\tfrac{1}{2}\left\{\sin[w_c + 2nw_s]t\right.$$

$$\left. + \sin[w_c - 2nw_s]t\right\}]. \qquad (15)$$

Since $\sin (-x) = - \sin x$, the sign of the second term in the first summation may be changed to yield" $- \sin [(2n-1)w_s + w_c]t$," whereupon

$$e_c = E_c \left\{ J_o(\beta)\sin w_c t + \sum_1^\infty J_n(\beta)[\sin(w_c + nw_s)t + (-1)^n \sin(w_c - nw_s)] \right\}. \tag{16}$$

The use of this expression in the analysis of the performance of f.m. detectors is a time-consuming procedure, and other, approximate, methods are generally used. This expression, however, with a table for evaluation of the Bessel functions, is valuable in determining the number of sidebands of significant amplitude in a frequency-modulated signal of given modulating frequency and deviation ratio. The amplitudes of the Bessel functions up to $J_{34}(24)$ are tabulated in Hund (15). From this tabulation, it may be seen that the number N of sidebands having amplitudes greater than 1% of the amplitude of the unmodulated carrier lies below the line $N = \beta + 5$, for $\beta \leq 25$. (See Figure 1.)

Discriminator Analysis

The actual method of analysis of a particular demodulator depends, of course, on the type of detector involved; however, analysis of the dual slope detector, the discriminator, and the ratio detector may be performed by a simple method. This method proceeds from consideration of the variation in impedance of a parallel tuned circuit with variations in the frequency of an applied signal.

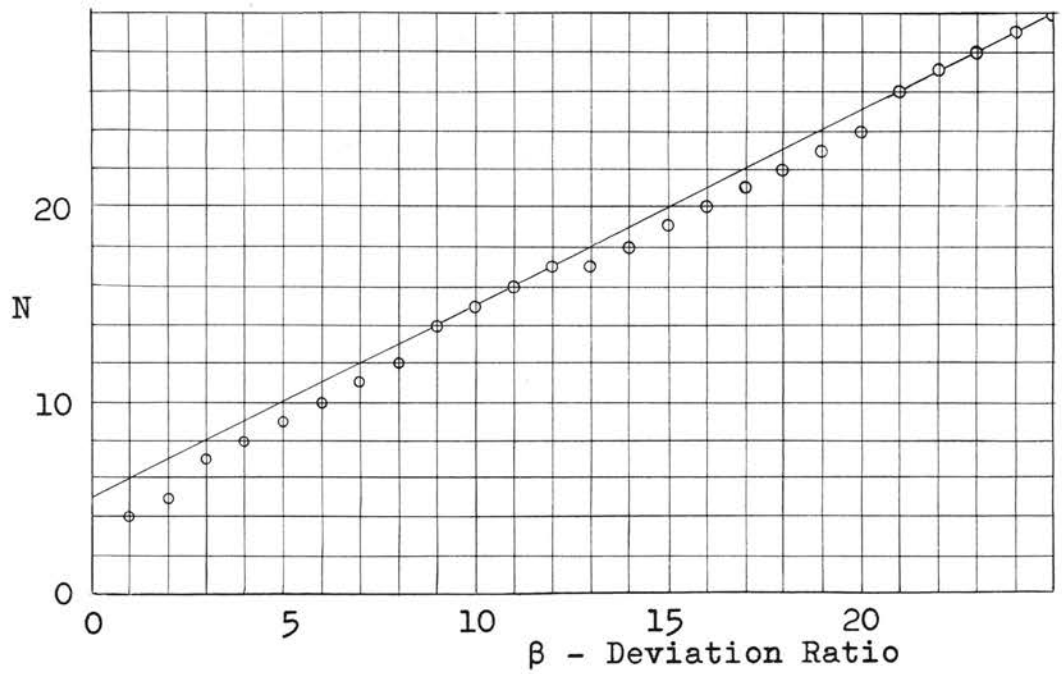

Figure 1. Number of Significant Sidebands
$N \leq \beta + 5$

The admittance of a parallel RLC circuit may be expressed as

$$Y = 1/Z = G + B_C + B_L = 1/R + jwC + 1/jwL. \tag{17}$$

The unloaded resonant frequency, as determined for unity-power-factor resonance, is obtained by setting the capacitive and inductive susceptances equal.

$$B_C = B_L$$

$$w_oC = 1/w_oL$$

$$w_o = 1/(\sqrt{LC}) \tag{18}$$

The susceptance at any frequency may be expressed as a function of the susceptance at resonance and the ratio of frequencies.

$$B_C = wC = (w/w_o)(w_oC) \quad (19)$$

$$B_L = 1/wL = (w_o/w)(1/w_oL) \quad (20)$$

Multiplying both sides of equation (17) by R and factoring B_o,

$$RY = R/Z = 1 + jR(wC - 1/wL), \quad (21)$$

$$= 1 + jRB\,(w/w_o - w_o/w). \quad (22)$$

The quantity RB_o is, by definition, the Q of the resonant circuit. The logarithmic frequency variable $w/w_o - w_o/w$ will be denoted by d. Making these substitutions,

$$R/Z = 1 + jQd. \quad (23)$$

If the parallel RLC circuit is connected to a constant-current source, the resultant voltage across the circuit, expressed as a function of frequency and of the voltage at resonance, is

$$E/E_o = Z/R = 1/(1 + jQd). \quad (24)$$

In the discriminator, the output voltage is derived as the difference in magnitude of two such functions. The magnitude of each is

$$\left| Z/R \right| = (1 + Q^2 d^2)^{-\frac{1}{2}}. \quad (25)$$

This function is symmetrical about d = 0 when plotted on a linear d scale, which corresponds to a logarithmic frequency scale. However, within the range $d = \pm 0.4$, ($p = .82$ to 1.22, where $p = \frac{w}{wo}$), the function is nearly

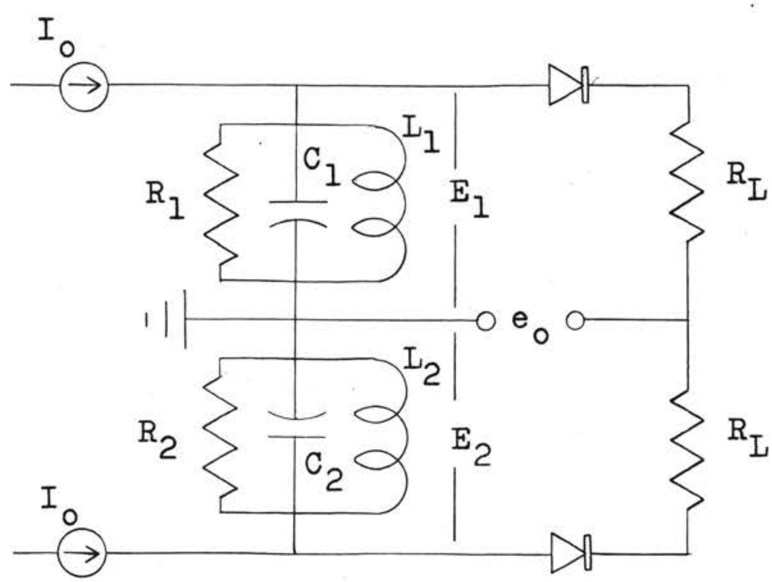

Figure 2. Simple Balanced Discriminator

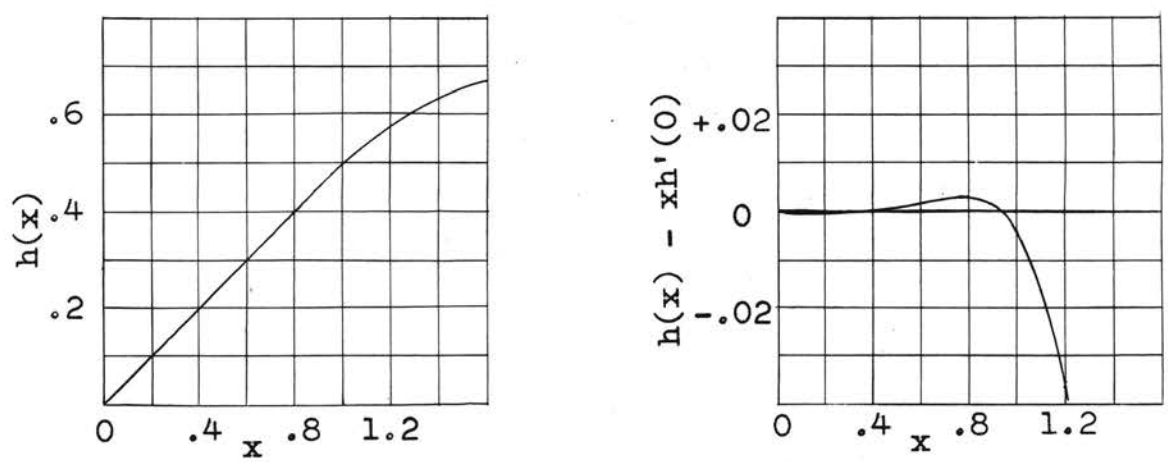

Figure 3. Output Linearity - Output and Error in Linearity of Output as a Function of x = Qd.

symmetrical when plotted on a linear frequency scale. The importance of this approximation will be shown later.

The output voltage of the simple balanced discriminator (Figure 2) may be written

$$e = e_1 - e_2 \cong \sqrt{2}\,(|E_1| - |E_2|). \qquad (26)$$

Neglecting the effect of the rectifiers and their loads for the moment, this may be normalized to

$$h = e/(\sqrt{2}\,E_o) = \frac{1}{\sqrt{1 + Q^2{}_1 d^2{}_1}} - \frac{1}{\sqrt{1 + Q^2{}_2 d^2{}_2}} \; . \qquad (27)$$

The linearity of the output function h depends upon the separation of the resonant frequencies of the two tuned circuits. Arguimbau (7) has shown that maximum linearity of this function with respect to the logarithmic variable d is obtained when the two circuits are each detuned from the center frequency by an amount equal to $\sqrt{1.5}$ half-bandwidths. Wider variations in the input frequency can be accommodated with a slight compromise of linearity near the center frequency if this detuning is increased slightly. A value of 1.4 offers a reasonable compromise. The output is linear within 1% for values of $d = \pm 1/Q$. (See Figure 3, page 16.)

Linearity With Frequency

The possibility of determing a set of values which would lead to an output linear in p rather than d was investigated. The derivation is shown in Appendix A, the results of which may be summarized as follows: The only linear output with respect to p is the solution of the trival case for which $E_o = 0$.

This does not eliminate the use of an approximation to linearity in p, but the limits of the approximation must be defined.

The distortion introduced into the output is primarily even-harmonic, since the discriminator does not produce equal positive and negative values of the output function $\underline{h}$ for equal frequency shifts above and below the center frequency. The symmetry of the output with respect to $\underline{d}$ is such that $h(d) = h(-d)$, so it will be assumed that the output may be expressed in a series

$$h = a_0 + a_1 \cos wt + a_2 \cos 2wt + \ldots, \tag{28}$$

when the input consists of a sinusoidal variation in frequency. If the assumption is made that the effect of harmonics higher than the second is negligible, only the terms shown in Equation (28) need be considered. The coefficients $\underline{a_n}$ of this expression may then be evaluated by the "three-point analysis" version of Fourier analysis. (2).

Choosing $h(+)$ at $wt = 0^o$, h_o at $wt = 90^o$ and 270^o, and $h(-)$ at $wt = 180^o$, and substituting in Equation (28),

$$h(+) = a_0 + a_1 + a_2 \tag{29}$$

$$h(0) = a_0 - a_2$$

$$h(-) = a_0 - a_1 + a_2 .$$

Simultaneous solution of these equations yields

$$a_0 = a_2 = \tfrac{1}{4}[h(+) + h(-)] \tag{30}$$

$$a_1 = \tfrac{1}{2}[h(+) - h(-)] .$$

The second harmonic distortion, expressed as a percentage, is

$$H_2 = a_2/a_1 \times 100\% = \frac{h(+) + h(-)}{2[h(+) - h(-)]} \times 100\% \ . \tag{31}$$

For a specific allowable harmonic distortion percentage X, this equation may be solved for the ratio of h(+) to h(-):

$$h(+)/h(-) = (1 + .02X)/(1 - .02X). \tag{32}$$

If, for example, the second harmonic distortion is to be limited to 2%, then the ratio h(+)/h(-) must not exceed

$$(1 + .04)/(1 - .04) = 1.04/.96 = 1.083. \tag{33}$$

It remains to establish the relationship of this requirement to the actual hardware involved in the circuit. It is necessary to review at this point some of the assumptions that have been made. First, it is assumed that the requirements for an output which is a linear function of d (over a limited range) have been met. This means that the following parameters of the two tuned circuits are equal:

$$Q_1 = Q_2 \tag{34}$$

$$R_1 = R_2$$

$$a_1 = a_2 \cong \sqrt{1.5}.$$

The assumption that the output function h is approximately linear in d may be written

$$h(+)/h(-) \cong d(+)/d(-) \tag{35}$$

$$= \frac{p(+) - 1/p(+)}{p(-) - 1/p(-)} . \tag{36}$$

Since equal shifts in frequency above and below the center frequency w_x are to be investigated,

$$w_x - w(+) = w(-) - w_x \tag{37}$$

$$1 - w(+)/w_x = w(-)/w_x - 1$$

$$1 - p(+) = p(-) - 1$$

$$p(+) = 2 - p(-). \tag{38}$$

Substitution of this into Equation (36) and dropping the (-) indicator leads to

$$d(+)/d(-) = \frac{(2 - p) - 1/(2 - p)}{p - 1/p}$$

$$= \frac{3p - 4p^2 + p^3}{-p^3 + 2p^2 + p - 2} = K. \tag{39}$$

Further manipulation leads to the following cubic, where $K = d(+)/d(-)$:

$$p^3 + p^2 \left[- \frac{2K + 4}{K + 1}\right] + p\left[\frac{3 - K}{K + 1}\right] + \frac{2K}{K + 1} = 0. \tag{40}$$

Solution of this cubic is possible, theoretically, by direct methods, but the term $K + 1$ in the denominator of the coefficients makes the manipulation very sensitive to slide-rule error, since K is very nearly -1 for the range of interest. (16).

Substitution of values of p into Equation (39), however, leads directly to the (approximate) relationship

$$K \cong -p. \quad (41)$$

The approximation is better than ½% for $.90 \leq p \leq 1.10$.

Therefore, the second-harmonic distortion in the output due to the discriminator being linear in d rather than p will be less than 2% if the frequency excursion p is limited to the range

$$.92 \leq p \leq 1.08. \quad (42)$$

One further point deserves mention in connection with the analysis of discriminators operating at low carrier frequencies. The diode detectors used to retrieve the signal from the amplitude modulated output of the tuned circuits must be carefully designed to avoid a form of output distortion known as "diagonal clipping." This form of distortion arises when the time constant of the filters following the diode rectifiers is too long, with respect to the period of the output waveform. This type of distortion may be eliminated by satisfying the relation

$$w_m RC \leq \sqrt{(1/M)^2 - 1} \quad (43)$$

in which w_m is the highest frequency with which the carrier is modulated. (17, p. 318). R represents the total load impedance and C the filter capacitance. M is the amplitude modulation index of the input to the detector.

In the case of the discriminator, the factor M may be related to the frequency deviation. From Equation (9), the peak deviation is

$$k_f E_s = \beta w_s. \tag{44}$$

The value of M may then be determined for a given detector by solving Equation (25) for the appropriate values of d, which gives the percentage change in output voltage for a given input.

The values of d may be obtained from

$$d_n = p_n - 1/p_n \qquad n = 1, 2. \tag{23}$$

and

$$p_1 = 1 + Bw_s/w_x = 1 + k_f E_s/w_x \tag{45}$$

$$p_2 = 1 - Bw_s/w_x = 1 - k_f E_s/w_x. \tag{46}$$

Another criterion, however, might also be used to select the filter wRC value. It has been shown that the output of the discriminator at a given frequency shift below the center frequency exceeds that for the same frequency shift above the center frequency [h(+) > h(-)]. A compensating effect can be shown in the characteristics of the diode detectors by considering their operation as

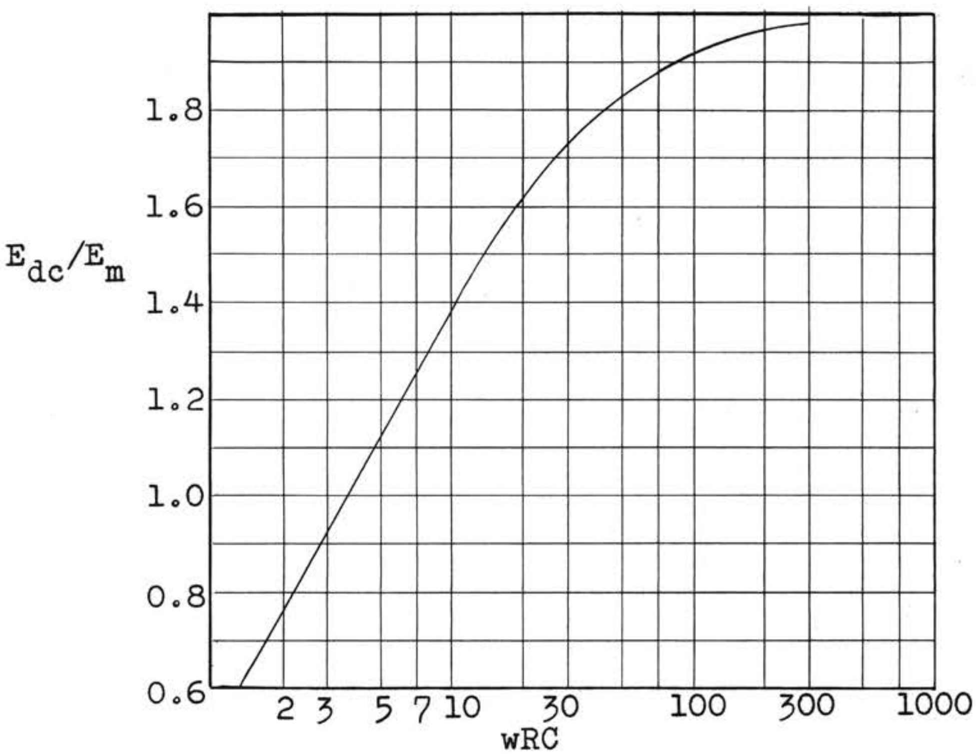

Figure 4. Output of Voltage Doubler

half-wave rectifiers. (17, p. 575-581). (See Figure 4.) Due to the action of the filters, their output increases with frequency. Taking this effect into account permits extending p to

$$.90 \leq p \leq 1.10, \tag{47}$$

while maintaining the h(+)/h(-) ratio as outlined above, if wRC = 100, a typical value for good power-supply operation. If wRC is reduced to 10, the permissable range of p becomes

$$.87 \leq p \leq 1.13. \tag{48}$$

These figures are for voltage doubler output circuits as used in the experimental device. Other charts are available for different types of rectifier connections. This effect has not been fully investigated, but is mentioned as a possible explanation of the reduced distortion indicated in the static test results.

CHAPTER IV

THE PROBLEM AND A SOLUTION

Commercial broadcast frequency modulation detectors operate at a carrier frequency of 10.7 megacycles. The maximum modulation frequency is 15 kilocycles, and the maximum deviation of the carrier is 75 kilocycles. In contrast, the system designed for this study operates at a carrier frequency near 10 kilocycles, with a maximum modulating frequency of 500 cycles, and a maximum expected carrier deviation of 20 to 25 cycles.

The narrow deviation of the carrier indicates that high sensitivity will be required -- an output voltage of 200 millivolts/cps was stated as an objective. The relatively high ratio of modulating to carrier frequency, however, indicates that a wide bandwidth will be required, which conflicts with the sensitivity requirement. Finally, the low carrier frequency indicates possible difficulty with variations in the Q of tuned circuits if conventional circuits were to be used. In this chapter, these difficulties and the steps taken to overcome or avoid them will be discussed.

The equations for the output of the frequency modulation discriminator developed so far have made use of the assumption that the Q of the tuned circuits involved is constant. For radio-frequency circuits, this assumption is justifiable, but at lower frequencies the equivalent series resistance is more nearly constant than the Q. Primarily due to the increase of equivalent resistance because of skin effect, the Q varies approximately logarithmically with frequency in the audio frequency range, in commercially available toroids suitable for discriminator use.

It is therefore necessary to consider what type of network, active or passive, might be used to provide a response equivalent to that of an RLC circuit through the audio range, with a constant Q as frequency is varied. If this Q were easily adjustable, so much the better.

At the suggestion of Dr. H. D. Crawford, a special frequency selective feedback amplifier circuit, with a twin-T feedback path, was investigated, and found to have more than ample adjustment provisions. The entire discriminator circuit has been built around the twin-T networks and the characteristics of feedback amplifiers.

A modification of the standard twin-T network (called "inside-out" from the original method of deriving its characteristics) was selected in order to provide ease in adjustment of the frequency of operation of the discriminator. (18)

The Twin-T Network

The normal twin-T circuit consists of two parallel branches between input and output. (See Figure 5.) One of these is two resistors in series, with a shunt capacitor from the junction point to ground. The other T is similar, with two capacitors in series and a resistor from junction to ground.

The modification of the circuit it consists in feeding the two-capacitor branch of the network from the arm of a potentiometer so that it is driven (at a low impedance level) with a specific fraction x of the input voltage as shown in Figure 6. Connected in this manner, the input potentiometer provides a means of varying the frequency in accordance with the relationship (derived in Appendix B),

$$w_x/w_o = 1/\sqrt{x} \tag{49}$$

where w_x is the null frequency at a setting x and w_o is the null frequency at $x = 1.0$. The frequency variation is accompanied by a slight effect on the Q of the circuit. (See Appendix B.)

There are several schemes for stating the null conditions for the twin-T network. For the commonly used symmetrical T, the following conditions will result in a null:

$$R_3 = \frac{R_1 R_2}{R_1 + R_2} \; ; \; C_n = 1/w_o R_n, \quad n = 1, 2, 3. \tag{50}$$

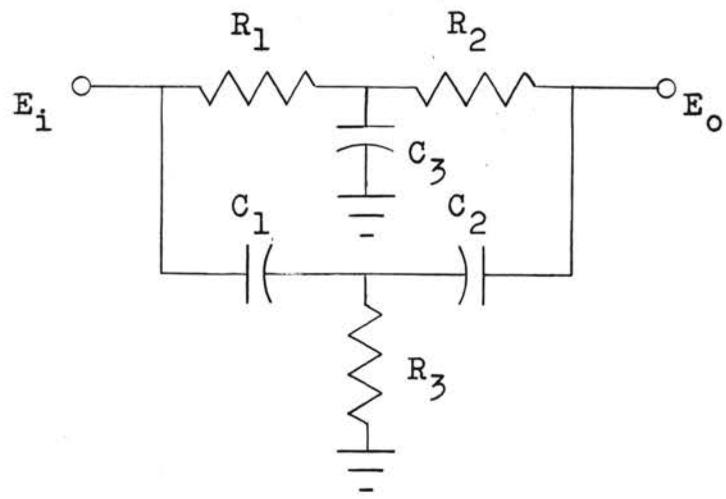

Figure 5. Conventional Twin-T

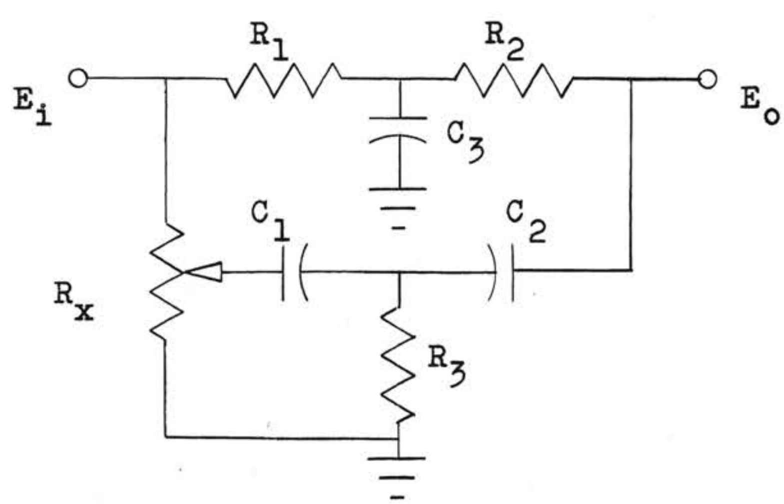

Figure 6. "Inside-Out" or Tunable Twin-T

If it is desired that the transfer function be symmetrical about the resonant frequency,

$$a = R_2/(R_1 + R_2) = 1/2, \text{ or } R_1 = R_2.(19) \qquad (51)$$

The expression for the transfer function as a function of frequency is:

$$\frac{E_o}{E_i} = \frac{1 - p^2}{(1-p^2) + j\,2p/a} = \frac{1}{1 - j\,2/ad} \qquad (52)$$

which becomes, for a = 1/2,

$$\frac{E_o}{E_i} = \frac{1}{1 - j\,4/d} \quad . \qquad (53)$$

In this expression, $d = p - \frac{1}{p}$, and $p = \frac{w}{w_o}$, where $w_o = \frac{1}{RC}$ for $R_1 = R_2 = 2R_3$ and $C_1 = C_2 = \frac{C_3}{2}$.

The addition of the tuning control modifies the transfer function to

$$\frac{E_o}{E_i} = \frac{1 - xp^2}{(1 - p^2) + j\,2p/a} \quad .$$

See Appendix B.

The restrictions on the source and load impedances are:

$$R_s \leq (1/6)(R_1 + R_2)(a - a^2) \qquad (54)$$

$$R_L \geq 3(R_1 + R_2). \qquad (55)$$

These may be readily realized by driving the network with a cathode follower and connecting the output directly to the grid of the following amplifier stage.

Twin-T Feedback Amplifier

When the twin-T network is used as the feedback path of a feedback amplifier (Figure 7), the resulting amplifier response roughly corresponds to the inverse of that of the network and is similar to that of the single-tuned

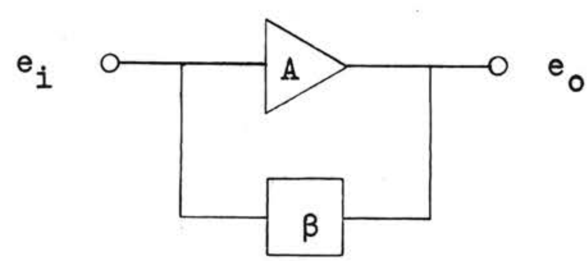

Figure 7. Feedback Amplifier

RLC network. (20). If G is the over-all gain of the amplifier, then

$$G = \frac{e_o}{e_i} = \frac{A}{1 + A\beta} . \qquad (56)$$

If the twin-T network constitutes the feeback path, then

$$\beta \equiv 1/(1-j4/d). \qquad (57)$$

Using this value of β, the gain expression becomes

$$\frac{G}{A} = \frac{1}{1 + \dfrac{A}{1 - j4/d}} = \frac{1 - j4/d}{(A + 1) - j4/d} \quad . \tag{58}$$

If

$$-0.4 \leq d \leq + 0.4, \tag{59}$$

$$\frac{G}{A} \cong \frac{-j4/d}{(A + 1) - j4/d} \tag{60}$$

$$= \frac{1}{1 + \dfrac{A + 1}{-j4/d}} \tag{61}$$

$$= \frac{1}{1 + j\dfrac{(A + 1)d}{4}} \quad . \tag{62}$$

Compare this with Equation (24):

$$\frac{Z}{R} = \frac{1}{1 + j(Q)d} \quad . \tag{24}$$

Thus, over the range of frequency indicated, the twin-T feedback amplifier simulates the response of an RLC circuit with a Q equal to $(A + 1)/4$.*

Solving for the value of p corresponding to the limiting value

$$d = 0.4 = p - 1/p = (p^2 - 1)/p \tag{63}$$

$$p^2 - .4p - 1 = 0$$

$$p = .2 \pm \sqrt{.04 + 1} = .2 \pm 1.0197.$$

---

*See Fleisher (20, p. 393) for a different derivation of this same approximation to the value of Q.

The limits on p for the approximation are;

$$.82 \leq p \leq 1.22 \; . \tag{64}$$

The second-harmonic distortion in the output when operating between these limits may be checked by Equations (31), (35), (39), and (41):

$$H_2 = \frac{h(+) + h(-)}{2[h(+) - h(-)]} \times 100\% \tag{31}$$

$$= \frac{h(+)/h(-) + 1}{2[h(+)/h(-) - 1]} \times 100\%$$

$$\cong \frac{K + 1}{2(K - 1)} \times 100\% \qquad (35), (39)$$

$$\cong \frac{-p + 1}{2(-p - 1)} \times 100\%; \tag{41}$$

The p to be used is that which corresponds to h(-), that is, p = 1.22. For this value of p

$$H_2 \cong .22/3.44 \times 100\% = 6.5\%. \tag{65}$$

Thus, the simulation of the response of the RLC circuit by the twin-T feedback amplifier is valid in this case to better than 1% in magnitude, and 6° in phase over a considerably wider frequency range than the assumption of a linear discriminator output with respect to frequency.

The twin-T feedback amplifier may be used as the frequency- to amplitude-modulation conversion element in a dual slope discriminator with the advantage over the RLC

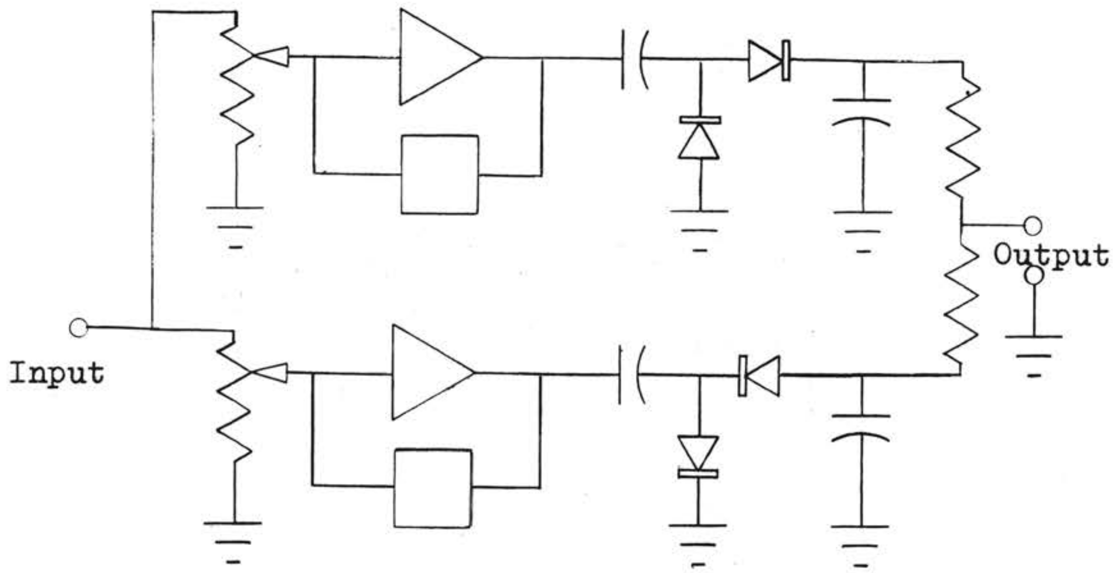

Figure 8. Feedback Amplifier Discriminator

circuit of having an adjustable Q which is independent of frequency. The gain requirements of the amplifier, A = 39 for Q = 10, are such that good sensitivity is obtained over the bandwidth required in this application. The amplifier has a constant maximum usable output voltage as the Q is varied, so that the sensitivity of the detector decreases as the bandwidth is increased. In addition, the networks are much less expensive than adjustable toroids. Figure 8 shows an arrangement of two feedback amplifiers in a discriminator circuit.

Meeting Design Requirements

The original statement of design requirements specified a carrier frequency between 3000 and 10,000 cps and a signal frequency range of 0 to 500 cps. For these requirements, the modulation index at the highest signal frequency is much less than unity, so the bandwidth of the discriminator needs only to be wide enough to include the first pair of sidebands of the modulated carrier. More sidebands will be significant at lower modulating frequencies, but their spacing decreases faster than their number increases. Thus, the discriminator curve must be linear over the range $f_x \pm f_s$, or $f_x \pm 500$ cps.

For the value of detuning $\underline{a} = 1.4$, the discriminator curve is satisfactorily linear to the limits $|Qd| = 1.0$ (Figure 3, page 16). Also, to remain within the 2% second-harmonic distortion limits, (Equation 42) $\underline{p}$ must be between .92 and 1.08. The value of $\underline{d}$ corresponding to these limits is .154. A minimum usable carrier frequency can be found from

$$p = (f_x + f_s)/f_x = 1.08$$

and the required signal frequency f_s. From this relation,

$$f_x = f_s/.08 = 500/.08 = 6250 \text{ cps.}$$

However, the output sensitivity improves as the Q is increased. In order to increase $\underline{Q}$, while remaining within the restrictions on $\underline{p}$ and $\underline{Qd}$, the carrier frequency must be increased. The calculation will be continued for a carrier frequency of 8500 cps. For this carrier frequency,

$$p = (f_x + f_s)/f_x = (8500 + 500)/8500 = 1.06$$

$$d = (p^2 - 1)/p \quad = (1.124 - 1.0)/1.06 = .117$$

$$Q = 1/d \qquad = 1/.117 = 8.58.$$

The requirement that the Q's of the two circuits be equal means that the detuning a of the two circuits in half-bandwidths will not be exactly equal. Nevertheless, the detuning may be calculated from the bandwidth of a circuit of the required Q tuned to the frequency f_x. The experimental work shows that the linearity of the output is much more sensitive to variations in Q than to variations in a.

$$BW = f_x/Q = 8500/8.58 = 990 \text{ cps}$$

$$1.4\ (BW/2) = .7\ BW = 693 \text{ cps} = f_x - f_1 = f_2 - f_x$$

$$f_1 = f_x - 693 = 8500 - 693 = 7807 \text{ cps}$$

$$f_2 = f_x + 693 = 8500 + 693 = 9193 \text{ cps}$$

Thus, to meet the requirements set forth, a discriminator with two tuned circuits, each having a Q of 8.6, tuned to 7807 and 9193 cps, may be used.

The gain of each amplifier is calculated from

$$Q = (A + 1)/4$$

$$A = 4Q - 1 = 4\ (8.58) - 1 = 33.3\ .$$

One final check should be made; that the entire operating range of the discriminator lies within the limits applicable to each of the amplifiers within which its approximation of the response of an RLC circuit is satisfactory.

At the upper resonant frequency f_2, the lower-frequency circuit is operating with

$$p_1 = 9193/7807 = 1.17$$

$$d_1 = 1.17 - 1/1.17 = .315 < .40.$$

This is within the limit, as stated in Equation (59).

An indication of the sensitivity of the discriminator can be obtained by calculating the response of each of the two circuits at the frequency $f_x + f_s$, subtracting, and dividing the result by f_s. In the numerical example,

$$p_1 = (f_x + f_s)/f_x = 1.15 \qquad p_2 = .98$$

$$d_1 = p_1 - 1/p_1 = .281 \qquad d_2 = -.04$$

$$Q_1 d_1 = 2.41 \qquad Q_2 d_2 = .343$$

$$|G/A|_1 = .383 \qquad |G/A|_2 = .950$$

In the latter expression, A represents the gain at the resonant frequency of the tuned circuits, or 33.3.

$$|G|_1 = 12.75 \qquad |G|_2 = 32$$

In the experimental unit, the output voltage is limited to 20 volts rms. For this gain setting, the input will be 20/33.3 = 0.6 volts.

$$E_1 = 7.65\text{v rms} \qquad E_2 = 19.2\text{v rms}$$

With the use of voltage doublers, an output dc voltage equal to 1.5 E_m can be obtained. The resistive mixing network gives an output $e_o = (e_1 + e_2)/2$.

$$e_1 = 16.2 \text{ volts} \qquad e_2 = 40.6 \text{ volts}$$

$$e_o = \frac{16.2 - 40.6}{2} = -12.2 \text{ volts.}$$

Since this voltage is due to a 500 cps shift in the input frequency, the theoretical sensitivity is about 24 mv/cps.

CHAPTER V

EQUIPMENT DESIGN AND OPERATION

Several amplifier designs were considered for use in the twin-T feedback circuit. In order to prevent changes in amplification with tube drift, an amplifier which itself was a feedback amplifier was used; that is, an amplifier was built which had more than the required gain as determined from the Q needed, with provisions for reducing the gain to a specific amount.

The dual-triode circuit known as the cascode amplifier (Figure 9) had been used by Dr. H. D. Crawford in some previous work, and is suggested by Fleisher (20) in a similar application. Higher gain can be realized with this circuit than with a single pentode stage (except under "starved-current" conditions) (21)*, and, being a single-stage amplifier, the cascode does not require the complicated coupling networks necessary for stability in multistage amplifiers of comparable gain. (22).

---

*Maximum gain for the 6AU6 pentode in a typical circuit is 371 at an output of 122 volts. See (21).

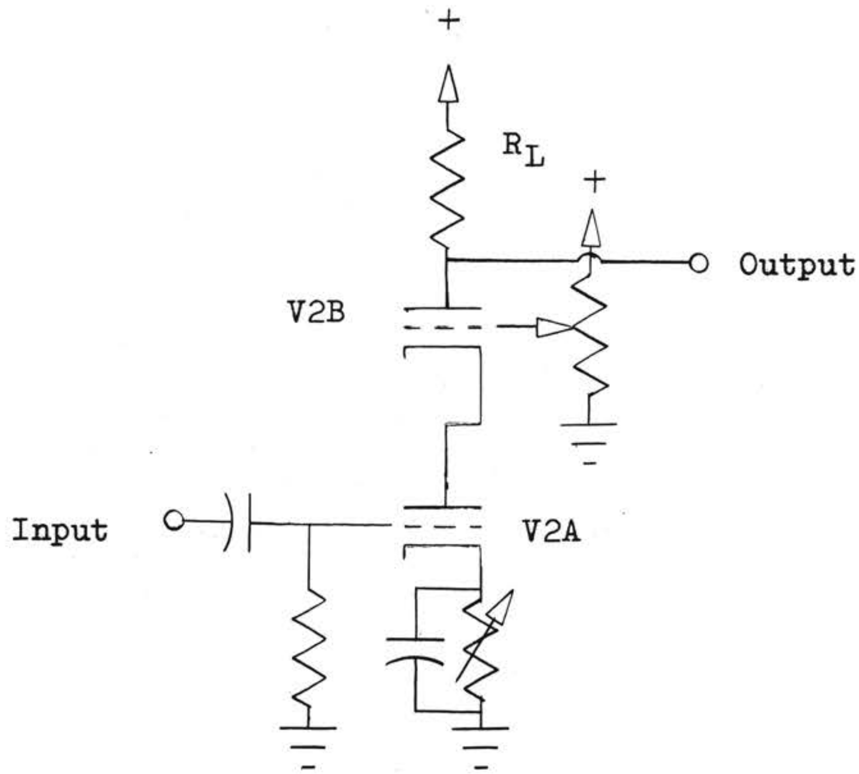

Figure 9. Cascode Amplifier

The Cascode Amplifier

The cascode circuit has some of the characteristics of a pentode, with the advantage that no screen supply is required. The cascode amplifier has somewhat lower noise than a pentode operating at the same gain, due to the elimination of noise arising from fluctuations in the screen current. However, since two tubes are used in series, the plate supply voltage requirement is higher than for normal triode amplifiers.

The upper triode, V2B, serves to isolate the plate of V2A from the output load, holding the plate voltage of V2A nearly constant while permitting the variations in plate current to appear as output voltage variations at the load. If the two triodes are identical, the voltage gain

$$A = \frac{uR_L}{r_p + \frac{r_p + R_L}{u + 1}} \quad (23) \tag{74}$$

$$= (u)(u + 1)\frac{R_L}{R_L + (u + 2)r_p}$$

$$= \frac{g_m R_L}{\frac{R_L}{(u + 1)r_p} + \frac{(u + 2)}{(u + 1)}} \; . \tag{75}$$

In the usual circuit, with $u \gg 1$, $R_L \gg (u + 1)r_p$, and the gain

$$A \cong g_m R_L \; . \; (23). \tag{76}$$

The cascode amplifier is discussed quantitatively in Fleisher's (20) article, but no direct design methods are given. Equation (76) indicates that a tube with high g_m is needed, as well as a high value of load resistance. The fact is, however, that the deciding characteristic for a triode in this application is the ratio of transconductance to plate current, which is, for low plate currents, nearly constant. For example, the 12AT7, for which the Tube Manual (21) lists a g_m of 4000 micromhos at 3.7 ma. plate current, has a g_m/I_b ratio of 1/2 at low plate currents, while the 12AX7, which has a listed g_m of 1250 at .5 ma., has a g_m/I_b ratio of nearly 3/2. *

When the approximation $g_m = 3/2 I_b$ is inserted in Equation (76), (and, since $R_L = E_L/I_b$):

$$A \cong g_m R_L \cong (3/2)(I_b)(E_L/I_b) = (3/2)E_L \quad . \qquad (77)$$

A point of diminishing return is reached as the load voltage drop is increased due to the need for a reasonably high output voltage swing. As the load voltage increases, tube drop must decrease (considering the supply voltages fixed), and the operating grid base of the input triode is progressively narrowed. The limiting value of E_L occurs when the input grid is driven from saturation to cutoff to produce the desired magnitude of output.

The simple, but effective, expedient of connecting the grid of V2B to a variable resistor and providing a variable cathode resistor for V2A, then adjusting both for maximum gain, resulted in the choice of operating parameters for the experimental discriminator. The resulting load voltage E_L was 225 volts, for which value Equation (77) gives a gain of 338. The measured gain was 368.

---

*The apparent discrepancy in the g_m and I_b values, and their ratios, arises because the ratios were determined from the RCA Tube Handbook, HB-3, while the values are from the RCA Receiving Tube Manual. The Handbook data is considered more nearly accurate.

Operational Amplifier Feedback

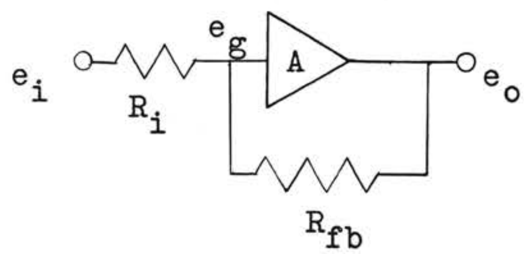

Figure 10. Operational Amplifier

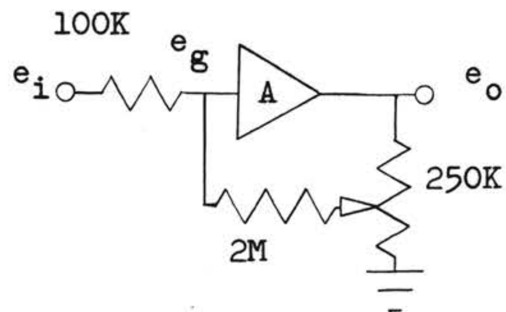

Figure 11. Amplifier as Used

The resistive mixing used to set the gain of the amplifier to be used with the twin-T feedback network is often used in analog computers, and thus is sometimes called the "operational amplifier" technique (Figure 10).

If the assumption is made that the input impedance of the amplifier is infinite (a good assumption when driving the grid directly), then equating the currents at this point, called the "summing point", $I_i = I_{fb}$. Since the voltages across the feedback resistor and input resistors must equal the product of their resistance and the current flowing,

$$\frac{(e_i - e_g)}{R_i} = \frac{(e_g - e_o)}{R_{fb}} .$$

Let

$$K = \frac{R_{fb}}{R_i} .$$

Then

$$Ke_i - Ke_g = e_g - e_o$$

$$Ke_i = (K + 1)e_g - e_o$$

$$e_o/e_i \equiv G = (K + 1)(e_g/e_i) - K.$$

But, since $A = e_o/e_g$; $e_g = e_o/A$, so

$$G = [(K + 1)/A](e_o/e_i) - K.$$

$$= -K/(1 + \frac{K + 1}{A})$$

Generally, $A \gg K + 1$, so

$$G \cong -K$$

$$\frac{e_o}{e_i} = - \frac{R_{fb}}{R_i} .$$

Because a gain of 39 is needed to realize a Q of 10, the resistive feedback network was modified by feeding a basic network with K = 20 from a 250K voltage divider. (See Figure 11, page 42.) With this scheme, a high resistance potentiometer is not needed, and satisfactory operation is obtained for gains of slightly over 20 to 50 or more. If K' is the ratio of the voltage divider, the gain of the amplifier is <u>approximately</u> equal to

$$\frac{e_o}{e_i} \cong \frac{1}{K - K'} .$$

In order to provide suitable driving and load impedances for the twin-T network, cathode followers of conventional design were connected to the input and output of the cascode amplifier. (See Figure 12, page 45.) The input impedance of the cascode stage with the resistance mixing network is 100,000 ohms, equal to the value of R_i chosen. Its output admittance is

$$Y_o = (R_i/(R_i + R_f)\, g_m + 1/R_L + 1/(R_i + R_f).$$ [41]

$$= 1250/21 + 1/.51 + 1/2.1$$

$$= 61.9 \text{ micromhos.}$$

The output impedance is thus

$$Z_o \cong 16{,}000 \text{ ohms}$$

which is relatively low. However, the output cathode follower is necessary to provide a lower impedance source for the diode detector networks.

The input cathode follower V1A couples the input signal into one of the summing inputs of the cascode amplifier. The opposite side of this tube, V1B, performs a similar function for the output of the twin-T network. The high value of load resistor is used to reduce current drain and to keep the gain high. The gain of this stage is .93, and the output impedance is 620 ohms.

Components for use in the twin-T networks were selected and matched from stock 5% components with the aid of an

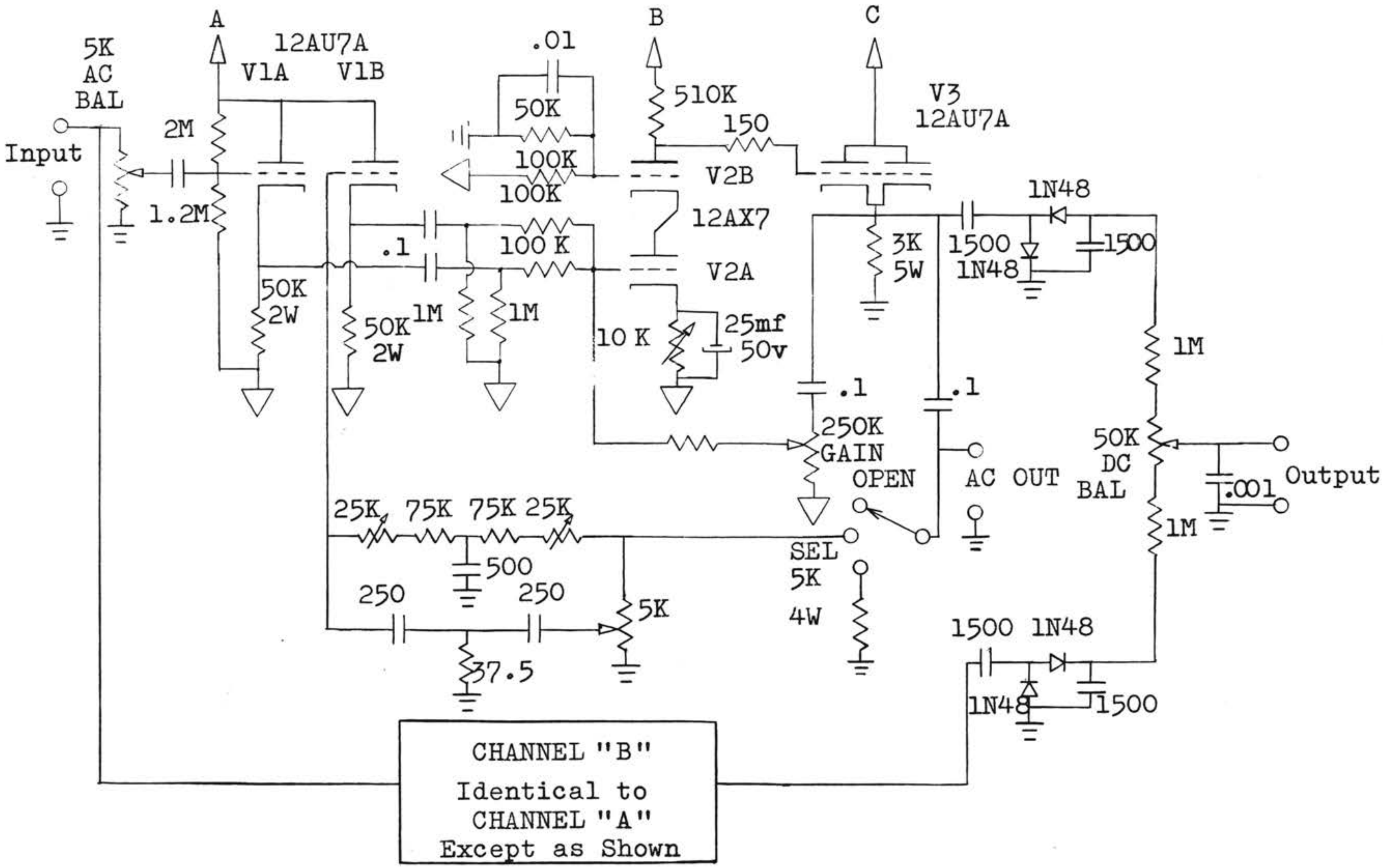

Figure 12. Complete Schematic

impedance bridge. Balanced pairs of 250 $\mu\mu$f capacitors were selected for the series capacitive arms of the network, and the parallel combinations making up the shunt arms were matched to be as near 500 $\mu\mu$f as possible. The resistors were selected on a similar basis, with care that the resistors in the series arms were below the nominal value, of 75K ohms, so that series trimmers could be added.

The nominal resonant frequency of the networks was 8500 cps, but the added resistance needed for trimming to null lowered this frequency to 7600 cps.

Final adjustment of the trimmers was done with the cascode amplifier tube removed from its socket. The frequency control was set to x = 1.0; that is, for minimum frequency. A signal was applied to the grid of V3, and an AC VTVM used to measure the signal at the cathode of V1B. The two trimmers and the frequency of the signal were alternately adjusted for minimum transmission.

This method of adjustment insures that the network will null under actual operating values of driving and load impedances, since the cathode followers isolate the network from the signal source and measuring instrument.

Precision silver-mica capacitors and carbon-film resistors should be used in the networks for best temperature stability, since these have low (and opposite) temperature characteristics. However, a null of -60 db was obtained in the experimental unit, in which 5% encapsulated silver-mica capacitors and 5%, ½watt carbon resistors were used. It was found necessary to use coaxial connection leads to the meter when making this null measurement.

In the output cathode follower circuit, two type 12AU7A tubes are used in parallel to provide a low output impedance. The load resistor is selected so that the operating point (as determined by the plate voltage of V2B) lies just under the maximum plate dissipation limit for the tube. This results in excellent linearity of the stage and a gain exceeding .99. The output impedance, $Z_o \cong r_p/2(u + 1)$, is 160 ohms.

The relatively low load impedance presented by the 5,000 ohms tuning control of the twin-T network could be increased to 10,000 ohms according to the restriction of Equation (55). The value used seems a reasonable compromise between the conflicting requirements of low source impedance for the twin-T and high load impedance for the cathode follower (so that high output voltage can be maintained).

The detector rectifier circuit uses 1N48 germanium diodes. The diodes were matched for output voltage (unloaded) in the circuit used. The circuit is a conventional half-wave voltage doubler with a resistive mixing circuit that gives an output equal to half the difference of the two rectified voltages. The dc output of each side is $1.5\ E_m$.

Adjustment Procedure

The first step in alignment of the discriminator is the adjustment of the twin-T networks as previously described. The resonant frequencies of the two networks are calculated as described on page 35. A preliminary setting of the two frequencies is made with the amplifiers set for maximum gain (maximum Q).

The adjustment of the Q of the two circuits on the experimental discriminator is done by repeatedly adjusting the gain and measuring the bandwidth of the resulting response between the 3-db points. In a unit built for inclusion in a system, a set of calibration curves of Q vs. A for various frequencies could be used to facilitate this adjustment. A single curve would not suffice, because the Q is a function, not only of the gain A, but also of the setting of the tuning potentiometer x. (See Appendix B.)

As the desired Q is approached, the tuning should be readjusted as necessary to maintain the desired resonant frequency.

When the Q is set, the input signal amplitude and the AC Balance control are set to give 20 volts rms at the AC Output of each amplifier at its resonant frequency. The DC Balance is then set to give zero output at the center frequency. This completes the alignment.

CHAPTER VI

TEST PROCEDURES AND RESULTS

A discriminator designed as described in the two preceding chapters was constructed and a number of tests performed both as a check on the design and in a search for areas in which further development might prove desirable. Unfortunately, the scope of this study did not permit investigation of a number of interesting effects which were observed during the testing. The test equipment used is indicated in Table I, page 50.

Tests were made to determine the adjustment range possible. The results are shown in curves A, B, and C, for wideband, low frequency operation, narrow-band low frequency operation, and narrow-band operation at a carrier frequency of 10,000 cps (Figure 13, page 51).

For the latter curve (C), the gains were equalized at the carrier frequency. The Q was 38.2 in each circuit, near the maximum obtainable. Each circuit was then detuned 1.4 half-bandwidths. A sensitivity of 50 mv/cps was obtained over a $\pm$ 100 cps band.

Curve A shows indirectly the effect of changing wRC product on the output of the voltage doubler rectifiers.

TABLE I

TEST EQUIPMENT

Equipment	Purpose
Kepco Model 400B Power Supplies (2)	Plate and Filament Supply
Hewlett-Packard Model 200AB Audio Oscillator	Signal Source
Berkeley Model 5500C Universal Counter and Timer	Measure Frequency of Input
Hewlett-Packard Model 400D (2) AC Vacuum-Tube Voltmeter	Measuring AC Voltage at Input and Output of Feedback Amplifiers
Hewlett-Packard Model 412A DC Vacuum Tube Voltmeter	Measuring DC Voltage Output of discriminator. (Also, driver amplifier for strip-chart recorder.)
Esterline-Angus Model AW Strip-Chart Recorder	Making Continuous Recordings of Output of Discriminator
Tektronix Type 545 Oscilloscope with Type 53/54C Dual-Trace Plug-In Unit	General Wave Form Observation
Tektronix Type 181 Time-Mark Generator	Source of Crystal-Controlled 10 kc Signal for Stability Checks

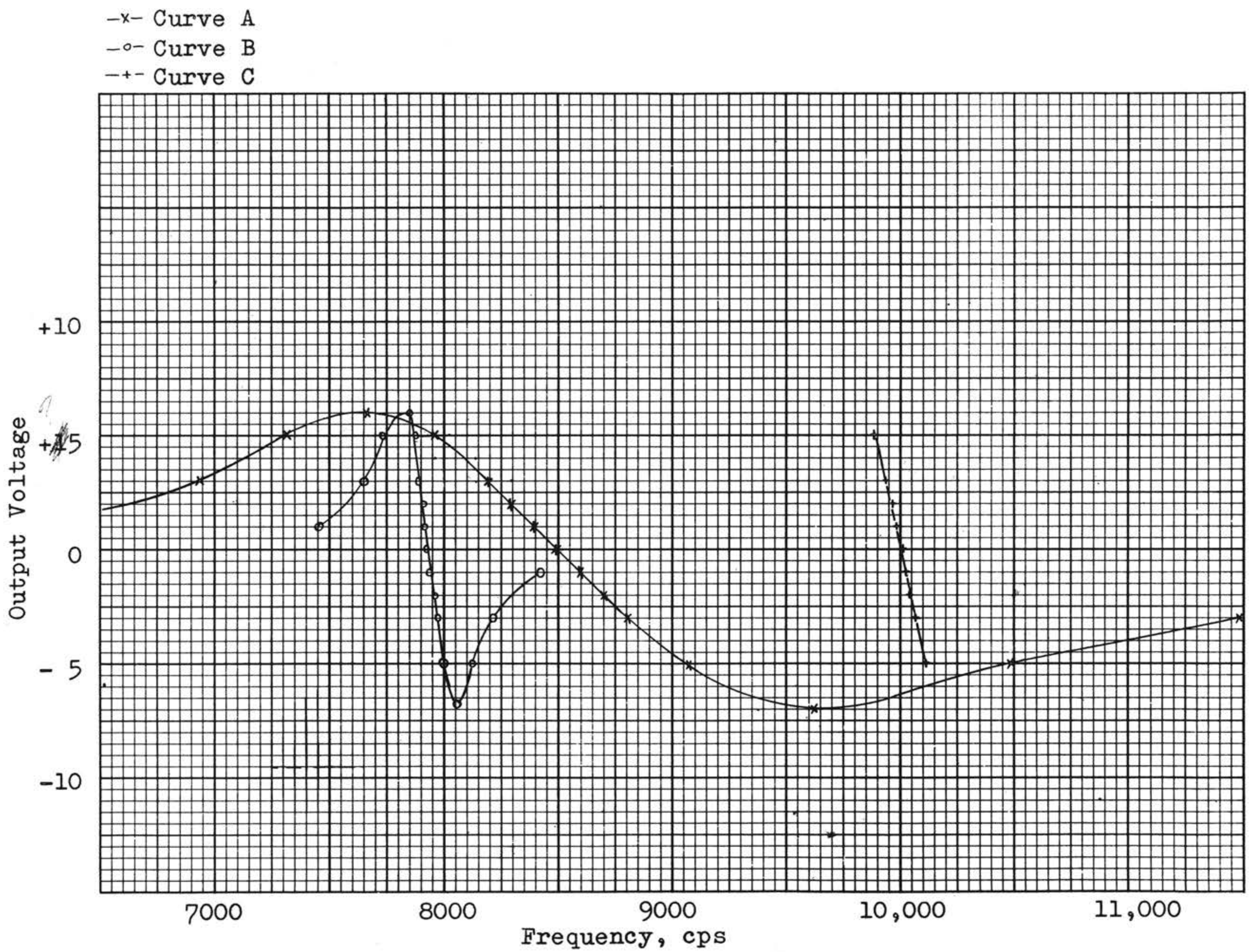

Figure 13. Performance - Available Adjustment Range

Note that the negative swing of the output voltage near 9600 cps is larger in magnitude than the positive swing at 7600 cps, in spite of the fact that the gain of the higher-frequency amplifier was lower than the other. The value of wRC was estimated at 8.0, as determined from the ratio of E_{dc}/E_m and Figure 4, page 23.

Curves D, E, and F were the results of checking more carefully the effect of differences in the Q of the two circuits. These were run with a center frequency of approximately 10,000 cps (Figure 14, page 53).

On Curve D, the Q's were set at 5 at the center frequency. The low value of Q was chosen to give as wide a deviation range as possible. After tuning the two circuits, the Q of the lower frequency one was 6.3, and the higher, 4.3.

For Curve E, $Q_1 = Q_2 = 5$ at the frequencies of operation of the two circuits.

For Curve F, $Q_1/Q_2 = w_1/w_2$, so that $Q_1 = 4.7$, and $Q_2 = 5.3$.

The setting $Q_1 \equiv Q_2$ gives best results. The second-harmonic distortion of the output as a function of input frequency as determined by a three-point analysis, is .42% over a 2000-cycle bandwidth from 8917 cps to 10917 cps. The output voltage over this range is $\pm$ 7 volts, for a sensitivity of 7.1 mv/cps.

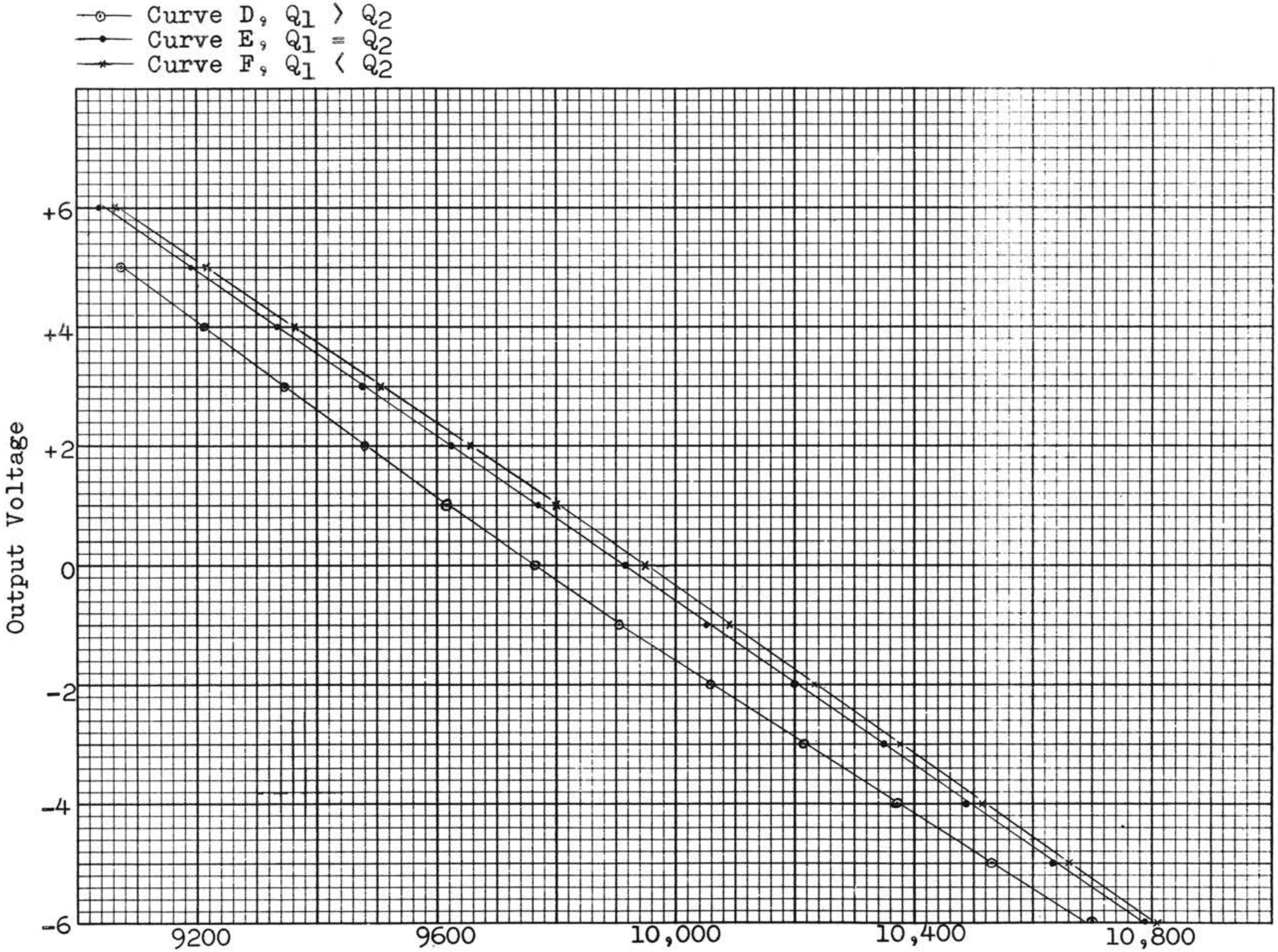

Figure 14. Variation of Output Linearity With Q

Following these tests, the load resistors in the diode rectifier circuits were changed from 200K to 1M to try to determine the effect of this increase in the wRC product. The discriminator was then adjusted by the procedure outlined in the previous chapter for a carrier frequency of 8500 cps and a maximum expected signal of 500 cps. A point-by-point curve was run (Figure 15, page 55).

The second-harmonic distortion, determined by the method previously outlined, is not measurable. The output sensitivity is slightly higher than predicted (page 37), due to the ratio E_{dc}/E_m being slightly higher than the 1.5 assumed. With a 20 volt rms signal applied, the diode voltage doubler output was:

Frequency	7800	9200
Channel "A"	+40.5	+41.5
Channel "B"	-45.1	-46.2

The difference between channels is primarily due to differences in the diode characteristics.

The average E_{dc}/E_m was 43.3/28.3, or 1.53, indicating a value of approximately 15 for wRC (Figure 4, page 23).

The sensitivity was 25.2 millivolts/cps. This sensitivity varies inversely with the bandwidth, since the same $\pm$ 12.6 volt dc output swing should be available with any desired bandwidth setting within the capability of the amplifiers.

In summary, the experimental unit can be adjusted to give the performance described in the specification of

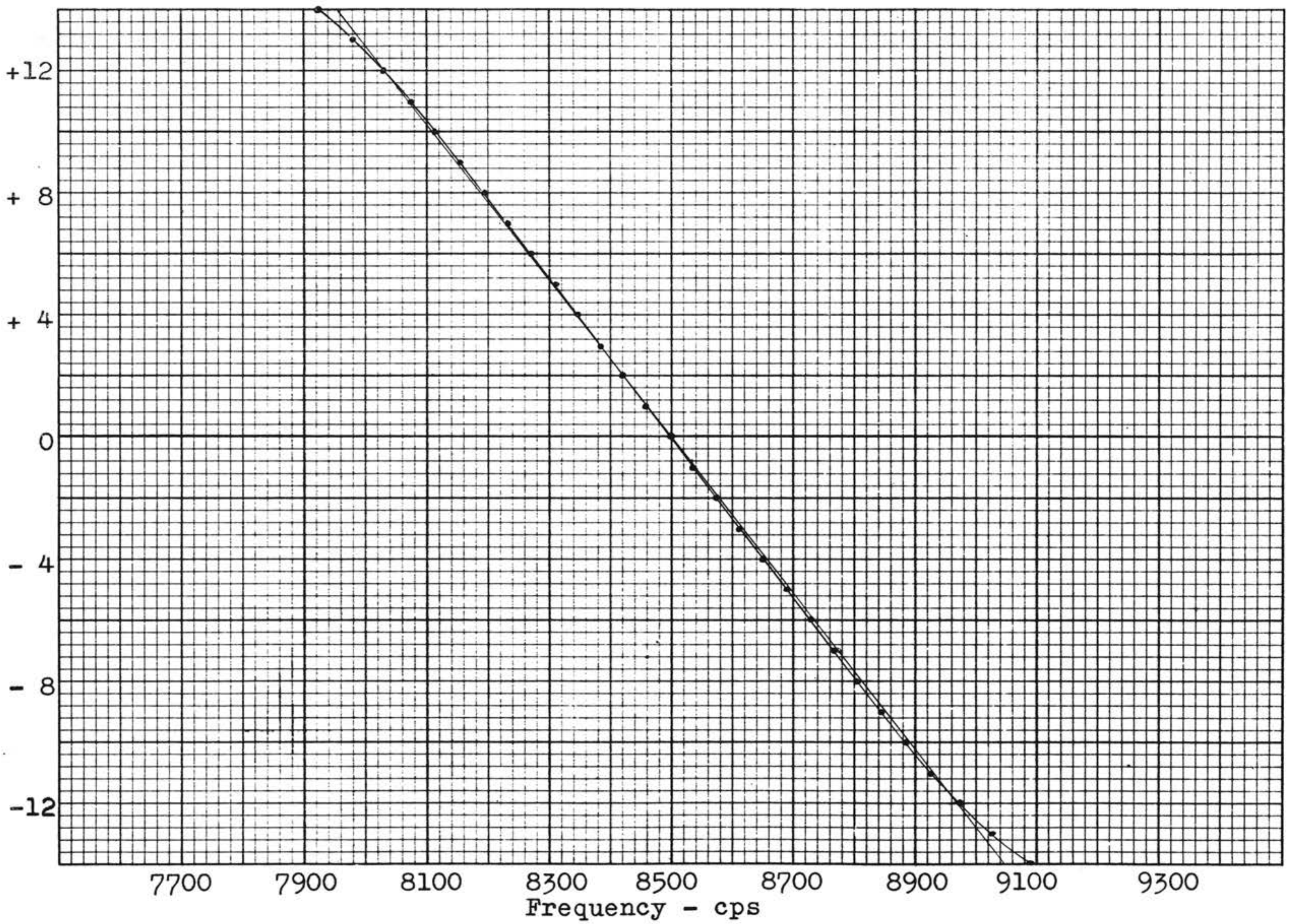

Figure 15. Static Test - Final Circuit and Alignment Procedure

requirements, except that of sensitivity.

The higher sensitivity, if desired, may effectively be obtained by use of a d.c. amplifier on the output of the discriminator.

CHAPTER VII

SUMMARY

This study investigates the design, construction, and operation of a frequency discriminator for use with carrier frequencies in the audio band. The requirements of narrow bandwidth and wide tuning range are met by the use of feedback amplifiers, having twin-T selective networks in the feedback path, to simulate the response of tuned circuits.

A discriminator was constructed which met all the original design requirements.

Investigation of the theory of discriminator response showed that the response, while actually linear in the logarithmic frequency variable, may be assumed linear with respect to frequency over the frequency range with 8% of the carrier frequency. In fact, the effect of variations in input frequency on the output of a capacitive-filtered diode rectifier circuit makes the assumption of frequency linearity often the more accurate.

The twin-T feedback amplifier was found to simulate the response of an RLC circuit to a good approximation within limits of ±20% of the resonant frequency. This amplifier was designed by combining three basic ideas into

an original circuit: the cascode twin-T amplifier, the use of operational amplifier techniques to set the gain, and the tunable twin-T to set the operating frequency.

While this combination was used to simulate the RLC circuit response needed for a frequency-modulation discriminator in this study, a number of other possible uses are recognized:

1. Since the tuning is performed by a single variable potentiometer, the potentiometer may be replaced with a variable gain amplifier, whereupon the feedback amplifier might be used in a swept-frequency analyzer or oscillator at audio frequencies.
2. As a rejection amplifier to eliminate undesired responses in the audio frequency region. It is necessary merely to take the output from the cathode of V1B.
3. As a discriminator for audio frequency shift keying, used to transmit teletype signals.

A number of areas where further study is indicated were found.

A random, erratic drift of low frequency was observed. The cause was not determined, but it was found not to be due to variations in power supply or heater voltages. It is possible that the amplifier tubes used had a slight tendency to drift in g_m. At least part of the drift could be traced to thermal variations in the components of the twin-T networks. It was attenuated over 10:1 by passing the discriminator output through a high-pass RC network having a time constant of 2.0 seconds.

It is possible that a conventional single-stage pentode amplifier might give as good results as the cascode amplifier used here. It would have the advantage of operating satisfactorily without the negative plate supply. The effect of noise due to fluctuations in screen current might, however, lead to problems.

Finally, the experimental unit built is in no way a finished model. It would be interesting to build one using fixed components throughout, to determine how well the feedback technique used eliminates problems due to variations in tube characteristics.

One point is critical in the layout of the circuit. The inductance in the grid circuit and capacitance in the cathode circuit of the output cathode follower must be kept low, to prevent high-frequency oscillation. Oscillation at 800kc was encountered. It caused a sharp break in the output response curve.

This trouble was eliminated by connecting a 100Ω resistor to the grid of V3 at the socket in series with the input from V2B.

SELECTED BIBLIOGRAPHY

(1) Eastman, A. J. Fundamentals of Vacuum Tubes, 3d ed. New York: McGraw-Hill, 1949.

(2) Seeley, S. W. and J. Avins. "The Ratio Detector." RCA Review, VIII (June, 1947), pp. 210-236.

(3) Carson, J. R. "Notes on the Theory of Modulation." Proceedings of the IRE, X (1922), p. 57-64.

(4) Armstrong, E. H. "A Method of Reducing Disturbances in Radio Signaling by a System of Frequency Modulation." Proceedings of the IRE, XXIV (May, 1936), p. 699.

(5) Chaffee, J. G. "The Detection of Frequency Modulated Waves." Proceedings of the IRE, XXIII (May, 1935), p. 517.

(6) Foster, D. E. and S. W. Seeley. "Automatic Tuning, Simplified Circuits and Design Practice." Proceedings of the IRE, XXV (1937), p. 289.

(7) Arguimbau, L. B. Vacuum-Tube Circuits and Transistors. New York: John Wiley and Sons, Inc., 1956.

(8) Bradley, W. E. "Single-Stage Frequency Modulation Detector." Electronics, (October, 1946), pp. 88-91.

(9) Giacolletto, L. D. "Experimental Tube for FM Detection." Electronics, (November, 1949), p. 87.

(10) Adler, Robert. "A Gated Beam Tube." Electronics, (February, 1950), p. 82.

(11) Schlesinger, K. "Sampling Detector for Intercarrier TV Sound." Electronics Engineering Manual, VIII (ca. 1956), New York: McGraw-Hill.

(12) Hulme, V. B. "Improved Frequency Modulation Discriminator." Electronic Engineering, XXIX (September, 1957), p. 416.

(13) Everitt, W. L. Communication Engineering. New York: 1937.

(14) Goldman, Stanford. Transformation Calculus and Electrical Transients. Englewood Cliffs, N.J.: Prentice-Hall, 1949.

(15) Hund, August. Frequency Modulation. New York: McGraw-Hill, 1942.

(16) Burington, R. S. Handbook of Mathematical Tables and Formulas. 3d ed., Sandusky, Ohio: Handbook Publishers, Inc., 1950.

(17) Reich, H. J. Theory and Applications of Electron Tubes, 2d ed. New York: McGraw-Hill, 1944.

(18) Howden, P. F. "Inside-Out Twin-T Varies Rejection Frequency." Electronic Design. (October 14, 1959), p. 198.

(19) Jungmeister, H. G., and H. Y. Konig. "Twin-T Filters." Archiv der Elektrischen Ubertragung, XIV (July, 1960), tr. E. Breuner. Electronic Design, (October 12, 1960), p. 214.

(20) Fleisher, Harold. "Low-Frequency Feedback Amplifiers." Vacuum Tube Amplifiers. Ed. Louis N. Ridenour. New York: McGraw-Hill, 1948, pp. 384-408.

(21) "Resistance Coupled Amplifiers." RCA Receiving Tube Manual, Series RC-20. New Jersey, 1960, p. 390.

(22) MacRae, Duncan Jr. "Low-Frequency Amplifiers With Stabilized Gain." Vacuum-Tube Amplifiers. Ed. Louis N. Ridenour. New York: McGraw-Hill, 1948, pp. 333-383.

(23) Westman, H. P., ed. Reference Data for Radio Engineers, 4th ed. New York: International Telephone and Telegraph Corp., 1956.

APPENDIX A

THE OUTPUT FUNCTION WITH RESPECT TO FREQUENCY

Let $h = (1 + Q^2{}_1 d^2{}_1)^{-\frac{1}{2}} - (1 + Q^2{}_2 d^2{}_2)^{-\frac{1}{2}} = |Z/R|_1 - |Z/R|_2$ where $d_n = p_n - 1/p_n$, and $p_n = w/w_n$, w_n being the resonant frequency of the circuit.

This function can be referred to the center frequency of the discriminator curve, w_x, by introducing the change of variable

$$p_1 = w/w_1 = (w/w_x)(w_x/w_1) \equiv p\gamma_1$$

and

$$p_2 = w/w_2 = (w/w_x)(w_x/w_2) \equiv p\gamma_2^{-1} \quad .$$

Substituting in h,

$$h = [1 + Q^2{}_1(p^2\gamma^2{}_1 + p^{-2}\gamma^{-2}{}_1 - 2)]^{-\frac{1}{2}}$$

$$- [1 + Q^2{}_2(p^2\gamma^{-2}{}_2 + p^{-2}\gamma^2{}_2 - 2)]^{-\frac{1}{2}} .$$

The derivatives with respect to the frequency variable p are as follows:

$$dh/dp = - Q^2{}_1\left|\frac{Z}{R}\right|^3_1 (p\gamma^2{}_1 - p^{-3}\gamma^{-2}{}_1)$$

$$+ Q^2_2\left|\frac{Z}{R}\right|^3_2(p\gamma^{-2}_2 - p^{-3}\gamma^2_2)$$

$$d^2h/dp^2 = - Q^2_1\left|\frac{Z}{R}\right|^3_1(\gamma^2_1 + 3p^{-4}\gamma^{-2}_1)$$

$$- 3 Q^2_1\left|\frac{Z}{R}\right|^5_1(p\gamma^2_1 - p^{-3}\gamma^{-2}_1)^2$$

$$+ Q^2_2\left|\frac{Z}{R}\right|^3_2(\gamma^{-2}_2 + 3p^{-4}\gamma^2_2)$$

$$- 3 Q^2_2\left|\frac{Z}{R}\right|^5_2(p\gamma^{-2}_2 - p^{-3}\gamma^2_2)^2 .$$

The condition for linearity at the center frequency f_x is that $d^2h/dp^2 = 0$ for $p = 0$. Substituting $p = 0$ into the above expression, expanding, and equating like powers of p to zero leads to the following conditions for linearity:

$$Q_1 = Q_2, \gamma_1 = \gamma^{-1}_2, \left|\frac{Z}{R}\right|_1 = \left|\frac{Z}{R}\right|_2 .$$

It may be noted that these are the same conditions as are required for linearity in d.

However, for these conditions,

$$d^2h/dp^2 = 2 Q^2 \left|\frac{Z}{R}\right|^3 (\gamma^2 - \gamma^{-2})$$

which vanishes only if $\gamma^2 = \gamma^{-2}$, or; $\gamma = 1$,

$$w_1 = w_2 = w_x .$$

For this condition, the output is indeed linear -- the straight line: $E_o = 0$.

Thus, the only solution which gives linearity with respect to frequency is the solution of the trivial case.

APPENDIX B

THE TUNABLE TWIN-T NETWORK

The nodal equations of the tunable twin-T circuit (Figure 16) are:

$$\begin{bmatrix} \frac{G_x}{1-x}+G & 0 & -G & 0 & -\frac{G_x}{1-x} \\ 0 & G+jwC & -G & -jwC & 0 \\ -G & -G & 2(G+jwC) & 0 & 0 \\ 0 & -jwC & 0 & 2(G+jwC) & -jwC \\ -\frac{G_x}{1-x} & 0 & 0 & -jwC & \frac{G_x}{x-x^2}+jwC \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix} = \begin{bmatrix} I_o \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{1}{R_x(1-x)} = \frac{G_x}{1-x}$$

$$\frac{1}{xR_x} = \frac{G_x}{x}$$

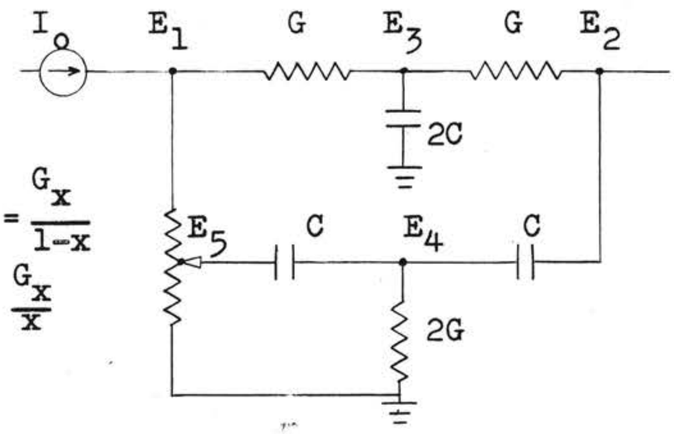

Figure 16. Tunable Twin-T

The output voltage $E_2 = \frac{I_o C_{12}}{\Delta}$, where Δ is the determinant of the coefficients, and C_{12} is the cofactor of the 1,2 term in the determinant of the coefficients. For $\Delta \neq 0$, $I_o \neq 0$, and $E_2 = 0$, the null condition is satisfied if, and only if, $C_{12} \equiv 0$.

Since $w_o = 1/RC = G/C$, $jwC = j(w/w_o)w_oC = jpG$, and

$$C_{12} = -G^4 \begin{vmatrix} 0 & -1 & -jp & 0 \\ -1 & 2(1+jp) & 0 & 0 \\ 0 & 0 & 2(1+jp) & -jp \\ \frac{-G_x}{G(1-x)} & 0 & -jp & \frac{G_x}{G(x-x^2)} + jp \end{vmatrix}$$

$$= -G^4[(G_x/G)(1/x-x^2) + jp] \begin{vmatrix} 0 & -1 & -jp \\ -1 & 2(1+jp) & 0 \\ 0 & 0 & 2(1+jp) \end{vmatrix}$$

$$+ jp \begin{vmatrix} 0 & -1 & -jp \\ -1 & 2(1+jp) & 0 \\ -(G_x/G)(1/1-x) & 0 & -jp \end{vmatrix}.$$

Since $G_x \gg G$, only terms containing the factor G_x/G are significant. Thus

$$C_{12} = -G^4 \left\{ \frac{G_x}{G} \frac{1}{x - x^2} - [-2(1+jp)] + jp[-\frac{G_x}{G} \frac{1}{1-x}(2)(1+jp)(jp)] \right\}$$

$$= -2G^4(G_x/G)\frac{1 + jp}{1 - x}(p^2 - 1/x)$$

At the null, $C_{12} = 0$, $p^2 - 1/x = 0$

$$p^2 = 1/x$$

$$p = w/w_o = 1/\sqrt{x}.$$

In this expression, w is the resonant frequency with $x < 1$, and w_o is the resonant frequency with $x = 1.0$.

The transfer function as a function of p is derived as follows:

$$\frac{E_o}{E_i} = B = \frac{E_2}{E_1} = \frac{I_o C_{12}}{\Delta} \cdot \frac{\Delta}{I_o C_{11}} = \frac{C_{12}}{C_{11}} .$$

$$C_{11} = G^4 \begin{vmatrix} 1 + jp & -1 & -jp & 0 \\ -1 & 2(1 + jp) & 0 & 0 \\ -jp & 0 & 2(1 + jp) & -jp \\ 0 & 0 & -jp & \frac{G_x}{G(x-x^2)} + jp \end{vmatrix}$$

Solution of this in a manner similar to that used for C_{12}, and solving for the transfer function gives

$$\frac{C_{12}}{C_{11}} = - x \frac{p^2 - 1/x}{1 - p^2 + 4jp} = \frac{xp^2 - 1}{(p^2 - 1) - 4jp}$$

$$= \frac{(xp^2 - 1)/(p^2 - 1)}{1 - 4j/d} .$$

If operation is restricted to the frequency range in which

$$|d| \leq 0.4,$$

a useful approximation to the transfer function can be written:

$$\frac{(xp^2-1)/(p^2-1)}{1-4j/d} \cong \frac{(xp^2-1)/(p^2-1)}{-4j/d}$$

$$= \frac{(xp^2-1)/(p^2-1)}{(-4jp)(p^2-1)}$$

$$= (xp^2-1)/(-4jp)$$

$$= j\,\frac{xp^2-1}{4p}$$

where p is the ratio w/w_o. Defining $p' \equiv w/w_x$, where w_x is the frequency at setting x,

$$B_T \cong j\,\frac{xp^2-1}{4p}$$

$$= j\,\frac{x(p'^2/x)-1}{4(p'/\sqrt{x})}$$

$$= j\,\frac{\sqrt{x}}{4}\cdot\frac{p'^2-1}{p'}\ .$$

Thus, in the frequency range for which this approximation is valid ($|\alpha| < 0.4$).

$$|B_{TTT}| = \sqrt{x}\cdot|B_{TT}|,$$

or, the transfer function of the tunable twin-T at a frequency other than its f_o is equal to $\sqrt{x}$ times the transfer function of a twin-T designed for that frequency. This explains, in part, the variation of Q with adjustment of the tuning control.

If, on the other hand, the range $|d'| \leq .4$ is considered, the approximation assumes a different form. For this range, the approximation takes the form:

$$B_{TTT} = \frac{(xp^2 - 1)/(p^2 - 1)}{1 + 4j/d}$$

$$d = p - 1/p = p'/\sqrt{x} - \sqrt{x}/p' = \frac{p'^2 - x}{(\sqrt{x})p'}$$

$$B_{TTT} = \frac{(p'^2 - 1)/(p'^2/x - 1)}{1 - 4j \dfrac{(\sqrt{x})p'}{p'^2 - x}}$$

$$\cong (j/4)\ \frac{p'^2 - 1}{p'}\quad \frac{p'^2 - x}{(1/x)(p'^2 - x)}$$

$$= jd'x/4,$$

provided

$$\frac{(\sqrt{x})p'}{p'^2 - x} \geq 2.5 \ .$$

Since p' may equal 1.0, this is equivalent to

$$\frac{\sqrt{x}}{1 - x} \geq 2.5 \ .$$

In the limit,

$$\sqrt{x} = 2.5 - 2.5x$$

$$x + .4\sqrt{x} - 2.5 = 0$$

from which $\sqrt{x} = .88$.

Thus, within the frequency range $w_x < w < 1.14\ w_x$, the tunable twin-T response varies from the response of the standard twin-T by a constant x. Outside these limits, the response varies in a more complex manner, but the experimental results show it is still usable.

VITA

David Allen Coryell

Candidate for the Degree of

Master of Science

Thesis: DESIGN OF A NARROW-BAND FREQUENCY MODULATION DISCRIMINATOR

Major Field: Electrical Engineering

Biographical:

Personal Data: Born in Emporia, Kansas, March 19, 1930, the son of Allen W. and Bonnie C. Coryell; married Patricia Lewis, of Tulsa, Oklahoma, August 10, 1950; Daughters Susan, Micaela, and Janet, and son Brian David.

Education: Attended grade school in Hitchcock, Oklahoma; graduated from Watonga High School in May, 1948; received the Bachelor of Science Degree from the Oklahoma State University, with a major in Electrical Engineering, in January, 1954; completed requirements for the Master of Science degree in August, 1961.

Professional Experience: Employed while attending the Oklahoma State University by Radio Station KSPI, Stillwater, Oklahoma, 1952-1953; employed by the Kennecott Copper Corporation, Hurley, New Mexico, as an Electronic Technician, 1954-1955; employed by Douglas Aircraft Company, Inc., Tulsa, Oklahoma, as an Electrical Engineer in the Electrical/Electronics Design Section, 1955-1960; employed as Instructor in Electrical Engineering by the Oklahoma State University during the 1960-1961 school year.