ANALYSIS OF CONTINUOUS BENT MEMBERS LOADED OUT OF PLANE BY THE CARRY OVER JOINT MOMENT METHOD

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PREFACE

The author wishes to express indebtness and appreciation to the following persons:

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W. H. C.

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NOMENCLATURE

i, j, k.	Letters designating joints of member
ω	Angle between transformed and basic axis
M_{jix}	Transformed moment at "j" facing "i" in the x direction
$M_{jix'}$	Basic moment at "j" facing "i" in the x' direction
$K_{jix'}$	Basic stiffness factor for span "ji" in x' direction
$C_{x'}K_{jix'}$	Basic carry over stiffness factor in x' direction
K_{jixx}	Transformed stiffness factor
CK_{jixx}	Transformed carry over stiffness factor
$\theta_{jix'}$	Basic end slope in the x' direction
θ_{jx}	Transformed end slope
$FM_{jix'}$	Basic fixed end moment
$EM_{jix'}$	Basic propped end moment
FM_{jix}	Transformed fixed end moment
V_{jiz}	Vertical shear at "j" facing "i"
x', y'	Basic coordinates
x, y	Transformed coordinates

L_{ij}. . . . . Length of span "ij"

r_{ijxx}. . . . . Carry over factor from JM_{ix} to JM_{jx}

JM_{jx} . . . . Joint moment at "j" in the x direction

m_{jx}. . . . . Starting value for JM_{jx}

$\sum$. . . . . Summation

INTRODUCTION

The purpose of this thesis is the demonstration of the derivation and application of the carry over joint moment method for analysis of continuous beams.

Several efforts are recorded in the literature and the oldest method for the analysis of continuous members in space is the method of virtual work. For the application of this method no special reference is given but it is generally accepted that this method was applied to this group of problems in the early part of the century.

The application of the modern philosophy of structural analysis, namely the application of successive approximations, has been reported in this country by Ferguson, Lothers, and Michalos (6, 8, 9). After developing the carry over moment method applied to planar frames, Tuma (2) extended the application of this method to continuous beams and frames in space (4). The derivation presented in the theoretical part of this thesis follows closely Tuma's lectures. The writer's contribution is the derivation of special formulas for special end conditions, the preparation of an example and the calculation of influence values.

The appendix material dealing with sign conventions and transformation matrices was prepared on the basis of Tuma's paper (1) dealing with transformation matrices. Additional references dealing with pipe line design (5) and general slope deflection equations (6, 7) are given.

CHAPTER I

STATEMENT OF THE PROBLEM

A continuous bent member in space is considered. The member lies in one plane and is loaded perpendicular to that plane. The supports are denoted by 0, 1, 2, 3,......i, j, k,.......n. The span lengths are L_1, L_2, L_3,.......L_i, L_j, L_k,.......L_n and the angles between the axes of spans and a selected coordinate system are $\omega_1, \omega_2, \omega_3, \ldots\ldots \omega_i$, $\omega_j, \omega_k, \ldots\ldots \omega_n$ (Fig. 1-1).

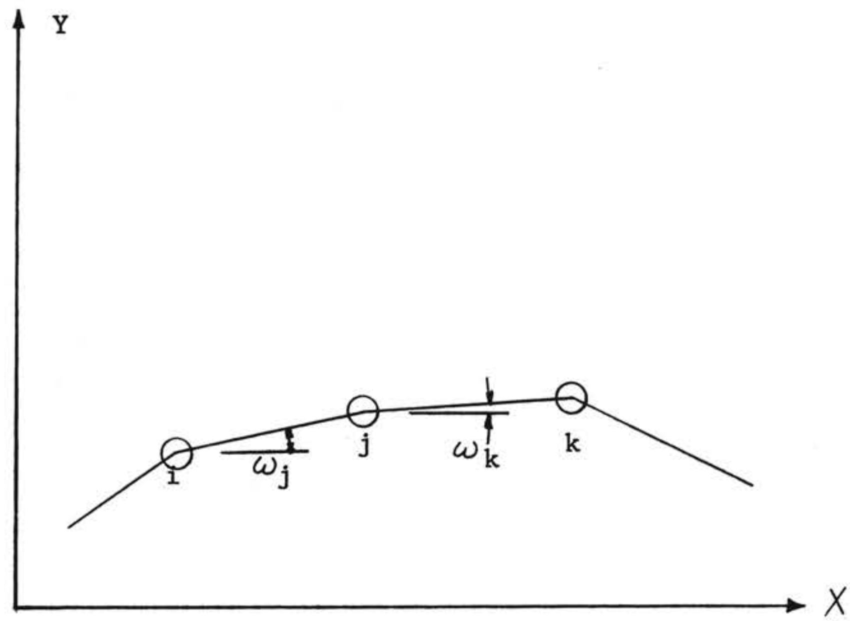

Fig. 1-1 Continuous Bent Member in Space

Vector notation for moments is used and the familiar "right hand rule" governs sign convention. Moments related to the principal axes of the spans will be referred to as basic moments and are denoted by the prime symbol. Moments related to the arbitrarily selected reference system are denoted as transformed moments. As the member is loaded perpendicular to the plane of the member only the vertical shears (V_z) exist and the moment in the vertical direction (M_z) is zero (Fig. 1-2).

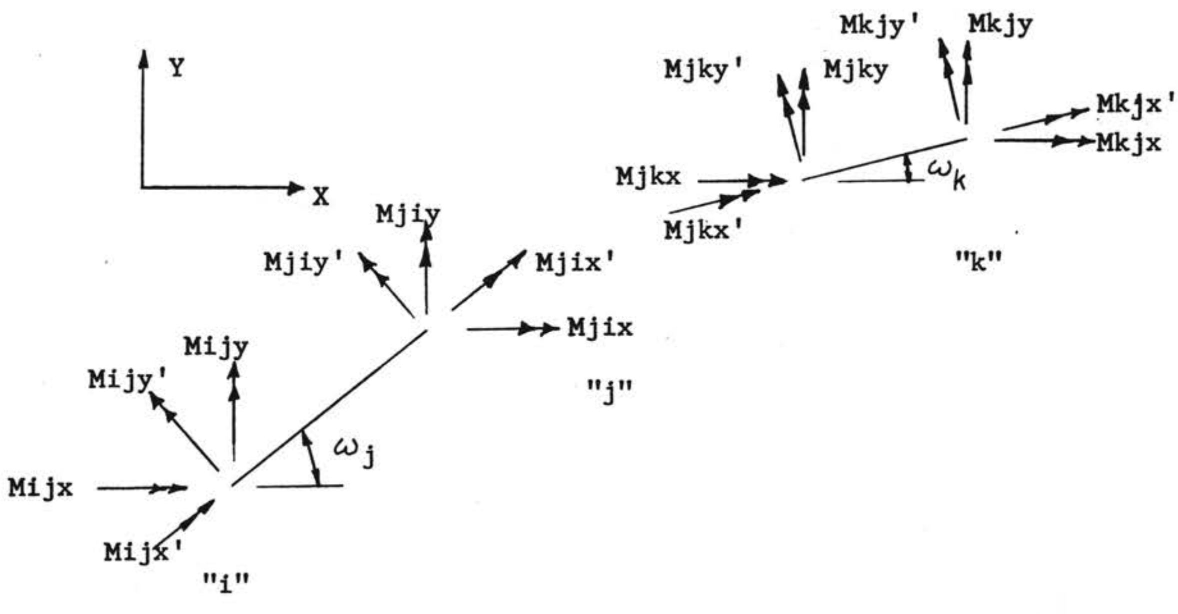

Fig. 1-2 Basic and Transformed Moments

The solution of this type of problem in this discussion will be by the "Carry Over Moment Method" derived by Tuma. The carry over method is a successive approximation which permits the rapid solution of a great number of unknowns to any degree of accuracy. It will become apparent to the reader that as the number of spans of the continuous member increase and consequently the number of unknowns, the solution by the slope deflection method becomes tedious while the labor involved in the carry over method is increased very little. This type of problem is one of many engineering problems to which the carry over method may be applied.

The "Transformation Matrix" applied to the analysis of space structures, as discussed by Tuma, is used extensively in this paper. The transformation matrix provides for a systematic transformation of moments, forces, slopes, conjugate moments, elastic weights, etc. from one coordinate system to another. It is readily seen that the transformation matrix is an invaluable tool in the analysis of space structures.

CHAPTER II

SLOPE DEFLECTION EQUATIONS AT JOINT "j"

2-1. Basic Slope Deflection Equations

The slope deflection equations related to the principal axes of the member "ij" at the end "j" may be expressed in terms of the stiffness factors, carry over stiffness factors, angular rotations, linear displacements, and fixed end moments. Because the supports of the member are rigid the linear displacement terms do not appear in the slope deflection equations. The same may be said about the slope deflection equations for the member "jk" at the end "j". These slope deflection equations will be denoted hereafter as the basic slope deflection equations and the terms in them will be denoted as basic, such as basic stiffness factors, basic fixed end moments, etc. The analytic expressions for these equations follow.

$$
\begin{aligned}
M_{jix'} &= K_{jix'}\theta_{jix'} + C_{x'}K_{ijx'}\theta_{ijx'} + FM_{jix'} \\
M_{jiy'} &= K_{jiy'}\theta_{jiy'} + C_{y'}K_{ijy'}\theta_{ijy'} + FM_{jiy'} \\
M_{jkx'} &= K_{jkx'}\theta_{jkx'} + C_{x'}K_{kjx'}\theta_{kjx'} + FM_{jkx'} \\
M_{jky'} &= K_{jky'}\theta_{jky'} + C_{y'}K_{kjy'}\theta_{kjy'} + FM_{jky'}
\end{aligned}
\qquad (2\text{-}1)
$$

2-2. Transformation of End Slopes and Moments

Because each system of basic slope deflection equations is related to a different set of axes, the direct solution of joint equilibrium is not possible. It is however possible to state the equilibrium of moments about any set of arbitrarily selected axes providing that all quantities in all equations are related to this new set of axes.

In many cases it becomes convenient to select one of the basic systems as the reference axes and to transfer the other systems to it. In order to make the discussion in this thesis completely general, the basic systems for the spans "ji" and "jk" are transferred to a new reference system defined by two transformation matrices (Table 2-1 and Table 2-2).

Table 2-1

	x'	y'
x	α_{jx}	α_{jy}
y	β_{jx}	β_{jy}

Transformation Matrix

for Span "ji"

Table 2-2

	x'	y'
x	α_{kx}	α_{ky}
y	β_{kx}	β_{ky}

Transformation Matrix

for Span "jk"

The first step in the procedure to relate all quantities of the slope deflection equations to the reference axes is to find the basic end slopes in terms of the transformed end slopes. This is accomplished by use of the transformation matrices for the spans "ji" and "jk" (Table 2-1 and Table 2-2). From these tables the basic end slopes in terms of the transformed end slopes are:

$$
\begin{aligned}
\theta_{ijx'} &= \theta_{ix}\alpha_{jx} + \theta_{iy}\beta_{jx} \\
\theta_{ijy'} &= \theta_{ix}\alpha_{jy} + \theta_{iy}\beta_{jy} \\
\theta_{jix'} &= \theta_{jx}\alpha_{jx} + \theta_{jy}\beta_{jx} \\
\theta_{jiy'} &= \theta_{jx}\alpha_{jy} + \theta_{jy}\beta_{jy} \\
\theta_{jkx'} &= \theta_{jx}\alpha_{kx} + \theta_{jy}\beta_{kx} \\
\theta_{jky'} &= \theta_{jx}\alpha_{ky} + \theta_{jy}\beta_{ky} \\
\theta_{kjx'} &= \theta_{kx}\alpha_{kx} + \theta_{ky}\beta_{kx} \\
\theta_{kjy'} &= \theta_{kx}\alpha_{ky} + \theta_{ky}\beta_{ky}
\end{aligned}
\qquad (2\text{-}2)
$$

The expressions for the basic end slopes are now substituted in the basic slope deflection equations. The basic end moments are then in terms of basic fixed end moments and transformed end slopes.

$$M_{jix'} = \begin{matrix} \theta_{jx}K_{jix'}\alpha_{jx} + \theta_{jy}K_{jix'}\beta_{jx} \\ \theta_{ix}C_{x'}K_{ijx'}\alpha_{jx} + \theta_{iy}C_{x'}K_{ijx'}\beta_{jx} \end{matrix} + FM_{jix'}$$

$$M_{jiy'} = \begin{matrix} \theta_{jx}K_{jiy'}\alpha_{jy} + \theta_{jy}K_{jiy'}\beta_{jy} \\ \theta_{ix}C_{y'}K_{ijy'}\alpha_{jy} + \theta_{iy}C_{y'}K_{ijy'}\beta_{jy} \end{matrix} + FM_{jiy'}$$

$$M_{jkx'} = \begin{matrix} \theta_{jx}K_{jkx'}\alpha_{kx} + \theta_{jy}K_{jkx'}\beta_{kx} \\ \theta_{kx}C_{x'}K_{kjx'}\alpha_{kx} + \theta_{ky}C_{x'}K_{kjx'}\beta_{kx} \end{matrix} + FM_{jkx'} \tag{2-3}$$

$$M_{jky'} = \begin{matrix} \theta_{jx}K_{jky'}\alpha_{ky} + \theta_{jy}K_{jky'}\beta_{ky} \\ \theta_{kx}C_{y'}K_{kjy'}\alpha_{ky} + \theta_{ky}C_{y'}K_{kjy'}\beta_{ky} \end{matrix} + FM_{jky'}$$

The second step in the procedure to relate all quantities in the slope deflection equations to the reference axes is to find the transformed moments in terms of the basic end moments. Again, this is accomplished by means of the transformation matrices for the spans "ji" and "jk" (Tables 2-1 and 2-2).

$$
\begin{aligned}
M_{jix} &= M_{jix}'\alpha_{jx} + M_{jiy}'\alpha_{jy} \\
M_{jiy} &= M_{jix}'\beta_{jx} + M_{jiy}'\beta_{jy} \\
M_{jkx} &= M_{jkx}'\alpha_{kx} + M_{jky}'\alpha_{ky} \\
M_{jky} &= M_{jkx}'\beta_{kx} + M_{jky}'\beta_{ky}
\end{aligned}
\qquad (2\text{-}4)
$$

The expressions for the basic end moments in terms of the basic fixed end moments and transformed end slopes (Eq. 2-3) are now substituted in the expressions for the transformed end moments (Eq. 2-4). All quantities in the resulting expressions are now related to the reference system chosen and solution for the unknown end slopes by joint equilibrium is now possible. These expressions are denoted as the transformed slope deflection equations (Eq. 2-5).

$$
\begin{aligned}
& \theta_{jx}(K_{jix}'\alpha_{jx}^2 + K_{jiy}'\alpha_{jy}^2) \\
& \theta_{jy}(K_{jix}'\alpha_{jx}\beta_{jx} + K_{jiy}'\alpha_{jy}\beta_{jy}) \\
M_{jix} = {} & \theta_{ix}(C_xK_{ijx}'\alpha_{jx}^2 + C_yK_{ijy}'\alpha_{jy}^2) \\
& \theta_{iy}(C_xK_{ijx}'\alpha_{jx}\beta_{jx} + C_yK_{ijy}'\alpha_{jy}\beta_{jy}) \\
& \alpha_{jx}FM_{jix}' + \beta_{jy}FM_{jiy}'
\end{aligned}
$$

$$
\begin{aligned}
& \theta_{jx}(K_{jix}'\alpha_{jx}\beta_{jx} + K_{jiy}'\alpha_{jy}\beta_{jy}) \\
& \theta_{jy}(K_{jix}'\beta_{jx}^2 + K_{jiy}'\beta_{jy}^2) \\
M_{jiy} = {} & \theta_{ix}(C_xK_{ijx}'\alpha_{jx}\beta_{jx} + C_yK_{ijy}'\alpha_{jy}\beta_{jy}) \\
& \theta_{iy}(C_xK_{ijx}'\beta_{jx}^2 + C_yK_{ijy}'\beta_{jy}^2) \\
& \beta_{jx}FM_{jix}' + \beta_{jy}FM_{jiy}'
\end{aligned}
$$

(2-5)

$$
\begin{aligned}
& \theta_{jx}(K_{jkx}'\alpha_{kx}^2 + K_{jky}'\alpha_{ky}^2) \\
& \theta_{jy}(K_{jkx}'\alpha_{kx}\beta_{kx} + K_{jky}'\alpha_{ky}\beta_{ky}) \\
M_{jkx} = {} & \theta_{kx}(C_xK_{kjx}'\alpha_{kx}^2 + C_yK_{kjy}'\alpha_{ky}^2) \\
& \theta_{ky}(C_xK_{kjx}'\alpha_{kx}\beta_{kx} + C_yK_{kjy}'\alpha_{ky}\beta_{ky}) \\
& \alpha_{kx}FM_{jkx}' + \alpha_{ky}FM_{jky}'
\end{aligned}
$$

$$
\begin{aligned}
& \theta_{jx}(K_{jkx}'\alpha_{kx}\beta_{kx} + K_{jky}'\alpha_{ky}\beta_{ky}) \\
& \theta_{jy}(K_{jkx}'\beta_{kx}^2 + K_{jky}'\beta_{ky}^2) \\
M_{jky} = {} & \theta_{kx}(C_xK_{kjx}'\alpha_{kx}\beta_{kx} + C_yK_{kjy}'\alpha_{ky}\beta_{ky}) \\
& \theta_{ky}(C_xK_{kjx}'\beta_{kx}^2 + C_yK_{kjy}'\beta_{ky}^2) \\
& \beta_{kx}FM_{jkx}' + \beta_{ky}FM_{jky}'
\end{aligned}
$$

2-3. Transformed Stiffness Factors, Carry Over Stiffness Factors, and Fixed End Moments

The coefficients of the end slopes in the transformed slope deflection equations (Eq. 2-5) are not merely a collection of algebraic terms. They are the transformed stiffness factors, carry over stiffness factors, and transformed fixed end moments. They have definite physical meanings similar to those of the basic stiffness factors, carry over stiffness factors, and basic fixed end moments. These transformed values are tabulated below.

Table 2-3 Transformed Stiffness and Carry over Stiffness Factors

$$
\begin{aligned}
K_{jixx} &= K_{jix}'\alpha_{jx}^2 + K_{jiy}'\alpha_{jy}^2 \\
K_{jiyy} &= K_{jix}'\beta_{jx}^2 + K_{jiy}'\beta_{jy}^2 \\
K_{jixy} &= K_{jix}'\alpha_{jx}\beta_{jx} + K_{jiy}'\alpha_{jy}\beta_{jy} = K_{jiyx}
\end{aligned}
$$

$$
\begin{aligned}
CK_{ijxx} &= C_x'K_{ijx}'\alpha_{jx}^2 + C_y'K_{ijy}'\alpha_{jy}^2 \\
CK_{ijyy} &= C_x'K_{ijx}'\beta_{jx}^2 + C_y'K_{ijy}'\beta_{jy}^2 \\
CK_{ijxy} &= C_x'K_{ijx}'\alpha_{jx}\beta_{jx} + C_y'K_{ijy}'\alpha_{jy}\beta_{jy} = CK_{ijyx}
\end{aligned}
$$

$$
\begin{aligned}
K_{jkxx} &= K_{jkx}'\alpha_{kx}^2 + K_{jky}'\alpha_{ky}^2 \\
K_{jkyy} &= K_{jkx}'\beta_{kx}^2 + K_{jky}'\beta_{ky}^2 \\
K_{jkxy} &= K_{jkx}'\alpha_{kx}\beta_{kx} + K_{jky}'\alpha_{ky}\beta_{ky} = K_{jkyx}
\end{aligned}
$$

$$
\begin{aligned}
CK_{kjxx} &= C_x'K_{kjx}'\alpha_{kx}^2 + C_y'K_{kjy}'\alpha_{ky}^2 \\
CK_{kjyy} &= C_x'K_{kjx}'\beta_{kx}^2 + C_y'K_{kjy}'\beta_{ky}^2 \\
CK_{kjxy} &= C_x'K_{kjx}'\alpha_{kx}\beta_{kx} + C_y'K_{kjy}'\alpha_{ky}\beta_{ky} = CK_{kjyx}
\end{aligned}
$$

Table 2-4 Transformed Fixed End Moments

$$FM_{jix} = FM_{jix}'\alpha_{jx} + FM_{jiy}'\alpha_{jy}$$

$$FM_{jiy} = FM_{jix}'\beta_{jx} + FM_{jiy}'\beta_{jy}$$

$$FM_{jkx} = FM_{jkx}'\alpha_{kx} + FM_{jky}'\alpha_{ky}$$

$$FM_{jky} = FM_{jkx}'\beta_{kx} + FM_{jky}'\beta_{ky}$$

Substituting the values given in Table 2-3 and Table 2-4, the transformed slope deflection equations are rewritten in a more meaningful form below, and these expressions will be used for the transformed slope deflection equations hereafter.

Transformed Slope Deflection Equations:

$$
\begin{aligned}
M_{jix} &= \theta_{jx}K_{jixx} + \theta_{jy}K_{jiyx} + \theta_{ix}CK_{ijxx} + \theta_{iy}CK_{ijyx} + FM_{jix} \\
M_{jiy} &= \theta_{jx}K_{jixy} + \theta_{jy}K_{jiyy} + \theta_{ix}CK_{ijxy} + \theta_{iy}CK_{ijyy} + FM_{jiy} \\
M_{jkx} &= \theta_{jx}K_{jkxx} + \theta_{jy}K_{jkyx} + \theta_{kx}CK_{kjxx} + \theta_{ky}CK_{kjyx} + FM_{jkx} \\
M_{jky} &= \theta_{jx}K_{jkxy} + \theta_{jy}K_{jkyy} + \theta_{kx}CK_{kjxy} + \theta_{ky}CK_{kjyy} + FM_{jky}
\end{aligned}
\tag{2-6}
$$

CHAPTER III

EQUILIBRIUM EQUATIONS AND CARRY OVER FUNCTIONS

3-1. Joint Equilibrium Equations.

In order to maintain equilibrium, the summation of moments about any set of axes at a joint must be zero. Using the transformed slope deflection equations the solution of joint equilibrium is now possible. The summation of moments at "j" are taken about the transformed x and y axes and are stated analytically below.

$$M_{jix} + M_{jkx} = 0$$

$$\left.\begin{array}{l} \theta_{ix}CK_{ijxx} + \theta_{jx}\sum K_{jxx} + \theta_{kx}CK_{kjxx} + FM_{jix} \\ \theta_{iy}CK_{ijyx} + \theta_{jy}\sum K_{jyx} + \theta_{ky}CK_{kjyx} + FM_{jkx} \end{array}\right. = 0 \qquad (3\text{-}1a)$$

$$M_{jiy} + M_{jky} = 0$$

$$\left.\begin{array}{l} \theta_{ix}CK_{ijxy} + \theta_{jx}\sum K_{jxy} + \theta_{kx}CK_{kjxy} + FM_{jiy} \\ \theta_{iy}CK_{ijyy} + \theta_{jy}\sum K_{jyy} + \theta_{ky}CK_{kjyy} + FM_{jky} \end{array}\right. = 0 \qquad (3\text{-}1b)$$

Equations (3-1a) and (3-1b) are the slope deflection joint equilibrium equations. Each equation is a six slope equation and there are two such equations at each joint. These equations may be put in matrix form and the matrix solved for the θ's. The end slopes may be transformed to the basic end slopes and the substitution of the values for the basic end slopes in the basic slope deflection equations will yield the final basic end moments.

3-2. Joint Moments.

A new term, the "joint moment", is now introduced in the joint equilibrium equations in order to put them in the carry over form. The "joint moment" is defined as the product of the rotation at a joint and the summation of stiffness factors related to the rotation at that joint.

$$
\begin{aligned}
JM_{ix} &= \theta_{ix}\sum K_{ixx} \\
JM_{iy} &= \theta_{iy}\sum K_{iyy} \\
JM_{jx} &= \theta_{jx}\sum K_{jxx} \\
JM_{jy} &= \theta_{jy}\sum K_{jyy} \\
JM_{kx} &= \theta_{kx}\sum K_{kxx} \\
JM_{ky} &= \theta_{ky}\sum K_{kyy}
\end{aligned}
\qquad (3\text{-}2)
$$

The joint moments are now substituted in the joint equilibrium equations (3-1a) and (3-1b) and the resulting expressions are shown below.

$$
JM_{jx} = \begin{array}{l} JM_{ix}\left(-\dfrac{CK_{ijxx}}{\sum K_{ixx}}\right) + \left(-\sum FM_{jx}\right) + JM_{kx}\left(-\dfrac{CK_{kjxx}}{\sum K_{kxx}}\right) \\ JM_{iy}\left(-\dfrac{CK_{ijyx}}{\sum K_{iyy}}\right) + JM_{jy}\left(-\dfrac{\sum K_{jyx}}{\sum K_{jyy}}\right) + JM_{ky}\left(-\dfrac{CK_{kjyx}}{\sum K_{kyy}}\right) \end{array} \qquad (3\text{-}3a)
$$

$$
JM_{jy} = \begin{array}{l} JM_{ix}\left(-\dfrac{CK_{ijxy}}{\sum K_{ixx}}\right) + JM_{jx}\left(-\dfrac{\sum K_{jxy}}{\sum K_{jxx}}\right) + JM_{kx}\left(-\dfrac{CK_{kjxy}}{\sum K_{kxx}}\right) \\ JM_{iy}\left(-\dfrac{CK_{ijyy}}{\sum K_{iyy}}\right) + \left(-\sum FM_{jy}\right) + JM_{ky}\left(-\dfrac{CK_{kjyy}}{\sum K_{kyy}}\right) \end{array} \qquad (3\text{-}3b)
$$

3-3. Carry Over Functions.

The joint moment equations (3-3a) and (3-3b) are now in carry over form. The coefficients of the joint moments are the influences they have on the joint moments on the left side of the equations and are defined as the carry over values. The summation of fixed end moments is defined as the starting value.

$$r_{ijxx} = -\frac{CKijxx}{\sum Kixx} \qquad r_{kjxx} = -\frac{CKkjxx}{\sum Kkxx}$$

$$r_{ijyy} = -\frac{CKijyy}{\sum Kiyy} \qquad r_{kjyy} = -\frac{CKkjyy}{\sum Kkyy}$$

$$(3\text{-}4)$$

$$r_{ijxy} = -\frac{CKijxy}{\sum Kixx} \qquad r_{kjxy} = -\frac{CKkjxy}{\sum Kkxx}$$

$$r_{ijyx} = -\frac{CKijyx}{\sum Kiyy} \qquad r_{kjyx} = -\frac{CKkjyx}{\sum Kkyy}$$

$$r_{jjxy} = -\frac{\sum Kjxy}{\sum Kjxx} \qquad r_{jjyx} = -\frac{\sum Kjyx}{\sum Kjyy}$$

$$m_{jx} = -(FMjix + FMjkx) \qquad (3\text{-}5a)$$

$$m_{jy} = -(FMjiy + FMjky) \qquad (3\text{-}5b)$$

3-4. Carry Over Joint Moment Equations.

The substitution of the carry over values (Eq. 3-4) and starting values (Eq. 3-5) in the joint moment equations (3-3) yields the joint moment equations in their final carry over form.

$$JM_{jx} = \begin{array}{l} JM_{ix}r_{ijxx} + m_{jx} + JM_{kx}r_{kjxx} \\ JM_{iy}r_{ijyx} + JM_{jy}r_{jjyx} + JM_{ky}r_{kjyx} \end{array} \qquad (3\text{-}6a)$$

$$JM_{jy} = \begin{array}{l} JM_{ix}r_{ijxy} + JM_{jx}r_{jjxy} + JM_{kx}r_{kjxy} \\ JM_{iy}r_{ijyy} + m_{jy} + JM_{ky}r_{kjyy} \end{array} \qquad (3\text{-}6b)$$

Two carry over joint moment equations may be written at each joint. As will be shown later, the most convenient method of solution for the joint moments is by use of a carry over table in which the joint moments, their carry over values, and their starting values are listed. The joint moments are then approximated in the table to the desired accuracy.

CHAPTER IV

MODIFIED CARRY OVER FUNCTIONS

One or more of the unknown joint moments may be eliminated from the carry over joint moment equations if these equations are modified to meet the requirements of known conditions at a joint. Three special cases are discussed in this chapter; a fixed end, a pinned end, and a member restrained against torsion but free to rotate in the y' direction.

4-1. Fixed End.

Consider the member at "i" fixed in all directions (Fig. 4-1). Then both the basic and transformed slopes at "i" are zero.

Conditions:

$\theta_{ijx'} = 0$

$\theta_{ijy'} = 0$

$\theta_{jx} = 0$

$\theta_{jy} = 0$

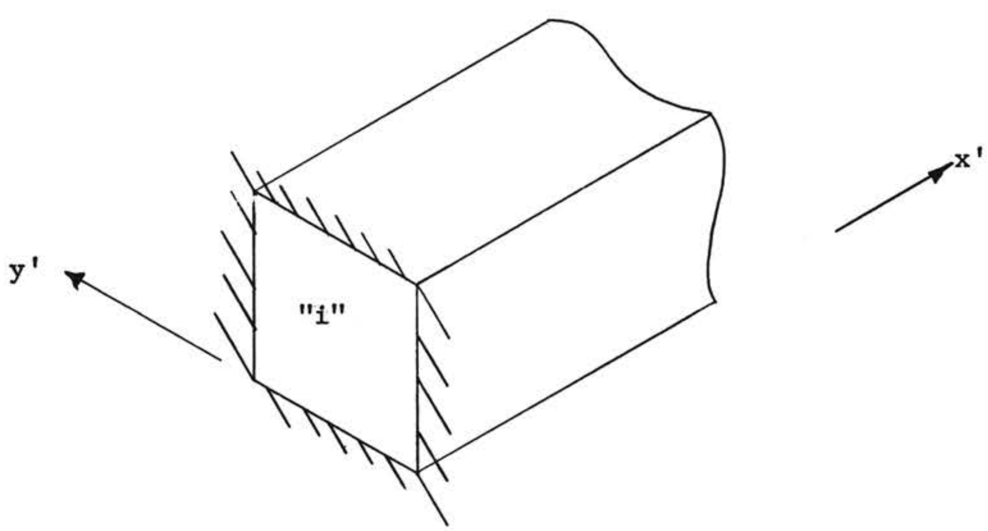

Fig. 4-1 Fixed End

The basic end slopes at "i" are eliminated and the basic slope deflection equations for $M_{jix'}$ and $M_{jiy'}$ become:

$$M_{jix'} = \theta_{jix'} K_{jix'} + FM_{jix'} \qquad (4\text{-}1a)$$

$$M_{jiy'} = \theta_{jiy'} K_{jiy'} + FM_{jiy'} \qquad (4\text{-}1b)$$

The procedure to find the modified carry over joint moment equations is exactly the same as was used to find the general equations. The effect on the carry over joint moment equations is to eliminate the joint moments at the fixed end.

$$JM_{jx} = \begin{array}{l} m_{jx} + JM_{kx} r_{kjxx} \\ JM_{jy} r_{jjyx} + JM_{ky} r_{kjyx} \end{array} \qquad (4\text{-}2a)$$

$$JM_{jy} = \begin{array}{l} JM_{jx} r_{jjxy} + JM_{kx} r_{kjxy} \\ m_{jy} + JM_{ky} r_{kjyy} \end{array} \qquad (4\text{-}2b)$$

4-2. Pinned End.

Consider the member at "i" free to rotate in all directions (Fig. 4-2). Then both the basic and transformed moments at "i" are zero.

Conditions:

$M_{ijx'} = 0$

$M_{ijy'} = 0$

$M_{ijx} = 0$

$M_{ijy} = 0$

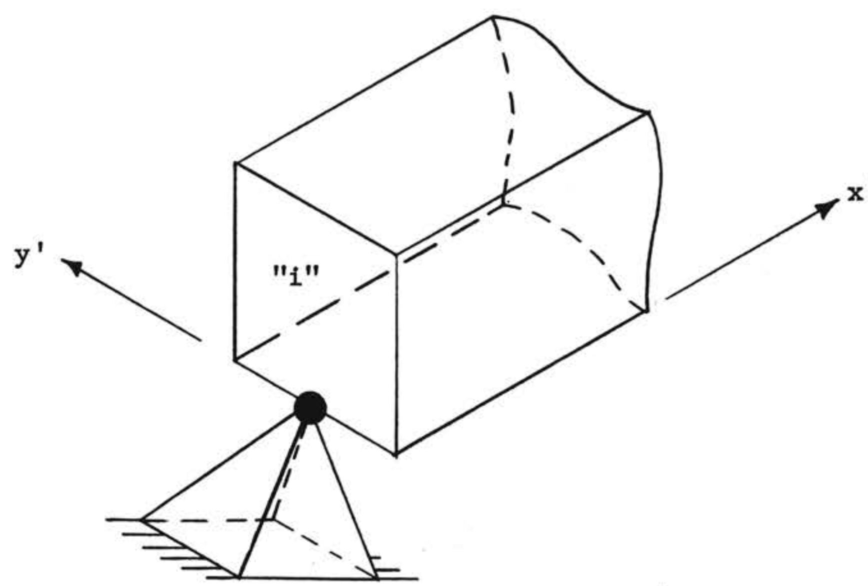

Fig. 4-2 Pinned End

The basic slope deflection equations are written for $M_{jix'}$ and $M_{jiy'}$ using the basic stiffness and carry over stiffness factors modified for a pinned end. These modified factors should be familiar to the reader and are simply stated below.

$$K'_{jix'} = K_{jix'}(1-C_{x'}^2) \qquad (4\text{-}3a)$$

$$K'_{jiy'} = K_{jiy'}(1-C_{y'}^2) \qquad (4\text{-}3b)$$

$$EM_{jix'} = FM_{jix'} - C_{x'}FM_{ijx'} \qquad (4\text{-}4a)$$

$$EM_{jiy'} = FM_{jiy'} - C_{y'}FM_{ijy'} \qquad (4\text{-}4b)$$

The modified basic slope deflection equations are:

$$M_{jix'} = \theta_{jix'}K'_{jix'} + EM_{jix'} \qquad (4\text{-}5a)$$

$$M_{jiy'} = \theta_{jiy'}K'_{jiy'} + EM_{jiy'} \qquad (4\text{-}5b)$$

Again, the same procedure as that used in deriving the general expressions is followed. The modified basic factors replace the regular basic factors in the determination of the transformed stiffness and carry over stiffness factors.

$$K'_{jixx} = K'_{jix'}\alpha_{jx}^2 + K'_{jiy'}\alpha_{jy}^2$$

$$K'_{jixy} = K'_{jiyx} = K'_{jix'}\alpha_{jx}\beta_{jx} + K'_{jiy'}\alpha_{jy}\beta_{jy} \qquad (4\text{-}6)$$

$$K'_{jiyy} = K'_{jix'}\beta_{jx}^2 + K'_{jiy'}\beta_{jy}^2$$

$$EM_{jix} = EM_{jix'}\alpha_{jx} + EM_{jiy'}\alpha_{jy}$$

$$EM_{jiy} = EM_{jix'}\beta_{jx} + EM_{jiy'}\beta_{jy} \qquad (4\text{-}7)$$

These modified transformed values are used to determine the carry over and starting values in the carry over joint moment equations. The joint moments at "i" are eliminated and the carry over joint moment equations become:

$$JM_{jx} = \frac{m'_{jx} + JM_{jy}r'_{jjyx}}{JM_{kx}r_{kjxx} + JM_{ky}r_{kjyx}} \qquad (4\text{-}8a)$$

$$JM_{jy} = \frac{JM_{jx}r'_{jjxy} + JM_{kx}r_{kjxy}}{m'_{jy} + JM_{ky}r_{kjyy}} \qquad (4\text{-}8b)$$

4-3. Torsional Restraint.

Consider the member at "i" fixed in the x' direction and pinned in the y' direction (Fig. 4-3). Then the rotation in the x' direction and the end moment in the y' direction at "i" are zero.

Conditions:

$M_{ijy'} = 0$

$\theta_{ijx'} = 0$

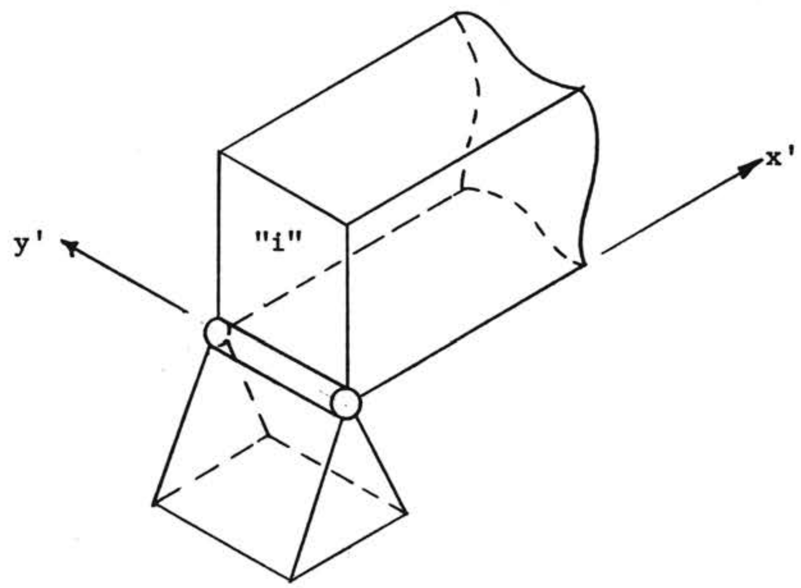

Fig. 4-3 Torsional Restraint

This case is simply a combination of the two previous cases and the modified basic slope deflection equations are:

$$M_{jix'} = \theta_{jix'}K_{jix'} + FM_{jix'} \qquad (4\text{-}9a)$$

$$M_{jiy'} = \theta_{jiy'}K'_{jiy'} + EM_{jiy'} \qquad (4\text{-}9b)$$

Again, those basic values which were modified are used in place of the regular basic values in the general expressions for the transformed values to find the modified transformed stiffness, carry over stiffness factors and end moments.

$$K''_{jixx} = K_{jix'}\alpha_{jx}^2 + K'_{jiy'}\alpha_{jy}^2$$
$$K''_{jixy} = K''_{jiyx} = K_{jix'}\alpha_{jx}\beta_{jx} + K'_{jiy'}\alpha_{jy}\beta_{jy} \qquad (4\text{-}10)$$
$$K''_{jiyy} = K_{jix'}\beta_{jx}^2 + K'_{jiy'}\beta_{jy}^2$$

$$FM'_{jix} = FM_{jix'}\alpha_{jx} + EM_{jiy'}\alpha_{jy}$$
$$FM'_{jiy} = FM_{jix'}\beta_{jx} + EM_{jiy'}\beta_{jy} \qquad (4\text{-}11)$$

These modified transformed values are used in place of the transformed values (Eq. 3-4 and 3-5) to find the carry over and starting values for the carry over joint moment equations.

$$JM_{jx} = \frac{m''_{jx} + JM_{kx}r_{kjxx}}{JM_{jy}r''_{jjyx} + JM_{ky}r_{kjyx}} \qquad (4\text{-}12a)$$

$$JM_{jy} = \frac{JM_{jx}r''_{jjxy} + JM_{kx}r_{kjxy}}{m''_{jy} + JM_{ky}r_{kjyy}} \qquad (4\text{-}12b)$$

It will be observed that if the basic slope deflection equations are modified to meet the requirements of known end conditions, the joint moments at that end are eliminated from the carry over joint moment equations. The procedure in calculating the modified transformed values is exactly the same as in the general derivation, but those basic values which were modified to meet special end conditions are used in place of the regular basic values. The modified transformed values are used in place of the regular transformed values to calculate the modified carry over and starting values. The proper modifications to meet the requirements of special end conditions will often greatly reduce the numerical calculations involved in the analysis of problems of this type.

CHAPTER V

FINAL MOMENTS

The values for the joint moments obtained from the solution of the carry over joint moment equations could be used to find the end slopes. The values for these end slopes could be substituted in the transformed slope deflection equations to find the transformed moments. It would be more desirable, however, to have the expressions for the transformed moments in terms of the joint moments. The values for the joint moments could then be used directly to find the final transformed moments. Substituting the expressions for the end slopes in terms of the joint moments (Eq. 3-2) in the transformed slope deflection equations, the final transformed moments become:

$$M_{jix} = JM_{jx}\frac{K_{jixx}}{\sum K_{jxx}} + JM_{jy}\frac{K_{jiyx}}{\sum K_{jyy}} + JM_{ix}\frac{CK_{ijxx}}{\sum K_{ixx}} + JM_{iy}\frac{CK_{ijyx}}{\sum K_{iyy}} + FM_{jix}$$

$$M_{jiy} = JM_{jx}\frac{K_{jixy}}{\sum K_{jxx}} + JM_{jy}\frac{K_{jiyy}}{\sum K_{jyy}} + JM_{ix}\frac{CK_{ijxy}}{\sum K_{ixx}} + JM_{iy}\frac{CK_{ijyy}}{\sum K_{iyy}} + FM_{jiy}$$

$$M_{jkx} = JM_{jx}\frac{K_{jkxx}}{\sum K_{jxx}} + JM_{jy}\frac{K_{jkyx}}{\sum K_{jyy}} + JM_{kx}\frac{CK_{kjxx}}{\sum K_{kxx}} + JM_{ky}\frac{CK_{kjyx}}{\sum K_{kyy}} + FM_{jkx}$$

$$M_{jky} = JM_{jx}\frac{K_{jkxy}}{\sum K_{jxx}} + JM_{jy}\frac{K_{jkyy}}{\sum K_{jyy}} + JM_{kx}\frac{CK_{kjxy}}{\sum K_{kxx}} + JM_{ky}\frac{CK_{kjyy}}{\sum K_{kyy}} + FM_{jky}$$

(Eq. 5-1) Final Moments in Terms of Joint Moments

The coefficients of the joint moments (Eq. 5-1) are the carry over values (Eq. 3-4) and the distribution factors similar to those used in the moment distribution method. The expressions for the final transformed moments may be rewritten using these values as:

$$M_{jix} = JM_{jx}D_{jixx} + JM_{jy}D_{jiyx} + JM_{ix}r_{ijxx} + JM_{iy}r_{ijyx} + FM_{jix} \qquad (5\text{-}2a)$$

$$M_{jiy} = JM_{jx}D_{jixy} + JM_{jy}D_{jiyy} + JM_{ix}r_{ijxy} + JM_{iy}r_{ijyy} + FM_{jiy} \qquad (5\text{-}2b)$$

$$M_{jkx} = JM_{jx}D_{jkxx} + JM_{jy}D_{jkyx} + JM_{kx}r_{kjxx} + JM_{ky}r_{kjyx} + FM_{jkx} \qquad (5\text{-}2c)$$

$$M_{jky} = JM_{jx}D_{jkxy} + JM_{jy}D_{jkyy} + JM_{kx}r_{kjxy} + JM_{ky}r_{kjyy} + FM_{jky} \qquad (5\text{-}2d)$$

For design purposes it is easier to work with the basic moments instead of the transformed moments. The basic moments are easily found by use of the transformation matrix. (Table 5-1)

Table 5-1

Transformation of Moments

	M_x'	M_y'
M_x	α_{jx}	α_{jy}
M_y	β_{jx}	β_{jy}

$$M_{jx}' = M_x\alpha_{jx} + M_y\beta_{jx}$$

$$M_{jy}' = M_x\alpha_{jy} + M_y\beta_{jy} \qquad (5\text{-}3)$$

Another way to determine the basic moments would be to find the transformed end slopes from the joint moments, transform them to the basic end slopes and substitute the basic end slopes in the basic slope deflection equations.

CHAPTER VI

NUMERICAL PROCEDURE

A systematic procedure for analysis will be outlined in the first part of this chapter. An example problem will be analyzed following the outlined procedure in the second part of this chapter.

6-1. Outline for Numerical Procedure.

a. Transformation Matrices.

A reference system is selected and transformation matrices for each span are established.

b. Basic Stiffness and Carry Over Stiffness Factors.

The basic values are calculated from the properties of the spans. They may be either relative or actual values.

c. Transformed Stiffness and Carry Over Stiffness Factors.

The transformed values are calculated from Table 2-3. Modified basic values are used in place of regular values in this table as they occur.

d. Carry Over Factors.

The carry over factors are calculated from Eq. 3-4.

e. Basic End Moments.

The basic fixed end moments are calculated and modified as required.

f. Transformed End Moments.

The transformed end moments are calculated from Table 2-4. Modified basic end moments are used for regular basic end moments as they occur.

g. Starting Values.

The starting values are calculated from Eq. 3-5.

h. Carry Over Procedure.

The joint moments, their carry over factors, and their starting values are listed in a table. The starting values are multiplied by their carry over factors and the resulting values are "carried over" to the joint moment to which the carry over factors apply. This procedure is repeated until the desired accuracy is obtained. Convergence occurs more rapidly if modified starting values are used as will be shown in the example.

i. Final Moments.

The final transformed moments may be calculated from the joint moments by use of Eq. 5-2 and transformed to the basic moments. Another method would be to find the transformed end slopes (Eq. 3-2), transform them to the basic end slopes, and substitute the basic end slopes in the basic slope deflection equations.

6-2. Example Problem.

A three span continuous bent member is considered. It is simply supported except at the ends it is restrained against torsion. Each span is of constant cross section. It will be analyzed for a uniform lateral load and influence values will be calculated. All dimensions are in feet, all moments are in kip-feet, and all forces are in kips, unless otherwise stated.

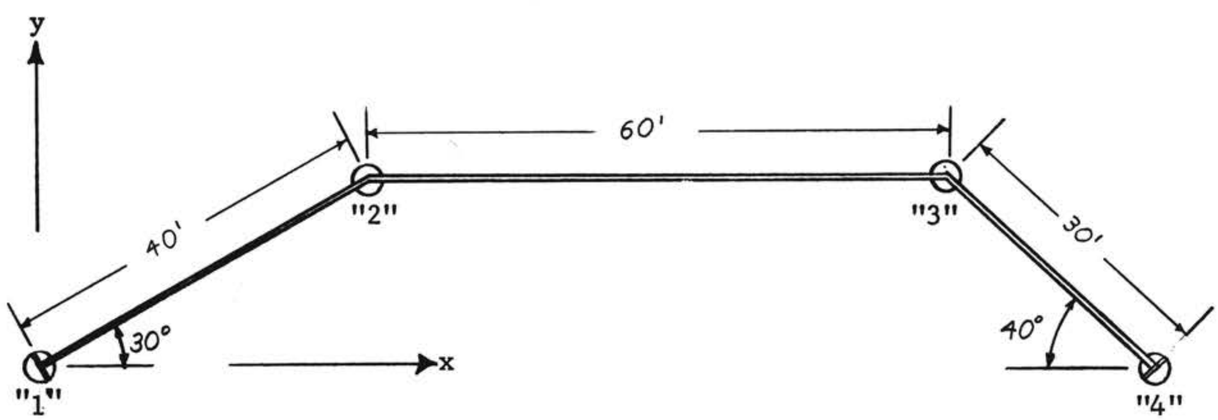

Fig. 6-1 Three Span Continuous Bent Member

a. Transformation Matrix.

The principal axes of span "23" coincide with the selected transformed axes and no transformation is necessary for this span. The transformation matrices for spans "12" and "34" are shown below.

$\underline{\omega_1 2 = 30^o}$

$\alpha_{2x} = 0.8660$

$\beta_{2x} = 0.5000$

$\alpha_{2y} = -0.5000$

$\beta_{2y} = 0.8600$

$\underline{\omega_1 4 = -40^o}$

$\alpha_{4x} = 0.7660$

$\beta_{4x} = -0.6428$

$\alpha_{4y} = 0.6428$

$\beta_{4y} = 0.7660$

	x'	y'
x	.8660	-.5000
y	.5000	.8660

Table 6-1

Transformation Matrix

for

Span "12"

	x'	y'
x	.7660	.6428
y	-.6428	.7660

Table 6-2

Transformation Matrix

for

Span "34"

b. Basic Stiffness and Carry Over Stiffness Factors.

Relative values of basic stiffness and carry over stiffness factors are shown. The basic values are modified as required to conform to known end conditions (4-3).

$$K_{12x'} = K_{21x'} = 1.0$$

$$K_{12y'} = K_{21y'} = 10.0$$

$$K_{23x'} = K_{32x'} = 1.4$$

$$K_{23y'} = K_{32y'} = 12.0$$

$$K_{34x'} = K_{43x'} = 0.8$$

$$K_{34y'} = K_{43y'} = 7.0$$

$$C_{x'} = -1.0$$

(For All Spans)

$$C_{y'} = 0.5$$

$$K'_{21y'} = 10.0(1-.5^2) = 7.50$$

$$K'_{34y'} = 7.0(1-.5^2) = 5.25$$

c. Transformed Stiffness and Carry Over Stiffness Factors.

The transformed values are calculated from Table 2-3. The modified basic values ($K'_{21y'}$ and $K'_{34y'}$) are used in place of the regular basic values.

$$K''_{21xx} = 1.0(.8660)^2 + 7.5(-.5000)^2 = 2.6250$$

$$K''_{21yy} = 1.0(.5000)^2 + 7.5(.8660)^2 = 5.8750$$

$$K''_{21xy} = K''_{21yx} = 1.0(.8660)(.5000) + 7.5(-.5000)(.8660) = -2.8145$$

$$K_{23xx} = K_{32xx} = 1.4$$

$$K_{23yy} = K_{32yy} = 12.0$$

$$K_{23xy} = K_{23yx} = K_{32xy} = K_{32yx} = 0$$

$$CK_{23xx} = CK_{32xx} = -1(1.4) = -1.4$$

$$CK_{23yy} = CK_{32yy} = .5(12.0) = 6.0$$

$$CK_{23xy} = CK_{23yx} = CK_{32xy} = CK_{32yx} = 0$$

$$K''_{34xx} = .8(.7660)^2 + 5.25(.6428)^2 = 2.6387$$

$$K''_{34yy} = .8(-.6428)^2 + 5.25(.7660)^2 = 3.4110$$

$$K''_{34xy} = K''_{34yx} = .8(.7660)(-.6428) + 5.25(.6428)(.7660) = 2.1911$$

$\sum K_{2xx} = 4.0250$ $\sum K_{3xx} = 4.0387$

$\sum K_{2yy} = 17.8750$ $\sum K_{3yy} = 15.4110$

$\sum K_{2xy} = -2.8145$ $\sum K_{3xy} = 2.1911$

$\sum K_{2yx} = -2.8145$ $\sum K_{3yx} = 2.1911$

d. Carry Over Factors.

The carry over factors are calculated from Eq. 3-4 and are shown below.

$$r_{22xy} = -\frac{-2.8145}{4.0250} = +0.6993$$

$$r_{22yx} = -\frac{-2.8145}{17.8750} = +0.1575$$

$$r_{32xx} = -\frac{-1.4}{4.0387} = +0.3466$$

$$r_{32yy} = -\frac{+6.0}{15.4110} = -0.3893$$

$$r_{32xy} = 0$$

$$r_{32yx} = 0$$

$$r_{33xy} = -\frac{+2.1911}{4.0387} = -0.5425$$

$$r_{33yx} = -\frac{+3.1911}{15.4110} = -0.1422$$

$$r_{23xx} = -\frac{-1.4}{4.0250} = +0.3478$$

$$r_{23yy} = -\frac{+6.0}{17.8750} = -0.3357$$

$$r_{23xy} = 0$$

$$r_{23yx} = 0$$

e. Basic Fixed and Propped End Moments.

Consider a uniform load of one kip per foot on all spans.

$$FM = \frac{wL^2}{12} \qquad\qquad EM = \frac{wL^2}{8}$$

$$FM_{21x'} = FM_{23x'} = FM_{32x'} = FM_{34x'} = 0$$

$$EM_{21y'} = \frac{+40^2}{8} = +200 \text{ kip-ft.}$$

$$FM_{23y'} = \frac{-60^2}{12} = -300 \text{ kip-ft.}$$

$$FM_{32y'} = \frac{+60^2}{12} = +300 \text{ kip-ft.}$$

$$EM_{34y'} = \frac{-30^2}{8} = -112.5 \text{ kip-ft.}$$

f., g. Transformed End Moments and Starting Values.

From Eq. (3-5) and Table 2-4 the starting values and transformed end moments are:

$$FM'_{21x} = -.5000(200) = -100 \text{ kip-ft.}$$

$$FM'_{21y} = .8660(200) = 173.2 \text{ kip-ft.}$$

$$FM_{23x} = 0$$

$$FM_{23y} = -300 \text{ kip-ft.}$$

$$FM_{32x} = 0$$

$$FM_{32y} = +300 \text{ kip-ft.}$$

$$FM'_{34x} = .6428(-112.5) = -72.3 \text{ kip-ft.}$$

$$FM'_{34y} = .7660(-112.5) = -86.2 \text{ kip-ft.}$$

$$m_{2x} = +100 \text{ kip-ft.}$$

$$m_{2y} = +126.8 \text{ kip-ft.}$$

$$m_{3x} = +72.3 \text{ kip-ft.}$$

$$m_{3y} = -213.8 \text{ kip-ft.}$$

h. Carry Over Procedure.

Table 6-3

JM2x	JM3x	JM2y	JM3y
.3478	.3466	-.3357	-.3893
.6993		.1575	
	-.5425		-.1422
100.0	72.3	126.8	-213.8
25.1			-39.2
20.0			-42.5
145.1	50.5	115.0	-295.5
	42.0	101.5	
32.1	92.5	216.5	-72.7
34.1			-50.2
66.2	23.0	47.8	-122.9
	17.5	46.3	
14.0	40.5	94.1	-31.6
14.8			-22.0
28.8	10.0	20.9	-53.6
	7.6	20.1	
6.1	17.6	41.0	-13.8
6.5			-9.5
12.6	4.4	9.1	-23.3
	3.3	8.8	
2.7	7.7	17.9	-6.0
2.8			-4.2
5.5	1.9	4.0	-10.2
	1.5	3.8	
1.2	3.4	7.8	-2.6
1.2			-1.8
2.4	.8	1.7	-4.4
	.6	1.7	
.5	1.4	3.4	-1.1
.5			-.8
1.0	.3	.7	-1.9
	.3	.7	
.2	.6	1.4	-.5
.2			-.3
.4	.1	.3	-.8
	.1	.3	
.1	.2	.6	-.2
.1			-.1
.2	.1	.1	-.3
	0	.1	
0	.1	.2	-.1
0			0
0			-.1
262.2	236.3	509.7	-513.0

$\sum$

i. Final Basic Moments.

From Eq. (3-2) the transformed end slopes are:

$$\theta_{2x} = \frac{262.2}{4.0250} = 65.413$$

$$\theta_{3x} = \frac{236.3}{4.0387} = 58.509$$

$$\theta_{2y} = \frac{509.7}{17.8750} = 28.515$$

$$\theta_{3y} = \frac{-513.0}{15.4110} = -33.288$$

From Transformation Matrix:

$$\theta_{21x'} = 65.413(.8660) + 28.515(.5000) = 70.905$$

$$\theta_{21y'} = 65.413(-.5000) + 28.515(.8660) = -8.013$$

$$\theta_{23x'} = 65.413$$

$$\theta_{23y'} = 28.515$$

$$\theta_{32x'} = 58.509$$

$$\theta_{32y'} = -33.288$$

$$\theta_{34x'} = 58.509(.7660) - 33.288(-.6428) = 66.215$$

$$\theta_{34y'} = 58.509(.6428) - 33.288(.7660) = 12.111$$

Substituting the values for the basic end slopes in the basic slopes deflection equations the final basic end moments are:

$M_{12y'} = 0$

$M_{12x'} = -1(1.0)(70.905) = -70.9$

$M_{21x'} = (1.0)(70.905) = +70.9$

$M_{21y'} = 7.50(-8.013) + 200 = +139.9$

$M_{23x'} = 1.4(65.413) - 1(1.4)(58.509) = +9.7$

$M_{23y'} = 12.0(28.515) + .5(12.0)(-33.288) - 300 = -157.6$

$M_{32x'} = 1.4(58.509) - 1(1.4)(65.413) = -9.7$

$M_{32y'} = 12.0(-33.288) + .5(12.0)(28.515) + 300 = +71.6$

$M_{34x'} = .8(66.215) = +53.0$

$M_{34y'} = 5.25(12.111) - 112.5 = -48.9$

$M_{43x'} = -1(.8)(66.215) = -53.0$

$M_{43y'} = 0$

6-2a. Influence Values.

Influence values for the moments and shears at each joint will be calculated for a one pound load moving across the continuous bent member. The influence values will be calculated at the tenth points of each span. The carry over procedure will be done for a starting moment of unity for each joint moment. The actual joint moments are then found by multiplying the joint moments due to unit starting values by the actual starting values. The final transformed and basic end moments are found as outlined (6-1). The shears at the joints are calculated by statics.

Table 6-4 Carry Over Table for $m_{2x} = 1.0000$

	JM2x	JM3x	JM2y	JM3y
	.3478	.3466	-.3357	-.3893
	.6993		.1575	
		-.5425		-.1422
	1.0000	0	0	0
		.3478	0	
		0	.6993	
	.1205	.3478	.6993	-.2348
	.1101			-.1887
	.2306	.0802	.1649	-.4235
		.0602	.1612	
	.0487	.1404	.3261	-.1095
	.0514			-.0762
	.1001	.0345	.0723	-.1857
		.0264	.0693	
	.0211	.0609	.1416	-.0475
	.0223			-.0330
	.0434	.0151	.0313	-.0805
		.0114	.0303	
	.0092	.0265	.0616	-.0207
	.0097			-.0144
	.0189	.0066	.0137	-.0351
		.0050	.0132	
	.0040	.0116	.0269	-.0090
	.0042			-.0063
	.0082	.0028	.0060	-.0153
		.0022	.0057	
	.0017	.0050	.0117	-.0039
	.0018			-.0027
	.0035	.0012	.0026	-.0066
		.0009	.0024	
	.0007	.0021	.0050	-.0017
	.0008			-.0011
	.0015	.0005	.0011	-.0028
		.0004	.0010	
	.0003	.0009	.0021	-.0007
	.0003			-.0005
	.0006	.0002	.0004	-.0012
		.0002	.0004	
	.0001	.0004	.0008	-.0003
	.0001			-.0002
	.0002	.0001	.0002	-.0005
		.0001	.0001	
	0	.0001	.0003	-.0001
	0			-.0001
	0	0	.0001	-.0002
		0	0	
		0	.0001	
Σ	1.4060	0.5957	1.2755	-.7514

Table 6-5 Carry Over Table for m_{3x} = 1.0000

JM2x	JM3x	JM2y	JM3y
.3478	.3466	-.3357	-.3893
.6993		.1575	
	-.5425		-.1422
0	1.0000	0	0
.3466			0
0			-.5425
.3466	.1205	.2112	-.5425
	.0771	.2423	
.0685	.1976	.4535	-.1522
.0714			-.1072
.1399	.0487	.1010	-.2594
	.0369	.0978	
.0297	.0856	.1988	-.0667
.0313			-.0464
.0610	.0212	.0440	-.1131
	.0161	.0427	
.0129	.0373	.0867	.0291
.0137			.0202
.0266	.0093	.0192	-.0493
	.0070	.0186	
.0056	.0163	.0378	-.0127
.0060			-.0088
.0116	.0040	.0084	-.0215
	.0031	.0081	
.0025	.0071	.0165	-.0055
.0026			-.0039
.0051	.0018	.0037	-.0094
	.0013	.0036	
.0011	.0031	.0073	-.0025
.0011			-.0017
.0022	.0008	.0016	-.0042
	.0006	.0015	
.0005	.0014	.0031	-.0010
.0005			-.0008
.0010	.0003	.0007	-.0018
	.0003	.0007	
.0002	.0006	.0014	-.0005
.0002			-.0003
.0004	.0001	.0003	-.0008
	.0001	.0003	
.0001	.0002	.0006	-.0002
.0001			-.0001
.0002	.0001	.0001	-.0003
	0	.0001	
0	.0001	.0002	-.0001
0			-.0001
0			-.0002
	0	.0001	
$\sum$ .5946	1.3493	.8060	-1.0025

Table 6-6 Carry Over Table for m_{2y} = 1.0000

	JM2x	JM3x	JM2y	JM3y
	.3478	.3466	-.3357	-.3893
	.6993		.1575	
		-.5425		-.1422
	0	0	1.0000	0
	0			-.3357
	.1575			0
	.1575	.0548	.1307	-.3357
		.0478	.1101	
	.0356	.1026	.2408	-.0808
	.0379			-.0557
	.0735	.0256	.0531	-.1365
		.0194	.0514	
	.0156	.0450	.1045	-.0351
	.0165			-.0244
	.0321	.0112	.0232	-.0595
		.0085	.0224	
	.0068	.0197	.0456	-.0153
	.0072			-.0107
	.0140	.0049	.0101	-.0260
		.0037	.0098	
	.0030	.0086	.0199	-.0067
	.0031			-.0047
	.0061	.0021	.0044	-.0114
		.0016	.0043	
	.0013	.0037	.0087	-.0029
	.0014			-.0020
	.0027	.0009	.0019	-.0049
		.0007	.0019	
	.0006	.0016	.0038	-.0013
	.0006			-.0009
	.0012	.0004	.0009	-.0022
		.0003	.0008	
	.0002	.0007	.0017	-.0006
	.0003			-.0004
	.0005	.0002	.0004	-.0010
		.0001	.0003	
	.0001	.0003	.0007	-.0002
	.0001			-.0002
	.0002	.0001	.0002	-.0004
		0	.0001	
	0	.0001	.0003	-.0001
	0			-.0001
	0	0	.0001	-.0002
		0	0	
		0	.0001	
$\sum$	.2878	.1823	1.4261	-.5778

Table 6-7 Carry Over Table for m_{3y} = 1.0000

JM2x	JM3x	JM2y	JM3y
.3478	.3466	-.3357	-.3893
.6993		.1575	
	-.5425		-.1422
0	0	0	1.0000
	0	-.3893	
	-.1422	0	
-.0493	-.1422	-.3893	.1307
-.0613			.0771
-.1106	-.0385	-.0809	.2078
	-.0295	-.0773	
-.0236	-.0680	-.1582	.0531
-.0249			.0369
-.0485	-.0169	-.0350	.0900
	-.0128	-.0339	
-.0103	-.0297	-.0689	.0231
-.0109			.0161
-.0212	-.0074	-.0153	.0392
	-.0056	-.0148	
-.0045	-.0130	-.0291	.0098
-.0046			.0071
-.0091	-.0032	-.0066	.0169
	-.0024	-.0064	
-.0019	-.0056	-.0130	.0044
-.0020			.0030
-.0039	-.0014	-.0029	.0074
	-.0011	-.0027	
-.0009	-.0025	-.0056	.0019
-.0009			.0014
-.0018	-.0006	-.0013	.0033
	-.0005	-.0013	
-.0004	-.0011	-.0026	.0009
-.0004			.0006
-.0008	-.0003	-.0006	.0015
	-.0002	-.0006	
-.0002	-.0005	-.0012	.0004
-.0002			.0003
-.0004	-.0001	-.0003	.0007
	-.0001	-.0003	
-.0001	-.0002	-.0006	.0002
-.0001			.0001
-.0002	-.0001	-.0001	.0003
	0	-.0001	
0	-.0001	-.0002	.0001
0			.0001
0	0	-.0001	.0002
	0	0	
	0	-.0001	
$\sum$ -.1965	-.2630	-.6688	1.3673

Actual Starting Values:

$$m''_{2x} = -(FM'_{21x} + FM_{23x})$$

$$m''_{2y} = -(FM'_{21y} + FM_{23y})$$

$$m''_{3x} = -(FM_{32x} + FM'_{34x})$$

$$m''_{3y} = -(FM_{32y} + FM'_{34y})$$

Actual Starting Values in Terms of Basic End Moments:

$$m''_{2x} = -EM_{21y'}\alpha_{2y} = +.5000\ EM_{21y'}$$

$$m''_{2y} = -(EM_{21y'}\beta_{2y} + FM_{23y'}) = -.8660EM_{21y'} - FM_{23y'}$$

$$m''_{3x} = -EM_{34y'}\alpha_{4y} = -.6428EM_{34y'}$$

$$m''_{3y} = -(FM_{32y'} + EM_{34y'}\beta_{4y}) = -FM_{32y'} - .76600EM_{34y'}$$

From Tables 6-4, 6-5, 6-6, and 6-7 the joint moments are found in terms of the actual starting values.

Table 6-8

Joint Moments in Terms of Starting Values

	m''_{2x}	m''_{3x}	m''_{2y}	m''_{3y}
JM_{2x}	1.4060	.5946	.2878	-.1965
JM_{2y}	1.2755	.8060	1.4261	-.6688
JM_{3x}	.5957	1.3493	.1823	-.2630
JM_{3y}	-.7514	-1.0025	-.5778	1.3673

Table 6-9

Joint Moments in Terms of Basic End Moments

	$EM_{21y'}$	$FM_{23y'}$	$FM_{32y'}$	$EM_{34y'}$
JM_{2x}	.4538	-.2878	.1965	-.2317
JM_{2y}	-.5973	-1.4261	.6688	-.0058
JM_{3x}	.1400	-.1823	.2630	-.6659
JM_{3y}	.1247	.5778	-1.3673	-.4029

Table 6-10

Transformed Moments in Terms of Joint Moments

	JM2x	JM3x	JM2y	JM3y	
M21x	.6522		-.1575		-.5EM21y'
M23x	.3478	-.3466			
M32x	-.3478	.3466			
M34x		.6534		.1422	.6428EM34y'
M21y	-.6993		.3287		.8660EM21y'
M23y			.6713	.3893	FM23y'
M32y			.3357	.7787	FM32y'
M34y		.5425		.2213	.7660EM34y'

Table 6-11

Transformed Moments in Terms of Basic Values

	EM21y'	FM23y'	FM32y'	EM34y'
M21x	-.1093	.0369	.0229	-.1502
M23x	.1093	-.0369	-.0229	.1502
M32x	-.1093	.0369	.0229	-.1502
M34x	.1093	-.0369	-.0229	.1502
M21y	.3524	-.2676	.0833	.1608
M23y	-.3524	.2676	-.0833	-.1608
M32y	-.1034	.0288	.1598	-.3156
M34y	.1034	.0288	-.1598	.3156

From Transformation Matrices:

$M_{21x'} = .8660M_{21x} + .5000M_{21y}$

$M_{23x'} = M_{23x}$

$M_{32x'} = M_{32x}$

$M_{34x'} = .7660M_{34x} - .6428M_{34y}$

$M_{21y'} = -.5000M_{21x} + .8660M_{21y}$

$M_{23y'} = M_{23y}$

$M_{32y'} = M_{32y}$

$M_{34y'} = .6428M_{34x} + .7660M_{34y}$

Table 6-12 Basic End Moments in Terms of Basic Fixed and Propped End Moments

	$EM_{21y'}$	$FM_{23y'}$	$FM_{32y'}$	$EM_{34y'}$
$M_{21x'}$	.081	-.102	.061	-.050
$M_{23x'}$	.109	-.037	-.023	.150
$M_{32x'}$	-.109	.037	.023	-.150
$M_{34x'}$	.017	-.046	.085	-.088
$M_{21y'}$	.360	-.250	.061	.214
$M_{23y'}$	-.352	.268	-.083	-.161
$M_{32y'}$	-.103	-.029	.160	-.316
$M_{34y'}$	.149	-.002	-.137	.338

Table 6-13

Basic Fixed and Propped End Moments

n	$EM_{21y'}$	$FM_{23y'}$	$FM_{32y'}$	$EM_{34y'}$
0.0	0	0	0	0
.1	1.980	-4.860	0.540	-2.565
.2	3.840	-7.680	1.920	-4.320
.3	5.464	-8.820	3.780	-5.355
.4	6.720	-8.640	5.760	-5.760
.5	7.500	-7.500	7.500	-5.625
.6	7.680	-5.760	8.640	-5.040
.7	7.140	-3.780	8.820	-4.098
.8	5.760	-1.920	7.680	-2.880
.9	3.420	-0.540	4.860	-1.485
1.0	0	0	0	0

Table 6-14

Final Basic End Moments

n	M_{12x}'& $-M_{21x}'$	M_{32x}'& $-M_{23x}'$	M_{34x}'& $-M_{43x}'$	M_{21y}'	M_{23y}'	M_{32y}'	M_{34y}'
1.0	0	0	0	0	0	0	0
.1	-.160	-.216	.034	.713	-.698	-.205	.295
.2	-.311	-.420	.065	1.382	-1.353	-.397	.572
.3	-.443	-.497	.093	1.967	-1.926	-.565	.814
.4	-.544	-.735	.114	2.419	-2.368	-.695	1.001
.5	-.608	-.820	.128	2.700	-2.643	-.776	1.118
.6	-.622	-.839	.131	2.765	-2.706	-.794	1.144
.7	-.578	-.780	.121	2.570	-2.516	-.738	1.064
.8	-.467	-.630	.098	2.074	-2.023	-.596	.858
.9	-.277	-.374	.058	1.231	-1.205	-.354	.510
2.0	0	0	0	0	0	0	0
.1	-.529	-.167	.270	1.248	-1.346	.226	-.064
.2	-.900	-.239	.516	2.037	-2.215	.528	-.248
.3	-1.131	-.239	.727	2.436	-2.675	.858	-.500
.4	-1.232	-.187	.887	2.511	-2.792	1.169	-.772
.5	-1.223	-.105	.983	2.333	-2.632	1.415	-1.013
.6	-1.115	-.015	.999	1.967	-2.261	1.547	-1.172
.7	-.924	.063	.924	1.483	-1.746	1.518	-1.200
.8	-.664	.105	.741	.948	-1.154	1.283	-1.048
.9	-.351	.091	.438	.431	-.549	.792	-.665
3.0	0	0	0	0	0	0	0
.1	-.128	.385	.226	-.549	.413	.810	-.867
.2	-.216	.649	.380	-.924	.695	1.363	-1.460
.3	-.268	.804	.471	-1.146	.861	1.690	-1.810
.4	-.288	.865	.507	-1.233	.926	1.818	-1.947
.5	-.281	.845	.495	-1.204	.905	1.775	-1.901
.6	-.252	.757	.444	-1.079	.810	1.591	-1.704
.7	-.205	.616	.361	-.877	.659	1.293	-1.385
.8	-.144	.433	.253	-.616	.463	.909	-.973
.9	-.074	.223	.131	-.318	.239	.469	-.502
4.0	0	0	0	0	0	0	0

Table 6-15

Final End Shears

Sta.	V12z	V21z	V23z	V32z	V34z	V43z
1.0	1.000	0	0	0	0	0
.1	.882	.118	.015	-.015	-.010	.010
.2	.765	.235	.029	-.029	-.019	.019
.3	.651	.349	.042	-.042	-.027	.027
.4	.540	.460	.051	-.051	-.033	.033
.5	.432	.568	.057	-.057	-.037	.037
.6	.331	.669	.058	-.058	-.038	.038
.7	.236	.764	.054	-.054	-.035	.035
.8	.148	.852	.044	-.044	-.020	.029
.9	.069	.931	.026	-.026	-.017	.017
2.0	0	1.000 / 0	0 / 1.000	0	0	0
.1	-.031	.031	.919	.081	.002	-.002
.2	-.051	.051	.828	.172	.008	-.008
.3	-.061	.061	.730	.270	.017	-.017
.4	-.063	.063	.627	.373	.026	-.026
.5	-.058	.058	.520	.480	.034	-.034
.6	-.049	.049	.412	.588	.039	-.039
.7	-.037	.037	.304	.696	.040	-.040
.8	-.024	.024	.198	.802	.035	-.035
.9	-.011	.011	.096	.904	.022	-.022
3.0	0	0	0	1.000 / 0	0 / 1.000	0
.1	.014	-.014	-.020	.020	.929	.071
.2	.023	-.023	-.034	.034	.849	.151
.3	.029	-.029	-.043	.043	.760	.240
.4	.031	-.031	-.046	.046	.665	.335
.5	.030	-.030	-.045	.045	.563	.437
.6	.027	-.027	-.040	.040	.457	.543
.7	.022	-.022	-.033	.033	.346	.654
.8	.015	-.015	-.023	.023	.232	.768
.9	.008	-.008	-.012	.012	.117	.883
4.0	0	0	0	0	0	1.000

CHAPTER VII

SUMMARY AND CONCLUSIONS

In this study the general procedure for the analysis of continuous bent members in one plane loaded perpendicular to that plane by the "Carry Over Method," is presented. This thesis may be extended for the analysis of continuous bent members not in one plane.

The presented procedure is adequate for application in engineering practice. It is suggested that the carry over procedure be used when the number of unknowns reaches four or more.

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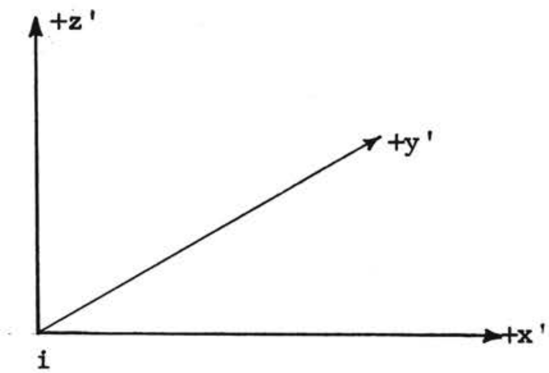
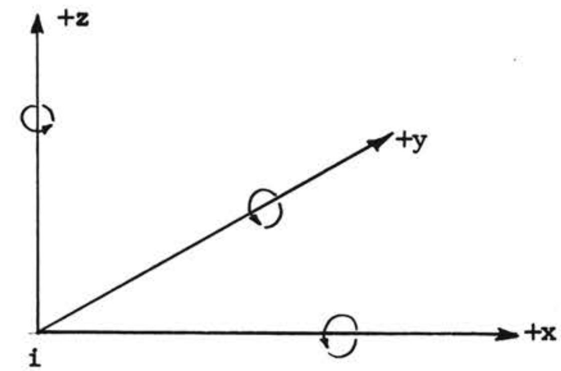
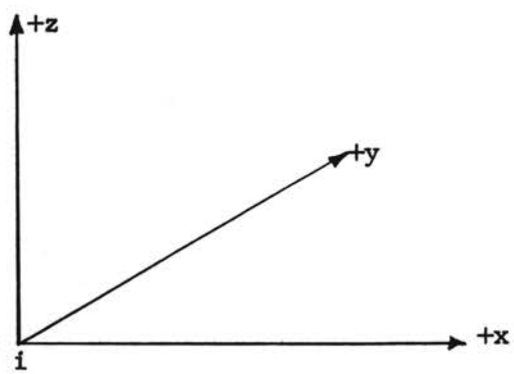
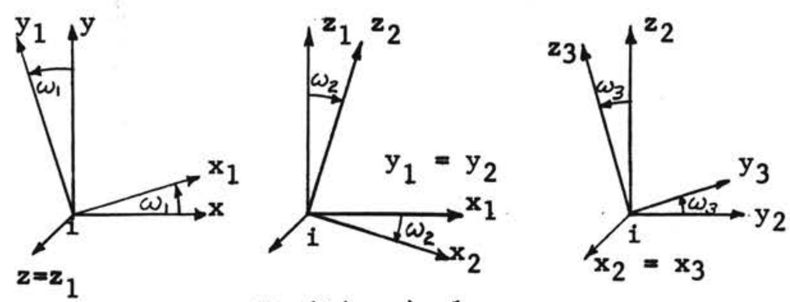
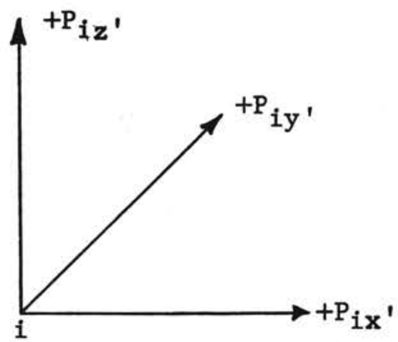
APPENDIX

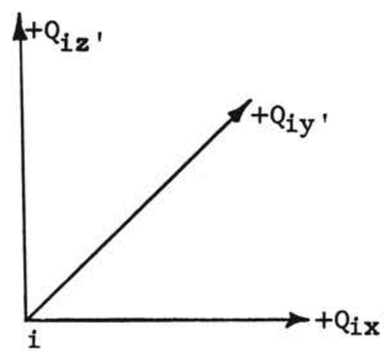
The application of the transformation matrix to the analysis of space structures is discussed by Tuma (1). Several tables from his paper are shown in this appendix. The tables are for a completely general space structure and apply equally well to the structure in one plane discussed in this thesis. In the case discussed in this thesis the "z" terms simply vanish from the general transformation matrices. "ω_2" and "ω_3" are zero and are used as such in the determination of the transformation matrices (Table F).

Geometry	Table A
 Basic Coordinates	 Positive Rotations
 Transformed Coordinates	 Positive Angles

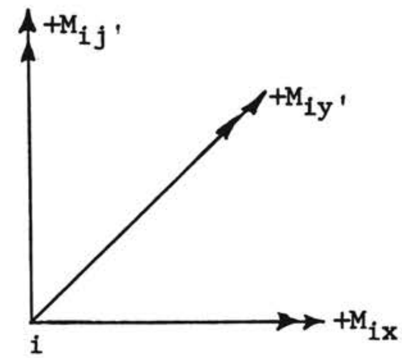
Real Forces and Moments   Table B



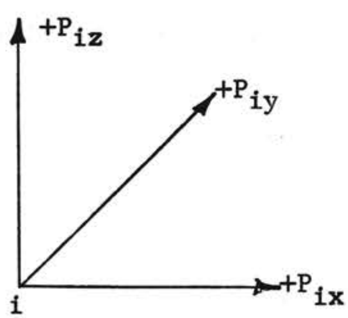
Basic Loads



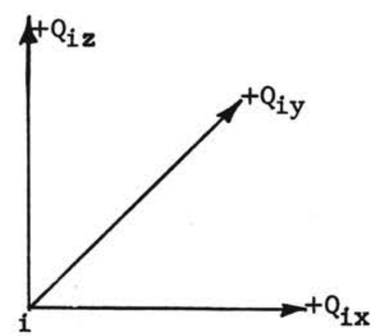
Basic Forces



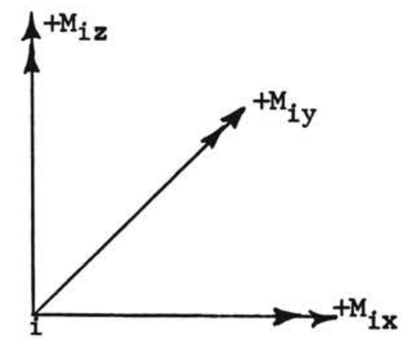
Basic Moments



Transformed Loads

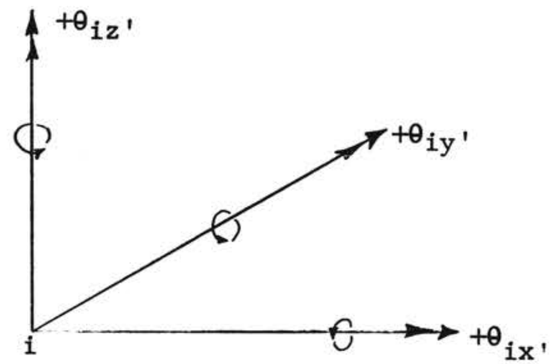


Transformed Forces

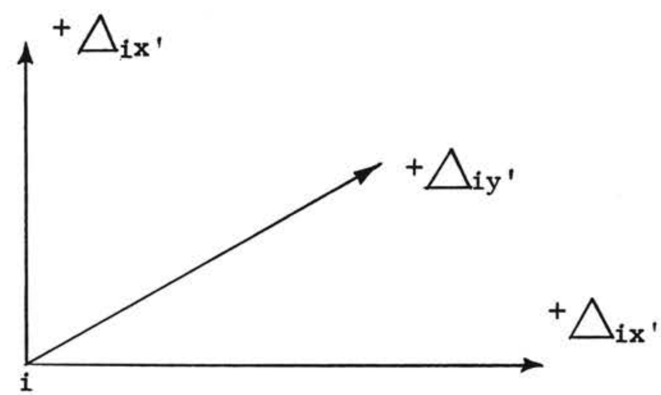


Transformed Moments

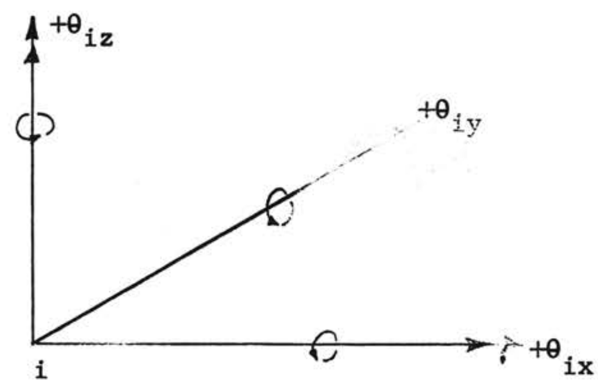
Rotations and Displacements Table C



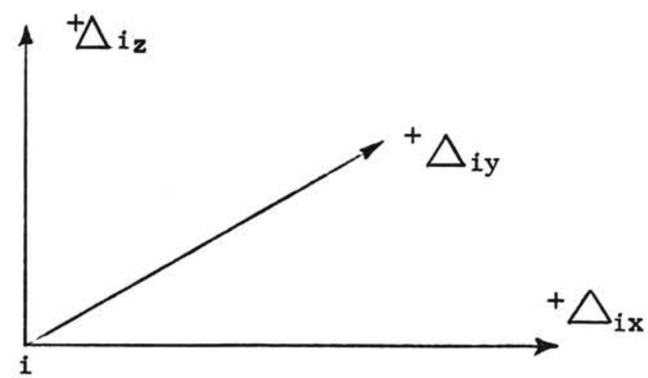
Basic Rotations



Basic Displacements

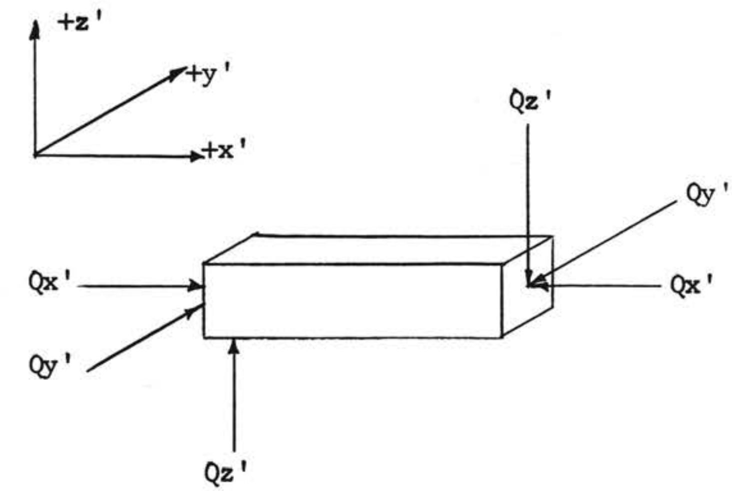


Transformed Rotations

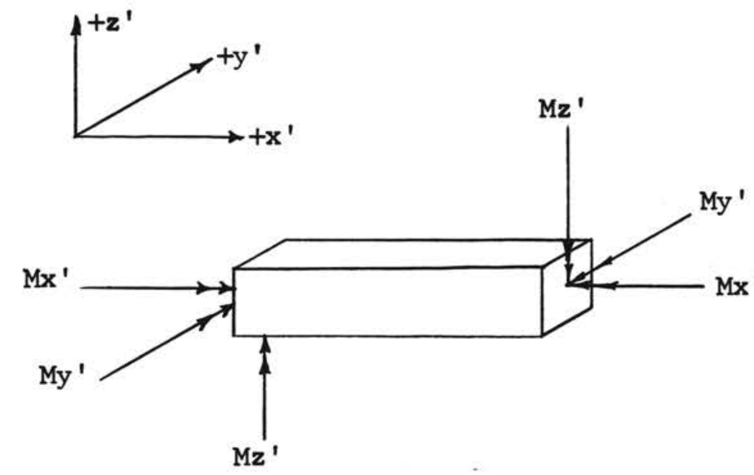


Transformed Displacements

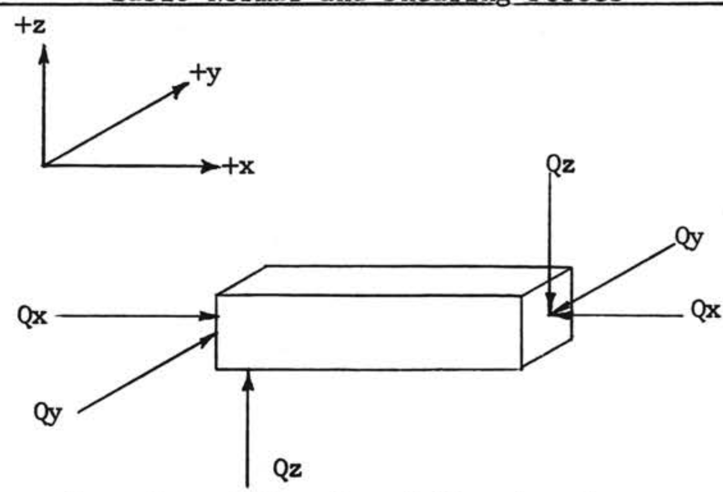
Cross Sectional Elements Table D



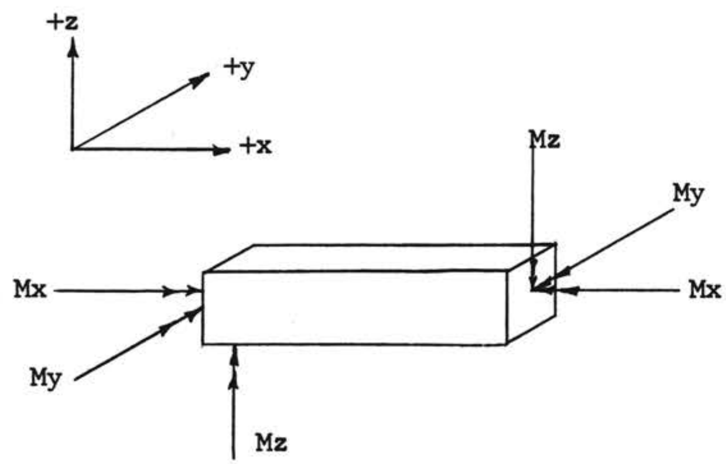
Basic Normal and Shearing Forces



Basic Bending and Twisting Moments

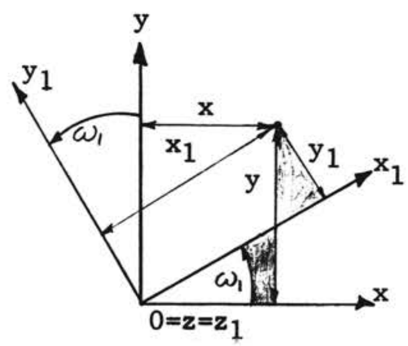


Transformed Normal and Shearing Forces

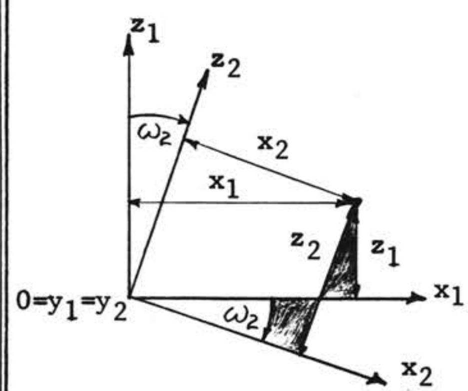


Transformed Bending and Twisting Moments

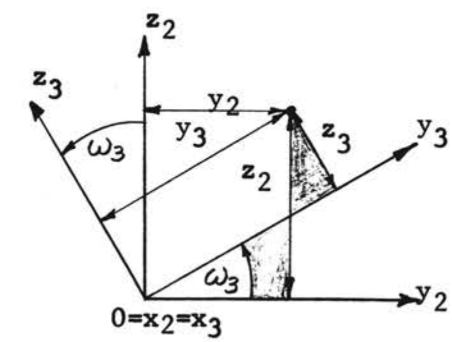
Transformation of Coordinates   Table E



Rotation ω_1



Rotation ω_2



Rotation ω_3

Rotation ω_1	Rotation ω_2	Rotation ω_3
$x = x_1\cos\omega_1 - y_1\sin\omega_1$	$x_1 = x_2\cos\omega_2 + z_2\sin\omega_2$	$x_2 = x_3$
$y = x_1\sin\omega_1 + y_1\cos\omega_1$	$y_1 = y_2$	$y_2 = y_3\cos\omega_3 - z_3\sin\omega_3$
$z = z_1$	$z_1 = -x_2\sin\omega_2 + z_2\cos\omega_2$	$z_2 = y_3\sin\omega_3 + z_3\cos\omega_3$
$x_1 = x\cos\omega_1 + y\sin\omega_1$	$x_2 = x_1\cos\omega_2 - z_1\sin\omega_2$	$x_3 = x_2$
$y_1 = -x\sin\omega_1 + y\cos\omega_1$	$y_2 = y_1$	$y_3 = y_2\cos\omega_3 + z_2\sin\omega_3$
$z_1 = z$	$z_2 = x_1\sin\omega_2 + z_1\cos\omega_2$	$z_3 = -y_2\sin\omega_3 + z_2\cos\omega_3$

Transformation Matrix --- Geometry --- Table F

$$\alpha_x = \cos\omega_1\cos\omega_2$$

$$\alpha_y = -\sin\omega_1\cos\omega_3 + \cos\omega_1\sin\omega_2\sin\omega_3$$

$$\alpha_z = \sin\omega_1\sin\omega_3 + \cos\omega_1\sin\omega_2\cos\omega_3$$

$$\beta_x = \sin\omega_1\cos\omega_2$$

$$\beta_y = \cos\omega_1\cos\omega_3 + \sin\omega_1\sin\omega_2\sin\omega_3$$

$$\beta_z = -\cos\omega_1\sin\omega_3 + \sin\omega_1\sin\omega_2\cos\omega_3$$

$$\gamma_x = -\sin\omega_2$$

$$\gamma_y = \cos\omega_2\sin\omega_3$$

$$\gamma_z = \cos\omega_2\cos\omega_3$$

$$x = x'\alpha_x + y'\alpha_y + z'\alpha_z$$

$$y = x'\beta_x + y'\beta_y + z'\beta_z$$

$$z = x'\gamma_x + y'\gamma_y + z'\gamma_z$$

	x'	y'	z'
x	α_x	α_y	α_z
y	β_x	β_y	β_z
z	γ_x	γ_y	γ_z

Transformation Matrix

$$x' = x\alpha_x + y\beta_x + z\gamma_x$$

$$y' = x\alpha_y + y\beta_y + z\gamma_y$$

$$z' = x\alpha_z + y\beta_z + z\gamma_z$$

General Transformation Matrices — Table G

	x'	y'	z'
x	α_x	α_y	α_z
y	β_x	β_y	β_z
z	γ_x	γ_y	γ_z

Coordinates

	θ_x'	θ_y'	θ_z'
θ_x	α_x	α_y	α_z
θ_y	β_x	β_y	β_z
θ_z	γ_x	γ_y	γ_z

Rotations

	Δ_x'	Δ_y'	Δ_z'
Δ_x	α_x	α_y	α_z
Δ_y	β_x	β_y	β_z
Δ_z	γ_x	γ_y	γ_z

Displacements

	P_x'	P_y'	P_z'
P_x	α_x	α_y	α_z
P_y	β_x	β_y	β_z
P_z	γ_x	γ_y	γ_z

Real Loads

	Q_x'	Q_y'	Q_z'
Q_x	α_x	α_y	α_z
Q_y	β_x	β_y	β_z
Q_z	γ_x	γ_y	γ_z

Real Forces

	M_x'	M_y'	M_z'
M_x	α_x	α_y	α_z
M_y	β_x	β_y	β_z
M_z	γ_x	γ_y	γ_z

Real Moments

	$\overline{P_x}'$	$\overline{P_y}'$	$\overline{P_z}'$
$\overline{P_x}$	α_x	α_y	α_z
$\overline{P_y}$	β_x	β_y	β_z
$\overline{P_z}$	γ_x	γ_y	γ_z

Conjugate Loads

	$\overline{Q_x}'$	$\overline{Q_y}'$	$\overline{Q_z}'$
$\overline{Q_x}$	α_x	α_y	α_z
$\overline{Q_y}$	β_x	β_y	β_z
$\overline{Q_z}$	γ_x	γ_y	γ_z

Conjugate Forces

	$\overline{M_x}'$	$\overline{M_y}'$	$\overline{M_z}'$
$\overline{M_x}$	α_x	α_y	α_z
$\overline{M_y}$	β_x	β_y	β_z
$\overline{M_z}$	γ_x	γ_y	γ_z

Conjugate Moments

VITA

William H. Childress

Candidate for the Degree of

Master of Science

Thesis: ANALYSIS OF CONTINUOUS BENT MEMBERS LOADED OUT OF PLANE BY THE CARRY OVER JOINT MOMENT METHOD

Major Field: Civil Engineering

Biographical:

Personal Data: Born in Glasgow, Montana, June 24, 1937, the son of Noah and Bonilee Childress.

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