

MODEL ANALYSIS OF DEFLECTIONS OF THIN PLATES,  
SUPPORTED AT THEIR CORNERS

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## CHAPTER I

### INTRODUCTION

#### 1.1 General

It is important in the flexibility analysis of plate-beam structures to be able to accurately predict the behavior of thin plates supported at their corners. For this reason, extensive analytical work has been conducted and several mathematical procedures have been developed to predict the behavior of such plates under both transverse and edge loadings. Thus far, however, no experimental work has been done to verify the theoretical solutions to the problem.

The purpose of this thesis is twofold: firstly, to experimentally study the behavior of thin plate models under the action of normal and edge loads, and to compare the results with the classical solutions; secondly, to determine the extent to which plaster models can be used to study the behavior of thin plates obeying the linear, small deflection theory.

#### 1.2 Historical Background

A number of authors in the past have been concerned with the use of plaster as a model material. Tests by Slater<sup>(1)</sup> and Peterson<sup>(2)</sup> show that the stress-strain diagram for plaster is practically a straight line. Seely<sup>(3, 4)</sup> found that plaster models yielded consistent and reliable values for the maximum stress in curved beams. Newmark and Lepper<sup>(5)</sup>

indicated that the plaster model behaves more or less as an elastic material whose ultimate strength can be predicted quite accurately.

A relatively limited amount of information is available in the literature on the theoretical analysis of a thin plate with these unusual boundary conditions. The problem attracted the attention of the early investigators, Hencky<sup>(6)</sup> and Nadai<sup>(7)</sup> and Galerkin<sup>(8)</sup>, who analyzed a uniformly loaded plate supported at its corners. Recently, Oden<sup>(9)</sup> successfully used trigonometric series to study corner supported plates under general loading. Parikh<sup>(10)</sup> and Patel<sup>(11)</sup> used Oden's approach to study some special problems of edge and transverse loading. Numerical techniques have been used by a number of authors to study related problems<sup>(12, 13, 14)</sup>. In particular, Reddy<sup>(14)</sup> presented influence coefficients for a square corner supported plate which were calculated using the finite difference method.

### 1.3 Scope of Study

The results of experiments conducted on small scale plaster models are presented in this thesis. Conclusions and recommendations are given concerning the desirability of using ordinary plaster-of-Paris for model analysis of thin plates.

Test results are compared with the finite difference solution of Reddy<sup>(14)</sup> and the theoretical results obtained by Oden. Significant conclusions drawn from the study are listed in the last chapter.



## CHAPTER II

### MAKING AND TESTING OF SPECIMENS

#### 2.1 General Properties of Material

The material used for the plaster models was a high grade ordinary plaster having nearly the same chemical composition as has plaster-of-Paris, but which was somewhat slower in setting. This permitted ample time for working and placing.

Since the strength and quality of the plaster may be greatly affected by the method of handling, particular care is necessary to secure uniform conditions in mixing and curing the plaster. Other investigators working with this material have found that the strength, density and rapidity of setting all increase with decreasing proportions of water used in mixing the plaster. The time of blending and the time of setting are also important factors affecting the quality of the mix.

The following mixing technique and time schedule of operations was maintained throughout the test:

Water-plaster ratio; 70 percent by weight.

Blending time; 6 min.

Hand stirring; 5 min.

High speed vibrating; 2 min.

The specimens were stacked in saturated air to cure for two days. In this curing period the specimens were not allowed to have direct contact with water, since repeated wetting of the plaster after it has set

causes the strength to decrease seriously and often results in the deterioration of specimens.

## 2.2 Method of Mixing, Pouring, and Curing

In the manufacture of the specimens, care was taken to secure uniformity in quality and strength of the plaster. Weighed quantities of plaster and water were mixed in a galvanized iron tub. The temperature of the mixing water when added averaged 65 deg. F. and the room temperature averaged about 80 deg. F. The plaster was carefully poured into the water and was totally immersed. The mixture was blended for a period of 6 minutes from the time the first plaster was added to the water. After this 6 minute period, stirring was begun slowly and evenly by hand so as to stir air bubbles toward the top. The stirring was continued steadily for 5 minutes.

Stirring alone was not sufficient and vibration was necessary to force out all of the entrapped air bubbles. The vibrator was the "elephant snout" type used for placing concrete. The end of the shaft was held in contact with the bottom and the sides of the tub. After about 2 minutes of vibration, the air had been removed and the mix was smooth, uniform in appearance, and ready to be poured into the molds.

The specimens were all poured to the correct size in aluminum molds supported on four adjustable supports to keep the mold in a level position (Fig. 2.1). The forms were covered with a film of clean oil to prevent sticking of the plaster to the mold.

Shortly after the plaster was cast into the molds it began to set. The side forms served as guides to give the proper thickness. About 15 minutes after it was placed the plaster was sufficiently hard to permit removal from the mold. At this time the plates were quite warm

to the touch and the surface in contact with the form was covered with a film of water that had been driven out of the plaster. This allowed the mold to be removed easily.

All specimens were kept in a saturated condition for two days. After this curing period, the plates were stacked in an open room to dry for about ten days before testing. The conditions in the drying room remained fairly constant during the drying period, a temperature of 85 deg. F. with a relative humidity of about 40 to 50 percent representing the usual state. By standing the specimens on edge so that both top and bottom surfaces were exposed to the air, uniform drying was obtained. After the minimum drying period, all specimens were given two coats of lacquer to insure a uniform moisture content.

### 2.3 Dimensions of Specimens

For ease in handling, plate dimensions of 1 ft. by 1 ft., 1 ft. by 1.5 ft., and 1 ft. by 2 ft. were chosen, with thicknesses of .625 in. These dimensions give thickness to length ratios well within the range to which the small deflection theory for thin plates is considered applicable. Furthermore, preliminary calculations showed that with these dimensions deflections of a magnitude easily measurable with available equipment could be expected.

Three control beams of 2 in. by 5/8 in. cross section, 12 inches long, were poured from each batch to determine properties of the plaster plates. The beams were subjected to the same curing and drying conditions as the plates.

### 2.4 Method of Testing

Eight batches of mix were poured and models corresponding to

each were classified as series A, B, C, D, E, F, G, and H. Series A, B, and C, were trial runs used to check the quality of control and to refine the mixing and casting procedure. General material properties were checked with series D to determine suitable loading techniques.

Material properties of the first six batches were determined by means of simple beam tests. The beams were loaded with a concentrated load and then unloaded and subjected to a uniform load. Center-line deflections were measured for each loading case. The modulus of elasticity was then calculated from the average of sixteen tests. Results were fairly consistent for both sets of tests.

A wide beam test was used for specimens from batches G and H. The individual plate models were simply supported on two opposite edges and loaded uniformly. This method of testing proved simpler than the small beam tests; it yielded more consistent results and allowed the effects of Poisson's ratio to be investigated.

Four adjustable aluminum supports were constructed to support the plate models. Small metal caps were fitted on each corner of the model to allow the plates to rotate freely over each support. Sets of extensimeters graduated in ten thousandths of an inch were mounted on aluminum angles and fixed under the specimen.

Ordinary masonry sand was used to obtain a uniform load on the plates from series D, E, and F. Dry sand was poured slowly and uniformly into a cardboard rack placed on each plate after the model had been carefully supported and leveled. To insure against nonuniformity in loading due to internal friction in the sand, paper rolls with a two inch inside diameter were filled with dried sand and used to apply a uniformly distributed load, sinusoidal edge forces, and edge moments, for specimens in series E, F, G, and H (Fig. 2-2, 3, 4).

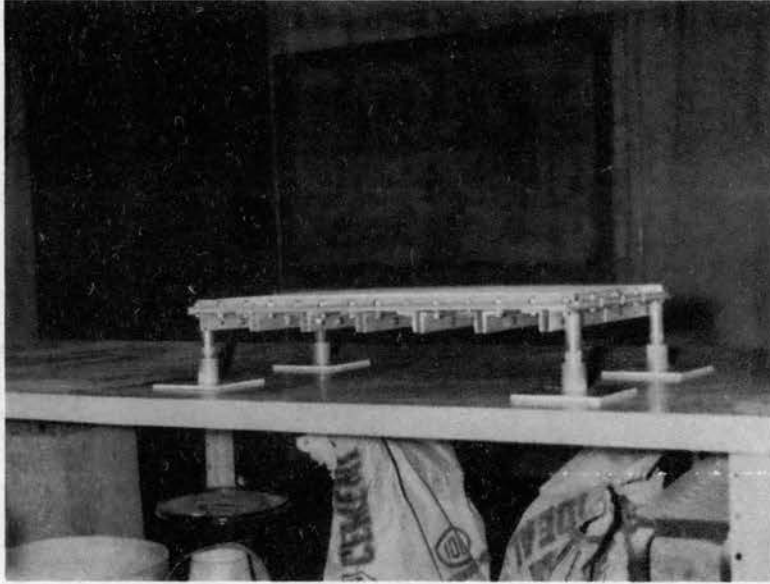


Fig. 2.1. Aluminum Form and Supports.

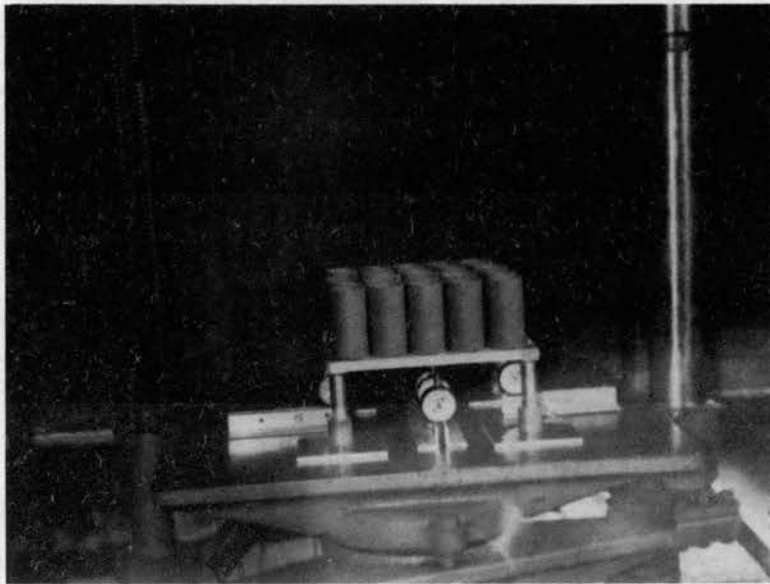


Fig. 2.2. Uniformly Loaded Plate.

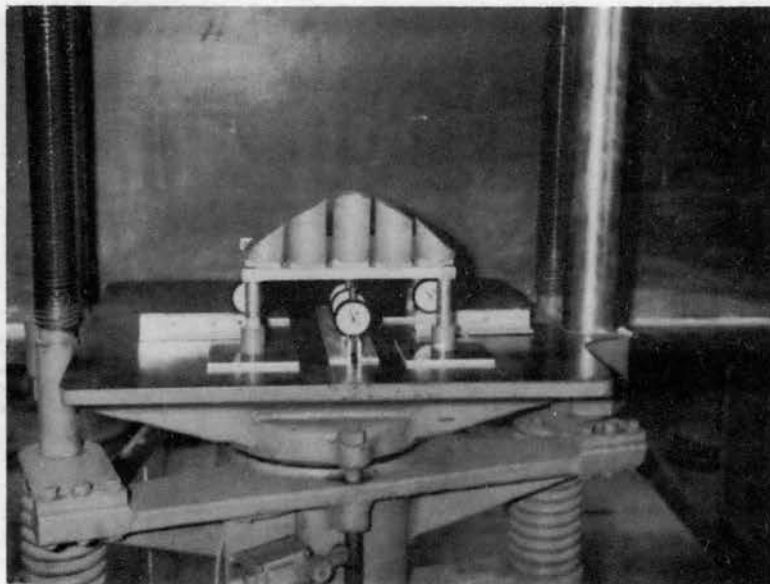


Fig. 2.3. Sinusoidal Edge Force Loading.

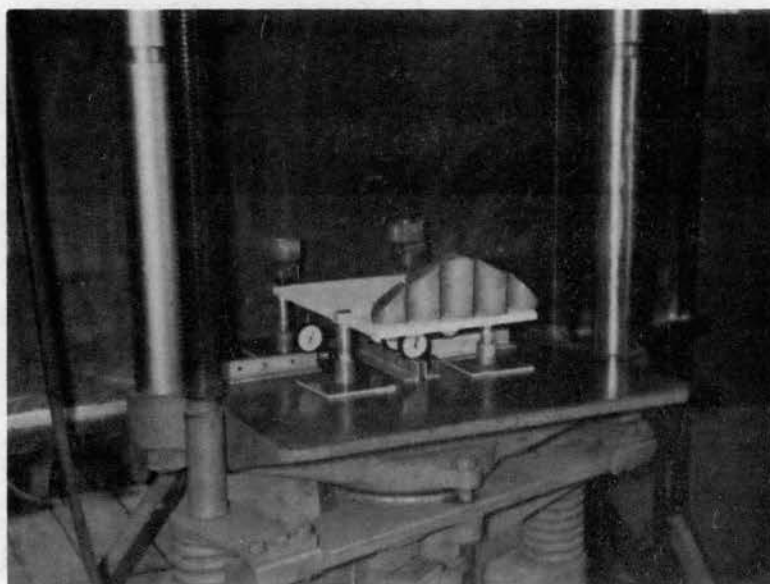


Fig. 2.4. Sinusoidal Edge Moment Loading.

## CHAPTER III

### TEST RESULTS

#### 3.1 Values of Modulus of Elasticity

The moduli of elasticity, determined from the sixteen beam tests, ranged from 820,000 lbs. per sq. in. to 1,210,000 lbs. per sq. in., with an average of all tests of 993,000 lbs. per sq. in. Averages for each batch were also computed.

A number of tests were made to determine the flexural strength of beams 2 in. wide and .625 in. deep loaded with a concentrated load applied along a transverse line at the center of an 11 in. span. The average value of the ultimate flexural strength computed by the flexure formula for a concentrated load was 835 lbs. per sq. in. The material from each batch tested behaved essentially as a linearly elastic material up to failure.

Wide beam tests were also made to determine the influence of Poisson's ratio. A group of wide beams, 12 in. wide, .625 in. deep, with a span of 18 in., were loaded over the full width along a transverse line at the center. Deflections measured along the centerline were found to be practically constant, varying a maximum of only .0003 in. from an average center deflection of .0021 in. This indicates that neglecting effects of Poisson's ratio may be justifiable for the material used.

### 3.2 Plate Deflections Due to a Uniform Load

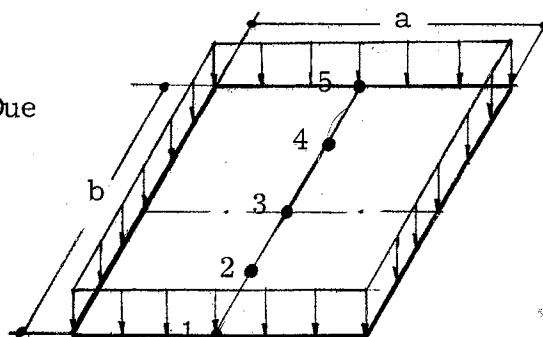
The results of tests performed on uniformly loaded square plates are given in Table 3.1. Two sets of theoretical data are included for comparison. One set is obtained by a finite difference solution to the problem. These values were calculated by using the deflection coefficients given by Reddy<sup>(14)</sup> who used a very fine sixty-four unit network resulting from an 8 by 8 grid and assumed Poisson's ratio to be zero. The second set of values in the table is obtained from the theoretical results of Oden<sup>(9)</sup>, Parikh<sup>(10)</sup>, and Patel<sup>(11)</sup>, who also assumed Poisson's ratio to be equal to zero. The later values result from a method of analysis which yields close to an exact solution to the plate differential equation and follows the classical Levy approach. Hence, the values are referred to as "classical" in the table.

The difference between the experimental values and those given by Oden ranges from 17 to zero percent, with an arithmetic mean of 4 percent. Finite difference values differed with the experimental results from 28 percent to 1 percent, with an average of 10 percent for the series. Individual results agree well with the average; though the specimens are from different batches. The deflection surface for each specimen is fairly smooth, as is indicated in Fig. 3.1. This figure shows the deflection surface of a plate along its centerline compared with averages of eight sets of test results.

The tests results of uniformly loaded plates with width to length ratios,  $a/b$ , of  $1/2$  and  $2$  are recorded in Table 3.2. The last two columns of the table give the ratio of theoretical to experimental values. It is seen that differences range from 30 percent to zero, and that the total range between the maximum and minimum difference is within



Table 3.1 Plate Deflections Due  
to Uniform Load,  
 $a/b = 1$

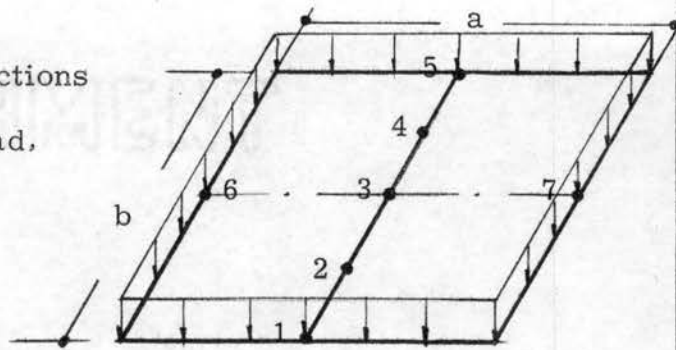


Specimen No.	Total Load lbs	Point	Deflection (inch)			Class. Exp.	F. D. Exp.
			Exper.	Classical	Finite D.		
E - 1	15	1	.00154	.00140	.00158	.91	1.03
		2	.00205	.00196	.00227	.96	1.11
		3	.00237	.00228	.00254	.96	1.07
		4	.00207	.00196	.00227	.95	1.09
		5	.00154	.00140	.00158	.91	1.03
	30	1	.00310	.00280	.00316	.90	1.02
		2	.00410	.00392	.00454	.95	1.11
		3	.00480	.00456	.00508	.95	1.06
		4	.00400	.00392	.00454	.98	1.14
		5	.00310	.00280	.00316	.90	1.02
E - 1	15	1	.00155	.00140	.00158	.90	1.02
		2	.00205	.00196	.00227	.96	1.10
		3	.00225	.00228	.00254	1.01	1.12
		4	.00205	.00196	.00227	.96	1.11
		5	.00156	.00140	.00158	.90	1.01
	30	1	.00285	.00280	.00316	.98	1.11
		2	.00455	.00392	.00454	.86	.99
		3	.00550	.00456	.00508	.83	.92
		4	.00440	.00392	.00454	.90	1.03
		5	.00300	.00280	.00316	.94	1.05
G - 1	20	1	.00181	.00186	.00212	1.03	1.17
		2	.00245	.00261	.00303	1.07	1.21
		3	.00290	.00295	.00339	1.02	1.17
		4	.00248	.00261	.00303	1.05	1.21
		5	.00180	.00186	.00212	1.03	1.17
H - 1	20	1	.00179	.00186	.00212	1.04	1.18
		2	.00247	.00261	.00303	1.05	1.21
		3	.00295	.00295	.00339	1.00	1.15
		4	.00235	.00261	.00303	1.11	1.28
		5	.00186	.00186	.00212	1.00	1.13

Table 3-2 Plate Deflections

Due to Uniform Load.

$$a/b = 1/2, 2$$



$$a/b = 1/2$$

Specimen No.	Total Load lbs.	Point	Deflection (inch)		Class. Exp.
			Exper.	Classical	
G - 1	19.2	1	.00120	.00090	.75
		2	.00591	.00594	1.00
		3	.00778	.00798	1.03
		4	.00590	.00594	1.00
		5	.00115	.00090	.78
		6	.00756	.00778	1.03
		7	.00756	.00778	1.03
H - 1	19.2	1	.00130	.00090	.70
		2	.00557	.00594	1.07
		3	.00738	.00798	1.08
		4	.00561	.00594	1.06
		5	.00110	.00090	.82
		6	.00726	.00778	1.07
		7	.00726	.00778	1.07
$a/b = 2$					
G - 1	19.2	1	.00756	.00750	.99
		2	.00730	.00761	1.05
		3	.00778	.00778	1.00
		4	.00731	.00761	1.05
		5	.00756	.00750	.99
		6	.00120	.00090	.75
		7	.00115	.00090	.78
H - 1	19.2	1	.00726	.00750	1.04
		2	.00732	.00761	1.04
		3	.00738	.00778	1.06
		4	.00731	.00761	1.04
		5	.00726	.00750	1.04
		6	.00130	.00090	.70
		7	.00110	.00090	.82

5 percent of the average. Deflection coefficients for the finite difference method are not given by Reddy for plates with these dimensions, and, hence, are not readily available for comparison.

Fig. 3.1 and 3.2 indicate the centerline of the deflection surfaces of plates with  $a/b$  ratios of  $1/2$  and  $2$ , respectively, plotted using the averages of eight sets of tests results.

### 3.3 Deflections Due to Edge Forces

Sinusoidal edge loads were applied on a single edge of a number of square plate models. The results of these tests are recorded in Table 3.3.

The difference between experimental and theoretical values is about 4 percent, on the average, but the variation is larger than that found for the uniformly loaded plates. The deflections due to edge loads are much more sensitive to small disturbances than that of the uniformly loaded plates, as is confirmed both analytically and experimentally. Thus, higher differences are to be expected. Also, the influence of Poisson's ratio is likely to be more significant in the case of edge loading than in that of transverse loading symmetrical to both centerlines of the plate.

The finite difference values are much higher than the experimental and classical values, with an average error of 35 percent.

Sinusoidal edge loads were applied on plates with  $a/b$  ratios of  $1/2$  and  $2$ . Tests results are listed in Table 3.4. The total edge loading amounted to 9.6 pounds. The experimental values shown in Table 3.4 represent the average of deflections obtained from four tests run on each plate. There is a larger deviation in these results than in the case of the square plate; the difference between the experimental and

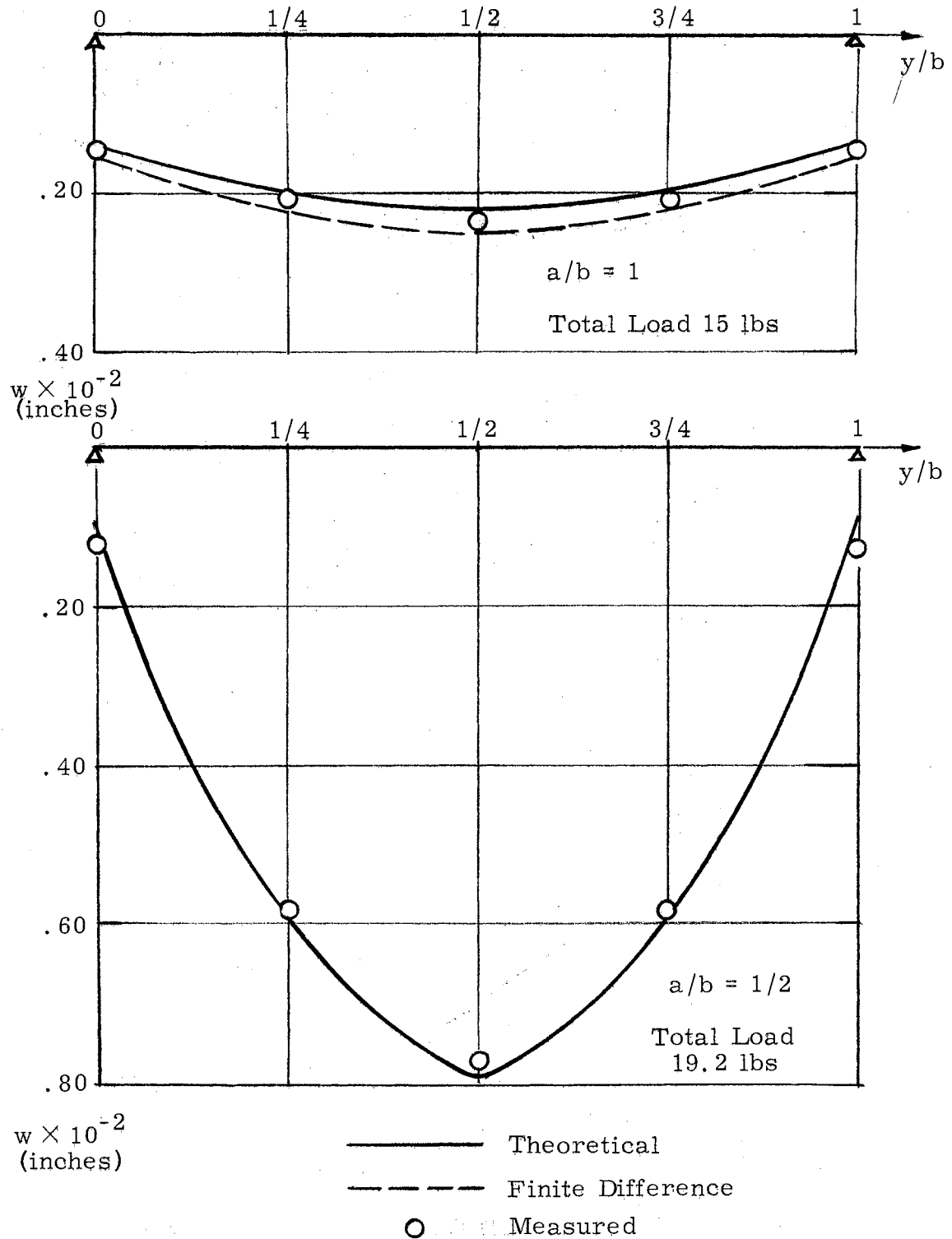
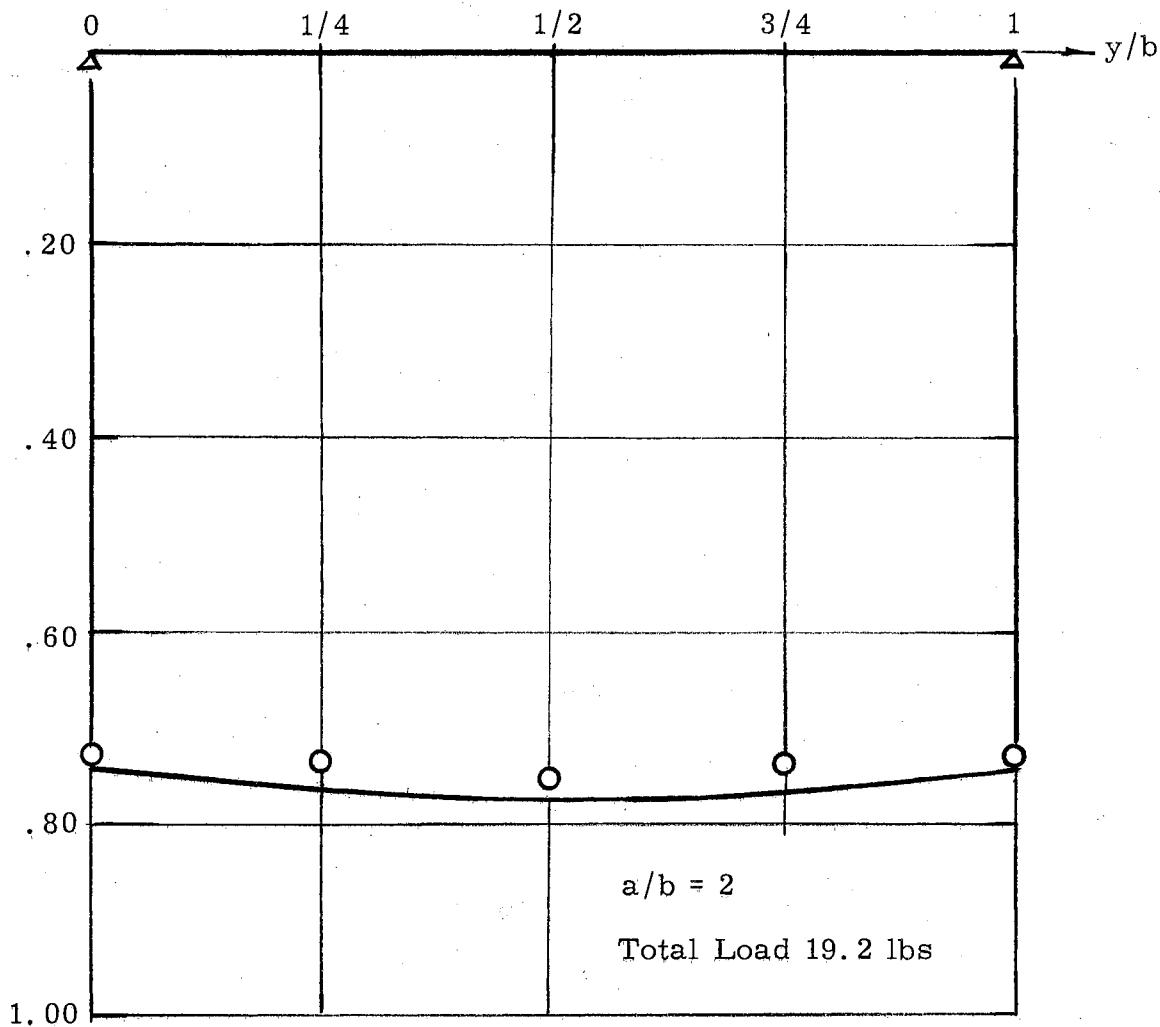


Fig. 3.1 Centerline Deflections of Uniformly Loaded Thin Plates.  $a/b = 1, 1/2$



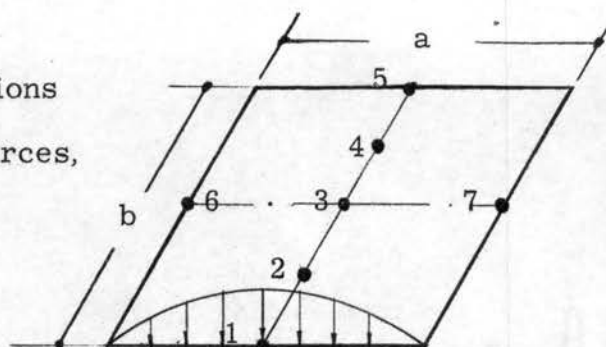
$w \times 10^{-2}$   
(inches)

———— Theoretical

○ Measured

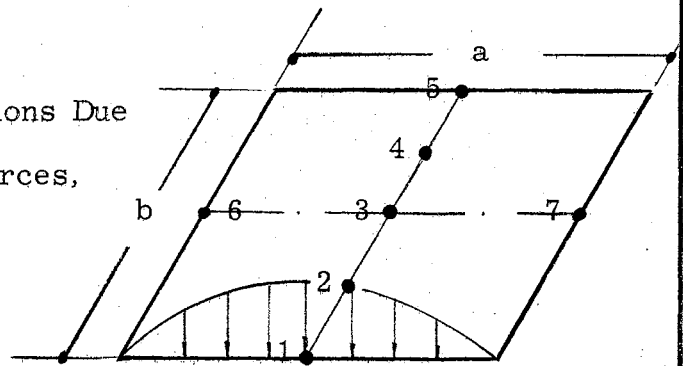
Fig. 3.2 Centerline Deflections of Uniformly Loaded Thin Plate.  $a/b = 2$

Table 3-3 Plate Deflections  
Due to Sinusoidal Edge Forces,  
 $a/b = 1$



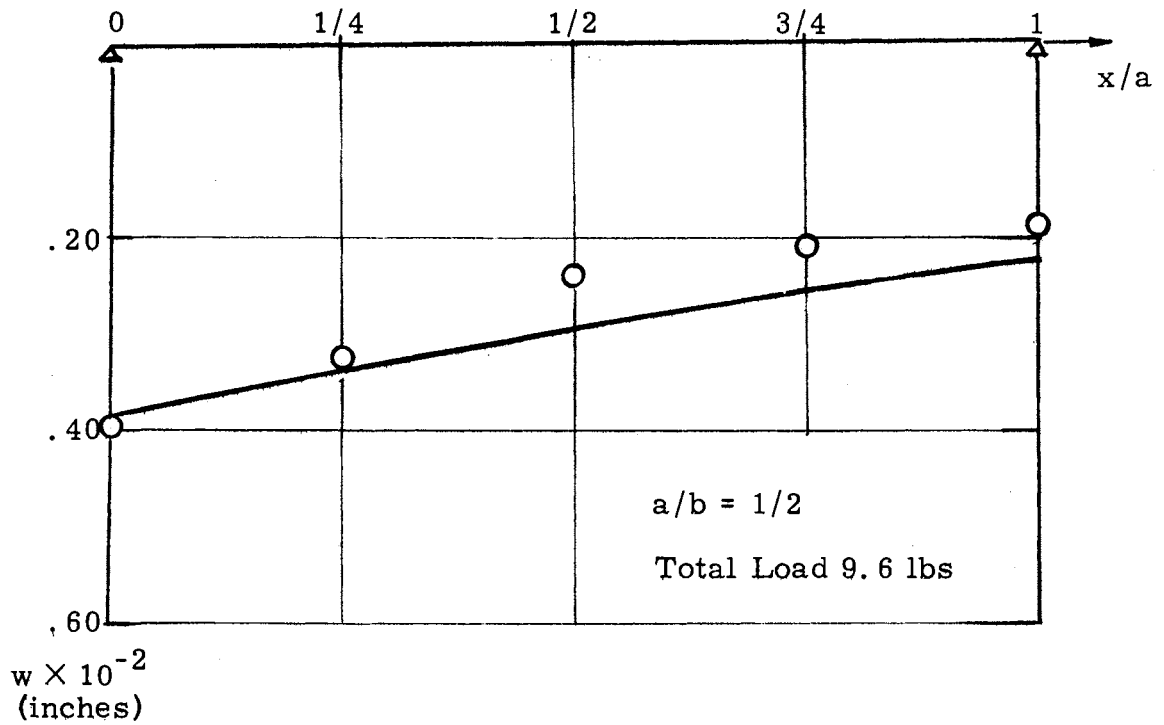
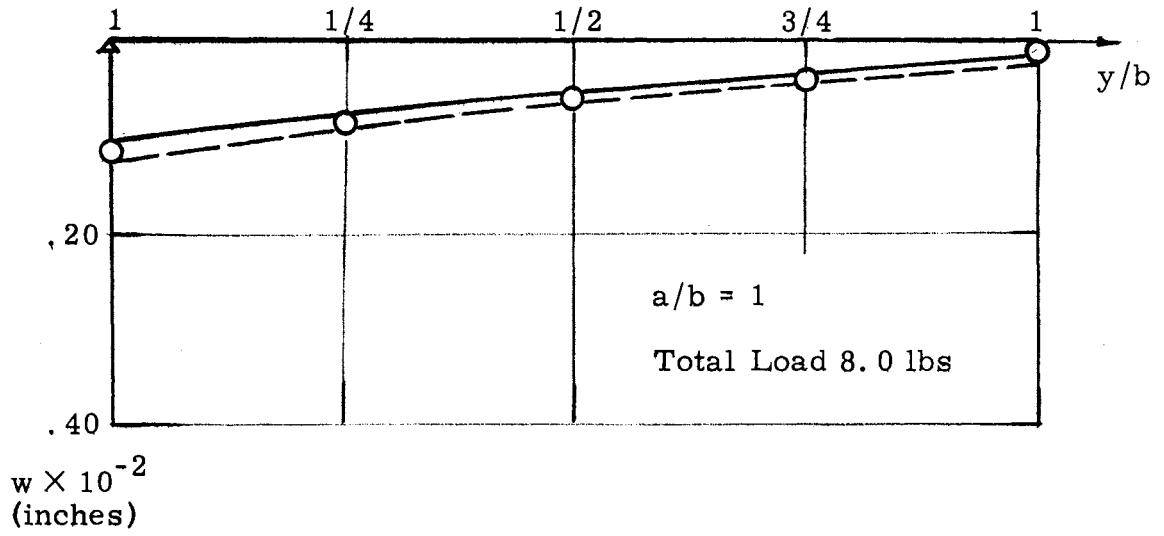
Specimen No.	Total Load lbs	Point	Deflection (inch)			Class. Exp.	F. D. Exp.
			Exper.	Classical	Finite D.		
E - 1	8.0	1	.00115	.00109	.00125	.95	1.09
		2	.00082	.00079	.00092	.96	1.11
		3	.00060	.00055	.00066	.92	1.10
		4	.00041	.00037	.00044	.90	1.08
		5	.00025	.00022	.00026	.88	1.03
		6	.00019	.00015	.00022	.79	1.15
		7	.00019	.00015	.00022	.79	1.15
F - 1	8.0	1	.00110	.00109	.00125	.99	1.14
		2	.00085	.00079	.00092	.93	1.08
		3	.00062	.00055	.00066	.89	1.07
		4	.00045	.00037	.00044	.82	.98
		5	.00021	.00022	.00026	1.05	1.24
		6	.00017	.00015	.00022	.89	1.30
		7	.00018	.00015	.00022	.84	1.22
G - 1	8.0	1	.00100	.00109	.00125	1.09	1.25
		2	.00075	.00079	.00092	1.05	1.23
		3	.00046	.00055	.00066	1.20	1.44
		4	.00032	.00037	.00044	1.15	1.38
		5	.00020	.00022	.00026	1.10	1.30
		6	.00011	.00015	.00022	1.36	2.00
		7	.00012	.00015	.00022	1.25	1.83
H - 1	8.0	1	.00100	.00109	.00125	1.09	1.25
		2	.00071	.00079	.00092	1.12	1.30
		3	.00045	.00055	.00066	1.22	1.47
		4	.00029	.00037	.00044	1.28	1.52
		5	.00018	.00022	.00026	1.22	1.45
		6	.00013	.00015	.00022	1.15	1.70
		7	.00012	.00015	.00022	1.25	1.83

Table 3.4 Plate Deflections Due  
to Sinusoidal Edge Forces,  
 $a/b = 1/2, 2$



$a/b = 1/2$

Spec. No.	Total Load lbs.	Point	Deflection (inch)		Class. Exp.
			Exper.	Classical	
G - 1	9.6	1	.00470	.00388	.82
		2	.00370	.00340	.92
		3	.00237	.00297	1.26
		4	.00218	.00257	1.13
		5	.00195	.00227	1.16
		6	.00162	.00019	1.17
		7	.00165	.00019	1.15
H - 1	9.6	1	.00310	.00388	1.25
		2	.00298	.00340	1.14
		3	.00228	.00297	1.30
		4	.00215	.00257	1.19
		5	.00192	.00227	1.18
		6	.00015	.00019	1.26
		7	.00015	.00019	1.26
$a/b = 2$					
G - 1	6.4	1	.00045	.00052	1.15
		2	.00027	.00031	1.15
		3	.00000	.00018	-
		4	.00000	.00000	1.00
		5	.00000	.00000	1.00
		6	.00000	.00000	-
		7	.00000	.00012	-
H - 1	6.4	1	.00044	.00052	1.18
		2	.00026	.00031	1.19
		3	.00000	.00018	-
		4	.00000	.00000	1.00
		5	.00000	.00000	1.00
		6	.00000	.00012	-
		7	.00000	.00012	-



——— Theoretical  
 - - - Finite Difference  
 ○ Measured

Fig. 3.3 Centerline Deflections Due to Sinusoidal Edge Forces.  $a/b = 1, 1/2$



the classical theoretical values on the average being 15 percent. Some difference, however, is to be expected as the formulation of classical theory is founded on assumptions which tend to magnify errors as  $a/b$  decreases<sup>(15)</sup>.

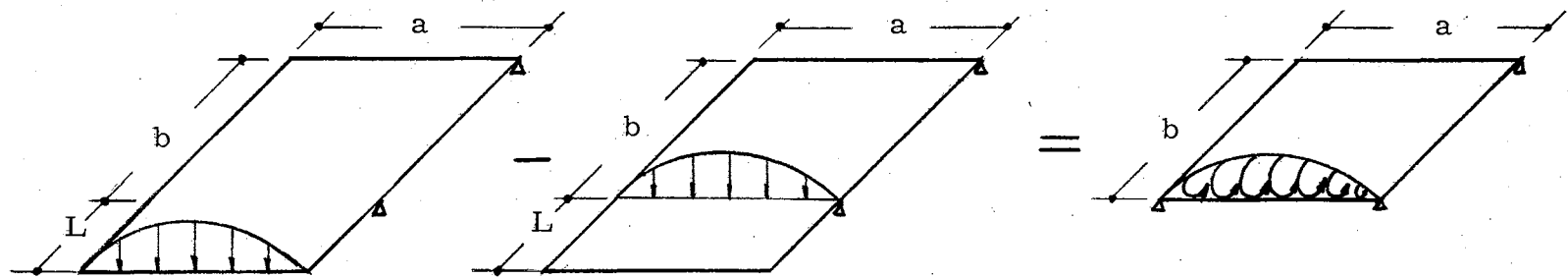
The deflection measurements of plates with  $a/b$  ratios of 2 are not reliable. The loading systems and apparatus were not originally prepared for the purpose of testing plates with these dimensions and it was found that deflections were too small to be accurately measured.

Fig. 3.3 shows the deflection surfaces of two sizes of plates measured along the centerline perpendicular to the loaded edge.

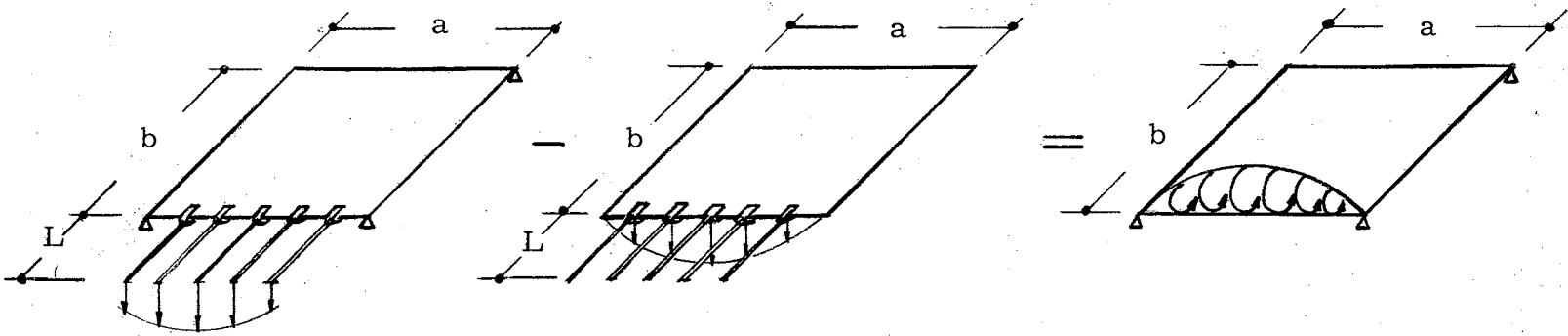
#### 3.4 Deflections Due to Edge Moments

The development of a pure edge moment for a corner supported plate is a difficult task experimentally, especially if the thickness of the plate is very small.

In the tests described in this study, edge moments were obtained by two different methods which are referred to as method A and B. In method A, a sinusoidal load was applied to a portion of the plate allowed to "overhang," as shown in Fig. 3.4a. Deflections were then measured and recorded. The load was then applied between supports and the resulting deflections subtracted from those of the "overhang" case to give the final values. To eliminate possible errors due to twisting moments and lateral bending in the overhang, method B was used. In this case the load was applied as concentrated forces at the end of steel rods clamped to the edge of the plate. The resulting deflections were measured and recorded. Final values were then obtained by subtracting from the recorded values those due to the concentrated loading applied between the supports. This process is illustrated in Fig. 3.4b. Deflections



(A)



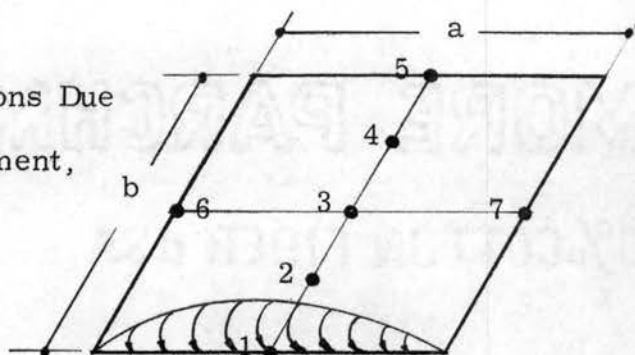
(B)

Fig. 3.4 Sinusoidal Edge Moment on Corner Supported Plate.

measured using both methods were practically the same.

Results are listed in Table 3.5. It is seen that there is an average difference of only 2 percent between the experimental and the classical solution. An average difference of 21 percent between the experimental and the finite difference values is found. Fig. 3.5 indicates the deflection surface of the plate along the centerline perpendicular to the loaded edge. The experimental results shown represent the averages of eight sets of test results.

Table 3.5 Plate Deflections Due  
to Sinusoidal Edge Moment,  
 $a/b = 1$



Spec. No.	Moment per inch	Point	Deflection (inch)			Class. Exp.	F. D. Exp.
			Exper.	Classical	Finite D.		
G - 1	4.7268	1	+.00060	+.00061	+.00080	1.02	1.33
		2	-.00070	-.00058	-.00090	.83	1.29
		3	-.00100	-.00092	-.00140	.92	1.40
		4	-.00076	-.00075	-.00090	.99	1.18
		5	-.00040	-.00037	-.00045	.92	1.12
		6	-.00079	-.00089	-.00085	1.13	1.08
		7	-.00077	-.00089	-.00085	1.15	1.10
H - 1	4.7268	1	+.00060	+.00061	+.00080	1.02	1.33
		2	-.00072	-.00068	-.00090	.80	1.25
		3	-.00095	-.00092	-.00140	.97	1.47
		4	-.00078	-.00075	-.00090	.96	1.22
		5	-.00041	-.00037	-.00045	.90	1.10
		6	-.00082	-.00089	-.00085	1.09	1.04
		7	-.00082	-.00089	-.00085	1.09	1.04

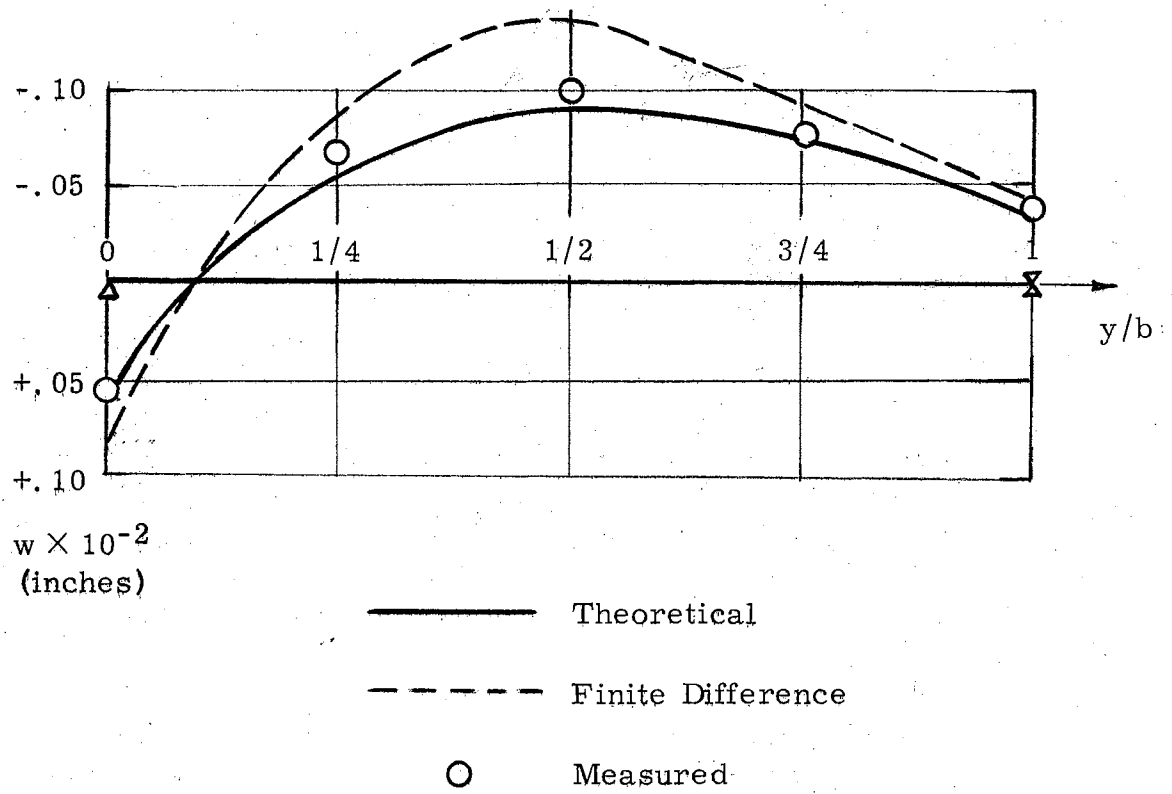



Fig. 3.5 Centerline Deflections of the Plate  
 Subjected to Edge Moment.  $a/b = 1$

① = 0



## CHAPTER IV

### SUMMARY AND CONCLUSIONS

#### 4.1 Summary

A model study of the deflections of thin elastic rectangular plates supported at their corners is presented in this thesis.

Plate models were constructed of ordinary plaster-of-Paris because of its relatively low Poisson's ratio and its similarity in behavior to plain concrete.

The plates were loaded uniformly and with sinusoidal edge forces and moments, and the experimental results for deflections of plates having length to width ratios of  $1/2$ ,  $1$ , and  $2$ , are recorded in tables. Tests results are compared with two available sources of theoretical data: the finite difference method and the classical Fourier series approach. Final centerline deflection curves for plates of various sizes under normal and edge loading are illustrated by scaled drawings.

#### 4.2 Conclusions

It is felt that results reported herein throw some light on the nature of the deflection surfaces of corner supported plates subjected to loads for which plaster behaves as an elastic material. The following conclusions may be drawn from the results given in the study:

1. Ordinary plaster, if properly cured and dried, has a linear stress-strain diagram up to the ultimate or breaking stress; its behavior

at working loads is approximately the same as an elastic material.

2. Best agreement between tests and theory is obtained from averages of three or more experiments for most loading cases. The necessity of a relatively large number of specimens and tests is not a serious objection to the plaster-model method of determining plate deflections, since many specimens may be made from one mold, and the testing is done quickly and with little expense.

3. Theoretical deflections obtained by the classical Fourier analysis are in good agreement with the experimental data. The finite difference method, using an 8 by 8 grid, gives values for deflections which, as a whole, do not agree with the experiments as well as the classical analysis. For some loading cases, the finite difference method may lead to deflections which are considerably in error.

4. Ordinary plaster-of-Paris may be used to construct inexpensive, linearly elastic, and fairly homogeneous small-scale models. The average difference throughout all the tests of only six percent between theoretical and experimental deflections indicates that the properties of the model closely parallel those assumed in the mathematical analysis, and that the effects of Poisson's ratio were practically negligible.



## BIBLIOGRAPHY

1. Slater, W. A., Committee Report, Proc. A. S. T. M., Vol. 14, Part 1, 1919, p. 349.
2. Peterson, R. E., "An Investigation of Plaster of Paris Specimens," M. S. Thesis, University of Illinois, Urbana, Illinois, Dec., 1928.
3. Seely, Fred B., and James, R. V., "The Plaster Model Method of Stress Determination Applied to Curved Beams," Eng. Exp. Sta. Bul., No. 195, University of Illinois, Urbana, Ill., 1929.
4. Seely, Fred B., and Dolan, Thomas J., "Stress Concentration at Fillets Holes, and Keyways as Found by the Plaster-Model Method," Eng. Exp. Sta. Bul., No. 276, University of Illinois, Urbana, Ill., 1935.
5. Newmark, Nathan M., and Lepper, Henry A., Jr., "Tests of Plaster-Model Slabs Subjected to Concentrated Loads," Eng. Exp. Sta. Bul., No. 313, University of Illinois, Urbana, Ill, 1939.
6. Hencky, H., "Der Spannungszustand in Rechteckigen Platten," Dissertation, München, 1913.
7. Nádai, A., "Über die Biegung durchlaufender Platten und der Rechteckigen Platten mit freien Rändern," Z. Angew. Math. u. Mech., Vol. 2, 1922, pp. 1-26.
8. Galerkin, B. G., Collected Papers, Vol. 2, Moscow, 1953, pp. 15, 29.
9. Oden, J. T., "Analysis of Thin Plates Supported at Their Corners," Eng. Exp. Sta. Bul., No. 134, Oklahoma State University, Stillwater, Oklahoma, 1963.
10. Parikh, S. K., "Deflections and Moments of Corner Supported Plates Subjected to Edge Loading," M. S. Report, Oklahoma State University, Stillwater, 1963.



11. Patel, B. B., "Effects of Normal and Hydrostatic Loads on Plates Supported at Their Corners," M. S. Report, Oklahoma State University, Stillwater, 1963.
12. Marcus, H., "Die Theorie elastischer gewebe," Berlin, J. Springer, 1932.
13. Benjamin, J. R., Scordelis, A. C., and others, "Discussion of Deflections in Gridworks and Slabs," Proceedings, ASCE Vol. 78, p. 89, Dec. 1952.
14. Reddy, V. A., "Analysis of Continuous Rectangular Plates on Flexible Beam Supports by Flexibility Methods," M. S. Thesis, Oklahoma State University, Stillwater, 1963.
15. Oden, J. T., "Analysis of Plate-Beam Structures," Doctoral Thesis, Oklahoma State University, Stillwater, Oklahoma, 1962.

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