MEASUREMENT OF TURBINE NOZZLE FLOW AREA BY CAPACITANCE METHODS

By

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MEASUREMENT OF TURBINE NOZZLE FLOW AREA

BY CAPACITANCE METHODS

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ACKNOWLEDGEMENT

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (CONCL'D)

FOREWORD

This investigation represents a portion of the duties of the author while assigned to the Nondestructive Testing Project as a graduate research assistant. The Nondestructive Testing Project was conducted for the Directorate Materiel Maintenance of the Oklahoma City Air Materiel Command under contract number AF 34(601)-9879.

CHAPTER I

INTRODUCTION

During the major overhaul of the J-57 jet engine, flow areas of the turbine nozzle guide vane sections must be determined in order to assure that the flow area available in the re-worked engine is such to provide optimum operating efficiency of the engine. It is therefore desirable to have an efficient, reliable method of measuring this flow area.

The method presently in general use is basically a direct measurement of the area of a trapezoid bounded by a) the trailing edge of one blade, b) the inner surface of the outer rim, c) the convex surface of the adjacent blade on a line which is the locus of points nearest the trailing edge of the first blade and d) the outer surface of the inner rim, as shown in Figure 1. Because of the short arc length involved, the curvatures of the inner and outer rims are not considered. This trapezoid actually represents the minimum flow area available in the direction of flow, due to the convergence of the nozzle from the leading edge to the trailing edge. The actual measurement is accomplished by measuring the distance between a) and c) above at three different points AA (root), BB (mean) and CC (tip) along the span of the blade. The average distance between all blades at each of the three locations is then multiplied by a weighting factor. After adding these three products, the sum is then multiplied by a correction factor representing the average deviation from nominal of the distance between b) and d). This deviation is determined by measuring the distance at four locations 90° apart on the nozzle section and taking the average. The formula used is

$$\text{Eq (1)} \qquad \text{Area} = (99.743K + 137.627L + 119.426M)\frac{H}{3.546}$$

where the symbols are as identified below:

SECTION	LOCATION	RADIUS	AVERAGE VALUE	NOMINAL
AA	Root	10.500	K	0.443
BB	Mean	11.926	L	0.540
CC	Tip	13.353	M	0.634
--	Spanwise	---	H	3.546

Figure 2 shows the range of measurements taken from the inspection sheet for the nozzle assembly used in this investigation.

The second stage nozzle section of the J-57 engine has 102 blades, and requires 306 measurements with a dial indicator, plus four measurements in the spanwise direction, making a total of 310 readings to be taken, recorded and operated on to arrive at the desired goal. From this it is apparent that the major disadvantage of this method is the large amount of time required for determining the available flow area. So it is desirable to have a method for measuring the area which would combine as many as possible of the individual measurements of the present method and give a direct reading and reliable operation.

The capacitance of two conductors is a function of the distance between the conductors and the specific inductive capacity of the dielectric separating the conductors; the specific function is determined by the geometry of the conductors. For parallel flat plate

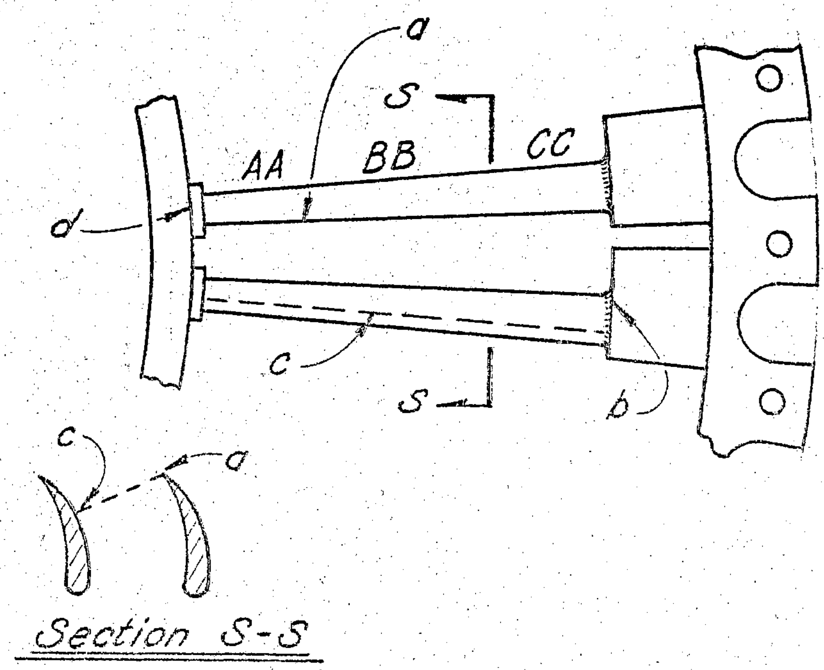

Figure 1. Turbine Nozzle Guide Vane Section.

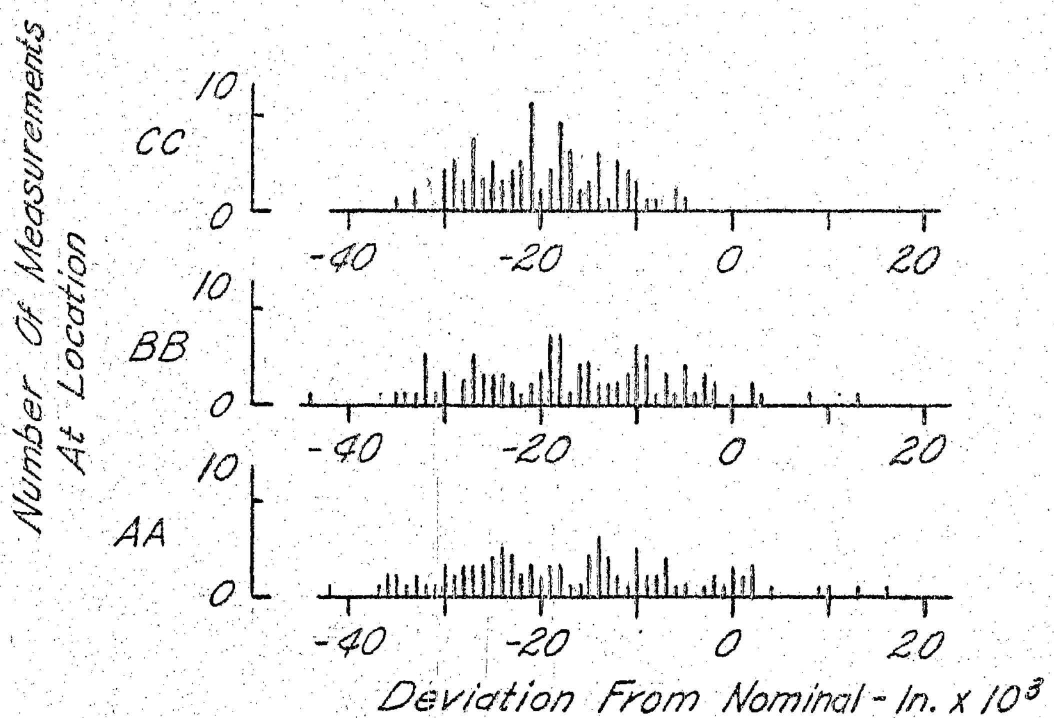

Figure 2. Range of Measurements for Nozzle Assembly.

conductors, (discounting edge effects) the capacitance is inversely proportional to the separation distance.

This study investigates the feasibility of using capacitance characteristics to measure the area, one plate of the capacitor to be referenced on the trailing edge of one blade and the other plate to be the convex surface of the adjacent blade. The parameter to be used is the change in capacitance associated with a change in distance, and not the actual or absolute capacitance.

Various transducers have been designed using the variation of capacitance associated with a change in the separation of the elements, such as the strain, pressure and acceleration pickups listed by Roberts (1).[1] However, in these, all elements of the transducer are separate from the system from which the measurement is obtained, whereas in this study the system (nozzle blades) forms a part of the transducer.

There are, of course, other methods which could prove to be feasible. Measurement of the trapezoid previously described could possibly be accomplished by submerging the nozzle section in water and aiming an ultrasonic beam so as to pick up reflections from the trailing edge and the adjacent blade; the distance between reflections would be representative of the minimum distance between the trailing edge and the next blade due to the perpendicularity requirements for a reflection. The total volume of space between nozzle blades might be determined by filling this space with pellets of proper size and weight and correlating the total weight of pellets used with total volume. Measurement of actual flow parameters such as volume rate and pressures while driving a fluid through the flow area would give an indication of flow area available.

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[1] Parentheses refer to Selected Bibliography.

Projecting light through the nozzle and measuring the intensity at exit would provide a measurement of some area, but this would be neither the total volume nor the area of the trapezoid. All of these methods seem either to be as involved in taking observations as the present method, to require elaborate equipment installation, or to be of doubtful reliability.

This study will be limited to the utilization of capacitance characteristics.

CHAPTER II

THEORETICAL CONSIDERATIONS

Capacitance Characteristics

The capacitance of any two conductors separated by a dielectric is a function of the distance between the conductors and the relative specific inductive capacity (dielectric constant) of the dielectric. The specific function is dependent upon the geometry of the conductors and the composition of the dielectric. Since the relative specific inductive capacity of a given dielectric is considered constant, only variations in geometry will be considered.

For a capacitor consisting of two flat parallel plates of equal area, the equation for the capacitance (not considering edge effects) is

Eq (2) $$C = \frac{0.2248KA}{d}, \text{ where}$$

K = Dielectric constant (dimensionless),

A = Area of one plate (in^2), and

d = Separation distance of the plates (in). (1)

This is a hyperbolic function, the eccentricity of which is dependent upon the area of the plates and the dielectric constant.

If the plates of a capacitor are separated by two dielectrics, air and some solid, with the thickness of the solid remaining constant while the air gap is varied, such as shown in Figure 3, the approximate capacitance is given by

Eq (3) $$C = 0.2248\,A\frac{(K_a d + K_s t)}{(d+t)^2}$$, where

d = Length of air gap (in),

t = Thickness of solid dielectric (in), and

A and K have same meaning as before with subscripts a and s denoting air and solid respectively.

For this equation, the average dielectric constant was deduced as

Eq (3a) $$K = \frac{(K_a d + K_s t)}{(d+t)}.$$

Typical curves of Equations (2) and (3) are shown in Figure 4 (values of constants used are shown in the figure). The hyperbola of Equation (3) intercepts the ordinate axis because the abcissa is not the distance separating the plates but is rather the distance separating one plate from the solid dielectric. Otherwise, the characteristics remain the same as for a single dielectric. Letting C_1 be the capacitance from Equation (2) at d = 0.002 inches and C_2 be the capacitance from Equation (3) at d = 0, so that the actual distance between plates is equal, the ratio C_2/C_1 is seen to be the ratio of the dielectric constants of the two dielectrics:

$$\frac{C_2}{C_1} = \frac{80.8}{18.8} = 4.3.$$

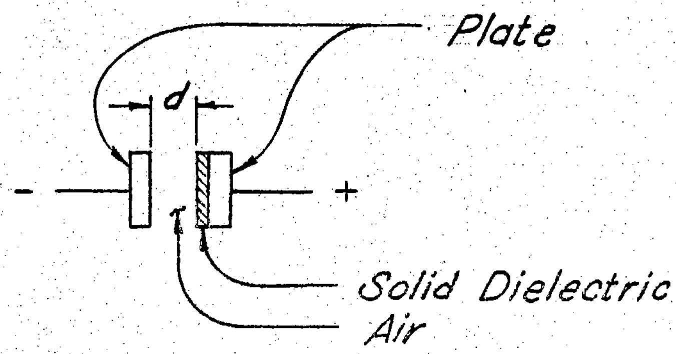

Figure 3. Parallel Plate Capacitor with Two Dielectrics.

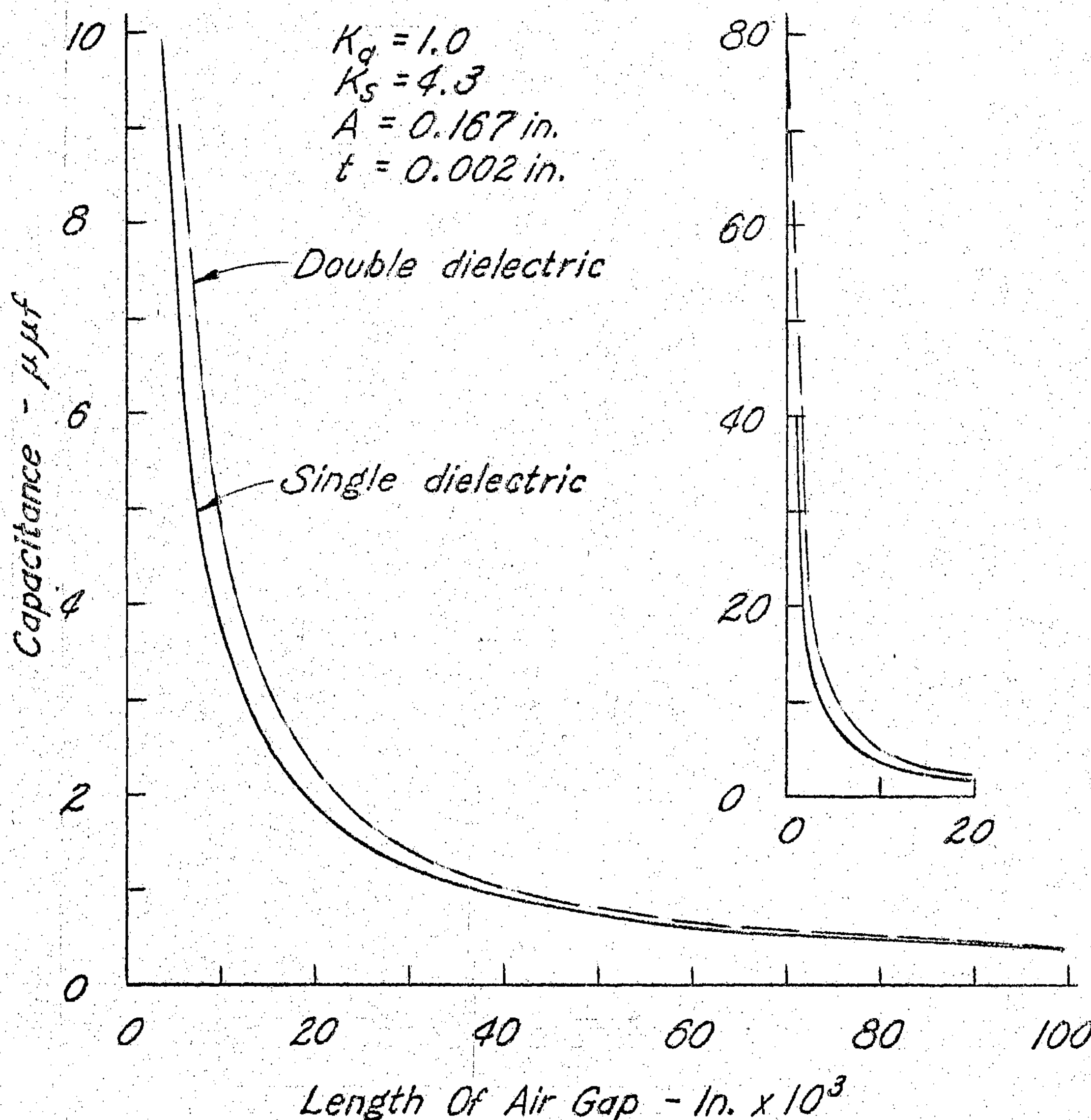

Figure 4. Capacitance Curves for Parallel Flat Plate, Single and Double Dielectric Capacitors.

If either or both of the plates of a capacitor are simple curves, convex towards the other, such as shown in Figure 5, it can be intuitively seen that the capacitance would be reduced from that of a parallel plate capacitor. On the premise that the capacitance is inversely proportional to the average distance separating the plates, the capacitance of the curved plate capacitor can be deduced as follows:

Let a = Width of capacitor section,

d = Minimum length of air gap,

b = Maximum length of air gap,

h = Length of air gap at any point,

r = Radius of curvature of the plate,

θ = Angle subtended by the arc of the plate,

A_t = Area of rectangle formed by sides of plates,

A_1 = Area of circular segment of curved plate, and

A_2 = Area bounded by the plates as two sides

$= A_t - A_1$.

Then

$$b = d + r(1 - \cos\tfrac{\theta}{2}),$$

$$A_t = ab, \quad A_1 = (\theta - \sin\theta)\frac{r^2}{2},$$

$$\theta = 2\sin^{-1}\left(\frac{a}{2r}\right),$$

and the average distance is given by

$$h_{av} = \frac{A_t - A_1}{a} = \frac{a[d + r(1 - \cos\frac{\theta}{2})] - [\theta - \sin\theta]\frac{r^2}{2}}{a}$$

$$= d + r(1 - \cos\tfrac{\theta}{2}) - (\theta - \sin\theta)\frac{r^2}{2a}.$$

For a given geometrical configuration, h_{av} is merely d plus a constant, or $d + h_{av_o}$, where $h_{av_o} = r(1 - \cos\theta/2) - (\theta - \sin\theta)r^2/2a = h_{av}$ at $d = 0$. Then for one curved plate, $h_{av} = d + h_{av_o}$, and for two curved plates with equal radii of curvature, $h_{av} = d + 2h_{av_o}$. The capacitances of these types are then approximated by

Eq (4) $$C = \frac{0.2248KA}{d + h_{av_o}}$$ (one plate curved),

and

Eq (5) $$C = \frac{0.2248KA}{d + 2h_{av_o}}$$ (both plates curved).

Combining these equations with Equation (3), the approximate capacitances for double dielectric, curved plate capacitors are found to be

Eq (6) $$C = 0.2248A\left[\frac{K_a(d + h_{av_o}) + K_s t}{(d + h_{av_o} + t)^2}\right]$$ (one plate curved),

and

Eq (7) $$C = 0.2248A\left[\frac{K_a(d + 2h_{av_o}) + K_s t}{(d + 2h_{av_o} + t)^2}\right]$$ (both plates curved).

All lineal and areal dimensions are in inches and square inches, respectively. Typical curves of Equations (4) through (7) are shown in Figure 6 (values of constants used are shown in the figure).

Consider a capacitor fashioned of two flat plates, equal to each other in width and length, but not parallel to each other. If the dielectric is air, the capacitance based upon the average distance between

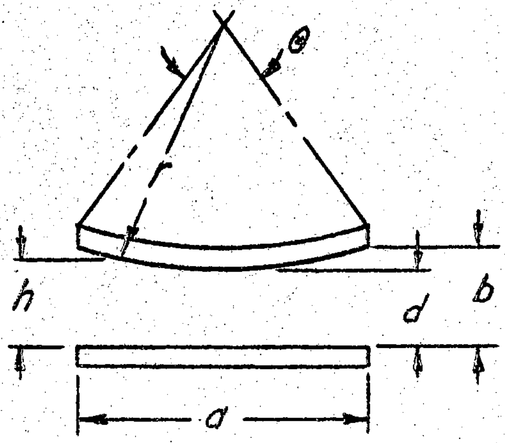

Figure 5. Curved Plate Capacitor.

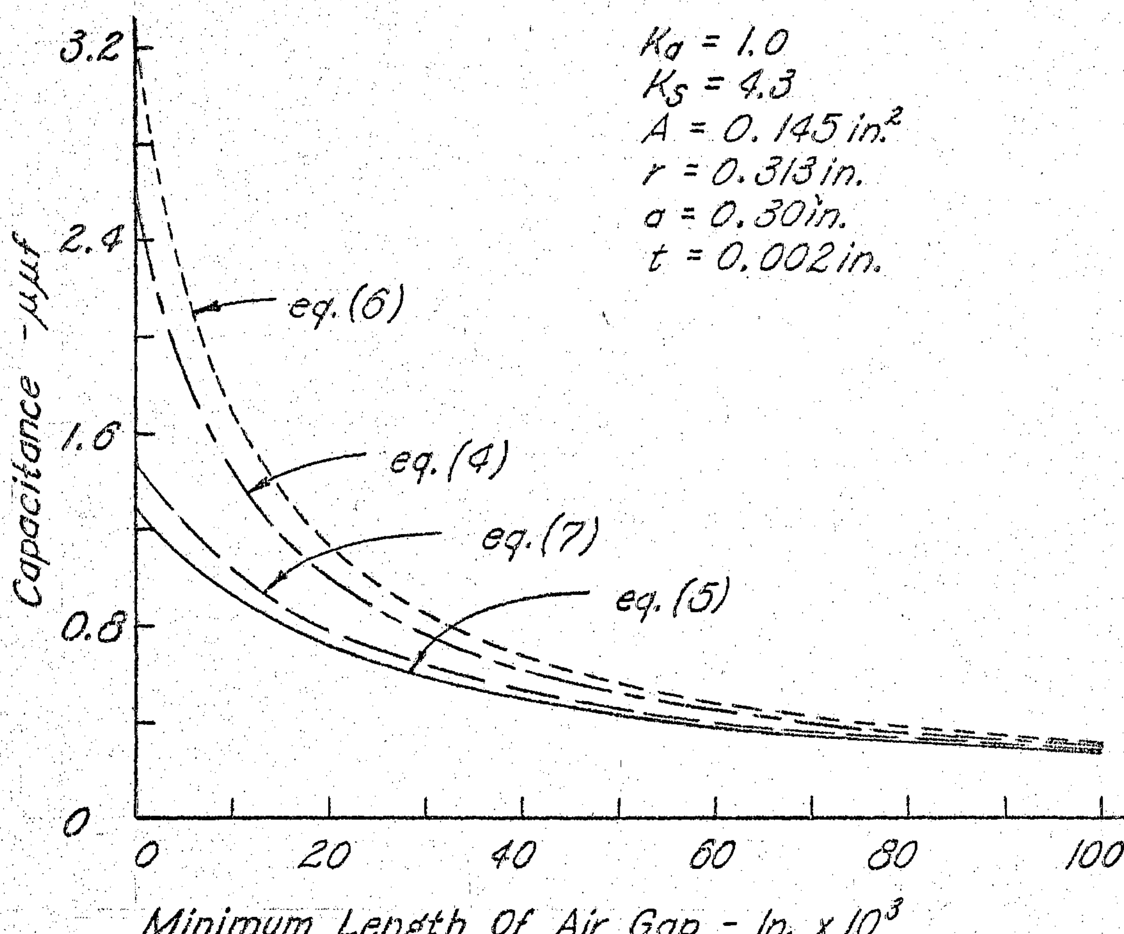

Figure 6. Capacitance Curves for Curved Plate, Single and Double Dielectric Capacitors.

them is given by

Eq (8) $$C = \frac{0.2248WLK}{d_{av}} = \frac{0.4496WLK}{(a+b)},$$

where a and b are the distances between plates at each end (in inches), the plates being parallel in the width direction, as shown in Figure 7. This expression is again a hyperbola. Taking an elemental portion, dl, of the capacitor, the differential expression for the capacitance is

$$dC = \frac{0.2248WK\,dl}{d};$$

d can be expressed as a function of l, such as d = a + kl, where k is the slope of the plate. Letting K' = 0.2248K,

$$dC = \frac{K'W\,dl}{a+kl},$$

and integration gives

Eq (9) $$C = K'W\int_0^L \frac{dl}{a+kl} = K'W\frac{1}{k}\ln\left(\frac{a+kL}{a}\right).$$

When k is zero, this equation results in the undefined quotient 0/0. Applying L'Hospital's rule,

$$C_{k=0} = K'W \lim_{k\to 0}\left[\frac{\ln\left(\frac{a+kL}{a}\right)}{k}\right] = \frac{K'WL}{a},$$

which is Equation (8) with K' = 0.2248K and a = d_{av}. Equation (9) is plotted in Figure 8 for various values of a. The average length of air gap is calculated as

$$d_{av} = a + \frac{kL}{2}.$$

Using this average, Equation (9) is plotted in Figure 9 along with Equation (8). Although the discrepancy of actual capacitance between the two equations is quite large, both exhibit the characteristic hyperbolic form. Combining Equations (7) and (8), an approximation of the capacitance of the capacitor shown in Figure 10, in which both plates are curved and a solid dielectric is applied to one plate, is given by

Eq (10)

$$C = 0.2248A\left[\frac{K_a(d_{av} + 2h_{av_o}) + K_s t}{(d_{av} + 2h_{av_o} + t)^2}\right].$$

Or, in the case of the plates having unequal radii of curvature,

$$C = 0.2248A\left[\frac{K_a(d_{av} + {}^1h_{av_o} + {}^2h_{av_o}) + K_s t}{(d_{av} + {}^1h_{av_o} + {}^2h_{av_o} + t)^2}\right],$$

where the superscripts on h_{av_o} denote the plate to which the correction for curvature applies. Differentiating Equation (10) with respect to d_{av} gives

$$\frac{dC}{dd_{av}} = 0.2248A\left\{\frac{K_a}{(d_{av} + 2h_{av_o} + t)^2} - \frac{2\left[K_a(d_{av} + 2h_{av_o}) + K_s t\right]}{(d_{av} + 2h_{av_o} + t)^3}\right\}.$$

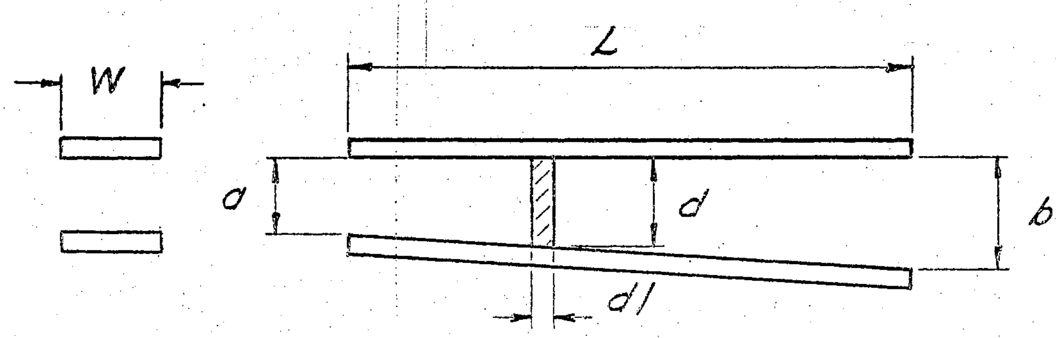

Figure 7. Capacitor with Non-parallel Flat Plates.

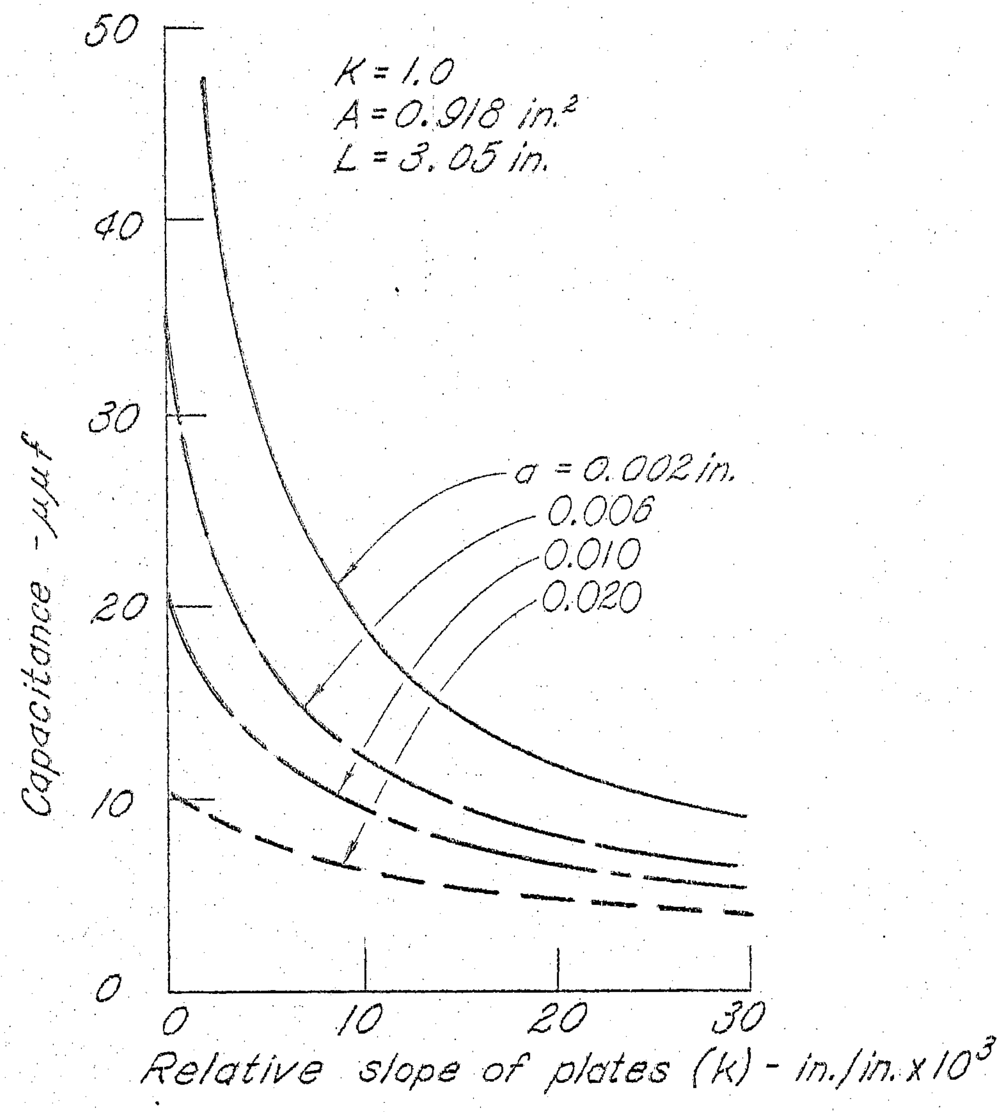

Figure 8. Capacitance Curves for Non-parallel Flat Plate Capacitor.

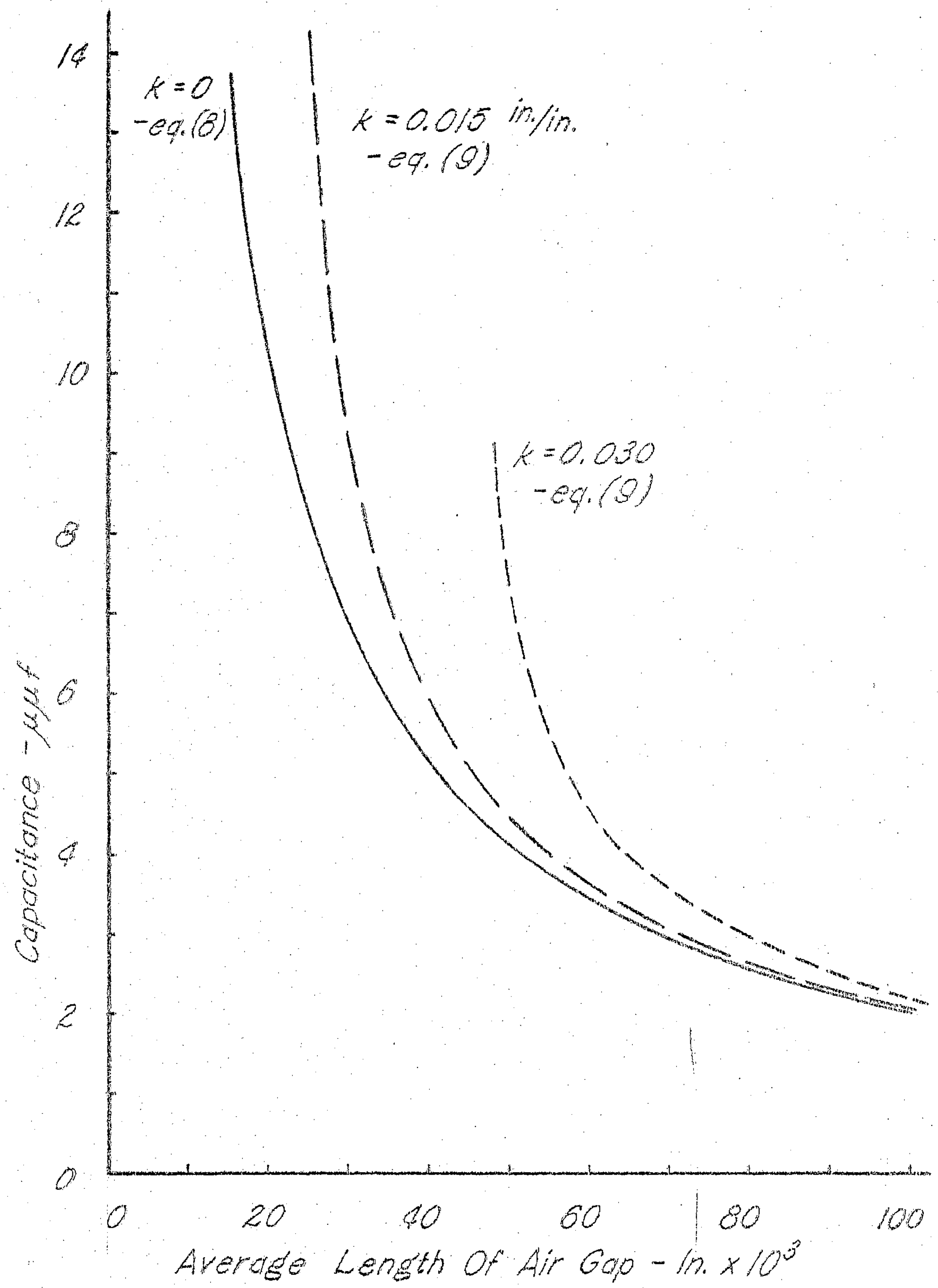

Figure 9. Combined Capacitance Curves for Non-parallel Flat Plate Capacitor.

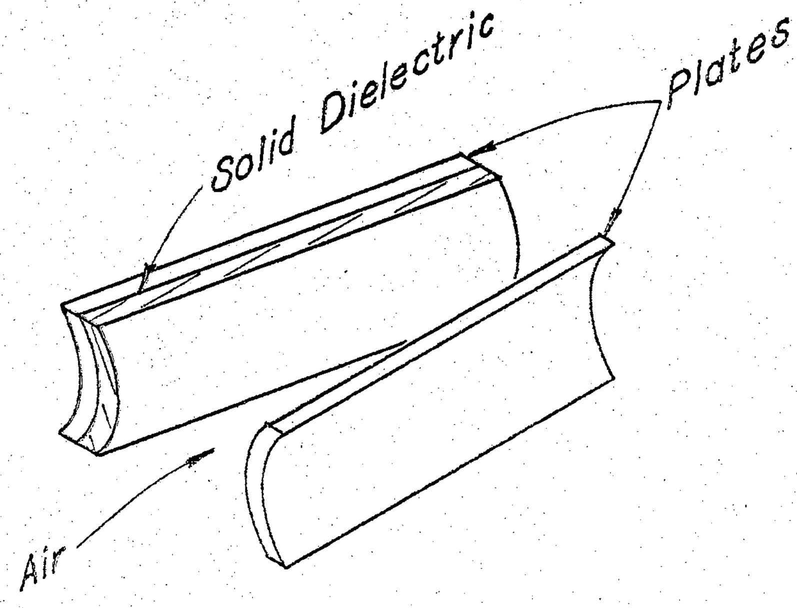

Figure 10. Capacitor with two non-parallel curved plates separated by two dielectrics.

This indicates that, all other factors being constant, the slope of the capacitance curve in magnitude is directly proportional to the plate area and inversely proportional to the square of the average distance. For illustration purposes, the simpler Equation (3) was differentiated and the slope of the capacitance plotted therefrom in Figure 11. In this curve

$$\Upsilon = \frac{1}{A}\frac{dC}{dd} = 0.2248\left[\frac{K_a}{(d+t)^2} - \frac{2(K_a d + K_s t)}{(d+t)^3}\right].$$

In the investigation of the relationship between capacitance and separating distance, the development beyond the flat parallel plate capacitor is not nor intended to be rigorous or even close to accurate as far as actual capacitance is concerned. Rather the purpose was to

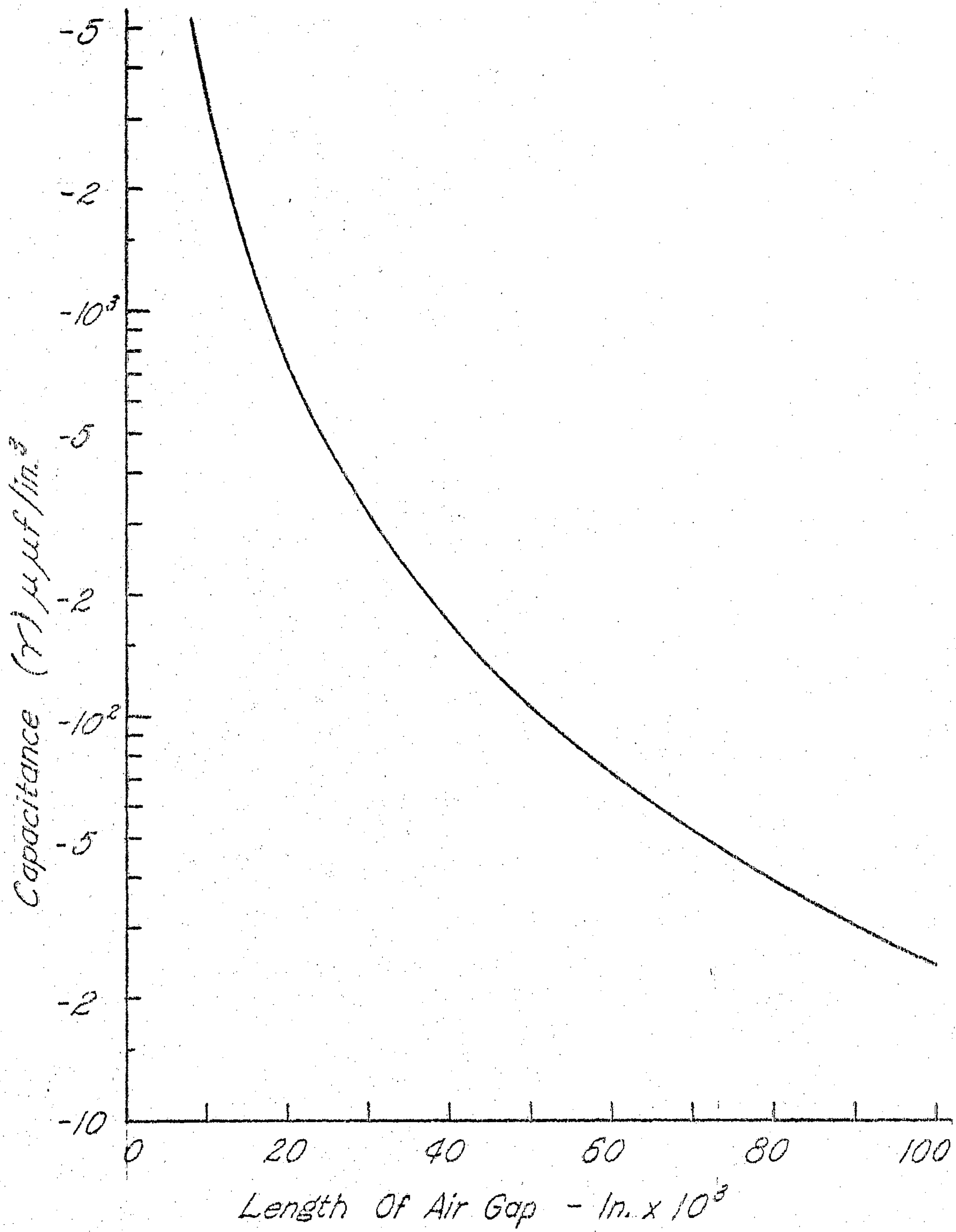

Figure 11. Capacitance Slope Curve.

investigate, on a qualitative basis, the relationship between the change in capacitance associated with a change in distance of separation of the plates.

Application to Area Measurement

The area of an annular segment of a circle is given as $A = (\theta/2)(r_2^2 - r_1^2)$, where the symbols are as denoted in Figure 12. If in a straight sided box having the shape of an annular segment is drawn the line AB parallel to one radial boundary (B_1) at a distance d away (see Figure 12), and the arc length S_o between the line AB and the other radial boundary (B_2) is known for any radius r_o, the area is given by

$$A = \frac{S_o + d}{2r_o}(r_2^2 - r_1^2).$$

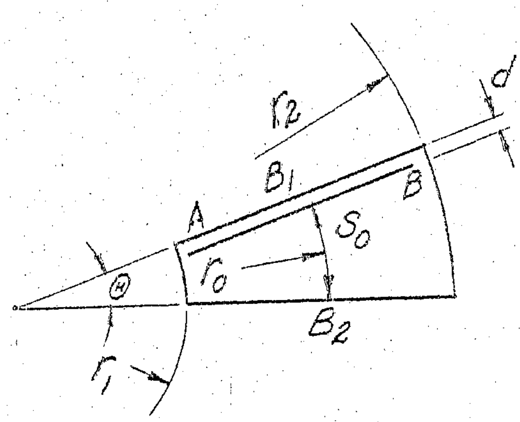

Figure 12. Area Diagram

Thus for a given geometry determining r_o, S_o, r_2 and r_1, the area becomes a function of d only. In the actual turbine nozzle, the area between any two blades is of the general shape of an annular segment, but the following deviations may and do occur:

1. B_1 and/or B_2 is cocked out of radial alignment, and
2. r_2 is not constant.

This makes it necessary to use the average distance, d_{av}, between AB and B_1, measure r_2 directly, and set $r_o = (r_2 + r_1)/2$. The area is then given by

$$A = \frac{S_o + d_{av}}{2r_o}(r_2^2 - r_1^2).$$

Summing the areas around the nozzle and taking the average of the r_2's measured, the total area is

$$A = (r_{2_{av}} + r_1)(102\, S_o + \sum d_{av}).$$

If AB and B_1 now formed the two plates of a capacitor, noting that d is a function of C [Equation (10)], the area available for flow is a function of the capacitance:

$$A = f(C).$$

CHAPTER III

EXPERIMENTAL PROCEDURE

Parallel Flat Plates

A capacitor was made, using air only as the dielectric, with one fixed plate having an area of 0.1665 $inches^2$ and the other plate of much larger area mounted so that the change in distance separating the two plates could be measured with a dial indicator. The resulting capacitance vs. distance curve with the capacitance of the lead, 36uuf, subtracted, is shown in Figure 13 along with the theoretical capacitance obtained from Equation (2). The large discrepancy between the theoretical and experimental results can be attributed to a) the fixed plate was not exactly flat, having been formed by filing the end of aluminum bar stock, and b) the movable plate was much larger than the fixed plate. The effect of the slight curvature of the fixed plate was to increase the average separation distance above that measured, since the measurement actually represented the minimum separation distance. The error caused by the excess area of the movable plate increases the edge effects, and is an additive, systematic error (Dummer) (2). By shifting the experimental curve horizontally and vertically without rotation, it was found that the corrections for the above errors were +0.002 inches and -4.8uuf. The resulting corrected curve is also shown in Figure 13. The scatter of the experimental data resulted from random error caused by a) errors in positioning the movable plate, and

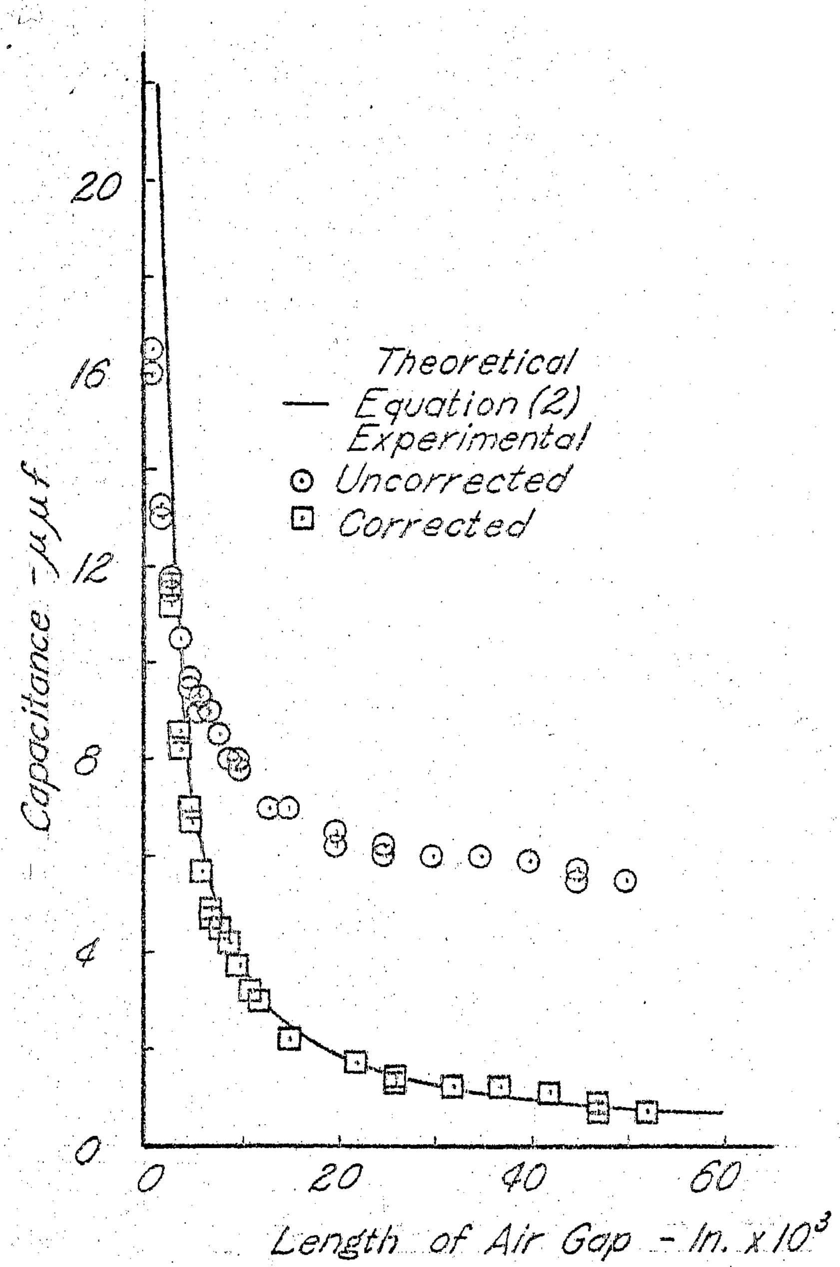

Figure 13. Capacitance Curves for Parallel Flat Plate, Single Dielectric Capacitor.

b) readout on impedance bridge, which had a claimed accuracy of luuf and a least division of 2uuf.

To the fixed plate of the previously used capacitor was added a layer of Mylar tape, covering the flat area and extending some distance back along the bar. The dielectric constant of the Mylar tape was determined experimentally to be approximately 4.3. The resulting capacitance vs. distance curves corresponding to the curves of Figure 13 are shown in Figure 14; corrections deduced from the single dielectric capacitor for edge effects and plate curvature were made. The discrepancy between experimental and theoretical curves in the range of 0 to 0.020 inches can be attributed to the reduction of edge effects by the extension of the Mylar tape along the sides, thus reducing the capacitance. Otherwise, the error falls well within the range expected from experimental determination of the dielectric constant, and the random error sources noted previously.

Details of the calibration jig used for these two capacitors are shown in Figure 15.

Short Curved Plate Transducer

Transducer A, having one curved plate as shown in Figure 16, was fabricated. The phenolic probe on which the plate was mounted was tapped for an adjusting screw (S_1) to allow the distance of separation to be changed. The width of the transducer plate in the spanwise direction was 0.5 inches. The point 'D' on the transducer was referenced on the trailing edge of one blade, and the plate was brought into position using the distance screw and the rotation screw (S_2) (see Figure 17). Several readings were then taken of change in distance vs. bridge output.

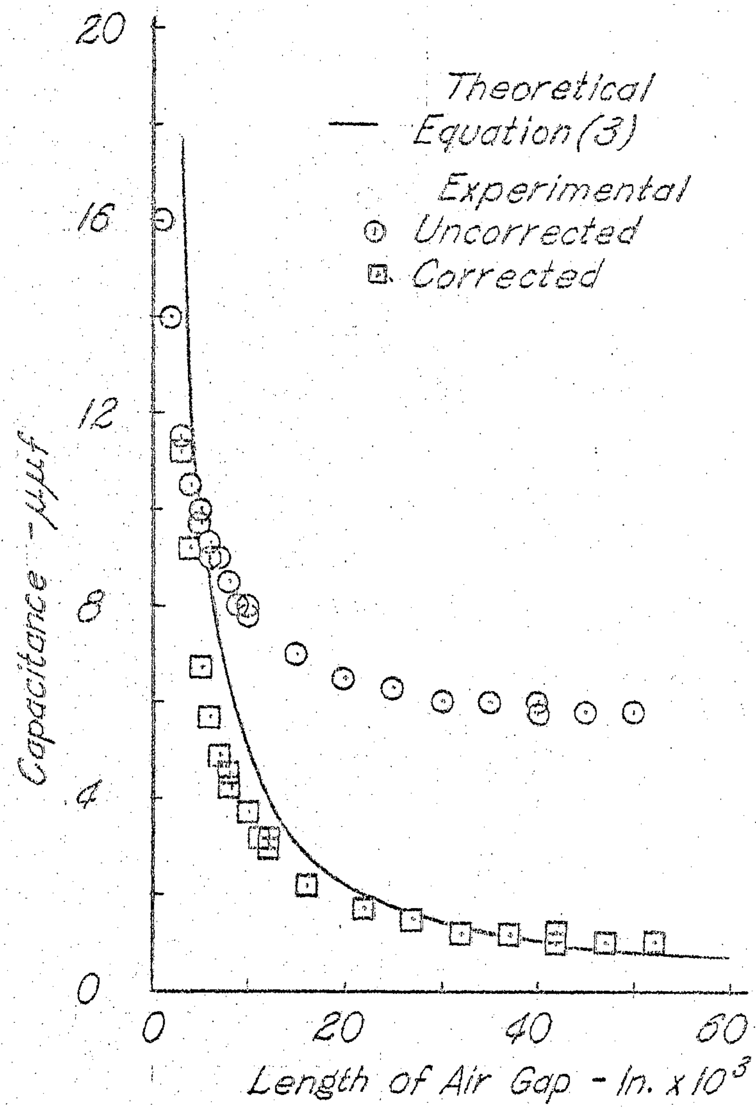

Figure 14. Capacitance Curves for Parallel Flat Plate, Double Dielectric Capacitor.

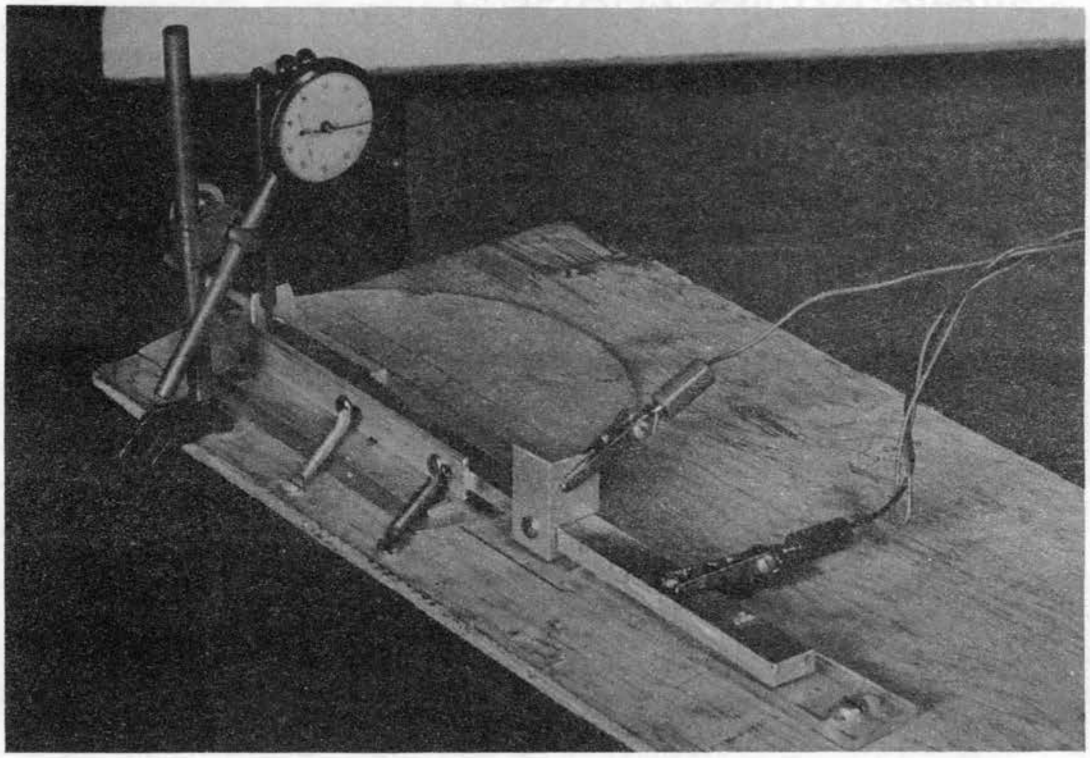

Figure 15. Parallel Flat Plate Capacitor in Calibration Jig

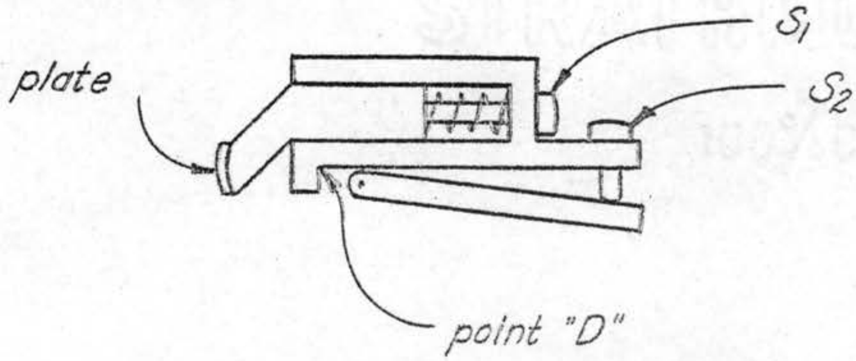

Figure 16. Transducer A.

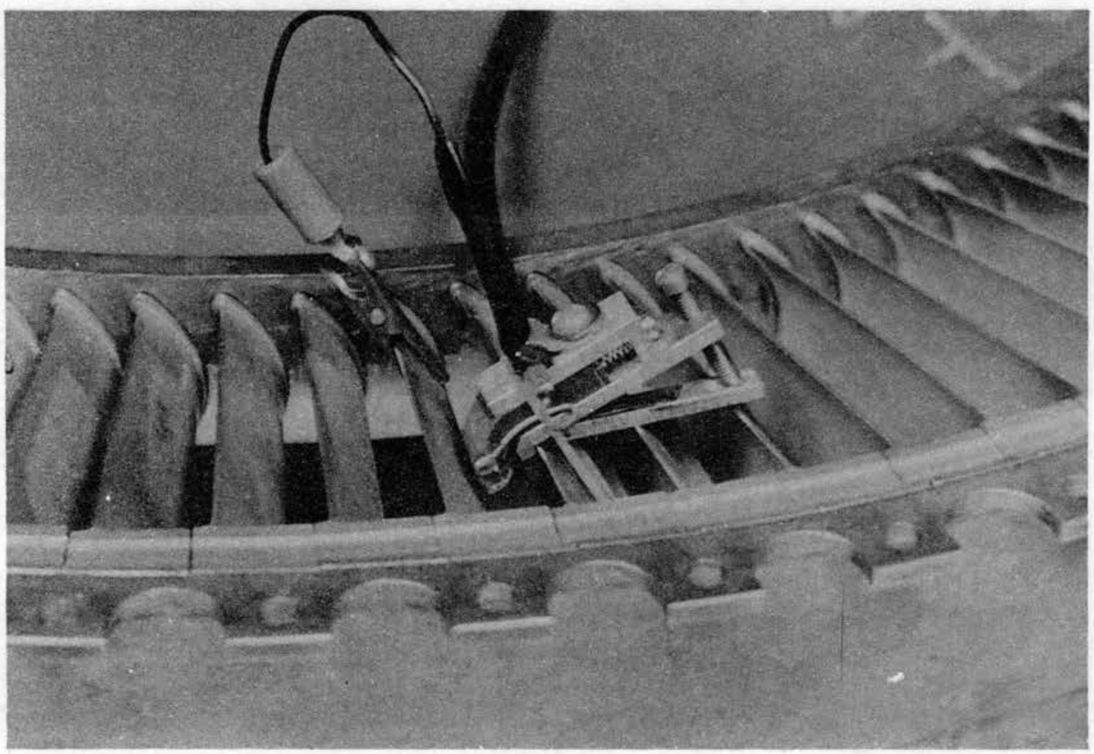

Figure 17. Transducer A Mounted in Nozzle Assembly.

The technique used here was to balance the impedance bridge at a gap set by thickness gage inserted between the transducer plate and the adjacent blade; then, using the bridge dial, drive the meter to one unit deflection. The probe was then moved through the range and meter deflection noted. The distance the probe moved was measured visually with a 20X index telescope. From Figure 1, it was estimated that the maximum change in distance to be encountered would not exceed .08 in. However, it was not possible to measure capacitance directly with any degree of accuracy outside the range of 0 to .01 in. with this transducer. Since the change in output of the unbalanced bridge is directly related to the change in the capacitance "seen" by the bridge, bridge output was used as the parameter, lending a much greater facility for detecting changes in capacitance. A typical curve of this data is shown in Figure 18. The measurable range available with this setup was less than 0.020 inches because of the limited travel of the transducer and the off-scale excursions of the bridge meter for larger distances. Also, accurate measurement of probe travel was difficult. The transducer was also rotated about reference point 'D' to determine the optimum angle to be used. This data is shown in Figure 19.

The transducer was then mounted in the bench calibration jig used previously and, using the circuit shown in Figure 20, bridge output vs. length of gap was measured. The bridge was balanced at a gap of 0.110 inches, and the data represents the amount of bridge unbalance caused by the change of capacitance of the probe associated with a change in distance. This curve is shown in Figure 21, and the recorded data are contained in Appendix A. For the purpose of investigation of data scatter, the amplification was reduced, and the curve of Figure 22 resulted. The data recorded are listed in Appendix B. At nearly all points, the

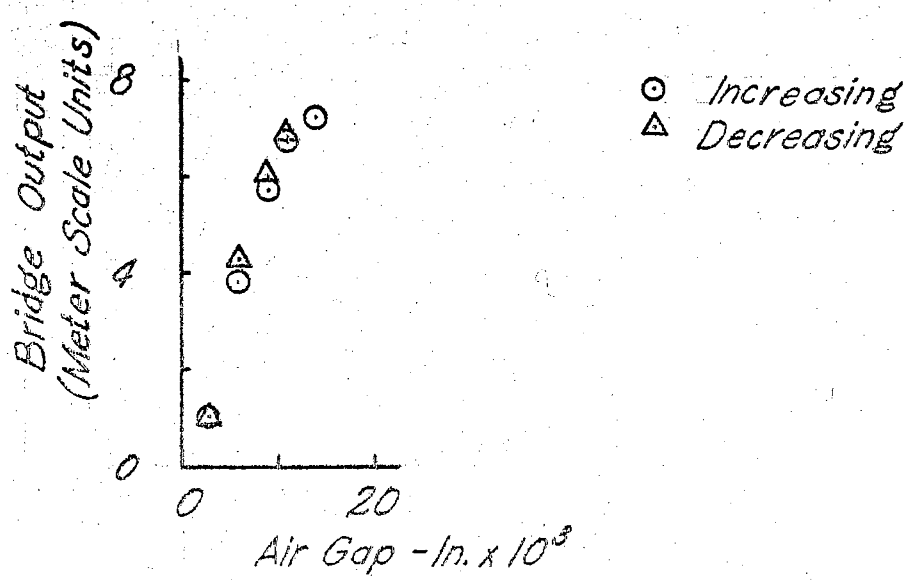

Figure 18. Bridge Output for Transducer A Mounted in Nozzle Assembly.

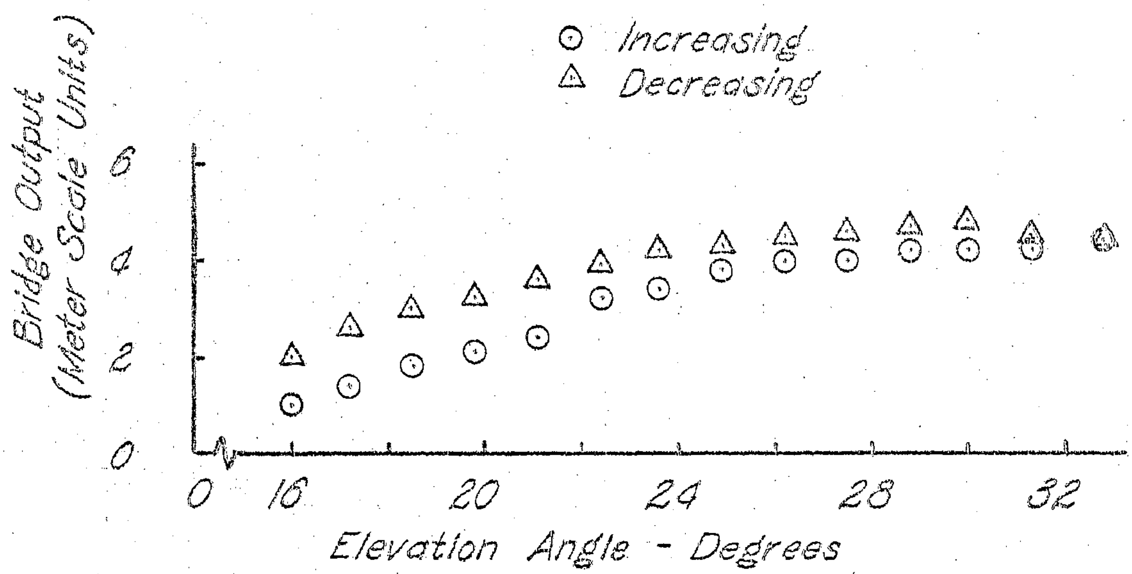

Figure 19. Bridge Output for Transducer A Mounted in Nozzle Assembly.

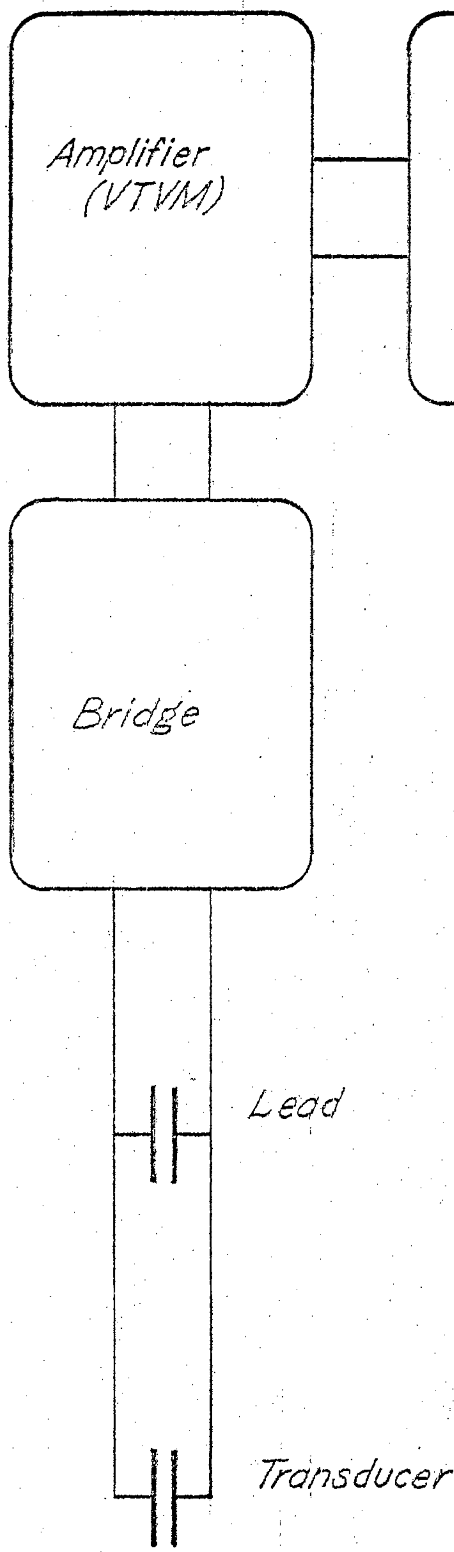

Figure 20. Circuit Schematic

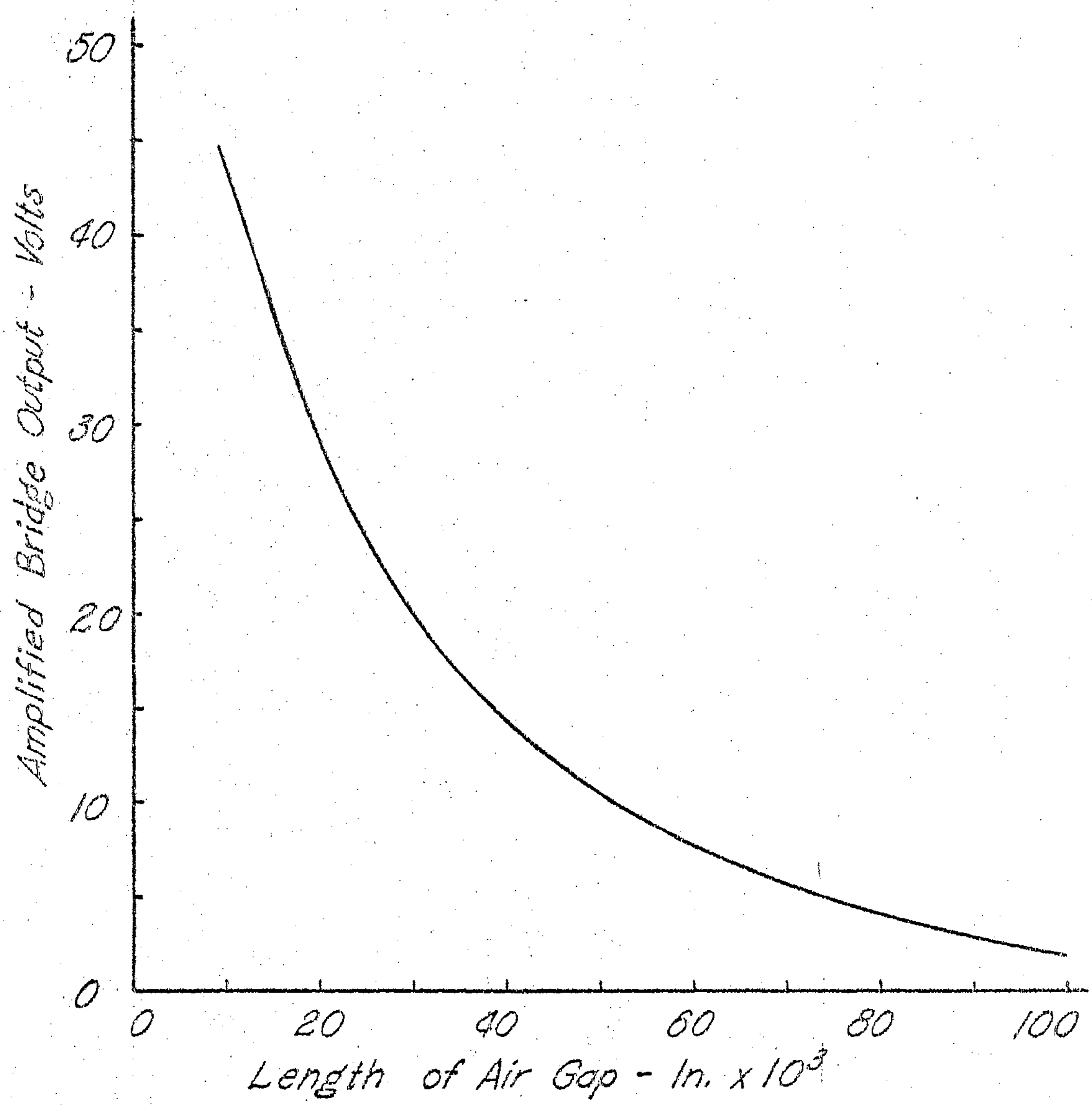

Figure 21. Amplified Bridge Output for Transducer A Mounted in Calibration Jig.

output recorded while decreasing the gap was greater than that occurring while increasing the gap. This can be attributed to the hysteresis of the mechanical linkage used to measure the gap.

Long Curved Plate Transducer

Transducer B shown in Figure 23 was made, having a plate radius of curvature of 0.55 inches, width of 0.30 inches and a length of 3.05 inches. The two projections on the side opposite the plate serve as lands with which to reference the transducer on the trailing edge of a blade, and the projection over the plate bears on the trailing edge of the adjacent blade. The spring clip serves to hold the transducer in position; the spanwise reference point is the outer surface of the inner rim. The transducer was connected in the circuit shown in Figure 20. The capacitance of the lead and the lead plus transducer was measured and the transducer then was positioned in the nozzle as shown in Figure 24. Since the blades of the nozzle were movable to some extent, the capacitance between the transducer and the adjacent blade was measured at the maximum and minimum separation distances available. The approximate difference in separation was 0.070 inches. Table I contains the data thus collected. In an effort to determine bridge output vs. change in separation distance, one thickness of Mylar tape (0.002 inches) was added to one land of the transducer at a time, and the capacitance measured at each addition. The bridge was first balanced with the blade separation at a maximum, then the blade separation reduced to the minimum and this separation maintained while taking the data, shown in Figure 25. Appendix C contains the complete data taken. Figure 26 shows the equipment used for measuring the changes in capacitance.

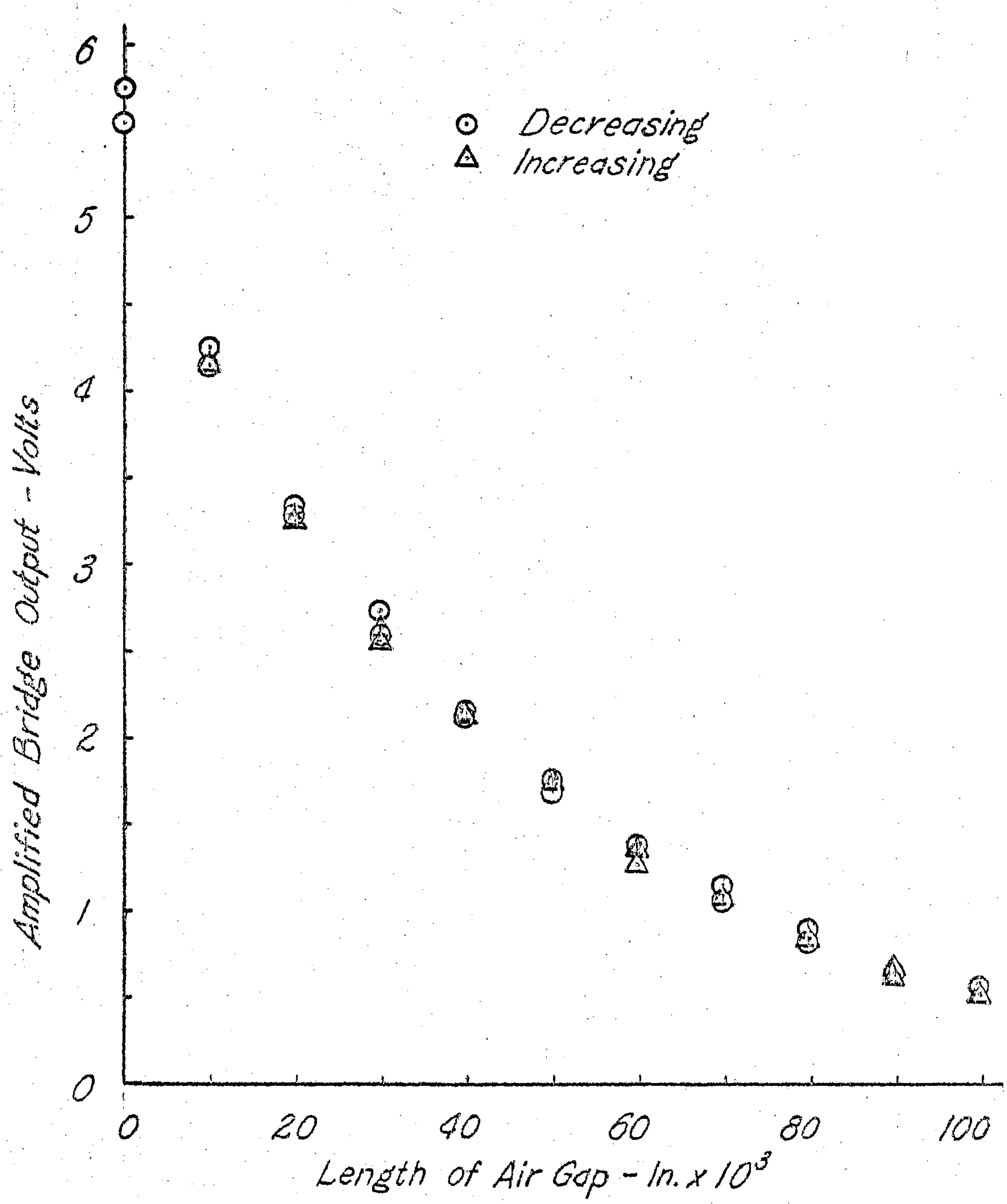

Figure 22. Amplified Bridge Output for Transducer A Mounted in Calibration Jig.

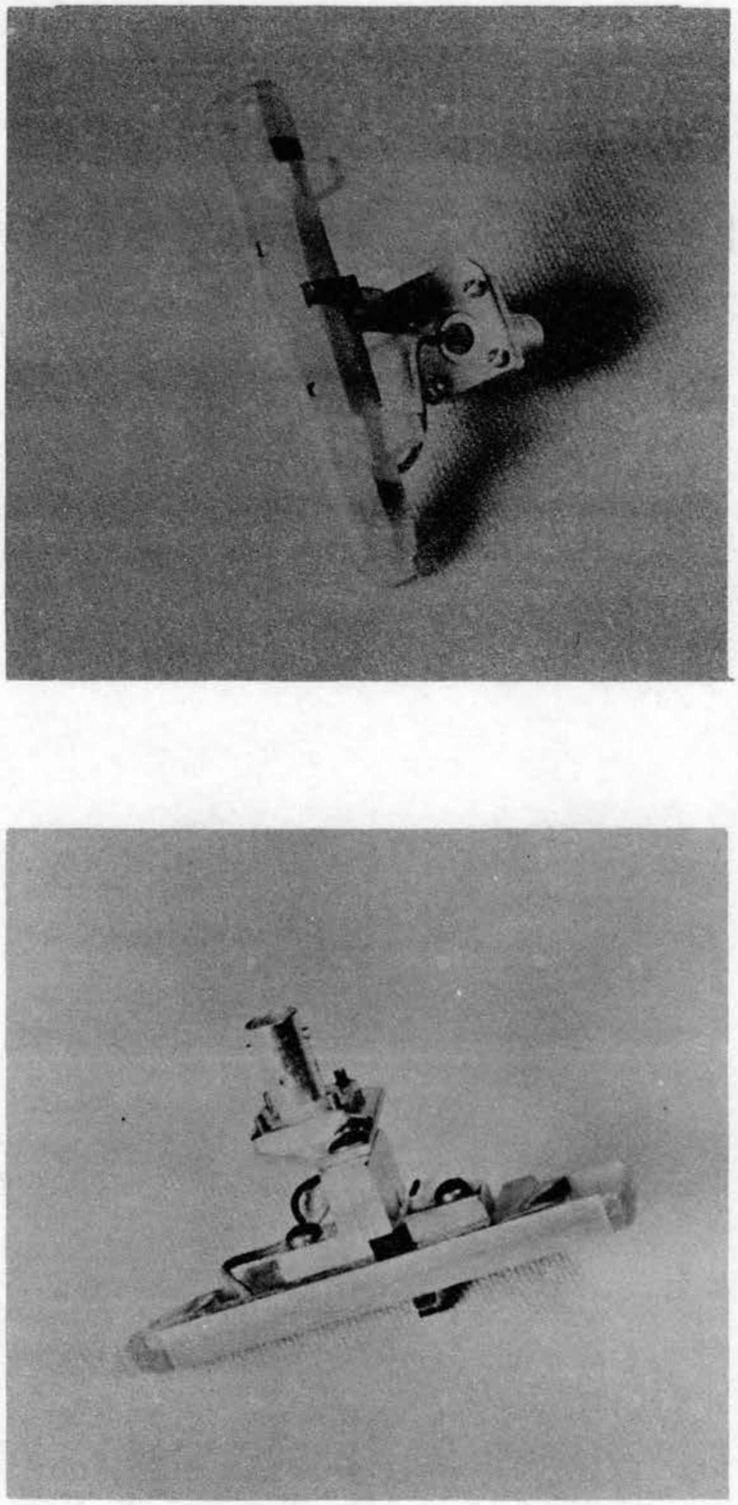

Figure 23. Transducer B.

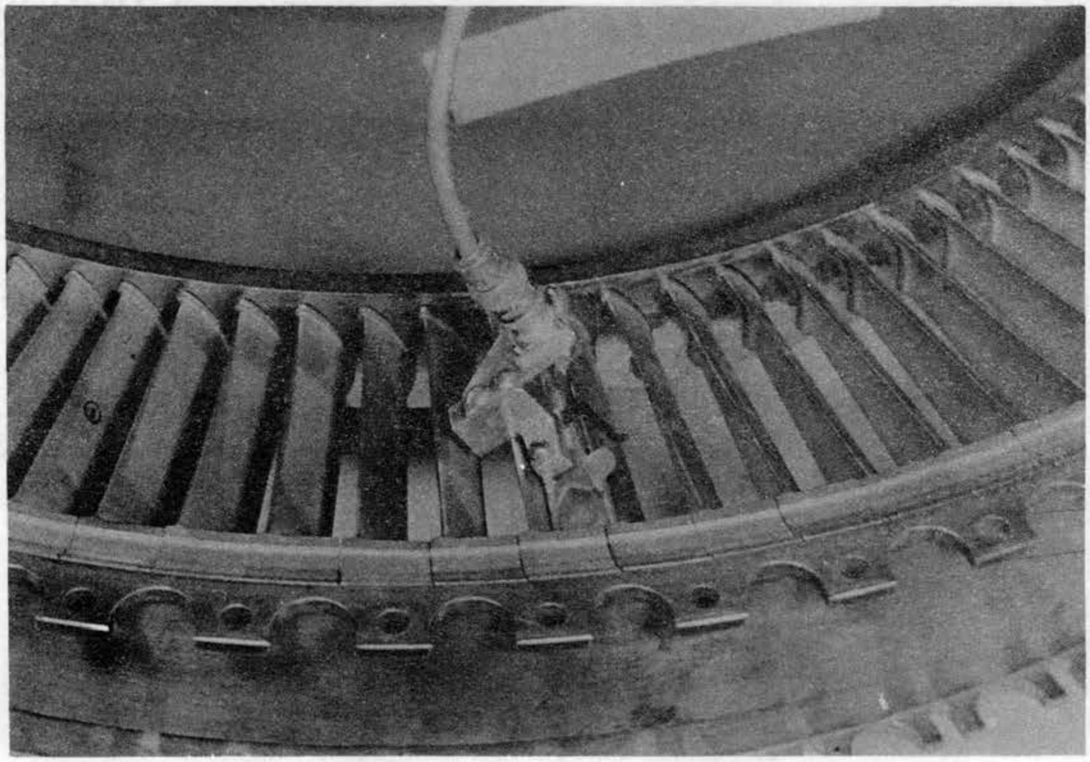

Figure 24. Transducer B Mounted in Nozzle.

TABLE I

CAPACITANCE OF TRANSDUCER B MOUNTED IN NOZZLE

Position	Capacitance
Lead	148.5uuf
Lead + Transducer	153.0uuf
Maximum Distance	162.5uuf
Minimum Distance	164.0uuf

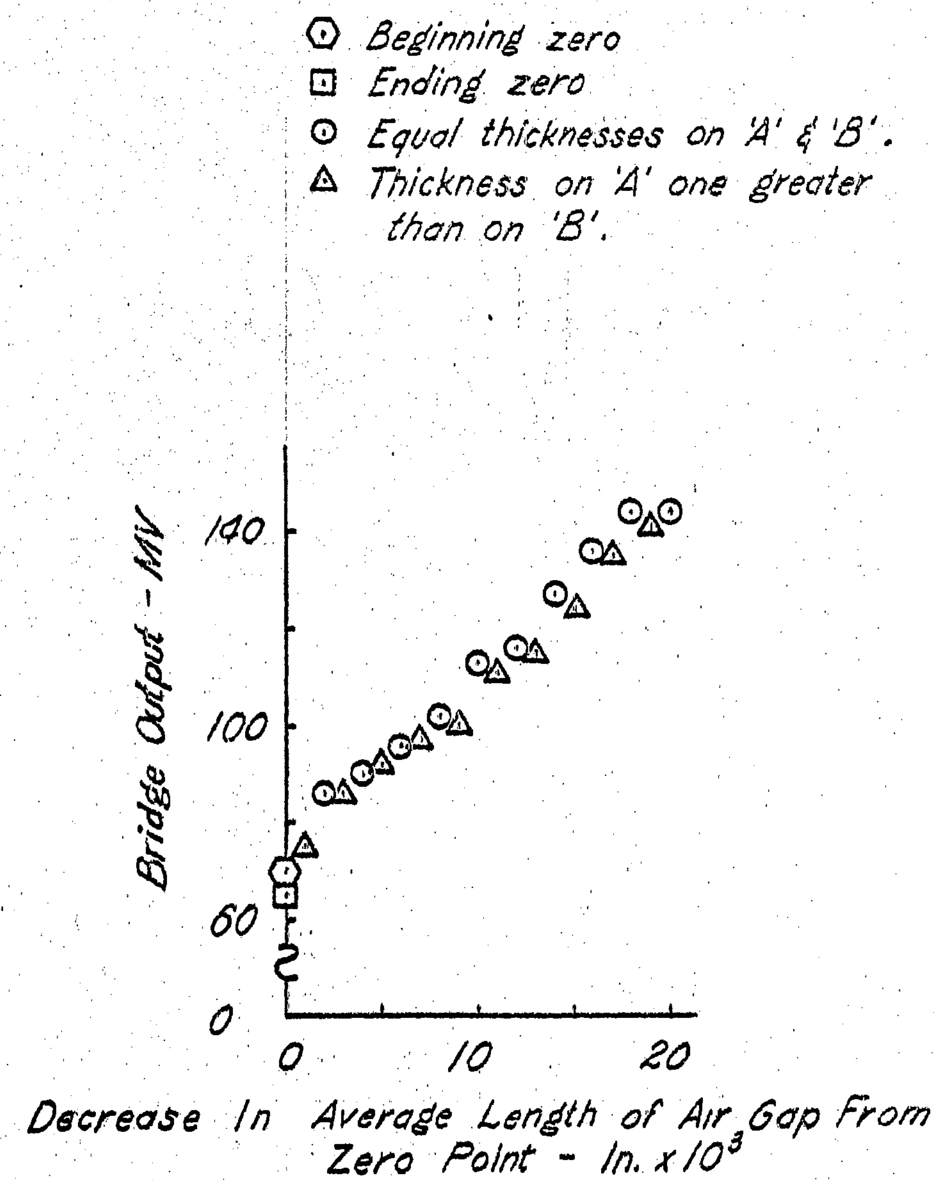

Figure 25. Bridge Output for Transducer B Mounted in Nozzle Assembly.

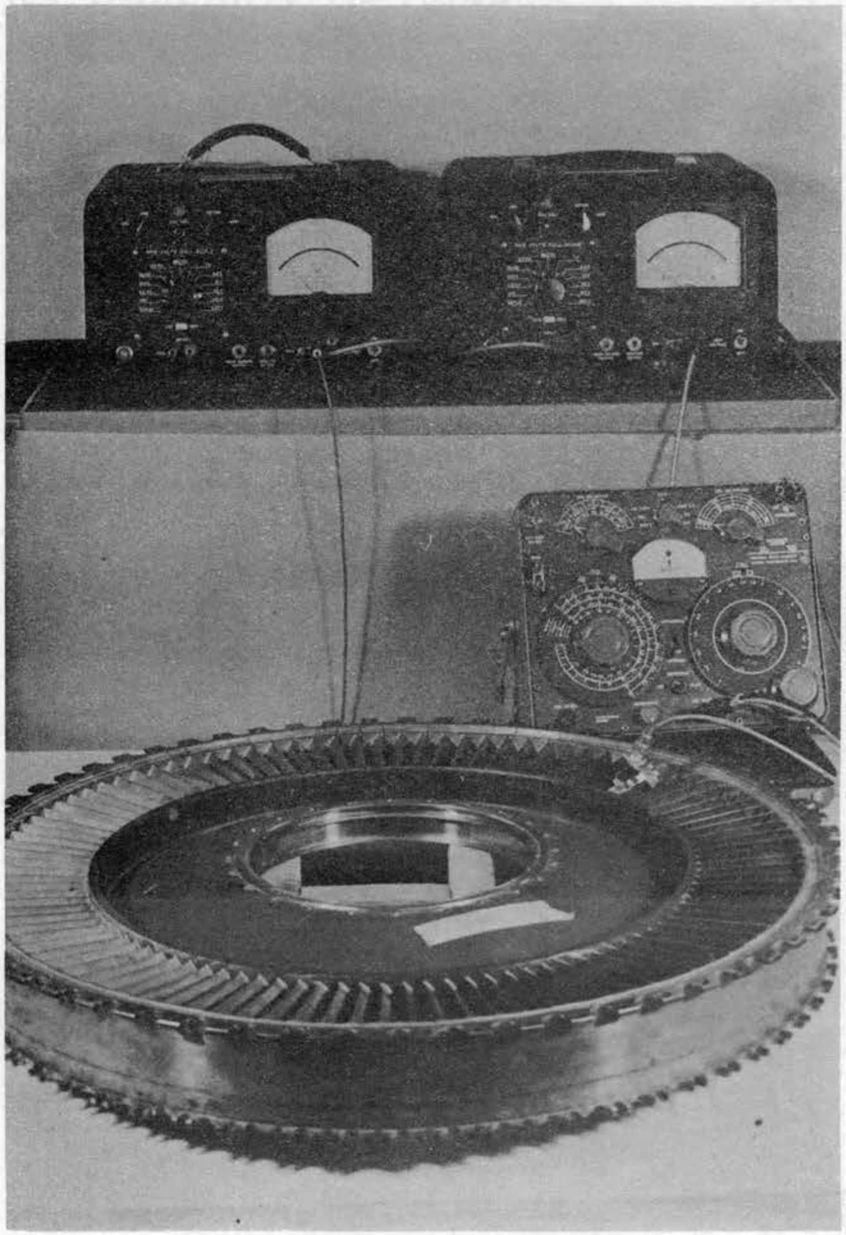

Figure 26. Equipment Used for Measuring Changes in Capacitance.

The inner land is referred to as 'A' and the outer one is 'B.' It was noted that at the zero position, the outer end of the transducer plate was somewhat closer to the adjacent blade than was the inner end of the plate. Since the slope of the capacitance vs. distance curve is greater for smaller distances of actual separation, a change in separation distance at the outer end would result in a larger change in capacitance than would an equal change at the inner end. Moreover, since some of the transducer plate was outboard of the 'B' land, a decrease in separation at the 'A' land would result in an increase in separation for that part of the plate outboard of the 'B' land. This could account for the fact that addition of one thickness to the 'B' land had more effect than did addition of one thickness to the 'A' land. The increase of separation outboard of 'B' could possibly change the capacitance more than the decrease in distance inboard of 'B.' However, since addition of tape to the lands made positioning of the transducer increasingly difficult, exact repeatable positioning was not possible. The probability of error was great, and the results cannot be regarded as being quantitatively representative.

CHAPTER IV

EVALUATION OF RESULTS

The purpose of this study was to investigate the feasibility of adapting capacitance methods to the measurement of available flow area of a turbine nozzle section of the J-57 jet engine. While the ultimate aim was to develop a system which would render a reliable direct reading of area, this study was confined to a preliminary investigation.

The effect of adding a layer of dielectric of different dielectric constant to a capacitor did not change the nature of the relationship between the change in capacitance vs. the change in distance, although the actual capacitance was increased over that of the single dielectric. That is, for a two dielectric capacitor, the capacitance was still inversely proportional to the distance separating the plates (Figure 14). These results are valid only when the change in separation distance is constant throughout the area of the plates concerned. Thus restricted, the distance of separation could be interpreted as the average distance of separation, and the validity of the relationship extended to a curved plate capacitor.

For transducer A, the change in capacitance is shown by Figure 18 and Figure 21 to be a function of the change in separation distance, although, as previously explained, the bridge output and not the actual capacitance was recorded. Figure 18 shows an increase in bridge output with an increase in separation distance, while the opposite was true in Figure 21. This was because the bridge was balanced at the initial gap for data taken with the transducer in the nozzle, while for data taken with the transducer mounted in the jig, the bridge was balanced with the gap set at a distance outside the range to be measured. The advantage gained by balancing the bridge at a gap outside the range to be measured was that the bridge meter did not undergo any off-scale excursion except in the lower end (0 to 0.008 inches) of the measured range. Figure 19 indicates the positioning angle of the transducer was not very critical, but should be in the range of 26^o to 30^o.

The manufacturer's specified accuracy of the VTVM was 3%, and the accuracy of reading the scales on the VTVM and the dial indicator was considered to be one half of the least division (3% on VTVM and 0.0005 inches on the dial indicator). Thus the over-all accuracy of output voltage was 6% and of gap distance was 0.0005 inches. The data of Figure 21 was within this range except for two points at the maximum gap range. At a gap of d = 0.085 inches, the accuracy of measurement of gap was found to be

$$A = \frac{(0.0905 - 0.0795)(3.35 - 3.5)}{(2.9 - 3.85)} = 0.00174 \text{ in}$$

and at a gap of d = 0.025 inches,

$$A = \frac{(0.0305 - 0.0195)(23.2 - 24.7)}{(29 - 20.25)} = -0.00189 \text{ in}$$

where $A = \frac{\text{Maximum deviation}}{\text{Minimum slope}}$.

The accuracy of the present system is at least $\pm$ 0.0005, which can be slightly bettered by an experienced operator.

The sources of errors in the measurement were system error of mechanical hysteresis in jig, random error of positioning the movable plate, plus the error due to drift of the circuitry. This drift was noted to be oscillatory and erratic over long periods of time. Considerable drift was also noted upon first activating the circuit; allowing time for the bridge elements to reach a stable temperature before taking data and making the time period of data collection as short as possible minimized the drift effects to a large extent. The bridge circuit was also very sensitive to persons in the vicinity; the sensitivity decreased with increased distance from the calibration jig. The bridge meter could be pegged by moving a hand within one or two inches of the transducer. An attempt was made to minimize the error introduced by this sensitivity by duplicating operator position as nearly as possible while reading data on the VTVM.

For the transducer B mounted in the nozzle, Figure 25 shows that the change in capacitance is a function of the change in average distance of separation, but the change of slope of one blade with respect to the other introduces error in the function as predicted by Equation (9). For varying amounts of slope and equal decrements of average separation distance, the resulting increase in capacitance or bridge output described a "step" function and not a smooth curve. The increasing bridge output of Figure 25 represents an increasing capacitance with decrease in distance, as indicated in Appendix C. Because of the difficulty of duplicating with any degree of precision the change in length of air gap, and the obviousness of the error introduced by slope changes, no attempt was made to collect extensive data for transducer B mounted in the nozzle, nor to evaluate the accuracy of measurement provided. However, it was noted that the sensitivity of the transducer to persons in the vicinity was much less than was the transducer in the calibration jig, due to the shielding effect of the nozzle assembly.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The use of capacitance characteristics presents a feasible method of measuring the available flow area of the nozzle section of the J-57 jet engine turbine. The relationship between average distance of separation of plates and capacitance for short length curved plate capacitors having two dielectrics between the plates provides a reproducible measurement of changes in distance through direct or indirect measurement of changes in capacitance, as long as the change in distance is constant over the area of the plates of the capacitor. However, when the change in distance is not constant, the capacitance becomes a function of not only the separation distance but also of the slope of one plate with respect to the other. This effect is minimized when one or both plates are short, but become pronounced as the length of the plates increases.

It is recommended that instead of using a long curved plate, a transducer having several short plates be used to measure the area directly. It is felt that a transducer modeled after Figure 27 (a) would serve the purpose of minimizing effects of changes in slope. It also is possible that, by adding plates as shown in Figure 27 (b), the spanwise curvature of the nozzle blades will be accounted for, and the direct linear spanwise measurement between inner and outer rims would be obviated. In both of these, the area of each of the radially oriented plates should be such as to automatically weight the measurement made at that point.

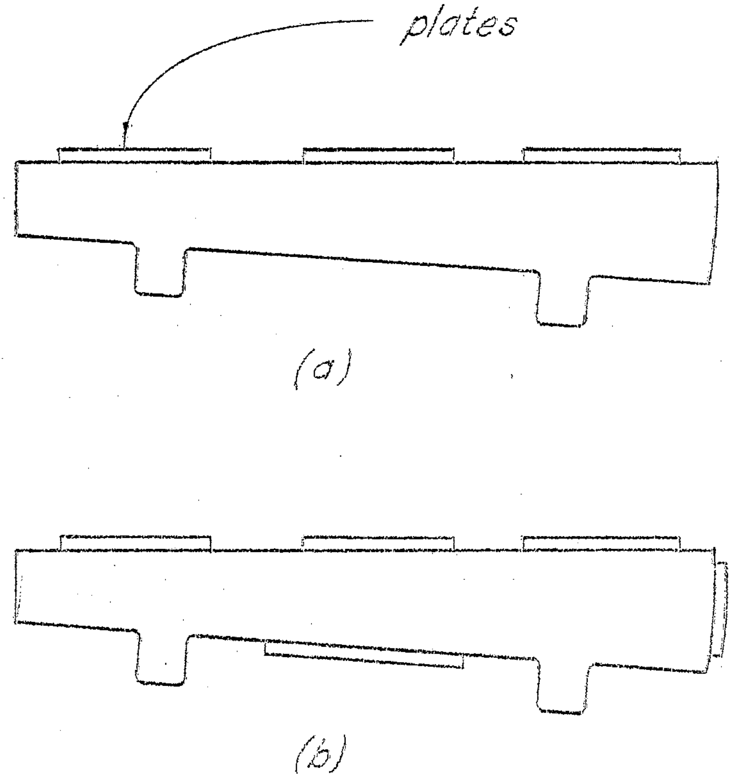

Figure 27. Proposed Transducers.

For instance, if the spanwise location of the plates coincide with the location of present measurements, the areas would be in proportion to the weighting factors shown in Equation (1).

The response of the transducer could be increased by using a dielectric with a higher dielectric constant to cover the transducer plates; for example, a ceramic (titania) dielectric with a constant of 100 would increase the absolute magnitude of the capacitance slope (Figure 11) by approximately 23 times.

The use of guard rings, as suggested by Gilbert (3), in the same plane as and completely surrounding the plates would decrease the effects of stray capacitance.

In order to establish a workable system, much care and precision would be necessary to compensate for or cancel the effects of circuitry drift and ambient conditions. An elaborate calibration system would also be required to attain the accuracy required. This would entail a high initial cost, but it is considered likely that this would be offset by the efficiency of the final system.

SELECTED BIBLIOGRAPHY

1. Roberts, Howard C., Mechanical Measurements by Electrical Methods, (The Instruments Publishing Co., Inc., Pittsburgh, 1946); Chapter 3.

2. Dummer, G. W. A., Variable Capacitors and Trimmers, (Sir Isaac Pitman and Sons, Ltd., London, 1957); Chapter 2.

3. Gilbert, Norman E., Electricity and Magnetism, (The Macmillan Company: New York, 1950) Sections 4.1 through 4.9.

APPENDIX A

BRIDGE OUTPUT FOR TRANSDUCER A IN CALIBRATION JIG

Length of Air Gap ($in \times 10^3$)	Bridge Output - Volts (Amplified)			
	Decreasing	Increasing	Decreasing	Increasing
100	1.7	1.8	1.85	1.9
95	2.15	2.3	2.35	2.35
90	2.9	2.8	2.75	2.9
85	3.5	3.35	3.45	3.4
80	3.85	4.0	4.15	3.95
75	4.7	4.75	4.85	4.9
70	5.6	5.6	5.75	5.6
65	6.7	6.6	6.7	6.65
60	7.9	7.5	7.8	7.75
55	9.1	8.85	9.1	8.9
50	10.5	10.25	10.6	10.25
45	12.2	11.8	12.1	12.0
40	14.2	14.0	14.25	14.1
35	17.25	16.4	17.0	16.8
30	20.25	19.4	20.1	19.2
25	24.7	23.6	24.3	23.2
20	29.5	29.0	29.6	29.0
15	36.8	36.0	36.5	35.5
10	44.0	43.3	44.0	
5	45.5			

APPENDIX B

BRIDGE OUTPUT FOR TRANSDUCER A IN CALIBRATION JIG

Length of Air Gap (in x 10^3)	Bridge Output - Volts (Amplified)					
	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.
100	0.57	0.525	0.505	0.535		0.535
90	0.65	0.675	0.65	0.64	0.645	0.62
80	0.90	0.85	0.83			
70	1.15	1.08	1.06			
60	1.38	1.36	1.39	1.27	1.37	1.31
50	1.76	1.74	1.69			
40	2.16	2.13	2.12			
30	2.73	2.63	2.70	2.55	2.71	2.61
20	3.34	3.25	3.29			
10	4.25	4.15	4.19			
0	5.75		5.55			

APPENDIX C

BRIDGE OUTPUT FOR TRANSDUCER B IN NOZZLE

Position	Number of Thicknesses of Tape on Land A	Number of Thicknesses of Tape on Land B	Decrease in Average Length of Air Gap from Zero Point (in x 10^3)	Bridge Output (mV)
1	0	0	0	70
2	1	0	1	75
3	1	1	2	86
4	2	1	3	86
5	2	2	4	90
6	3	2	5	92
7	3	3	6	96
8	4	3	7	97
9	4	4	8	102
10	5	4	9	100
11	5	5	10	113
12	6	5	11	111
13	6	6	12	116
14	7	6	13	117
15	7	7	14	127
16	8	7	15	124
17	8	8	16	136
18	9	8	17	137
19	9	9	18	144
20	10	9	19	141
21	10	10	20	144
22	0	0	0	65

APPENDIX D

LIST OF APPARATUS USED

1. True RMS Electronic Voltmeter (VTVM)
   Ballantine Laboratories, Inc.
   Model 320
   100uV to 320 Volts RMS
   (2 Units)

2. Impedance Bridge
   General Radio Company
   Type 1650-A
   luuf to 1000 uf

3. Dial Indicator and Accessories
   Brown and Sharpe Manufacturing Company
   0 to 0.1 inches

VITA

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