MATRIX ANALYSIS OF OPEN SPANDREL ARCHES
BY THE STRING POLYGON METHOD

By

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M. N.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

d_j, d_k . . . . .Segment Lengths, or Panel Lengths.

$[f_j]_{3x3}$ . . . . $[G_j]_{3x8}\ [C_2]_{8x3}$.

h_i, h_j . . . . .Height of Spandrel Columns.

i, j, k . . . . .Deck Joints.

i', j', k' . . . . .Arch Joints

ℓ_1 . . . . .Span AB.

n . . . . .Number of Closed Arch Panels.

x_{1j}, x_{2j}, x_{3j} . . . . .j th Panel Redundants.

$\bar{x}$, $\bar{y}$ . . . . .Coordinate Axes.

$\bar{x}_j$, $\bar{y}_j$ . . . . .Coordinates of Arch Joint of Panel j with Respect to Axes $\bar{x}$, $\bar{y}$.

A, B, C, D . . . . . .Arch Supporting Column Ends.

$[B_j]$ . . . . .Column Vector of Bending Moments Due to Loads of Panel j.

BM_{ij}, BM_{ji} . . . . . .Bending Moment of Basic Structure Due to Load.

$[C_1]_{8x9}$ . . . . .Coefficient Matrix of the Vector [X] .

$[C_2]_{8x3}$ . . . . .Coefficient Matrix of the Vector [Y] .

EI . . . . .Flexural Rigidity.

F_{ji}, F_{jk} . . . . .Angular Flexibilities.

ΣF_j . . . . $F_{ji} + F_{jk}$.

$[F_j]_{3x9}$ . . . . $[G_j]_{3x8}[C_1]_{8x9}$.

G_{ij}, G_{kj} . . . . Angular Carry-Over Values.

$[G_j]_{3x8}$ . . . . Matrix of Angular Carry-Over Values.

M_{AB}, M_{BA} . . . . . . Panel End Moments.

M_i, M_j, M_k . . . . . . Moments at i, j, and k.

$[M_j]_{8x1}$ . . . . Moment Matrix of Panel j.

$\overline{P}_j$ . . . . Joint Elastic Weights at j.

$\overline{P}_{ji}$, $\overline{P}_{jk}$ . . . . Segmental Elastic Weights at j in ijk.

$\overline{P}_i^j$, $\overline{P}_j^j$ . . . . Joint Elastic Weights for Panel j at i, j .

$[T_j]_{3x1}$ . . . . Column Vector Containing Load Functions, τ .

$[X_{ijk}]$ . . . . Column Vector of Redundants of Panels i, j, and k .

Y_1, Y_2, Y_3 . . . . . . Column Redundants.

$[Y_{1,2,3}]$ . . . . Column Vector of Column Redundants.

[**f**] . . . . All [f] Matrices of the Structure.

[**B**] . . . . All [B] Matrices of the Structure.

[**E**] . . . . Additional Equations in Matrix Form to Solve for the Redundants.

[**F**] . . . . All [F] Matrices of the Structure.

[**G**] . . . . All [G] Matrices of the Structure.

[**T**] . . . . Column Vector Containing the Load Functions, τ, for the Whole Structure.

ν_j . . . . $\frac{h_j}{h_i}$.

τ_{ji}, τ_{jk} Angular Load Functions.

$\Sigma\, \tau_j$ $\tau_{ji} + \tau_{jk}$.

CHAPTER I

INTRODUCTION

1-1. General

The open spandrel arch, whether totally fixed at the springing or continuous over elastic piers, is externally and internally indeterminate. For an accurate prediction of stresses in such structures, the interaction between the deck, the spandrel columns and the arch rib must be considered. If complete continuity of all elements is considered, the open spandrel arch becomes a complex system for which the mathematical analysis by classical methods is so complicated that it has been seldom undertaken. The existence of interaction between the various parts of the open spandrel arch, however, have long been recognized, and the complexity of the standard methods of analysis led to the extensive experimental studies of Finlay[1], Wilson[2], and Newmark[3].

An experimental analysis carried out by Wilson and Kluge consisted of tests of three span arches on high piers, each span being composed of a rib with spandrel columns and a deck. These experiments were usually limited to the determination of influence lines for the fixed-end reactions at the springings and the moment and thrust at the arch crown.

A theoretical analysis of open spandrel arches using equivalent elastic systems was developed by Beaufoy[4]. In his approach, Beaufoy considered full continuity between the arch rib, the spandrel columns and the deck. Comparison with some experimental results of Wilson and Kluge showed close agreement, and the shear resistance of the spandrel columns is shown to be an important item in deck participation. Beaufoy did not consider multi-span open spandrel arches.

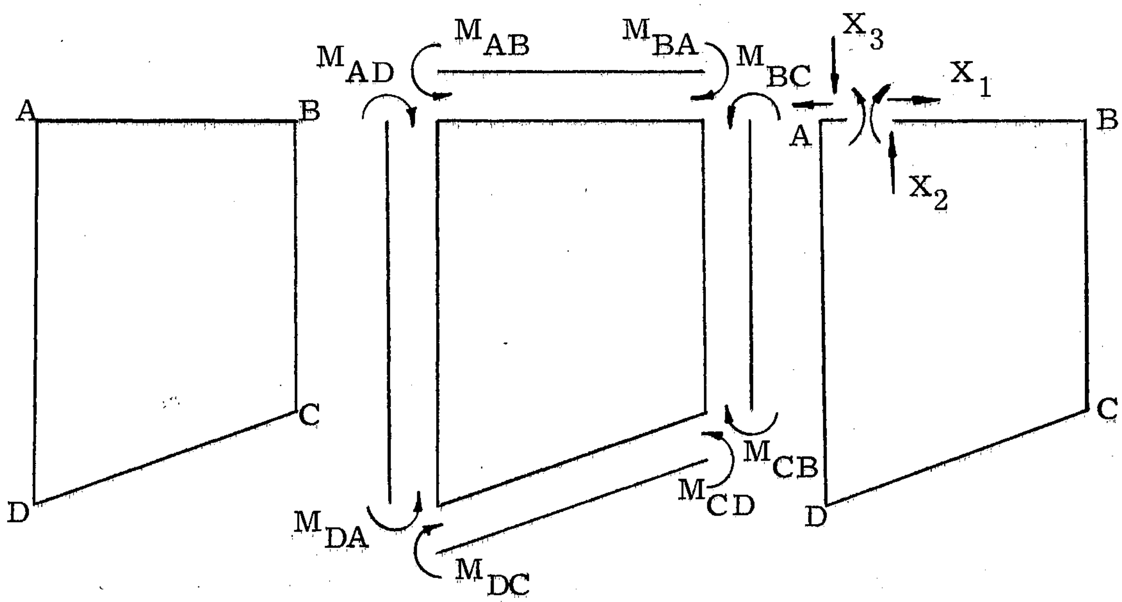
A mathematical solution for the open spandrel arch as a monolithic structure, assuming full continuity, was presented by Diwan[5]. The solution was based on the principle of balancing the panel moments. This is done by imposing a special type of panel distortion which produces chord moments of a known pattern only in the deformed panel. It was then possible to study, analytically, deck participation.

The purpose of this thesis is to show the application of the string polygon method to the analysis of open spandrel arches. It is shown that the method provides an efficient means for matrix formulation of the problem, and is sufficiently broad in scope to allow open spandrel arches of more than one span to be analyzed.

The string polygon method is based on the concept of joint elastic weights. Representation of deformations as conjugate forces and moments was first presented by Mohr[6], and extensions of this concept were investigated by several authors. The development of joint elastic weights may be found in the work of Kaufman[7]. The string polygon method is based on a generalization of the joint elastic weight expression. The approach was developed by Tuma in his lectures at Oklahoma State University, and extended by Chu[8], Oden[9], Boecker[10], and others to the solution of many special problems. The possibility of the application of the string polygon method to the analysis of complex frames was recorded by Tuma and Oden[11].

1-2. Scope of Study

For each closed panel of an open spandrel arch (Fig. 1-1a), either the end bending moments (Fig. 1-1b), or any three panel redundants (Fig. 1. 1c) may be selected as unknowns.



(a) Closed Panel of an Open Spandrel Arch

(b) End Moments as Unknowns

(c) Panel Redundants as Unknowns

Fig. 1-1

Unknowns of Typical Arch Panel

The selection of unknowns distinguishes between two ways in which the string polygon method is used in analysis. The two standard approaches are:

1. The direct approach in which end moments are selected as unknowns, and

2. The <u>indirect approach</u> in which only panel redundants are selected as unknowns.

In this study, a combination of the two methods is adopted, and the relationship between the two approaches is illustrated by a series of matrix operations.

Joint elastic weights are computed for each panel in terms of end moments. Three elasto-static equations for each panel are then written in terms of end moments. The end bending moments, in turn, are expressed in matrix notation in terms of panel redundants. Additional elasto-static equations are obtained by taking a suitable conjugate structure for each span.

In addition to the usual assumptions of linear structural analysis, the following assumptions are made:

1. The spandrel columns are vertical.
2. The bottom member of each panel is straight.
3. Loads are applied normally to the arch deck.

A brief discussion of the string polygon method is given in Chapter II. Chapter III deals with the matrix formulation of the problem and in Chapter IV application of the theory is illustrated numerically. The study is summarized and pertinent conclusions are drawn and listed in Chapter V.

CHAPTER II

STRING POLYGON METHOD

Basic principles of the string polygon concept are fully discussed elsewhere and are not repeated here. The classification of elastic weights is restated for completeness.

2-1. Classification of Elastic Weights

It was shown by Tuma and Oden that there are three types of elastic weights:

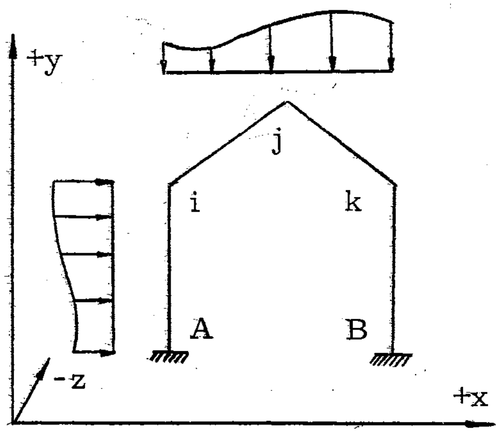
1. Elemental elastic weights.
2. Segmental elastic weights.
3. Joint elastic weights.

The representation of these elastic weights as loads on the conjugate structure is illustrated in Fig. 2-1.

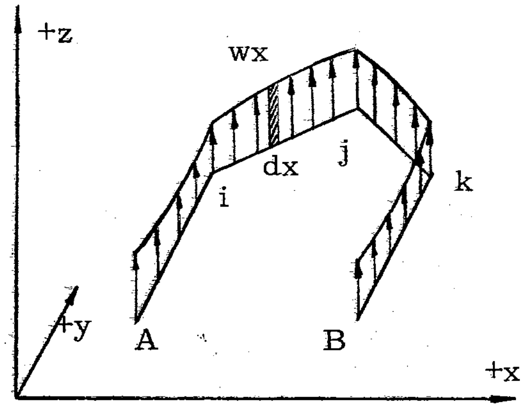
The application of the elemental elastic weights to the analysis of a closed ring is well known under the names of "column analogy" developed by Cross[12] and the "conjugate frame method" discussed by Kinney[13].

Segmental elastic weights may be interpreted as reactions of each segment of the conjugate structure. The segmental elastic weights (denoted $\overline{P}_{ji}$, $\overline{P}_{jk}$, · · · , etc.) are defined as the changes in the slope of of the elastic curve between the respective ends of each segment.

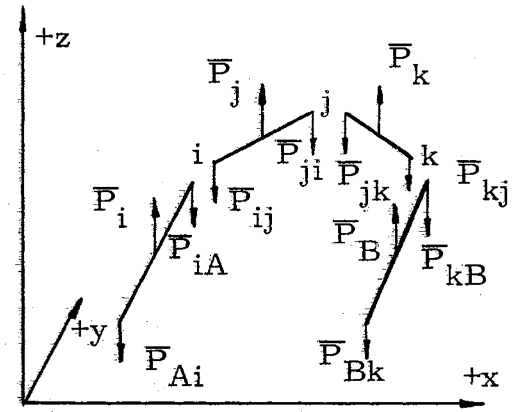
Joint elastic weights may be defined as the change of the change in slope of the polygonal line, ijk, at j .



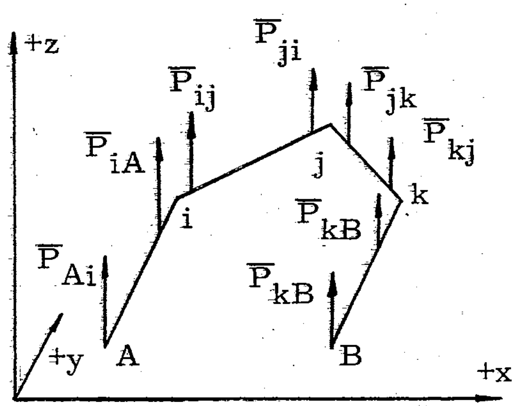
Real Frame with Real Loads



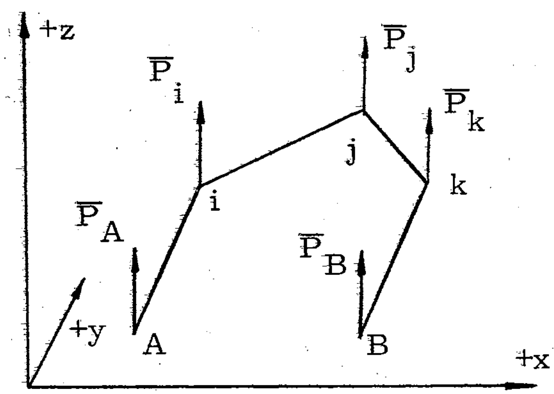
Conjugate Frame with Elemental Elastic Weights



Segmental Conjugate Beams with Segmental Elastic Weights



Conjugate Frame with Segmental Elastic Weights



Conjugate Frame with Joint Elastic Weights

Fig. 2-1 Elastic Weights

2-2. Equations for Elastic Weights

The joint elastic weight, $\overline{P}_j$, is related to the segmental elastic weights by the general formula,

$$\overline{P}_j = \overline{P}_{ji} + \overline{P}_{jk} . \qquad (2\text{-}1)$$

Segmental elastic weights may be expressed in terms of end moments as follows:

$$\begin{aligned} \overline{P}_{ji} &= M_{ij} G_{ij} + M_{ji} F_{ji} + \tau_{ji} \\ P_{jk} &= M_{kj} G_{kj} + M_{jk} F_{jk} + \tau_{jk} , \end{aligned} \qquad (2\text{-}2)$$

where

F_{ji}, F_{jk} = Angular flexibilities;

G_{ij}, G_{kj} = Angular carry-over values;

M_i, M_j, M_k = Moments at i, j and k; and

τ_{ji}, τ_{jk} = Angular load functions.

These quantities are defined in Table 2-1. Therefore, the joint elastic weight is given by

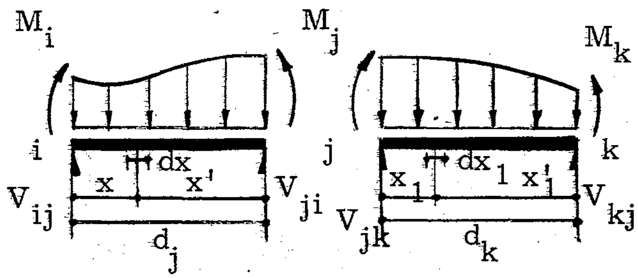
$$\overline{P}_j = M_i G_{ij} + M_j \sum F_j + M_k G_{kj} + \sum \tau_j , \qquad (2\text{-}3)$$

in which

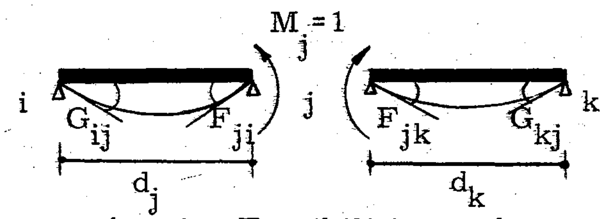
$$\sum F_j = F_{ji} + F_{jk} ,$$

$$\sum \tau_j = \tau_{ji} + \tau_{jk} ,$$

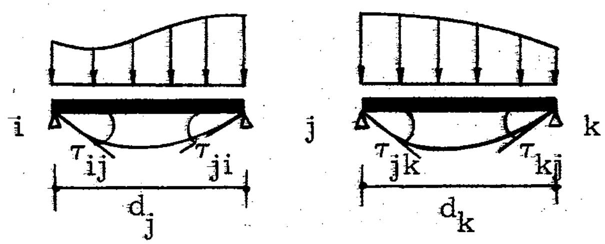
TABLE 2-1 ANGULAR FLEXIBILITIES AND LOAD FUNCTIONS



Segments ij, jk with Loads



Angular Flexibilities and Carry Over Values



Angular Load Functions

Quantity	Algebraic Definition	Physical Interpretation
F_{ji}	$\int_i^j \frac{x^2\,dx}{d_j^2\,EI_x}$	End slope of j of the simple beam ij due to a unit moment applied at j.
F_{jk}	$\int_j^k \frac{x_1'^2\,dx_1}{d_k^2\,EI_{x_1}}$	End slope of j of the simple beam jk due to a unit moment applied at j.
G_{ij}	$\int_i^j \frac{xx'\,dx}{d_j^2\,EI_x}$	End slope of the simple beam ij at i due to unit moment applied at j.
G_{kj}	$\int_j^k \frac{x_1 x_1'\,dx_1}{d_k^2\,EI_{x_1}}$	End slope of the simple beam jk at k due to unit moment applied at j.
τ_{ji}	$\int_i^j \frac{BM_x\,x\,dx}{d_j\,EI_x}$	End slope of the simple beam ij at j due to loads.
τ_{jk}	$\int_j^k \frac{BM_{x_1}\,x_1'\,dx_1}{d_k\,EI_{x_1}}$	End slope of the simple beam jk at j due to loads.

These elastic weights may be used for the calculation of bending moments and displacements of joints of the polygon by considering the equations of "elasto-static" equilibrium of the conjugate structure. Unknown end moments for any closed panel can be evaluated by solving the equations of static equilibrium and elasto-static equilibrium of the real and conjugate structures, respectively.

CHAPTER III

MATRIX FORMULATION

3-1. Selection of Redundants

A continuous, multispan, open spandrel arch subjected to a general system of deck loads is considered (Fig. 3-1). The arch spandrel contains 'n' closed panels and the supporting columns may have any degree of fixity at ends A, B, C, D, ...

There are three redundants corresponding to each closed ring of the structure. The 'panel' redundants for the j th panel are designated as X_{ij}, X_{2j}, and X_{3j}; and the 'column' redundants developed in the supporting arch are denoted Y_1, Y_2, Y_3, ...

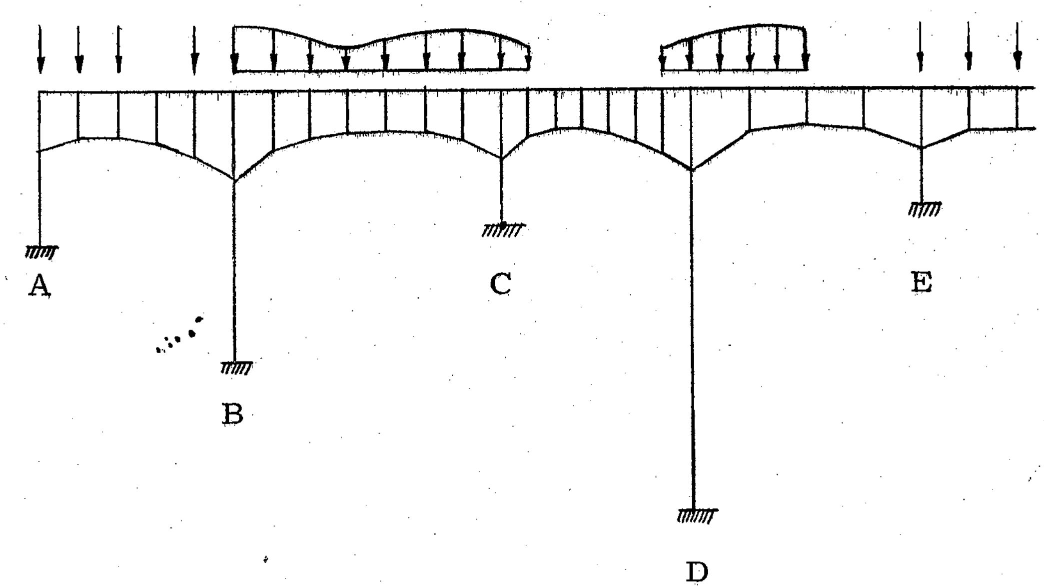


Fig. 3-1

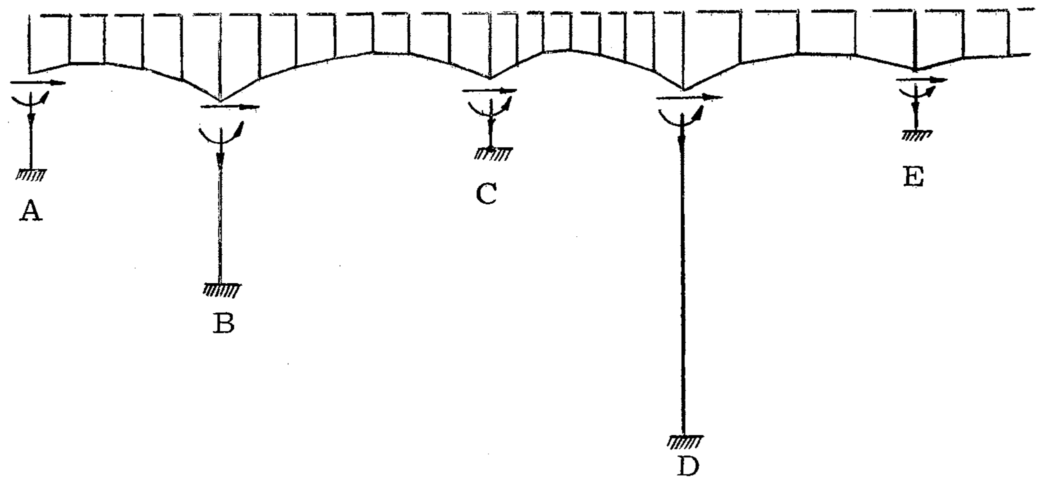
General Continuous Open Spandrel Arch

The choice of redundants is completely arbitrary in the general flexibility approach to the analysis of complex structures. There are, in fact, an infinite number of different choices for just the panel redundants of a single closed ring. With computer facilities available, the most time consuming part of matrix analysis of complex structures is the process of obtaining the necessary algebraic relationships between unknowns, rather than the process of solving the algebraic equations. Thus, the selection of redundants must be made with this in mind. The goal of this investigation is to establish a systematic and simple procedure for obtaining a set of simultaneous equations for the redundants.

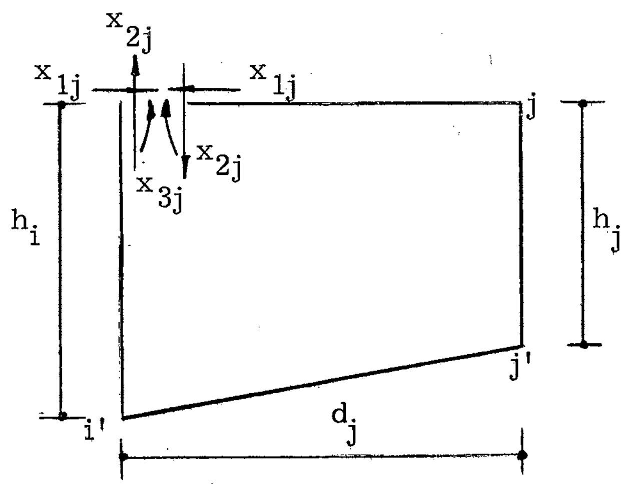
Redundants of each panel are selected near the top left joint of the panel (Fig. 3-2). The bending moment due to unit redundants can be obtained easily in terms of the dimensions of the panel under consideration. The column redundants are selected at the top of each column so that the redundants of any column will only influence the panels of a single span. With this choice of redundants, coefficients in equations relating end moments and redundants are simply lengths of members and coordinates of joints.

Releasing the panel and column redundants, the basic structure for the arch becomes the structure shown in Fig. 3-2. At each support, in general, three column redundants are considered. Modifications can be easily made for other support conditions.

The redundants of one panel will influence only the adjacent panels. In general, X_{1j}, X_{2j}, and X_{3j} will effect panels i, j, and k. Therefore, without loss of generality, the panels i, j, and k with the corresponding three column redundants are considered for the general matrix formulation of the problem.



(a) Basic Structure



(b) A Typical Panel of Basic Structure

Fig. 3-2

Geometry of Basic Structure

3-2. Static Relationships

Directions of end bending moments of each panel are selected in such a way that panel deformations are compatible with those of adjacent panels. For example, if end moments of i and k cause tension on the outside fibers of those two panels, end moments of panel j must produce tension inside the panel j.

Fig. 3-3 shows panel redundants for panels i, j, and k along with the column redundants, Y_1, Y_2, and Y_3 which influence deformations of these panels. End moments of the j th panel are assumed to produce tension inside the panel. The bending moment diagrams produced by unit redundants of panels i, j, and k are given in Fig. 3-4.

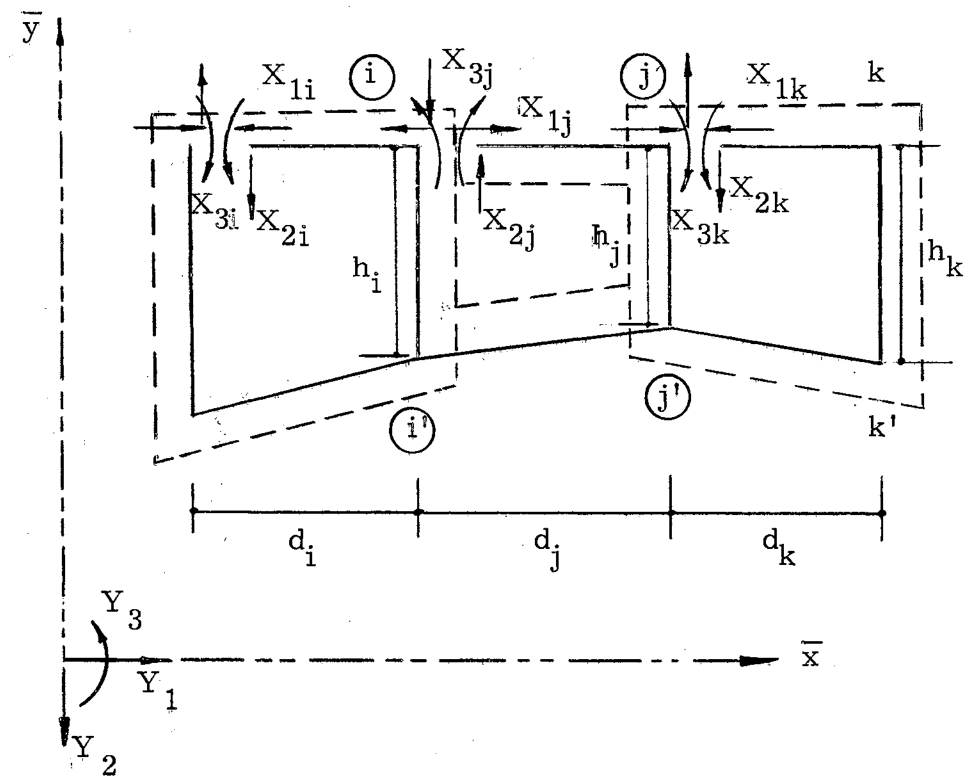


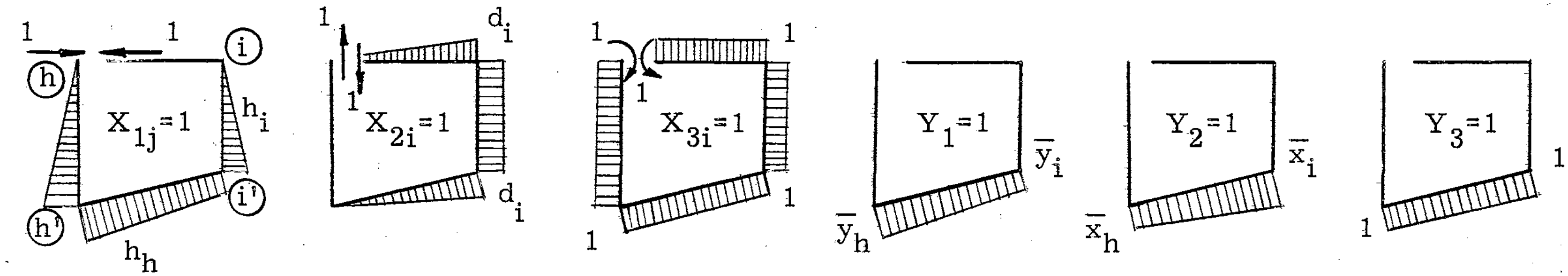
Fig. 3-3

Typical Panels i, j, and k

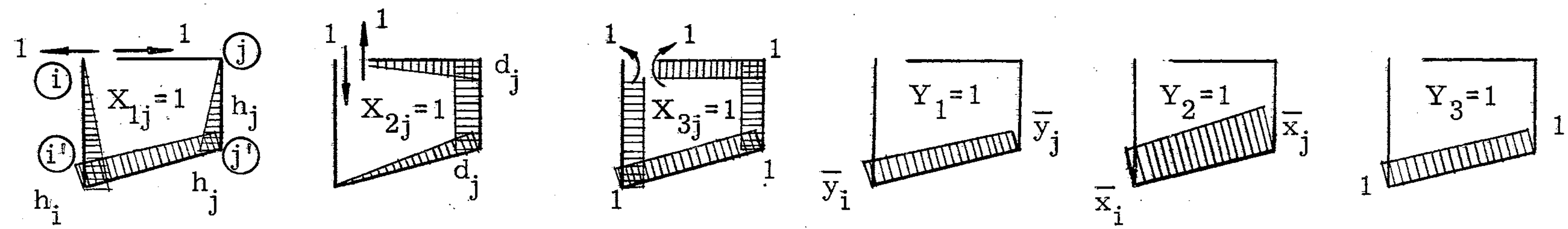
Positive end moments of panels i, j, and k causing compatible deformations of those panels are shown in Fig. 3-5.

Using the equations of statics, end moments for panel j can be written in terms of the twelve redundants as follows:

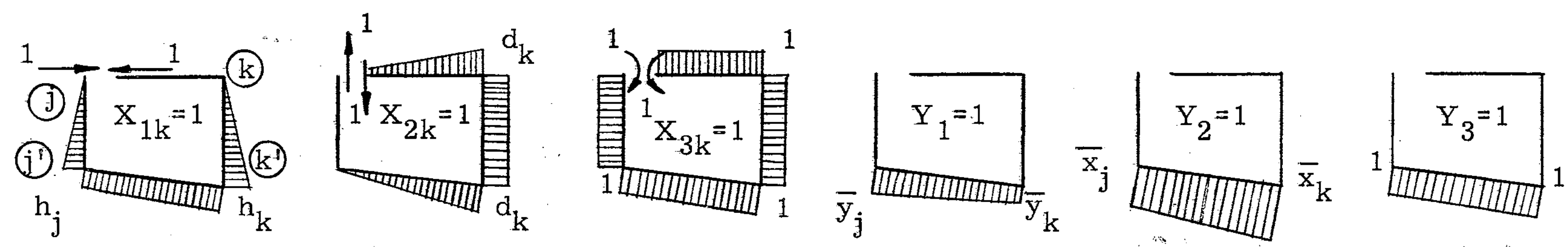
$$M_{ij} = X_{3j} + BM_{ij}$$



(a) Panel i



(b) Panel j



(c) Panel k

Fig. 3-4. Bending Moment Diagrams Due to Unit Redundants

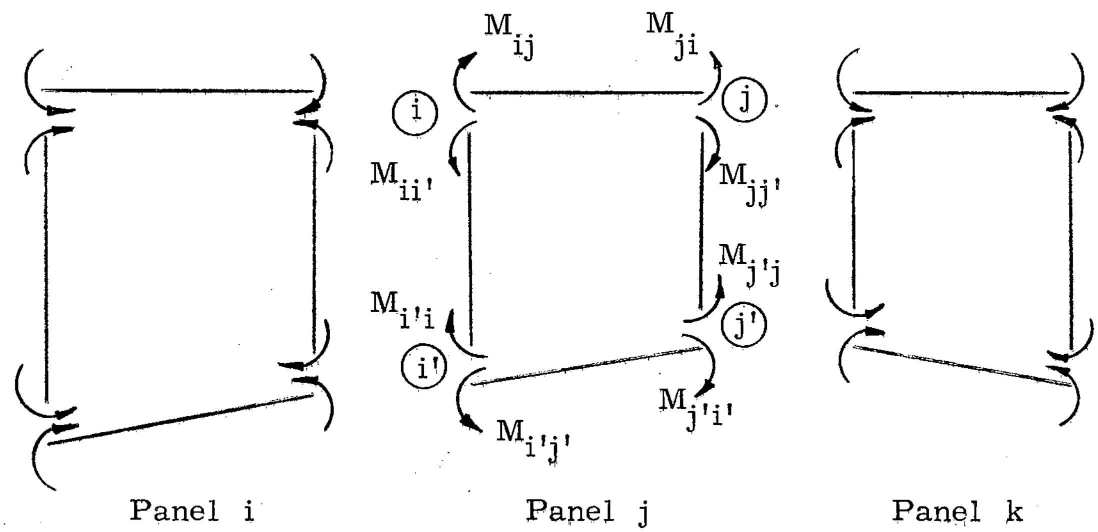


Fig. 3-5

Directions of End Moments

$$M_{ji} = d_j X_{2j} + X_{3j} + BM_{ji}$$

$$M_{jj'} = d_j X_{2j} + X_{3j} + X_{3k} + BM_{jj'}$$

$$M_{j'j} = h_j X_{1j} + d_j X_{2j} + X_{3j} + h_j X_{1k} + X_{3k} + BM_{j'j}$$

$$M_{j'i'} = h_j X_{1j} + d_j X_{2j} + X_{3j} + \bar{y}_j Y_1 + \bar{x}_j Y_2 + Y_3 + BM_{j'i'}$$

$$M_{i'j'} = h_i X_{1j} + X_{3j} + \bar{y}_i Y_1 + \bar{x}_j Y_2 + Y_3 + BM_{i'j'}$$

$$M_{i'i} = h_i X_{1i} + d_i X_{2i} + X_{3i} + h_i X_{1j} + X_{3j} + BM_{i'i}$$

$$M_{ii'} = d_i X_{2i} + X_{3i} + X_{3j} + BM_{ii'} \quad ,$$

(3-1)

where BM is the bending moment of the basic structure due to loads.

After some rearranging, Eq's. (3-1) can be written in the matrix form:

$$
\begin{bmatrix} M_{ij} \\ M_{ji} \\ M_{jj'} \\ M_{j'j} \\ M_{j'i'} \\ M_{i'j'} \\ M_{i'i} \\ M_{ii'} \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & d_j & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & d_j & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & h_i & d_j & 1 & h_j & 0 & 1 \\
0 & 0 & 0 & h_i & d_j & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & h_i & 0 & 1 & 0 & 0 & 0 \\
h_i & d_i & 1 & h_i & 0 & 1 & 0 & 0 & 0 \\
0 & d_i & 1 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} X_{1i} \\ X_{2i} \\ X_{3i} \\ X_{1j} \\ X_{2j} \\ X_{3j} \\ X_{1k} \\ X_{2k} \\ X_{3k} \end{bmatrix}
+
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\bar{y}_j & \bar{x}_j & 1 \\
\bar{y}_i & \bar{x}_i & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}
+
\begin{bmatrix} BM_{ij} \\ BM_{ji} \\ BM_{jj'} \\ BM_{j'j} \\ BM_{j'i'} \\ BM_{i'j'} \\ BM_{i'i} \\ BM_{ii'} \end{bmatrix}
\tag{3-2}
$$

or

$$
[M_j]_{8\times1} = [C_1]_{8\times9}[X_{ijk}]_{9\times1} + [C_2]_{8\times3}[Y_{1,2,3}]_{3\times1} + [B_j]_{8\times1} \tag{3-2a}
$$

where $[C_1]$ and $[C_2]$ are coefficient matrices of the unknown vectors $[X]$ and $[Y]$, respectively; $[B_j]$ is a column vector of bending moments due to loads; and the subscripts indicate the order of the matrices. For the panel i or k, all corresponding panel redundants and column redundants cause tension outside the panels. Since the moments causing tension inside are taken as positive, for any outside tension panel j the end moments can be represented in matrix form

$$[M_j]_{8\times1} = [-C_1]_{8\times9}[X_{ijk}]_{9\times1} + [C_2]_{8\times3}[Y_{1,2,3}]_{3\times1} + [B_j]_{8\times1} \quad .$$

(3-3)

3-3. Elasto Static Equations

For any panel j, elasto-static equations can be written in many different forms. In this study, three moment equilibrium equations are utilized, as this procedure is systematic and renders a set of equations which are relatively simple in form.

The conjugate of panel j loaded by joint elastic weights is shown in Fig. 3-6. The joint elastic weights for panel j at j, j', i and i' are denoted by $\overline{P}_j^j$, $\overline{P}_{j'}^j$, $\overline{P}_i^j$, and $\overline{P}_{i'}^j$, respectively. Since loads are acting only on the arch deck and are assumed to act toward the deck, τ values for all members except the deck members are zero. For panels assumed in tension on the inside, τ values are positive, and for those in tension on the outside τ values are negative. Also, since panel members are constant in cross-section,

$$F_{ij} = 2G_{ij} = 2G_{ji}$$

$$F_{jj'} = 2G_{jj'} = 2G_{j'j} \quad .$$

Thus, $\overline{P}_j^j$ may be written

$$\overline{P}_j^j = M_{ij}G_{ij} + M_{ji}(2G_{ji}) + M_{jj'}(2G_{jj'}) + M_{j'j}G_{j'j} + \tau_{ji} \ ,$$

or

$$\overline{P}_j^j = G_{ij}(M_{ij} + 2M_{ji}) + G_{jj'}(M_{j'j} + 2M_{jj'}) + \tau_{ji} \quad . \tag{3-4a}$$

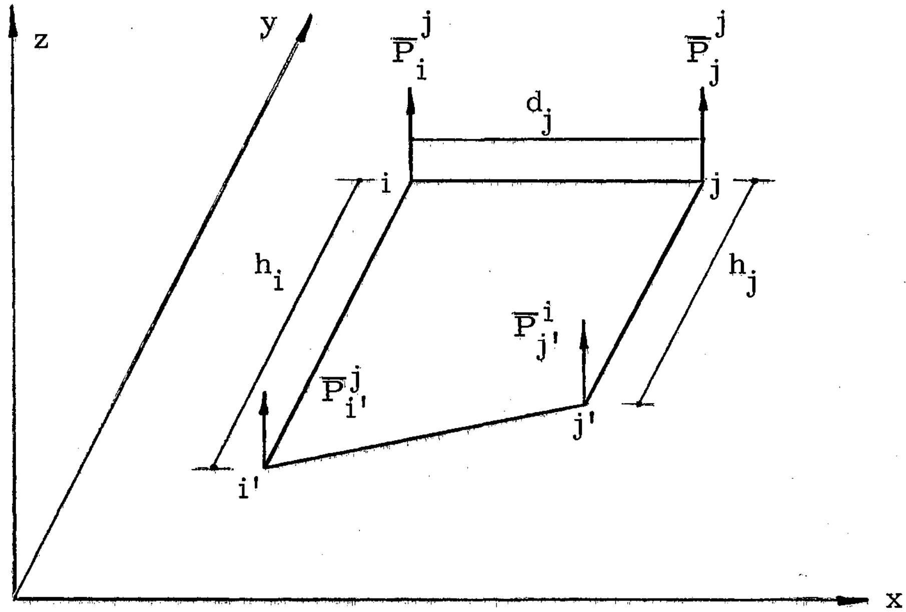


Fig. 3-6

Conjugate of Panel j

Similarly,

$$\overline{P}^{j}_{j'} = G_{i'j'}(M_{i'j'} + 2M_{j'i'}) + G_{jj'}(M_{jj'} + 2M_{j'j}) \; , \qquad (3\text{-}4b)$$

$$\overline{P}^{j}_{i} = G_{ii'}(M_{i'i} + 2M_{ii'}) + G_{ij}(M_{ji} + 2M_{ij}) + \tau_{ij}, \qquad (3\text{-}4c)$$

$$\overline{P}^{j}_{i'} = G_{ii'}(M_{ii'} + 2M_{i'i}) + G_{i'j'}(M_{j'i'} + 2M_{i'j'}) \; . \qquad (3\text{-}4d)$$

Since the conjugate structure must be in elasto-static equilibrium, the static moments of the elastic weights about ii', ij, and jj' must vanish independently. Thus,

$$ii' \; ; \qquad d_j(\overline{P}^{j}_{j} + \overline{P}^{j}_{j'}) = 0 \qquad (3\text{-}5a)$$

$$ij; \qquad h_i \, \overline{P}^j_{i'} + h_j \, \overline{P}^j_{j'} = 0 \tag{3-5b}$$

$$jj'; \qquad d_j(\overline{P}^j_i + \overline{P}^j_{i'}) = 0 \tag{3-5c}$$

Substituting Eq's. (3-4) into Eq's. (3-5) and simplifying, gives

$$(G_{ij})M_{ij} + (2G_{ij})M_{ji} + (3G_{jj'})M_{jj'} + (3G_{jj'})M_{j'j} +$$

$$+ (2G_{i'j'})M_{j'i'} + (G_{i'j'})M_{i'j'} = -\tau_{ji} , \tag{3-6a}$$

$$[\nu_j G_{jj'}]M_{jj'} + [2\nu_j G_{jj'}]M_{j'j} + [(1 \quad 2\nu_j)G_{i'j'}]M_{j'i'} +$$

$$+ [(2 + \nu_j)G_{i'j'}]M_{i'j'} + [2G_{ii'}]M_{i'i} + [G_{ii'}]M_{ii'} = 0 , \tag{3-6b}$$

$$(2G_{ij})M_{ij} + (G_{ij})M_{ji} + (G_{i'j'})M_{j'i'} + (2G_{i'j'})M_{i'j'} +$$

$$+ (3G_{ii'})M_{i'i} + (3G_{ii'})M_{ii'} = -\tau_{ij} , \tag{3-6c}$$

in which $\nu_j = h_j/h_i$.

Eq's. (3-6) may be represented in the matrix form:

$$\begin{bmatrix} G_{ij} & 2G_{ij} & 3G_{jj'} & 3G_{jj'} & 2G_{i'j'} & G_{i'j'} & 0 & 0 \\ 0 & 0 & \nu_j G_{jj'} & 2\nu_j G_{jj'} & (1+2\nu_j)G_{i'j'} & (2+\nu_j)G_{i'j'} & 2G_{ii'} & G_{ii'} \\ 2G_{ij} & G_{ij} & 0 & 0 & G_{i'j'} & 2G_{i'j'} & 3G_{ii'} & 3G_{ii'} \end{bmatrix} \begin{bmatrix} M_{ij} \\ M_{ji} \\ M_{jj'} \\ M_{j'j} \\ M_{j'i'} \\ M_{i'j'} \\ M_{i'i} \\ M_{ii'} \end{bmatrix} = \begin{bmatrix} -\tau_{ji} \\ 0 \\ -\tau_{ij} \end{bmatrix}, \qquad (3\text{-}7)$$

or

$$[G_j]_{3\times 8}\,[M_j]_{8\times 1} = [-T_j]_{3\times 1} \quad , \qquad (3\text{-}8)$$

in which $[G_j]$ is the coefficient matrix of moment vector $[M_j]$ and $[-T_j]$ is a column vector containing the load functions, τ.

Substituting $[M_j]$ from Eq. (3-2a) into Eq. (3-8) yields

$$\Big[[G_j]_{3\times 8}[C_1]_{8\times 9}\Big]\;[X_{ijk}]_{9\times 1} + \Big[[G_j]_{3\times 8}[C_2]_{8\times 3}\Big]\;[Y_{1,2,3}]_{3\times 1} + $$

$$+ \;[G_j]_{3\times 8}[B_j]_{8\times 1} = [-T_j]_{3\times 1} \qquad (3\text{-}9)$$

Denoting,

$$[G_j]_{3\times8}[C_1]_{8\times9} = [F_j]_{3\times9}$$

and

$$[G_j]_{3\times8}[C_2]_{8\times3} = [f_j]_{3\times3} \quad ,$$

Eq. (3-9) becomes

$$[F_j]_{3\times9}[X_{ijk}]_{9\times1} + [f_j]_{3\times3}[Y_{1,2,3}]_{3\times1} +$$

$$+ [G_j]_{3\times8}[B_j]_{8\times1} = [-T_j]_{3\times1} \qquad (3\text{-}10)$$

Similary, for 'outside tension' panels,

$$[-F_j]_{3\times9}[X_{ijk}]_{9\times1} + [-f_j]_{3\times3}[Y_{1,2,3}]_{3\times1} +$$

$$+ [G_j]_{3\times8}[B_j]_{8\times1} = [T_j]_{3\times1} \quad . \qquad (3\text{-}11)$$

3-4. Flexibility Matrices

Given a loading condition, the matrices $[G_j][B_j]$ and $[T_j]$ can be determined for each panel. $[B_j]$ is a column vector and the product of $[G_j]$ and $[B_j]$ can be evaluated easily for each panel once the loading is specified.

The final flexibility matrix, considering each panel in turn, can be easily obtained with the aid of general expressions for the matrices $[F_j]$ and $[f_j]$. The general formulation of matrices $[F_j]$ and $[f_j]$ is presented in this article.

Substituting for $[C_1]$ and $[G_j]$ from Eq's. (3-2a) and(3-8), the matrices $[F_j]$ and $[f_j]$ for any 'inside tension' panel become

$$
[F_j] = \left[\begin{array}{ccccc}
0 & 0 & 0 & 3h_jG_{jj'} + G_{i'j'}(2h_j+h_i) & 2d_j(G_{ij} + 3G_{jj'} + G_{i'j'}) \\
2h_iG_{ii'} & 3d_iG_{ii'} & 3G_{ii'} & 2h_i\nu_jG_{jj'} + G_{i'j'}[h_j(1+2\nu_j) + h_i(2+\nu_j)] + 2h_iG_{ii'} & d_j[3\nu_jG_{jj'} + (1+2\nu_j)G_{i'j'}] \\
3h_iG_{ii'} & 6d_iG_{ii'} & 6G_{ii'} & G_{i'j'}(h_j+2h_i) + 3h_iG_{ii'} & d_j(G_{ij} + G_{i'j'})
\end{array}\right.
$$

$$
\left.\begin{array}{cccc}
3(G_{ij} + 2G_{jj'} + G_{i'j'}) & 3h_jG_{jj'} & 0 & 6G_{jj'} \\
3[\nu_jG_{jj'} + (1+\nu_j)G_{i'j'} + G_{ii'}] & 2h_j\nu_jG_{jj'} & 0 & 3\nu_jG_{jj'} \\
3(G_{ij} + G_{i'j'} + 2G_{ii'}) & 0 & 0 & 0
\end{array}\right]_{3\times 9}
$$

(3-12)

$$
[f_j] = G_{i'j'}\begin{bmatrix}
(2\overline{y}_j + \overline{y}_i) & (2\overline{x}_j + \overline{x}_i) & 3 \\
[(1+2\nu_j)\overline{y}_j + (2+\nu_j)\overline{y}_i] & [(1+2\nu_j)\overline{x}_j + (2+\nu_j)\overline{x}_i] & 3(1+\nu_j) \\
(\overline{y}_j + 2\overline{y}_i) & (\overline{x}_j + 2\overline{x}_i) & 3
\end{bmatrix}_{3\times 3}
$$

(3-13)

3-5. Final Matrix

Additional equations may be necessary, depending upon the end conditions of the supporting arch columns. These are obtained by considering elasto static equilibrium of any other set of closed rings.

For example, considering the first span of the general continuous arch of Fig. 3-1, the conjugate structure shown in Fig. 3-7b is selected. $\overline{P}_A$, $\overline{P}_{0'}$, $\overline{P}_0$, $\cdots$, $\overline{P}_B$ are joint elastic weights acting at A, 0', 0, $\cdots$, B, respectively. The static moments of these elastic weights about A0, 05 and 5B must vanish independently. Thus,

$$\text{A0} \; ; \quad \overline{P}_1(d_1) + \overline{P}_2(d_1 + d_2) + \overline{P}_3(d_1 + d_2 + d_3) + \\ + \overline{P}_4(d_1 + d_2 + d_3 + d_4) + (\overline{P}_5 + \overline{P}_{5'} + P_B)\ell_1 = 0 \tag{3-14a}$$

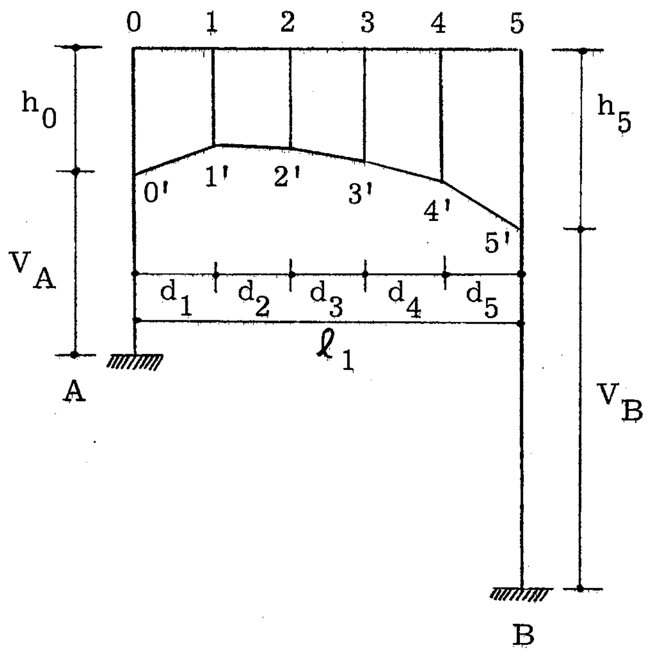
$$\text{05} \; ; \quad \overline{P}_{0'}(h_0) + \overline{P}_A(h_0 + v_A) + \overline{P}_{5'}(h_5) + \overline{P}_B(h_5 + v_B) = D \tag{3-14b}$$

$$\text{5B} \; ; \quad (\overline{P}_0 + \overline{P}_{0'} + \overline{P}_A)\ell_1 + \overline{P}_1(d_2 + d_3 + d_4 + d_5) + \\ + \overline{P}_2(d_3 + d_4 + d_4) + \overline{P}_3(d_4 + d_5) + \overline{P}_4(d_5) = 0 \tag{3-14c}$$

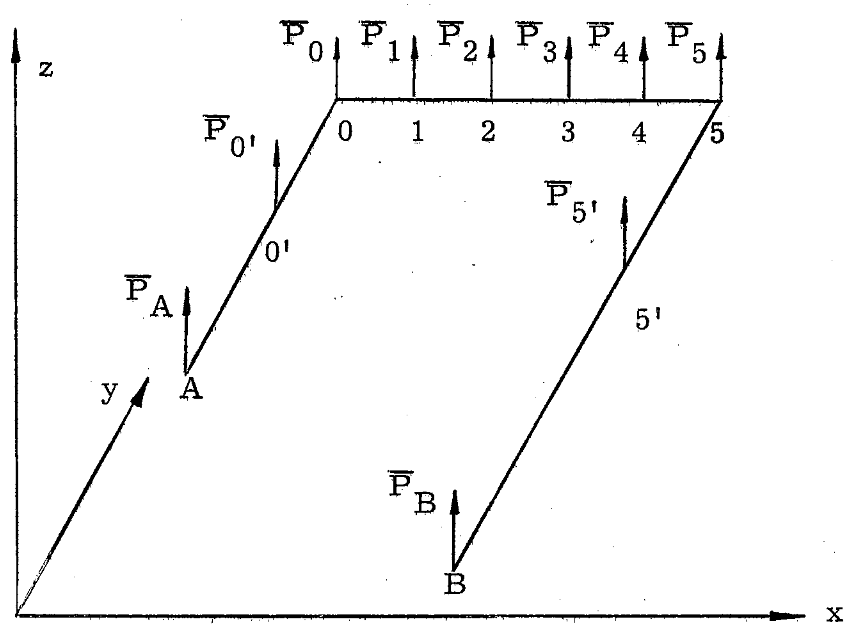
Similar considerations for other spans give the additional independent equations.

For n panels, a total of 3n elasto static equations can be written. $[F_j]$ matrices for the first and last panel of the system will be of order of 3×6 as there are only six 'panel redundants' in each case.

In general, the final matrix for the system can be written:



(a)



(b)

Fig. 3-7
Span AB and Its Conjugate Structure

$$\begin{bmatrix} [f] & [F] \\ [\quad E & \quad] \end{bmatrix} \begin{bmatrix} [Y] \\ [X] \end{bmatrix} = [T] - \begin{bmatrix} [G] & [B] \end{bmatrix}, \qquad (3\text{-}15)$$

in which

$\begin{bmatrix} f \end{bmatrix}$ represents all [f] matrices of the structure,

$\begin{bmatrix} F \end{bmatrix}$ represents all [F] matrices of the structure,

$\begin{bmatrix} E \end{bmatrix}$ represents the additional equations in matrix form required to to solve for the unknown column redundants,

$\begin{bmatrix} Y \end{bmatrix}$ represents the column redundant vector,

$\begin{bmatrix} X \end{bmatrix}$ represents the panel redundant vector,

$\begin{bmatrix} T \end{bmatrix}$ represents the column vector containing the load functions, τ for the whole structure,

$\begin{bmatrix} G \end{bmatrix} \begin{bmatrix} B \end{bmatrix}$ represents the product of [G] and [B] for the complete structure.

The general form of these matrices is illustrated by a numerical example.

With computer facilities available, Eq's. (3-15) may be solved for the panel and column redundants. Once these redundants are computed, final bending moments at the ends of each member are obtained by direct substitution into Eq. (3-2a) or Eq. (3-3).

CHAPTER IV

NUMERICAL EXAMPLE

4-1. Example

To illustrate the use of the method, a single span, reinforced concrete, open spandrel arch is considered. The details of the arch along with the loading is shown in Fig. 4-1. A considerable amount of experimental data is available for this particular structure. It was constructed and tested by Wilson and Kluge[2] and their staff and later analyzed theoretically by Beaufoy[4] and Diwan[5].

The actual structure is idealized by considering the geometry as being defined by the centerlines of each member. It is also assumed that the arch rib between any two spandrel columns is prismatic with a constant cross section identical with that at the middle of its length. Centerline dimensions of the model and the relative elastic properties ("the elastic areas", $\frac{L}{EI}$) of all members are shown in Fig. 4-2.

Since there are nine panels and the ends of the arch are fixed, the structure is statically indeterminate to the thirtieth degree. Twenty-seven equations are obtained for the nine panels and the additional three equations are formed by considering the conjugate structure of span AB.

4-2. Analysis

Steps in the procedure of analysis of this structure are listed as follows:

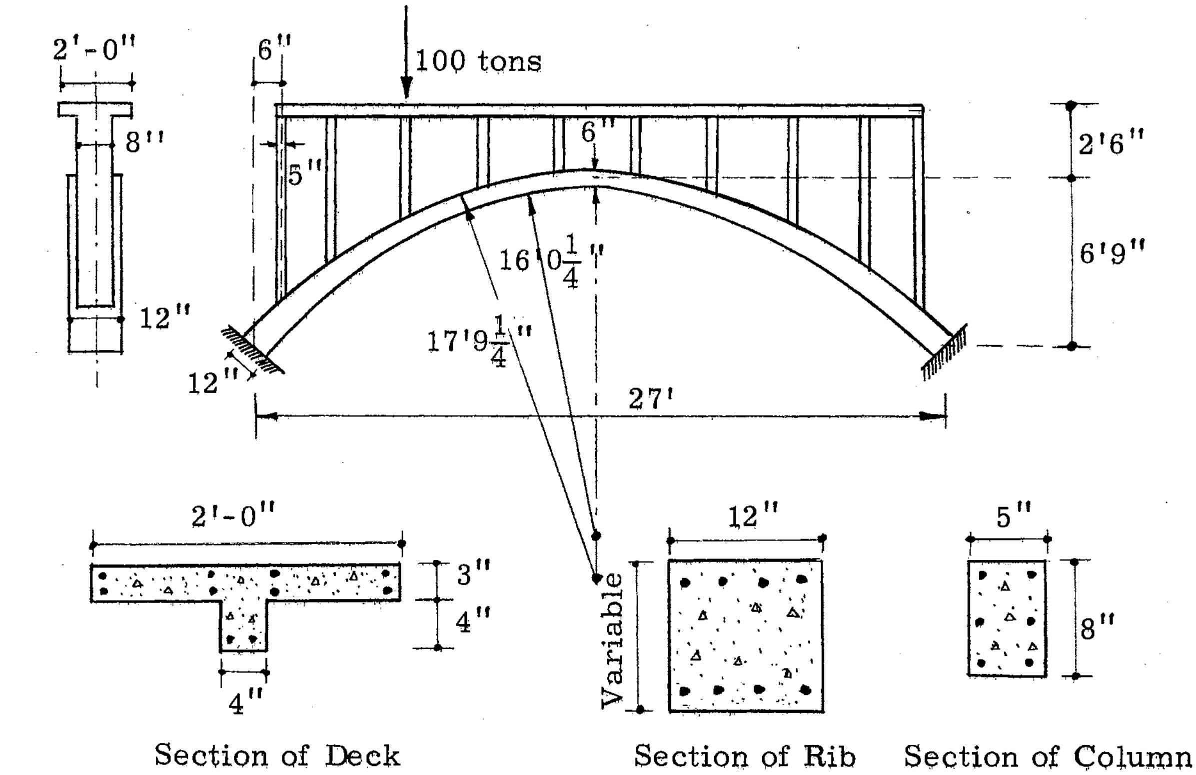


Fig. 4.1 Details of Span

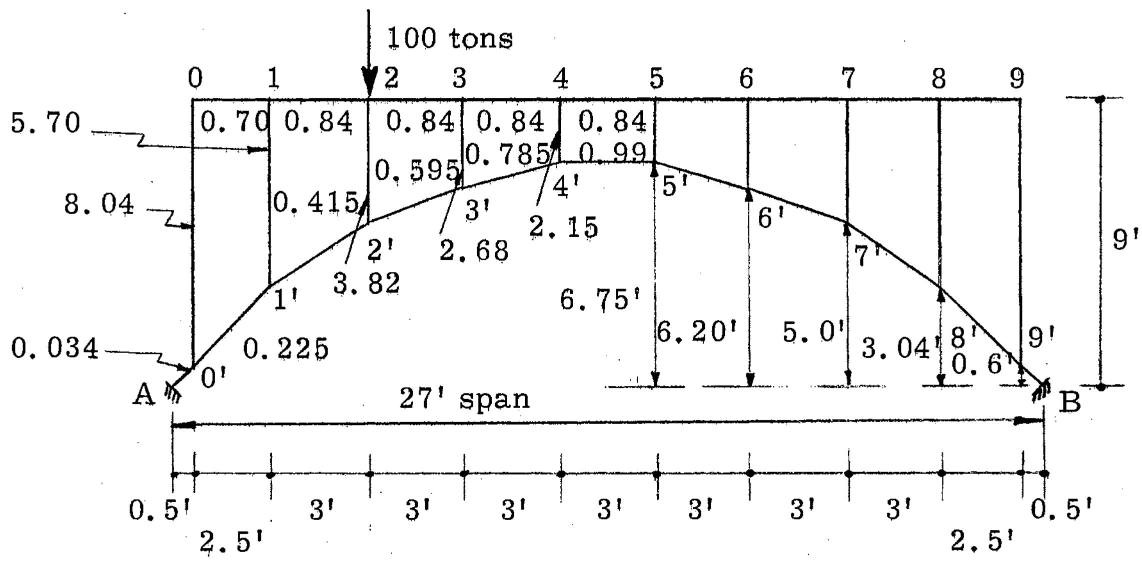


Fig. 4.2 Centerline Dimensions of the Structure with Relative Elastic Areas

1. Bending moments due to loads:

   After choosing Y_1, Y_2, and Y_3 as three column redundants at A, bending moments due to the given load are calculated (Fig. 4-3).

2. [F] matrices:

   From the relative elastic areas of the members, angular carry-over values are calculated and [F] matrices for the nine panels are obtained from Eq. (3-12) (Tables 4-1a, b).

3. [f] matrices:

   [f] matrices for the nine panels are calculated using Eq. (3-13) (Table 4-2).

4. [G] [B] matrices:

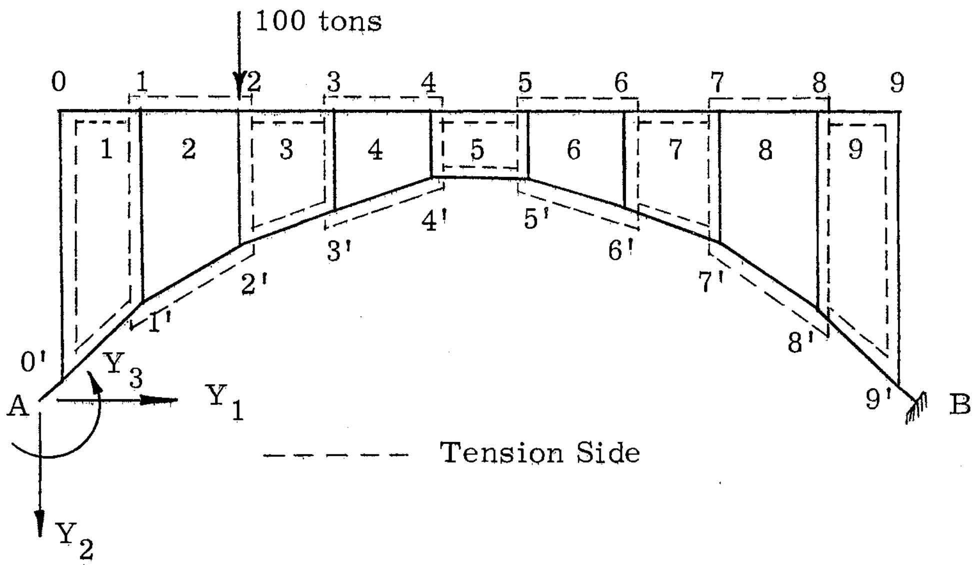
   Using Eq. (3-7), the product of the [G] and [B] matrices for each panel are evaluated and recorded in Table 4-3.

5. Equations for column redundants:

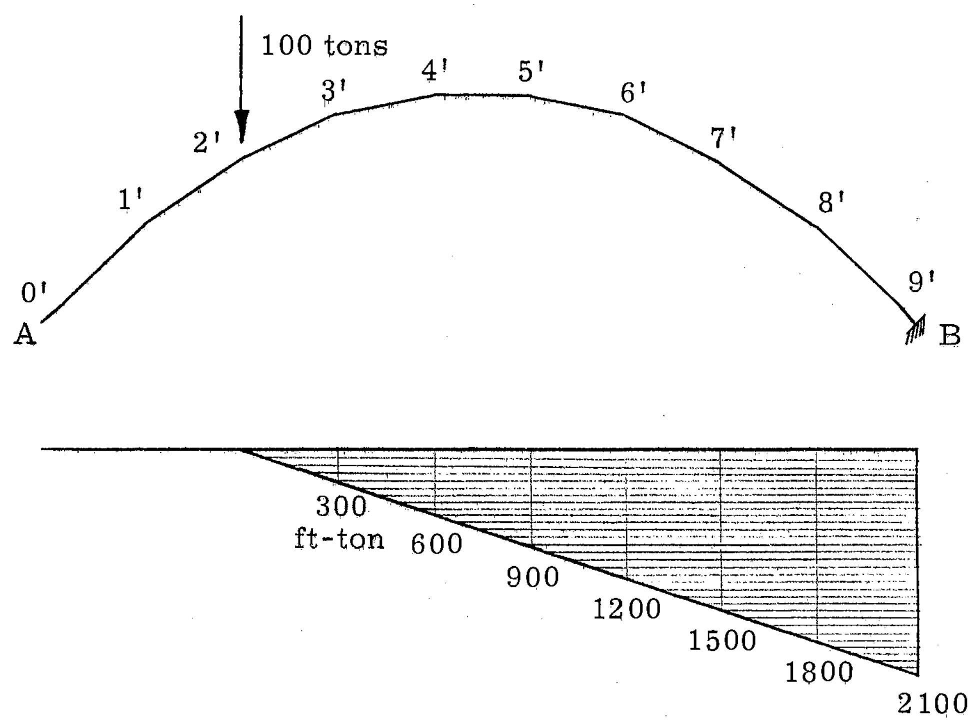
   Three additional equations are necessary because of the presence of column redundants. These equations are obtained by considering the elasto static equilibrium of the conjugate structure of span AB (Fig. 4-4). Static moments about 0'0, 9'9, and 09 yield the following equations:

0'0 :

$$\begin{aligned}
&-0.2652\, Y_1 + 0.3003\, Y_2 + 1.4583\, X_2 + 0.8750\, X_3 - 5.6700\, X_5 + \\
&- 3.3600\, X_6 + 9.4500\, X_8 + 5.8800\, X_9 - 13.2300\, X_{11} - 7.4000\, X_{12} + \\
&+ 17.0100\, X_{14} + 8.8200\, X_{15} - 20.7900\, X_{17} - 13.440\, X_{18} + \\
&+ 20.5700\, X_{20} + 15.9600\, X_{21} - 20.3500\, X_{23} - 18.4800\, X_{24} + \\
&+ 612.9650\, X_{25} + 389.6200\, X_{26} + 110.3650\, X_{27} = -27.5250
\end{aligned} \tag{4-1}$$



(a) Basic Structure



(b) Bending Moments Due to Load

Fig. 4-3 Basic Structure and Bending Moments Due to Load

TABLE 4-1a - [F] Matrices

Panel 1				17.748000	15.021000	6.162600	16.986000	0.000000	5.700000
				31.940710	5.281233	6.234095	8.033250	0.000000	2.021800
				34.621500	0.385500	8.502600	0.000000	0.000000	0.000000
Panel 2	0.000000	0.000000	0.000000	- 8.606030	-11.050000	-4.447500	- 7.640000	0.000000	-3.820000
	-11.324000	- 7.125000	-2.850000	-15.844800	- 2.560000	-4.479460	- 3.420000	0.000000	-1.282500
	-16.986000	-14.250000	-5.700000	-15.100000	- 0.627600	-6.327600	0.000000	0.000000	0.000000
Panel 3	0.000000	0.000000	0.000000	4.704000	9.474999	3.397500	3.752000	0.000000	2.680000
	5.093330	5.730000	1.910000	8.581667	3.527999	3.353750	1.750933	0.000000	0.938000
	7.639990	11.460000	3.820000	8.711000	0.717499	4.537500	0.000000	0.000000	0.000000
Panel 4	0.000000	0.000000	0.000000	- 3.373833	- 8.074999	-2.962500	- 2.418750	0.000000	-2.150000
	- 2.501333	- 4.020000	-1.340000	- 5.591611	- 3.614821	-2.911741	- 1.295759	0.000000	-0.863839
	- 3.751999	- 8.040000	-2.680000	- 4.779041	- 0.812500	-3.492500	0.000000	0.000000	0.000000
Panel 5	0.000000	0.000000	0.000000	3.532500	8.280000	3.065000	2.418750	0.000000	2.150000
	1.612500	3.225000	1.075000	5.452500	4.710000	3.140000	1.612500	0.000000	1.075000
	2.418750	6.450000	2.150000	3.532500	0.915000	3.065000	0.000000	0.000000	0.000000

TABLE 4. 1b [F] Matrices

Panel 6	0. 000000	0. 000000	0. 000000	- 4. 779041	- 9. 665000	-3. 492500	- 3. 752000	0. 000000	-2. 680000
	- 1. 612500	- 3. 225000	-1. 075000	- 6. 958449	- 6. 372055	-3. 623499	- 3. 112770	0. 000000	-1. 667554
	- 2. 418750	- 6. 450000	-2. 150000	- 3. 373833	- 0. 812500	-2. 962500	0. 000000	0. 000000	0. 000000
Panel 7	0. 000000	0. 000000	0. 000000	8. 711000	12. 895000	4. 532500	7. 640000	0. 000000	3. 820000
	2. 501333	4. 020000	1. 340000	12. 259524	9. 313928	4. 778928	7. 276190	0. 000000	2. 728571
	3. 751999	8. 040000	2. 680000	4. 704000	0. 712500	3. 392500	0. 000000	0. 000000	0. 000000
Panel 8	0. 000000	0. 000000	0. 000000	-18. 087330	-18. 355000	-6. 327500	-16. 986000	0. 000000	-5. 700000
	- 5. 093333	- 5. 730000	-1. 910000	-24. 564104	-13. 565350	-6. 673175	-16. 872760	0. 000000	-4. 246500
	- 7. 640000	-11. 460000	-3. 820000	- 8. 605566	- 0. 627500	-4. 447500	0. 000000	0. 000000	0. 000000
Panel 9	0. 000000	0. 000000	0. 000000	34. 621500	20. 870833	8. 502500			
	11. 324000	8. 550000	2. 850000	45. 017242	14. 522950	8. 786829			
	16. 986000	17. 100000	5. 700000	17. 748000	0. 385500	6. 162500			

TABLE 4.2 [f] MATRICES

Panel 1	0.250360	0.243750	0.112500	Panel 6	-2.505458	- 6.672498	-0.392499
	0.336705	0.322916	0.192307		-5.695173	-14.583550	-0.880944
	0.159000	0.131250	0.112500		-2.577416	- 6.279998	-0.392499
Panel 2	-0.901934	- 1.037500	-0.207500	Panel 7	1.606500	5.950002	0.198333
	-1.371835	- 1.526474	-0.346795		4.020501	14.152504	0.722500
	-0.766367	- 0.830000	-0.207500		1.725500	5.652502	0.198333
Panel 3	1.725501	2.180000	0.297500	Panel 8	-0.766367	- 4.772502	-0.207500
	2.814351	3.895000	0.505750		-2.050737	-11.676031	-0.516675
	1.606500	2.330250	0.297500		-0.901934	- 4.565002	-0.207500
Panel 4	-2.511999	- 4.317499	-0.392499	Panel 9	0.159000	2.925000	0.112500
	-4.576595	- 7.394417	-0.707901		0.512094	7.234508	0.271057
	-2.505458	- 3.924999	-0.392499		0.250360	2.812500	0.112500
Panel 5	3.341250	6.930000	0.495000				
	6.682500	13.365000	0.990000				
	3.341250	6.435000	0.495000				

TABLE 4.3 PRODUCT OF [G] AND [B] MATRICES

Panel 1	0.000000	Panel 4	-205.999999	Panel 7	426.000000
	0.000000		-344.250000		974.850000
	0.000000		-183.850000		396.499999
Panel 2	0.000000	Panel 5	392.600000	Panel 8	-341.000000
	0.000000		736.500000		-825.000000
	0.000000		341.700000		-314.749999
Panel 3	39.749980	Panel 6	-380.259999	Panel 9	205.875000
	80.225000		-817.794200		533.865000
	49.500000		-345.750000		215.250000

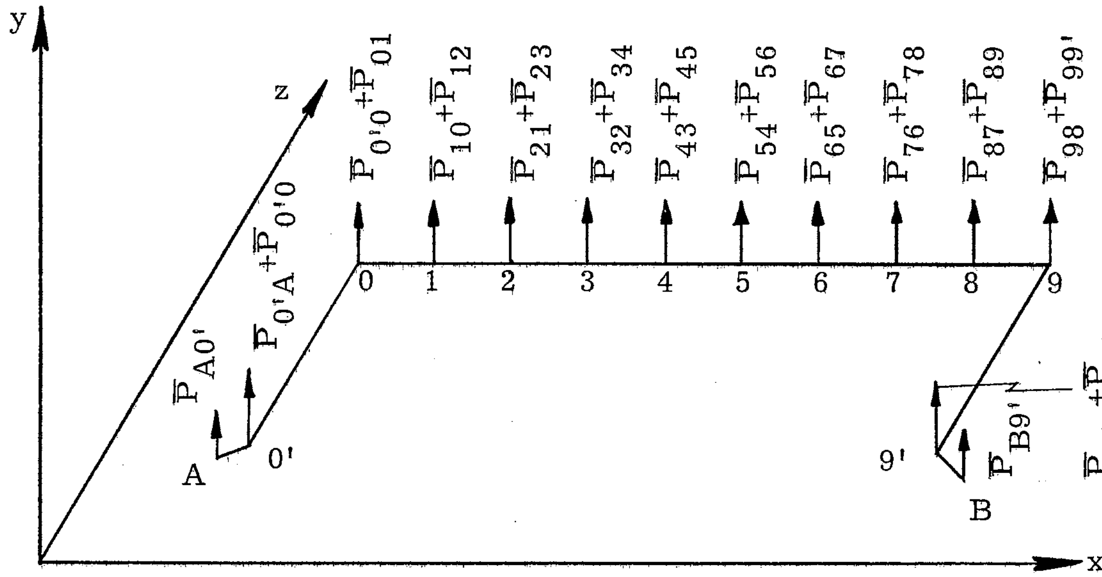


Fig. 4-4

Conjugate Structure

9'9 :

$$- 0.2652\, Y_1 - 0.3003\, Y_2 + 827.9680\, X_1 + 22.8033\, X_2 + 226.3650\, X_3 +$$

$$- 27.0900\, X_5 - 18.4800\, X_6 + 23.3100\, X_8 + 13.9600\, X_9 - 19.5300\, X_{11} +$$

$$- 11.4400\, X_{12} + 15.7500\, X_{14} + 10.9200\, X_{15} - 11.9700\, X_{17} +$$

$$- 8.4000\, X_{18} + 8.1900\, X_{20} + 7.8800\, X_{21} - 4.4100\, X_{23} - 3.3600\, X_{24} +$$

$$+ 0.7292\, X_{26} + 0.8750\, X_{27} = - 47.6500 \qquad (4\text{-}2)$$

09 :

$$- 0.1754\, Y_1 + 0.5118\, Y_2 + 169.1008\, X_1 + 33.7680\, X_3 +$$

$$+ 139.1008\, X_{25} + 64.4200\, X_{26} + 23.7680\, X_{27} = 20.2200\,. \qquad (4\text{-}3)$$

In matrix form;

$$\begin{bmatrix} & & \\ & \mathbf{E} & \\ & & \end{bmatrix}_{3x30} \begin{bmatrix} \mathbf{Y} \\ \mathbf{X} \end{bmatrix}_{30x1} = \begin{bmatrix} -27.5250 \\ -47.6500 \\ 20.2200 \end{bmatrix}_{3x1} \qquad (4\text{-}4)$$

6. Final Matrix:

The final 30x30 matrix needed to solve for the redundants is of the form:

$$\begin{bmatrix}
[& & & & \mathbf{E} & & & & &]_{3x30} \\
[f_1]_{3x3} & [F_1]_{3x6} & & & & & & & & \\
[f_2]_{3x3} & & [F_2]_{3x9} & & & & & & & \\
[f_3]_{3x3} & & & [F_3]_{3x9} & & & & & & \\
[f_4]_{3x3} & & & & [F_4]_{3x9} & & & & & \\
[f_5]_{3x3} & & & & & [F_5]_{3x9} & & & & \\
[f_6]_{3x3} & & & & & & [f_6]_{3x9} & & & \\
[f_7]_{3x3} & & & & & & & [F_7]_{3x9} & & \\
[f_8]_{3x3} & & & & & & & & [F_8]_{3x9} & \\
[f_g]_{3x3} & & & & & & & & & [F_9]_{3x9}
\end{bmatrix}
\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ X_1 \\ X_2 \\ \\ \cdot \\ \cdot \\ \cdot \\ X_{27} \end{bmatrix}
= -\begin{bmatrix} [\mathbf{G}][\mathbf{B}] \end{bmatrix} \qquad (4\text{-}5)$$

7. Solution for the redundants:

   The final set of 30 equations is solved for the unknowns. The values of the redundants are recorded in Table 4-4.

8. End moments:

   Each panel end moments can be obtained on substitution of these values in Eq's. (3-2) or 3-3).

A comparison of the panel end moments recorded in Table 4-4 with those presented by Beaufoy and Diwan reveals that the results are in close agreement with experimental data with the exception of Y_3. This value is slightly over ten percent in error and considerably influences some of the arch joint moments.

TABLE 4-4 VALUES OF REDUNDANTS

Y_1	Y_2	Y_3	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
46.89	-83.29	98.42	1.11	-1.20	-5.36	- 6.85	-25.78	25.47	17.56	-7.17	31.45	-6.74	13.28	-24.07
X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}
-11.15	- 5.43	5.11	15.39	2.67	4.99	-10.83	5.73	-21.17	21.34	-8.53	19.62	0.04	1.33	1.18

CHAPTER V

SUMMARY AND CONCLUSIONS

5-1. Summary

The analysis of a general, continuous, open spandrel arch subjected to a general system of deck loading is presented in this study. The panel and column redundants are selected in such a way that a simple and systematic procedure is established for obtaining a set of simultaneous equations for the redundants.

Panel redundants are selected near the top-left joint of each panel. The column redundants are chosen at the top of each column so that the redundants of any column will influence the panels of a single span. Thus, coefficients in equations relating end moments and redundants are simply lengths of the members and coordinates of joints.

A combination of the direct and indirect approach of the string polygon method is adopted in this study to obtain the final equations. The relationship between the two approaches is illustrated by a series of matrix operations.

5-2. Conclusions

General expressions for matrices $[F_j]$ and $[f_j]$ are derived in this work. With the aid of these expressions, three equations can be obtained easily for each panel. If support conditions are such that additional equations are necessary, they may be obtained by considering elasto-static equilibrium of any other set of closed rings. In the analysis of multi-span arches the formulation of these additional equations is complex and time consuming. In addition, the resulting matrix is poorly conditioned for inversion. For the analysis of Vierendeel trusses this approach has proved to be easy and accurate.

It is shown that the method provides an efficient means for matrix formulation of the problem, and is sufficiently broad in scope to allow open spandrel arches of more than one span to be analyzed.

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