ANALYSIS OF TWO HINGED CIRCULAR ARCHES

BY ELECTRONIC COMPUTER

By

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Dean of the Graduate School

PREFACE

The analysis of a two hinged circular arch structure, while not particularly complex, follows a lengthy procedure. The idea of utilization of the electronic computer to analyze such structures was conceived by Professor A. A. Orr of the School of Architecture at Oklahoma State University, and formalized into a research project by him. The author had the fortunate experience to work under Professor Orr as a Graduate Assistant for a portion of the project, and this thesis is an attempt to enumerate and expand upon the research study.

I wish to express my appreciation to the following persons who gave of their talent and understanding, both when I was a resident graduate student at the University and during the course of completion of this thesis since leaving the campus:

To Professor Orr, whose technical ability and counseling made the period of study rewarding academically and personally.

To Professor F. Cuthbert Salmon, Head of the School of Architecture, whose encouragement and cooperation made it possible for the Graduate Assistantship I was awarded during my resident work.

To Professor Emeritus Raymond E. Means, whose knowledge and advice during my undergraduate studies was instrumental in the seeking of advanced study.

To my wife, Anne, and our fine children, who were patient and encouraging to the pursuit of this work although often denied a normal home life.

To my late father, John O. Bradley, Architect, and to my mother, whose consultation and advice were always available.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

φ	Angle of inclination of the arch axis at the hinge.
β	Angle of inclination of the arch axis at any point.
α	Angle of inclination of the arch axis at the point of concentrated load.
L	Span of the arch.
R	Radius of the arch.
h	Rise or the arch.
x	Any point on the arch axis measured horizontally from the hinge.
y	Any point on the arch axis measured vertically from the hinge.
$\bar{x}$	Horizontal distance from any point on the arch axis to dead load centroid.
k	Kip.
w	Uniform load.
P	Concentrated load.
H	Horizontal reaction.
V_L, V_R	Vertical reactions.
M'	Bending moment at any point on the arch axis due to given loads and vertical reactions only.
M	Bending moment at any point on the arch axis due to given loads and reactions.
S_β	Shear at any point on the arch axis defined by the position angle β.
T_β	Thrust at any point on the arch axis defined by the position angle β.
ds	Differential arch element.

$\Delta x, \Delta y$	Horizontal and vertical deflections.
E	Modulus of elasticity.
I	Moment of inertia.
d	Depth of the arch section.
$d\varphi$	Differential position angle.
$\Delta d\varphi$	Change of the differential angle.
f	Unit stress.
dA	Differential area.
Σ	Summation.
D	Denominator, horizontal reaction equation.
N	Numerator, horizontal reaction equation.

CHAPTER I

INTRODUCTION

This thesis deals with the analysis of two hinged circular arch structures of constant cross section and radius. For values of an angle φ in the range of 30° to 45° as shown in Figure 1, analyses were made for dead load, concentrated load, full live load and drift or snow load.

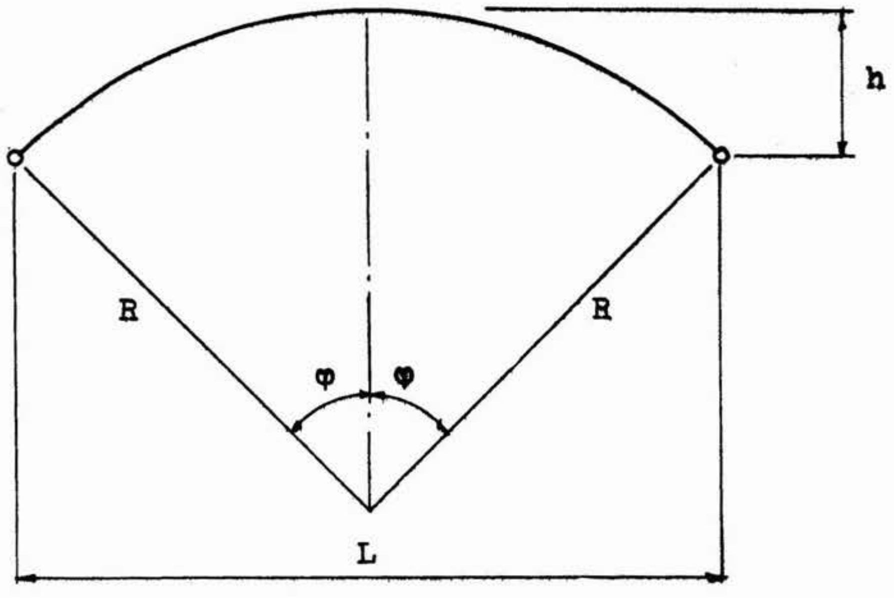


Figure 1

General Structure

The electronic computer was utilized to make calculations of data, which was then tabulated.

Initially, it was necessary to write out reaction and moment equations, the derivations of which are shown in Appendix A. These equations were converted to computer machine language and arranged in the proper sequence for calculations. Data for reaction and moment coefficients originally was fed from the computer in eight significant digits, but rounded off for tabulation. Constants (w, R and P) were factored from equations prior to machine calculations, and are shown as multipliers in the tables of coefficients. It will be noted in the example problems in Chapter III that for design purposes, once the geometry of the structure is exactly defined, the coefficients may be rounded off to three or four places and result in sufficient accuracy.

1. Shear and Thrust

Evaluation of equations for shear and thrust is depen-

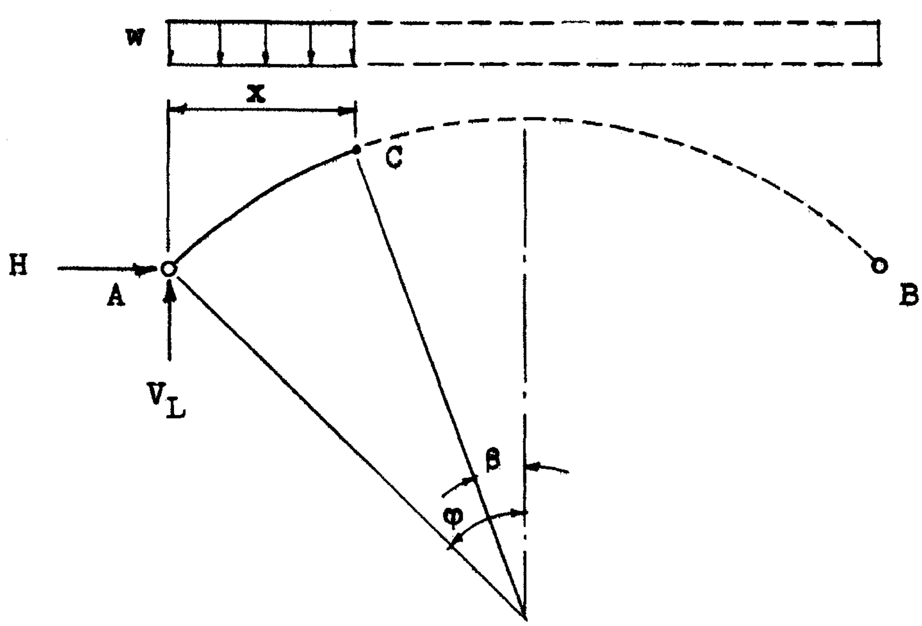
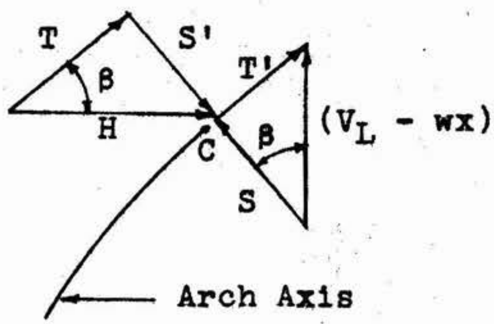


Figure 2

Partial Structure for Shear and Thrust

dent on the values of horizontal and vertical reactions. Derivation of shear and thrust equations for the full live load condition is as illustrated below (derivations for the other loading conditions are similar).

From Figure 2, at any point, C, on the arch axis, there are force diagrams:



$$S = (V_L - wx)\text{Cos}\beta$$
$$S' = H\text{Sin}\beta$$
$$T = H\text{Cos}\beta$$
$$T' = (V_L - wx)\text{Sin}\beta$$

from which

$$S_\beta = S - S' = (V_L - wx)\text{Cos}\beta - H\text{Sin}\beta$$
$$T_\beta = T + T' = (V_L - wx)\text{Sin}\beta + H\text{Cos}\beta$$

2. Load Conditions and General Equations

The figures that follow indicate the particular loading conditions, geometry of the structure, and equations for reactions, moment, and shear and thrust. The use of the tabulated coefficients is based on selection of the angle φ within the range of 30° to 45°. It is possible to use the equations directly for any value of φ that will follow from a given span and rise. The vertical reactions are obtained from simple statics. Derivation of the horizontal reaction equations is shown in Appendix A.

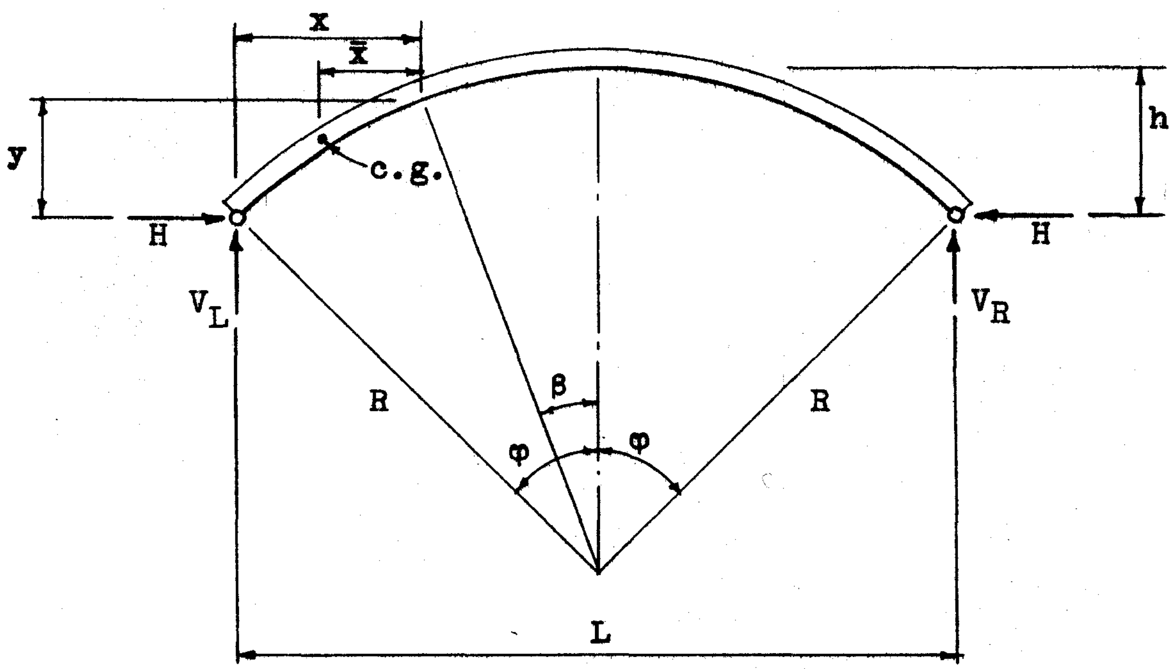


Figure 3

Case I: Dead Load

a. Geometry

$L = 2R\text{Sin}\varphi$

Total Arc Length $= 2R\varphi$

Partial Arc Length $= R(\varphi - \beta)$

$x = R(\text{Sin}\varphi - \text{Sin}\beta)$

$y = R(\text{Cos}\beta - \text{Cos}\varphi)$

$\overline{x} = R\left(\frac{\text{Cos}\beta - \text{Cos}\varphi}{\varphi - \beta} - \text{Sin}\beta\right)$, c.g. of dead load

b. Reactions

$V_L = V_R = wR\varphi$

$$H = 2wR\left[\frac{\text{Cos}\varphi(2\text{Sin}\varphi - 2\varphi\text{Cos}\varphi - \varphi^2\text{Sin}\varphi) - \frac{\varphi}{4} + \frac{\text{Sin}\varphi\text{Cos}\varphi}{4} + \frac{\varphi\text{Sin}}{2}}{\varphi - 3\text{Sin}\varphi\text{Cos}\varphi + 2\varphi\text{Cos}^2\varphi}\right.$$

c. Moments

$M = V_L x - Hy - wR(\varphi - \beta)\overline{x}$

d. Shear and Thrust

$S_\beta = \left[V_L - wR(\varphi - \beta)\right]\text{Cos}\beta - H\text{Sin}\beta$

$T_\beta = \left[V_L - wR(\varphi - \beta)\right]\text{Sin}\beta + H\text{Cos}\beta$

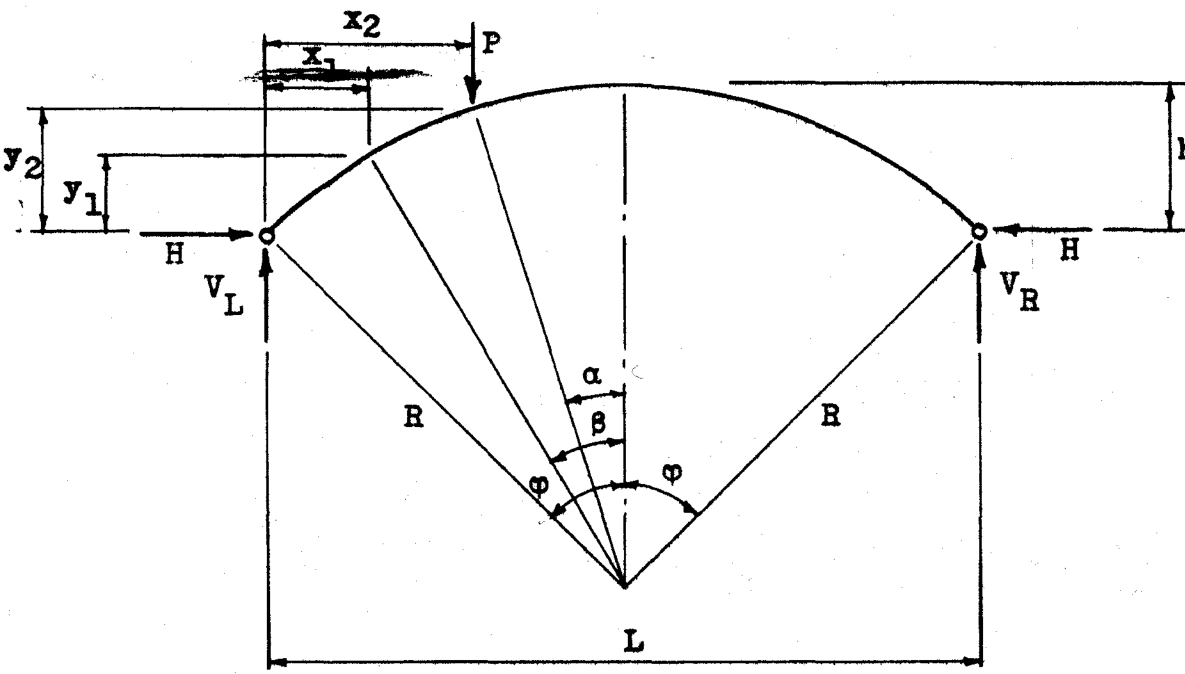


Figure 4

Case II: Concentrated Load

a. Geometry

$L = 2R\sin\varphi$

$x_1 = R(\sin\varphi - \sin\beta)$ $\qquad y_1 = R(\cos\beta - \cos\varphi)$

$x_2 = R(\sin\varphi - \sin\alpha)$ $\qquad y_2 = R(\cos\alpha - \cos\varphi)$

b. Reactions

$$V_L = \frac{P(L - x_2)}{L} = \frac{P\left[2R\sin\varphi - R(\sin\varphi - \sin\alpha)\right]}{2R\sin\varphi} = \frac{P(\sin\varphi + \sin\alpha)}{2\sin\varphi}$$

$$V_R = \frac{Px_2}{L} = \frac{PR(\sin\varphi - \sin\alpha)}{2R\sin\varphi} = \frac{P(\sin\varphi - \sin\alpha)}{2\sin\varphi}$$

$$H = P\left[\frac{\cos\varphi(\alpha\sin\alpha + \cos\alpha - \varphi\sin\varphi - \cos\varphi) + \frac{\sin^2\varphi}{2} - \frac{\sin^2\alpha}{2}}{\varphi - 3\sin\varphi\cos\varphi + 2\varphi\cos^2\varphi}\right]$$

c. Moments

(1) $\beta \geq \alpha$: $\quad M = V_L x_1 - Hy_1$

(2) $\beta \leq \alpha$: $\quad M = V_L x_1 - Hy_1 - P(x_1 - x_2)$

(3) $\beta < \varphi$: $M = V_R x_1 - H y_1$

d. Shear and Thrust

(1) $\beta \geq \alpha$: $S_\beta = P\mathrm{Cos}\beta\left(\dfrac{\mathrm{Sin}\varphi + \mathrm{Sin}\alpha}{2\mathrm{Sin}\varphi}\right) - H\mathrm{Sin}\beta$

$$T_\beta = P\mathrm{Sin}\beta\left(\frac{\mathrm{Sin}\varphi + \mathrm{Sin}\alpha}{2\mathrm{Sin}\varphi}\right) + H\mathrm{Cos}\beta$$

(2) $\beta \leq \alpha$: $S_\beta = P\mathrm{Cos}\beta\left(\dfrac{\mathrm{Sin}\varphi + \mathrm{Sin}\alpha}{2\mathrm{Sin}\varphi} - 1\right) - H\mathrm{Sin}\beta$

$$T_\beta = P\mathrm{Sin}\beta\left(\frac{\mathrm{Sin}\varphi + \mathrm{Sin}\alpha}{2\mathrm{Sin}\varphi} - 1\right) + H\mathrm{Cos}\beta$$

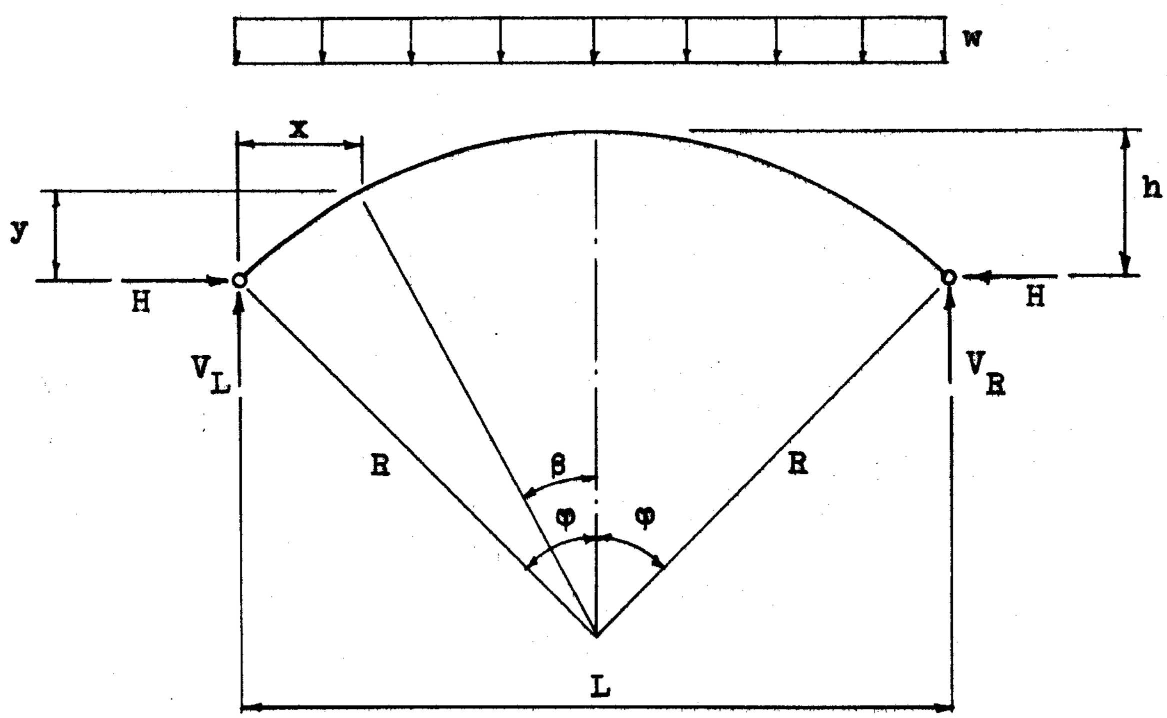


Figure 5

Case III: Full Live Load

a. Geometry

$L = 2R\mathrm{Sin}\varphi$
$x = R(\mathrm{Sin}\varphi - \mathrm{Sin}\beta)$
$y = R(\mathrm{Cos}\varphi - \mathrm{Cos}\beta)$

b. Reactions

$$V_L = V_R = \frac{wL}{2} = wR\mathrm{Sin}\varphi$$

$$H = \frac{wR}{6}\left[\frac{4\sin^3\varphi + 3\varphi\cos\varphi(1 - 2\sin^2\varphi) - 3\sin\varphi\cos^2\varphi}{\varphi - 3\sin\varphi\cos\varphi + 2\varphi\cos^2\varphi}\right]$$

c. Moments

$$M = V_L x - Hy - \frac{wx^2}{2}$$

d. Shear and Thrust

$$S_\beta = wR\sin\beta\cos\beta - H\sin\beta$$

$$T_\beta = wR\sin\beta\cos\beta + H\cos\beta$$

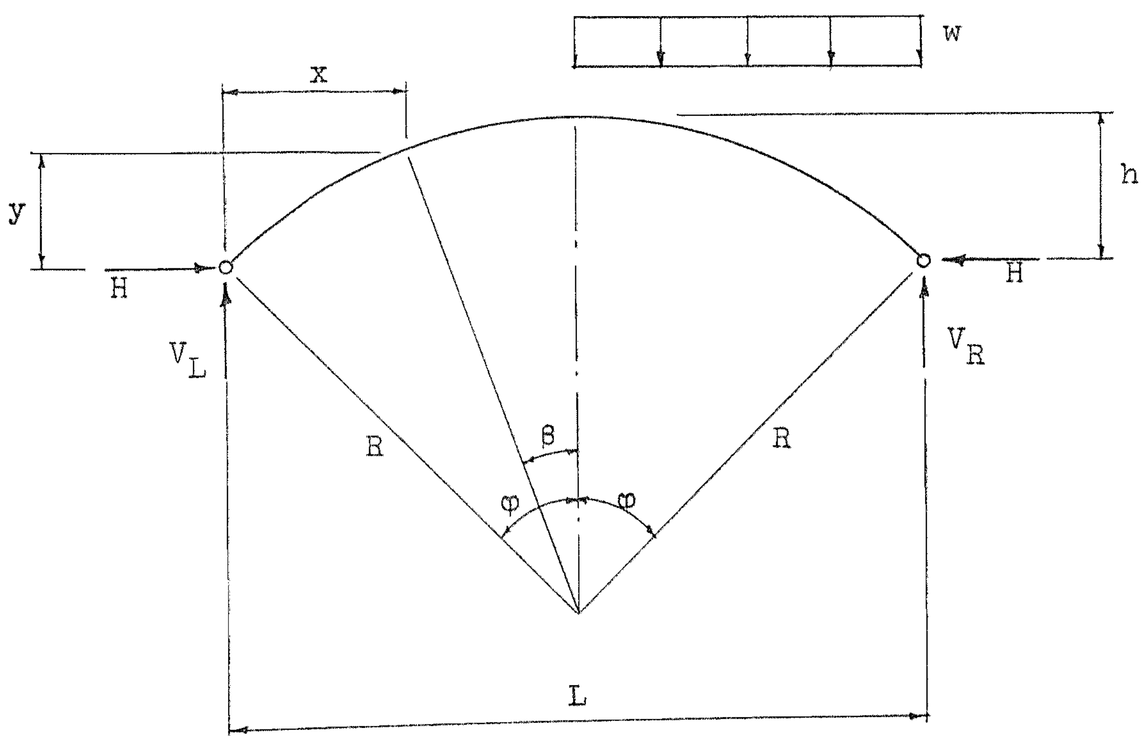


Figure 6

Case IV: Drift Load

a. Geometry

$$L = 2R\sin\varphi$$
$$x = R(\sin\varphi - \sin\beta)$$
$$y = R(\cos\beta - \cos\varphi)$$

b. Reactions

$$V_L = \frac{wL}{8} = \frac{wRSin\varphi}{4}$$

$$V_R = \frac{3wL}{8} \quad \frac{3wRSin\varphi}{4}$$

$$H = \frac{wR}{12}\left[\frac{4Sin^3\varphi + 3\varphi Cos\varphi(1 - 2Sin^2\varphi) - 3Sin\varphi Cos^2\varphi}{\varphi - 3Sin\varphi Cos\varphi + 2\varphi Cos^2\varphi}\right]$$

c. Moments

(1) Left Side: $M = V_L x - Hy$

(2) Right Side: $M = V_R x - Hy - \frac{wx^2}{2}$

d. Shear and Thrust

(1) Left Side: $S_\beta = \frac{wRSin\varphi Cos\beta}{4} - HSin\beta$

$T_\beta = \frac{wRSin\varphi Sin\beta}{4} + HCos\beta$

(2) Right Side: $S_\beta = wRCos\beta\left(Sin\beta - \frac{Sin\varphi}{4}\right) - HSin\beta$

$T_\beta = wRSin\beta\left(Sin\beta - \frac{Sin\varphi}{4}\right) + HCos\beta$

CHAPTER II

SAMPLE COMPUTER PROGRAM FOR CONCENTRATED LOAD

The program for the concentrated load will be used as an illustration of the machine language equations and their corresponding meanings. Calculations were made by the computer in response to the listed statements and equations for the tabulated coefficients. Data cards for values of the angle φ, in radians, from 30° to 45° were fed into the machine. Statements were set up so that cycling to the next angle was automatic once calculations were complete for a particular angle.

The angles φ, α and β are indicated on the card listing as A, D and B, respectively. The first column is the statement number and the second column is a continuation number for equations too lengthy to be punched on one data card. It is noted that the equation for the horizontal reaction (statement 13) required two continuation cards.

It should be noted that statement 13 differs slightly from the corresponding derivation as shown in Appendix A. The derived equation was modified for simplification of computer programming, although the value of each equation is exactly the same.

Statement			Significance
1	0	Read,A	Read φ (in radians)
2	0	Punch,A	Record φ
3	0	SA = SinF(A)	Sinφ = Sin function of φ
4	0	CA = CosF(A)	Cosφ = Cos function of φ
5	0	D = A	$\alpha = \varphi$
6	0	D = D - .1*A	$\alpha = \alpha - .1\varphi$
7	0	SD = SinF(D)	Sinα = Sin function of α
8	0	CD = CosF(D)	Cosα = Cos function of α
9	0	VL = (SA + SD)/(2.*SA)	$V_L = \frac{\sin\varphi + \sin\alpha}{2\sin\varphi}$
10	0	Punch,VL	Record V_L
11	0	VR = (SA - SD)/(2.*SA)	$V_R = \frac{\sin\varphi - \sin\alpha}{2\sin\varphi}$
12	0	Punch,VR	Record V_R
13	0	HP = (4.*CA*(CD + D*SD - CA - A*SA) - (2	$H = \left[4\cos\varphi(\cos\alpha + \alpha\sin\alpha - \cos\varphi - \varphi\sin\varphi) - 2\right.$
13	1	.*SD**2.) + (2.*SA**2.))/2.*(6.*	$\left.\sin^2\alpha + 2\sin^2\varphi\right] \Big/ \left[2(6\right.$
13	2	A - 6.*SA*CA - 4.*A*SA**2.)	$\left.\varphi - 6\sin\varphi\cos\varphi - 4\varphi\sin^2\varphi)\right]$

Statement			Significance
14	0	Punch,HP	Record H
15	0	B = A	$\beta = \varphi$
16	0	B = B - .1*A	$\beta = \beta - .1\varphi$
17	0	SB = SinF(B)	$\sin\beta$ = Sin function of β
18	0	CB = CosF(B)	$\cos\beta$ = Cos function of β
19	0	BML = (VL*(SA - SB)) - (HP*(CB - CA))	Moment from $\beta = \varphi$ to $\beta = \alpha$ $= V_L(\sin\varphi - \sin\beta) - H(\cos\beta - \cos\alpha)$
20	0	Punch,BML	Record the above moment
21	0	X = D - B	$X = \alpha - \beta$
22	0	If(X)16,23,23	If X is negative ($\beta > \alpha$), re-cycle to statement 16
			If X is zero ($\beta = \alpha$), go to statement 23
			If X is positive ($\beta < \alpha$), go to statement 23
23	0	B = B - .1*A	$\beta = \beta - .1\varphi$
24	0	SB = SinF(B)	$\sin\beta$ = Sin function of β
25	0	CB = CosF(B)	$\cos\beta$ = Cos function of β
26	0	BMR = (VL*(SA - SB)) - (HP*(CB - CA)) -	Moment from $\beta = \alpha$ to $\beta = 0$ $= V_L(\sin\varphi - \sin\beta) - H(\cos\beta - \cos\varphi) -$
26	1	((SA - SB) - (SA - SD))	$[(\sin\varphi - \sin\beta) - (\sin\varphi - \sin\alpha)]$

Statement	Significance
27 0 Punch,BMR	Record the above moment
28 0 If(B)29,29,23	If β is negative, go to statement 29
	If β is zero, go to statement 29
	If β is positive, re-cycle to statement 23
29 0 B = A	$\beta = \varphi$
30 0 B = B - .1*A	$\beta = \beta - .1\varphi$
31 0 SB = SinF(B)	$\sin\beta$ = Sin function of β
32 0 CB = CosF(B)	$\cos\beta$ = Cos function of β
33 0 BMRR = (VR*(SA - SB)) - (HP*(CB - CA))	Moment on unloaded side, $\beta = \varphi$ to $\beta = 0$, $= V_R(\sin\varphi - \sin\beta) - H(\cos\beta - \cos\varphi)$
34 0 Punch,BMRR	Record the above moment
35 0 If(B)36,36,30	If β is negative, go to statement 36
	If β is zero, go to statement 36
	If β is positive, re-cycle to statement 30
36 0 If(D)37,37,6	If α is negative, go to statement 37
	If α is zero, go to statement 37
	If α is positive, re-cycle to statement 6
37 0 Continue	Continue to next statement

Statement	Significance
38 0 End	Computer has completed calculations for all values of φ from 30° to 45°

Reaction and moment coefficients are listed below as an example of computer calculations. All calculations are based on points on the arch axis that correspond to angle increments in tenths of φ. The last two digits in the coefficient column establish the decimal. The particular listing is for the case of φ = to 30° with the load at α = 0.7 φ (refer to Table I-B in Appendix B).

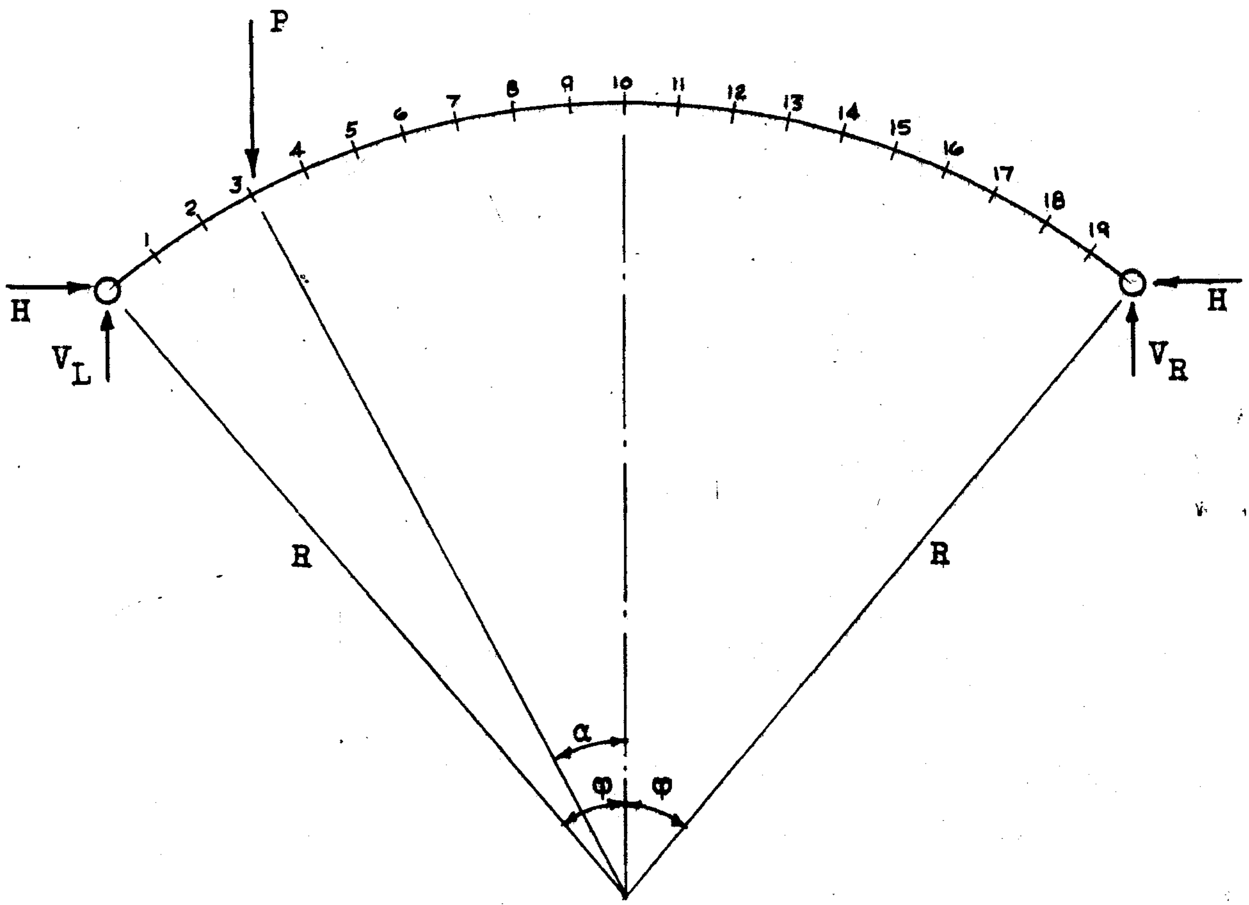


Figure 7

Location Points on the Arch Axis

Coefficient	Significance		
5235988050 +	φ	=	0.52359880 radians
8583679850 +	V_L/P	=	0.85836798
1416320650 +	V_R/P	=	0.14163206
6343033550 +	H/P	=	0.63430335
2364746849 +	$M_1/PRx10^{-3}$	=	23.647468
4991214249 +	$M_2/PRx10^{-3}$	=	49.912142
7872204049 +	$M_3/PRx10^{-3}$	=	78.722040
6064722049 +	$M_4/PRx10^{-3}$	=	60.647220
4410593049 +	$M_5/PRx10^{-3}$	=	44.105930
2914350049 +	$M_6/PRx10^{-3}$	=	29.143500
1580096049 +	$M_7/PRx10^{-3}$	=	15.800960
4114850048 +	$M_8/PRx10^{-3}$	=	4.1148500
5879370048 -	$M_9/PRx10^{-3}$	=	- 5.8793700
1416449049 -	$M_{10}/PRx10^{-3}$	=	-14.164490
2071105549 -	$M_{11}/PRx10^{-3}$	=	-20.711055
2549431549 -	$M_{12}/PRx10^{-3}$	=	-25.494315
2851132049 -	$M_{13}/PRx10^{-3}$	=	-28.511320
2975042849 -	$M_{14}/PRx10^{-3}$	=	-29.750428
2920822649 -	$M_{15}/PRx10^{-3}$	=	-29.208226
2688621249 -	$M_{16}/PRx10^{-3}$	=	-26.886212
2279074849 -	$M_{17}/PRx10^{-3}$	=	-22.790748
1693305849 -	$M_{18}/PRx10^{-3}$	=	-16.933058
9329194048 -	$M_{19}/PRx10^{-3}$	=	- 9.3291940

CHAPTER III

ILLUSTRATIVE NUMERICAL EXAMPLES

The utilization of the tabulated data is based on selecting an angle φ within the range of 30° to 45° after establishing a desired span. Values of the radius and heighth of the arch structure are dependent on this angle and span. For a constant radius, h varies as φ and L vary.

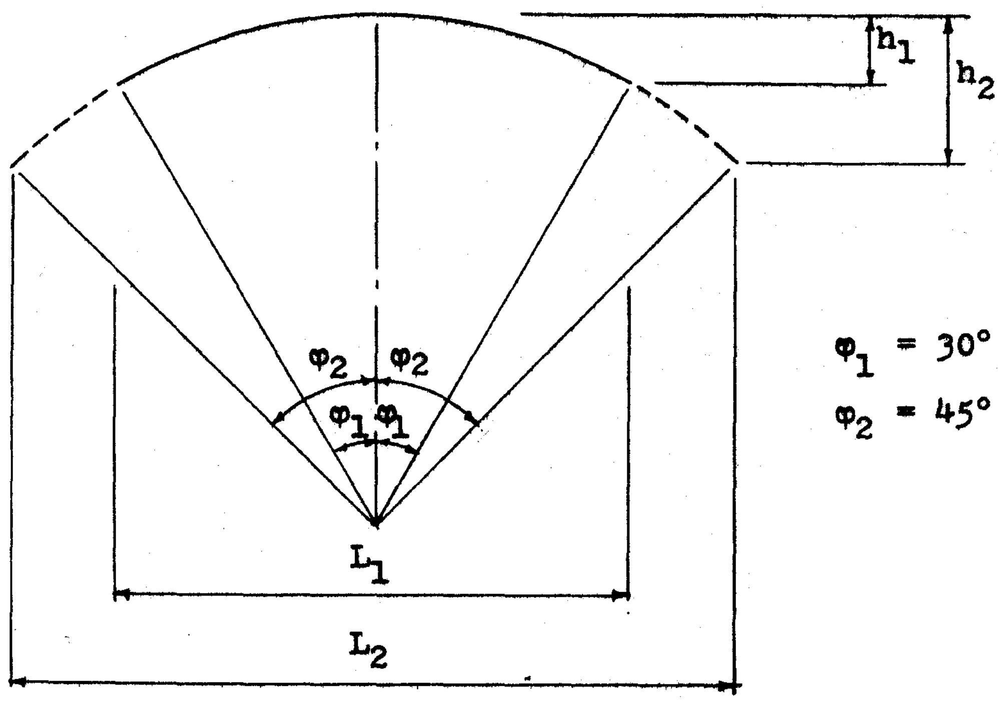


Figure 8

Variation of Rise with Span and Central Angle

1. Example A

Assume a two hinged arch of constant cross section and radius as shown in Figure 9. For purposes of determining maximum shear and thrust as well as maximum moment, the arch will be analyzed for dead load plus full live load, and for dead load plus drift load.

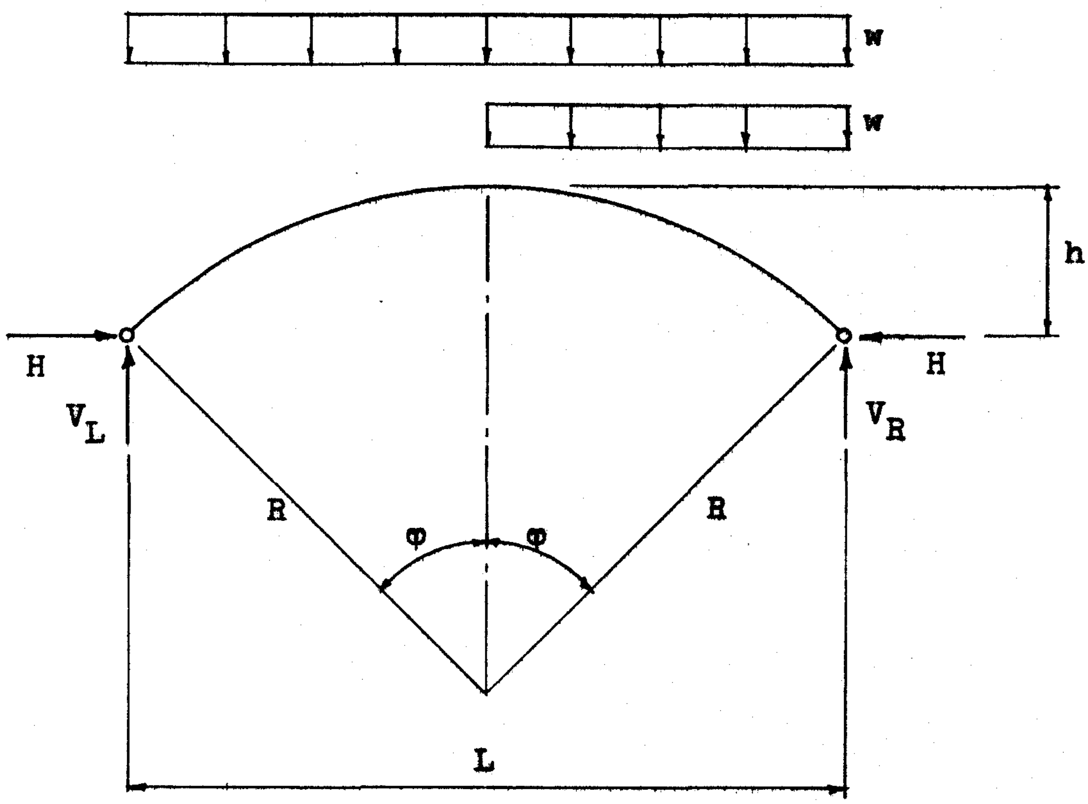


Figure 9

Full Live Load and Drift Load Conditions

a. Geometry and Loading

Arch spacing @ 30 feet o.c.
Dead load @ 10 psf: $w = 10 \times 30 = 0.3$ k/ft
Live load @ 30 psf: $w = 30 \times 30 = 0.9$ k/ft
Span L = 120 feet
$\varphi = 35°$
$\sin\varphi = 0.573576$

$$L = 2R\sin\varphi$$

$$R = \frac{L}{2\sin\varphi} = \frac{120}{2(0.573576)} = 104.606887 \text{ feet}$$

$$R = \frac{4h^2 + L^2}{8h} = 104.606887$$

$4h^2 - 836.855096\ h + 14400 = 0$, quadratic in h

$$h = \frac{-(-836.855096) \pm \sqrt{(836.855096)^2 - 4(4)14400}}{2(4)}$$

$$= \frac{836.855096 \pm \sqrt{469926.451701}}{8}$$

$$= \frac{836.855096 - 685.511817}{8} = \frac{151.343279}{8}$$

$$= 18.917910 \text{ feet}$$

For purposes of fabrication, complete accuracy in the above calculations is required. The same calculations performed using a slide rule are as follows:

$$R = \frac{L}{2\sin\varphi} = \frac{120}{2(0.573576)} = 104.6 \text{ feet}$$

$$R = \frac{4h^2 + L^2}{8h} = 104.6$$

$$4h^2 - 837\ h + 14400 = 0$$

$$h = \frac{-(-837) \pm \sqrt{(837)^2 - 4(4)14400}}{2(4)}$$

$$= \frac{837 \pm \sqrt{470000}}{8} = \frac{837 \pm 687}{8} = \frac{150}{8}$$

$$= 18.75 \text{ feet}$$

The difference in the two radius values is slight, but there is a discrepancy of approximately two inches in the two calculations for the rise. For calculations that follow, slide rule accuracy is sufficient and values for R and h may be rounded off.

b. <u>Reactions</u> (refer to Table VI-A, Appendix B)

Dead + Full Live Load:

$$V_L = V_R = 0.61087(0.3)104.61 + 0.57358(0.9)104.61$$
$$= 19.2 + 54.0 = 73.2 \text{ k}$$

$$H = 0.92839(0.3)104.61 + 0.89654(0.9)104.61$$
$$= 29.2 + 84.5 = 113.7 \text{ k}$$

Dead + Drift Load:

$$V_L = 0.61087(0.3)104.61 + 0.14340(0.9)104.61$$
$$= 19.2 + 13.5 = 32.7 \text{ k}$$

$$V_R = 0.61087(0.3)104.61 + 0.43018(0.9)104.61$$
$$= 19.2 + 40.5 = 59.7 \text{ k}$$

$$H = 0.92839(0.3)104.61 + 0.44827(0.9)104.61$$
$$= 29.2 + 42.3 = 71.5 \text{ k}$$

c. <u>Moments</u>

$$\text{Dead Load Multiplier} = wR^2 \times 10^{-3}$$
$$= 0.3(104.61)^2 10^{-3} = 3.28 \text{ k-ft}$$

$$\text{Live and Drift Load Multiplier} = wR^2 \times 10^{-3}$$
$$= 0.9(104.61)^2 10^{-3} = 9.83 \text{ k-ft}$$

The moment coefficients from Table VI-A for the various points on the arch axis, times the above multipliers, can be tabulated for final moment values, either maximum positive or negative, in kip-feet:

Point	Dead Load	Live Load	Drift Load	Maximum Moment
1	-4.77	-20.00	- 75.50	- 80.27
2	-6.80	-28.50	-134.20	-141.00
3	-6.75	-28.60	-176.00	-182.75
4	-5.37	-22.70	-201.00	-206.37
5	-3.15	-13.38	-208.00	-211.15
6	-0.65	- 2.70	-198.50	-199.15
7	1.73	7.51	-171.50	-169.77
8	3.66	15.85	-127.20	-123.54
9	4.91	21.30	- 66.20	- 61.29
10	5.37	23.10	11.55	28.47
11	4.91	21.30	87.50	92.41
12	3.66	15.85	143.00	146.66
13	1.73	7.51	179.00	180.73
14	-0.65	- 2.70	196.00	195.35
15	-3.15	-13.38	195.00	191.85
16	-5.37	-22.70	178.00	172.63
17	-6.75	-28.60	147.50	140.75
18	-6.80	-28.50	105.80	99.00
19	-4.77	-20.00	55.60	50.83

For concrete design, maximum and minimum positive moments and maximum and minimum negative moments can be readily determined from the above tabulation.

d. Shear and Thrust

The greatest shearing stress will be found at one of the supports under the particular loading condition that produces the largest reaction values, which in this example is the dead plus full live load condition. The shear calculations are made separately for dead load and live load and added for the total shear value.

Dead Load (refer to Case I, page 4):

$V_L = 19.2$ k

$H = 29.2$ k

$\varphi = \beta = 35° = 0.610866$ radians
$\sin\varphi = \sin\beta = 0.573576$
$\cos\varphi = \cos\beta = 0.819152$

$$S_\beta = \left[V_L - wR(\varphi - \beta)\right]\cos\beta - H\sin\beta$$
$$= 19.2(0.819152) - 29.2(0.573576$$
$$= 15.72 - 16.72 = -1.0 \text{ k}$$

Live Load (refer to Case III, page 6):

$H = 84.5$ k

$$S_\beta = wR\sin\beta\cos\beta - H\sin\beta$$
$$= 0.9(104.61)0.573576(0.819152) - 84.5(0.573576)$$
$$= 44.1 - 48.4 = -4.3 \text{ k}$$

Total $S_\beta = -1.0 - 4.3 = -5.3$ k

Thrust would be computed at the point of maximum moment and the combined bending and axial stress checked at that point. Maximum moment occurs under the dead plus drift load condition at point 5 on the arch axis. Separate calculations are made for the dead and drift load influences and added for the total thrust.

Dead Load (refer to Case I, page 4):

$V_L = 19.2$ k

$H = 29.2$ k

$\varphi = 0.610866$ radians
$\beta = 0.5\varphi = 17.5° = 0.305433$ radians
$\text{Sin}\beta = 0.300706$
$\text{Cos}\beta = 0.953717$

$$T_\beta = \left[V_L - wR(\varphi - \beta)\right] \text{Sin}\beta + H\text{Cos}\beta$$

$$= \left[19.2 - 0.3(104.61)(0.610866 - 0.305433)\right] 0.300706 + 29.2(0.953717)$$

$$= 2.9 + 27.8 = 30.7 \text{ k}$$

Drift Load (refer to Case IV, page 7):

$H = 42.3$ k

$$T_\beta = \frac{wR\sin\varphi\text{Sin}\beta}{4} + H\text{Cos}\beta$$

$$= \frac{0.9(104.61)0.573576(0.300706)}{4} + 42.3(0.953717)$$

$$= 4.1 + 40.3 = 44.4 \text{ k}$$

Total $T_\beta = 30.7 + 44.4 = 75.1$ k

2. Example B

This example will illustrate use of the tables for concentrated load as well as dead load. Assume an arch loaded as shown in Figure 10.

a. Geometry and Loading

Arch spacing @ 20 feet o.c.

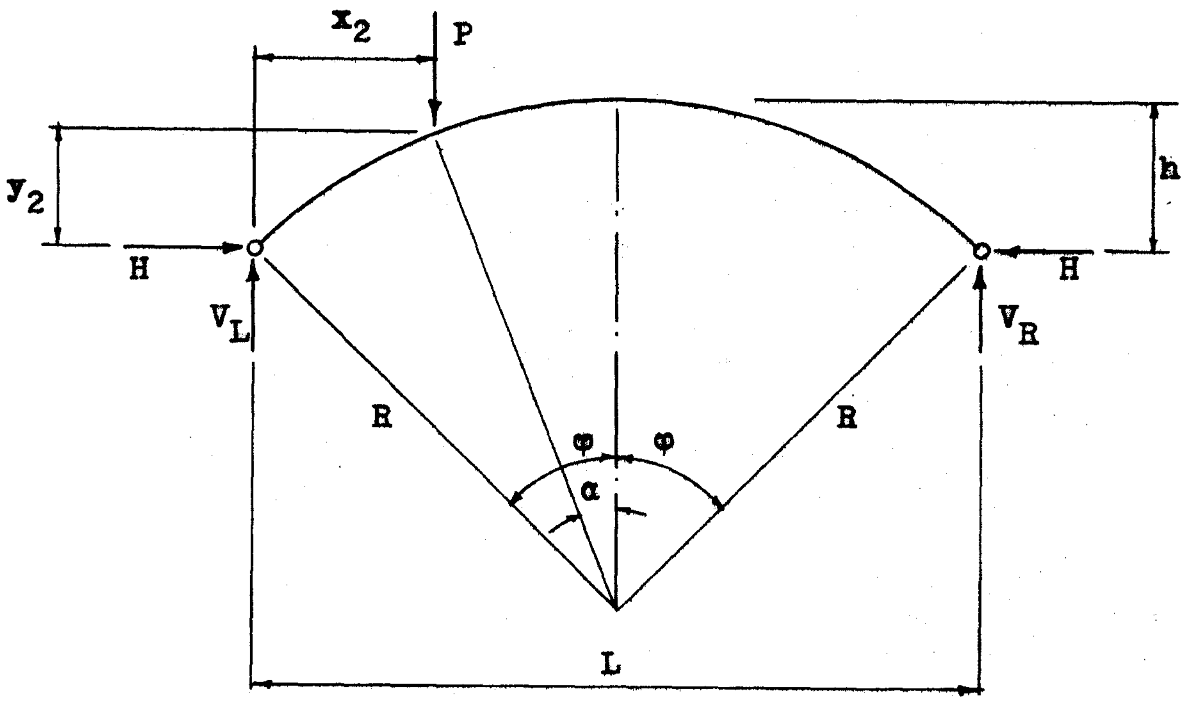


Figure 10

Concentrated Load Condition

Dead load @ 10 psf: $w = 10 \times 20 = 0.2$ k/ft
Assume P of 10 k placed at $\alpha = 0.6\varphi$
Span L = 80 feet
$\varphi = 42°$
$\sin\varphi = 0.669131$
$\cos\varphi = 0.743145$
$\alpha = 25.2°$
$\sin\alpha = 0.425779$
$\cos\alpha = 0.904827$

$L = 2R\sin\varphi$

$$R = \frac{L}{2\sin\varphi} = \frac{80}{2(0.669131)} = 59.779027 \text{ feet}$$

$$R = \frac{4h^2 + L^2}{8h} = 59.779027$$

$4h^2 - 478.232216\,h + 6400 = 0,$ quadratic in h

$$h = \frac{-(-478.232216) \pm \sqrt{(478.232216)^2 - 4(4)6400}}{2(4)}$$

$$= \frac{478.232216 \pm \sqrt{126306.052420}}{8}$$

$$= \frac{478.232216 - 355.395628}{8} = \frac{122.836588}{8}$$

$$= 15.354574 \text{ feet}$$

For the equations to locate exact load coordinates, refer to Case II, page 5.

$$x_2 = R(\text{Sin}\varphi - \text{Sin}\alpha)$$
$$= 59.779027(0.669131 - 0.425779)$$
$$= 14.547346 \text{ feet}$$

$$y_2 = R(\text{Cos}\alpha - \text{Cos}\varphi)$$
$$= 59.779027(0.904827 - 0.743145)$$
$$= 9.665193 \text{ feet}$$

Slide rule accuracy is sufficient for the remaining calculations. Note that values for R and h are rounded off.

b. Reactions (refer to Table XIII-A for dead load and to Table XIII-B for concentrated load, Appendix B)

$$V_L = 0.73304(0.2)59.78 + 0.81816(10)$$
$$= 8.8 + 8.2 = 17.0 \text{ k}$$

$$V_R = 0.73304(0.2)59.78 + 0.18184(10)$$
$$= 8.8 + 1.8 = 10.6 \text{ k}$$

$$H = 0.89633(0.2)59.78 + 0.55057(10)$$
$$= 10.7 + 5.5 = 16.2 \text{ k}$$

c. Moments

Dead Load Multiplier = $wR^2 \times 10^{-3}$

$= 0.2(59.78)^2 10^{-3} = 0.72$ k-ft

Concentrated Load Multiplier = $PR \times 10^{-3}$

$= 10(59.78)10^{-3} = 0.60$ k-ft

The moment coefficients from Tables XIII-A and XIII-B for the various points on the arch axis, times the respective multiplier, can be tabulated for final moment values, either maximum positive or negative, in kip-feet:

Point	Dead Load	Concentrated Load	Maximum Moment
1	-2.16	12.10	9.94
2	-3.08	27.20	24.12
3	-3.06	45.20	42.14
4	-2.43	66.00	63.57
5	-1.42	49.20	47.78
6	-0.29	33.80	33.51
7	0.80	19.90	20.70
8	1.69	7.65	9.34
9	2.25	- 2.96	- 0.71
10	2.45	-11.80	- 9.35
11	2.25	-18.90	-16.65
12	1.69	-24.30	-22.61
13	0.80	-27.70	-26.90
14	-0.29	-29.30	-29.59
15	-1.42	-29.00	-30.42
16	-2.43	-26.80	-29.23
17	-3.06	-22.90	-25.96

18	-3.08	-17.10	-20.18
19	-2.16	- 9.40	-11.56

d. Shear and Thrust

Shear calculations are made separately for dead load and concentrated load and added for the total shear value.

Dead Load (refer to Case I, page 4):

$V_L = 8.8$ k

$H = 10.7$ k

$\varphi = \beta = 42° = 0.733039$ radians
$\sin\varphi = \sin\beta = 0.669131$
$\cos\varphi = \cos\beta = 0.743145$

$$S_\beta = \left[V_L - wR(\varphi - \beta)\right]\cos\beta - H\sin\beta$$
$$= 8.8(0.743145) - 10.7(0.669131)$$
$$= 6.5 - 7.2 = -0.7 \text{ k}$$

Concentrated Load (refer to Case II, page 5):

$H = 5.5$ k

$\sin\alpha = 0.425779$

$$S_\beta = P\cos\beta\left(\frac{\sin\varphi + \sin\alpha}{2\sin\varphi}\right) - H\sin\beta$$

$$= 10(0.743145)\left[\frac{0.669131 + 0.425779}{2(0.669131)}\right] - 5.5(0.669131)$$

$$= 6.1 - 3.7 = 2.4 \text{ k}$$

Total $S_\beta = -0.7 + 2.4 = 1.7$ k

The point of maximum moment as tabulated above is at point 4 on the arch axis which coincides with the point of application of the concentrated load. Separate calculations are made for the dead and concentrated load influences and added for the total thrust.

Dead Load (refer to Case I, page 4):

$V_L = 8.8$ k

$H = 10.7$ k

$\beta = \alpha = 25.2° = 0.439823$ radians
$\sin\beta = \sin\alpha = 0.425779$
$\cos\beta = \cos\alpha = 0.904827$

$$T_\beta = \left[V_L - wR(\varphi - \beta)\right] \sin\beta + H\cos\beta$$
$$= \left[8.8 - 0.2(59.78)(0.733039 - 0.439823)\right] 0.425779 + 10.7(0.904827)$$
$$= 2.3 + 9.7 = 12.0 \text{ k}$$

Concentrated Load (Refer to Case II, page 5):

$H = 5.5$ k

$$T_\beta = P\sin\beta\left(\frac{\sin\varphi + \sin\alpha}{2\sin\varphi}\right) + H\cos\beta$$

$$= 10(0.425779)\left[\frac{0.669131 + 0.425779}{2(0.669131)}\right] + 5.5(0.904827)$$

$$= 3.5 + 5.0 = 8.5 \text{ k}$$

Total $T_\beta = 12.0 + 8.5 = 20.5$ k

CHAPTER IV

SUMMARY AND CONCLUSIONS

A method of analysis of two hinged circular arches of constant cross section and radius, based on the position angle of the hinges, is presented. Calculations of reaction and moment coefficients have been made by electronic computer for dead load, concentrated load, full live load and drift or snow load. Utilization of the tabulated coefficients for design is based on selection of the hinge position angle in the range from 30° to 45° after establishing a desired span.

The major problem in analysis of a two hinged arch is determination and evaluation of the horizontal reaction equation. The vertical reactions are written and evaluated by statics. The derivation of horizontal reaction equations for the various loading conditions is as shown in Appendix A. In addition, there are summary pages that list geometry, reaction, moment, shear and thrust equations for the individual load conditions on pages 4-8 of Chapter I.

1. Suggestions for Future Study

It is intended that the material in this study be of practical aid to the structural designer. Further study and research is needed, however, in wind load analysis which often, in combination with dead load, governs in structural design. The concept of horizontal wind loading on a curved member is not sufficiently exact, in that such members may be subjected to normal pressure and suction forces due to wind rather than pure horizontal forces.

It would be advantageous to include programming equations that would establish x and y coordinates for the various points on the arch axis. This could be done, for example, with appropriate equations inserted between statements 17 and 18 in the card listing for concentrated load on page 11 in Chapter II. Consideration could be given to graphing of the tabulated moment coefficients. Graphs or moment diagrams often present a clearer picture of structural action under load than tabulated data.

A SELECTED BIBLIOGRAPHY

1. Roark, Raymond J. Formulas for Stress and Strain. New York: McGraw-Hill Co., 1938, p.133.

2. Johnson, J.B., C.W. Bryan and F.E. Turneaure. Modern Framed Structures, Part II. New York: John Wiley and Sons, 1917.

3. Timoshenko, S. Strength of Materials, Part I. Princeton: Van Norstrand and Co., 1955.

4. Timoshenko, S. Strength of Materials, Part II. Princeton: Van Norstrand and Co., 1956.

5. Olsen, Gerner A. Elements of Mechanics of Materials. Englewood Cliffs: Prentice Hall Co., 1958.

6. Scofield, W. Fleming and W.H. O'Brien. "Design of a Two Hinged Glued Laminated Wooden Arch." (Unpublished, 1952).

APPENDIX A

DERIVATIONS

This appendix sets out the derivation of the horizontal reaction equations of a statically indeterminate two hinged arch. The vertical reactions of such a structure are readily determined by statics. Once the reactions, both horizontal and vertical, are determined, the arch may be analyzed at any point for moment, shear and thrust. The shape of the arch is of constant cross section and constant circular radius.

The theory of determining the horizontal reaction equation is based on the strain or deformation of the arch. When a structure is loaded, it acquires strain energy that is stored in the stressed member or members. This strain energy is known in various other terminology as elastic energy or potential energy of deformation. Stress within the elastic limit is implied, and therefore the strain energy is equal to the work done by the external forces in producing the stress, and is recoverable upon release of the external forces.

Assume a two hinged arch as shown in Figure 11. When loads are applied to the structure, it will assume a deformed or strained shape. At the hinges, neglecting displace-

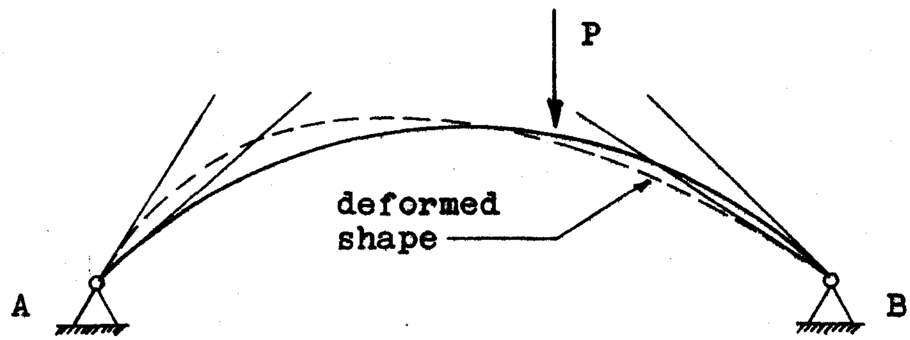


Figure 11

Assumed Deformed Shape Under Load

ment of supports, there will be no horizontal or vertical deflection, but the end tangents at A and B do not remain the same after as before load. There is a change in slope of each element ds of the arch as the end points rotate to a certain degree about the hinge. Determination of an equation for the change in slope of the element ds as the arch is deformed is the basis for the derivation of the horizontal reaction equation.

Consider an arch as shown in Figure 12, hinged at one end A, and supported by a roller at end B. As loads are applied to the arch, it would assume the shape AB', with a resultant deflection Δx.

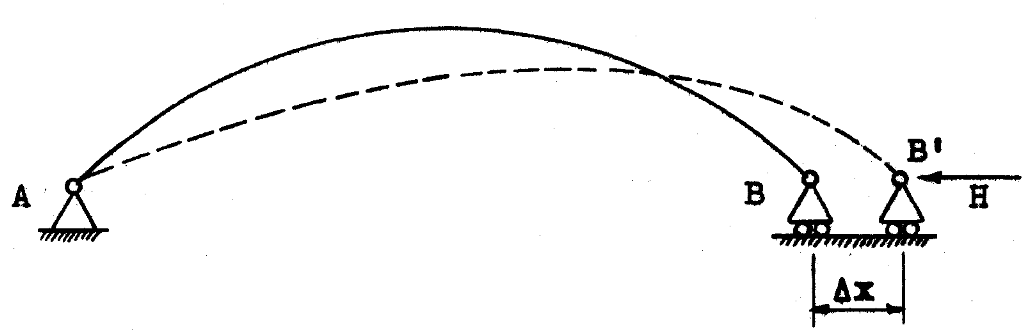


Figure 12

Horizontal Deflection of Roller Support

If a horizontal force H were applied at B' of sufficient magnitude, the displaced end of the arch would be returned to its original position at B and Δx would equal zero, the arch assuming the strained or deformed shape as shown in Figure 11. This is the condition at each hinge support of a two hinged arch as the horizontal reactions prevent displacement, Δx being equal to zero.

Assumptions

In determination of the basic equation for H, the derivation that follows is based on the following conditions and/or assumptions:

1. The arch is of constant cross section and homogeneous material (EI = a constant), and of a constant circular radius.
2. The material of the arch conforms to Hooke's Law, stating that stress is proportional to strain, and that all deformation and stress is within the elastic limit.
3. The basic equation for H neglects influences due to change of temperature, displacement of supports, or shortening of the center-line of the arch due to longitudinal compression.
4. The radius of curvature R is large as compared to the depth of the cross section, d. The ratio R/d equal to or greater than ten allows adapting the ordinary flexure theory of straight beams to the

curved section (1). The neutral axis and the centroid of the arch coincide in pure bending, and the neutral axis does not displace from its original position when the arch is deformed. In the curved beam theory, the neutral axis does displace under deformation relative to centroid, but the R/d ratio is in the range approaching or less than one.

Deformation of a Plane Element

In Figure 13, CDEF represents an element of an arch, ds in length, whose end faces CD and FE are perpendicular to tangents at N and N', the neutral axis of the element. Extensions of CD and FE make an angle dφ, a differential of the arch angle φ.

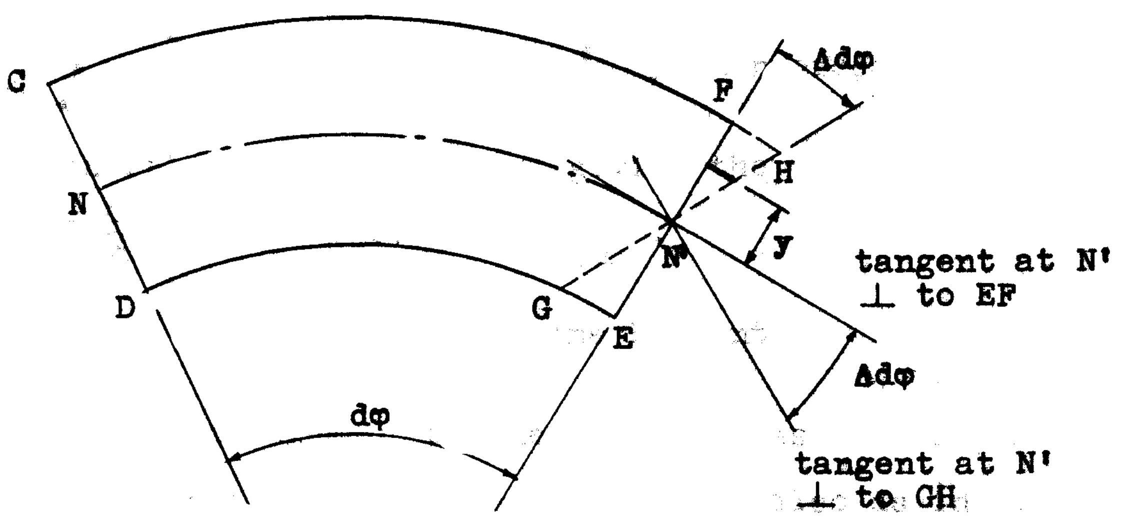


Figure 13

Two Dimensional Arch Element

When a load is applied, moments are induced on the arch element causing relative movement of CD and FE due to bending and a resulting change in angle of the two faces. This angle change can be represented by $\Delta d\varphi$ and a new position of FE by GH. At a distance y from the neutral axis NN', there will be a change in the element fiber length equal to $y\Delta d\varphi$. This value represents the elongation or strain of the fiber, so that the unit strain or unit elongation will be equal to $\frac{y\Delta d\varphi}{ds}$. From Hooke's Law, unit stress is proportional to unit strain, E being constant, and the corresponding stress per unit area will be $f = E\,\frac{y\Delta d\varphi}{ds}$. If dA represents a differential area of the cross section, the moment M on the cross section is equal to $\int_E^F fydA$, and substituting the value of f found above, is equal to $\int_E^F Ey^2\,\frac{\Delta d\varphi}{ds}\,dA$. At any particular section of the arch, E and $\frac{\Delta d\varphi}{ds}$ are constant so that $M = E\,\frac{\Delta d\varphi}{ds}\int_E^F y^2 dA$. The integral $\int_E^F y^2 dA$ represents the moment of inertia of the section, giving the value for M equal to $EI\,\frac{\Delta d\varphi}{ds}$, from which $\Delta d\varphi = \frac{Mds}{EI}$.

Deformation of a Line Element

Rather than an element of the arch such as CDEF shown in Figure 13, let a curved line segment NN', also ds in length, represent a linear element of the arch in an unstrained form. The element may be assumed to have a deformed shape NN" upon application of loads, as shown in Figure 14.

The angle $\Delta d\varphi$ is equivalent to the change in slope from

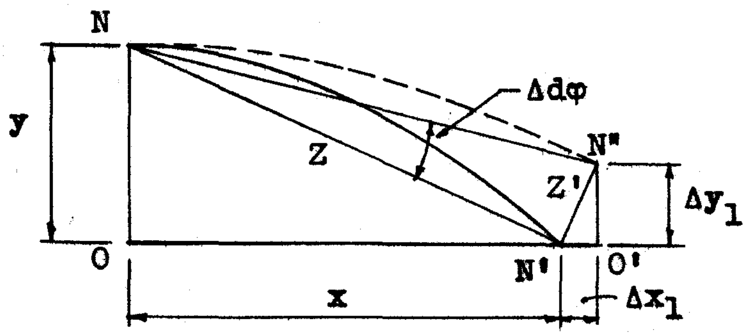


Figure 14

Linear Arch Element

a tangent at N' to a tangent at N". Using coordinate axes, the end of the element at N', relative to N, has moved to the right a distance Δx_1 and up a distance Δy_1 to a new position at N". Denoting the line connecting N to N' as length Z and the line connecting N' to N" as length Z', and using the similar triangles ONN' and O'N'N", $\Delta x_1 = \frac{yZ'}{Z}$. Further, Tan $\Delta d\varphi = \frac{Z'}{Z}$ and $Z' = Z\Delta d\varphi$. $\Delta d\varphi$ is a differential angle and in the range where the tangent of a small angle equals the angle itself. Substituting this value of Z' into the value for Δx_1 above, $\Delta x_1 = y\Delta d\varphi$. From Figure 13, $\Delta d\varphi = \frac{Mds}{EI}$, so that finally $\Delta x_1 = \frac{Myds}{EI}$. It can be shown similarly that $\Delta y_1 = \frac{Mxds}{EI}$.

Total Deformation

If the whole arch was held at end A as shown in Figure 15 but allowed to displace at end B, much as the linear element in Figure 14 above, there would be a total displacement equal to the sum of the displacements of the individual elements.

Figure 15

Structure with One Support Removed

This summation, then, is $\Sigma\Delta x_n = \int_A^B \frac{Myds}{EI}$ and $\Sigma\Delta y_n = \int_A^B \frac{Mxds}{EI}$. The hinges, however, allow rotation at A and B but no horizontal or vertical displacement, so that $\int_A^B \frac{Myds}{EI} = 0$ and $\int_A^B \frac{Mxds}{EI} = 0$.

Basic Horizontal Reaction Equation

In determining the basic equation for H, use only the summation for Δx_n, since there is no displacement in the y direction, where M = the total moment at any point on the arch axis.

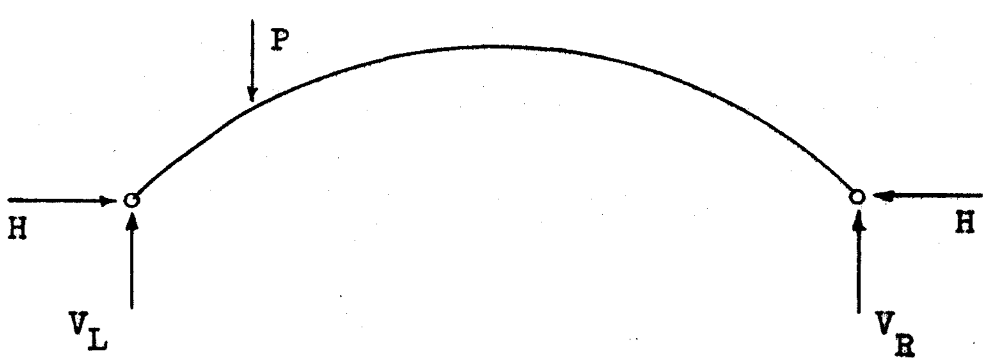


Figure 16

Structure with Concentrated Load

Consider an arch loaded as in Figure 16, of which the vertical reactions can be determined by statics. Assuming a free-body of the arch, the loads and reactions can be represented by the principles of superposition as in Figure 17.

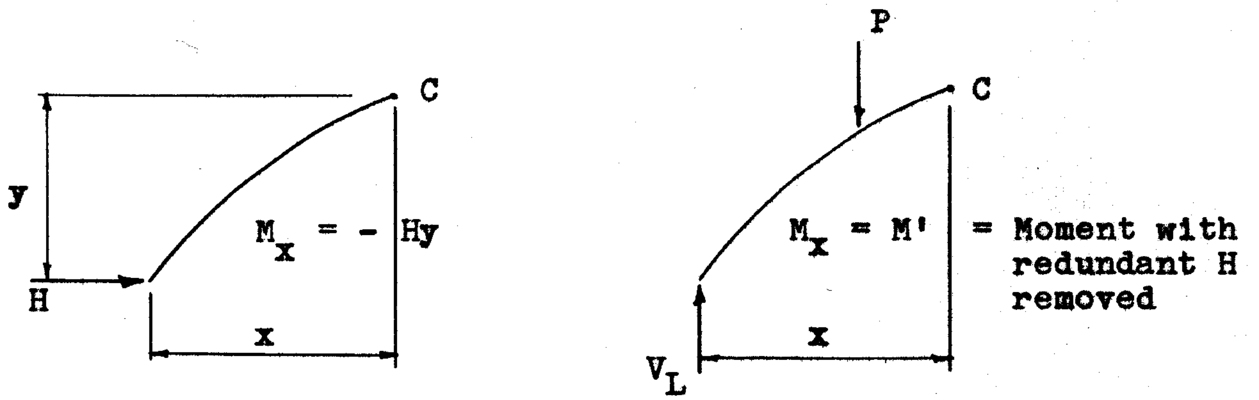


Figure 17

Free Body Segments

At any point C on the arch axis, the total moment can be separated into a simple beam moment M' due to vertical loads and reactions only and a moment equal to Hy, or M = M' - Hy. Substituting,

$$\int_A^B \frac{Myds}{EI} = \int_A^B \frac{(M' - Hy)yds}{EI} = \int_A^B \frac{M'yds}{EI} - H\int_A^B \frac{y^2ds}{EI} = 0$$

from which, $H = \dfrac{\int_A^B M'yds}{\int_A^B y^2ds}$, EI cancelling as a constant.

In this expression for H, the denominator remains the same regardless of the loading condition. The numerator depends on whether M' is the simple beam moment for dead load, live load, drift load, concentrated load, or combinations of these or other loading conditions.

Horizontal Reaction Equations for Various Loads

1. General

As stated above, the integral $\int y^2 ds$ as the denominator is constant for all loading conditions. Figure 18 represents a general sketch of the arch for purposes of evaluating this integral.

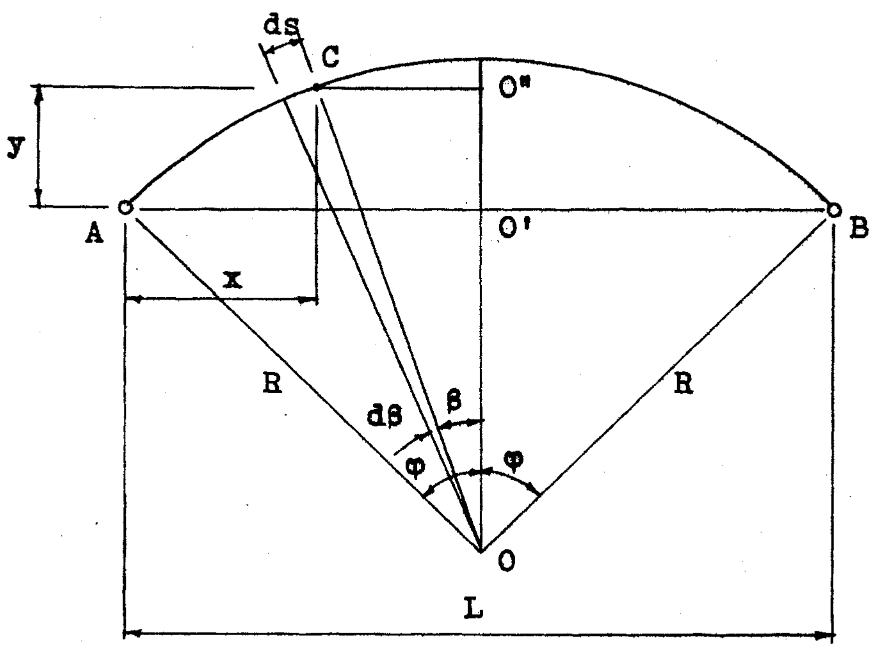


Figure 18

Geometry of the General Structure

The angle φ defines the position angle of the hinge with reference to a vertical through the radius point, O, of the arch, and the angle β defines the position angle of any point C on the arch axis with reference to the same line. From inspection:

$$\operatorname{Sin}\varphi = \frac{AO'}{R}, \quad AO' = R\operatorname{Sin}\varphi$$

$$\text{Sin}\beta = \frac{CO''}{R}, \quad CO'' = R\text{Sin}\beta$$

$$\text{Cos}\varphi = \frac{OO'}{R}, \quad OO' = R\text{Cos}\varphi$$

$$\text{Cos}\beta = \frac{OO''}{R}, \quad OO'' = R\text{Cos}\beta$$

$$x = AO' - CO'' \quad = R(\text{Sin}\varphi - \text{Sin}\beta)$$

$$y = OO'' - OO' \quad = R(\text{Cos}\beta - \text{Cos}\varphi)$$

These are expressions for x and y in polar coordinates. -ther, $\text{Sin}\varphi = \frac{L/2}{R}$ so that $L = 2R\text{Sin}\varphi$, and $ds = Rd\beta$.

Substituting the above values into the integral and in- grating from zero to φ so that the integral is doubled to egrate along the entire arch, the denominator

$$D = 2\int_0^{\varphi} y^2 ds$$

$$= 2\int_0^{\varphi} \left[R(\text{Cos}\beta - \text{Cos}\varphi)\right]^2 Rd\beta$$

$$= 2R^3 \int_0^{\varphi} (\text{Cos}\beta - \text{Cos}\varphi)^2 d\beta$$

$$= 2R^3 \int_0^{\varphi} (\text{Cos}^2\beta - 2\text{Cos}\beta\text{Cos}\varphi + \text{Cos}^2\varphi)d\beta$$

$$= 2R^3 \left[\frac{\beta}{2} + \frac{\text{Sin}\beta\text{Cos}\beta}{2} - 2\text{Sin}\beta\text{Cos}\varphi + \beta\text{Cos}^2\varphi\right]_0^{\varphi}$$

$$= 2R^3 \left[\frac{\varphi}{2} + \frac{\text{Sin}\varphi\text{Cos}\varphi}{2} - 2\text{Sin}\varphi\text{Cos}\varphi + \varphi\text{Cos}^2\varphi\right]$$

$$= R^3 \left[\varphi + \text{Sin}\varphi\text{Cos}\varphi - 4\text{Sin}\varphi\text{Cos}\varphi + 2\varphi\text{Cos}^2\varphi\right]$$

$$= R^3 \left[\varphi - 3\text{Sin}\varphi\text{Cos}\varphi + 2\varphi\text{Cos}^2\varphi\right] \qquad \underline{\text{EQ. 1}}$$

This equation is used in determining H for all loading conditions that follow.

From the basic integral to the resulting equation 1 above, all steps of integration were included and no attempt made to omit even simple operations.

For each of the following loading conditions, the particular integrals will be evaluated for the numerator,

$$N = \int M'yds$$

keeping in mind that M' is the simple beam moment due to vertical loads and vertical reactions only, with the redundant horizontal reaction H removed.

2. Dead Load

Consider this for the first condition, as it is present in any structural design problem regardless of other loadings.

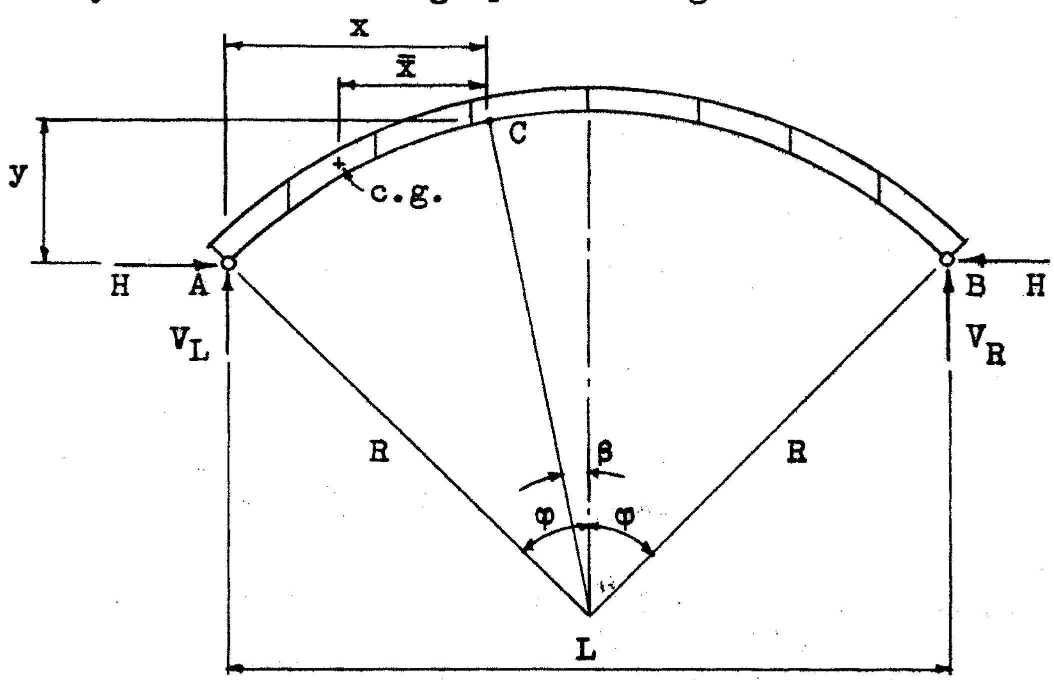


Figure 19

Geometry of the Structure for Dead Load

For any point C on the arch axis as shown in Figure 19,

$$x = R(\sin\varphi - \sin\beta)$$

$$y = R(\cos\beta - \cos\varphi)$$

and the total arch length along the axis is $\frac{2\pi R\varphi}{180°}$ which equals $2R\varphi$ for 180° in radians. The partial arch length along the axis from the hinge to any point C is $R(\varphi - \beta)$. The horizontal distance $\bar{x}$ from point C to the center of gravity of the dead load is $R\left[\frac{\cos\beta - \cos\varphi}{\varphi - \beta} - \sin\beta\right]$. This latter expression is not derived herein.

a. Reactions

The total load on the arch with dead load only is $2wR\varphi$, so that $V_L = V_R = wR\varphi$. At any point C, the simple moment M' is seen to be $V_L x - wR(\varphi - \beta)\bar{x}$. Observing the limits set on the integral and substituting the above values into M', the dead load numerator is

$$N = 2\int_0^{\varphi} M'y\,ds$$

$$= 2\int_0^{\varphi}\left[V_L x - wR(\varphi - \beta)\bar{x}\right]y\,ds$$

$$= 2\int_0^{\varphi}\left\{wR\varphi\left[R(\sin\varphi - \sin\beta)\right] - wR(\varphi - \beta)\left[R\left(\frac{\cos\beta - \cos\varphi}{\varphi - \beta} - \sin\beta\right)\right]\right\}R(\cos\beta - \cos\varphi)R\,d\beta$$

$$= 2\int_0^{\varphi}\left\{wR\varphi\left[R(\sin\varphi - \sin\beta)\right] - wR(\varphi - \beta)\left[\frac{R}{\varphi - \beta}\left(\cos\beta - \cos\varphi - \sin\beta(\varphi - \beta)\right)\right]\right\}R(\cos\beta - \cos\varphi)R\,d\beta$$

$$= 2wR^4 \int_0^\varphi \Big\{\varphi(\text{Sin}\varphi - \text{Sin}\beta) - \Big[\text{Cos}\beta - \text{Cos}\varphi$$

$$- \text{Sin}\beta(\varphi - \beta)\Big]\Big\}(\text{Cos}\beta - \text{Cos}\varphi)d\beta$$

$$= 2wR^4 \int_0^\varphi (\varphi\text{Sin}\varphi - \varphi\text{Sin}\beta - \text{Cos}\beta + \text{Cos}\varphi + \varphi\text{Sin}\beta$$

$$-\beta\text{Sin}\beta)(\text{Cos}\beta - \text{Cos}\varphi)d\beta$$

$$= 2wR^4 \int_0^\varphi (\varphi\text{Sin}\varphi\text{Cos}\beta - \text{Cos}^2\beta + \text{Cos}\varphi\text{Cos}\beta - \beta\text{Sin}\beta\text{Cos}\beta$$

$$- \varphi\text{Sin}\varphi\text{Cos}\varphi + \text{Cos}\varphi\text{Cos}\beta - \text{Cos}^2\varphi + \beta\text{Sin}\beta\text{Cos}\varphi)d\beta$$

Evaluation of the above is fairly straightforward. The term $\beta\text{Sin}\beta\text{Cos}\beta$ is integrated by parts. Integration yields

$$2wR^4 \Big[\varphi\text{Sin}\varphi\text{Sin}\beta - \frac{\beta}{2} - \frac{\text{Sin}\beta\text{Cos}\beta}{2} + 2\text{Cos}\varphi\text{Sin}\beta - \frac{\beta\text{Sin}^2\beta}{2}$$

$$+ \frac{\beta}{4} - \frac{\text{Sin}\beta\text{Cos}\beta}{4} - \beta\varphi\text{Sin}\varphi\text{Cos}\varphi - \beta\text{Cos}^2\varphi + \text{Cos}\varphi\text{Sin}\beta$$

$$- \beta\text{Cos}\beta\text{Cos}\varphi\Big]_0^\varphi$$

$$= 2wR^4 \Big[\varphi\text{Sin}^2\varphi - \frac{\varphi}{2} - \frac{\text{Sin}\varphi\text{Cos}\varphi}{2} + \frac{2\text{Sin}\varphi\text{Cos}\varphi}{2} - \frac{\varphi\text{Sin}^2\varphi}{2} + \frac{\varphi}{4}$$

$$- \frac{\text{Sin}\varphi\text{Cos}\varphi}{4} - \varphi^2\text{Sin}\varphi\text{Cos}\varphi - \varphi\text{Cos}^2\varphi + \text{Sin}\varphi\text{Cos}\varphi$$

$$- \varphi\text{Cos}^2\varphi\Big]$$

Cancelling like terms and combining

$$N_{DL} = 2wR^4 \Big[\text{Cos}\varphi(2\text{Sin}\varphi - 2\varphi\text{Cos}\varphi - \varphi^2\text{Sin}\varphi) - \frac{\varphi}{4} + \frac{\text{Sin}\varphi\text{Cos}\varphi}{4}$$

$$+ \frac{\varphi \mathrm{Sin}^2\varphi}{2}\Bigg] \qquad \underline{\text{EQ. 2}}$$

The final equation for the horizontal reactions due to dead load is:

$$H = \frac{N_{DL}}{D} = \frac{\text{EQ. 2}}{\text{EQ. 1}}$$

$$= \frac{2wR^4\left[\mathrm{Cos}\varphi(2\mathrm{Sin}\varphi - 2\varphi\mathrm{Cos}\varphi - \varphi^2\mathrm{Sin}\varphi) - \frac{\varphi}{4} + \frac{\mathrm{Sin}\varphi\mathrm{Cos}\varphi}{4} + \frac{\varphi\mathrm{Sin}^2\varphi}{2}\right]}{R^3(\varphi - 3\mathrm{Sin}\varphi\mathrm{Cos}\varphi + 2\varphi\mathrm{Cos}^2\varphi)}$$

$$= 2wR\left[\frac{\mathrm{Cos}\varphi(2\mathrm{Sin}\varphi - 2\varphi\mathrm{Cos}\varphi - \varphi^2\mathrm{Sin}\varphi) - \frac{\varphi}{4} + \frac{\mathrm{Sin}\varphi\mathrm{Cos}\varphi}{4} + \frac{\varphi\mathrm{Sin}^2\varphi}{2}}{\varphi - 3\mathrm{Sin}\varphi\mathrm{Cos}\varphi + 2\varphi\mathrm{Cos}^2\varphi}\right]$$

3. Concentrated Load

Consider this loading condition which is commonly encountered in any structure.

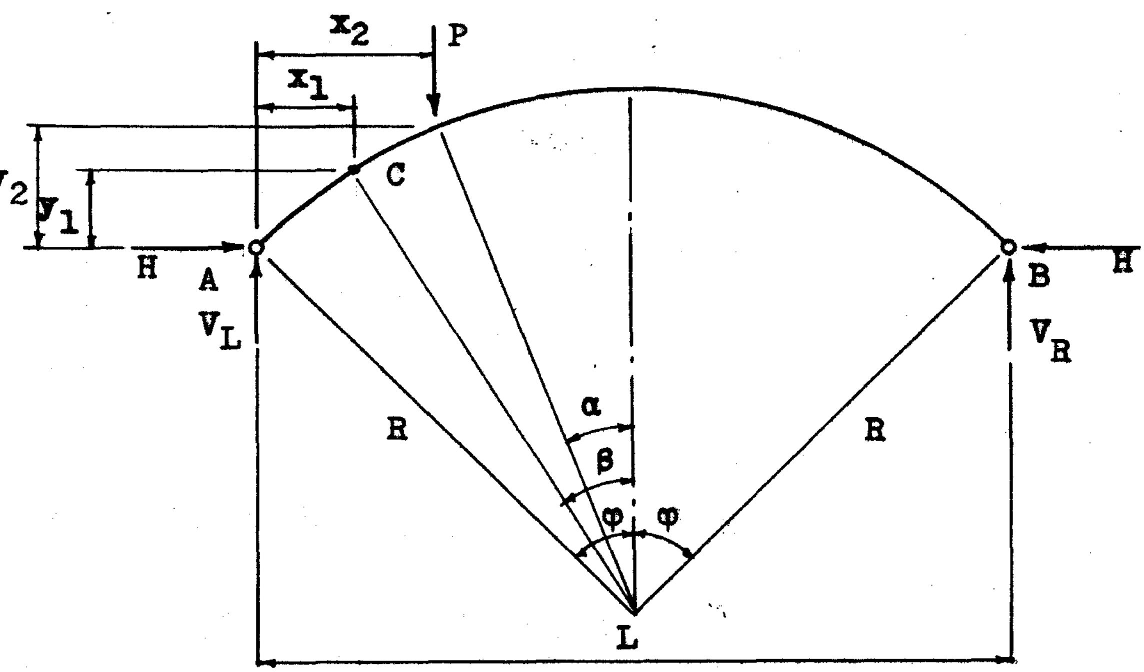


Figure 20

Geometry of the Structure for Concentrated Load

In Figure 20, the same relationships exist as before,

$$L = 2R\sin\varphi$$

$$x_1 = R(\sin\varphi - \sin\beta)$$

$$y_1 = R(\cos\beta - \cos\varphi)$$

$$ds = Rd\beta$$

In this case, α is the position angle of the concentrated load point with reference to a vertical through the radius point, and coordinate values can be shown to be

$$x_2 = R(\sin\varphi - \sin\alpha)$$

$$y_2 = R(\cos\alpha - \cos\varphi)$$

$$x_1 - x_2 = R(\sin\alpha - \sin\beta)$$

a. Reactions

Through statics, the vertical reactions are

$$V_L = P\,\frac{L - x_2}{L} = P\,\frac{2R\sin\varphi - R(\sin\varphi - \sin\alpha)}{2R\sin\varphi} = P\,\frac{\sin\varphi + \sin\alpha}{2\sin\varphi}$$

$$V_R = P\,\frac{x_2}{L} = P\,\frac{R(\sin\varphi - \sin\alpha)}{2R\sin\varphi} = P\,\frac{\sin\varphi - \sin\alpha}{2\sin\varphi}$$

Evaluation of the integral $\int M'yds$ for the concentrated load is quite lengthy, and results in three separate integrals dependent on the angles φ, β and α. Close observance of the limits is maintained with the particular simple beam moment M' for each integral. The numerator is

$$N_P = \int M'yds$$

$$= \int_\alpha^\varphi M'yds + \int_0^\alpha M'yds + \int_0^\varphi M'yds$$

Note the segments of the arch α to φ, 0 to α and 0 to φ as limits. The load is indicated on the left side and the first two integrals are for that half of the arch. The third integral is a summation for the right unloaded half. When the load is on the right side, α (Figure 20) is indicated accordingly so that the same three integrals result.

(1) $\int_{\alpha}^{\varphi} M'yds \quad (M' = V_L x_1)$

$$= \int_{\alpha}^{\varphi} (V_L x_1) yds$$

$$= \int_{\alpha}^{\varphi} \left[V_L R(\text{Sin}\varphi - \text{Sin}\beta)\right] \left[R(\text{Cos}\beta - \text{Cos}\varphi)\right] Rd\beta$$

$$= V_L R^3 \int_{\alpha}^{\varphi} (\text{Sin}\varphi - \text{Sin}\beta)(\text{Cos}\beta - \text{Cos}\varphi) d\beta$$

$$= V_L R^3 \int_{\alpha}^{\varphi} (\text{Sin}\varphi\text{Cos}\beta - \text{Sin}\beta\text{Cos}\beta - \text{Sin}\varphi\text{Cos}\varphi + \text{Sin}\beta\text{Cos}\varphi) d\beta$$

$$= V_L R^3 \left[\text{Sin}\varphi\text{Sin}\beta - \frac{\text{Sin}^2\beta}{2} - \beta\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}\beta\text{Cos}\varphi\right]_{\alpha}^{\varphi}$$

$$= V_L R^3 \left[(\text{Sin}^2\varphi - \frac{\text{Sin}^2\varphi}{2} - \varphi\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}^2\varphi) - (\text{Sin}\varphi\text{Sin}\alpha - \frac{\text{Sin}^2\alpha}{2} - \alpha\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}\varphi\text{Cos}\alpha)\right]$$

$$= V_L R^3 (\text{Sin}^2\varphi - \frac{\text{Sin}^2\varphi}{2} - \varphi\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}^2\varphi - \text{Sin}\varphi\text{Sin}\alpha + \frac{\text{Sin}^2\alpha}{2} + \alpha\text{Sin}\varphi\text{Cos}\varphi + \text{Cos}\varphi\text{Cos}\alpha)$$

<u>EQ. 3a</u>

(2) $\int_{0}^{\alpha} M'yds \quad \left[M' = V_L x_1 - P(x_1 - x_2)\right]$

$$= \int_0^{\alpha} \left[V_L x_1 - P(x_1 - x_2)\right] y ds$$

$$= \int_0^{\alpha} \left[V_L R(\mathrm{Sin}\varphi - \mathrm{Sin}\beta) - PR(\mathrm{Sin}\alpha - \mathrm{Sin}\beta)\right] R(\mathrm{Cos}\beta - \mathrm{Cos}\varphi) R d\beta$$

$$= R^3 \int_0^{\alpha} \left[V_L(\mathrm{Sin}\varphi - \mathrm{Sin}\beta) - P(\mathrm{Sin}\alpha - \mathrm{Sin}\beta)\right] (\mathrm{Cos}\beta - \mathrm{Cos}\varphi) d\beta$$

$$= R^3 \int_0^{\alpha} \left[V_L(\mathrm{Sin}\varphi - \mathrm{Sin}\beta)(\mathrm{Cos}\beta - \mathrm{Cos}\varphi) - P(\mathrm{Sin}\alpha - \mathrm{Sin}\beta)\right.$$
$$\left.(\mathrm{Cos}\beta - \mathrm{Cos}\varphi)\right] d\beta$$

$$= R^3 \int_0^{\alpha} \left[V_L(\mathrm{Sin}\varphi\mathrm{Cos}\beta - \mathrm{Sin}\beta\mathrm{Cos}\beta - \mathrm{Sin}\varphi\mathrm{Cos}\varphi + \mathrm{Sin}\beta\mathrm{Cos}\varphi)\right.$$
$$\left. - P(\mathrm{Sin}\alpha\mathrm{Cos}\beta - \mathrm{Sin}\beta\mathrm{Cos}\beta - \mathrm{Sin}\alpha\mathrm{Cos}\varphi + \mathrm{Sin}\beta\mathrm{Cos}\varphi)\right] d\beta$$

$$= \left[V_L R^3(\mathrm{Sin}\varphi\mathrm{Sin}\beta - \frac{\mathrm{Sin}^2\beta}{2} - \beta\mathrm{Sin}\varphi\mathrm{Cos}\varphi - \mathrm{Cos}\beta\mathrm{Cos}\varphi)\right.$$
$$\left. - PR^3(\mathrm{Sin}\alpha\mathrm{Sin}\beta - \frac{\mathrm{Sin}^2\beta}{2} - \beta\mathrm{Sin}\alpha\mathrm{Cos}\varphi - \mathrm{Cos}\beta\mathrm{Cos}\varphi)\right]_0^{\alpha}$$

$$= V_L R^3(\mathrm{Sin}\varphi\mathrm{Sin}\alpha - \frac{\mathrm{Sin}^2\alpha}{2} - \alpha\mathrm{Sin}\varphi\mathrm{Cos}\varphi - \mathrm{Cos}\alpha\mathrm{Cos}\varphi)$$
$$- PR^3(\mathrm{Sin}^2\alpha - \frac{\mathrm{Sin}^2\alpha}{2} - \alpha\mathrm{Sin}\alpha\mathrm{Cos}\varphi - \mathrm{Cos}\alpha\mathrm{Cos}\varphi)$$

<u>EQ. 3b</u>

(3) $$\int_0^{\varphi} M' y ds \qquad (M' = V_R x_1)$$

$$= \int_0^{\varphi} (V_R x_1) y ds$$

$$= \int_0^{\varphi} \left[(P - V_L) R(\mathrm{Sin}\varphi - \mathrm{Sin}\beta)\right] R(\mathrm{Cos}\beta - \mathrm{Cos}\varphi) R d\beta$$

$$= (P - V_L)R^3 \int_0^{\varphi} (\text{Sin}\varphi - \text{Sin}\beta)(\text{Cos}\beta - \text{Cos}\varphi)d\beta$$

$$= (P - V_L)R^3 \int_0^{\varphi} \Big[(\text{Sin}\varphi\text{Cos}\beta - \text{Sin}\beta\text{Cos}\beta - \text{Sin}\varphi\text{Cos}\varphi$$

$$+ \text{Sin}\beta\text{Cos}\varphi)\Big] d\beta$$

$$= (P - V_L)R^3 \left[\text{Sin}\varphi\text{Sin}\beta - \frac{\text{Sin}^2\beta}{2} - \beta\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}\beta\text{Cos}\varphi\right]_0^{\varphi}$$

$$= (P - V_L)R^3 \left[\text{Sin}^2\varphi - \frac{\text{Sin}^2\varphi}{2} - \varphi\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}^2\varphi\right]$$

$$= PR^3 \left[\text{Sin}^2\varphi - \frac{\text{Sin}^2\varphi}{2} - \varphi\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}^2\varphi\right]$$

$$- V_LR^3 \left[\text{Sin}^2\varphi - \frac{\text{Sin}^2\varphi}{2} - \varphi\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}^2\varphi\right]$$

EQ. 3c

N_P, then, is the sum of equations 3a, 3b and 3c:

EQ.3a $$V_LR^3 \left[\text{Sin}^2\varphi - \frac{\text{Sin}^2\varphi}{2} - \varphi\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}^2\varphi - \text{Sin}\varphi\text{Sin}\alpha + \frac{\text{Sin}^2\alpha}{2} + \alpha\text{Sin}\varphi\text{Cos}\varphi + \text{Cos}\varphi\text{Cos}\alpha\right]$$

EQ.3b $$+ V_LR^3 \left[\text{Sin}\varphi\text{Sin}\alpha - \frac{\text{Sin}^2\alpha}{2} - \alpha\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}\alpha\text{Cos}\varphi\right]$$

$$- PR^3 \left[\text{Sin}^2\alpha - \frac{\text{Sin}^2\alpha}{2} - \alpha\text{Sin}\alpha\text{Cos}\varphi - \text{Cos}\alpha\text{Cos}\varphi\right]$$

EQ.3c $$+ PR^3 \left[\text{Sin}^2\varphi - \frac{\text{Sin}^2\varphi}{2} - \varphi\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}^2\varphi\right]$$

$$- V_LR^3 \left[\text{Sin}^2\varphi - \frac{\text{Sin}^2\varphi}{2} - \varphi\text{Sin}\varphi\text{Cos}\varphi - \text{Cos}^2\varphi\right]$$

Cancelling like terms,

$$N_P = V_L R^3(0) + PR^3\left[\mathrm{Sin}^2\varphi - \frac{\mathrm{Sin}^2\varphi}{2} - \varphi\mathrm{Sin}\varphi\mathrm{Cos}\varphi - \mathrm{Cos}^2\varphi - \mathrm{Sin}^2\alpha + \frac{\mathrm{Sin}^2\alpha}{2} + \alpha\mathrm{Sin}\alpha\mathrm{Cos}\varphi + \mathrm{Cos}\alpha\mathrm{Cos}\varphi\right]$$

$$= PR^3\left[\alpha\mathrm{Sin}\alpha\mathrm{Cos}\varphi + \mathrm{Cos}\varphi\mathrm{Cos}\alpha - \varphi\mathrm{Sin}\varphi\mathrm{Cos}\varphi - \mathrm{Cos}^2\varphi - \frac{\mathrm{Sin}^2\alpha}{2} + \frac{\mathrm{Sin}^2\varphi}{2}\right]$$

$$= PR^3\left[\mathrm{Cos}\varphi(\alpha\mathrm{Sin}\alpha + \mathrm{Cos}\alpha - \varphi\mathrm{Sin}\varphi - \mathrm{Cos}\varphi) + \frac{\mathrm{Sin}^2\varphi}{2} - \frac{\mathrm{Sin}^2\alpha}{2}\right]$$

EQ. 3

The final equation for the horizontal reaction due to a concentrated load,

$$H = \frac{N_P}{D} = \frac{\text{EQ. 3}}{\text{EQ. 1}}$$

$$= \frac{PR^3\left[\mathrm{Cos}\varphi(\alpha\mathrm{Sin}\alpha + \mathrm{Cos}\alpha - \varphi\mathrm{Sin}\varphi - \mathrm{Cos}\varphi) + \frac{\mathrm{Sin}^2\varphi}{2} - \frac{\mathrm{Sin}^2\alpha}{2}\right]}{R^3\left[\varphi - 3\mathrm{Sin}\varphi\mathrm{Cos}\varphi + 2\varphi\mathrm{Cos}^2\varphi\right]}$$

$$= P\left[\frac{\mathrm{Cos}\varphi(\alpha\mathrm{Sin}\alpha + \mathrm{Cos}\alpha - \varphi\mathrm{Sin}\varphi - \mathrm{Cos}\varphi) + \frac{\mathrm{Sin}^2\varphi}{2} - \frac{\mathrm{Sin}^2\alpha}{2}}{\varphi - 3\mathrm{Sin}\varphi\mathrm{Cos}\varphi + 2\varphi\mathrm{Cos}^2\varphi}\right]$$

4. Live Load

As before, β is the position angle of any point on the arch axis, $ds = Rd\beta$, and

$$L = 2R\mathrm{Sin}\varphi$$

$$x = R(\mathrm{Sin}\varphi - \mathrm{Sin}\beta)$$

$$y = R(\mathrm{Cos}\beta - \mathrm{Cos}\varphi)$$

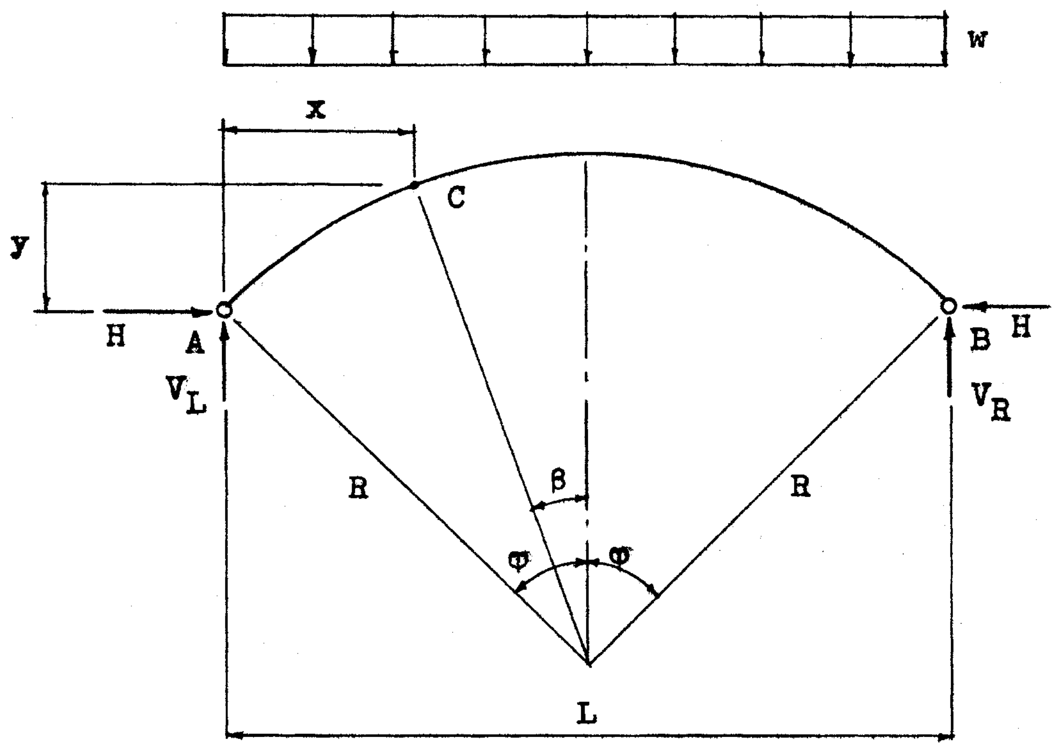


Figure 21

Geometry of the Structure for Full Live Load

a. Reactions

Inspection of this loading condition in Figure 21 indicates

$$V_L = V_R = \frac{wL}{2} \quad \frac{w(2RSin\varphi)}{2} = wRSin\varphi$$

from which the live load numerator

$$N = 2\int_0^{\varphi} M'yds \qquad (M' = V_Lx - \frac{wx^2}{2})$$

$$= 2\int_0^{\varphi} (V_Lx - \frac{wx^2}{2})yds$$

$$= 2\int_0^{\varphi} \left[\frac{wLx}{2} - \frac{wx^2}{2}\right]yds$$

$$= 2\int_0^{\varphi}\left[\frac{w}{2}(2R\sin\varphi)R(\sin\varphi - \sin\beta) - \frac{w}{2}R^2(\sin\varphi - \sin\beta)^2\right]$$

$$\left[R(\cos\beta - \cos\varphi)\right] Rd\beta$$

$$= wR^4\int_0^{\varphi}\left[2\sin\varphi(\sin\varphi - \sin\beta) - (\sin\varphi - \sin\beta)^2\right]$$

$$\left[\cos\beta - \cos\varphi\right] d\beta$$

$$= wR^4\int_0^{\varphi}\left[2\sin^2\varphi - 2\sin\varphi\sin\beta - \sin^2\varphi + 2\sin\varphi\sin\beta\right.$$

$$\left. - \sin^2\beta\right]\left[\cos\beta - \cos\varphi\right] d\beta$$

$$= wR^4\int_0^{\varphi}(\sin^2\varphi - \sin^2\beta)(\cos\beta - \cos\varphi)d\beta$$

$$= wR^4\int_0^{\varphi}(\sin^2\varphi\cos\beta - \sin^2\beta\cos\beta - \sin^2\varphi\cos\varphi$$

$$+ \sin^2\beta\cos\varphi)d\beta$$

$$= wR^4\int_0^{\varphi}\left[\sin^2\varphi\cos\beta - (1 - \cos^2\beta)\cos\beta - \sin^2\varphi\cos\varphi\right.$$

$$\left. + \sin^2\beta\cos\varphi\right] d\beta$$

$$= wR^4\int_0^{\varphi}\left[\sin^2\varphi\cos\beta - \cos\beta + \cos^3\beta - \sin^2\varphi\cos\varphi\right.$$

$$\left. + \sin^2\beta\cos\varphi\right] d\beta$$

$$= wR^4\left[\sin^2\varphi\sin\beta - \sin\beta + \sin\beta - \frac{\sin^3\beta}{3} - \beta\sin^2\varphi\cos\varphi\right.$$

$$\left. + \left(\frac{\beta}{2} - \frac{\sin\beta\cos\beta}{2}\right)\cos\varphi\right]_0^{\varphi}$$

$$= wR^4 \left[\sin^3\varphi - \sin\varphi + \sin\varphi - \frac{\sin^3\varphi}{2} - \varphi\sin^2\varphi\cos\varphi + \frac{\varphi}{2}\cos\varphi - \frac{\sin\varphi\cos^2\varphi}{2} \right]$$

$$= \frac{wR^4}{6} \left[6\sin^3\varphi - 2\sin^3\varphi - 6\varphi\sin^2\varphi\cos\varphi + 3\varphi\cos\varphi - 3\sin\varphi\cos^2\varphi \right]$$

$$= \frac{wR^4}{6} \left[4\sin^3\varphi + 3\varphi\cos\varphi(1 - 2\sin^2\varphi) - 3\sin\varphi\cos^2\varphi \right] \qquad \underline{\text{EQ. 4}}$$

The final equation for the horizontal reaction due to live load,

$$H = \frac{N_{LL}}{D} = \frac{\text{EQ. 4}}{\text{EQ. 1}}$$

$$= \frac{\frac{wR^4}{6} \left[4\sin^3\varphi + 3\varphi\cos\varphi(1 - 2\sin^2\varphi) - 3\sin\varphi\cos^2\varphi \right]}{R^3 \left[\varphi - 3\sin\varphi\cos\varphi + 2\varphi\cos^2\varphi \right]}$$

$$= \frac{wR}{6} \left[\frac{4\sin^3\varphi + 3\varphi\cos\varphi(1 - 2\sin^2\varphi) - 3\sin\varphi\cos^2\varphi}{\varphi - 3\sin\varphi\cos\varphi + 2\varphi\cos^2\varphi} \right]$$

<u>5. Drift Load</u> (refer to Figure 22)

This type of loading may be encountered as snow load on half of an arch span on the leeward side. The angle β is defined as before, and also

$$L = 2R\sin\varphi$$

$$x = R(\sin\varphi - \sin\beta)$$

$$y = R(\cos\beta - \cos\varphi)$$

$$ds = Rd\beta$$

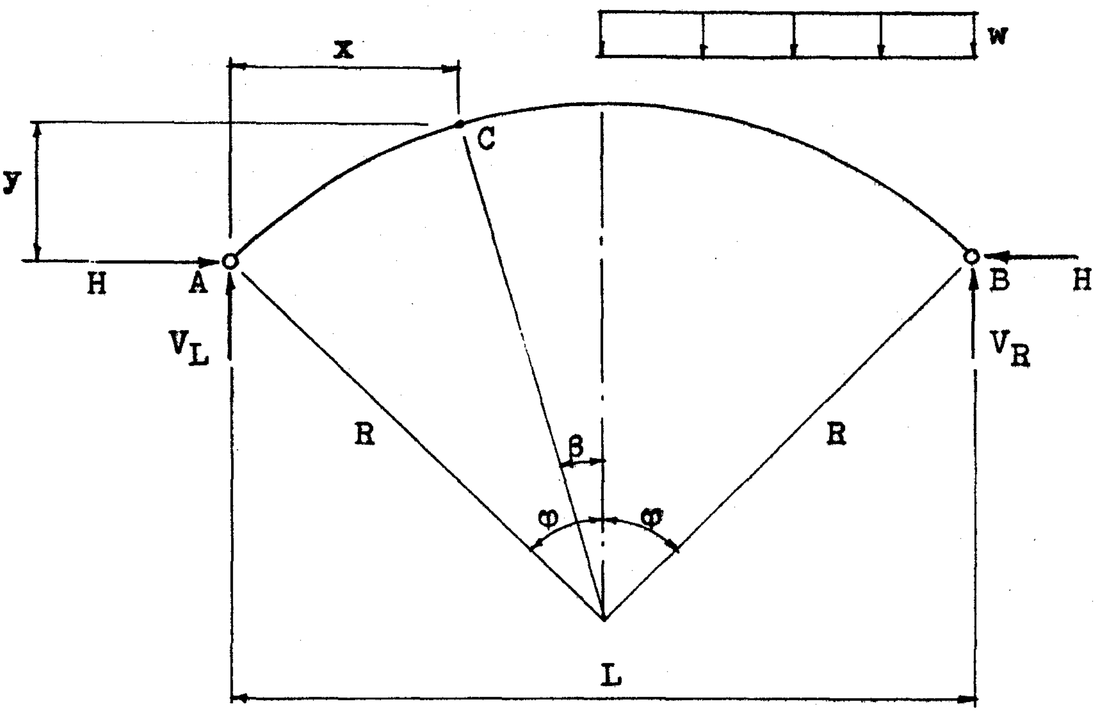


Figure 22

Geometry of the Structure for Drift Load

a. Reactions

Utilizing statics and summing moments about the hinges, the vertical reactions are:

$$V_L = \frac{1}{8}\,wL = \frac{1}{8}\,w(2R\sin\varphi) = \frac{wR\sin\varphi}{4}$$

$$V_R = \frac{3}{8}\,wL = \frac{3}{8}\,w(2R\sin\varphi) = \frac{3wR\sin\varphi}{4}$$

Rather than repeat essentially the integration process for the live load numerator, it can be shown that

$$N_{DRL} = \frac{1}{2}\,N_{LL} = \frac{1}{2}\,(EQ.\ 4)$$

$$= \frac{wR^4}{12} \left[4\sin^3\varphi + 3\varphi\cos\varphi(1 - 2\sin^2\varphi) - 3\sin\varphi\cos^2\varphi\right] \quad \underline{\text{EQ. 5}}$$

The final equation for the horizontal reactions due to drift load,

$$H = \frac{N_{DRL}}{D} = \frac{\text{EQ. 5}}{\text{EQ. 1}}$$

$$= \frac{\frac{wR^4}{12}\left[4\sin^3\varphi + 3\varphi\cos\varphi(1 - 2\sin^2\varphi) - 3\sin\varphi\cos^2\varphi\right]}{R^3\left[\varphi - 3\sin\varphi\cos\varphi + 2\varphi\cos^2\varphi\right]}$$

$$= \frac{wR}{12}\left[\frac{4\sin^3\varphi + 3\varphi\cos\varphi(1 - 2\sin^2\varphi) - 3\sin\varphi\cos^2\varphi}{\varphi - 3\sin\varphi\cos\varphi + 2\varphi\cos^2\varphi}\right]$$

This latter equation will be noted as equal to half the value of the horizontal reaction equation due to full live load.

APPENDIX B

TABULATED REACTION AND MOMENT COEFFICIENTS

TABLES I-A to XVI-A
for
Dead Load, Live Load and Drift Load

TABLES I-B to XVI-B
for
Concentrated Load

TABLE I-A

$\varphi = 30°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
V_L		0.52360	0.50000	0.12500
V_R		0.52360	0.50000	0.37500
H		0.94738	0.92355	0.46167
MOMENTS	1	-0.78627	-1.11999	- 5.78197
	2	-1.11355	-1.59496	-10.28089
	3	-1.10445	-1.59065	-13.48444
	4	-0.86883	-1.25919	-15.38384
	5	-0.50309	-0.73663	-15.97387
	6	-0.08976	-0.14155	-15.25294
	7	0.30309	0.42665	-13.22300
	8	0.62204	0.88986	- 9.88963
	9	0.82758	1.18931	- 5.26494
	10	0.90034	1.29468	0.64734
	11	0.82758	1.18931	6.45427
	12	0.62204	0.88986	10.77949
	13	0.30309	0.42665	13.64966
	14	-0.08976	-0.14155	15.11139
	15	-0.50309	-0.73663	15.23725
	16	-0.86883	-1.25919	14.12465
	17	-1.10445	-1.59065	11.89379
	18	-1.11355	-1.59496	8.68593
	19	-0.78627	-1.11999	4.66198

Multiply reaction coefficients by (wR)

Multiply moment coefficients by $(wR^2 \times 10^{-3})$

TABLE II-A

$\varphi = 31°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
V_L		0.54105	0.51504	0.12876
V_R		0.54105	0.51504	0.38628
H		0.94401	0.91831	0.45916
MOMENTS	1	-0.90193	-1.27268	- 6.14718
	2	-1.28055	-1.81397	-10.93043
	3	-1.27529	-1.81099	-14.33574
	4	-1.01111	-1.43577	-16.35315
	5	-0.59787	-0.84277	-16.97677
	6	-0.12953	-0.16661	-16.20474
	7	0.31614	0.47936	-14.03936
	8	0.67831	1.00600	-10.48693
	9	0.91173	1.34673	- 5.56128
	10	0.99440	1.46699	0.73339
	11	0.91173	1.34673	6.90801
	12	0.67831	1.00600	11.49294
	13	0.31614	0.47936	14.51872
	14	-0.12953	-0.16661	16.03813
	15	-0.59787	-0.84277	16.13400
	16	-1.01111	-1.43577	14.91739
	17	-1.27529	-1.81099	12.52475
	18	-1.28055	-1.81397	9.11646
	19	-0.90193	-1.27268	4.87450

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE III-A

$\varphi = 32°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
V_L		0.55851	0.52992	0.13248
V_R		0.55851	0.52992	0.39744
H		0.94032	0.91315	0.45657
MOMENTS	1	-1.02370	-1.44035	- 6.52116
	2	-1.45392	-2.05496	-11.59560
	3	-1.44849	-2.05398	-15.20748
	4	-1.14902	-1.63138	-17.34555
	5	-0.68013	-0.96143	-18.00315
	6	-0.14855	-0.19682	-17.17821
	7	0.35748	0.53440	-14.87330
	8	0.76874	1.13083	-11.09563
	9	1.03385	1.51705	- 5.86081
	10	1.12776	1.65254	0.82636
	11	1.03385	1.51705	7.37786
	12	0.76874	1.13083	12.22647
	13	0.35748	0.53440	15.40770
	14	-0.14855	-0.19682	16.98139
	15	-0.68013	-0.96143	17.04174
	16	-1.14902	-1.63138	15.71418
	17	-1.44849	-2.05398	13.15350
	18	-1.45392	-2.05496	9.54063
	19	-1.02370	-1.44035	5.08082

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE IV-A

$\varphi = 33°$

	REACTIONS	DEAD LOAD	LIVE LOAD	DRIFT LOAD
	V_L	0.57596	0.54464	0.13616
	V_R	0.57596	0.54464	0.40848
	H	0.93643	0.90776	0.45388
	1	-1.15498	-1.62121	- 6.90231
	2	-1.63976	-2.31419	-12.27340
	3	-1.63229	-2.31404	-16.09545
	4	-1.29250	-1.83836	-18.35580
	5	-0.76114	-1.08332	-19.04694
	6	-0.15892	-0.22082	-18.16658
	7	0.41433	0.60458	-15.71764
	8	0.88018	1.27800	-11.70824
	9	1.18051	1.71407	- 6.15602
MOMENTS	10	1.28677	1.86703	0.93359
	11	1.18051	1.71407	7.87009
	12	0.88018	1.27800	12.98624
	13	0.41433	0.60458	16.32221
	14	-0.15892	-0.22082	17.94577
	15	-0.76114	-1.08332	17.96362
	16	-1.29250	-1.83836	16.51743
	17	-1.63229	-2.31404	13.78142
	18	-1.63976	-2.31419	9.95921
	19	-1.15498	-1.62121	5.28111

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE V-A

$\varphi = 34°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
V_L		0.59341	0.55919	0.13980
V_R		0.59341	0.55919	0.41939
H		0.93245	0.90224	0.45112
MOMENTS	1	-1.29951	-1.81832	- 7.29146
	2	-1.84886	-2.59719	-12.96546
	3	-1.83603	-2.59856	-17.00201
	4	-1.45277	-2.06551	-19.38692
	5	-0.85357	-1.21800	-20.11178
	6	-0.17440	-0.24906	-19.17404
	7	0.47207	0.67887	-16.57701
	8	0.99572	1.43581	-12.32983
	9	1.33628	1.92639	- 6.45232
	10	1.45615	2.09861	1.04941
	11	1.33628	1.92639	8.37872
	12	0.99572	1.43581	13.76565
	13	0.47207	0.67887	17.25589
	14	-0.17440	-0.24906	18.92500
	15	-0.85357	-1.21800	18.89377
	16	-1.45277	-2.06551	17.32141
	17	-1.83603	-2.59856	14.40345
	18	-1.84489	-2.59719	10.36826
	19	-1.29951	-1.81832	5.47314

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE VI-A

$\varphi = 35°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
V_L		0.61087	0.57358	0.14340
V_R		0.61087	0.57358	0.43018
H		0.92839	0.89654	0.44827
MOMENTS	1	-1.45875	-2.03088	- 7.68747
	2	-2.07173	-2.90214	-13.66964
	3	-2.06268	-2.90449	-17.92419
	4	-1.63309	-2.30853	-20.43524
	5	-0.96077	-1.36004	-21.19345
	6	-0.19840	-0.27468	-20.19596
	7	0.52755	0.76519	-17.44650
	8	1.11769	1.61390	-12.95534
	9	1.49821	2.16397	- 6.74466
	10	1.63300	2.35757	1.17868
	11	1.49821	2.16397	8.90863
	12	1.11769	1.61390	14.56924
	13	0.52755	0.76519	18.21169
	14	-0.19840	-0.27468	19.92126
	15	-0.96077	-1.36004	19.83340
	16	-1.63309	-2.30853	18.12671
	17	-2.06268	-2.90449	15.01970
	18	-2.07173	-2.90214	10.76750
	19	-1.45875	-2.03088	5.65659

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE VII-A

$\varphi = 36°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
V_L		0.62832	0.58779	0.14695
V_R		0.62832	0.58779	0.44084
H		0.92418	0.89079	0.44539
MOMENTS	1	-1.63115	-2.26390	- 8.09214
	2	-2.31693	-3.23824	-14.38940
	3	-2.30698	-3.24450	-18.86694
	4	-1.82636	-2.58289	-21.50709
	5	-1.07385	-1.52674	-22.29943
	6	-0.22029	-0.31690	-21.24082
	7	0.59261	0.84331	-18.33546
	8	1.25354	1.79055	-13.59478
	9	1.67972	2.40447	- 7.04362
	10	1.83063	2.62053	1.31040
	11	1.67972	2.40447	9.44811
	12	1.25354	1.79055	15.38534
	13	0.59261	0.84331	19.17877
	14	-0.22029	-0.31690	20.92393
	15	-1.07385	-1.52674	20.77269
	16	-1.82636	-2.58289	18.92420
	17	-2.30698	-3.24450	15.62245
	18	-2.31693	-3.23824	11.15116
	19	-1.63115	-2.26390	5.82823

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE VIII-A

$\varphi = 37°$

	REACTIONS	DEAD LOAD	LIVE LOAD	DRIFT LOAD
	V_L	0.64577	0.60182	0.15046
	V_R	0.64577	0.60182	0.45136
	H	0.91985	0.88481	0.44241
MOMENTS	1	-1.81834	-2.51240	- 8.50223
	2	-2.58331	-3.59575	-15.11865
	3	-2.57250	-3.60431	-19.82167
	4	-2.03652	-2.87006	-22.59168
	5	-1.19691	-1.69605	-23.41715
	6	-0.24429	-0.34986	-22.29462
	7	0.66316	0.94155	-19.22878
	8	1.40106	1.99658	-14.23241
	9	1.87694	2.68087	- 7.33308
	10	2.04543	2.92130	1.46065
	11	1.87694	2.68087	10.01395
	12	1.40106	1.99658	16.22898
	13	0.66316	0.94155	20.17032
	14	-0.24429	-0.34986	21.94476
	15	-1.19691	-1.69605	21.72109
	16	-2.03652	-2.87006	19.72162
	17	-2.57250	-3.60431	16.21736
	18	-2.58331	-3.59575	11.52290
	19	-1.81834	-2.51240	5.98983

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE IX-A

$\varphi = 38°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
V_L		0.66323	0.61566	0.15392
V_R		0.66323	0.61566	0.46174
H		0.91534	0.87871	0.43936
MOMENTS	1	-2.01891	-2.78046	- 8.91907
	2	-2.86780	-3.98217	-15.85992
	3	-2.85440	-3.99417	-20.79203
	4	-2.25691	-3.18242	-23.69371
	5	-1.32158	-1.88189	-24.55220
	6	-0.26045	-0.38906	-23.36374
	7	0.75044	1.04357	-20.13353
	8	1.57250	2.21525	-14.87579
	9	2.10268	2.97480	- 7.62112
	10	2.29030	3.24191	1.62097
	11	2.10268	2.97480	10.59592
	12	1.57250	2.21525	17.09103
	13	0.75044	1.04357	21.17710
	14	-0.26045	-0.38906	22.97468
	15	-1.32158	-1.88189	22.67032
	16	-2.25691	-3.18242	20.51128
	17	-2.85440	-3.99417	16.79787
	18	-2.86780	-3.98217	11.87775
	19	-2.01891	-2.78046	6.13861

Multiply reaction coefficients by (wR)

Multiply moment coefficients by $(wR^2 \times 10^{-3})$

TABLE X-A

$\varphi = 39°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
V_L		0.68068	0.62932	0.15733
V_R		0.68068	0.62932	0.47199
H		0.91082	0.87251	0.43625
MOMENTS	1	-2.24043	-3.06943	- 9.34259
	2	-3.18443	-4.39979	-16.61314
	3	-3.17215	-4.41690	-21.77798
	4	-2.51157	-3.52298	-24.81318
	5	-1.47556	-2.08071	-25.70470
	6	-0.29934	-0.43751	-24.44839
	7	0.82166	1.14749	-21.05007
	8	1.73353	2.44416	-15.52549
	9	2.32171	3.28483	- 7.90851
	10	2.53007	3.58057	1.79083
	11	2.32171	3.28483	11.19334
	12	1.73353	2.44416	17.96964
	13	0.82166	1.14749	22.19755
	14	-0.29934	-0.43751	24.01089
	15	-1.47556	-2.08071	23.61759
	16	-2.51157	-3.52298	21.29020
	17	-3.17215	-4.41690	17.36109
	18	-3.18443	-4.39979	12.21335
	19	-2.24043	-3.06943	6.27315

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE XI-A

$\varphi = 40°$

	REACTIONS	DEAD LOAD	LIVE LOAD	DRIFT LOAD
	V_L	0.69813	0.64279	0.16070
	V_R	0.69813	0.64279	0.48209
	H	0.90610	0.86615	0.43307
MOMENTS	1	-2.47606	-3.37834	- 9.77156
	2	-3.51972	-4.84646	-17.37601
	3	-3.50608	-4.86913	-22.77630
	4	-2.77509	-3.88696	-25.94611
	5	-1.62838	-2.30578	-26.87001
	6	-0.32617	-0.48688	-25.54349
	7	0.91512	1.26120	-21.97302
	8	1.92497	2.69295	-16.17600
	9	2.57644	3.62124	- 8.18977
	10	2.80707	3.94759	1.97403
	11	2.57644	3.62124	11.81099
	12	1.92497	2.69295	18.86894
	13	0.91512	1.26120	23.23503
	14	-0.32617	-0.48688	25.05662
	15	-1.62838	-2.30578	24.56423
	16	-2.77509	-3.88696	22.05915
	17	-3.50608	-4.86913	17.90716
	18	-3.51972	-4.84646	12.52955
	19	-2.47606	-3.37834	6.93222

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE XII-A

$\varphi = 41°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
	V_L	0.71559	0.65606	0.16402
	V_R	0.71559	0.65606	0.49204
	H	0.90126	0.85962	0.42981
MOMENTS	1	-2.72986	-3.70706	-10.20519
	2	-3.88114	-5.32171	-18.14706
	3	-3.86630	-5.34974	-23.78495
	4	-3.05971	-4.27251	-27.09001
	5	-1.79386	-2.53492	-28.04532
	6	-0.35591	-0.53416	-26.64560
	7	1.01503	1.39083	-22.89919
	8	2.13053	2.96702	-16.82408
	9	2.85022	3.98934	- 8.46179
	10	3.10514	4.34969	2.17489
	11	2.85022	3.98934	12.45113
	12	2.13053	2.96702	19.79111
	13	1.01503	1.39083	24.29002
	14	-0.35591	-0.53416	26.11185
	15	-1.79386	-2.53492	25.51039
	16	-3.05971	-4.27251	22.81750
	17	-3.86630	-5.34974	18.43521
	18	-3.88114	-5.32171	12.82534
	19	-2.72986	-3.70706	6.49814

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE XIII-A

$\varphi = 42°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
V_L		0.73304	0.66913	0.16728
V_R		0.73304	0.66913	0.50185
H		0.89633	0.85300	0.42650
MOMENTS	1	-3.00443	-4.05952	-10.64472
	2	-4.27322	-5.83287	-18.92869
	3	-4.25872	-5.86872	-24.80742
	4	-3.37220	-4.69175	-28.24934
	5	-1.97932	-2.78828	-29.23597
	6	-0.39630	-0.59384	-27.76200
	7	1.11348	1.51942	-23.83534
	8	2.34220	3.25013	-17.47710
	9	3.13509	4.37344	- 8.73239
	10	3.41586	4.76927	2.38468
	11	3.13509	4.37344	13.10582
	12	2.34220	3.25013	20.72722
	13	1.11348	1.51942	25.35476
	14	-0.39630	-0.59384	27.16817
	15	-1.97932	-2.78828	26.44768
	16	-3.37220	-4.69175	23.55760
	17	-4.25872	-5.86872	18.93870
	18	-4.27322	-5.83287	13.09582
	19	-3.00443	-4.05952	6.58519

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE XIV-A

$\varphi = 43°$

	REACTIONS	DEAD LOAD	LIVE LOAD	DRIFT LOAD
	V_L	0.75049	0.68200	0.17050
	V_R	0.75049	0.68200	0.51150
	H	0.89126	0.84622	0.42311
	1	-3.29809	-4.43292	-11.08798
	2	-4.69229	-6.37423	-19.71683
	3	-4.67765	-6.41769	-25.83798
	4	-3.70484	-5.13346	-29.41696
	5	-2.17511	-3.05210	-30.43362
	6	-0.43580	-0.64998	-28.88225
	7	1.22352	1.66488	-24.77156
	8	2.57422	3.56189	-18.12471
MOMENTS	9	3.44590	4.79355	- 8.99114
	10	3.75466	5.22781	2.61372
	11	3.44590	4.79355	13.78468
	12	2.57422	3.56189	21.68661
	13	1.22352	1.66488	26.43645
	14	-0.43580	-0.64998	28.23228
	15	-2.17511	-3.05210	27.38152
	16	-3.70484	-5.13346	24.28351
	17	-4.67765	-6.41769	19.42029
	18	-4.69229	-6.37423	13.34260
	19	-3.29809	-4.43292	6.65506

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE XV-A

$\varphi = 44°$

REACTIONS		DEAD LOAD	LIVE LOAD	DRIFT LOAD
V_L		0.76794	0.69466	0.17367
V_R		0.76794	0.69466	0.52099
H		0.88606	0.83933	0.41966
MOMENTS	1	-3.61268	-4.83082	-11.53605
	2	-5.14152	-6.95247	-20.51357
	3	-5.12699	-7.00580	-26.87964
	4	-4.06190	-5.60887	-30.59673
	5	-2.38558	-3.33907	-31.64294
	6	-0.47873	-0.71623	-30.01210
	7	1.34093	1.81344	-25.71382
	8	2.82247	3.88713	-18.77343
	9	3.77874	5.23419	- 9.24495
	10	4.11749	5.70968	2.85468
	11	3.77874	5.23419	14.47916
	12	2.82247	3.88713	22.66057
	13	1.34093	1.81344	27.52726
	14	-0.47873	-0.71623	29.29587
	15	-2.38558	-3.33907	28.30388
	16	-4.06190	-5.60887	24.98787
	17	-5.12699	-7.00580	19.87382
	18	-5.14152	-6.95247	13.56110
	19	-3.61268	-4.83082	6.70523

Multiply reaction coefficients by (wR)

Multiply moment coefficients by $(wR^2 \times 10^{-3})$

TABLE XVI-A

$\varphi = 45°$

	REACTIONS	DEAD LOAD	LIVE LOAD	DRIFT LOAD
	V_L	0.78540	0.70711	0.17678
	V_R	0.78540	0.70711	0.53033
	H	0.88074	0.83232	0.41616
MOMENTS	1	-3.94930	-5.25359	-11.98838
	2	-5.62258	-7.56809	-21.31790
	3	-5.60863	-7.63332	-27.93104
	4	-4.44517	-6.11788	-31.78704
	5	-2.61218	-3.64848	-32.86211
	6	-0.52614	-0.79100	-31.14963
	7	1.46517	1.96673	-26.66017
	8	3.08683	4.22950	-19.42137
	9	4.13367	5.69904	- 9.49216
	10	4.50436	6.21787	3.10893
	11	4.13367	5.69904	15.19121
	12	3.08683	4.22950	23.65087
	13	1.46517	1.96673	28.62688
	14	-0.52614	-0.79100	30.35865
	15	-2.61218	-3.64848	29.21365
	16	-4.44517	-6.11788	25.66917
	17	-5.60863	-7.63332	20.29773
	18	-5.62258	-7.56809	13.74981
	19	-3.94930	-5.25359	6.73480

Multiply reaction coefficients by (wR)

Multiply moment coefficients by ($wR^2 \times 10^{-3}$)

TABLE I-B

$\varphi = 30^{\circ}$

	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
	0.95399	0.90674	0.85837	0.80902	0.75882	0.70791	0.65643	0.60453	0.55236	0.50000
	0.04601	0.09326	0.14163	0.19098	0.24118	0.29209	0.34357	0.39547	0.44764	0.50000
	0.21401	0.42767	0.63430	0.82768	1.00197	1.15204	1.27353	1.36289	1.41753	1.43595
1	38.5465	31.0348	23.6475	16.5462	9.8827	3.7914	- 1.6120	- 6.2325	- 9.9978	-12.8668
2	31.5489	64.2424	49.9121	36.1205	23.1565	11.2772	0.7031	- 8.3842	-15.8464	-21.6046
3	25.0359	51.1630	78.7220	58.6691	39.7853	22.4368	6.9389	- 6.4494	-17.5297	-26.1894
4	19.0253	39.0863	60.6472	84.1303	59.7233	37.2398	17.0782	- 0.4333	-15.0432	-26.6087
5	13.5336	28.0455	44.1059	62.2364	82.9159	55.6454	31.0934	9.6476	- 8.3936	-22.8612
6	8.5758	18.0709	29.1435	42.3983	58.3922	77.6033	48.9460	23.7656	2.4008	-14.9574
7	4.1656	9.1896	15.8010	24.6704	36.4174	51.5761	70.5870	41.8821	17.3104	- 2.9187
8	0.3150	1.4262	4.1149	9.1013	17.0517	28.5425	44.0511	63.9473	36.2944	13.2216
9	- 2.9643	- 5.1959	- 5.8794	- 4.2618	0.3539	8.5725	20.8966	37.7179	59.2875	33.4076
0	- 5.6666	-10.6653	-14.1645	-15.3960	-13.6474	- 8.3001	1.1623	15.1433	33.9065	57.6196
1	- 7.7824	-14.9625	-20.7111	-24.2615	-24.9025	-22.0150	-15.0815	- 3.6958	12.4107	33.4076
2	- 9.3036	-18.0712	-25.4943	-30.8251	-33.3689	-32.5206	-27.7736	-18.7287	- 5.1193	13.2216
3	-10.2294	-19.9896	-28.5113	-35.0822	-39.0406	-39.8092	-36.9040	-29.9427	-18.6677	- 2.9187
4	-10.5560	-20.7102	-29.7504	-37.0169	-41.8965	-43.8538	-42.4394	-37.2975	-28.1867	-14.9574
5	-10.2827	-20.2311	-29.2082	-36.6237	-41.9285	-44.6433	-44.3646	-40.7730	-33.6501	-22.8612
6	- 9.4102	-18.5536	-26.8862	-33.9037	-39.1368	-42.1755	-42.6744	-40.3597	-35.0429	-26.6087
7	- 7.9408	-15.6822	-22.7908	-28.8643	-33.5289	-36.4571	-37.3734	-36.0586	-32.3614	-26.1894
8	- 5.8786	-11.6249	-16.9331	-21.5195	-25.1201	-27.5039	-28.4761	-27.8816	-25.6129	-21.6046
9	- 3.2293	- 6.3927	- 9.3292	-11.8892	-13.9336	-15.3405	-16.0069	-15.8511	-14.8159	-12.8668

Multiply reaction coefficients by (P)

Multiply moment coefficients by $(PR \times 10^{-3})$

TABLE II-B

$\varphi = 31°$

α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L	0.95427	0.90720	0.85895	0.80965	0.75944	0.70847	0.65689	0.60485	0.55253	0.50000
V_R	0.04573	0.09280	0.14105	0.19035	0.24056	0.29153	0.34311	0.39515	0.44747	0.50000
H	0.20553	0.41116	0.61038	0.79706	0.96554	1.11077	1.22843	1.31502	1.36799	1.38585
1	39.4871	31.8007	24.2286	16.9405	10.0939	3.8299	- 1.7293	- 6.4840	-10.3578	-13.3071
2	32.3349	65.9072	51.2123	37.0514	23.7251	11.5030	0.6182	- 8.7384	-16.4205	-22.3449
3	25.6726	52.5144	80.8720	60.2738	40.8539	22.9968	7.0358	- 6.7567	-18.1705	-27.0872
4	19.5197	40.1393	62.3336	86.5397	61.4300	38.2778	17.5046	- 0.5446	-15.6028	-27.5199
5	13.8943	28.8180	45.3565	64.0514	85.3932	57.3012	31.9939	9.8798	- 8.7248	-23.6417
6	8.8127	18.5837	29.9906	43.6622	60.1704	80.0113	50.4615	24.4858	2.4434	-15.4642
7	4.2898	9.4664	16.2807	25.4315	37.5563	53.2101	72.8532	43.2308	17.8691	- 3.0111
8	0.3389	1.4926	4.2669	9.4130	17.6171	29.4792	45.4991	66.0599	37.5071	13.6810
9	- 3.0272	- 5.3117	- 6.0117	- 4.3417	0.4175	8.8957	21.6197	38.9961	61.2854	34.5502
10	- 5.8023	-10.9341	-14.5364	-15.8072	-14.0112	- 8.5033	1.2578	15.6919	35.0729	59.5749
11	- 7.9760	-15.3531	-21.2747	-24.9399	-25.6141	-22.6514	-15.5089	- 3.7637	12.8640	34.5502
12	- 9.5393	-18.5510	-26.1995	-31.7032	-34.3445	-33.4919	-28.6132	-19.2929	- 5.2527	13.6810
13	-10.4914	-20.5257	-29.3075	-36.0922	-40.1960	-41.0161	-38.0441	-30.8815	-19.2595	- 3.0111
14	-10.8282	-21.2690	-30.5859	-38.0892	-43.1450	-45.1943	043.7647	-38.4853	-29.1036	-15.4642
15	-10.5489	-20.7788	-30.0310	-37.6882	-43.1829	-46.0142	-45.7583	-42.0818	-34.7564	-23.6417
16	- 9.6541	-19.0564	-27.6443	-34.8904	-40.3096	-43.4735	-44.0192	-41.6607	-36.2011	-27.5199
17	- 8.1465	-16.1069	-23.4329	-29.7041	-34.5336	-37.5796	-38.5524	-37.2231	-33.4335	-27.0872
18	- 6.0305	-11.9390	-17.4091	-22.1443	-25.8717	-28.3498	-29.3739	-28.7821	-26.4619	-22.3449
19	- 3.3124	- 6.5648	- 9.5905	-12.2333	-14.3492	-15.8110	-16.5105	-16.3622	-15.3065	-13.3071

Multiply reaction coefficients by (P)

Multiply moment coefficients by ($PR \times 10^{-3}$)

TABLE III-B

$\rho = 32^{\circ}$

	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L	0.95455	0.90769	0.85956	0.81030	0.76007	0.70904	0.65735	0.60518	0.55270	0.50000
V_R	0.04545	0.09231	0.14044	0.18970	0.23993	0.29096	0.34265	0.39482	0.44730	0.50000
	0.19762	0.39567	0.58793	0.76836	0.93142	1.07212	1.18619	1.27020	1.32161	1.33895
1	40.3923	32.5383	24.7870	17.3159	10.2888	3.8548	- 1.8582	- 6.7454	-10.7259	-13.7540
2	33.0907	67.5215	52.4720	37.9488	24.2648	11.7046	0.5127	- 9.1105	-17.0098	-23.0978
3	26.2837	53.8251	82.9686	61.8345	41.8844	23.5250	7.1054	- .0879	-18.8323	-28.0024
4	19.9924	41.1599	63.9781	88.8983	63.0927	39.2791	17.8992	- 0.6840	-16.1874	-28.4523
5	14.2365	29.5652	46.5750	65.8268	87.8236	58.9177	32.8604	10.0813	- 9.0836	-24.4463
6	9.0339	19.0773	30.8136	44.8954	61.9110	82.3797	51.9425	25.1745	2.4571	-15.9967
7	4.4009	9.7288	16.7430	26.1695	38.6651	54.8120	75.0860	44.5484	18.3986	- 3.1300
8	0.3519	1.5490	4.4071	9.7075	18.1584	30.3896	46.9188	68.1427	38.6913	14.1138
9	- 3.0992	- 5.4341	- 6.1515	- 4.4338	0.4618	9.1970	22.3184	40.2477	63.2557	35.6663
10	- 5.9455	-11.2066	-14.9121	-16.2268	-14.3905	- 8.7249	1.3319	16.2160	36.2129	61.5039
11	- 8.1756	-15.7452	-21.8393	-25.6236	-26.3381	-23.3035	-15.9556	- 3.8546	13.2915	35.6663
12	- 9.7798	-19.0304	-26.9034	-32.5841	-35.3300	-34.4764	-29.4702	-19.8787	- 5.4111	14.1138
13	-10.7571	-21.0560	-30.1007	-37.1030	-41.3589	-42.2341	-39.1997	-31.8406	-19.8754	- 3.1300
14	-11.1031	-21.8249	-31.4171	-39.1607	-44.3991	-46.5439	-45.1036	-39.6916	-30.0435	-15.9967
15	-10.8168	-21.3229	-30.8487	-38.7507	-44.4411	-47.3924	-47.1636	-43.4071	-35.8835	-24.4463
16	- 9.8991	-19.5554	-28.3972	-35.8745	-41.4848	-44.7770	-45.3733	-42.9756	-37.3772	-28.4523
17	- 8.3527	-16.5280	-24.0701	-30.5408	-35.5393	-38.7057	-39.7383	-38.3984	-34.5201	-28.0024
18	- 6.1826	-12.2501	-17.8811	-22.7664	-26.6233	-29.1976	-30.2761	-29.6899	-27.3210	-23.0978
19	- 3.3954	- 6.7350	- 9.8494	-12.5756	-14.7645	-16.2823	-17.0162	-16.8771	-15.8023	-13.7540

Multiply reaction coefficients by (P)

Multiply moment coefficients by ($PR \times 10^{-3}$)

TABLE IV-B

$\varphi = 33^\circ$

α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L	0.95485	0.90819	0.86018	0.81097	0.76074	0.70964	0.65784	0.60552	0.55288	0.50000
V_R	0.04515	0.09181	0.13982	0.18903	0.23926	0.29036	0.34216	0.39448	0.44712	0.50000
H	0.19009	0.38099	0.56667	0.74119	0.89913	1.03556	1.14626	1.22785	1.27780	1.29464
1	41.2645	33.2503	25.3260	17.6772	10.4745	3.8738	- 1.9905	- 7.0080	-11.0934	-14.1985
2	33.8220	69.0905	53.6979	38.8220	24.7892	11.8967	0.4020	- 9.4841	-17.5974	-23.8460
3	26.8777	55.1035	85.0214	63.3645	42.8964	24.0422	7.1695	- 7.4199	-19.4904	-28.9105
4	20.4545	42.1592	65.5936	91.2231	64.7363	40.2700	18.2896	- 0.8223	-16.7662	-29.3751
5	14.5737	30.3007	47.7768	67.5829	90.2363	60.5262	33.7254	10.2868	- 9.4338	-25.2383
6	9.2548	19.5672	31.6303	46.1215	63.6475	84.7437	53.4256	25.8705	2.4825	-16.5139
7	4.5155	9.9943	17.2075	26.9101	39.7805	56.4203	77.3251	45.8772	18.9432	- 3.2308
8	0.3716	1.6137	4.5563	10.0124	18.7145	31.3146	48.3525	70.2405	39.8936	14.5670
9	- 3.1620	- 5.5439	- 6.2769	- 4.5096	0.5270	9.5190	23.0366	41.5191	65.2468	36.8045
10	- 6.0776	-11.4634	-15.2696	-16.6257	-14.7445	- 8.9221	1.4287	16.7625	37.3753	63.4556
11	- 8.3628	-16.1193	-22.3829	-26.2838	-27.0341	-23.9287	-16.3776	- 3.9221	13.7417	36.8045
12	-10.0072	-19.4905	-27.5845	-33.4397	-36.2859	-35.4329	-30.3018	-20.4410	- 5.5476	14.5670
13	-11.0095	-21.5745	-30.8705	-38.0878	-42.4920	-43.4241	-40.3302	-32.7771	-20.4711	- 3.2308
14	-11.3651	-22.3614	-32.2254	-40.2067	-45.6243	-47.8665	-46.4187	-40.8770	-30.9652	-16.5139
15	-11.0726	-21.8487	-31.6447	-39.7893	-45.6723	-48.7456	-48.5471	-44.7136	-36.9949	-25.2383
16	-10.1332	-20.0381	-29.1305	-36.8370	-42.6359	-46.0582	-46.7083	-44.2743	-38.5404	-29.3751
17	- 8.5500	-16.9354	-24.6911	-31.3596	-36.5251	-39.8134	-40.9084	-39.5607	-35.5964	-28.9105
18	- 6.3281	-12.5512	-18.3411	-23.3753	-27.3603	-30.0319	-31.1667	-30.5882	-28.1729	-23.8460
19	- 3.4749	- 6.8998	-10.1016	-12.9105	-15.1718	-16.7461	-17.5155	-17.3868	-16.2942	-14.1985

Multiply reaction coefficients by (P)

Multiply moment coefficients by ($PR \times 10^{-3}$)

TABLE V-B

$\varphi = 34°$

α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L	0.95516	0.90871	0.86083	0.81167	0.76142	0.71025	0.65834	0.60587	0.55306	0.50000
V_R	0.04484	0.09129	0.13917	0.18833	0.23858	0.28975	0.34166	0.39413	0.44694	0.50000
H	0.18295	0.36712	0.54656	0.71552	0.86864	1.00106	1.10859	1.18790	1.23647	1.25286
1	42.1023	33.9341	25.8434	18.0215	10.6466	3.8822	- 2.1307	- 7.2764	-11.4648	-14.6454
2	34.5271	70.6090	54.8855	39.6655	25.2897	12.0705	0.2775	- 9.8676	-18.1920	-24.5985
3	27.4526	56.3430	87.0241	64.8561	43.8779	24.5360	7.2162	- 7.7645	-20.1577	-29.8242
4	20.9037	43.1297	67.1729	93.5043	66.3456	41.2350	18.6609	- 0.9745	-17.3551	-30.3043
5	14.9033	31.0157	48.9543	69.3092	92.6138	62.1085	34.5712	10.4785	- 9.7940	-26.0369
6	9.4727	20.0437	32.4324	47.3290	65.3604	87.0831	54.8913	26.5542	2.4989	-17.0371
7	4.6310	10.2523	17.6655	27.6412	40.8817	58.0135	79.5496	47.1960	19.4804	- 3.3366
8	0.3952	1.6759	4.7055	10.3150	19.2639	32.2317	49.7784	72.3312	41.0906	15.0164
9	- 3.2183	- 5.6521	- 6.3971	- 4.5819	0.5914	9.8384	23.7519	42.7874	67.2343	37.9398
10	- 6.2014	-11.7154	-15.6178	-17.0169	-15.0951	- 9.1177	1.5260	17.3084	38.5359	65.4051
11	- 8.5403	-16.4864	-22.9143	-26.9329	-27.7235	-24.5497	-16.7972	- 3.9888	14.1904	37.9398
12	-10.2240	-19.9418	-28.2514	-34.2821	-37.2331	-36.3831	-31.1295	-21.0018	- 5.6856	15.0164
13	-11.2509	-22.0791	-31.6249	-39.0581	-43.6150	-44.6066	-41.4562	-33.7119	-21.0687	- 3.3366
14	-11.6161	-22.8875	-33.0178	-41.2377	-46.8386	-49.1812	-47.7288	-42.0606	-31.8892	-17.0371
15	-11.3182	-22.3642	-32.4255	-40.8131	-46.8926	-50.0906	-49.9255	-46.0184	-38.1089	-26.0369
16	-10.3582	-20.5111	-29.8498	-37.7859	-43.7767	-47.3317	-48.0384	-45.5715	-39.7060	-30.3043
17	- 8.7395	-17.3346	-25.3000	-32.1667	-37.5019	-40.9142	-42.0742	-40.7214	-36.6749	-29.8242
18	- 6.4679	-12.8460	-18.7920	-23.9752	-28.0902	-30.8607	-32.0538	-31.4853	-29.0262	-24.5985
19	- 3.5513	- 7.0610	-10.3488	-13.2404	-15.5749	-17.2066	-18.0126	-17.8955	-16.7869	-14.6454

Multiply reaction coefficients by (P)

Multiply moment coefficients by $(PR \times 10^{-3})$

TABLE VI-B

$\varphi = 35^\circ$

α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L	0.95547	0.90925	0.86150	0.81240	0.76213	0.71089	0.65886	0.60624	0.55325	0.50000
V_R	0.04453	0.09075	0.13850	0.18760	0.23787	0.28911	0.34114	0.39376	0.44675	0.50000
H	0.17613	0.35389	0.52743	0.69113	0.83968	0.96830	1.07285	1.15001	1.19728	1.21324
MOMENTS 1	42.9053	34.5914	26.3408	18.3508	10.8087	3.8842	- 2.2747	- 7.5463	-11.8359	-15.0900
2	35.2062	72.0807	56.0384	40.4833	25.7737	12.2338	0.1472	-10.2532	-18.7853	-25.3466
3	28.0093	57.5497	88.9820	66.3149	44.8392	25.0178	7.2568	- 8.1105	-20.8222	-30.7314
4	21.3413	44.0795	68.7233	95.7495	67.9339	42.1884	19.0273	- 1.1262	-17.9390	-31.2243
5	15.2272	31.7206	50.1164	71.0149	94.9718	63.6816	35.4150	10.6735	-10.1465	-26.8236
6	9.6898	20.5188	33.2305	48.5287	67.0681	89.4172	56.3588	27.2447	2.5262	-17.5456
7	4.7497	10.5161	18.1288	28.3749	41.9891	59.6128	81.7804	48.5256	20.0319	- 3.4250
8	0.4254	1.7498	4.8674	10.6286	19.8282	33.1636	51.2189	74.4368	42.3053	15.4857
9	- 3.2654	- 5.7441	- 6.4987	- 4.6369	0.6773	10.1791	24.4876	44.0758	69.2422	39.0967
10	- 6.3138	-11.9478	-15.9431	-17.3860	-15.4194	- 9.2877	1.6474	17.8772	39.7187	67.3771
11	- 8.7052	-16.8313	-23.4200	-27.5569	-28.3838	-25.1425	-17.1907	- 4.0315	14.6615	39.0967
12	-10.4273	-20.3697	-28.8909	-35.0974	-38.1493	-37.3040	-31.9304	-21.5386	- 5.8020	15.4857
13	-11.4787	-22.5599	-32.3513	-40.0007	-44.7068	-45.7598	-42.5557	-34.6236	-21.6463	- 3.4250
14	-11.8537	-23.3903	-33.7829	-42.2415	-48.0227	-50.4674	-49.0138	-43.2228	-32.7954	-17.5456
15	-11.5511	-22.8579	-33.1804	-41.8113	-48.0846	-51.4093	-51.2809	-47.3041	-39.2076	-26.8236
16	-10.5719	-20.9647	-30.5462	-38.7118	-44.8923	-48.5818	-49.3483	-46.8522	-40.8589	-31.2243
17	- 8.9198	-17.7177	-25.8899	-32.9545	-38.4577	-41.9957	-43.2233	-41.8688	-37.7434	-30.7314
18	- 6.6009	-13.1290	-19.2290	-24.5610	-28.8048	-31.6753	-32.9287	-32.3726	-29.8726	-25.3466
19	- 3.6240	-7.2157	-10.5883	-13.5625	-15.9696	-17.6594	-18.5030	-18.3990	-17.2758	-15.0900

Multiply reaction coefficients by (P)

Multiply moment coefficients by $(PR \times 10^{-3})$

TABLE VII-B

$\varphi = 36^\circ$

	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
L	0.95580	0.90980	0.86219	0.81315	0.76287	0.71155	0.65940	0.60661	0.55345	0.50000
R	0.04420	0.09020	0.13781	0.18685	0.23713	0.28845	0.34060	0.39339	0.44655	0.50000
	0.16973	0.34140	0.50939	0.66810	0.81236	0.93739	1.03914	1.11427	1.16033	1.17588
1	43.6688	35.2169	26.8112	18.6586	10.9523	3.8708	- 2.4318	- 7.8272	-12.2161	-15.5422
2	35.8512	73.4954	57.1429	41.2629	26.2247	12.3698	- 0.0066	-10.6587	-19.3955	-26.1088
3	28.5366	58.7099	90.8756	67.7238	45.7570	25.4635	7.2660	- 8.4833	-21.5097	-31.6581
4	21.7538	44.9920	70.2213	97.9368	69.4720	43.1003	19.3574	- 1.3095	-18.5504	-32.1683
5	15.5295	32.3958	51.2360	72.6751	97.2762	65.2104	36.2198	10.8343	-10.5294	-27.6372
6	9.8885	20.9710	33.9944	49.6932	68.7327	91.7067	57.7866	27.9001	2.5218	-18.0829
7	4.8528	10.7626	18.5648	29.0817	43.0617	61.1761	83.9728	49.8207	20.5516	- 3.5430
8	0.4424	1.8111	5.0080	10.9220	20.3646	34.0660	52.6268	76.5096	43.4889	15.9252
9	- 3.3235	- 5.8446	- 6.6169	- 4.7065	0.7407	10.4955	25.1952	45.3348	71.2203	40.2240
10	- 6.4355	-12.1855	-16.2809	-17.7652	-15.7619	- 9.4782	1.7439	18.4189	40.8729	69.3196
11	- 8.8777	-17.1790	-23.9347	-28.1873	-29.0585	-25.7524	-17.6064	- 4.0994	15.1049	40.2240
12	-10.6367	-20.7980	-29.5365	-35.9161	-39.0771	-38.2392	-32.7511	-22.0989	- 5.9453	15.9252
13	-11.7112	-23.0394	-33.0814	-40.9444	-45.8074	-46.9249	-43.6729	-35.5573	-22.2499	- 3.5430
14	-12.0950	-23.8906	-34.5497	-43.2445	-49.2132	-51.7633	-50.3143	-44.4051	-33.7261	-18.0829
15	-11.7867	-23.3483	-33.9356	-42.8074	-49.2810	-52.7355	-52.6494	-48.6074	-40.3286	-27.6372
16	-10.7873	-21.4146	-31.2415	-39.6347	-46.0105	-49.8374	-50.6687	-48.1477	-42.0312	-32.1683
17	- 9.1010	-18.0971	-26.4781	-33.7391	-39.4146	-43.0806	-44.3802	-43.0278	-38.8275	-31.6581
18	- 6.7344	-13.4090	-19.6641	-25.1437	-29.5194	-32.4918	-33.8086	-33.2678	-30.7298	-26.1088
19	- 3.6967	- 7.3687	-10.8264	-13.8825	-16.3639	-18.1127	-18.9957	-18.9063	-17.7703	-15.5422

Multiply reaction coefficients by (P)

Multiply moment coefficients by ($PR \times 10^{-3}$)

TABLE VIII-B

$\varphi = 37^\circ$

	α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
	V_L	0.95614	0.91038	0.86290	0.81392	0.76362	0.71223	0.65995	0.60701	0.55365	0.50000
	V_R	0.04386	0.08962	0.13710	0.18608	0.23638	0.28777	0.34005	0.39299	0.44635	0.50000
	H	0.16357	0.32945	0.49213	0.64611	0.78629	0.90793	1.00702	1.08024	1.12515	1.14031
	1	44.3965	35.8146	27.2612	18.9512	11.0856	3.8505	- 2.5925	- 8.1094	-12.5953	-15.9914
	2	36.4703	74.8606	58.2121	42.0166	26.6585	12.4947	- 0.1658	-11.0657	-20.0031	-26.8653
	3	29.0467	59.8354	92.7237	69.0999	46.6540	25.8964	7.2700	- 8.8565	-22.1925	-32.5763
	4	22.1566	45.8824	71.6910	100.0882	70.9885	43.9999	19.6838	- 1.4911	-19.1544	-33.1006
	5	15.8287	33.0598	52.3419	74.3162	99.5607	66.7296	37.0240	10.9999	-10.9015	-28.4361
	6	10.0896	21.4211	34.7570	50.8524	70.3926	93.9907	59.2183	28.5645	2.5319	-18.6021
	7	4.9629	11.0148	19.0097	29.7946	44.1418	62.7460	86.1741	51.1293	21.0897	- 3.6397
	8	0.4702	1.8842	5.1654	11.2306	20.9179	34.9844	54.0526	78.6002	44.6946	16.3888
	9	- 3.3679	- 5.9285	- 6.7118	- 4.7540	0.8282	10.8347	25.9269	46.6173	73.2234	41.3774
MOMENTS	10	- 6.5412	-12.4028	-16.5908	-18.1174	-16.0753	- 9.6413	1.8685	18.9872	42.0542	71.2891
	11	- 9.0327	-17.5037	-24.4183	-28.7873	-29.7009	-26.3322	-17.9919	- 4.1396	15.5756	41.3774
	12	-10.8280	-21.2019	-30.1494	-36.7026	-39.9707	-39.1428	-33.5410	-22.6316	- 6.0623	16.3888
	13	-11.9254	-23.4941	-33.7785	-41.8552	-46.8737	-48.0583	-44.7597	-36.4643	-22.8291	- 3.6397
	14	-12.3186	-24.3666	-35.2843	-44.2154	-50.3704	-53.0288	-51.5860	-45.5627	-34.6349	-18.6021
	15	-12.0057	-23.8159	-34.6607	-43.7733	-50.4463	-54.0334	-53.9915	-49.8887	-41.4306	-28.4361
	16	-10.9882	-21.8442	-31.9101	-40.5308	-47.1011	-51.0680	-51.9660	-49.4243	-43.1878	-33.1006
	17	- 9.2703	-18.4597	-27.0441	-34.5013	-40.3486	-44.1449	-45.5181	-44.1714	-39.8991	-32.5763
	18	- 6.8591	-13.6766	-20.0830	-25.7100	-30.2172	-33.2930	-34.6747	-34.1519	-31.5783	-26.8653
	19	- 3.7647	- 7.5148	-11.0558	-14.1936	-16.7489	-18.5576	-19.4809	-19.4076	-18.2601	-15.9914

Multiply reaction coefficients by (P)

Multiply moment coefficients by ($PR \times 10^{-3}$)

TABLE IX-B

$\varphi = 38^\circ$

α		0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L		0.95649	0.91097	0.86364	0.81471	0.76441	0.71293	0.66052	0.60741	0.55386	0.50000
V_R		0.04351	0.08903	0.13636	0.18529	0.23559	0.28707	0.33948	0.39259	0.44614	0.50000
H		0.15770	0.31810	0.47572	0.62522	0.76151	0.87995	0.97652	1.04794	1.09176	1.10656
MOMENTS	1	45.0855	36.3800	27.6860	19.2237	11.2033	3.8180	- 2.7630	- 8.3989	-12.9798	-16.4401
	2	37.0589	76.1682	59.2366	42.7348	27.0653	12.5981	- 0.3421	-11.4855	-20.6203	-27.6281
	3	29.5335	60.9152	94.5130	70.4298	47.5162	26.3018	7.2520	- 9.2463	-22.8879	-33.5029
	4	22.5425	46.7375	73.1167	102.1870	72.4660	44.8688	19.9859	- 1.6910	-19.7727	-34.0427
	5	16.1166	33.6974	53.4162	75.9192	101.8051	68.2175	37.8036	11.1471	-11.2883	-29.2451
	6	10.2840	21.8525	35.4984	51.9856	72.0255	96.2452	60.6267	29.2115	2.5280	-19.1312
	7	5.0704	11.2546	19.4420	30.4913	45.2056	64.2968	88.3550	52.4228	21.6154	- 3.7455
	8	0.4987	1.9504	5.3175	11.5309	21.4632	35.8919	55.4656	80.6790	45.8900	16.8444
	9	- 3.4089	- 6.0148	- 6.8062	- 4.8033	0.9143	11.1691	26.6513	47.8923	75.2181	42.5236
	10	- 6.6414	-12.6190	-16.8958	-18.4665	-16.3852	- 9.8046	1.9895	19.5508	43.2288	73.2520
	11	- 9.1806	-17.8245	-24.8936	-29.3805	-30.3362	-26.9089	-18.3785	- 4.1828	16.0402	42.5236
	12	-11.0110	-21.5998	-30.7512	-37.4794	-40.8546	-40.0411	-34.3301	-23.1661	- 6.1851	16.8444
	13	-12.1308	-23.9413	-34.4628	-42.7546	-47.9285	-49.1850	-45.8447	-37.3729	-23.4143	- 3.7455
	14	-12.5331	-24.8343	-36.0054	-45.1739	-51.5153	-54.2866	-52.8551	-46.7215	-35.5500	-19.1312
	15	-12.2161	-24.2748	-35.3723	-44.7267	-51.5992	-55.3233	-55.3306	-51.1708	-42.5388	-29.2451
	16	-11.1812	-22.2655	-32.5661	-41.4148	-48.1799	-52.2907	-53.2601	-50.7013	-44.3498	-34.0427
	17	- 9.4328	-18.8150	-27.5992	-35.2530	-41.2723	-45.2021	-46.6529	-45.3151	-40.9753	-33.5029
	18	- 6.9788	-13.9386	-20.4937	-26.2681	-30.9070	-34.0886	-35.5379	-35.0357	-32.4300	-27.6281
	19	- 3.8299	- 7.6578	-11.2803	-14.4999	-17.1293	-18.9991	-19.9642	-19.9085	-18.7515	-16.4440

Multiply reaction coefficients by (P)

Multiply moment coefficients by ($PR \times 10^{-3}$)

TABLE X-B

$\phi = 39^{\circ}$

Moments

α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L	0.95685	0.91158	0.86440	0.81554	0.76521	0.71366	0.66112	0.60783	0.55408	0.50000
V_R	0.04315	0.08842	0.13560	0.18446	0.23479	0.28634	0.33888	0.39217	0.44592	0.50000
H	0.15211	0.30724	0.46005	0.60529	0.73791	0.85330	0.94749	1.01720	1.05999	1.07445
1	45.7343	36.9143	28.0862	19.4769	11.3057	3.7738	- 2.9421	- 8.6947	-13.3685	-16.8988
2	37.6149	77.4205	60.2176	43.4193	27.4453	12.6816	- 0.5333	-11.9163	-21.2449	-28.3947
3	29.9945	61.9535	96.2454	71.7162	48.3441	26.6820	7.2153	- 9.6499	-23.5928	-34.4346
4	22.9085	47.5630	74.5011	104.2366	73.9053	45.7101	20.2676	- 1.9060	-20.4012	-34.9903
5	16.3896	34.3159	54.4629	77.4888	104.0104	69.6800	38.5633	11.2795	-11.6849	-30.0594
6	10.4681	22.2735	36.2235	53.0983	73.6331	98.4745	62.0176	29.8457	2.5157	-19.6647
7	5.1714	11.4914	19.8674	31.1783	46.2551	65.8338	90.5220	53.7064	22.1347	- 3.8543
8	0.5240	2.0197	5.4704	11.8301	22.0031	36.7940	56.8726	82.7512	47.0815	17.2985
9	- 3.4504	- 6.0930	- 6.8937	- 4.8470	1.0018	11.5048	27.3757	49.1658	77.2112	43.6695
10	- 6.7400	-12.8234	-17.1890	-18.8046	-16.6885	- 9.9618	2.1145	20.1159	44.4039	75.2154
11	- 9.3251	-18.1307	-25.3536	-29.9590	-30.9612	-27.4764	-18.7585	- 4.2229	16.5057	43.6695
12	-11.1893	-21.9812	-31.3354	-38.2389	-41.7257	-40.9279	-35.1110	-23.6968	- 6.3072	17.2985
13	-12.3307	-24.3709	-35.1282	-43.6352	-48.9889	-50.2989	-46.9207	-38.2773	-23.9995	- 3.8543
14	-12.7417	-25.2842	-36.7072	-46.1131	-52.6451	-55.5311	-54.1150	-47.8762	-36.4656	-19.6647
15	-12.4204	-24.7168	-36.0651	-45.6612	-52.7371	-56.6002	-56.6607	-52.4492	-43.6479	-30.0594
16	-11.3684	-22.6714	-33.2048	-42.2815	-49.2447	-53.5013	-54.5459	-51.9750	-45.5131	-34.9903
17	- 9.5904	-19.1573	-28.1397	-35.9898	-42.1839	-46.2487	-47.7804	-46.4558	-42.0327	-34.4346
18	- 7.0948	-14.1910	-20.8932	-26.8151	-31.5874	-34.8761	-36.3956	-35.9172	-33.2825	-28.3947
19	- 3.8930	- 7.7953	-11.4988	-14.8000	-17.5044	-19.4360	-20.4441	-20.4080	-19.2433	-16.8988

Multiply reaction coefficients by (P)

Multiply moment coefficients by ($PR \times 10^{-3}$)

TABLE XI-B

$\varphi = 40^\circ$

	α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
	V_L	0.95722	0.91220	0.86518	0.81638	0.76604	0.71441	0.66173	0.60826	0.55431	0.50000
	V_R	0.04278	0.08780	0.13482	0.18362	0.23396	0.28559	0.33827	0.39174	0.44569	0.50000
	H	0.14676	0.29686	0.44510	0.58626	0.71538	0.82789	0.91981	0.98790	1.02972	1.04385
MOMENTS	1	46.3426	37.4166	28.4601	19.7099	11.3927	3.7176	- 3.1302	- 8.9970	-13.7614	-17.3558
	2	38.1387	78.6156	61.1522	44.0685	27.7987	12.7440	- 0.7401	-12.3584	-21.8769	-29.1655
	3	30.4307	62.9483	97.9169	72.9568	49.1381	27.0354	7.1588	-10.0679	-24.3070	-35.3715
	4	23.2561	48.3573	75.8401	106.2343	75.3069	46.5220	20.5278	- 2.1365	-21.0398	-35.9437
	5	16.6500	34.9134	55.4773	79.0223	106.1775	71.1090	39.3019	11.3970	-12.0913	-30.8792
	6	10.6444	22.6824	36.9274	54.1883	75.2169	100.6766	63.3897	30.4667	2.4950	-20.2027
	7	5.2687	11.7236	20.2810	31.8533	47.2923	67.3550	92.6737	54.9798	22.6480	- 3.9662
	8	0.5490	2.0906	5.6192	12.1262	22.5399	37.6893	58.2727	84.8167	48.2696	17.7511
	9	- 3.4893	- 6.1648	- 6.9791	- 4.8867	1.0934	11.8404	28.0991	50.4378	79.2033	44.8152
	10	- 6.8337	-13.0173	-17.4753	-19.1336	-16.9825	-10.1144	2.2425	20.6825	45.5796	77.1789
	11	- 9.4631	-18.4234	-25.8030	-30.5242	-31.5730	-28.0358	-19.1328	- 4.2598	16.9726	44.8152
	12	-11.3599	-22.3471	-31.9063	-38.9824	-42.5808	-41.8042	-35.8845	-24.2233	- 6.4280	17.7511
	13	-12.5220	-24.7840	-35.7786	-44.4981	-49.9918	-51.4010	-47.9883	-39.1773	-24.5839	- 3.9662
	14	-12.9415	-25.7174	-37.3932	-47.0340	-53.7569	-56.7632	-55.3663	-49.0268	-37.3812	-20.2027
	15	-12.6162	-25.1426	-36.7423	-46.5778	-53.8575	-57.8648	-57.9823	-53.7237	-44.7577	-30.8792
	16	-11.5477	-23.0624	-33.8291	-43.1316	-50.2932	-54.7003	-55.8236	-53.2451	-46.6773	-35.9437
	17	- 9.7413	-19.4871	-28.6677	-36.7123	-43.0814	-47.2853	-48.9009	-47.5934	-43.1309	-35.3715
	18	- 7.2057	-14.4340	-21.2833	-27.3512	-32.2573	-35.6557	-37.2477	-36.7961	-34.1356	-29.1655
	19	- 3.9533	- 7.9277	-11.7118	-15.0939	-17.8734	-19.8683	-20.9209	-20.9058	-19.7352	-17.3558

Multiply reaction coefficients by (P)

Multiply moment coefficients by $(PR \times 10^{-3})$

TABLE XII-B

$\varphi = 41^{\circ}$

α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L	0.95760	0.91285	0.86599	0.81726	0.76690	0.71518	0.66236	0.60870	0.55454	0.50000
V_R	0.04240	0.08715	0.13401	0.18274	0.23310	0.28482	0.33764	0.39130	0.44546	0.50000
H	0.14162	0.28690	0.43074	0.56802	0.69379	0.80355	0.89332	0.95987	1.00076	1.01458
1	46.9102	37.8864	28.8100	19.9248	11.4661	3.6520	- 3.3245	- 9.3026	-14.1553	-17.8116
2	38.6310	79.7526	62.0445	44.6861	28.1288	12.7909	- 0.9569	-12.8057	-22.5102	-29.9337
3	30.8435	63.8995	99.5333	74.1573	49.9028	27.3699	7.0907	-10.4913	-25.0219	-36.3044
4	23.5875	49.1202	77.1419	108.1874	76.6768	47.3145	20.7769	- 2.3714	-21.6775	-36.8910
5	16.9003	35.4905	56.4696	80.5290	108.3136	72.5226	40.0318	11.5126	-12.4942	-31.6905
6	10.8160	23.0800	37.6222	55.2662	76.7855	102.8651	64.7569	31.0897	2.4810	-20.7295
7	5.3659	11.9523	20.6961	32.5284	48.3272	68.8757	94.8255	56.2594	23.1716	- 4.0642
8	0.5777	2.1644	5.7780	12.4319	23.0843	38.5942	59.6823	86.8932	49.4715	18.2203
9	- 3.5214	- 6.2284	- 7.0475	- 4.9092	1.2003	12.1930	28.8389	51.7265	81.2123	45.9791
10	- 6.9182	-13.1989	-17.7393	-19.4396	-17.2554	-10.2447	2.3914	21.2692	46.7744	79.1619
11	- 9.5901	-18.7010	-26.2264	-31.0626	-32.1599	-28.5696	-19.4837	- 4.2750	17.4589	45.9791
12	-11.5184	-22.6958	-32.4491	-39.6965	-43.4086	-42.6530	-36.6332	-24.7279	- 6.5301	18.2203
13	-12.7008	-25.1788	-36.3998	-45.3306	-50.9866	-52.4750	-49.0313	-40.0561	-25.1510	- 4.0642
14	-13.1287	-26.1320	-38.0504	-47.9247	-54.8408	-57.9679	-56.5938	-50.1575	-38.2816	-20.7295
15	-12.8000	-25.5505	-37.3922	-47.4658	-54.9514	-59.1037	-59.2820	-54.9803	-45.8545	-31.6905
16	-11.7163	-23.4373	-34.4288	-43.9560	-51.3179	-55.8764	-57.0820	-54.4998	-47.8309	-36.8910
17	- 9.8832	-19.8033	-29.1751	-37.4134	-43.9590	-48.3026	-50.0053	-48.7185	-44.2009	-36.3044
18	- 7.3100	-14.6669	-21.6583	-27.8715	-32.9122	-36.4211	-38.0880	-37.6659	-34.9828	-29.9337
19	- 4.0100	- 8.0545	-11.9166	-15.3790	-18.2342	-20.2928	-21.3910	-21.3986	-20.2240	-17.8116

Multiply reaction coefficients by (P)

Multiply moment coefficients by $(PR \times 10^{-3})$

TABLE XIII-B

$\varphi = 42^\circ$

	α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
	V_L	0.95799	0.91352	0.86682	0.81816	0.76779	0.71598	0.66300	0.60916	0.55478	0.50000
	V_R	0.04201	0.08648	0.13318	0.18184	0.23221	0.28402	0.33700	0.39084	0.44522	0.50000
	H	0.13668	0.27736	0.41701	0.55057	0.67316	0.78030	0.86802	0.93311	0.97312	0.98665
MOMENTS	1	47.4362	38.3222	29.1323	20.1175	11.5222	3.5726	- 3.5295	- 9.6164	-14.5550	-18.2710
	2	39.0906	80.8287	62.8879	45.2649	28.4288	12.8137	-11.9253	-13.2675	-23.1539	-30.7088
	3	31.2316	64.8035	101.0855	75.3072	50.6289	27.6738	6.9983	-10.9335	-25.7504	-37.2464
	4	23.9014	49.8481	78.3955	110.0830	78.0034	48.0729	20.9991	- 2.6271	-22.3306	-37.8488
	5	17.1394	36.0429	57.4275	81.9941	110.4053	73.9015	40.7345	11.6072	-12.9129	-32.5128
	6	10.9818	23.4620	38.2941	56.3160	78.3242	105.0208	66.0986	31.6929	2.4522	-21.2670
	7	5.4617	12.1731	21.0982	33.1865	49.3439	70.3753	96.9552	57.5220	23.6821	- 4.1718
	8	0.6089	2.2366	5.9320	12.7298	23.6201	39.4871	61.0783	88.9560	50.6628	18.6810
	9	- 3.5479	- 6.2883	- 7.1142	- 4.9322	1.3061	12.5407	29.5713	53.0067	83.2130	47.1358
	10	- 6.9946	-13.3728	-17.9965	-19.7405	-17.5239	-10.3749	2.5372	21.8507	47.9630	81.1380
	11	- 9.7073	-18.9676	-26.6391	-31.5916	-32.7382	-29.0994	-19.8347	- 4.2936	17.9396	47.1358
	12	-11.6657	-23.0313	-32.9781	-40.3981	-44.2249	-43.4952	-37.3802	-25.2345	- 6.6375	18.6810
	13	-12.8677	-25.5590	-37.0056	-46.1485	-51.9678	-53.5407	-50.0712	-40.9365	-25.7239	- 4.1718
	14	-13.3041	-26.5316	-38.6912	-48.7999	-55.9101	-59.1633	-57.8174	-51.2894	-39.1879	-21.2670
	15	-12.9725	-25.9438	-38.0259	-48.3381	-56.0307	-60.3329	-60.5772	-56.2378	-46.9572	-32.5128
	16	-11.8746	-23.7987	-35.0133	-44.7656	-52.3289	-57.0430	-58.3359	-55.7550	-48.9899	-37.8488
	17	-10.0165	-20.1078	-29.6696	-38.1016	-44.8245	-49.3115	-51.1054	-49.8436	-45.2753	-37.2464
	18	- 7.4080	-14.8911	-22.0234	-28.3818	-33.5579	-37.1799	-38.9246	-38.5353	-35.8332	-30.7088
	19	- 4.0632	- 8.1765	-12.1158	-15.6585	-18.5896	-20.7132	-21.8589	-21.8910	-20.7144	-18.2710

Multiply reaction coefficients by (P)

Multiply moment coefficients by $(PR \times 10^{-3})$

TABLE XIV-B

$\varphi = 43^\circ$

α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L	0.95839	0.91420	0.86768	0.81909	0.76870	0.71680	0.66367	0.60963	0.55502	0.50000
V_R	0.04161	0.08580	0.13232	0.18091	0.23130	0.28320	0.33633	0.39037	0.44498	0.50000
H	0.13193	0.26818	0.40380	0.53380	0.65336	0.75799	0.84377	0.90746	0.94664	0.95989
1	47.9195	38.7247	29.4285	20.2905	11.5632	3.4823	- 3.7422	- 9.9352	-14.9572	-18.7306
2	39.5164	81.8456	63.6853	45.8094	28.7029	12.8181	- 1.4411	-13.7375	-23.8015	-31.4839
3	31.5939	65.6637	102.5775	76.4131	51.3224	27.9547	6.8903	-11.3854	-26.4833	-38.1879
4	24.1964	50.5457	79.6067	111.9294	79.2944	48.8070	21.2052	- 2.8923	-22.9874	-38.8051
5	17.3656	36.5768	58.3583	83.4283	112.4615	75.2575	41.4229	11.6940	-13.3334	-33.3318
6	11.1401	23.8355	38.9519	57.3500	79.8438	107.1574	67.4297	32.2915	2.4242	-21.7990
7	5.5548	12.3937	21.4970	33.8413	50.3548	71.8691	99.0790	58.7841	24.1968	- 4.2715
8	0.6412	2.3157	6.0916	13.0345	24.1604	40.3844	62.4781	91.0228	51.8617	19.1520
9	- 3.5702	- 6.3356	- 7.1682	- 4.9404	1.4245	12.9007	30.3145	54.2969	85.2242	48.3044
10	- 7.0643	-13.5297	-18.2355	-20.0205	-17.7737	-10.4873	2.6984	22.4456	49.1643	83.1270
11	- 9.8158	-19.2141	-27.0296	-32.0955	-33.2937	-29.6079	-20.1676	- 4.2969	18.4336	48.3044
12	-11.8031	-23.3446	-33.4823	-41.0718	-45.0156	-44.3138	-38.1073	-25.7253	- 6.7322	19.1520
13	-13.0240	-25.9162	-37.5851	-46.9373	-52.9223	-54.5819	-51.0908	-41.8013	-26.2853	- 4.2715
14	-13.4686	-26.9082	-39.3058	-49.6460	-56.9528	-60.3345	-59.0213	-52.4068	-40.0844	-21.7990
15	-13.1345	-26.3151	-38.6345	-49.1828	-57.0845	-61.5391	-61.8541	-57.4821	-48.0517	-33.3318
16	-12.0234	-24.1403	-35.5752	-45.5502	-53.3166	-58.1890	-59.5733	-56.9987	-50.1424	-38.8051
17	-10.1417	-20.3959	-30.1449	-38.7687	-45.6704	-50.3030	-52.1918	-50.9593	-46.3447	-38.1879
18	- 7.5000	-15.1031	-22.3743	-28.8765	-34.1890	-37.9256	-39.7510	-39.3978	-36.6800	-31.4839
19	- 4.1130	- 8.2917	-12.3071	-15.9293	-18.9369	-21.1264	-22.3210	-22.3795	-21.2027	-18.7306

Multiply reaction coefficients by (P)

Multiply moment coefficients by ($PR \times 10^{-3}$)

TABLE XV-B

$\varphi = 44°$

α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L	0.95880	0.91490	0.86856	0.82004	0.76963	0.71764	0.66436	0.61012	0.55528	0.50000
V_R	0.04120	0.08510	0.13144	0.17996	0.23037	0.28236	0.33564	0.38988	0.44472	0.50000
H	0.12739	0.25937	0.39114	0.51774	0.63439	0.73664	0.82055	0.88292	0.92130	0.93430
MOMENTS										
1	48.3578	39.0909	29.6956	20.4401	11.5855	3.3774	- 3.9662	-10.2624	-15.3653	-19.1940
2	39.9050	82.7973	64.4311	46.3128	28.9441	12.7969	- 1.7095	-14.2224	-24.4600	-32.2659
3	31.9257	66.4722	104.0017	77.4655	51.9733	28.2028	6.7569	-11.8567	-27.2303	-39.1388
4	24.4671	51.2036	80.7665	113.7147	80.5375	49.5045	21.3831	- 3.1791	-23.6600	-39.7720
5	17.5731	37.0813	59.2519	84.8180	114.4682	76.5762	42.0829	11.7591	-13.7702	-34.1620
6	11.2844	24.1888	39.5848	58.3535	81.3289	109.2586	68.7342	32.8700	2.3810	-22.3417
7	5.6379	12.6018	21.8809	34.4772	51.3432	73.3398	101.1800	60.0290	24.6982	- 4.3809
8	0.6670	2.3888	6.2448	13.3299	24.6882	41.2683	63.8639	93.0761	53.0499	19.6147
9	- 3.5959	- 6.3837	- 7.2215	- 4.9503	1.5382	13.2545	31.0504	55.5791	87.2272	49.4658
10	- 7.1349	-13.6830	-18.4683	-20.2962	-18.0224	-10.6003	2.8568	23.0360	50.3595	85.1095
11	- 9.9231	-19.4535	-27.4096	-32.5903	-33.8436	-30.1130	-20.4999	- 4.3028	18.9223	49.4658
12	-11.9378	-23.6483	-33.9729	-41.7332	-45.7974	-45.1261	-38.8318	-26.2174	- 6.8320	19.6147
13	-13.1764	-26.2619	-38.1492	-47.7114	-53.8655	-55.6149	-52.1065	-42.6668	-26.8521	- 4.3809
14	-13.6285	-27.2725	-39.9039	-50.4762	-57.9828	-61.4962	-60.2204	-53.5244	-40.9865	-22.3417
15	-13.2915	-26.6741	-39.2267	-50.0114	-58.1253	-62.7355	-63.1258	-58.7264	-49.1519	-34.1620
16	-12.1673	-24.4703	-36.1216	-46.3196	-54.2919	-59.3253	-60.8055	-58.2422	-51.3001	-39.7720
17	-10.2626	-20.6739	-30.6069	-39.4226	-46.5053	-51.2858	-53.2732	-52.0744	-47.4185	-39.1388
18	- 7.5885	-15.3075	-22.7151	-29.3611	-34.8114	-38.6644	-40.5732	-40.2596	-37.5298	-32.2659
19	- 4.1609	- 8.4026	-12.4927	-16.1943	-19.2791	-21.5355	-22.7806	-22.8673	-21.6926	-19.1940

Multiply reaction coefficients by (P)

Multiply moment coefficients by $(PR \times 10^{-3})$

TABLE XVI-B

$\varphi = 45^\circ$

α	0.9φ	0.8φ	0.7φ	0.6φ	0.5φ	0.4φ	0.3φ	0.2φ	0.1φ	0
V_L	0.95923	0.91563	0.86946	0.82102	0.77060	0.71851	0.66507	0.61062	0.55554	0.50000
V_R	0.04077	0.08437	0.13054	0.17898	0.22940	0.28149	0.33493	0.38938	0.44446	0.50000
H	0.12299	0.25090	0.37896	0.50230	0.61617	0.71614	0.79829	0.85939	0.89701	0.90976
1	48.7527	39.4214	29.9340	20.5669	11.5904	3.2586	- 4.2009	-10.5974	-15.7787	-19.6601
2	40.2601	83.6852	65.1260	46.7761	29.1549	12.7516	- 1.9963	-14.7209	-25.1276	-33.0530
3	32.2331	67.2320	105.3590	78.4659	52.5855	28.4203	6.6001	-12.3452	-27.9891	-40.0962
4	24.7213	51.8258	81.8769	115.4410	81.7374	50.1682	21.5355	- 3.4847	-24.3456	-40.7464
5	17.7710	37.5617	60.1112	86.1664	116.4312	77.8613	42.7176	11.8058	-14.2195	-34.9994
6	11.4251	24.5277	40.1960	59.3307	82.7863	111.3287	70.0160	33.4321	2.3267	-22.8907
7	5.7227	12.8041	22.2542	35.0991	52.3173	74.7925	103.2623	61.2608	25.1910	- 4.4949
8	0.6989	2.4631	6.3964	13.6213	25.2122	42.1444	65.2406	95.1204	54.2325	20.0745
9	- 3.6120	- 6.4246	- 7.2692	- 4.9560	1.6563	13.6085	31.7840	56.8582	89.2272	50.6256
10	- 7.1933	-13.8247	-18.6898	-20.5615	-18.2603	-10.7075	3.0178	23.6268	51.5539	87.0910
11	-10.0163	-19.6779	-27.7740	-33.0702	-34.3781	-30.6082	-20.8265	- 4.3061	19.4110	50.6256
12	-12.0571	-23.9346	-34.4448	-42.3760	-46.5606	-45.9257	-39.5484	-26.7058	- 6.9318	20.0745
13	-13.3129	-26.5890	-38.6926	-48.4651	-54.7883	-56.6335	-53.1131	-43.5281	-27.4196	- 4.4949
14	-13.7727	-27.6177	-40.4807	-51.2852	-58.9919	-62.6429	-61.4100	-54.6380	-41.8900	-22.8907
15	-13.4336	-27.0146	-39.7981	-50.8191	-59.1455	-63.9169	-64.3880	-59.9669	-50.2539	-34.9994
16	-12.2979	-24.7833	-36.6489	-47.0696	-55.2481	-60.4477	-62.0287	-59.4821	-52.4597	-40.7464
17	-10.3724	-20.9376	-31.0526	-40.0599	-47.3238	-52.2565	-54.3467	-53.1864	-48.4939	-40.0962
18	- 7.6690	-15.5012	-23.0436	-29.8331	-35.4214	-39.3939	-41.3893	-41.1187	-38.3809	-33.0530
19	- 4.2044	- 8.5077	-12.6715	-16.4523	-19.6143	-21.9392	-23.2365	-23.3534	-22.1829	-19.6601

Multiply reaction coefficients by (P)

Multiply moment coefficients by ($PR \times 10^{-3}$)

VITA

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