# AN EVALUATIVE AND PREDICTIVE GROWTH 

PHASE CAPABILITY - MIX MODEL FOR R\&D LAUNCH VEHICLE PROGRAMS

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## PREFACE

The aerospace corporations have continually experienced organizational restructuring as existing R\&D launch vehicle programs are completed, and new ones are initiated. Existing capabilities which support the mature program are inadequate to achieve new program objectives. Therefore, the aerospace corporations are required to establish an embryonic R\&D organizational structure which rapidly evolves as the program progresses. This evolution or growth has been characterized by a continually changing technology, and hence, a continually changing organizational capability. It is this changing organizational capabilitymix which is the basis for this dissertation.

The analysis of the effort expenditures recorded during the growth phase of selected R\&D launch vehicle organizations resulted in an evaluative and predictive growth phase capability-mix model. With this model, future R\&D launch vehicle organizations may be staffed more efficiently and effectively during the crucial period known as growth phase.

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## CHAPTER I

## FORMULATION OF THE PROBLEM

## Introduction

The statement that "man now stands at the threshold of space" instills the mind with glamorous visions of adventure. However, behind these visions lie a multitude of problems that must be conquered. One of these colossal problems is that of determining the optimum organizational staffing pattern to accomplish this adventure。

Man's pursuit of technological advances has enabled him to realistically attempt space exploration. These same technical advances and complexities have generated much of the difficulty in determining the proper R\&D Iaunch vehicle capability-mix which can accomplish the desired objectives in the most efficient manner. As new materials and prom pulsion systems are developed, so must we develop new approaches to the R\&D launch vehicle capabilitymmix problem.

R\&D launch vehicle organizational aspects are many and varied, depending upon specific variables such as organizational type, management, capabilities, and environment, to mention a few. The interrelations between these different Variables determine the organizational structure which is most appropriate for a particular situation. This
investigation will be concerned with only one of the above mentioned variables, specifically, the capability-mix which exists at any discrete time during the growth phase of an R\&D launch vehicle program.

Barnard (1) defines an organization as a system of consciously coordinated personal activities or forces. This is the achievement of objectives in a collective fashion. It means that the sequences of activity necessary to achieve the objectives are too much for one individual, and thus, are divided into smaller segments which may be accomplished by the individual contributors of the organization. At the individual level these segments may be viewed as roles. At the group level the segments may be viewed as departments. These segments are integrated or organized in a particular sequence or pattern designed to achieve the organizational objectives. The resulting pattern constitutes the organizational structure. Organizations, therefore, have an initial or intended structure which is simply a static picture of the pattern of the segments as planned by manage ment, in order for the contributors to assist in achieving the organizational objectives. This intended ox proposed organizational structure is of vital concern when thousands of contributors are involved. The organizations which have evolved in the support of the Apollo Program are typical examples.

In the structuring of organizations to cope with the development of large launch vehicles for the Apollo Program,
certain events are qbserved as taking place. Primary among these events is a continual change in the capability-mix as the organization evolves. It is this capability-mix, especially during the organization's period of growth, which can cause an organization to ultimately succeed or fail. It may be theorized that the efficiency and effectiveness of an organization are directly related to the changing capabilitymix. Hence, knowledge of the proper capability-mix for a specific time period of a program can be extremely beneficial.

When speaking of organizations in a general fashion, the terms growth and size are frequently utilized interchangeably, which can lead to considerable confusion. Since this investigation will consider only the growth phase of R\&D launch vehicle programs, averall definitions are desirable. Organizational growth may be defined as an internal process of the organization, which brings about certain directions of development. From a biological point of view, growth has a natural connotation. Penrose (2) describes growth as a process which occurs under "normal" conditions or when nothing restricts or inhibits it. Size is a resultant characteristic of growth. However, it should. be noted that other results of growth exist. Size possesses the advantage of being easily observed and measured, and henceforth, receives considerable attention in organizational analysis.

The external conditions for organizational growth are numerous in today's society. Among the most important,
according to Litterer (3), are: the demand for the organization's output; the possibility of obtaining a special opportunity, such as a monopoly through patents or franchise; and the high cost of entry to the field which may keep other organizations from being established to exploit developing demand.

However, organizational growth is not spontaneous. It is the result of management decisions, decisions to increase production in response to demand, decisions to stimulate demand, or decisions to create a demand. The relationships between specific decisions and the ultimate growth of the organization may not be recognizable, but organizational growth is necessarily dependent upon some decisions and the actions which follow them. These decisions are also functions of the goals pursued by the members of the organization. Hence, organizational growth ordinarily takes place when the increased size is viewed positively as related to the achievement of the organization ${ }^{\circ}$ s goals, together with the goals of the individual members of the organizationa The difficulty is to achieve organizational growth in the desired effort expenditure categorieso

History of the Effort Expenditure Problem

The field of launch vehicle effort expenditures is very lucrative to the researcher. However, prior to plunging into a full scale investigation, it is necessary to isolate the problem areas, specify the constraints to be
imposed upon the investigation, and finally, suggest an alternative approach or technique toward a satisfactory solution of the problem.

A study by Peck and Scherer (4) indicates that ten of eleven R\&D launch vehicle programs exceeded original effort expenditure expectations by a factor of 2.2. Although the entire R\&D program was considered by Peck and Scherer, it appears that the effort expenditure expectations were the least accurate during the early portions of the program, commonly known as the growth phase. The assumption that a definite problem exists is valid since the growth phase of R\&D launch vehicle programs have consistently exceeded their original estimates。

The classical approach in the analysis of R\&D launch vehicle effort expenditures is to consider the total R\&D program from initiation to completion. The total effort for a specified effort category is then divided by the total launch vehicle weight which was produced during the R\&D program. The result is the generally employed hours of effort required for each pound produced. Hence, if the total weight for a proposed program is known, the above process may be reversed and the resulting value is an estimate of the total effort required for a particular effort category, within a certain program. However, the fallacy in this approach is the uncertainty associated with total weight values prior to design completion and the failure to consider other program parameters. Hence, the
usual assumption is that the weight data is normally of such a major concern that reasonably good weights are ordinarily available early in the program. Thus, it is difficult to develop a concrete approach to the estimation problem with a structurally unsound foundation.

The Research Problem

The point of real concern in analyzing data from past R\&D launch vehicle programs is not only the total cost, total effort expenditure, or total time elapsed. Even more significant are the relationships which exist between the major categories of effort during incremental time periods of the R\&D program. Furthermore, does a pattern appear for these relationships as a function of time, which might be useful for predictive and evaluative purposes?

The proposed investigation will attempt to establish the above mentioned patterns and relationships. Once established, these may serve as a basis to assist in the evaluation of future contractor proposals on R\&D launch vehicle programs. Furthermore, it appears that a better method can be developed to lend some validity to the enormous problem of estimating what the capability-mix should be during the various periods of an R\&D launch vehicle program. The proposed investigation will not attempt to be all encompassing. Instead, itwill concentrate on the most crucial portion of the program, the growth phase. It is theorized at this time that if the proper
capability-mix is achieved during the growth phase of the program, the major obstacles to objective accomplishment will have been achieved. Furthermore, the contractor will be less inclined to over-staff his work force on this particular progrem, since reduction in effort expenditures would be already under way. The natural tendency for the individuals possessing the desired capabilities is to leave a program which is in the negative slope portion of the effort expenditure curve, and to seek employment on a "going" program. Hence, the capability-mix beyond the growth phase of the program should be easier to control if the proper capability-mix is achieved during the growth phase.

## Objectives to be Attained

The major objective of this investigation is to develop a logical, systematic, step-by-step approach for determining the proper capability-mix at a specific point during the growth phase of an R\&D launch vehicle program. This objective will be accomplished through the development of a model which represents four growth phase effort expenditure categories through six interdependent ratios.

Another objective is to develop within the model an acceptable range of ratio values associated with specified time intervals of the growth phase. This would permit the use of the model in the evaluation of R\&D launch vehicle program contractor proposals.

A third objective is that the model possess the inherent ability to provide estimates or predictions of the effort expenditure ratio for a selected time interval of the growth phase. Attainment of this objective would provide a means for tracking and adjusting the capabilitymix once the program is underway.

A fourth objective is that the model permit a retracing of actions at any time in the future. Fulfillment of this objective will assure consistency in all capability-mix estimates, since all elements will have been considered in a similar fashion.

## Phases of the Investigation

The first phase of the investigation was a literature survey. This phase served as a period of orientation for the researcher and proved extremely valuable. The results of the literature survey are noted as references within the text. It should be noted that little research has been accomplished with respect to R\&D launch vehicle growth phase capability-mix.

The second phase of the investigation was a critical analysis of other techniques utilized in determining effort expenditures to ascertain if any were directly applicable to the situation being studied. In addition to the classical approach, the techniques were of two general classifications. These were the determination of total effort expenditures as a function of total cost when: (a) first R\&D unit cost
is known; and, (b) first operational unit cost is available. Once the total effort expenditures are determined, the total is distributed to the various time intervals of the program. However, it is virtually impossible to determine the first R\&D unit cost. Hence, this portion of the investigation verified the need for an alternative method. The determination of the capability-mix during the growth phase of R\&D launch vehicle programs, may be considered an alternative method. The developed alternative should be readily applicable to the average, space-related R\&D launch vehicle program.

The third phase of the research was an investigation of the manpower expenditures for each of four effort categories, during the growth phase of five R\&D launch vehicle programs, This data was analyzed and synthesized to formulate an alternative solution to the problem.

The fourth phase of the investigation was the testing of the developed model with a two-part test case, to ascertain whether the objectives of the investigation had been attained. The test case illustrates the evaluative features of the developed model. The fifth and final phase of the investigation illustrates the use of the model as a predictive devíce.

## CHAPTER II

## MODEL FORMULATION

## Backgraund

The researcher is frequently confronted with the model development problem. Should the model be based upon measurement and experiment, pure mathematics, or a combination of both? The methodology of physical measurements is somewhat easier to comprehend than a pure mathematical approach. Basically, this is because one can repeat a physical experiment many times under controlled conditions and arrive rather easily at an objective measurement, as well as a calculation of the expected experimental error. In a management environment, it is difficult to simulate an exact situation and to carry out experiments due to many interdependent factors, and the cost involved. Therefore, we must content ourselves with statistical measurements, obtained by observing a number of similar situations. Analysis of these statistics permits the development of functional relationships or theoretical distributions. Starting from such analyses, and proceeding through a synthesis process, a mathematical model may be developed.

Bursk and Chapman (5) describe a model as a simplified
representation of an operation, containing only those aspects which are of primary importance to the problem under study, Manipulating the model itself makes it possible to determine the effects of changes in the system, rather than imposing changes on the modeled entity,

There are several different general types of research models which might be utilized in this investigation. Most of these models are mathematical in form, consisting of a set of equations relating significant programmatic variables in the operation under study, to the outcome. The model to be developed is of this type, being specifically designated as a symbolic model. The primary purpose of this model is to represent the system under study through symbols. The secondary purpose is to aid in the analysis and synthesis of the four effort categories.

## System Structure

The analysis and synthesis of the data, and the subsequent development of the model, will depend to a large extent upon the system structure. The structure has been conveniently broken dowf into workable constituent categories. The following list of effort categories represents in total, all.the possible combinations and types of direct labor effort which occur at any one time during the growth phase of an R\&D launch vehicle program. Obviously, some very distinct effort categories have been contained within the four categories which follow. However, additional
detail may be added at any time, once the original concept has been developed, tested, and proven. The symbolic notation in parenthesis following each effort category title shall be used throughout the investigation. The effort categories and their respective descriptions are:

1. Engineering (E) - Includes all effort associated with the design and development of the stage, ground test articles, stage support equipment, models, mockups, and component tests.
2. Manufacturing (M) - Supports all effort associated with the fabrication, assembly, in-plant test, system tests, program planning, documentation, and sustaining manufacturing for the stage.
3. Tooling (T) - All effort associated with the design, fabrication, installation, and check-out of the basic tooling, and
i.the sustaining tooling effort.
4. Quality Assurance (Q) - Includes all effort utilized for the quality
inspection and reliability assurance during stage manufacture, test, and acceptance.

Functional relationships will not be developed directly for the engineering, manufacturing, tooling, and quality assurance categories. Instead, six unitless ratios will be developed from the four effort categories given above, where six is the number of combinations of four different things taken two at a time, without regard to the assignment of the things in a group. The resulting ratio groups, together with the symbolic notation, are shown below:

# Engineering/Manufacturing 

Engineering/Tooling
Engineering/quality Assurance (E/Q)
Manufacturing/Tooling ( $M / \mathbb{T}$ )
Manufacturing/Quality Assurance (M/Q)
Quality Assurance/Tooling
$(Q / \mathbb{T})$
The above six ratio groups are the nucleus of the total model. It is felt that a detailed mathematical analysis of these ratio groups will facilitate the actual development of the total model. Each of these ratio groups will be developed as a functional relationship where the value of the effort ratio will be dependent upon a time value.

Algebraically, the dependent variables as a function of the independent variable are:

$$
(E / \mathbb{N})_{n}=f(t)_{n}
$$

$$
\begin{aligned}
& (E / T)_{n}=f(t)_{n} \\
& (E / Q)_{n}=f(t)_{n} \\
& (\mathbb{M} / T)_{n}=f(t)_{n} \\
& (\mathbb{M} / Q)_{n}=f(t)_{n} \\
& (Q / T)_{n}=f(t)_{n}
\end{aligned}
$$

where
$t=a$ value of time between 0.0 and 1.0 , and
$n=$ a selected time value during the growth phase.
Prediction limits will then be developed for individual values of the independent variable. This will permit the use of the model in an evaluative fashion. Substitution of a discrete time value into the developed functional relationships will permit the model to be utilized in a predictive fashion.

## Growth Phase

The growth phase is defined as the time beginning with program initiation and continuing until the effort summation for the four effort categories achieves a maximum. It is entirely possible that this maximum point for the total expenditures is beyond the maximum effort expenditure point for at least one of the effort categories. This situation is illustrated in Figure 1 where the engineering effort is shown declining prior to the total peak effort expenditure. Furthermore, it should be noted that the engineering or manufacturing category generally has achieved maximum effort expenditure prior to termination of the growth phase.


Figure 1. Distribution of Effort for a Hypothetical R\&D Launch Vehicle Program

Intuition would cause one to anticipate that manufacturing effort tends to predominate toward the end of an R\&D launch vehicle program growth phase. However, it appears that the proper expenditure of engineering effort early in the growth phase causes an earlier decrease in engineering effort during the final quarters of the growth phase.

In an effort to simplify the handling of the independent variable, time, within the defined growth phase, program initiation is assigned a value of 0.0 , while the termination of the growth phase is assigned a value of 1.0. Hence, the only possible values of time which are available for the model lie within these bounds.

## Model Assumptions

The development of a predictive or evaluative model must, of necessity, be based upon certain general assumptions. The statement of these assumptions is necessary to establish a reasonable bound upon the nebulous areas. The assumptions associated with this investigation are:

1. Empirical information accounts for average delays, average changes, and average effort increases. Therefore, it is not necessary to modify the data to compensate for this aspect.
2. The engineering, manufacturing, tooling, and quality assurance effort categories adequately represent the overall direct effort expenditures for a particular time period.
3. R\&D launch vehicle programs receive similar national priority ratings during the growth phase of the program.
4. The contractors are and have been conscientiously and accurately reporting the direct effort hours which occur for their particular program, on the NASA FORM 533 (Contractor Financial Management Report), Budget Bureau. No. 104-R011.1, as required by NASA Management Manual 6-2-4.
5. Consideration will be limited to large launch vehicles/stages, $14,000<$ Large $<290,000$, where the unit of the limitation is vehicle/stage dry weight in pounds.
6. Each program must accomplish a similar number of state-of-the-art advances to achieve the program objective.
7. The derived results will be in terms of effort ratios. The model will not, nor is it designed to provide the required effort expenditures for a particular effort category at a specified time.
8. The model considers only the R\&D prime contractor direct effort expenditures on the stage, thereby excluding launch effort and any effort expended on operational stages.

The above assumptions narrow the field of investigation to a specific area, yet are flexible enough to permit the development of a feasible and workable growth phase effort

CHAPTER III

## DATA COLLECTION

## General

The collection of empirical data on launch vehicles is very time-consuming, and in some isolated instances, impossible. The methods in which records of past programs have been maintained are, in many instances, difficult to comprehend. Only a continuing aognizance of govermment and industry financial management systems permits one to extract the desired information from the mountain of reports available. In general, government agencies are somewhat consistent in record keeping, from the standpoint of providing conversion codes when changing from one method of record maintenance to another. Industry is far from consistent, when comparing one company with several others. Each company has definite peculiar aspects suitable to top management's desires or old line company policies. However, some uniformity among contractor reporting of effort expenditures has been achieved with the development and required use of Budget Bureau Form No. 104-R011.1, as prescribed by NASA Management Manual 6-2-4.

## Data Sources/Constraints

The empirical information collected and utilized in this investigation was available directly from within government agencies. Although industry is reluctant to provide empirical data so as not to jeopardize their compatitive position, the present financial management arrangements for R\&D launch vehicle programs serve this purpose quite adequately.

Since only the growth phase of each program was of interest, it was of the utmost importance that empirical data be available for the early phases of the program. In most cases, this necessitated examining the original letter contract agreement between the government and the contractor, and extracting the appropriate data. Since the phasing-out of the letter contract, and the phasing-in of the prime contract, represent overlapping areas when depicted on a time scale, it was necessary to combine the two sets of effort values to acquire a true representation of the original situation. Furthermore, it was necessary to collect data considerably beyond the maximum point of total effort expenditures to ascertain that the maximum point had been achieved.

The data was available for discrete time periods of three months, or one quarter. Hence, the point of maximum total effort expenditures was recorded in terms of quarters. It should be noted that the number of quarters required to achieve the peak of the growth phase varies between programs,
as would be expected. However, this difference in the number of quarters to growth phase peak is not a problem. The quarter in which the growth phase reaches peak will be represented on the time scale by the value of 1.0. Each other quarter value will then be divided by the value of the original maximum quarter to achieve some positive decimal value less than 1.0 .

It should be noted, however, that the actual direct effort data is not presented, nor utilized, in any fashion within the scope of this investigation. Instead, a coding process, which produces no adverse effects and does not distort the empirical data, has been utilized to protect the original data from unnecessary exposure.

For the purpose of this investigation, the collected direct manhour values will serve as the basis for the dependent variables, namely, the various developed ratios. The stability associated with the hour as compared to the monetary aspect, dollars, suggests that manhours will provide more valid results. It is recognized that the various manhours can readily be converted to the monetary unit with little difficulty, if so desired. However, the end result of this investigation is not concerned with any conversion method to arrive at monetary values.

The Empirical Data

The results of the data collection phase of the investigation, together with the imposed constraints and coding
effect, are shown in Tables I through V. In each case, three quarters of effort beyond the growth phase peak are provided to definitely establish the quarter of maximum effort expenditure. The quarter of peak expenditures is designated by an asterisk in the totals column.

It is immediately apparent that only a limited number of $R \& D$ launch vehicle systems were considered. However, it must be noted that the universe of R\&D launch vehicle programs, which are available for consideration in this investigation, is relatively small. Hence, the sample size is limited. Calculation of an exact value for the sample size is not necessary, since a larger sample size cannot be obtained. In a controlled experiment, the desired sample size is more readily obtainable. However, when dealing with real life situations the desired amount of raw data may be difficult to obtain.

Al though the number of R\&D programs utilized is small, the manner in which the data is analyzed results in 75 data sets from the five programs. Hence; the sample size is quite adequate when considered in this fashion. This approach to the data analysis phase of the investigation is considered to utilize the available data to the fullest extent possible.

Sample Population Similarity

A common problem in the collection of data is to determine whether several samples should be regarded as

TABLE I
PROGRAM "A" DIRECT EFFORT EXPENDITURES (Thousands of Manhours)

| Qtr. | E | M | T | Q | Totals |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 128.44 | 148.72 | 23.65 | 30.41 | 331.22 |
| 2 | 253.50 | 300.82 | 47.31 | 59.53 | 661.16 |
| 3 | 377.90 | 446.15 | 68.90 | 94.63 | 947. 58 |
| 4 | 500.23 | 598.26 | 93.33 | 125.05 | 1316.87 |
| 5 | 602.94 | 754.38 | 114.91 | 155.48 | 1627.71 |
| 6 | 726.70 | 902.45 | 136.50 | 189.27 | 1954.92 |
| 7 | 819.26 | 1061.32 | 158.86 | 219.70 | 2259.14 |
| 8 | 896.34 | 1172.86 | 189.27 | 244.65 | 2503.12 |
| 9 | 948.47 | 1262.04 | 216.32 | 260.26 | 2687.09 |
| 10 | 999.17 | 1354.07 | 235.30 | 270.40 | 2858.94 |
| 11 | 1049.88 | 1453.40 | 260.26 | 271.70 | 3035.24 |
| 12 | 1090.43 | 1537.90 | 269.10 | 277.15 | 3174.58 |
| 13 | 1124.88 | 1612.26 | 273.77 | 288.60 | 3299.51 |
| 14 | 1153.22 | 1667.63 | 280.53 | 305.50 | 3406.88 |
| 15 | 1166. 10 | 1730.56 | 285.22 | 327.85 | 3509.73 |
| 16 | 1184.30 | 1777.87 | 296.91 | 358.27 | '3617.35* |
| 17 | 1165.45 | 1715.74 | 290.68 | 354.90 | 3526.77 |
| 18 | 1128.92 | 1616.29 | 283.92 | 348.14 | 3377.27 |
| 19 | 1087.71 | 1524.38 | 276.51 | 338.00 | 3226.60 |

* Denotes growth phase peak.


## TABLE II

## PROGRAM "B" DIRECT EFFORT EXPENDITURES (Thousands of Manhours)

| Qtr. | E | M | T | Q | Totals |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 685.09 | 128.70 | 42.90 | 26.00 | 882.69 |
| 2 | 717.59 | 143.00 | 58.50 | 49.40 | 968.49 |
| 3 | 721.50 | 211.89 | 61.09 | 76.70 | 1071.18 |
| 4 | 705.90 | 352.29 | 150.80 | 331.50 | 1540.49 |
| 5 | 937.30 | 536.90 | 224.89 | 261.30 | 1960.39 |
| 6 | 1071.20 | 887.90 | 291.20 | 257.40 | 2507.70 |
| 7 | 1010.09 | 1049.10 | 301.60 | 276.90 | 2637.69 |
| 8 | 1427.40 | 1353.30 | 365.29 | 347.10 | 3493.09 |
| 9 | 1281.80 | 2044.90 | 305.50 | 426.40 | 4058.60 |
| 10 | 1514.50 | 2355.60 | 352.29 | 468.00 | 4690.39 |
| 11 | 1610.69 | 2468.70 | 362.70 | 542.10 | 49'84.19 |
| 12 | 1688.69 | 2431.00 | 413.40 | 542.10 | 5075.19 |
| 13 | 1764.10 | 2187.90 | 386.10 | 531.70 | 4869.80 |
| 14 | 2572.70 | 3053.70 | 430.29 | 585.00 | 6641.69 |
| 15 | 2083.90 | 3534.70 | 387.40 | 692.90 | 6698.90 |
| 16 | 2314.00 | 3506.10 | 416.00 | 799.50 | 7035.60* |
| 17 | 1795.30 | 3052.40 | 265.20 | 824.20 | 5937.10 |
| 18 | 1740.70 | 2356.90 | 367.90 | 747.50 | 5213.00 |
| 19 | 1433.90 | 2548.00 | 245.70 | 629.20 | 4856.80 |

*Denotes growth phase peak.

TABLE III
PROGRAM "C" DIRECT EFFORT EXPENDITURES (Thousands of Manhours)

| Qtr. | E | $M$ | T | Q | Totals |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 1 | 139.36 | 70.33 | 38.73 | 10.92 | 259.34 |
| 2 | 212.28 | 81.25 | 45.50 | 13.00 | 352.03 |
| 3 | 568.75 | 463.70 | 89.30 | 26.90 | 1148.65 |
| 4 | 878.40 | 509.85 | 132.60 | 79.03 | 1599.88 |
| 5 | 1048.70 | 675.47 | 145.86 | 112.71 | 1982.74 |
| 6 | 1102.65 | 858.00 | 231.52 | 131.30 | 2323.47 |
| 7 | 1107.60 | 1054.82 | 299.77 | 155.35 | 2617.54 |
| 8 | 1229.66 | 1715.73 | 205.92 | 150.80 | 3302.11 |
| 9 | 1383.71 | 1909.56 | 320.19 | 180.30 | 3793.76 |
| 10 | 1406.07 | 2074.27 | 327.98 | 223.72 | 4032.04 |
| 11 | 1437.80 | 2088.45 | 238.02 | 200.45 | 3964.72 |
| 12 | 1384.50 | 2444.00 | 131.30 | 254.80 | $4214.60 *$ |
| 13 | 1160.90 | 2620.80 | 93.60 | 166.40 | 4041.70 |
| 14 | 1137.50 | 2592.20 | 67.60 | 318.50 | 4115.80 |
| 15 | 999.70 | 1775.80 | 52.00 | 301.60 | 3129.10 |

*Denotes growth phase peak.

TABLE IV
PROGRAM "D" DIRECT EFFORT EXPENDITURES (Thousands of Manhours)

| Qtr. | E | M | T | Q | Totals |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 47.31 | 7.01 | 11.56 | 2.07 | 67.95 |
| 2 | 192.26 | 28.46 | 18.20 | 8.57 | 247.49 |
| 3 | 361.26 | 37.44 | 92.94 | 13.64 | 505.28 |
| 4 | 376.86 | 52.00 | 97.50 | 20.80 | 547.16 |
| 5 | 436.53 | 98.80 | 162.50 | 28.60 | 726.43 |
| 6 | 520.91 | 468.00 | 247.00 | 39.00 | 1274.91 |
| 7 | 588.64 | 435.50 | 156.00 | 52.00 | 1232.14 |
| 8 | 557.95 | 494.00 | 162.50 | 91.00 | 1305.45 |
| 9 | 522.20 | 747.50 | 130.00 | 84.50 | 1484.20 |
| 10 | . 401.56 | 845.00 | 240.50 | 84.50 | 1571.56 |
| 11 | 459.94 | 858.00 | 234.00 | 130.00 | 1681.94 |
| 12 | 830.05 | 923.00 | 200.97 | 156.00 | 2110.02 |
| 13 | 643.10 | 838.88 | 178.75 | 130.00 | 1790.73 |
| 14 | 722.80 | 987.60 | 190.70 | 156.00 | 2057.10 |
| 15 | 930.93 | 882.95 | 168.08 | 188.50 | 2170.46* |
| 16 | 658.84 | 601.90 | 104.00 | 165.10 | 1529.84 |
| 17 | 683.67 | 658.45 | 88.40 | 161.20 | 1591.72 |
| 18 | 495.95 | 343.07 | 71.50 | 131.30 | 1041.82 |

* Denotes growth phase peak.


## TABLE V

PROGRAM "F" DIRECT EFFORT EXPENDITURES (Thousands of Manhours)

| Qtr. | E | M | T | Q | Totals |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 108.16 | 16.25 | 3.90 | 2.46 | 130.77 |
| 2 | 216.32 | 35.36 | 8.84 | 5.32 | 265.84 |
| 3 | 310.96 | 54.08 | 16.25 | 10.01 | 391.30 |
| 4 | 439.40 | 108.16 | 28.60 | 18.45 | 594.61 |
| 5 | 567.83 | 199.42 | 43.93 | 27.04 | 840.22 |
| 6 | 676.00 | 277.41 | 81.11 | 33.80 | 1068.32 |
| 7 | 811.20 | 381.94 | 125.05 | 60.83 | 1379.02 |
| 8 | 973.43 | 473.20 | 155.48 | 67.60 | 1669.71 |
| 9 | 1061.32 | 517.14 | 152.10 | 87.87 | 1818.43 |
| 10 | 1183.00 | 550.94 | 152.10 | 121.67 | 2007.71 |
| 11 | 1284.40 | 574.60 | 148.72 | 135.20 | 2142.92 |
| 12 | 1372.27 | 605.02 | 145.33 | 169.00 | 2291.62 |
| 13 | 1439.87 | 645.57 | 145.33 | 182.51 | 2413.28 |
| 14 | 1487.19 | 686.13 | 145.33 | 189.27 | 2507.92 |
| 15 | 1548.03 | 753.73 | 145.33 | 196.03 | 2643. 12 |
| 16 | 1622.40 | 807.81 | 145.33 | 202.80 | 2778.34* |
| 17 | 1541.28 | 814.58 | 158.86 | 202.80 | 2717.52 |
| 18 | 1480.44 | 824.72 | 162.24 | 202.80 | 2670.20 |
| 19 | 1426.36 | 827.84 | 165.62 | 202.80 | 2622.62 |

*Denotes growth phase peak.
coming from the same population. In this investigation, the question arises as to whether the five R\&D launch vehicle programs are derived from the same or similar populations. Almost invariably, samples will differ, and the question is whether the differences signify differences among the populations, or are merely the chance variations to be expected among random samples from the same population. When this problem arises, one tends to assume that the samples are of approximately the same form, in the sense that if they differ it is merely due to shift or translation.

Friedman (6) states that the "method of ranks" can be applied to data classified by two or more criteria to determine whether the factors used as criteria of classification have a significant influence on the variate classified. Stated differently, the "method of ranks" tests the hypothesis that the values of the variate, corresponding to each subdivision by one of the factors, are homogeneous, that is, from the same universe.

The "method of ranks" utilizes information based solely on "order" and makes no use of the quantitative values of the variate as such. For this reason, an assumption need not be made as to the nature of the underlying universe. Since the nature of the underlying universe of R\&D launch vehicle programs is unknow at this time, it appears that a non-parametric rank test will provide the necessary information to make a determination of sample population similarity.

The rank test to be utilized for this determination is the Kruskal-Wallis (7) H-Test. The H-Test requires that all the observations be ranked together, that is, to array the $N$ observations in order of magnitude and replace the smallest by one, the next to the smallest by two, and so on, the largest being replaced by $N$, and then, the sum of the ranks obtained for each sample. The test statistic to be computed, provided there are no ties, is:

$$
H=\frac{12}{N(N+1)} \quad \sum_{i=1}^{c} \frac{R_{i}^{2}}{n_{i}}-3(N+1)
$$

where

$$
\begin{aligned}
c= & \text { the number of samples, } \\
n_{i}= & \text { the number of abservations in the ith sample, } \\
\mathbb{N}= & \sum n_{i}, \text { the number of observations in all } \\
& \text { samples combined, and } \\
R_{i}= & \text { the sum of the ranks in the ith sample. }
\end{aligned}
$$

The null hypothesis associated with the H-Test is that the samples all come from the same or identical populations. Large values of $H$ lead to the rejection of the null hypothesis, while small values lead to acceptance. Since the $n_{i}$ are not too small, and the samples come from the continuous populations, $H$ is distributed as chi-square, permitting use of readily available tables of chi-square.

To acquire the necessary values for the H-Test, it is necessary to sum the growth phase effort expenditure values, by effort category, for each of the five R\&D launch vehicle programs. This results in a $4 \times 5$ matrix as shown in

Table VI. The values in Table VI are now ranked for all N values, with $\mathrm{N}=20$, as shown in Table VII.

The necessary mathematical operations, which permit direct substitution in the H-Test equation above, are also shown in Table VII。 Substituting the calculated values into the equation results in the following:

$$
H_{c a l}=\frac{12(2323)}{20(21)}-3(21)=3.36
$$

Entering the chi-square table with $c-1=4$ degrees of freedom, with an $\alpha=0.05$, we note the table value of $H$ is 9.488. Since the calculated $H$ velue is less than the table $H$ value, we accept the null hypothesis. We conclude the five R\&D launch vehicle programs are not significantly different from one another, and furthermore, are derived from the same population. If the programs are, in fact, from different populations, this difference in population sources is not detectable from the sample data, at the specified value of alpha. Hence, from a statistical point of view, we are confident that the five programs possess similar characteristics and attributes.

Thus, the data collection phase of the investigation resulted in the acquisition of data on five R\&D launch vehicle programs, with a growth phase total of 75 observations for each effort category. The nonparametric H-Test resulted in the acceptance of the null hypothesis that the samples were derived from the same population. The chapter which follows initiates the data analysis phase of the

TABLE VI
TOTALS OF GROWTH PHASE DIRECT EFFORT EXPENDITURES (Thousands of Manhours)

| Effort <br> Category | Program |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| E | 13021.76 | 22106.45 | 11899.48 | 7592.30 | 15101.78 |
| WI | 18780.69 | 26245.68 | 13945.43 | 7704.14 | 6686.76 |
| T | 2950.14 | 4449.95 | 2206.69 | 2291. 20 | 1642.73 |
| Q | 3478.54 | 6214.00 | 1539.28 | 1185.18 | 1509.86 |

TABLE VII
H-TEST ON TOTALS OF GROWTH PHASE DIRECT EFFORT EXPENDITURES


## DATA ANALYSIS

The Approach

The data analysis phase of any investigation must be thoroughly outlined from initiation through completion if unnecessary computations and operations are to be avoided. Such a procedure would undoubtedly begin with the desired objective and then establish the sequence of basic mathematical and statistical operations necessary to attain this objective. Then the logic and methodology may be fully developed toward the achievement of the overall objective.

Since the desired objective is a predictive and evaluative growth phase model for R\&D launch vehicle programs, the logic and rationale shall be developed toward this goal. Functional relationships are considered to depict the change of the capabilitymix as a function of time. Since 75 data points are available for each ratio group, with a corresponding time value, it appears that regression analysis or the method of least squares would provide the desired results. The regression equation would serve as a predictive model; the regression equation with applicable prediction limits would serve as the evaluative model.

However, prior to subjecting the data to regression analysis, determination of the underlying distribution for each ratio group is necessary. Regression analysis and the method of least squares assumes the dependent variable data is derived from a population with a normal distribution. Therefore, it will be necessary to perform a normality test in an effort to make this determination.

Essentially the same input is required for both the normality test and the regression analysis. Thus, the four categories of collected empirical data will be converted into this input, namely, six distinct, yet interrelated ratios. The first phase of the data analysis may now be initiated。

Rapid turn-around time and easy manipulation of the data is necessary if a predictive or evaluative technique is to be useful. With this concept in mind, computer programs have been developed and utilized during the data analysis phase of the investigation for those procedures which lend themselves most readily to computer operations.

## Ratio Calculations

The collected empirical growth phase deta for the four effort categories shown in Tables I through V must now be placed in a fashion which will permit the other phases of the analysis to proceed. In essence, this implies the calculation of the six effort expenditure ratios as a function of time, as defined in Chapter II. The Ratio

Program, described in Appendix A-1, was utilized in making these computations. The resulting effort expenditure ratios, together with the corresponding growth phase time values, for the five considered programs, are show in Tables VIII through XII. The associated growth phase time values represent the 75 increments of time being considered for the five programs, for each of the six ratio groups.

It should be noted that the magnitudes of these ratios vary considerably between ratio groups. The largest ratio value is observed for the $E / Q$ ratio group, while the smallest ratio value is noted for the $Q / T$ ratio group. These ratios depict the capability-mix which exists or existed during the specified period of time. As an example, an $E / M$ ratio of 6.7 at atime of 0.0666 indicates that during the first quarter of a 15 quarter program, six and seventenths hours of engineering effort were expended for each hour of manufacturing effort expended. The interrelationships between the six ratio groups become obvious, since a change in any one of the four basic effort expenditure categories will cause an incremental change in three of the six ratio groups. It becomes immediately evident that indiscriminate basic effort expenditure value changes cannot occur without detection。 Furthermore, any attempt to compensate for one change in the basic effort expenditure category, with still another change, will cause at least five ratio values to be involved. Thus, additional constraints are being placed upon the estimator, forcing him to
table VIII
PROGRAM 'A' DIRECT EFFORT EXPENDITURE RATIOS

| $\begin{gathered} \text { TIME } \\ 0.0625 \end{gathered}$ | $\begin{gathered} E / M \\ 0.8636 \end{gathered}$ | $\begin{gathered} E / T \\ 5.4308 \end{gathered}$ | $\begin{gathered} E / Q \\ 4.2236 \end{gathered}$ | $\begin{aligned} & M / T \\ & 6.2883 \end{aligned}$ | $\begin{gathered} M / Q \\ 4.8904 \end{gathered}$ | $\begin{aligned} & Q / T \\ & 1.2858 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1250 | 0.8426 | 5.3582 | 4.2583 | 6.3584 | 5.0532 | 2.2582 |
| 0.1875 | 0.8470 | 5.4847 | 3.9934 | 6.4753 | 4.7146 | 1.3734 |
| 0.2500 | 0.8361 | 5.3597 | 4.0002 | 6.4101 | 4.7841 | 1.3398 |
| 0.3125 | 0.7992 | 5.2470 | 3.8779 | 6.5649 | 4.8519 | 1.3530 |
| 0.3750 | 0.8052 | 5.3238 | 3.8394 | 6.6113 | 4.7680 | 1.3865 |
| 0.4375 | 0.7719 | 5.1571 | 3.7289 | 6.6808 | 4.8307 | 1.3829 |
| 0.5000 | 0.7642 | 4.7357 | 3.6637 | $6.296 ?$ | 4.7940 | 1.2925 |
| 0.5625 | 0.7515 | 4.3845 | 3.6443 | 5.8342 | 4.8491 | 2.2032 |
| 0.6250 | 0.7379 | 4.2463 | 3.6951 | 5.7546 | 5.0076 | 1.1491 |
| 0.6875 | 0.7223 | 4.0339 | 3.8641 | 5.5844 | 5.3492 | 1.0439 |
| 0.7500 | 0.7090 | 4.0521 | 3.9344 | 5.7149 | 5.5489 | 1.0299 |
| 0.8125 | 0.6977 | 4.1088 | 3.8977 | 5.8891 | 5.5864 | 1.0541 |
| 0.8750 | 0.6915 | 4.1108 | 3.7748 | 5.9445 | 5.4586 | 2.0890 |
| 0.9375 | 0.6738 | 4.0884 | 3.5568 | 6.0674 | 5.2785 | 1.1494 |
| 2.0000 | 0.6661 | 3.9887 | 3.3056 | 5.9879 | 4.9623 | 1.2066 |

## TABLE IX

PROGRAM 'B' DIRECT EFFORT EXPENDITURE RATIOS

| TIME | $E / M$ | $E / T$ | $E / Q$ | $M / T$ | $M / Q$ | $0 / T$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0625 | 5.3231 | 15.9694 | 26.3496 | 3.0000 | 4.9499 | 0.6060 |
| 0.1250 | 5.0181 | 12.2664 | 14.5261 | 2.4444 | 2.8947 | 0.8444 |
| 0.1875 | 3.4050 | 11.8104 | 9.4067 | 3.4684 | 2.7625 | 1.2555 |
| 0.2500 | 2.0037 | 4.6810 | 2.1294 | 2.3361 | 1.0627 | 2.1982 |
| 0.3125 | 1.7457 | 4.1678 | 3.5870 | 2.3873 | 2.0547 | 1.1619 |
| 0.3750 | 1.2064 | 3.6785 | 4.1616 | 3.0491 | 3.4494 | 0.8839 |
| 0.4375 | 0.5628 | 3.3491 | 3.6478 | 3.4784 | 3.7887 | 0.9181 |
| 0.5000 | 1.0547 | 3.9075 | 4.1123 | 3.7047 | 3.8988 | 0.9502 |
| 0.5625 | 0.6268 | 4.1957 | 3.0060 | 6.6936 | 4.7957 | 1.3957 |
| 0.6250 | 0.6429 | 4.2990 | 3.2361 | 6.6865 | 5.0333 | 1.3284 |
| 0.6875 | 0.6524 | 4.4408 | 2.9712 | 6.8064 | 4.5539 | 1.4946 |
| 0.7500 | 0.6946 | 4.0848 | 3.1150 | 5.8805 | 4.4844 | 1.3113 |
| 0.8125 | 0.8062 | 4.5690 | 3.3178 | 5.6666 | 4.1149 | 1.3371 |
| 0.8750 | 0.8424 | 5.9789 | 4.3977 | 7.0968 | 5.2200 | 1.3595 |
| 0.9375 | 0.5895 | 5.3791 | 3.0075 | 9.1241 | 5.2013 | 1.7885 |
| 1.0000 | 0.6599 | 5.5625 | 2.8943 | 8.4281 | 4.3853 | 1.9218 |

TABLE X
PROGRAM 'C' DIRECT EFFORT EXPENDITURE RATIOS

| TIME | $E / M$ | $E / T$ | $E / Q$ | $M / T$ | $M / Q$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0833 | 1.9815 | 3.5982 | 12.7619 | 1.8159 | 6.4404 | $0 / T$ |
| 0.1666 | 2.6126 | 4.6654 | 16.3292 | 1.7857 | 6.2500 | 0.2857 |
| 0.2500 | 1.2265 | 6.3689 | 21.1431 | 5.1926 | 17.2379 | 0.3012 |
| 0.3333 | 1.7228 | 6.6244 | 11.2147 | 3.8450 | 6.4513 | 0.5960 |
| 0.4166 | 1.5525 | 7.1897 | 9.3044 | 4.6309 | 5.9929 | 0.7727 |
| 0.5000 | 1.2851 | 4.7626 | 8.3979 | 3.7059 | 6.5346 | 0.5671 |
| 0.5833 | 1.0500 | 3.6948 | 7.1297 | 3.5187 | 6.7899 | 0.5182 |
| 0.6666 | 0.7166 | 5.9715 | 8.1542 | 8.3320 | 11.3775 | 0.7323 |
| 0.7500 | 0.7246 | 4.3215 | 7.6744 | 5.9638 | 10.5910 | 0.5631 |
| 0.8333 | 0.6778 | 4.2870 | 6.2849 | 6.3243 | 9.2717 | 0.6821 |
| 0.9166 | 0.6884 | 6.0406 | 7.2728 | 8.7742 | 10.4188 | 0.8421 |
| 1.0000 | 0.5664 | 10.5445 | 5.4336 | 18.6138 | 9.5918 | 1.9405 |

TABLE XI
PROGRAM 'D' DIRECT EFFORT EXPENDITURE RATIOS

| TIME | E/M |
| :--- | :--- |
| 0.0666 | 6.7489 |
| 0.1333 | 6.7554 |
| 0.2000 | 9.6490 |
| 0.2666 | 7.2473 |
| 0.3333 | 4.4183 |
| 0.4000 | 1.1130 |
| 0.4666 | 1.3516 |
| 0.5333 | 1.1294 |
| 0.6000 | 0.6985 |
| 0.6666 | 0.4752 |
| 0.7333 | 0.5360 |
| 0.8000 | 0.8992 |
| 0.8666 | 0.7666 |
| 0.9333 | 0.7318 |
| 1.0000 | 1.0543 |

$E / T$
4.0925
10.5637
3.8870
3.8652
2.6863
2.1089
3.7733
3.4335
4.0169
1.6696
1.9655
4.1302
3.5977
3.7902
5.5386
$E / Q$
22.8550
22.4340
26.4853
28.1182
15.2632
13.3566
11.3199
6.1313
6.1798
4.7521
3.5379
5.3208
4.0469
4.6333
4.9386
$M / T$
0.6064
1.5637
0.4028
0.5333
0.6080
1.8947
2.7916
3.0400
5.7500
3.5135
3.6666
4.5927
4.6930
5.2788
5.2531
$M / Q$
3.3864
3.3208
2.7448
2.5000
3.4545
12.0000
8.3750
5.4285
8.8461
10.0000
6.6000
5.9166
6.4529
6.3307
4.6840

Q/T 0.1790
0.4708
0.1467
0.2233
0.1759
0.1578
0.3333
0.5600
0.6500
0.3513
0.5555
0.7762
0.7272
0.8180
1.1214

TABLE XII
PROGRAM 'F' DIRECT EFFORT EXPENDITURE RATIOS

| $T I M E$ | $E / M$ | $E / T$ | $E / Q$ |  | M/T | $M / Q$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.0625 | 6.6560 | 27.7333 | 43.9674 | 4.1666 | 6.6056 | 0.6307 |
| 0.1250 | 6.2176 | 24.4705 | 40.6616 | 4.0000 | 6.6466 | 0.6018 |
| 0.1875 | 5.7500 | 19.1359 | 31.0649 | 3.3280 | 5.4025 | 0.6159 |
| 0.2500 | 4.0625 | 15.3636 | 23.8157 | 3.7818 | 5.8623 | 0.6451 |
| 0.3125 | 2.8474 | 12.9257 | 20.9996 | 4.5394 | 7.3750 | 0.6155 |
| 0.3750 | 2.4368 | 8.3343 | 20.0000 | 3.4201 | 8.2073 | 0.4167 |
| 0.4375 | 2.1238 | 6.4870 | 13.3355 | 3.0542 | 6.2788 | 0.4864 |
| 0.5000 | 2.0571 | 6.2608 | 14.3998 | 3.0434 | 7.0000 | 0.4347 |
| 0.5625 | 2.0522 | 6.9777 | 12.0782 | 3.4000 | 5.8852 | 0.5777 |
| 0.6250 | 2.1472 | 7.7777 | 9.7230 | 3.6222 | 4.5281 | 0.7999 |
| 0.6875 | 2.2352 | 8.6363 | 9.5000 | 3.8636 | 4.2500 | 0.9090 |
| 0.7500 | 2.2681 | 9.4424 | 8.1199 | 4.1630 | 3.5799 | 1.1628 |
| 0.8125 | 2.2303 | 9.9075 | 7.8892 | 4.4420 | 3.5371 | 1.2558 |
| 0.8750 | 2.1675 | 10.2331 | 7.8575 | 4.7211 | 3.6251 | 1.3023 |
| 0.9375 | 2.0538 | 10.6518 | 7.8969 | 5.1863 | 3.8449 | 1.3488 |
| 1.0000 | 2.0083 | 11.1635 | 8.0000 | 5.5584 | 3.9832 | 1.3954 |

exercise logic and rationale in arriving at the estimated capability-mix requirements for a proposed program, rather than merely distributing the manpower effort in a haphazard fashion to fulfill a total manpower estimate of some type.

## Establishing Ratio Data Normality

The technique to be utilized in the analysis of the collected data is regression analysis, which assumes the sample dependent variable data is derived from a population with a normal distribution. Thus, it is important to determine the distribution from which the sample dependent variable data was obtained. In this investigation, the calculated ratios for the six effort ratio groups are the sample dependent variable data. Each effort ratio group will be analyzed individually, since it is assumed that each effort ratio group will possess a different functional relationship, and therefore may possess a different underlying distribution。

Ostle (8) states that the assumption of independence, or granting normality to the dependent variable data, is a crucial assumption and its importance should not be overlooked. A definite determination as to the validity of the normality assumption should be made if at all possible. If the data is found to be non-normal in nature, it is permissible to apply a transformation, i.e., logarithmic, square root, cosine, exponential, or some other logical function, to cause the dependent variable data distribution
to approach normality. This procedure does not have an adverse effect upon the data, but must be considered when using the regression results for model building purposes.

Bennett and Franklin (9) suggest the use of normal probability graph paper in the testing of moderately large samples ( $n>50$ ) for non-normality. The size of the dependent variable sample (75) is observed to be sufficiently large to utilize this procedure. However, in the interest of utilizing a more rapid and precise technique, recourse had been made to one of the simplest methods for testing normality, namely, the "Chi Square" test. The grouping of the dependent sample variable data into class intervals is necessary for this procedure to achieve valid results. The "Chi Square" test does not prove normality as such, but gives no reason to suspect the data is non-normal in nature, provided the results are acceptable.

A computer program was developed which performs the ordinary "Chi Square" or "goodness of fit" procedure. A complete description of this computer program together with its precise formulation is given in Appendix A-2. The initial results for the dependent variable data of each effort ratio group, with the data in its original fashion, are shown in Table XIII. It is readily apparent that the original effort ratio distribution for each effort ratio group is non-normal in nature. The calculated value exceeds the table value for each of the six ratio groups by a considerable amount. Hence, the hypothesis of normality is

TABLE XIII
NORMALITY TEST RESULTS FOR THE ORIGINAI
EFFORT RATIO DISTRIBUTIONS

| Ratio | $x^{2} \mathrm{cal}$ | $x^{2} \operatorname{table}(0.05)$ | d.f. | Hypothesis of Normality |
| :--- | :--- | :--- | :--- | :--- |
| $E / M$ | 27.29 | 3.84 | 1 | Reject |
| $E / T$ | 12.51 | 5.99 | 2 | Reject |
| $E / Q$ | 9.42 | 5.99 | 2 | Reject |
| $M / T$ | 9.63 | 3.84 | 17.41 | 7.82 |

rejected and a transformation of some type becomes necessary. Scatter diagrams for the original ratio data as a function of time suggest that perhaps a natural logarithmic transformation would cause certain ratio groups to approach normality. However, experience dictates that a simple natural logarithmic transformation may create more problems than it solves. Since the natural logarithm of one is zero, the natural logarithm of a decimal number is a negative value, and the natural logarithm of zero is negative infinity, it was judged best to add unity to each ratio value before the transformation was accomplished. Hence, the first transformation utilized on all ratio groups was the natural logarithm $[Y(I)+1]$, where $Y(I)$ represents the various values of the dependent variable. Table XIV indicates that this transformation was satisfactory for the $E / Q$ and $Q / T$ ratio groups. However, the other ratio groups did not approach normality with the application of the logarithmic transformation.

The square root is a common transformation frequently utilized when transforming data。 This particular transformation does not cause any unusual problems within the realm of this investigation since the input ratios are always positive. Thus, the double root aspect need not be considered, since all values of the square root for this investigation are obviously positive. Referring to Table XIV, we note that the $\mathbb{M} / \mathbb{T}$ ratio group was the only group of the remaining four to approach normality with this

TABLE XIV
NORMALITY TEST RESULTS FOR THE TRANSFORNED EFFORT RATIO DISTRIBUTIONS

| Ratio | Transformation | $X^{2}$ cal. | $X^{2}$ table(0.05) | d.f. | Hypothesis of <br> Normality |
| :--- | :--- | :--- | :---: | :---: | :---: |
| $E / M$ | $\operatorname{SQRT}[Y(I)+2] / Y(I)$ | 5.76 | 5.99 | 2 | Accept |
| $E / T$ | $Y(I) /[Y(I)+1]$ | 5.15 | 5.99 | 2 | Accept |
| $E / Q$ | $\operatorname{LN}[Y(I)+1]$ | 5.98 | 5.99 | 2 | Accept |
| $M / T$ | $\operatorname{SQRT}[Y(I)]$ | 2.58 | 5.99 | 2 | Accept |
| $M / Q$ | $\operatorname{IN}[\operatorname{IN}[Y(I)+1]]$ | 5.22 | 5.99 | 2 | Accept |
| $Q / T$ | $\operatorname{LN}[Y(I)+1]$ | 4.86 | 7.82 | 3 | Accept |

transformation. A variation of the square root transformation was found applicable for the $E / \mathbb{M}$ ratio group. It should be noted that only the numerator of the expression $[Y(I)+2] / Y(I)$ has the square root operation performed upon it. The denominator performs in its true state, as a variable, and has no mathematical operation performed upon it prior to utilization.

Many other transformations were attempted upon the remaining two ratio groups. The E/T ratio group was observed to approach normality with a $Y(I) /[Y(I)+1]$ transformation. The $\mathbb{M} / Q$ ratio group was not as simple. After attempting a large number of single operation transformations upon this category without success, a double operation transformation in the form of $\operatorname{LN}[\operatorname{IN}[Y(I)+1]]$ was attempted. The dependent variable data was observed to approach normality with this transformation.

The dependent variable data was classified into five class intervals for the first five ratio groups, and into six class intervals for the $Q / T$ ratio group during the goodness of fit procedure. Since three degrees of freedom are lost for the estimated parameters, the respective degrees of freedom are show in Table XIV.

Thus, the hypothesis of normality is found acceptable for each of the ratio groups within the constraints of the specified transformation. Any manipulations with the dependent variable data must now be accomplished with the dependent variable data in its transformed state. An
analysis of regression may now be performed upon the transformed ratio data as a function of time, with reasonable confidence that the data analyzed possesses an underlying distribution which approaches a normal distribution.

## Functional Relationship Development

In most physical sciences, relationships are commonly determined through controlled experiments. In the social sciences and in certain physical sciences, like astronomy, controlled experiments may be impossible, or at least very difficult. Relationships must in such cases be discovered by analyzing the data as it becomes available. The tool which was devised to accomplish this is regression analysis. Often, laboratory conditions cannot be set up that will exactly reproduce conditions within a controlled environment. Consequently, the researcher is frequently in the position of the social scientist and astronomer, in that he must take the data as he finds them. Hence, regression analysis is a very useful tool of both management and industrial research. Regression analysis or the method of least squares fits a line or a curve to a set of sample points such that the sum of the squares of the deviations of the sample points from the fitted line or curve is a minimum. The method of least squares is mathematical and impersonal. Duncan (9) states that if the variations around the regression are random, the method of least squares permits the computation of sampling errors and hence the determination of the
reliability of dependent variable estimates from the fitted line. Furthermore, if the distribution of points around the regression is not only random, but normal in form, then the least squares method gives the maximum likelihood estimate of the universe regression. Hence, lines and curves of regression are commonly estimated from sample data by the method of least squares.

The computer program developed for the regression analysis portion of the investigation solves the normal equations by the least squares method. The program is designed to transform the effort ratio data into the form specified by the normality portion of the investigation. This capability permitted the use of the same input data as utilized with the Chi-square program. The large amount of core storage necessary for the Regression-Limit Program prevented one continuous program from being utilized.

In addition to the regression coefficients, the computer program provides an analysis of regression variance. This analysis partitions the total dependent variable variation into that portion due to the regression upon the independent variable, and that portion attributable to other causes (about the regression or erfor). The appropriate degrees of freedom are provided together with the mean square values. Thus, the values necessary for an "F" test are readily available。

The null hypothesis associated with the analysis of regression variance "F" test may be stated as follows:
$H_{0}: Y(I)$ is independent of $X(I)$,
where
$Y(I)$ represents the effort ratio groups, and $X(I)$ is the corresponding time value. The "F" values provided with the regression coefficients will permit the acceptance or rejection of this null hypothesis. If the calculated "F" value is smaller than the appropriate table "F" value, the null hypothesis is accepted; a calculated "F" value larger than the appropriate table "F" value will cause the null hypothesis to be rejected. It is desired that the null hypothesis be rejected, permitting the assumption that $Y(I)$ is dependent upon $X(I)$. This would not prove that $Y(I)$ is dependent upon $X(I)$, but would leave little or no reason to believe the two variables are independent.

Table XV summarizes the regression analysis original results, that is, the dependent variable effort ratio data within the transformation constraints determined by the normality procedure。 It is noted that the $E / \mathbb{M}, E / Q, M / T$, and $Q / T$ effort ratio groups reject the established null hypothesis. In fact, the test of the null hypothesis that $Y(I)$ is dependent upon $X(I)$ is clearly significant at the 99 per cent confidence level for these four effort ratio groups. However, the $E / T$ and $\mathbb{M} / Q$ effort ratio groups display functional relationships which accept the null hypothesis even at the 95 per cent confidence level. It should be noted that the $M / Q$ effort ratio group rejects the

TABLE XV
REGRESSION ANALYSIS ORIGINAI RESULTS

| Ratio | Functional Relationship | $\mathrm{F}_{\mathrm{cal}}$ 。 | $\mathrm{F}_{\text {table }}(0.05)$ | $F_{\text {table }}(0.01)$ | Null Hypothesis |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{E} / \mathrm{M}$ | $Y=0.7721+1.5665 X$ | 38.89 |  | 6.99 | Reject |
| $\mathrm{E} / \mathrm{T}$ | $Y=0.8584-0.0398 \mathrm{X}$ | 2.44 | 3.97 |  | Accept |
| E/Q | $Y=2.7904-1.2443 X$ | 31.37 |  | 6.99 | Reject |
| $M / T$ | $Y=1.5237+1.1075 X$ | 33.59 |  | 6.99 | Reject |
| M/Q | $Y=0.5180+0.1367 X$ | 2.91 | 3.97 |  | Accept |
| $Q / T$ | $\mathrm{Y}=0.4342+0.3538 \mathrm{X}$ | 15.76 |  | 6.99 | Reject |

null hypothesis when compared with a table $F_{0,10}=2.77$. However, the $\mathrm{E} / \mathbb{T}$ effort ratio group continues to accept the null hypothesis even at this confidence level.

Since it is desirable to reject the null hypothesis with at least 95 per cent confidence, one of two approaches immediately present themselves. The first, and perhaps the least desirable, is to eliminate the $E / \mathbb{T}$ and $\mathbb{M} / Q$ effort ratio groups from the total model. Obviously, this would result in a model which would not be as effective, since two of the six interdependencies would be eliminated. Hence, this approach is immediately rejected. A more feasible solution to the existing problem is to retain these two effort ratio groups within the realm of the overall model. In an effort to acquire functional relationships which reject the established null hypothesis, the independent variable will be studied. An attempt will be made to determine the type of transformation which will convert the originally developed functional relationship into such a fashion which causes rejection of the null hypothesis. Until this point of the investigation, the independent variable has not been transformed in any fashion. Hence, the independent variable has maintained a linear or untransformed condition in the form of input values of time. This linear condition for the independent variable will be maintained for the four effort ratio groups which have rejected the null hypothesis. During the search for a suitable transformation for the respective $E / T$ and $M / Q$
independent variable, the dependent variable must maintain the transformation constraints which were established previously.

The search for the $M / Q$ independent variable transformation proved more challenging and tedious than any of the prior transformations. After exhausting all known "simple" transformations with little or no success, an attempt was made to utilize double operation transformations similar to the type utilized for the dependent variable of this group. When this failed, the following procedure was utilized to arrive at an acceptable transformation.

Let

$$
\ln (y+1)=a^{x}
$$

where

$$
\begin{aligned}
& y=Y(I), \\
& x=X(I)
\end{aligned}
$$

But

$$
\ln (y+1)=Y^{\prime}
$$

$$
Y^{\prime}=a^{x}
$$

$$
\ln Y^{\prime}=x \ln a
$$

$$
\ln Y^{\prime}=Y^{\prime \prime}
$$

where

$$
Y^{\prime \prime}=b_{1} x+b_{0}
$$

When

$$
b_{0} \rightarrow 0,
$$

the resulting expression is:

$$
\ln a=b_{1}
$$

Taking the natural logarithm of both sides gives

$$
a=e^{b_{1}}
$$

Utilizing the developed regression program, the dependent variable was constrained to the transformation $\operatorname{LN}[\operatorname{LN}[Y(I)+1]]$, and the coefficient $b_{1}$ was obtained.

Thus,

$$
\begin{aligned}
b_{1} & =0.1371 \text { and } \\
a & =e^{0.1371} .
\end{aligned}
$$

Locating this value in natural logarithm tables we find:

$$
a=1.1469 \text {. }
$$

Substituting in the original equation above, we find:

$$
\ln (y+1)=1.1469^{x}
$$

However, the constraining transformation on the dependent variable is $\operatorname{LN}[\operatorname{IN}[Y(I)+1]]$. Hence, we must revert to this acceptable dependent variable transformation, and try various combinations of the independent variable transformation developed above. The $X(I)$ values were transformed into the following forms:
(a)

$$
\mathrm{a}^{\mathrm{X}} \text { where } \mathrm{x}=\mathrm{X}(\mathrm{I})
$$

$$
a=1.1469
$$

(b)
(c)


$$
\left[\frac{\frac{x}{a^{x}}}{\frac{x}{a^{x}}-0.5367}\right] \text { where } 0.5367 \text { approximates } \bar{x}
$$

(d)

$$
\left|\frac{\frac{x}{a^{x}}}{\frac{x}{a^{x}}-0.5367}\right|
$$

(e)

$$
\text { LN }\left|\frac{\frac{x}{a^{x}}}{\frac{x}{a^{x}}-0.5367}\right|
$$

Combination (e) resulted in a calculated $F$ value of 4.056, which rejects the null hypothesis at the 95 per cent confidence level. Since considerable manipulation was
required to achieve this $F$ value, it was considered best to cease the search for a transformation which would provide a higher $F$ value. Hence, this independent variable transformation was accepted as is designated in Table XVI.

The search for an independent variable transformation for the $E / T$ effort ratio group was not exceedingly difficult. Referring to Table XVII, we note the transformation to be $\operatorname{SQRT}[X(I)+0.6]$. With the $E / T$ independent variable within the constraint of this transformation, the null hypothesis is observed as rejected with 99 per cent confidence, as shown in Table XVI.

Referring to Table XVI, we note a summary of the developed functional relationships, together with the appropriate values of $F$. The $M / Q$ effort ratio group is the sole group which is not acceptable at the 99 per cent confidence level. However, this is not expected to affect the total model in an adverse fashion. We may conclude, with at least 95 per cent confidence, that the effort ratio data varies as a function of growth phase time while within the constraints of the dependent and independent variable transformations summarized in Table XVII.

## Prediction Limits for Individual Values

The use of the functional relationships developed in the previous section depends, to a large extent, upon the ease with which an acceptable range of values can be stated for a given value of the independent variable. Since the

## TABLE XVI

REGRESSION ANALYSIS FINAI RESULTS

| Ratio | Functional Relationship | $F_{c a l}$ | $F_{\text {table }}(0.05)$ | $\left.F_{\text {table }} 0.01\right)$ | Null Hypothesis |
| :--- | :--- | :---: | :---: | :---: | :---: |
| $E / M$ | $Y=0.7721+1.5665 X$ | 38.89 | 6.99 | Reject |  |
| $E / T$ | $Y=1.0163-0.2973 X$ | 8.63 | 6.99 | Reject |  |
| $E / Q$ | $Y=2.7904-1.2443 X$ | 31.37 | 6.99 | Reject |  |
| $M / T$ | $Y=1.5237+1.1075 X$ | 33.59 | 6.99 | Reject |  |
| $M / Q$ | $Y=0.5615+0.0319 X$ | 4.06 | 3.97 | Reject |  |
| $Q / T$ | $Y=0.4342+0.3538 X$ | 15.76 |  | Reject |  |

TABLE XVII
SUMMARY OF DEPENDENT AND INDEPENDENT
VARIABLE TRANSFORMATIONS

| Ratio | Variable Transformation | Variable Transformation |
| :--- | :--- | :--- |
| $E / M$ | $S Q R T[Y(I)+2] / Y(I)$ | $\operatorname{LINEAR}$ |
| $E / T$ | $Y(I) /[Y(I)+1]$ | $\operatorname{SQRT}[X(I)] /[X(I)+0.6]$ |
| $E / Q$ | $\operatorname{LN}[Y(I)+1]$ | $\operatorname{LINEAR}$ |
| $M / T$ | $\operatorname{SQRT}[Y(I)]$ | $\operatorname{LINEAR}$ |
| $M / Q$ | $\operatorname{LN}[\operatorname{LN}[Y(I)+1]]$ | $\operatorname{LN}\left[\operatorname{ABS}\left[\left(X / a^{X}\right) /\left[\left(X / a^{x}\right)-0.5367\right]\right]\right]$ |
| $Q / T$ | $\operatorname{LN}[Y(I)+1]$ | $\operatorname{LINEAR}$ |

values necessary to establish prediction limits for individual values were readily available within the regression computer program, a subroutine entitled "Limit" was developed to be utilized with the Regression Program.

Duncan (9) states that if we use the sample line of regression to estimate a particular value of $Y$, we must add to the error in the same line of regression some measure of the possible deviation of the individual value from the regression value. Likewise, Ezekiel and Fox (10) define the standard error of an individual forecast as composed of the error of points along the calculated regression line, plus that of individual estimates around that line.

The confidence limits for the regression estimate are given by Bartee (11) as:

$$
Y=b_{0}+b_{1} X_{0} \pm t_{\frac{\alpha}{2}} S_{e} \sqrt{\frac{1}{n}+\frac{\left(X_{0}-\bar{X}\right)^{2}}{\left(X_{i}-\bar{X}\right)^{2}}}
$$

where

$$
\begin{aligned}
\mathrm{b}_{0}= & \text { constant term of the regression equation, } \\
\mathrm{b}_{1}= & \text { coefficient of the independent variable of the } \\
& \text { regression equation, } \\
\mathrm{X}_{0}= & \text { individual value selected, } \\
t^{\frac{\alpha}{2}}= & \text { confidence interval for a two tail test statistic, } \\
\mathrm{S}_{\mathrm{e}}= & \text { standard error of the estimate, } \\
\mathrm{X}_{\mathrm{i}}= & \text { an independent variable value, and } \\
\tilde{\mathrm{X}}= & \text { mean of all independent variable values. }
\end{aligned}
$$

Expanding the above

$$
Y=b_{0}+b_{1} X_{0} \pm t \sqrt{\frac{\alpha}{2}} \sqrt{\frac{s_{e}^{2}}{n}+\frac{s_{e}^{2}\left(X_{0}-\bar{X}\right)^{2}}{\left(X_{i}-\bar{X}\right)^{2}}}
$$

The terms within the radical of the above expression account for the error of points along the calculated regression line. We must now add the standard error of individual estimates around the regression line, which is designated as $S_{e}$. Adding this term to the previous equation gives:

$$
Y=b_{0}+b_{1} X_{0} \pm t_{\frac{\alpha}{2}} \sqrt{S_{e}^{2}+\frac{S_{e}^{2}}{n}+\frac{S_{e}^{2}\left(X_{0}-\bar{X}\right)^{2}}{\left(X_{i}-\bar{X}\right)^{2}}}
$$

Simplifying yields:

$$
Y=b_{0}+b_{1} X_{0} \pm t_{\frac{\alpha}{2}} S_{e} \sqrt{1+\frac{1}{n}+\frac{\left(X_{0}-\bar{X}\right)^{2}}{\left(X_{i}-\bar{X}\right)^{2}}}
$$

The relationship with the regression program outputs are as follows:

$$
\begin{aligned}
S_{e} & =\sqrt{\text { MS about }} \\
\sum\left(X_{i}-\bar{X}\right)^{2} & =\frac{S_{\text {about }}}{b_{1}^{2}}
\end{aligned}
$$

Utilizing the above expression in the Limit Program, prediction loci are developed for the selected individual values. Five different values have been selected for which prediction limits for individual values are calculated. It should be noted that the prediction limits for $Y(I)$ get wider as $X(I)$ deviates from its mean, both positively and
negatively. This means that predictions of the dependent variable are subject to the least error when the independent variable is near its mean, and are subject to the greatest error when the independent variable is distant from its mean.

Prior to the utilization of the computer calculated prediction intervals, a definite rationale must be est tablished which corresponds with the overall objectives. The direct application of prediction limits to the developed functional relationships may cause situations to exist which are not logical. Hence, it is necessary to observe the values of the prediction limits, and then to exercise the necessary judgement to maintain a feasible model. Thus, the acceptable limits to be placed on individual values are based upon experience and judgement. Since the developed model is based upon data which exists within definite bounds, it is desirable to maintain acceptability values within these bounds. Therefore, if a calculated upper or lower predicted limit exceeds the input ratio value for that particular point in time, the predicted limit will be redefined and will assume the value of the extreme input ratio. It should be noted that this will cause the loci of points associated with the prediction limits for individual values to assume a constant value during certain time periods. When the upper or lower prediction limits remain within the extreme ratio values, the prediction limit values will be considered valid. The 95 per cent prediction interval for the $E / M$ ratio group has been selected to
illustrate graphically the effects of the aforementioned adjustments, and is shown in Figure 2. It is noted that the 95 per cent prediction interval, in this case, includes $a_{1}, a_{2}, b_{1}, b_{2}$, and $c_{2}$. The area depicted by $c_{2}$ contains non-feasible solutions, since the effort expenditure ratio can never be negative. The $b_{2}$ area contains solutions which are below the smallest observed effort expenditure ratio for the $E / \mathbb{M}$ category. In a similar fashion, the $b_{1}$ area contains solutions which are beyond the upper limit of the empirical data. Hence, the only acceptable solutions for this case are located within areas $a_{1}$ and $a_{2}$. Through the adjustment of the prediction limits for individual values non-feasible solutions have been greatly reduced, if not entirely eliminated.

The application of the above rationale to the six effort ratio groups results in adjustment to the prediction loci for individual values in nine of the twelve cases, at the 95 per cent prediction level. These adjustments to the prediction intervals for individual values are shown in a numeric fashion for each effort ratio group and for each of five prediction levels, in Tables XVIII through XXIII. The $X(I)$ values for which prediction limits have been calculated are the respective $X(I)$ values for the test case, to be illustrated in the next chapter, plus the smallest increment of growth phase time anticipated, namely 0.01 . The value 0.01 was selected rather than zero since theoretically, a capability-mix should not exist at time zero


Growth Phase Time in Per Cent (X)
Figure 2. Adjusted 95 Per Cent Prediction Limits for Individual Values of the E/M Effort Ratio Group

## TABLE XVIII

PREDICTION INTERVALS FOR INDIVIDUAL VALUES OF THE E/M EFFORT RATIO GROUP LOWER LIMITS IN PERCENT

UPPER LIMITS IN PERCENT

| X(I) | 99 | 98 | 95 | 90 | 80 | Y(I) | 80 | 90 | 95 | 98 | 99 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0100 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 0.7877 | 1.6085 | 1.8409 | 2.0425 | 2.2769 | 2.4369 |
| 0.0769 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 0.8925 | 1.7091 | 1.9403 | 2.1410 | 2.3741 | 2.5334 |
| 0.1428 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 0.9957 | 1.8088 | 2.0390 | 2.2388 | 2.4709 | 2.6295 |
| 0.1538 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 1.0129 | 1.8255 | 2.0555 | 2.2552 | 2.4872 | 2.6456 |
| 0.2307 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 1.1334 | 1.9426 | 2.1717 | 2.3705 | 2.6015 | 2.7593 |
| 0.2857 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 0.4123 | 1.2195 | 2.0268 | 2.2554 | 2.4537 | 2.6842 | 2.8416 |
| 0.3076 | 0.3537 | 0.3537 | 0.3537 | 0.3537 | 0.4473 | 1.2539 | 2.0604 | 2.2888 | 2.4870 | 2.7173 | 2.8746 |
| 0.3846 | 0.3537 | 0.3537 | 0.3537 | 0.03537 | 0.5697 | 1.3745 | 2.1792 | 2.4071 | 2.6048 | 2.8345 | 2.9914 |
| 0.4285 | 0.3537 | 0.3537 | 0.3537 | 0.4116 | 0.6392 | 1.4432 | 2.2472 | 2.4749 | 2.6724 | 2.9020 | 3.0588 |
| 0.4615 | 0.3537 | 0.3537 | 0.3537 | 0.4638 | 0.6913 | 1.4949 | 2.2986 | 2.5261 | 2.7236 | 2.9530 | 3.1097 |
| $0 \cdot 5384$ | 0.3537 | 0.3537 | 0.3873 | 0.5846 | 0.8121 | 1.6154 | 2.4187 | 2.6461 | 2.8435 | 3.0729 | 3.2295 |
| 0.5714 | 0.3537 | 0.3537 | 0.4388 | 0.6362 | 0.8637 | 1.6671 | 2.4705 | 2.6980 | 2.8954 | 3.1247 | 3.2814 |
| 0.6153 | 0.3537 | 0.3537 | 0.5071 | 0.7045 | 0.9321 | 1.7359 | 2.5396 | 2.7672 | 2.9646 | 3.1941 | 3.3107 |
| 0.6923 | 0.3537 | 0.3961 | 0.6259 | 0.8236 | 1.0515 | 1.8565 | 2.6614 | 2.8 .893 | 3.0871 | 3.3107 | 3.3107 |
| 0.7142 | 0.3537 | 0.4295 | 0.6594 | 0.857 .3 | 1.0854 | 1.8908 | 2.6962 | 2.9242 | 3.1221 | 3.3107 | 3.3107 |
| 0.7692 | 0.3556 | 0.5130 | 0.7433 | 0.9416 | 1.1701 | 1.9769 | 2.7838 | 3.0123 | 3.2105 | 3.3107 | 3.3107 |
| 0.8461 | 0.4706 | 0.6285 | 0.8597 | 1.0586 | 1.2878 | 2.0974 | 2.9070 | 3.1362 | 3.3107 | 3.3107 | 3.3107 |
| 0.8571 | 0.4870 | 0.6449 | 0.8762 | 1.0752 | 1.3046 | 2.1146 | 2.9247 | 3.1540 | 3.3107 | 3.3107 | 3.3107 |
| 0.9230 | 0.5842 | 0.7427 | 0.9748 | 1.1746 | 1.4048 | 2.2179 | 3.0309 | 3.2611 | 3.3107 | 3.3107 | 3.3107 |
| 1.0000 | 0.6964 | 0.8557 | 1.0890 | 1.2898 | 1.5212 | 2.3385 | 3.1557 | 3.3107 | 3.3107 | 3.3107 | 3.3107 |

TABLE XIX

## PREDICTION INTERVALS FOR INDIVIDUAL VALUES OF THE E/T EFFORT RATIO GROUP

 LOWER LIMITS IN PERCENTUPPER LIMITS IN PERCENT

| X (1) | 99 | 98 | 95 | 90 | 80 | Y(1) | 80 | 90 | 95 | 98 | 99 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1428 | 0.7761 | 0.7953 | 0.8234 | 0.8475 | 0.8754 | 0.9738 | 0.9651 | 0.9651 | 0.9651 | 0.9651 | 0.9651 |
| 0.4096 | 0.7294 | 0.7454 | 0.7688 | 0.7890 | 0.8123 | 0.8945 | 0.9651 | 0.9651 | 0.9651 | 0.9651 | 0.9651 |
| 0.5087 | 0.7058 | 0.7213 | 0.7439 | 0.7634 | 0.7858 | 0.8650 | 0.9442 | 0.9651 | 0.9651 | 0.9651 | 0.9651 |
| 0.5202 | 0.7029 | 0.7183 | 0.7408 | 0.7602 | 0.7826 | 0.8616 | 0.9406 | 0.9630 | 0.9651 | 0.9651 | 0.9651 |
| 0.5782 | 0.6869 | 0.7022 | 0.7246 | 0.7438 | 0.7660 | 0.8443 | 0.9227 | 0.9449 | 0.9641 | 0.9651 | 0.9651 |
| 0.6034 | 0.6796 | 0.6948 | 0.7172 | 0.7364 | 0.7586 | 0.8368 | 0.9151 | 0.9373 | 0.9565 | 0.9651 | 0.9651 |
| 0.6110 | 0.6773 | 0.6926 | 0.7149 | 0.7341 | 0.7563 | 0.8346 | 0.9129 | 0.9350 | 0.9543 | 0.9651 | 0.9651 |
| 0.6250 | 0.6730 | 0.6883 | 0.7107 | 0.7299 | 0.7521 | 0.8304 | 0.9088 | 0.9309 | 0.9502 | 0.9651 | 0.9651 |
| 0.6298 | 0.6716 | 0.6868 | 0.7092 | 0.7285 | 0.7506 | 0.8290 | 0.9073 | 0.9295 | 0.9488 | 0.9651 | 0.9651 |
| 0.6308 | 0.6712 | 0.6865 | 0.7089 | 0.7282 | 0.7503 | 0.8287 | 0.9071 | 0.9292 | 0.9485 | 0.9651 | 0.9651 |
| 0.6353 | 0.6698 | 0.6851 | 0.7075 | 0.7268 | 0.7490 | 0.8274 | 0.9057 | 0.9279 | 0.9472 | 0.9651 | 0.9651 |
| 0.6360 | 0.6696 | 0.6849 | 0.7073 | 0.7266 | 0.7488 | 0.8272 | 0.9055 | 0.9277 | 0.9470 | 0.9651 | 0.9651 |
| 0.6364 | 0.6695 | 0.6848 | 0.7072 | 0.7264 | 0.7486 | 0.8270 | 0.9054 | 0.9276 | 0.9469 | 0.9651 | 0.9651 |
| 0.6399 | 0.6684 | 0.6837 | 0.7061 | 0.7254 | 0.7476 | 0.8260 | 0.9044 | 0.9266 | 0.9459 | 0.9651 | 0.9651 |
| 0.6405 | 0.6682 | 0.6835 | 0.7059 | 0.7252 | 0.7474 | 0.8258 | 0.9042 | 0.9265 | 0.9457 | 0.9651 | 0.9651 |
| 0.6430 | 0.6674 | 0.6827 | 0.7051 | 0.7244 | 0.7466 | 0.8251 | 0.9035 | 0.9257 | 0.9450 | 0.9651 | 0.9651 |
| 0.6438 | 0.6672 | 0.6825 | 0.7049 | 0.7242 | 0.7464 | 0.8248 | 0.9033 | 0.9255 | 0.9448 | 0.9651 | 0.9651 |
| 0.6445 | 0.6670 | 0.6823 | 0.7047 | 0.7239 | 0.7462 | 0.8246 | 0.9032 | 0.9253 | 0.9446 | 0.9651 | 0.9351 |
| 0.6453 | 0.6667 | 0.6820 | 0.7044 | 0.7237 | 0.7459 | 0.8244 | 0.9029 | 0.9251 | 0.9444 | 0.9651 | 0.9651 |
| 0.6464 | 0.6667 | 0.6820 | 0.7044 | 0.7237 | 0.7459 | 0.8244 | 0.9028 | 0.9250 | 0.9443 | 0.9651 | 0.9651 |

PREDICTION INTERVALS FOR INDIVIDUAL VALUES OF THE E/Q EFFORT RATIO GROUP. LOWER LIMITS IN PERCENT

UPPER LIMITS IN PERCENT

| X (1) | 99 | 98 | 95 | 90 | 80 | Y(1) | 80 | 90 | 95 | 98 | 99 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0100 | 1.3192 | 1.4608 | 1.6680 | 1.8464 | 2.0520 | 2.7779 | 3.5039 | 3.7094 | 3.8059 | 3.8059 | 3.8059 |
| 0.0769 | 1.2433 | 1.3842 | 1.5904 | 1.7679 | 1.9724 | 2.6947 | 3.4170 | 3.6215 | 3.7990 | 3.8059 | 3.8059 |
| 0.1428 | 1.1677 | 1.3079 | 1.5132 | 1.6899 | 1.8935 | 2.6127 | 3.3318 | 3.5355 | 3.7122 | 3.8059 | 3.8059 |
| 0.1538 | 1.1549 | 1.2951 | 1.5003 | 1.6768 | 1.8803 | 2.5990 | 3.3177 | 3.5212 | 3.6977 | 3.8059 | 3.8059 |
| 0.2307 | 1.1408 | 1.2048 | 1.4091 | 1.5850 | 1.7876 | 2.5033 | 3.2190 | 3.4217 | 3.5975 | 3.8018 | 3.8059 |
| 0.2857 | 1.1408 | 1.1408 | 1.3433 | 1.5187 | 1.7209 | 2.4349 | 3.1489 | 3.3510 | 3.5265 | 3.7303 | 3.8059 |
| 0.3076 | 1.1408 | 1.1408 | 1.3169 | 1.4922 | 1.6942 | 2.4076 | 3.1210 | 3.3230 | 3.4983 | 3.7020 | 3.8059 |
| 0.3846 | 1.1408 | 1.1408 | 1.2236 | 1.3985 | 1.6001 | 2.3118 | 3.0236 | 3.2251 | 3.4000 | 3.6032 | 3.7420 |
| 0.4285 | 1.1408 | 1.1408 | 1.1700 | 1.3447 | 1.5461 | 2.2572 | 2.9683 | 3.1697 | 3.3444 | 3.5474 | 3.6861 |
| 0.4615 | 1.1408 | 1.1408 | 1. 1408 | 1.3041 | 1.5053 | 2.2161 | 2.9269 | 3.1282 | 3.3028 | 3.5058 | 3.6444 |
| 0.5384 | 1.1408 | 1.1408 | 1.1408 | 1.2088 | 1.4100 | 2.1204 | 2.8309 | 3.0321 | 3.2067 | 3.4095 | 3.5481 |
| 0.5714 | 1.1408 | 1.1408 | 1.1408 | 1.1676 | 1.3688 | 2.0794 | 2.7900 | 2.9912 | 3.1658 | 3.3686 | 3.5072 |
| 0.6153 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.3139 | 2.0248 | 2.7356 | 2.9369 | 3.1116 | 3.3145 | 3.4532 |
| 0.6923 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.2170 | 1.9289 | 2.6409 | 2.8425 | 3.0174 | 3.2207 | 3.3595 |
| 0.7142 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.1893 | 1.9017 | 2.6141 | 2.8158 | 2.9908 | 3.1942 | 3.3331 |
| 0.7692 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.8333 | 2.5469 | 2.7490 | 2.9244 | 3.1281 | 3.2673 |
| 0.8461 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.7376 | 2.4536 | 2.6564 | 2.8323 | 3.0368 | 3.1764 |
| 0.8571 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.7239 | 204403 | 2.6432 | 2.8192 | 3.0238 | 3.1635 |
| 0.9230 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.6419 | 2.3610 | 2.5646 | 2.7413 | 2.9466 | 3.0868 |
| 1.0000 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.1408 | 1.5461 | 2.2689 | 2.4736 | 2.6512 | 2.8575 | 2.9985 |

TABLE XXI

PREDICTION INTERVALS FOR INDIVIDUAL VALUES OF THE M/T EFFORT RATIO GROUP LOWER LIMITS IN PERCENT

UPPER LIMITS IN PERCENT

| X (1) | 99 | 98 | 95 | 90 | - 80 | Y(1) | 80 | 90 | 95. | 98 | 99 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0100 | 0.6346 | 0.6346 | 0.6346 | 0.7336 | 0.9104 | 1.5347 | 2.1590 | 2.3358 | 2.4892 | 2.6675 | 2.7892 |
| 0.0769 | 0.6346 | 0.6346 | 0.6591 | 0.8117 | 0.9876 | 1.6088 | 2.2300 | 2.4059 | 2.5585 | 2.7358 | 2.8570 |
| 0.1428 | 0.6346 | 0.6346 | 0.7362 | 0.8882 | 1.0633 | 1.6818 | 2.3003 | 2.4754 | 2.6273 | 2.8039 | 2.9245 |
| 0.1538 | 0.6346 | 0.6346 | 0.7490 | 0.9009 | 1.0759 | 1.6940 | 2.3120 | 2.4870 | 2.6389 | 2.8153 | 2.9359 |
| 0.2307 | 0.6346 | 0.6624 | 0.8381 | 0.9893 | 1.1636 | 1.7791 | 2.3946 | 2.5689 | 2.7202 | 2.8959 | 3.0159 |
| 0.2857 | 0.6346 | 0.7260 | 0.9013 | 1.0521 | 1.2260 | 1.8400 | 2.4541 | 2.6279 | 2.7788 | 2.9541 | 3.0739 |
| 0.3076 | .0.6346 | 0.7511 | 0.9263 | 1.0770 | 1.2508 | 1.8643 | 2.4778 | 2.6515 | 2.8023 | 2.9774 | 3.0971 |
| 0.3846 | 0.7196 | 0.8390 | 1.0137 | 1.1641 | 1.3375 | 1.9496 | 2.5617 | 2.7350 | 2.8854 | 3.0602 | 3.1795 |
| 0.4285 | 0.7693 | 0.8886 | 1.0632 | 1.2134 | 1.3866 | 1.9982 | 2.6098 | 2.7829 | 2.9332 | 3.1078 | 3.2271 |
| 0.4615 | 0.8064 | 0.9256 | 1.1002 | 1.2504 | 1.4234 | 2.0347 | 2.6460 | 2.8191 | 2.9693 | 3.1438 | 3.2630 |
| 0.5384 | 0.8921 | 1.0113 | 1.1857 | 1.3358 | 1.5089 | 2.1199 | 2.7309 | 2.9039 | 3.0541 | 3.2285 | 3.3477 |
| 0.5714 | 0.9285 | 1.0477 | 1.2222 | 1.3723 | 1.5453 | 2.1564 | 2.7675 | 2.9406 | 3.0907 | 3.2652 | 3.3844 |
| 0.6153 | 0.9766 | 1.0958 | 1.2704 | 1.4206 | 1.5937 | 2.2051 | 2.8164 | 2.9895 | 3.1397 | 3.3143 | 3.4335 |
| 0.6923 | 1.0601 | 1.1795 | 1.3543 | 1.5047 | 1.6781 | 2.2903 | 2.9026 | 3.0760 | 3.2264 | 3.4012 | 3.5206 |
| 0.7142 | 1.0836 | 1.2031 | 1.3780 | 1.5285 | 1.7020 | 2.3146 | 2.9272 | 3.1007 | 3.2512 | 3.4261 | 3.5456 |
| 0.7692 | 1.1422 | 1.2619 | 1.4372 | 1.5880 | 1.7617 | 2.3755 | 2.9893 | 3.1630 | 3.3139 | 3.4892 | 3.6088 |
| 0.8461 | 1. 2233 | 1.3434 | 1.5192 | 1.6705 | 1.8448 | 2.4607 | 3.0765 | 3.2508 | 3.4022 | 3.5780 | 3.6981 |
| 0.8571 | 1.2348 | 1.3549 | 1.5308 | 1.6822 | 1.8567 | 2.4728 | 3.0890 | 3.2635 | 3.4149 | 3.5908 | 3.7109 |
| 0.9230 | 1.3032 | 1.4238 | 1.6003 | 1.7523 | 1.9274 | 2.5458 | . 3.1643 | 3.3394 | 3.4913 | 3.6679 | 3.7885 |
| 1.0000 | 1.3820 | 1.5032 | 1.6807 | 1.8335 | 2.0095 | 2.6311 | 3.2527 | 3.4288 | 3.5815 | 3.7590 | 3.8802 |

PREDICTION INTERVALS FOR INDIVIDUAL VALUES OF THE M/Q EFFORT RATIO GROUP LOWER LIMITS IN PERCENT

UPPER LIMITS IN PERCENT

| X ( I ) | 99 | 98 | 95 | 90 | 80 | Y(I) | 80 | 90 | 95 | 98 | 99 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.9654 | -0.1139. | -0.0606 | 0.0172 | 0.0843 | 0.1616 | 0.4347 | 0.7078 | 0.7852 | 0.8523 | 0.9302 | 0.9835 |
| -1.8005 | -0.0190 | 0.0316 | 0.1059 | 0.1699 | 0.2436 | 0.5039 | 0.7642 | 0.8379 | 0.9019 | 0.9762 | 1.0270 |
| -1.0412 | 0.0108 | 0.0610 | 0.1345 | 0.1978 | 0.2707 | 0.5282 | 0.7856 | 0.8585 | 0.9218 | 0.9953 | 1.0455 |
| -0.9415 | 0.0146 | 0.0648 | 0.1382 | 0.2014 | 0.2742 | 0.5314 | 0.7885 | 0.8613 | 0.9245 | 0.9979 | 1.0481. |
| $-0.3372$ | 0.0370 | 0.0868 | 0.1598 | 0.2226 | 0.2950 | 0.5507 | 0.8063 | 0.8787 | 0.9415 | 1.0145 | 1.0643 |
| 0.0475 | 0.0506 | 0.1003 | 0.1731 | 0.2358 | 0.3080 | 0.5630 | 0.8179 | 0.8901 | 0.9528 | 1.0256 | 1.0659 |
| 0.1985 | 0.0558 | 0.1055 | 0.1783 | 0.2409 | 0.3130 | 0.5678 | 0.8226 | 0.8947 | 0.9573 | 1.0300 | 1.0659 |
| 0.7528 | 0.0744 | 0.1240 | 0.1966 | 0.2591 | 0.3311 | 0.5855 | 0.8399 | 0.9119 | 0.9744 | 1.0470 | 1.0659 |
| 0.9559 | 0.0809 | 0.1305 | 0.2031 | 0.2656 | 0.3376 | 0.5920 | 0.8463 | 0.9183 | 0.9808 | 1.0535 | 1.0659 |
| 1.0784 | 0.0848 | 0.1344 | 0.2070 | 0.2695 | 0.3415 | 0.5959 | . 0.8503 | 0.9223 | 0.9848 | 1.0574 | 1.0659 |
| 1.1139 | 0.0859 | 0.1355 | 0.2081 | 0.2706 | 0.3426 | 0.5970 | 0.8514 | 0.9234 | 0.9859 | 1.0586 | 1.0659 |
| 1.2181 | 0.0891 | 0.1387 | 0.2114 | 0.2739 | 0.3459 | 0.6004 | 0.8548 | 0.9268 | 0.9893 | 1.0620 | 1.0659 |
| 1.2459 | 0.0900 | 0.1396 | 0.2 .122 | 0.2748 | 0.3468 | 0.6012 | 0.8557 | 0.9277 | 0.9902 | 1.0629 | 1.0659 |
| 2.4317 | 0.0957 | 0.1453 | 0.2180 | 0.2805 | 0.3526 | 0.6072 | 0.8617 | 0.9338 | 0.9964 | 1.0659 | 1,0659 |
| 1.4930 | 0.0975 | 0.1472 | 0.2199 | 0.2824 | 0.3545 | 0.6091 | 0.8637 | 0.9358 | 0.9984 | 1.0659 | 100659 |
| 1.7646 | 0.1056 | 0.1553 | 0.2281 | 0.2907 | 0.3629 | 0.6178 | 0.8727 | 0.9449 | 1.0076 | 1.0659 | 1.0659 |
| 1.9132 | 0.1099 | 0.1596 | 0.2325 | 0.2952 | 0.3674 | 0.6226 | 0.8777 | 0.9499 | 1.0126 | 1.0659 | 1.0659 |
| 2.6147 | 0.1292 | 0.1793 | 0.2526 | 0.3156 | 0.3883 | 0.6450 | 0.9016 | 0.9743 | 1.0374 | 1.0659 | 1.0659 |
| 2.9761 | 0.1386 | 0.1889 | 0.2625 | 0.3258 | 0.3988 | 0.6565 | 0.9143 | 0.9873 | 1.0506 | 1.0659 | 1.0659 |
| 4.1483 | 0.1662 | 0.2174 | 0.2924 | 0.3569 | 0.4313 | 0.6940 | 0.9566 | 1.0310 | 1.0659 | 1.0659 | 1.0659 |

TABLE XXIII

PREDICTION INTERVALS FOR INDIVIDUAL VALUES OF THE Q/T EFFORT RATIO GROUP LOWER LIMITS IN PERCENT

UPPER LIMITS IN PERCENT

| $x(1)$ | 99 | 98 | 95 | 90 | 80 | Y(I) | 80 | 90 | 95 | 98 | 99 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0100 | 0.1368 | 0.1368 | 0.1368 | 0.1368 | 0.1464 | 0.4377 | 0.7289 | 0.8114 | 0.8830 | 0.9661 | 1.0229 |
| 0.0769 | 0.1368 | 0.1368 | 0.1368 | 0.1368 | 0.1716 | 0.4614 | 0.7511 | 0.8332 | 0.9044 | 0.9871 | 1.0436 |
| 0.1428 | 0.1368 | 0.1368 | 0.1368 | 0.1368 | 0.1962 | 0.4847 | 0.7732 | 0.8549 | 0.9258 | 1.0082 | 1.0044 |
| 0.1538 | 0.1368 | 0.1368 | 0.1368 | 0.1368 | 0.2003 | 0.4886 | 0.7769 | 0.8585 | 0.9294 | 1.0117 | 1.0679 |
| 0.2307 | 0.1368 | 0.1368 | 0.1368 | 0.1474 | 0.2287 | 0.5158 | 0.8029 | 0.8842 | 0.9548 | 1.0368 | 1.0927 |
| 0.2857 | 0.1368 | 0.1368 | 0.1368 | 0.1677 | 0.2488 | 0.5353 | 0.8217 | 0.9028 | 0.9732 | 1.0550 | 1.1108 |
| 0.3076 | 0.1368 | 0.1368 | 0.1368 | 0.1758 | 0.2568 | 0.5430 | 0.8292 | 0.9102 | 0.9806 | 1.0623 | 1.1181 |
| 0.3846 | 0.1368 | 0.1368 | 0.1368 | 0.2038 | 0.2847 | 0.5702 | 0.8558 | 0.9367 | 1.0068 | 1.0883 | 1.1440 |
| 0,4285 | 0.1368 | 0.1368 | 0.1496 | 0.2197 | 0.3005 | 0.5858 | 0.8711 | 0.9519 | 1.0220 | 1.1034 | 1.1590 |
| 0.4615 | 0.1368 | 0.1368 | 0.1615 | 0.2316 | 0.3123 | 0.5975 | 0.8826 | 0.9634 | 1.0334 | 1.1148 | 1.1625 |
| 0.5384 | 0. 1368 | 0.1368 | 0.1889 | 0.2589 | 0.3396 | 0.6247 | 0.9097 | 0.9904 | 1.0605 | 2.1418 | 1.1625 |
| 0.5714 | 0.1368 | 0.1368 | 0.2005 | 0.2706 | 0.3513 | 0.6363 | 0.9214 | 1.0021 | 1.0722 | d. 1536 | 1.1625 |
| 0.6153 | 0.1368 | 0.1368 | 0.2159 | 0.2859 | 0.3667 | 0.6519 | 0.9371 | 1.0178 | 1.0879 | 1.1625 | 1. 1625 |
| 0.6923 | 0.1368 | 0.1609 | 0.2425 | 0.3126 | 0.3935 | 0.6791 | 0.9648 | 1.0456 | 1.1158 | 1.1625 | 1.1625 |
| 0.7142 | 0.1368 | 0.1683 | 0.2499 | 0.3202 | 0.4011 | 0.6869 | 0.9727 | 1.0536 | 1.1238 | 1.1625 | 1.1625 |
| 0.7692 | 0.1368 | 0.1869 | 0.2686 | 0.3390 | 0.4200 | 0.7063 | 0.9927 | 1.0737 | 1.1441 | 1.1625 | 1.1625 |
| 0.8461 | 0.1563 | 0.2123 | 0.2943 | 0.3649 | 0.4463 | 0.7336 | 1. 0208 | 1.1022 | 1.1625 | 1.1625 | 1.1625 |
| 0.8571 | 0.1599 | 0.2159 | 0.2980 | 0.3686 | 0.4500 | 0.7374 | 1.0249 | 1.1063 | 2, 1625 | 1.1625 | 1.1625 |
| 0.9230 | 0.1811 | 0.2373 | 0.3197 | 0.3906 | 0.4723 | 0.7608 | 1.0493 | . 1.1310 | 1.1625 | 1.1625 | 1.1625 |
| 1.0000 | 0.2053 | 0.2619 | 0.3447 | 0.4159 | 0.4980 | 0.7880 | 1.0780 | 1.1601 | 1.1625 | 1.1625 | 1.1625 |

of the growth phase. Hence, a value close to zero provides sufficient capability-mix information as we approach the limiting value of zero.

The data analysis phase of the investigation is now complete. Functional relationships have been developed for the six effort ratio groups as originally stated, together with prediction limits for individual values of the independent variable. The chapter which follows will illustrate the feasibility of the developed model.

# CHAPTER V <br> MODEL VALIDATION AND APPLICATIONS 

General

The data analysis portion of the investigation resulted in a functional relationship for each of the effort ratio groups. These results may be utilized in two fashions, first as an evaluative model, and second, as a predictive model. During the model validation portion of the investigation, the results shall be utilized in an evaluative fashion。

A thorough validation of the developed model demands that a test case be readily available which consists of two parts; first, the original contractor proposal for the growth phase of the specified program, and second, the empirical data as it actually occurred for the growth phase of the program. The designation, original contractor proposal, does not refer to the contractor response to a government agency request for quotation on a particular program. Instead, it refers to the proposal prepared by the contractor after the contractor is already expending effort under a letter contract for the particular program.

A comparison between the results of both parts of the test case, after subjecting each part to evaluation by the
developed model, will provide a measure of the model's efficiency at various levels of prediction. The true efficiency of the model cannot be accurately determined until future contractor proposals are evaluated and adjusted within the constraints of the model, and then the actual program results are compared with the adjusted contractor proposal.

The empirical test data associated with the original contractor proposal has been designated "TEST-P", while the actual results for the same program are designated "TEST-A". During the remainder of this investigation, the above designations will hold for the two portions of the test case.

## Testing the Model

Any attempt at testing the validity or applicability of the developed model must begin with empirical data in the same fashion as that utilized for the developed model. Hence, the growth phase data associated with TEST-P and TEST-A was collected under the same effort breakdown as previously. The empirical data was then coded in the same fashion as the total model data. This coded empirical data is shown for TEST-P in Table XXIV, while Table XXV presents the coded empirical data for TEST-A.

It is immediately evident that the contractor viewed the original task as considerably less complex than the final results depict. This type of optimism early in a program appears to be prevalent among the launch vehicle

TABLE XXIV
PROGRAM "TEST-P" DIREGT EFFORT EXPENDITURES (Thousands of Manhours)

| Qtr。 | $E$ | $M$ | $T$ | $Q$ | Totals |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 132.73 | 260.13 | 116.47 | 4.28 | 513.61 |
| 2 | 250.11 | 298.35 | 209.42 | 5.58 | 763.46 |
| 3 | 293.92 | 325.52 | 285.35 | 6.76 | 911.55 |
| 4 | 354.38 | 432.25 | 330.72 | 8.57 | 1125.92 |
| 5 | 372.84 | 605.41 | 179.13 | 11.43 | 1168.81 |
| 6 | 509.98 | 993.06 | 168.60 | 24.18 | 1695.82 |
| 7 | 503.61 | 1062.88 | 113.75 | 29.63 | $1709.87 *$ |
| 8 | 441.74 | 935.35 | 43.55 | 24.70 | 1445.34 |
| 9 | 409.63 | 571.87 | 61.36 | 20.41 | 1063.27 |
| 10 | 375.18 | 432.64 | 30.42 | 19.76 | 858.00 |

* Denotes growth phase peak.

TABLE XXV
PROGRAM "TEST-A" DIRECT EFFORT EXPENDITURES (Thousands of Manhours)

| Qtro | $E$ | $M$ | $T$ | $Q$ | Totals |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 1 | 102.69 | 16.11 | 3.90 | 3.11 | 125.81 |
| 2 | 486.20 | 74.75 | 237.89 | 14.95 | 813.79 |
| 3 | 631.80 | 98.80 | 364.00 | 19.50 | 1114.10 |
| 4 | 782.59 | 117.12 | 513.50 | 50.70 | 1463.91 |
| 5 | 1085.50 | 306.80 | 300.30 | 68.90 | 1761.50 |
| 6 | 1565.19 | 971.09 | 501.79 | 149.50 | 3187.57 |
| 7 | 1807.00 | 1017.90 | 594.10 | 248.30 | 3667.30 |
| 8 | 1571.69 | 599.30 | 609.70 | 166.40 | 2947.09 |
| 9 | 1721.19 | 1090.70 | 406.90 | 293.80 | 3512.59 |
| 10 | 2112.50 | 1528.80 | 449.79 | 336.70 | 4427.79 |
| 11 | 2336.10 | 1790.10 | 440.70 | 391.29 | 4958.19 |
| 12 | 2494.70 | 1826.50 | 399.10 | 403.00 | 5123.30 |
| 13 | 2507.70 | 1925.30 | 309.40 | 412.10 | $5154.50 *$ |
| 14 | 2460.90 | 1723.80 | 288.60 | 409.50 | 4882.80 |
| 15 | 2519.40 | 1730.30 | 288.60 | 430.30 | 4968.60 |
| 16 | 2346.50 | 1554.80 | 169.00 | 403.00 | 4473.30 |

*Denotes growth phase peak.
contractors. However, the original contention still holds, that is, a "proper" capability-mix early in a program will cause this uncalled for optimism to be detected more rapidly, thereby permitting a rapid re-assessment of the existing situation. In most cases, this will entail a slowing-down in the effort build-up for the manufacturing, tooling, and quality areas, while doing detailed "homework" in the critical engineering and design area. The above logic and rationale is in keeping with the developed model. Processing the coded empirical data for TEST-P and TEST-A through the Ratio Program results in the corresponding ratio values as a function of time, which are shown in Tables XXVI and XXVII, respectively。 It is not necessary to conduct a test of normality on the TEST data, since the model constraints have been fully established previously, with the TEST data excluded from consideration. However, it is necessary to transform the effort ratio values given in TABLES XXVI and XXVII into the same form as utilized in the model development. This is accomplished with the aid of the "Transform T-A, T-P" Computer Program, which is described in Appendix $A-4$ 。 The transformed data for TEST-P and TEST-A is shown in Tables XXVIII and XXIX, respectively. It should be noted that the independent variable for the $E / T$ and $M / Q$ effort ratio groups also required transformation. The independent variable for the $E / M, E / Q, M / T$, and $Q / T$ effort ratio groups remains in its origingl state. Hence, the corresponding dependent variable ratio value is shown with

TABLE XXVI
PROGRAM 'TEST-P' DIRECT EFFORT EXPENDITURE RATIOS

| $T I M E$ | $E / M$ | $E / T$ | $E / Q$ | $M / T$ | $M / Q$ | $0 / T$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1428 | 0.5102 | 1.1396 | 31.0116 | 2.2334 | 60.7780 | 0.0367 |
| 0.2857 | 0.8383 | 1.1942 | 44.8225 | 2.4246 | 53.4677 | 0.0266 |
| 0.4285 | 0.9029 | 1.0300 | 43.4792 | 1.1407 | 48.1538 | 0.0236 |
| 0.5714 | 0.8198 | 1.0715 | 42.3512 | 1.3069 | 50.4375 | 0.0259 |
| 0.7142 | 0.6158 | 2.0813 | 32.6194 | 3.3797 | 52.9667 | 0.0638 |
| 0.8571 | 0.5135 | 3.0247 | 21.0909 | 5.8900 | 41.0694 | 0.1434 |
| 1.0000 | 0.4738 | 4.4273 | 16.9966 | 9.3439 | 35.8717 | 0.2604 |

TABLE XXVII
PROGRAM 'TEST-A' DIRECT EFFORT EXPENDITURE RATIOS

| $\begin{aligned} & \text { TIME } \\ & 0.0769 \end{aligned}$ | $\begin{aligned} & E / M \\ & 6.3743 \end{aligned}$ | $\begin{gathered} E / T \\ 26.3307 \end{gathered}$ | $\begin{gathered} E / Q \\ 33.0192 \end{gathered}$ | $\begin{gathered} M / T \\ 4.1307 \end{gathered}$ | $\begin{gathered} M / 0 \\ 5.1800 \end{gathered}$ | $\begin{gathered} \text { Q/T } \\ 0.7974 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1538 | 6.5043 | 2.0438 | 32.5217 | 0.3142 | 5.0000 | 0.0628 |
| 0.2307 | 6.3947 | 1.7357 | 32.4000 | 0.2714 | 5.0666 | 0.0535 |
| 0.3076 | 6.6819 | 1.5240 | 25.4357 | 0.2280 | 2.3100 | 0.0987 |
| 0.3846 | 3.5381 | 3.6147 | 15.7547 | 1.0216 | 4.4528 | 0.2294 |
| 0.4615 | 1.6117 | 3.1192 | 10.4694 | 1.9352 | 6.4955 | 0.2979 |
| 0.5384 | 1.7752 | 3.0415 | 7.2774 | 1.7133 | 4.0994 | 0.4179 |
| 0.6153 | 2.6225 | 2.5778 | 9.4452 | 0.9829 | 3.6015 | 0.2729 |
| 0.6923 | 1.5780 | 4.2300 | 5.8583 | 2.6805 | 3.7123 | 0.7220 |
| 0.7692 | 1.3818 | 4.6966 | 6.2741 | 3.3989 | 4.5405 | 0.7485 |
| 0.8461 | 1.3050 | 5.3008 | 5.9702 | 4.0619 | 4.5748 | 0.8878 |
| 0.9230 | 1.3658 | 6.2508 | 6.1903 | 4.5765 | 4.5322 | 1.0097 |
| 1.0000 | 1.3024 | 8.1050 | 6.0851 | 6.2226 | 4.6719 | 1.3319 |

TABLE XXVIII
"TEST-P" TRANSFORNED EFFORT RATIO VAIUES

| $\mathrm{X}(\mathrm{I})$ | $\mathrm{E} / \mathrm{M}$ | $\mathrm{E} / \mathrm{T}$ | $\mathrm{E} / \mathrm{Q}$ | $\mathrm{M} / \mathrm{T}$ | $\mathrm{M} / \mathrm{Q}$ | $\mathrm{Q} / \mathrm{T}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| -1.0412 |  |  |  |  | 1.4167 |  |
| 0.0475 |  |  | 3.4660 | 1.4944 | 1.3858 |  |
| 0.1428 | 3.1053 |  | 3.8247 | 1.1935 |  | 0.0360 |
| 0.2857 | 2.0096 |  | 3.7950 | 1.0680 | 0.0262 |  |
| 0.4285 | 1.8870 |  |  |  | 0.0233 |  |
| 0.5087 |  | 0.5326 |  |  |  |  |
| 0.5714 | 2.0483 |  | 3.7459 | 1.1431 |  | 0.0255 |
| 0.6034 |  | 0.5442 |  |  |  |  |
| 0.6250 |  | 0.8157 |  |  |  |  |
| 0.6353 |  | 0.7515 |  |  |  |  |
| 0.6364 |  | 0.5073 |  |  |  |  |
| 0.6430 |  | 0.6754 |  |  |  |  |
| 0.6453 |  | 0.5172 |  |  |  |  |
| 0.7142 | 2.6264 |  | 3.5151 | 1.8383 |  | 0.0618 |
| 0.8571 | 3.0874 |  | 3.0951 | 2.4269 |  | 0.1340 |
| 0.9559 |  |  |  |  | 1.2829 |  |
| 1.0000 | 3.3196 |  | 2.8901 | 3.0567 |  | 0.2314 |
| 1.1139 |  |  |  |  | 1.3596 |  |
| 1.2181 |  |  |  |  | 1.3189 |  |
| 1.7646 |  |  |  |  | 1.3833 |  |
| 4.1483 |  |  |  |  |  |  |

TABLE XXIX
"TEST-A" TRANSFORMED EFFORT RATIO VALUES

| X (I) | E/M | $E / T$ | $E / Q$ | M/T | M/Q | Q/T |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1.8005 |  |  |  |  | 0.5995 |  |
| -0.9415 |  |  |  |  | 0.5831 |  |
| -0.3372 |  |  |  |  | 0.5893 |  |
| 0.0769 | 0.4539 |  | 3.5269 | 2.0324 |  | 0.5863 |
| 0.1538 | 0.4483 |  | 3.5121 | 0.5605 |  | 0.0609 |
| 0.1985 |  |  |  |  | 0.1797 |  |
| 0.2307 | 0.4530 |  | 3.5085 | 0.5209 |  | 0.0521 |
| 0.3076 | 0.4409 |  | 2.7994 | 0.4774 |  | 0.0941 |
| 0.3846 | 0.6651 |  | 2.8186 | 1.0107 |  | 0.2065 |
| 0.4096 |  | 0.9634 |  |  |  |  |
| 0.4615 | 1.1791 |  | 2.4396 | 1.3911 |  | 0.2607 |
| 0.5202 |  | 0.6714 |  |  |  |  |
| 0.5384 | 1.0945 |  | 2.1135 | 1.3089 |  | 0.3491 |
| 0.5782 |  | 0.6344 |  |  |  |  |
| 0.6110 |  | 0.6038 |  |  |  |  |
| 0.6153 | 0.8198 |  | 2.3461 | 0.9914 |  | 0.2412 |
| 0.6250 |  | 0.8901 |  |  |  |  |
| 0.6298 |  | 0.7833 |  |  |  |  |
| 0.6308 |  | 0.8620 |  |  |  |  |
| 0.6360 |  | 0.8412 |  |  |  |  |
| 0.6399 |  | 0.7572 |  |  |  |  |
| 0.6405 |  | 0.8244 |  |  |  |  |
| 0.6438 |  | 0.8087 |  |  |  |  |
| 0.6445 |  | 0.7525 |  |  |  |  |
| 0.6454 |  | 0.7204 |  |  |  |  |
| 0.6923 | 1.1987 |  | 1.9254 | 1.6372 |  | 0.5434 |
| 0.7528 |  |  |  |  | 0.5283 |  |
| 0.7692 | 1.3308 |  | 1.9843 | 1.8436 |  | $0.5587$ |
| 0.8461 0.9230 | 1.3930 1.3432 |  | 1.9816 1.9727 | 2.0154 2.1392 |  | $\begin{aligned} & 0.6354 \\ & 0.6979 \end{aligned}$ |
| 0.9230 | 1.3432 |  | 1.9727 | 2.1392 |  | 0.6979 |
|  |  |  |  |  | 0.5513 |  |
| $\begin{aligned} & 1.0000 \\ & 1.0784 \end{aligned}$ | 1.3953 |  | 1.9579 | 2.4945 | 0.5368 | 0.8466 |
| 1.2459 |  |  |  |  | 0.5413 |  |
| 1.4317 |  |  |  |  | 0.7002 |  |
| 1.4930 |  |  |  |  | 0.5377 |  |
| 1.9132 |  |  |  |  | 0.4383 |  |
| 2.6147 |  |  |  |  | 0.4880 |  |
| 2.9761 |  |  |  |  | 0.4229 |  |

the appropriate values of the independent variable in the tables mentioned above.

The TEST-P transformed effort ratio values shall be utilized first in the developed model. Referring to Tables XVIII through XXIII, we may now determine whether the TEST-P values are acceptable, based upon the model criteria. If the TEST-P value lies within the prediction interval, it is acceptable; if outside the prediction interval, it is unacceptable. This procedure is accomplished manually, since the effort required to mechanize the operation is greater than the manual effort required.

Figure 3 portrays in a graphical fashion the results for the $M / T$ effort ratio group at the 90 per cent prediction level. It is immediately noted that only five of the possible seven $\mathbb{M} / T$ ratio values given in Table XXVIII lie within the acceptable interval. Hence, based upon the developed model, the TEST-P ratio values are rejected which correspond with $X(I)$ values of 0.4285 and 0.5714 .

The TEST-A transformed effort ratio values are tested in a similar fashion. A summary of acceptability is presented in Table XXX for both TEST-P and TEST-A.

Interpretation of Acceptability Results

The acceptability summary presented in Table XXX represents, in an overall fashion, the similarity between the actual effort expenditure ratios and the predicted model limits, shown under TEST-A. The disparity between the


Figure 3. $M / T$ Effort Ratio Values for TEST-P with 90 Per Cent Prediction Limits

TABLE XXX
ACCEPTABILITY SUMMARY

| Ratio <br> Group | TEST-P |  |  |  |  | TEST-A |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Individual Value Prediction Limits |  |  |  |  | Individual Value Prediction Limits |  |  |  |  |
|  | 99\% | 98\% | 95\% | 90\% | 80\% | 99\% | 98\% | 95\% | 90\% | 80\% |
| E/M | 5 | 5 | 5 | 5 | 4 | 13 | 13 | 13 | 13 | 9 |
| $\mathrm{E} / \mathrm{T}$ | 3 | 2 | 2 | 2 | 2 | 10 | 10 | 10 | 9 | 9 |
| $E / Q$ | 3 | 1 | 1 | 1 | 0 | 13 | 13 | 13 | 12 | 10 |
| M/T | 7 | 7 | 6 | 5 | 4 | 10 | 9 | 8 | 7 | 5 |
| M/Q | 0 | 0 | 0 | 0 | 0 | 13 | 13 | 13 | 12 | 12 |
| $Q / T$ | 1 | 0 | 0 | 0 | 0 | 10 | 10 | 10 | 9 | 7 |
| Totals | 19 | 15 | 14 | 13 | 10 | 69 | 68 | 67 | 62 | 52 |
| Per Cent | 45.24 | 35.71 | 33.33 | 30.95 | 23.81 | 88.46 | 87.18 | 85.90 | 79.49 | 66.67 |

contractor proposed effort expenditure ratios and the predicted model results are shown under TEST-P. The integer value located at the intersection of each effort ratio row and prediction limit column represents the number of test values which are acceptable, that is, the test value remained within the model developed prediction limits.

It should be noted that the maximum number of TEST-P values which could lie within the developed prediction limits for any effort ratio group is seven, the same value as the number of proposed quarters of effort for the TEST-P growth phase. The maximum value for the TEST-A portion, considering only the growth phase, is thirteen. Hence, the maximum possible number of test values for the entire growth phase of TEST-P is 42, for all six effort ratio groups. A comparable value for the TEST-A portion is 78. Thus, the overall growth phase for both TEST-P and TEST-A may be best evaluated on a percentage basis. Any comparison, other than percentage of growth phase totals tends to be erroneous, due to the differences in the number of effort ratio values being compared.

Comparing the percentages, at the same prediction level for both TEST-P and TEST-A, it is observed that the model, when used as an evaluative device, is a good approximation of the true situation. The wide disparity between the TEST $-P$ and TEST-A percentages further substantiates this conclusion. Furthermore, the relatively low percentage values for the TEST-P portion suggests that the contractor
had indeed proposed a growth phase capability-mix which was considerably less than optimum.

The reliability with which the developed model approaches the true situation varies with the specified prediction limit. As would be expected, the larger the prediction interval, the larger the number of test values which lie within the interval. However, the model does not achieve the theoretical number of test values expected within the specified interval. Hence, we may conclude that the difference between the achieved percentage and the theoretical percentage may be attributable to sampling error or less than optimum functional relationships. Even with this in mind, the developed model is considered an excellent device for proposal evaluation.

The most desirable prediction interval to be utilized when using the developed model as an evaluative device is a decision which the user must render. For purposes of this investigation, it is noted that the least disparity, between the actual and theoretical number of values contained within a prediction interval, occurs for the 95 per cent prediction interval. Hence, it appears desirable to utilize 95 per cent prediction intervals if only one prediction interval is to be considered. However, the similarity between the theoretical and actual results associated with the 99 per cent, 98 per cent, and 90 per cent prediction intervals are very near the value for 95 per cent. The only prediction interval which may be rather restrictive is the 80 per cent
interval. However, the tight bounds which the 80 per cent interval places upon the individual point being evaluated suggests that the program under consideration should be well-defined and perhaps only requires minor advances in the state-of-the-art. If this is the case, the 80 per cent interval should be considered as the limiting case. Only those individuals well acquainted with the program under consideration can exercise a valid judgement concerming the value of the prediction interval.

When utilizing the developed model as an evaluative device, it is anticipated that the rejection of a contractor proposal on the basis of an unacceptable capability-mix will undoubtedly lead to one question on the contractor's part; namely, how does a contractor achieve an acceptable capability-mix? Since the developed model does not estimate or predict effort expenditures as such, a general response to this query would be to furnish the contractor with the acceptable prediction interval for specified individual values. Thus, the contractor will know the range in which the ratio is permitted to vary and still remain acceptable. Attempting to remain within acceptable effort ratio expenditure intervals may very well cause the contractor to reconsider his total estimate, if the unacceptable ratio values are considerably beyond the acceptable limits. However, it still remains a contractor/government representative judgement as to the total effort expenditure required for the growth phase of a program.

Based upon the ratio group interdependencies which exist, it appears that the acceptance of any three contractor proposed ratio group values for a specified time value would permit the calculation of the remaining ratio group values. This conclusion is valid, only under certain circumstances. Unfortunately, the correct circumstances do not.exist within the developed model. The necessary circumstances are that each ratio group possess the same transformation. Likewise, the associated time variate for each ratio group must possess the same transformation, although it may be different than the ratio group transformation, Hence, discretion must be exercised when utilizing the evaluative model, to avoid illogical pitfalls such as the above.

Prediction with the Model

It is frequently desirable to estimate the capabilitymix required at some point during the growth phase of a program, once the program is underway. When this situation occurs, the determination of the program time value associated with the growth phase is extremely critical, since the number of quarters of effort required to attain effort expenditure peak may have increased. Thus, care must be exercised to assure that the best estimate of the independent variable is being utilized. It is only then that the developed model can function as a predictive device, and provide results which are meaningful.

The utilization of the developed model as a predictive
device assumes the user has available the growth phase time value for which the estimate is desired. Then, availability of the tabulated values included in Appendix $B$ is desirable, to eliminate the need for detailed mathematical operations relating to transformations. The user has only to substitute the independent variable value into the developed functional relationships for the $E / \mathbb{M}, E / Q, \mathbb{M} / \mathbb{T}$, and $Q / T$ groups, perform the indicated mathematical operations, and then search for the resulting value in the transformed column of the appropriate table in Appendix B. It should be noted that the data is arranged in rows from left to right in Appendix B. Once the transformed value has been located, or a value near the transformed value, the real or untransformed value immediately to its left is read or secured by interpolation. This is the predicted or estimated value for the effort expenditure ratio in its natural or untransformed state.

The $E / T$ and $\mathbb{M} / Q$ groups utilize a slightly different approach in arriving at a predictive value. Since the independent variable for these two groups was also transformed in developing the overall model, it is necessary to follow the same technique in this predictive process. Thus, the independent variable value associated with each of these two groups is located in the untransformed column of the appropriate independent variable tables of Appendix B. The respective transformed independent variable value is located immediately to the right of this value. This transformed
value is then substituted into the respective functional relationship to arrive at some dependent variable value. This value is then located in the appropriate dependent variable tables of Appendix B. The desired value is located immediately to the left of the above located value and represents the predicted capability-mix for the particular effort ratio group, for the selected growth phase time value.

## Limiting Values

The tables developed in Appendix B, for use with the model as a predictive device, have been developed with the extreme input effort ratios as the bounding values. Obviously, some bound was necessary to reduce the number of acceptable values. Several of the extreme predicted limit values resulted in solutions which were not feasible. Since it was desired to eliminate these non-feasible solutions, the extreme input ratio values were selected as upper and lower bounds. Hence, each set of ratios, i.e., E/M, will possess its own unique extreme input ratio values. This is essentially the same logic and rationale utilized in Chapter IV in adjusting the prediction limits for individual values. The primary difference between the two methods is the range of interest. In Chapter IV, we were concerned.with only twenty discrete values of $X(I)$. Now, we are concerned with every possible value of $X(I)$ within the specified bounds. Since it is impossible to provide every value for the specified bounds, values are provided
for selected increments of $X(I)$ in Appendix $B$ 。
The ranges for the six effort ratio groups, together with the incremental value between the untransformed table values, are shown in Table XXXI. It should be noted that the untransformed values and the incremental value for each effort ratio group are the inputs to the Predictive Value Program described in Appendix A. The Predictive Value Program provides the capability to develop exact tables with any degree of sensitivity desired, by simply changing the incremental input value. Hence, Appendix B is only a representative sample of the values which can be generated, and furthermore, are desirable to utilize the developed model in a predictive fashion.

As mentioned earlier in the investigation, the developed model does not estimate or predict the actual effort expenditure required by a particular effort category during a specific time period. Instead, the developed model establishes the relationships which exist for the four effort expenditure categories in the form of six interrelated ratios.

TABLE XXXI
PREDICTIVE MODEL RANGES AND INCREMENTS

| Ratio Group | Untransformed Values |  | Transformed Values |  | Increment |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Lower | Upper | Lower | Upper |  |
| E/Min (Dep.) | 0.4752 | 9.6490 | 0.3537 | 3.3101 | 0.036549 |
| E/T (Dep.) | 1.6696 | 27.7332 | 0.6254 | 0.9651 | 0.103839 |
| E/Q (Dep.) | 2.1293 | 43.9673 | 1.1408 | 3.8059 | 0.166685 |
| $M / T$ (Dep.) | 0.4028 | 18.6137 | 0.6347 | 4.3143 | 0.072553 |
| M/Q (Dep.) | 1.0627 | 17.2378 | -0.3229 | 1.0659 | 0.064443 |
| Q/T (Dep.) | 0.1466 | 2.1983 | 0.1368 | 1.1626 | 0.008174 |
| E/TT (Ind.) | 0.1428 | 0.6464 | 0.5088 | 0.6450 | 0.002006 |
| M/Q ( Ind.) | -3.9654 | 4.1483 | -0.1269 | 0.1385 | 0.032436 |

## CHAPTER VI

## CONCLUSIONS AND RECOMMENDATIONS

The concluding chapter is divided into two principal sections. The first section consists of general remarks and conclusions about the developed model, together with the model applications. The second section proposes possible areas for future investigations concerning the developed model, as well as extensions pertaining to the general topic of growth phase capability-mix.

General Remarks and Conclusions

The ability to determine or evaluate a proposed R\&D launch vehicle growth phase capability-mix quickly and with reasonable accuracy, is highly desirable for current aerospace management. The effect of rising manpower costs and curtailed budgets makes this ability even more paramount. This investigation has resulted in a mathematical model which expedites the evaluation and estimation processes, yet is easily manipulated. The ability to reconstruct any evaluations or predictions at any future time is also inherent. As mentioned earlier, little research appears in the literature concerning the capability-mix which should exist for a particular time period of an R\&D launch vehicle
growth phase. This investigation has formulated this problem in terms of six effort expenditure ratios derived from the four major effort expenditure categories which exist in an R\&D launch vehicle program.

In this dissertation, two distinct applications have been developed as part of an overall capability-mix model. The resulting applications consist of, first, an evaluative model, and second, a predictive model. Both of the resulting applications are based upon functional relationships which have been developed in the overall model. The functional relationships, together with the prediction limits for individual values, provide a baseline for the evaluation of a contractor proposed capability-mix for various time periods of the growth phase of an R\&D launch vehicle program. Hence, the designation, evaluative model.

Utilizing the functional relationships, together with the values provided in Appendix B, results in a predicted capability-mix for a particular growth phase time period. Thus, the designation, predictive model.

Two desirable qualities of a model are practicality, and ease of implementation. It is believed that the developed model possesses these qualities since the restrictions and conditions of the model are not considered to be so stringent as to make the model impractical for government or industrial applications. Furthermore, the final decision concerning the evaluative model can be readily understood and easily implemented. Although the
calculations to arrive at a solution in the evaluative model are not difficult, the transformation of several values for use within the model can become quite laborious. Consequently, the use of the Transform T-P, T-A Program is recommended when utilizing the evaluative model. The predictive model requires only the basic mathematical operations, and in some cases, interpolation within the tables of Appendix $B$, to be implemented.

## Proposals for Future Investigations

The model developed through this investigation considers all direct effort expenditures as occurring in one of four effort categories, namely, engineering, manufacturing, tooling, or quality assurance. Future investigations might well be oriented toward the breakdown of the four effort expenditure categories into as many meaningful and factual categories as might become available. For example, test operations appears to be a significant effort expenditure category, but is contained within the manufacturing category in the developed model. The difficulty, of course, is the ability to acquixe empirical data in any additional categories which are specified.

With the progression of time, additional R\&D launch vehicle programs will have attained the completion of program growth phase. It is recommended that the additional empirical data be integrated with the data provided within this investigation, and a new model be developed. The
entire procedure must be retraced, since additional data will undoubtedly require different transformations to achieve the dual constraint of normality and variable dependence. The new model approach and concept would be identical to that outlined in this investigation. The benefits to be derived from the inclusion of additional growth phase data will more than offset the additional development effort required.

As pointed out in Chapter I, relatively little progress has been made in the mathematical analysis of the R\&D launch vehicle growth phase capability-mix problem. It is possible that the application of the concepts presented in this dissertation may result in the solution of other unresolved capability-mix problems. The expenditure of additional research effort in extending the presented concept beyond the growth phase of an R\&D launch vehicle program also appears justified at this time.

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APPENDIXES

## FOREWORD

Appendix A contains the computational procedures, together with a brief description of each of the five developed computer programs, utilized in this investigation. The computational procedures were written in FORTRAN IV for the IBM 1130 computing system.

Appendix $B$ consists of eight sets of tabular values utilized with the predictive model. Each of the six dependent variable ratio transformations require one set. Then, one set is required for each of the $E / T$ and $\mathbb{M} / Q$ independent variable value transformations.

The computer program descriptions, together with the tabular values of Appendix B, are included as support material for the text and to eliminate redundant research for future investigations.

## APPENDIX A

COMPUTATIONAL PROCEDURES FOR BASIC MODEI.

## APPENDIX A-1

RATIO PROGRAM

## RATIO PROGRAM

## Program Description

The Ratio Program performs the simple mathematical operation of division for the six desired ratio groups. The present maximum is 25 quarters of empirical data for each of the four basic effort expenditure categories.

Data Cards

The first data card contains a real number in columns one through three which specifies the number of quarters of empirical data which will be processed. Card two identifies the source of the data to be processed. Card three begins the data sequence. Data is entered in fields of ten columns with two digits beyond the decimal, four fields per card. The order of variables must be constant for each card, and is referred to in this order in the program.

## Program Output

The capability-mix for selected increments of time, for the particular program being processed, is provided by the ratio program.

## FORTRAN IV LISTING OF RATIO PROGRAM

DIMENSION E(25), XM(25),T(25):Q(25),R(25)
60 READ (2,4)RN
NPT=RN+.I
4 FORMAT(F3.0)
READ (2.7)
7 FORMAT(16HIPROGRAM A )
WRITE $(3,7)$
WRITE(3,1)
1 FORMAT('ODIRECT MANHOUR RATIOS AS A FUNCTION OF TIME!) WRITE(3,2)
2 FORMAT('O TIME $\quad, 3 X,{ }^{\prime} E / M^{\prime}, I O X, ' E / T{ }^{\prime}, 10 X{ }^{\circ} E / Q^{\prime}$.
610X.'M/T',10X.'M/Q',10X,'Q/T'/I
READ(2,3)(E(I),XM(I),T(I),Q(I),I=1,NPT)
3 FORMAT(4F10.1)
$K=0$
DO $10 \mathrm{l}=1, \mathrm{NPT}$
IF(XM(I)-0)20,20,30
20 REM $=0.0$
GO TO 40
30 REM=E(I)/XM(I)
40 IF(T(I)-0)21,21,31
21 RET=0.0
GO TO 41
31 RET=E(1)/T(1)
41 IF(Q(I)-0)22,22,32
22 REQ $=0.0$
GO TO 42
$32 R E Q=E(1) / Q(1)$
$42 \operatorname{IF}(T(I)-0) 23,23,33$
$23 \mathrm{RMT}=0.0$
GO TO 43
33 RMT=XM(I)/T(I)
43 IF(Q(I)-0)24,24,34
$24 \mathrm{RMQ}=0.0$
GO TO 44
34 RMO=XM(I)/Q(I)
44 IF(T(I)-0)25.25.35
25 RQT $=0.0$
GO TO 45
35 RQT=Q(I)/T(I)
$45 K=K+1$
$X K=K$
R(I) $=X K / R N$
WRITE(3.5)R(I),REM,RET,REQ.RMT,RMQ,RQT
5 FORMAT(7(F11.4,2X)/)
10 CONTINUE
GO TO 60
50 STOP
END

APPENDIX A-2

CHI-SQUARE PROGRAM

# CHI-SQUARE PROGRAM 

## Program Description

The Chi-Square Program compares a set of sample frequencies with a set of frequencies that would be expected on the basis of some hypothesis. If the two sets compare well, the hypothesis is accepted; if they compare poorly, the hypothesis is rejected. The formulation of the test is as follows: Let $F_{1}, F_{2}, \ldots, F_{k}$ be the sample frequencies of the classes, and let $f_{1}, f_{2}, \ldots, f_{k}$ be the frequencies that would be expected on the basis of hypothesis $H_{0}$. Then, if $H_{o}$ is true, sample values of the quantity

$$
\sum_{i=1}^{k} \frac{\left(F_{i}-f_{i}\right)^{2}}{f_{i}}
$$

will tend to approximate the chi-square distribution with the degrees of freedom equal to $k$ minus the number of $F_{i}$ parameters that are utilized in the determination of the theoretical distribution. The developed theoretical distribution utilizes the sample mean, sample standard deviation, and sample size as estimates of the theoretical distribution parameters. Hence, a degree of freedom is lost for each estimated theoretical parameter. The program also possesses the capability to transform the input data into any desired form while conducting a search for a dependent variable
distribution which approaches normality.

Data Cards

The first data card may contain any desired numeric or al phameric characters in the first 72 columns. This is usually an identification card. The second data card consists of an integer number in columns nine and ten which designates the k class intervals into which the data is to be classified, and an integer number in columns 19 and 20 which specifies the number of observations which follow. The data cards which follow contain the dependent variable data in eight fields of ten columns per card, with four digits beyond the decimal. The program is presently limited to 1000 observations of the dependent variable.

## Program Output

The output consists of the empirical and theoretical frequencies grouped into appropriate class intervals. The value of $\chi^{2}$ is provided, together with the mean and variance. If any class interval contains less than five observations, the grouping of class intervals to achieve the $\chi^{2}$ criteria must be accomplished manually.

## FORTRAN IV LISTING OF CHI SQUARE PROGRAM

```
        COMMON X(1000),NN(10),CPT(10),EN(10),CHI(10),ILYA(72)
        COMMON K,MOR
        DO 2 IRMA=1,25
        I RMA = I RMA
        READ(2,99)ILYA
        WRITE(3,89)ILYA
        READ(2,3)K,MOR
        READ(2,4)(X(I),I=1,MOR)
        DO 6 I=1,MOR
        X(I)=X(I)
    6 CONTINUE
        WRITE(3,9)(XII),I=1,MOR)
        CALL NCHI
    3 FORMAT(2I20)
    4 \mp@code { F O R M A T ~ ( 8 F 1 0 . 4 ) }
    9 FORMAT(//1X,10(1X,F10.4))
    89 FORMAT(1H1,1X,72A1)
    99 FORMAT (72A1)
    2 CONTINUE
        CALL EXIT
        END
        SUBROUTINE NCHI
        COMMON X(1000),NN(10),CPT(10),EN(10),CHI(10),ILYA(72)
        COMMON K,MOR
    20 FORMAT(//E 15.7)
    18 FORMAT(//I10,F15.2,E15.7)
209 FORMAT(///7X,4HO(I),10X,4HE(I),6X,6HCHI SQ)
    KK=K-1
    B=X(1)
    DO 33 l=1,MOR
        IF(B-X(I))4,4,33
    4 B=X (1)
33 CONTINUE
        WRITE( 3,20)B
        A=X|1|
        DO }7\textrm{I}=1,MO
        IF(X(I)-A)88,88,7
    88 A=X(I)
    7 CONTINUE
        WRITE( 3.20)A
        WRITE( 3.209)
        RNC= (B-A)/KK
        DO 15 I=1,K
15 NN(I)=0.
        DO 16 I= 1,MOR
```

FORTRAN IV LISTING OF CHI SQUARE PROGRAM ICONTINUED)

```
    1F(X(1)-A)10,10,9
    9 IF(B-X(I))11,11,12
    10 NN(1)=NN(1)+1
    GO TO 16
    11 NN(K)=NN(K)+1
    GO TO 16
    12 J=(X(I)-A)/RNC
    NN(J+1)=NN(J+1)+1
    16 CONTINUE
    CPT(1)=RNC+A
    CPT}(K)=
    DO 27 1=2,KK
    27CPT(I)=CPT(I-1)+RNC
    SUM=0.0
    DO 13 I=1,MOR
    13 SUM=SUM+X(I)
    AVE=SUM/MOR
    SUM=SUM**2/MOR
    EXSQ=0.
    DO 14 I=1,MOR
14 EXSQ=EXSQ+X(I)**2
    SIG=SQRT((EXSQ-SUM)/(MOR-1))
    TWOPI=SQRT (2*3.1416)
    TWOPI=1./TWOPI
    DO 23 I=1,KK
    Z=(CPT(I)-AVE)/SIG
    IF(Z)25,26,26
25 LL=-1
    GO TO 28
26 LL=1
28 Z=ABS(Z)
    1F(2-3.)30,30,31
    31 Z=3.
    30 F=1.
    SUM=Z
    DO 24 J=1:10
    F=F*J
    L=2*J+1
    24 SUM=SUM+((Z**L.)*((-1)**J))/{(2**J)*L*F)
    EN(I)=(.5+(LL*TWOPI*SUM))*MOR
    1F(EN(I))207,23,23
207 EN(I)=0.
    23 CONTINUE
    EN(K)=FLOAT(MOR)-EN(KK)
    IF(EN(K))210,211,211
210 EN(K)=0.
211 SUM=EN(1)
    DO 29 I=20KK
```

```
    EN(I)=EN(1)-SUM
29 SUM=EN(1)+SUM
    DO 17 I=1:K
    F=EN(I)
    CHI(I)=((NN(I)-EN(\))**2)/EN(\)
27 WRITE( 3,18)NN(I),F,CHI(I)
    SUM=0.
    DO 19 I=1,K
29 SUM=SUM+CHI(1)
WRITE( 3.20) SUM
WRITEI 3.201 AVE
WRITEI 3.20: SIG
RETURN
END
```


## APPENDIX A-3

REGRESSION-IIMIT PROGRAM

## REGRESSION-LTMIT PROGRAM

## Program Description

The Regression-Limit Program provides a "least squares" fit of up to 100 observations for each of the dependent and independent variables. The program consists of two main subroutines, first, the regression analysis, and second, the prediction limits for individual values. The mathematical procedure utilized in the regression subroutine is the standard correlation method. Although equations finally solved by the program are linear, many transcendental functions may be included. The program operator's intuition and initiative are the upper bound for the number of possible transformations which may be utilized within the program. With the transformation option, non-linear terms are transformed and handled as linear variables.

The limit subroutine calculates the prediction interval for up to 51 selected values of $X(I)$, for the 99 per cent, 98 per cent, 95 per cent, 90 per cent, or 80 per cent prediction limits for individual values.

The regression analysis subroutine may be utilized without the limit subroutine, but the limit subroutine can be utilized only in conjunction with the regression subroutine。

## Data Cards

The first data card may contain any desired numeric or alphameric characters in the first 72 columns. This is usually an identification card. The second card consists of an integer number in column ten, which designates the number of independent variables, and an integer number in columns 19 and 20 which specifies the number of dependent/ independent pair observations which follow. The data cards which immediately follow contain the dependent variable data in eight fields of ten columns per card, with four digits beyond the decimal. The independent variable data cards follow the dependent variable data cards, in the same format as the dependent variable. The card immediately after the last independent variable data card possesses a one, two, three, four, or five in column ten which designates the desired prediction interval for the limit subroutine.

## Program Output

The output of the Regression-Limit Program consists of:
a. The regression coefficients $b_{0}$ and $b_{1}$.
b. Table of regression variance analysis including the "F" ratio。
c. Prediction intervals for the selected individual values of $X(I)$ about the corresponding $Y(I)$ 。

## FORTRAN IV LISTING OF REGRESSION-LIMIT PROGRAM

```
    COMMON A(2,3),COEF(2),KON(2),N1,NPT,ILYA(72),B,O
    COMMONSSREG,DSERR,T(5),UL(51),DL(51),AL(119),BARX
        COMMON Y(100),X(2,100)
    3 FORMAT(2I10)
    4 \text { FORMAT(8F10.4)}
    9 FORMAT(//10(1X,F10.4))
89 FORMAT(1H1,1X,72A1)
9 9 ~ F O R M A T ( 7 2 A 1 )
    READ(2,99)ILYA
    WRITE(3,89)ILYA
    READ(2,3)N1,NPT
    READ(2,4)(Y(I),I=1,NPT)
    DO 5 I=1,N1
    5 READ(2,4)(X(I,J),J=2,NPT)
        DO 6 J=1,NPT
        Y(J)=ALOG(Y(J)+1.0)
    6 ~ C O N T I N U E
    B=X(1,1)
    DO 33 I=1,NPT
    IF(B-X(1,1) 18,8,33
    8 B=x(1,I)
33 CONTINUE
    Q=X(1,1)
    DO 7 I= L,NPT
    IF(X(1,I)-Q)88,88,7
88 Q=x(1,I)
    CONTINUE
        WRITE(3,9)(Y(1),1=1,NPT)
    DO 10 I=1,N1
10 WRITE(3,9)(X(I,J),J=1,NPT)
    CALL MLREG
    CALL LIMIT
    STOP
    END
    SUBROUTINE MLREG
    COMMON A(2,3),GOEF(2),KON(2),N2,NPT,ILYA(72),B,Q
    COMMONSSREG,DSERR,T(5),UL(51),DL(51),AL(119),BARX
        COMMON Y(100),X(2,100)
    2 FORMAT(///2X,18HCURVE COEFFICIENTS)
    31 FORMAT(//2X,2HB(,11,1H),3X,E15,7)
    62 FORMAT(//2X6HSOURCE9X4HS.S.9X4HD.F.9X4HM.S.9X1HF)
    6 3 \text { FORMAT(//2X6HDUE TO,4X,E1O.4,4X,E1O.4,4X,E1O.4,4X,E1O.4)}
    6 4 ~ F O R M A T ( / / 2 X 5 H A B O U T , 4 X , E 1 0 . 4 , 4 X , E 1 0 . 4 , 4 X , E 1 0 . 4 ) ~
    6 5 \text { FORMAT(//2X5HTOTAL,4X,E10.4,4X,E1O.4)}
    6 9 \text { FORMAT(//2X,2OHANOVA FOR CURVE WITH,I3,13HIND VARIABLES)}
900 FORMAT(//4X15HSINGULAR MATRIX/4X2OHCURVE FIT IMPOSSIBLE)
```

```
    N=Nl+1
    M=N+1
    ANPT = NPT
    DO 10 I=1,N
    DO 10 J=1,M
10 A(I,J)=0.
    DO 80 l=2,N
    DO 80 J=I:N
    DO 80 K=1,NPT
80 A(I,J)=A(I*J)+X(I-I*K)*X(J-I,K)
    A(1,1)=NPT
    DO 81 J=2,N
    DO 81 K=1,NPT
81 A(1,J)=A(1,J)+X(J-1;K)
    DO 82 K=1,NPT
82 A(1,M)=A(1,M)+Y(K)
    DO 83 I =2,N
    DO 83 K=1,NPT
83 A(I,M)=A(I,M)+X(I-1,K)*Y(K)
    DO 84 I=1,N
    DO 84 J=1,N
84 A(J.I)=A(I:J)
    BARX=A(1,N)/NPT
    DO 417 J=1,M
    DO 417 I=1,N
417 A(I,J)=A(I,J)/ANPT
    IERR=0
    M=N+1
    DO 25 I=1,N
    1F(A(IgI)) 40,41:40
41 IERR=1
    GO TO 210
40 TEMP = 1.0/A(1.1)
    IP I=I +1
    DO 51 J=IPI,M
51 A(I,\) =A(I,J)*TEMP
    DO 24 K=1,N
    IF(I-K) 1:24:1
    1 DO 50 J=IPI*M
50 A(K,J)=A(K,J)-A(K,I)*A(IOJ)
24 CONTINUE
25 CONTINUE
    N=Nl+1
    M=N+1
210 IF(IERR) 21:20,21
21 WRITE(3.900)
    CALL EXIT
20 CONTINUE
    DO 13 K=1,N
```

```
13 COEF(K)=A(K,M)
    SUMR2=0.0
    DO 15 I=1,NPT
    SUMX=0.
    DO 14 K=1.N1
14 SUMX=SUMX+COEF(K+1)*X(K,I)
    YC=COEF(1)+SUMX
    R=Y(1)-YC
15 SUMR2=SUMR2+R*R
    SIGMA=SQRT (SUMR2/ANPT )
    SSERR=SUMR2
    SUMR2=Y(1)
    DO 60 I=2,NPT
60 SUMR2=SUMR2+Y(I)
    BARY1=SUMR2/NPT
    SUMR2=0.0
    DO 61 I=1,NPT
    R=Y(I)-BARYI
61 SUMR2=SUMR2+R*R
    SSTOT=SUMR2
    SSREG=SSTOT-SSERR
    DSREG=SSREG/N1
    DSERR=SSERR/(NPT-(N1+1))
    FRATO=DSREG/DSERR
    DEGFT=N1
    DEGFB=NPT-(N1+1)
    DEGRE=NPT-1
    WRITE(3,69)NI
    WRITE (3,62)
    WRITE(3,63)SSREG,DEGFT,DSREG,FRATO
    WRITE(3,64)SSERR,DEGFB,DSERR
    WRITE(3,65)SSTOT,DEGRE
    WRITE(3,2)
    MM=N1+1
    DO 800 I=1:10
800 KON(1)=1-1
    WRITE(3,31)(KON(I),COEF(I),I=1:MM)
    RETURN
    END
```

SUBROUTINE LIMIT
COMMON A(2,3), COEF (2),KON(2),N1,NPT,ILYA(72),B,Q COMMONSSREG,DSERR,T(5),UL(51),DL(51),AL(119),BARX COMMON Y(100):X(2,100)
DSERR=SQRT(DSERR)
SUMK=SSREG/(COEF(2)**2)
$T(1)=2.576$
$T(2)=2.326$
$T(3)=1.960$
$T(4)=1.645$
$T(5)=1.282$
READ (2.1)MAX
READ (2,4)NX
4 FORMAT (IIO)
$\operatorname{READ}(2,5)(X(1, I), I=1, N X)$
5 FORMAT (8F10.4)
WRITE $(3,50)$
1 FORMAT (I10)
DO $2 I=1, N X$
$Y(1)=\operatorname{COEF}(1)+\operatorname{COEF}(2) * X(1,1)$
2 CONTINUE
DO $3 \mathrm{I}=2$, NX
SHERI = SQRT ( $1+1 /$ NPT+( $(\{X(1, I)-B A R X) * * 2) /$ SUMK $) \mid$
SHERI $=T(M A X) * D S E R R * S H E R I$
UL (I) $=Y(I)+$ SHERI
3 DL(I) $=Y(I)-$ SHERI
WRITE(3,51)(X(1,I),DL(I),Y(I),UL(I), $1=1, N X)$
51 FORMAT (//4(10X,F10.4))
50 FORMAT (1HI, $14 \mathrm{X}, 4 \mathrm{HX}(\mathrm{I}), 16 \mathrm{X}, 5 \mathrm{HLOWER}, 16 \mathrm{X}, 4 \mathrm{HY}(\mathrm{I}), 15 \mathrm{X}, 5 \mathrm{HUPPER})$ RETURN END

## TRANSFORM $T-A, T-P$ PROGRAM

Program Description


#### Abstract

The Transform T-A, T-P Program performs only one operation, namely, the transformation of the program input data into any form desired by the user. The desired transformation is designated within loop six of the program deck.


## Data Cards

The first data card may contain any desired numeric or alphameric characters in the first 72 columns. This is usually an identification card. The second card consists of an integer value in column ten which defines the number of independent variables, and an integer value in columns 19 and 20 which designates the number of dependent/independent variable pair observations which follow. The data cards which immediately follow contain the dependent variable data in eight fields of ten columns per card, with four digits beyond the decimal. The independent variable data cards follow the dependent variable data cards, in the same format as the dependent variable.

## Program Output

The output consists of the input data values transformed according to the specified transformation for both the dependent and independent variable.

FORTRAN IV LISTING OF TRANSFORM T-A,T-P PROGRAM

COMMON A(10,11),X(10,150),Y(150), COEF(10),KON(10)
COMMON NI,NPT,ILYA(72)
2 READ (2,991ILYA
WRITE (3,89) ILYA
READ $(2,3) N 1, N P T$
$\operatorname{READ}(2,4)(Y(1), 1=1, N P T)$
DO $51=1, N 1$
$5 \operatorname{READ}(2,4)(X(1, J), J=2, N P T)$
DO $51=1, N 1$
DO $6 \mathrm{~J}=1$,NPT
$Y(J)=A L O G(Y(J)+1.0)$
6 CONTINUE
WRITE(3,9)(Y(I),I=1,NPT)
DO $10 \quad 1=1, N 1$
10 WRITE $(3,9)(X(I, J), J=1, N P T)$
3 FORMAT(2I10)
4 FORMAT (8F10.4)
9 FORMAT(//1X.10(1X,F10.4))
89 FORMAT(1H1,1X,72A1)
99 FORMAT(72A1)
GO TO 2
END

## APPENDIX A-5

PREDICTIVE VAUUE PROGRAM

PREDICTIVE VALUE PROGRAM

## Program Description

The Predictive Value Program generates tables of real values, and the corresponding transformed values, for specified increments of a designated real number range.

Data Cards

Only one data card is required, since the program is a generator. Three values are required on this card, entered in fields of ten columns with six digits beyond the decimal. The first value is the upper limiting value, second is the lower limiting value, and last is the desired increment.

## Program Output

The output consists of both the real and the transformed values, read left to right for the specified range and increment.

## FORTRAN IV LISTING OF PREDICTIVE VALUE PROGRAM

```
    DIMENSION R(1000),RT(1000)
    READ(2,1)U,XL,D
1 FORMAT(3F10.6)
XLM=XL-D
N=(U-XL)/D+1
IF(126-N)91,91,93
91K=126
    GO TO 8
93K=N
    | J=1
    DO 10 I= 1,K
    R(I)=XLM+J*D
10 J=J+1
    DO 20 I=1,K
20 RT(I)=SQRT(R(1)+2)/R(!)
    WRITE(3,2)
2 FORMAT(1H1,47X:28HE/M DEPENDENT VARIABLE GROUP)
    WRITE(3,3)
3 FORMAT (1HO,19X,13HR=BASIC VALUE;44X:4OHRT=TRANSFORM VALUE
    6 SQRT(R+2)/R)
        WRITE(3.5)
5 FORMAT (1HO,6X,5(1HR,9X,2HRT:8X),1HR:9X:2HRT/)
    WRITE(3,4)(R(I),RT(I),I=1,K)
4 FORMAT(12F10.4/)
    IP=(R(I)-XL)/D+1
    IL=N-IP
    XLM=R(I)
    IF(IL-0)9,9,94
94 IF(126-IL)8,8,92
92 K=IL
    GO TO 8
9 STOP
    END
```


## APPENDIX B

PREDICTIVE MODEL TABULAR VALUES

## APPENDIX B-1

## TABULAR VALUES FOR THE E/M

DEPENDENT VARIABLE

E/M DEPENDENT VARIABLE GROUP


E/M DEPENDENT VARIABLE GROUP

| - $R$ | RT | R | RT | R | RT | R | RT | R | RT | R | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5.0804 | 0.5237 | 5.1170 | 0.5213 | 5.1535 | 0.5189 | 5.1901 | 0.5166 | 5.2266 | 0.5143 | 5.2632 | 0.5120 |
| 5.2997 | 0.5097 | 5.3363 | 0.5075 | 5.3728 | 0.5053 | 5.4094 | 0.5032 | 5.4459 | 0.5010 | 5.4825 | 0.4989 |
| 5.5190 | 0.4968 | 5.5556 | 0.4947 | 5.5921 | 0.4927 | 5.6287 | 0.4907 | 5.6652 | 0.4887 | 5.7018 | 0.4867 |
| 5.7383 | 0.4847 | 5.7749 | 0.4828 | 5.8114 | 0.4809 | 5.8480 | 0.4790 | 5.8845 | 0.4771 | 5.9210 | 0.4753 |
| 5.9576 | 0.4734 | 5.9941 | 0.4716 | 6.0307 | 0.4699 | 6.0672 | 0.4681 | 6.1038 | 0.4663 | 6.1403 | 0.4646 |
| 6.1769 | 0.4629 | 6.2134 | 0.4612 | 6.2500 | 0.4595 | 6.2865 | 0.4579 | 6.3231 | 0.4562 | 6.3596 | 0.4546 |
| 6.3962 | 0.4530 | 6.4327 | 0.4514 | 6.4693 | 0.4498 | 6.5058 | 0.4482 | 6.5424 | 0.4467 | 6.5789 | 0.4452 |
| 6.6155 | 0.4436 | 6.6520 | 0.4421 | 6.6886 | 0.4406 | 6.7251 | 0.4392 | 6.7617 | 0.4377 | 6.7982 | 0.4363 |
| 6.8348 | 0.4348 | 6.8713 | 0.4334 | 6.9079 | 0.4320 | 6.9444 | 0.4306 | 6.9810 | 0.4292 | 7.0175 | 0.4279 |
| 7.0541 | 0.4265 | 7.0906 | 0.4252 | 7.1272 | 0.4238 | 7.1637 | 0.4225 | 7.2003 | 0.4212 | 7.2368 | $0.4199$ |
| 7.2734 | 0.4186 | 7.3099 | 0.4174 | 7.3465 | 0.4161 | .7.3830 | 0.4148 | 7.4196 | 0.4136 | 7.4561 | 0.4124 |
| 7.4927 | 0.4112 | 7.5292 | 0.4099 | 7.5658 | 0.4087 | 7.6023 | 0.4076 | 7.6389 | 0.4064 | 7.6754 | 0.4052 |
| 7.7120 | 0.4040 | 7.7485 | 0.4029 | 7.7850 | 0.4018 | 7.8216 | 0.4006 | 7.8581 | 0.3995 | 7.8947 | 0.3984 |
| 7.9312 | 0.3973 | 7.9678 | 0.3962 | 8.0043 | 0.3951 | 8.0409 | 0.3940 | 8.0774 | 0.3930 | 8.1140 | 0.3919 |
| $8 \cdot 1505$ | 0.3908 | 8.1871 | 0.3898 | 8.2236 | 0.3888 | 8.2602 | 0.3877 | 8.2967 | 0.3867 | $8 \cdot 3333$ | 0.3857 |
| 8.3698 | 0.3847 | 8.4064 | 0.3837 | 8.4429 | 0.3827 | 8.4795 | 0.3817 | 8.5160 | 0.3807 | 8.5526 | 0.3798 |
| 8.5891 | 0.3788 | 8.6257 | 0.3779 | 8.6622 | 0.3769 | 8.6988 | 0.3760 | 8.7353 | 0.3750 | 8.7719 | 0.3741 |
| 8.8084 | 0.3732 | 8.8450 | 0.3723 | 8.8815 | 0.3714 | 8.9181 | 0.3705 | 8.9546 | 0.3696 | 8.9912 | 0.3687 |
| 9.0277 | 0.3678 | 9.0643 | 0.3669 | 9.1008 | 0.3660 | 9.1374 | 0.3652 | 9.1739 | 0.3643 | 9.2105 | 0.3635 |
| 9.2470 | 0.3626 | 9.2836 | 0.3618 | 9.3201 | 0.3609 | 9.3567 | 0.3601 | 9.3932 | 0.3593 | 9.4298 | 0.3585 |
| 9.4663 | 0.3577 | 9.5029 | 0.3569 | 9.5394 | 0.3560 | 9.5760 | 0.3553 | 9.6125 | 0.3545 | 9.6490 | 0.3537 |

## APPENDIX B-2

TABULAR VALUES FOR THE E/T DEPENDENT VARIABLE

E/T DEPENDENT VARIABLE GROUP



## APPENDIX B-3

TABULAR VALUES FOR THE E/Q
DEPENDENT VARIABLE

E/O DEPENDENT VARIABLE GROUP

| R | RT | R | RT | R | RT | R | RT | R | RT | R | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.1293 | 1.1408 | 2.2960 | 1.1927 | 2.4627 | 1.2420 | 2.6294 | 1.2890 | 2.7961 | 1.3339 | 2.9628 | 1.3769 |
| 3.1295 | 1.4181 | 3.2961 | 1.4577 | 3.4628 | 1.4957 | 3.6295 | 1.5324 | 3.7962 | 1.5678 | 3.9629 | 1.6019 |
| 4.1296 | 1.6350 | 4.2963 | 1.6670 | 4.4629 | 1.6979 | 4.6296 | 1.7280 | 407963 | 1.7572 | 4.9630 | 1.7855 |
| $5 \cdot 1297$ | 1.8131 | 5.2964 | 1.8399 | 5.4630 | 1.8661 | 5.6297 | 1.8915 | 5.7964 | 1.9164 | 5.9631 | 1.9406 |
| 6.1298 | 1.9642 | 6.2965 | 1.9873 | 6.4632 | 2.0099 | 6.6298 | 2.0320 | 6.7965 | 2.0536 | 6.9632 | 2.0748 |
| 7.1299 | 2.0955 | 7.2966 | 2.1158 | 7.4633 | 2.1357 | 7.6300 | 2. 1552 | 7.7966 | 2.1743 | 7.9633 | 201931 |
| 8.1300 | 2.2115 | 8.2967 | 2.2296 | 8.4634 | 2.2474 | 8.6301 | $2 \cdot 2648$ | 8.7968 | 2.2820 | 8.9634 | 2.2989 |
| 9.1301 | 2.3155 | 9.2968 | 2.3318 | 9.4635 | 2.3478 | 9.6302 | 2.3637 | 9.7969 | 2.3792 | 9.9635 | 2.3945 |
| 10.1302 | 2.4096 | 10.2969 | 2.4245 | 10.4636 | 2.4391 | 10.6303 | 2.4536 | 10.7970 | 2.4678 | 10.9637 | 204818 |
| 11.1303 | 2.4957 | 11.2970 | 2.5093 | 11.4637 | 2.5228 | 11.6304 | 2.5361 | 11.7971 | 2.5492 | 11.9638 | 2.5621 |
| 12.1304 | 2.5749 | 12.2971 | 2.5875 | 12.4638 | 2.6000 | 12.6305 | 2.6123 | 12.7972 | 2.6244 | 12.9639 | 2.6364 |
| 13.1306 | 2.6483 | 13.2972 | 2.6600 | 13.4639 | 2.6716 | 13.6306 | 2.6831 | 13.7973 | 2.6944 | 13.9640 | 2.7056 |
| 14.1307 | 2.7167 | 14.2974 | 2.7276 | 14.4640 | 2.7385 | 14.6307 | 2.7492 | 14.7974 | 2.7598. | 14.9641 | 2.7703 |
| 15.1308 | 2.7807 | 15.2975 | 2.7910 | 15.4641 | 2.8011 | 15.6308 | 2.8112 | 15.7975. | 2.8212 | 15.9642 | 2.8311 |
| 16.1309 | 2.8408 | 16.2976 | 2.8505 | 16.4643 | 2.8601 | 16.6309 | 2.8696 | 16.7976 | 2.8790 | 16.9643 | 288883 |
| 17.1310 | 2.8976 | 17.2977 | 2.9067 | 17.4644 | 2.9158 | 17.6311 | 2.9248 | 17.7977 | 2.9337 | 17.9644 | 2.9425 |
| 18.1311 | 2.9513 | 18.2978 | 2.9599 | 18.4645 | 2.9685 | 18.6312 | 2.9771 | 18.7979 | 2.9855 | 18.9645 | 2.9939 |
| 19.1312 | 3.0022 | 19.2979 | 3.0105 | 19.4646 | 3.0186 | 19.6313 | 3.0268 | 19.7980 | 3.0348 | 19.9646 | 3.0428 |
| 20.1313 | 3.0507 | 20.2980 | 3.0586 | 20.4647 | 3.0664 | 20.6314 | 3.0741 | 20.7981 | 3.0818 | 20.9648 | 3.0894 |
| 21.1314 | 3.0970 | 21.2981 | 3.1045 | 21.4648 | 3.1119 | 21.6315 | 3.1193 | 21.7982 | 3.1266 | 21.9649 | $3 \cdot 1339$ |
| 22.1315 | 3.1411 | 22.2982 | 3.1483 | 22.4649 | 3.1555 | 22.6316 | 3.1625 | 22.7983 | 3.1696 | 22.9650 | 3.1765 |

E/Q DEPENDENT VARIABLE GROUP

| R | RT | R | RT | R | RT | R | RT | R | RT | R | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 23.1317 | 3.1835 | 23.2983 | 3.1904 | 23.4650 | 3.1972 | 23.6317 | 3.2040 | 23.7984 | 3.2107 | 23.9651 | 3.2174 |
| 24.1318 | 3.2241 | 24.2985 | 3.2307 | 24.4651 | 3.2373 | 24.6318 | 3.2438 | 24.7985 | 3.2503 | 24.9652 | 3.2567 |
| 25.1319 | 3.2631 | 25.2986 | 3. 2695 | 25.4652 | 3.2758 | 25.6319 | 3.2821 | 25.7986 | 3.2883 | 25.9653 | 3.2945 |
| 26.1320 | 3.3007 | 26.2987 | 3.3068 | 26.4654 | 3.3129 | 26.6320 | 3.3189 | 26.7987 | 3.3249 | 26.9654 | 3.3309 |
| 27.1321 | 3.3369 | 27.2988 | 3.3428 | 27.4655 | 3.3486 | 27.6322 | 3.3545 | 27.7988 | 3.3603 | 27.9655 | 3.3661 |
| 28.1322 | 3.3718 | 28.2989 | 3.3775 | 28.4656 | 3.3832 | 28.6323 | 3.3888 | 28.7989 | 3.3944 | 28.9656 | 3.4000 |
| 29.1323 | 3.4055 | 29.2990 | 3.4111 | 29.4657 | 3.4166 | 29.6324 | 3.4220 | 29.7991 | 3.4274 | 29.9657 | 3.4328 |
| 30.1324 | 3.4382 | 30.2991 | 3.4435 | 30.4658 | 3.4489 | 30.6325 | 3.4541 | 30.7992 | 3.4594 | 30.9659 | 3.4646 |
| 31.1325 | 3.4698 | 31.2992 | 3.4750 | 31.4659 | 3.4801 | 31.6326 | 3.4853 | 31.7993 | 3.4904 | 31.9660 | 3.4954 |
| 32.1326 | 3.5005 | 32.2993 | 3.5055 | 32.4660 | 3.5105 | 32.6327 | 3.5154 | 32.7994 | 3.5204 | 32.9661 | 3.5253 |
| 33.1328 | 3.5302 | 33.2994 | 3.5351 | 33.4661 | 3.5399 | 33.6328 | 3.5448 | 33.7995 | 3.5496 | 33.9662 | 3.5543 |
| 34.1329 | 3.5591 | 34.2995 | 3.5638 | 34.4662 | 3.5685 | 34.6329 | 3.5732 | 34.7996 | 3.5779 | 34.9663 | 3.5825 |
| 35.1330 | 3.5872 | 35.2997 | 3.5918 | 35.4664 | 3.5963 | 35.6330 | 3.6009 | 35.7997 | 3.6054 | 35,9664 | 3.6100 |
| 36.1331 | 3.6145 | 36.2998 | 3.6189 | 36.4665 | 3.6234 | 36.6331 | 3.6278 | 36.7998 | 3.6323 | 36.9665 | 3.6367 |
| 37.1332 | 3.6410 | 37.2999 | 3.6454 | 37.4666 | 3.6497 | 37.6333 | 3.6541 | 37.7999 | 3.6584 | 37.9666 | 3.6627 |
| 38.1333 | 3.6669 | 38.3000 | 3.6712 | 38.4667 | 3.6754 | 38.6334 | 3.6796 | 38.8000 | 3.6838 | 38.9667 | 3.6880 |
| 39.1334 | 3.6922 | 39.3001 | 3.6963 | 39.4668 | 3.7004 | 39.5335 | 3.7045 | 39.8002 | 3.7086 | 39.9668 | 3.7127 |
| 40.1335 | 3.7168 | 40.3002 | 3.7208 | 40.4669 | 3.7248 | 40.6336 | 3.7289 | 40.8003 | 3.7329 | 40.9670 | 3.7368 |
| 41.1336 | 3.7408 | 41.3003 | 3.7447 | 41.4670 | 3.7487 | 41.6337 | 3.7526 | 41.8004 | 3.7565 | 41.9671 | 3.7604 |
| 42.1337 | 3.7643 | 42.3004 | 3.7681 | 42.4671 | 3.7720 | 42.6338 | 3.7758 | 42.8005 | 3.7796 | 42.9672 | 3.7834 |
| 43.1339 | 3.7872 | 43.3005 | 3.7909 | 43.4672 | 3.7947 | 43.6339. | 3.7984 | 43.8006 | 3.8022 | 43.9673 | 3.8059 |

## APPENDIX B-4

## TABULAR VALUES FOR THE M/T <br> DEPENDENT VARIABLE

## M/T DEPENDENT VARIABLE GROUP

| R | RT | R | RT | $R$ | RT | R | RT | $R$ | RT | R | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.4028 | 0.6347 | 0.4754 | 0.6895 | 0.5480 | 0.7402 | 0.6205 | 0.7877 | 0.6931 | 0.8325 | 0.7656 | 0.8750 |
| 0.8382 | 0.9155 | 0.9107 | 0.9543 | 0.9833 | 0.9916 | 1.0558 | 1.0275 | 1.1284 | 1.0622 | 1.2009 | 1.0958 |
| 1.2735 | 1.1285 | 1.3460 | 1.1602 | 1.4186 | 1.1910 | 1.4911 | 1.2211 | 1.5637 | 1.2504 | 1.6363 | 1.2791 |
| 10.7088 | 103072 | 1.781 .4 | 1.3346 | 1.8539 | 1.3616 | 1.9265 | 1.3879 | 1.9990 | 1.4138 | 2.0716 | 1.4393 |
| 2.1441 | 1.4642 | 2.21 .67 | 1.4888 | 2.2892 | 1.5130 | 2.3618 | 1.5368 | 2.4343 | 1.5602 | 2.5069 | 1.5833 |
| 2.5794 | 1.6060 | 2.6520 | 1.6285 | 2.7245 | 1.6506 | 2.7971 | 1.6724 | 2.86 .97 | 1.6940 | 2.9422 | 1.7153 |
| 3.0148 | 1.7363 | 3.0873 | 1.7570 | 3.1599 | 1. 7776 | 3.2324 | 1.7979 | 3.3050 | 1.8179 | 3.3775 | 1.8378 |
| 3.4501 | 2.8574 | 3.5226 | 1.8768 | 3.5952 | 1.8961 | 3.6677 | 1.9151 | 3.7403 | 1.9339 | 3.8128 | 1.9526 |
| 3.8854 | 1.9711 | 3.95.79 | 1.989.4 | 4:0305 | 2.0076 | 4.103 .1 | 2.0256 | 4.1756 | 2.0434 | 4.2482 | 2.0611 |
| $4 \cdot 3207$ | 2.0786 | 4.3933 | 2.0960 | 4.46 .58 | 2.1.132 | 4.5384 | 2.1303 | 4.6109 | 2.1473 | 4.6835 | 2.1641 |
| 4.7560 | 2. 2808 | 4.8286 | 2.1974 | 4.90.11 | 2.2138 | 4.9737 | 2.2301 | 5.0462 | 2.2463 | 5.1188 | 2.2624 |
| 5.1913 | 2.2784 | 5.2639 | $2 \cdot 2943$ | $5 \cdot 3365$ | 2.3100 | 5.4090 | 2.3257 | 5.4816 | 2.3412 | 5.5541 | 2.3567 |
| 5.6267 | 2.3720 | 5.6992 | 2.3873 | 5.7718 | 2.4024 | 5.8443 | 2.4175 | 5.9169 | 2.4324 | 5.9894 | 2.4473 |
| 6.0620 | 2.4622 | 6.1345 | 2.4768 | 6.2071 | 2.4914 | 6.2796 | 2.5059 | 6.3522 | 2.5203 | 6.4247 | 2.5347 |
| 6.4973 | 2.5489 | 6.5699 | 2.5631 | 6.6424 | 2.5772 | 6.7150 | 2.5913 | 6.7875 | 2.6052 | 6.8601 | 2.6191 |
| 6.9326 | 2.6329 | 7.0052 | 2.6467 | 7.0777 | 2.6604 | 7.1503 | 2.6740 | 7.2228 | 2.6875 | 7.2954 | 2.7010 |
| 7-36.79 | 2.7144 | 7.4405 | 2.7277 | 7.5130 | 2.7410 | 7.5856 | 2.7542 | 7.6582 | 2.7673 | 7.7307 | 2.7804 |
| 7.8033 | 2.7934 | 7.8758 | 2.8063 | 7.9484 | 2.8192 | 8.0209 | 2.8321 | 8.0935 | 2.8449 | 8.1660 | 2.8576 |
| 8. 2386 | 2.8703 | 8.3111 | 2.8829 | 8.3837 | 2.8954 | 8.4562 | 2.9079 | 8.5288 | 2.9204 | 8.6013 | 2.9328 |
| 8.6739 | 2.9451 | 8.97464 | 2.9574 | 8.8190 | 2.9696 | 8.8916 | 2.9818 | 8.9641 | 2.9940 | 9.0367 | 3.0061 |
| 9.1092 | 3.0181 | 9.1818 | 3.0301 | 9.2543 | 3.0420 | 9.3269 | 3.0540 | 9.3994 | 3.0658 | 9.4720 | 3.0776 |

M/T DEPENDENT VARIABLE GROUP

## R=BASIC VALUE

$R T=$ IRANSFORMED. VALUE=SORT(R)

| R | RT | R | RT | R | RT | R | RT | $R$ | RT | $R$ | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9.5 .445 | 3.0894 | 9.6171 | 3.1011 | 9.6896 | 3.1128 | 9.7622 | 3.1244 | 9.8347 | 3.1360 | 9.9073 | 3.1475 |
| 9.9798 | 3.1590 | 10.0524 | 3.1705 | 10.1250 | 3.1819 | 10.1975 | 3.1933 | 10.2701 | 3.2047 | 10.3426 | 3.2160 |
| 10.4152 | 3.2272 | 10.4877 | 3.2384 | 10.5603 | 3.2496 | 10.6328 | 3.2608 | 10.7054 | 3.2719 | 10.7779 | 3.2829 |
| 10.8505 | 3.2940 | 10.9230 | 3.3050 | 10.9956 | 3.3159 | 11.0681 | 3.3268 | 11.1407 | 3.3377 | 11.2132 | 3.3486 |
| 11.2858 | 3.3594 | 11.3584. | 3.3702 | 1.1 .4309 | 3.3809 | 11.5035 | 3.3916 | 11.5760 | 3.4023 | 11.6486 | 3.4130 |
| 11.7211 | 3.4236 | 11.7937 | 3.4341 | 11.8662 | 3.4447 | 11.9388 | 3.4552 | 12.0113 | 3.4657 | 12.0839 | 3047.61 |
| 12.1564 | 3.4866 | 12.2290 | 3.4970 | 12.3015 | 3.5073 | 12.3741 | 3.5176 | 12.4466 | 3.5279 | 12.5192 | 3.5382 |
| 12.5918 | 3.5484 | 12.6643 | 3.5587 | 12.7369 | 3.5688 | 12.8094 | 3.5790 | 12.8820 | 3.5891 | 12.9545 | 3.5992 |
| 13.0271 | 3.6093 | 13.0996 | 3.6193 | 13.172 .2 | 3.6293 | 13.2447 | 3.63 .93 | 13.3173 | 3.6492 | 13.3898 | 3.6592 |
| 13.4624 | 3.6691 | 13.5349 | 3.6789 | 13.6075 | 3.68888 | 13.6800 | 3.6986 | 13.7526 | 3.7084 | 13.8252 | 3.7182 |
| 13.8977 | 3.7279 | 13.9703 | 3.7376 | 14.0428 | 3.7473 | 14.1154 | 3.7570 | 14.1879 | 3.7666 | 14.2605 | 3.7763 |
| 14.3330 | 3.7859 | 14.4056 | 3.7954 | 14.4781 | 3.8050 | 14.5507 | 3.8145 | 14.6232 | 3.8240 | 14.6958 | 3.8335 |
| 14.7683 | 3.8429 | 14.8409 | 3.8523 | 14.9134 | 3.86 .17 | 14.9860 | 3.8711 | 15.0586 | 3.8805 | 15.1311 | 3.8898 |
| 15.2037 | 3.8 .991 | 15.2762 | 3.9084 | 15.3488 | 3.9177 | 15.4213 | 3.9270 | 15.4939 | 3.9362 | 15.5664 | 3.9454 |
| 15.6390 | 3.9546 | 15.7115 | 3.9637 | 15.7841 | 3.9729 | 15.8566 | 3.9820 | 15.9292 | 3.9911 | 16.0017 | 4.0002 |
| 16.0743 | 4.0092 | 16.146 .9 | 4.0183 | 16.2194 | 4.0273 | 16.2920 | 4.0363 | 16.3645 | 4.0453 | 16.4371 | 4.0542 |
| 16.5096 | . 4.0632 | 16.5822 | 4.0721 | 16.6547 | 4.0810 | 16.7273 | 4.0899 | 16.7998 | 4.0987 | 16.8724 | 4.1076 |
| 16.9449 | 4.11 .64 | 17.0175 | 4.1252 | 17.0900 | 4.1340 | 17.1626 | 4.1427 | 17.2351 | 4.1515 | 17.3077 | 4.1602 |
| 17.3803 | 4.1689 | 17.4528 | 4.1776 | 17.5254 | 4.1863 | 17.5979 | 4.1949 | 17.670 .5 | 4.2036 | 17.7430 | 4.2122 |
| 17.8156 | 4.2208 | 17.8881 | 4.2294 | 17.9607 | 4.2380 | 18.0332 | 4.2465 | 18.1058 | 4.2550 | 18.1783 | 4.2636 |
| 18.2509 | 4.2721 | 18.3234 | 4.2805 | 18.3960 | 4.2890 | 18.4685 | 4.2975 | 18.5411 | 4.3059 | 18.6137 | 4.3143 |

## APPENDIX B-5

TABULAR VALUES FOR THE M/Q
DEPENDENT VARIABLE

M/Q DEPENDENT VARIABLE GROUP

| R | RT | $R$ | RT | R | RT | R | RT | $R$ | RT | R | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0627 | -0.3229 | 2.1271 | -0.2813 | 1.1915 | -0.2425 | 2.2560 | -0.2062 | 1.3204 | -0.1722 | 1.3849 | -0.1402 |
| 1.4493 | -0.1100 | 2.5138 | -0.0814 | 1.5782 | -0.0543 | 1.6426 | -0.0286 | 1.7071 | -0.0041 | 1.7715 | 0.0192 |
| 1.8360 | 0.0415 | 1.9004 | 0.0628 | 1.9649 | 0.0832 | 2.0293 | 0.1028 | 2.0937 | 0.1216 | 2.1582 | 0.1397 |
| 2.2226 | 0.1571 | 2.2871 | 0.1739 | 2.3515 | 0.1901 | 2.4160 | 0.2057 | 2.4804 | 0.2208 | 2.5448 | 0.2354 |
| 2.6093 | 0.2496 | 2.6737 | 0.2633 | 2.7382 | 0.2765 | 2.8026 | 0.2894 | 2.8671 | 0.3019 | 2.9315 | 0.3141 |
| 2.9959 | 0.3259 | 3.0604 | 0.3373 | 3.1248 | 0.3485 | 3.1893 | 0.3594 | 3.2537 | 0.3700 | 3.3182 | 0.3803 |
| 3.3826 | 0.3904 | 3.4470 | 0.4002 | 3.5115 . | 0.4098 | 3.5759 | 0.4192 | 3.6404 | 0.4284 | 3.7048 | 0.4373 |
| 3.7693 | 0.4460 | 3.8337 | 0.4546 | 3.8981 | 0.4630 | 3.9626 | 0.4712 | 4.0270 | 0.4792 | 4.0915 | 0.4870 |
| 4.1559 | 0.4947 | 4.2204 | 0.5023 | 4.2848 | 0.5097 | 4.3492 | 0.5169 | 4.4137 | 0.5241 | 4.4781 | 0.5310 |
| 4.5426 | 0.5379 | 4.6070 | 0.5446 | 4.6715 | 0.5512 | 4.7359 | 0.5577 | 4.8003 | 0.5641 | 4.8648 | 0.5703 |
| 4.9292 | 0.5765 | 4.9937 | 0.5826 | 5.0581 | 0.5885 | 5.1226 | 0.5944 | 5.1870 | 0.6001 | 5.2514 | 0.6058 |
| 5.3159 | 0.5114 | 5.3803 | 0.6169 | 5.4448 | 0.6223 | 5.5092 | 0.6276 | 5.5737 | 0.6329 | 5.6381 | 0.6380 |
| 5.7025 | 0.6431 | 5.7670 | 0.6481 | 5.8314 | 0.6531 | 5.8959 | 0.6580 | 5.9603 | 0.6628 | 6.0248 | 0.6675 |
| 6.0892 | 0.6722 | 6.1536 | 0.6768 | 6.2181 | 0.6813 | 6.2825 | 0.6858 | 6.3470 | 0.6902 | 6.4114 | 0.6946 |
| 6.4759 | 0.6989 | 6.5403 | 0.7032 | 6.6047 | 0.7074 | 6.6692 | 0.7115 | 6.7336 | 0.7156 | 6.7981 | 0.7197 |
| 6.8625 | 0.7237 | 6.9270 | 0.7276 | 6.9914 | 0.7315 | 7.0558 | 0.7354 | 7.1203 | 0.7392 | 7.1847 | 0.7430 |
| 7.2492 | 0.7467 | 7.3136 | 0.7504 | 7.3781 | 0.7540 | 7.4425 | 0.7576 | 7.5069 | 0.7612 | 7.5714 | 0.7647 |
| 7.6358 | 0.7682 | 7.7003 | 0.7716 | 7.7647 | 0.7750 | 7.8292 | 0.7784 | 7.8936 | 0.7817 | 7.9581 | 0.7850 |
| 8.0225 | 0.7883 | 8.0869 | 0.7915 | 8.1514 | 0.7947 | 8.2158 | 0.7979 | 8.2803 | 0.8010 | 8.3447 | 0.8041 |
| 8.4092 | 0.8072 | 8.4736 | 0.8102 | 8.5380 | 0.8132 | 8.6025 | 0.8162 | 8.6669 | 0.8192 | 8.7314 | 0.8221 |
| 8.7958 | 0.8250 | 8.8603 | 0.8279 | 8.9247 | 0.8307 | 8.9891 | 0.8335 | 9.0536 | 0.8363 | 9.1180 | 0.8391 |

# M/O DEPENDENT VARIABLE GROUP 

| $R$ | RT | R | RT | R | RT | R | RT | R | RT | R | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9.1825 | 0.8418 | 9.2469 | 0.8445 | 9.3114 | 0.8472 | 9.3758 | 0.8499 | 9.4402 | 0.8525 | 9.5047. | 0.8551 |
| 9.5691 | 0.8577 | 9.6336 | 0.8603 | 9.6980 | 0.8629 | 9.7625 | 0.8654 | 9.8269 | 0.8679 | 9,8913 | 0.8704 |
| 9.9558 | 0.8729 | 10.0202 | 0.8753 | 10.0847 | 0.8777 | 10.1491 | 0.8801 | 10.2136 | 0.8825 | 10.2780 | 0.8849 |
| 10.3424 | 0.8872 | 10.4069 | 0.8896 | 10.4713 | 0.8919 | 10.5358 | 0.8942 | 10.6002 | 0.8965 | 10.6647 | 0.8987 |
| 10.7291 | 0.9010 | 10.7935 | 0.9032 | 10.8580 | 0.9054 | 10.9224 | 0.9076 | 10.9869 | 0.9097 | 11.0513 | 0.9119 |
| 11.1158 | 0.91 .40 | 11.1802 | 0.9162 | 11.2446 | 0.9183 | 11.3091 | 0.9204 | 11.3735 | 0.9224 | 11.4380 | 0.9245 |
| 11.5024 | 0.9266 | 11.5669 | 0.9286 | 11.6313 | 0.9306 | 11.6957 | 0.9326 | 11.7602 | 0.9346 | 11.8246 | 0.9366 |
| 11.8891 | 0.9385 | 11.9535 | 0.9405 | 12.0180 | 0.9424 | 12.0824 | 0.9444 | 12.1468 | 0.9463 | 12.2113 | 0.9482 |
| 12.2757 | 0.9500 | 12.3402 | 0.9519 | 12.4046 | 0.9538 | 12.4691 | 0.9556 | 12.5335 | 0.9574 | 12.5979 | 0.9593 |
| 12.662 .4 | 0.9611 | 12.7268 | 0.9629 | 12.7913 | 0.9647 | 12.8557 | 0.9664 | 12.9202 | 0.9682 | 12.9846 | 0.9700 |
| 13.0490 | 0.9717 | 13.1135 | 0.9734 | 13.1779 | 0.9751 | 13.2424 | 0.9769 | . 13.3068 | 0.9786 | 13.3713 | 0.9802 |
| 13.4357 | 0.9819 | 13.5001 | 0.9836 | 13.5646 | 0.9852 | 13.6290 | 0.9869 | 13.6935 | 0.9885 | 13.7579 | 0.9902 |
| 13.8224 | 0.9918 | 13.8868 | 0.9534 | 13.9512 | 0.9950 | 14.0157 | 0.9966 | 14.0801 | 0.9981 | 14.1446 | 0.9997 |
| 14.2090 | 1.0013 | 14.2735 | 1.0028 | 14.3379 | 1.0044 | 14.4023 | 1.0059 | 14.4668 | 1.0074 | 14.5312 | 1.0089 |
| 14.5957 | 1.0105 | 14.6601 | 1.0120 | 14.7246 | 1.0134 | 14.7890 | 1.0149. | 14.8535 | 1.0164 | 14.9179 | 1.0179 |
| 14.9823 | 1.0193 | 15.0468 | 1.0208 | 15.1112 | 1.0222 | 15.1757 | 1.0237 | 15.2401 | 1.0251 | 15.3046 | 1.0265 |
| 15.3690 | 1.0279 | 15.4334 | 1.0273 | 15.4979 | 1.0307 | 15.5623 | 2.0321 | 15.6268 | 1.0335 | 15.6912 | 1.0349 |
| 15.7557 | 1.0362 | 15.8201 | 1.0376 | 15.8845 | 1.0390 | 15.9490 | 1.0403 | 16.0134 | 1.0416 | 16.0779 | 1.0430 |
| 16.1423 | 1.0443 | 16.2068 | 1.0456 | 16.2712 | 1.0469 | 16.3356 | 1.0482 | 16.4001 | 1.0495 | 16.4645 | 1.0508 |
| 16.5290 | 1.0521 | 16.5934 | 1.0534 | 16.6579 | 1.0547 | 16.7223 | 1.0559 | 16.7867 | 1.0572 | 16.8512 | 1.0585 |
| 16.9156 | 1.0597 | 16.9801 | 1.0610 | 17.0445 | 1.0622 | 17.1090 | 1.0634 | 17.1734 | 1.0646 | 17.2378 | 1.0659 |

## APPENDIX B-6

TABULAR VALUES FOR THE $Q / T$
DEPENDENT VARIABLE

Q/T DEPENDENT VARIABLE GROUP
R=BASIC VALUE
$R T=T R A N S F O R M E D$ VALUE $=L N(R+1)$

| R | RT | R | RT | R | RT | R | RT | R | RT | $R$ | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1466 | 0.1368 | 0.1548 | 0.143 .9 | 0.1630 | 0.1510 | 0.1712 | 0.1580 | 0.1793 | 0.1650 | 0.1875 | 0.1719 |
| 0.1957 | 0.1787 | 0.2039 | 0.1855 | 0.2120 | 0.1923 | 0.2202 | 0.1990 | 0.2284 | 0.2057 | 0.2366 | 0.2123 |
| 0.2447 | 0.2189 | 0.2529 | 0.2255 | 0.2611 | 0.2320 | 0.2693 | 0.2384 | 0.2774 | 0.2448 | 0.2856 | 0.2512 |
| 0.2938 | 0.2576 | 0.3020 | 0.2639 | 0.3101 | 0.2701 | 0.3183 | 0.2763 | 0.3265 | 0.2825 | 0.3347 | 0.2887 |
| 0.3428 | 0.2948 | 0.3510 | 0.3008 | 0.3592 | 0.3069 | 0.3673 | 0.3129 | 0.3755 | 0.3188 | 0.3837 | 0.3247 |
| 0.3919 | 0.3306 | 0.4000 | 0.3365 | 0.4082 | 0.3423 | 0.4164 | 0.3482 | 0.4246 | 0.3539 | 0.4327 | 0.3596 |
| 0.4409 | 0.3653 | 0.4491 | 0.3709 | 0.4573 | 0.3755 | 0.4654 | 0.3821 | 0.4736 | 0.3877 | 0.4818 | 0.3932 |
| 0.4900 | 0.3987 | 0.4981 | 0.4042 | 0.5063 | 0.4096 | 0.5145 | 0.4151 | 0.5227 | 0.4204 | 0.5308 | 0.4258 |
| 0.5390 | 0.4311 | 0.5472 | 0.4364 | 0.5554 | 0.4417 | 0.5635 | 0.4469 | 0.5717 | 0.4521 | 0.5799 | 0.4573 |
| 0.5880 | 0.4625 | 0.5952 | 0.4676 | 0.6044 | 0.4727 | 0.6126 | 0.4778 | 0.6207 | 0.4829 | 0.6289 | 0.4879 |
| 0.6371 | 0.4929 | 0.6453 | 0.4979 | 0.6534 | 0.5028 | 0.6616 | 0.5078 | 0.6698 | 0.5127 | 0.6780 | 0.5176 |
| 0.6851 | 0.5224 | 0.6943 | 0.5273 | 0.7025 | 0.5321 | 0.7107 | 0.5369 | 0.7188 | 0.5416 | 0.7270 | 0.5464 |
| 0.7352 | 0.5511 | 0.7434 | 0.5558 | 0.7515 | 0.5605 | 0.7597 | 0.5651 | 0.7679 | 0.5698 | 0.7760 | 0.5744 |
| 0.7842 | 0.5790 | 0.7924 | 0.5835 | 0.8006 | 0.5881 | 0.8087 | 0.5926 | 0.8169 | 0.5971 | 0.8251 | 0.6016 |
| 0.8333 | 0.5051 | 0.8414 | 0.6105 | 0.8496 | 0.6150 | 0.8578 | 0.6194 | 0.8660 | 0.6238 | 0.8741 | 0.6281 |
| 0.8823 | 0.5325 | 0.8905 | 0.6368 | 0.8987 | 0.6411 | 0.9068 | 0.6454 | 0.9150 | 0.6497 | 0.9232 | 0.6540 |
| 0.9314 | 0.5582 | 0.9395 | 0.5624 | 0.9477 | 0.5656 | 0.8559 | 0.6708 | 0.9640 | 0.6750 | 0.9722 | 0.6791 |
| 0.9804 | 0.6833 | 0.9886 | 0.5874 | 0.99 .57 | 0.6915 | 1.0049 | 0.6956 | 1.0131 | 0.6996 | 1.0213 | 0.7037 |
| 1.0294 | 0.7077 | 1.0376 | 0.7118 | 1.0458 | 0.7158 | 1.0540 | 0.7197 | 1.0621 | 0.7237 | 1.0703 | 0.7277 |
| 1.0785 | 0.7316 | 1.0867 | 0.7355 | 1.0948 | 0.7394 | 1.1030 | 0.7433 | 1.1112 | 0.7472 | 1.1194 | 0.7512 |
| 1.1275 | 0.7549 | 1.1357 | 0.7588 | 1.1439 | 0.7626 | 1.1521 | 0.7664 | 1.1602 | 0.7702 | 1.1684 | 0.7740 |

Q/T DEPENDENT VARIABLE GROUP
R=BASIC VALLUE
$R T=T R A N S F O R M E D \quad V A L U E=L N(R+1)$

| R | RT | R | RT | $R$ | RT | R | RT | R | RT | R | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.1766 | 0.7777 | 1.1847 | 0.7815 | 1.1929 | 0.7852 | 1.2011 | 0.7889 | 1.2093 | 0.7926 | 1.2174 | 0.7963 |
| 1.2256 | 0.8000 | 1.2338 | 0.8037 | 1.2420 | 0.8073 | 1.2501 | 0.8110 | 1.2583 | 0.8146 | 1.2665 | 0.8182 |
| . 1.2747 | 0.8218 | 1.2828 | 0.88254 | 1.2910 | 0.8290 | 1.2992 | 0.8325 | 1.3074 | 0.8361 | So3155 | 0.8396 |
| 1.3237 | 0.8431 | 1.3319 | 0.8466 | 1.3401 | 0.8501 | 2.3482 | 0.8536 | 1.3564 | 0.8571 | 1.3646 | 0.8606 |
| 1.3727 | 0.8640 | 1.3809 | 0.8675 | 1.38891 | 0.8709 | 1.3973 | 0.8743 | 1.4054 | 0.8777 | 1.4136 | 0.8811 |
| 1.4218 | 0.8845 | 1.4300 | 0.8878 | 1.4381 | 0.8912 | 1.4463 | 0.8946 | 1.4545 | 0.8979 | 1.4627 | 0.9012 |
| 1.4708 | 0.9045 | 1.4790 | 0.9078 | 1.4872 | 0.9111 | 1.4954 | 0.9144 | 1.5035 | 0.9177 | 1.5117 | 0.9209 |
| 1.5199 | 0.9242 | 1.5281 | 0.9274 | 1.5362 | 0.9306 | 2.5444 | 0.9339 | 1.5526 | 0.9371 | 1.56008 | 0.9403 |
| 1.5689 | 0.9435 | 1.5771 | 0.9466 | 1.5853 | 0.9498 | 1.5934 | 0.9530 | 1.6016 | 0.9561 | 1.6098 | 0.9592 |
| 1.6180 | 0.9624 | 1.6261 | 0.9655 | 1.6343 | 0.9686 | 1.6425 | 0.9717 | 1.6507 | 0.9748 | 1.6588 | 0.9779 |
| 2.6670 | 0.9809 | 1.6752 | 0.9340 | 1.6834 | 0.9870 | 1.6915 | 0.9901 | 1.6997 | 0.9931 | 1.7079 | 0.9961 |
| 1.7161 | 0.9991 | 1.7242 | 1.0022 | 1.7324 | 1.00052 | 1.7406 | 1.0081 | 1.7488 | 1.0111 | 1.7569 | 1.0141 |
| 1.7651 | 1.0170 | 2.7733 | 1.0200 | 1.7814 | 1.0229 | 1.7896 | 1.0259 | 1.7978 | 1.0288 | 1.8060 | 1.0317 |
| 1.8141 | 4.0346 | 1.8223 | 1.0375 | 1.8305 | 1.0404 | 1.8387 | 1.0433 | 1.8468 | 1.0462 | 1.8550 | 1.0490 |
| 1.8632 | 1.0519 | 1.8714 | 1.0548 | 1.8795 | 1.0576 | 1.8877 | 1.0604 | 1.8959 | 1.0633 | 1.9041 | 1.0661 |
| 1.9122 | 1.0689 | 1.9204 | 1.0717 | 1.9286 | 1.0745 | 1.9368 | 1.0773 | 1.9449 | 1.0801 | 1.9531 | 1.0828 |
| 1.9613 | 1.0855 | 1.9695 | 1.0883 | 1.9776 | 1.0911 | 1.9858 | 1.0938 | 1.9940 | 1,0966 | 2.0021 | 1.0993 |
| 2.0103 | 1.1020 | 2.0185 | 1.1047 | 2.0267 | 1.1074 | 2.0348 | 1.1101 | 2.0430 | 1.1128 | 2.0512 | 1.1155 |
| 2.0594 | 1.1182 | 2.0675 | 1.1208 | 2.0757 | 1.1235 | 2.0839 | 1.1262 | 200921 | 1.1288 | 2.1002 | 1.1314 |
| 2.1084 | 1.1341 | 2.1166 | 1.1367 | 2.1248 | 1.1393 | 2.1329 | 1.1419 | 2.1411 | 1.1445 | 2.1493 | 1.1471 |
| 2.1575 | 1.14 .97 | 2.1656 | 1.1523 | 2.1738 | 1.1549 | 2.1820 | 1.1575 | 2.1901 | 1.1600 | $2 \cdot 1983$ | 1.1626 |

APPENDIX $\mathrm{B}-7$

TABULAR VALUES FOR THE E/T INDEPENDENT VARIABLE

## e/t independent variable group



E/T INDEPENDENT VARIABLE GROUP

|  | R=BASIC VALUE |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $R$ | RT | $R$ | RI | R | RT | R | RT | R | RT | R | RT |
| 0.3956 | 0.6317 | 0.3976 | 0.6320 | 0.3996 | 0.6324 | 0.4016 | 0.6327 | 0.4036 | 0.6330 | 0.4056 | 0.6333 |
| 0.4076 | 0.6336 | 0.4096 | 0.6339 | 0.4117 | 0.6342 | 0.4137 | 0.6345 | 0.4157 | 0.6347 | 0.4 .177 | 0.6350 |
| 0.4197 | 0.6353 | 0.4217 | 0.6355 | 0.4237 | 0.6358 | 0.4257 | 0.6361. | 0.4277 | 0.6363 | 0.4297 | 0.6366 |
| 0.4317 | 0.6368 | 0.4337 | 0.6370 | 0.4357 | 0.6373 | 0.4377 | 0.6375 | 0.4397 | 0.6377 | 0.4417 | 0.6380 |
| 0.4437 | 0.6382 | 0.4458 | 0.6384 | 0.4478 | 0.6386 | 0.4498 | 0.6388 | 0.4518 | 0.6390 | 0.4538 | 0.6392 |
| 0.4558 | 0.6394 | 0.4578 | 0.6396 | 0.4598 | 0.6398 | 0.4618 | 0.6400 | 0.4638 | 0.6401 | 0.4658 | 0.6403 |
| 0.4678 | 0.6405 | 0.4698 | 0.6407 | . 0.4718 | 0.6408 | 0.4738 | 0.6410 | 0.4758 | 0.6411 | 0.4779 | 0.6413 |
| 0.4799 | 0.6414 | 0.4819 | 0.6416 | 0.4839 | 0.6417 | 0.4859 | 0.6419 | 0.4879 | 0.6420 | 0.4899 | 0.6421 |
| 0.4919 | 0.6423 | 0.4939 | 0.6424 | 0.4959 | 0.6425 | 0.4979 | 0.6427 | 0.4999 | 0.6428 | 0.5019 | 0.6429 |
| 0.5039 | 0.6430 | 0.5059 | 0.6431 | 0.5079 | 0.6432 | 0.5099 | 0.6433 | 0.5120 | 0.6434 | 0.5140 | 0.6435 |
| 0.5160 | 0.6 .436 | 0.5180 | 0.6437 | 0.5200 | 0.6438 | 0.5220 | 0.6439 | 0.5240 | 0.6440 | 0.5260 | 0.6441 |
| 0.5280 | 0.6442 | 0.5300 | 0.6442 | 0.5320 | 0.6443 | 0.5340 | 0.6444 | 0.5360 | 0.6444 | 0.5380 | 0.6445 |
| 0.5400 | 0.6446 | 0.5420 | 0.6446 | 0.5441 | 0.6447 | 0.5461 | 0.6447 | 0.5481 | 0.644 .8 | 0.5501 | 0.6448 |
| 0.5521 | 0.6449 | 0.5541 | 0.6449 | 0.5561 | 0.6450 | 0.5581 | 0.6450 | 0.5601 | 0.6451 | 0.5621 | 0.6451 |
| 0.5641 | 0.6451 | 0.5661 | 0.6452 | 0.5681 | 0.6452 | 0.5701 | 0.6452 | 0.5721 | 0.6453 | 0.5741 | 0.6453 |
| 0.5761 | 0.6453 | 0.5782 | 0.6453 | 0.5802 | 0.6454 | 0.5822 | 0.6454 | 0.5842 | 0.6454 | 0.5862 | 0.6454 |
| 0.5882 | 0.6454 | 0.5902 | 0.6454 | 0.5922 | 0.6454 | 0.5942 | 0.6454 | 0.5962 | 0.6454 | 0.5982 | 0.6454 |
| 0.6002 | 0.6454 | 0.6022 | 0.6454 | 0.6042 | 0.6454 | 0.6062 | 0.6454 | 0.6082 | 0.6454 | 0.6102 | 0.6454 |
| 0.6123 | 0.6454 | 0.6143 | 0.6454 | 0.6163 | 0.6454 | 0.6183 | 0.6454 | 0.6203 | 0.6454 | 0.6223 | 0.6453 |
| 0.6243 | 0.6453 | 0.6263 | 0.6453 | 0.6283 | 0.6453 | 0.6303 | 0.6453 | 0.6323 | 0.6452 | 0.6343 | 0.6452 |
| 0.6363 | 0.6452 | 0.6383 | 0.6451 | 0.6403 | 0.6451 | 0.6423 | 0.6451 | 0.6444 | 0.6450 | 0.6464 | 0.6450 |

## APPENDIX B-8

TABULAR VALUES FOR THE $M / Q$ INDEPENDENT VARIABLE

M/O INDEPENDENT VARIABLE GROUP
r=basic value

| $R$ | RT | R | RT | R | RT | $R$ | RT | R | RT | R | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.9654 | -0.1269 | -3.9331 | -0.1279 | -3.9008 | -0.1289 | -3.8685 | -0.1299 | -3.8361 | -0.1309 | -3.8038 | -0.1319 |
| -3.7715 | -0.1330 | -3.7392 | -0.1341 | -3.7068 | -0.1352 | -3.6745 | -0.1363 | -3.6422 | -0.1374 | -3.6099 | -0.1386 |
| -3.5775 | -0.1397 | -3.5452 | -0.1409 | -3.5129 | -0.1421 | -3.4806 | -0.1434 | -3.4482 | -0.1446 | -3.4159 | -0.1459 |
| -3.3836 | -0.1472 | -3.3513 | -0.1485 | -3.3189 | -0.1498 | -3.2866 | -0.1512 | -3.2543 | -0.1526 | -3.2220 | -0.1540 |
| -3.1896 | -0.1555 | -3.1573 | -0.1569 | -3.1250 | -0.1584 | -3.0926 | -0.1600 | -3.0603 | -0.1615 | -3.0280 | -0.1631 |
| -2.9957 | -0.1647 | -2.9633 | -0.1664 | -2.9310 | -0.1681 | -2.8987 | -0.1698 | -2.8664 | -0.1716 | -2.8340 | -0.1734 |
| -2.8017 | -0.1752 | -2.7694 | -0.1771 | -2.7371 | -0.1790 | -2.7047 | -0.1810 | -2.6724 | -0.1830 | -2.6401 | -0.1850 |
| -2.6078 | -0.1871 | -2.5754 | -0.1892 | -2.5431 | -0.1914 | -2.5108 | -0.1937 | -2.4785 | -0.1960 | -2.4461 | -0.1983 |
| -2.4138 | -0.2007 | -2.3815 | -0.2032 | -2.3491 | -0.2057 | -2.3168 | -0.2083 | -2.2845 | -0.2110 | -2.2522 | -0.2137 |
| -2.2198 | -0.2165 | -2.1875 | -0.2194 | -2.1552 | -0.2223 | -2.1229 | -0.2253 | -2.0905 | -0. 2285 | -2.0582 | -0.2317 |
| -2.0259 | -0.2350 | -1.9936 | -0.2383 | -1.9612 | -0.2418 | -1.9289 | -0.2454 | -1.8966 | -0.2491 | -1.8643 | -0.2529 |
| -1.8319 | -0.2569 | -1.7996 | -0.2609 | -1.7673 | -0.2651 | -1.7350 | -0.2695 | -1.7026 | -0.2739 | -1.6703 | -0.2786 |
| -1.6380 | -0.2834 | -1.6057 | -0.2883 | -1.5733 | -0.2935 | -1.5410 | -0.2988 | -1.5087 | -0.3043 | -1.4763 | -0.3100 |
| -1.4440 | -0.3160 | -2.4117 | -0.3222 | -1.3794 | -0.3286 | -1.3470 | -0.3353 | -1.3147 | -0.3423 | -1.2824 | -0.3495 |
| -1.2501 | -0.3571 | -1.2177 | -0.3651 | -1.1854 | -0.3734 | -1.1531 | -0.3821 | -1.1208. | -0.3912 | -1.0884 | -0.4008 |
| -1.0561 | -0.4108 | -1.0238 | -0.4214 | -0.9915 | -0.4326 | -0.9591 | -0.4443 | -0.9268 | -0.4568 | -0.8945 | -0.4699 |
| -0.8622 | -0.4839 | -0.8298 | -0.4987 | -0.7975 | -0.5145 | -0.7652 | -0.5314 | -0.7328 | -0.5494 | -0.7005 | -0.5687 |
| -0.6682 | -0.5895 | -0.6359 | -0.6119 | -0.6035 | -0.6361 | -0.5712 | -0.6624 | -0.5389 | -0.6910 | -0.5066 | -0.7224 |
| -0.4742 | -0.7568 | -0.4419 | -0.794? | -0.4096 | -0.8373 | -0.3773 | -0.8847 | -0.3449 | -0.9383 | -0.3126 | -0.9993 |
| -0.2803 | -1.0696 | -0.2480 | -1.1518 | -0.2156 | -1.2494 | -0.1833 | -1.3678 | -0.1510 | -1.5159 | -0.2187 | -1.7086 |
| -0.0863 | -1.9759 | -0.0540 | -2.3914 | -0.0217 | -3.2465 | 0.0105 | -3.9048 | 0.0429 | -2.4426 | 0.0752 | -1.8135 |

M/O INDEPENDENT VARIABLE GROUP

| R | RT | R | RT | R. | RT | R | RT | R | RT | R | RT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1075 | -1.3835 | 0.1399 | -1.0424 | 0.1722 | -0.7496 | 0.2045 | -0.4847 | 0.2368 | -0.2356 | 0.2692 | 0.0063 |
| 0.3015 | 0.2485 | 0.3338 | 0.4982 | 0.3661 | 0.7643 | 0.3985 | 2.0591 | 0.4308 | 1.4035 | 0.4631 | 2.8402 |
| 0.4954 | 2.4868 | 0.5278 | 4.0842 | 0.5601 | 3.1737 | 0.5924 | 2.3631 | 0.6247 | 1.9590 | 0.6571 | 2.6968 |
| 0.6884 | 1.5071 | 0.7217 | 1.3609 | 0.7540 | 1.2437 | 0.7864 | 1.1471 | 0.8187 | 1.0656 | 0.8510 | 0.9959 |
| 0.8834 | 0.9353 | 0.9157 | 0.9821 | 0.9480 | 0.8349 | 0.9803 | 0.7928 | 1.0127 | 0.7549 | 1.0450 | 0.7206 |
| 1.0773 | 0.6894 | 1.1096 | 0.6609 | 1.1420 | 0.6348 | 1.1743 | 0.6106 | 1.2066 | 0.5883 | 1.2389 | 0.5677 |
| 1.2713 | 0.5484 | 1.3036 | 0.5305 | 1.3359 | 0.5137 | 1.3682 | 0.4979 | 1.4006 | 0.4831 | 1.4329 | 0.4692 |
| 1.4652 | 0.4561 | 1.4975 | 0.4437 | 1.5299 | 0.4320 | 1.5622 | 0.4208 | 1.5945 | 0.4103 | 1.6268 | 0.4003 |
| 2.6592 | 0.3907 | 1.5915 | 0.3916 | 1.7238 | 0.3729 | 1.7562 | 0.3647 | 1.7885 | 0.3567 | 1.8208 | 0.3492 |
| 1.8531 | 0.3419 | 1.8855 | 0.3349 | . 1.9178 | 0.3282 | 1.9501 | 0.3218 | 1.9824 | 0.3156 | 2.0148 | 0.3097 |
| 2.0471 | 0.3040 | 2.0794 | 0.2985 | 2.1117 | 0.2932 | 2.1441 | 0.2880 | 2.1764 | 0.2831 | 2.2087 | 0.2783 |
| 2.2410 | 0.2737 | 2.2734 | 0.2692 | 2.3057 | 0.2649 | 2.3380 | 0.2607 | 2.3703 | 0.2567 | 2.4027 | 0.2527 |
| 2.4350 | 0.2489 | 2.4673 | 0.2452 | 2.4997 | 0.2416 | 2.5320 | 0.2382 | 2.5643 | 0.2348 | 2.5966 | 0.2315 |
| 2.6290 | 0.2283 | 2.6613 | 0.2252 | 2.6936 | 0.2222 | 2.7259 | 0.2192 | 2.7583 | 0.2163 | 2.7906 | 0.2135 |
| 2.8229 | 0.2108 | 2.8552 | 0.2082 | 2.8876 | 0.2056 | 2.9199 | 0.2031 | 2.9522 | 0.2006 | 2.9845 | 0.1982 |
| 3.0169 | 0.1958 | 3.0492 | 0.1935 | 3.0815 | 0.1913 | 3.1138 | 0.1891 | 3.2462 | 0.1870 | 3.1785 | 0.1849 |
| 3.2108 | 0.1829 | 3.2431 | 0.1809 | 3.2755 | 0.1789 | 3.3078 | 0.1770 | 3.3401 | 0.1751 | 3.3725 | 0.1733 |
| 3.4048 | 0.1715 | 3.4371 | 0.1697 | 3.4694 | 0.1680 | 3.5018 | 0.1663 | 3.5341 | 0.1647 | 3.5664 | 0.1630 |
| 3.5987 | 0.1615 | 3.6311 | 0.1599 | 3.6634 | 0.1584 | 3.6957 | 0.1569 | 3.7280 | 0.1554 | 3.7604 | 0.1539 |
| 3.7927 | 0.1525 | 3.8250 | 0.1511 | 3.8573 | 0.1498 | 3.8897 | 0.1484 | 3.9220 | 0.1471 | 3.9543 | 0.1458 |
| 3.9866 | 0.1445 | 4.0190 | 0.1433 | 4.0513 | 0.1421 | 4.0836 | 0.1409 | 4.1160 | 0.1397 | 4.1483 | 0.1385 |

VITA<br>Richard Martin Wyskida<br>Candidate for the Degree of<br>Doctor of Philosophy

$\begin{aligned} \text { Thesis: } & \text { AN EVALUATIVE AND PREDICTIVE GROWTH PHASE } \\ & \text { CAPABIIITY-MIX MODEI FOR R\&D LAUNCH VEHICLE } \\ & \text { PROGRAMS }\end{aligned}$
Major Field: Engineering
Biographical:
Personal Data: Born in Perrysburg, New York, September 2, 1935, the son of Martin J. and Mary Wyskida.

Education: Attended grade school in Perrysburg, New York; graduated from Gowanda High School, Gowanda, New York, in 1952; received the Bachelor of Science degree from Tri-State College, Angola, Indiana, with a major in Electrical Engineering, in June, 1960; received the Master of Science degree from the University of Alabama, with a major in Industrial Engineering, in August, 1964; completed requirements for the Doctor of Philosophy degree in July, 1968.

Professional Experience: Employed by the Philco
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