PREDICTING PRODUCTIVITY OF ONE OR TWO ELEVATORS FOR CONSTRUCTION OF HIGH-RISE BUILDINGS

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PREFACE

Examples of material handling systems that require vertical transportation are numerous on construction projects. One such example is that of vertical transportation systems used to transfer materials during the construction of high rise buildings. The optimal design of material handling systems requires careful consideration of the associated waiting and interference problems.

The construction manager can schedule and expedite materials and subcontractors on a project, but he cannot altogether prevent waiting since delays can usually be attributed to a series of chance occurrences beyond his control. It is the responsibility of the construction manager who designs the vertical transportation system to evaluate properly the demand for lifting service, to establish the appropriate level of lifting service, to estimate the various costs associated with the satisfaction of demand, and to determine the optimum combination of equipment for the system. It is the purpose of this research to develop mathematical models and analytical procedures which can be useful as the basis for making such decisions.

The writer has attempted to give credit to all sources from which material has been taken. He apologizes for any omissions of this character which may, unknowingly, have

occurred.

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Finally, the writer wishes to acknowledge the tremendous moral support of his wife, Betty Ann, and his two sons, Kelly and Ken. It is hoped that the effort represented on the following pages is equal to theirs.

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CHAPTER I

INTRODUCTION

High-rise construction requires the assembling, transporting, and fastening of various materials within selected time periods according to a preconceived schedule. These requirements are met by grouping the necessary materials and men at the various stations along the vertical profile. Frequently, at any given instant, there is more than one demand to vertically transport these materials from the receiving point to designated stations. It is customary for the general contractor to assume the responsibility for providing vertical transportation to his subcontractors. This suggests that the general contractor is faced with the selection and provision of appropriate vertical transportation equipment to satisfy their demands as well as his own.

In some states it is unlawful to lift men on the same equipment used to lift materials. This study presumes that separate transportation will be provided for no other purpose than that of lifting men; therefore, the transportation of men will not be considered as contributing to the demand for lifting service.

The quantity and type of materials vary during the construction period and among the selected time periods.

Because the time intervals between arrivals of materials to be vertically transported are random, a decision must be made as to the service capacity level of lifting equipment necessary to avoid excessive waiting time for materials during periods of heavy demand. The decision rests on economically balancing the cost of waiting time against the cost of providing lifting equipment.

Problems of the type posed here commonly arise in construction systems where materials form waiting lines for some type of servicing. Approaches to the solution of material handling problems in high-rise construction vary with the many types of equipment available and the structural framing system of the building.

The types of equipment suitable for lifting purposes can be grouped into "families" of equipment having similar characteristics but varying in capacity, reach, and cost. For example, a family of erection cranes would include 8-ton cranes, 18-ton cranes, 60-ton cranes, tower cranes, and climbing cranes. One family of equipment would include all hoists, of which a construction elevator would be one class within this group. Most high-rise building projects require a combination of lifting equipment to achieve compatibility between cost and efficiency. There are a considerable number of possible combinations of lifting equipment available to any given project, but most high-rise buildings employ an elevator as one device. Using the elevator as the basic lifting machine, the remainder of the vertical transportation equipment is selected, or provisions are made to supplement the elevator during periods of heavy demand. Since the elevator is used as a basis for the selection of the lifting equipment to be employed on a particular project, it would seem reasonable and appropriate to know the performance capability of one elevator.

The nature of the problem to be considered here is to determine the expected service that could be contributed by an elevator to the vertical transportation equipment in the building process and those times in the process when it would be more economical to augment the elevator capacity by supplementary equipment.

Decisions on the type or types of equipment to be employed on a particular high-rise project seem to have been predicated on intuitive judgment and experience. A decision made on this basis may not be the "best" decision. Management's objective in a problem of this kind should be to select from among alternative operating schemes the "one" that most nearly maintains an economic balance between waiting times and transport capacity. Any queueing system, meeting this objective requires a practical and effective analytical method of solution which will predict delays produced at specified arrival and service capacity levels. Such a method is developed in this treatise to determine:

1. The relationship between the height of a building and the capacity requirements of one elevator.
2. The relationship between the area of a building

and the capacity requirements of one elevator.

3. The relationship between the configuration of the area of a building and the capacity requirements of one elevator.

4. The relative economy of one, or more elevators.

The result of developing this treatise is a technique that will enable construction managers to predict the productivity that may be expected from an elevator so that supplementary equipment requirements, if needed, can also be predicted.

CHAPTER II

LITERATURE SEARCH

This chapter reviews the development of queueing or waiting-line theory and establishes the extent to which queueing models have been applied to construction engineering problems.

In 1961 Cox and Smith (1) stated that a recent bibliography listed some 600 papers on queueing and allied subjects. Since that date many additional studies have been published, indicating the interest in queueing theory.

Any review of queueing literature must begin with A. K. Erlang, who developed models to study telephone problems as early as 1905. Others in addition to Erlang continued to study mass communication problems with little or no attention given to other areas of application until around 1947. The work with queueing during this period of time was based on the assumption that each unit of demand on the system, such as the placing of a call, was independent of other units of demand and therefore not susceptible to control or manipulation by the system. The significance of this assumption was that it enabled a system input to be described by the Poisson process, which is characterized by a negative exponential distribution of intervals between arrivals (such as calls) demanding service. The variation in the lengths of these intervals contributes to the measure of congestion or utilization of a system which is characteristic of many real-world problems.

The emphasis placed on so-called operations research methods during World War II gave impetus to the extension of both mathematical theories and use of the models, including those of queueing. Following World War II, an impressive number of publications on queueing appeared in the journals. Considerable attention has been given to modifying the original assumption by Erlang in an attempt to describe more accurately the behavior of the systems under study. Unfortunately for the practitioner, this attention has been directed for the most part toward the theoretical aspects of queueing theory rather than toward useful applications of the theory to practice.

In most cases application to a practical problem involves an economic model in order to provide a basis for choosing among alternative systems. Economic models dealing with industrial type problems are discussed in Morse (2) and Bowman and Fetter (3). The economic models presented in these two studies deal with such problems as docing facilities in a harbor and the number of machines assigned to an operator (as in a textile mill). Mangelsdorf (4) indicated procedures that could be employed in the application of waiting-line theory to machine assignment both with a finite and infinite population. Of particular interest in Mangelsdorf's work is his attention to the problem of determining cost of an operator, attendant, or repairman, etc.; cost of excess or idle machine capacity; and costs associated with a delay in performing services.

Considerable research effort has been directed to the study of vehicular traffic by the use and development of queueing models. Prominent among the publications in this area are those resulting from studies of waiting-line problems encountered by The Port of New York Authority, which is charged with the responsibility of operating such public facilities as airports, tunnels, bridges, land and marine terminals. One such problem studied by the Port Authority was the problem of waiting-lines at toll booths at the Port Authority's bridge and tunnel facilities (5). Other problems analyzed by the Port Authority were telephone and lobby information services, motorized police patrols, and elevator service (6). Shelton (7) provides what appears to be a summary of the solution methods used for waiting-line problems analyzed by the Port Authority's Management Engineering Group. In his article, Shelton graphically represents the results of some properties of the systems studied.

As noted previously, the limited number of articles and publications on the topics of practical application of waiting-line theory to problem solving is in marked contrast to the treatment afforded the theoretical approach. One's first encounter with the mathematical sophistication and elegance found in the technical literature can be a frustrating experience causing one to despair of ever solving a practical problem by queueing theory. The difficulty is that the mathematics necessary to describe precisely the behavior of waiting-line systems is unfamiliar to the practitioner. Too few attempts have been made to translate the theory into any form suitable for application to the real-world environment. One exception to this is the work by Hillier (8) although he warns of the danger of attempting mathematical short-cuts and taking liberties with the theory (9). In (9) it is pointed out that invalid results may be obtained for waiting-line predictions unless valid waiting-line equations or valid Monte Carlo simulation are used. This article presents a broad conceptual framework of the general approach to many industrial waiting-line problems.

In (8), economic models for industrial waiting-line problems are developed and some basic results derived for the case where the study is based upon fundamental cost considerations and the assumption of an infinite population. Included are a number of economic models and accompanying procedures for determining the level of service which minimizes the total of the expected cost of service and the expected cost of waiting for that service. The first model presented in Hillier's article is for the case in which both the arrival rate and the service rate are fixed and the number of service channels must be determined. The second model developed is for the case in which both the arrival rate and the number of service channels must be determined, i.e., where both the number of service facilities to distribute among the entire population and the number of service channels to assign each facility must be determined. The inclusion of travel time costs in this model is a feature that has many counterparts in practical problems and is therefore of considerable value to the model. The third model is for the case in which both the service rate and the number of service channels must be determined. Several special cases of the two latter models are also analyzed. Considerable attention is given in (8) and (9) to the determination of cost coefficients and to defending the cost of obtaining information necessary for accurate cost figures for the various measures of effectiveness. The great difficulty in the determination of these costs is one serious disadvantage in the application of any economic model, including an economic model based on waiting-line behavior. However, it is pointed out in (8) that the solution to an economic waiting-line model is generally not very sensitive to the cost assigned to waiting time.

It has probably not escaped the reader's attention that all of the references cited have been in areas other than construction. Few attempts at applying waiting-line theory to practical problems have been published, and those that have been published deal primarily with the areas cited above.

Review of the literature has revealed only two significant applications of waiting-line theory to construction engineering problems. Motion and time studies were conducted in an effort to test a mathematical model which approximated the probability that a given number of earth movers would be waiting in line at a loader during hauling operations (10). In this earth moving application, if there was at least one earth mover waiting in line at all times then the loader could ideally work to its full capacity. If the probability of this occurrence is known, the production rate of the loader can be modified and job production calculated. A total cost analysis can then be developed to reflect the optimum number of earth movers to be used.

The other example of the application of waiting-line theory to construction is given in (11). A simulation approach was used to predict the productivity of an earth moving system involving one pusher working with a fleet of scrapers. Four models were developed for several possible system arrangements and compared to the results of a computer simulation model. The comparison of the predictions of the waiting-line model with those of the simulation program for a wide range of systems resulted in an average error of -3 per cent. In all cases, it was assumed that non-delay cycle times and machine efficiency were known.

The problem contemplated in the present study differs from the problems referred to in the references cited in several significant respects. The type of unit or material to be serviced varies considerably in physical characteristics, thereby imposing particular requirements on the service facility. The character of the demand gradually changes as the building process steps through the various stages of construction. None of the references cited considered the consequences of abnormal delays or demands.

It is this author's opinion that a queueing model developed for a construction material handling system should incorporate means for up-dating the schedule and smoothing the demand with associated costs for alternative corrective procedures. Furthermore, graphical computational aids to enable the practitioner to use the information without possessing an intimate knowledge of the development is of great benefit to the construction industry. The absence of some or all of the above desired information in the problems attacked in the literature encouraged the author's desire to make this study.

CHAPTER III

THE BUILDING CONSTRUCTION PROCESS

Current Practice

The construction of a building is generally viewed as consisting of three broad stages: 1) initial stage, 2) intermediate stage, and 3) finishing stage.

The initial stage begins with the foundation and progressively includes the frame and floors. The intermediate stage, after allowing the initial stage to progress sufficiently to avoid interference, follows the initial stage and proceeds concurrently with the initial stage. The finishing stage begins after the intermediate stage is well advanced and proceeds concurrently with the initial and intermediate stages.

This process forms a pattern of repetitive operations unique to reinforced concrete frame construction since the pattern of operations differs for structural steel frame construction. It is because of this difference in construction procedure that only the case of a reinforced concrete frame building is considered in this research effort. It should be recognized, however, that many similarities exist between the procedures for constructing a reinforced concrete frame building and a structural frame building; so the method presented here, with appropriate modifications, could be applied to the determination of vertical transportation equipment for the structural steel frame building.

Foundations, which are included in the initial stage of construction, are typically piling or reinforced concrete spread footings, raft footings, slabs, or combinations of these. For a variety of reasons, foundation work is carried on below finished ground level, frequently to considerable depth. The nature of this work precludes the use of elevators and must, of necessity, depend upon some other mode of vertical transportation. Only after the foundation work is concluded and the surrounding area backfilled and compacted, is the installation of an elevator practical. It is at this point in the building process that construction of the first floor slab and columns can begin. The materials for the first floor slab and columns either do not require vertical transportation equipment or can be better transported by other means. Therefore, only those materials required for the second and succeeding floors are considered as contributing to the demand for vertical transportation service provided by an elevator.

The reinforced concrete frame and floors included in the initial stage involve material such as forming lumber, reinforcing steel, concrete, imbedded items, etc.

When a floor is determined to have gained the desired strength to accommodate additional construction, it is released to the intermediate stage. The materials necessary to accomplish construction in this stage typically include exterior masonry units, mortar, window sash, interior masonry units for partitions, rough electrical, rough plumbing, ducts, etc.

As soon as interference from the intermediate stage is seen to be negligible, construction activities of the finishing stage can commence. Materials included in the construction activities of this stage are floor coverings, wall coverings, electrical and plumbing fixtures, finished hardware, etc.

The initial phase, beginning first, will terminate first; then the intermediate stage will terminate, and finally the finishing stage terminates marking the completion of the structure.

The operations or tasks to be performed on each floor are basically the same for the same stage. The repetitive characteristics of this type construction establish a recurring pattern of construction procedures early so that an entire project can be viewed as a series of time periods of equal length. The materials required for each period can be determined with reasonable accuracy and confidence in much the same manner used to prepare a CPM network.

With the material requirements for each time period established and the on-station date for each specified by the schedule, the requirements to be imposed on the elevator for the general case can be related to the performance capability of the elevator subject to the limitation of effective loads assigned to each material.

Effective Load Capacity

Effective load capacity is defined here to be the number of units of a material that an elevator can lift in one load. The units are expressed in commonly accepted terms of measurement for that material such as cubic yards, lineal feet, square feet, etc.

Not all materials can be transported by elevator either because of the weight, volume, or length of the material to be transported. In order to determine those materials susceptible to transport by elevator it is convenient to classify materials corresponding to the capacity of the elevator based on the material's weight, volume or length so that the number of units of each material to be transported by elevator can be determined. This will identify those materials that can be transported by elevator without exceeding its capacity. Further, the total required number of elevator trips to each floor can be determined by considering the number of units of each material contained in an elevator load.

Several materials for which the effective load capacity needs to be determined may be considered. Reinforcing steel, which is a major material requirement in the initial stage, comes in assorted diameters and varying lengths. The weight of this material is a function of the bar diameter and length of bar. If the elevator under consideration has a clear platform size of 4'-6" x 4'-6", that number of reinforcing bars requiring a clear dimension of less than 4'-6" whose total weight does not exceed the weight capacity of the elevator can be transported. However, many of the reinforcing bars required will exceed the 4'-6" clear dimension of the elevator platform. In this case, it is not the weight capacity of the elevator that is the limiting factor, but the length capacity of the elevator. It would be necessary in this case to transport those reinforcing bars exceeding 4'-6" by means other than an elevator. The effective load capacity for bars less than 4'-6" would be determined either in number of bars of a certain size or in pounds. The effective load capacity would then be based on the weight capacity of the elevator (weight of the bars) or the volume capacity of the elevator (number of bars).

Another example is lightweight accoustical material packaged in large bags or cartons with negligible weight but considerable volume per bag or carton. The effective load capacity in this case would be based on the volume capacity of the elevator rather than the weight or length capacity of the elevator.

The Basic Problem

Some observations can be made from the foregoing description of the building process.

1. The time of arrival at the lifting site for each material is not exactly known. Contributing to this situation are delays in shipment, weather, inadequate manpower, non-availability of hauling or unloading equipment, oversights in ordering, defective material returned for replacement, delays in approving shop drawings, prolonged laboratory tests, incorrect fabrication, strikes, disputes, work stoppages, and human frailties in general.
2. The materials arriving at the lifting site do not all possess the same characteristics. In addition to variations in size, weight, and physical properties, the handling times required for loading and unloading can vary.
3. The successive floor destinations of each trip load of material depends on the order of material arrivals. This fact prevents an orderly sequence of floor by floor deliveries if idle time of the elevator(s) is to be kept at a minimum.
4. The elevators serving the construction of the building are expected to lift all material capable of being lifted as determined by the effective load capacity.
5. Because different materials have different handling times and different floor destinations, longer or shorter times are required to transport

each elevator load.

6. On those days when an elevator's capacity is devoted exclusively to the transportation of concrete, other materials must await service or else supplementary lifting equipment must be provided.

7. Since the arrival times of material to the lifting site are indefinite and the time to handle and lift a material varies, it is reasonable to expect that a build-up of materials waiting to be lifted will occur at times.

8. The nature of the building process is such that some materials must be in place, fastened, and finished before others. This gives rise to the question of the cost associated with <u>not</u> having a material in place at a given time.

The above observations suggest that the determination of the productivity of one or more elevators must take into account that material arrivals may fluctuate considerably, placing a varying demand on the elevator service capacity. The elevator service capacity must be selected to insure that the demand be advantageously satisfied thereby avoiding persistent and recurring bottlenecks. If the elevator service capacity is not selected so that its mean capacity is at least as large as the average demand imposed on it by materials arriving to be lifted, a build-up of materials occurs until the demand decreases or the elevator service capacity is increased. But even if the mean elevator service capacity is selected to satisfy average demand, transient build-ups of materials waiting to be lifted may nevertheless occur due to the probabilistic nature of material arrivals or the variability of the actual elevator service capacity available to perform the required service.

The approach to be used in determining the contribution of an elevator to the vertical transportation system is that of waiting-line theory. Waiting-line theory is a technique that relies on probability theory to analyze bottleneck situations such as the one posed here.

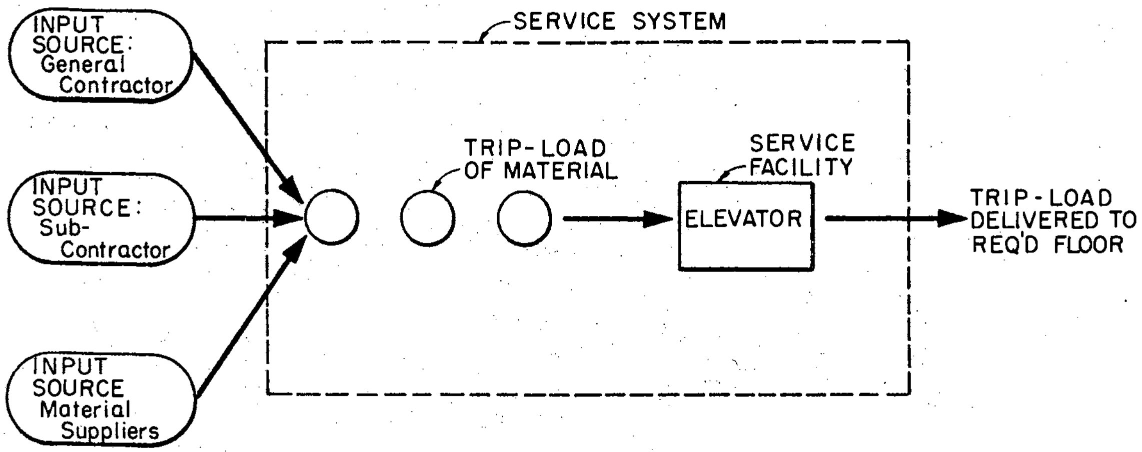
A waiting-line problem arises in this study when materials arriving for service at the elevator find the elevator not immediately available to provide the required service, thus resulting in a waiting-line of materials and its associated costs of delays. The objective of this research is to develop economic models capable of determining the level of elevator service that will result in the minimum sum of two opposing costs: 1) the cost of waiting time of material to be lifted, and 2) the cost of providing additional elevator service capacity.

CHAPTER IV

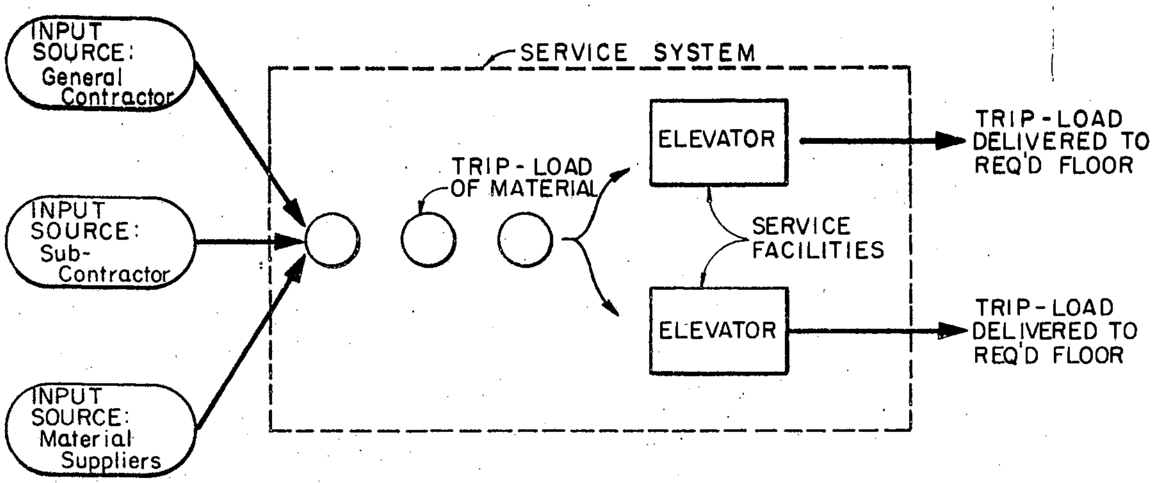
WAITING-LINE PROCESS

Description

A waiting-line process typically involves a service system which has one or more service facilities. The service system is subjected to varying demands for service by those items requiring service. Items requiring service are generated at different times by an input source, generally referred to as a population. The fact that items requiring service arrive at different times accounts for the varying demand imposed on the service system. In some waiting-line processes, items arriving for service may not enter the service system because of the number of items waiting for service. This line of items waiting for service is called a queue, or waiting-line. An item, in order to be serviced, must enter the service system by joining the waiting-line even though the waiting-line is of zero length. After an item enters the service system, it is selected for servicing by some decision rule called service discipline. The item is considered to be free of the service system after it has been serviced. See Figure 1 for a schematic diagram of the waiting-line process as applied to the problem under consideration.



SINGLE QUEUE - ONE ELEVATOR



SINGLE QUEUE - TWO ELEVATORS

Figure 1. Vertical Transportation System as a Waiting Line Process

Input Source

An input source is characterized by:

1. The size.
2. The arrival time distribution of arrivals seeking service.
3. The freedom of arrivals to accept or reject service.

The size of an input source is considered finite or infinite depending on whether or not it affects the rate at which the source generates arrivals for service. It is general practice to consider the input source infinite if the ratio of arrivals to be serviced to potential arrivals is very small. In this study, the input source will be considered infinite since the combined requirements for lifting service of the general contractor, subcontractor, and miscellaneous suppliers will be quite large when compared to the trip-loads of material in the systems at any given time.

Arrival Time Distribution

Material arriving at the elevator to be lifted has been previously shown to arrive in a more or less irregular pattern. The time interval between successive arrivals will be considered as independent random variables which will be assumed to have a statistical distribution that can be approximated from actual observation. Arrival time distributions that have been studied in practical problems are found to be exponential in many cases. It can be shown that an exponential arrival time distribution may be taken as characterizing Poisson-type arrivals if the number of arrivals during any i^{th} time interval is independent of the number of arrivals that occurred prior to the beginning of the time interval. Material arriving on a construction project exhibits this characteristic; therefore, the assumption of Poisson arrivals will be made throughout this study.

Freedom of Arrivals to Accept or Reject Service

The nature of building construction is such that no material is delivered to the lift site unless it is to be used in the construction of the building. It is, therefore, reasonable to assume that all arrivals both join and remain in the queue until served.

Queue

The material waiting in line to be served by the elevator is the queue. It is assumed that sufficient space is available at the job site in the vicinity of the elevator so that a restriction of the queue length is not necessary. The queue is assumed to be in a steady state condition.

Service Facility

The service facility in this study is either one elevator or two elevators, as the case may be. If one elevator is being used it will be referred to as a single channel service facility; in the case of two elevators, the service facility will be referred to as having two channels. Where two elevators are being analyzed, it will be assumed they operate in parallel such that one queue serves both elevators. Arriving material may be serviced by either of the elevators available. If an elevator is not immediately available, the arriving material joins the queue and is served in turn on a "first-come, first-served" basis.

Service-Time Distribution

The service time for each trip-load of material will be considered as the sum of the time required to load the elevator, travel time of the elevator up and down, and the time to unload the elevator at the prescribed floor. Clearly, the service time for each trip-load of material may vary and the service time for any particular trip-load does not depend in any way on the service time of the preceding trip-load. It is reasonable to assume that the service times are random and that each service time has a constant probability of terminating in the next small increment of time regardless of how long service has already taken place. The exponential distribution will be assumed to best approximate these conditions.

Mathematical Proofs of Queueing Formulae

The queueing theory applicable to single-channel and multi-channel problems is adequately developed in a number of sources.* The mathematical proofs of the queueing

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*C. W. Churchman, R. L. Ackoff, and E. L. Arnoff, formulae used in developing the economic models which follow are well known. The inclusion of these proofs here would be repetitious and serve no useful purpose.

Introduction to Operations Research (New York: John Wiley & Sons, Inc., 1957).

T. L. Saaty, *Mathematical Methods of Operations Research* (New York: McGraw-Hill Book Company, Inc., 1959).

W. R. Van Voorhis (edited by R. L. Ackoff), "Waiting-Line Theory as a Management Tool," *The Journal of Operations Research Society of America*, Vol. 4 (1956), p. 221.

P. J. Burke, "The Output of a Queueing System," *The Journal of Operations Research Society of America*, Vol. 4 (1956), p. 699.

G. Luchak, "The Solution of the Single-Channel Queueing Equations Characterized by a Time-Dependent Poisson-Distribution Arrival Rate and a General Class of Holding Times," *The Journal of Operations Research Society of America*, Vol. 4 (1956), p. 711.

CHAPTER V

DEVELOPMENT OF ECONOMIC MODELS

Method of Analysis

As previously noted, it is convenient to consider the time required to construct a building as a series of time periods of equal length. This length shall be arbitrarily taken as one week. For each time period the quantity of each material required for each floor as well as the tools and equipment necessary in the construction of the building is reasonably known. The number of trip-loads to be lifted in that time period is determined by the effective load capacity of the elevator. The service time required for each trip-load is determined by the loading and unloading time plus travel time of elevators for that particular item transported. The mean arrival rate and mean service rate can be derived from these data.

The cost of providing an elevator and its associated full-time operating cost is known. As the measure of effectiveness, the average waiting time per trip-load will be determined. The cost associated with waiting (see pages 68 and 73) is assumed to be obtainable.

To effect an economic balance between having material wait for service and having sufficient elevator capacity to handle all demand for service, a model is developed to represent these opposing policies.

The total cost of waiting will be assumed to be proportional to the total time that all trip-loads spend in the system, both waiting and in service. The cost of service provided by one elevator is considered to be a linear function of the number of elevators employed.

Economic Model No. 1

Symbols

ρ = elevator(s) occupancy ratio = percentage of time in use = $\frac{\lambda}{k\mu}$

λ = average number of elevator trips per week

μ = average service rate per elevator per week

k = number of elevators

W = mean waiting time per trip in system

W_q = mean waiting time per trip in queue

C_k = cost of providing an elevator per week

C_w = cost of waiting per week

C_o = cost of operation per elevator per week

P_o = probability that no trips are in the system in a small time interval

θ = ρk

The fixed cost of providing elevator capacity per week is

$$kC_k$$

while the operating cost per week for the elevator capacity provided is

$$\frac{\lambda}{k\mu}(C_o) \text{ .}$$

The cost associated with a trip-load waiting is

$$\lambda(W)\left(\frac{C_w}{\lambda}\right) \text{ .}$$

The total cost associated with a variation in the number of elevators provided is the sum of these three costs and is

$$TC_i = kC_{k_i} + \rho_i kC_o + \lambda_i(W_i)\left(\frac{C_{w_i}}{\lambda_i}\right)$$

where

TC_i = total cost for i^{th} time period.

The operating cost is independent of the number of elevators provided since

$$\rho kC_o = \left(\frac{\lambda}{k\mu}\right)(k)(C_o) = \left(\frac{\lambda}{\mu}\right) C_o \text{ .}$$

Also,

$$(\lambda)(W)\left(\frac{C_w}{\lambda}\right) = W(C_w) \text{ .}$$

It can be shown (see Appendix B) that

$$W = W_q + \frac{1}{\mu}$$

since if μ = service rate then $\frac{1}{\mu}$ = time to service one trip-load. Also, for $k > 1$

$$W_q = \left[\frac{\mu\left(\frac{\lambda}{\mu}\right)^k}{(k-1)!\ (k\mu-\lambda)^2}\right]\left[P_o\right] \qquad (5\text{-}1)$$

where

$$P_o = \frac{1}{\left[\sum_{n=0}^{k-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \frac{k\mu}{k\mu-\lambda}} \text{ .}$$

Therefore, TC_i can be written for the case $k > 1$ as

$$TC_i = kC_{k_i} + \frac{\lambda_i}{\mu_i} C_o$$

$$+ \left[\left(\frac{\mu_i \left(\frac{\lambda_i}{\mu_i}\right)^k}{(k-1)!\ (k\mu_i - \lambda_i)^2} \right) \left(\frac{1}{\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda_i}{\mu_i}\right)^n + \frac{1}{k!} \left(\frac{\lambda_i}{\mu_i}\right)^k \frac{k\mu_i}{k\mu_i - \lambda_i}} \right) + \frac{1}{\mu_i} \right] C_{w_i} \qquad (5\text{-}2)$$

which, for the case $k = 1$, reduces to

$$TC_i = kC_{k_i} + \left(\frac{\lambda_i}{\mu_i}\right) C_o + \left(\frac{1}{\mu_i - \lambda_i}\right) C_{w_i} . \qquad (5\text{-}3)$$

To determine the i^{th} time period at which the relative economy of providing one or two elevators is the same, the computer program (see Appendix A, Program 1, which has been written for $k = 1$-8) may be used to calculate the cumulative TC_i's for $k = 1,2$, given cost coefficients C_{k_i}, C_o, and C_{w_i}. These cumulative TC_i's may be plotted against the time periods to determine that time period at which the break-even point occurs for successive values of k. The time periods can be related to the floor on which the concrete frame is being constructed by referring to the CPM network. It is this floor that determines the maximum economic height that one elevator can serve for the parameters inserted in Computer Program No. 1.

Figure 2 is a representative cumulative plot of Equations (5-2) and (5-3) using fictitious data and assumed cost coefficients.

Computer Program No. 1 includes a plot subroutine;

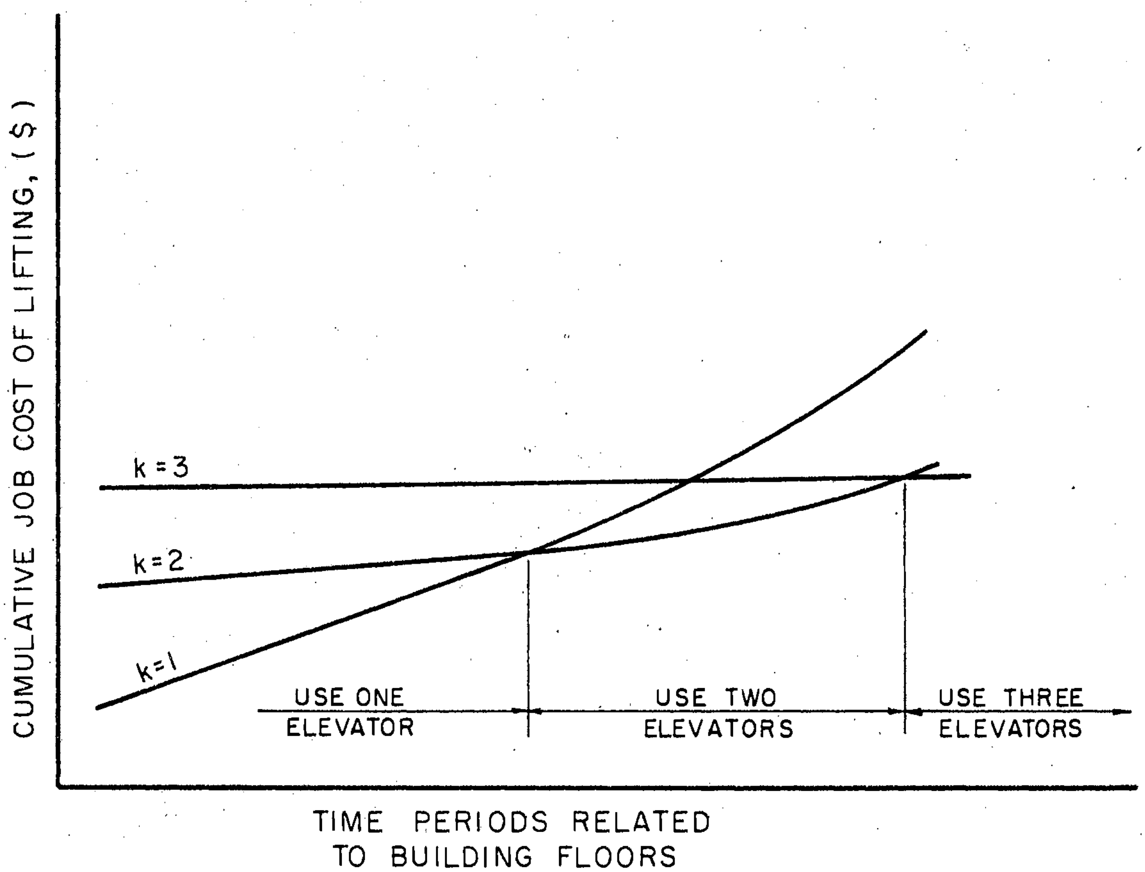


Figure 2. Relationship Between Cumulative Cost of Lifting and Number of Building Floors for Various Levels of Service

therefore, a plot similar to that of Figure 2 will be included as part of the output from the Program.

Manipulation of Model 1

Calculation of Optimum k*. From the Total Cost Equation, a Total Variable Cost (TVC) equation can be written for the ith time period:

$$TVC_i = kC_{k_i} + \lambda_i W_i (C'_{w_i}) \qquad (5\text{-}4)$$

where

$$C'_{w_i} = \frac{C_{w_i}}{\lambda_i}$$

or

$$TVC_i = C_{k_i}\left[k + \lambda_i W_i \left(\frac{C'_{w_i}}{C_{k_i}}\right)\right] . \qquad (5\text{-}5)$$

In order to minimize Equation (5-5), values of the average waiting time per trip, W_i, are required. For ease of computation by graphical aids, W_i can be stated in terms of $\frac{\lambda_i}{\mu_i}$ by expressing W_i as a multiple, f_i, of the average service time, $\frac{1}{\mu_i}$, in the following manner:

$$f_i = \frac{W_i}{\frac{1}{\mu_i}} \quad \text{and} \quad W_i = \frac{f_i}{\mu_i} .$$

Equation (5-5) can be written as

$$TVC_i = C_{k_i}\left[k + \left(\frac{\lambda_i}{\mu_i}\right)\left(f_i\right)\left(\frac{C'_{w_i}}{C_{k_i}}\right)\right] . \qquad (5\text{-}6)$$

*This development generally follows that of Manglesdorf (4).

As presently defined, $\rho_i = \frac{\lambda_i}{k\mu_i} < 1.0$. It is convenient to relax this upper bound restriction on ρ so that the ratio $\left(\frac{\lambda_i}{\mu_i}\right)$ can be expressed in terms of the number of elevators, k. Therefore, let

$$\theta_i = \left(\frac{\lambda_i}{\mu_i}\right) = \left(\frac{\lambda_i}{k\mu_i}\right) k = \rho_i k \ .$$

Making the appropriate substitution in Equation (5-6), the expression to be minimized now becomes

$$TVC_i = C_{k_i}\left[k + (\theta_i)(f_i)\left(\frac{C'_{w_i}}{C_{k_i}}\right)\right] . \tag{5-7}$$

Taking the first difference with respect to k and setting it equal to zero since we seek the k for which $\frac{\Delta TVC}{\Delta k}$ is as near zero as possible, the following equation is obtained:

$$\frac{\Delta TVC_i}{\Delta k} = \left[(k+1) + (\theta_i)(f_{i_{k+1}})\left(\frac{C'_{w_i}}{C_{k_i}}\right)\right] - \left[k + (\theta_i)(f_{i_k})\left(\frac{C'_{w_i}}{C_{k_i}}\right)\right] = 0 \tag{5-8}$$

Since the number of elevators, k, can only be a positive integer, generally for any given values of θ and $\frac{C'_{w_i}}{C_{k_i}}$ Equation (5-8) cannot be perfectly satisfied. The optimum solution then is the value of k which most nearly satisfies Equation (5-8).

If, however, Equation (5-8) is solved for $\frac{C'_{w_i}}{C_{k_i}}$, then

$$\frac{C'_{w_i}}{C_{k_i}} = \frac{1}{\theta_i(f_{i_k} - f_{i_{k+1}})} . \tag{5-9}$$

Since

$$f_k = \mu W_k \quad \text{and} \quad W_k = W_{q_k} + \frac{1}{\mu}$$

then

$$f_k = \mu \left(W_{q_k} + \frac{1}{\mu}\right) = \mu W_{q_k} + 1 .$$

Hence,

$$\frac{C'_{w_i}}{C_{k_i}} = \frac{1}{\theta_i[(\mu W_{q_k} + 1) - (\mu W_{q_{k+1}} + 1)]} .$$

Therefore,

$$\frac{C'_{w_i}}{C_{k_i}} = \frac{1}{\theta_i(\mu W_{q_k} - \mu W_{q_{k+1}})} . \qquad (5\text{-}10)$$

Results from Equation (5-10) give particular values of $\frac{C'_{w_i}}{C_{k_i}}$ for which exact solutions can be calculated. If for each value of k, various values of θ are inserted in Equation (5-10), the resulting values of $\frac{C'_{w_i}}{C_{k_i}}$ can be plotted in a useful form. The results of several such calculations are shown in Figure 4.

The calculation of μW_q for various values of θ and k is very tedious. A Computer Program (Appendix B, Program No. 2, which has been written for $k = 1\text{-}6$) calculates values of μW_q as a function of ρ and the results are graphically summarized in Figure 3. The use of Figure 3 permits calculations required in the solution of the Model to be made easily and rapidly by hand.

The lines in Figure 4 are the loci of combinations of values of θ and $\frac{C'_{w_i}}{C_{k_i}}$ for which Equation (5-8) is exactly satisfied. For values of θ and $\frac{C'_{w_i}}{C_{k_i}}$ whose intersection in Figure 4 falls between the lines, the optimum solution to

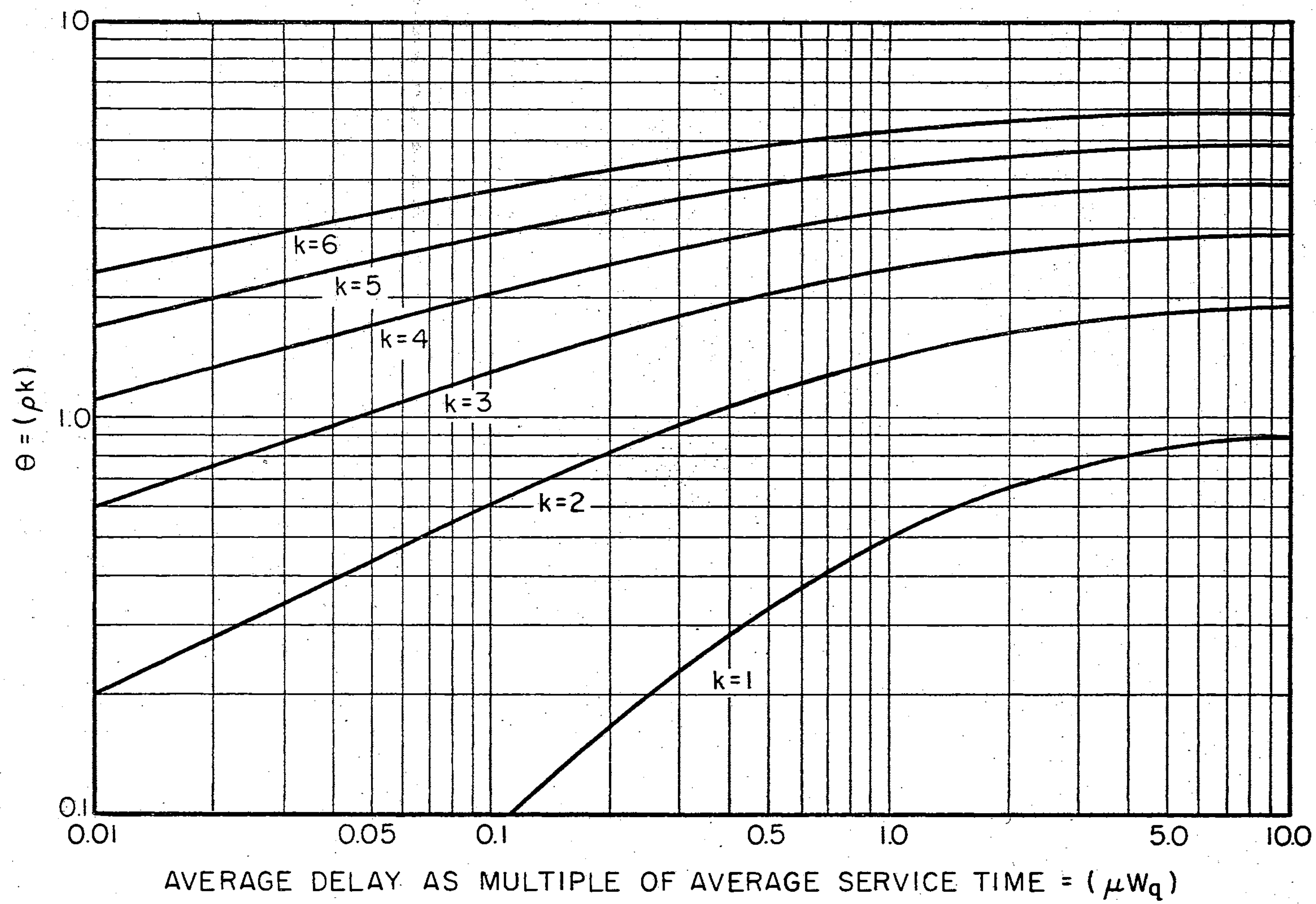


Figure 3. Average Delay with Service Times Exponentially Distributed

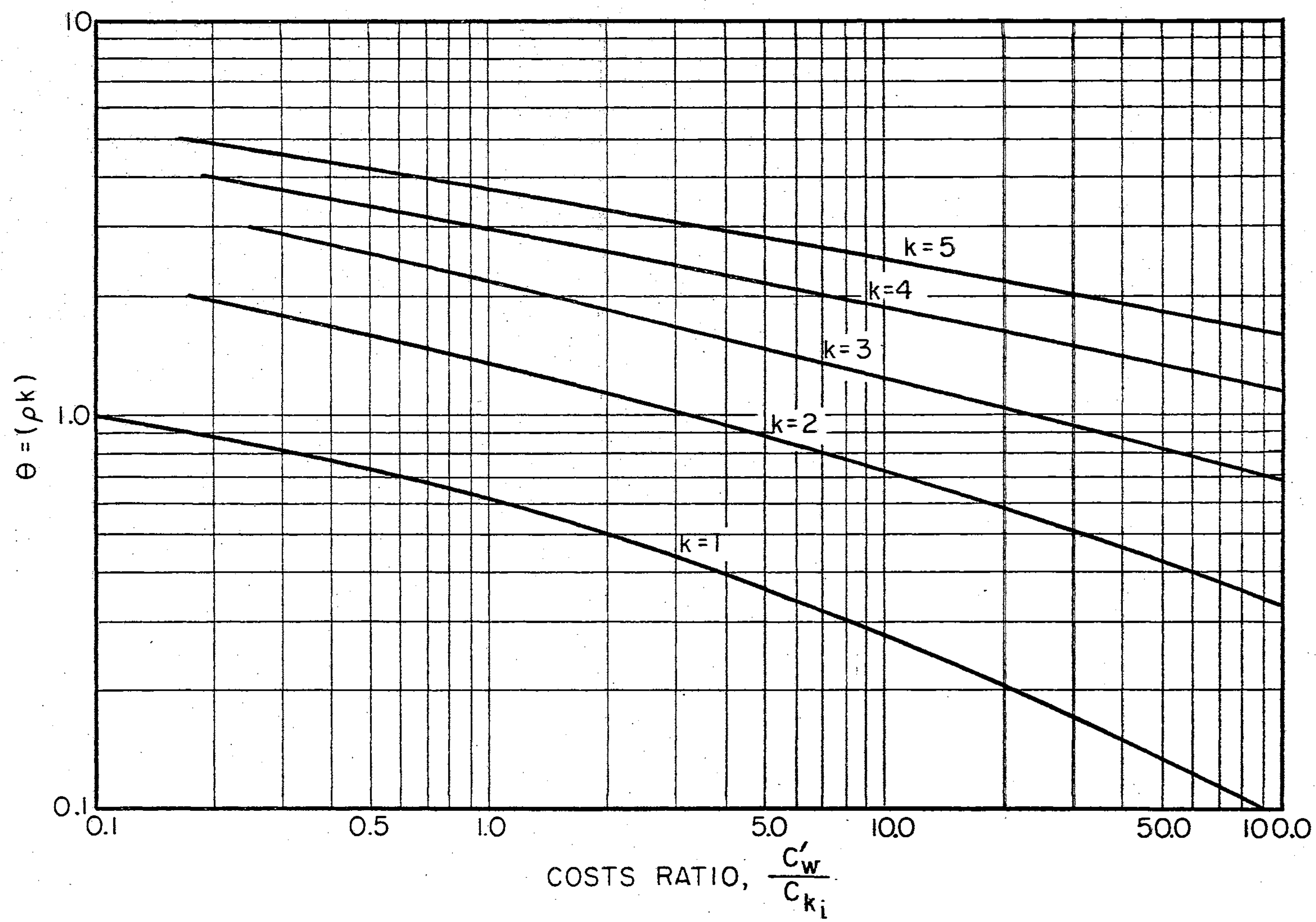


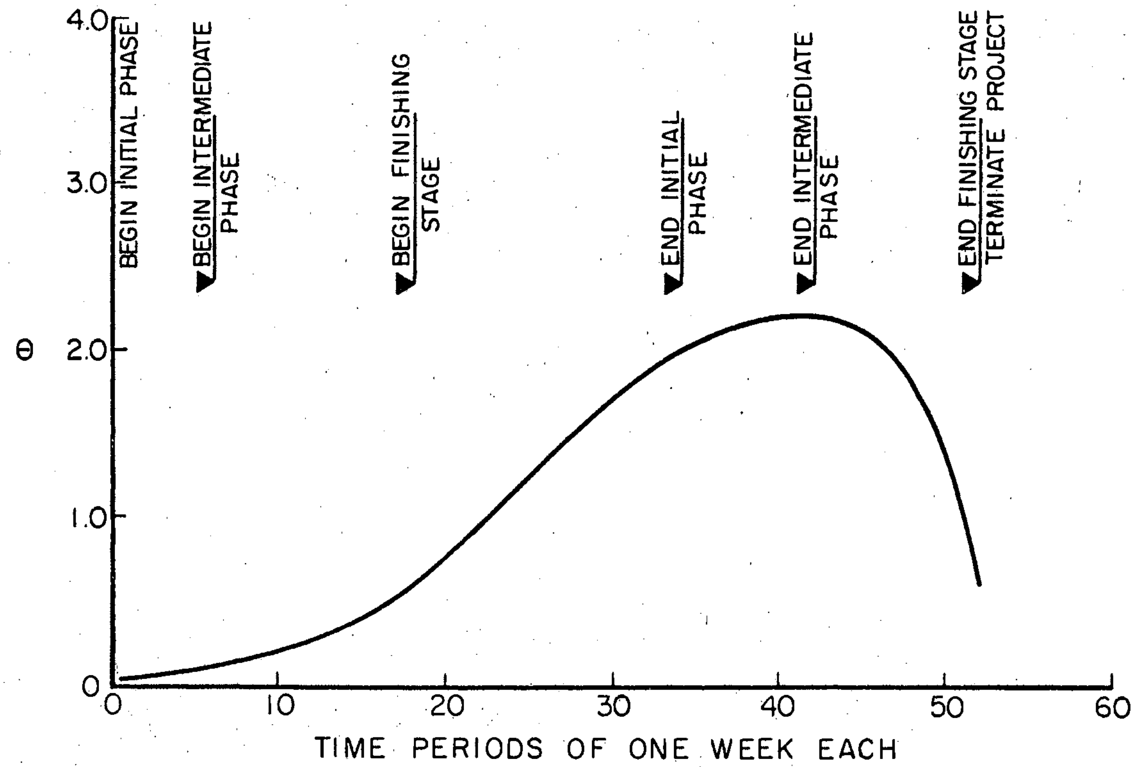
Figure 4. Optimum Number of Elevators (or Machines) with Service Time Exponentially Distributed

Equation (5-8) is obtained by using the value of k in the zone between the lines. Because the optimum solution of Equation (5-8) results in the minimum TVC for any given values of θ and $\frac{C_{w_i}}{C_{k_i}}$, Figure 4 is useful for rapidly noting whether the economic service of one elevator has been exceeded for the i^{th} time period. It can be similarly noted from Figure 4 the time period in which the economic service capacity of the second elevator will be exceeded, since increasing demand will be correspondingly reflected in the value of θ.

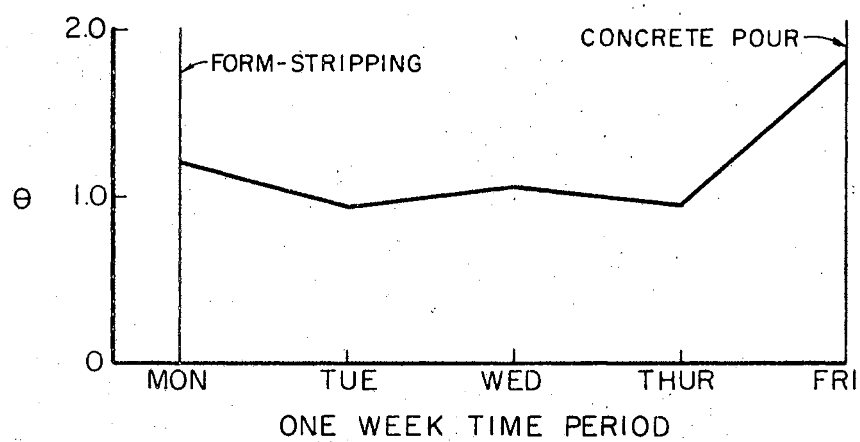
Economy of Supplementary Equipment

It is quite likely that θ will assume a wide range of values over the time periods. See Figures 5(a) and (b). Typically, the value of θ tends to increase with each successive time period until the decreased demand for elevator service offsets the travel time of the elevator. At this point, the value of θ levels off and decreases over time. The plot of the variation of θ confirms what is intuitively obvious. That is, the demand for elevator service varies with the degree of construction activity. As some lower floors are being completed, other floors are in the intermediate stage while still others may be in the initial stage.

The variability of θ is even more noticeable in Figure 5(b). This plot shows the value of θ by days in a time period of one week during which a concrete pour is scheduled.



(a)



(b)

Figure 5. Typical Variation of θ Over Time

The increase in the value of θ is apparent for the day on which the elevator is to transport concrete and the day that forms are stripped and relocated.

By previous definition, the value of θ must lie in the closed interval, $0 < \theta < 1$ for $k = 1$. It becomes apparent, then, if $\theta \geq 1.0$, one of two alternatives can be selected. The system may work out the demand by the deferring of some activities or, alternatively, the elevator capacity may be augmented for the day or days on which $\theta \geq 1.0$. There is a cost associated with each alternative. The cost can be calculated in the following manner. Since λ_i = number of trips the elevator makes in i^{th} time period of five days and λ_{id} = number of trips the elevator makes on d^{th} day in i^{th} time period, then

$$\lambda_i = \sum_{d=1}^{d=5} \lambda_{id} .$$

The value of θ_i for i^{th} time period is determined by

$$\theta_i = \frac{\lambda_i}{\mu_i} = \frac{\Sigma\lambda_{id}}{\mu_i} .$$

θ_i can be reduced to a revised value, θ_{i_r}, which assures that the system will not be overloaded. This reduction takes the following form

$$\theta_{i_r} = \frac{\sum_{d=1}^{d=5} \lambda_{id} - \sum_{d=1}^{d=5} \lambda'_{id}}{\mu_i}$$

where λ'_{id} is defined as the number of deferred trips on the d^{th} day during the i^{th} time period. Equation (5-10) can be restated in a form such that the cost of additional supplementary equipment to service certain λ'_{id}'s is equal to the cost of deferring the servicing of these λ'_{id}'s. Equation (5-10) restated is

$$C_{k_i} = [(\theta_i)(\mu W_{q_k})(C'_{w_i})] - [(\theta_{ir})(\mu W_{q_{k+1}})(C'_{w_i})].$$

Consider this example. Suppose from the output of Computer Program No. 1 that for some time period i, $\theta_i = 1.0$, $k = 1$, $C'_{w_i} = 1000$ and $C_k = 500$. Entering Figure 4 with $\frac{C'_{w_i}}{C_{k_i}} = 2.0$ and $\theta_i = 1.0$, it is noted that θ_{ir} must be 0.5 for $k = 1$ to be economical. Assume that $\Sigma\lambda'_{id}$'s $= \frac{1}{2}\Sigma\lambda_i$. Since additional equipment beyond the one elevator should be considered, enter Figure 3 with $k = 2$ and note the average delay as a multiple of the average service time (μW_q), for $\theta_i = 1.0$ and $\theta_{ir} = 0.5$. The cost of additional equipment to service the λ'_{id}'s of interest is

$$= [(\theta_i)(\mu W_{q_k})(C'_{w_i})] - [(\theta_{ir})(\mu W_{q_{k+1}})(C'_{w_i})]$$

$$= [(1.0)(0.33)(1000)] - [(0.5)(0.066)(1000)]$$

$$= \$297.$$

The \$297 figure also represents the waiting cost incurred if the λ'_{id}'s are deferred.

Updating Schedule

For a variety of reasons, the work may fall behind schedule. Weather, equipment breakdown, under-estimating the demand for elevator service, etc., has the effect of increasing the demand in following time periods or days within a time period. Increasing λ_i or λ_{id} may increase θ_i to a value such that the economic capacity of the elevator provided is temporarily, or even permanently, exceeded. Examination of the revised schedule will give a preview of the expected daily demand from which revisions in θ_i can be calculated. The break-even cost of bringing in supplementary equipment to augment the service capacity already available can be calculated using the same procedure outlined in the preceding section.

It may well be that the system is already operating at or near its maximum economic capacity. Should this be the case, delays in the early time periods could necessitate the installation of the second elevator at an earlier date. Perhaps a second elevator was never intended on a project but delays greatly increased demand in subsequent time periods. The procedure outlined could be used to evaluate the relative economy of providing a second elevator or using supplementary equipment. Furthermore, this procedure could also be used to evaluate the advisability of two shifts, overtime work, or additional manpower, by comparing this cost with the cost of a second elevator or supplementary equipment.

Schedule Smoothing

Referring again to Figure 5(b), it can be noted that the value of θ_i varies from day to day. The value of θ_i on some days indicates that the service capacity will be exceeded. Examination of the activities to be performed on these days may reveal that some of the activities could be deferred to subsequent days having smaller θ_i values. The result of re-scheduling activities in this fashion is to level the demand imposed on the elevator.

The determination of C_w will be discussed in more detail; however, it can be pointed out here that some activities, as scheduled, do not contribute to the costs included in C_w because of slack in the schedule. Therefore, some activities may be deferred, or handled earlier, without penalty.

The work may be ahead of schedule. This condition implies that the productivity of either men or machines, or both, was underestimated. Under-estimating the productivity of manpower has the effect of increasing demand on the service facility by requiring material at a faster rate. Under-estimating productivity of the service facility has the effect of reducing the value of θ_i. In either case it may prove to be advantageous to reschedule activities affected in an attempt to maintain values of θ_i within tolerable limits for the service capacity level presently in use on the job.

Economic Model No. 2

Purpose of Model No. 2

Model No. 1 establishes a criterion for determining the maximum number of floors that one, or two, elevators can economically service without regard to the configuration of the building. The dimensions of a building are of concern since the cost of laterally transporting materials from the threshold of the elevator to the various work stations may exceed the cost of an additional elevator.

Consider buildings having identical floor areas per floor but different configurations. For a building with 6400 square feet per floor several different configurations follow:

1. Square: 80 ft. long x 80 ft. wide = 6400 square feet
2. Rectangular: 128 ft. long x 50 ft. wide = 6400 square feet
3. Rectangular: 256 ft. long x 25 ft. wide = 6400 square feet
4. Circular: 90 ft. in diameter = 6400 square feet
5. Trapezoidal: 128 ft. long with one end 60 ft. wide and the other end 40 ft. wide = 6400 square feet.

It is clear from the above that the configuration of a building may have the effect of varying the distances that materials have to be laterally transported from the elevator. Therefore, a model is needed to serve as a restriction on Model 1 to reflect the effect of lateral dimensions on the decision between one or two elevators.

Basis of Economic Model 2

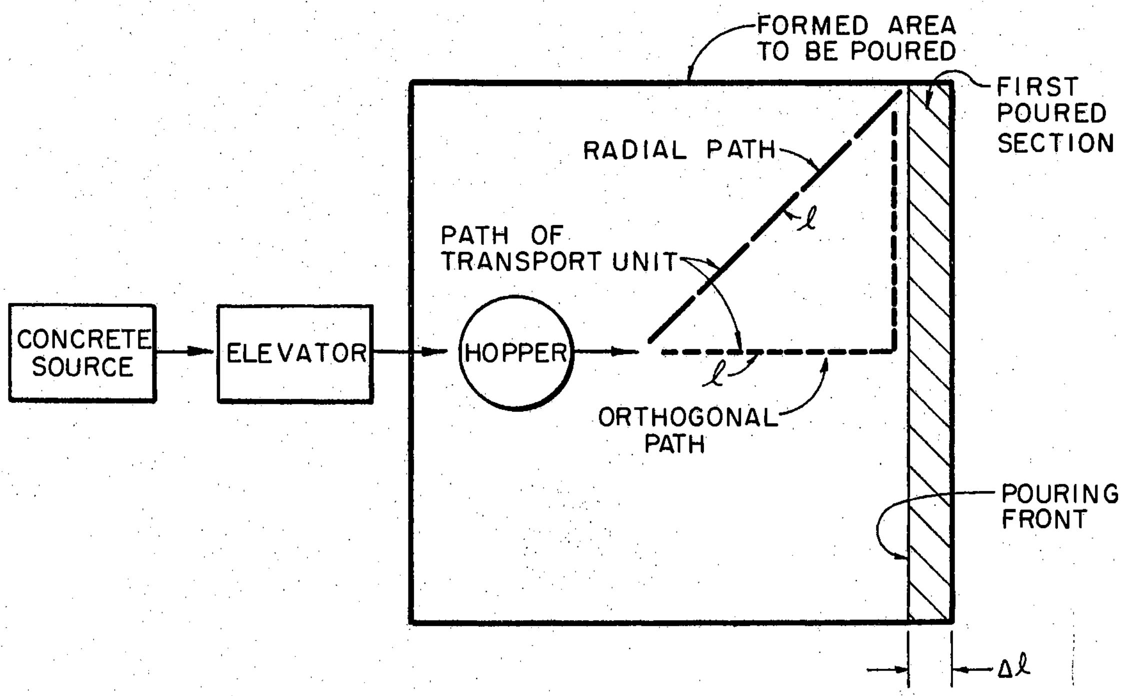
In Chapter III it was pointed out that the rate of concrete placing controlled the construction schedule of a reinforced concrete frame building. Of these concreting operations, placement of concrete for floors is the most critical. For this reason, Model 2 is based on the premise that placement of floor concrete controls the maximum distance that can be economically served from one elevator. This distance is the limiting length, width, or diameter of a building that can be served by one elevator which is equivalent to taking into account the configuration of a building.

The approach to determine the limiting distance for one elevator will be to write a total cost equation using one elevator to supply the concrete and a total cost equation using two elevators to supply the concrete. Solving these two equations simultaneously will result in determining the distance for which the cost of using one or two elevators is the same.

The rate at which the concrete can be placed on a floor cannot exceed the service capacity of the elevator. This service capacity is known for each floor as determined from Model 1.

The mechanics of placing concrete on a floor follows a sequence of events (see Figure 6):

1. The elevator discharges concrete into a hopper located at floor being poured.
2. Transport units (usually referred to as concrete buggies) carry the concrete from the hopper to the placement point over a system of runways and return. Runways may be radial or orthogonal.
3. The concreting begins at the outer edge of the



NOTE: RUNWAY LAYOUT MAY BE RADIAL OR ORTHOGONAL.
THE RUNWAYS DETERMINE PATH OF TRANSPORT UNIT.
ℓ^*= MAXIMUM ECONOMIC DISTANCE FROM HOPPER DISCHARGE TO POINT OF DEPOSIT MEASURED ALONG RUNWAY, IN FEET.
ℓ= TRAVEL DISTANCE OF A TRANSPORT UNIT, IN FEET.

Figure 6. Diagram of Floor Concrete Placement

formwork the greatest distance from the elevator and is placed uniformly along its edge for the full width. The process is repeated row by row until the entire form area has been covered.

4. As the distance from the point of placement to the hopper recedes, the number of transport units required to meet the pouring rate decreases from a maximum number to theoretically zero. The transport units and their operators not required are dismissed and removed from the pouring area.

Development of Economic Model 2

Symbols.

CY – cubic yards

r – service capacity of elevator in CY/hour (This is the pouring rate that must be maintained.)

ℓ – travel distance of a transport unit, in feet

v – velocity of a transport unit, in feet/minute

y – volume of a transport unit, in CY

n – number of transport unit operators, one operator for each transport unit

C_m – cost of an operator, in \$/hr.

C_k – cost of an elevator, in \$/hr.

k – number of elevators

C_b – cost of a transport unit, in \$/hr.

w – width of pouring front, in feet

ℓ^* – maximum economic distance from hopper discharge to point of deposit measured along runway

U - labor utilization factor, percentage expressed as a decimal

d - depth of slab, in feet

H - time to load and dump one transport unit, in hours.

Time needed to pour a $\Delta\ell$ (see Figure 6) section while maintaining the pouring rate, r,

$$= \frac{\text{volume of } \Delta\ell \text{ row}}{\text{pouring rate}} = \frac{(w)(\Delta\ell)(d)}{27\ r} .$$

Number of transport unit trips to pour a $\Delta\ell$ section

$$= \frac{\text{volume of } \Delta\ell \text{ row}}{\text{volume of a transport unit}} = \frac{(w)(\Delta\ell)(d)}{27\ y} .$$

Number of trips per hour required to meet pouring rate

$$= \frac{\frac{w(\Delta\ell)d}{27\ y}}{\frac{w(\Delta\ell)d}{27\ r}} = \frac{r}{y} .$$

Cycle time needed to meet pouring rate

$$= \begin{array}{c}\text{travel time of}\\ \text{transport unit}\end{array} + \begin{array}{c}\text{load and dump time}\\ \text{of transport unit}\end{array} .$$

Maximum distance traveled by transport unit = $2\ell + \frac{w}{2}$ with one elevator.

Maximum distance traveled by transport unit = $\frac{2\ell}{k} + \frac{w}{2}$ with k=1,2 elevators.

Travel time of transport unit in hours = $\dfrac{\frac{2\ell}{k} + \frac{w}{2}}{60v}$.

Cycle time in hours = $H + \dfrac{\frac{2\ell}{k} + \frac{w}{2}}{60v}$.

Number of operators needed for maximum travel distance

$$= H + \frac{\frac{2\ell}{k} + \frac{w}{2}}{60v} \quad \frac{r}{y} .$$

Cost of Operators = $C_m \quad H + \dfrac{\frac{2\ell}{k} + \frac{w}{2}}{60v} \quad \dfrac{r}{y}$.

Cost of transport units $= C_b \left[H + \dfrac{\frac{2\ell}{k} + \frac{w}{2}}{60v} \right] \left[\dfrac{r}{y} \right]$.

Cost of elevators $= (k)(C_k)$.

Therefore, the total cost to maintain r

= (Cost of elevators) + (Cost of operators) + (Cost of transport units)

$$= kC_k + C_m \left[H + \frac{\frac{2\ell}{k} + \frac{w}{2}}{60v} \right] + C_b \left[H + \frac{\frac{2\ell}{k} + \frac{w}{2}}{60v} \right] \left[\frac{r}{y} \right] . \qquad (5\text{-}10)$$

Therefore,

$$TC(k=1) = C_k + C_m \left[\frac{Hr}{y} + \frac{\ell r}{30vy} + \frac{wr}{120vy} \right] + C_b \left[\frac{Hr}{y} + \frac{\ell r}{30vy} + \frac{wr}{120vy} \right] \qquad (5\text{-}11)$$

$$TC(k=2) = 2C_k + C_m \left[\frac{Hr}{y} + \frac{\ell r}{60vy} + \frac{wr}{120vy} \right] + C_b \left[\frac{Hr}{y} + \frac{\ell r}{60vy} + \frac{wr}{120vy} \right] . \qquad (5\text{-}12)$$

Setting $TC(k=1) = TC(k=2)$ and solving for ℓ^*:

$$\ell^*_{1-2} = \frac{60vy}{r} \left[\frac{C_k}{C_m + C_b} \right] . \qquad (5\text{-}13)$$

This value of ℓ^* does not reflect the fact that the number of operators used in the pouring operations varies linearly from a maximum of n to zero with an average requirement of n/2. This means that the value of ℓ^* as shown is based on the assumption that the operators are used at 100 per cent productivity during the entire pouring operation. This is not consistent with field practice. It is customary field practice when operators are no longer needed on the pour to either send them to other work or detail them to cleaning and conditioning the transport units for the next pour. The cost of operators engaged in cleaning and conditioning the transport units is a legitimate charge against the pouring operation. The cost of operators detailed to other work should be deducted. This modification can be accomplished by multiplying C_m by a factor. This factor would be the percentage of time an operator is engaged in the pouring operations which is equivalent to applying this factor to the hourly cost, C_m. If all operators are engaged in the pouring operation 100 per cent of the time, the factor would be 1.0 since the entire cost should be assessed against the work. On the other hand, if the operators are assigned other work as they become unnecessary to the pouring operation, on the average only n/2 or 50 per cent of the total operator's cost should be charged to the pouring operation. This would result in a factor of 0.5 being applied to C_m. It follows then that this factor, U, reflects the operator utilization and can take on values from 0.5 to 1.0; i.e., $0.5 \leq U \leq 1.0$. Therefore, Equation (5-13) should be stated as

$$\ell^*_{1-2} = \frac{60vy}{r} \left[\frac{C_k}{U(C_m) + C_b} \right]. \qquad (5\text{-}14)$$

This value of ℓ^*_{1-2} is the travel distance from one elevator at which the cost of providing two elevators is the same as the cost of providing one elevator.

The interpretation of ℓ^*_{1-2} in Equation (5-14) can be explained by referring to Figures 7(a)(b)(c)(d).

Given the configuration of the building and the layout of proposed system of runways to accommodate the transport

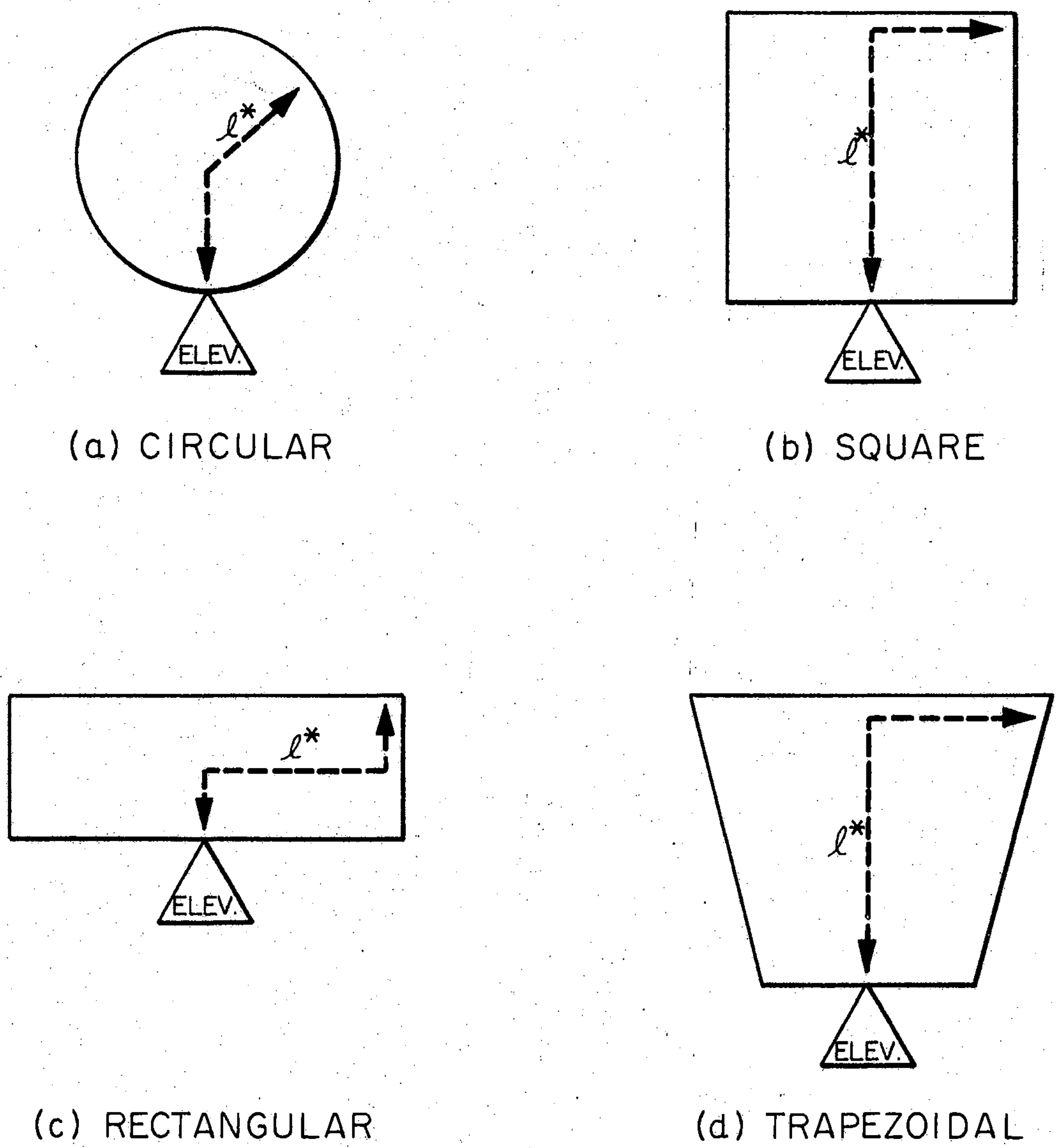


Figure 7. Interpretation of ℓ^* for Various Floor Configurations

units, ℓ^* is the distance to the extreme point of the pouring area measured along the runways. If the dimensions of the building are such that the value of ℓ^* will be exceeded, then it will be more economical to provide two elevators for the construction.

Although beyond the scope of this study, if the dimensions of a building are such that Model 2 indicates more than two elevators are required, then it is this writer's opinion that an entirely different problem arises and a new approach should be taken. The approach presently envisioned is to test the cost of three elevators against the cost of combinations of equipment.

Graphical Relationships for Model 2

Equation (5-14) can be written in the following form:

$$\ell^*_{1-2} = \frac{60vy}{r} \left[\frac{\frac{C_k}{C_b}}{U\left(\frac{C_m}{C_b}\right) + 1} \right] . \qquad (5\text{-}15)$$

Observed data indicates that velocity, v, of a transport unit varies from 10 feet per minute to 40 feet per minute, including the time for passing on runways, rest time, etc. A figure of 10 ft/min is used in these calculations. The volume, y, of a transport unit is taken to be nine cubic feet, or 0.33 CY. These values of v and y will be considered constant since they are representative values characteristic of a specific firm. As previously noted, the values of U range from 0.5 to 1.0.

For various values of C_k, C_b, C_m and C_b, a range of values for $\frac{C_k}{C_b}$ and $\frac{C_m}{C_b}$ can be calculated. Using the values of these ratios, values of

$$\left[\frac{\left(\frac{C_k}{C_b}\right)}{U\left(\frac{C_m}{C_b}\right) + 1}\right]$$

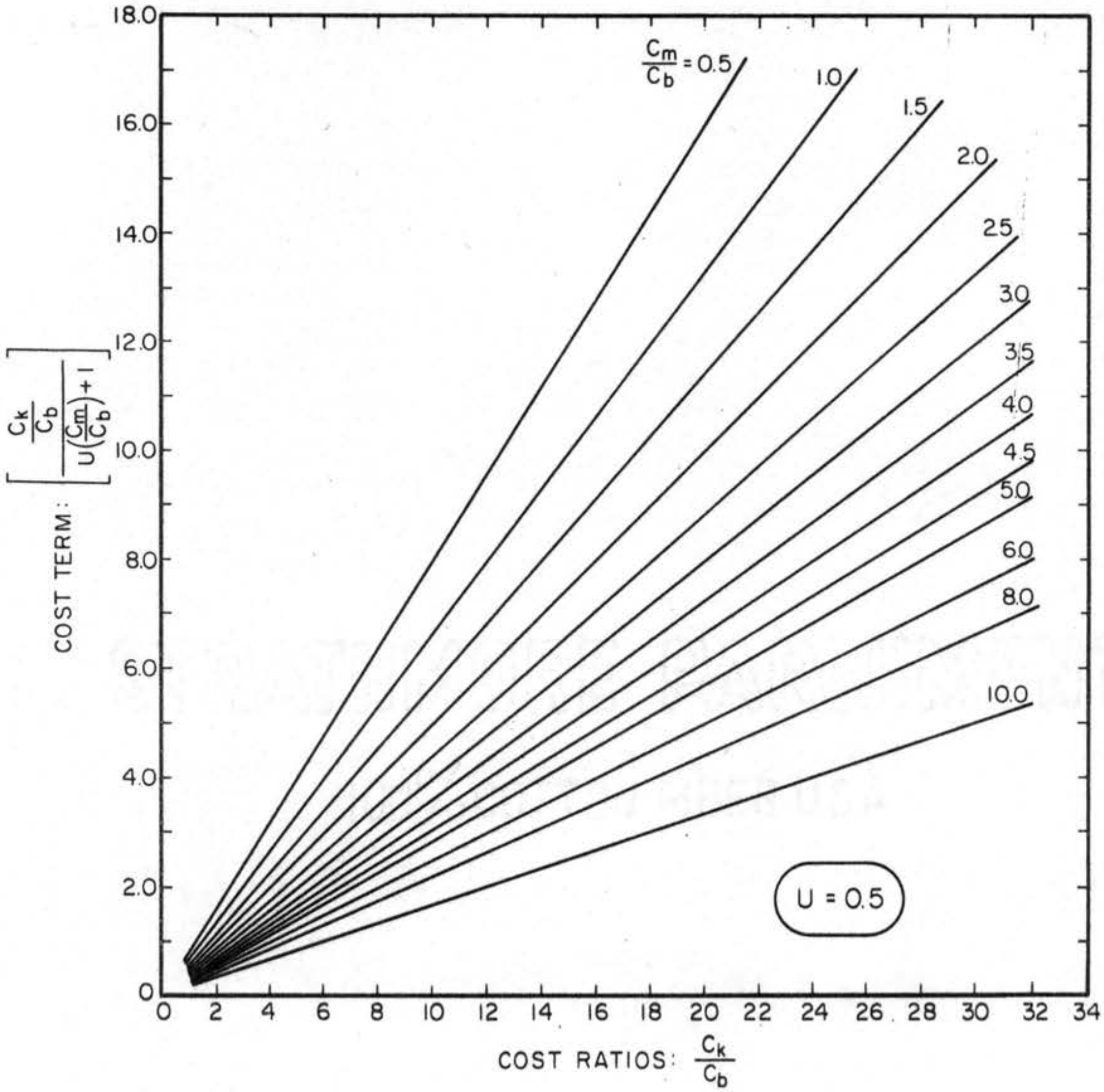
can be determined for values of U = 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0. Computer Program No. 3, Appendix C, was written to perform these calculations. The results are graphically summarized by Figures 8(a) through 8(f).

With the values of

$$\left[\frac{\left(\frac{C_k}{C_b}\right)}{U\left(\frac{C_m}{C_b}\right) + 1}\right]$$

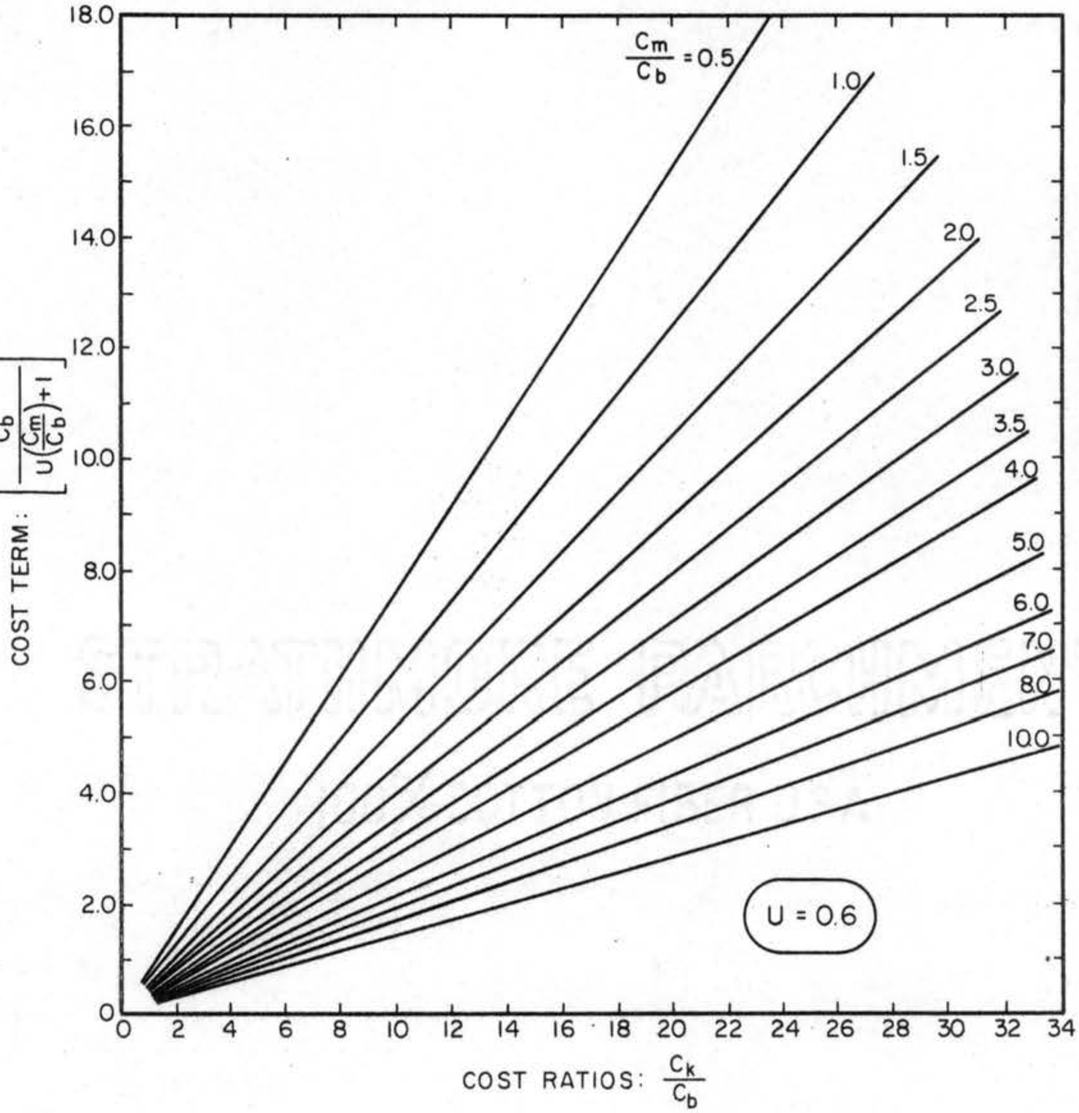
obtained from Computer Program No. 3, values of ℓ^*_{1-2} were calculated for various values of r. Computer Program No. 4, Appendix C, was written to perform these calculations. These results are graphically summarized by Figures 9(a) through 9(d).

The upper and lower limits of $\frac{C_k}{C_b}$ and $\frac{C_m}{C_b}$ were established as follows: minimum and maximum realistic values of C_k, C_b and C_m were assumed. The upper limit of $\frac{C_k}{C_b}$ was calculated by using the maximum assumed value of C_k and the minimum assumed value of C_b. The lower limit of $\frac{C_k}{C_b}$ was calculated by using the minimum assumed value of C_k and the maximum



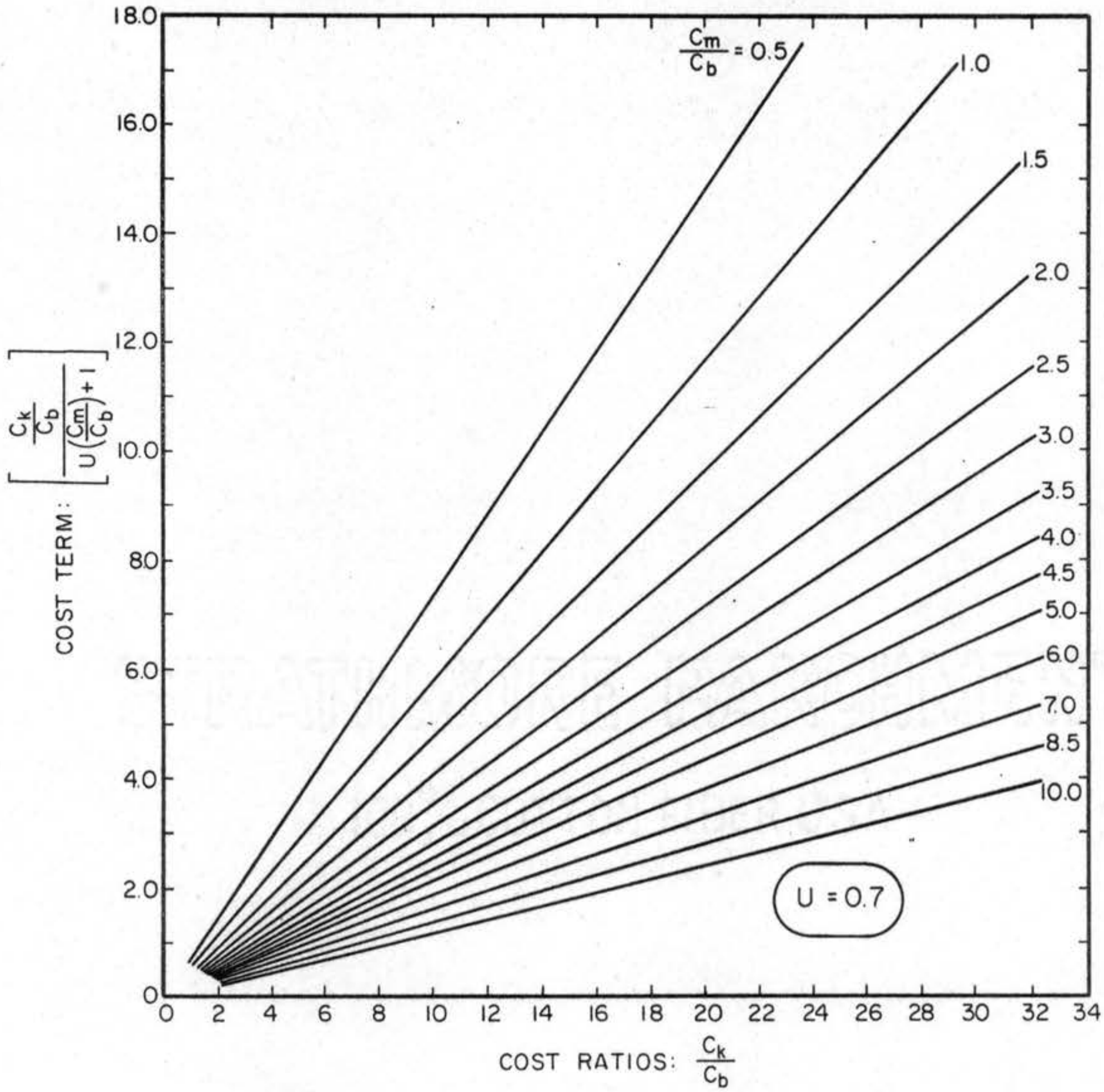
(a)

Figure 8. Values of Cost Term for Values of $\left(\frac{C_k}{C_b}\right)$ and $\left(\frac{C_m}{C_b}\right)$ for U = 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0



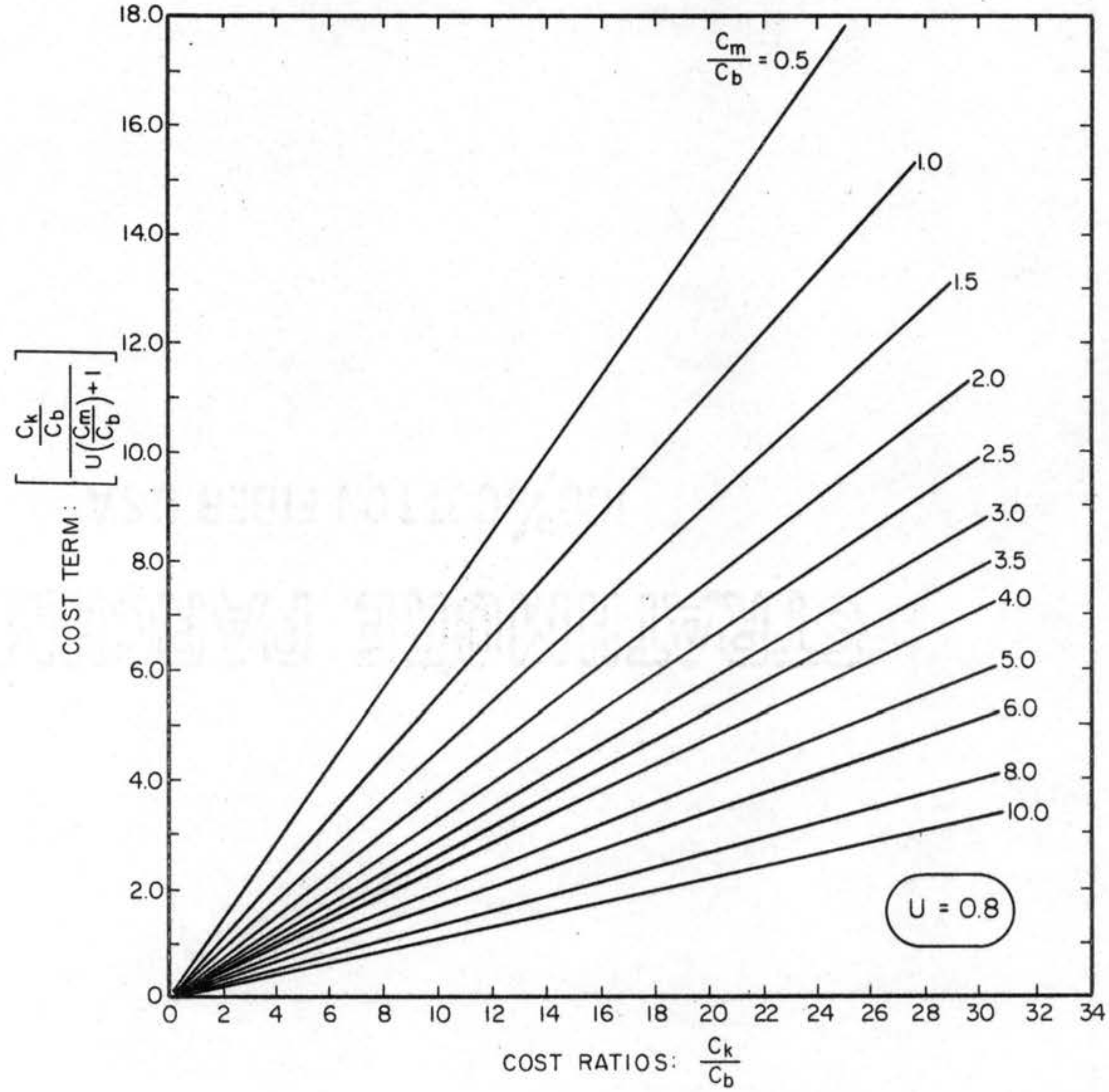
(b)

Figure 8. (Continued)



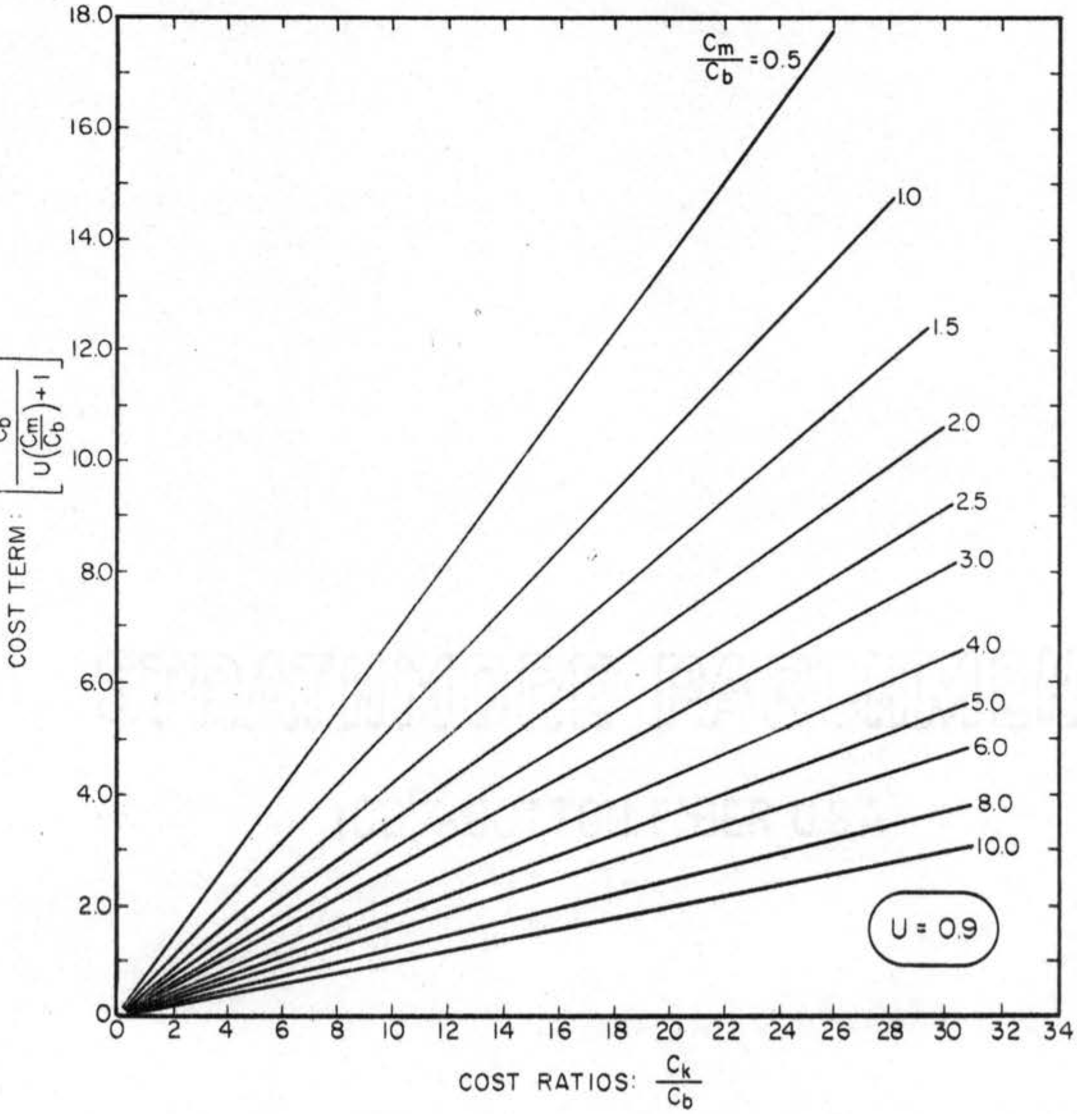
(c)

Figure 8. (Continued)



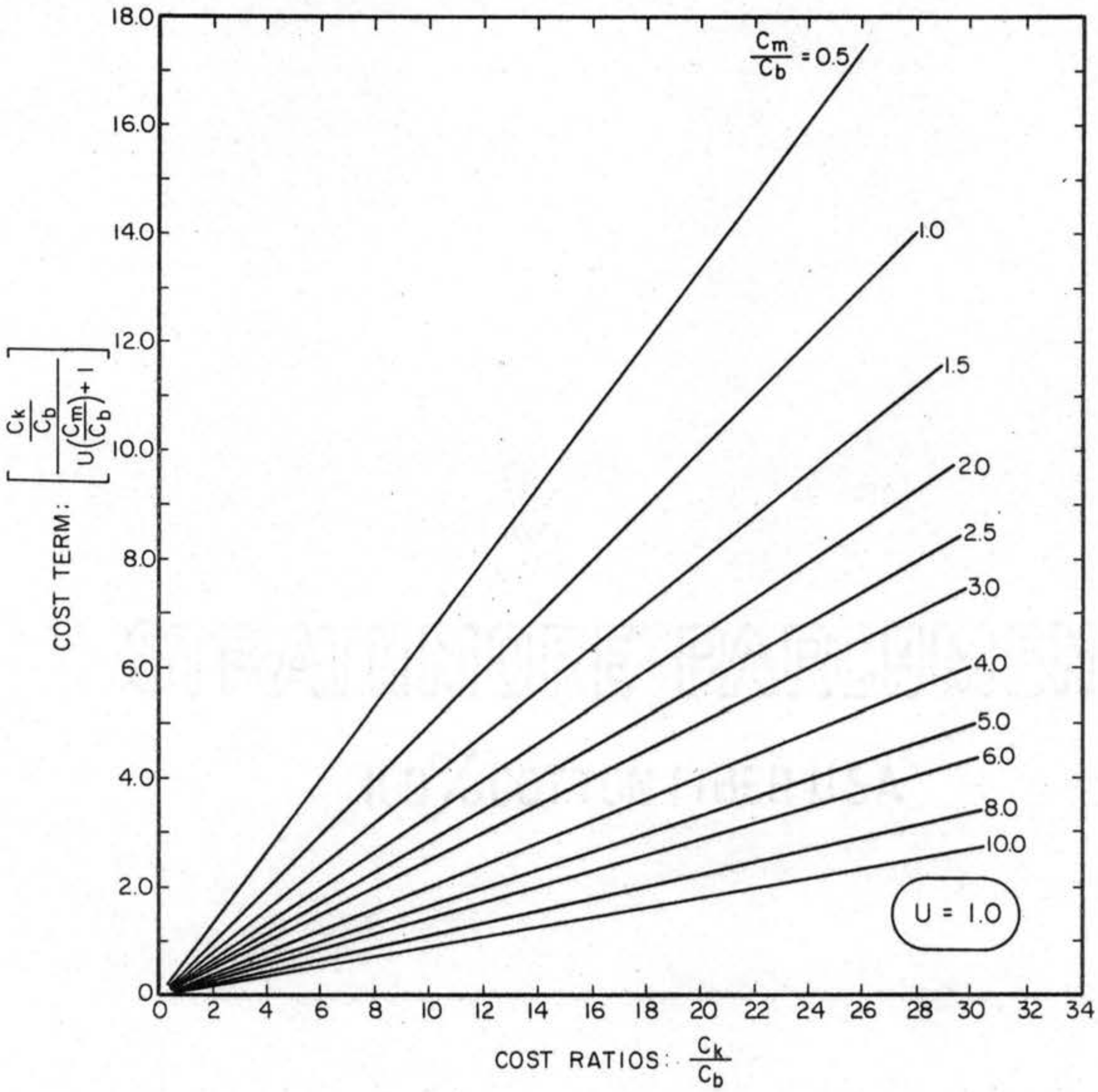
(d)

Figure 8. (Continued)



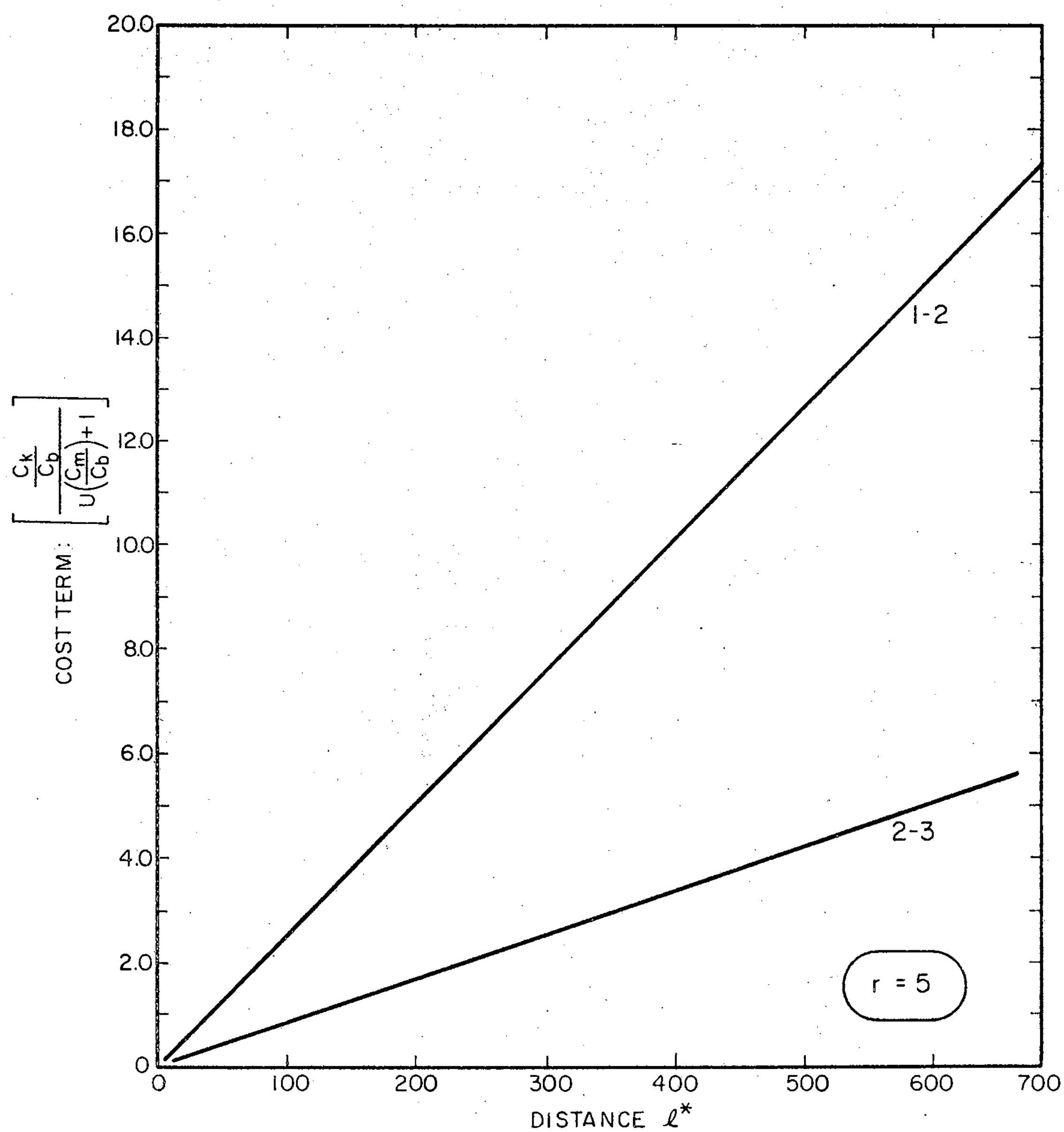
(e)

Figure 8. (Continued)



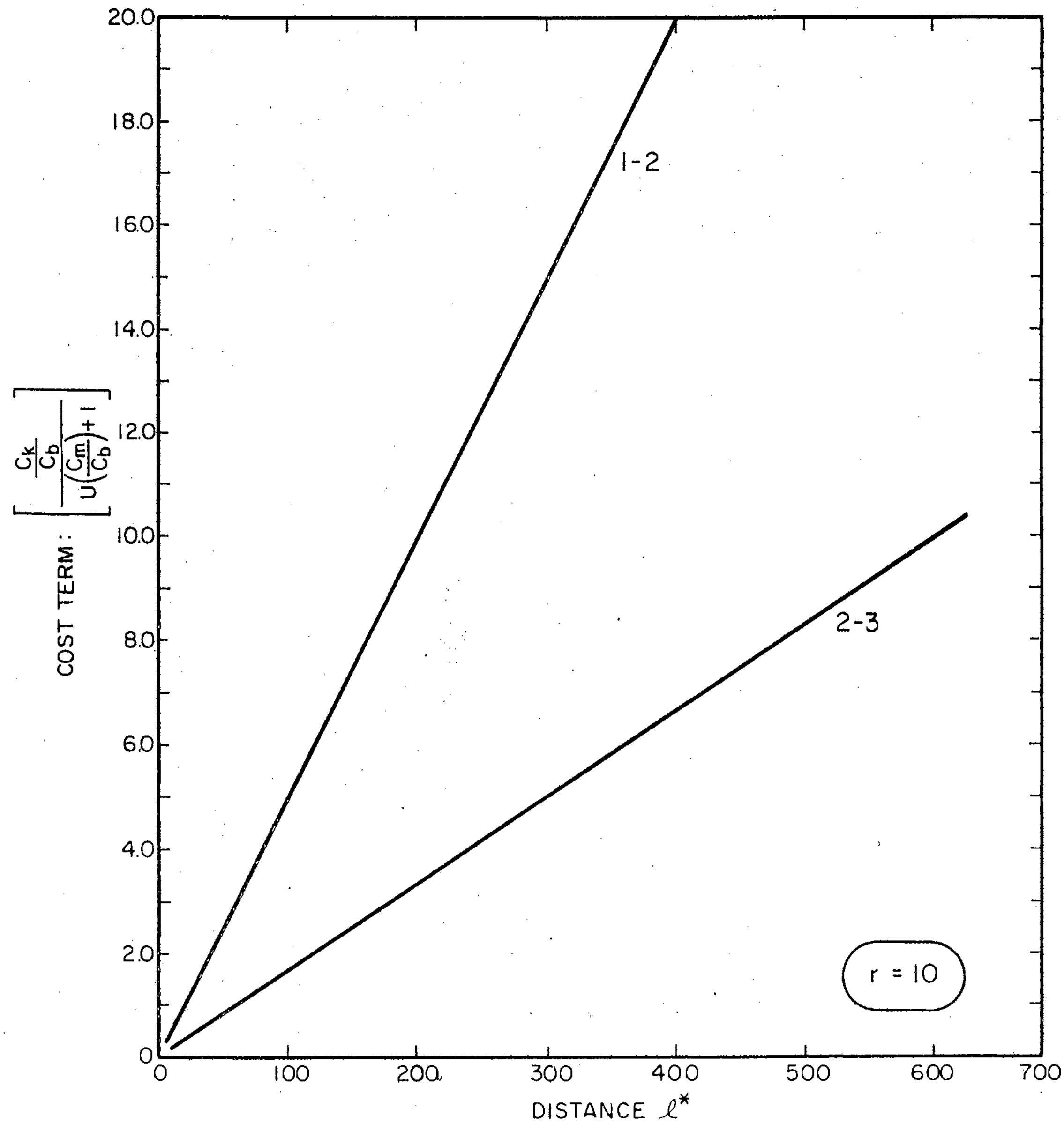
(f)

Figure 8. (Continued)



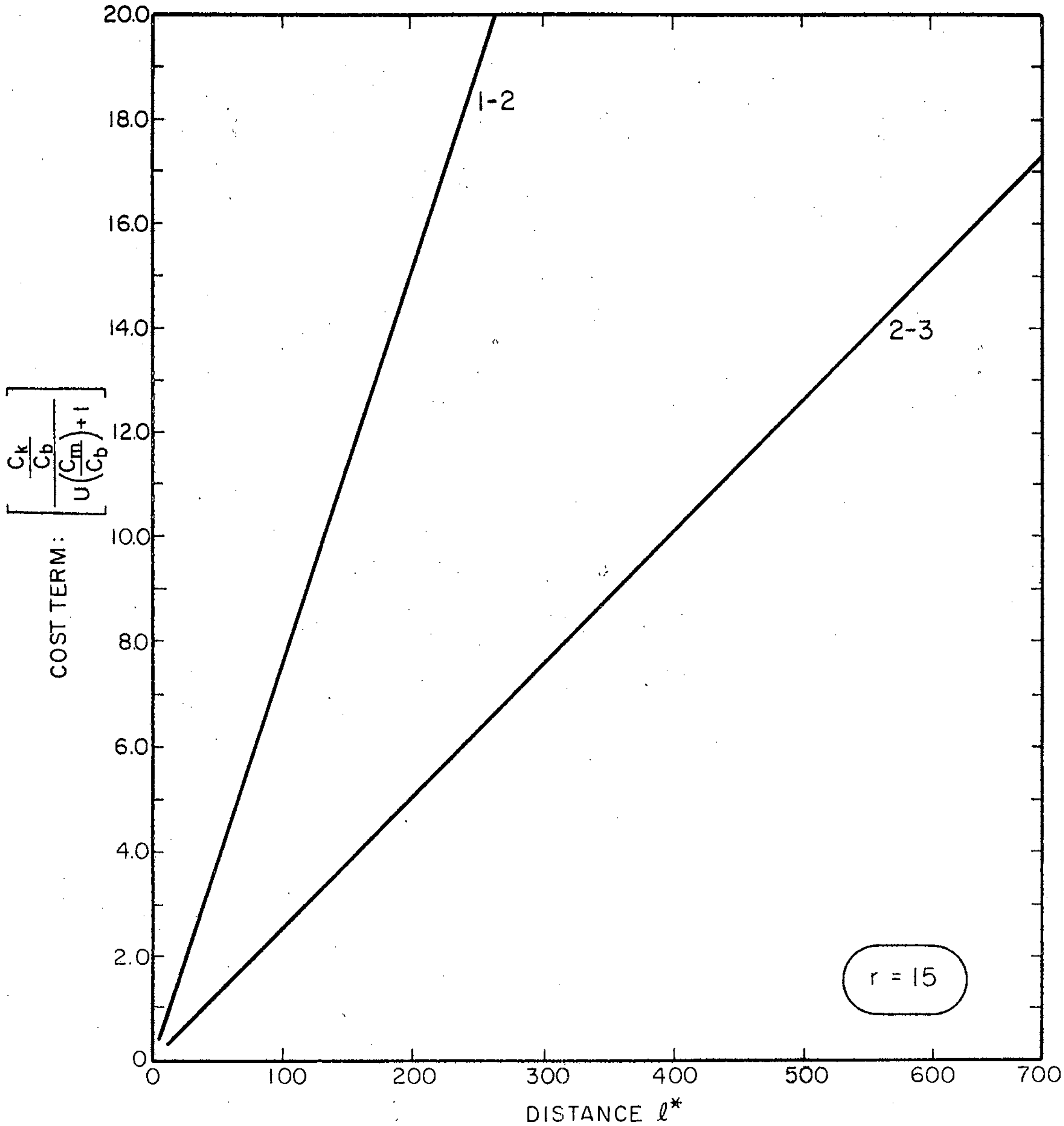
(a)

Figure 9. Distance ℓ^* vs. Cost Term for
r = 5, 10, 15 and 20



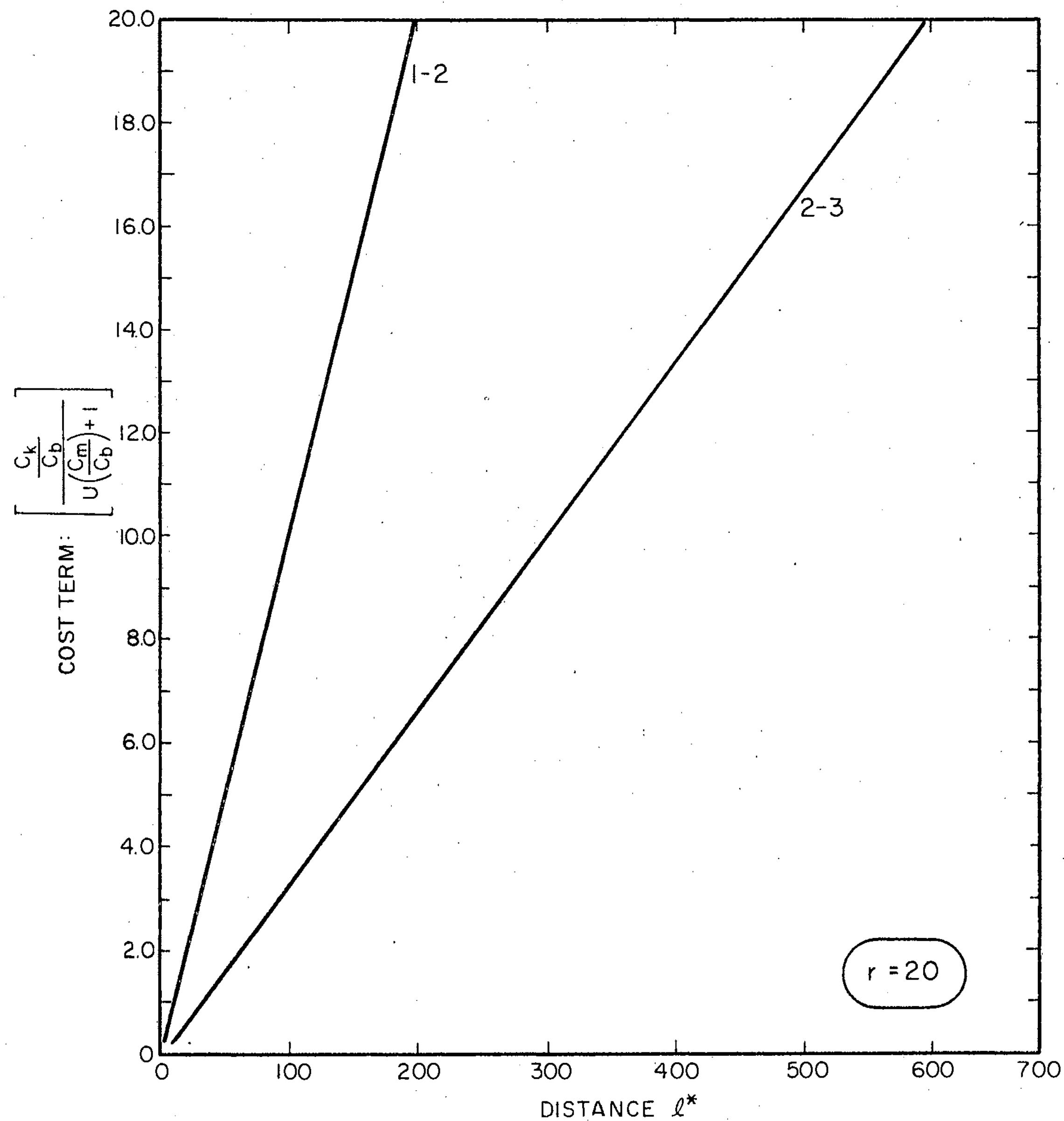
(b)

Figure 9. (Continued)



(c)

Figure 9. (Continued)



(d)

Figure 9. (Continued)

assumed value of C_b. Similar calculations were made for $\frac{C_m}{C_b}$.

The upper and lower limits of the pouring rate, r, are based on the maximum and minimum quantity of concrete likely to be placed in one day. Records from several sources for many jobs revealed that the minimum quantity of concrete per day was 40 CY (2160 square feet of floor area 6" thick) and the maximum 160 CY per day (8640 square feet of floor area 6" thick). The average was approximately 92 CY per day (5000 square feet of floor area 6" thick) or 11.4 CY/hour. Therefore, the values of r used in the calculations were 5, 10, 15 and 20.

Economic Model No. 3

Purpose of Model No. 3

Model 2 was developed to determine the maximum economic distance that could be served from an elevator given certain cost information and the configuration of the building. The basis of Model 2 is the assumption that the time between adjacent concrete deposits during concreting operations for the floor slabs controls the economic selection of elevators. The development of Model 2 ignored the behavior of concrete, namely, the "initial set" time.

The "initial set" time of concrete is the time required for the chemical reaction of the ingredients to cause the concrete to harden. This hardening process contributes to the strength-gaining property of concrete and continues indefinitely at a diminishiong rate with the highest rate beginning immediately after the concrete is placed in the form. The degree to which concrete is allowed to harden before fresh concrete is placed against it is critical. Concrete that has set too long will not bond to fresh concrete, causing unsightly and unsafe "cold-joints."

The time of hardening varies depending on a number of factors. Some of the more important factors are ambient temperature, wind velocity at exposed surfaces, humidity, and tightness of forms. But, in general, concrete in a floor slab that hardens beyond one hour will not bond. Times of fifteen, thirty, and forty-five minutes are frequently used as the maximum allowable time between "old" and "new" concrete, i.e., before fresh concrete is placed next to concrete previously poured.

It is not uncommon for contract specifications to state the maximum allowable time interval between concrete loads. This specified time interval does not relieve the contractor from responsibility of noting conditions that affect the "initial set" time of the concrete and, if required in his judgment, reducing the time interval to one that is appropriate.

The cost of not meeting the contract requirement in this respect cannot be ignored since "cold-joints" resulting from concrete not bonding is cause for rejection of the slab. To replace a floor slab that has been condemned is a prohibitively expensive operation, due not only to the expense of tearing out and replacing but also for the loss of construction time that may extend the project into the penalty period.

The purpose of Model 3 is to recognize that the "initial set" time of concrete in floor slabs is the factor governing the number of elevators with respect to the lateral dimension of a building. Model 3 also serves as a restriction on Model 1.

Basis of Model 3

The method generally used to place concrete for a floor slab was outlined in the development of Model 2. The successive dumping of fresh concrete along the width of the formed area creates a line of hardening concrete called a pouring front. The maximum length that the pouring front can attain is the maximum dimension of the formed area measured parallel to the pouring front.

The maintenance of the pouring front within the allowable time interval between loads while also maintaining a rate of pour equal to the delivery rate of the elevator is the basis of Model 3.

Development of Economic Model 3

Symbols.

L = a dimension in the formed area to be poured, in feet

w = width of pour front expressed as some fraction of L

y = volume of a transport unit, in CY

d = thickness of concrete floor to be poured, in feet

T_{pf} = maximum allowable time interval between placing fresh concrete against previously poured concrete, in minutes

v = velocity of a transport unit, in feet per minute

n = number of transport units.

For the general case, consider the area to be poured to have a trapezoidal configuration. Figure 10 shows the area and its dimensions. The analysis that follows is valid for any square, rectangular, or trapezoidal area where the transport units follow orthogonal travel patterns. A similar analysis could be made for circular areas. Models using other travel patterns could be developed, but it is felt that models with orthogonal travel patterns best simulate actual field conditions.

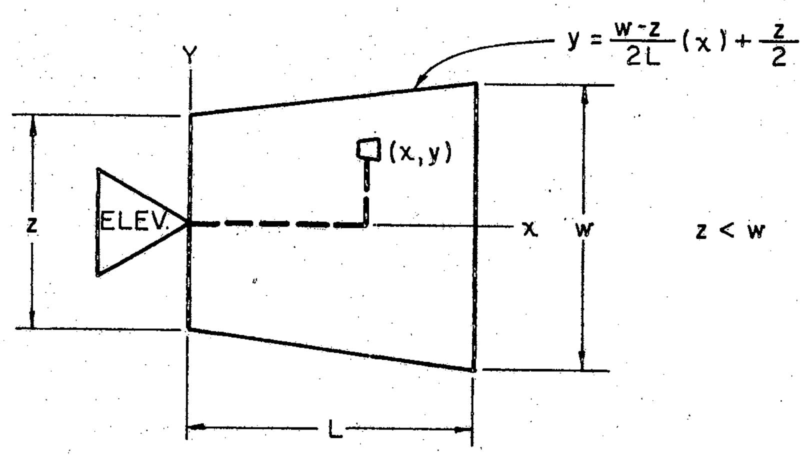


Figure 10. Dimensioned Floor Area with Orthogonal Travel Pattern

Maximum distance traveled by a transport unit = (2)(average distance) from elevator and return.

$$= 2 \left[\frac{4 \displaystyle\int_{\text{area}} (x + y)\, dy\, dx}{\frac{L}{2}\,(w + z)} \right]$$

$$= \frac{16}{L(w + z)} \int_0^L \int_0^{\frac{w - z}{2L}x + \frac{z}{2}} (x + y)\, dy\, dx$$

$$= \frac{8wL + 4wL + 2w^2 + 2wz + 2z^2}{3(w + z)} \quad . \qquad (5\text{-}16)$$

For the case of a square area, $z = w = L$, Equation (5-16) becomes $3L$; for a rectangle whose width is $\frac{1}{2}L$, Equation (5-16) becomes $\frac{5}{2}L$.

Assume for the sake of simplicity that the area to be poured is a square such that $z = w = L$. The maximum distance traversed by a transport unit is $3L$.

Maximum travel time, in minutes, per unit $= \frac{3L}{v}$

Number of trips/hr one unit can make, assuming no down time, $= \dfrac{60}{\frac{3L}{v}} = \dfrac{20v}{L}$

and, Number of trips/hr n units can make $= n\left(\frac{20v}{L}\right)$. (5-17)

Each transport unit can place y CY per trip. The length of the pouring front is L feet. The square feet of floor area covered per load is $\frac{27y}{d}$; and, assuming a square area of coverage for each load, the pouring front is reduced by an amount of $\sqrt{\frac{27y}{d}}$ for each transport unit load.

Number of trips required to complete the pouring front is then

$$= \frac{L}{\sqrt{\frac{27y}{d}}}$$

and the number of trips required per hour to maintain the pouring front and not exceed the maximum allowable time interval T_{pf}, is

$$= \left(\frac{60}{T_{pf}}\right) \left(\frac{L}{\sqrt{\frac{27y}{d}}}\right) . \qquad (5\text{-}18)$$

Equating (5-17) and (5-18) and solving for L gives

$$L = \left[(n)(v)(T_{pf})\right]^{\frac{1}{2}} \left[\frac{3y}{d}\right]^{\frac{1}{4}} . \qquad (5\text{-}19)$$

<u>Manipulation of Model 3</u>

The terms v and y may be considered as constants. If, for various values of T_{pf} the value of L is calculated for various combinations of n and d, the results can be summarized graphically in useful form. This permits the determination of the effect of three of these variables on the fourth variable or the selection of values for some combination of these variables that meets a specific requirement. As an example, the value of L may be fixed and it is desired to know the number of transport units, n, necessary to meet some time interval, T_{pf}.

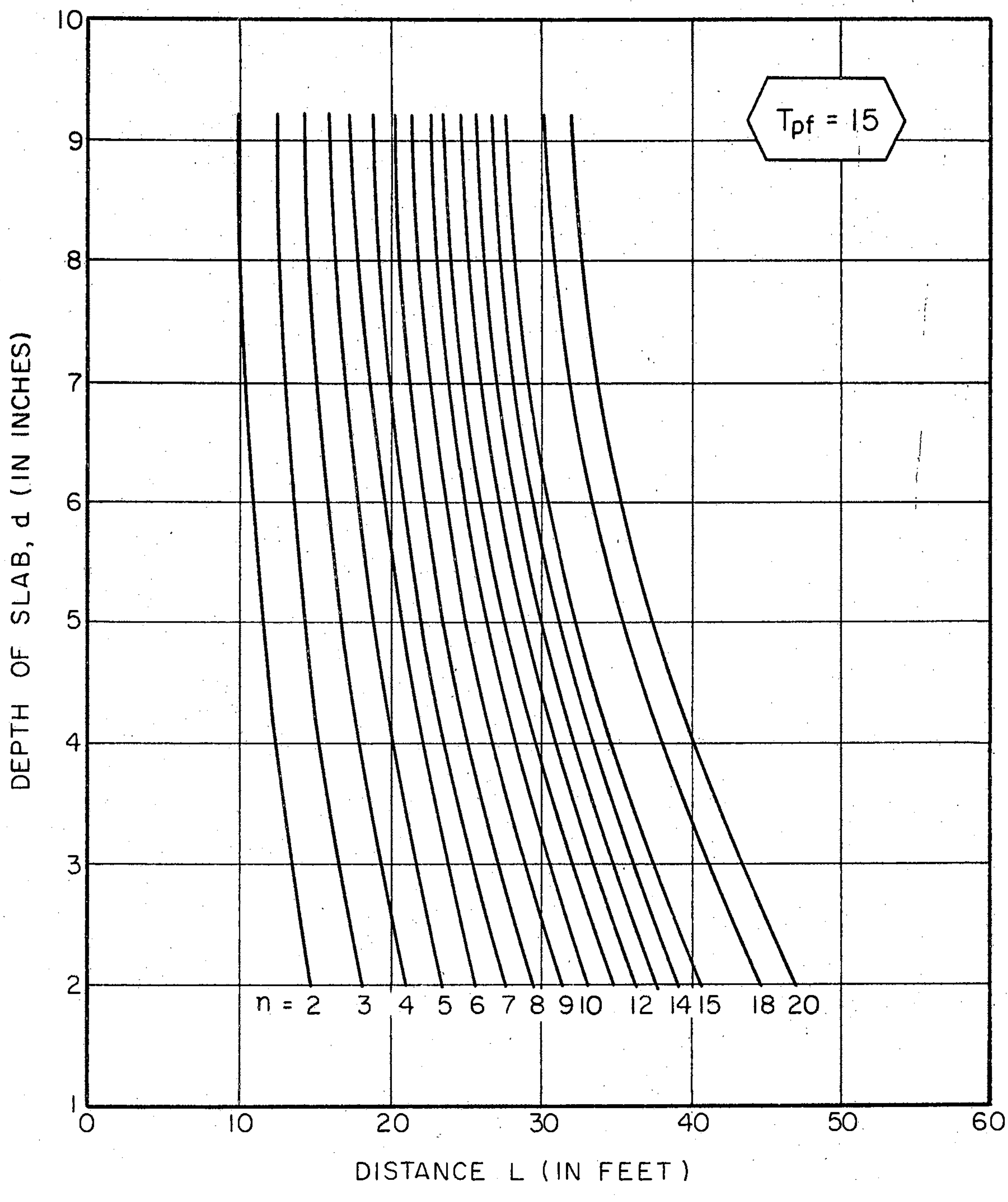
Computer Program Number 5, Appendix C, was written to perform the calculations for a range of values of variables n, d, and L for various values of T_{pf}. These calculations assumed fixed values for v and y. The graphical representation of these relationships are shown by Figures 11(a) through 11(d).

Discussion of Cost Coefficients and Variables Used in Models

The extent to which the three models developed simulate actual field conditions depends upon the accuracy of the values selected for the cost coefficients and variables used in the model. The behavior of the models is more sensitive to changes in the values of some cost coefficients and variables than others. Reasonable results are, therefore, obtainable even though the values selected for some cost coefficients and variables are in error.

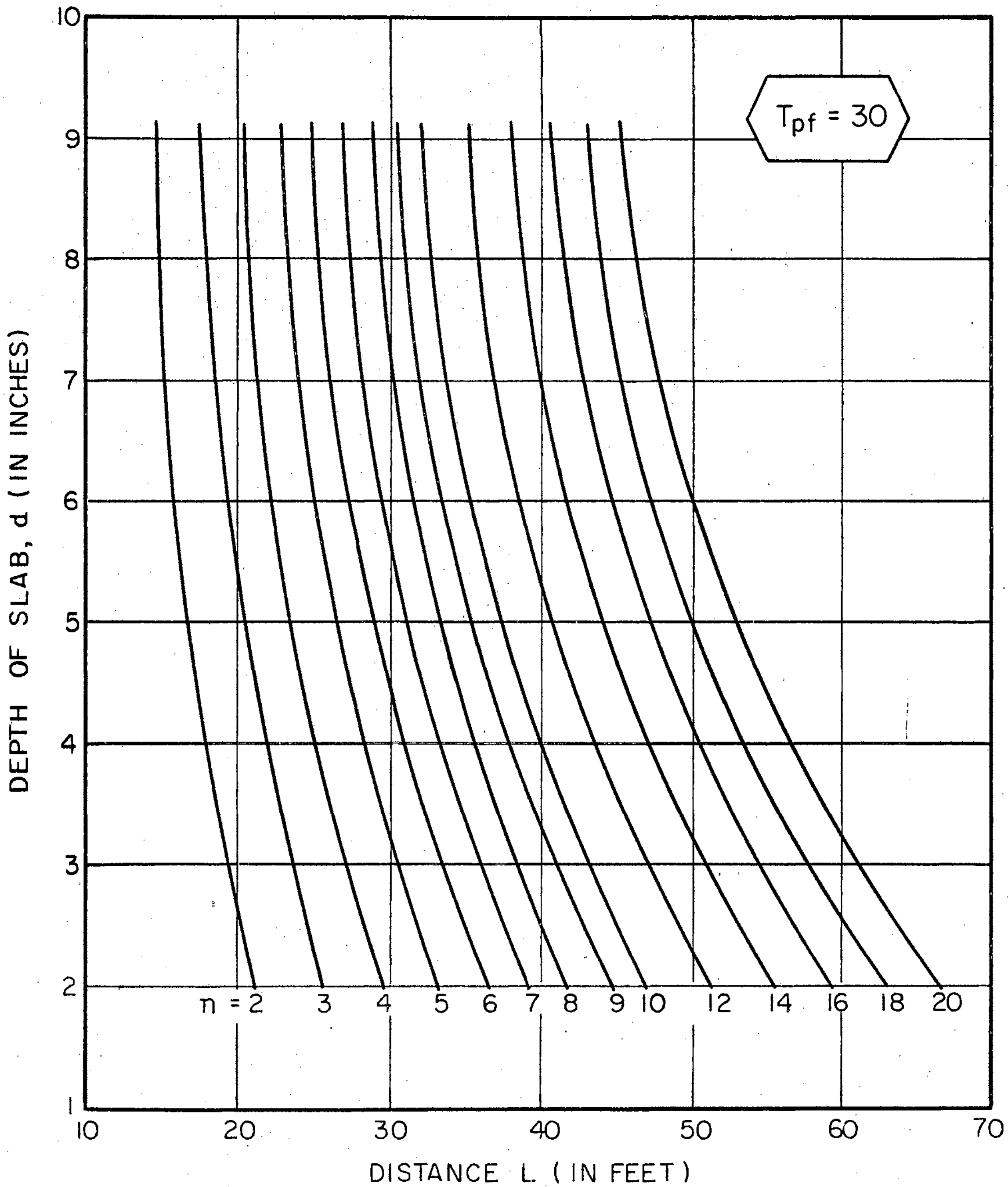
Cost Coefficients C_k, C_m, C_b, C_o

The determination of appropriate values for C_k, C_m, C_b and C_o is relatively easy. C_k is simply the average weekly rental cost or ownership cost of one elevator including an allowance for maintenance plus the employer's weekly cost of wages for the hoisting engineer. Rental rates are quoted as the sum of the costs of the basic unit plus the cost per linear foot of tower, cable, and accessories as specified. Accessories would include concrete bucket, attachment, hopper, tripping devices, etc. C_o is the weekly operating



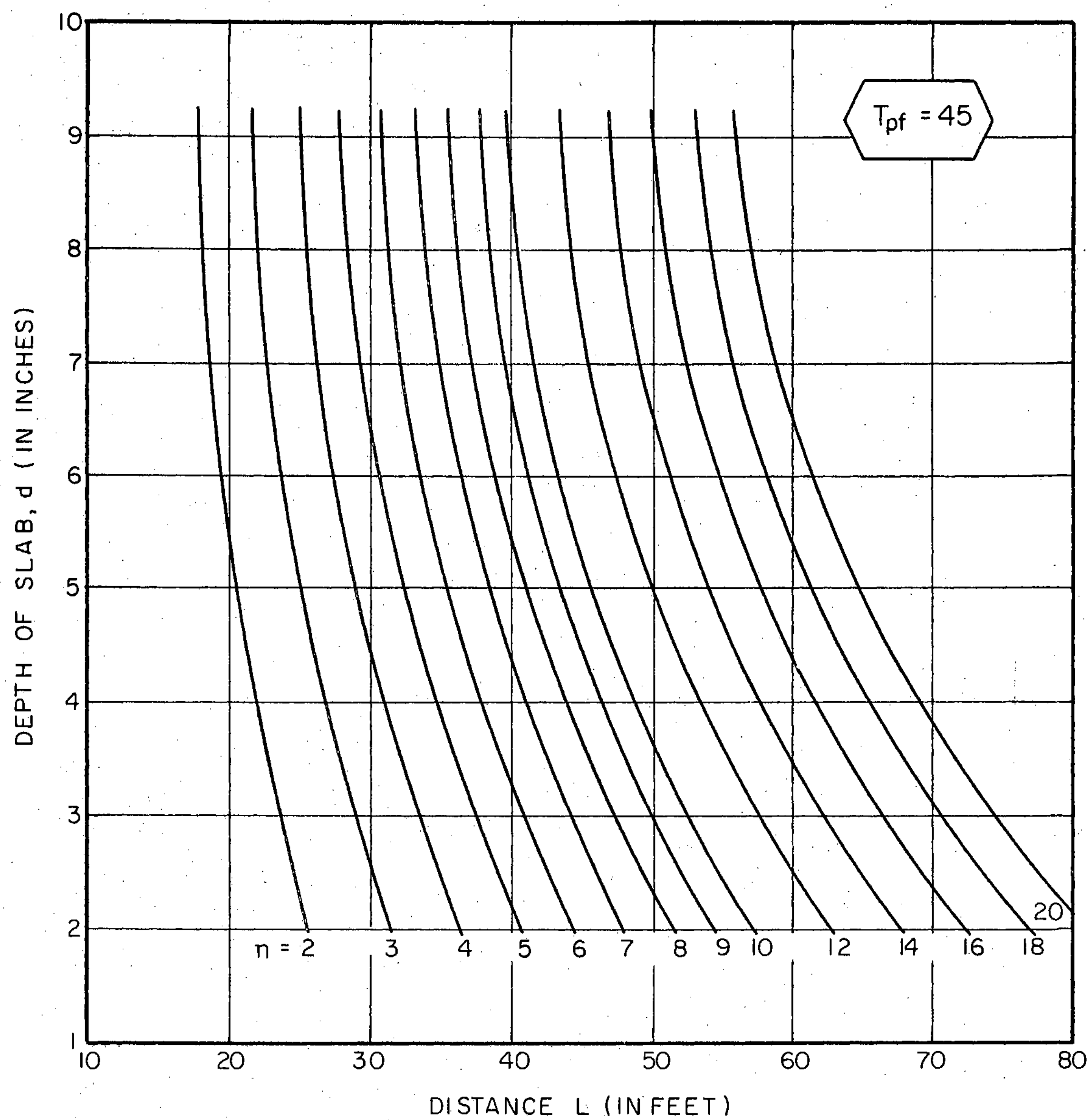
(a)

Figure 11. Pouring Front Restriction Relationship of d, n and L When T_{pf} = 15, 30, 45 and 60 Minutes



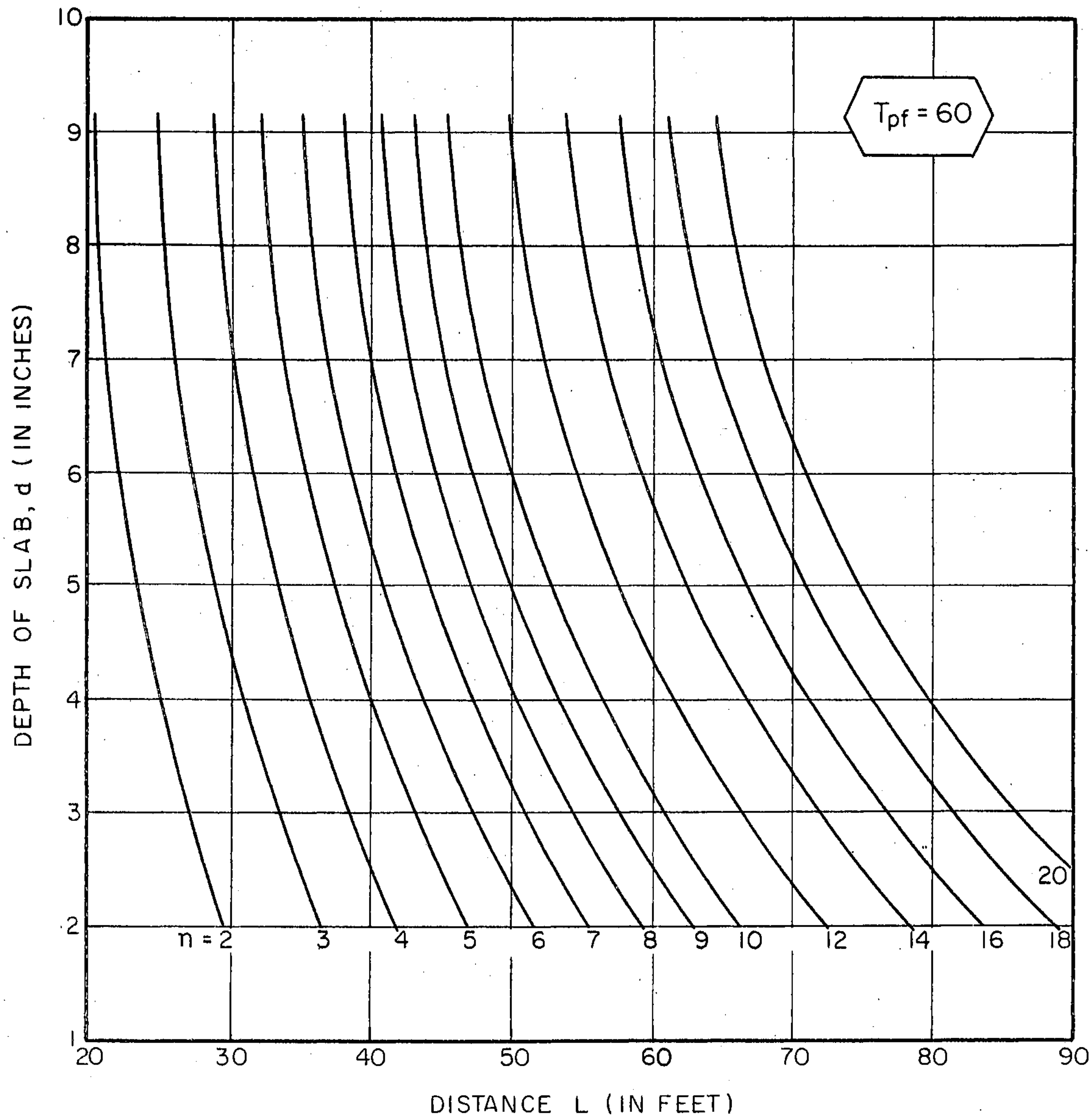
(b)

Figure 11. (Continued)



(c)

Figure 11. (Continued)



(d)

Figure 11. (Continued)

cost assuming full time operation for forty hours per week. The model accounts for the percentage of time the elevator is not operating. It is obviously necessary to know the price of fuel required—whether natural gas, gasoline, or diesel—or, if electric, power rates. Good estimates of fuel consumption are available so that the value of C_o should be without appreciable error.

The value of both C_k and C_o may vary among time periods. Provision is made in Computer Program No. 1, as part of the input, to permit this variability. C_m is merely the employer's cost of hourly wages for transport unit operators. C_b is the rental, or ownership, cost per hour of a transport unit. It should also include the cost of runways and the men necessary to relocate them.

Cost Coefficients C_{w_i}, C'_{w_i}

Determining a value for C_{w_i} is admittedly difficult. Fortunately the behavior of the model is not too sensitive to changes in C_{w_i} thereby permitting a margin of error in the choice of C_{w_i} without invalidating the results. Note from Figure 4 that for a value of $\theta = 0.5$, an error fifteen times the true value of C'_{w_i}, $C'_{w_i} = C_{w_i}/\lambda_i$, can be incurred before the optimum number of elevators changes from one to two.

C_{w_i} is the composite weekly cost incurred by not having material on-station as required to maintain continuous and uninterrupted work. Not all materials scheduled to be lifted during a weekly time period will contribute to the value of C_{w_i} since a delay in their on-station arrival date will not interrupt the work.

It is, therefore, necessary to determine which activities are critical in the time period. The material, tools, equipment, etc., required by these activities must then be evaluated in terms of the cost incurred by their delay. A portion of this delay cost would obviously be the cost of wages for the men waiting for the material. This cost is easily determined. However, there are other costs involved that are not so readily evaluated. Costs associated with general overhead, job overhead, investment, idled equipment, penalties, reduced performance bond capacity, to mention a few, should be considered. Although not precise, a "quick and dirty" approach to evaluating the latter costs is to sum the average weekly general overhead assigned to the project, weekly project overhead, interest on average weekly investment, appropriate charges for any equipment on the project idled as a result of the delay in material, and weekly penalty for exceeding the contract time. Actually, this approach is not as bad as it may at first seem. Since only those activities that are critical are being considered as candidates to contribute to C_{w_i}, it is reasonable to expect the project to be delayed by an amount corresponding to the delay in these critical activities.

A value of C_{w_i} must be specified for each i^{th} time period for Computer Program No. 1 since the value of C_{w_i} may not be the same for each time period.

Variables

The volume, y, of a transport unit depends on the size of unit selected. Popular sizes that are not motorized have volumes of six cubic feet and nine cubic feet. It may not always be good practice to consider a unit fully loaded although the assumption in these calculations is that the value of y is nine cubic feet and fully loaded.

The value of v has been observed to vary from ten feet per minute to forty feet per minute. This variance can be attributed to the transport unit operators and the condition of the runways. Wide, stable runways designed to be easily relocated can noticeably increase the value of v. The value for v of ten feet per minute assumed in these calculations gives a very conservative result.

Selection of appropriate values for n, T_{pf}, and d have either already been explained or are self-explanatory.

CHAPTER VI

APPLICATION OF THE ECONOMIC MODELS

From the construction schedule, the number of weeks required for construction and the type and quantity of material by floors for each week is obtained. Each material is translated into the effective load capacity of the elevator so that the number of trips per week required to lift that material can be determined. The total number of trips per week required to lift all materials scheduled for that week is the value of λ_i for the i^{th} time period where a time period is one week's duration.

The average time required per trip is the value $\frac{1}{\mu}$ for the assumption of exponential service time. This time is the sum of the loading time, travel time, and unloading time for each elevator load. The travel time is given by $\frac{\text{distance}}{\text{velocity}}$ where distance is the sum of the distances from the point of departure to the point of delivery and from the point of delivery to the point of return. The load time is the time to place the material on the elevator platform assuming the material to be at or near the elevator threshold. The unloading time is the time required to remove the material from the elevator platform onto the landing.

The value of μ, the service rate, may be regarded as the output of the elevator over the time period divided by the portion of the time period that the elevator is actually in operation. Expressed another way, μ is the number of trips per week an elevator could make each week if continuously operated. The calculation of μ for a time period of one week is:

$$\mu_i = \frac{\Sigma \text{ trips made in time period } i}{\Sigma \text{ time for each trip expressed as a fraction of a week}} .$$

In testing the model, it was found convenient to prepare the data in tabular form by recording for each time period the material to be lifted and the floor to which delivery was to be made. The form used for this purpose was designed as shown in Figure 12. The information shown on this form was summarized on a form designed as shown in Figure 13. Also included on this second form were the number of trips and time per trip, in minutes, for each material for each time period. Data cards can then be readily keypunched using the code sheet shown in Appendix A. One data card is required for each material to be lifted during a time period. This is the extent of the data required for Computer Program No. 1.

Computer Program No. 1 (Appendix A) calculates, by time period, the following: total time in minutes that elevator is used in time period, λ, μ, W, p, W_q and P_o. The calculations are repeated for the number of elevators specified.

DATA RECORDING FORM SHOWING PERIOD ACTIVITY BY FLOORS

PROJECT NO. ________ DATE:
TIME PERIOD 1 WEEK

PERIOD	TRANSPORTABLE ITEMS												
	CON COL	CON SLB	DR FMBE	BLD TIL	MAS UNT	DUR WAL	MAS MOR	AC UNT	PNT	CEI SYS	FLR COV	PMB RGH	PMB FIN
1	2												
2	3	2										2	
3	4	3	2	2		2	2					3	
4	5	4		2			2					4	
5	6	5			2		2					5	
6	7	6	3	3		3	3			2		6	
7	8	7		3			3	2				7	
8	9	8			3		3		2			8	
9	10	9	4	4		4	4			3	2	9	2
10		10		4			4					10	
11					4		4		3				
12			5	5		5	5			4	3		3
13				5			5	4					
14					5		5		4				
15			6	6		6	6			5	4		4
16				6			6						
17					6		6		5				
18			7	7		7	7			6	5		5
19				7			7	6					
20					7		7		6				
21			8	8		8	8			7	6		6
22				8			8						
23					8		8		7				
24			9	9		9	9			8	7		7
25				9			9	8					
26					9		9		8				
27			10	10		10	10			9	8		8
28				10			10						
29					10		10		9				
30	FLOOR DESTINATION									10	9		9
31								10					
32									10				
33											10		10
34													
35													

REVISED: __YES __NO UPDATE NO.: _______ SHEET ____ OF ____

Figure 12. Data Recording Form Showing Period Activity By Floors

DATA SUMMARY FORM OF PERIOD ACTIVITY BY FLOORS

PROJECT NO. ________ DATE ________ TIME PERIOD ________

PERIOD	TRANSPORTABLE ITEMS												
	CON COL	CON SLB	DR FMBE	BLD TIL	MAS UNT	DUR WAL	MAS MOR	AC UNT	PNT	CEI SYS	FLR COV	PMB RGH	PMB FIN
1	2 30 0.67												
2	3 30 1.0	2 150 0.67										2 4 40.33	
3	4 30 1.33	3 150 1.0	2 4 9.33	2 26 11.33		2 2 10.33	2 10 4.33					3 4 40.67	
4	5 30 1.67	4 150 1.33		2 26 11.33			2 10 433					4 4 41.0	
5	6 30 2.0	5 150 1.67			2 14 3.33		2 10 4.33					5 4 41.33	
6	7 30 2.33	6 150 2.0	3 4 9.67	3 26 11.67		3 2 1067	3 10 4.67			2 10 9.33		6 4 41.67	
7	8 30 2.67	7 150 2.33		3 26 11.67			3 10 4.67	2 2 40.33				7 4 42.0	
8	9 30 3.0	8 150 2.67			3 14 3.67		3 10 4.67		2 8 12.33			8 4 42.33	
9	10 30 3.33	9 150 3.0	4 4 10.0	4 26 12.0		4 2 11.0	4 10 5.0			3 10 9.67	2 8 8.33	9 4 42.67	2 6 10.33
10		10 150 3.33		4 26 12.0			4 10 5.0					10 4 43.0	
11					4 14 4.0		4 10 5.0		3 8 12.67				
12			5 4 10.33	5 26 12.33		5 2 11.33	5 10 5.33			4 10 10.0	3 8 8.67		3 6 10.67
13				5 26 12.33			5 10 5.33	4 2 41.0					
14					5 14 4.33		5 10 5.33		4 8 13.0				
15			6 4 10.67	6 26 12.67		6 2 11.67	6 10 5.67			5 10 10.33	4 8 9.0		4 6 11.0
16				6 26 12.67			6 10 5.67						
17					6 14 4.67		6 10 5.67		5 8 13.33				
18			7 4 11.00	7 26 13.0		7 2 12.0	7 10 6.0			6 10 10.67	5 8 9.33		5 6 11.33
19				7 26 13.0			7 10 6.0	6 2 41.67					
20					7 14 5.0		7 10 6.0		6 8 13.67				
21			8 4 11.33	8 26 13.33		8 2 12.33	8 10 6.33			7 10 11.0	6 8 9.67		6 6 11.67
22				8 26 13.33			8 10 6.33						
23					8 14 5.33		8 10 6.33		7 8 14.0				
24			8 9 11.67	9 26 13.67		9 2 12.67	9 10 6.67			8 10 11.33	7 8 10.0		7 6 12.0
25				9 26 13.67			9 10 6.67	8 2 42.33					
26					9 14 5.67		9 10 6.67		8 8 14.33				
27			10 9 12.67	10 26 14.0		10 2 13.0	10 10 7.0			9 10 1167	8 8 10.33		8 6 12.33
28				10 26 14.0			10 10 7.0						
29					10 14 6.0		10 10 7.0		9 8 14.67				
30										10 10 12.0	9 8 10.67		9 6 12.67
31	FLOOR DESTINATION							10 2 43.0					
32	NUMBER OF TRIPS								10 8 15.00				
33	TIME (IN MINUTES) PER TRIP										10 8 11.00		10 6 13.0
34													
35													

REVISED: ___YES ___NO UPDATE NO. ________ SHEET ______ OF ______

Figure 13. Data Summary Form of Period Activity By Floors

Computer Program No. 1 calculates and accumulates the TC_i's. The output will include a plot of the Cumulative TC_i's, one curve for each elevator specified. This plot will show the time period during which two elevators are as economical as one. The ΣTC_i's for the project duration time is the estimated cost of elevator service level indicated, provided the restrictions imposed on Model 1 by Models 2 and 3 have not been violated.

The restriction placed on the system by Model 2 should now be checked. From the data used to determine λ_i, one may find the λ_{id} where d is the day scheduled for the concrete pour. If all trips not connected with the concrete pour are subtracted from λ_{id}, the resulting value is the value of r_i. With a value of U, and the values of $\left(\frac{C_k}{C_b}\right)$ and $\left(\frac{C_m}{C_b}\right)$, read the value of

$$\left[\frac{\left(\frac{C_k}{C_b}\right)}{\mu\left(\frac{C_m}{C_b}\right) + 1}\right]$$

from the appropriate chart, Figures 8(a) through 8(d). With the value of r_i, and

$$\left[\frac{\left(\frac{C_k}{C_b}\right)}{\mu\left(\frac{C_m}{C_b}\right) + 1}\right],$$

read the value of ℓ^* from the chart, Figure 9. If this value of ℓ^* exceeds the distance to the extreme point of the pouring area as measured along the runways, then this restriction has been violated. A violation indicates the need of an additional elevator or supplementary equipment for that time period.

Not all time periods have concrete pours scheduled since the majority of concrete work is scheduled in the early time periods. Therefore, a comparison should be made of the cost of providing supplementary equipment for only those time periods with scheduled concrete pours and the cost of providing two elevators for the entire project duration time.* The analysis for this procedure was explained on pages 36, 38 and 39, Chapter V.

The second restriction on Model 1 is that represented by Model 3. Again, v and y are considered constants. Knowing d and T_{pf}, enter the appropriate chart, Figures 11(a) through 11(d), and select that value of L given by the number of transport units, n, to be used. If the value of L obtained exceeds the L value for the building under consideration, the "cold-joints" restriction has been violated.

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*The presumption is that no other material would affect the behavior of Model 2 as does that of concrete. Model 2 can easily be used for any material other than concrete. In the case of masonry, for instance, r would be the number of masonry units per hour, y would be the number of masonry units per load, C_b would be the cost per hour for the conveyance used to transport the masonry units. The other terms would remain the same.

The analysis for judging whether to provide a second elevator or bring in supplementary equipment as needed is the same as that explained for Model 2.

If the configuration of the building is such that the restrictions of Model 2 and Model 3 are satisfied by one elevator but Model 1 indicates two elevators are required, it is possible by examining the ρ_i values (more specifically, the θ_i values) for each time period to eliminate the second elevator by the use of supplementary equipment on certain days at reduced cost. This analysis can be accomplished by following the procedure outlined on pages 36, 38 and 39, Chapter V.

If a resource leveling technique is used in conjunction with the CPM network, the advantage of "Smoothing of Demand" discussed in Chapter V will be realized. However, in the event that delays are experienced in the course of the job, the demand should be reviewed.

A weekly review of the progress of the job is recommended. This may permit rescheduling lifting requirements in a manner that will not exceed the service capacity level of available equipment. If not, supplementary equipment can be scheduled.

CHAPTER VII

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The vertical transportation of materials as discrete entities by a sequence of scheduled activities has been idealized as mathematical models to represent a system, and the flow of the material has been described as a function of several parameters which determine the flow rate or productivity of the system in terms of the productivity of the individual components. Calculations have been made to analyze the behavior of the system in response to changes in the parameters. The values of the parameters used for each of the calculations were representative of the range in values most likely to be encountered in practice for construction of high-rise buildings.

Conflicts among desirable attributes of a vertical material handling system for high-rise construction were presented to show why construction managers exhibit indecision in the selection of a system for a specific project. Various equipment suitable for lifting has been delineated by type into "families" according to the particular lifting service provided. From the "family" of hoists, elevators were selected to be studied and evaluated since they are assumed to be the basic lifting machines for vertical transportation systems in high-rise construction.

The problem of determining the type and quantity of materials susceptible to transport by an elevator resulted in defining a term "effective load capacity." The effective load capacity of an elevator identifies those materials, and the quantity of each, that can be transported by one trip-load.

Eight observations were made of current practice in high-rise construction establishing the basic problem of this study. A discussion of these observations demonstrated the value of waiting-line theory as an approach to the solution of the problem.

The waiting-line process as it applies to building construction procedures was outlined. The input sources were described and the queue discipline defined in accordance with construction practice. On the basis of a description of the arrival of material for service and the servicing procedure, an explanation was given for the assumption of probability distributions used in the study, that of Poisson arrivals and exponential service.

Waiting-line theory is a useful method for analyzing the effect of fluctuations in demand on material handling systems that operate at varying per cent utilizations of capacity. Although waiting-line models are not necessarily precise predictors, they can be regarded as a framework in which to identify the basic functional relationship between variables of a problem.

Using measures of cost associated with states of the system and the concept of state probability varying with load and other system parameters, it has been demonstrated for concrete construction that

1. The optimal range of service capacity for one or two elevators can be determined.
2. The effect, on this optimal range, of changes in the system parameters can be evaluated.
3. The total cost for a vertical transportation system, including supplementary equipment, can be estimated for bidding purposes.
4. The vertical transportation requirements can be updated and the need for and the cost of supplementary equipment, if indicated, can be predicted in advance.
5. The smoothing of demand for service may reduce the cost of the vertical transportation system.
6. The maximum economic height that can be served by elevators is limited by the floor area and configuration of the building.
7. The solution given by Model 1 is not sensitive to the cost coefficients; therefore, the estimate of C_w need not be a precise one.
8. Much of the information required to use the methods presented in this study is repetitive from job to job.

The results obtained through this study can be used to evaluate the contribution to a vertical transportation system of one or two elevators. However, when the demand on such a system exceeds the capacity of two elevators, it is this writer's opinion that systems articulated in other ways should be considered. The contribution of other "families" of equipment, or combinations, to a vertical transportation system should be studied to obtain eventually the "best" line-up of equipment at the least cost.

Future studies of cost coefficients would also be fruitful since the reluctance to evaluate a cost-of-waiting has been a deterrent in the use of many economic waiting-line models. The similarities of high-rise construction projects and of their associated costs should serve as a challenge to arrive at a fairly narrow range of standard values of cost coefficients that could be used, with slight modification, in any geographical location.

Both the designers of vertical transportation systems handling materials as discrete entities and the manufacturers of the building materials these systems handle must face the problem of packaging. Frequently, identical material, but from different manufacturers, will be packaged in different sizes and shapes. Future studies could profitably be conducted with the purpose of standardizing the packaging of many materials used in building construction.

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APPENDIX A

COMPUTER PROGRAM NO. 1

Instructions for Computer Program No. 1

Arrangement

Following is an ordered listing of the necessary control cards, source deck, "Tape" subroutine decks, binary subroutine deck, and data deck, for the Oklahoma State University IBM 7040 System.

```
$ID

$JOB

$IBJOB

$IBFTC   MAIN
```

"MAIN" PROGRAM FORTRAN DECK

```
$IBFTC   TAPE
```

"TAPE" SUBROUTINE FORTRAN DECK

```
$IBFTC   PRTOUT
```

"PRTOUT" SUBROUTINE FORTRAN DECK

BINARY SUBROUTINE DECK

$ENTRY

First Data Card "Control Card"

DATA CARD DECK

$END

$IBSYS

First Data Card "Control Card"

By punching the word CARD in columns 15-18, the deck of input data cards will be read and then written on Tape Unit # 4. This tape can be saved and used for future runs. When using tape input from a previous run, punching the word TAPE will read data card images from TAPE UNIT # 4.

Other control information that appears on the First Data Card is as follows:

Columns 1-3	Maximum number of weeks per period specifies various lengths of time periods
Columns 4-6	Maximum number of elevators to be tested
Columns 7-9	Starting number of week per period (used only for discontinuous runs)

Columns 10-12	Starting number of elevators
Columns 15-18	The word CARD or TAPE. If a rerun, instruct computer operator to mount tape containing card images on tape drive # 4. Mount scratch tapes on tape drives 0, 1, and 2.
Columns 20-28	Number of floors

Special Restrictions on Data Deck

All data cards pertaining to one time period must be placed together and the time periods must be sorted and arranged into ascending order. If there is only one card for the last time period, it must be followed by another card with the same time period and the balance of the columns blank. The cost coefficients C_{w_i}, C_o, and C_{k_i} are punched into columns 31-60 of the first card of the set of cards for each time period. Any values punched into these columns on the other cards of a set will be ignored.

The card format for entering C_{w_i}, C_o, and C_{k_i} on the first card of the set of cards for each time period is as follows:

Column 31-40	C_o punch anywhere in field with decimal point
Column 41-50	C_{w_i} punch anywhere in field with decimal point
Column 51-60	C_{k_i} punch anywhere in field with decimal point

For multiple time period analysis (two or more time periods taken as one) the cost coefficient is averaged over the periods. In the process of reading in the data cards and writing them on tape the cards are checked to be sure that the period is in ascending sequence. If a card is found to be out of sequence the following message is printed:

"PERIOD OUT OF SEQUENCE" (card image of out-of-sequence card).

This card image may be of the card following the actual out-of-sequence card if two cards are interchanged. After this message the program continued to check the balance of the data deck and after the last card is read and there was a sequence error, this message is printed:

"PERIOD SEQUENCE ERROR—JOB TERMINATED"

Following this message a CALL EXIT is executed since out-of-sequence cards would give erroneous results.

The test on UTIL (utilization factor, rho) is for 0.98 and all operations involved are set to zero on output when UTIL exceeds this value.

The FACTL(K) Function Subprogram

This program yields answers of K! for arguments in the range greater than or equal to zero to K = 33. For arguments in the range 1 to 20 the method is table lookup. 0 is treated as a special case. For arguments in the range 21 to 33 the method is arithmetic expansion, starting with the table value for K = 20. For arguments greater than 33, floating point overflow will occur because K! becomes so large. The argument is a fixed point quantity and the result is floating point.

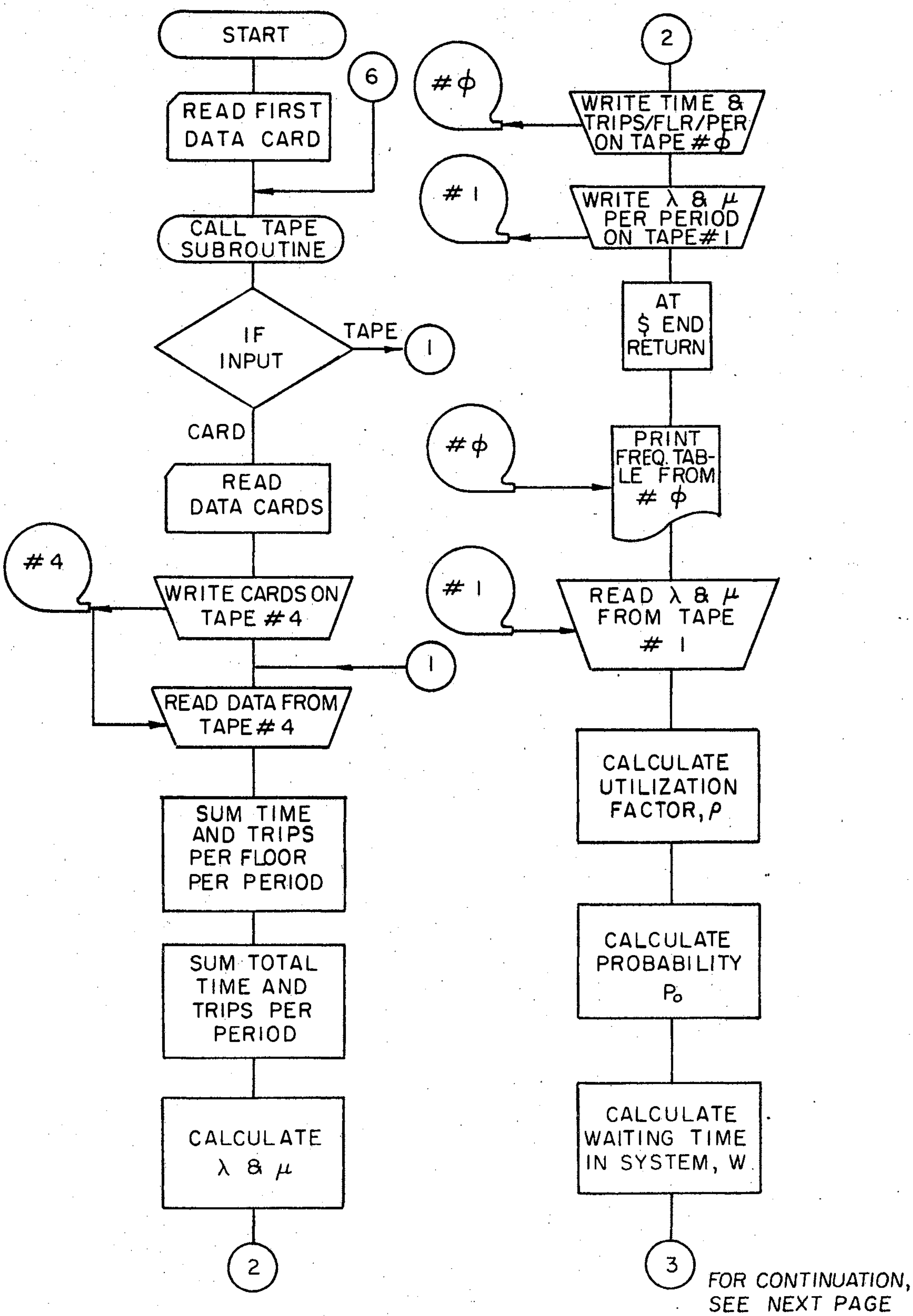

Figure 14. Flowchart for Program No. 1, Calculations of λ, μ, W, ρ, W_q and P_o

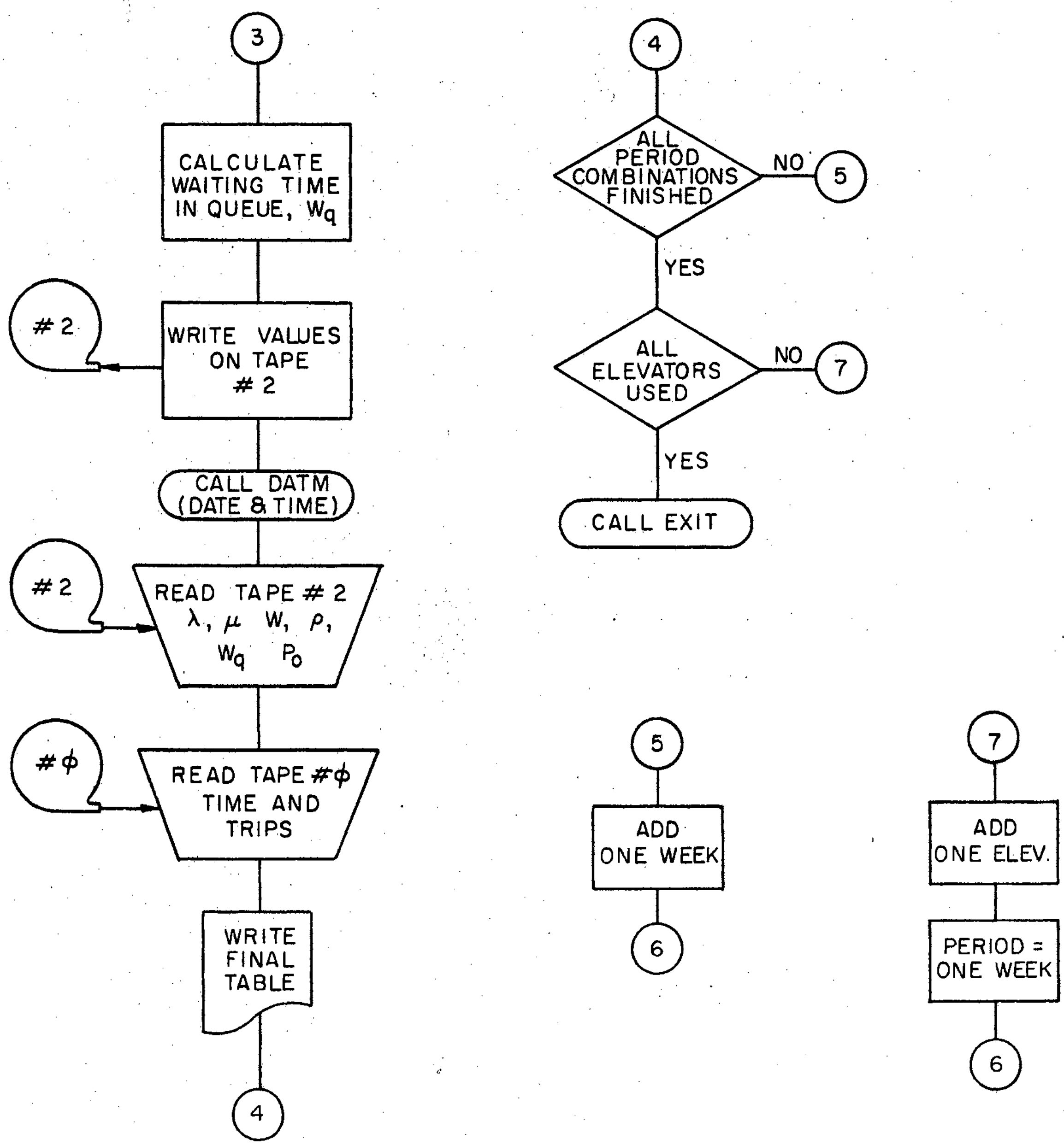

Figure 14. (Continued)

DATA CARD CODE SHEET

Project Number:________________ Date:______________

Time Period:________________ Update No.:________

Revised:__________yes______no No. of Cards:______

Initial Column	Final Column	No. of Columns	Item	Remarks
1	6	6	Material	Alphameric
7	8	2	Blank	
9	11	3	Time Period	Right Justified
12	13	2	Blank	
14	15	2	Floor	Right Justified
16	17	2	Blank	
18	20	3	Number of Trips	Right Justified
21	22	2	Blank	
23	28	6	Time per Trip, in minutes	Decimal Point in Column 26

THE FOLLOWING PUNCHED ON FIRST CARD ONLY
OF SET OF TIME PERIOD CARDS

31	40	10	Cost Coefficient, C_o	In field
41	50	10	Cost Coefficient, C_{w_i}	In field
51	60	10	Cost Coefficient, C_{k_i}	In field

Figure 15. Data Card Code Sheet

APPENDIX B

COMPUTER PROGRAM NO. 2

Explanation of Computer Program No. 2

Equation (5-1), page 28, states the expression for average delay time (average time waiting in queue) as

$$W_q = \left[\frac{\mu\left(\frac{\lambda}{\mu}\right)^k}{(k-1)!\ (k\mu-\lambda)^2}\right]\left[P_o\right]$$

where

$$P_o = \frac{1}{\left[\sum_{n=0}^{k-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k\frac{k\mu}{k\mu-\lambda}}\ .$$

Equation (5-10), page 33, includes the term W_q in the following form

$$\frac{C_{w_i}}{C_{k_i}} = \frac{1}{\theta_i(\mu W_{q_k} - \mu W_{q_{k+1}})}$$

which was obtained by expressing the average time spent in the system, W, as a multiple, f, of the average service time $\frac{1}{\mu}$ such that $f_k = \mu W_{q_k} + 1$ since $W = W_q + \frac{1}{\mu}$.

The expression for W_q can be restated in the following form

$$W_q = \frac{1}{\mu}\frac{(P > 0)}{k(1 - \rho)} \quad \text{or} \quad \mu W_q = \frac{(P > 0)}{k(1 - \rho)}$$

where

$$(P > 0) = \frac{\left[\dfrac{(k\rho)^k}{k!(1 - \rho)}\right]}{\left[\displaystyle\sum_{i=0}^{k-1} \frac{(k\rho)^i}{i!} + \frac{(k\rho)^k}{k!(1 - \rho)}\right]} .$$

It can be seen that the average delay expressed as a multiple of the average service time is a function of ρ only for various values of k.

Computer Program No. 2 computes values for μW_q as a function of ρ in the manner indicated by the flowchart which follows.

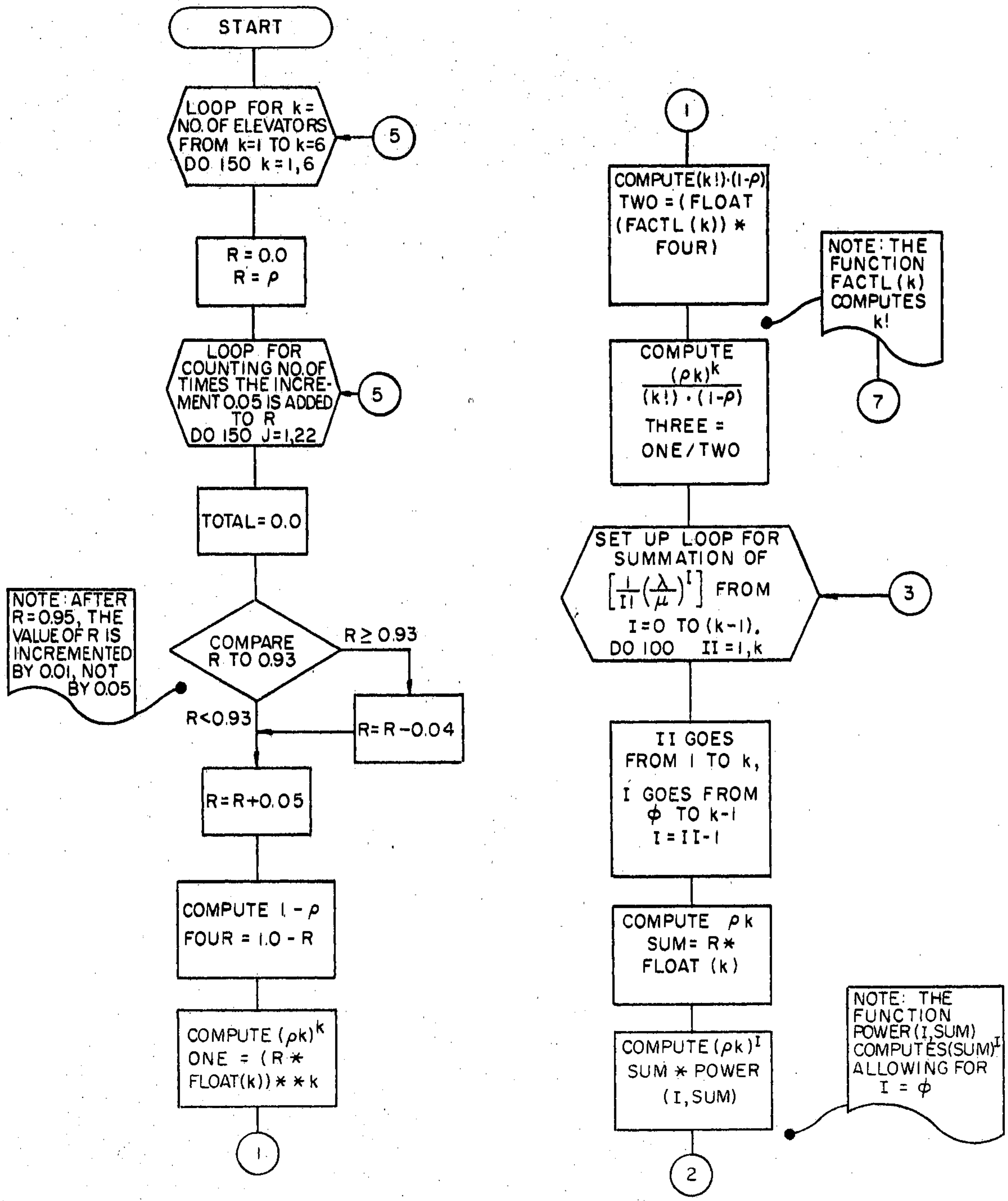

Figure 16. Flowchart for Program No. 2, μW_q as a Function of ρ

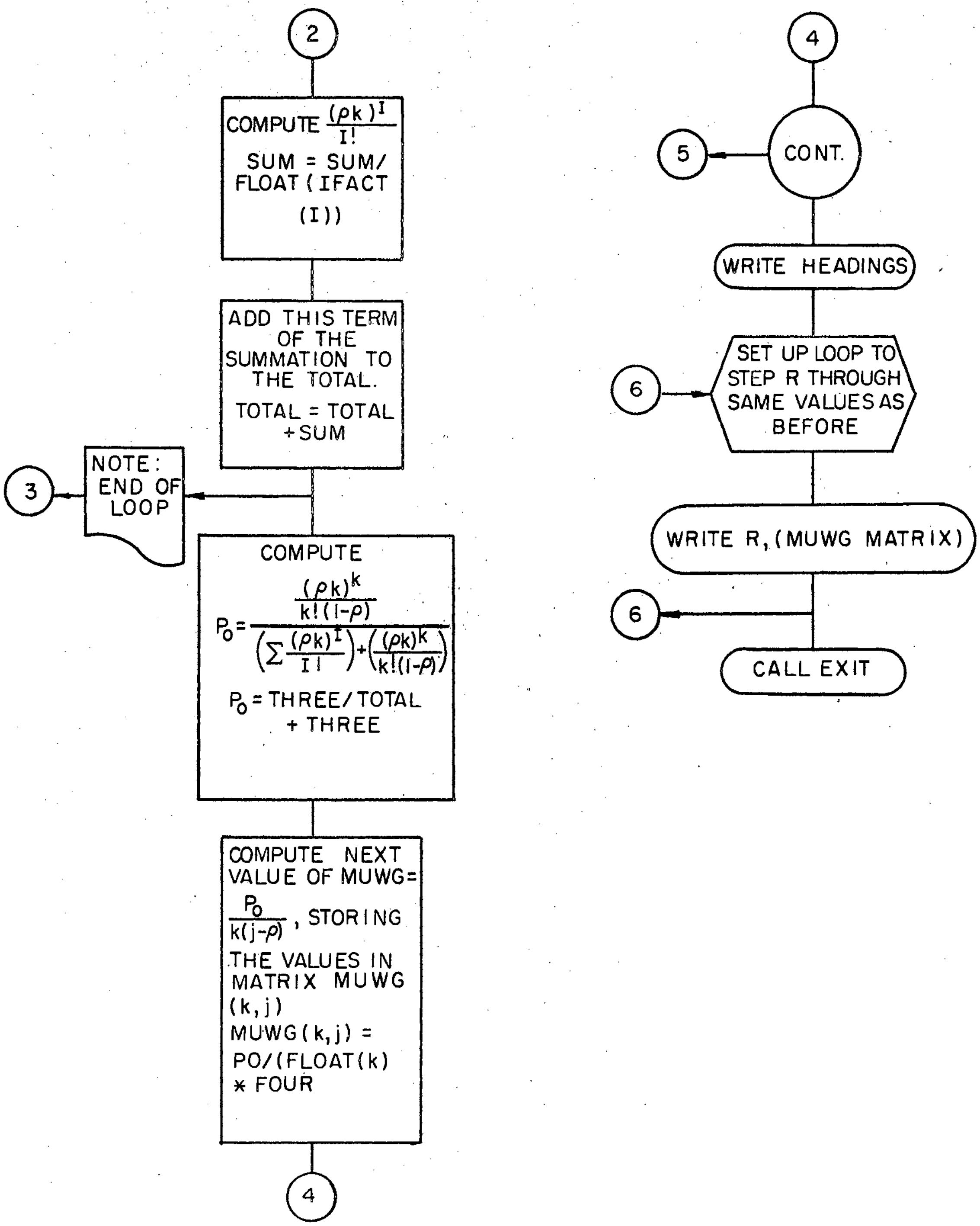

Figure 16. (Continued)

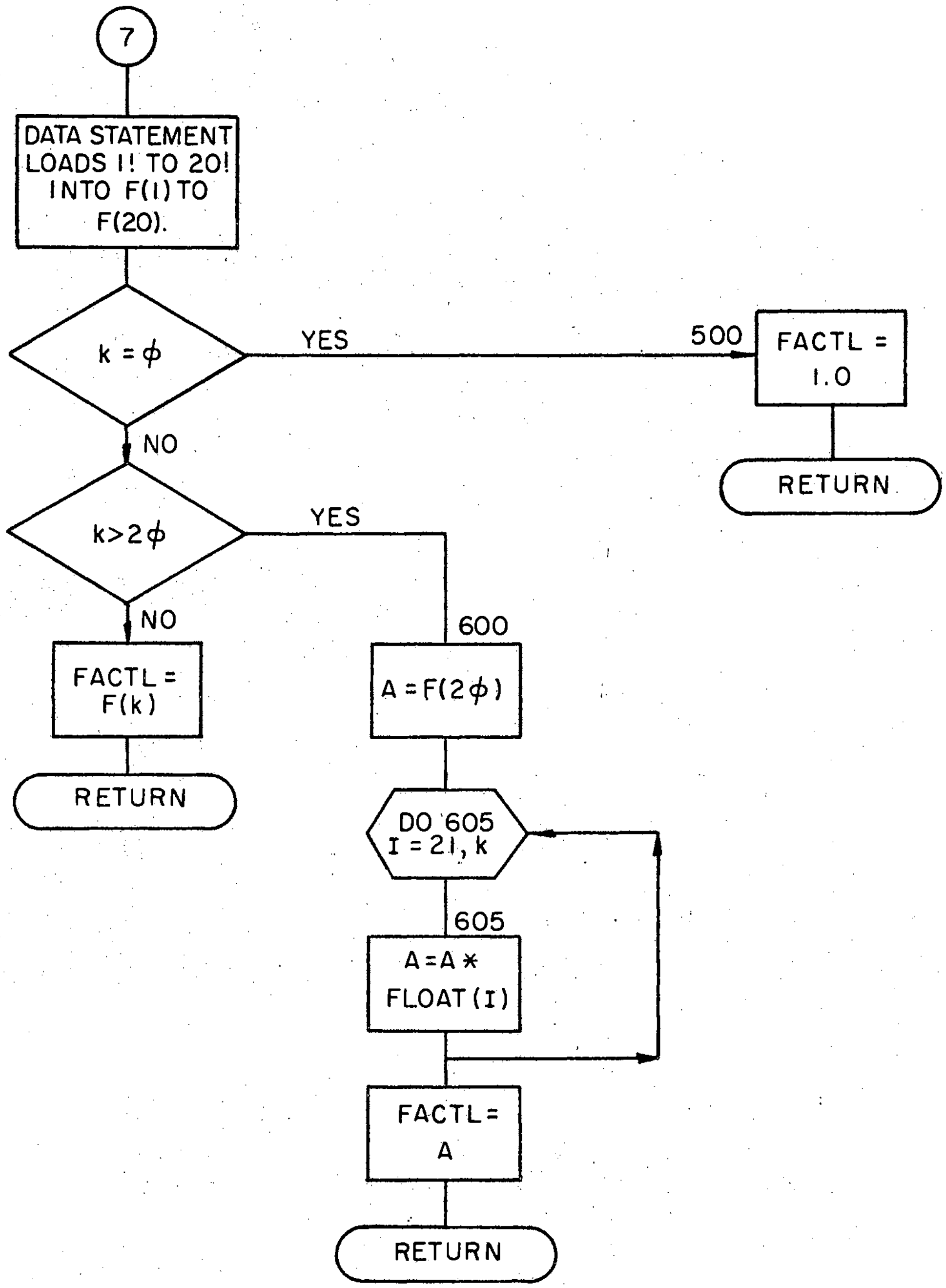

Figure 17. Flowchart for FACTL(K)

APPENDIX C

COMPUTER PROGRAMS NOS. 3, 4 AND 5

Explanation of Computer Programs Nos. 3, 4 and 5

Computer Program No. 3

For various values of C_k, C_b, C_m and C_b, a range of values for $\frac{C_k}{C_b}$ and $\frac{C_m}{C_b}$ were calculated. Using the values of these ratios, values of

$$\left[\frac{\left(\frac{C_k}{C_b}\right)}{U\left(\frac{C_m}{C_b}\right) + 1}\right]$$

can be calculated for values of U = 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0. Computer Program No. 3 performs these calculations in the manner indicated by the flowchart, page 104.

Computer Program No. 4

With the values of

$$\left[\frac{\left(\frac{C_k}{C_b}\right)}{U\left(\frac{C_m}{C_b}\right) + 1}\right]$$

obtained from Computer Program 3, values of ℓ^*_{1-2} and ℓ^*_{2-3} were calculated for various values of r. Computer Program No. 4 was written to perform these calculations according to the flowchart, page 105.

Computer Program No. 5

Equation (5-19), page 67, is stated as follows:

$$L = [(n)(v)(T_{pf})]^{\frac{1}{2}} \left[\frac{3y}{d}\right]^{\frac{1}{4}} .$$

Computer Program No. 5 was written to perform calculations for a range of values of the variables n, d and L for various values of T_{pf}. These calculations assumed fixed values for v and y. The procedure the program follows is indicated by the flowchart, page 106.

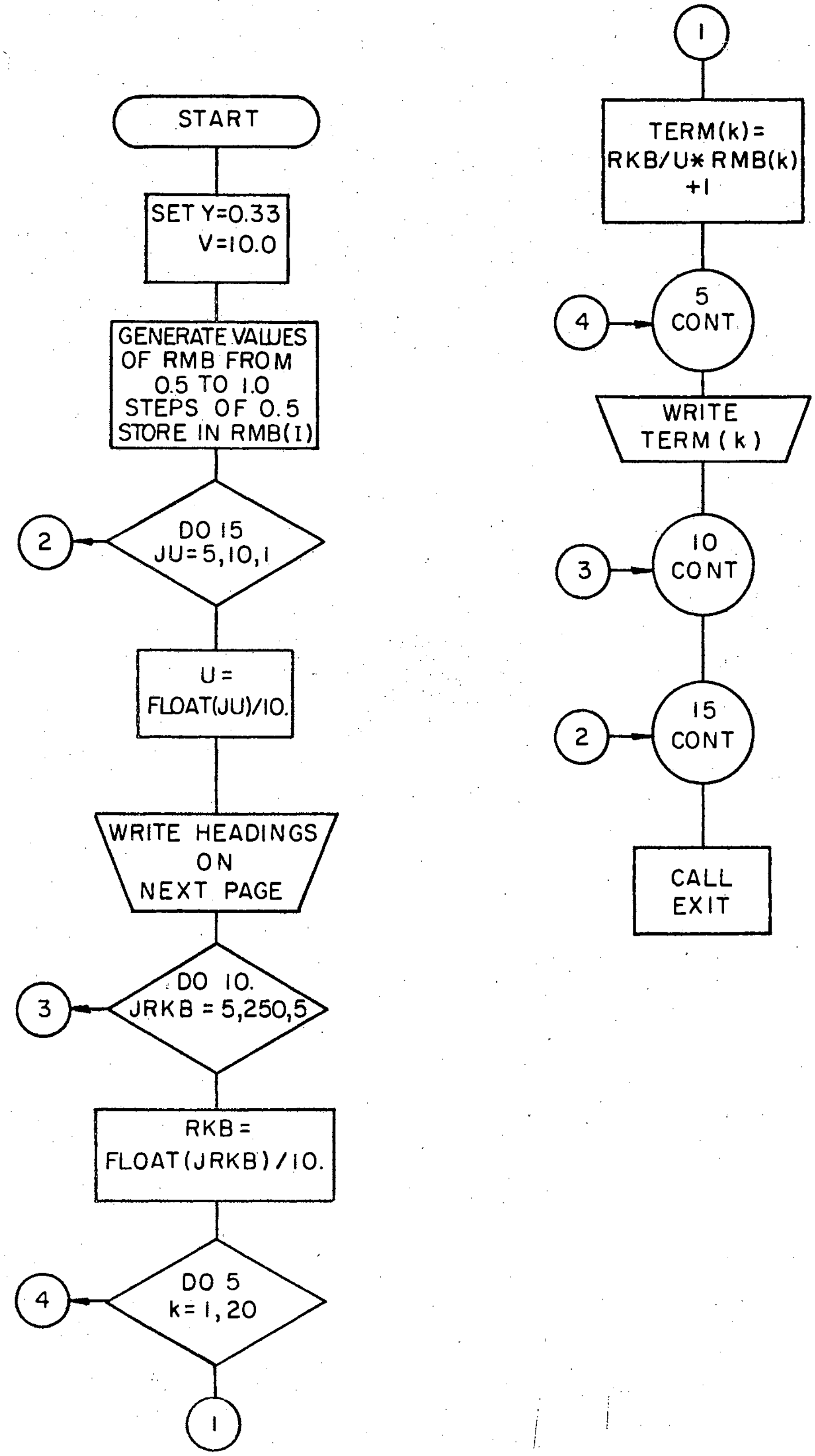

Figure 18. Flowchart for Program No. 3, Calculations of $\dfrac{\frac{C_k}{C_b}}{U\left(\frac{C_m}{C_b}\right)+1}$ for Various Values of (U)

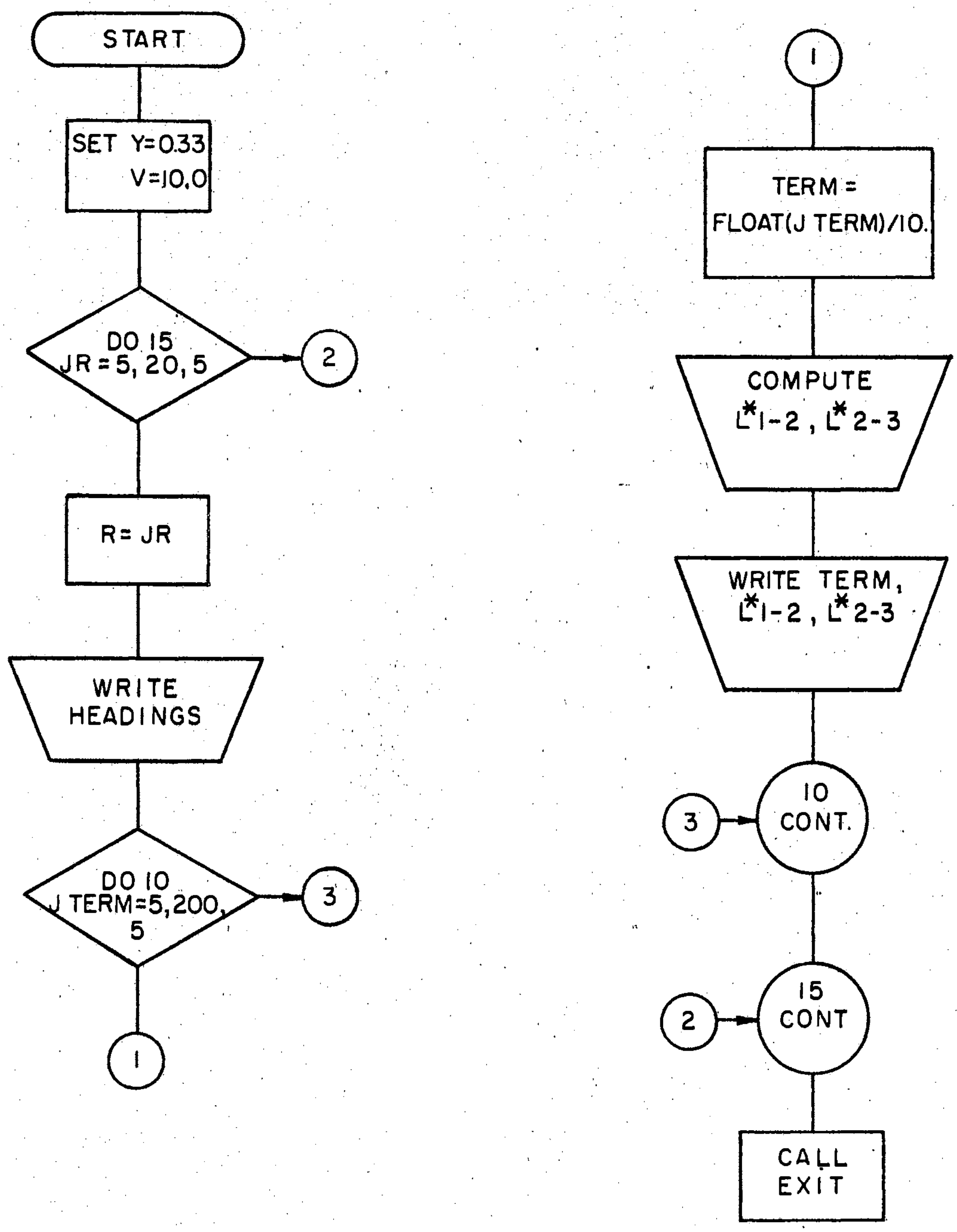

Figure 19. Flowchart for Program No. 4, Calculation of ℓ for Various Values of r

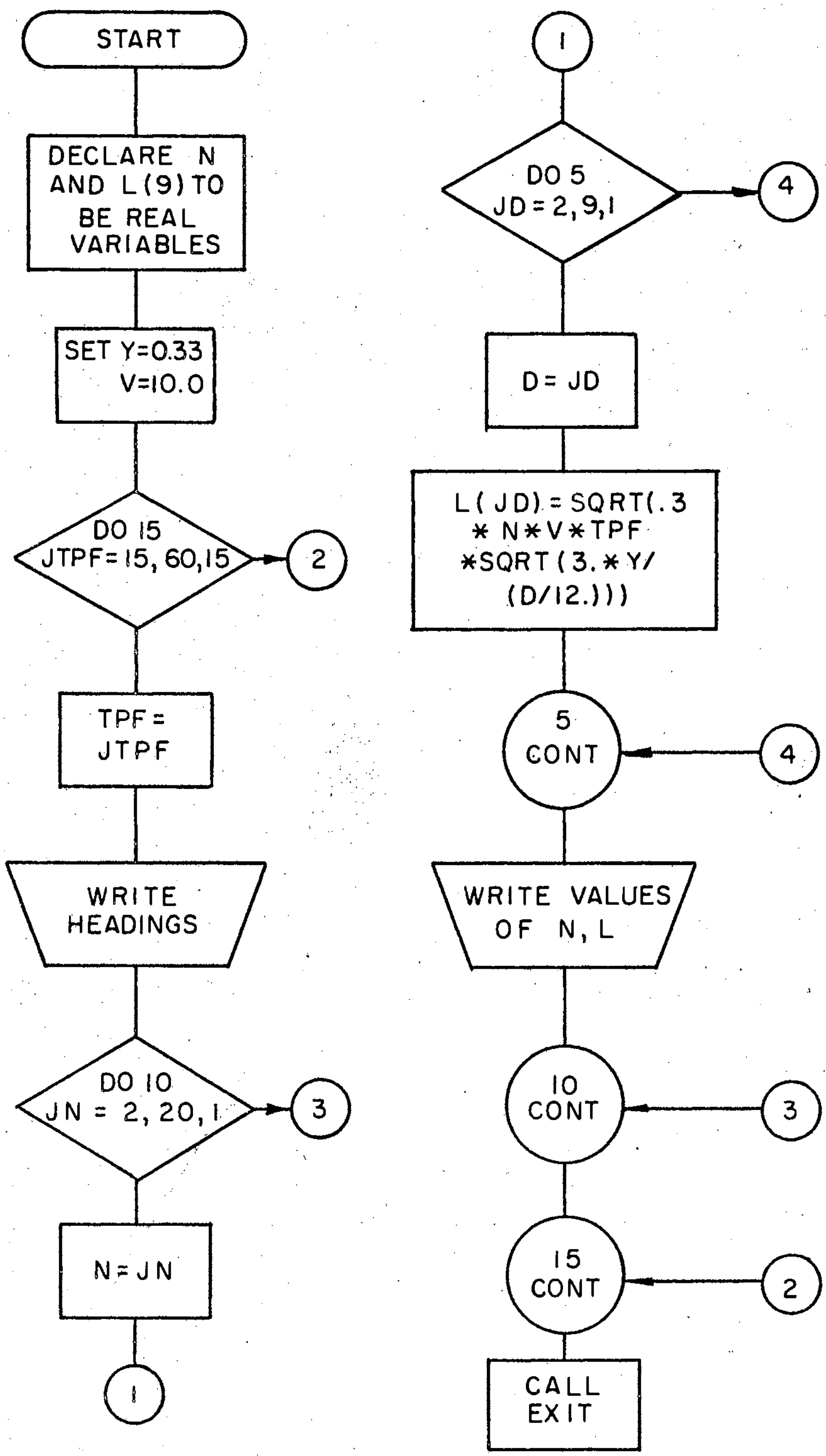

Figure 20. Flowchart for Program No. 5, Calculations of L and n for Various Values of T_{pf}

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