LAMINAR NATURAL-CONVECTION OF A FLUID
IN THE SUPERCRITICAL REGION

By

THOMAS EDWARD MULLIN

Bachelor of Science
University of Kentucky
Lexington, Kentucky
1951

Master of Science
University of Louisville
Louisville, Kentucky
1960

Submitted to the faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the degree of
DOCTOR OF PHILOSOPHY
May, 1968

Name: Thomas E. Mullin Date of Degree: May, 1968

Institution: Oklahoma State University

Location: Stillwater, Oklahoma

Title of Study: LAMINAR NATURAL-CONVECTION OF A FLUID IN THE SUPERCRITICAL REGION

Pages in Study: 168

Candidate for Degree of Doctor of Philosophy

Major Field: Engineering

Scope and Method of Study: A practical method for solving the problem of natural-convection in the supercritical region where the physical properties are highly variable has been developed. The fluid chosen is Freon 114. Analytical methods for obtaining the necessary thermodynamic and transport properties are also presented. The solution of the reduced differential equations for natural-convection along a vertical flat plate using the iterative method of Newton-Raphson is demonstrated. Four cases of natural-convection in the supercritical region were solved for the following fluid states:

$P = 540.00$ psia; $T_w - T_\infty = 5^{\circ}F$

Case (1): $T_w = 320.00^{\circ}F$

Case (2): $T_w = 315.00^{\circ}F$

Case (3): $T_w = 310.00^{\circ}F$

Case (4): $T_w = 305.00^{\circ}F$.

Findings and Conclusions: The analytical procedure presented provides a practical method for solving the problem of natural-convection in the supercritical region for laminar flow. For the particular cases considered it was estimated that the maximum coefficient of heat transfer occurs at a wall temperature near the maximum Prandtl number.

ADVISER'S APPROVAL ______________________________

LAMINAR NATURAL-CONVECTION OF A FLUID IN THE SUPERCRITICAL REGION

Thesis Approved:

Gerald D. Parker
Thesis Adviser

J. A. Wiebelt

K. C. Chao

A. M. Rowe

N. Durham
Dean of the Graduate College

ACKNOWLEDGMENT

The author would like to express his sincere appreciation and extend his gratitude to the following individuals who were most helpful to me during my graduate program.

Dr. J. D. Parker, who served as committee chairman and thesis adviser, for his helpful assistance and guidance throughout the entire Ph.D. program.

Dr. J. A. Wiebelt, Dr. A. M. Rowe and Dr. K. C. Chao, for their constructive technical discussions and suggestions during this investigation.

Dr. H. R. Sebesta and Mr. Dennis Unruh, for their invaluable assistance in solving the two point boundary value problem.

Mr. Harvey P. Metzler, for his encouragement, assistance, and technical discussions during this investigation.

Dr. Grosvenor and his staff, in particular Mr. Milton Boydston, Mr. Ron Sanders, Mr. Earl Westfall, Mr. Jim McGee and Mrs. Nancy Norton for their assistance regarding the development of the various computer programs.

Mrs. Margaret Estes, for invaluable assistance and advice in preparation of the manuscript.

The National Science Foundation, for their financial support during this investigation.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

Symbols	Descriptions
A	defined by Equation (22), Chapter III
A1	defined by Equation (35), Chapter III
A2	defined by Equation (36), Chapter III
A3	defined by Equation (37), Chapter III
a_0 thru a_5	defined by Equations (3) through (8), Chapter IV
B	defined by Equation (31), Chapter III
B1	defined by Equation (38), Chapter III
B2	defined by Equation (39), Chapter III
b	A constant defined in Equation (1), Chapter IV.
C_v^o	specific heat at constant volume, low pressure
C_v	specific heat at constant volume
$\bar{C}$	constant of the reference temperature equation
C	defined by Equation (5), Chapter II, or as defined by Equation (19), Chapter II, or as defined by Equation (9), Chapter III
C_w	defined by Equation (19), Chapter II, evaluated at the wall
C_{ref}	defined by Equation (19), Chapter II, evaluated at the reference temperatures
C_p	specific heat at constant pressure
C_{pw}	specific heat at constant pressure evaluated at the wall
$F(\eta)$	similarity variable

$Gr_{x,w}$	Grashof Number
g	acceleration due to gravity
g_c	a constant 32.2(Lbm/Lbf)(Ft/Sec^2)
h	coefficient of heat transfer
k	thermal conductivity or a constant defined by Equation (1), Chapter IV
k_w	thermal conductivity evaluated at the wall
k_∞	thermal conductivity evaluated at the main stream
k_{ref}	thermal conductivity evaluated at the reference temperature
M	molecular weight
$Nu_{x,w}$	local Nusselt Number evaluated at the wall
P	pressure
Pr	Prandtl Number
Pc	critical pressure
p	pressure
q	heat transfer
q_w	heat transfer at the wall
q_{ref}	heat transfer at the wall using the reference temperature
R	perfect gas constant or gas constant for Equation (1), Chapter IV
S	entropy
T	temperature
T_w	temperature evaluated at the wall
T_∞	temperature evaluated at the main stream
Tr	reduced temperature
T_c	critical temperature
t	the independent variable time

U	unknown wall or free stream condition
u	velocity component along the plate
v	velocity component perpendicular to the plate
X	dependent state variable defined by Equation 1, Chapter V
x	coordinate along the plate
y	coordinate perpendicular to the plate
z	defined by Equation (2), Chapter IV
Z_c	defined by Equation (40), Chapter IV
ΔT	$T_w - T_\infty$
$\mathcal{K}$	defined by Equation (33), Chapter IV
η	similarity coordinate as defined by Equation (3), Chapter II, or as defined by Equation (7), Chapter III
$\theta(\eta)$	dimensionless temperature
λ	defined by Equation (40), Chapter IV
μ	viscosity
μ_w	viscosity evaluated at the wall
μ_∞	viscosity evaluated at the main stream
ν	kinematic viscosity μ/ρ
ν_w	kinematic viscosity evaluated at the wall
ν_{ref}	kinematic viscosity evaluated at the reference temperature
ρ	density
ρ_w	density evaluated at the wall
ρ_∞	density evaluated at the main stream
ρ_{ref}	density evaluated at the reference temperature

ρ_r	reduced temperature
ρ_c	critical density

CHAPTER I

INTRODUCTION

Statement of the Problem

Convection heat transfer occurs whenever a fluid comes into contact with a body that is at a different temperature than the fluid. This transfer of heat takes place by a combination of conduction and mass transport. If the body surface is at a higher temperature than the fluid, heat flows by conduction from the body to the fluid next to the surface. This transmitted energy which increased the internal energy of the fluid is then transported away by the motion of the fluid. The reverse process occurs if the body surface is at a lower temperature than the fluid. If the motion of the fluid is caused solely by the density changes resulting from the temperature gradient between the body surface and the fluid, then the heat-transfer mechanism is defined as natural or free-convection.

The natural-convection heat transfer coefficients are relatively low; however, this information is essential for the successful design of many devices such as, high speed gas turbines, electrical transformers and transmission lines, and boilers. The effective application of insulation to such equipment as pipes, towers, and heat exchangers also requires a knowledge of natural-convection heat transfer coefficients.

Evaluation of natural-convection heat transfer coefficients from boundary layer theory is long and difficult. This is particularly true if the fluid properties are highly variable which is the case when the fluid is in the supercritical state.

The primary purpose of this work was to develop a practical method to study the natural-convection characteristics of a fluid in a supercritical state. Of particular interest was that portion of the supercritical region in which phase-like changes occur; i.e., where the density, specific heat at constant pressure, dynamic viscosity, and thermal conductivity change rapidly with respect to temperature at constant pressure. The development included the following phases:

1. The application of the continuity, momentum, and energy equations to the problem of natural convection along a vertical flat plate with constant wall and main stream temperatures and their reduction from a set of nonlinear simultaneous, partial differential equations with variable coefficients to two ordinary nonlinear differential equations with variable coefficients. The variable coefficients were expressed in dimensionless form in terms of the thermodynamic and transport properties. In this form it was easier to see the effects of the role played by the variation

of the physical properties and their derivatives on natural-convection.

2. The development of analytical procedures for determining the thermodynamic and transport properties including their derivatives. The physical properties determined were the variation of density, specific heat at constant pressure, dynamic viscosity, and thermal conductivity with temperature at constant pressure.

3. The numerical solution of the reduced ordinary nonlinear differential equations by the method of Runga Kutta. Since two of the wall boundary conditions are unknown; namely, $F''(0)$ and $\theta'(0)$, the iterative process of Newton Raphson was used for their determination.

CHAPTER II

SELECTIVE REVIEW OF THE LITERATURE

Of the many papers in the literature covering the problem of natural convection there are four which closely relate to this work. They are:

1. Pohlhausen, E. "The Heat Exchange Between Solid Bodies and Fluids With Small Friction and Small Conduction of Heat." ZAMM 1, 115, 1921.
2. Ostrach, S. "An Analysis of Laminar Free Convection Flow and Heat Transfer About a Flat Plate Parallel to the Direction of the Generating Body Force." NACA Report 1111, 1953.
3. Sparrow, E. M., and R. J. Gregg. "The Variable Fluid Property Problem in Free Convection." Transaction of ASME, Vol. 80, 1958, pp. 879-886.
4. Fritsch, C. A., and R. J. Grosh. "Free-Convection Heat Transfer to a Supercritical Fluid." A.I.Ch.E. Journal (June, 1955), pp. 1010-1016.

During this review of the above papers, it should be kept in mind that the equations obtained from the application of the continuity, momentum, and energy equations to the problem of natural-convection along a vertical flat plate are a set of nonlinear, simultaneous, partial differential equations with variable coefficients. Also the constant property problem of natural-convection is defined as one in which all fluid variations are neglected with the exception of the role that density variations play in producing a buoyancy force.

E. Pohlhausen demonstrated by introducing the stream functions

$$u = \frac{\partial \Psi}{\partial y} \tag{1}$$

$$v = -\frac{\partial \Psi}{\partial x} \tag{2}$$

and the similarity variables

$$\eta = C \frac{y}{x^{1/4}} \tag{3}$$

$$F(\eta) = \frac{\Psi}{4\nu C x^{3/4}} \tag{4}$$

$$C = \sqrt[4]{g \frac{(T_w - T_\infty)}{4\nu^2 \, T_\infty}} \tag{5}$$

that the set of partial differential equations for the constant property problem can be reduced to a set of ordinary nonlinear differential equations of the form

$$F''' = -3FF'' + 2(F')^2 - \theta \tag{6}$$

$$\theta'' = -3PrF\theta' \tag{7}$$

with the boundary conditions

$$\begin{aligned} F(0) &= 0 & & \\ F'(0) &= 0 & F'(\infty) &= 0 \\ \theta(0) &= 1 & \theta(\infty) &= 0 \ . \end{aligned} \tag{8}$$

Pohlhausen solved Equations (6), (7) and (8) for a value of the Prandtl Number of 0.73. Schlichting [13] showed that the theoretical solutions for the velocity and temperature profiles at Pr = 0.73 were in very good agreement with experimental data for air.

S. Ostrach presented a detail treatment of the constant property problem for natural convection along a vertical flat plate. His paper contained a thorough coverage of the derivations of the equations and presents numerically computed cases for the range of Prandtl Numbers from 0.01 to 1000.

Although the constant property problem has been thoroughly solved, it still leaves open the important question as to the range of the temperature difference between the wall and the fluid for which the constant property solution would be valid. Since the temperature difference does not influence the reduced differential equations of the constant property problem, the application of constant property results to a practical problem must be done with care.

The work of Sparrow and Gregg is concerned with the variable property problem of natural convection along a vertical flat plate. They demonstrated without placing any restrictions on the nature of the property variation that by using the stream functions

$$u = \frac{\rho_w}{\rho} \frac{\partial \Psi}{\partial y} \tag{9}$$

$$v = -\frac{\rho_w}{\rho} \frac{\partial \Psi}{\partial x} \tag{10}$$

and the similarity variables

$$C = \left[\frac{g(\rho_\infty - \rho_w)/\rho_w}{4\nu^2}\right]^{1/4}, \quad \eta = CX^{-1/4} \int_0^y \frac{\rho}{\rho_w}\, dy \tag{11}$$

$$F(\eta) = \frac{1}{4\nu_w C} \left(\frac{\Psi}{x^{3/4}}\right) \tag{12}$$

along with the dimensionless temperature

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{13}$$

that the partial differential equations for natural-convection along a vertical flat plate can be reduced to ordinary differential equations of the form:

$$\frac{d}{d\eta}\left[\frac{\rho\mu}{\rho_w\mu_w} F''\right] + 3FF'' - 2(F')^2 + \frac{\rho_\infty - \rho}{\rho_\infty - \rho_w} \cdot \frac{\rho_w}{\rho} = 0 \tag{14}$$

$$\frac{d}{d\eta}\left[\frac{\rho k}{\rho_w k_w} \theta'\right] + 3Pr_w \frac{C_p}{C_{pw}} F \theta' = 0 . \tag{15}$$

They also proposed a system whereby the variable property problem of natural convection can be computed using constant property results by evaluating the properties at a reference temperature defined as

$$T_{ref} = T_w - 0.38(T_w - T_\infty) . \qquad (16)$$

Equation (16) is essentially based on a perfect gas state with power-law and Sutherland-type formula to describe the viscosity and thermal conductivity variations. Mercury was also used as a fluid medium. In effect Sparrow and Gregg found a reference temperature for evaluating k and ν such that the constant property solution would yield the correct value for h. It is of interest to note that the constant 0.38 of the reference temperature Equation (16) can be computed directly from the information given in his paper. To aid in this computation the following reasoning is presented:

1. The heat transfer for the variable property case is equal to the heat transfer for the constant property case using the reference temperature; i.e.:

$$q_w = q_{ref} . \qquad (17)$$

2. Continuing

$$k_w C_w \theta'(0)_w = k_{ref} C_{ref} \theta'(0)_{ref} . \qquad (18)$$

3. The term C is defined as

$$C = \left[\frac{g}{4\nu^2}\left(\frac{\rho_\infty - \rho_w}{\rho_w}\right)\right]^{1/4} \qquad (19)$$

and therefore

$$\frac{C_{ref}}{C_w} = \left[\frac{\nu_w}{\nu_{ref}}\right]^{1/2} . \qquad (20)$$

4. The substitution of Equation (20) into Equation (18) gives the necessary equation to be satisfied if Equation (17) is to be true; i.e.:

$$\left[\frac{k_{ref}}{k_w}\right]\left[\frac{\mu_w}{\mu_{ref}}\right]^{\frac{1}{2}}\left[\frac{\rho_{ref}}{\rho_w}\right]^{\frac{1}{2}} = \frac{\theta'(0)_w}{\theta'(0)_{ref}} \tag{21}$$

where

$\theta'(0)_w$ = the dimensionless temperature slope at the wall for the variable property case

$\theta'(0)_{ref}$ = the dimensionless temperature slope at the wall for the constant property case.

Using Equation (21) and the expressions for the physical properties of a given fluid including the values of $\theta'(0)_w$ and $\theta'(0)_{ref}$ one can find an expression to compute the reference temperature constant directly.

Sparrow and Gregg considered the fluid (Gas A)

$$\begin{aligned} P &= \rho RT \\ k &\sim T^{3/4} \\ \mu &\sim T^{3/4} \\ C_p &= \text{Constant} \\ Pr &= \text{Constant} . \end{aligned} \tag{22}$$

The value of $\bar{C}$ will now be evaluated for this type of fluid. The various ratios that are required by Equation (21) are formed as follows:

$$\frac{\rho_{ref}}{\rho_w} = \frac{T_w}{T_{ref}} \tag{23}$$

$$\frac{\mu_w}{\mu_{ref}} = \frac{k_w}{k_{ref}} = \left[\frac{T_w}{T_{ref}}\right]^{3/4} . \tag{24}$$

Substituting Equations (23) and (24) into Equation (21) yields

$$\frac{T_{ref}}{T_w} = \left[\frac{\theta'(0)_{ref}}{\theta'(0)_w}\right]^8 . \tag{25}$$

The reference temperature Equation (16) may be rearranged as follows:

$$\frac{T_{ref}}{T_w} = 1 - \bar{C}\left[1 - \frac{T_\infty}{T_w}\right] . \tag{26}$$

The combining of Equations (25) and (26) and then solving for $\bar{C}$ gives

$$\bar{C} = \frac{1 - [\theta'(0)_{ref}/\theta'(0)_w]^8}{1 - (T_\infty/T_w)} \tag{27}$$

The application of Equation (27) to Gas A with a $Pr = 0.7$ yields the following:

$$Nu_{x,w}/Gr_{x,w}^{1/4} = -[\theta'(0)_{ref}/\sqrt{2}] = 0.353 \quad \text{(reference temperature solution)}$$

T_w/T_∞	$-\theta'(0)_w/\sqrt{2}$	$\theta'(0)_{ref}/\theta'(0)_w$	$[\theta'(0)_{ref}/\theta'(0)_w]^8$
4	0.371	0.95148	0.672
3	0.368	0.95920	0.716
5/2	0.366	0.96448	0.749
2	0.363	0.97245	0.799
3/4	0.348	1.01436	1.122
1/2	0.339	1.04129	1.383
1/3	0.330	1.06969	1.714
1/4	0.323	1.09287	2.035

$1-[\theta'(0)_{ref}/\theta'(0)_w]^8$	T_∞/T_w	$1-(T_\infty/T_w)$	$\bar{C}$
0.328	1/4	3/4	0.438
0.284	1/3	2/3	0.427
0.251	2/5	3/5	0.418
0.201	1/2	1/2	0.402
−0.122	4/3	−1/3	0.366
−0.383	2/1	−1	0.383
−0.714	3/1	−2	0.357
−1.035	4/1	−3	0.345

Another fluid considered by Sparrow and Gregg was (Gas B)

$$P = \rho RT$$
$$k \sim T^{2/3}$$
$$\mu \sim T^{2/3} \qquad (28)$$
$$C_p = \text{Constant}$$
$$Pr = \text{Constant} \, .$$

If the same procedure that was used for Gas A is applied to Gas B, the following is obtained:

$$\bar{C} = \frac{1 - [\theta'(0)_{ref}/\theta'(0)_w]^6}{1 - T_\infty/T_w} \, . \qquad (29)$$

The application of Equation (29) to case B for Pr = 0.7 gives the following results:

$$Nu_{x,w}/Gr_{x,w}^{1/4} = -[\theta'(0)_{ref}/\sqrt{2}] = 0.353 \quad \text{(reference temperature solution)}$$

$$T_w/T_\infty = 3$$

$$-\theta'(0)_{ref}/2 = 0.373$$

$$\bar{\bar{C}} = 0.432.$$

The value of T_{ref}/T_w for various values of $(T_w-T_\infty)/T_w$ is plotted in Figure 1. The slope of the lines represents the reference temperature constant $\bar{\bar{C}}$. The table of $\bar{\bar{C}}$ values confirms the choice of $\bar{\bar{C}} = 0.38$ as a good estimate of the constant used for the reference temperature equation. The graph demonstrates how T_{ref} depends on the choice of $\bar{\bar{C}}$ with larger error for large or small values of T_∞/T_w.

In view of the types of fluid states assumed by Sparrow and Gregg, they have restricted their results to that portion of the property diagram where their physical property assumptions are valid which is primarily in the perfect gas region.

C. A. Fritsch and R. J. Grosh's work relates to the problem of natural-convection along a vertical flat plate in which the fluid is in the supercritical state. The fluid was water and the assumptions made regarding the physical properties were that C_v, μ and k were constant; however, they are evaluated at the mean temperature $(T_w + T_\infty)/2$. The results obtained were reported to be appreciably different from those predicted by constant property analysis.

Fritsch and Grosh utilized the assumption of constant

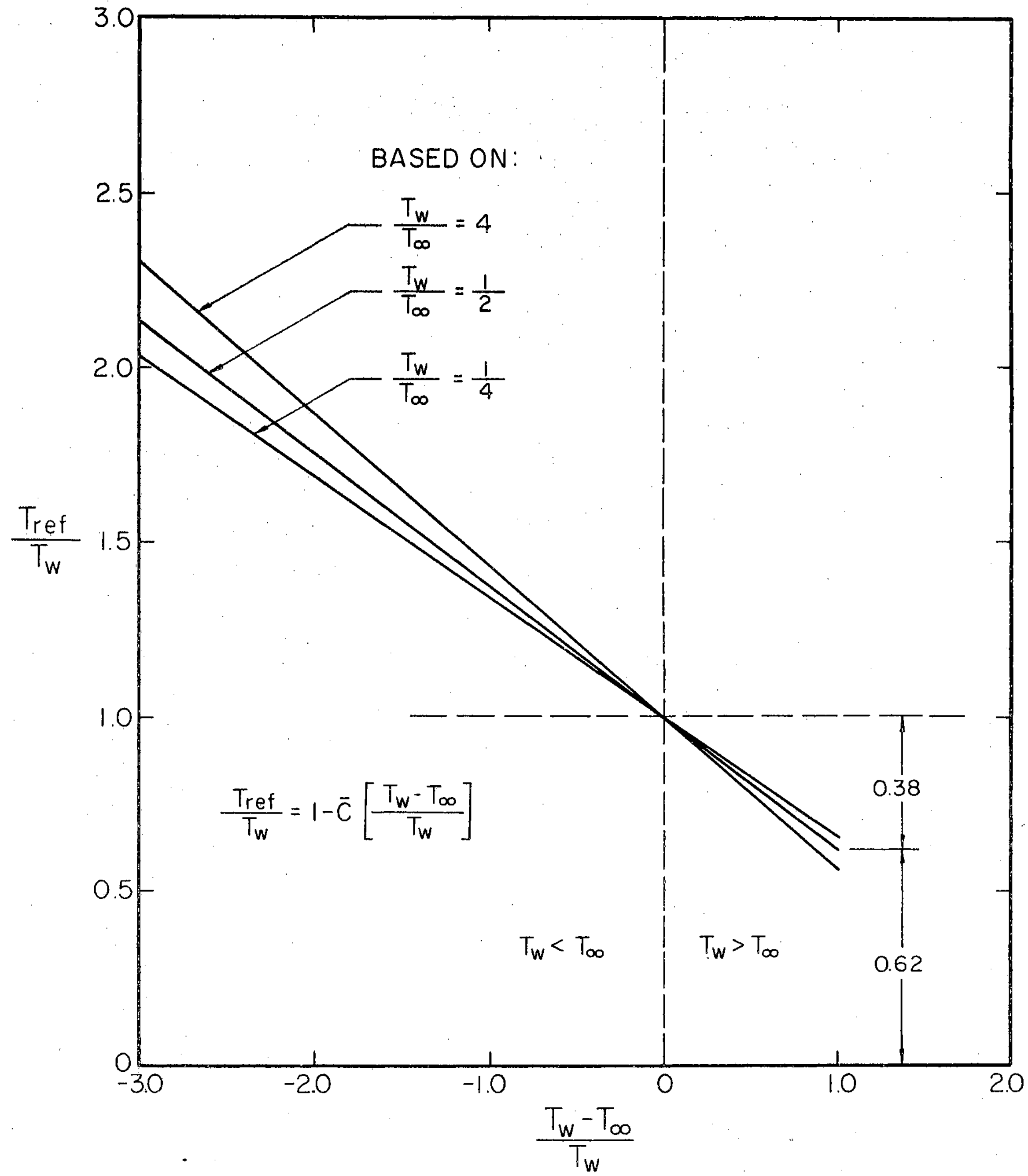

Figure 1. The Reference Temperature

dynamic viscosity and thermal conductivity to reduce the set of nonlinear partial differential equations to a set of nonlinear ordinary differential equations. This procedure has the effect of eliminating the role that the derivatives of these properties play in their analysis; however, in certain portions of the supercritical regions these properties do have strong variations.

This whole question regarding what physical properties may or may not be considered constant does not appear as important as the question of which dimensionless property groups may be considered constant. This, of course, indicates the need to know what are the dimensionless property groups. Also, in view of the uncertain temperature ranges for which the constant property solutions are valid and the fluid state restriction placed on the variable property solutions presented in the literature indicates a need for further investigation into the variable property problem of natural-convection.

CHAPTER III

THE FUNDAMENTAL EQUATIONS FOR LAMINAR NATURAL-CONVECTION

The Reduced Differential Equations

The equations for the conservation of mass, momentum, and energy were used to analyse the natural-convection of a fluid along a vertical flat plate. If the conditions of steady state, two dimensional flow, with constant wall and main stream temperatures are assumed, the conservation equations reduce to the following:

Continuity

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{1}$$

Momentum

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = (\rho_\infty - \rho)\, g + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

Energy

$$\rho\, C_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) \tag{3}$$

with the boundary conditions

$$u(x,0) = 0 \qquad u(x,\infty) = 0$$
$$v(x,0) = 0 \tag{4}$$
$$T(x,0) = T_w \qquad T(x,\infty) = T_\infty \, .$$

Equations (1), (2) and (3) form a set of nonlinear partial differential equations with variable coefficients which can be reduced to a set of two nonlinear ordinary differential equations by means of the compressible flow stream function and similarity variables. The stream function reduces the number of dependent variables and the similarity variables transforms the partial differential equations to ordinary differential equations. The stream function and similarity variables are defined as follows:

<u>Stream Function</u>

$$u = \frac{\rho_w}{\rho} \left(\frac{\partial \Psi}{\partial y}\right)_x \tag{5}$$

$$v = -\frac{\rho_w}{\rho} \left(\frac{\partial \Psi}{\partial x}\right)_y \tag{6}$$

<u>Similarity Variables</u>

$$\eta = C\, x^{-1/4} \int_0^y \frac{\rho}{\rho_w}\, dy \tag{7}$$

$$F(\eta) = \frac{1}{4\nu_w C} \left(\frac{\Psi}{x^{3/4}}\right) \tag{8}$$

$$C = \left[\frac{g}{4\nu_w^{\,2}} \left(\frac{\rho_\infty - \rho_w}{\rho_w}\right)\right]^{1/4} \tag{9}$$

<u>Dimensionless Temperature</u>

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \, . \tag{10}$$

It will be noted that Equations (5) and (6) satisfy the equation of continuity, Equation (1). Equations (5) through (10) will give the following set of equations:

$$u = 4\nu_w C^2 x^{1/2} F'(\eta) \tag{11}$$

$$v = \frac{\rho_w}{\rho} C \nu_w x^{-1/4} [\eta F'(\eta) - 3F(\eta)] \tag{12}$$

$$\left(\frac{\partial u}{\partial x}\right)_y = \nu_w C^2 x^{-1/2} [2F'(\eta) - \eta F''(\eta)] \tag{13}$$

$$\left(\frac{\partial u}{\partial y}\right)_x = 4\nu_w C^3 x^{1/4} \frac{\rho}{\rho_w} F''(\eta) \tag{14}$$

$$\left(\frac{\partial T}{\partial x}\right)_y = -(T_w - T_\infty) \frac{x^{-1}}{4} \eta\theta'(\eta) \tag{15}$$

$$\left(\frac{\partial T}{\partial y}\right)_x = C(T_w - T_\infty) x^{-1/4} \frac{\rho}{\rho_w} \theta'(\eta) \tag{16}$$

$$\frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right)_x = C^2 x^{-1/2} (T_w - T_\infty) \frac{\rho}{\rho_w} k \frac{\partial}{\partial \eta}\left(\frac{\rho}{\rho_w} \theta'(\eta)\right)_x + C^2 x^{-1/2} (T_w - T_\infty) \frac{\rho^2}{\rho_w^2} \theta'(\eta) \left(\frac{\partial k}{\partial \eta}\right)_x . \tag{18}$$

The introduction of Equations (11) through (18) into Equations (2) and (3) will give the set of ordinary differential equations, previously published by Sparrow and Gregg [15].

$$\frac{d}{d\eta}\left[\frac{\rho\mu}{\rho_w \mu_w} F''\right] + 3FF'' - 2[F']^2 + \frac{\rho_\infty - \rho}{\rho_\infty - \rho_w} \frac{\rho_w}{\rho} = 0 \tag{19}$$

$$\frac{d}{d\eta}\left[\frac{\rho k}{\rho_w k_w}\ \theta'\right] + 3Pr_w \frac{C_p}{C_{pw}} F\theta' = 0 \qquad (20)$$

with the boundary conditions

$$\begin{aligned} &F(0) = 0 \\ &F'(0) = 0 \qquad F'(\infty) = 0 \\ &\theta(0) = 1 \qquad \theta(\infty) = 0 \;. \end{aligned} \qquad (21)$$

Further Development of the Reduced Ordinary Differential Equations

It should be noted that before Equations (19) and (20) can be solved it will be necessary to determine either C_p, μ, k and ρ as functions of the independent variable η or as functions of the dependent variables. Since the physical properties are expressed as functions of temperature at a constant pressure and the temperature is a function of $\theta(\eta)$, it is an easy matter to obtain the physical properties as functions of $\theta(\eta)$ once the state equations for the physical properties are known. Also, it will be noted that the first terms of Equations (19) and (20), written in this form, do not show clearly the dimensionless physical property groups. In order to show how this can be accomplished the following is presented:

1. Define the function

$$A = \frac{\rho\mu}{\rho_w \mu_w} = f(\eta) = f(p,T) \qquad (22)$$

2. The derivative of A with respect to η is

$$\frac{dA}{d\eta} = \frac{\mu}{\rho_w \mu_w} \frac{d\rho}{d\eta} + \frac{\rho}{\rho_w \mu_w} \frac{d\mu}{d\eta} \; . \tag{22}$$

3. Note that

$$\rho = \rho(p,T) = f(\eta) \tag{23}$$

$$\frac{d\rho}{d\eta} = \left(\frac{\partial \rho}{\partial T}\right)_p \frac{dT}{d\eta} + \left(\frac{\partial \rho}{\partial p}\right)_T \frac{dp}{d\eta} \; . \tag{24}$$

4. Since pressure is held constant this reduces to

$$\frac{d\rho}{d\eta} = \left(\frac{\partial \rho}{\partial T}\right)_p \frac{dT}{d\eta} \; . \tag{25}$$

5. The definition of the dimensionless temperature

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{26}$$

gives

$$\frac{dT}{d\eta} = \Delta T \; \theta'(\eta) \; . \tag{27}$$

6. The substitution of Equation (27) into (25) gives

$$\frac{d\rho}{d\eta} = \left(\frac{\partial \rho}{\partial T}\right)_p \Delta T \; \theta'(\eta) \; . \tag{28}$$

7. The same reasoning as above when applied to the viscosity gives

$$\frac{d\mu}{d\eta} = \left(\frac{\partial \mu}{\partial T}\right)_p \Delta T \; \theta'(\eta) \; . \tag{29}$$

8. The substitution of Equations (28) and (29) into Equation (22) yields

$$\frac{dA}{d\eta} = \frac{\mu}{\rho_w \mu_w} \left(\frac{\partial \rho}{\partial T}\right)_p \Delta T \; \theta'(\eta) + \frac{\rho}{\rho_w \mu_w} \left(\frac{\partial \mu}{\partial T}\right)_p \Delta T \; \theta'(\eta) . \tag{30}$$

9. If an identical line of reasoning as the above is applied to the term

$$B = \frac{\rho k}{\rho_w k_w} \tag{31}$$

the following expression is obtained:

$$\frac{dB}{d\eta} = \frac{k}{\rho_w k_w} \left(\frac{\partial \rho}{\partial T}\right)_p \Delta T \ \theta'(\eta)$$

$$+ \frac{\rho}{\rho_w \mu_w} \left(\frac{\partial k}{\partial T}\right)_p \Delta T \ \theta'(\eta) \ . \tag{32}$$

10. The substitution of Equations (30) and (32) into the reduced ordinary differential Equations (19) and (20) yields the following:

Momentun Equation

$$F''' = -(A1)\theta' F'' - 3(A2)FF''$$
$$+ 2(A2)(F')^2 - (A3) \tag{33}$$

Energy Equation

$$\theta'' = -(B1)[\theta']^2 - 3(B2)F\theta' \tag{34}$$

Dimensionless Property Coefficients

$$A1 = \left[\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p + \frac{1}{\mu}\left(\frac{\partial \mu}{\partial T}\right)_p\right](\Delta T) \tag{35}$$

$$A2 = \frac{\rho_w \mu_w}{\rho \mu} \tag{36}$$

$$A3 = (A2)\left[\frac{\rho_\infty - \rho}{\rho_\infty - \rho_w}\right]\frac{\rho_w}{\rho} \tag{37}$$

$$B1 = \left[\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p + \frac{1}{k}\left(\frac{\partial k}{\partial T}\right)_p\right](\Delta T) \tag{38}$$

$$B2 = (A2)\,\frac{C_p \mu}{k} \quad . \tag{39}$$

Equations (35) through (39) show that once the equations of state for the physical properties C_p, μ, k and ρ are known as functions of temperature and pressure these coefficients may be expressed as functions of the dependent variable $\theta(\eta)$ since $T = T_\infty - \Delta T\ \theta(\eta)$.

Two interesting points are brought out in the coefficients A1 and B1 which do not appear in the constant property solution. These points are

1. The magnitude of ΔT can have a strong influence on the natural convection problem, and
2. The terms in the brackets are additive in the supercritical region and subtractive in the perfect gas region.

An examination of the property diagrams shown in Figures (2), (4) and (5), Chapter IV, will show the additive effect of the terms of the coefficients A1 and B1 in the supercritical region. This means that for the constant property solutions to be accurate the coefficients A1 and B1 must be small.

Heat Transfer at the Wall

The Fourier Law of Heat Transfer is used to compute the heat transfer from the wall to the fluid and is expressed as follows:

$$q = -k_w \left(\frac{\partial T}{\partial y}\right)_{y=0} . \tag{40}$$

When the equation is expressed in terms of the similarity variables it has the form

$$q = -k_w (T_w - T_\infty) Cx^{-1/4} \theta'(0) . \tag{41}$$

The coefficient of heat transfer is defined by the expression

$$h = - q/(T_\infty - T_w) . \tag{42}$$

In laminar natural-convection on a vertical plate this is usually presented in the form of the Nusselt Number, the Grashof Number and $\theta'(0)$ as follows:

$$\frac{Nu_{x,w}}{[Gr_{x,w}/4]^{1/4}} = -\theta'(0) \tag{43}$$

where

$$Gr_{x,w} = \left[g\ x^3\ \frac{(\rho_\infty - \rho_w)/\rho_w}{\nu_w^{\ 2}}\right] \tag{44}$$

$$Nu_{x,w} = \frac{h_x x}{k_w} . \tag{45}$$

Shearing Stress at the Wall

Stokes Law of Viscosity is used to compute the shearing stress at the wall. Considering the x-direction only, it may be expressed as follows:

$$\tau_{yx,w} = \mu_w \left(\frac{\partial u}{\partial y}\right)_{y=0} = f(x) \quad . \tag{46}$$

The equation may be expressed in terms of the similarity variables in the form:

$$\tau_{yx,w} = 4 \frac{\mu_w}{g_c} C^3 x^{1/4} \nu_w F''(0) \quad . \tag{47}$$

Before computations can be performed regarding the heat transfer or shearing stress it is, of course, necessary to solve the reduced differential equations for $F''(0)$ and $\theta'(0)$. Before this can be carried out thermodynamic and transport properties must be determined for all values of temperature over the range specified by the physical problem. It will be the purpose of Chapter IV to present the procedure for their determination.

CHAPTER IV

THE THERMODYNAMIC AND TRANSPORT PROPERTIES

Introduction

Before the solutions to the reduced form of the momentum and energy equations can be obtained it was necessary to determine the following thermodynamic and transport properties over the temperature range of interest.

	Thermodynamic Properties	Symbols
1.	Density	ρ
2.	Coefficient of volume expansion	$1/\rho(\partial\rho/\partial T)_p$
3.	Specific heat at constant pressure	C_p

	Transport Properties	Symbols
1.	Viscosity	μ
2.	The Term	$1/\mu(\partial\mu/\partial T)_p$
3.	Thermal Conductivity	k
4.	The Term	$1/k(\partial k/\partial T)_p$

It will be the purpose of this chapter to describe the derivations of the equations and the procedures used to determine the value of the above properties. The example fluid chosen is symmetrical dichlortetrafluoroethane ($C\,C\ell\,F_2 - C\,C\ell\,F_2$) which is also known as Freon 114. The fluid state was assumed to be supercritical.

It will be noted that the units used in the various equations of this chapter are inconsistant in the sense that different units were used to express the same property. This is due to the different choices made by different investigators. Since the coefficients of the reduced differential equations are dimensionless property groups, it was decided to develop the equations in the same system of units as the work reported in the literature and make the necessary unit changes in the computer program.

Density

Martin [8] developed an equation of state for Freon 114. This equation, which is valid in the supercritical region, was used to determine the density as a function of temperature at constant pressure. The equation of state is as follows:

Equation of State

$$p = \frac{RT}{(v-b)} + \frac{A_2 + B_2T + C_2 \operatorname{Exp}(-kTr)}{(v-b)^2} + \frac{A_3 + B_3T + C_3 \operatorname{Exp}(-kTr)}{(v-b)^3} + \frac{A_4}{(v-b)^4} + \frac{A_5 + B_5T + C_5 \operatorname{Exp}(-kTr)}{(v-b)^5} . \tag{1}$$

In using this equation of state the following units and constants must be used:

<u>Description</u>

p = pressure = psia

T = temperature = ^{o}R

v = specific volume = CuFt/lb

R = gas constant = 10.7351/170.936 CuFt/^{o}R SqIn

T_r = reduced temperature

$k = 3.0$

$b = 5.9149070 \times 10^{-3}$

$A_2 = -2.3856704$

$B_2 = 1.0801207 \times 10^{-3}$

$C_2 = -6.5643648$

$A_3 = 3.4055687 \times 10^{-2}$

$B_3 = -5.3336494 \times 10^{-6}$

$C_3 = 1.6366057 \times 10^{-1}$

$A_4 = -3.8574810 \times 10^{-4}$

$A_5 = 1.6017659 \times 10^{-6}$

$B_5 = 6.2632340 \times 10^{-10}$

$C_5 = -1.0165314 \times 10^{-5}$

To obtain value of $\rho = \rho(T)$ it was necessary to use numerical techniques due to the fact that the equation of state is explicit only in pressure. The procedure is as follows:

1. Define the following term:

$$Z = (v - b) \tag{2}$$

$$a_0 = p \qquad (3)$$

$$a_1 = RT \qquad (4)$$

$$a_2 = A_2 + B_2T + C_2 \text{ Exp}(-kTr) \qquad (5)$$

$$a_3 = A_3 + B_3T + C_3 \text{ Exp}(-kTr) \qquad (6)$$

$$a_4 = A_4 \qquad (7)$$

$$a_5 = A_5 + B_5T + C_5 \text{ Exp}(-kTr) \qquad (8)$$

2. The equation of state was rewritten as follows:

$$f(Z) = a_0Z^5 - a_1Z^4 - a_2Z^3 - a_3Z^2 - a_4Z - a_5 = 0 \qquad (9)$$

3. The first derivative of the 5^{th} degree polynomial is:

$$f'(Z) = 5a_0Z^4 - 4a_1Z^3 - 3a_2Z^2$$

$$- 2a_3Z - a_4 \qquad (10)$$

4. The problem was then reduced to finding the proper root of the polynomial once the pressure and temperature were specified. The Newton-Raphson iteration method was used to obtain the roots. This equation has the form:

$$Z_{n+1} = Z_n - \frac{f(Z)}{f'(Z)} \qquad (11)$$

5. After the proper root was obtained, the density was computed from the following expression:

$$\rho = \frac{1}{(Z + b)} \qquad (12)$$

The entire procedure was programed for the IBM 7040 computer and the computed values are shown in Figure 2 for a pressure of 540.00 psia over the temperature range from

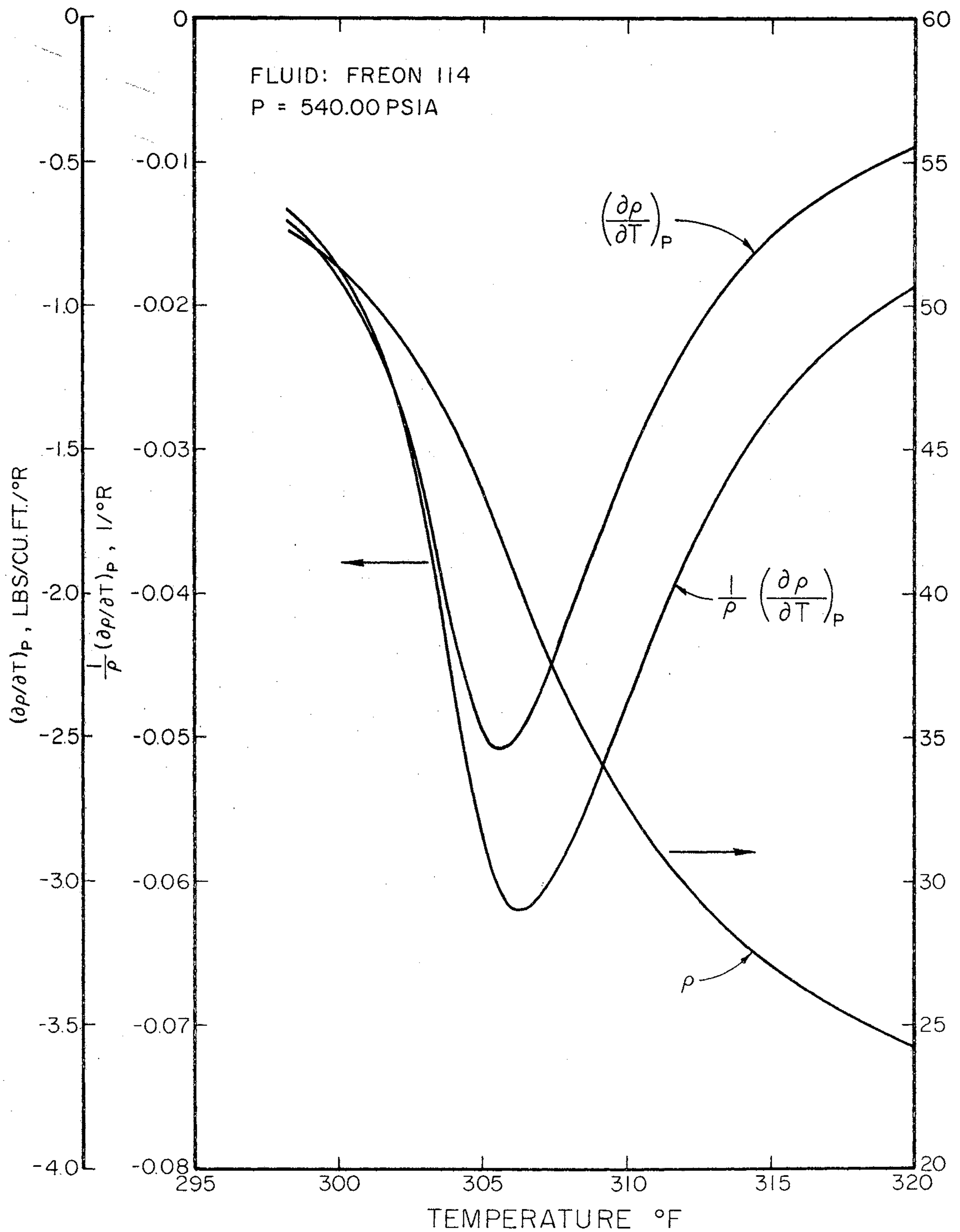

Figure 2. Thermodynamic Properties in the Supercritical Region

$300^\circ F$ to $320^\circ F$.

Coefficient of Volume Expansion

An expression for the coefficient of volume expansion was obtained as follows:

1. Since the equation of state was implicit in volume and temperature it was helpful to use the techniques of implicit differentiation by starting with the expression:

$$\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial P}\right)_v \left(\frac{\partial P}{\partial v}\right)_T = -1 \qquad (13)$$

2. Since $v = 1/\rho$, Equation (13) was rearranged to obtain an expression for the coefficient of volume expansion in terms of the derivatives of the equation of state. The advantage of this rearrangement was that the dependent variable of the derivatives was pressure. The resulting equation was

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p = -\rho\,\frac{(\partial P/\partial T)_v}{(\partial P/\partial v)_T} \qquad (14)$$

where

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v-b} \qquad (15)$$

$$+ \frac{B_2 - (k/T_c)\ C_2\ \mathrm{Exp}(-\,kTr)}{(v-b)^2}$$

$$+ \frac{B_3 - (k/T_c)\ C_3\ \mathrm{Exp}(-\,kTr)}{(v-b)^3}$$

$$+ \frac{B_5 - (k/T_c)\ C_5\ \mathrm{Exp}(-\,kTr)}{(v-b)^5}$$

$$\left(\frac{\partial P}{\partial v}\right)_T = - \frac{RT}{(v - b)^2} \tag{16}$$

$$- \frac{2[A_2 + B_2T + C_2 \operatorname{Exp}(- kTr)]}{(v - b)^3}$$

$$- \frac{3[A_3 + B_3T + C_3 \operatorname{Exp}(- kTr)]}{(v - b)^4}$$

$$- \frac{4A_5}{(v - b)^5}$$

$$- \frac{5[A_5 + B_5T + C_5 \operatorname{Exp}(- kTr)}{(v - b)^6} .$$

These equations were programed for the IBM 7040 computer and the results are shown in Figure 2 for a pressure range of 540.00 psia over the temperature range from 300.00°F to 320.00°F.

Specific Heat at Constant Pressure

Along with an equation of state for Freon 114, Martin [8] also developed an expression for the specific heat at constant volume and zero pressure. With this information and the thermodynamic potential functions, it was possible to derive an equation for the specific heat at constant pressure. An examination of the thermodynamic relation

$$C_p - C_v = T\left(\frac{\partial P}{\partial T}\right)_v\left(\frac{\partial v}{\partial T}\right)_p \tag{17}$$

shows that expressions for the right hand side may be obtained from the equation of state. The value of the specific heat at constant volume and zero pressure can be obtained from the expression, where C_v^o=BTU/Lb°F and T=°R:

$$C_v^o = 0.0175 + 3.49 \times 10^{-4}\ T - 1.67 \times 10^{-7}\ T^2. \tag{18}$$

The problem was then reduced to obtaining an expression for the effects of pressure on the value of the specific heat at constant volume. This was done as follows:

1. Start with a functional expression for the specific heat at constant volume and form its differential:

$$C_v = C_v(T,\ v) \tag{19}$$

$$dC_v = \left(\frac{\partial C_v}{\partial T}\right)_v dT + \left(\frac{\partial C_v}{\partial v}\right)_T dv\ . \tag{20}$$

2. From the definition of C_v and the thermodynamic potential function $(\partial u/\partial s)_v = T$, an expression for the second coefficient of the differential equation was obtained as follows:

$$C_v = \left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{\partial u}{\partial s}\right)_v \left(\frac{\partial s}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v\ . \tag{21}$$

The derivative was taken of the expression with respect to the volume holding the temperature constant which resulted in

$$\left(\frac{\partial C_v}{\partial v}\right)_T = T\ \frac{\partial^2 s}{\partial v_T \partial T_v}\ . \tag{22}$$

3. The derivative of the Maxwell Equation (23) with respect to volume holding the temperature constant

was obtained as follows:

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial s}{\partial v}\right)_T \tag{23}$$

$$\left(\frac{\partial^2 P}{\partial T^2}\right)_V = \frac{\partial^2 s}{\partial v_T \partial T_V} \ . \tag{24}$$

4. Equating Equations (22) and (24) gave

$$\left(\frac{\partial C_V}{\partial v}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V \ . \tag{25}$$

5. The substitution of step (4) into step (1) and assuming the temperature constant gave:

$$dC_V = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V dv \ . \tag{26}$$

6. The second derivative of the equation of state was obtained and substituted into Equation (26). The result of this operation was

$$\int_{C_V^o}^{C_V} dC_V = \int_{V_{o=\infty}}^{V} T\left(\frac{k}{T_c}\right)^2 \mathrm{Exp}(-\,kTr)\left[\frac{C_2}{(v-b)^2} + \frac{C_3}{(v-b)^3} + \frac{C_5}{(v-b)^5}\right] dv_T \ . \tag{27}$$

7. The integration of Equation (27) gave

$$C_V = C_V^o - T\left(\frac{k}{T_c}\right)^2 \mathrm{Exp}(-\,kTr)\left[\frac{C_2}{(v-b)} + \frac{C_3}{2(v-b)^2} + \frac{C_5}{4(v-b)^4}\right] \tag{28}$$

which was used to calculate the effects of pressure change on the specific heat at constant volume for a given temperature.

To summarize, the procedure used to compute the specific heat at constant pressure, the following set of equations were used:

$$C_v^o = 0.0175 + 3.49 \times 10^{-4}\, T - 1.67 \times 10^{-7}\, T^2 \qquad (29)$$

$$C_v = C_v^o - T\left(\frac{k}{T_c}\right)^2 \mathrm{Exp}(-\,kTr)\left[\frac{C_2}{(v-b)} + \frac{C_3}{2(v-b)^2} + \frac{C_5}{4(v-b)^4}\right] \qquad (30)$$

$$C_p = C_v + \left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial T}\right)_p \quad . \qquad (31)$$

These equations were programed for the IBM 7040 computer and the computed results are shown in Figure 3 for a pressure range of 540.00 psia over the temperature range from 300.00°F to 320.00°F.

Viscosity

Experimental data of viscosity for Freon-114 in the supercritical region is not available; however, it is available [1] at low pressure and for a temperature range from 50°F to 300°F. This fact made it necessary to develop a Sutherland-type viscosity equation [12] for the low pressure and temperature range available and then employ the residual viscosity equation to correct for the pressure. The residual viscosity equations, which were presented by Jossi, Stiel, and Thodos [6], were developed especially for the supercritical region. The procedure used to obtain the necessary equations for viscosity in the supercritical

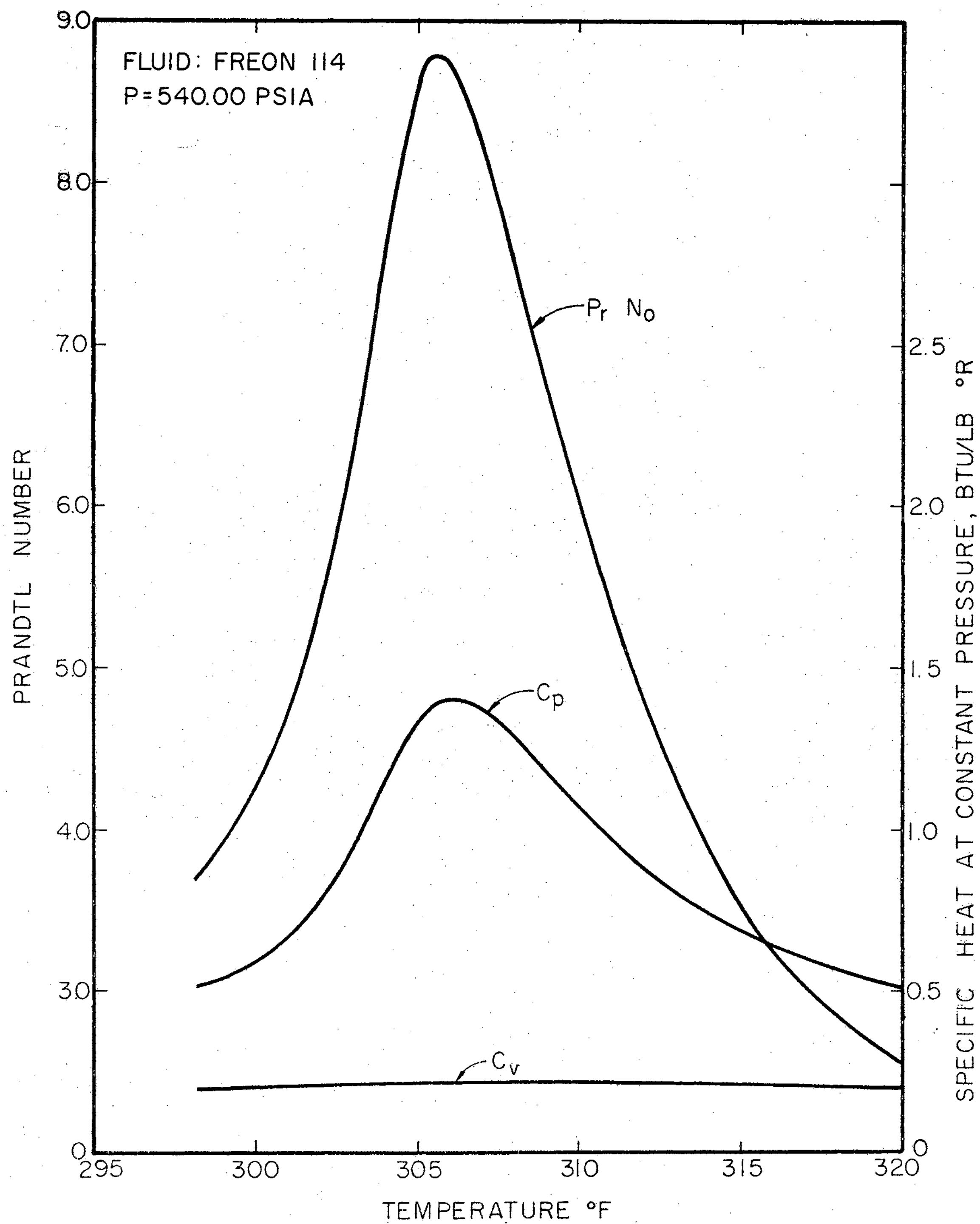

Figure 3. Thermodynamic Properties in the Supercritical Region

region was as follows:

1. The experimental data [1] used to determine the constants of the Sutherland-type viscosity equation for a pressure of one atmosphere was:

$$T_1 = 100^{\circ}F = 559.69^{\circ}R$$
$$T_2 = 300^{\circ}F = 759.69^{\circ}R$$
$$\mu_1^{\circ} = 0.01173 \text{ Centipoises}$$
$$\mu_2^{\circ} = 0.0158 \text{ Centipoises} .$$

2. The resulting Sutherland=type equation for the viscosity at low pressure was

$$\mu^{\circ} = 1.0167 \times 10^{-3} \frac{T^{3/2}}{T + 586.55} \qquad (32)$$

where

μ° = low pressure viscosity = Centipoises

T = temperature = $^{\circ}R$.

3. The residual viscosity, for $0.3 \leq \rho_r \leq 2.0$, was

$$(\mu - \mu^{\circ})\mathcal{K} = [23.12 \text{ Exp}(1.079\rho_r) - 25.0] \times 10^{-5} \qquad (33)$$

where

$$\mathcal{K} = \frac{T_c^{1/6}}{M^{1/2} P_c^{2/3}}$$

ρ_r = reduced density = ρ/ρ_c

μ° = low pressure viscosity = centipoises

μ = viscosity = centipoises

ρ = density = gm/cc

T_c = critical temperature = $418.88^{\circ}K$

M = molecular weight = 170.936

P_c = critical pressure = 32.19 atm .

This combination of equations was programed for the IBM 7040 computer and the results of such computations are shown in Figure 4.

The Term $1/\mu\ (\partial\mu/\partial T)_p$

An expression for the term $1/\mu\ (\partial\mu/\partial T)_p$ was obtained by substituting the Sutherland-type equation into the residual viscosity equation, taking the derivative with respect to temperature while holding the pressure constant and dividing by the viscosity. The equation of state was also employed. To facilitate this operation the following equation was used:

$$\mu = \mu(\rho, T) \tag{34}$$

$$\frac{1}{\mu}\left(\frac{\partial\mu}{\partial T}\right)_p = \frac{1}{\mu}\left[\left(\frac{\partial\mu}{\partial\rho}\right)_T\left(\frac{\partial\rho}{\partial T}\right)_p + \left(\frac{\partial\mu}{\partial T}\right)_\rho\right] . \tag{35}$$

Performing the operation indicated by Equation (35) on the Sutherland viscosity equation, the equation of state $p=f(T,v)$, and the residual viscosity equation resulted in the following equations:

$$\left(\frac{\partial\mu}{\partial\rho}\right)_T = \frac{1.079}{\rho_c}\left[(\mu - \mu^o) + \frac{2.5 \times 10^{-4}}{\mathcal{K}}\right] \tag{36}$$

$$\left(\frac{\partial\rho}{\partial T}\right)_p = \text{(See Equation (14))} \tag{37}$$

$$\left(\frac{\partial\mu}{\partial T}\right)_\rho = \frac{3}{2}\frac{\mu^o}{T} - \frac{\mu^o}{T + 586.55} . \tag{38}$$

The equations were programed for the IBM 7040 computer and the computed results are shown in Figure 4 for a pressure of 540.00 psia over a temperature range from 300.00°F

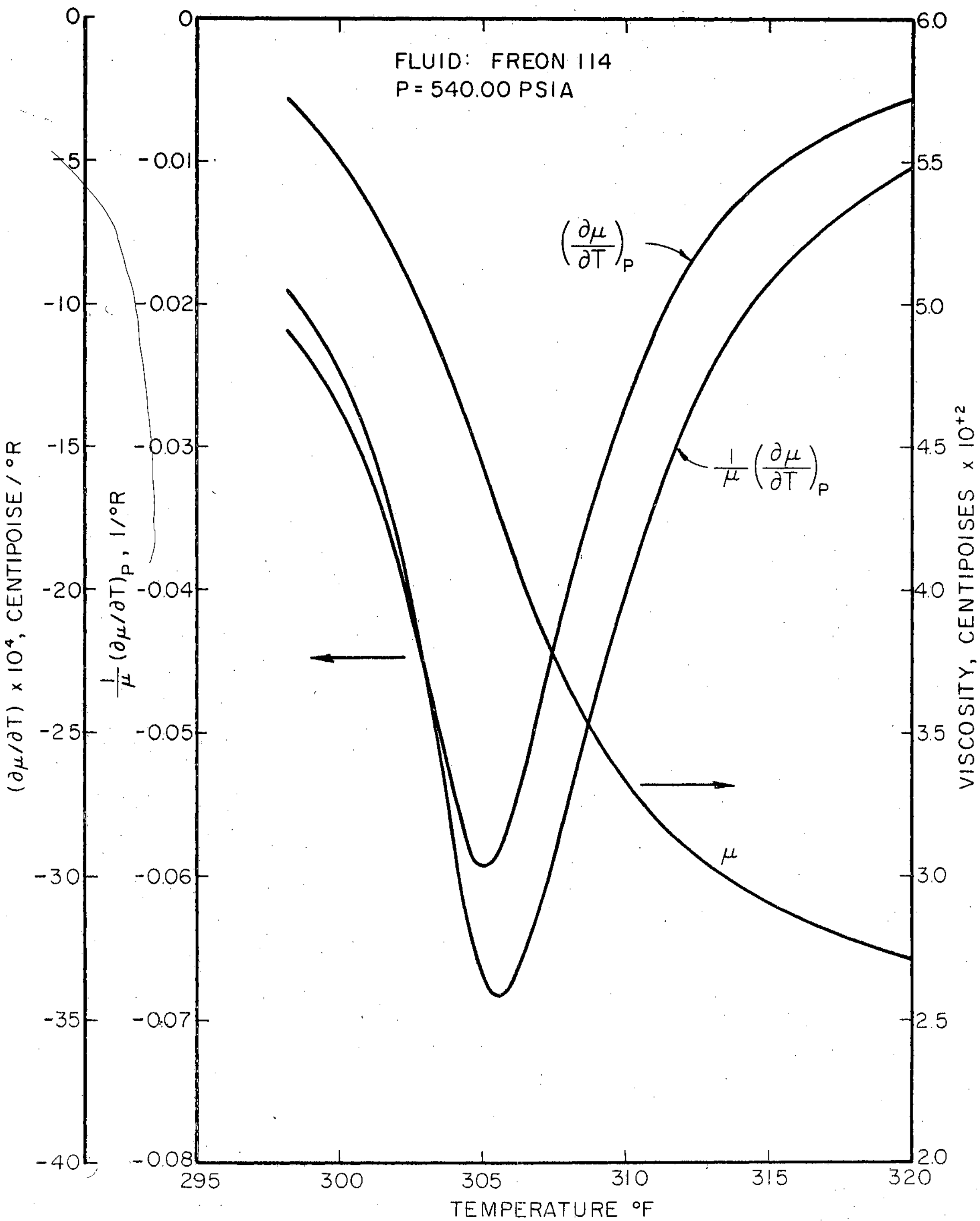

Figure 4. Transport Properties in the Supercritical Region

to 320.00°F.

Thermal Conductivity

Downing [1] reported that the Bromley equation was valid for determining the thermal conductivity of Freon 114 at low pressure. The Bromley equation may be expressed as follows:

$$\frac{Mk^o}{\mu^o} = 0.0132\ C_v^{\ o} + 0.034 - \frac{0.007}{T_r} \tag{39}$$

where

$$\mu^o = 1.0167 \times 10^{-3} \frac{T^{3/2}}{T + 586.55}$$

T = temperature = °R

μ^o = low pressure viscosity = centipoises

$C_v^{\ o}$ = low pressure specific heat at constant volume = Cal/gm mole °C

k^o = low pressure thermal conductivity = Cal/sec cm °K

Tr = reduced temperature = T/T_c

T_c = critical temperature

M = molecular weight .

Stiel and Thodos [16] developed a residual thermal conductivity equation for nonpolar gases which is applicable in the supercritical region. The equation which is valid for $0.5 \leq \rho_r \leq 2.0$, was

$$(k - k^o)\ \lambda Zc^5 = 13.10 \times 10^{-8}\ [\mathrm{Exp}(0.67\ \rho_r) - 1.069] \tag{40}$$

where

$$\lambda = \frac{M^{1/2}\ T_c^{\ 1/6}}{P_c^{\ 2/3}}$$

$$Z_c = \frac{P_c v_c}{RT_c}$$

$$\rho_r = \rho/\rho_c$$

M = molecular weight; v_c = cc./gm mole

T_c = critical temperature = $^{\circ}$K

P_c = critical pressure = atm

R = perfect gas constant

ρ_r = reduced density; R = 1.987 Cal/gm mole $^{\circ}$K

k^{o} = low pressure thermal conductivity

= Cal/sec cm $^{\circ}$K

k = thermal conductivity = Cal/sec cm $^{\circ}$K.

This combination of equations was also programed for the IBM 7040 computer and the calculated results are shown in Figure 5.

The Term $1/k\ (\partial k/\partial T)_p$

An expression for the term $1/k\ (\partial k/\partial T)_p$ was obtained in a manner similar to that used to obtain $1/\mu\ (\partial\mu/\partial T)_p$; that is, by substituting the Bromley equation into the residual thermal conductivity equation, taking the derivative of the thermal conductivity with respect to temperature while holding the pressure constant and then dividing by the thermal conductivity. To facilitate this operation the following equation was used:

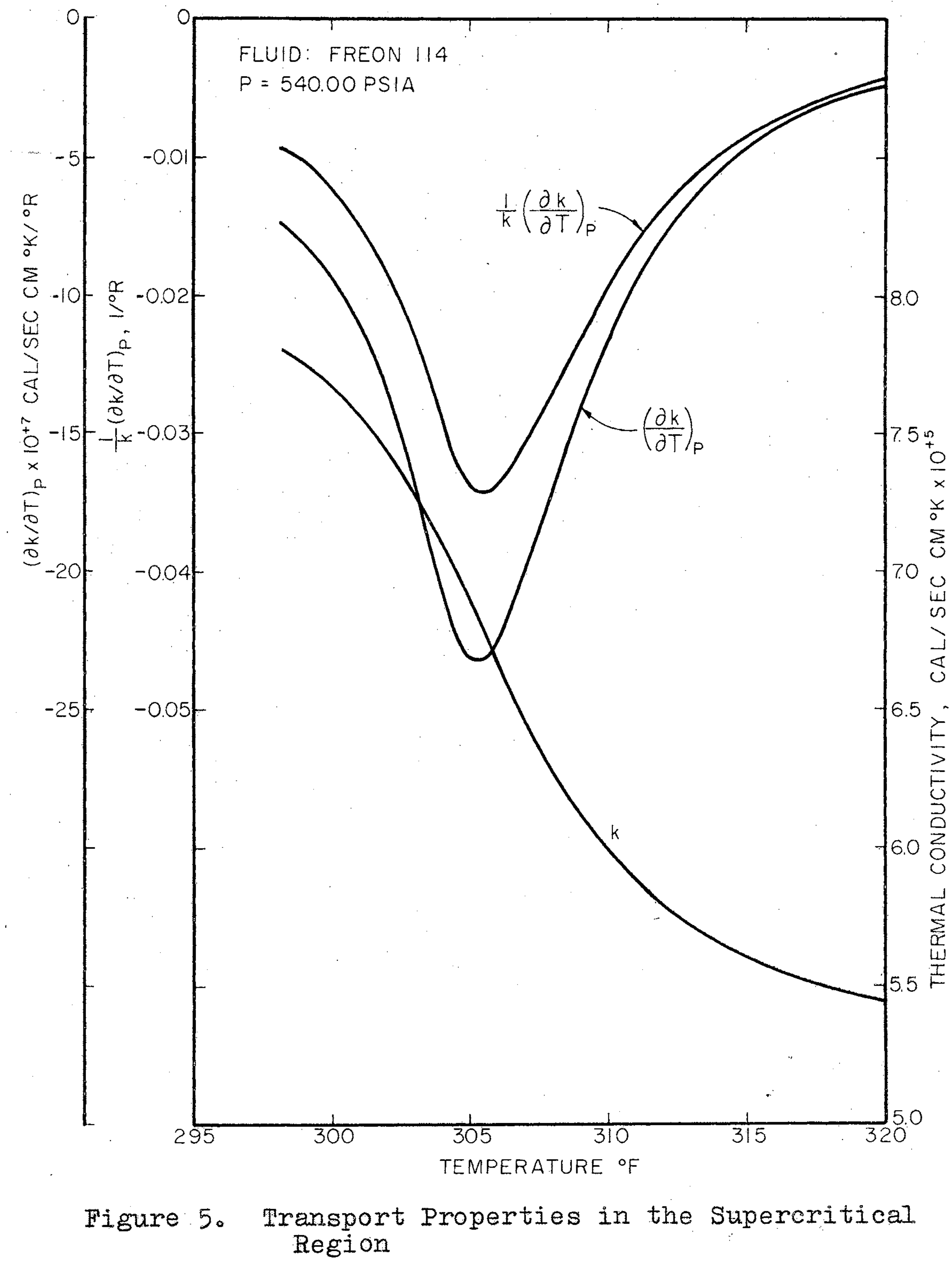

Figure 5. Transport Properties in the Supercritical Region

$$k = k(\rho, T) \tag{41}$$

$$\frac{1}{k}\left(\frac{\partial k}{\partial T}\right)_p = \frac{1}{k}\left[\left(\frac{\partial k}{\partial \rho}\right)_T\left(\frac{\partial \rho}{\partial T}\right)_p + \left(\frac{\partial k}{\partial T}\right)_\rho\right] \; . \tag{42}$$

Performing the operation indicated by Equation (42) on the Bromley equation, the equation of state and the residual thermal conductivity equation resulted in the following equations:

$$\left(\frac{\partial k}{\partial \rho}\right)_T = \frac{0.67}{\rho_c}\left[(k - k^o) + \frac{14.003 \times 10^{-8}}{\lambda Z_c^5}\right] \tag{43}$$

$$\left(\frac{\partial \rho}{\partial T}\right)_p = \text{(See Equation (14))} \tag{44}$$

$$\begin{aligned}\left(\frac{\partial k}{\partial T}\right)_\rho &= 0.0132\, C_v^{\,o}\left(\frac{\partial \mu^o}{\partial T}\right)_\rho + 0.0132\, \mu^o\left(\frac{\partial C_v^{\,o}}{\partial T}\right)_\rho \\ &+ \frac{0.034}{M}\left(\frac{\partial \mu^o}{\partial T}\right)_\rho - \frac{0.007}{MTr}\left(\frac{\partial \mu^o}{\partial T}\right)_\rho \\ &+ \frac{0.007}{MTr}\,\frac{\mu^o}{T}\end{aligned} \tag{45}$$

where

$$\left(\frac{\partial \mu^o}{\partial T}\right)_\rho = \frac{3}{2}\,\frac{\mu^o}{T} - \frac{\mu^o}{T + 586.55} \tag{46}$$

$$\left(\frac{\partial C_v^{\,o}}{\partial T}\right)_\rho = 3.49 \times 10^{-4} - 3.34 \times 10^{-7}\, T \; . \tag{47}$$

Even though it was stated after each section of this chapter that the expressions were programed for IBM 7040 computer, it was not intended to imply that each section was programed separately. The computer program is located in Appendix A.

The computed results agreed with all the data the author could find on densities and specific heats at constant pressure [17].

CHAPTER V

THE NUMERICAL SOLUTION

Introduction

The introduction of the stream functions and similarity variables into the continuity, momentum and energy equations for natural-convection along a vertical flat plate resulted in a set of reduced ordinary nonlinear differential equations with variable coefficients. Except for a few types, closed solutions of nonlinear differential equations are unknown at the present time and one must use either the analog computer or the techniques of numerical analysis and the digital computer. It will be the purpose of this chapter to explain how the set of reduced ordinary nonlinear differential equations were solved. The solutions for five different fluid states are also presented. The fluid chosen was Freon 114 and was assumed to be in a supercritical state where there are strong variations in the fluid properties.

The method of Runga-Kutta was used to solve the set of reduced ordinary nonlinear differential equations. To employ the method of Runga-Kutta it was necessary to know the starting boundary conditions. In this situation only three of the necessary five starting boundary conditions are known, namely:

$$F(0) = 0$$

$$F'(0) = 0$$

$$F''(0) = \text{unknown}$$

$$\theta(0) = 1$$

$$\theta'(0) = \text{unknown}$$

To overcome this difficulty the iterative method of Newton-Raphson [9], [14] was used. An explanation of the iterative method of Newton-Raphson is given in Appendix E. This method required the linearization of the reduced ordinary nonlinear differential equations and a starting solution. The starting solution used was the constant property solution for Pr = 2. The linearization of a set of differential equations requires a knowledge of all dependent variables. In the case of the set of reduced ordinary differential equations, the dimensionless property groups are functions of $\theta(\eta)$ and the functions must be obtained before a solution is possible. To obtain these functions, the equations for the physical properties C_p, μ, k and ρ must be known as functions of pressure and temperature.

If the equations are explicit in C_p, μ, k and ρ, the expression for the dimensionless coefficients as functions of $\theta(\eta)$ are readily obtained by performing the mathematical operations indicated by the dimensionless property groups. These operations also yield another set of dimensionless property groups which may be used for correlation purposes for that particular fluid system. If the equations are not explicit in C_p, μ, k and ρ, the obtaining of the dimensionless property groups as functions of $\theta(\eta)$ is more difficult. In this work the equation of state $p = f(T, v)$ was explicit in pressure.

The expressions for the dimensionless groups as functions of $\theta(\eta)$ were obtained by computing the numerical values of the dimensionless groups over the temperature range of interest at constant pressure, then this information was fitted to polynomials. This operation gave the dimensionless property groups as functions of temperature. Replacing the temperature in these expressions with the dimensionless temperature gave the expressions for the dimensionless property groups as functions of $\theta(\eta)$. The substitution of these expressions into the reduced ordinary nonlinear differential equations made the linearization of these equations possible.

As stated previously, five solutions of the reduced differential equations were solved and presented in this chapter. The fluid states of these systems and the computed results for the heat transfer coefficients and shearing stresses at the wall are shown below:

Freon : 114

$P = 540.00$ psia

$T_w > T_\infty$

$\Delta T = 5^{\circ}F$

Case	$T_w(^oF)$	$hx^{1/4}$[BTU/Hr Ft^2 $^oF/Ft^{1/4}$]
1	320.00	12.90
2	315.00	18.34
3	310.00	28.16
4	305.00	23.56
5	305.00	23.56

$\tau_{xy,w}\ x^{-1/4}(Lbs_f/Ft^2\ Ft^{1/4})$	η
0.0014	10.0
0.0022	10.0
0.0043	10.0
0.0044	10.0
0.0044	8.0

The above information shows that the coefficient of heat transfer increases as the fluid state at the wall approaches the region of maximum C_p variation and then decreases.

It will be noted that cases (4) and (5) are identical with the exception that in case (4) the numerical solution was carried out to an $\eta=10$ where as case (5) the solution was carried out to $\eta=8$. The purpose of this operation was to establish a practical value for $\eta=\infty$.

The Linearization of the Reduced Differential Equations

The differential equations to be linearized are as follows:

The Differential Equations

$$F''' = -(A1)\theta' F'' - 3(A2)FF'' + 2(A2)[F']^2 - (A3) \tag{18}$$

$$\theta'' = -(B1)[\theta']^2 - 3(B2)F\theta' \quad . \tag{19}$$

Boundary Conditions

$$\begin{array}{ll} F(0) = 0 & F(\infty) = u_1{}^3 \\ F'(0) = 0 & F'(\infty) = 0 \\ F''(0) = u_3{}^1 & F''(\infty) = u_2{}^4 \\ \theta(0) = 1.0 & \theta(\infty) = 0 \\ \theta'(0) = u_5{}^2 & \theta'(\infty) = u_5{}^5 \end{array} \tag{20}$$

Coefficients

$$A1 = \left[\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p + \frac{1}{\mu}\left(\frac{\partial\mu}{\partial T}\right)_p\right]\Delta T \tag{21}$$

$$A2 = \frac{\rho_w \mu_w}{\rho\mu} \tag{22}$$

$$A3 = (A2)\left[\frac{\rho_\infty - \rho}{\rho_\infty - \rho_w}\right]\frac{\rho_w}{\rho} \tag{23}$$

$$B1 = \left[\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p + \frac{1}{k}\left(\frac{\partial k}{\partial T}\right)_p\right]\Delta T \tag{24}$$

$$B2 = (A2)\frac{C_p\mu}{k} \quad . \tag{25}$$

At this point it is necessary to make a decision as to what method should be used to evaluate the coefficients once $\theta(\eta)$ is determined for a given η. It appears that two possible methods are available. They are:

1. Write the numerical program in such a way that for a given η it permits the method of Runga-Kutta to evaluate $\theta(\eta)$ at an η which in turn allows the evaluation of the temperature. Then go to a subroutine and evaluate the property coefficients using the equations developed in Chapter IV. This would be the best method since it would guarantee the exact value of the coefficients at the boundary conditions. However, it was found that the computer time required for this procedure was prohibitive. It required approximately forty-five minutes to an hour of 7040 time, depending on the integration interval chosen, to perform a single iteration.
2. The second possibility is to first compute the values of the coefficients for a given temperature range. Then, using a polynomial, curve fit the data by method of least square. It is then possible to obtain an expression for the coefficients as a function of $\theta(\eta)$. This may be expressed in mathematical terms as follows:

$$\text{Coeff.} = a + bT + cT^2 + dT^3 \qquad (26)$$

where

$$T = T_\infty + \Delta T \ \theta(\eta) \ . \qquad (27)$$

This method will introduce some error due to the imperfection of the least square fit; however, it has the advantage of converging rapidly. It is possible to obtain four iterations in twenty minutes on the 7040 computer.

In view of the above experience the second method was used and the expressions for the coefficients as functions of $\theta(\eta)$ were obtained by first assuming that the physical properties shown below may be expressed as follows:

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p + \frac{1}{\mu}\left(\frac{\partial \mu}{\partial T}\right)_p = A_1 + B_1 T + C_1 T^2 + D_1 T^3 \tag{28}$$

$$\frac{1}{\rho\mu} = A_2 + B_2 T + C_2 T^2 + D_2 T^3 \tag{29}$$

$$\frac{1}{\rho^2\mu} = A_3 + B_3 T + C_3 T^2 + D_3 T^3 \tag{30}$$

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p + \frac{1}{k}\left(\frac{\partial k}{\partial T}\right)_p = A_4 + B_4 T + C_4 T^2 + D_4 T^3 \tag{31}$$

$$\frac{C_p\mu}{\rho} \times 10^{-2} = A_5 + B_5 T + C_5 T^2 + D_5 T^3 \,. \tag{32}$$

The coefficients of the polynomial were determined by the method of least squares and plots of these equations are shown in Figures 6, 7, 8 and 9. The substitution of the expression

$$T = T_\infty + \Delta T\ \theta(\eta) \tag{33}$$

into Equations (28) through (32), and the substitution of Equations (28) through (32) into Equations (21) through (25) will give the desired equations for the coefficients as functions of $\theta(\eta)$ which are as follows:

$$A1 = CA + CB\theta + CC\theta^2 + CD\theta^3 \tag{34}$$

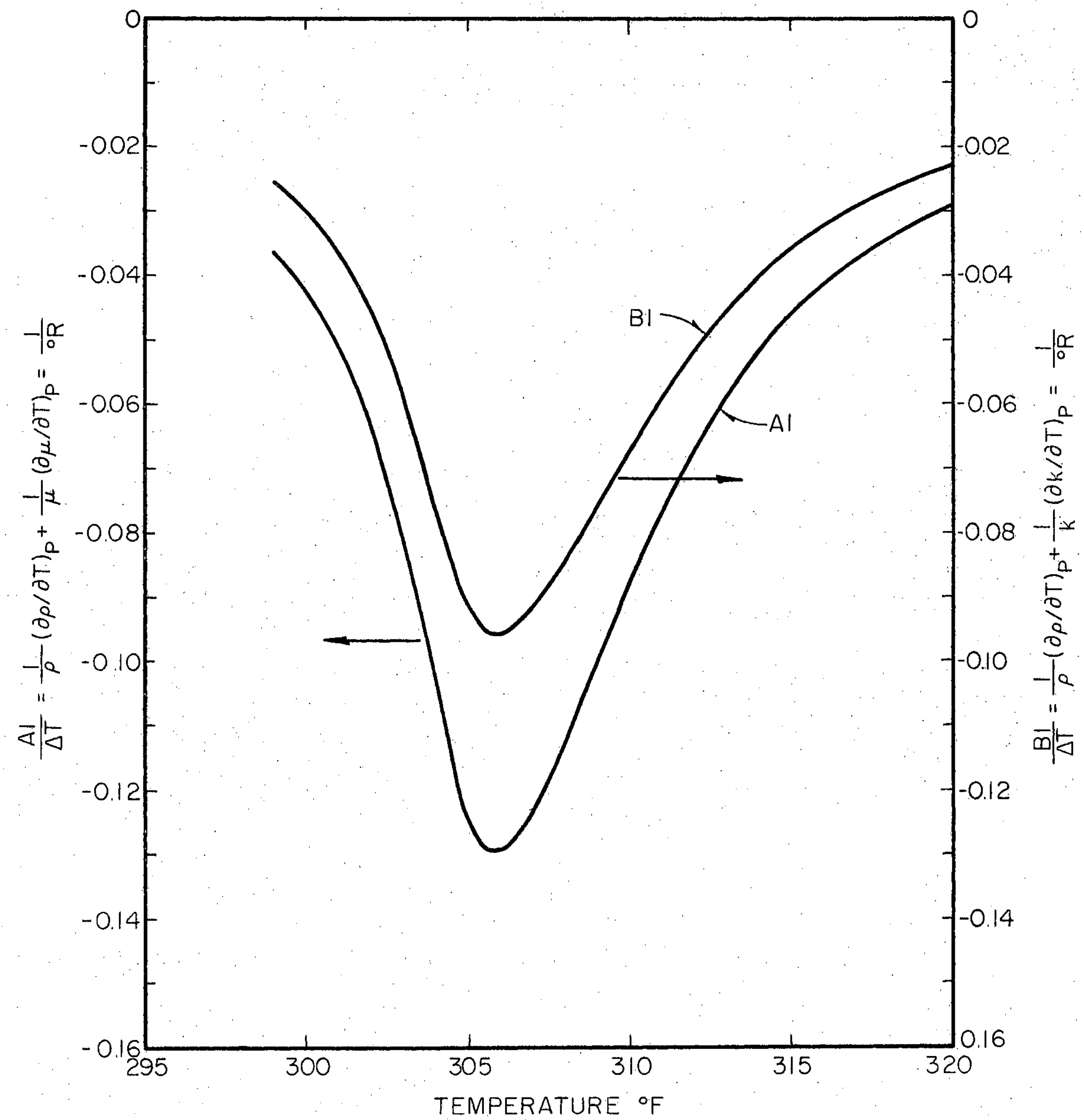

Figure 6. The Terms A1 and B1

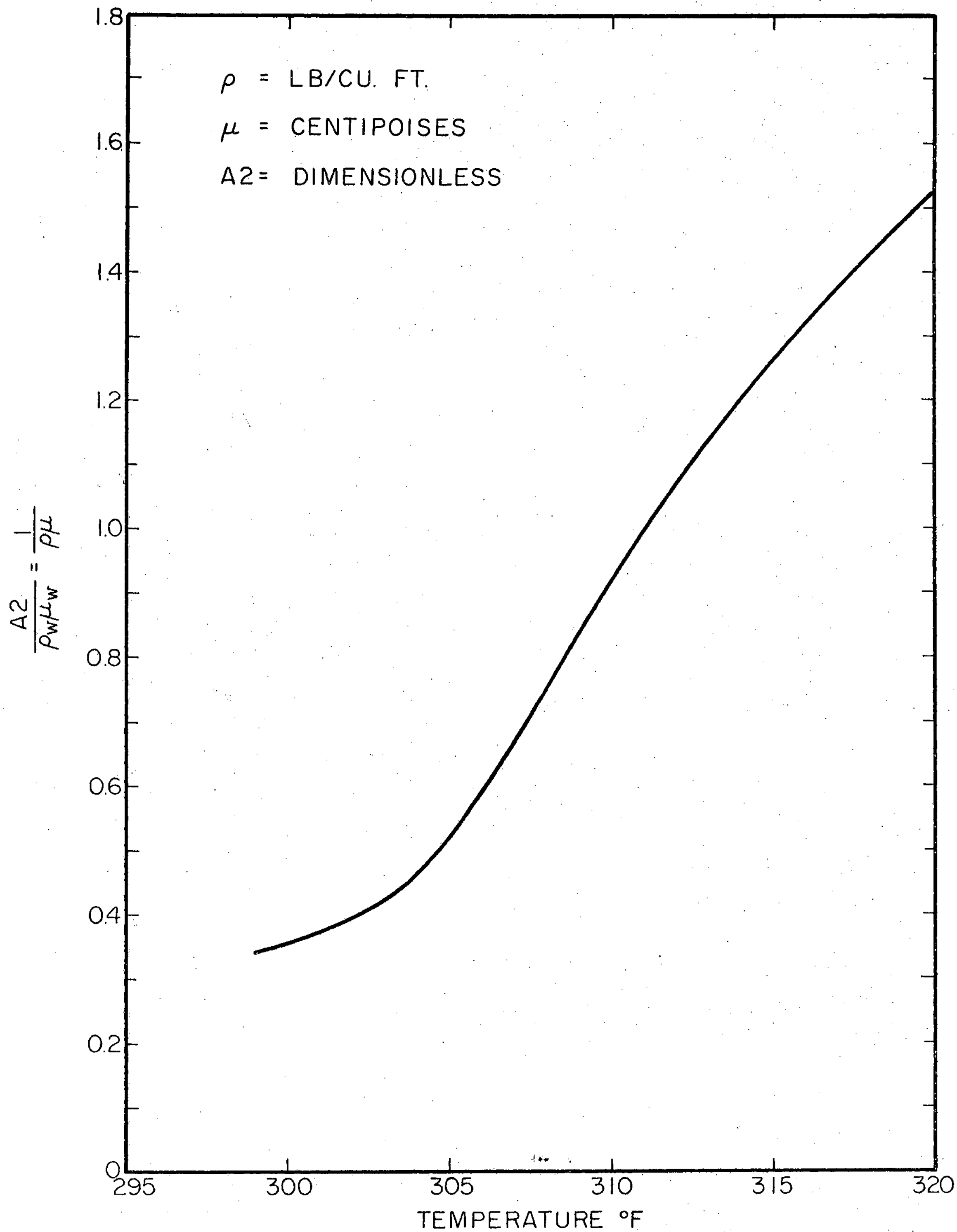

Figure 7. The Term $\frac{1}{\rho\mu}$

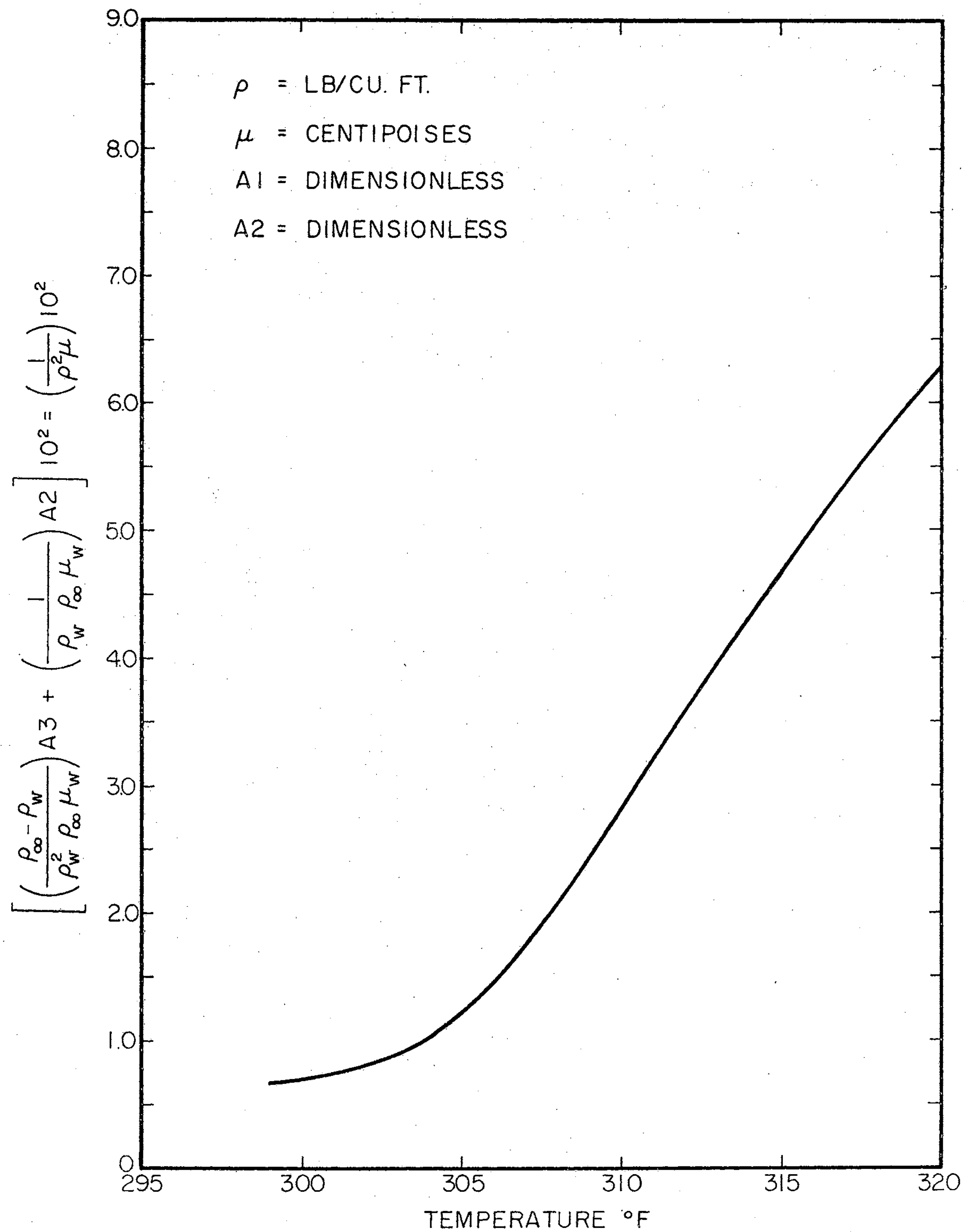

Figure 8. The Term $\frac{1}{\rho^2 \mu}$

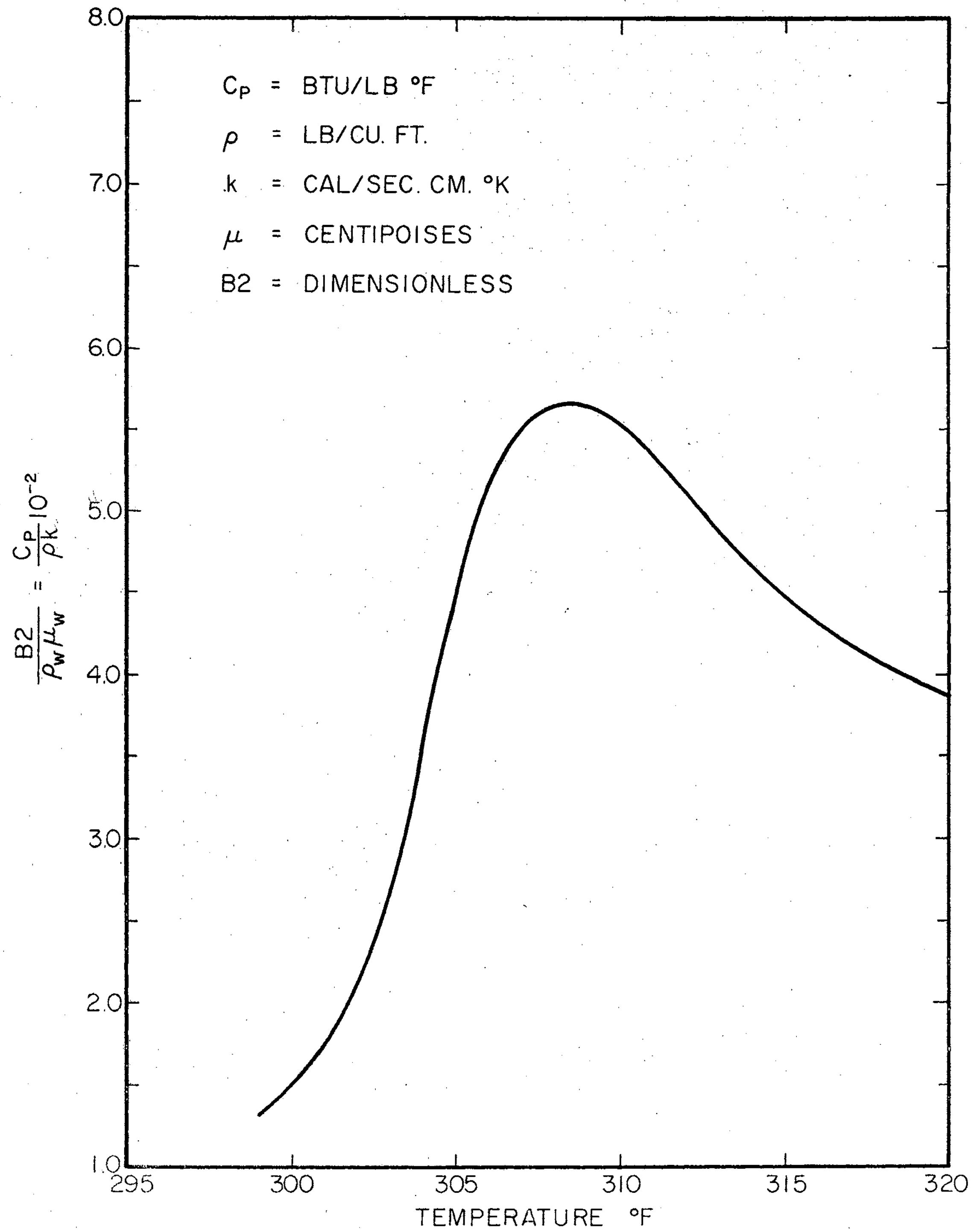

Figure 9. The Term $\frac{C_p}{\rho k}$

where

$$CA = [A_1 + B_1T_\infty + C_1T_\infty^2 + D_1T_\infty^3]\Delta T \quad (35)$$

$$CB = [B_1 + 2C_1T_\infty + 3D_1T_\infty^2](\Delta T)^2$$

$$CC = [C_1 + 3D_1T_\infty](\Delta T)^3$$

$$CD = [D_1](\Delta T)^4$$

$$3(A2) = CE + CF\theta + CG\theta^2 + CH\theta^3 \quad (36)$$

where

$$CE = 3[A_2 + B_2T_\infty + C_2T_\infty^2 + D_2T_\infty^3]\rho_w\mu_w \quad (37)$$

$$CF = 3[B_2 + 2C_2T_\infty + 3D_2T_\infty^2]\Delta T\ \rho_w\mu_w$$

$$CG = 3[C_2 + 3D_2T_\infty](\Delta T)^2\ \rho_w\mu_w$$

$$CH = 3[D_2](\Delta T)^3\ \rho_w\mu_w$$

$$2(A2) = CI + CJ\theta + CK\theta^2 + CL\theta^3 \quad (38)$$

where

$$CI = 2[A_2 + B_2T_\infty + C_2T_\infty^2 + C_2T_\infty^3]\ \rho_w\mu_w \quad (39)$$

$$CJ = 2[B_2 + 2C_2T_\infty + 3D_2T_\infty^2]\Delta T\ \rho_w\mu_w$$

$$CK = 2[C_2 + 3D_2T_\infty](\Delta T)^2\ \rho_w\mu_w$$

$$CL = 2[D_2](\Delta T)^3\ \rho_w\mu_w$$

$$A3 = [CM + CN\theta + CP\theta^2 + CR\theta^3] - [CS + CT\theta + CU\theta^2 + CW\theta^3] \quad (40)$$

where

$$CM = [A_3 + B_3T_\infty + C_3T_\infty^2 + D_3T_\infty^3] \left[\frac{\rho_w^2 \rho_\infty \mu_w}{\rho_\infty - \rho_w}\right] \quad (41)$$

$$CN = [B_3 + 2C_3T_\infty + 3D_3T_\infty^2] \left[\frac{\rho_w^2 \rho_\infty \mu_w}{\rho_\infty - \rho_w}\right] \Delta T$$

$$CP = [C_3 + 3D_3T_\infty] \left[\frac{\rho_w^2 \rho_\infty \mu_w}{\rho_\infty - \rho_w}\right] (\Delta T)^2$$

$$CR = [D_3] \left[\frac{\rho_w^2 \rho_\infty \mu_w}{\rho_\infty - \rho_w}\right] (\Delta T)^3$$

$$CS = [A_2 + B_2T_\infty + C_2T_\infty^2 + D_2T_\infty^3] \left[\frac{\rho_w^2 \mu_w}{\rho_\infty - \rho_w}\right]$$

$$CT = [B_2 + 2C_2T_\infty + 3D_3T_\infty^2] \left[\frac{\rho_w^2 \mu_w}{\rho_\infty - \rho_w}\right] \Delta T$$

$$CU = [C_2 + 3D_2T_\infty] \left[\frac{\rho_w^2 \mu_w}{\rho_\infty - \rho_w}\right] (\Delta T)^2$$

$$CW = [D_2] \left[\frac{\rho_w^2 \mu_w}{\rho_\infty - \rho_w}\right] (\Delta T)^3$$

$$B1 = CAA + CBB\theta + CCC\theta^2 + CDD\theta^3 \quad (42)$$

where

$$CAA = [A_4 + B_4T_\infty + C_4T_\infty^2 + D_4T_\infty^3]\Delta T \quad (43)$$

$$CBB = [B_4 + 2C_4T_\infty + 3D_4T_\infty^2](\Delta T)^2$$

$$CCC = [C_4 + 3D_4T_\infty](\Delta T)^3$$

$$CDD = [D_4](\Delta T)^4$$

$$3B2 = CEE + CFF\theta + CGG\theta^2 + CHH\theta^3 \quad (44)$$

where

$$CEE = 3[A_5 + B_5T_\infty + C_5T_\infty^2 + D_5T_\infty^3]\ \rho_w\mu_w \tag{45}$$

$$CFF = 3[B_5 + 2C_5T_\infty + 3D_5T_\infty^2]\Delta T\ \rho_w\mu_w$$

$$CGG = 3[C_5 + 3D_5T_\infty](\Delta T)^2\ \rho_w\mu_w$$

$$CHH = 3[D_5](\Delta T)^3\ \rho_w\mu_w \ .$$

It will be noted that the above method of determining the coefficients could be simplified if the physical properties ρ, μ, k and C_p were expressed as functions of temperature instead of those shown in Equations (28) through (32). The advantage of the method used is that no derivatives of the polynomials are necessary to evaluate the property coefficients of the reduced differential equations.

The substituting of Equations (34), (36), (38), (40), (42) and (44) into Equations (18) and (19) permit the reduced differential equations to be expressed as follows:

$$\begin{aligned} F''' = &- [CA + CB\theta + CC\theta^2 + CD\theta^3]\ \theta'F'' \\ &- [CE + CF\theta + CG\theta^2 + CH\theta^3]\ FF'' \\ &+ [CI + CJ\theta + CK\theta^2 + CL\theta^3]\ [F']^2 \\ &- [CM + CN\theta + CP\theta^2 + CR\theta^3] \\ &+ [CS + CT\theta + CU\theta^2 + CW\theta^3] \end{aligned} \tag{46}$$

$$\begin{aligned} \theta'' = &- [CAA + CBB\theta + CCC\theta^2 + CDD\theta^3][\theta']^2 \\ &- [CEE + CFF\theta + CGG\theta^2 + CHH\theta^3]\ F\ \theta' \ . \end{aligned} \tag{47}$$

The next step in this development will be to form a set of first order differential equations to represent the reduced third order momentum differential equation and the reduced second order energy differential equation. Using state variable notation one proceeds as follows:

1. Define:

$$\begin{aligned} X_1 &= F \\ X_2 &= F' \\ X_3 &= F'' \\ X_4 &= \theta \\ X_5 &= \theta' \end{aligned} \tag{48}$$

2. The set of differential equations become:

$$\begin{aligned} \mathring{X}_1 &= X_2 \\ \mathring{X}_2 &= X_3 \\ \mathring{X}_3 &= -[CA + CBX_4 + CCX_4^2 + CDX_4^3]\, X_5X_3 \\ &\quad - [CE + CFX_4 + CGX_4^2 + CHX_4^3]\, X_1X_3 \\ &\quad + [CI + CJX_4 + CKX_4^2 + CLX_4^3]\, X_2^2 \\ &\quad - [CM + CNX_4 + CPX_4^2 + CRX_4^3] \\ &\quad + [CS + CTX_4 + CUX_4^2 + CWX_4^3] \\ \mathring{X}_4 &= X_5 \\ \mathring{X}_5 &= -[CAA + CBBX_4 + CCCX_4^2 + CDDX_4^3]\, X_5^2 \\ &\quad - [CEE + CFFX_4 + CGGX_4^2 + CHHX_4^3]\, X_1X_5. \end{aligned} \tag{49}$$

At this point, it is now possible to linearize the above set of first order differential equations in the neighborhood of a specific solution. This is achieved by expanding each of the above first order differential equations into a first order truncated Taylor series. The results of this operation are the following:

<u>The Linearized Set of Differential Equations</u>

1. $\mathring{X}_{1(s+1)} = X_{2(s+1)}$ (50)

2. $\mathring{X}_{2(s+1)} = X_{3(s+1)}$ (51)

3. $\mathring{X}_{3(s+1)} = J_1X_{1(s+1)} + J_2X_{2(s+1)} + J_3X_{3(s+1)} + J_4X_{4(s+1)} + J_5X_{5(s+1)} + H_1 + H_2 + H_3 + H_4 + H_5$ (52)

where

$$J_1 = -[CE + CFX_{4s} + CGX_{4s}^2 + CHX_{4s}^3]\ X_{3s}$$

$$J_2 = 2\ [CI + CJX_{4s} + CKX_{4s}^2 + CLX_{4s}^3]\ X_{2s}$$

$$J_3 = -[CA + CBX_{4s} + CCX_{4s}^2 + CDX_{4s}^3]\ X_{5s} - [CE + CFX_{4s} + CGX_{4s}^2 + CHX_{4s}^3]\ X_{1s}$$

$$J_4 = -[CB + 2CCX_{4s} + 3CDX_{4s}^2]\ X_{3s}X_{5s} - [CF + 2CGX_{4s} + 3CHX_{4s}^2]\ X_{1s}X_{3s} - [CJ + 2CKX_{4s} + 3CLX_{4s}^2]\ X_{2s}^2 - [CN + 2CPX_{4s} + 3CRX_{4s}^2] + [CT + 2CUX_{4s} + 3CWX_{4s}^2]$$

$$J_5 = -[CA + CBX_{4s} + CCX_{4s}^2 + CDX_{4s}^3]\ X_{3s}$$

$$H_1 = [CA + 2CBX_{4s} + 3CCX_{4s}^2 + 4CDX_{4s}^3]\ X_{3s}X_{5s}$$

$$H_2 = [CE + 2CFX_{4s} + 3CGX_{4s}^2 + 4CHX_{4s}^3]\ X_{1s}X_{3s}$$

$$H_3 = -[CI + 2CJX_{4s} + 3CKX_{4s}^2 + 4CLX_{4s}^3]\ X_{2s}^2$$

$$H_4 = -[CM - CPX_{4s}^2 - 2CRX_{3s}^3]$$

$$H_5 = +[CS - CUX_{4s}^2 - 2CWX_{4s}^3]$$

4. $\mathring{X}_{4(s+1)} = X_{5(s+1)}$ (53)

5. $\mathring{X}_{5(s+1)} = J_{11}X_{1(s+1)} + J_{44}X_{4(s+1)} + J_{55}X_{5(s+1)} + H_{11}$ (54)

where

$$J_{11} = -[CEE + CFFX_{4s} + CGGX_{4s}^2 + CHHX_{4s}^3]\ x_{5s}^2$$

$$J_{44} = -[CBB + 2CCX_{4s} + 3CDDX_{4s}^2]X_{5s}^2$$
$$-[CFF + 2CGGX_{4s} + 3CHHX_{4s}^2]X_{1s}X_{5s}$$
$$J_{55} = -2[CAA + CBBX_{4s} + CCCX_{4s}^2 + CDDX_{4s}^3]X_{5s}$$
$$-[CEE + CFFX_{4s} + CGGX_{4s}^2 + CHHX_{4s}^3]X_{1s}$$
$$H_{11} = [CAA + 2CBBX_{4s} + 3CCX_{4s}^2 + 4CDDX_{4s}^3]X_{5s}^2$$
$$+[CEE + 2CFFX_{4s} + 3CGGX_{4s}^2 + 4CHHX_{4s}^3]X_{1s}X_{5s}$$

The Boundary Conditions

Wall	Main Stream	
$X_1(0) = 0$		
$X_2(0) = 0$	$X_2(\infty) = 0$	
$X_3(0) = U_3^1$		(55)
$X_4(0) = 1$	$X_4(\infty) = 0$	
$X_5(0) = U_5^2$		

Equations (50) through (54) including the boundary condition are the linearized set of first order differential equations of the reduced momentum and energy differential equations. The procedure and the various computer programs used to solve this set of equations will now be explained.

Procedure for Solving the Linearized Set of Nonlinear Differential Equations

The procedure used in solving the linearized set of differential equations consisted of four separate computer programs. The are defined as follows:

1. The Physical Property Program

2. The Least Square Program

3. The Iterative Program

4. The Heat Transfer Program

The Physical Property Program was used to compute physical property information versus temperature at constant pressure in the supercritical region. This program is a Fortran IV representation of the physical property equations developed in Chapter IV. For a specified pressure, initial temperature, final temperature and a first estimate as to the value of the specific volume, this program will print out thirty-seven pieces of information on the physical properties. It will also punch a set of data cards of the following:

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p + \frac{1}{\mu}\left(\frac{\partial \mu}{\partial T}\right)_p \text{ versus } T$$

$$\frac{1}{\rho\mu} \text{ versus } T$$

$$\frac{1}{\rho^2\mu} \text{ versus } T$$

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p + \frac{1}{k}\left(\frac{\partial k}{\partial T}\right)_p \text{ versus } T$$

$$\frac{C_p\mu}{\rho}\, 10^{-2} \text{ versus } T\ .$$

The data cards obtained from the Physical Property Program provided the necessary information to compute the coefficients for Equations (28) through (32) with the Least

Square Program. The coefficients are designated as

$$\begin{matrix} A_1, & B_1, & C_1, & D_1 \\ A_2, & B_2, & C_2, & D_2 \\ A_3, & B_3, & C_3, & D_3 \\ A_4, & B_4, & C_4, & D_4 \\ A_5, & B_5, & C_5, & D_5 \end{matrix}$$

In order to use the Iterative Program the coefficient information, which is obtained from the Least Square Program, along with the wall and main stream temperatures, which is obtained from a statement of the problem, the wall viscosity, the wall and main stream density, which is obtained from the Physical Property Program are needed. The Iterative Program serves two purposes. The first purpose is the determination of the coefficients for Equations (34), (36), (38), (40), (42) and (44). These equations represent the property coefficients of the reduced differential equations as functions of $\theta(\eta)$. These coefficients are designated as

CA ,	CB ,	CC ,	CD
CE ,	CF ,	CG ,	CH
CI ,	CJ ,	CK ,	CL
CM ,	CN ,	CP ,	CR
CS ,	CT ,	CV ,	CW
CAA,	CBB,	CCC,	CDD
CEE,	CFF,	CGG,	CHH

and are evaluated from Equations (35), (37), (39), (41), (43) and (45). The second purpose is to solve Equations (50) through (54) which are the linearized set of first order differential equations of the reduced momentum and energy equations. The numerical methods used were the method of Runga-Kutta and the iterative method of Newton-Raphson. The solutions of the differential equations are designated as

$$F(\eta) \qquad \theta(\eta)$$
$$F'(\eta) \qquad \theta'(\eta)$$
$$F''(\eta) \qquad \theta''(\eta)$$
$$F'''(\eta)$$

The Heat Transfer Program was used to evaluate the heat transfer, the coefficient of heat transfer, and the wall shearing stress. The necessary information to perform these calculations is obtained from the Physical Property and Iterative programs. All computer programs described in this section are located in the Appendix.

Solutions of the Reduced Differential Equations of Natural-Convection in the Supercritical Region

Five different problems relating to natural-convection along a vertical flat plate in the supercritical were solved using the equations and procedures presented in this work. The specifications of these problems are as follows:

Fluid: Freon 114

$P = 540.00$ psia

$\Delta T = 5^{\circ}F$

$T_W > T_\infty$

Case	T_w (°R)	T_∞ (°R)	ρ_w (Lbs/CuFt)	ρ_∞ (Lbs/CuFt)
1	779.69	774.69	24.27	27.15
2	774.69	769.69	27.15	36.61
3	769.69	764.69	32.61	43.45
4	764.69	759.69	43.45	51.31
5	764.69	759.69	43.45	51.31

$C_{pw}\left(\frac{BTU}{lb^oF}\right)$	$C_{p\infty}\left(\frac{BTU}{lb^oR}\right)$	μ_w (CP)	μ_∞ (CP)
0.51	0.68	0.027	0.029
0.68	1.08	0.029	0.033
1.08	1.33	0.033	0.044
1.33	0.59	0.044	0.055
1.33	0.59	0.044	0.055

$k_w\left(\frac{Cal}{SecCm^oK}\right)$	$k_\infty\left(\frac{Cal}{SecCm^oK}\right)$	Pr_w	Pr_∞
5.4×10^{-5}	5.6×10^{-5}	2.55	3.55
5.6×10^{-5}	5.9×10^{-5}	3.55	6.04
5.9×10^{-5}	6.9×10^{-5}	6.04	8.60
6.9×10^{-5}	7.7×10^{-5}	8.60	4.26
6.9×10^{-5}	7.7×10^{-5}	8.60	4.26

The pertinent results from the solutions of the above cases are shown in the following tables and in

Figures 10, 11, 12 and 13.

Case	$F''(0)$	$\theta'(0)$	$qX^{\frac{1}{4}}$(BTU/HrFt^2/Ft$^{\frac{1}{4}}$)
1	0.555	-0.86	64.44
2	0.536	-1.02	91.72
3	0.617	-1.26	140.80
4	0.738	-1.07	117.85
5	0.738	-1.07	117.85

$hX^{\frac{1}{4}}$(BTU/HrFt^2 ^{o}F/Ft$^{\frac{1}{4}}$)	$\tau_{yx,w}X^{-\frac{1}{4}}$(Lbs$_f$/Ft^2/Ft$^{-\frac{1}{4}}$)	η_w
12.90	0.0014	10.0
18.34	0.0022	10.0
28.16	0.0043	10.0
23.56	0.0044	10.0
23.56	0.0044	8.0

Cases (4) and (5) are identical problems with the exception that the solutions were carried out to η=10 in Case (4) and to η=8 in Case (5). The purpose in doing this is to establish an acceptable value for $\eta=\infty$. Cases (4) and (5) are essentially identical solutions with the computed boundary conditions as follows:

Case No.	η_∞	$F''(0)$	$\theta'(0)$
4	10.0	0.738067	-1.06679
5	8.0	0.738208	-1.06686

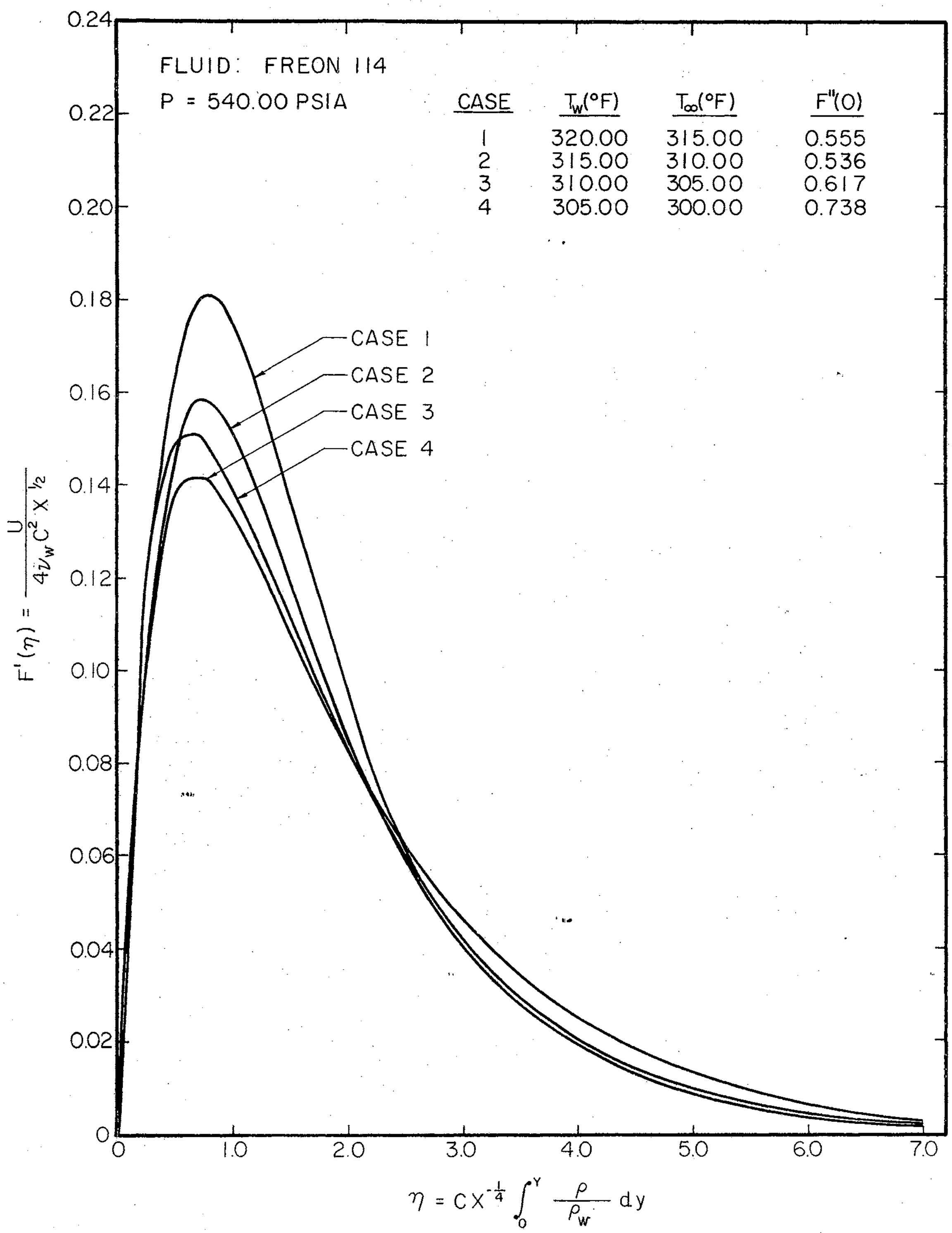

Figure 10. The Velocity Profiles for Four Different Combinations of Wall and Stream Temperatures

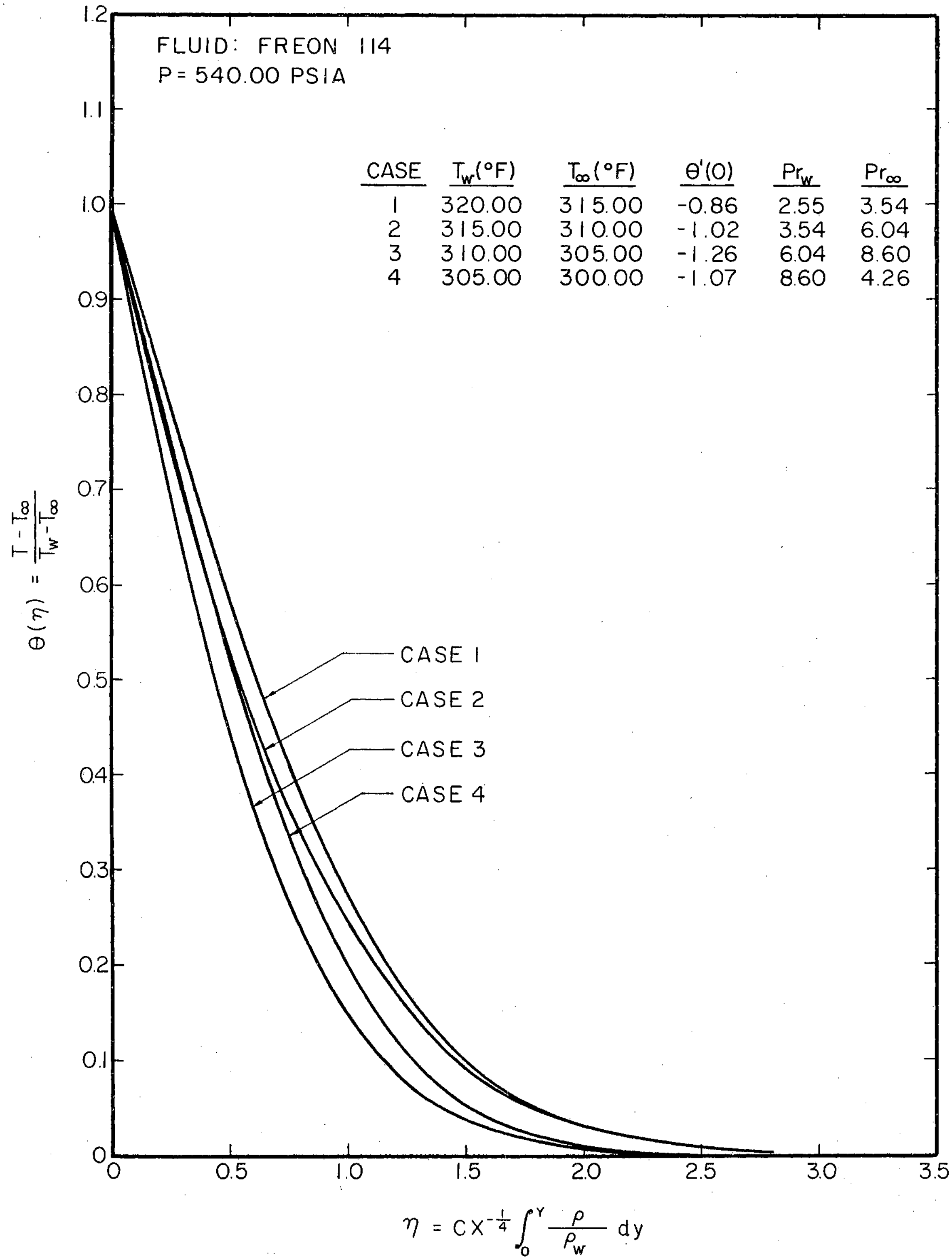

Figure 11. The Temperature Profiles for Four Different Combinations of Wall and Stream Temperatures

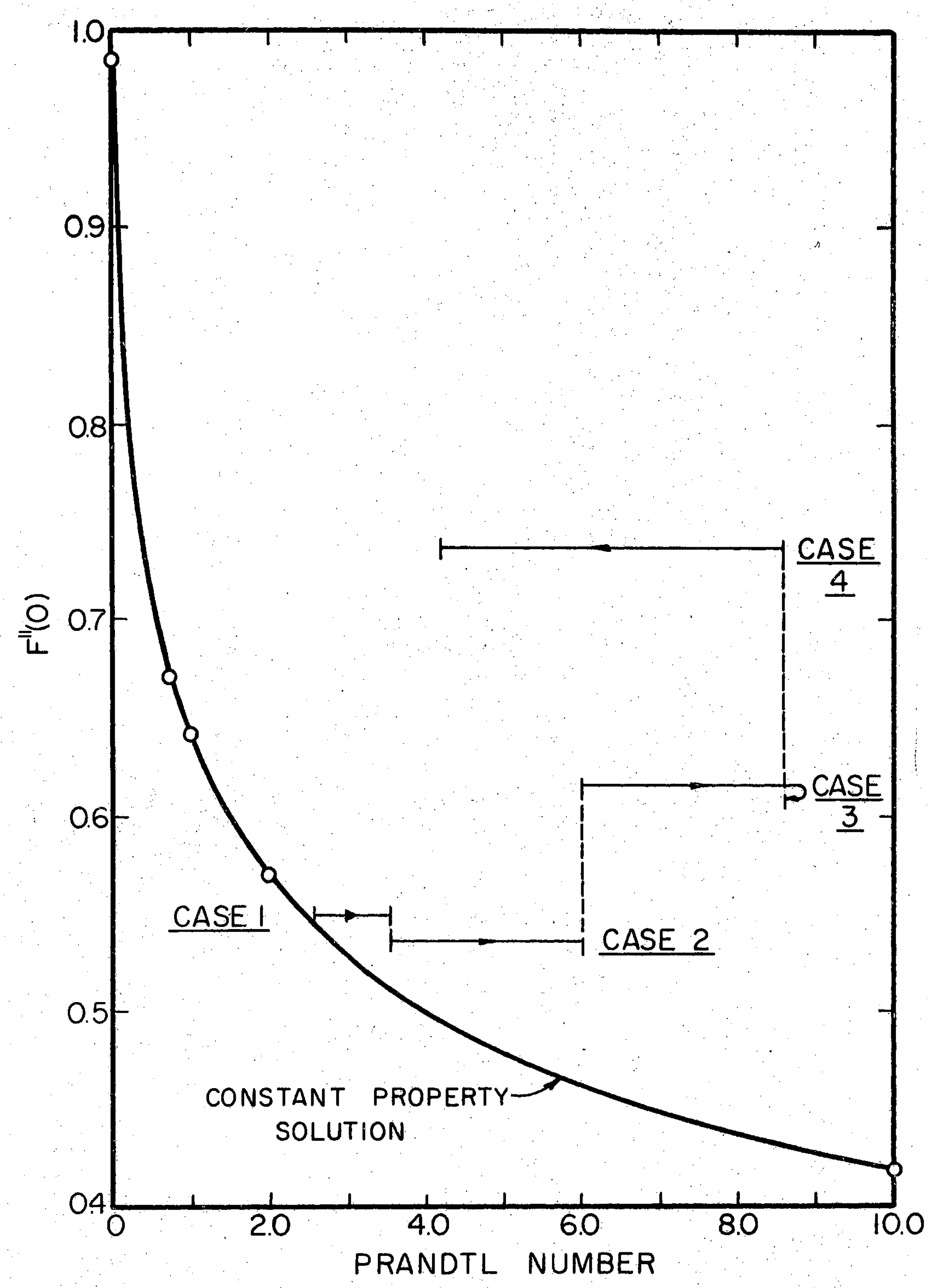

Figure 12. Constant Property and Variable Property Solutions

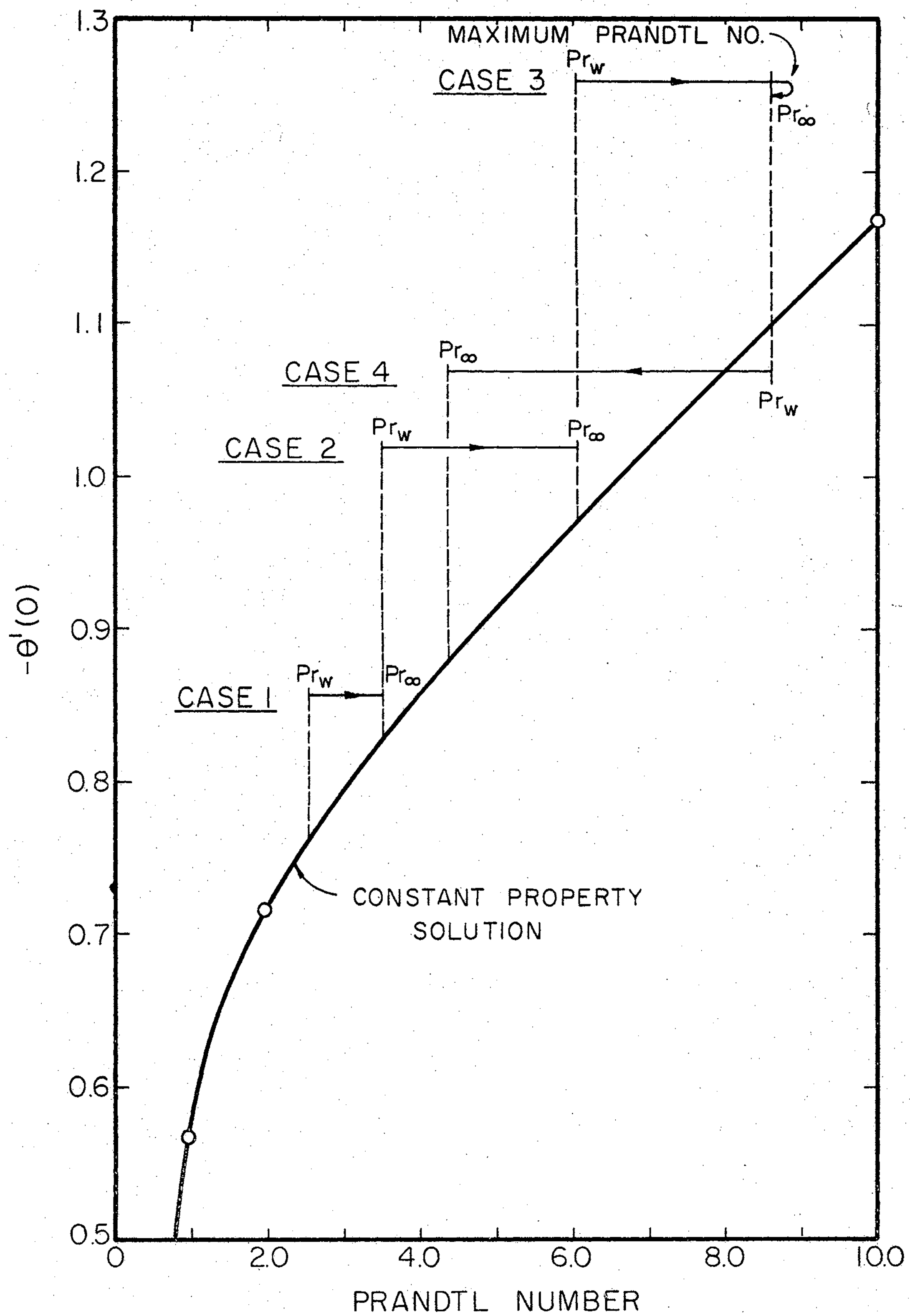

Figure 13. Constant Property and Variable Property Solutions

In view of the results it was possible to assume that infinity is established at $\eta=8.0$; however, to be on the safe side infinity was defined as $\eta=10$.

Cases (1) through (4) are the solutions for a situation in which the state of the fluid is in that portion of the supercritical region in which the density, specific heat at constant pressure, viscosity, thermal conductivity and Prandtl Number experience their maximum variation with respect to temperature at constant pressure as shown in Figures 2, 3, 4 and 5 of Chapter IV. The region was crossed in five degree increments.

As the plate temperature, which is $5^{\circ}F$ above the main stream temperature, is increased in the region of maximum property variation, the velocity gradient at the wall continues to increase in all cases while the temperature gradient at the wall increases very rapidly in Cases (1) through (3) and then decreases in Case (4). This variation is shown in Figures 10 and 11.

A comparison between the variable property solutions of Cases (1), (2), (3) and (4) and the constant property solution is shown in Figures 12 and 13. It will be noted that the results are different between the two types of solutions even though the temperature difference between the wall and main streem is only $5^{\circ}F$. This difference is due to the assumptions made to obtain the constant property solutions.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The primary purpose of this work was to develop an analytical method to study the natural-convection in the supercritical state where the physical properties vary strongly for small temperature changes. As a result of this work specific conclusions may be stated as follows:

1. The reduced differential equations for natural convection along a vertical flat plate (see Equations (33) through (39), Chapter III) were developed with no simplifying assumptions made regarding the physical properties. The property coefficients were expressed as dimensionless groups which may be expressed as functions of $\theta(\eta)$ once the equations of state for the physical properties C_p, μ, k and ρ are known. The reduced differential equations expressed in this form has the advantage of rapid formulation to a specific group of problems once the specification of these problems are formulated. Consider the simple case in which the physical properties of a system are specified as follows:

$$P = \rho RT$$
$$C_p = \text{constant}$$
$$\mu = C_1 \; T^{3/4} \qquad (1)$$
$$k = C_2 \; T^{3/4}$$
$$Pr = \text{constant} \; .$$

Then employing Equations (33) through (39, located in Chapter III, the property coefficients become

$$A1 = - \frac{\Delta T}{4T} = - \frac{\Delta T}{4T_\infty + 4\Delta T \; \theta} \qquad (2)$$

$$A2 = \left[\frac{T}{T_w} \right]^{1/4} = \left[\frac{T_\infty}{T_w} + \frac{\Delta T}{T_w} \; \theta \right] \qquad (3)$$

$$A3 = \left[\frac{T_\infty}{T_w} + \frac{\Delta T}{T_w} \; \theta \right]^{1/4} \theta \qquad (4)$$

$$B1 = - \frac{\Delta T}{4T} = - \frac{\Delta T}{4T_\infty + 4\Delta T \; \theta} \qquad (5)$$

$$B2 = \left[\frac{T_\infty}{T_w} + \frac{\Delta T}{T_w} \; \theta \right]^{1/4} Pr \qquad (6)$$

The reduced differential will then have the form

$$F''' = \left[\frac{\Delta T}{4T_\infty + 4\Delta T \; \theta} \right] \theta' F'' - 3 \left[\frac{T_\infty}{T_w} + \frac{\Delta T}{T_w} \; \theta \right] FF'' \qquad (7)$$
$$+ 2 \left[\frac{T_\infty}{T_w} + \frac{\Delta T}{T_w} \; \theta \right]^{1/4} (F')^2 - \left[\frac{T_\infty}{T_w} + \frac{\Delta T}{T_w} \; \theta \right]^{1/4} \theta$$

$$\theta'' = \left[\frac{\Delta T}{4T_\infty + 4\Delta T \; \theta} \right] (\theta')^2 \qquad (8)$$
$$- 3 \left[\frac{T_\infty}{T_w} + \frac{\Delta T}{T_w} \; \theta \right]^{1/4} Pr \; F \; \theta' \; .$$

In this form the equations show the dimensionless groups necessary for correlation purposes which for this particular case are as follows:

$$\frac{Nu}{[Gr/4]^{1/4}} = -\theta'(0) = f\left(Pr, \frac{T_\infty}{T_w}\right) . \qquad (9)$$

The conclusion made from the above was that the reduced differential equations in the form shown in Equations (33) through (39), Chapter III, provides an easy recipe for obtaining the final form of the set of reduced differential equations so that they may be numerically integrated and for obtaining the dimensionless groups for correlating purposes. It should be kept in mind, however, that as the expressions for the physical properties become more complicated, the process described above becomes more difficult.

2. If the physical properties of a given fluid state are such that the property coefficients over the temperature range considered have the following values:

$$\begin{aligned} A1 &= 0 \\ A2 &= 1 \\ A3 &= \theta(\eta) \\ B1 &= 0 \\ B2 &= Pr = \text{constant} . \end{aligned} \qquad (10)$$

The reduced differential equations reduces to

$$F''' = -3FF'' + 2(F')^2 - \theta \qquad (11)$$

$$\theta''' = -\beta Pr\, F\, \theta' . \qquad (12)$$

Equations (11) and (12) also represent the case in which all fluid properties are considered constant with the exception of the part that density variation plays in producing a buoyancy force. This means that if the problem under consideration meet, in an engineering sense, the conditions of Equation (10) then the constant property solution of Ostrach may be used and are available in reference [10]. It should also be noted that Sparrow and Gregg [15] developed a system whereby the variable property problem in which the fluid is in a perfect gas state may be solved using a defined reference temperature and the constant property solutions of Ostrach.

Although the constant property solution may be applied to any fluid, it is of interest to note that even for such a simple fluid state as a perfect gas with Sutherland-type expression for the dynamic viscosity and power law equation for the thermal conductivity that a difference exist between the constant proper solution of Ostrasch [10] and the variable property solution of Sparrow and Gregg [15]. The magnitude of this difference, of course, depends on the temperature at which the physical properties are measured for the constant property solution and the temperature ratio between the wall and main stream. The purpose of this discussion was to point out the need for determining when the constant property solutions may be applied to a given problem.

It was concluded from an examination of Equations (10) and (11) that the constant property solutions of Ostrach are not only valid for those cases in which the fluid property may be considered constant, but are also valid for the variable property cases in which the conditions of Equations (10) are satisfied.

3. In the supercritical region, simplifying assumptions regarding the physical properties would be difficult to justify because of their strong variations with temperature at constant pressure. Also, many of the equations of state developed in the literature do not apply when the fluid is assumed to be in the supercritical state. Even the equation of state used in this work, although valid in the supercritical region, is applicable only to a single fluid. This means that the possibility of solving the problem of natural-convection for many fluid systems at one time seems remote and that each fluid system must be investigated separately.

4. The numerical solution of the reduced differential equations is complicated by the fact that two of the five wall boundary conditions are unknown. These two unknown boundary conditions are $F''(0)$ and $\theta'(0)$ and their values must be obtained in order to evaluate the heat transfer and shearing stress. The iterative method of Newton-Raphson which is described in Appendix E was demonstrated to be a very effective and practical numerical method for overcoming this difficulty.

5. Four cases of natural-convection in the super-critical region were computed for a fluid state in which the pressure is 540.00 psia (P_c = 473 psia) over a temperature range from 320°F to 300°F in five degree increments. Under these conditions it is estimated that the maximum coefficient of heat transfer occurs at a wall temperature of 309°F, which is very close to the maximum Prandtl Number.

The important questions as to what is the maximum value of the coefficient of heat transfer and what must the temperature of the vertical wall be in order to obtain this maximum value for a given pressure and temperature difference has not been answered. To obtain answers to these questions it is recommended that, by using the method of solution presented in this work, a systematic set of solutions be made. This will also permit the development of a corrolation formula.

SELECTIVE BIBLIOGRAPHY

1. Downing, R. C. "Transport Properties of Freon Fluorocarbons." Dupont Technical Bulletin C-30, 1965.

2. Fritsch, C. A., and R. J. Grosh. "Free Convection Heat Transfer to a Supercritical Fluid." A.I.Ch.E. Journal (June, 1955), pp. 1010-1016.

3. Griffith, J. D., and R. H. Sabersky. "Convection in a Fluid at Supercritical Pressures." ARS Journal (March, 1960), pp. 288-291.

4. Hildebrand, F. B. Introduction to Numerical Analysis. McGraw Hill Co., Inc., 1956.

5. Holman, J. P., S. N. Rea and C. E. Howard. "Force Convection Heat Transfer to Freon 12 Near the Critical State in a Vertical Annulus." International Journal of Heat Mass Transfer, Vol. 8 (1965), pp. 1095-1102.

6. Jossi, J. A., L. I. Stiel and G. Thodus. "The Viscosity of Pure Substances in the Dense Gaseous and Liquid Phase." A.I.Ch.E. Journal, Vol. 8, No. 1 (March, 1962), pp. 11-15.

7. Kazimierz, Brodowicz, and Jerzy Blalokoz. "Free Convection Heat Transfer From a Vertical Plate to Freon 12 Near the Critical State." Archiwum Budowy Maszyn (1963), pp. 289-303.

8. Martin, Joseph J. "Thermodynamic Properties of Dichlortetrafluoroethane." Journal of Chemical and Engineering Data, Vol. 5, No. 3 (July, 1960), pp. 334-336.

9. McGill, R., and P. Kenneth. "Solution of the Veriational Problem by Means of a Generalized Newton-Raphson Operator." AIAA Journal, Vol. 2, No. 10 (October, 1964), pp. 26-64.

10. Ostrach, Simon. "An Analysis of Laminar Free Convection Flow and Heat Transfer About a Flat Plate Parallel to the Direction of the Generating Body Force." NASA, Technical Note 2635 (February, 1952).

11. Pohlhausen, E. "The Heat Exchange Between Solid Bodies and Fluids With Small Friction and Small Conduction of Heat." ZAMM 1, 115, 1921.

12. Reid and Sherwood. Properties of Gases and Liquids. McGraw Hill Co., Inc., 1950.

13. Schlichting, H. Boundary Layer Theory, Fourth Edition. McGraw Hill Co., Inc., 1960.

14. Sebesta, H., and D. Unruh. "Solution of the Two Point Boundary Value Problem by the Newton-Raphson Method." Oklahoma State University. Unpublished.

15. Sparrow, E. M., and J. L. Gregg. "The Variable Fluid Property Problem in Free Convection." Transactions of the ASME, Vol. 80 (1958), pp. 879-886.

16. Stiel, L. I., and G. Thodus. "The Thermal Conductivity of Nonpolar Substances in the Dense Gaseous and Liquid Region." A.I.Ch.E. Journal, Vol. 10, No. 1 (January, 1964), pp. 26-64.

17. Van Wie, N. H., and R. A. Ebel. "Some Properties of Freon 114 Volume II The Critical Temperature to 400°F." Report No. K-1430, Vol. II, Distributed by: Clearinghouse for Federal Scientific and Technical Information, U. S. Department of Commerce, National Bureau of Standards (September 3, 1959).

APPENDIX A

THE PHYSICAL PROPERTY PROGRAM

The computer program that was used to compute the values of the various physical properties is presented in this appendix. It is a Fortran IV representation of the physical property equations developed in Chapter IV. Also, the computed results of the physical property for a pressure of 540.00 psia over a temperature range from 320°F to 300°F are presented.

As an aid for using this program the following terms are defined:

Input Information

Program Symbols	Descriptions	Units
P	Pressure	psia
T	Initial temperature	°R
V	Initial specific value	CuFt/Lb
DTEMP	Temperature increment	°R
MDOLP	Maximum number of DOLOOP calculations	--

The initial value of the specific volume needs only to be an estimated value since its only purpose is to start the iteration process of Newton-Raphson on the equation of states (see Chapter IV). The term DTEMP specifies the temperature increment for which the physical properties are to be computed. The term MDOLP determines the temperature range over which the physical properties are to be computed. The output terms (see Table I) are defined as follows:

Output Information

Program Symbols	Descriptions	Units
PRESSURE	Pressure	psia
TEMP	Temperature	^{o}R
SPVL	Specific volume	CuFt/Lb
DEN	Density	Lbs/CuFt
DERPTV	$(\partial P/\partial T)_v$	psia/^{o}R
DERPVT	$(\partial P/\partial v)_T$	psia/(CuFt/Lb)
DERVTP	$(\partial v/\partial T)_p$	(CuFt/Lb)/^{o}R
DERDTP	$(\partial \rho/\partial T)_p$	(Lbs/CuFt)/^{o}R
CVLP	Specific heat at constant volume - low pressure	BTU/Lb^{o}F
CVHP	Specific heat at constant volume	BTU/Lb^{o}F
CPHP	Specific heat at constant pressure	BTU/Lb^{o}F
VISLP	Viscosity - low pressure	Centipoises
XI	$\mathcal{K}$ - (see Equation 33, Chapter IV)	--
VISHP	Viscosity	Centipoises
DERUDT	$(\partial \mu/\partial \rho)_T$	Centipoises/(Lbs/CuFt)
DERUTD	$(\partial \mu/\partial T)_\rho$	Centipoises/^{o}R
DERUTP	$(\partial \mu/\partial T)_p$	Centipoises/^{o}R
CONLP	Thermal conductivity - low pressure	Cal/SecCm ^{o}K

CONZC	Z_c – (see Equation 40, Chapter IV)	--
CONLA	λ – (see Equation 40, Chapter IV)	--
CONHP	Thermal conductivity	Cal/SecCm ^oK
DERKDT	$(\partial K/\partial \rho)_T$	(Cal/SecCm ^oK)/(Lbs/CuFt)
DERCVT	$(\partial C_v{}^o/\partial T)_p$ – low pressure	(BTU/Lb^{o}R)/^{o}R
DERKTD	$(\partial k/\partial T)_\rho$	(Cal/SecCm^{o}K)/^{o}R
DERKTP	$(\partial k/\partial T)_p$	(Cal/SecCm^{o}K)/^{o}R
DCOEFF	$1/\rho(\partial \rho/\partial T)_p$	$1/^oR$
ECOEFF	$1/\mu(\partial \mu/\partial T)_p$	$1/^oK$
HCOEFF	$1/k(\partial k/\partial T)_p$	$1/^oR$
PRNO	Prandtl Number	dimensionless
ITR	Number of Newton-Raphson iterations to compute the specific volume	--
A111	$1/\rho(\partial \rho/\partial T)_p + 1/\mu(\partial \mu/\partial T)_p$	$1/^oR$
A222	$1/\rho\mu$	1/(CuFt)(Centi-poises)
A333	$1/\rho^2\mu$	1/(CuFt)^2(Centi-poises)
B111	$1/\rho(\partial \rho/\partial T) + 1/k(\partial k/\partial T)_p$	$1/^oR$
B222	$(C_p/\rho k)\,10^{-2}$	$\frac{BTU/Lb^oR}{(Lb/CuFt)(Cal/SecCm^oK)}$

The program will also punch out data cards for the following:

A111 Versus T

A222 Versus T

A333 Versus T

B111 Versus T

B222 Versus T

These cards provide the necessary data that is to be used in the least square program (see Appendix B).

```
$ID          C-0001 THOMAS E MULLIN                    2555-41002
$JOB                THOMAS E MULLIN                    2555-41002
$IBJOB NAMEPR  MAP
$IBFTC           NODECK
C       NAME  THOMAS E MULLIN
C       SUBJECT  THERMODYNAMIC AND TRANSPORT PROPERTIES OF FREON-114
C
C       DATA INPUT  P=PRESSURE(PSIA)
C                   T=TEMPERATURE(DEG.R)
C                   V=SPECIFIC VOLUME (CUFT/LBM)
C                   DTEMP = TEMP INCREMENT (DEG.R)
C                   NDOLP = MAX NO. OF DOLOOP ITERATION
C
      DOUBLE PRECISION  A2,A3,A4,A5,A00,A11,A22,A33,A44,A55,B2,B3,B5,BV,
     1C2,C3,C5,CVLP,CVHP,CPHP,CONC,CON1,CON2,CON3,CONLP,CON5,CON6,CONZC,
     2CONL1,CONL2,CONLA,CON7,CONHP,DERPTV,DERPVT,DERVTP,DERDTP,DENR,
     3DERUDT,DERUTD,DERUTP,DERKDT,DERCVT,DER1,DER2,DER3,DER4,DER5,
     4DERKTD,DERKTP,EXPNEG,FN,FNDER,F11,F22,F33,F55,F66,F77,F88,F99,F10,
     5H,P,R,SPVOL,TC,TR,T,V,VISLP,VISHP,W22,W33,W55,XIN,XID,XI,XIC,Z,
     6ZNEW,ZDIFF,DCOEFF,ECOEFF,HCOEFF,PRNO,DTEMP,DENSTY,
     7A111,A222,A333,B111,B222
C
   40 FORMAT (D16.2,D16.2,D16.9,D16.2,I4)
      WRITE(6,30)
   30 FORMAT(1H1,4X,14HPRESSURE(PSIA),5X,13HSPVL(CUFT/LB),9X,12HDEN(LB/C
     1UFT),9X,13HDERPTV(PSI/R),10X,21HDERPVT(PSI/(CUFT/LB)),2X,17HDERVTP
     2(CUFT/LB)/R)
      WRITE(6,32)
   32 FORMAT(1H ,4X,7HTEMP(R),12X,17HDERDTP(LB/CUFT)/R,5X,14HCVLP(BTU/LB
     1-R),7X,14HCVHP(BTU/LB-R),9X,14HCPHP(BTU/LB-R),9X,9HVISLP(CP))
      WRITE(6,34)
   34 FORMAT(1H ,23X,2HXI,20X,9HVISHP(CP),12X,20HDERUDT(CP/(LB/CUFT)),3X
     1,12HDERUTD(CP/R),11X,12HDERUTP(CP/R))
      WRITE(6,36)
   36 FORMAT(1H ,23X,19HCONLP(CAL/SEC-CM-K),3X,5HCONZC,16X,5HCONLA,18X,1
     19HCONHP(CAL/SEC-CM-K),4X,6HDERKDT)
      WRITE(6,38)
   38 FORMAT(1H ,23X,18HDERCVT(BTU/LB-R)/R,4X,6HDERKTD,15X,22HDERKTP(CAL
     1/SEC-CM-K)/R,1X,6HDCOEFF,17X,6HECOEFF)
      WRITE(6,42)
   42 FORMAT(1H ,23X,6HHCOEFF,16X,4HPRNO,17X,3HITR)
      WRITE(6,150)
  150 FORMAT(1H ,23X,4HA111,18X,4HA222,17X,4HA333,
     1        19X,4HB111,19X,4HB222,//)
C
      KKX = 0
C
      READ(5,40)P,T,V,DTEMP,NDOLP
C
C       THE THERMODYNAMIC PROPERTIES OF FREON-114
C       CONSTANTS FOR THE MARTIN AND HOU EQ. OF STATE
      R  =(1.07351D+01)/(1.70936D+02)
      H  =  3.0D+00
      A2 = -2.3856704D+00
      B2 =  1.0801207D-03
      C2 = -6.5643648D+00
      A3 =  3.4055687D-02
      B3 = -5.3336494D-06
      C3 =  1.6366057D-01
      A4 = -3.8574810D-04
      A5 =  1.6017659D-06
      B5 =  6.2632340D-10
```

```
      C5 = -1.0165314D-05
      TC =  7.5395D+02
      BV =  5.9149070D-03
C
      ITR = 1
      Z = V-BV
      DO 50 J=1,NDOLP,1
      TR = T/TC
      EXPNEG = (1.0D+00)/(DEXP(H*TR))
C     COEFF'S FOR 5TH DEGREE POLY.
      A00 = P
      A11 = R*T
      A22 = (A2)+(B2*T)+(C2*EXPNEG)
      A33 = (A3)+(B3*T)+(C3*EXPNEG)
      A44 = (A4)
      A55 = (A5)+(B5*T)+(C5*EXPNEG)
C     NEWTON RAPHSON ITERATION
   43 FN = ((((A00*Z-A11)*Z-A22)*Z-A33)*Z-A44)*Z-A55
      FNDER = (((5.0D+00*A00*Z-4.0D+00*A11)*Z-3.0D+00*A22)*Z
     1-2.0D+00*A33)*Z-A44
      ZNEW = Z - (FN/FNDER)
      ZDIFF = ZNEW - Z
      IF(DABS(ZDIFF).LE. 1.0D-12)GO TO 45
      Z = ZNEW
      ITR = ITR + 1
      GO TO 43
C
C     COMPUTING THE DENSITY (CUFT/LBM)
   45 SPVOL = ZNEW + BV
      DENSTY = (1.0D+00)/SPVOL
C
C     COMPUTING THE DERIVATIVES (PSI/DEGR)(PSI/CUFT)(CUFT/LB/DEGR)(LB/CUFR/DEGR)
      F11 = R/ZNEW
      F22 =((B2)-(H/TC)*(C2)*(EXPNEG))/(ZNEW**2.0)
      F33 =((B3)-(H/TC)*(C3)*(EXPNEG))/(ZNEW**3.0)
      F55 =((B5)-(H/TC)*(C5)*(EXPNEG))/(ZNEW**5.0)
      F66 =(A11)/(ZNEW**2.0)
      F77 = ((2.0D+00)*(A22))/(ZNEW**3.0)
      F88 = ((3.0D+00)*(A33))/(ZNEW**4.0)
      F99 = ((4.0D+00)*(A44))/(ZNEW**5.0)
      F10 = ((5.0D+00)*(A55))/(ZNEW**6.0)
      DERPTV = (F11)+(F22)+(F33)+(F55)
      DERPVT =-(F66)-(F77)-(F88)-(F99)-(F10)
      DERVTP =-(DERPTV/DERPVT)
      DERDTP =-(DENSTY**2.0)*(DERVTP)
C
C     COMPUTING THE SPECIFIC HEAT AT CONSTANT VOLUME (BTU/LBM*DEGR)
      CVLP =(1.75D-02)+(3.49D-04)*(T)-(1.67D-07)*(T**2.0)
      W22 = ((T)*((H/TC)**2.0)*(C2)*(EXPNEG))/(ZNEW)
      W33 = ((T)*((H/TC)**2.0)*(C3)*(EXPNEG))/(2.0D+00*(ZNEW**2.0))
      W55 = ((T)*((H/TC)**2.0)*(C5)*(EXPNEG))/(4.0D+00*(ZNEW**4.0))
      CVHP = (CVLP) - ((W22+W33+W55)*(1.44D+02/7.78D+02))
C
C     COMPUTING THE SPECIFIC HEAT AT CONSTANT PRESSURE (BTU/LBM*DEGR)
      CPHP =(CVHP)+(T)*(DERPTV)*(DERVTP)*(1.44D+02/7.78D+02)
C
C     COMPUTING THE DYNAMIC VISCOSITY (CENTIPOISES)
      VISLP =((1.0167D-03)*(T**1.5))/(T+5.8655D+02)
      XIN = ((4.18878D+02)**0.16667)
      XID =  (1.70936D+02**0.5)*(3.2189D+01**0.66667)
      XI = (XIN)/(XID)
      DENR = (DENSTY)/(3.632D+01)
```

```
      XIC = (2.5D-04)/(XI)
      VISHP = (VISLP)+((2.312D-04)*(DEXP(1.079D+00*DENR)))/(XI) -(XIC)
C
C     COMPUTING THE DERIVATIVE (CENTIPOISES/LB/CUFT)
      DERUDT = (1.079D+00/3.632D+01)*(VISHP-VISLP+XIC)
      DERUTD = (1.5)*(VISLP/T)-(VISLP)/(T+5.8655D+02)
      DERUTP = (DERUDT)*(DERDTP)+(DERUTD)
C
C     COMPUTING THE THERMAL CONDUCTIVITY (CAL/SEC CM DEGK)
      CONC = ((1.32D-02)*(2.52D+02)*(1.8D+00))/(4.536D+02)
      CON1 = (CONC)*(CVLP)*(VISLP)
      CON2 = ((3.40D-02)*(VISLP))/(1.70936D+02)
      CON3 = ((7.0D-03)*(VISLP))/((TR)*(1.70936D+02))
      CONLP= (CON1)+(CON2)-(CON3)
      CON5 = (4.73187D+02)*(1.70936D+02)
      CON6 = (3.632D+01)*(1.07315D+01)*(7.5395D+02)
      CONZC = (CON5)/(CON6)
      CONL1 =  (1.70936D+02**0.5)*(4.18877D+02**0.16667)
      CONL2 = (3.2189D+01**0.66667)
      CONLA =  CONL1/CONL2
      CON7  =  (CONLA)*(CONZC**5.0)
      CONHP = (CONLP)+((1.31D-07*DEXP(6.7D-01*DENR))/CON7)
     1- (1.40039D-07/CON7)
C
C     COMPUTING THE DERIVATIVE (CAL/SEC CM DEGK)/DEGR
      DERKDT =(6.7D-01/3.632D+01)* (CONHP-CONLP+(1.40039D-07/CON7))
      DERCVT = (3.49D-04) -(3.34D-07*T)
      DER1 = (CONC)*(CVLP)*(DERUTD)
      DER2 = (CONC)*(VISLP)*(DERCVT)
      DER3 =  (3.40D-02/1.70936D+02)*(DERUTD)
      DER4 = ((7.0D-03)*(DERUTD))/((1.70936D+02)*(TR))
      DER5 = ((7.0D-03)*(VISLP))/((1.70936D+02)*(TR)*(T))
      DERKTD = (DER1)+(DER2)+(DER3)-(DER4)+(DER5)
      DERKTP = (DERKDT)*(DERDTP) + (DERKTD)
C
C     COEFF'S OF THE DIFF. EQ.S
      DCOEFF = (DERDTP)/(DENSTY)
      ECOEFF = (DERUTP)/(VISHP)
      HCOEFF = (DERKTP)/(CONHP)
      PRNO = (CPHP*VISHP*1.0D-02)/(CONHP)
      A111 = DCOEFF + ECOEFF
      A222 = 1.0D+00/(DENSTY*VISHP)
      A333 = 1.0D+00/(DENSTY*DENSTY*VISHP)
      B111 = DCOEFF + HCOEFF
      B222 = (CPHP)/(DENSTY*CONHP*1.0D+02)
C
      KKX = KKX + 1
      IF(KKX.LE.5)GO TO 143
      WRITE(6,30)
   30 FORMAT(1H1,4X,14HPRESSURE(PSIA),5X,13HSPVL(CUFT/LB),9X,12HDEN(LB/C
     1UFT),9X,13HDERPTV(PSI/R),10X,21HDERPVT(PSI/(CUFT/LB)),2X,17HDERVTP
     2(CUFT/LB)/R)
      WRITE(6,32)
   32 FORMAT(1H ,4X,7HTEMP(R),12X,17HDERDTP(LB/CUFT)/R,5X,14HCVLP(BTU/LB
     1-R),7X,14HCVHP(BTU/LB-R),9X,14HCPHP(BTU/LB-R),9X,9HVISLP(CP))
      WRITE(6,34)
   34 FORMAT(1H ,23X,2HXI,20X,9HVISHP(CP),12X,20HDERUDT(CP/(LB/CUFT)),3X
     1,12HDERUTD(CP/R),11X,12HDERUTP(CP/R))
      WRITE(6,36)
   36 FORMAT(1H ,23X,19HCONLP(CAL/SEC-CM-K),3X,5HCONZC,16X,5HCONLA,18X,1
     19HCONHP(CAL/SEC-CM-K),4X,6HDERKDT)
      WRITE(6,38)
```

```
 38 FORMAT(1H ,23X,18HDERCVT(BTU/LB-R)/R,4X,6HDERKTD,15X,22HDERKTP(CAL
   1/SEC-CM-K)/R,1X,6HDCOEFF,17X,6HECOEFF)
    WRITE(6,42)
 42 FORMAT(1H ,23X,6HHCOEFF,16X,4HPRNO,17X,3HITR)
    WRITE(6,150)
150 FORMAT(1H ,23X,4HA111,18X,4HA222,17X,4HA333,
   1        19X,4HB111,19X,4HB222,//)
    KKX = 0
143 WRITE(6,60)P,SPVOL,DENSTY,DERPTV,DERPVT,DERVTP
 60 FORMAT(3X,D15.8,5X,D15.8,7X,D15.8,7X,D15.8,7X,D15.8,7X,D15.8)
    WRITE(6,60)T,DERDTP,CVLP,CVHP,CPHP,VISLP
    WRITE(6,62)XI,VISHP,DERUDT,DERUTD,DERUTP
 62 FORMAT(23X,D15.8,7X,D15.8,7X,D15.8,7X,D15.8,7X,D15.8)
    WRITE(6,62)CONLP,CONZC,CONLA,CONHP,DERKDT
    WRITE(6,62)DERCVT,DERKTD,DERKTP,DCOEFF,ECOEFF
    WRITE(6,63)HCOEFF,PRNO,ITR
 63 FORMAT(23X,D15.8,7X,D15.8,7X,I15)
    WRITE(6,66) A111,A222,A333,B111,B222
 66 FORMAT(23X,D15.8,7X,D15.8,7X,D15.8,7X,D15.8,7X,D15.8,//)
    WRITE(7,97) T,A111
    WRITE(7,97) T,A222
    WRITE(7,97) T,A333
    WRITE(7,97) T,B111
    WRITE(7,97) T,B222
 97 FORMAT(D20.8,D20.8)
C
    T = T - DTEMP
 50 ITR=1
C
    STOP
    END
```

TABLE I

THERMODYNAMIC AND TRANSPORT PROPERTIES OF FREON 114

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.41205912D-01	0.24268362D 02	0.28467528D 01	-0.37455300D 04	0.76004006D-03
0.77969000D 03	-0.44762817D 00	0.18808976D 00	0.20061111D 00	0.51285295D 00	0.16201259D-01
	0.20675842D-01	0.27105049D-01	0.68314470D-03	0.19310373D-04	-0.28648444D-03
	0.42805194D-04	0.27524379D 00	0.35342444D 01	0.54435446D-04	0.67723872D-06
	0.88583540D-04	0.70786765D-07	-0.23236437D-06	-0.18444927D-01	-0.10569413D-01
	-0.42686225D-02	0.25536494D 01	4		
	-0.29014340D-01	0.15202302D 01	0.62642471D-01	-0.22713550D-01	0.38821348D 01
0.54000000D 03	0.41053166D-01	0.24358657D 02	0.28611721D 01	-0.37281354D 04	0.76745389D-03
0.77949000D 03	-0.45536430D 00	0.18807203D 00	0.20062734D 00	0.51743065D 00	0.16197397D-01
	0.20675842D-01	0.27162954D-01	0.68497970D-03	0.19312030D-04	-0.29260327D-03
	0.42791036D-04	0.27524379D 00	0.35342444D 01	0.54482490D-04	0.67836773D-06
	0.88650340D-04	0.70796418D-07	-0.23810803D-06	-0.18694146D-01	-0.10772145D-01
	-0.43703588D-02	0.25797178D 01	4		
	-0.29466291D-01	0.15113660D 01	0.62046361D-01	-0.23064505D-01	0.38988979D 01
0.54000000D 03	0.40898919D-01	0.24450524D 02	0.28758524D 01	-0.37105338D 04	0.77505086D-03
0.77929000D 03	-0.46334721D 00	0.18805430D 00	0.20064388D 00	0.52214227D 00	0.16193535D-01
	0.20675842D-01	0.27222104D-01	0.68685170D-03	0.19313686D-04	-0.29893713D-03
	0.42776875D-04	0.27524379D 00	0.35342444D 01	0.54530702D-04	0.67951832D-06
	0.88717140D-04	0.70806061D-07	-0.24404686D-06	-0.18950400D-01	-0.10981412D-01
	-0.44754028D-02	0.26065704D 01	4		
	-0.29931812D-01	0.15024158D 01	0.61447180D-01	-0.23425803D-01	0.39161524D 01
0.54000000D 03	0.40743133D-01	0.24544013D 02	0.28908023D 01	-0.36927243D 04	0.78283728D-03
0.77909000D 03	-0.47158790D 00	0.18803654D 00	0.20066073D 00	0.52699329D 00	0.16189672D-01
	0.20675842D-01	0.27282544D-01	0.68876200D-03	0.19315342D-04	-0.30549649D-03
	0.42762713D-04	0.27524379D 00	0.35342444D 01	0.54580122D-04	0.68069123D-06
	0.88783940D-04	0.70815693D-07	-0.25019006D-06	-0.19213969D-01	-0.11197507D-01
	-0.45839043D-02	0.26342406D 01	4		
	-0.30411476D-01	0.14933774D 01	0.60844872D-01	-0.23797873D-01	0.39339153D 01
0.54000000D 03	0.40585771D-01	0.24639177D 02	0.29060305D 01	-0.36747068D 04	0.79081969D-03
0.77889000D 03	-0.48009798D 00	0.18801878D 00	0.20067789D 00	0.53198951D 00	0.16185808D-01
	0.20675842D-01	0.27344319D-01	0.69071199D-03	0.19316999D-04	-0.31229243D-03
	0.42748549D-04	0.27524379D 00	0.35342444D 01	0.54630792D-04	0.68188724D-06
	0.88850740D-04	0.70825314D-07	-0.25654737D-06	-0.19485146D-01	-0.11420743D-01
	-0.46960215D-02	0.26627640D 01	4		
	-0.30905889D-01	0.14842487D 01	0.60239379D-01	-0.24181168D-01	0.39522041D 01

TABLE I (Continued)

PRESSURE(PSIA)	SPVL(CUFT/LB)	DEN(LB/CUFT)	DERPTV(PSI/R)	DERPVT(PSI/(CUFT/LB))	DERVTP(CUFT/LB)/R
TEMP(R)	DERDTP(LB/CUFT)/R	CVLP(BTU/LB-R)	CVHP(BTU/LB-R)	CPHP(BTU/LB-R)	VISLP(CP)
	XI	VISHP(CP)	DERUDT(CP/(LB/CUFT))	DERUTD(CP/R)	DERUTP(CP/R)
	CONLP(CAL/SEC-CM-K)	CONZC	CONLA	CONHP(CAL/SEC-CM-K)	DERKDT
	DERCVT(BTU/LB-R)/R	DERKTD	DERKTP(CAL/SEC-CM-K)/R	DCOEFF	ECOEFF
	HCOEFF	PRNO	ITR		
	A111	A222	A333	B111	B222
0.54000000D 03	0.40426792D-01	0.24736071D 02	0.29215462D 01	-0.36564811D 04	0.79900487D-03
0.77869000D 03	-0.48888969D 00	0.18800100D 00	0.20069538D 00	0.53713697D 00	0.16181945D-01
	0.20675842D-01	0.27407477D-01	0.69270310D-03	0.19318656D-04	-0.31933674D-03
	0.42734383D-04	0.27524379D 00	0.35342444D 01	0.54682756D-04	0.68310715D-06
	0.88917540D-04	0.70834924D-07	-0.26312911D-06	-0.19764241D-01	-0.11651446D-01
	-0.48119212D-02	0.26921777D 01	4		
	-0.31415687D-01	0.14750278D 01	0.59630642D-01	-0.24576163D-01	0.39710370D 01
0.54000000D 03	0.40266155D-01	0.24834753D 02	0.29373592D 01	-0.36380476D 04	0.80739987D-03
0.77849000D 03	-0.49797594D 00	0.18798321D 00	0.20071319D 00	0.54244205D 00	0.16178081D-01
	0.20675842D-01	0.27472071D-01	0.69473684D-03	0.19320313D-04	-0.32664191D-03
	0.42720215D-04	0.27524379D 00	0.35342444D 01	0.54736060D-04	0.68435180D-06
	0.88984340D-04	0.70844523D-07	-0.26994621D-06	-0.20051576D-01	-0.11889963D-01
	-0.49317801D-02	0.27225209D 01	4		
	-0.31941540D-01	0.14657124D 01	0.59018602D-01	-0.24983356D-01	0.39904326D 01
0.54000000D 03	0.40103817D-01	0.24935282D 02	0.29534794D 01	-0.36194070D 04	0.81601196D-03
0.77829000D 03	-0.50737037D 00	0.18796541D 00	0.20073133D 00	0.54791142D 00	0.16174217D-01
	0.20675842D-01	0.27538152D-01	0.69681480D-03	0.19321971D-04	-0.33422121D-03
	0.42706045D-04	0.27524379D 00	0.35342444D 01	0.54790751D-04	0.68562210D-06
	0.89051140D-04	0.70854112D-07	-0.27701022D-06	-0.20347489D-01	-0.12136661D-01
	-0.50557844D-02	0.27538349D 01	4		
	-0.32484149D-01	0.14563002D 01	0.58403198D-01	-0.25403273D-01	0.40104104D 01
0.54000000D 03	0.39939735D-01	0.25037722D 02	0.29699174D 01	-0.36005603D 04	0.82484869D-03
0.77809000D 03	-0.51708737D 00	0.18794759D 00	0.20074981D 00	0.55355209D 00	0.16170352D-01
	0.20675842D-01	0.27605778D-01	0.69893865D-03	0.19323628D-04	-0.34208872D-03
	0.42691874D-04	0.27524379D 00	0.35342444D 01	0.54846881D-04	0.68691896D-06
	0.89117940D-04	0.70863689D-07	-0.28433343D-06	-0.20652332D-01	-0.12391924D-01
	-0.51841312D-02	0.27861633D 01	4		
	-0.33044257D-01	0.14467889D 01	0.57784367D-01	-0.25836464D-01	0.40309902D 01
0.54000000D 03	0.39773862D-01	0.25142140D 02	0.29866843D 01	-0.35815091D 04	0.83391783D-03
0.77789000D 03	-0.52714213D 00	0.18792976D 00	0.20076862D 00	0.55937141D 00	0.16166487D-01
	0.20675842D-01	0.27675008D-01	0.70111016D-03	0.19325286D-04	-0.35025942D-03
	0.42677700D-04	0.27524379D 00	0.35342444D 01	0.54904502D-04	0.68824338D-06
	0.89184740D-04	0.70873256D-07	-0.29192882D-06	-0.20966479D-01	-0.12656163D-01
	-0.53170289D-02	0.28195517D 01	4		
	-0.33622642D-01	0.14371762D 01	0.57162047D-01	-0.26283507D-01	0.40521925D 01
0.54000000D 03	0.39606152D-01	0.25248603D 02	0.30037916D 01	-0.35622555D 04	0.84322744D-03
0.77769000D 03	-0.53755070D 00	0.18791192D 00	0.20078779D 00	0.56537710D 00	0.16162622D-01
	0.20675842D-01	0.27745904D-01	0.70333116D-03	0.19326944D-04	-0.35874921D-03
	0.42663524D-04	0.27524379D 00	0.35342444D 01	0.54963671D-04	0.68959638D-06
	0.89251540D-04	0.70882812D-07	-0.29981020D-06	-0.21290315D-01	-0.12929808D-01
	-0.54546975D-02	0.28540485D 01	4		
	-0.34220123D-01	0.14274594D 01	0.56536175D-01	-0.26745012D-01	0.40740385D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.39436555D-01	0.25357185D 02	0.30212513D 01	-0.35428021D 04	0.85278579D-03
0.77749000D 03	-0.54833002D 00	0.18789406D 00	0.20080731D 00	0.57157725D 00	0.16158756D-01
	0.20675842D-01	0.27818530D-01	0.70560360D-03	0.19328602D-04	-0.36757503D-03
	0.42649347D-04	0.27524379D 00	0.35342444D 01	0.55024447D-04	0.69097904D-06
	0.89318340D-04	0.70892357D-07	-0.30799219D-06	-0.21624247D-01	-0.13213316D-01
	-0.55973701D-02	0.28897045D 01	4		
	-0.34837563D-01	0.14176362D 01	0.55906687D-01	-0.27221617D-01	0.40965496D 01
0.54000000D 03	0.39265020D-01	0.25467961D 02	0.30390763D 01	-0.35231524D 04	0.86260144D-03
0.77729000D 03	-0.55949798D 00	0.18787619D 00	0.20082718D 00	0.57798035D 00	0.16154891D-01
	0.20675842D-01	0.27892957D-01	0.70792953D-03	0.19330261D-04	-0.37675488D-03
	0.42635167D-04	0.27524379D 00	0.35342444D 01	0.55086889D-04	0.69239250D-06
	0.89385140D-04	0.70901891D-07	-0.31649031D-06	-0.21968700D-01	-0.13507169D-01
	-0.57452929D-02	0.29265731D 01	4		
	-0.35475869D-01	0.14077038D 01	0.55273516D-01	-0.27713993D-01	0.41197480D 01
0.54000000D 03	0.39091496D-01	0.25581011D 02	0.30572798D 01	-0.35033103D 04	0.87268314D-03
0.77709000D 03	-0.57107348D 00	0.18785831D 00	0.20084741D 00	0.58459529D 00	0.16151024D-01
	0.20675842D-01	0.27969257D-01	0.71031112D-03	0.19331920D-04	-0.38630792D-03
	0.42620986D-04	0.27524379D 00	0.35342444D 01	0.55151065D-04	0.69383796D-06
	0.89451940D-04	0.70911414D-07	-0.32532104D-06	-0.22324117D-01	-0.13811876D-01
	-0.58987264D-02	0.29647108D 01	4		
	-0.36135993D-01	0.13976595D 01	0.54636599D-01	-0.28222843D-01	0.41436561D 01
0.54000000D 03	0.38915929D-01	0.25696419D 02	0.30758758D 01	-0.34832807D 04	0.88303987D-03
0.77689000D 03	-0.58307647D 00	0.18784041D 00	0.20086802D 00	0.59143140D 00	0.16147158D-01
	0.20675842D-01	0.28047507D-01	0.71275064D-03	0.19333579D-04	-0.39625455D-03
	0.42606803D-04	0.27524379D 00	0.35342444D 01	0.55217041D-04	0.69531667D-06
	0.89518740D-04	0.70920927D-07	-0.33450186D-06	-0.22690962D-01	-0.14127978D-01
	-0.60579462D-02	0.30041769D 01	4		
	-0.36818940D-01	0.13875005D 01	0.53995870D-01	-0.28748908D-01	0.41682969D 01
0.54000000D 03	0.38738262D-01	0.25814271D 02	0.30948788D 01	-0.34630694D 04	0.89368084D-03
0.77669000D 03	-0.59552801D 00	0.18782250D 00	0.20088899D 00	0.59849844D 00	0.16143291D-01
	0.20675842D-01	0.28127787D-01	0.71525048D-03	0.19335238D-04	-0.40661646D-03
	0.42592618D-04	0.27524379D 00	0.35342444D 01	0.55284890D-04	0.69682997D-06
	0.89585540D-04	0.70930428D-07	-0.34405134D-06	-0.23069720D-01	-0.14456042D-01
	-0.62232436D-02	0.30450339D 01	4		
	-0.37525762D-01	0.13772240D 01	0.53351262D-01	-0.29292964D-01	0.41936936D 01
0.54000000D 03	0.38558437D-01	0.25934661D 02	0.31143043D 01	-0.34426832D 04	0.90461542D-03
0.77649000D 03	-0.60845035D 00	0.18780458D 00	0.20091033D 00	0.60580662D 00	0.16139424D-01
	0.20675842D-01	0.28210182D-01	0.71781320D-03	0.19336897D-04	-0.41741680D-03
	0.42578431D-04	0.27524379D 00	0.35342444D 01	0.55354688D-04	0.69837924D-06
	0.89652340D-04	0.70939919D-07	-0.35398918D-06	-0.23460895D-01	-0.14796671D-01
	-0.63949268D-02	0.30873474D 01	4		
	-0.38257566D-01	0.13668269D 01	0.52702710D-01	-0.29855821D-01	0.42198696D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.38376395D-01	0.26057685D 02	0.31341684D 01	-0.34221299D 04	0.91585313D-03
0.77629000D 03	-0.62186695D 00	0.18778664D 00	0.20093205D 00	0.61336661D 00	0.16135556D-01
	0.20675842D-01	0.28294784D-01	0.72044146D-03	0.19338557D-04	-0.42868018D-03
	0.42564242D-04	0.27524379D 00	0.35342444D 01	0.55426513D-04	0.69996596D-06
	0.89719140D-04	0.70949399D-07	-0.36433630D-06	-0.23865012D-01	-0.15150502D-01
	-0.65733217D-02	0.31311867D 01	4		
	-0.39015514D-01	0.13563063D 01	0.52050148D-01	-0.30438334D-01	0.42468483D 01
0.54000000D 03	0.38192075D-01	0.26183443D 02	0.31544880D 01	-0.34014188D 04	0.92740360D-03
0.77609000D 03	-0.63580256D 00	0.18776869D 00	0.20095416D 00	0.62118956D 00	0.16131688D-01
	0.20675842D-01	0.28381687D-01	0.72313809D-03	0.19340216D-04	-0.44043284D-03
	0.42550051D-04	0.27524379D 00	0.35342444D 01	0.55500451D-04	0.70159168D-06
	0.89785940D-04	0.70958868D-07	-0.37511492D-06	-0.24282619D-01	-0.15518205D-01
	-0.67587725D-02	0.31766242D 01	4		
	-0.39800824D-01	0.13456591D 01	0.51393512D-01	-0.31041392D-01	0.42746533D 01
0.54000000D 03	0.38005412D-01	0.26312042D 02	0.31752809D 01	-0.33805602D 04	0.93927655D-03
0.77589000D 03	-0.65028328D 00	0.18775073D 00	0.20097664D 00	0.62928710D 00	0.16127820D-01
	0.20675842D-01	0.28470992D-01	0.72590609D-03	0.19341876D-04	-0.45270271D-03
	0.42535858D-04	0.27524379D 00	0.35342444D 01	0.55576589D-04	0.70325803D-06
	0.89852740D-04	0.70968326D-07	-0.38634861D-06	-0.24714284D-01	-0.15900490D-01
	-0.69516431D-02	0.32237365D 01	4		
	-0.40614774D-01	0.13348819D 01	0.50732738D-01	-0.31665927D-01	0.43033075D 01
0.54000000D 03	0.37816342D-01	0.26443594D 02	0.31965657D 01	-0.33595662D 04	0.95148169D-03
0.77569000D 03	-0.66533658D 00	0.18773275D 00	0.20099952D 00	0.63767132D 00	0.16123952D-01
	0.20675842D-01	0.28562805D-01	0.72874860D-03	0.19343536D-04	-0.46551957D-03
	0.42521664D-04	0.27524379D 00	0.35342444D 01	0.55655022D-04	0.70496674D-06
	0.89919540D-04	0.70977773D-07	-0.39806239D-06	-0.25160596D-01	-0.16298104D-01
	-0.71523175D-02	0.32726034D 01	4		
	-0.41458700D-01	0.13239716D 01	0.50067763D-01	-0.32312913D-01	0.43328339D 01
0.54000000D 03	0.37624797D-01	0.26578217D 02	0.32183620D 01	-0.33384505D 04	0.96402868D-03
0.77549000D 03	-0.68099140D 00	0.18771476D 00	0.20102277D 00	0.64635482D 00	0.16120083D-01
	0.20675842D-01	0.28657238D-01	0.73166899D-03	0.19345197D-04	-0.47891509D-03
	0.42507467D-04	0.27524379D 00	0.35342444D 01	0.55735848D-04	0.70671963D-06
	0.89986340D-04	0.70987209D-07	-0.41028279D-06	-0.25622163D-01	-0.16711837D-01
	-0.73612011D-02	0.33233089D 01	4		
	-0.42334000D-01	0.13129247D 01	0.49398526D-01	-0.32983364D-01	0.43632545D 01
0.54000000D 03	0.37430707D-01	0.26716033D 02	0.32406905D 01	-0.33172288D 04	0.97692706D-03
0.77529000D 03	-0.69727818D 00	0.18769675D 00	0.20104642D 00	0.65535064D 00	0.16116213D-01
	0.20675842D-01	0.28754411D-01	0.73467077D-03	0.19346857D-04	-0.49292304D-03
	0.42493269D-04	0.27524379D 00	0.35342444D 01	0.55819171D-04	0.70851862D-06
	0.90053140D-04	0.70996634D-07	-0.42303794D-06	-0.26099615D-01	-0.17142519D-01
	-0.75787213D-02	0.33759409D 01	4		
	-0.43242134D-01	0.13017379D 01	0.48724970D-01	-0.33678337D-01	0.43945902D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.37234002D-01	0.26857172D 02	0.32635728D 01	-0.32959186D 04	0.99018610D-03
0.77509000D 03	-0.71422886D 00	0.18767874D 00	0.20107045D 00	0.66467230D 00	0.16112344D-01
	0.20675842D-01	0.28854451D-01	0.73775771D-03	0.19348518D-04	-0.50757932D-03
	0.42479068D-04	0.27524379D 00	0.35342444D 01	0.55905101D-04	0.71036574D-06
	0.90119940D-04	0.71006049D-07	-0.43635766D-06	-0.26593599D-01	-0.17591024D-01
	-0.78053282D-02	0.34305911D 01	4		
	-0.44184622D-01	0.12904076D 01	0.48047038D-01	-0.34398927D-01	0.44268607D 01
0.54000000D 03	0.37034608D-01	0.27001771D 02	0.32870316D 01	-0.32745400D 04	0.10038147D-02
0.77489000D 03	-0.73187694D 00	0.18766071D 00	0.20109487D 00	0.67433372D 00	0.16108474D-01
	0.20675842D-01	0.28957489D-01	0.74093375D-03	0.19350179D-04	-0.52292215D-03
	0.42464866D-04	0.27524379D 00	0.35342444D 01	0.55993754D-04	0.71226312D-06
	0.90186740D-04	0.71015453D-07	-0.45027350D-06	-0.27104776D-01	-0.18058270D-01
	-0.80414952D-02	0.34873553D 01	4		
	-0.45163046D-01	0.12789302D 01	0.47364680D-01	-0.35146271D-01	0.44600841D 01
0.54000000D 03	0.36832451D-01	0.27149972D 02	0.33110907D 01	-0.32531155D 04	0.10178214D-02
0.77469000D 03	-0.75025753D 00	0.18764266D 00	0.20111966D 00	0.68434925D 00	0.16104604D-01
	0.20675842D-01	0.29063668D-01	0.74420311D-03	0.19351841D-04	-0.53899215D-03
	0.42450662D-04	0.27524379D 00	0.35342444D 01	0.56085252D-04	0.71421303D-06
	0.90253540D-04	0.71024845D-07	-0.46481886D-06	-0.27633824D-01	-0.18545221D-01
	-0.82877198D-02	0.35463332D 01	4		
	-0.46179045D-01	0.12673022D 01	0.46677847D-01	-0.35921543D-01	0.44942759D 01
0.54000000D 03	0.36627454D-01	0.27301925D 02	0.33357752D 01	-0.32316702D 04	0.10322140D-02
0.77449000D 03	-0.76940727D 00	0.18762461D 00	0.20114484D 00	0.69473355D 00	0.16100733D-01
	0.20675842D-01	0.29173137D-01	0.74757023D-03	0.19353502D-04	-0.55583246D-03
	0.42436456D-04	0.27524379D 00	0.35342444D 01	0.56179726D-04	0.71621785D-06
	0.90320340D-04	0.71034227D-07	-0.48002899D-06	-0.28181429D-01	-0.19052886D-01
	-0.85445236D-02	0.36076284D 01	4		
	-0.47234315D-01	0.12555199D 01	0.45986496D-01	-0.36725953D-01	0.45294491D 01
0.54000000D 03	0.36419539D-01	0.27457789D 02	0.33611114D 01	-0.32102326D 04	0.10469993D-02
0.77429000D 03	-0.78936437D 00	0.18760653D 00	0.20117038D 00	0.70550160D 00	0.16096862D-01
	0.20675842D-01	0.29286055D-01	0.75103981D-03	0.19355164D-04	-0.57348890D-03
	0.42422248D-04	0.27524379D 00	0.35342444D 01	0.56277311D-04	0.71828011D-06
	0.90387140D-04	0.71043598D-07	-0.49594112D-06	-0.28748286D-01	-0.19582320D-01
	-0.88124524D-02	0.36713480D 01	4		
	-0.48330606D-01	0.12435795D 01	0.45290593D-01	-0.37560739D-01	0.45656132D 01
0.54000000D 03	0.36208627D-01	0.27617727D 02	0.33871269D 01	-0.31888345D 04	0.10621834D-02
0.77409000D 03	-0.81016854D 00	0.18758845D 00	0.20119629D 00	0.71666857D 00	0.16092991D-01
	0.20675842D-01	0.29402590D-01	0.75461686D-03	0.19356826D-04	-0.59201001D-03
	0.42408039D-04	0.27524379D 00	0.35342444D 01	0.56378152D-04	0.72040246D-06
	0.90453940D-04	0.71052958D-07	-0.51259445D-06	-0.29335091D-01	-0.20134621D-01
	-0.90920762D-02	0.37376026D 01	4		
	-0.49469712D-01	0.12314775D 01	0.44590109D-01	-0.38427167D-01	0.46027733D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.35994639D-01	0.27781915D 02	0.34138507D 01	-0.31675112D 04	0.10777707D-02
0.77389000D 03	-0.83186092D 00	0.18757035D 00	0.20122254D 00	0.72824972D 00	0.16089120D-01
	0.20675842D-01	0.29522920D-01	0.75830666D-03	0.19358488D-04	-0.61144718D-03
	0.42393827D-04	0.27524379D 00	0.35342444D 01	0.56482401D-04	0.72258772D-06
	0.90520740D-04	0.71062307D-07	-0.53003018D-06	-0.29942533D-01	-0.20710932D-01
	-0.93839881D-02	0.38065058D 01	4		
	-0.50653465D-01	0.12192100D 01	0.43885022D-01	-0.39326521D-01	0.46409298D 01
0.54000000D 03	0.35777492D-01	0.27950534D 02	0.34413131D 01	-0.31463025D 04	0.10937642D-02
0.77369000D 03	-0.85448397D 00	0.18755224D 00	0.20124914D 00	0.74026031D 00	0.16085248D-01
	0.20675842D-01	0.29647234D-01	0.76211481D-03	0.19360150D-04	-0.63185473D-03
	0.42379614D-04	0.27524379D 00	0.35342444D 01	0.56590219D-04	0.72483885D-06
	0.90587540D-04	0.71071645D-07	-0.54829153D-06	-0.30571293D-01	-0.21312435D-01
	-0.96888039D-02	0.38781738D 01	4		
	-0.51883728D-01	0.12067734D 01	0.43175324D-01	-0.40260097D-01	0.46800769D 01
0.54000000D 03	0.35557106D-01	0.28123774D 02	0.34695458D 01	-0.31252523D 04	0.11101650D-02
0.77349000D 03	-0.87808129D 00	0.18753412D 00	0.20127606D 00	0.75271541D 00	0.16081376D-01
	0.20675842D-01	0.29775731D-01	0.76604725D-03	0.19361812D-04	-0.65328994D-03
	0.42365399D-04	0.27524379D 00	0.35342444D 01	0.56701775D-04	0.72715898D-06
	0.90654340D-04	0.71080973D-07	-0.56742372D-06	-0.31222029D-01	-0.21940350D-01
	-0.10007160D-01	0.39527248D 01	4		
	-0.53162379D-01	0.11941640D 01	0.42461016D-01	-0.41229189D-01	0.47202017D 01
0.54000000D 03	0.35333399D-01	0.28301834D 02	0.34985821D 01	-0.31044096D 04	0.11269718D-02
0.77329000D 03	-0.90269744D 00	0.18751598D 00	0.20130328D 00	0.76562972D 00	0.16077503D-01
	0.20675842D-01	0.29908622D-01	0.77011026D-03	0.19363475D-04	-0.67581308D-03
	0.42351182D-04	0.27524379D 00	0.35342444D 01	0.56817249D-04	0.72955141D-06
	0.90721140D-04	0.71090289D-07	-0.58747390D-06	-0.31895369D-01	-0.22595928D-01
	-0.10339710D-01	0.40302778D 01	4		
	-0.54491297D-01	0.11813783D 01	0.41742113D-01	-0.42235079D-01	0.47612830D 01
0.54000000D 03	0.35106290D-01	0.28484924D 02	0.35284567D 01	-0.30838287D 04	0.11441805D-02
0.77309000D 03	-0.92837762D 00	0.18749783D 00	0.20133077D 00	0.77901737D 00	0.16073630D-01
	0.20675842D-01	0.30046133D-01	0.77431049D-03	0.19365138D-04	-0.69948739D-03
	0.42336963D-04	0.27524379D 00	0.35342444D 01	0.56936829D-04	0.73201962D-06
	0.90787940D-04	0.71099595D-07	-0.60849104D-06	-0.32591894D-01	-0.23280447D-01
	-0.10687125D-01	0.41109523D 01	4		
	-0.55872341D-01	0.11684129D 01	0.41018644D-01	-0.43279020D-01	0.48032899D 01
0.54000000D 03	0.34875700D-01	0.28673259D 02	0.35592059D 01	-0.30635696D 04	0.11617839D-02
0.77289000D 03	-0.95516738D 00	0.18747966D 00	0.20135853D 00	0.79289164D 00	0.16069757D-01
	0.20675842D-01	0.30188499D-01	0.77865498D-03	0.19366801D-04	-0.72437904D-03
	0.42322742D-04	0.27524379D 00	0.35342444D 01	0.57060714D-04	0.73456726D-06
	0.90854740D-04	0.71108889D-07	-0.63052580D-06	-0.33312131D-01	-0.23995199D-01
	-0.11050086D-01	0.41948666D 01	4		
	-0.57307331D-01	0.11552645D 01	0.40290658D-01	-0.44362218D-01	0.48461804D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.34641551D-01	0.28867068D 02	0.35908675D 01	-0.30436986D 04	0.11797710D-02
0.77269000D 03	-0.98311216D 00	0.18746149D 00	0.20138649D 00	0.80726465D 00	0.16065884D-01
	0.20675842D-01	0.30335970D-01	0.78315116D-03	0.19368465D-04	-0.75055696D-03
	0.42308519D-04	0.27524379D 00	0.35342444D 01	0.57189111D-04	0.73719819D-06
	0.90921540D-04	0.71118173D-07	-0.65363033D-06	-0.34056530D-01	-0.24741485D-01
	-0.11429279D-01	0.42821362D 01	4		
	-0.58798016D-01	0.11419299D 01	0.39558223D-01	-0.45485810D-01	0.48898994D 01
0.54000000D 03	0.34403767D-01	0.29066584D 02	0.36234807D 01	-0.30242888D 04	0.11981265D-02
0.77249000D 03	-0.10122567D 01	0.18744330D 00	0.20141465D 00	0.82214703D 00	0.16062010D-01
	0.20675842D-01	0.30488812D-01	0.78780689D-03	0.19370128D-04	-0.77809271D-03
	0.42294294D-04	0.27524379D 00	0.35342444D 01	0.57322241D-04	0.73991646D-06
	0.90988340D-04	0.71127446D-07	-0.67785798D-06	-0.34825446D-01	-0.25520598D-01
	-0.11825392D-01	0.43728727D 01	4		
	-0.60346044D-01	0.11284063D 01	0.38821428D-01	-0.46650838D-01	0.49343771D 01
0.54000000D 03	0.34162277D-01	0.29272053D 02	0.36570864D 01	-0.30054205D 04	0.12168302D-02
0.77229000D 03	-0.10426447D 01	0.18742509D 00	0.20144293D 00	0.83754749D 00	0.16058135D-01
	0.20675842D-01	0.30647303D-01	0.79263046D-03	0.19371792D-04	-0.80706017D-03
	0.42280068D-04	0.27524379D 00	0.35342444D 01	0.57460333D-04	0.74272630D-06
	0.91055140D-04	0.71136708D-07	-0.70326295D-06	-0.35619118D-01	-0.26333808D-01
	-0.12239103D-01	0.44671811D 01	4		
	-0.61952926D-01	0.11146912D 01	0.38080389D-01	-0.47858221D-01	0.49795273D 01
0.54000000D 03	0.33917014D-01	0.29483728D 02	0.36917267D 01	-0.29871815D 04	0.12358562D-02
0.77209000D 03	-0.10743177D 01	0.18740687D 00	0.20147131D 00	0.85347237D 00	0.16054261D-01
	0.20675842D-01	0.30811736D-01	0.79763058D-03	0.19373456D-04	-0.83753517D-03
	0.42265840D-04	0.27524379D 00	0.35342444D 01	0.57603628D-04	0.74563216D-06
	0.91121940D-04	0.71145958D-07	-0.72989984D-06	-0.36437647D-01	-0.27182343D-01
	-0.12671074D-01	0.45651579D 01	4		
	-0.63619990D-01	0.11007823D 01	0.37335248D-01	-0.49108721D-01	0.50252450D 01
0.54000000D 03	0.33667916D-01	0.29701868D 02	0.37274454D 01	-0.29696680D 04	0.12551724D-02
0.77189000D 03	-0.11073144D 01	0.18738864D 00	0.20149971D 00	0.86992518D 00	0.16050386D-01
	0.20675842D-01	0.30982422D-01	0.80281646D-03	0.19375120D-04	-0.86959507D-03
	0.42251610D-04	0.27524379D 00	0.35342444D 01	0.57752378D-04	0.74863868D-06
	0.91188740D-04	0.71155198D-07	-0.75782316D-06	-0.37280966D-01	-0.28067369D-01
	-0.13121938D-01	0.46668882D 01	4		
	-0.65348335D-01	0.10866780D 01	0.36586182D-01	-0.50402905D-01	0.50714045D 01
0.54000000D 03	0.33414928D-01	0.29926744D 02	0.37642873D 01	-0.29529847D 04	0.12747399D-02
0.77169000D 03	-0.11416698D 01	0.18737040D 00	0.20152807D 00	0.88690597D 00	0.16046511D-01
	0.20675842D-01	0.31159685D-01	0.80819774D-03	0.19376785D-04	-0.90331815D-03
	0.42237378D-04	0.27524379D 00	0.35342444D 01	0.57906847D-04	0.75175071D-06
	0.91255540D-04	0.71164428D-07	-0.78708664D-06	-0.38148813D-01	-0.28989964D-01
	-0.13592290D-01	0.47724427D 01	4		
	-0.67138777D-01	0.10723769D 01	0.35833399D-01	-0.51741103D-01	0.51178576D 01

TABLE I (Continued)

PRESSURE(PSIA)	SPVL(CUFT/LB)	DEN(LB/CUFT)	DERPTV(PSI/R)	DERPVT(PSI/(CUFT/LB))	DERVTP(CUFT/LB)/R
TEMP(R)	DERDTP(LB/CUFT)/R	CVLP(BTU/LB-R)	CVHP(BTU/LB-R)	CPHP(BTU/LB-R)	VISLP(CP)
	XI	VISHP(CP)	DERUDT(CP/(LB/CUFT))	DERUTD(CP/R)	DERUTP(CP/R)
	CONLP(CAL/SEC-CM-K)	CONZC	CONLA	CONHP(CAL/SEC-CM-K)	DERKDT
	DERCVT(BTU/LB-R)/R	DERKTD	DERKTP(CAL/SEC-CM-K)/R	DCOEFF	ECOEFF
	HCOEFF	PRNO	ITR		
	A111	A222	A333	B111	B222
0.54000000D 03	0.33158006D-01	0.30158629D 02	0.38022985D 01	-0.29372450D 04	0.12945118D-02
0.77149000D 03	-0.11774141D 01	0.18735214D 00	0.20155632D 00	0.90441081D 00	0.16042635D-01
	0.20675842D-01	0.31343866D-01	0.81378454D-03	0.19378449D-04	-0.93878290D-03
	0.42223144D-04	0.27524379D 00	0.35342444D 01	0.58067306D-04	0.75497330D-06
	0.91322340D-04	0.71173646D-07	-0.81774253D-06	-0.39040702D-01	-0.29951089D-01
	-0.14082667D-01	0.48818747D 01	4		
	-0.68991791D-01	0.10578786D 01	0.35077146D-01	-0.53123369D-01	0.51644310D 01
0.54000000D 03	0.32897113D-01	0.30397804D 02	0.38415260D 01	-0.29225718D 04	0.13144334D-02
0.77129000D 03	-0.12145713D 01	0.18733387D 00	0.20158437D 00	0.92243117D 00	0.16038760D-01
	0.20675842D-01	0.31535320D-01	0.81958743D-03	0.19380114D-04	-0.97606725D-03
	0.42208908D-04	0.27524379D 00	0.35342444D 01	0.58234040D-04	0.75831167D-06
	0.91389140D-04	0.71182853D-07	-0.84984074D-06	-0.39955890D-01	-0.30951557D-01
	-0.14593539D-01	0.49952162D 01	4		
	-0.70907447D-01	0.10431831D 01	0.34317714D-01	-0.54549429D-01	0.52109252D 01
0.54000000D 03	0.32632227D-01	0.30644553D 02	0.38820179D 01	-0.29090969D 04	0.13344409D-02
0.77109000D 03	-0.12531583D 01	0.18731559D 00	0.20161211D 00	0.94095324D 00	0.16034883D-01
	0.20675842D-01	0.31734419D-01	0.82561745D-03	0.19381779D-04	-0.10152476D-02
	0.42194671D-04	0.27524379D 00	0.35342444D 01	0.58407341D-04	0.76177123D-06
	0.91455940D-04	0.71192049D-07	-0.88342790D-06	-0.40893346D-01	-0.31992002D-01
	-0.15125289D-01	0.51124745D 01	4		
	-0.72885348D-01	0.10282913D 01	0.33555435D-01	-0.56018635D-01	0.52571130D 01
0.54000000D 03	0.32363336D-01	0.30899163D 02	0.39238227D 01	-0.28969616D 04	0.13544614D-02
0.77089000D 03	-0.12931833D 01	0.18729729D 00	0.20163944D 00	0.95995738D 00	0.16031007D-01
	0.20675842D-01	0.31941550D-01	0.83188610D-03	0.19383445D-04	-0.10563977D-02
	0.42180431D-04	0.27524379D 00	0.35342444D 01	0.58587513D-04	0.76535756D-06
	0.91522740D-04	0.71201235D-07	-0.91854634D-06	-0.41851724D-01	-0.33072838D-01
	-0.15678193D-01	0.52336283D 01	4		
	-0.74924563D-01	0.10132049D 01	0.32790692D-01	-0.57529918D-01	0.53027380D 01
0.54000000D 03	0.32090447D-01	0.31161922D 02	0.39669893D 01	-0.28863164D 04	0.13744125D-02
0.77069000D 03	-0.13346444D 01	0.18727898D 00	0.20166624D 00	0.97941753D 00	0.16027130D-01
	0.20675842D-01	0.32157114D-01	0.83840528D-03	0.19385110D-04	-0.10995878D-02
	0.42166190D-04	0.27524379D 00	0.35342444D 01	0.58774865D-04	0.76907636D-06
	0.91589540D-04	0.71210409D-07	-0.95523305D-06	-0.42829335D-01	-0.34194231D-01
	-0.16252407D-01	0.53586242D 01	4		
	-0.77023566D-01	0.99792683D 00	0.32023918D-01	-0.59081742D-01	0.53475149D 01
0.54000000D 03	0.31813582D-01	0.31433116D 02	0.40115666D 01	-0.28773203D 04	0.13942023D-02
0.77049000D 03	-0.13775287D 01	0.18726065D 00	0.20169237D 00	0.99930074D 00	0.16023253D-01
	0.20675842D-01	0.32381526D-01	0.84518731D-03	0.19386776D-04	-0.11448830D-02
	0.42151947D-04	0.27524379D 00	0.35342444D 01	0.58969713D-04	0.77293350D-06
	0.91656340D-04	0.71219573D-07	-0.99351851D-06	-0.43824122D-01	-0.35356055D-01
	-0.16847946D-01	0.54873733D 01	4		
	-0.79180178D-01	0.98246087D 00	0.31255599D-01	-0.60672068D-01	0.53911295D 01

TABLE I (Continued)

PRESSURE(PSIA)	SPVL(CUFT/LB)	DEN(LB/CUFT)	DERPTV(PSI/R)	DERPVT(PSI/(CUFT/LB))	DERVTP(CUFT/LB)/R
TEMP(R)	DERDTP(LB/CUFT)/R	CVLP(BTU/LB-R)	CVHP(BTU/LB-R)	CPHP(BTU/LB-R)	VISLP(CP)
	XI	VISHP(CP)	DERUDT(CP/(LB/CUFT))	DERUTD(CP/R)	DERUTP(CP/R)
	CONLP(CAL/SEC-CM-K)	CONZC	CONLA	CONHP(CAL/SEC-CM-K)	DERKDT
	DERCVT(BTU/LB-R)/R	DERKTD	DERKTP(CAL/SEC-CM-K)/R	DCOEFF	ECOEFF
	HCOEFF	PRNO	ITR		
	A111	A222	A333	B111	B222
0.54000000D 03	0.31532783D-01	0.31713027D 02	0.40576035D 01	-0.28701407D 04	0.14137298D-02
0.77029000D 03	-0.14218108D 01	0.18724231D 00	0.20171766D 00	0.10195668D 01	0.16019375D-01
	0.20675842D-01	0.32615212D-01	0.85224489D-03	0.19388442D-04	-0.11923425D-02
	0.42137702D-04	0.27524379D 00	0.35342444D 01	0.59172380D-04	0.77693491D-06
	0.91723140D-04	0.71228725D-07	-0.10334257D-05	-0.44833652D-01	-0.36557866D-01
	-0.17464664D-01	0.56197479D 01	4		
	-0.81391518D-01	0.96681217D 00	0.30486279D-01	-0.62298315D-01	0.54332407D 01
0.54000000D 03	0.31248115D-01	0.32001931D 02	0.41051483D 01	-0.28649520D 04	0.14328855D-02
0.77009000D 03	-0.14674518D 01	0.18722396D 00	0.20174196D 00	0.10401679D 01	0.16015497D-01
	0.20675842D-01	0.32858610D-01	0.85959102D-03	0.19390108D-04	-0.12420183D-02
	0.42123456D-04	0.27524379D 00	0.35342444D 01	0.59383192D-04	0.78108660D-06
	0.91789940D-04	0.71237867D-07	-0.10749691D-05	-0.45855103D-01	-0.37798869D-01
	-0.18102245D-01	0.57555803D 01	4		
	-0.83653972D-01	0.95098710D 00	0.29716554D-01	-0.63957348D-01	0.54734826D 01
0.54000000D 03	0.30959662D-01	0.32300094D 02	0.41542484D 01	-0.28619350D 04	0.14515523D-02
0.76989000D 03	-0.15143989D 01	0.18720560D 00	0.20176508D 00	0.10610491D 01	0.16011619D-01
	0.20675842D-01	0.33112170D-01	0.86723901D-03	0.19391774D-04	-0.12939540D-02
	0.42109207D-04	0.27524379D 00	0.35342444D 01	0.59602477D-04	0.78539462D-06
	0.91856740D-04	0.71246997D-07	-0.11181537D-05	-0.46885277D-01	-0.39077898D-01
	-0.18760189D-01	0.58946605D 01	4		
	-0.85963175D-01	0.93499345D 00	0.28947081D-01	-0.65645465D-01	0.55114690D 01
0.54000000D 03	0.30667535D-01	0.32607773D 02	0.42049501D 01	-0.28612752D 04	0.14696070D-02
0.76969000D 03	-0.15625844D 01	0.18718722D 00	0.20178680D 00	0.10821478D 01	0.16007741D-01
	0.20675842D-01	0.33376345D-01	0.87520239D-03	0.19393441D-04	-0.13481842D-02
	0.42094957D-04	0.27524379D 00	0.35342444D 01	0.59830563D-04	0.78986502D-06
	0.91923540D-04	0.71256117D-07	-0.11629747D-05	-0.47920612D-01	-0.40393404D-01
	-0.19437803D-01	0.60367373D 01	4		
	-0.88314016D-01	0.91884041D 00	0.28178570D-01	-0.67358415D-01	0.55467982D 01
0.54000000D 03	0.30371868D-01	0.32925205D 02	0.42572986D 01	-0.28631615D 04	0.14869223D-02
0.76949000D 03	-0.16119266D 01	0.18716883D 00	0.20180691D 00	0.11033952D 01	0.16003862D-01
	0.20675842D-01	0.33651598D-01	0.88349488D-03	0.19395107D-04	-0.14047338D-02
	0.42080705D-04	0.27524379D 00	0.35342444D 01	0.60067775D-04	0.79450382D-06
	0.91990340D-04	0.71265226D-07	-0.12094166D-05	-0.48957221D-01	-0.41743449D-01
	-0.20134200D-01	0.61815192D 01	4		
	-0.90700670D-01	0.90253866D 00	0.27411785D-01	-0.69091421D-01	0.55790601D 01
0.54000000D 03	0.30072824D-01	0.33252614D 02	0.43113371D 01	-0.28677846D 04	0.15033685D-02
0.76929000D 03	-0.16623292D 01	0.18715042D 00	0.20182516D 00	0.11247162D 01	0.15999983D-01
	0.20675842D-01	0.33938394D-01	0.89213031D-03	0.19396774D-04	-0.14636175D-02
	0.42066451D-04	0.27524379D 00	0.35342444D 01	0.60314436D-04	0.79931695D-06
	0.92057140D-04	0.71274324D-07	-0.12574536D-05	-0.49990934D-01	-0.43125715D-01
	-0.20848303D-01	0.63286775D 01	4		
	-0.93116649D-01	0.88610037D 00	0.26647540D-01	-0.70839237D-01	0.56078435D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.29770587D-01	0.33590201D 02	0.43671076D 01	-0.28753356D 04	0.15188166D-02
0.76909000D 03	-0.17136832D 01	0.18713201D 00	0.20184129D 00	0.11460311D 01	0.15996103D-01
	0.20675842D-01	0.34237201D-01	0.90112256D-03	0.19398441D-04	-0.15248401D-02
	0.42052195D-04	0.27524379D 00	0.35342444D 01	0.60570861D-04	0.80431023D-06
	0.92123940D-04	0.71283411D-07	-0.13070495D-05	-0.51017355D-01	-0.44537524D-01
	-0.21578850D-01	0.64778502D 01	4		
	-0.95554879D-01	0.86953917D 00	0.25886692D-01	-0.72596205D-01	0.56327445D 01
0.54000000D 03	0.29465372D-01	0.33938143D 02	0.44246501D 01	-0.28860049D 04	0.15331402D-02
0.76889000D 03	-0.17658671D 01	0.18711357D 00	0.20185503D 00	0.11672564D 01	0.15992223D-01
	0.20675842D-01	0.34548486D-01	0.91048552D-03	0.19400109D-04	-0.15883963D-02
	0.42037937D-04	0.27524379D 00	0.35342444D 01	0.60837357D-04	0.80948933D-06
	0.92190740D-04	0.71292486D-07	-0.13581581D-05	-0.52031930D-01	-0.45975859D-01
	-0.22324410D-01	0.66286478D 01	4		
	-0.98007789D-01	0.85287014D 00	0.25130136D-01	-0.74356340D-01	0.56533758D 01
0.54000000D 03	0.29157414D-01	0.34296594D 02	0.44840032D 01	-0.28999815D 04	0.15462179D-02
0.76869000D 03	-0.18187486D 01	0.18709513D 00	0.20186607D 00	0.11883062D 01	0.15988343D-01
	0.20675842D-01	0.34872714D-01	0.92023301D-03	0.19401776D-04	-0.16542707D-02
	0.42023678D-04	0.27524379D 00	0.35342444D 01	0.61114221D-04	0.81485973D-06
	0.92257540D-04	0.71301551D-07	-0.14107234D-05	-0.53030007D-01	-0.47437396D-01
	-0.23083391D-01	0.67806579D 01	4		
	-0.10046740D 00	0.83610970D 00	0.24378797D-01	-0.76113397D-01	0.56693738D 01
0.54000000D 03	0.28846975D-01	0.34665679D 02	0.45452042D 01	-0.29174536D 04	0.15579354D-02
0.76849000D 03	-0.18721855D 01	0.18707667D 00	0.20187411D 00	0.12090932D 01	0.15984463D-01
	0.20675842D-01	0.35210347D-01	0.93037875D-03	0.19403444D-04	-0.17224381D-02
	0.42009417D-04	0.27524379D 00	0.35342444D 01	0.61401739D-04	0.82042669D-06
	0.92324340D-04	0.71310605D-07	-0.14646803D-05	-0.54006889D-01	-0.48918522D-01
	-0.23854053D-01	0.69334506D 01	4		
	-0.10292541D 00	0.81927553D 00	0.23633621D-01	-0.77860942D-01	0.56804064D 01
0.54000000D 03	0.28534338D-01	0.35045495D 02	0.46082895D 01	-0.29386108D 04	0.15681864D-02
0.76829000D 03	-0.19260257D 01	0.18705820D 00	0.20187881D 00	0.12295295D 01	0.15980582D-01
	0.20675842D-01	0.35561840D-01	0.94093624D-03	0.19405112D-04	-0.17928623D-02
	0.41995154D-04	0.27524379D 00	0.35342444D 01	0.61700181D-04	0.82619520D-06
	0.92391140D-04	0.71319648D-07	-0.15199535D-05	-0.54957868D-01	-0.50415341D-01
	-0.24634507D-01	0.70865806D 01	4		
	-0.10537321D 00	0.80238644D 00	0.22895566D-01	-0.79592375D-01	0.56861762D 01
0.54000000D 03	0.28219805D-01	0.35436106D 02	0.46732950D 01	-0.29636483D 04	0.15768724D-02
0.76809000D 03	-0.19801063D 01	0.18703971D 00	0.20187982D 00	0.12495263D 01	0.15976701D-01
	0.20675842D-01	0.35927639D-01	0.95191876D-03	0.19406780D-04	-0.18654936D-02
	0.41980889D-04	0.27524379D 00	0.35342444D 01	0.62009802D-04	0.83216998D-06
	0.92457940D-04	0.71328680D-07	-0.15764564D-05	-0.55878215D-01	-0.51923634D-01
	-0.25422696D-01	0.72395858D 01	4		
	-0.10780185D 00	0.78546228D 00	0.22165593D-01	-0.81300911D-01	0.56864216D 01

TABLE I (Continued)

PRESSURE(PSIA)	SPVL(CUFT/LB)	DEN(LB/CUFT)	DERPTV(PSI/R)	DERPVT(PSI/(CUFT/LB))	DERVTP(CUFT/LB)/R
TEMP(R)	DERDTP(LB/CUFT)/R	CVLP(BTU/LB-R)	CVHP(BTU/LB-R)	CPHP(BTU/LB-R)	VISLP(CP)
	XI	VISHP(CP)	DERUDT(CP/(LB/CUFT))	DERUTD(CP/R)	DERUTP(CP/R)
	CONLP(CAL/SEC-CM-K)	CONZC	CONLA	CONHP(CAL/SEC-CM-K)	DERKDT
	DERCVT(BTU/LB-R)/R	DERKTD	DERKTP(CAL/SEC-CM-K)/R	DCOEFF	ECOEFF
	HCOEFF	PRNO	ITR		
	A111	A222	A333	B111	B222
0.54000000D 03	0.27903700D-01	0.35837542D 02	0.47402567D 01	-0.29927736D 04	0.15839009D-02
0.76789000D 03	-0.20342504D 01	0.18702122D 00	0.20187680D 00	0.12689931D 01	0.15972819D-01
	0.20675842D-01	0.36308180D-01	0.96333924D-03	0.19408449D-04	-0.19402648D-02
	0.41966622D-04	0.27524379D 00	0.35342444D 01	0.62330839D-04	0.83835536D-06
	0.92524740D-04	0.71337702D-07	-0.16340871D-05	-0.56763113D-01	-0.53438780D-01
	-0.26216350D-01	0.73919798D 01	4		
	-0.11020189D 00	0.76852378D 00	0.21444657D-01	-0.82979463D-01	0.56809123D 01
0.54000000D 03	0.27586361D-01	0.36249797D 02	0.48092112D 01	-0.30262183D 04	0.15891819D-02
0.76769000D 03	-0.20882609D 01	0.18700270D 00	0.20186937D 00	0.12878354D 01	0.15968937D-01
	0.20675842D-01	0.36703881D-01	0.97521013D-03	0.19410118D-04	-0.20170831D-02
	0.41952354D-04	0.27524379D 00	0.35342444D 01	0.62663504D-04	0.84475529D-06
	0.92591540D-04	0.71346712D-07	-0.16927228D-05	-0.57607521D-01	-0.54955580D-01
	-0.27012897D-01	0.75432356D 01	4		
	-0.11256310D 00	0.75159248D 00	0.20733702D-01	-0.84620417D-01	0.56694392D 01
0.54000000D 03	0.27268150D-01	0.36672822D 02	0.48801965D 01	-0.30642532D 04	0.15926218D-02
0.76749000D 03	-0.21419105D 01	0.18698418D 00	0.20185716D 00	0.13059500D 01	0.15965055D-01
	0.20675842D-01	0.37115141D-01	0.98754322D-03	0.19411786D-04	-0.20958174D-02
	0.41938084D-04	0.27524379D 00	0.35342444D 01	0.63007985D-04	0.85137322D-06
	0.92658340D-04	0.71355711D-07	-0.17522095D-05	-0.58405936D-01	-0.56467990D-01
	-0.27809325D-01	0.76927580D 01	4		
	-0.11487393D 00	0.73469072D 00	0.20033656D-01	-0.86215262D-01	0.56517979D 01
0.54000000D 03	0.26949442D-01	0.37106519D 02	0.49532514D 01	-0.31072107D 04	0.15941150D-02
0.76729000D 03	-0.21949271D 01	0.18696564D 00	0.20183980D 00	0.13232189D 01	0.15961172D-01
	0.20675842D-01	0.37542324D-01	0.10003494D-02	0.19413455D-04	-0.21762805D-02
	0.41923812D-04	0.27524379D 00	0.35342444D 01	0.63364431D-04	0.85821192D-06
	0.92725140D-04	0.71364699D-07	-0.18123479D-05	-0.59152060D-01	-0.57968720D-01
	-0.28601975D-01	0.78398420D 01	4		
	-0.11712078D 00	0.71784162D 00	0.19345431D-01	-0.87754035D-01	0.56277648D 01
0.54000000D 03	0.26630642D-01	0.37550729D 02	0.50284159D 01	-0.31555136D 04	0.15935333D-02
0.76709000D 03	-0.22469731D 01	0.18694709D 00	0.20181690D 00	0.13395006D 01	0.15957290D-01
	0.20675842D-01	0.37985751D-01	0.10136381D-02	0.19415125D-04	-0.22582024D-02
	0.41909538D-04	0.27524379D 00	0.35342444D 01	0.63732950D-04	0.86527334D-06
	0.92791940D-04	0.71373676D-07	-0.18728722D-05	-0.59838335D-01	-0.59448672D-01
	-0.29386247D-01	0.79836155D 01	4		
	-0.11928701D 00	0.70106925D 00	0.18669924D-01	-0.89224582D-01	0.55970673D 01
0.54000000D 03	0.26312178D-01	0.38005215D 02	0.51057299D 01	-0.32097125D 04	0.15907125D-02
0.76689000D 03	-0.22976195D 01	0.18692852D 00	0.20178811D 00	0.13546187D 01	0.15953406D-01
	0.20675842D-01	0.38445677D-01	0.10274170D-02	0.19416794D-04	-0.23411966D-02
	0.41895262D-04	0.27524379D 00	0.35342444D 01	0.64113582D-04	0.87255827D-06
	0.92858740D-04	0.71382643D-07	-0.19334242D-05	-0.60455373D-01	-0.60896224D-01
	-0.30156234D-01	0.81229643D 01	4		
	-0.12135160D 00	0.68439889D 00	0.18008025D-01	-0.90611607D-01	0.55593477D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.25994520D-01	0.38469646D 02	0.51852311D 01	-0.32705326D 04	0.15854394D-02
0.76669000D 03	-0.23463134D 01	0.18690995D 00	0.20175306D 00	0.13683490D 01	0.15949523D-01
	0.20675842D-01	0.38922265D-01	0.10416910D-02	0.19418464D-04	-0.24247150D-02
	0.41880985D-04	0.27524379D 00	0.35342444D 01	0.64506289D-04	0.88006596D-06
	0.92925540D-04	0.71391598D-07	-0.19935190D-05	-0.60991292D-01	-0.62296349D-01
	-0.30904258D-01	0.82564415D 01	4		
	-0.12328764D 00	0.66785734D 00	0.17360631D-01	-0.91895550D-01	0.55141251D 01
0.54000000D 03	0.25678184D-01	0.38943563D 02	0.52669520D 01	-0.33389301D 04	0.15774370D-02
0.76649000D 03	-0.23923428D 01	0.18689135D 00	0.20171142D 00	0.13804046D 01	0.15945639D-01
	0.20675842D-01	0.39415547D-01	0.10564609D-02	0.19420134D-04	-0.25079963D-02
	0.41866706D-04	0.27524379D 00	0.35342444D 01	0.64910917D-04	0.88779359D-06
	0.92992340D-04	0.71400542D-07	-0.20525060D-05	-0.61431019D-01	-0.63629621D-01
	-0.31620352D-01	0.83821654D 01	4		
	-0.12506064D 00	0.65147349D 00	0.16728656D-01	-0.93051371D-01	0.54607585D 01
0.54000000D 03	0.25363750D-01	0.39426346D 02	0.53509145D 01	-0.34161568D 04	0.15663551D-02
0.76629000D 03	-0.24347999D 01	0.18687275D 00	0.20166291D 00	0.13904232D 01	0.15941755D-01
	0.20675842D-01	0.39925379D-01	0.10717224D-02	0.19421804D-04	-0.25900078D-02
	0.41852425D-04	0.27524379D 00	0.35342444D 01	0.65327161D-04	0.89573555D-06
	0.93059140D-04	0.71409475D-07	-0.21095273D-05	-0.61755656D-01	-0.64871213D-01
	-0.32291734D-01	0.84977170D 01	4		
	-0.12662687D 00	0.63527888D 00	0.16113055D-01	-0.94047391D-01	0.53984202D 01
0.54000000D 03	0.25051876D-01	0.39917170D 02	0.54371236D 01	-0.35038306D 04	0.15517655D-02
0.76609000D 03	-0.24725529D 01	0.18685413D 00	0.20160730D 00	0.13979566D 01	0.15937870D-01
	0.20675842D-01	0.40451376D-01	0.10874642D-02	0.19423474D-04	-0.26693893D-02
	0.41838142D-04	0.27524379D 00	0.35342444D 01	0.65754524D-04	0.90388264D-06
	0.93125940D-04	0.71418398D-07	-0.21634793D-05	-0.61942090D-01	-0.65990075D-01
	-0.32902364D-01	0.86000574D 01	4		
	-0.12793216D 00	0.61930837D 00	0.15514837D-01	-0.94844454D-01	0.53260876D 01
0.54000000D 03	0.24743311D-01	0.40414963D 02	0.55255580D 01	-0.36040052D 04	0.15331715D-02
0.76589000D 03	-0.25042352D 01	0.18683550D 00	0.20154443D 00	0.14024702D 01	0.15933985D-01
	0.20675842D-01	0.40992845D-01	0.11036657D-02	0.19425145D-04	-0.27444132D-02
	0.41823857D-04	0.27524379D 00	0.35342444D 01	0.66192257D-04	0.91222108D-06
	0.93192740D-04	0.71427309D-07	-0.22129888D-05	-0.61963070D-01	-0.66948590D-01
	-0.33432744D-01	0.86854940D 01	4		
	-0.12891166D 00	0.60360072D 00	0.14935080D-01	-0.95395814D-01	0.52425705D 01
0.54000000D 03	0.24438910D-01	0.40918355D 02	0.56161605D 01	-0.37192295D 04	0.15100333D-02
0.76569000D 03	-0.25282665D 01	0.18681685D 00	0.20147424D 00	0.14033565D 01	0.15930100D-01
	0.20675842D-01	0.41548711D-01	0.11202948D-02	0.19426816D-04	-0.28129771D-02
	0.41809571D-04	0.27524379D 00	0.35342444D 01	0.66639314D-04	0.92073155D-06
	0.93259540D-04	0.71436209D-07	-0.22564185D-05	-0.61788079D-01	-0.67703113D-01
	-0.33860170D-01	0.87497378D 01	4		
	-0.12949119D 00	0.58819900D 00	0.14374942D-01	-0.95648249D-01	0.51465870D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.24139636D-01	0.41425645D 02	0.57088261D 01	-0.38525816D 04	0.14818183D-02
0.76549000D 03	-0.25429248D 01	0.18679820D 00	0.20139684D 00	0.13999690D 01	0.15926215D-01
	0.20675842D-01	0.42117444D-01	0.11373063D-02	0.19428486D-04	-0.28726558D-02
	0.41795283D-04	0.27524379D 00	0.35342444D 01	0.67094296D-04	0.92938824D-06
	0.93326340D-04	0.71445099D-07	-0.22919193D-05	-0.61385280D-01	-0.68205843D-01
	-0.34159674D-01	0.87880968D 01	4		
	-0.12959112D 00	0.57315055D 00	0.13835646D-01	-0.95544954D-01	0.50369025D 01
0.54000000D 03	0.23846552D-01	0.41934784D 02	0.58033919D 01	-0.40076584D 04	0.14480755D-02
0.76529000D 03	-0.25464785D 01	0.18677952D 00	0.20131244D 00	0.13916826D 01	0.15922329D-01
	0.20675842D-01	0.42697007D-01	0.11546394D-02	0.19430158D-04	-0.29208344D-02
	0.41780993D-04	0.27524379D 00	0.35342444D 01	0.67555423D-04	0.93815832D-06
	0.93393140D-04	0.71453977D-07	-0.23175460D-05	-0.60724732D-01	-0.68408411D-01
	-0.34305847D-01	0.87958418D 01	4		
	-0.12913314D 00	0.55850640D 00	0.13318452D-01	-0.95030580D-01	0.49125340D 01
0.54000000D 03	0.23560794D-01	0.42443392D 02	0.58996299D 01	-0.41885044D 04	0.14085290D-02
0.76509000D 03	-0.25373826D 01	0.18676084D 00	0.20122146D 00	0.13779763D 01	0.15918443D-01
	0.20675842D-01	0.43284839D-01	0.11722183D-02	0.19431829D-04	-0.29549345D-02
	0.41766701D-04	0.27524379D 00	0.35342444D 01	0.68020532D-04	0.94700188D-06
	0.93459940D-04	0.71462844D-07	-0.23314433D-05	-0.59782748D-01	-0.68267194D-01
	-0.34275581D-01	0.87687465D 01	4		
	-0.12804994D 00	0.54431978D 00	0.12824606D-01	-0.94058333D-01	0.47730021D 01
0.54000000D 03	0.23283528D-01	0.42948817D 02	0.59972445D 01	-0.43994647D 04	0.13631759D-02
0.76489000D 03	-0.25145156D 01	0.18674214D 00	0.20112451D 00	0.13585300D 01	0.15914556D-01
	0.20675842D-01	0.43877892D-01	0.11899523D-02	0.19433501D-04	-0.29727200D-02
	0.41752408D-04	0.27524379D 00	0.35342444D 01	0.68487116D-04	0.95587269D-06
	0.93526740D-04	0.71471701D-07	-0.23320851D-05	-0.58546795D-01	-0.67749837D-01
	-0.34051443D-01	0.87037442D 01	4		
	-0.12629663D 00	0.53064373D 00	0.12355258D-01	-0.92598238D-01	0.46185873D 01
0.54000000D 03	0.23015888D-01	0.43448247D 02	0.60958769D 01	-0.46449662D 04	0.13123624D-02
0.76469000D 03	-0.24774123D 01	0.18672343D 00	0.20102236D 00	0.13333143D 01	0.15910669D-01
	0.20675842D-01	0.44472733D-01	0.12077394D-02	0.19435172D-04	-0.29726332D-02
	0.41738112D-04	0.27524379D 00	0.35342444D 01	0.68952418D-04	0.96471989D-06
	0.93593540D-04	0.71480546D-07	-0.23185284D-05	-0.57019845D-01	-0.66841703D-01
	-0.33625049D-01	0.85995724D 01	4		
	-0.12386155D 00	0.51752808D 00	0.11911368D-01	-0.90644894D-01	0.44505202D 01
0.54000000D 03	0.22758900D-01	0.43938855D 02	0.61951292D 01	-0.49292444D 04	0.12568111D-02
0.76449000D 03	-0.24264285D 01	0.18670470D 00	0.20091592D 00	0.13026457D 01	0.15906782D-01
	0.20675842D-01	0.45065711D-01	0.12254711D-02	0.19436844D-04	-0.29540812D-02
	0.41723815D-04	0.27524379D 00	0.35342444D 01	0.69413568D-04	0.97349053D-06
	0.93660340D-04	0.71489380D-07	-0.22906157D-05	-0.55222842D-01	-0.65550528D-01
	-0.32999539D-01	0.84572307D 01	4		
	-0.12077337D 00	0.50501588D 00	0.11493606D-01	-0.88222381D-01	0.42710358D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DFRKTP(CAL/SFC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.22513409D-01	0.44417972D 02	0.62945665D 01	-0.52560583D 04	0.11975831D-02
0.76429000D 03	-0.23627791D 01	0.18668596D 00	0.20080621D 00	0.12671876D 01	0.15902894D-01
	0.20675842D-01	0.45653167D-01	0.12430388D-02	0.19438517D-04	-0.29175876D-02
	0.41709517D-04	0.27524379D 00	0.35342444D 01	0.69867753D-04	0.98213271D-06
	0.93727140D-04	0.71498204D-07	-0.22490644D-05	-0.53194212D-01	-0.63907672D-01
	-0.32190307D-01	0.82800898D 01	4		
	-0.11710188D 00	0.49314015D 00	0.11102266D-01	-0.85384519D-01	0.40832447D 01
0.54000000D 03	0.22280026D-01	0.44883252D 02	0.63937631D 01	-0.56284410D 04	0.11359741D-02
0.76409000D 03	-0.22884270D 01	0.18666721D 00	0.20069426D 00	0.12278880D 01	0.15899007D-01
	0.20675842D-01	0.46231655D-01	0.12603402D-02	0.19440189D-04	-0.28647562D-02
	0.41695216D-04	0.27524379D 00	0.35342444D 01	0.70312385D-04	0.99059871D-06
	0.93793940D-04	0.71507016D-07	-0.21954058D-05	-0.50986211D-01	-0.61965253D-01
	-0.31223600D-01	0.80735842D 01	4		
	-0.11295146D 00	0.48192143D 00	0.10737222D-01	-0.82209811D-01	0.38908333D 01
0.54000000D 03	0.22059086D-01	0.45332794D 02	0.64923185D 01	-0.60485299D 04	0.10733713D-02
0.76389000D 03	-0.22058448D 01	0.18664845D 00	0.20058111D 00	0.11858698D 01	0.15895118D-01
	0.20675842D-01	0.46798143D-01	0.12772850D-02	0.19441862D-04	-0.27980506D-02
	0.41680914D-04	0.27524379D 00	0.35342444D 01	0.70745250D-04	0.99884767D-06
	0.93860740D-04	0.71515817D-07	-0.21317871D-05	-0.48658921D-01	-0.59789779D-01
	-0.30133290D-01	0.78445559D 01	4		
	-0.10844870D 00	0.47136669D 00	0.10397918D-01	-0.78792211D-01	0.36976623D 01
0.54000000D 03	0.21850654D-01	0.45765220D 02	0.65898806D 01	-0.65175010D 04	0.10111054D-02
0.76369000D 03	-0.21177152D 01	0.18662967D 00	0.20046767D 00	0.11423008D 01	0.15891230D-01
	0.20675842D-01	0.47350149D-01	0.12937996D-02	0.19443534D-04	-0.27204555D-02
	0.41666610D-04	0.27524379D 00	0.35342444D 01	0.71164602D-04	0.10068474D-05
	0.93927540D-04	0.71524607D-07	-0.20606913D-05	-0.46273462D-01	-0.57454000D-01
	-0.28956690D-01	0.76004241D 01	4		
	-0.10372746D 00	0.46146959D 00	0.10083413D-01	-0.75230152D-01	0.35073646D 01
0.54000000D 03	0.21654544D-01	0.46179685D 02	0.66861597D 01	-0.70356052D 04	0.95033186D-03
0.76349000D 03	-0.20266428D 01	0.18661087D 00	0.20035478D 00	0.10982761D 01	0.15887341D-01
	0.20675842D-01	0.47885809D-01	0.13098286D-02	0.19445207D-04	-0.26351095D-02
	0.41652304D-04	0.27524379D 00	0.35342444D 01	0.71569198D-04	0.10145749D-05
	0.93994340D-04	0.71533387D-07	-0.19846476D-05	-0.43886026D-01	-0.55029027D-01
	-0.27730471D-01	0.73483903D 01	4		
	-0.98915053D-01	0.45221213D 00	0.97924473D-02	-0.71616497D-01	0.33230312D 01
0.54000000D 03	0.21470361D-01	0.46575836D 02	0.67809342D 01	-0.76022828D 04	0.89196026D-03
0.76329000D 03	-0.19349370D 01	0.18659207D 00	0.20024309D 00	0.10547341D 01	0.15883452D-01
	0.20675842D-01	0.48403876D-01	0.13253349D-02	0.19446881D-04	-0.25449927D-02
	0.41637996D-04	0.27524379D 00	0.35342444D 01	0.71958287D-04	0.10220164D-05
	0.94061140D-04	0.71542155D-07	-0.19059952D-05	-0.41543795D-01	-0.52578283D-01
	-0.26487501D-01	0.70948352D 01	4		
	-0.94122078D-01	0.44356697D 00	0.95235429D-02	-0.68031296D-01	0.31470346D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.21297556D-01	0.46953744D 02	0.68740485D 01	-0.82163249D 04	0.83663299D-03
0.76309000D 03	-0.18444863D 01	0.18657325D 00	0.20013315D 00	0.10124140D 01	0.15879562D-01
	0.20675842D-01	0.48903663D-01	0.13402982D-02	0.19448554D-04	-0.24527132D-02
	0.41623687D-04	0.27524379D 00	0.35342444D 01	0.72331556D-04	0.10291661D-05
	0.94127940D-04	0.71550912D-07	-0.18267320D-05	-0.39283051D-01	-0.50153978D-01
	-0.25254979D-01	0.68449730D 01	4		
	-0.89437029D-01	0.43550022D 00	0.92750904D-02	-0.64538031D-01	0.29809873D 01
0.54000000D 03	0.21135478D-01	0.47313811D 02	0.69654072D 01	-0.88760476D 04	0.78474198D-03
0.76289000D 03	-0.17567208D 01	0.18655442D 00	0.20002534D 00	0.97184923D 00	0.15875672D-01
	0.20675842D-01	0.49384960D-01	0.13547122D-02	0.19450228D-04	-0.23604009D-02
	0.41609376D-04	0.27524379D 00	0.35342444D 01	0.72689046D-04	0.10360248D-05
	0.94194740D-04	0.71559658D-07	-0.17484467D-05	-0.37129134D-01	-0.47795947D-01
	-0.24053784D-01	0.66027466D 01	4		
	-0.84925080D-01	0.42797398D 00	0.90454346D-02	-0.61182918D-01	0.28258038D 01
0.54000000D 03	0.20983418D-01	0.47656679D 02	0.70549656D 01	-0.95794558D 04	0.73646831D-03
0.76269000D 03	-0.16726366D 01	0.18653557D 00	0.19991994D 00	0.93338581D 00	0.15871782D-01
	0.20675842D-01	0.49847931D-01	0.13685818D-02	0.19451902D-04	-0.22696882D-02
	0.41595063D-04	0.27524379D 00	0.35342444D 01	0.73031079D-04	0.10425984D-05
	0.94261540D-04	0.71568394D-07	-0.16723198D-05	-0.35097634D-01	-0.45532245D-01
	-0.22898742D-01	0.63708975D 01	4		
	-0.80629878D-01	0.42094862D 00	0.88329407D-02	-0.57996375D-01	0.26818205D 01
0.54000000D 03	0.20840648D-01	0.47983153D 02	0.71427193D 01	-0.10324382D 05	0.69183017D-03
0.76249000D 03	-0.15928580D 01	0.18651671D 00	0.19981712D 00	0.89721420D 00	0.15867892D-01
	0.20675842D-01	0.50293021D-01	0.13819202D-02	0.19453576D-04	-0.21817491D-02
	0.41580749D-04	0.27524379D 00	0.35342444D 01	0.73358173D-04	0.10488964D-05
	0.94328340D-04	0.71577118D-07	-0.15991659D-05	-0.33196193D-01	-0.43380752D-01
	-0.21799423D-01	0.61511364D 01	4		
	-0.76576945D-01	0.41438449D 00	0.86360413D-02	-0.54995616D-01	0.25489355D 01
0.54000000D 03	0.20706450D-01	0.48294131D 02	0.72286945D 01	-0.11108598D 05	0.65072970D-03
0.76229000D 03	-0.15177119D 01	0.18649784D 00	0.19971696D 00	0.86340443D 00	0.15864001D-01
	0.20675842D-01	0.50720869D-01	0.13947464D-02	0.19455250D-04	-0.20973679D-02
	0.41566432D-04	0.27524379D 00	0.35342444D 01	0.73670978D-04	0.10549308D-05
	0.94395140D-04	0.71585831D-07	-0.15294952D-05	-0.31426425D-01	-0.41351183D-01
	-0.20761164D-01	0.59443520D 01	4		
	-0.72777608D-01	0.40824320D 00	0.84532673D-02	-0.52187589D-01	0.24267413D 01
0.54000000D 03	0.20580132D-01	0.48590554D 02	0.73129399D 01	-0.11929890D 05	0.61299307D-03
0.76209000D 03	-0.14473023D 01	0.18647895D 00	0.19961951D 00	0.83193924D 00	0.15860110D-01
	0.20675842D-01	0.51132238D-01	0.14070830D-02	0.19456924D-04	-0.20170175D-02
	0.41552114D-04	0.27524379D 00	0.35342444D 01	0.73970222D-04	0.10607151D-05
	0.94461940D-04	0.71594533D-07	-0.14635810D-05	-0.29785673D-01	-0.39447081D-01
	-0.19786083D-01	0.57508161D 01	3		
	-0.69232754D-01	0.40248839D 00	0.82832640D-02	-0.49571756D-01	0.23146367D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.20461043D-01	0.48873365D 02	0.73955193D 01	-0.12786120D 05	0.57840217D-03
0.76189000D 03	-0.13815748D 01	0.18646006D 00	0.19952475D 00	0.80274201D 00	0.15856218D-01
	0.20675842D-01	0.51527961D-01	0.14189548D-02	0.19458599D-04	-0.19409335D-02
	0.41537795D-04	0.27524379D 00	0.35342444D 01	0.74256668D-04	0.10662634D-05
	0.94528740D-04	0.71603224D-07	-0.14015194D-05	-0.28268460D-01	-0.37667578D-01
	-0.18873987D-01	0.55703629D 01	3		
	-0.65936039D-01	0.39708621D 00	0.81247980D-02	-0.47142447D-01	0.22119143D 01
0.54000000D 03	0.20348577D-01	0.49143485D 02	0.74765060D 01	-0.13675254D 05	0.54671788D-03
0.76169000D 03	-0.13203686D 01	0.18644114D 00	0.19943263D 00	0.77569878D 00	0.15852326D-01
	0.20675842D-01	0.51908900D-01	0.14303874D-02	0.19460274D-04	-0.18691783D-02
	0.41523473D-04	0.27524379D 00	0.35342444D 01	0.74531085D-04	0.10715898D-05
	0.94595540D-04	0.71611904D-07	-0.13432816D-05	-0.26867622D-01	-0.36008822D-01
	-0.18023106D-01	0.54025338D 01	3		
	-0.62876444D-01	0.39200556D 00	0.79767553D-02	-0.44890728D-01	0.21178233D 01
0.54000000D 03	0.20242178D-01	0.49401798D 02	0.75559787D 01	-0.14595388D 05	0.51769631D-03
0.76149000D 03	-0.12634574D 01	0.18642222D 00	0.19934307D 00	0.75067468D 00	0.15848434D-01
	0.20675842D-01	0.52275917D-01	0.14414064D-02	0.19461949D-04	-0.18016936D-02
	0.41509150D-04	0.27524379D 00	0.35342444D 01	0.74794228D-04	0.10767083D-05
	0.94662340D-04	0.71620573D-07	-0.12887544D-05	-0.25575129D-01	-0.34465079D-01
	-0.17230666D-01	0.52466892D 01	3		
	-0.60040207D-01	0.38721804D 00	0.78381365D-02	-0.42805795D-01	0.20316127D 01
0.54000000D 03	0.20141337D-01	0.49649137D 02	0.76340182D 01	-0.15544752D 05	0.49109939D-03
0.76129000D 03	-0.12105781D 01	0.18640328D 00	0.19925599D 00	0.72752558D 00	0.15844541D-01
	0.20675842D-01	0.52629853D-01	0.14520368D-02	0.19463625D-04	-0.17383403D-02
	0.41494825D-04	0.27524379D 00	0.35342444D 01	0.75046823D-04	0.10816322D-05
	0.94729140D-04	0.71629231D-07	-0.12377710D-05	-0.24382661D-01	-0.33029550D-01
	-0.16493316D-01	0.51020900D 01	3		
	-0.57412211D-01	0.38269795D 00	0.77080484D-02	-0.40875977D-01	0.19525594D 01
0.54000000D 03	0.20045592D-01	0.49886280D 02	0.77107054D 01	-0.16521711D 05	0.46670139D-03
0.76109000D 03	-0.11614522D 01	0.18638433D 00	0.19917131D 00	0.70610612D 00	0.15840648D-01
	0.20675842D-01	0.52971516D-01	0.14623027D-02	0.19465300D-04	-0.16789293D-02
	0.41480498D-04	0.27524379D 00	0.35342444D 01	0.75289559D-04	0.10863743D-05
	0.94795940D-04	0.71637878D-07	-0.11901339D-05	-0.23281996D-01	-0.31694946D-01
	-0.15807422D-01	0.49679546D 01	3		
	-0.54976942D-01	0.37842209D 00	0.75856947D-02	-0.39089418D-01	0.18799838D 01
0.54000000D 03	0.19954524D-01	0.50113949D 02	0.77861195D 01	-0.17524764D 05	0.44429240D-03
0.76089000D 03	-0.11157999D 01	0.18636536D 00	0.19908891D 00	0.68627482D 00	0.15836755D-01
	0.20675842D-01	0.53301673D-01	0.14722267D-02	0.19466976D-04	-0.16232428D-02
	0.41466170D-04	0.27524379D 00	0.35342444D 01	0.75523086D-04	0.10909465D-05
	0.94862740D-04	0.71646514D-07	-0.11456309D-05	-0.22265247D-01	-0.30453880D-01
	-0.15169282D-01	0.48434986D 01	3		
	-0.52719127D-01	0.37436956D 00	0.74703663D-02	-0.37434528D-01	0.18132584D 01

TABLE I (Continued)

PRESSURE(PSIA)	SPVL(CUFT/LB)	DEN(LB/CUFT)	DERPTV(PSI/R)	DERPVT(PSI/(CUFT/LB))	DERVTP(CUFT/LB)/R
TEMP(R)	DERDTP(LB/CUFT)/R	CVLP(BTU/LB-R)	CVHP(BTU/LB-R)	CPHP(BTU/LB-R)	VISLP(CP)
	XI	VISHP(CP)	DERUDT(CP/(LB/CUFT))	DERUTD(CP/R)	DERUTP(CP/R)
	CONLP(CAL/SEC-CM-K)	CONZC	CONLA	CONHP(CAL/SEC-CM-K)	DERKDT
	DERCVT(BTU/LB-R)/R	DERKTD	DERKTP(CAL/SEC-CM-K)/R	DCOEFF	ECOEFF
	HCOEFF	PRNO	ITR		
	A111	A222	A333	B111	B222
0.54000000D 03	0.19867755D-01	0.50332813D 02	0.78603370D 01	-0.18552536D 05	0.42367991D-03
0.76069000D 03	-0.10733473D 01	0.18634638D 00	0.19900869D 00	0.66789725D 00	0.15832862D-01
	0.20675842D-01	0.53621046D-01	0.14818303D-02	0.19468652D-04	-0.15710500D-02
	0.41451840D-04	0.27524379D 00	0.35342444D 01	0.75748006D-04	0.10953600D-05
	0.94929540D-04	0.71655139D-07	-0.11040465D-05	-0.21325002D-01	-0.29299130D-01
	-0.14575255D-01	0.47279593D 01	3		
	-0.50624131D-01	0.37052159D 00	0.73614322D-02	-0.35900257D-01	0.17518110D 01
0.54000000D 03	0.19784944D-01	0.50543484D 02	0.79334312D 01	-0.19603770D 05	0.40468905D-03
0.76049000D 03	-0.10338364D 01	0.18632739D 00	0.19893056D 00	0.65084780D 00	0.15828968D-01
	0.20675842D-01	0.53930310D-01	0.14911337D-02	0.19470328D-04	-0.15221179D-02
	0.41437508D-04	0.27524379D 00	0.35342444D 01	0.75964885D-04	0.10996251D-05
	0.94996340D-04	0.71663752D-07	-0.10651687D-05	-0.20454395D-01	-0.28223793D-01
	-0.14021856D-01	0.46206118D 01	4		
	-0.48678188D-01	0.36686130D 00	0.72583302D-02	-0.34476251D-01	0.16951236D 01
0.54000000D 03	0.19705782D-01	0.50746527D 02	0.80054716D 01	-0.20677317D 05	0.38716201D-03
0.76029000D 03	-0.99702350D 00	0.18630838D 00	0.19885442D 00	0.63501039D 00	0.15825073D-01
	0.20675842D-01	0.54230095D-01	0.15001554D-02	0.19472004D-04	-0.14762182D-02
	0.41423174D-04	0.27524379D 00	0.35342444D 01	0.76174241D-04	0.11037516D-05
	0.95063140D-04	0.71672355D-07	-0.10287939D-05	-0.19647128D-01	-0.27221383D-01
	-0.13505798D-01	0.45207768D 01	4		
	-0.46868511D-01	0.36337354D 00	0.71605598D-02	-0.33152925D-01	0.16427307D 01
0.54000000D 03	0.19629991D-01	0.50942459D 02	0.80765240D 01	-0.21772127D 05	0.37095705D-03
0.76009000D 03	-0.96268328D 00	0.18628936D 00	0.19878016D 00	0.62027866D 00	0.15821179D-01
	0.20675842D-01	0.54520985D-01	0.15089129D-02	0.19473681D-04	-0.14331316D-02
	0.41408839D-04	0.27524379D 00	0.35342444D 01	0.76376556D-04	0.11077481D-05
	0.95129940D-04	0.71680947D-07	-0.99472966D-06	-0.18897464D-01	-0.26285871D-01
	-0.13024018D-01	0.44278251D 01	4		
	-0.45183335D-01	0.36004469D 00	0.70676739D-02	-0.31921482D-01	0.15942149D 01
0.54000000D 03	0.19557319D-01	0.51131752D 02	0.81466504D 01	-0.22887243D 05	0.35594722D-03
0.75989000D 03	-0.93060835D 00	0.18627033D 00	0.19870771D 00	0.60655568D 00	0.15817284D-01
	0.20675842D-01	0.54803521D-01	0.15174223D-02	0.19475358D-04	-0.13926505D-02
	0.41394502D-04	0.27524379D 00	0.35342444D 01	0.76572275D-04	0.11116231D-05
	0.95196740D-04	0.71689527D-07	-0.96279618D-06	-0.18200205D-01	-0.25411697D-01
	-0.12573692D-01	0.43411780D 01	4		
	-0.43611902D-01	0.35686246D 00	0.69792731D-02	-0.30773897D-01	0.15492035D 01
0.54000000D 03	0.19487540D-01	0.51314840D 02	0.82159089D 01	-0.24021788D 05	0.34201904D-03
0.75969000D 03	-0.90060894D 00	0.18625128D 00	0.19863698D 00	0.59375352D 00	0.15813389D-01
	0.20675842D-01	0.55078206D-01	0.15256984D-02	0.19477035D-04	-0.13545805D-02
	0.41380163D-04	0.27524379D 00	0.35342444D 01	0.76761806D-04	0.11153839D-05
	0.95263540D-04	0.71698097D-07	-0.93282659D-06	-0.17550653D-01	-0.24593767D-01
	-0.12152223D-01	0.42603060D 01	4		
	-0.42144419D-01	0.35381581D 00	0.68949997D-02	-0.29702876D-01	0.15073636D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRNO A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03 0.75949000D 03	0.19420446D-01 -0.87251203D 00 0.20675842D-01 0.41365822D-04 0.95330340D-04 -0.11757241D-01 -0.40772001D-01	0.51492122D 02 0.18623222D 00 0.55345502D-01 0.27524379D 00 0.71706655D-07 0.41847268D 01 0.35089475D 00	0.82843541D 01 0.19856787D 00 0.15337550D-02 0.35342444D 01 -0.90466706D-06 4 0.68145328D-02	-0.25174961D 05 0.58179254D 00 0.19478712D-04 0.76945526D-04 -0.16944573D-01 -0.28701814D-01	0.32907118D-03 0.15809493D-01 -0.13187410D-02 0.11190375D-05 -0.23827428D-01 0.14683987D 01
0.54000000D 03 0.75929000D 03	0.19355852D-01 -0.84616031D 00 0.20675842D-01 0.41351480D-04 0.95397140D-04 -0.11386582D-01 -0.39486598D-01	0.51663961D 02 0.18621315D 00 0.55605840D-01 0.27524379D 00 0.71715203D-07 0.41140016D 01 0.34809028D 00	0.83520369D 01 0.19850032D 00 0.15416049D-02 0.35342444D 01 -0.87817628D-06 4 0.67375839D-02	-0.26346029D 05 0.57060080D 00 0.19480389D-04 0.77123784D-04 -0.16378154D-01 -0.27764736D-01	0.31701313D-03 0.15805597D-01 -0.12849645D-02 0.11225904D-05 -0.23108444D-01 0.14320439D 01
0.54000000D 03 0.75909000D 03	0.19293587D-01 -0.82141094D 00 0.20675842D-01 0.41337136D-04 0.95463940D-04 -0.11038280D-01 -0.38280931D-01	0.51830693D 02 0.18619407D 00 0.55859615D-01 0.27524379D 00 0.71723739D-07 0.40477323D 01 0.34539420D 00	0.84190052D 01 0.19843425D 00 0.15492598D-02 0.35342444D 01 -0.85322485D-06 4 0.66638932D-02	-0.27534318D 05 0.56011334D 00 0.19482067D-04 0.77296899D-04 -0.15847964D-01 -0.26886244D-01	0.30576407D-03 0.15801701D-01 -0.12530969D-02 0.11260485D-05 -0.22432967D-01 0.13980633D 01
0.54000000D 03 0.75889000D 03	0.19233497D-01 -0.79813436D 00 0.20675842D-01 0.41322791D-04 0.95530740D-04 -0.10710549D-01 -0.37148414D-01	0.51992624D 02 0.18617497D 00 0.56107196D-01 0.27524379D 00 0.71732264D-07 0.39855581D 01 0.34279912D 00	0.84853037D 01 0.19836960D 00 0.15567307D-02 0.35342444D 01 -0.82969446D-06 4 0.65932260D-02	-0.28739212D 05 0.55027153D 00 0.19483744D-04 0.77465168D-04 -0.15350915D-01 -0.26061464D-01	0.29525179D-03 0.15797804D-01 -0.12229966D-02 0.11294173D-05 -0.21797499D-01 0.13662458D 01
0.54000000D 03 0.75869000D 03	0.19175442D-01 -0.77621310D 00 0.20675842D-01 0.41308443D-04 0.95597540D-04 -0.10401763D-01 -0.36083098D-01	0.52150037D 02 0.18615585D 00 0.56348923D-01 0.27524379D 00 0.71740779D-07 0.39271516D 01 0.34029828D 00	0.85509740D 01 0.19830629D 00 0.15640278D-02 0.35342444D 01 -0.80747708D-06 4 0.65253699D-02	-0.29960142D 05 0.54102243D 00 0.19485422D-04 0.77628864D-04 -0.14884229D-01 -0.25285993D-01	0.28541167D-03 0.15793908D-01 -0.11945334D-02 0.11327016D-05 -0.21198869D-01 0.13364029D 01
0.54000000D 03 0.75849000D 03	0.19119292D-01 -0.75554063D 00 0.20675842D-01 0.41294094D-04 0.95664340D-04 -0.10110450D-01 -0.35079598D-01	0.52303192D 02 0.18613673D 00 0.56585111D-01 0.27524379D 00 0.71749282D-07 0.38722161D 01 0.33788556D 00	0.86160555D 01 0.19824426D 00 0.15711603D-02 0.35342444D 01 -0.78647413D-06 4 0.64601326D-02	-0.31196585D 05 0.53231825D 00 0.19487101D-04 0.77788240D-04 -0.14445402D-01 -0.24555852D-01	0.27618585D-03 0.15790010D-01 -0.11675883D-02 0.11359064D-05 -0.20634197D-01 0.13083659D 01

TABLE I (Continued)

PRESSURE(PSIA) TEMP(R)	SPVL(CUFT/LB) DERDTP(LB/CUFT)/R XI CONLP(CAL/SEC-CM-K) DERCVT(BTU/LB-R)/R HCOEFF A111	DEN(LB/CUFT) CVLP(BTU/LB-R) VISHP(CP) CONZC DERKTD PRND A222	DERPTV(PSI/R) CVHP(BTU/LB-R) DERUDT(CP/(LB/CUFT)) CONLA DERKTP(CAL/SEC-CM-K)/R ITR A333	DERPVT(PSI/(CUFT/LB)) CPHP(BTU/LB-R) DERUTD(CP/R) CONHP(CAL/SEC-CM-K) DCOEFF B111	DERVTP(CUFT/LB)/R VISLP(CP) DERUTP(CP/R) DERKDT ECOEFF B222
0.54000000D 03	0.19064930D-01	0.52452330D 02	0.868058460 01	-0.32448061D 05	0.26752245D-03
0.75829000D 03	-0.73602031D 00	0.18611759D 00	0.19818347D 00	0.52411579D 00	0.15786113D-01
	0.20675842D-01	0.56816052D-01	0.15781369D-02	0.19488779D-04	-0.11420520D-02
	0.41279744D-04	0.27524379D 00	0.35342444D 01	0.77943529D-04	0.11390357D-05
	0.95731140D-04	0.71757774D-07	-0.76659566D-06	-0.14032176D-01	-0.20100869D-01
	-0.98352701D-02	0.38204826D 01	4		
	-0.34133045D-01	0.33555534D 00	0.63973391D-02	-0.23867446D-01	0.12819834D 01
0.54000000D 03	0.19012248D-01	0.52597672D 02	0.87445957D 01	-0.33714123D 05	0.25937485D-03
0.75809000D 03	-0.71756444D 00	0.18609844D 00	0.19812385D 00	0.51637594D 00	0.15782215D-01
	0.20675842D-01	0.57042019D-01	0.15849657D-02	0.19490458D-04	-0.11178246D-02
	0.41265391D-04	0.27524379D 00	0.35342444D 01	0.78094948D-04	0.11420937D-05
	0.95797940D-04	0.71766255D-07	-0.74775959D-06	-0.13642513D-01	-0.19596512D-01
	-0.95750060D-02	0.37717070D 01	4		
	-0.33239025D-01	0.33330252D 00	0.63368303D-02	-0.23217519D-01	0.12571194D 01
0.54000000D 03	0.18961148D-01	0.52739422D 02	0.88081210D 01	-0.34994361D 05	0.25170115D-03
0.75789000D 03	-0.70009331D 00	0.18607927D 00	0.19806535D 00	0.50906332D 00	0.15778317D-01
	0.20675842D-01	0.57263263D-01	0.15916543D-02	0.19492137D-04	-0.10948144D-02
	0.41251037D-04	0.27524379D 00	0.35342444D 01	0.78242697D-04	0.11450841D-05
	0.95864740D-04	0.71774725D-07	-0.72989098D-06	-0.13274573D-01	-0.19118966D-01
	-0.93285509D-02	0.37256674D 01	4		
	-0.32393539D-01	0.33112239D 00	0.62784607D-02	-0.22603124D-01	0.12336519D 01

APPENDIX B

THE LEAST SQUARE PROGRAM

The purpose of the least square program is to determine the regression coefficients for Equations (28) to (32), Chapter V. The program is a standard IBM share program and is presented below. All instruction necessary to operate the program are stated within the program. All computations must be double precision. The information obtained from this program is to be part of the input data for the iterative program (see Appendix C).

```
$ID          C-0001 THOMAS E MULLIN                    2555-41002
$JOB                THOMAS E MULLIN                    2555-41002
$IBJOB NAMEPR  MAP,DECK
$IBFTC
C
C     ...............................................................
C
C        SAMPLE MAIN PROGRAM FOR POLYNOMIAL REGRESSION - POLRG
C
C        PURPOSE
C           (1) READ THE PROBLEM PARAMETER CARD FOR A POLYNOMIAL REGRES-
C           SION, (2) CALL SUBROUTINES TO PERFORM THE ANALYSIS, (3)
C           PRINT THE REGRESSION COEFFICIENTS AND ANALYSIS OF VARIANCE
C           TABLE FOR POLYNOMIALS OF SUCCESSIVELY INCREASING DEGREES,
C           AND (4) OPTIONALLY PRINT THE TABLE OF RESIDUALS AND A PLOT
C           OF Y VALUES AND Y ESTIMATES.
C
C        REMARKS
C           THE NUMBER OF OBSERVATIONS, N, MUST BE GREATER THAN M+1,
C           WHERE M IS THE HIGHEST DEGREE POLYNOMIAL SPECIFIED.
C           IF THERE IS NO REDUCTION IN THE RESIDUAL SUM OF SQUARES
C           BETWEEN TWO SUCCESSIVE DEGREES OF THE POLYNOMIALS, THE
C           PROGRAM TERMINATES THE PROBLEM BEFORE COMPLETING THE ANALY-
C           SIS FOR THE HIGHEST DEGREE POLYNOMIAL SPECIFIED.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           GDATA
C           ORDER
C           MINV
C           MULTR
C           PLOT   (A SPECIAL PLOT SUBROUTINE PROVIDED FOR THE SAMPLE
C                  PROGRAM.)
C
C        METHOD
C           REFER TO B. OSTLE, 'STATISTICS IN RESEARCH', THE IOWA STATE
C           COLLEGE PRESS', 1954, CHAPTER 6.
C
C     ...............................................................
C
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO THE
C     PRODUCT OF N*(M+1), WHERE N IS THE NUMBER OF OBSERVATIONS AND M
C     IS THE HIGHEST DEGRFE POLYNOMIAL SPECIFIED..
C
      DIMENSION X(1100)
C
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO THE
C     PRODUCT OF M*M..
C
      DIMENSION DI(100)
C
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO
C     (M+2)*(M+1)/2..
C
      DIMENSION D(66)
C
C     THE FOLLOWING DIMENSIONS MUST BE GREATER THAN OR EQUAL TO M..
C
      DIMENSION B(10),E(10),SB(10),T(10)
C
C     THE FOLLOWING DIMENSIONS MUST BE GREATER THAN OR EQUAL TO (M+1)..
C
      DIMENSION XBAR(11),STD(11),COE(11),SUMSQ(11),ISAVE(11)
```

```
C                                                                       POLRG059
C     THE FOLLOWING DIMENSION MUST BE GREATER THAN OR EQUAL TO 10..     POLRG060
C                                                                       POLRG061
         DIMENSION ANS(10)                                              POLRG062
C                                                                       POLRG063
C     THE FOLLOWING DIMENSION WILL BE USED IF THE PLOT OF OBSERVED DATA POLRG064
C     AND ESTIMATES IS DESIRED.  THE SIZE OF THE DIMENSION, IN THIS     POLRG065
C     CASE, MUST BE GREATER THAN OR EQUAL TO N*3.  OTHERWISE, THE SIZE  POLRG066
C     OF DIMENSION MAY BE SET TO 1.                                     POLRG067
C                                                                       POLRG068
         DIMENSION P(300)                                               POLRG069
C                                                                       POLRG070
C     ..........................................................POLRG071
C                                                                       POLRG072
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE  POLRG073
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION      POLRG074
C        STATEMENT WHICH FOLLOWS.                                       POLRG075
C                                                                       POLRG076
C     DOUBLE PRECISION X,XBAR,STD,D,SUMSQ,DI,E,B,SB,T,ANS,DET,COE       POLRG077
C                                                                       POLRG078
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS    POLRG079
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS      POLRG080
C        ROUTINE.                                                       POLRG081
C                                                                       POLRG082
C        .......................................................POLRG083
C                                                                       POLRG084
    1 FORMAT(A4,A2,I5,I2,I1)                                            POLRG085
    2 FORMAT(D20.8,D20.8)                                               POLRG086
    3 FORMAT(27H1POLYNOMIAL REGRESSION......,A4,A2/)                    POLRG087
    4 FORMAT(23H0NUMBER OF OBSERVATIONS,I6//)                           POLRG088
    5 FORMAT(32H0POLYNOMIAL REGRESSION OF DEGREE,I3)                    POLRG089
    6 FORMAT(12H0    INTERCEPT,D25.16)
    7 FORMAT(26H0    REGRESSION COEFFICIENTS/(3D25.16))
    8 FORMAT(1H0/24X,24HANALYSIS OF VARIANCE FOR,I4,19H   DEGREE POLYNOMIPOLRG092
     1AL/)                                                              POLRG093
    9 FORMAT(1H0,5X,19HSOURCE OF VARIATION,7X,9HDEGREE OF,7X,6HSUM OF,9XPOLRG094
     1,4HMEAN,10X,1HF,9X,20HIMPROVEMENT IN TERMS/33X,7HFREEDOM,8X,7HSQUAPOLRG095
     2RES,7X,6HSQUARE,7X,5HVALUE,8X,17HOF SUM OF SQUARES)               POLRG096
   10 FORMAT(20H0   DUE TO REGRESSION,12X,I6,F17.5,F14.5,F13.5,F20.5)   POLRG097
   11 FORMAT(32H    DEVIATION ABOUT REGRESSION    ,I6,F17.5,F14.5)      POLRG098
   12 FORMAT(8X,5HTOTAL,19X,I6,F17.5///)                                POLRG099
   13 FORMAT(17H0    NO IMPROVEMENT)                                    POLRG100
   14 FORMAT(1H0//27X,18HTABLE OF RESIDUALS//16H OBSERVATION NO.,5X,7HX POLRG101
     1VALUE,7X,7HY VALUE,7X,10HY ESTIMATE,7X,8HRESIDUAL/)               POLRG102
   15 FORMAT(1H0,3X,I6,F18.5,F14.5,F17.5,F15.5)                         POLRG103
C                                                                       POLRG104
C     ..........................................................POLRG105
C                                                                       POLRG106
C     READ PROBLEM PARAMETER CARD                                       POLRG107
C                                                                       POLRG108
  100 READ (5,1) PR,PR1,N,M,NPLOT                                       POLRG109
C                                                                       POLRG110
C        PR....PROBLEM NUMBER (MAY BE ALPHAMERIC)                       POLRG111
C        PR1...PROBLEM NUMBER (CONTINUED)                               POLRG112
C        N.....NUMBER OF OBSERVATIONS                                   POLRG113
C        M.....HIGHEST DEGREE POLYNOMIAL SPECIFIED                      POLRG114
C        NPLOT.OPTION CODE FOR PLOTTING                                 POLRG115
C              0  IF PLOT IS NOT DESIRED.                               POLRG116
C              1  IF PLOT IS DESIRED.                                   POLRG117
C                                                                       POLRG118
C     PRINT PROBLEM NUMBER AND N.                                       POLRG119
C                                                                       POLRG120
```

```
      WRITE (6,3) PR,PR1
      WRITE (6,4) N
C
C     READ INPUT DATA
C
      L=N*M
      DO 110 I=1,N
      J=L+I
C
C        X(I) IS THE INDEPENDENT VARIABLE, AND X(J) IS THE DEPENDENT
C        VARIABLE.
C
  110 READ (5,2) X(I),X(J)
C
      CALL GDATA (N,M,X,XBAR,STD,D,SUMSQ)
C
      MM=M+1
      SUM=0.0
      NT=N-1
C
      DO 200 I=1,M
      ISAVE(I)=I
C
C     FORM SUBSET OF CORRELATION COEFFICIENT MATRIX
C
      CALL ORDER (MM,D,MM,I,ISAVE,DI,E)
C
C     INVERT THE SUBMATRIX OF CORRELATION COEFFICIENTS
C
      CALL MINV (DI,I,DET,B,T)
C
      CALL MULTR (N,I,XBAR,STD,SUMSQ,DI,E,ISAVE,B,SB,T,ANS)
C
C     PRINT THE RESULT OF CALCULATION
C
      WRITE (6,5) I
      SUMIP=ANS(4)-SUM
      IF(SUMIP) 140, 140, 150
  140 WRITE (6,13)
      GO TO 210
  150 WRITE (6,6) ANS(1)
      WRITE (6,7) (B(J),J=1,I)
      WRITE (6,8) I
      WRITE (6,9)
      SUM=ANS(4)
      WRITE (6,10) I,ANS(4),ANS(6),ANS(10),SUMIP
      NI=ANS(8)
      WRITE (6,11) NI,ANS(7),ANS(9)
      WRITE (6,12) NT,SUMSQ(MM)
C
C     SAVE COEFFICIENTS FOR CALCULATION OF Y ESTIMATES
C
      COE(1)=ANS(1)
      DO 160 J=1,I
  160 COE(J+1)=B(J)
      LA=I
  200 CONTINUE
C
C     TEST WHETHER PLOT IS DESIRED
C
  210 IF(NPLOT) 100, 100, 220
C
```

```
C         CALCULATE ESTIMATES                                            POLRG183
C                                                                        POLRG184
  220 NP3=N+N                                                            POLRG185
      DO 230 I=1,N                                                       POLRG186
      NP3=NP3+1                                                          POLRG187
      P(NP3)=COE(1)                                                      POLRG188
      L=I                                                                POLRG189
      DO 230 J=1,LA                                                      POLRG190
      P(NP3)=P(NP3)+X(L)*COE(J+1)                                        POLRG191
  230 L=L+N                                                              POLRG192
C                                                                        POLRG193
C         COPY OBSERVED DATA                                             POLRG194
C                                                                        POLRG195
      N2=N                                                               POLRG196
      L=N*M                                                              POLRG197
      DO 240 I=1,N                                                       POLRG198
      P(I)=X(I)                                                          POLRG199
      N2=N2+1                                                            POLRG200
      L=L+1                                                              POLRG201
  240 P(N2)=X(L)                                                         POLRG202
C                                                                        POLRG203
C     PRINT TABLE OF RESIDUALS                                           POLRG204
C                                                                        POLRG205
      WRITE (6,3) PR,PR1                                                 POLRG206
      WRITE (6,5) LA                                                     POLRG207
      WRITE (6,14)                                                       POLRG208
      NP2=N                                                              POLRG209
      NP3=N+N                                                            POLRG210
      DO 250 I=1,N                                                       POLRG211
      NP2=NP2+1                                                          POLRG212
      NP3=NP3+1                                                          POLRG213
      RESID=P(NP2)-P(NP3)                                                POLRG214
  250 WRITE (6,15) I,P(I),P(NP2),P(NP3),RESID                            POLRG215
C                                                                        POLRG216
      CALL PLOT (LA,P,N,3,0,1)                                           POLRG217
C                                                                        POLRG218
      GO TO 100                                                          POLRG219
      END                                                                POLRG220
$IBFTC ORDER
C                                                                        ORDER001
C     ...................................................................ORDER002
C                                                                        ORDER003
C        SUBROUTINE ORDER                                                ORDER004
C                                                                        ORDER005
C        PURPOSE                                                         ORDER006
C           CONSTRUCT FROM A LARGER MATRIX OF CORRELATION COEFFICIENTS   ORDER007
C           A SUBSET MATRIX OF INTERCORRELATIONS AMONG INDEPENDENT       ORDER008
C           VARIABLES AND A VECTOR OF INTERCORRELATIONS OF INDEPENDENT   ORDER009
C           VARIABLES WITH DEPENDENT VARIABLE.  THIS SUBROUTINE IS       ORDER010
C           NORMALLY USED IN THE PERFORMANCE OF MULTIPLE AND POLYNOMIAL  ORDER011
C           REGRESSION ANALYSES.                                         ORDER012
C                                                                        ORDER013
C        USAGE                                                           ORDER014
C           CALL ORDER (M,R,NDEP,K,ISAVE,RX,RY)                          ORDER015
C                                                                        ORDER016
C        DESCRIPTION OF PARAMETERS                                       ORDER017
C           M     - NUMBER OF VARIABLES AND ORDER OF MATRIX R.           ORDER018
C           R     - INPUT MATRIX CONTAINING CORRELATION COEFFICIENTS.    ORDER019
C                   THIS SUBROUTINE EXPECTS ONLY UPPER TRIANGULAR        ORDER020
C                   PORTION OF THE SYMMETRIC MATRIX TO BE STORED (BY     ORDER021
C                   COLUMN) IN R.  (STORAGE MODE OF 1)                   ORDER022
C           NDEP  - THE SUBSCRIPT NUMBER OF THE DEPENDENT VARIABLE.      ORDER023
```

```
C          K       - NUMBER OF INDEPENDENT VARIABLES TO BE INCLUDED        ORDER024
C                    IN THE FORTHCOMING REGRESSION.                        ORDER025
C          ISAVE  - INPUT VECTOR OF LENGTH K+1 CONTAINING, IN ASCENDING   ORDER026
C                    ORDER, THE SUBSCRIPT NUMBERS OF K INDEPENDENT         ORDER027
C                    VARIABLES TO BE INCLUDED IN THE FORTHCOMING REGRES-   ORDER028
C                    SION.                                                 ORDER029
C                    UPON RETURNING TO THE CALLING ROUTINE, THIS VECTOR    ORDER030
C                    CONTAINS, IN ADDITION, THE SUBSCRIPT NUMBER OF        ORDER031
C                    THE DEPENDENT VARIABLE IN K+1 POSITION.               ORDER032
C          RX      - OUTPUT MATRIX (K X K) CONTAINING INTERCORRELATIONS    ORDER033
C                    AMONG INDEPENDENT VARIABLES TO BE USED IN FORTH-      ORDER034
C                    COMING REGRESSION.                                    ORDER035
C          RY      - OUTPUT VECTOR OF LENGTH K CONTAINING INTERCORRELA-    ORDER036
C                    TIONS OF INDEPENDENT VARIABLES WITH DEPENDENT         ORDER037
C                    VARIABLES.                                            ORDER038
C                                                                          ORDER039
C       REMARKS                                                            ORDER040
C          NONE                                                            ORDER041
C                                                                          ORDER042
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                      ORDER043
C          NONE                                                            ORDER044
C                                                                          ORDER045
C       METHOD                                                             ORDER046
C          FROM THE SUBSCRIPT NUMBERS OF THE VARIABLES TO BE INCLUDED      ORDER047
C          IN THE FORTHCOMING REGRESSION, THE SUBROUTINE CONSTRUCTS THEORDER048
C          MATRIX RX AND THE VECTOR RY.                                    ORDER049
C                                                                          ORDER050
C                                                                          ORDER052
C     ..................................................................ORDER051
      SUBROUTINE ORDER (M,R,NDEP,K,ISAVE,RX,RY)                            ORDER053
      DIMENSION R(1),ISAVE(1),RX(1),RY(1)                                  ORDER054
C                                                                          ORDER055
C                                                                          ORDER057
C        ...............................................................ORDER056
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE     ORDER058
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION         ORDER059
C        STATEMENT WHICH FOLLOWS.                                          ORDER060
C                                                                          ORDER061
      DOUBLE PRECISION R,RX,RY                                             ORDER062
C                                                                          ORDER063
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS       ORDER064
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS         ORDER065
C        ROUTINE.                                                          ORDER066
C                                                                          ORDER067
C                                                                          ORDER069
C        ...............................................................ORDER068
C     COPY INTERCORRELATIONS OF INDEPENDENT VARIABLES                      ORDER070
C     WITH DEPENDENT VARIABLE                                              ORDER071
C                                                                          ORDER072
      MM=0                                                                 ORDER073
      DO 130 J=1,K                                                         ORDER074
      L2=ISAVE(J)                                                          ORDER075
      IF(NDEP-L2) 122, 123, 123                                            ORDER076
  122 L=NDEP+(L2*L2-L2)/2                                                  ORDER077
      GO TO 125                                                            ORDER078
  123 L=L2+(NDEP*NDEP-NDEP)/2                                              ORDER079
  125 RY(J)=R(L)                                                           ORDER080
C                                                                          ORDER081
C     COPY A SUBSET MATRIX OF INTERCORRELATIONS AMONG                      ORDER082
C     INDEPENDENT VARIABLES                                                ORDER083
C                                                                          ORDER084
      DO 130 I=1,K                                                         ORDER085
```

```
      L1=ISAVE(I)                                                       ORDER086
      IF(L1-L2) 127, 128, 128                                           ORDER087
  127 L=L1+(L2*L2-L2)/2                                                 ORDER088
      GO TO 129                                                         ORDER089
  128 L=L2+(L1*L1-L1)/2                                                 ORDER090
  129 MM=MM+1                                                           ORDER091
  130 RX(MM)=R(L)                                                       ORDER092
C                                                                       ORDER093
C     PLACE THE SUBSCRIPT NUMBER OF THE DEPENDENT                       ORDER094
C     VARIABLE IN ISAVE(K+1)                                            ORDER095
C                                                                       ORDER096
      ISAVE(K+1)=NDEP                                                   ORDER097
      RETURN                                                            ORDER098
      END                                                               ORDER099
$IBFTC MINV
C                                                                       MINV 001
C                                                                       MINV 003
C     ..................................................................MINV 002
C                                                                       MINV 004
C        SUBROUTINE MINV                                                MINV 005
C                                                                       MINV 006
C        PURPOSE                                                        MINV 007
C           INVERT A MATRIX                                             MINV 008
C                                                                       MINV 009
C        USAGE                                                          MINV 010
C           CALL MINV(A,N,D,L,M)                                        MINV 011
C                                                                       MINV 012
C        DESCRIPTION OF PARAMETERS                                      MINV 013
C           A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY  MINV 013
C               RESULTANT INVERSE.                                      MINV 014
C           N - ORDER OF MATRIX A                                       MINV 015
C           D - RESULTANT DETERMINANT                                   MINV 016
C           L - WORK VECTOR OF LENGTH N                                 MINV 017
C           M - WORK VECTOR OF LENGTH N                                 MINV 018
C                                                                       MINV 019
C        REMARKS                                                        MINV 020
C           MATRIX A MUST BE A GENERAL MATRIX                           MINV 021
C                                                                       MINV 022
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  MINV 023
C           NONE                                                        MINV 024
C                                                                       MINV 025
C        METHOD                                                         MINV 026
C           THE STANDARD GAUSS-JORDAN METHOD IS USED. THE DETERMINANT   MINV 027
C           IS ALSO CALCULATED. A DETERMINANT OF ZERO INDICATES THAT    MINV 028
C           THE MATRIX IS SINGULAR.                                     MINV 029
C                                                                       MINV 030
C     ..................................................................MINV 031
C                                                                       MINV 032
      SUBROUTINE MINV(A,N,D,L,M)                                        MINV 033
      DIMENSION A(1),L(1),M(1)                                          MINV 034
C                                                                       MINV 035
C                                                                       MINV 037
C        ...............................................................MINV 036
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE  MINV 038
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION      MINV 039
C        STATEMENT WHICH FOLLOWS.                                       MINV 040
C                                                                       MINV 041
      DOUBLE PRECISION A,D,BIGA,HOLD                                    MINV 042
C                                                                       MINV 043
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS    MINV 044
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS      MINV 045
C        ROUTINE.                                                       MINV 046
C                                                                       MINV 047
```

```
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO      MINV 048
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  ABS IN STATEMENT  MINV 049
C        10 MUST BE CHANGED TO DABS.                                    MINV 050
C                                                                       MINV 051
C                                                                       MINV 053
C        ...............................................................MINV 052
C        SEARCH FOR LARGEST ELEMENT                                     MINV 054
C                                                                       MINV 055
      D=1.0                                                             MINV 056
      NK=-N                                                             MINV 057
      DO 80 K=1,N                                                       MINV 058
      NK=NK+N                                                           MINV 059
      L(K)=K                                                            MINV 060
      M(K)=K                                                            MINV 061
      KK=NK+K                                                           MINV 062
      BIGA=A(KK)                                                        MINV 063
      DO 20 J=K,N                                                       MINV 064
      IZ=N*(J-1)                                                        MINV 065
      DO 20 I=K,N                                                       MINV 066
      IJ=IZ+I                                                           MINV 067
   10 IF(DABS(BIGA)-DABS(A(IJ))) 15,20,20                               MINV 068
   15 BIGA=A(IJ)                                                        MINV 069
      L(K)=I                                                            MINV 070
      M(K)=J                                                            MINV 071
   20 CONTINUE                                                          MINV 072
C                                                                       MINV 073
C        INTERCHANGE ROWS                                               MINV 074
C                                                                       MINV 075
      J=L(K)                                                            MINV 076
      IF(J-K) 35,35,25                                                  MINV 077
   25 KI=K-N                                                            MINV 078
      DO 30 I=1,N                                                       MINV 079
      KI=KI+N                                                           MINV 080
      HOLD=-A(KI)                                                       MINV 081
      JI=KI-K+J                                                         MINV 082
      A(KI)=A(JI)                                                       MINV 083
   30 A(JI) =HOLD                                                       MINV 084
C                                                                       MINV 085
C        INTERCHANGE COLUMNS                                            MINV 086
C                                                                       MINV 087
   35 I=M(K)                                                            MINV 088
      IF(I-K) 45,45,38                                                  MINV 089
   38 JP=N*(I-1)                                                        MINV 090
      DO 40 J=1,N                                                       MINV 091
      JK=NK+J                                                           MINV 092
      JI=JP+J                                                           MINV 093
      HOLD=-A(JK)                                                       MINV 094
      A(JK)=A(JI)                                                       MINV 095
   40 A(JI) =HOLD                                                       MINV 096
C                                                                       MINV 097
C        DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS        MINV 098
C        CONTAINED IN BIGA)                                             MINV 099
C                                                                       MINV 100
   45 IF(BIGA) 48,46,48                                                 MINV 101
   46 D=0.0                                                             MINV 102
      RETURN                                                            MINV 103
   48 DO 55 I=1,N                                                       MINV 104
      IF(I-K) 50,55,50                                                  MINV 105
   50 IK=NK+I                                                           MINV 106
      A(IK)=A(IK)/(-BIGA)                                               MINV 107
   55 CONTINUE                                                          MINV 108
C                                                                       MINV 109
```

```
C        REDUCE MATRIX                                               MINV 110
C                                                                    MINV 111
      DO 65 I=1,N                                                    MINV 112
      IK=NK+I                                                        MINV 113
      IJ=I-N                                                         MINV 114
      DO 65 J=1,N                                                    MINV 115
      IJ=IJ+N                                                        MINV 116
      IF(I-K) 60,65,60                                               MINV 117
   60 IF(J-K) 62,65,62                                               MINV 118
   62 KJ=IJ-I+K                                                      MINV 119
      A(IJ)=A(IK)*A(KJ)+A(IJ)                                        MINV 120
   65 CONTINUE                                                       MINV 121
C                                                                    MINV 122
C        DIVIDE ROW BY PIVOT                                         MINV 123
C                                                                    MINV 124
      KJ=K-N                                                         MINV 125
      DO 75 J=1,N                                                    MINV 126
      KJ=KJ+N                                                        MINV 127
      IF(J-K) 70,75,70                                               MINV 128
   70 A(KJ)=A(KJ)/BIGA                                               MINV 129
   75 CONTINUE                                                       MINV 130
C                                                                    MINV 131
C        PRODUCT OF PIVOTS                                           MINV 132
C                                                                    MINV 133
      D=D*BIGA                                                       MINV 134
C                                                                    MINV 135
C        REPLACE PIVOT BY RECIPROCAL                                 MINV 136
C                                                                    MINV 137
      A(KK)=1.0/BIGA                                                 MINV 138
   80 CONTINUE                                                       MINV 139
C                                                                    MINV 140
C        FINAL ROW AND COLUMN INTERCHANGE                            MINV 141
C                                                                    MINV 142
      K=N                                                            MINV 143
  100 K=(K-1)                                                        MINV 144
      IF(K) 150,150,105                                              MINV 145
  105 I=L(K)                                                         MINV 146
      IF(I-K) 120,120,108                                            MINV 147
  108 JQ=N*(K-1)                                                     MINV 148
      JR=N*(I-1)                                                     MINV 149
      DO 110 J=1,N                                                   MINV 150
      JK=JQ+J                                                        MINV 151
      HOLD=A(JK)                                                     MINV 152
      JI=JR+J                                                        MINV 153
      A(JK)=-A(JI)                                                   MINV 154
  110 A(JI) =HOLD                                                    MINV 155
  120 J=M(K)                                                         MINV 156
      IF(J-K) 100,100,125                                            MINV 157
  125 KI=K-N                                                         MINV 158
      DO 130 I=1,N                                                   MINV 159
      KI=KI+N                                                        MINV 160
      HOLD=A(KI)                                                     MINV 161
      JI=KI-K+J                                                      MINV 162
      A(KI)=-A(JI)                                                   MINV 163
  130 A(JI) =HOLD                                                    MINV 164
      GO TO 100                                                      MINV 165
  150 RETURN                                                         MINV 166
      END                                                            MINV 167
$IBFTC PLOT
C                                                                    PLOT 001
C     ...............................................................PLOT 002
C                                                                    PLOT 003
```

```
C        SUBROUTINE PLOT                                                PLOT 004
C                                                                       PLOT 005
C        PURPOSE                                                        PLOT 006
C           PLOT SEVERAL CROSS-VARIABLES VERSUS A BASE VARIABLE         PLOT 007
C                                                                       PLOT 008
C        USAGE                                                          PLOT 009
C           CALL PLOT (NO,A,N,M,NL,NS)                                  PLOT 010
C                                                                       PLOT 011
C        DESCRIPTION OF PARAMETERS                                      PLOT 012
C           NO - CHART NUMBER (3 DIGITS MAXIMUM)                        PLOT 013
C           A  - MATRIX OF DATA TO BE PLOTTED. FIRST COLUMN REPRESENTS  PLOT 014
C                BASE VARIABLE AND SUCCESSIVE COLUMNS ARE THE CROSS-    PLOT 015
C                VARIABLES (MAXIMUM IS 9).                              PLOT 016
C           N  - NUMBER OF ROWS IN MATRIX A                             PLOT 017
C           M  - NUMBER OF COLUMNS IN MATRIX A (EQUAL TO THE TOTAL      PLOT 018
C                NUMBER OF VARIABLES). MAXIMUM IS 10.                   PLOT 019
C           NL - NUMBER OF LINES IN THE PLOT. IF 0 IS SPECIFIED, 50     PLOT 020
C                LINES ARE USED.                                        PLOT 021
C           NS - CODE FOR SORTING THE BASE VARIABLE DATA IN ASCENDING   PLOT 022
C                ORDER                                                  PLOT 023
C                  0  SORTING IS NOT NECESSARY (ALREADY IN ASCENDING    PLOT 024
C                     ORDER).                                           PLOT 025
C                  1  SORTING IS NECESSARY.                             PLOT 026
C                                                                       PLOT 027
C        REMARKS                                                        PLOT 028
C           NONE                                                        PLOT 029
C                                                                       PLOT 030
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  PLOT 031
C           NONE                                                        PLOT 032
C                                                                       PLOT 033
C     ..................................................................PLOT 034
C                                                                       PLOT 035
      SUBROUTINE PLOT(NO,A,N,M,NL,NS)                                   PLOT 036
      DIMENSION OUT(101),YPR(11),ANG(9),A(1)                            PLOT 037
C                                                                       PLOT 038
    1 FORMAT(1H1,60X,7H CHART ,I3,//)                                   PLOT 039
    2 FORMAT(1H ,F11.4,5X,101A1)                                        PLOT 040
    3 FORMAT(1H )                                                       PLOT 041
    4 FORMAT(10H 123456789)                                             PLOT 042
    5 FORMAT(10A1)                                                      PLOT 043
    7 FORMAT(1H ,16X,101H.         .         .         .         .      PLOT M01
     1    .         .         .         .         .         .)          PLOT 045
    8 FORMAT(1H0,9X,11F10.4)                                            PLOT 046
C                                                                       PLOT 047
C     ..................................................................PLOT 048
C                                                                       PLOT 049
      NLL=NL                                                            PLOT 050
C                                                                       PLOT 051
      IF(NS) 16, 16, 10                                                 PLOT 052
C                                                                       PLOT 053
C        SORT BASE VARIABLE DATA IN ASCENDING ORDER                     PLOT 054
C                                                                       PLOT 055
   10 DO 15 I=1,N                                                       PLOT 056
      DO 14 J=I,N                                                       PLOT 057
      IF(A(I)-A(J)) 14, 14, 11                                          PLOT 058
   11 L=I-N                                                             PLOT 059
      LL=J-N                                                            PLOT 060
      DO 12 K=1,M                                                       PLOT 061
      L=L+N                                                             PLOT 062
      LL=LL+N                                                           PLOT 063
      F=A(L)                                                            PLOT 064
      A(L)=A(LL)                                                        PLOT 065
```

```
   12 A(LL)=F                                                           PLOT 066
   14 CONTINUE                                                          PLOT 067
   15 CONTINUE                                                          PLOT 068
C                                                                       PLOT 069
C         TEST NLL                                                      PLOT 070
C                                                                       PLOT 071
   16 IF(NLL) 20, 18, 20                                                PLOT 072
   18 NLL=50                                                            PLOT 073
C                                                                       PLOT 074
C         PRINT TITLE                                                   PLOT 075
C                                                                       PLOT 076
   20 WRITE(6,1)NO                                                      PLOT 077
C                                                                       PLOT 078
C         DEVELOP BLANK AND DIGITS FOR PRINTING                         PLOT 079
C                                                                       PLOT 080
      REWIND 0
      WRITE (0 ,4)                                                      PLOT 082
      REWIND 0                                                          PLOT 083
      READ (0 ,5) BLANK,(ANG(I),I=1,9)                                  PLOT 084
      REWIND 0                                                          PLOT 083
C                                                                       PLOT 086
C         FIND SCALE FOR BASE VARIABLE                                  PLOT 087
C                                                                       PLOT 088
      XSCAL=(A(N)-A(1))/(FLOAT(NLL-1))                                  PLOT 089
C                                                                       PLOT 090
C         FIND SCALE FOR CROSS-VARIABLES                                PLOT 091
C                                                                       PLOT 092
      M1=N+1                                                            PLOT M02
      YMIN=A(M1)                                                        PLOT M03
      YMAX=YMIN                                                         PLOT M04
      M2=M*N                                                            PLOT 096
      DO 40 J=M1,M2                                                     PLOT 097
      IF(A(J)-YMIN) 28,26,26                                            PLOT 098
   26 IF(A(J)-YMAX) 40,40,30                                            PLOT 099
   28 YMIN=A(J)                                                         PLOT 100
      GO TO 40                                                          PLOT 101
   30 YMAX=A(J)                                                         PLOT 102
   40 CONTINUE                                                          PLOT 103
      YSCAL=(YMAX-YMIN)/100.0                                           PLOT 104
C                                                                       PLOT 105
C         FIND BASE VARIABLE PRINT POSITION                             PLOT 106
C                                                                       PLOT 107
      XB=A(1)                                                           PLOT 108
      L=1                                                               PLOT 109
      MY=M-1                                                            PLOT 110
      I=1                                                               PLOT M05
   45 F=I-1                                                             PLOT M06
      XPR=XB+F*XSCAL                                                    PLOT 113
      IF(A(L)-XPR) 50,50,70                                             PLOT 114
C                                                                       PLOT 115
C         FIND CROSS-VARIABLES                                          PLOT 116
C                                                                       PLOT 117
   50 DO 55 IX=1,101                                                    PLOT 118
   55 OUT(IX)=BLANK                                                     PLOT 119
      DO 60 J=1,MY                                                      PLOT 120
      LL=L+J*N                                                          PLOT 121
      JP=((A(LL)-YMIN)/YSCAL)+1.0                                       PLOT 122
      OUT(JP)=ANG(J)                                                    PLOT 123
   60 CONTINUE                                                          PLOT 124
C                                                                       PLOT 125
C         PRINT LINE AND CLEAR, OR SKIP                                 PLOT 126
C                                                                       PLOT 127
```

```
      WRITE(6,2)XPR,(OUT(IZ),IZ=1,101)                                   PLOT 128
      L=L+1                                                              PLOT 129
      GO TO 80                                                           PLOT 130
   70 WRITE(6,3)                                                         PLOT 131
   80 I=I+1                                                              PLOT M07
      IF(I-NLL) 45, 84, 86                                               PLOT M08
   84 XPR=A(N)                                                           PLOT M09
      GO TO 50                                                           PLOT M10
C                                                                        PLOT 133
C        PRINT CROSS-VARIABLES NUMBERS                                   PLOT 134
C                                                                        PLOT 135
   86 WRITE(6,7)                                                         PLOT M11
      YPR(1)=YMIN                                                        PLOT 137
      DO 90 KN=1,9                                                       PLOT 138
   90 YPR(KN+1)=YPR(KN)+YSCAL*10.0                                       PLOT 139
      YPR(11)=YMAX                                                       PLOT 140
      WRITE(6,8)(YPR(IP),IP=1,11)                                        PLOT 141
      RETURN                                                             PLOT 142
      END                                                                PLOT 143
$IBFTC GDATA
C                                                                        GDATA001
C                                                                        GDATA003
C     ...................................................................GDATA002
C        SUBROUTINE GDATA                                                GDATA004
C                                                                        GDATA005
C        PURPOSE                                                         GDATA006
C           GENERATE INDEPENDENT VARIABLES UP TO THE M-TH POWER (THE     GDATA007
C           HIGHEST DEGREE POLYNOMIAL SPECIFIED) AND COMPUTE MEANS,      GDATA008
C           STANDARD DEVIATIONS, AND CORRELATION COEFFICIENTS.  THIS     GDATA009
C           SUBROUTINE IS NORMALLY CALLED BEFORE SUBROUTINES ORDER,      GDATA010
C           MINV AND MULTR IN THE PERFORMANCE OF A POLYNOMIAL            GDATA011
C           REGRESSION.                                                  GDATA012
C                                                                        GDATA013
C        USAGE                                                           GDATA014
C           CALL GDATA (N,M,X,XBAR,STD,D,SUMSQ)                          GDATA015
C                                                                        GDATA016
C        DESCRIPTION OF PARAMETERS                                       GDATA017
C           N     - NUMBER OF OBSERVATIONS.                              GDATA018
C           M     - THE HIGHEST DEGREE POLYNOMIAL TO BE FITTED.          GDATA019
C           X     - INPUT MATRIX (N BY M+1) .  WHEN THE SUBROUTINE IS    GDATA020
C                   CALLED, DATA FOR THE INDEPENDENT VARIABLE ARE        GDATA021
C                   STORED IN THE FIRST COLUMN OF MATRIX X, AND DATA FORGDATA022
C                   THE DEPENDENT VARIABLE ARE STORED IN THE LAST        GDATA023
C                   COLUMN OF THE MATRIX.  UPON RETURNING TO THE         GDATA024
C                   CALLING ROUTINE, GENERATED POWERS OF THE INDEPENDENTGDATA025
C                   VARIABLE ARE STORED IN COLUMNS 2 THROUGH M.          GDATA026
C           XBAR  - OUTPUT VECTOR OF LENGTH M+1 CONTAINING MEANS OF      GDATA027
C                   INDEPENDENT AND DEPENDENT VARIABLES.                 GDATA028
C           STD   - OUTPUT VECTOR OF LENGTH M+1 CONTAINING STANDARD      GDATA029
C                   DEVIATIONS OF INDEPENDENT AND DEPENDENT VARIABLES.   GDATA030
C           D     - OUTPUT MATRIX (ONLY UPPER TRIANGULAR PORTION OF THE  GDATA031
C                   SYMMETRIC MATRIX OF M+1 BY M+1) CONTAINING CORRELA-  GDATA032
C                   TION COEFFICIENTS.  (STORAGE MODE OF 1)              GDATA033
C           SUMSQ - OUTPUT VECTOR OF LENGTH M+1 CONTAINING SUMS OF       GDATA034
C                   PRODUCTS OF DEVIATIONS FROM MEANS  OF INDEPENDENT    GDATA035
C                   AND DEPENDENT VARIABLES.                             GDATA036
C                                                                        GDATA037
C        REMARKS                                                         GDATA038
C           N MUST BE GREATER THAN M+1.                                  GDATA039
C           IF M IS EQUAL TO 5 OR GREATER, SINGLE PRECISION MAY NOT BE   GDATA040
C           SUFFICIENT TO GIVE SATISFACTORY COMPUTATIONAL RESULTS.       GDATA041
C                                                                        GDATA042
```

```
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  GDATA043
C           NONE                                                        GDATA044
C                                                                       GDATA045
C        METHOD                                                         GDATA046
C           REFER TO B. OSTLE, 'STATISTICS IN RESEARCH', THE IOWA STATE GDATA047
C           COLLEGE PRESS, 1954, CHAPTER 6.                             GDATA048
C                                                                       GDATA049
C     ..................................................................GDATA050
C                                                                       GDATA051
      SUBROUTINE GDATA (N,M,X,XBAR,STD,D,SUMSQ)                         GDATA052
      DIMENSION X(1),XBAR(1),STD(1),D(1),SUMSQ(1)                       GDATA053
C                                                                       GDATA054
C        ...............................................................GDATA055
C                                                                       GDATA056
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE  GDATA057
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION      GDATA058
C        STATEMENT WHICH FOLLOWS.                                       GDATA059
C                                                                       GDATA060
      DOUBLE PRECISION X,XBAR,STD,D,SUMSQ,T1,T2                         GDATA061
C                                                                       GDATA062
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS    GDATA063
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS      GDATA064
C        ROUTINE.                                                       GDATA065
C                                                                       GDATA066
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO      GDATA067
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  SQRT AND ABS IN   GDATA068
C        STATEMENT 180 MUST BE CHANGED TO DSQRT AND DABS.               GDATA069
C                                                                       GDATA070
C                                                                       GDATA072
C     GENERATE INDEPENDENT VARIABLES                                    GDATA073
C                                                                       GDATA074
      IF(M-1) 105, 105, 90                                              GDATAM01
   90 L1=0                                                              GDATAM02
      DO 100 I=2,M                                                      GDATA076
      L1=L1+N                                                           GDATA077
      DO 100 J=1,N                                                      GDATA078
      L=L1+J                                                            GDATA079
      K=L-N                                                             GDATA080
  100 X(L)=X(K)*X(J)                                                    GDATA081
C                                                                       GDATA082
C     CALCULATE MEANS                                                   GDATA083
C                                                                       GDATA084
  105 MM=M+1                                                            GDATAM03
      DF=N                                                              GDATA086
      L=0                                                               GDATA087
      DO 115 I=1,MM                                                     GDATA088
      XBAR(I)=0.0                                                       GDATA089
      DO 110 J=1,N                                                      GDATA090
      L=L+1                                                             GDATA091
  110 XBAR(I)=XBAR(I)+X(L)                                              GDATA092
  115 XBAR(I)=XBAR(I)/DF                                                GDATA093
C                                                                       GDATA094
      DO 130 I=1,MM                                                     GDATA095
  130 STD(I)=0.0                                                        GDATA096
C                                                                       GDATA097
C     CALCULATE SUMS OF CROSS-PRODUCTS OF DEVIATIONS                    GDATA098
C                                                                       GDATA099
      L=((MM+1)*MM)/2                                                   GDATA100
C        ...............................................................GDATA071
      DO 150 I=1,L                                                      GDATA101
  150 D(I)=0.0                                                          GDATA102
      DO 170 K=1,N                                                      GDATA103
```

```
      L=0                                                               GDATA104
      DO 170 J=1,MM                                                     GDATA105
      L2=N*(J-1)+K                                                      GDATA106
      T2=X(L2)-XBAR(J)                                                  GDATA107
      STD(J)=STD(J)+T2                                                  GDATA108
      DO 170 I=1,J                                                      GDATA109
      L1=N*(I-1)+K                                                      GDATA110
      T1=X(L1)-XBAR(I)                                                  GDATA111
      L=L+1                                                             GDATA112
  170 D(L)=D(L)+T1*T2                                                   GDATA113
      L=0                                                               GDATA114
      DO 175 J=1,MM                                                     GDATA115
      DO 175 I=1,J                                                      GDATA116
      L=L+1                                                             GDATA117
  175 D(L)=D(L)-STD(I)*STD(J)/DF                                        GDATA118
      L=0                                                               GDATA119
      DO 180 I=1,MM                                                     GDATA120
      L=L+I                                                             GDATA121
      SUMSQ(I)=D(L)                                                     GDATA122
  180 STD(I)=DSQRT(DABS(D(L)))                                          GDATA123
C                                                                       GDATA124
C     CALCULATE CORRELATION COEFFICIENTS                                GDATA125
C                                                                       GDATA126
      L=0                                                               GDATA127
      DO 190 J=1,MM                                                     GDATA128
      DO 190 I=1,J                                                      GDATA129
      L=L+1                                                             GDATA130
  190 D(L)=D(L)/(STD(I)*STD(J))                                         GDATA131
C                                                                       GDATA132
C     CALCULATE STANDARD DEVIATIONS                                     GDATA133
C                                                                       GDATA134
      DF=SQRT(DF-1.0)                                                   GDATA135
      DO 200 I=1,MM                                                     GDATA136
  200 STD(I)=STD(I)/DF                                                  GDATA137
      RETURN                                                            GDATA138
      END                                                               GDATA139
$IBFTC MULTR
C                                                                       MULTR001
C                                                                       MULTR003
C     ..................................................................MULTR002
C        SUBROUTINE MULTR                                               MULTR004
C                                                                       MULTR005
C        PURPOSE                                                        MULTR006
C           PERFORM A MULTIPLE LINEAR REGRESSION ANALYSIS FOR A DEPEN-  MULTR007
C           DENT VARIABLE AND A SET OF INDEPENDENT VARIABLES.  THIS     MULTR008
C           SUBROUTINE IS NORMALLY USED IN THE PERFORMANCE OF MULTIPLE  MULTR009
C           AND POLYNOMIAL REGRESSION ANALYSES.                         MULTR010
C                                                                       MULTR011
C        USAGE                                                          MULTR012
C           CALL MULTR (N,K,XBAR,STD,D,RX,RY,ISAVE,B,SB,T,ANS)          MULTR013
C                                                                       MULTR014
C        DESCRIPTION OF PARAMETERS                                      MULTR015
C           N     - NUMBER OF OBSERVATIONS.                             MULTR016
C           K     - NUMBER OF INDEPENDENT VARIABLES IN THIS REGRESSION. MULTR017
C           XBAR  - INPUT VECTOR OF LENGTH M CONTAINING MEANS OF ALL    MULTR018
C                   VARIABLES. M IS NUMBER OF VARIABLES IN OBSERVATIONS.MULTR019
C           STD   - INPUT VECTOR OF LENGTH M CONTAINING STANDARD DEVI-  MULTR020
C                   ATIONS OF ALL VARIABLES.                            MULTR021
C           D     - INPUT VECTOR OF LENGTH M CONTAINING THE DIAGONAL OF MULTR022
C                   THE MATRIX OF SUMS OF CROSS-PRODUCTS OF DEVIATIONS  MULTR023
C                   FROM MEANS FOR ALL VARIABLES.                       MULTR024
C           RX    - INPUT MATRIX (K X K) CONTAINING THE INVERSE OF      MULTR025
```

```
C                INTERCORRELATIONS AMONG INDEPENDENT VARIABLES.        MULTR026
C          RY    - INPUT VECTOR OF LENGTH K CONTAINING INTERCORRELA-   MULTR027
C                TIONS OF INDEPENDENT VARIABLES WITH DEPENDENT          MULTR028
C                VARIABLE.                                              MULTR029
C          ISAVE - INPUT VECTOR OF LENGTH K+1 CONTAINING SUBSCRIPTS OF  MULTR030
C                INDEPENDENT VARIABLES IN ASCENDING ORDER.  THE         MULTR031
C                SUBSCRIPT OF THE DEPENDENT VARIABLE IS STORED IN       MULTR032
C                THE LAST, K+1, POSITION.                               MULTR033
C          B     - OUTPUT VECTOR OF LENGTH K CONTAINING REGRESSION      MULTR034
C                COEFFICIENTS.                                          MULTR035
C          SB    - OUTPUT VECTOR OF LENGTH K CONTAINING STANDARD        MULTR036
C                DEVIATIONS OF REGRESSION COEFFICIENTS.                 MULTR037
C          T     - OUTPUT VECTOR OF LENGTH K CONTAINING T-VALUES.       MULTR038
C          ANS   - OUTPUT VECTOR OF LENGTH 10 CONTAINING THE FOLLOWING  MULTR039
C                INFORMATION..                                          MULTR040
C                ANS(1)  INTERCEPT                                      MULTR041
C                ANS(2)  MULTIPLE CORRELATION COEFFICIENT               MULTR042
C                ANS(3)  STANDARD ERROR OF ESTIMATE                     MULTR043
C                ANS(4)  SUM OF SQUARES ATTRIBUTABLE TO REGRES-         MULTR044
C                        SION (SSAR)                                    MULTR045
C                ANS(5)  DEGREES OF FREEDOM ASSOCIATED WITH SSAR        MULTR046
C                ANS(6)  MEAN SQUARE OF SSAR                            MULTR047
C                ANS(7)  SUM OF SQUARES OF DEVIATIONS FROM REGRES-      MULTR048
C                        SION (SSDR)                                    MULTR049
C                ANS(8)  DEGREES OF FREEDOM ASSOCIATED WITH SSDR        MULTR050
C                ANS(9)  MEAN SQUARE OF SSDR                            MULTR051
C                ANS(10) F-VALUE                                        MULTR052
C                                                                       MULTR053
C        REMARKS                                                        MULTR054
C           N MUST BE GREATER THAN K+1.                                 MULTR055
C                                                                       MULTR056
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  MULTR057
C           NONE                                                        MULTR058
C                                                                       MULTR059
C        METHOD                                                         MULTR060
C           THE GAUSS-JORDAN METHOD IS USED IN THE SOLUTION OF THE      MULTR061
C           NORMAL EQUATIONS.  REFER TO W. W. COOLEY AND P. R. LOHNES,  MULTR062
C           'MULTIVARIATE PROCEDURES FOR THE BEHAVIORAL SCIENCES',      MULTR063
C           JOHN WILEY AND SONS, 1962, CHAPTER 3, AND B. OSTLE,         MULTR064
C           'STATISTICS IN RESEARCH', THE IOWA STATE COLLEGE PRESS,     MULTR065
C           1964, CHAPTER 8.                                            MULTR066
C                                                                       MULTR067
C                                                                       MULTR069
C     ..................................................................MULTR068
      SUBROUTINE MULTR (N,K,XBAR,STD,D,RX,RY,ISAVE,B,SB,T,ANS)          MULTR070
      DIMENSION XBAR(1),STD(1),D(1),RX(1),RY(1),ISAVE(1),B(1),SB(1),    MULTR071
     1          T(1),ANS(1)                                             MULTR072
C                                                                       MULTR073
C                                                                       MULTR075
C        ...............................................................MULTR074
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE  MULTR076
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION      MULTR077
C        STATEMENT WHICH FOLLOWS.                                       MULTR078
C                                                                       MULTR079
      DOUBLE PRECISION XBAR,STD,D,RX,RY,B,SB,T,ANS,RM,BO,SSAR,SSDR,SY,  MULTR080
     1                 FN,FK,SSARM,SSDRM,F                              MULTR081
C                                                                       MULTR082
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS    MULTR083
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS      MULTR084
C        ROUTINE.                                                       MULTR085
C                                                                       MULTR086
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO      MULTR087
```

```
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  SQRT AND ABS IN    MULTR088
C        STATEMENTS 122, 125, AND 135 MUST BE CHANGED TO DSQRT AND DABS.MULTR089
C                                                                       MULTR090
C                                                                       MULTR092
C        ...............................................................MULTR091
      MM=K+1                                                            MULTR093
C                                                                       MULTR094
C        BETA WEIGHTS                                                   MULTR095
C                                                                       MULTR096
      DO 100 J=1,K                                                      MULTR097
  100 B(J)=0.0                                                          MULTR098
      DO 110 J=1,K                                                      MULTR099
      L1=K*(J-1)                                                        MULTR100
      DO 110 I=1,K                                                      MULTR101
      L=L1+I                                                            MULTR102
  110 B(J)=B(J)+RY(I)*RX(L)                                             MULTR103
      RM=0.0                                                            MULTR104
      BO=0.0                                                            MULTR105
      L1=ISAVE(MM)                                                      MULTR106
C                                                                       MULTR107
C        COEFFICIENT OF DETERMINATION                                   MULTR108
C                                                                       MULTR109
      DO 120 I=1,K                                                      MULTR110
      RM=RM+B(I)*RY(I)                                                  MULTR111
C                                                                       MULTR112
C        REGRESSION COEFFICIENTS                                        MULTR113
C                                                                       MULTR114
      L=ISAVE(I)                                                        MULTR115
      B(I)=B(I)*(STD(L1)/STD(L))                                        MULTR116
C                                                                       MULTR117
C        INTERCEPT                                                      MULTR118
C                                                                       MULTR119
  120 BO=BO+B(I)*XBAR(L)                                                MULTR120
      BO=XBAR(L1)-BO                                                    MULTR121
C                                                                       MULTR122
C        SUM OF SQUARES ATTRIBUTABLE TO REGRESSION                      MULTR123
C                                                                       MULTR124
      SSAR=RM*D(L1)                                                     MULTR125
C                                                                       MULTR126
C        MULTIPLE CORRELATION COEFFICIENT                               MULTR127
C                                                                       MULTR128
  122 RM=DSQRT(DABS(RM))                                                MULTR129
C                                                                       MULTR130
C        SUM OF SQUARES OF DEVIATIONS FROM REGRESSION                   MULTR131
C                                                                       MULTR132
      SSDR=D(L1)-SSAR                                                   MULTR133
C                                                                       MULTR134
C        VARIANCE OF ESTIMATE                                           MULTR135
C                                                                       MULTR136
      FN=N-K-1                                                          MULTR137
      SY=SSDR/FN                                                        MULTR138
C                                                                       MULTR139
C        STANDARD DEVIATIONS OF REGRESSION COEFFICIENTS                 MULTR140
C                                                                       MULTR141
      DO 130 J=1,K                                                      MULTR142
      L1=K*(J-1)+J                                                      MULTR143
      L=ISAVE(J)                                                        MULTR144
  125 SB(J)=DSQRT(DABS((RX(L1)/D(L))*SY))                               MULTR145
C                                                                       MULTR146
C        COMPUTED T-VALUES                                              MULTR147
C                                                                       MULTR148
  130 T(J)=B(J)/SB(J)                                                   MULTR149
```

```
C
C        STANDARD ERROR OF ESTIMATE
C
  135 SY=DSQRT(DABS(SY))
C
C        F VALUE
C
      FK=K
      SSARM=SSAR/FK
      SSDRM=SSDR/FN
      F=SSARM/SSDRM
C
      ANS(1)=BO
      ANS(2)=RM
      ANS(3)=SY
      ANS(4)=SSAR
      ANS(5)=FK
      ANS(6)=SSARM
      ANS(7)=SSDR
      ANS(8)=FN
      ANS(9)=SSDRM
      ANS(10)=F
      RETURN
      END
```

APPENDIX C

THE ITERATIVE PROGRAM

The two main purposes of this program are as follows:

1. To compute and print-out the values of Equations (34), (36), (38), (40), (42) and (44), Chapter V.
2. To solve the set of linearized differential equations, Equations (50) through (54), Chapter V.

The purpose of computing and printing out the value of Equations (50) through (54) is due to the fact that these equations were obtained by the method of least square and contain slight imperfections. The degree of imperfection should, of course, be examined to determine if the least square fit is acceptable. Equation (43), Chapter V, gave the most trouble with this regard. Although this problem presents no obstacle to the solution of the set of differential equations, it may require a minor program change in the subroutine call DERFUN.

The read statements are defined as follows:

Program Symbols	Equation Symbols
Coefficients of Equation (28), Chapter V	
A1	A_1
B1	B_2
C1	C_2

D1	D_2

Coefficients of Equation (29), Chapter V

A2	A_2
B2	B_2
C2	C_2
D2	D_2

Coefficient of Equation (30), Chapter V

A3	A_3
B3	B_3
C3	C_3
D3	D_3

Coefficients of Equation (31), Chapter V

A4	A_4
B4	B_4
C4	C_4
D4	D_4

Coefficients of Equation (32), Chapter V

A5	A_5
B5	B_5
C5	C_5
D5	D_5
TI	Wall Temperature - ^{o}R
DENI	Wall Density - CuFt/Lb
VISHPI	Wall Viscosity
TF	Main Stream Temperature - ^{o}R
DELX	Interval of Integration
X2M	Maximum Value of $F'(\eta)$

X4M	Maximum Value of $\theta(\eta)$
KMAX	Maximum number of columns in the starting solution matrix.

```
$ID          C-0001 THOMAS E MULLIN                    2555-41002
$JOB                THOMAS E MULLIN                    2555-41002
$IBJOB NAMEPR  MAP
$IBFTC
C
      DOUBLE PRECISION DELT1,DELT2,DELT3,DELT4,TF1,TF2,TF3,
     1                 DVWAL,DCOEF1,DCOEF2,TI,TF,DENI,VISHPI,DENF,
     2                 A1,B1,C1,D1,
     3                 A2,B2,C2,D2,
     4                 A3,B3,C3,D3,
     5                 A4,B4,C4,D4,
     6                 A5,B5,C5,D5
C
      DOUBLE PRECISION X2M,X4M,X2P,X4P,X2H1,X4H1,X2H2,X4H2,
     1                 DENOM,ZUM,U31,U52
C
C     DIMENSION STATEMENT
      COMMON/SXARE/A(5,501),KKP,M
      COMMON Y(202)
      COMMON/SCOEF/CA,CB,CC,CD,
     1             CE,CF,CG,CH,
     2             CI,CJ,CK,CL,
     3             CM,CN,CP,CR,
     4             CS,CT,CU,CW,
     5             CAA,CBB,CCC,CDD,
     6             CEE,CFF,CGG,CHH
C
C     MATRIX INDEXER - MAIN PROGRAM = N
C                    - SUBROUTINE DERFUN = M
C
C
C     DATA INPUT
      READ(5,1) A1
      READ(5,1) B1
      READ(5,1) C1
      READ(5,1) D1
      READ(5,1) A2
      READ(5,1) B2
      READ(5,1) C2
      READ(5,1) D2
      READ(5,1) A3
      READ(5,1) B3
      READ(5,1) C3
      READ(5,1) D3
      READ(5,1) A4
      READ(5,1) B4
      READ(5,1) C4
      READ(5,1) D4
      READ(5,1) A5
      READ(5,1) B5
      READ(5,1) C5
      READ(5,1) D5
      READ(5,1) TI
      READ(5,1) DENI
      READ(5,1) VISHPI
      READ(5,1) TF
      READ(5,1) DENF
      READ(5,1) X2M
      READ(5,1) X4M
      READ(5,2) DELX
      READ(5,3) KXMAT
    1 FORMAT(D25.16)
```

```
    2 FORMAT(E17.8)
    3 FORMAT(I15)
C
C     THE WRITE STATEMENTS FOR THE INPUT DATA
      WRITF(6,7000)
      WRITE(6,7001)
      WRITE(6,7002)
      WRITE(6,7003)
      WRITE(6,7004)
      WRITE(6,7005)
      WRITE(6,7006)
      WRITE(6,7007)
      WRITE(6,7008)
 7000 FORMAT(1H1,12X,14HTHE INPUT DATA,/)
 7001 FORMAT(1H ,12X,2HA1,18X,2HB1,18X,2HC1,18X,2HD1)
 7002 FORMAT(1H ,12X,2HA2,18X,2HB2,18X,2HC2,18X,2HD2)
 7003 FORMAT(1H ,12X,2HA3,18X,2HB3,18X,2HC3,18X,2HD3)
 7004 FORMAT(1H ,12X,2HA4,18X,2HB4,18X,2HC4,18X,2HD4)
 7005 FORMAT(1H ,12X,2HA5,18X,2HB5,18X,2HC5,18X,2HD5)
 7006 FORMAT(1H ,12X,2HTI,18X,4HDENI,16X,6HVISHPI,14X,2HTF)
 7007 FORMAT(1H ,12X,4HDENF,16X,4HDELX,16X,3HX2M,17X,3HX4M)
 7008 FORMAT(1H ,12X,5HKXMAT,/)
C
      WRITE(6,7009) A1,B1,C1,D1
      WRITE(6,7009) A2,B2,C2,D2
      WRITE(6,7009) A3,B3,C3,D3
      WRITE(6,7009) A4,B4,C4,D4
      WRITE(6,7009) A5,B5,C5,D5
      WRITE(6,7009) TI,DENI,VISHPI,TF
      WRITE(6,7009) DENF,DELX,X2M,X4M
      WRITE(6,7010) KXMAT
 7009 FORMAT(12X,E15.8,5X,E15.8,5X,E15.8,5X,E15.8)
 7010 FORMAT(12X,I15)
C
C     PURPOSE-TO COMPUTE THE PROPERTY COEFFICIENTS
C               FOR THE DIFFERENTIAL EQUATIONS
      DELT1 = TI-TF
      DELT2 = DELT1*DELT1
      DELT3 = DELT2*DELT1
      DELT4 = DELT3*DELT1
      TF1 = TF
      TF2 = TF1*TF1
      TF3 = TF2*TF1
      DVWAL = DENI*VISHPI
      DCOEF1 = (DENI*DENI*DENF*VISHPI)/(DENF-DENI)
      DCOEF2 = (DCOEF1)/(DENF)
C
      CA = (A1+B1*TF1+C1*TF2+D1*TF3)*(DELT1)
      CB = (B1+2.0D+00*C1*TF1+3.0D+00*D1*TF2)*(DELT2)
      CC = (C1+3.0D+00*D1*TF1)*(DELT3)
      CD = (D1)*(DELT4)
C
      CE = 3.0D+00*(A2+B2*TF1+C2*TF2+D2*TF3)*(DVWAL)
      CF = 3.0D+00*(B2+2.0D+00*C2*TF1+3.0D+00*D2*TF2)*(DELT1)*(DVWAL)
      CG = 3.0D+00*(C2+3.0D+00*D2*TF1)*(DELT2)*(DVWAL)
      CH = 3.0D+00*(D2)*(DELT3)*(DVWAL)
C
      CI = 2.0D+00*(A2+B2*TF1+C2*TF2+D2*TF3)*(DVWAL)
      CJ = 2.0D+00*(B2+2.0D+00*C2*TF1+3.0D+00*D2*TF2)*(DELT1)*(DVWAL)
      CK = 2.0D+00*(C2+3.0D+00*D2*TF1)*(DELT2)*(DVWAL)
      CL = 2.0D+00*(D2)*(DELT3)*(DVWAL)
C
```

```
      CM = (A3+B3*TF1+C3*TF2+D3*TF3)*(DCOEF1)
      CN = (B3+2.0D+00*C3*TF1+3.0D+00*D3*TF2)*(DELT1)*(DCOEF1)
      CP = (C3+3.0D+00*D3*TF1)*(DELT2)*(DCOEF1)
      CR = (D3)*(DELT3)*(DCOEF1)
C
      CS = (A2+B2*TF1+C2*TF2+D2*TF3)*(DCOEF2)
      CT = (B2+2.0D+00*C2*TF1+3.0D+00*D2*TF2)*(DELT1)*(DCOEF2)
      CU = (C2+3.0D+00*D2*TF1)*(DELT2)*(DCOEF2)
      CW = (D2)*(DELT3)*(DCOEF2)
C
      CAA = (A4+B4*TF1+C4*TF2+D4*TF3)*(DELT1)
      CBB = (B4+2.0D+00*C4*TF1+3.0D+00*D4*TF2)*(DELT2)
      CCC = (C4+3.0D+00*D4*TF1)*(DELT3)
      CDD = (D4)*(DELT4)
C
      CEE = 3.0D+00*(A5+B5*TF1+C5*TF2+D5*TF3)*(DVWAL)
      CFF = 3.0D+00*(B5+2.0D+00*C5*TF1+3.0D+00*D5*TF2)*(DELT1)*(DVWAL)
      CGG = 3.0D+00*(C5+3.0D+00*D5*TF1)*(DELT2)*(DVWAL)
      CHH = 3.0D+00*(D5)*(DELT3)*(DVWAL)
C
C     THE WRITE STATEMENTS FOR THE CALCULATED COEFFICIENTS
      WRITE(6,7100)
      WRITE(6,7200)
      WRITE(6,7300)
      WRITE(6,7400)
      WRITE(6,7500)
      WRITE(6,7600)
      WRITE(6,7700)
      WRITE(6,7800)
      WRITE(6,8200)
      WRITE(6,8300)
 7100 FORMAT(1H1,12X,27HTHE CALCULATED COEFFICIENTS,/)
 7200 FORMAT(1H ,12X,2HCA,18X,2HCB,18X,2HCC,18X,2HCD)
 7300 FORMAT(1H ,12X,2HCE,18X,2HCF,18X,2HCG,18X,2HCH)
 7400 FORMAT(1H ,12X,2HCI,18X,2HCJ,18X,2HCK,18X,2HCL)
 7500 FORMAT(1H ,12X,2HCM,18X,2HCN,18X,2HCP,18X,2HCR)
 7600 FORMAT(1H ,12X,2HCS,18X,2HCT,18X,2HCU,18X,2HCW)
 7700 FORMAT(1H ,12X,3HCAA,17X,3HCBB,17X,3HCCC,17X,3HCDD)
 7800 FORMAT(1H ,12X,3HCEE,17X,3HCFF,17X,3HCGG,17X,3HCHH)
 8200 FORMAT(1H ,12X,5HDELT1,15X,5HDELT2,15X,5HDELT3,15X,
     1        5HDELT4,15X,3HTF1)
 8300 FORMAT(1H ,12X,3HTF2,17X,3HTF3,17X,5HDVWAL,15X,
     1        6HDCOEF1,14X,6HDCOEF2,/)
C
      WRITE(6,712) CA,CB,CC,CD
      WRITE(6,712) CE,CF,CG,CH
      WRITE(6,712) CI,CJ,CK,CL
      WRITE(6,712) CM,CN,CP,CR
      WRITE(6,712) CS,CT,CU,CW
      WRITE(6,712) CAA,CBB,CCC,CDD
      WRITE(6,712) CEE,CFF,CGG,CHH
      WRITE(6,711) DELT1,DELT2,DELT3,DELT4,TF1
      WRITE(6,711) TF2,TF3,DVWAL,DCOEF1,DCOEF2
  711 FORMAT(10X,E15.6,5X,E15.6,5X,E15.6,5X,E15.6,5X,E15.6)
  712 FORMAT(10X,E15.6,5X,E15.6,5X,E15.6,5X,E15.6)
C
C     THE COMPUTING OF THE PROPERTY COEFFICIENT AS
C     A FUNCTION OF THETA
C
      T1 = 0.0
      T2 = T1*T1
      T3 = T2*T1
```

```
C
      WRITE(6,3000)
      WRITE(6,3001)
      WRITE(6,3002)
 3000 FORMAT(1H1,6X,28HCOEFFS AS FUNCTIONS OF THETA,/)
 3001 FORMAT(1H ,6X,5HTHETA,10X,4HA111,11X,4HA222,11X,4HA333,11X,
     1       4HA444,11X,4HB111,11X,4HB222)
 3002 FORMAT(1H ,21X,5HDA111,10X,5HDA222,10X,5HDA333,10X,5HDA444,10X,
     1       5HDB111,10X,5HDB222,/)
C
      DO 61 KT = 1,11
      A111 = CA+CB*T1+CC*T2+CD*T3
      A222 = CE+CF*T1+CG*T2+CH*T3
      A333 = CI+CJ*T1+CK*T2+CL*T3
      A441 = CM+CN*T1+CP*T2+CR*T3
      A442 = CS+CT*T1+CU*T2+CW*T3
      A444 = A441-A442
      B111 = CAA+CBB*T1+CCC*T2+CDD*T3
      B222 = CEE+CFF*T1+CGG*T2+CHH*T3
C     THE DERIVATIVES
      DA111 = CB+2.0*CC*T1+3.0*CD*T2
      DA222 = CF+2.0*CG*T1+3.0*CH*T2
      DA333 = CJ+2.0*CK*T1+3.0*CL*T2
      DA441 = CN+2.0*CP*T1+3.0*CR*T2
      DA442 = CT+2.0*CU*T1+3.0*CW*T2
      DA444 = DA441-DA442
      DB111 = CBB+2.0*CCC*T1+3.0*CDD*T2
      DB222 = CFF+2.0*CGG*T1+3.0*CHH*T2
C
      WRITE(6,111) T1,A111,A222,A333,A444,B111,B222
      WRITE(6,112) DA111,DA222,DA333,DA444,DB111,DB222
  111 FORMAT(6X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,
     1       2X,E13.6)
  112 FORMAT(21X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,/)
C
      T1 = T1+0.1
      T2 = T1*T1
      T3 = T2*T1
   61 CONTINUE
C
C
C     CONSTANT PROPERTY SOLUTION (PRNO = 2.0)
C     PURPOSE-TO PROVIDE A STARTING SOLUTION FOR THE
C               ITERATION PROCESS
C
      KKP =  1
C
      Y(1) = 0.0
      Y(2) = 0.0
      Y(3) = 0.5713
      Y(4) = 1.0
      Y(5) =-0.7165
      Y(6) = 0.0
      Y(7) = DELX
C
      U31L = Y(3)
      U52L = Y(5)
C
      A(1,1) = Y(1)
      A(2,1) = Y(2)
      A(3,1) = Y(3)
      A(4,1) = Y(4)
```

```
      A(5,1) = Y(5)
C
      WRITE(6,707)
  707 FORMAT(1H1,11X,30HTHE CONSTANT PROPERTY SOLUTION,/)
      WRITE(6,708)
  708 FORMAT(1H ,11X,3HETA,17X,6HF(ETA),14X,7HF'(ETA),13X,8HF''(ETA),
     1       12X,6HT(ETA),14X,7HT'(ETA))
      WRITE(6,709) Y(6),A(1,1),A(2,1),A(3,1),A(4,1),A(5,1)
C
      CALL START(5,1,1,DELX,0.0,0.0,0.0,0.0,0.0)
C
      A(1,2) = Y(1)
      A(2,2) = Y(2)
      A(3,2) = Y(3)
      A(4,2) = Y(4)
      A(5,2) = Y(5)
C
      KKK = 9
      KXX = KXMAT - 2
      DO 100 KK = 1,KXX
      N = KK+2
      CALL KAMSUB (1)
      A(1,N) = Y(1)
      A(2,N) = Y(2)
      A(3,N) = Y(3)
      A(4,N) = Y(4)
      A(5,N) = Y(5)
C
      IF(KKK.NE.KK)GO TO 100
      KKK = KKK + 10
      WRITE(6,709) Y(6),A(1,N),A(2,N),A(3,N),A(4,N),A(5,N)
  709 FORMAT(9X,E15.6,5X,E15.6,5X,E15.6,5X,E15.6,5X,E15.6,5X,E15.6)
  100 CONTINUE
C
C     THE PARTICULAR SOLUTION
 2000 KKP = 2
      Y(1) = 0.0
      Y(2) = 0.0
      Y(3) = 0.5713
      Y(4) = 1.0
      Y(5) =-0.7165
      Y(6) = 0.0
      Y(7) = DELX
      M = 1
      CALL START (5,1,1,DELX,0.0,0.0,0.0,0.0,0.0)
      KXX = KXMAT - 2
      DO 101 KK =1,KXX
      M = KK + 1
      CALL KAMSUB (1)
  101 CONTINUE
C     STORE FINAL VALUES
      X2P = Y(2)
      X4P = Y(4)
C
C     THE WRITE STATEMENTS FOR THE PARTICULAR SOLUTION
      WRITE(6,713)
      WRITE(6,7131)
      WRITE(6,715) X2P,X4P
  713 FORMAT(1H1,12X,36HTHE PARTICULAR SOLUTION FINAL VALUES,/)
 7131 FORMAT(1H ,12X,3HX2P,17X,3HX4P)
  715 FORMAT(12X,E15.8,5X,E15.8,/)
C
```

```
C       THE FIRST HOMOGENEOUS SOLUTION
        KKP =  3
        Y(1) = 0.0
        Y(2) = 0.0
        Y(3) = 1.0
        Y(4) = 0.0
        Y(5) = 0.0
        Y(6) = 0.0
        Y(7) = DELX
        M = 1
        CALL START(5,1,1,DELX,0.0,0.0,0.0,0.0,0.0)
        KXX = KXMAT - 2
        DO 102 KK = 1,KXX
        M = KK + 1
        CALL KAMSUB (1)
  102   CONTINUE
C       STORE FINAL VALUES
        X2H1 = Y(2)
        X4H1 = Y(4)
C
C       THE WRITE STATEMENTS FOR THE FIRST
C       HOMOGENEOUS SOLUTION
        WRITE(6,716)
        WRITE(6,7161)
        WRITE(6,717) X2H1,X4H1
  716   FORMAT(1H ,12X,43HTHE FIRST HOMOGENEOUS SOLUTION FINAL VALUE,/)
 7161   FORMAT(1H ,12X,4HX2H1,16X,4HX4H1)
  717   FORMAT(12X,E15.8,5X,F15.8,/)
C
C       THE SECOND HOMOGENEOUS SOLUTION
        KKP =  4
        Y(1) = 0.0
        Y(2) = 0.0
        Y(3) = 0.0
        Y(4) = 0.0
        Y(5) = 1.0
        Y(6) = 0.0
        Y(7) = DELX
        M = 1
        CALL START(5,1,1,DELX,0.0,0.0,0.0,0.0,0.0)
        KXX = KXMAT - 2
        DO 103 KK=1,KXX
        M = KK + 1
        CALL KAMSUB (1)
  103   CONTINUE
C       STORE FINAL VALUES
        X2H2 = Y(2)
        X4H2 = Y(4)
C
C       THE WRITE STATEMENTS FOR THE SECOND
C       HOMOGENEOUS SOLUTION
        WRITE(6,718)
        WRITE(6,7181)
        WRITE(6,719) X2H2,X4H2
  718   FORMAT(1H ,12X,44HTHE SECOND HOMOGENEOUS SOLUTION FINAL VALUES,/)
 7181   FORMAT(1H ,12X,4HX2H2,16X,4HX4H2)
  719   FORMAT(12X,E15.8,5X,E15.8,/)
C
C       COMPUTING THE NEW INITIAL CONDITIONS
        DENOM = (X2H1*X4H2)-(X2H2*X4H1)
        ZUM = (X2M-X2P)*(X4H2)-(X4M-X4P)*(X2H2)
        U31 = ZUM/DENOM
```

```
      ZUM = (X4M-X4P)*(X2H1)-(X2M-X2P)*(X4H1)
      U52 = ZUM/DENOM
      U31 = 0.5713+U31
      U52 =-0.7165+U52
C
C     THE WRITE STATEMENTS FOR THE
C     BOUNDARY CONDITIONS
      WRITE(6,720)
  720 FORMAT(1H ,12X,21HVALUES OF U31 AND U52,/)
      WRITE(6,7201)
 7201 FORMAT(1H ,12X,3HU31,17X,3HU52)
      WRITE(6,721) U31,U52
  721 FORMAT(12X,E15.8,5X,E15.8)
C
C     THE NEXT STARTING SOLUTION
C
      KKP =  5
C
      Y(1) = 0.0
      Y(2) = 0.0
      Y(3) = U31
      Y(4) = 1.0
      Y(5) = U52
      Y(6) = 0.0
      Y(7) = DELX
C
      Y1 = Y(1)
      Y2 = Y(2)
      Y3 = Y(3)
      Y4 = Y(4)
      Y5 = Y(5)
C
      WRITE(6,200)
  200 FORMAT(1H1,5X,35HTHE NEXT STARTING OR FINAL SOLUTION,/)
      WRITE(6,201)
  201 FORMAT(1H ,5X,3HETA,12X,6HF(ETA),9X,7HF'(ETA),8X,8HF''(ETA),7X,
     1       9HF'''(ETA),6X,6HT(ETA),9X,7HT'(ETA),8X,8HT''(ETA))
C
      M = 1
      CALL DERFUN
      WRITE(6,202) Y(6),Y(1),Y(2),Y(3),Y(10),Y(4),Y(11),Y(12)
C
      CALL START(5,1,1,DELX,0.0,0.0,0.0,0.0,0.0)
C
      A(1,1) = Y1
      A(2,1) = Y2
      A(3,1) = Y3
      A(4,1) = Y4
      A(5,1) = Y5
C
      Y1 = Y(1)
      Y2 = Y(2)
      Y3 = Y(3)
      Y4 = Y(4)
      Y5 = Y(5)
C
      KKK=9
      KXX = KXMAT - 2
      DO 105 KK = 1,KXX
      M = KK + 1
      N = KK + 1
      CALL KAMSUB (1)
```

```
      A(1,N) = Y1
      A(2,N) = Y2
      A(3,N) = Y3
      A(4,N) = Y4
      A(5,N) = Y5
C
      Y1 = Y(1)
      Y2 = Y(2)
      Y3 = Y(3)
      Y4 = Y(4)
      Y5 = Y(5)
C
      IF(KKK.NE.KK)GO TO 105
      KKK = KKK + 10
      WRITE(6,202) Y(6),Y(1),Y(2),Y(3),Y(10),Y(4),Y(11),Y(12)
  202 FORMAT(5X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,2X,E13.6,2X,
     1         E13.6,2X,E13.6,2X,E13.6)
  105 CONTINUE
C
      A(1,KXMAT) = Y1
      A(2,KXMAT) = Y2
      A(3,KXMAT) = Y3
      A(4,KXMAT) = Y4
      A(5,KXMAT) = Y5
C
C     TESTING THE NEW BOUNDARY CONDITIONS
      DELU31 = U31L - U31
      DELU52 = U52L - U52
C
      IF(ABS(DELU31).LT.1.0E-03)GO TO 500
      U31L = U31
      U52L = U52
      GO TO 2000
C
  500 IF(ABS(DELU52).LE.1.0E-03)GO TO 501
      U31L = U31
      U52L = U52
      GO TO 2000
C
  501 STOP
      END
$IBFTC DERFUN
      SUBROUTINE DERFUN
C
C
      COMMON/SXARF/A(5,501),KKP,M
      COMMON Y(202)
      COMMON/SCOEF/CA,CB,CC,CD,
     1             CE,CF,CG,CH,
     2             CI,CJ,CK,CL,
     3             CM,CN,CP,CR,
     4             CS,CT,CU,CW,
     5             CAA,CBB,CCC,CDD,
     6             CEE,CFF,CGG,CHH
C
C     LOGIC
      IF(KKP.EQ.1)GO TO 1001
      IF(KKP.EQ.2)GO TO 1002
      IF(KKP.EQ.3)GO TO 1003
      IF(KKP.EQ.4)GO TO 1003
      IF(KKP.EQ.5)GO TO 1002
C
```

```
C     THE CONSTANT PROPERTY EQUATIONS
 1001 Y(8) =  Y(2)
      Y(9) =  Y(3)
      Y(10)=-3.0*Y(1)*Y(3)+2.0*Y(2)*Y(2)-Y(4)
      Y(11)= Y(5)
      Y(12)= -3.0*2.0*Y(1)*Y(5)
      RETURN
C
C     THE NON-HOMOGENEOUS LINEAR EQUATIONS
 1002 Y(8) =  Y(2)
      Y(9) =  Y(3)
C
      SJ1 = -CE*A(3,M)
     1      -CF*A(3,M)*A(4,M)
     2      -CG*A(3,M)*A(4,M)*A(4,M)
     3      -CH*A(3,M)*A(4,M)*A(4,M)*A(4,M)
      SJ2 = +2.0*CI*A(2,M)
     1      +2.0*CJ*A(2,M)*A(4,M)
     2      +2.0*CK*A(2,M)*A(4,M)*A(4,M)
     3      +2.0*CL*A(2,M)*A(4,M)*A(4,M)*A(4,M)
      SJ3 = -CA*A(5,M)
     1      -CB*A(4,M)*A(5,M)
     2      -CC*A(4,M)*A(4,M)*A(5,M)
     3      -CD*A(4,M)*A(4,M)*A(4,M)*A(5,M)
     4      -CE*A(1,M)
     5      -CF*A(1,M)*A(4,M)
     6      -CG*A(1,M)*A(4,M)*A(4,M)
     7      -CH*A(1,M)*A(4,M)*A(4,M)*A(4,M)
      SJ41 = -CB*A(3,M)*A(5,M)
     1       -2.0*CC*A(3,M)*A(4,M)*A(5,M)
     2       -3.0*CD*A(3,M)*A(4,M)*A(4,M)*A(5,M)
      SJ42 = -CF*A(1,M)*A(3,M)
     1       -2.0*CG*A(1,M)*A(3,M)*A(4,M)
     2       -3.0*CH*A(1,M)*A(3,M)*A(4,M)*A(4,M)
      SJ43 = +CJ*A(2,M)*A(2,M)
     1       +2.0*CK*A(2,M)*A(2,M)*A(4,M)
     2       +3.0*CL*A(2,M)*A(2,M)*A(4,M)*A(4,M)
      SJ44 = -CN
     1       -2.0*CP*A(4,M)
     2       -3.0*CR*A(4,M)*A(4,M)
      SJ45 = +CT
     1       +2.0*CU*A(4,M)
     2       +3.0*CW*A(4,M)*A(4,M)
      SJ4 =  SJ41+SJ42+SJ43+SJ44+SJ45
      SJ5 =  -CA*A(3,M)
     1       -CB*A(3,M)*A(4,M)
     2       -CC*A(3,M)*A(4,M)*A(4,M)
     3       -CD*A(3,M)*A(4,M)*A(4,M)*A(4,M)
C
      HJ1 = +CA*A(3,M)*A(5,M)
     1      +2.0*CB*A(3,M)*A(4,M)*A(5,M)
     2      +3.0*CC*A(3,M)*A(4,M)*A(4,M)*A(5,M)
     3      +4.0*CD*A(3,M)*A(4,M)*A(4,M)*A(4,M)*A(5,M)
      HJ2 = +CE*A(1,M)*A(3,M)
     1      +2.0*CF*A(1,M)*A(3,M)*A(4,M)
     2      +3.0*CG*A(1,M)*A(3,M)*A(4,M)*A(4,M)
     3      +4.0*CH*A(1,M)*A(3,M)*A(4,M)*A(4,M)*A(4,M)
      HJ3 = -CI*A(2,M)*A(2,M)
     1      -2.0*CJ*A(2,M)*A(2,M)*A(4,M)
     2      -3.0*CK*A(2,M)*A(2,M)*A(4,M)*A(4,M)
     3      -4.0*CL*A(2,M)*A(2,M)*A(4,M)*A(4,M)*A(4,M)
      HJ4 = -CM
```

```
     1        -0.0
     2        +CP*A(4,M)*A(4,M)
     3        +2.0*CR*A(4,M)*A(4,M)*A(4,M)
      HJ5  =  +CS
     1        +0.0
     2        -CU*A(4,M)*A(4,M)
     3        -2.0*CW*A(4,M)*A(4,M)*A(4,M)
C     REMOVING END POINT ERROR ON DENSITY RATIO
      IF(ABS(HJ4).LE.ABS(HJ5))GO TO 18
      GO TO 19
   18 HJ4  =  0.0
      HJ5  =  0.0
C
   19 HJ  =  HJ1+HJ2+HJ3+HJ4+HJ5
C
      Y(10)  =  +(SJ1)*Y(1)
     1          +(SJ2)*Y(2)
     2          +(SJ3)*Y(3)
     3          +(SJ4)*Y(4)
     4          +(SJ5)*Y(5)
     5          +(HJ)
C
      Y(11)  =  Y(5)
C
      ST1  =  -CEE*A(5,M)
     1        -CFF*A(4,M)*A(5,M)
     2        -CGG*A(4,M)*A(4,M)*A(5,M)
     3        -CHH*A(4,M)*A(4,M)*A(4,M)*A(5,M)
      ST4  =  -CBB*A(5,M)*A(5,M)
     1        -2.0*CCC*A(4,M)*A(5,M)*A(5,M)
     2        -3.0*CDD*A(4,M)*A(4,M)*A(5,M)*A(5,M)
     3        -CFF*A(1,M)*A(5,M)
     4        -2.0*CGG*A(1,M)*A(4,M)*A(5,M)
     5        -3.0*CHH*A(1,M)*A(4,M)*A(4,M)*A(5,M)
      ST5  =  -2.0*CAA*A(5,M)
     1        -2.0*CBB*A(4,M)*A(5,M)
     2        -2.0*CCC*A(4,M)*A(4,M)*A(5,M)
     3        -2.0*CDD*A(4,M)*A(4,M)*A(4,M)*A(5,M)
     4        -CEE*A(1,M)
     5        -CFF*A(1,M)*A(4,M)
     6        -CGG*A(1,M)*A(4,M)*A(4,M)
     7        -CHH*A(1,M)*A(4,M)*A(4,M)*A(4,M)
      HT  =  +CAA*A(5,M)*A(5,M)
     1       +2.0*CBB*A(4,M)*A(5,M)*A(5,M)
     2       +3.0*CCC*A(4,M)*A(4,M)*A(5,M)*A(5,M)
     3       +4.0*CDD*A(4,M)*A(4,M)*A(4,M)*A(5,M)*A(5,M)
     4       +CEE*A(1,M)*A(5,M)
     5       +2.0*CFF*A(1,M)*A(4,M)*A(5,M)
     6       +3.0*CGG*A(1,M)*A(4,M)*A(4,M)*A(5,M)
     7       +4.0*CHH*A(1,M)*A(4,M)*A(4,M)*A(4,M)*A(5,M)
C
      Y(12)  =  +(ST1)*Y(1)
     1          +(ST4)*Y(4)
     2          +(ST5)*Y(5)
     3          +(HT)
      RETURN
C     THE HOMOGENEOUS LINEAR EQUATIONS
 1003 Y(8)  =   Y(2)
      Y(9)  =   Y(3)
C
      SJ1  =  -CE*A(3,M)
     1        -CF*A(3,M)*A(4,M)
```

```
     2        -CG*A(3,M)*A(4,M)*A(4,M)
     3        -CH*A(3,M)*A(4,M)*A(4,M)*A(4,M)
      SJ2  =  +2.0*CI*A(2,M)
     1        +2.0*CJ*A(2,M)*A(4,M)
     2        +2.0*CK*A(2,M)*A(4,M)*A(4,M)
     3        +2.0*CL*A(2,M)*A(4,M)*A(4,M)*A(4,M)
      SJ3  =  -CA*A(5,M)
     1        -CB*A(4,M)*A(5,M)
     2        -CC*A(4,M)*A(4,M)*A(5,M)
     3        -CD*A(4,M)*A(4,M)*A(4,M)*A(5,M)
     4        -CE*A(1,M)
     5        -CF*A(1,M)*A(4,M)
     6        -CG*A(1,M)*A(4,M)*A(4,M)
     7        -CH*A(1,M)*A(4,M)*A(4,M)*A(4,M)
      SJ41  =  -CB*A(3,M)*A(5,M)
     1          -2.0*CC*A(3,M)*A(4,M)*A(5,M)
     2          -3.0*CD*A(3,M)*A(4,M)*A(4,M)*A(5,M)
      SJ42  =  -CF*A(1,M)*A(3,M)
     1          -2.0*CG*A(1,M)*A(3,M)*A(4,M)
     2          -3.0*CH*A(1,M)*A(3,M)*A(4,M)*A(4,M)
      SJ43  =  +CJ*A(2,M)*A(2,M)
     1          +2.0*CK*A(2,M)*A(2,M)*A(4,M)
     2          +3.0*CL*A(2,M)*A(2,M)*A(4,M)*A(4,M)
      SJ44  =  -CN
     1          -2.0*CP*A(4,M)
     2          -3.0*CR*A(4,M)*A(4,M)
      SJ45  =  +CT
     1          +2.0*CU*A(4,M)
     2          +3.0*CW*A(4,M)*A(4,M)
      SJ4  =   SJ41+SJ42+SJ43+SJ44+SJ45
      SJ5  =   -CA*A(3,M)
     1         -CB*A(3,M)*A(4,M)
     2         -CC*A(3,M)*A(4,M)*A(4,M)
     3         -CD*A(3,M)*A(4,M)*A(4,M)*A(4,M)
C
      Y(10)  =  +(SJ1)*Y(1)
     1          +(SJ2)*Y(2)
     2          +(SJ3)*Y(3)
     3          +(SJ4)*Y(4)
     4          +(SJ5)*Y(5)
C
      Y(11)  =  Y(5)
C
      ST1  =  -CEE*A(5,M)
     1        -CFF*A(4,M)*A(5,M)
     2        -CGG*A(4,M)*A(4,M)*A(5,M)
     3        -CHH*A(4,M)*A(4,M)*A(4,M)*A(5,M)
      ST4  =  -CBB*A(5,M)*A(5,M)
     1        -2.0*CCC*A(4,M)*A(5,M)*A(5,M)
     2        -3.0*CDD*A(4,M)*A(4,M)*A(5,M)*A(5,M)
     3        -CFF*A(1,M)*A(5,M)
     4        -2.0*CGG*A(1,M)*A(4,M)*A(5,M)
     5        -3.0*CHH*A(1,M)*A(4,M)*A(4,M)*A(5,M)
      ST5  =  -2.0*CAA*A(5,M)
     1        -2.0*CBB*A(4,M)*A(5,M)
     2        -2.0*CCC*A(4,M)*A(4,M)*A(5,M)
     3        -2.0*CDD*A(4,M)*A(4,M)*A(4,M)*A(5,M)
     4        -CEE*A(1,M)
     5        -CFF*A(1,M)*A(4,M)
     6        -CGG*A(1,M)*A(4,M)*A(4,M)
     7        -CHH*A(1,M)*A(4,M)*A(4,M)*A(4,M)
C
```

```
      Y(12) = +(ST1)*Y(1)
     1         +(ST4)*Y(4)
     2         +(ST5)*Y(5)
      RETURN
      END
$IBFTC START
      SUBROUTINE START(M1,M2,M3,A1,A2,A3,A4,A5,A6)
      COMMON/SHARE/NN,SPACE,MODE,KKA,E1MAX,E1MIN,E2MAX,E2MIN,FACT
C
C   NO INFORMATION IS REQUIRED IN THIS SUBROUTINE
C
      NN=M1
      MODE=M2
      KKA=M3
      E1MAX=A2
      E1MIN=A3
      E2MAX=A4
      E2MIN=A5
      FACT=A6
      SPACE=A1
      CALL KAMSUB(0)
      END
C
$IBFTC KAMSUB
      SUBROUTINE KAMSUB(NSTART)
      DIMENSION DELY(4,100),BET(4),XV(5),FV(4,100),YU(5,100)
      COMMON /SHARE/NN,SPACE,MODE,KKA,E1MAX,E1MIN,E2MAX,E2MIN,FACT
      COMMON Y(202)
      COMMON/INTDAT/Z(5,202),IERR
      DOUBLE PRECISION YU
C
C   NO INFORMATION IS REQUIRED IN THIS SUBROUTINE
C
      IF(NSTART.LE.0)GO TO 9977
      GO TO (1001,2000,2000),MODE
C      RUNGE-KUTTA
1000  LL=1
1001  DO 1034 K=1,4
      DO 1350 I=1,NN
      DELY(K,I)=Y(N2)*FV(MM,I)
      Q=YU(MM,I)
1350  Y(I)=Q+BET(K)*DELY(K,I)
      Y(NP1)=BET(K)*Y(N2)+XV(MM)
      CALL DERFUN
      DO 1100 I=1,NN
      IPN2=I+N2
1100  FV(MM,I)=Y(IPN2)
1034  CONTINUE
      DO 1039 I=1,NN
      DEL=(DELY(1,I)+2.0*DELY(2,I)+2.0*DELY(3,I)+DELY(4,I))/6.0
      YU(MM+1,I)=YU(MM,I)+DEL
1039  CONTINUE
      MM=MM+1
      XV(MM)=XV(MM-1)+Y(N2)
      DO 1400 I=1,NN
1400  Y(I)=YU(MM,I)
      Y(NP1)=XV(MM)
      CALL DERFUN
      GO TO (42,100,100),MODE
100   DO 150 I=1,NN
      IPN2=I+N2
150   FV(MM,I)=Y(IPN2)
```

```
      GO TO (1001,1001,1001,2000),MM
C     ADAMS-MOULTON
2000  DO 2048 I=1,NN
      DEL=Y(N2)*(55.0*FV(4,I)-59.0*FV(3,I)
     1+37.0*FV(2,I)-9.0*FV(1,I))/24.0
      Y(I)=YU(4,I)+DEL
2048  DELY(1,I)=Y(I)
      Y(NP1)=XV(4)+Y(N2)
      CALL DERFUN
      XV(5)=Y(NP1)
      DO 2051 I=1,NN
      IPN2=I+N2
      DEL=Y(N2)*(9.0*Y(IPN2)+19.0*FV(4,I)
     1-5.0*FV(3,I)+FV(2,I))/24.0
      YU(5,I)=YU(4,I)+DEL
2051  Y(I)=YU(5,I)
      CALL DERFUN
      GO TO (42,42,3000),MODE
C     ERROR ANALYSIS
3000  SSE=0.0
      DO 3033 I=1,NN
      EPSIL=R*ABS(Y(I)-DELY(1,I))
      GO TO (3301,3307),KKA
3301  IF(Y(I))3650,3307,3650
3650  EPSIL=EPSIL/ABS(Y(I))
3307  IF(SSE-EPSIL)3032,3033,3033
3032  SSE=EPSIL
3033  CONTINUE
      IF(E1MAX-SSE)3034,3034,3035
3034  IF(ABS(Y(N2))-E2MIN)42,42,4340
3035  IF(SSE-E1MIN)3036,42,42
3036  IF(E2MAX-ABS(Y(N2)))42,42,5360
4340  LL=1
      IERR = 1
      MM=1
      Y(N2)=Y(N2)*FACT
      GO TO 1001
5360  GO TO (42,5361),LL
5361  XV(2)=XV(3)
      XV(3)=XV(5)
      DO 5363 I=1,NN
      FV(2,I)=FV(3,I)
      IPN2=I+N2
      FV(3,I)=Y(IPN2)
      YU(2,I)=YU(3,I)
5363  YU(3,I)=YU(5,I)
      Y(N2)=2.0*Y(N2)
      IERR = 2
      LL=2
      MM=3
      GO TO 1001
C     EXIT ROUTINE
   42 GO TO(43,44,44),MODE
   44 DO 707 K=1,4
      Z(K,NP1)= XV(K)
      Z(K,N2)=XV(K+1)-XV(K)
      DO 707 I=1,NN
      Z(K,I)= YU(K,I)
      IPN2 = N2 + I
  707 Z(K,IPN2)= FV(K,I)
   43 Z(5,NP1) = XV(5)
      DO 708 I= 1,NN
```

```
      Z(5,I)= YU(5,I)
      IPN2  = N2 + I
  708  Z(5,IPN2)= Y(IPN2)
      Z(5,N2)= Y(N2)
      DO 12 K=1,3
      XV(K)=XV(K+1)
      DO 12 I=1,NN
      FV(K,I)=FV(K+1,I)
12    YU(K,I)=YU(K+1,I)
      LL=2
      MM=4
      XV(4)=XV(5)
      DO 52 I=1,NN
      IPN2=I+N2
      FV(4,I)=Y(IPN2)
52    YU(4,I)=YU(5,I)
      GO TO (70,70,73),MODE
9977  CONTINUE
      IERR = 3
      ALPHA=Y(NN+1)
      EPM=0.0
      GO TO (7,9,9),MODE
7     MM=4
      GO TO 8
9     MM=1
8     BET(1)=0.5
      BET(2)=0.5
      BET(3)=1.0
      BET(4)=0.0
5     N2=NN+2
      Y(N2)=SPACE
      NP1=NN+1
      R=19.0/270.0
      XV(MM)=Y(NP1)
      IF(E1MIN)2,2,1
2     E1MIN=E1MAX/55.0
1     IF(FACT)4,4,3
4     FACT=1.0/2.0
3     CALL DERFUN
      DO 320 I=1,NN
      IPN2=I+N2
      FV(MM,I)=Y(IPN2)
320   YU(MM,I)=Y(I)
      GO TO 1000
73    E=ABS(XV(4)-ALPHA)
      IF(E-EPM)2000,2000,71
71    EPM=E
70    RETURN
      END
```

TABLE II

COMPUTER INPUT DATA FOR CASE 1

A1	B1	C1	D1
A2	B2	C2	D2
A3	B3	C3	D3
A4	B4	C4	D4
A5	B5	C5	D5
TI	DENI	VISHPI	TF
DENF	DELX	X2M	X4M
KXMAT			

-0.10153561E 05	0.38964079E 02	-0.49843979E-01	0.21255052E-04
-0.90019949E 03	0.22697023E 01	-0.14277368E-02	0.
-0.65521389E 03	0.24862754E 01	-0.31476978E-02	0.13297636E-05
-0.13237356E 03	0.33796765E 00	-0.21575205E-03	0.
0.55669496E 04	-0.14193787E 02	0.90533624E-02	0.
0.77969000E 03	0.24268362E 02	0.27105049E-01	0.77468999E 03
0.27149972E 02	0.20000000E-01	0.50000000E-05	0.50000000E-05
501			

TABLE III

POLYNOMIAL REGRESSION COEFFICIENTS FOR CASE 1

CA	CB	CC	CD	
CE	CF	CG	CH	
CI	CJ	CK	CL	
CM	CN	CP	CR	
CS	CT	CU	CW	
CAA	CBB	CCC	CDD	
CEE	CFF	CGG	CHH	
DELT1	DELT2	DELT3	DELT4	TF1
TF2	TF3	DVWAL	DCOEF1	DCOEF2
-0.230849E 00	0.128255E 00	-0.557186E-01	0.132844E-01	
0.250154E 01	0.568289E 00	-0.704369E-01	0.	
0.166769E 01	0.378860E 00	-0.469579E-01	0.	
0.702032E 01	0.259201E 01	-0.215209E 00	0.250006E-01	
0.702249E 01	0.159534E 01	-0.197735E 00	0.	
-0.179143E 00	0.921434E-01	-0.269690E-01	0.	
0.886390E 01	-0.164470E 01	0.446644E 00	0.	
0.500000E 01	0.250000E 02	0.125000E 03	0.625000E 03	0.774690E 03
0.600145E 06	0.464926E 09	0.657795E 00	0.150406E 03	0.553982E 01

TABLE IV

COEFFICIENTS OF THE REDUCED DIFFERENTIAL EQUATIONS AS FUNCTIONS OF $\theta(\eta)$ FOR CASE 1

THETA	A111	A222	A333	A444	B111	B222
	DA111	DA222	DA333	DA444	DB111	DB222
0.	-0.230849E 00	0.250154E 01	0.166769E 01	-0.217748E-02	-0.179143E 00	0.886390E 01
	0.128255E 00	0.568289E 00	0.378860E 00	0.996669E 00	0.921434E-01	-0.164470E 01
0.100000E 00	-0.218568E 00	0.255766E 01	0.170511E 01	0.973397E-01	-0.170199E 00	0.870390E 01
	0.117510E 00	0.554202E 00	0.369468E 00	0.993925E 00	0.867495E-01	-0.155537E 01
0.200000E 00	-0.207321E 00	0.261238E 01	0.174159E 01	0.196657E 00	-0.161793E 00	0.855283E 01
	0.107562E 00	0.540115E 00	0.360076E 00	0.992680E 00	0.813557E-01	-0.146604E 01
0.300000E 00	-0.197029E 00	0.266569E 01	0.177712E 01	0.295926E 00	-0.153928E 00	0.841069E 01
	0.984110E-01	0.526027E 00	0.350685E 00	0.992935E 00	0.759619E-01	-0.137671E 01
0.400000E 00	-0.187612E 00	0.271759E 01	0.181172E 01	0.395294E 00	-0.146601E 00	0.827749E 01
	0.900571E-01	0.511940E 00	0.341293E 00	0.994691E 00	0.705681E-01	-0.128738E 01
0.500000E 00	-0.178991E 00	0.276808E 01	0.184538E 01	0.494914E 00	-0.139814E 00	0.815322E 01
	0.825001E-01	0.497852E 00	0.331902E 00	0.997946E 00	0.651743E-01	-0.119805E 01
0.600000E 00	-0.171085E 00	0.281716E 01	0.187810E 01	0.594934E 00	-0.133566E 00	0.803788E 01
	0.757403E-01	0.483765E 00	0.322510E 00	0.100270E 01	0.597805E-01	-0.110872E 01
0.700000E 00	-0.163816E 00	0.286483E 01	0.190989E 01	0.695504E 00	-0.127858E 00	0.793147E 01
	0.697775E-01	0.459678E 00	0.313118E 00	0.100896E 01	0.543867E-01	-0.101939E 01
0.800000E 00	-0.157103E 00	0.291109E 01	0.194073E 01	0.796775E 00	-0.122689E 00	0.783400E 01
	0.646117E-01	0.455590E 00	0.303727E 00	0.101671E 01	0.489929E-01	-0.930065E 00
0.900000E 00	-0.150867E 00	0.295595E 01	0.197063E 01	0.898896E 00	-0.118059E 00	0.774546E 01
	0.602431E-01	0.441503E 00	0.294335E 00	0.102597E 01	0.435991E-01	-0.840736E 00
0.100000E 01	-0.145028E 00	0.299939E 01	0.199959E 01	0.100202E 01	-0.113969E 00	0.766585E 01
	0.556715E-01	0.427416E 00	0.284944E 00	0.103672E 01	0.382053E-01	-0.751407E 00

TABLE V

SOLUTIONS OF THE REDUCED DIFFERENTIAL EQUATIONS AS FUNCTIONS OF η FOR CASE 1

ETA	F(ETA)	F'(ETA)	F''(ETA)	F'''(ETA)	T(ETA)	T'(ETA)	T''(ETA)
0.	0.	0.	0.555436E 00	-0.110581E 01	0.100000E 01	-0.857689E 00	0.838390E-01
0.200000E 00	0.970626E-02	0.904165E-01	0.356041E 00	-0.888291E 00	0.830373E 00	-0.836405E 00	0.148192E 00
0.400000E 00	0.337981E-01	0.145293E 00	0.199742E 00	-0.677728E 00	0.666949E 00	-0.792899E 00	0.295090E 00
0.600000E 00	0.660146E-01	0.173011E 00	0.838402E-01	-0.485718E 00	0.515376E 00	-0.717407E 00	0.456920E 00
0.800000E 00	0.101704E 00	0.181212E 00	0.362308E-02	-0.322018E 00	0.381920E 00	-0.613243E 00	0.573646E 00
0.100000E 01	0.137636E 00	0.176420E 00	-0.472187E-01	-0.192328E 00	0.271137E 00	-0.493306E 00	0.612123E 00
0.120000E 01	0.171755E 00	0.163821E 00	-0.755948E-01	-0.969392E-01	0.184570E 00	-0.373661E 00	0.573439E 00
0.140000E 01	0.202902E 00	0.147243E 00	-0.880181E-01	-0.318753E-01	0.120762E 00	-0.267462E 00	0.482896E 00
0.160000E 01	0.230564E 00	0.129310E 00	-0.899583E-01	0.898646E-02	0.761987E-01	-0.181864E 00	0.372392E 00
0.180000E 01	0.254648E 00	0.111677E 00	-0.856062E-01	0.320756E-01	0.465471E-01	-0.118171E 00	0.267004E 00
0.200000E 01	0.275318E 00	0.952865E-01	-0.779338E-01	0.430346E-01	0.276374E-01	-0.738190E-01	0.180253E 00
0.220000E 01	0.292876E 00	0.805921E-01	-0.689032E-01	0.462887E-01	0.160117E-01	-0.445844E-01	0.115785E 00
0.240000E 01	0.307678E 00	0.677344E-01	-0.597167E-01	0.450318E-01	0.908409E-02	-0.261692E-01	0.713880E-01
0.260000E 01	0.320089E 00	0.556703E-01	-0.510462E-01	0.414130E-01	0.506347E-02	-0.149954E-01	0.425540E-01
0.280000E 01	0.330456E 00	0.472594E-01	-0.432174E-01	0.367876E-01	0.278118E-02	-0.842142E-02	0.246713E-01
0.300000E 01	0.339091E 00	0.393193E-01	-0.363443E-01	0.319535E-01	0.150945E-02	-0.465073E-02	0.139803E-01
0.320000E 01	0.346269E 00	0.326582E-01	-0.304206E-01	0.273432E-01	0.811692E-03	-0.253273E-02	0.777451E-02
0.340000E 01	0.352228E 00	0.270923E-01	-0.253781E-01	0.231630E-01	0.433744E-03	-0.136338E-02	0.425698E-02
0.360000E 01	0.357168E 00	0.224545E-01	-0.211217E-01	0.194866E-01	0.231200E-03	-0.726882E-03	0.230139E-02
0.380000E 01	0.361261E 00	0.185980E-01	-0.175497E-01	0.163154E-01	0.123612E-03	-0.384455E-03	0.123116E-02
0.400000E 01	0.364651E 00	0.153955E-01	-0.145641E-01	0.136147E-01	0.668819E-04	-0.202003E-03	0.652944E-03
0.420000E 01	0.367456E 00	0.127392E-01	-0.120758E-01	0.113341E-01	0.371495E-04	-0.105559E-03	0.343823E-03
0.440000E 01	0.369777E 00	0.105374E-01	-0.100061E-01	0.941947E-02	0.216449E-04	-0.549119E-04	0.179985E-03
0.460000E 01	0.371696E 00	0.871340E-02	-0.828711E-02	0.781855E-02	0.135933E-04	-0.284585E-04	0.937617E-04
0.480000E 01	0.373283E 00	0.720304E-02	-0.686095E-02	0.648375E-02	0.942646E-05	-0.147035E-04	0.486491E-04
0.500000E 01	0.374595E 00	0.595278E-02	-0.567868E-02	0.537310E-02	0.727613E-05	-0.757734E-05	0.251587E-04
0.520000E 01	0.375678E 00	0.491807E-02	-0.469919E-02	0.445034E-02	0.616904E-05	-0.389673E-05	0.129753E-04
0.540000E 01	0.376574E 00	0.406189E-02	-0.388808E-02	0.368454E-02	0.560017E-05	-0.200045E-05	0.667682E-05
0.560000E 01	0.377313E 00	0.335353E-02	-0.321666E-02	0.304953E-02	0.530832E-05	-0.102550E-05	0.342940E-05
0.580000E 01	0.377923E 00	0.276752E-02	-0.266102E-02	0.252332E-02	0.515879E-05	-0.525092E-06	0.175875E-05
0.600000E 01	0.378427E 00	0.228274E-02	-0.220131E-02	0.208748E-02	0.508226E-05	-0.268605E-06	0.900839E-06
0.620000E 01	0.378842E 00	0.188170E-02	-0.182104E-02	0.172664E-02	0.504312E-05	-0.137295E-06	0.460939E-06
0.640000E 01	0.379184E 00	0.154994E-02	-0.150651E-02	0.142798E-02	0.502313E-05	-0.701319E-07	0.235654E-06
0.660000E 01	0.379466E 00	0.127547E-02	-0.124641E-02	0.118086E-02	0.501291E-05	-0.358058E-07	0.120395E-06
0.680000E 01	0.379697E 00	0.104838E-02	-0.103133E-02	0.976417E-03	0.500770E-05	-0.182732E-07	0.614756E-07
0.699999E 01	0.379888E 00	0.860463E-03	-0.853488E-03	0.807313E-03	0.500504E-05	-0.932254E-08	0.313764E-07
0.719999E 01	0.380044E 00	0.704935E-03	-0.706453E-03	0.667457E-03	0.500368E-05	-0.475498E-08	0.160085E-07
0.739999E 01	0.380172E 00	0.576185E-03	-0.584893E-03	0.551804E-03	0.500299E-05	-0.242486E-08	0.816549E-08
0.759999E 01	0.380276E 00	0.469575E-03	-0.484398E-03	0.456173E-03	0.500264E-05	-0.123644E-08	0.416414E-08
0.779999E 01	0.380361E 00	0.381267E-03	-0.401320E-03	0.377104E-03	0.500246E-05	-0.630429E-09	0.212328E-08
0.799999E 01	0.380429E 00	0.308088E-03	-0.332643E-03	0.311733E-03	0.500237E-05	-0.321436E-09	0.108256E-08
0.819999E 01	0.380485E 00	0.247416E-03	-0.275872E-03	0.257689E-03	0.500232E-05	-0.163898E-09	0.551927E-09
0.839999E 01	0.380529E 00	0.197084E-03	-0.228944E-03	0.213011E-03	0.500230E-05	-0.835783E-10	0.281397E-09
0.859999E 01	0.380564E 00	0.155297E-03	-0.190152E-03	0.176077E-03	0.500228E-05	-0.426266E-10	0.143478E-09
0.879999E 01	0.380592E 00	0.120575E-03	-0.158086E-03	0.145546E-03	0.500228E-05	-0.217450E-10	0.731652E-10
0.899999E 01	0.380613E 00	0.916925E-04	-0.131581E-03	0.120308E-03	0.500227E-05	-0.110959E-10	0.373162E-10
0.919999E 01	0.380629E 00	0.676367E-04	-0.109672E-03	0.994459E-04	0.500227E-05	-0.566390E-11	0.190367E-10
0.939999E 01	0.380640E 00	0.475709E-04	-0.915615E-04	0.822012E-04	0.500227E-05	-0.289243E-11	0.971441E-11
0.959999E 01	0.380648E 00	0.308030E-04	-0.765919E-04	0.679467E-04	0.500227E-05	-0.147790E-11	0.495912E-11
0.979999E 01	0.380652E 00	0.167613E-04	-0.642181E-04	0.561641E-04	0.500227E-05	-0.755634E-12	0.253277E-11
0.999999E 01	0.380655E 00	0.497289E-05	-0.539901E-04	0.464248E-04	0.500227E-05	-0.386655E-12	0.129429E-11

TABLE VI

COMPUTER INPUT DATA FOR CASE 2

A1	B1	C1	D1
A2	B2	C2	D2
A3	B3	C3	D3
A4	B4	C4	D4
A5	B5	C5	D5
TI	DENI	VISHPI	TF
DENF	DELX	X2M	X4M
KXMAT			
-0.43506594E 03	0.11182717E 01	-0.71865062E-03	0.
-0.14583423E 04	0.37103767E 01	-0.23574024E-02	0.
-0.29808781E 02	0.73594019E-01	-0.45250884E-04	0.
-0.30841414E 03	0.79239742E 00	-0.50901766E-03	0.
0.20657112E 04	-0.51201616E 01	0.31747486E-02	0.
0.77468999E 03	0.27149972E 02	0.29063668E-01	0.76968999E 03
0.32607773E 02	0.20000000E-01	0.50000000E-05	0.50000000E-05
501			

TABLE VII

POLYNOMIAL REGRESSION COEFFICIENTS FOR CASE 2

CA	CB	CC	CD	
CE	CF	CG	CH	
CI	CJ	CK	CL	
CM	CN	CP	CR	
CS	CT	CU	CW	
CAA	CBB	CCC	CDD	
CEE	CFF	CGG	CHH	
DELT1	DELT2	DELT3	DELT4	TF1
TF2	TF3	DVWAL	DCOEF1	DCOEF2

-0.441519E 00	0.299884E 00	-0.898313E-01	0.	
0.217512E 01	0.963922E 00	-0.139513E 00	0.	
0.145008E 01	0.642614E 00	-0.930087E-01	0.	
0.360286E 01	0.251876E 01	-0.144797E 00	0.	
0.360674E 01	0.159835E 01	-0.231337E 00	0.	
-0.336922E 00	0.220645E 00	-0.636272E-01	0.	
0.131787E 02	-0.275803E 01	0.187884E 00	0.	
0.500000E 01	0.250000E 02	0.125000E 03	0.625000E 03	0.769690E 03
0.592423E 06	0.455982E 09	0.789078E 00	0.127995E 03	0.392529E 01

TABLE VIII

COEFFICIENTS OF THE REDUCED DIFFERENTIAL EQUATIONS AS FUNCTIONS OF $\theta(\eta)$ FOR CASE 2

THETA	A111 DA111	A222 DA222	A333 DA333	A444 DA444	B111 DB111	B222 DB222
0.	-0.441519E 00 0.299884E 00	0.217512E 01 0.963922E 00	0.145008E 01 0.642614E 00	-0.387192E-02 0.920404E 00	-0.336922E 00 0.220645E 00	0.131787E 02 -0.275803E 01
0.100000E 00	-0.412429E 00 0.281917E 00	0.227012E 01 0.936019E 00	0.151341E 01 0.624013E 00	0.890339E-01 0.937712E 00	-0.315494E 00 0.207920E 00	0.129048E 02 -0.272045E 01
0.200000E 00	-0.385136E 00 0.263951E 00	0.236233E 01 0.908116E 00	0.157488E 01 0.605411E 00	0.183671E 00 0.955020E 00	-0.295338E 00 0.195194E 00	0.126346E 02 -0.268287E 01
0.300000E 00	-0.359639E 00 0.245985E 00	0.245174E 01 0.880214E 00	0.163450E 01 0.586809E 00	0.280038E 00 0.972328E 00	-0.276455E 00 0.182469E 00	0.123682E 02 -0.264530E 01
0.400000E 00	-0.335939E 00 0.228019E 00	0.253837E 01 0.852311E 00	0.169225E 01 0.568208E 00	0.378136E 00 0.989636E 00	-0.258844E 00 0.169744E 00	0.121056E 02 -0.260772E 01
0.500000E 00	-0.314035E 00 0.210052E 00	0.262221E 01 0.824409E 00	0.174814E 01 0.549606E 00	0.477965E 00 0.100694E 01	-0.242506E 00 0.157018E 00	0.118467E 02 -0.257014E 01
0.600000E 00	-0.293928E 00 0.192085E 00	0.270325E 01 0.796506E 00	0.180217E 01 0.531004E 00	0.579525E 00 0.102425E 01	-0.227441E 00 0.144293E 00	0.115915E 02 -0.253257E 01
0.700000E 00	-0.275618E 00 0.174120E 00	0.278151E 01 0.768603E 00	0.185434E 01 0.512402E 00	0.682816E 00 0.104156E 01	-0.213648E 00 0.131567E 00	0.113402E 02 -0.249499E 01
0.800000E 00	-0.259104E 00 0.156153E 00	0.285697E 01 0.740701E 00	0.190465E 01 0.493801E 00	0.787837E 00 0.105887E 01	-0.201127E 00 0.118842E 00	0.110925E 02 -0.245741E 01
0.900000E 00	-0.244387E 00 0.138187E 00	0.292965E 01 0.712798E 00	0.195310E 01 0.475199E 00	0.894589E 00 0.107618E 01	-0.189879E 00 0.106116E 00	0.108487E 02 -0.241984E 01
0.100000E 01	-0.231467E 00 0.120221E 00	0.299953E 01 0.684896E 00	0.199969E 01 0.456597E 00	0.100307E 01 0.109348E 01	-0.179904E 00 0.933909E-01	0.106086E 02 -0.238226E 01

TABLE IX

SOLUTIONS OF THE REDUCED DIFFERENTIAL EQUATIONS
AS FUNCTIONS OF η FOR CASE 2

ETA	F(ETA)	F'(ETA)	F''(ETA)	F'''(ETA)	T(ETA)	T'(ETA)	T''(ETA)
0.	0.	0.	0.536268E 00	-0.121972E 01	0.100000E 01	-0.101697E 01	0.186060E 00
0.200000E 00	0.919698E-02	0.848245E-01	0.321853E 00	-0.926649E 00	0.800718E 00	-0.972347E 00	0.289282E 00
0.400000E 00	0.314579E-01	0.132535E 00	0.164252E 00	-0.656713E 00	0.613385E 00	-0.893717E 00	0.505276E 00
0.600000E 00	0.604561E-01	0.153865E 00	0.566374E-01	-0.428772E 00	0.446210E 00	-0.771197E 00	0.708595E 00
0.800000E 00	0.918544E-01	0.157881E 00	-0.106511E-01	-0.253961E 00	0.307006E 00	-0.617586E 00	0.806067E 00
0.100000E 01	0.122925E 00	0.151571E 00	-0.484046E-01	-0.132163E 00	0.199589E 00	-0.457700E 00	0.773303E 00
0.120000E 01	0.152125E 00	0.139831E 00	-0.664003E-01	-0.542584E-01	0.122775E 00	-0.314712E 00	0.646013E 00
0.140000E 01	0.178709E 00	0.125818E 00	-0.722035E-01	-0.816573E-02	0.716725E-01	-0.201821E 00	0.481546E 00
0.160000E 01	0.202427E 00	0.111409E 00	-0.710550E-01	0.168683E-01	0.398614E-01	-0.121474E 00	0.326333E 00
0.180000E 01	0.223316E 00	0.976325E-01	-0.663172E-01	0.288440E-01	0.212138E-01	-0.690789E-01	0.204044E 00
0.200000E 01	0.241556E 00	0.849845E-01	-0.600210E-01	0.331762E-01	0.108528E-01	-0.373571E-01	0.119145E 00
0.220000E 01	0.257397E 00	0.736498E-01	-0.533223E-01	0.333166E-01	0.536184E-02	-0.193304E-01	0.656323E-01
0.240000E 01	0.271105E 00	0.636410E-01	-0.468320E-01	0.313550E-01	0.256990E-02	-0.962527E-02	0.344027E-01
0.260000E 01	0.282937E 00	0.548835E-01	-0.408388E-01	0.284911E-01	0.120052E-02	-0.463580E-02	0.172860E-01
0.280000E 01	0.293134E 00	0.472649E-01	-0.354514E-01	0.253742E-01	0.549464E-03	-0.216961E-02	0.837822E-02
0.300000E 01	0.301911E 00	0.406615E-01	-0.306836E-01	0.223314E-01	0.248104E-03	-0.990793E-03	0.393834E-02
0.320000E 01	0.309458E 00	0.349522E-01	-0.265035E-01	0.195116E-01	0.111762E-03	-0.443142E-03	0.180386E-02
0.340000E 01	0.315944E 00	0.300243E-01	-0.228601E-01	0.169695E-01	0.512565E-04	-0.194777E-03	0.808331E-03
0.360000E 01	0.321513E 00	0.257762E-01	-0.196964E-01	0.147134E-01	0.248320E-04	-0.843997E-04	0.355665E-03
0.380000E 01	0.326294E 00	0.221175E-01	-0.169564E-01	0.127296E-01	0.134397E-04	-0.361647E-04	0.154165E-03
0.400000E 01	0.330394E 00	0.189689E-01	-0.145878E-01	0.109956E-01	0.857642E-05	-0.153714E-04	0.660336E-04
0.420000E 01	0.333911E 00	0.162608E-01	-0.125431E-01	0.948584E-02	0.651418E-05	-0.650209E-05	0.280348E-04
0.440000E 01	0.336924E 00	0.139328E-01	-0.107801E-01	0.817509E-02	0.564257E-05	-0.274726E-05	0.118345E-04
0.460000E 01	0.339506E 00	0.119325E-01	-0.926132E-02	0.703955E-02	0.527394E-05	-0.116439E-05	0.498446E-05
0.480000E 01	0.341716E 00	0.102142E-01	-0.795397E-02	0.605749E-02	0.511722E-05	-0.497525E-06	0.210281E-05
0.500000E 01	0.343607E 00	0.873872E-02	-0.682934E-02	0.520934E-02	0.504990E-05	-0.215557E-06	0.892664E-06
0.520000E 01	0.345225E 00	0.747198E-02	-0.586241E-02	0.447770E-02	0.502048E-05	-0.953076E-07	0.383364E-06
0.540000E 01	0.346608E 00	0.638469E-02	-0.503147E-02	0.384716E-02	0.500733E-05	-0.432857E-07	0.167575E-06
0.560000E 01	0.347789E 00	0.545159E-02	-0.431767E-02	0.330420E-02	0.500128E-05	-0.203100E-07	0.750374E-07
0.580000E 01	0.348798E 00	0.465091E-02	-0.370470E-02	0.283699E-02	0.499839E-05	-0.988380E-08	0.346308E-07
0.600000E 01	0.349657E 00	0.396393E-02	-0.317848E-02	0.243520E-02	0.499696E-05	-0.499492E-08	0.165507E-07
0.620000E 01	0.350390E 00	0.337455E-02	-0.272683E-02	0.203984E-02	0.499623E-05	-0.261720E-08	0.821088E-08
0.640000E 01	0.351013E 00	0.286893E-02	-0.233927E-02	0.179312E-02	0.499584E-05	-0.141637E-08	0.422689E-08
0.660000E 01	0.351542E 00	0.243517E-02	-0.200677E-02	0.153828E-02	0.499563E-05	-0.787658E-09	0.225136E-08
0.680000E 01	0.351991E 00	0.206307E-02	-0.172154E-02	0.131948E-02	0.499551E-05	-0.447726E-09	0.123502E-08
0.699999E 01	0.352371E 00	0.174385E-02	-0.147690E-02	0.113166E-02	0.499544E-05	-0.258881E-09	0.694103E-09
0.719999E 01	0.352691E 00	0.146999E-02	-0.126709E-02	0.970491E-03	0.499540E-05	-0.151654E-09	0.397664E-09
0.739999E 01	0.352961E 00	0.123503E-02	-0.108717E-02	0.832204E-03	0.499537E-05	-0.897247E-10	0.231236E-09
0.759999E 01	0.353188E 00	0.103342E-02	-0.932887E-03	0.713574E-03	0.499536E-05	-0.534860E-10	0.135996E-09
0.779999E 01	0.353377E 00	0.860405E-03	-0.800604E-03	0.611820E-03	0.499535E-05	-0.320686E-10	0.806792E-10
0.799999E 01	0.353533E 00	0.711917E-03	-0.687187E-03	0.524551E-03	0.499535E-05	-0.193142E-10	0.481824E-10
0.819999E 01	0.353663E 00	0.584453E-03	-0.589949E-03	0.449713E-03	0.499534E-05	-0.116741E-10	0.289247E-10
0.839999E 01	0.353768E 00	0.475013E-03	-0.506586E-03	0.385541E-03	0.499534E-05	-0.707655E-11	0.174352E-10
0.859999E 01	0.353854E 00	0.381026E-03	-0.435119E-03	0.330518E-03	0.499534E-05	-0.429977E-11	0.105444E-10
0.879999E 01	0.353922E 00	0.300286E-03	-0.373852E-03	0.283342E-03	0.499534E-05	-0.261771E-11	0.639422E-11
0.899999E 01	0.353975E 00	0.230903E-03	-0.321330E-03	0.242896E-03	0.499534E-05	-0.159630E-11	0.388622E-11
0.919999E 01	0.354015E 00	0.171255E-03	-0.276306E-03	0.208222E-03	0.499534E-05	-0.974809E-12	0.236640E-11
0.939999E 01	0.354044E 00	0.119952E-03	-0.237709E-03	0.178496E-03	0.499534E-05	-0.595999E-12	0.144326E-11
0.959999E 01	0.354063E 00	0.758042E-04	-0.204622E-03	0.153014E-03	0.499534E-05	-0.364771E-12	0.881446E-12
0.979999E 01	0.354074E 00	0.377888E-04	-0.176259E-03	0.131169E-03	0.499534E-05	-0.223451E-12	0.538962E-12
0.999999E 01	0.354079E 00	0.503074E-05	-0.151946E-03	0.112443E-03	0.499534E-05	-0.136986E-12	0.329885E-12

TABLE X

COMPUTER INPUT DATA FOR CASE 3

A1	B1	C1	D1
A2	B2	C2	D2
A3	B3	C3	D3
A4	B4	C4	D4
A5	B5	C5	D5
TI	DENI	VISHPI	TF
DENF	DELX	X2M	X4M
KXMAT			
0.28297524E 06	-0.11051106E 04	0.14385884E 01	-0.62422477E-03
0.26524352E 06	-0.10363133E 04	0.13495275E 01	-0.58575432E-03
0.95202585E 02	-0.25143390E 00	0.16601635E-03	0.
0.20484292E 06	-0.79988818E 03	0.10411450E 01	-0.45171753E-03
-0.50445313E 07	0.19650555E 05	-0.25515495E 02	0.11043553E-01
0.76968999E 03	0.32607773E 02	0.33376345E-01	0.76469000E 03
0.42448247E 02	0.20000000E-01	0.50000000E-05	0.50000000E-05

TABLE XI

POLYNOMIAL REGRESSION COEFFICIENTS FOR CASE 3

CA	CB	CC	CD	
CE	CF	CG	CH	
CI	CJ	CK	CL	
CM	CN	CP	CR	
CS	CT	CU	CW	
CAA	CBB	CCC	CDD	
CEE	CFF	CGG	CHH	
DELT1	DELT2	DELT3	DELT4	TF1
TF2	TF3	DVWAL	DCOEF1	DCOEF2
-0.623739E 00	-0.252935E 00	0.821630E 00	-0.390140E 00	
0.168830E 01	0.107930E 01	0.470651E 00	-0.239060E 00	
0.112553E 01	0.719533E 00	0.313767E 00	-0.159373E 00	
0.180586E 01	0.188919E 01	0.635354E 00	0.	
0.186480E 01	0.119214E 01	0.519855E 00	-0.264053E 00	
-0.456367E 00	-0.209799E 00	0.609171E 00	-0.282323E 00	
0.145644E 02	0.138273E 02	-0.147586E 02	0.450713E 01	
0.500000F 01	0.250000E 02	0.125000E 03	0.625000E 03	0.764690E 03
0.584751E 06	0.447153E 09	0.108833E 01	0.153082E 03	0.360633E 01

TABLE XII

COEFFICIENTS OF THE REDUCED DIFFERENTIAL EQUATIONS AS FUNCTIONS OF $\theta(\eta)$ FOR CASE 3

THETA	A111 DA111	A222 DA222	A333 DA333	A444 DA444	B111 DB111	B222 DB222
0.	-0.623739E 00 -0.252935E 00	0.168830E 01 0.107930E 01	0.112553E 01 0.719533E 00	-0.589399E-01 0.697050E 00	-0.456367E 00 -0.209799E 00	0.145644E 02 0.138273E 02
0.100000E 00	-0.641207E 00 -0.100313E 00	0.180069E 01 0.116626E 01	0.120046E 01 0.777505E 00	0.121842E-01 0.728072E 00	-0.471538E 00 -0.964341E-01	0.158041E 02 0.110108E 02
0.200000E 00	-0.644582E 00 0.289001E-01	0.192107E 01 0.123887E 01	0.128071E 01 0.825915E 00	0.872026E-01 0.774936E 00	-0.476219E 00 -0.910088E-05	0.167756E 02 0.846474E 01
0.300000E 00	-0.636207E 00 0.134705E 00	0.204799E 01 0.129714E 01	0.136533E 01 0.864763E 00	0.167700E 00 0.837644E 00	-0.472104E 00 0.794765E-01	0.175060E 02 0.618909E 01
0.400000E 00	-0.618421E 00 0.217102E 00	0.218002E 01 0.134107E 01	0.145335E 01 0.894048E 00	0.255259E 00 0.916195E 00	-0.460888E 00 0.142023E 00	0.180224E 02 0.418387E 01
0.500000E 00	-0.593567E 00 0.276090E 00	0.231573E 01 0.137066E 01	0.154382E 01 0.913770E 00	0.351467E 00 0.101059E 01	-0.444264E 00 0.187630E 00	0.183518E 02 0.244907E 01
0.600000E 00	-0.563984E 00 0.311670E 00	0.245367E 01 0.138590E 01	0.163578E 01 0.923931E 00	0.457905E 00 0.112083E 01	-0.423927E 00 0.216297E 00	0.185212E 02 0.984708E 00
0.700000E 00	-0.532013E 00 0.323841E 00	0.259243E 01 0.138679E 01	0.172829E 01 0.924528E 00	0.576160E 00 0.124691E 01	-0.401570E 00 0.228025E 00	0.185578E 02 -0.209232E 00
0.800000E 00	-0.499996E 00 0.312604E 00	0.273055E 01 0.137335E 01	0.182037E 01 0.915564E 00	0.707814E 00 0.138883E 01	-0.378887E 00 0.222814E 00	0.184884E 02 -0.113274E 01
0.900000E 00	-0.470273E 00 0.277958E 00	0.286662E 01 0.134556E 01	0.191108E 01 0.897037E 00	0.854454E 00 0.154660E 01	-0.357572E 00 0.200663E 00	0.183402E 02 -0.178583E 01
0.100000E 01	-0.445185E 00 0.219904E 00	0.299919E 01 0.130342E 01	0.199946E 01 0.868948E 00	0.101766E 01 0.172021E 01	-0.339319E 00 0.161573E 00	0.181403E 02 -0.216848E 01

TABLE XIII

SOLUTIONS OF THE REDUCED DIFFERENTIAL EQUATIONS
AS FUNCTIONS OF η FOR CASE 3

ETA	F(ETA)	F'(ETA)	F''(ETA)	F'''(ETA)	T(ETA)	T'(ETA)	T''(ETA)
0.	0.	0.	0.617572E 00	-0.206712E 01	0.100000E 01	-0.126181E 01	0.540250E 00
0.200000E 00	0.990154E-02	0.883248E-01	0.295611E 00	-0.120119E 01	0.759057E 00	-0.114149E 01	0.715600E 00
0.400000E 00	0.321035E-01	0.127791E 00	0.117511E 00	-0.641210E 00	0.547000E 00	-0.969952E 00	0.984907E 00
0.600000E 00	0.592867E-01	0.140965E 00	0.246269E-01	-0.325245E 00	0.373679E 00	-0.760207E 00	0.107656E 01
0.800000E 00	0.876072E-01	0.140707E 00	-0.217205E-01	-0.158293E 00	0.242736E 00	-0.552674E 00	0.974786E 00
0.100000E 01	0.115138E 00	0.133880E 00	-0.436703E-01	-0.711478E-01	0.150449E 00	-0.376845E 00	0.776396E 00
0.120000E 01	0.140965E 00	0.124083E 00	-0.527714E-01	-0.248619E-01	0.891728E-01	-0.242976E 00	0.565282E 00
0.140000E 01	0.164703E 00	0.113224E 00	-0.550031E-01	-0.120303E-03	0.506035E-01	-0.148776E 00	0.383629E 00
0.160000E 01	0.186253E 00	0.102322E 00	-0.535974E-01	0.126855E-01	0.275198E-01	-0.866885E-01	0.244714E 00
0.180000E 01	0.205665E 00	0.919047E-01	-0.503742E-01	0.187029E-01	0.143613E-01	-0.481394E-01	0.147266E 00
0.200000E 01	0.223064E 00	0.822233E-01	-0.463681E-01	0.208913E-01	0.720565E-02	-0.255264E-01	0.838283E-01
0.220000E 01	0.238609E 00	0.733707E-01	-0.421544E-01	0.210007E-01	0.348511E-02	-0.129590E-01	0.452788E-01
0.240000E 01	0.252468E 00	0.653547E-01	-0.380371E-01	0.200553E-01	0.163022E-02	-0.631884E-02	0.232976E-01
0.260000E 01	0.264805E 00	0.581394E-01	-0.341635E-01	0.186340E-01	0.740550E-03	-0.296970E-02	0.114694E-01
0.280000E 01	0.275774E 00	0.516690E-01	-0.305941E-01	0.170494E-01	0.328569E-03	-0.135003E-02	0.542669E-02
0.300000E 01	0.285518E 00	0.458805E-01	-0.273438E-01	0.154615E-01	0.143741E-03	-0.595691E-03	0.247842E-02
0.320000E 01	0.294167E 00	0.407107E-01	-0.244046E-01	0.139462E-01	0.631383E-04	-0.255940E-03	0.109699E-02
0.340000E 01	0.301839E 00	0.360991E-01	-0.217582E-01	0.125362E-01	0.288660E-04	-0.107390E-03	0.472275E-03
0.360000E 01	0.308640E 00	0.319894E-01	-0.193824E-01	0.112419E-01	0.146173E-04	-0.441220E-04	0.198410E-03
0.380000E 01	0.314665E 00	0.283297E-01	-0.172537E-01	0.100634E-01	0.881064E-05	-0.177926E-04	0.815745E-04
0.400000E 01	0.319999E 00	0.250729E-01	-0.153496E-01	0.899582E-02	0.648582E-05	-0.705730E-05	0.329059E-04
0.420000E 01	0.324718E 00	0.221763E-01	-0.136485E-01	0.803207E-02	0.556954E-05	-0.275852E-05	0.130525E-04
0.440000E 01	0.328891E 00	0.196013E-01	-0.121304E-01	0.716441E-02	0.521338E-05	-0.106436E-05	0.510127E-05
0.460000E 01	0.332578E 00	0.173132E-01	-0.107768E-01	0.638494E-02	0.507664E-05	-0.405997E-06	0.196783E-05
0.480000E 01	0.335833E 00	0.152807E-01	-0.957103E-02	0.568596E-02	0.502471E-05	-0.153310E-06	0.750402E-06
0.500000E 01	0.338705E 00	0.134760E-01	-0.849758E-02	0.506013E-02	0.500517E-05	-0.573793E-07	0.283269E-06
0.520000E 01	0.341237E 00	0.118738E-01	-0.754257E-02	0.450055E-02	0.499789E-05	-0.213083E-07	0.105983E-06
0.540000E 01	0.343467E 00	0.104519E-01	-0.669338E-02	0.400079E-02	0.499519E-05	-0.785939E-08	0.393449E-07
0.560000E 01	0.345428E 00	0.919017E-02	-0.593866E-02	0.355492E-02	0.499420E-05	-0.288186E-08	0.145074E-07
0.580000E 01	0.347152E 00	0.807081E-02	-0.526818E-02	0.315749E-02	0.499383E-05	-0.105143E-08	0.531793E-08
0.600000E 01	0.348665E 00	0.707790E-02	-0.467276E-02	0.280353E-02	0.499370E-05	-0.382005E-09	0.193965E-08
0.620000E 01	0.349991E 00	0.619725E-02	-0.414417E-02	0.248849E-02	0.499365E-05	-0.138319E-09	0.704513E-09
0.640000E 01	0.351151E 00	0.541627E-02	-0.367505E-02	0.220827E-02	0.499364E-05	-0.499490E-10	0.255024E-09

TABLE XIV

COMPUTER INPUT DATA FOR CASE 4

A1	B1	C1	D1
A2	B2	C2	D2
A3	B3	C3	D3
A4	B4	C4	D4
A5	B5	C5	D5
TI	DENI	VISHPI	TF
DENF	DELX	X2M	X4M
KXMAT			

-0.12906121E 06	0.50647826E 03	-0.66249913E 00	0.28884763E-03
-0.27309015E 06	0.10785347E 04	-0.14198646E 01	0.62308177E-03
-0.10690617E 05	0.42202856E 02	-0.55534683E-01	0.24359615E-04
-0.86208233E 05	0.33815024E 03	-0.44210669E 00	0.19266394E-03
0.57764179E 05	-0.15215576E 03	0.10020045E 00	0.
0.76469000E 03	0.43448247E 02	0.44727330E-01	0.75968999E 03
0.51314840E 02	0.20000000E-01	0.50000000E-05	0.50000000E-05
501			

TABLE XV

POLYNOMIAL REGRESSION COEFFICIENTS FOR CASE 4

CA	CB	CC	CD	
CE	CF	CG	CH	
CI	CJ	CK	CL	
CM	CN	CP	CR	
CS	CT	CU	CW	
CAA	CBB	CCC	CDD	
CEE	CFF	CGG	CHH	
DELT1	DELT2	DELT3	DELT4	TF1
TF2	TF3	DVWAL	DCOEF1	DCOEF2
-0.216281E 00	-0.680136E-01	-0.524395E 00	0.180530E 00	
0.206154E 01	0.474633E 00	0.265797E-01	0.454069E 00	
0.137436E 01	0.316422E 00	0.177198E-01	0.302712E 00	
0.379045E 01	0.132720E 01	-0.239796E 00	0.167708E 01	
0.379538E 01	0.873821E 00	0.489343E-01	0.835961E 00	
-0.152568E 00	-0.509803E-01	-0.376510E 00	0.120415E 00	
0.899407E 01	0.253019E 01	0.146041E 02	0.	
0.500000E 01	0.250000E 02	0.125000E 03	0.625000E 03	0.759690E 03
0.577129E 06	0.438439E 09	0.194332E 01	0.550774E 03	0.107332E 02

TABLE XVI

COEFFICIENTS OF THE REDUCED DIFFERENTIAL EQUATIONS AS FUNCTIONS OF $\theta(\eta)$ FOR CASE 4

THETA	A111 DA111	A222 DA222	A333 DA333	A444 DA444	B111 DB111	B222 DB222
0.	-0.216281E 00	0.206154E 01	0.137436E 01	-0.493777E-02	-0.152568E 00	0.899407E 01
	-0.680136E-01	0.474633E 00	0.316422E 00	0.453376E 00	-0.509803E-01	0.253019E 01
0.100000E 00	-0.228146E 00	0.210972E 01	0.140648E 01	0.383536E-01	-0.161311E 00	0.939313E 01
	-0.167477E 00	0.493571E 00	0.329047E 00	0.420863E 00	-0.122670E 00	0.545102E 01
0.200000E 00	-0.249416E 00	0.216116E 01	0.144077E 01	0.809171E-01	-0.176861E 00	0.100843E 02
	-0.255108E 00	0.539753E 00	0.359835E 00	0.438818E 00	-0.187134E 00	0.837185E 01
0.300000E 00	-0.279007E 00	0.221858E 01	0.147905E 01	0.127800E 00	-0.198497E 00	0.110675E 02
	-0.333907E 00	0.613180E 00	0.408785E 00	0.507240E 00	-0.244374E 00	0.112927E 02
0.400000E 00	-0.315836E 00	0.228470E 01	0.152314E 01	0.184047E 00	-0.225495E 00	0.123428E 02
	-0.400875E 00	0.713850E 00	0.475900E 00	0.626130E 00	-0.294389E 00	0.142135E 02
0.500000E 00	-0.358821E 00	0.236226E 01	0.157484E 01	0.254708E 00	-0.257134E 00	0.139102E 02
	-0.457011E 00	0.841764E 00	0.561176E 00	0.795486E 00	-0.337179E 00	0.171343E 02
0.600000E 00	-0.406877E 00	0.245397E 01	0.163598E 01	0.344827E 00	-0.292690E 00	0.157697E 02
	-0.502315E 00	0.996923E 00	0.654615E 00	0.101531E 01	-0.372744E 00	0.200552E 02
0.700000E 00	-0.458923E 00	0.256255E 01	0.170837E 01	0.459452E 00	-0.331442E 00	0.179212E 02
	-0.536788E 00	0.117933E 01	0.786217E 00	0.128560E 01	-0.401084E 00	0.229760E 02
0.800000E 00	-0.513874E 00	0.269074E 01	0.179383E 01	0.603629E 00	-0.372666E 00	0.203649E 02
	-0.550428E 00	0.138897E 01	0.925982E 00	0.160636E 01	-0.422199E 00	0.258968E 02
0.900000E 00	-0.570647E 00	0.284125E 01	0.189417E 01	0.782406E 00	-0.415641E 00	0.231006E 02
	-0.573237E 00	0.162586E 01	0.108391E 01	0.197759E 01	-0.436089E 00	0.288177E 02
0.100000E 01	-0.628160E 00	0.301682E 01	0.201121E 01	0.100083E 01	-0.459643E 00	0.261284E 02
	-0.575214E 00	0.189030E 01	0.126000E 01	0.239928E 01	-0.442755E 00	0.317385E 02

TABLE XVII

SOLUTIONS OF THE REDUCED DIFFERENTIAL EQUATIONS AS FUNCTIONS OF η FOR CASE 4

ETA	F(ETA)	F'(ETA)	F''(ETA)	F'''(ETA)	T(ETA)	T'(ETA)	T''(ETA)
0.	0.	0.	0.738057E 00	-0.289387E 01	0.100000E 01	-0.106679E 01	0.523089E 00
0.200000E 00	0.114254E-01	0.100198E 00	0.313464E 00	-0.147039E 01	0.796788E 00	-0.963340E 00	0.568845E 00
0.400000E 00	0.361098E-01	0.139885E 00	0.108852E 00	-0.692496E 00	0.616461E 00	-0.835642E 00	0.695519E 00
0.600000E 00	0.655043E-01	0.150919E 00	0.136723E-01	-0.318615E 00	0.463574E 00	-0.692379E 00	0.721484E 00
0.800000E 00	0.956124E-01	0.148720E 00	-0.300386E-01	-0.145739E 00	0.339287E 00	-0.552134E 00	0.672733E 00
0.100000E 01	0.124595E 00	0.140458E 00	-0.499491E-01	-0.649738E-01	0.241788E 00	-0.425673E 00	0.588497E 00
0.120000E 01	0.151618E 00	0.129488E 00	-0.584248E-01	-0.245785E-01	0.167782E 00	-0.317652E 00	0.490825E 00
0.140000E 01	0.176323E 00	0.117482E 00	-0.608976E-01	-0.225396E-02	0.113401E 00	-0.229457E 00	0.391781E 00
0.160000E 01	0.198604E 00	0.105357E 00	-0.599154E-01	0.109730E-01	0.747125E-01	-0.160507E 00	0.299486E 00
0.180000E 01	0.218495E 00	0.936522E-01	-0.568721E-01	0.187585E-01	0.480433E-01	-0.108852E 00	0.219500E 00
0.200000E 01	0.236114E 00	0.826854E-01	-0.526596E-01	0.228788E-01	0.302038E-01	-0.717051E-01	0.154619E 00
0.220000E 01	0.251629E 00	0.726250E-01	-0.478913E-01	0.244673E-01	0.185996E-01	-0.459884E-01	0.105025E 00
0.240000E 01	0.265229E 00	0.635377E-01	-0.429857E-01	0.243715E-01	0.112416E-01	-0.287897E-01	0.690455E-01
0.260000E 01	0.277109E 00	0.554217E-01	-0.382123E-01	0.232338E-01	0.668194E-02	-0.176370E-01	0.440961E-01
0.280000E 01	0.287460E 00	0.482332E-01	-0.337305E-01	0.215194E-01	0.391363E-02	-0.105994E-01	0.274548E-01
0.300000E 01	0.295460E 00	0.419046E-01	-0.296217E-01	0.195449E-01	0.226303E-02	-0.626329E-02	0.167184E-01
0.320000E 01	0.304274E 00	0.363575E-01	-0.259162E-01	0.175131E-01	0.129440E-02	-0.364665E-02	0.998593E-02
0.340000E 01	0.311050E 00	0.315113E-01	-0.226121E-01	0.155450E-01	0.733828E-03	-0.209591E-02	0.586567E-02
0.360000E 01	0.316920E 00	0.272873E-01	-0.196895E-01	0.137063E-01	0.413321E-03	-0.119114E-02	0.339597E-02
0.380000E 01	0.322001E 00	0.236120E-01	-0.171188E-01	0.120279E-01	0.231995E-03	-0.670368E-03	0.194170E-02
0.400000E 01	0.326397E 00	0.204185E-01	-0.148670E-01	0.105190E-01	0.130346E-03	-0.374096E-03	0.109829E-02
0.420000E 01	0.330197E 00	0.176462E-01	-0.129001E-01	0.917645E-02	0.738141E-04	-0.207236E-03	0.615476E-03
0.440000E 01	0.333480E 00	0.152416E-01	-0.111859E-01	0.799038E-02	0.425892E-04	-0.114075E-03	0.342157E-03
0.460000E 01	0.336315E 00	0.131570E-01	-0.969438E-02	0.694786E-02	0.254447E-04	-0.624505E-04	0.188904E-03
0.480000E 01	0.338761E 00	0.113508E-01	-0.839815E-02	0.603483E-02	0.160794E-04	-0.340269E-04	0.103674E-03
0.500000E 01	0.340871E 00	0.978640E-02	-0.727274E-02	0.523734E-02	0.109863E-04	-0.184643E-04	0.566078E-04
0.520000E 01	0.342690E 00	0.843181E-02	-0.629639E-02	0.454216E-02	0.822705E-05	-0.998421E-05	0.307727E-04
0.540000E 01	0.344256E 00	0.725920E-02	-0.544987E-02	0.393710E-02	0.673714E-05	-0.538238E-05	0.166650E-04
0.560000E 01	0.345504E 00	0.624434E-02	-0.471627E-02	0.341111E-02	0.593491E-05	-0.289400E-05	0.899552E-05
0.580000E 01	0.346763E 00	0.536615E-02	-0.408081E-02	0.295429E-02	0.550401E-05	-0.155255E-05	0.484203E-05
0.600000E 01	0.347758E 00	0.460634E-02	-0.353053E-02	0.255785E-02	0.527305E-05	-0.831293E-06	0.260004E-05
0.620000E 01	0.348612E 00	0.394901E-02	-0.305416E-02	0.221404E-02	0.514948E-05	-0.444366E-06	0.139325E-05
0.640000E 01	0.349344E 00	0.338040E-02	-0.264186E-02	0.191602E-02	0.508347E-05	-0.237197E-06	0.745262E-06
0.660000E 01	0.349969E 00	0.288857E-02	-0.228510E-02	0.165781E-02	0.504826E-05	-0.126458E-06	0.398039E-06
0.680000E 01	0.350504E 00	0.246316E-02	-0.197644E-02	0.143417E-02	0.502949E-05	-0.673495E-07	0.212312E-06
0.699999E 01	0.350959E 00	0.209522E-02	-0.170943E-02	0.124054E-02	0.501950E-05	-0.358373E-07	0.113120E-06
0.719999E 01	0.351345E 00	0.177699E-02	-0.147848E-02	0.107294E-02	0.501419E-05	-0.190549E-07	0.602128E-07
0.739999E 01	0.351672E 00	0.150175E-02	-0.127875E-02	0.927894E-03	0.501136E-05	-0.101251E-07	0.320249E-07
0.759999E 01	0.351948E 00	0.126369E-02	-0.110602E-02	0.802396E-03	0.500986E-05	-0.537723E-08	0.170211E-07
0.779999E 01	0.352180E 00	0.105778E-02	-0.956660E-03	0.693828E-03	0.500907E-05	-0.285442E-08	0.904135E-08
0.799999E 01	0.352373E 00	0.879677E-03	-0.827512E-03	0.599918E-03	0.500864E-05	-0.151465E-08	0.480027E-08
0.819999E 01	0.352533E 00	0.725612E-03	-0.715847E-03	0.518698E-03	0.500842E-05	-0.803463E-09	0.254753E-08
0.839999E 01	0.352665E 00	0.592331E-03	-0.619301E-03	0.448459E-03	0.500830E-05	-0.426094E-09	0.135152E-08
0.859999E 01	0.352771E 00	0.477020E-03	-0.535829E-03	0.387722E-03	0.500824E-05	-0.225918E-09	0.716806E-09
0.879999E 01	0.352857E 00	0.377246E-03	-0.463664E-03	0.335204E-03	0.500820E-05	-0.119763E-09	0.380083E-09
0.899999E 01	0.352923E 00	0.290904E-03	-0.401273E-03	0.289796E-03	0.500819E-05	-0.634792E-10	0.201499E-09
0.919999E 01	0.352974E 00	0.216174E-03	-0.347335E-03	0.250537E-03	0.500818E-05	-0.336429E-10	0.106807E-09
0.939999E 01	0.353010E 00	0.151483E-03	-0.300704E-03	0.216595E-03	0.500817E-05	-0.178287E-10	0.566076E-10
0.959999E 01	0.353035E 00	0.954712E-04	-0.260390E-03	0.187251E-03	0.500817E-05	-0.944758E-11	0.299992E-10
0.979999E 01	0.353049E 00	0.469628E-04	-0.225538E-03	0.161884E-03	0.500817E-05	-0.500615E-11	0.158970E-10

TABLE XVIII

COMPUTER INPUT DATA FOR CASE 5

A1	B1	C1	D1
A2	B2	C2	D2
A3	B3	C3	D3
A4	B4	C4	D4
A5	B5	C5	D5
TI	DENI	VISHPI	TF
DENF	DELX	X2M	X4M
KXMAT			
-0.12906121E 06	0.50647826E 03	-0.66249913E 00	0.28884763E-03
-0.27309015E 06	0.10785347E 04	-0.14198646E 01	0.62308177E-03
-0.10690617E 05	0.42202856E 02	-0.55534683E-01	0.24359615E-04
-0.86208233E 05	0.33815024E 03	-0.44210669E 00	0.19266394E-03
0.57764179E 05	-0.15215576E 03	0.10020045E 00	0.
0.76469000E 03	0.43448247E 02	0.44727330E-01	0.75968999E 03
0.51314840E 02	0.20000000E-01	0.50000000E-05	0.50000000E-05

TABLE XIX

POLYNOMIAL REGRESSION COEFFICIENTS FOR CASE 5

CA	CB	CC	CD	
CE	CF	CG	CH	
CI	CJ	CK	CL	
CM	CN	CP	CR	
CS	CT	CU	CW	
CAA	CBB	CCC	CDD	
CEE	CFF	CGG	CHH	
DELT1	DELT2	DELT3	DELT4	TF1
TF2	TF3	DVWAL	DCOEF1	DCOEF2
-0.216281E 00	-0.680136E-01	-0.524395E 00	0.180530E 00	
0.206154E 01	0.474633E 00	0.265797E-01	0.454069E 00	
0.137436E 01	0.316422E 00	0.177198E-01	0.302712E 00	
0.379045E 01	0.132720E 01	-0.239796E 00	0.167708E 01	
0.379538E 01	0.873821E 00	0.489343E-01	0.835961E 00	
-0.152568E 00	-0.509803E-01	-0.376510E 00	0.120415E 00	
0.899407E 01	0.253019E 01	0.146041E 02	0.	
0.500000E 01	0.250000E 02	0.125000E 03	0.625000E 03	0.759690E 03
0.577129E 06	0.438439E 09	0.194332E 01	0.550774E 03	0.107332E 02

TABLE XX

COEFFICIENTS OF THE REDUCED DIFFERENTIAL EQUATIONS AS FUNCTIONS OF $\theta(\eta)$ FOR CASE 5

THETA	A111	A222	A333	A444	B111	B222
	DA111	DA222	DA333	DA444	DB111	DB222
0.	-0.216281E 00	0.206154E 01	0.137436E 01	-0.493777E-02	-0.152568E 00	0.899407E 01
	-0.680136E-01	0.474633E 00	0.316422E 00	0.453376E 00	-0.509803E-01	0.253019E 01
0.100000E 00	-0.228146E 00	0.210972E 01	0.140648E 01	0.383536E-01	-0.161311E 00	0.939313E 01
	-0.157477E 00	0.493571E 00	0.329047E 00	0.420863E 00	-0.122670E 00	0.545102E 01
0.200000E 00	-0.249416E 00	0.216116E 01	0.144077E 01	0.809171E-01	-0.176861E 00	0.100843E 02
	-0.256108E 00	0.539753E 00	0.359835E 00	0.438818E 00	-0.187134E 00	0.837185E 01
0.300000E 00	-0.279007E 00	0.221858E 01	0.147905E 01	0.127800E 00	-0.198497E 00	0.110675E 02
	-0.333907E 00	0.613180E 00	0.408786E 00	0.507240E 00	-0.244374E 00	0.112927E 02
0.400000E 00	-0.315836E 00	0.228470E 01	0.152314E 01	0.184047E 00	-0.225495E 00	0.123428E 02
	-0.400875E 00	0.713850E 00	0.475900E 00	0.626130E 00	-0.294389E 00	0.142135E 02
0.500000E 00	-0.358821E 00	0.236226E 01	0.157484E 01	0.254708E 00	-0.257134E 00	0.139102E 02
	-0.457011E 00	0.841764E 00	0.561176E 00	0.795486E 00	-0.337179E 00	0.171343E 02
0.600000E 00	-0.406877E 00	0.245397E 01	0.163598E 01	0.344827E 00	-0.292690E 00	0.157697E 02
	-0.502315E 00	0.996923E 00	0.664615E 00	0.101531E 01	-0.372744E 00	0.200552E 02
0.700000E 00	-0.458923E 00	0.256255E 01	0.170837E 01	0.459452E 00	-0.331442E 00	0.179212E 02
	-0.536788E 00	0.117933E 01	0.786217E 00	0.128560E 01	-0.401084E 00	0.229760E 02
0.800000E 00	-0.513874E 00	0.269074E 01	0.179383E 01	0.603629E 00	-0.372666E 00	0.203649E 02
	-0.550428E 00	0.138897E 01	0.925982E 00	0.160636E 01	-0.422199E 00	0.258958E 02
0.900000E 00	-0.570647E 00	0.284125E 01	0.189417E 01	0.782406E 00	-0.415641E 00	0.231006E 02
	-0.573237E 00	0.162586E 01	0.108391E 01	0.197759E 01	-0.436089E 00	0.288177E 02
0.100000E 01	-0.628160E 00	0.301682E 01	0.201121E 01	0.100083E 01	-0.459643E 00	0.261284E 02
	-0.575214E 00	0.189000E 01	0.126000E 01	0.239928E 01	-0.442755E 00	0.317385E 02

TABLE XXI

SOLUTIONS OF THE REDUCED DIFFERENTIAL EQUATIONS AS FUNCTIONS OF η FOR CASE 5

ETA	F(ETA)	F'(ETA)	F''(ETA)	F'''(ETA)	T(ETA)	T'(ETA)	T''(ETA)
0.	0.	0.	0.738280E 00	-0.289404E 01	0.100000E 01	-0.106686E 01	0.523158E 00
0.200000E 00	0.114292E-01	0.100232E 00	0.313575E 00	-0.147109E 01	0.796776E 00	-0.963391E 00	0.568971E 00
0.400000E 00	0.361216E-01	0.139927E 00	0.108824E 00	-0.693126E 00	0.616441E 00	-0.835661E 00	0.695724E 00
0.600000E 00	0.655233E-01	0.150944E 00	0.135434E-01	-0.318997E 00	0.463555E 00	-0.692360E 00	0.721671E 00
0.800000E 00	0.956334E-01	0.148713E 00	-0.302200E-01	-0.145905E 00	0.339274E 00	-0.552088E 00	0.672845E 00
0.100000E 01	0.124612E 00	0.140413E 00	-0.501488E-01	-0.650110E-01	0.241786E 00	-0.425617E 00	0.588526E 00
0.120000E 01	0.151621E 00	0.129403E 00	-0.586252E-01	-0.245611E-01	0.167791E 00	-0.317599E 00	0.490790E 00
0.140000E 01	0.176306E 00	0.117358E 00	-0.610926E-01	-0.222372E-02	0.113420E 00	-0.229418E 00	0.391710E 00
0.160000E 01	0.198557E 00	0.105194E 00	-0.601045E-01	0.109994E-01	0.747373E-01	-0.160485E 00	0.299405E 00
0.180000E 01	0.218412E 00	0.934521E-01	-0.570566E-01	0.187782E-01	0.480708E-01	-0.108847E 00	0.219430E 00
0.200000E 01	0.235988E 00	0.824487E-01	-0.528406E-01	0.228939E-01	0.302309E-01	-0.717131E-01	0.154570E 00
0.220000E 01	0.251452E 00	0.723524E-01	-0.480696E-01	0.244809E-01	0.186243E-01	-0.460047E-01	0.105000E 00
0.240000E 01	0.264994E 00	0.632297E-01	-0.431612E-01	0.243859E-01	0.112625E-01	-0.288096E-01	0.690394E-01
0.260000E 01	0.276809E 00	0.550789E-01	-0.383847E-01	0.232504E-01	0.669883E-02	-0.176571E-01	0.441031E-01
0.280000E 01	0.287087E 00	0.478563E-01	-0.338993E-01	0.215387E-01	0.392666E-02	-0.106176E-01	0.274689E-01
0.300000E 01	0.296009E 00	0.414943E-01	-0.297864E-01	0.195671E-01	0.227273E-02	-0.627846E-02	0.167350E-01
0.320000E 01	0.303737E 00	0.359148E-01	-0.260762E-01	0.175380E-01	0.130139E-02	-0.365860E-02	0.100019E-01
0.340000E 01	0.310421E 00	0.310370E-01	-0.227669E-01	0.155721E-01	0.738726E-03	-0.210492E-02	0.587950E-02
0.360000E 01	0.315194E 00	0.267826E-01	-0.198387E-01	0.137354E-01	0.416672E-03	-0.119769E-02	0.340707E-02
0.380000E 01	0.321171E 00	0.230781E-01	-0.172521E-01	0.120584E-01	0.234240E-03	-0.674973E-03	0.195015E-02
0.400000E 01	0.325457E 00	0.198565E-01	-0.150040E-01	0.105506E-01	0.131823E-03	-0.377255E-03	0.110446E-02
0.420000E 01	0.329142E 00	0.170575E-01	-0.130308E-01	0.920869E-02	0.747690E-04	-0.209355E-03	0.619829E-03
0.440000E 01	0.332305E 00	0.146274E-01	-0.113101E-01	0.802298E-02	0.431972E-04	-0.115470E-03	0.345145E-03
0.460000E 01	0.335014E 00	0.125186E-01	-0.981205E-02	0.698053E-02	0.258261E-04	-0.633535E-04	0.190907E-03
0.480000E 01	0.337331E 00	0.106896E-01	-0.850931E-02	0.606731E-02	0.163152E-04	-0.346030E-04	0.104991E-03
0.500000E 01	0.339306E 00	0.910354E-02	-0.737745E-02	0.526939E-02	0.111297E-04	-0.188271E-04	0.574586E-04
0.520000E 01	0.340986E 00	0.772864E-02	-0.639475E-02	0.457359E-02	0.831264E-05	-0.102101E-04	0.313144E-04
0.540000E 01	0.342410E 00	0.653698E-02	-0.554202E-02	0.396774E-02	0.678687E-05	-0.552166E-05	0.170055E-04
0.560000E 01	0.343612E 00	0.550429E-02	-0.480240E-02	0.344082E-02	0.596263E-05	-0.297910E-05	0.920714E-05
0.580000E 01	0.344621E 00	0.460947E-02	-0.416109E-02	0.298296E-02	0.551834E-05	-0.160413E-05	0.497223E-05
0.600000E 01	0.345463E 00	0.383417E-02	-0.360520E-02	0.258539E-02	0.527929E-05	-0.862328E-06	0.267944E-05
0.620000E 01	0.346161E 00	0.316245E-02	-0.312344E-02	0.224038E-02	0.515087E-05	-0.462920E-06	0.144129E-05
0.640000E 01	0.346734E 00	0.258051E-02	-0.270600E-02	0.194112E-02	0.508196E-05	-0.248224E-06	0.774118E-06
0.660000E 01	0.347199E 00	0.207634E-02	-0.234434E-02	0.168164E-02	0.504503E-05	-0.132977E-06	0.415261E-06
0.680000E 01	0.347569E 00	0.163955E-02	-0.203104E-02	0.145672E-02	0.502525E-05	-0.711847E-07	0.222532E-06
0.699999E 01	0.347858E 00	0.126113E-02	-0.175965E-02	0.126181E-02	0.501467E-05	-0.380837E-07	0.119153E-06
0.719999E 01	0.348077E 00	0.933268E-03	-0.152458E-02	0.109293E-02	0.500901E-05	-0.203655E-07	0.637578E-07
0.739999E 01	0.348235E 00	0.649199E-03	-0.132098E-02	0.946641E-03	0.500598E-05	-0.108869E-07	0.340987E-07
0.759999E 01	0.348339E 00	0.403061E-03	-0.114462E-02	0.819928E-03	0.500436E-05	-0.581848E-08	0.182295E-07
0.779999E 01	0.348398E 00	0.189777E-03	-0.991875E-03	0.710184E-03	0.500350E-05	-0.310923E-08	0.974297E-08
0.799999E 01	0.348417E 00	0.494921E-05	-0.859571E-03	0.615142E-03	0.500304E-05	-0.166137E-08	0.520629E-08

APPENDIX D

THE HEAT TRANSFER PROGRAM

The purpose of the program, which is presented below, is to compute the values of the coefficient of heat transfer, the heat transfer and shearing stress. The read statements are defined as follows:

Program Symbols	Description	Units
TW	Wall temperature	^oR
TS	Main stream temperature	^oR
DENW	Wall density	Lbs/CuFt
VISW	Wall viscosity	Centipoises
CONDW	Wall thermal conductivity	Cal/SecCm ^oK
FX2D	$F''(\eta)$	Dimensionless
TX1D	$\theta'(\eta)$	Dimensionless

```
$JOB   WATFOR       THOMAS E MULLIN                       2555-41002
$JOB                THOMAS F MULLIN                       2555-41002
$IBJOB NAMEPR  MAP
$IBFTC
C      THOMAS E. MULLIN
C      THE HEAT TRANSFER AND SHEARING STRESS
C
       WRITE(6,11)
       WRITE(6,12)
       WRITE(6,13)
       WRITE(6,14)
   11  FORMAT(1H1,12X,38HQX+(1/4)=HEAT TRANSFER=BTU/HR FT+(7/4))
   12  FORMAT(1H ,12X,
     1        49HHX+(1/4)=COEFF. OF HEAT TRANSFER=BTU/HR FT(7/4) R)
   13  FORMAT(1H ,12X,38HTAU-(1/4)=SHEARING STRESS=LBSF/FT(9/4),/)
   14  FORMAT(1H ,12X,8HQX+(1/4),12X,8HHX+(1/4),12X,9HTAU-(1/4))
C
C      KX = THE NUMBER OF SETS TO BE RUN
       READ(5,25) KX
   25  FORMAT(I25)
C
       DO 100 K=1,KX
C
       READ(5,10) TW
       READ(5,10) TS
       READ(5,10) DENW
       READ(5,10) DENS
       READ(5,10) VISW
       READ(5,10) CONDW
       READ(5,10) FX2D
       READ(5,10) TX1D
   10  FORMAT(E25.8)
C
C      CONVERTING UNITS
       VISW=VISW*2.42
       CONDW=CONDW*((3600.00*2.54*12.0)/(252.00*1.8))
C
C      COMPUTING THE TERM C
       G=32.2*3600.00*3600.00
       VISKM=VISW/DENW
       VISKM2=VISKM*VISKM
       DENR=(DENS-DENW)/DENW
       C1=(G*DENR)/(4.0*VISKM2)
       C=C1**0.25
C
C      COMPUTING THE LOCAL HEAT TRANSFER
       QX14=-(CONDW)*(TW-TS)*(C)*(TX1D)
C
C      COMPUTING THE LOCAL COEFFICIENT OF HEAT TRANSFER
       HX14=QX14/(TW-TS)
C
C      COMPUTING THE LOCAL SHEAR STRESS
       C3=C*C*C
       TAUX14=(4.0*VISW*C3*VISKM*FX2D)/G
C
       WRITE(6,15) QX14,HX14,TAUX14
   15  FORMAT(12X,E15.8,5X,E15.8,5X,E15.8)
  100  CONTINUE
       STOP
       END
```

TABLE XXII

THE HEAT TRANSFER AND SHEARING STRESS

QX+(1/4)=HEAT TRANSFER=BTU/HR FT+(7/4)
HX+(1/4)=COEFF. OF HEAT TRANSFER=BTU/HR FT(7/4) R
TAU-(1/4)=SHEARING STRESS=LBSF/FT(9/4)

QX+(1/4)	HX+(1/4)	TAU-(1/4)
0.11784961E 03	0.23569922E 02	0.43832586E-02
0.14079813E 03	0.28159627E 02	0.43418033E-02
0.91724492E 02	0.18344899E 02	0.22013317E-02
0.64440493E 02	0.12888099E 02	0.14026083E-02

APPENDIX E

SOLUTION OF TWO POINT BOUNDARY VALUE PROBLEMS BY THE NEWTON-RAPHSON METHOD

The general problem is to solve a set of n first order linear or nonlinear differential equations in which at least n boundary conditions are known. These boundary conditions may be either initial, final or a combination of each such that if there are R known initial boundary conditions there must be known at least (n - R) final boundary conditions. This may be expressed in functional form using state variable notation as follows:

The Set of Differential Equations

$$\underline{\dot{X}} = \underline{f}(\underline{X}, t) \tag{1}$$

$$\underline{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \qquad \underline{f}(\underline{X}, t) = \begin{bmatrix} f_1(X_1, X_2 \cdots X_n, t) \\ \vdots \\ f_n(X_1, X_2 \cdots X_n, t) \end{bmatrix}$$

Initial Boundary Conditions / Final Boundary Conditions

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_{n-1} \\ X_n \end{bmatrix}_{(t_1)} = \begin{bmatrix} C_1^{\,1} \\ U_2^{\,1} \\ C_3^{\,2} \\ \vdots \\ C_{n-1}^{R} \\ U_n^{n-R} \end{bmatrix} \qquad \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_{n-1} \\ X_n \end{bmatrix}_{(t_2)} = \begin{bmatrix} U_1^{(n-R)+1} \\ C_2^{R+1} \\ C_3^{R+2} \\ \vdots \\ U_{n-1}^{n} \\ C_n^{n} \end{bmatrix} \tag{2}$$

The independent variable is t and X_1, X_2, ... , X_n are the dependent variables.

The independent variable time is assumed to be a constant, i.e., $t = t_a$, then the set of first order differential equations will be functions of the state variables. The expansion of the set of differential equations into a first order truncated Taylor series will give at any instant of time:

$$\mathring{X}_{(s+1)} = \mathring{X}_s + J(X_s)[X_{(s+1)} - X_{ns}] \tag{3}$$

where $J(X_s)$, the Jacobian matrix, is:

$$J(X_s) = \begin{bmatrix} \partial f_1/\partial X_1 & \cdots & \partial f_1/\partial X_n \\ \vdots & & \vdots \\ \partial f_n/\partial X_1 & \cdots & \partial f_n/\partial X_n \end{bmatrix} . \tag{4}$$

For a given iteration, which implies that $C(t_a)$ and $H(t_a)$ are known at each instant of time, the above equations are of the form of a set of first-order linear differential equations, i.e.:

$$\overset{\circ}{X}_{(s+1)} + C(t_a)\, X_{(s+1)} = H(t_a) \tag{5}$$

where

$$C(t_a) = -J(X_s) \tag{6}$$

$$H(t_a) = X_s - J(X_s)X_{ns} \; . \tag{7}$$

Since Equation (5) is linear, the well-known superposition properties may be used. The fact that $J(X_s)$ is a function of the dependent variable prevents easy determination of a closed form analytical solution. However, this presents no obstacle as far as numerical integration is concerned. If the set converges, then:

$$\overset{\circ}{X}_{(s+1)} = \overset{\circ}{X}_s \; . \tag{8}$$

That is, the solution for the linearized set of differential equations converges to the solution of the original set of differential equations.

The procedure for using the iteration method of Newton-Raphson is as follows:

1. Specify a starting solution. This may be obtained by taking an arbitrary set of initial boundary conditions and solving the original set of differential equations numerically or by a crude guess. The purpose of this is to be able to initially compute the values of $C(t_a)$ and $H(t_a)$

for the linearized set of differential equations. The equations are:

$$C(t_a) = - J(X) \tag{9}$$

$$H(t_a) = \dot{X}_s - J(X_s)\ X_s \ . \tag{10}$$

2. Obtain a particular solution for the linearized set of differential equations using all known initial boundary conditions and arbitrarily specifying the unknown initial boundary conditions; for example:

$$\dot{X}_{(s+1)} + C(t_a)\ X_{(s+1)} = H(t_a) \tag{11}$$

with the initial conditions

<u>Initial</u>

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}_{(t_1)} = \begin{bmatrix} k_1 \\ k_2 \\ U_1^{\ 1} \\ k_3 \\ U_5^{\ 2} \end{bmatrix} \tag{12}$$

where

k_1, k_2, k_3 = known boundary condition

U = unknown boundary condition .

3. The operation of step (2) will yield values at the final boundary defined as:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}_{(t_2)}, P \; . \qquad (13)$$

4. The homogeneous part of the set of ordinary linearized differential equations are solved once for each unknown initial boundary condition. All of the initial boundary conditions, both known and unknown, are set equal to zero with the exception of one of the unknown boundary conditions which is set equal to the value of one. Thus, for a fifth-order example with two unknown initial boundaries on X_3 and X_5 the equation

$$\dot{X}_{(s+1)} + C(t_a)\, X_{(s+1)} = 0 \qquad (14)$$

is solved with the initial boundary conditions

<u>First Solution</u>   <u>Second Solution</u>

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}_{(t_1)} = \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \\ 0.0 \\ 0.0 \end{bmatrix} \qquad \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}_{(t_1)} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 1.0 \end{bmatrix} \qquad (15)$$

5. This operation of step (4) will yield values at the final boundary defined as:

First Solution / Second Solution

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}_{(t_2),H1} \qquad \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}_{(t_2),H2} \qquad (16)$$

6. The values of the unknown boundary conditions U_3 and U_5 can now be determined from a set of simultaneous linear equations which represents the superposition of the solutions to the linearized set of differential equations at t_2. The equation for the fifth-order case with $X_3(t_1)$ and $X_5(t_1)$, unknown, are

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}_{(t_2)} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}_{(t_2),P} + U_3^{'} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}_{(t_2),H1} + U_5^{2} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}_{(t_2),H2} \qquad (17)$$

The third and fifth equations, of the above set, represent two simultaneous linear algebraic equations in two unknowns $U_3^{\,1}$ and $U_5^{\,2}$. These may be easily solved.

7. At this point all the initial boundary conditions are known. The equations shown in step (2) are now solved with the new initial boundary conditions and the resulting solution, $\overset{\circ}{X}_{(s+1)}$, now becomes the next starting solution to the system of equations. This procedure is repeated until convergence is obtained.

VITA

Thomas Edward Mullin

Candidate for the Degree of

Doctor of Philosophy

Thesis: LAMINAR NATURAL CONVECTION OF A FLUID IN THE SUPERCRITICAL REGION

Major Field: Engineering

Biographical:

Personal Data: Born in Louisville, Kentucky, February 29, 1928, the son of Mr. and Mrs. Michael Joseph Mullin.

Education: Attended Louisville Male High School and graduated in June, 1946; received the Bachelor of Science degree from the University of Kentucky, Lexington, Kentucky in 1951; received the Master of Science degree from the University of Louisville, Louisville, Kentucky, in December, 1960; completed requirements for the Doctor of Philosophy degree in May, 1968.

Professional Experience: Served in the United States Army from June, 1946, to November, 1947; employed from August, 1951, to December, 1952, by the Standard Oil Development Company, New Jersey, in the Cost Estimating Division; employed from December, 1952, to October, 1958, by the Girdler Construction Company, Gas Process Division as a design engineer; employed from October, 1958, to the present time by the University of Louisville, Louisville, Kentucky, as an Associate Professor of Mechanical Engineering.

Organizations: Pi Tau Sigma, Tau Beta Pi, Sigma Nu, Phi Kappa Phi, American Society of Mechanical Engineers, American Society of Engineering Education, Kentucky State Engineer License No. 3412M.