EVALUATION OF EXPLORATORY DRILLING VENTURES BY STATISTICAL DECISION METHODS

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NOMENCLATURE

C = initial investment, cost of a venture

D = present value factor

f = fractional participation in a venture

i = interest rate

M = total risk capital

p = probability, fractional

n = number of ventures in a group

r = rate of return, used as an interest rate

V_I = value of information decreasing probability of failure

v = fraction of cost; vC is the expectancy per venture

w = fraction of cost; wC is the lowest acceptable return per venture

$\overline{X}_A$ = expectancy of the undeferred net operating income

X_E = expectancy of a venture, including the initial investment

y = the total present value of a group of ventures

z = number of standard deviations, measured from the mean; z is negative for values less than the mean

σ = standard deviation of the parent distribution

σ_A = standard deviation of the parent probability distribution of undeferred net operating income

σ^2 = variance of a probability distribution

CHAPTER I

INTRODUCTION

Statement of the Problem

The problem which has been investigated in this research is the problem of appraisal of uncertain economic ventures. More particularly, this investigation has been concerned with the appraisal of economic undertakings wherein the probability of complete failure of the individual venture is high and the probability of success is low and the magnitude of the occasional success is itself variable. The drilling of exploratory oil wells furnishes an example which satisfies this description. For an individual well, even when it is located with geologic guidance, there is quite a low probability of discovering sufficient quantities of oil to repay the drilling and completion costs. However, where a drilling program consists of a score of such wells, there is often a high probability that the better discoveries will repay the costs of the entire program as well as the necessary profits.

One class of man's endeavors is characterized by nearly complete certainty. The construction of a building serves as an example, for almost all buildings can be completed essentially as planned, although minor variations commonly occur because of changes in materials and the performance of the craftsmen. Any small risks which are inherent in all such activities are accepted as part of ordinary existence and quite generally ignored. The attractiveness of economic participation in such undertakings can be described adequately in terms of an expected rate of return, and in many cases the variations in this quantity will be small. A continuum of increasing relative uncertainty extends to those ventures where the probability of success of the individual trial is low.

Another class of man's endeavors is characterized by high uncertainty. The probability of failure of an individual trial often exceeds the probability of its success. Producing a highly successful theatrical play would serve as an example of this class, as would the exploratory well drilled for the discovery of petroleum. As a general class, such fields of endeavor are characterized by high profits in a relatively few fortunate cases which serve to attract new entrants into the field. They are also characterized by losses of fortunes in many cases where individuals are eliminated from the field. Even though based upon accurate information, expected rates of return serve as an insufficient guide for participation in such areas.

A solution to the problem of appraisal of highly uncertain economic ventures is presented in this dissertation. It was necessary for the author to provide a framework which will serve to guide the conduct of an economic unit through a series of such highly uncertain ventures. Suitable guidance implies formulating rules which can be followed in order to impart great confidence in the survival of the economic unit. In order to provide for such assurance, it was necessary to formulate guide lines for partial participation in relatively large uncertain ventures. Another requirement which was accomplished was to provide a means of appraisal of an individual venture which is commensurate with the financial position of the particular economic unit. Another facet of the problem for which a solution was found involves the placing of an appraisal value on information which increases the chances of success of a prospective undertaking.

State of the Art

The present state of the art of appraisal of economic participation in uncertain endeavors is believed to be typified by the current practices and valuation methods of the petroleum production industry. Additional information can be gained from a study of the publications by the more eminent business schools. The findings from both of these sources will be discussed.

The petroleum producing industry is an extractive industry and it depends for its survival upon discovering new deposits through continuous exploration efforts. New leases must be bought for drilling sites and new wildcat wells must be drilled. The probability of discovery of a new deposit is low, generally less than one-half and often around one-tenth, depending upon the area, even after improving the odds by expending a considerable amount of geophysical effort in selecting favorable drilling locations.

Between ten and fifteen years ago the industry became extremely conscious of the internal rate of return concept of evaluating an investment. Before that time, most evaluations of prospective ventures had been made on a present value basis in which present value factors based on interest rates usually in the 6 through 10 per cent range were used with the expected cash flow to give an expected present value. But at the present time, there is generally an attempt made to reduce the evaluation to an expected internal rate of return basis. Previous to this present investigation, no satisfactory methods have been described for combining statistical probabilities, risk and rate of return on invested capital.

The drilling of exploratory wells, it has been noted, involves risk. Money spent in drilling unsuccessful wells is lost. The risk is commonly handled by insisting upon a high expected rate of return, perhaps expressed as a return of two for one on money invested. In the case of wells drilled for purposes of extending an already discovered field, there is also some risk and this is commonly handled by insisting upon an expected rate of return of about twenty per cent per year on the unamortized investment.[1] There is admittedly much reliance upon experience in the area and situations which are familiar to the parties concerned. The rules of thumb and the scintillations of intuition which have been generated by such experience will, undoubtedly, in most cases never reach the stage of mathematical formulation.

The risk inherent in investing in oil exploration ventures appears to have been not well understood by the oil producers. Many have seen their capital disappear in successive unfavorable ventures. Without outside capital it has been estimated that the oil producing industry in the United States would disappear in about twenty years.[2] Much exploratory work and indeed the financing of secondary recovery projects is arranged by promoters who attract tax dollars into the industry. This money if not reinvested would otherwise be taxed as profit of the potential investor and is commonly counted as a net loss of only about fifty cents on the dollar invested if indeed the project

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[1]J. J. Arps, "Profitability of Capital Expenditures for Development Drilling and Producing Property Appraisal," presented at the Annual Fall Meeting of the Society of Petroleum Engineers of AIME, Dallas, Tex., Oct. 6-9, 1957.

[2]William C. Mitchell, Jr., promoter of oil exploration deals, in lecture given before the Oklahoma City Section of Society of Petroleum Engineers of AIME, Nov. 13, 1967.

fails. For his part, the investor is trying to turn his tax dollars into hard dollars in the ground, several fold. The objectives of all parties are well understood by all concerned. In the arranging of these deals in the financial centers, the true speculative nature of the proposed undertakings is probably better recognized than anywhere else.

The use of decision trees for evaluating alternate courses of action has recently been introduced by some oil operators.[3] The method used present values which were obtained according to fifteen per cent interest rate factors. Essentially, the expected present values of alternate choices, such as undertaking the drilling of a well versus selling the lease, are calculated according to the best information and estimates available, and the most favorable choice, according to the comparison of expected present values, becomes evident in the developed solution. The traditional approaches of the petroleum industry of basing an evaluation on the most probable values of factors entering into the evaluation were recently discussed by Campbell.[4] He also discussed some new management

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[3]E. Murray Gullatt, Executive V.P. Production and Acquisition, Livingston Oil Company, Tulsa, Oklahoma, "The Present State of the Art," lecture delivered before the Oklahoma City Section of the Society of Petroleum Engineers of AIME, Dec. 12, 1967.

[4]John M. Campbell, Professor, University of Oklahoma, "Decision Theory - Its Problems, Use and Future," lecture delivered before the Oklahoma City Section of the Society of Petroleum Engineers of AIME, Dec. 19, 1967.

techniques, particularly utility theory, which might be applicable to the oil producing industry..

Among the more recent papers of considerable immediate interest which have been published by the business schools is one by John F. Magee which describes the use of decision trees.[5] The fact that this technique is beginning to be used to some extent in the petroleum producing industry was noted previously. The technique is being utilized essentially as described by Magee. The branches of the decision tree diagram show possible alternative decisions and the various possible outcomes which might follow each decision are traced to the end result and evaluated. By reversing directions, the values inherent in each of the previous decisions can be constituted. The method as illustrated deals with the probabilities of various possible outcomes by combining them into an expected value. The technique is directed toward maximizing expected value.

A review of the current status of the use of cardinal utility theory as an aid in decision making has been published by Ralph O. Swalm.[6] His account was characterized by the author as a progress report and he stated that

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[5]John F. Magee, "Decision Trees for Decision Making," Harvard Business Review, July-August, 1964.

[6]Ralph O. Swalm, "Utility Theory - Insights into Risk Taking," Harvard Business Review, November-December, 1966.

research in the use of this theory has raised as many questions as it has answered and much remains to be done. Methods of constructing utility functions were described and their applications to simple gambling-type situations were illustrated. The idea of "planning horizon" was mentioned, and men were not asked to make decisions involving amounts greater than twice their total annual corporate recommendations. It was pointed out that apparently the human mind has limited information handling capabilities and that it is necessary to simplify the complex fabric of the environment into workable concepts for a decision. Thus, one tends to use expected values. He pointed out clearly that business men do not tend to maximize the expected value in their transactions. In his use of utility functions, he converted expected dollars into an expected utility. He stated that despite the fact that utility theory is urged on business men by the textbook writers nevertheless it essentially is not being used.

A recent paper by David B. Hertz has described the use of simulation methods in order to obtain the probability density function of the rate of return which could be expected from a single business venture.[7] Simulation methods were used to combine all of the major variable quantities which would enter into a rate of return

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[7]David B. Hertz, "Risk Analysis in Capital Investment," Harvard Business Review, January-February, 1964.

calculation, and repeated random trials produced the probability distribution of the combined variables, converted into a rate of return index. The method described in this paper offers certainly one of the best methods of analyzing the potentialities and risk within a single venture. It deals with a single relatively large venture of several facets but wherein the probability of some degree of success is very high and conversely the probability of complete failure is very low. Within this area, it offers the means of comparing two or more such potential economic endeavors from the standpoint of their respective probability distributions of internal rate of return.

Objective of the Investigation

Before proceeding further, a hypothesis is here stated which may serve as a guide and lend coherence to further theoretical developments.

Hypothesis:

The best theoretical basis for making choices for participation in uncertain economic ventures is to be found by combining utility theory with statistical decision theory. Utility theory furnishes a basis for policy. Statistical decision methods permit an interpretation of both the favorable possibilities as well as the risks of participation so that the policy can be implemented.

The accomplishment of the objectives of the investigation was pursued by manipulating and analyzing a model which was analogous in operational responses to the physical system. A particular physical system of immediate concern was the drilling of exploratory oil wells. In this system, the responses or rewards of nature to the moves of the entrepreneur spring from the unknown disperse state of nature, in which the responses of the existing deterministic state of nature is analogous to the drawing of numbers from the unknown locations in an existing predetermined table of random numbers.

Model:

The Model which was used to derive rules for participation in uncertain economic ventures is a sampling distribution from an infinite population of various possible financial outcomes. A probability distribution of the resources of a particular development area, as described by best obtainable estimates, was converted to a probability distribution of financial outcomes. From the infinite population, the sampling distribution was obtained either by the methods of mathematical probability or by the methods of the Monte Carlo process, whichever was more convenient in a particular case. In general, the sampling distributions were found to be approximately normally distributed about a mean value which corresponded to the expectancy of the infinite population.

The values of the financial outcomes in the infinite population were present values as of the beginning of the financial venture. Thus, the sampling distributions corresponding to ten, twenty, or thirty combined individual ventures could be obtained by adding the present values of the individual ventures. It is evident that a number of such distributions could be obtained which would correspond to various different interest rates used for discounting the anticipated future incomes from the individual ventures. Two rates of interest should be considered for discounting. One is the minimum expected rate of return which would be acceptable to a company as an over-all average return on their investments. A second lower rate of interest should also be considered which essentially represents the cost of capital and overhead. The lower rate should represent the lower limit at which the company could stay in business temporarily until the arrival of better days. In this investigation, 8 per cent was used as the lower rate of return for placing a value on the statistical cut-off point where 95 per cent of the cases should be of higher value. In general, where a particular interest rate enters into the decision process, the critical points in a present value probability distribution of financial outcomes should be calculated according to that interest rate.

Accomplishments:

The controlling position of utility theory has been accepted in this investigation as the basis for over-all guidance policy. Motivation to enter into or to persist in an industry is related to the concepts of utility.

Consequently, the selection of the confidence limits under which an individual will participate in an industry must be made by the methods which have been adopted by the utility theory approach to evaluation. More specifically, the acceptance of such guide lines as 95 per cent confidence that the outcome will be equal to or better than an 8 per cent rate of return on the investment, such decisions as these might well be made by the utility judgment of the entrepreneur or the investor. Given such policy guide lines, the role of statistical decision methods is to provide an interpretation of the physical system so that such guide lines can be followed.

The investigation of the behavior of the uncertain ventures consisting of oil exploration efforts and the drilling of wild cat exploratory wells was implemented by studying the probability distributions of the sizes of oil fields in Oklahoma, Texas, and Louisiana. An economic decision model which was proposed by Louis F. Davis to the Society of Petroleum Engineers was used in conjunction with the sampling distribution to develop an appraisal evaluation of oil explorations in the Oklahoma area.[8]

Some original developments have been achieved during this investigation and are reported herein. The model which was adopted in this investigation has been used to

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[8]Louis F. Davis, Vice President, Domestic Production, Atlantic Richfield, Dallas, Texas, Distinguished Lecturer of AIME, lecture before the Oklahoma City Section of Petroleum Engineers of AIME, May 1967.

develop the following tools for evaluating uncertain ventures:

(1) A solution which gives the lowest point which would be reached by cumulative losses in a series of uncertain ventures, according to a pre-selected confidence limit, before an upswing in fortunes occurs in response to expectancy.

(2) An appraisal equation which gives the maximum advisable degree of fractional participation in a relatively large economic venture, according to the nature of the enterprise and the financial resources of the individual company.

(3) An appraisal equation which gives the maximum price which should be paid by an individual entrepreneur or company for a particular uncertain economic venture, commensurate with the probabilistic description of the venture, the stipulated rate of return on investment and the financial resources of the economic unit.

(4) An adaptation of the appraisal equations in order to place a value on information which would reduce the extent or the relative number of economic failures and, thereby, permit

a more favorable description of the economic venture.

CHAPTER II

REVIEW OF SOME PREVIOUSLY PROPOSED APPLICATIONS OF PROBABILITY, STATISTICS, AND UTILITY THEORY IN OIL EXPLORATION

Historical Trends

The concepts which have been developed as a basis for choice and formulating effective procedures for decision making under conditions of uncertainty can be broadly classified into two groups. Those groups include, first, those methods which lean strongly on probability theory and statistical interpretations and, second, those methods which lean heavily upon utility theory. The first group requires further subdivision into two subgroups, first, those which rely for interpretation upon the use of the expected value only and, second, those which rely for interpretation upon the entire probability spectrum of possible values. At the present time there does not seem to be any clear basis for subclassifications of utility evaluation methods. Parenthetically, the present investigation embraces utility theory as the basis for determining an over-all policy and presents statistical methods based upon the entire probability spectrum for governing individual operations in order to implement the policy.

All of the concepts for the appraisal of uncertain ventures have been refined by mathematical approaches and methodology. There has been builing of one upon the other as investigations have followed over periods of time. In the following discussion, individual topics are arranged chronologically within the general area.

Applications of Probability and Statistics

(a) Interpretations Based Upon a Lottery

The study of probability teaches that in such a game as tossing a coin for heads or tails, the proportion of heads in a long series of trials will be 0.5, or nearly so. However, the actual difference between the total number of heads and of tails tends to increase. Whitworth gave stern warning regarding this, and he felt that the individual gambler must eventually lose if he continued playing, since the resources of the world would be greater than that of any individual so that his fortune must eventually be extinguished.[1] He then proceeded to formulate a means of surviving through a series of uncertain ventures. His equation is noteworthy because it takes into account the present wealth of the person.

Whitworth reasoned in this manner. If there are m

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[1] William A. Whitworth, Choice and Chance (Cambridge, 1878).

tickets in a lottery, there must be m - 1 failures for 1 success. Thus, the probability of winning is $^1/m$. Let A be the prize, let M be the available capital, let X be the break-even cost. If a man either wins or loses, he re-invests in another lottery for a chance again costing $^X/M$, so that this ratio remains constant. Every successful venture multiplies the capital by

$$(1 + \frac{A}{M} - \frac{X}{M}).$$

Every failure multiplies the capital by

$$(1 - \frac{X}{M}).$$

If it is specified that the losses must balance the gains, the average multiplier must equal unity, which gives the relation

$$(1 + \frac{A}{M} - \frac{X}{M})(1 - \frac{X}{M})^{m-1} = 1.$$

This relation can be solved for X. It, thus, becomes an appraisal equation which gives the maximum amount which could be invested, at the break-even point. It has disadvantages, of course, such as requiring a successively smaller investment with each successive loss. Even though it may not overcome such objections, it is nevertheless noteworthy as a relatively early attempt to account for both the probability of success and the present position of the player.

Hayward used Whitworth's Equation as an appraisal equation for calculating the maximum investment which should be made in a potential economic venture.[2] He gave the following approximate solution:

$$X = \frac{M(M + A)}{M + (1 - p)A} \left[\left(1 + \frac{A}{M}\right)^{p} - 1 \right]$$

where

X = the break-even appraisal cost

A = the reward if the venture is successful, present value of the operating profit

M = the available capital

p = the probability of success.

This equation would have the disadvantages inherent in the original equation.

A paper dealing with applications of probability theory to oil exploration was written by Pirson in 1941.[1] At that time Pirson was particularly interested in placing values on combinations of geophysical methods. The following type of relation was illustrated. Assuming that the chance of finding a well by random drilling is 1/25 and by geologic structural methods is 1/10, then the chance of a discovery because both forces are acting becomes

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[2] J. T. Hayward, "Probabilities and Wildcats," API Drilling and Production Practice (1934).

[3] Sylvain J. Pirson, "Probability Theory Applied to Oil Exploration Ventures." The Petroleum Engineer, Vol. 12, Nos. 5, 6, 7, 8 (Feb.-May 1941).

1 - (24/25)(9/10) = 34/250. He also carried out a mathematical development and simplification of Whitworth's Equation. Where the investment in a single venture is small compared to the total available capital, the following approximate relation gives the maximum investment which can be made in a venture where probability of success is p,

$$\text{Investment} = [1 - (\frac{1 - p}{2})(\frac{\text{Reward}}{\text{Capital Available}})][\text{Expectancy}]$$

where Expectancy = (p)(Reward).

Pirson included an example which shows that where the present worth of the royalty under a 40 acre tract is $1000 per acre and where the probability of success is 17/125, an operator with resources of $100,000 would be able to pay $4499; whereas, an operator with resources of $1,000,000 would be able to pay $5346 for this same royalty, where both are the maximum or break-even prices for these two operators with different financial resources.

(b) Applications of the Binomial Distribution to Oil Exploration

The application of the binomial distribution to oil exploration was described by Hayward in 1934. It was again discussed by Pirson in 1941. More recently it has been mentioned by Arps.[4] These authors have recognized the value of this equation for estimating the probability of drilling various numbers of dry wells in succession, the "gambler's ruin" concept, wherein the expected one good well in say five does not hit regularly and one could face ruin by drilling say eight or so dry wells in a row. This distribution is adapted for describing an accept or reject type of operation. It can be used to calculate the probability of exactly x occurrences in a total of n trials of an event that has a constant probability of occurrence p.[5]

$$p(x) = \frac{n!}{x!\,(n - x)!}\, p^{x}\, q^{n-x}$$

where $0 \leq x \leq n$

$q = 1 - p.$

If one considers a group of eight wells where the probability of success is 0.2, setting x = 0 and solving gives p(0) = 0.168. That is, about one time out of six there would be no discoveries out of the group of eight exploratory wells. Thus, where there is insufficient financial backing, the normal runs of misfortune act to force the operator out of business.

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[4] J. J. Arps, "The Profitability of Exploratory Ventures," Economics of Petroleum Exploration, Development, and Property Evaluation (Englewood Cliffs, N. J., 1961).

[5] W. J. Fabrycky and P. E. Torgersen, Operations Economy (Englewood Cliffs, N. J., 1966).

(c) Descriptions of Size Distributions of Resource Deposits

The size distributions of several kinds of mineral deposits have been observed in various regions to follow the lognormal distribution. It was pointed out by Arps that while the binomial distribution can be used to describe the probability of drilling a dry well, the size of a discovery would be described by some other distribution.[6] There tend to be more small deposits than larger ones, and the marginal wells generally cost more to complete. It was Arps' observation that dry holes in some very favorable areas may be as low as 50 per cent to as high as 95 per cent of the total wildcat wells. Then, about 60 per cent of the so-called successful wildcats are marginal.

The method of evaluation proposed by Arps consisted of entering a single value of the estimate of the oil reserves which could be discovered. The one successful well was charged with drilling several dry wells and several marginal wells, according to the history of the particular area. Average operating profits per barrel, or per thousand cubic feet of natural gas, were used to convert estimated reserves to an undiscounted future profit and also to an average annual rate of return, which is approximately the same as the internal rate of return.

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[6] Arps.

(d) Simulation of a Single Venture

A method of analyzing risks and profits has been described by D. B. Hertz.[7] It is a statistical method wherein estimates of probability distributions of component parts are combined by simulation, the Monte Carlo technique. The several experts within a company are called upon to make probability estimates of the range of values which might occur within their respective spheres of responsibility. For example, the marketing estimate would include several demands with a probability estimate of each. In the Monte Carlo process, randomly chosen values of each component are combined, and the profitability result of that particular combination is computed. The process is repeated many times until the pattern of the combinations becomes evident. An example described by Hertz was for a manufacturing company evaluating the profitability of undertaking the manufacture of a new item. Aspects which were considered were the following:

A. Market analysis

1. Market size
2. Selling prices
3. Market growth rate
4. Share of the total market

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[7] David B. Hertz, "Risk Analysis in Capital Investment," Harvard Business Review, Vol. 42, No. 1 (Jan-Feb. 1964).

B. Investment cost analysis

5. Investment required

6. Residual value

C. Operating and fixed costs

7. Operating costs

8. Fixed costs

9. Useful life of facilities.

The results can be portrayed in a number of ways, such as undiscounted value, present value or internal rate of return. The latter was used by Hertz. From plots of the probability distributions of the rates of return which might be obtained from different prospective ventures, the management would have a basis for comparison and choice.

(e) Study of Petroleum Recovery from a Reservoir

The amount of oil which will be recovered from a particular petroleum reservoir can be estimated from information which becomes available after the discovery well and several development wells have been drilled. Information which becomes available at such time comes from electric and nuclear logs which are run inside the drilled wells and from the analysis of fluid samples from the reservoir. It then becomes possible through interpretation of the logs to give estimates of such quantities as fractional porosity of the rock, fraction of the pore space filled with water, thickness of the oil zones, areal extent of the reservoir and other quantities of importance in determining the ultimate recovery of oil from the reservoir. In general, a single value is not obtained for a particular quantity, but rather a range of values is determined during the analysis. The relative frequency with which values occur within a range permit an estimate of a probability distribution of the various quantities. The use of uniform and triangular probability distributions for describing such values was discussed by Walstrom et al.[8]

Total ultimate recovery of oil from a reservoir depends upon the values of such physical quantities. For the most part, they are independent of each other, but this is only an incidental detail. Combining different values of the quantities in random fashion by the Monte Carlo technique produces a probability distribution of the total ultimate recovery of oil. In turn, probability distributions of this nature can be used as the basis for economic decisions regarding whether or not to install facilities such as pipe lines and processing plants. The use of the Monte Carlo technique in this area of decision making promises to increase very rapidly.

Applications of Utility Theory

Utility is the power of satisfying, directly or

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[8]J. E. Walstrom, T. O. Mueller, and R. C. McFarlane, "Evaluating Uncertainty in Engineering Calculations," paper presented at the Annual Fall Meeting, Society of Petroleum Engineers of AIME, Houston, Texas, Oct. 4, 1967.

indirectly, human needs and desires. All things considered as wealth possess utility, although not all things possessing utility can be considered as wealth. The total utility of a substance relates to the total amount of the substance which one possesses, and the marginal utility relates to the change in satisfaction associated with possessing one more unit of the substance. These are commonly recognized concepts in economics.[9]

The origins of the utility concept were concerned with consumer goods, things which directly satisfy human wants. It is based upon the fact that acquisition of a given increment of a substance gives different amounts of satisfaction, depending upon the amount of the substance in possession as compared to the amount needed. The measure of human satisfaction is not so much by logic as by sensation.

The first mathematical formulations of utility were published by Fisher.[10] The great work in this area, however, was published by Von Neuman and Morgenstern.[11] They noted that it is impossible to compare the utility preferences of different people except through the use of indifference curves. Investigations along these lines

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[9] R. H. Leftwich, The Price System and Resource Allocation (New York, 1965).

[10] Irving Fisher, Mathematical Investigations in the Theory of Values and Prices (New Haven, Conn., 1925).

[11] John Von Newman and Oskar Morgenstern, Theory of Games and Economic Behavior (Princeton, N.J., 1947).

were made by Davidson, Supples, and Siegel.[12] These authors found that consistent choices were made when individuals were given choices between winning and losing certain sums of money in well known games of chance. However, other tests, such as choices based upon artistic preferences, were unsatisfactory. They felt that subjective motives and values could not be replaced with objective values.

Techniques were developed by Grayson for determining an oil operator's preferences regarding whether or not to drill an oil well, where the probability of success is of the order of one-tenth.[13] A plot of utility versus dollars is made by this method, both measured from the present position of the operator. Utility is plotted on an arbitrary scale. For example, the prospects of a 50-50 chance of losing $30,000 might exactly balance the 50-50 chance of winning $100,000. When several such choices have been stated, it is possible to construct the operator's utility curve. In using such a curve, the negative utility corresponding to the cost of a well would be multiplied by the estimated probability of a dry well. The positive utilities corresponding to various possible

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[12] Donald Davidson, Patrick Supples, and Sidney Siegel. Decision Making: An Experimental Approach (Stanford, Calif., 1957).

[13] C. Jackson Grayson, Jr., Decision Under Uncertainty (Cambridge, Mass., 1960).

favorable outcomes would be multiplied by the estimated probabilities of each such favorable event. Then, the various positive and negative utility increments are added, and the greater the sum is above zero the more attractive is the venture. Thus, it becomes evident that judgment is finally restricted to comparisons based upon a single characterization, an expectancy of utility.

An intriguing procedure for calculating the optimum relative participation in a venture has been outlined by Grayson. His method is based on the use of the utility function of the investor. The essential features of his example can be crystallized in two tables. The first table gives the probabilities and the expectations. The various possible outcomes of a particular venture are shown in the following table, which was given by Grayson.

The second table analyzes the same venture in terms of expected utility. As a result, an optimum degree of participation appears, which reflects the financial resources of the entrepreneur. This method requires that every monetary value in the following table be converted to its corresponding utility value (or relative preference value) of the particular investor. The utility function which was used for this purpose, in this case, is shown on Figure 1. This permits calculation of the values shown in Table II.

TABLE I

THE MONETARY EXPECTATIONS OF PARTICIPATION IN A DRILLING VENTURE (AFTER GRAYSON)

Possible Discovery, 10^3 bbl.	Probability	Per Cent Participation 0	100	75	50	25
		Monetary outcome, 10^3 dollars				
0	0.60	0	-50	-38	-25	-13
100	0.20	0	100	75	50	25
200	0.10	0	200	150	100	50
500	0.07	0	500	375	250	125
1000	0.03	0	1000	750	500	250
Expected Monetary Value, 10^3 dollars		0	75	56	38	19

TABLE II

THE EXPECTED UTILITY VALUES OF PARTICIPATION IN A DRILLING VENTURE (AFTER GRAYSON)

Possible Discovery, 10^3 bbl.	Probability	Possible Per Cent Participation 0	100	75	50	25
		Utility Corresponding to Outcome				
0	0.60	0	-14	-10	-4.2	-2.0
100	0.20	0	5	3.8	2.5	1.2
200	0.10	0	10	7.5	5.0	2.5
500	0.07	0	40	26.0	12.5	6.2
1000	0.03	0	80	75.0	40.0	12.5
Expected Utility Value		0	-1.2	-0.4	0.6	0.1

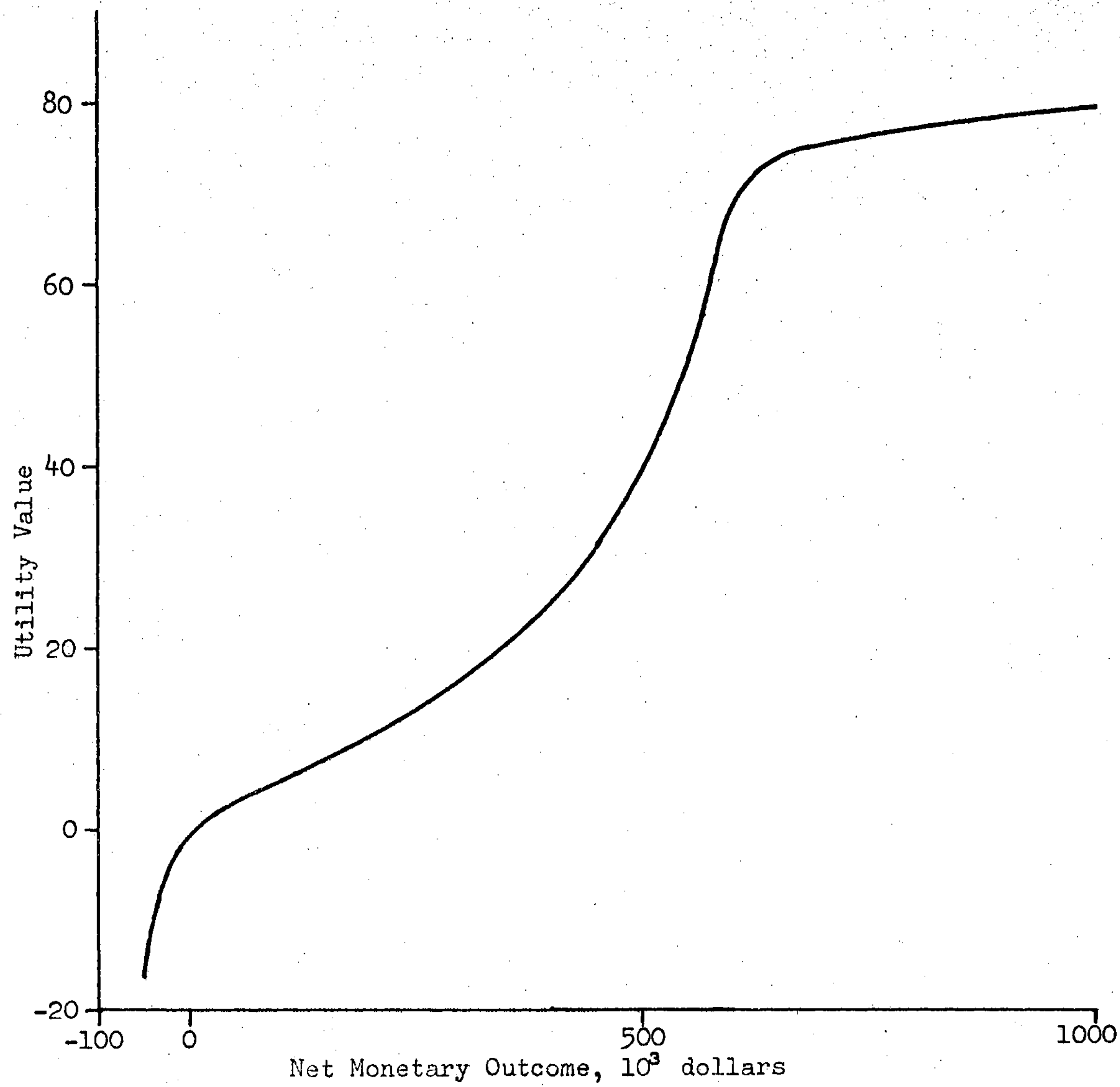

Figure 1. Utility Function of an Individual (After Grayson)

It is evident that if one judged the venture solely in terms of the expected monetary value, one would choose the course of drilling with 100 per cent interest. By this course, the expectations are maximized. In the long-run, the expectations would be realized, assuming, of course, that the estimates of reserves and probabilities were sufficiently accurate. The position of the large operator who could drill many wells would tend towards this decision.

In the case where one judges according to a utility scale, it is seen that the decision may not be the same. For the case illustrated, the utility is maximized by retaining a 50 per cent interest in the venture. This, it must be born in mind, stems from the financial position of one particular operator or investor, and it reflects his financial reserves. Thus, the 50 per cent figure would not apply to every individual nor to every company. For some, it might be more; for others it might be less, depending upon their financial position at the time. In the utility curve approach, there will always remain the problem of adjusting the curve to changing circumstances and justifying its position and shape. There may be an apparent lack of objectivity in the method. Nevertheless, the method clearly points to the desirability, in some cases, of fractional participation in a venture.

The chief criticisms of this type of application of utility theory have centered around four objections.

First, the individual tends to exhibit a different utility curve at different times, because of changes in his internal mood and changes due to his interpretation of variations in the economic environment. Second, where a corporation is concerned, there should be a corporate utility curve for governing actions rather than individual utility curves. Cardinal utility values are not additive because the index is unique to its position on the scale. Hence, a cumulative or average utility function is not meaningful. Third, the utility curves of many people tend to be logarithmic in shape. On the negative side, they decrease rapdily, since loss of some particular amount of capital spells disaster. On the positive side, the leveling off of the utility curve (decreasing marginal utility) is alternately accepted as theoretically correct or sometimes used to point out the lack of imagination and ambition upon the part of that individual. Fourth, the same utility curve might not apply at different times when the individual's investment status is different, both with respect to the total amount invested and with respect to the remaining uncertainty in the present investments. To these might be added another criticism. Fifth, the conversion of dollar expectancy into utility expectancy is not adapted for displaying the complete range of possible unfavorable as well as favorable outcomes, which may contain a wide variation in the actual return from the venture.

Both statistical and utility measures of value were employed by Kaufman.[14] This author presents the management problem of being forced to choose from an array of opportunities which arise during a fiscal year. He assumed that money not reinvested would be subjected to higher taxes than if reinvested during the fiscal year. After considerable study from a mathematical point of view, the lognormal distribution was accepted as the best means of describing the size distribution of oil fields. After establishing a size distribution, it was then used as a probability distribution to aid in predicting the sizes of the drilling opportunities which would come during the remainder of a fiscal year. This problem which Kaufman attempted to solve centers around the idea that some good deals may currently be available, but there is nevertheless the possibility that far more attractive ones may appear in the near future and there is only so much money to invest. And, as the end of the fiscal year approaches, the money must be invested. In one approach, the size of the oil field was assumed to be known before the investment was made. In another, the expected size was used in connection with a utility function curve as described by Grayson.

Refinements in the methods of obtaining utility

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[14] Gordon M. Kaufman, Statistical Decision and Related Techniques in Oil and Gas Exploration (Englewood Cliffs, 1963).

curves from oil operators and executives were made by Newendorp.[15] The theory is basically the same as described by Grayson and Swalm. The questions and descriptions of hypothetical deals which were presented to the operators for determining their indifference points were representative of actual deals which must be judged in daily transactions. Newendorp stated the amounts of the required investments, which ranged between $40,000 and $200,000. Decisions were made from the standpoint of an owner of 100 per cent of the working interest. The type of geologic or geophysical control was given and sections of maps were provided for interpretation. An index of the fractional reliability of the information was included, as were both the costs of a dry hole and the costs of a completed producer. The participants who took the prepared tests were asked to state the expectancy, either in dollars or in barrels, which would be sufficient for them to recommend drilling the well. From this information, the utility curves of the individuals were drawn, and during subsequent consultations the participants generally accepted the curves as representative of their preferences. Some mentioned, however, that at different times they would probably have different utility curves. In addition to their possible use in actual decision situations, the

[15]Paul D. Newendorp, "Applications of Utility Theory to Drilling Investment Decisions," (Dr. of Engineering Dissertation, University of Oklahoma, 1967).

prepared hypothetical questions and deals were considered by Newendorp as having possible value for training of future decision makers.

Summary

The petroleum industry prefers to reduce evaluations to a statement of the expected rate of return. Risk is recognized in the sense that its presence requires a greater expected rate of return, but no relations have been developed between these two quantities. Methods of evaluation which are based on success probability ratios and which incorporate only the expected return have been proposed but are not being used. Reliance is commonly placed in the binomial distribution for predicting long runs of unsuccessful trials. Cardinal utility theory has been used for portraying the fear of gambler's ruin in a new cover, but it also is tied to expected values and has not been adopted by the industry.

At the forefront of the developing evaluation concepts in the petroleum industry are the following two methods. A formalized procedure for choosing between alternative courses of action based on the use of decision trees has very recently been introduced in petroleum exploration evaluations. Inherently, this method appears to be restrained to the use of expected values. A more flexible tool for evaluation purposes is the simulation of a probability distribution. In the petroleum industry, this has been used for evaluating the ultimate recovery of oil from a reservoir at a time early in the development drilling stage.

In the following chapters, a method of evaluation is proposed which is intended to be applicable in cases of high uncertainty, such as the case of deciding whether or not to drill an exploratory well. Interpretations are developed which permit both risk and rate of return to be jointly considered in an evaluation. Statistical solutions have been found which show relations between the appraisal value, the risk and the financial position of the investor. Attention is now directed toward the development of these new evaluation tools.

CHAPTER III

DEVELOPMENT OF A STATISTICAL FINANCIAL DECISION MODEL

A Model of Combined Independent Ventures

Interpretation of the expectancy to be gained from participation in a single economic undertaking can readily be stated as an anticipated amount or as an expected rate of return. However, even where the expectancy is high, in those field of endeavor where the chance of success of an individual venture is relatively low, interpretation of the actual proceeds from a single venture is most often a distressing experience. It becomes evident in such cases that the proper interpretation should be based upon participation in some number of individual undertakings. In this manner one can survive and take advantage of the expectancies. These two objectives, survival and profit, can be achieved in highly uncertain ventures only by making careful decisions. In order to be able to outline the rules which should govern such decisions, a model is herein described which can be used to portray the probable outcomes of participation in several uncertain ventures.

A model must have correspondence to the real world if the answers which are obtained from the model are to have meaning in the real world. It, therefore, is necessary to examine the characteristics of the real world before constructing a model. The economic area of special interest in this investigation is the exploration for new oil fields. Probable good locations for the drilling of successful exploratory wells are selected by geologic and geophysical methods. These methods are not 100 per cent effective or reliable, for a number of reasons, so that the success of an exploratory well is probabilistic. An exploratory well is not simply successful or unsuccessful, although usually 75 per cent or more are completely unsuccessful. Geologic conditions and reservoir rock characteristics are variable, and among those exploratory wells which are classified as successful, some discover only small reserves of petroleum while others with decreasing probability discover large and profitable new oil fields. The outcomes of the exploratory wells, thus, can be considered as a sampling distribution taken at selected locations which are a subset of all possible locations and whose magnitudes are drawn from a continuous distribution where most of the values are zero, some are small, some medium, and a few are large.

A question which must be answered for each proposed application concerns whether or not a model can be constructed which has the necessary correspondence to the real world which it represents. Within the intelligence which is enjoyed by most people, there is the capacity to describe and apparently understand simple probabilistic patterns and events. Gambling in various forms and using various devices is indulged in by savages as well as by highly civilized and educated men. Such devices as dice, cards, drawing straws, flipping coins, and lottery tickets are widely used. The notion of a set of equally likely events, of which only a limited number will occur, is readily grasped. Ideas regarding independent events and dependent events are readily understood with respect to common phenomena. The study of mathematical probability starts with simple models and proceeds to those which are quite complex. However, as long as the descriptions and models are at least relatively simple, they can be widely used and understood. The ideas of win or lose which are reinforced through experiences in gambling and games of chance make it relatively easy for most men to use the concepts of probability for describing to some degree the possible outcomes of uncertain economic ventures. Concepts which are almost instinctively formed in the mind relate to the array of possible results of a single trial and to the average result of all such trials. It is the natural, or parent, distribution which is traced by the mind.

Petroleum reservoir properties such as areal extent, formation thickness, porosity of the rock and fluid properties such as viscosity and amount of gas in solution have been described by probability distributions in recent papers.[1,2] In the majority of cases, the probability distributions which were used were very simple. The continuous rectangular distribution was used in cases where upper and lower limits of values could be estimated but within these limits one value seemed as likely as another. A distribution which is apparently developing into wide usage is the continuous triangular distribution. This can be described in terms of extreme upper and lower values, neither of which is very likely to happen, and a most probable value. The histogram has been used some, particularly as a discrete distribution, in which form it was used by both Grayson and Hertz. Thus, there exist cases where the use of probability distributions has been demonstrated for adequately describing the sizes and qualities of petroleum reservoirs.

Simulation procedures in this research related to the financial outcomes from drilling a group of exploratory wells and were accomplished by the Monte Carlo process. Random numbers, which were in decimal form, were generated in the computer. If it were assumed for a particular region that three wells out of four were dry, then numbers

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[1] J. E. Walstron, T. D. Mueller, and R. C. McFarlane, "Evaluating Uncertainty in Engineering Calculations," paper presented at the Annual Fall Meeting of the Society of Petroleum Engineers of AIME, Houston, Texas, Oct. 4, 1967.

[2] Marvin B. Smith, "Estimate Reserves by Using Computer Simulation Method," <u>Oil and Gas Journal</u>, Vol. 66, No. 11 (March 11, 1968), pp. 81-84.

between 0 and 0.75 were taken as outcome cases where the cost of drilling a dry hole was lost. Numbers between 0.75 and 1.00 would represent favorable outcomes, from minimum to the maximum possible for the region. The favorable outcomes were handled algebraically so that they would correspond to sampling from the previously assigned probability distribution.

The model was built up by combining the outcomes of a group of trials. These were considered to represent economic trials or ventures. Combined values of a group were recorded and analyzed at several different group size levels. By way of illustration, the case may be taken where a total of 100 series of trials were formed. Each series consisted ultimately of 30 consecutive independent samplings chosen by random numbers from the parent distribution. For convenience, the running total values as formed within each series were recorded when there had been completed totals of 3, 5, 10, 15, 20, and 30 individual trials. These respective group size levels were later individually sorted and divided into percentage units. The average of the fifth and sixth highest values was recorded as that value which was exceeded by 5 per cent of the groups where the group size level, for example, consisted of a total of 10 individual ventures. Similar calculations gave values which were exceeded by 80 per cent of the cases after a total of 20 ventures in each group, and all other points of interest were calculated for constructing a percentage probability distribution chart.

Examples of Economic Distributions

A rectangular probability distribution of the outcomes of a single venture is shown in Figure 2. This outcome distribution includes the initial cost; where the returns are low there could be a net loss of $4000 and the highest anticipated returns would result in a net gain of $6000. Any result between these limits is indicated to be equally likely by the rectangular distribution. An infinite number of trials would result in there being equal numbers of outcomes of all values within the range. The distribution of the results of an infinite number of trials defines the probability distribution of the outcome of a single trial. The average of all these results is the expectancy, which in the case at hand is $1000 per venture.

Results obtained from combining several random trials into groups are shown on Figure 3. The lines tend to slope upward to the right, which is a response in accord with the idea that the total expectancy is proportional to the total number of ventures in a group. This distribution tends to be symmetrical about the 50 per cent line, which is the expectancy. Dispersion or spreading of the values continues to increase as the size of the group increases, but it increases ever more slowly. This is entirely in accordance with the principles of statistics and will be examined more in detail later.

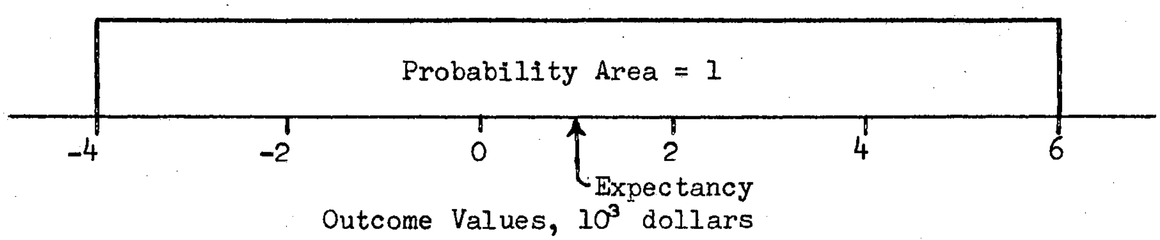

Figure 2. A Uniform Probability Distribution of the Outcome Values of a Single Venture

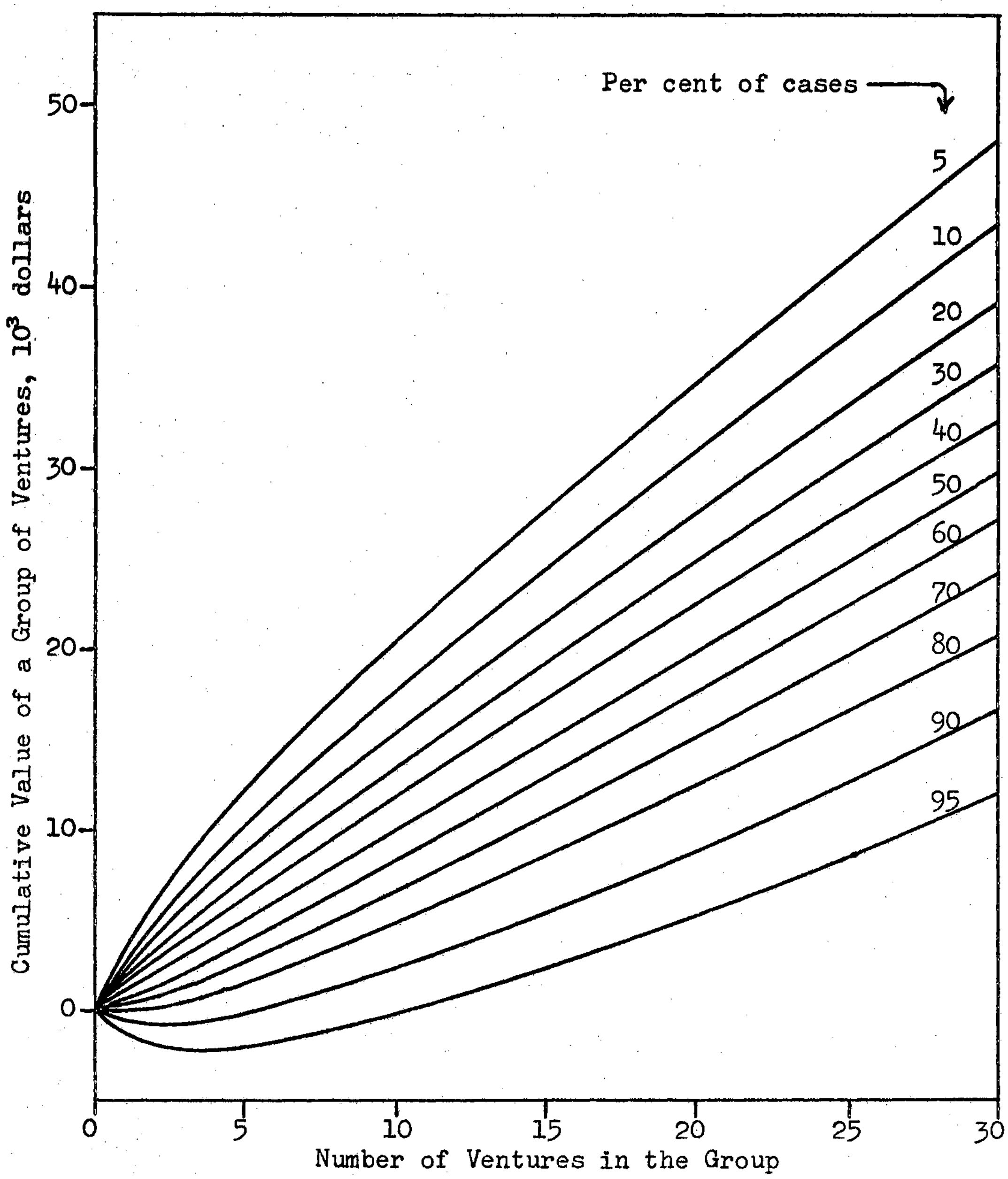

Figure 3. Probability Distribution of the Cumulative Value of a Group of Ventures (Simulation of the distribution shown in Figure 2)

A second example which approximates the economics of many wildcat wells is shown on Figure 4. In this particular example, it is assumed that there is a 60 per cent chance of drilling a dry well which would cost \$150,000. If oil were found, mechanical and financial uncertainties would preclude completing the well unless there were sufficient reserves to reduce the net loss to \$50,000; the total net profit if completed would most likely be \$200,000, but there is a decreasing probability that it could go to an upper limit of \$1,000,000.

The expectancy can be calculated as follows:

$$X_E = p_1 C + (1 - p_1)\left(\frac{X_L + X_M + X_H}{3}\right)$$

$$= (0.6)(-150{,}000) +$$

$$(0.4)\left(\frac{-50{,}000 + 200{,}000 + 1{,}000{,}000}{3}\right)$$

$$= 63{,}300.$$

The results from combining several random trials into groups are shown on Figure 5. The same features are apparent on this figure as are displayed on Figure 2. The dispersion is considerably greater, which makes some difference in appearance. Some values which may be read from Figure 5 include the following. If one were to drill 10 such wells, as described by the probability diagram of Figure 4, there is a 50 per cent chance of making more than \$700,000; there is a 20 per cent chance of making over \$1,500,000; but there is also a 25 per cent chance of losing some money, and indeed a 5 per cent chance of

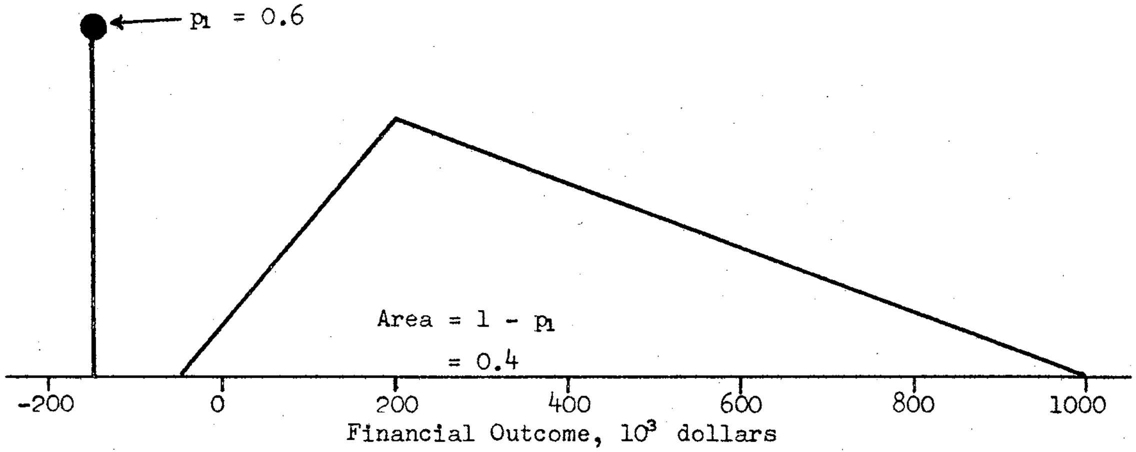

Figure 4. Probability Distribution of Possible Outcomes From Drilling an Exploratory Oil Well

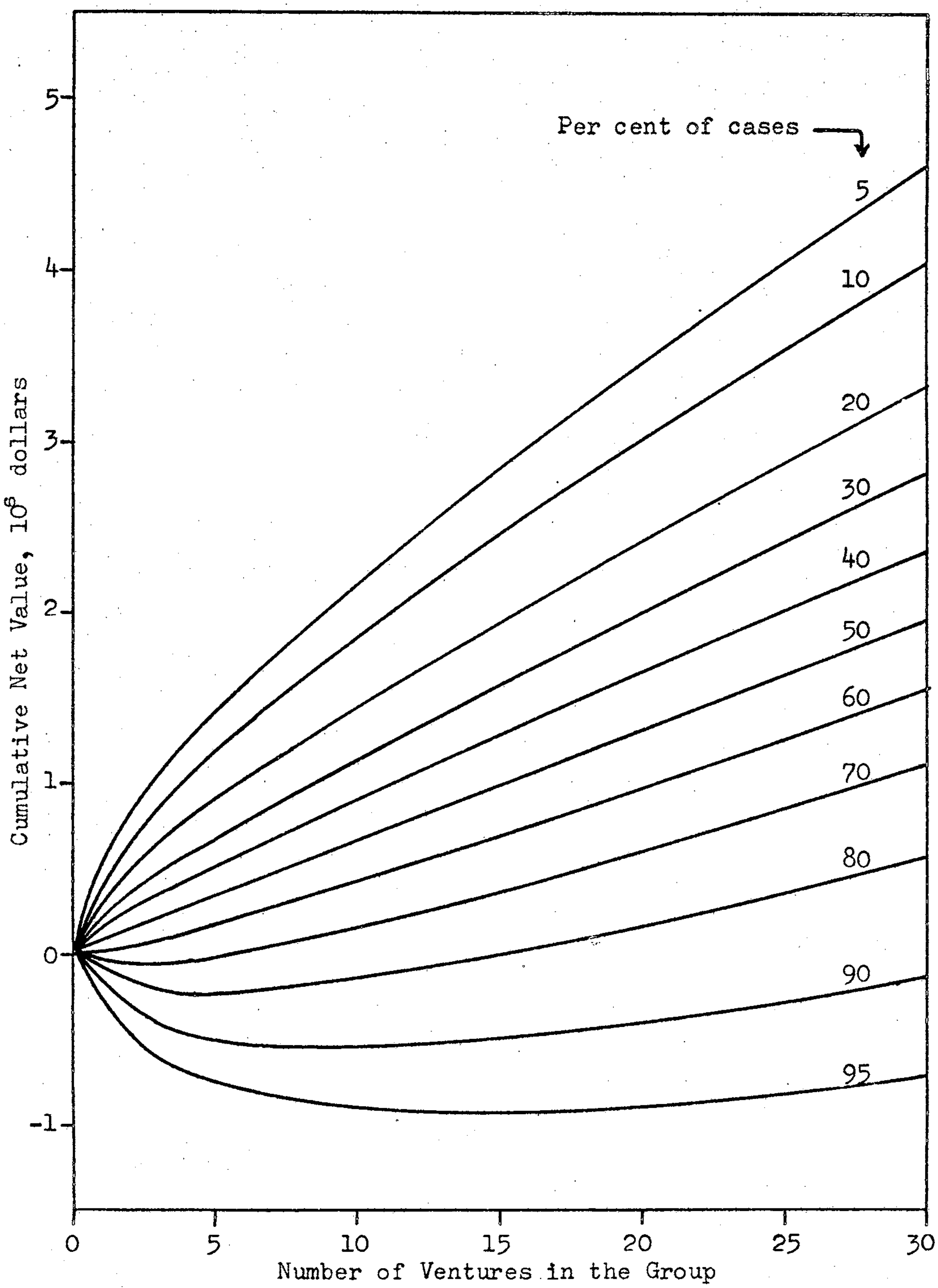

Figure 5. Probability Distribution of the Cumulative Value of a Group of Ventures (Individual venture is shown on Figure 4)

losing more than $900,000.

Comparisons Between Cumulative Values of Groups Taken From Different Parent Distributions

The percentage distributions of cumulative group values in the examples have been seen to lie fairly symmetrically about the 50 per cent line, which coincides with the cumulative expectancy of the number of ventures in the group. This provides one relationship whereby such distributions, which are drawn from different parent distributions, can be compared to each other. For any number of ventures in the group, the position of the expectancy of the group can be calculated by multiplying the expectancy of a single venture, which can be calculated from the parent distribution, by the number of ventures in the group. This relationship can be expressed by the following equation:

$$y_E - y_o = n\ X_E \qquad (1)$$

where $y_E - y_o$ = the expected change in financial position after completing a total of n ventures

n = the number of independent trials, or ventures

X_E = the mean or expected outcome of a single venture.

The dispersion of values above and below the expected value can be expressed in terms of the number of standard deviations of the particular parent distribution. Procedures for calculating standard deviations of some types of distribution are well known and are given in an appendix. Further standardizations for comparisons between the sampling distributions are suggested by rules observed in the study of statistics. First, sampling distributions tend to be normally distributed regardless of the character of the parent distribution. Second, the standard deviation of a normal sampling distribution increases according to the square root of the size of the sample. These concepts are expressed in the following equation:

$$y - y_E = z \sigma \sqrt{n} \qquad (2)$$

where $y - y_E$ = the difference between the cumulative group value and the expected cumulative group value

z = the number of standard deviations above the mean; z is negative for outcomes below the mean

σ = the standard deviation of the parent distribution

n = the number of ventures in the group.

Combining Equations (1) and (2) gives the following:

$$y - y_o = n X_E + z \sigma \sqrt{n}. \qquad (3)$$

Equation (3) expresses the net financial change after completing a group of n ventures.

Solving Equation (3) for z, the number of standard deviations from the mean, gives a way of comparing all of the percentage distributions of cumulative group values to each other and to the normal distribution. This solution is

$$z = \frac{(y - y_o) - n X_E}{\sigma \sqrt{n}}. \tag{4}$$

The results of the Monte Carlo simulation of the examples which have been cited were used with Equation (4) to calculate the corresponding z-values. These z-values were plotted versus n as shown in Figures 6 and 7. There were two reasons for doing this. First, for the smaller sample sizes the distribution might not be normal and Equation (2) might not hold. Second, the Monte Carlo process might not converge rapidly to its best values and, therefore, require the running of a very large number of series of trials before a reliable answer was produced by the process. This aspect relates to the efficiency of the computer programming and to the quality of the random numbers which are generated within the computer. If all these conditions could be met perfectly, the lines on the graphs would be a series of straight horizontal lines. There is, of course, some irregularity and the horizontal traces on Figures 6 and 7 are shown as broken lines. Nevertheless, there is no trend in these other than horizontal and the data tend to become smoother as the number

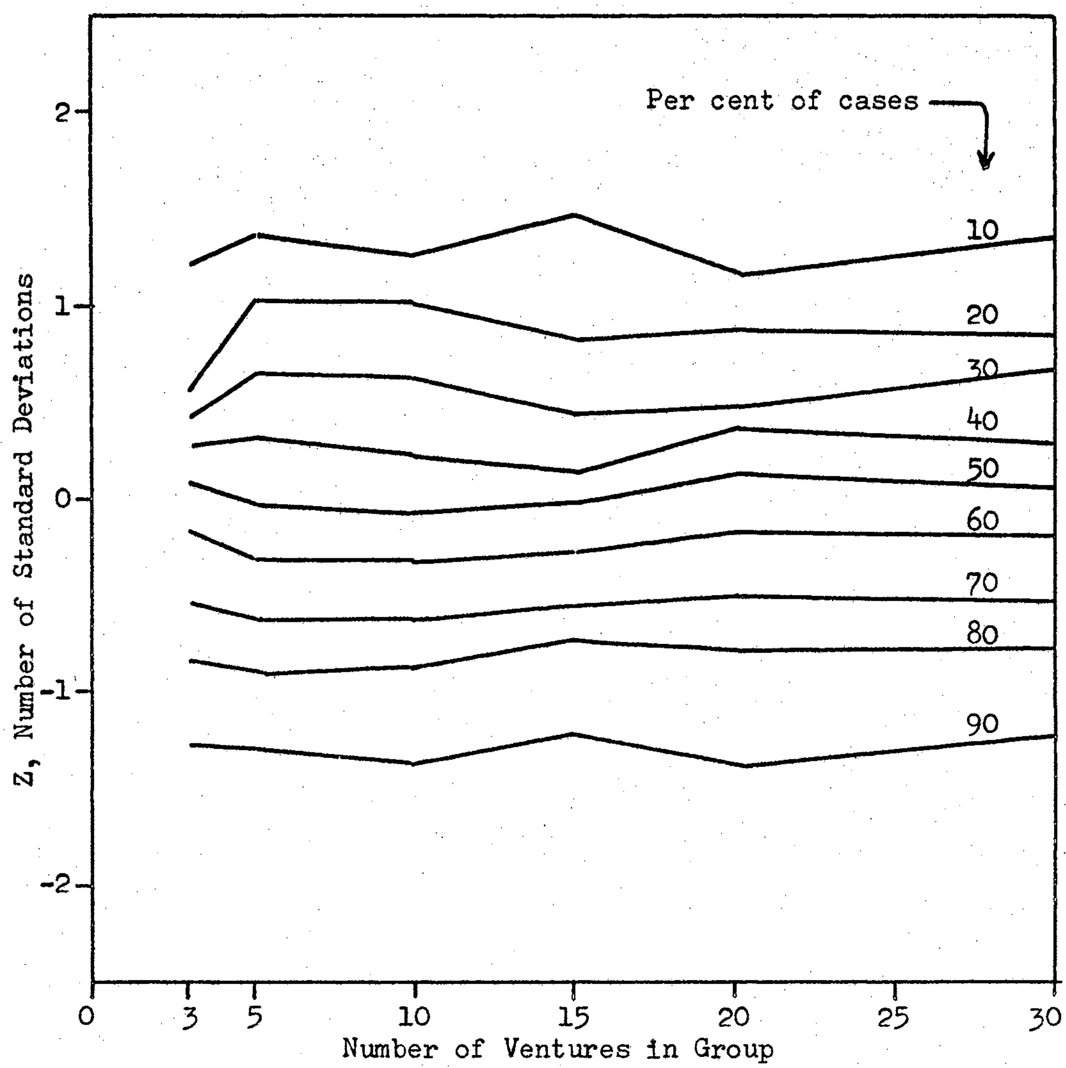

Figure 6. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 2

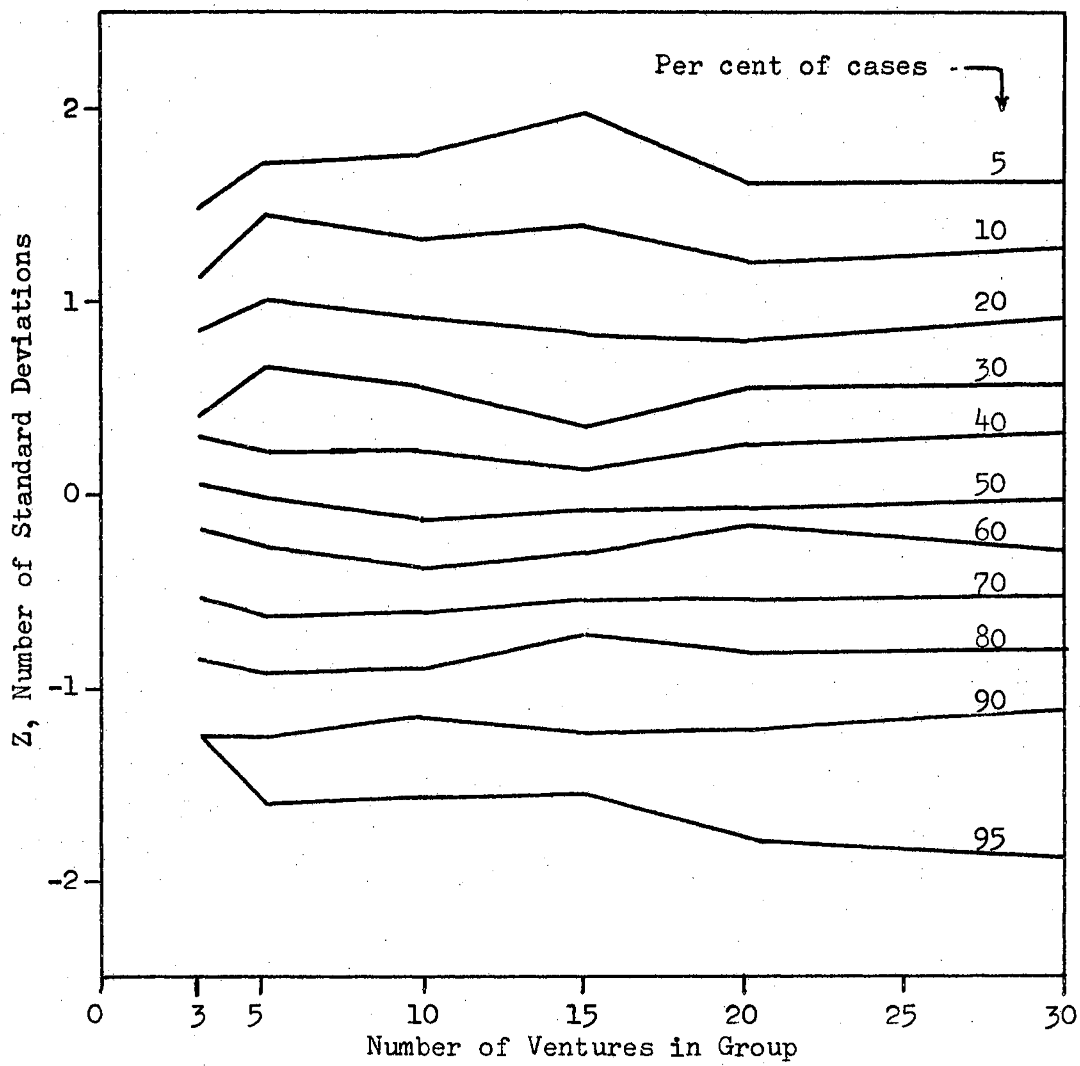

Figure 7. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 4

of ventures per group increases. It is concluded, therefore, that sampling distributions taken from natural distributions such as those shown in Figures 2 and 4 follow the relation that the standard deviation of the group is proportional to the square root of the number of ventures in the group.

Another method of analysis of such distributions is shown on Figures 8 and 9. These are plots made as normal probability graphs, in which the horizontal scale is divided in such manner that the data will plot as a straight line if the data points constitute a normal distribution. In plotting these graphs, the only data used was from the largest size groups. The justification for this is the conclusion made previously that there was no trend in the standard deviation with group size other than the anticipated proportionality to the square root of group size. The Monte Carlo process gives better results as the number of trials are increased, and there were for example three times as many individual trials in the data for groups of thirty as there were in the data for groups of ten. From Figures 7 and 8, it may be seen that the sampling distributions are normal in those cases which have been illustrated.

The relations which have been verified here facilitate further interpretations and the development of appraisal equations. For this purpose, the relations developed herein would not be absolutely necessary, of course, since

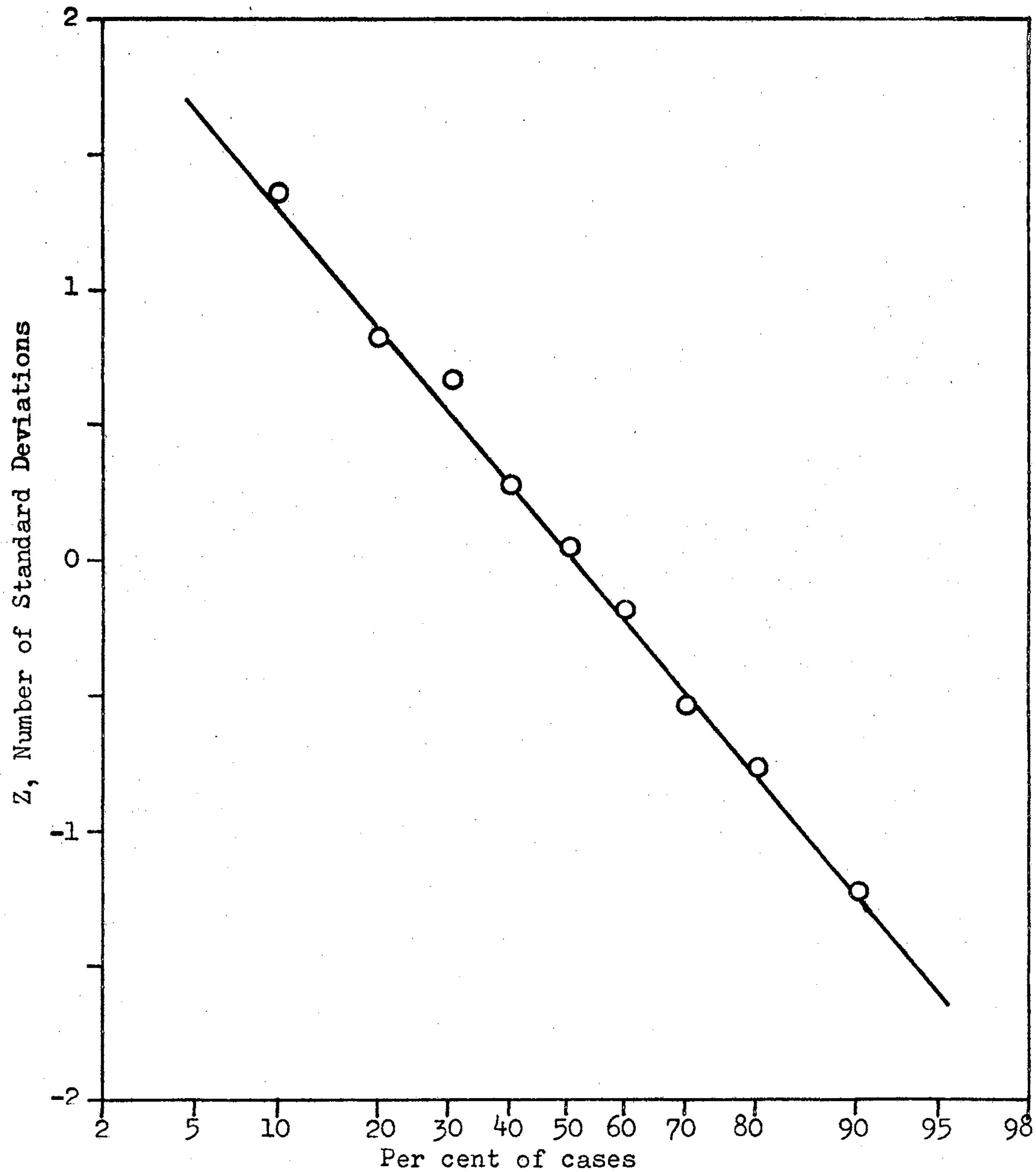

Figure 8. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 2

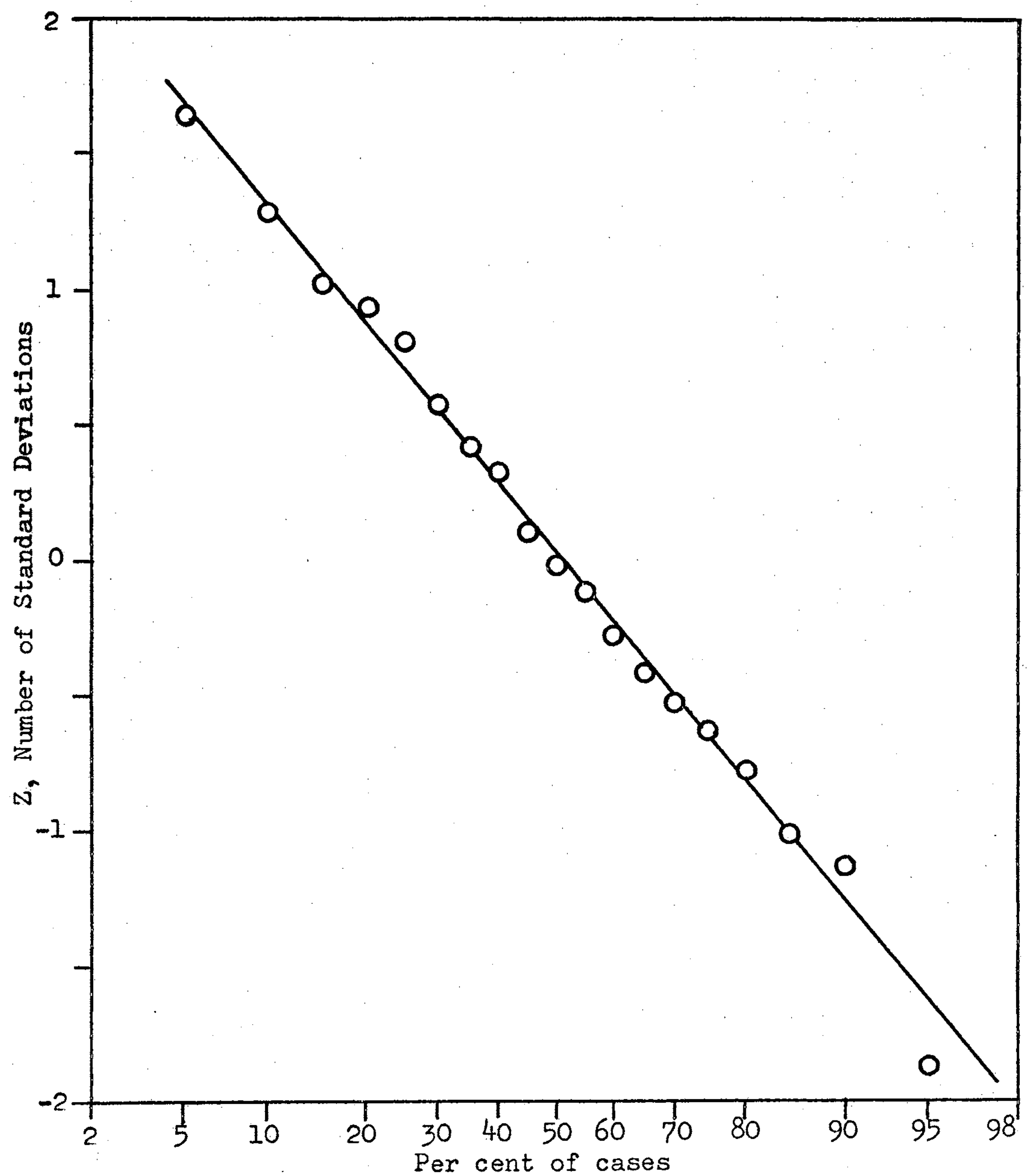

Figure 9. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 4

graphical means are always available for interpretations. Even if such relations were only approximately correct they would be useful, for it will be shown that other quantities have considerable effect upon appraisals under uncertainty.

CHAPTER IV

THEORETICAL DEVELOPMENTS AND INTERPRETATIONS

Rates of Return and Present Values

Methods of calculating rates of return are widely known and have been described in technical articles and many textbooks. Details of various methods differ slightly but they give approximately the same answer. Where the major portion of the necessary investment is made at the beginning of a venture, one straight forward method is to find the interest rate which will reduce the anticipated net earnings to a present value which equals the investment. This interest rate is said to be the internal rate of return, and this method will serve to provide a basis for the following discussion.

Specifying the rate of return which should be earned on an absolutely safe investment is outside of the scope of this research. Suffice it to say that the investor requires a time rate of return for the use of his capital. Somewhere around three per cent per year is required to keep abreast of inflation. Conditions in the environment determine what is acceptable. For purposes of illustration, a rate of eight per cent per year has been assumed as the minimum acceptable rate of return. Also imposed upon the decision environment are further conditions arising from the influence of probability distributions.

As the study of relations between rates of return and the probability distributions of financial outcomes is pursued, it becomes evident that a single distribution of possible sizes of resource discoveries gives rise to many interpretations of financial outcomes. Each resource distribution can be converted to a corresponding undeferred or undiscounted return cash flow distribution based on net operating profits. A family of probability distributions of the present values of the return cash flow can then be formed. Each member of the family is produced by using a different interest rate for discounting to a present value. Subsequently, the initial investment can be included into each member, thereby producing the completed distribution corresponding to each different interest rate.

Further development of the relations between resource distributions and financial outcome distributions is attained through recourse to an example. The top most diagram on Figure 10 illustrates the estimate of the probable resource discoveries from drilling an exploratory well. There was a probability of 0.7 that the well would be completely unsuccessful. The triangular distribution was used to estimate the probable size of a discovery where the well is successful. It was assumed that the well

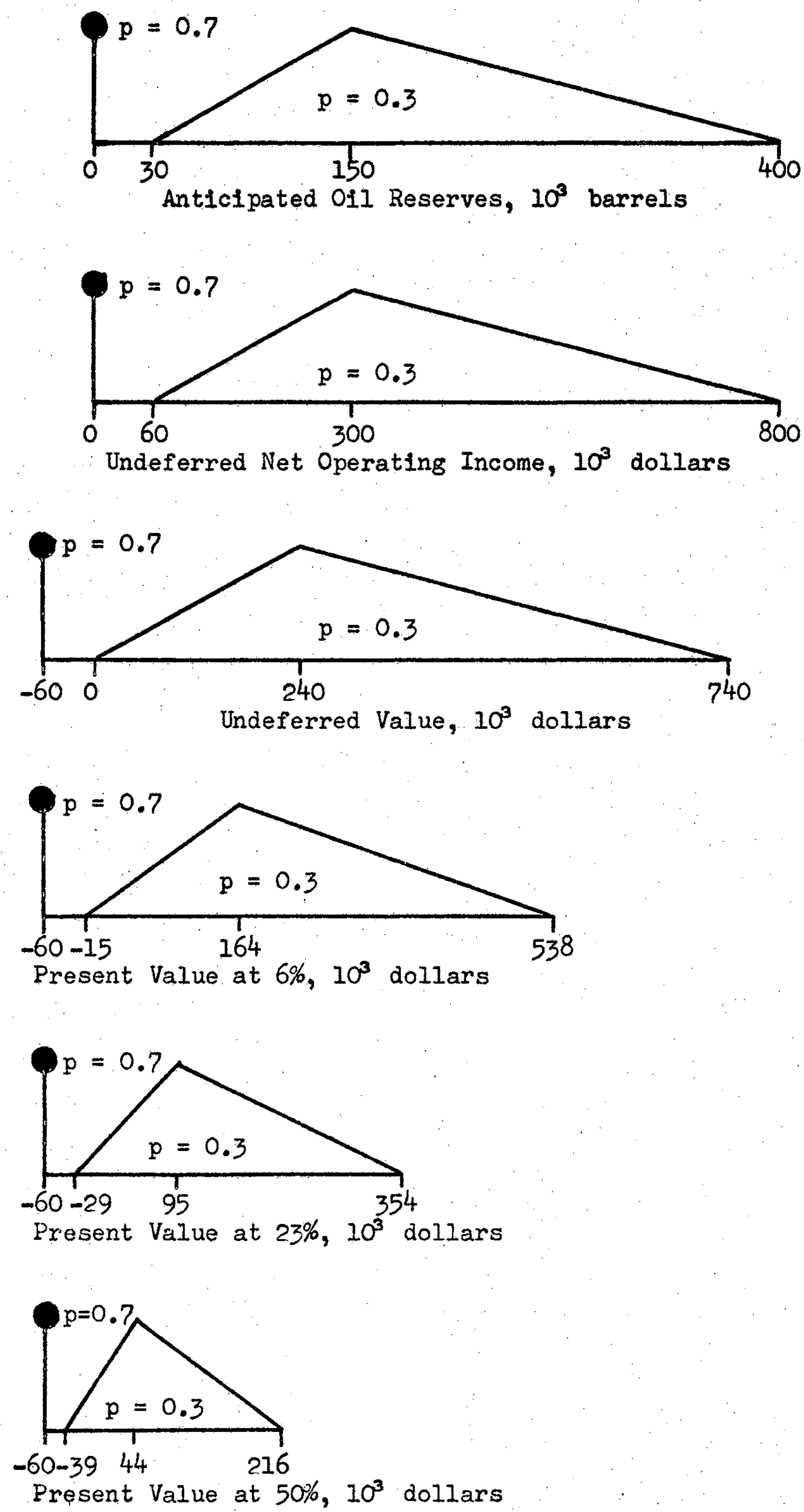

Figure 10. Expressions of Income Distribution

would not be completed where the discovered reserves were less than 30,000 barrels. The most probable discovery size was 150,000 barrels and there was a vanishing probability of discovering a maximum of 400,000 barrels. The second diagram from the top of Figure 10 shows the resource discoveries converted to dollars. A net operating income of $2.00 per barrel was assumed to be representative of the venture, and multiplying barrels by this figure gave the probability distribution of the undeferred net operating income. This represents money as it would be received some time in the future, not reduced to a present value. The third diagram from the top on Figure 10 shows the undeferred value where the cost of the completed well was $60,000. The burden of the initial investment shifts the distribution of net operating profits downward by the amount of the investment. It is worthy of note that the shape of the distribution remains constant throughout this shift of values.

A second type of modification of the probability distribution of outcomes from drilling such a well results from discounting to a present value. Such modifications result from differences in time between investment and return cash flow from the venture. Present values in this example were estimated by using the commonly assumed exponential decline relations, the initial production rate was taken as 20 times the final rate and the decline extended over 15 years. These widely known relations are given in the Appendix. The results of these calculations are shown in Table III. An interest rate of 23 per cent was found to give the present value factor which reduced the future expected earnings to a present value equal to the initial investment. Accordingly, then, 23 per cent is the expected internal rate of return.

TABLE III

PRESENT VALUE FACTORS FOR ANTICIPATED FUTURE INCOME

Interest Rate, per cent	Present Value Factor
0	1.000
8	0.748
20	0.549
23	0.518
30	0.453
50	0.345

Foundations for producing the family of various present value distributions have been set by calculating the present value factors. Returning again to inspection of Figure 10, it may be recalled that the upper three diagrams, which have already been discussed, refer to, first, the amounts of oil discovered, second, the values of the oil in terms of undeferred net operating income, and third, the undeferred values corrected to include the initial investment. The lower three diagrams represent the net present values according to different rates of discount. In all of these cases, the initial investment retains the same present value of $60,000. But the present value of the probable future income shrinks by varyingly severe amounts as higher rates are used for discounting.

For each different present value probability distribution of an individual venture, there is a corresponding sampling distribution. Some of these are illustrated in Figure 11. The present value of the group of ventures is plotted versus the number of ventures in the group. Where eight per cent is used for discounting, the expectancy is positive and the distribution appears to become favorable as the number of ventures increases. Where the expected internal rate of return of 23 per cent is used for discounting, the expectancy is zero and results appear to spread both favorably and unfavorably as the number of ventures increases. Where 50 per cent is used for discounting, the expectancy is negative and increasing the number of ventures appears to be unfavorable. The same identical prospective venture may look good or bad according to the discount rate imposed upon its anticipated earnings. An observation may be made, however, that imposing a particular discount rate does not eliminate the tendency for results to disperse both above and below the

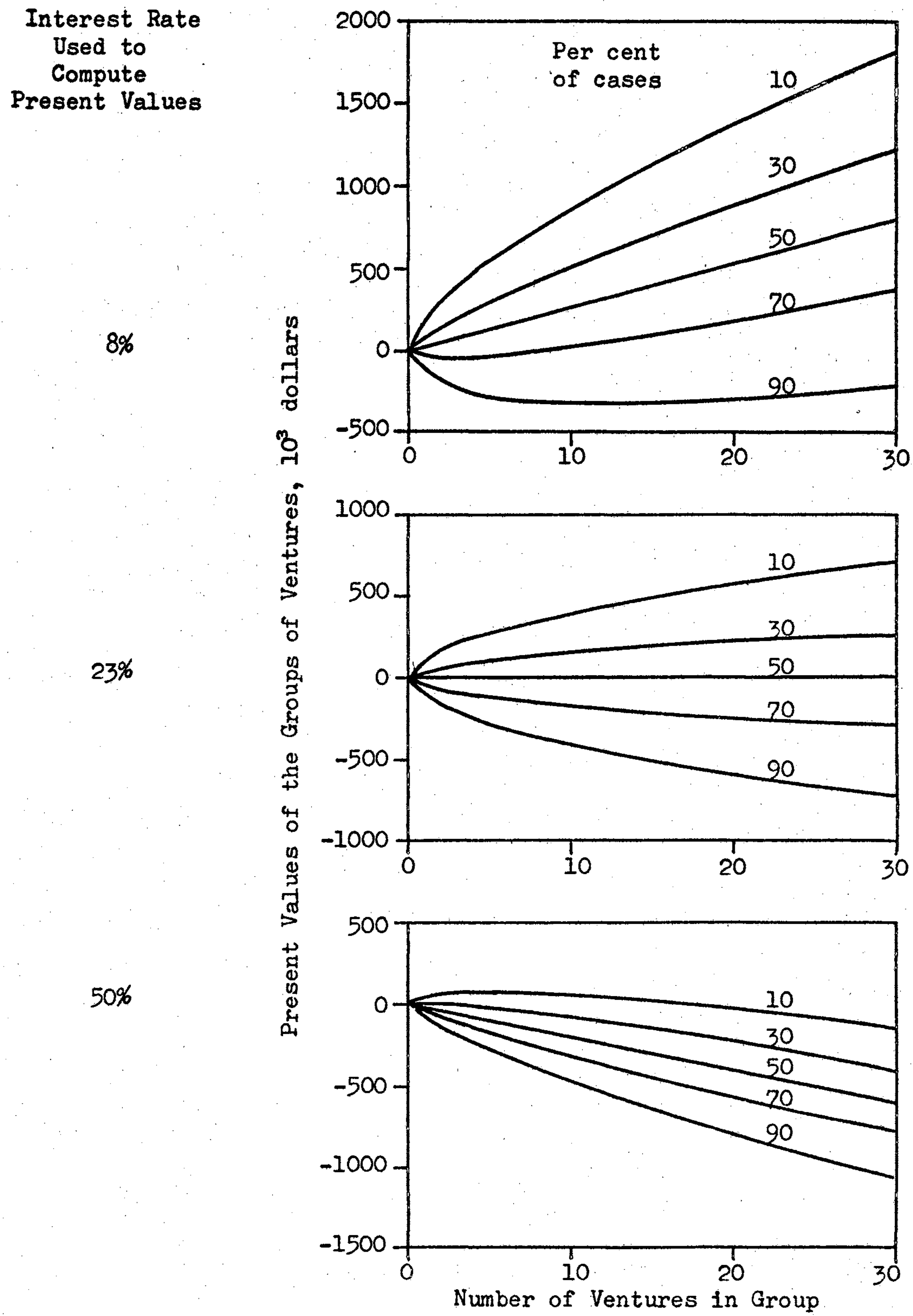

Figure 11. Present Value Distributions for Various Interest Discount Rates

expected value.

Another way of displaying the same information is shown on Figure 12. Here, the internal rate of return is plotted versus the number of ventures in the group. One-half of the cases produced a combined higher rate of return than the expected rate of 23 per cent and one-half of the cases produced less. Calculations were made by determining, for example, the value of the undeferred net operating income which would be exceeded by say 30 per cent of the cases where there were 15 ventures in the group. Then, it was necessary to determine the interest rate which would reduce this to a present value equal to the required initial investment. This graph shows the expected type of relationship wherein the spread of values for the 5 per cent to 95 per cent of total cases becomes less as the number of ventures increases.

Interpretation of the relations generated by discounting at different rates of interest are as follows. When the present value is used as a criterion, possible outcomes are centered around the expected value. When rate of return is used as a criterion, possible outcomes are centered around the expected rate of return. In either case, the expected value is only a point on the spectrum of possible outcomes. Results are equally probable both above and below the expected value. The actual magnitude of the difference between the expected values and probable results depends upon the variance of the parent or natural

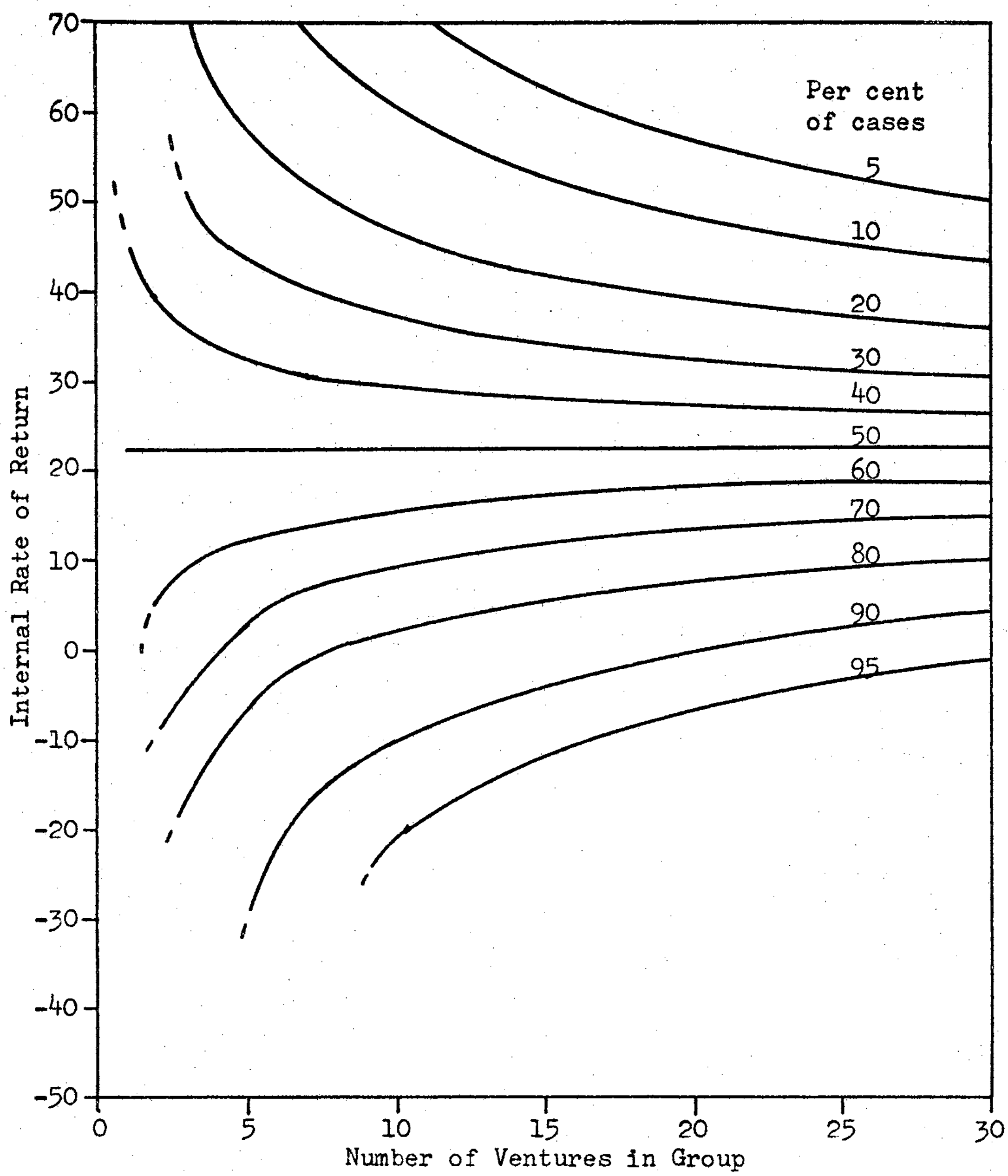

Figure 12. Internal Rate of Return for Groups of Ventures

distribution, the number of ventures grouped together and the reducing influence of present value factors.

Solution for Group Size Giving Greatest Possible Loss

Most relationships if traced far enough exhibit unfavorable areas. So it is with the relation between cumulative present value and group size. On Figure 13 it may be observed that the lower curved lines which outline the statistical results pass through a minimum as the number of ventures increases. For example, the curve labeled 90 shows the boundary where 90 per cent of the cases are more favorable than the position of the line. Conversely, there is the risk that 10 per cent of the cases will be worse off than the position of the line. Indeed, there is the knowledge that 10 per cent will be under that line. While it is desirable in the first sense to avoid the region where the 80 and 90 per cent lines pass through a minimum, there is a valuable interpretation associated with the presence of this minimum region.

The first sense in which the relations between cumulative present value and group size should be interpreted is as a group of concurrent ventures. Avoiding the minimum region can be managed by engaging in a larger number of concurrent independent ventures as is discussed in later sections. However, where events are independent, whether they be economic ventures or tossing coins,

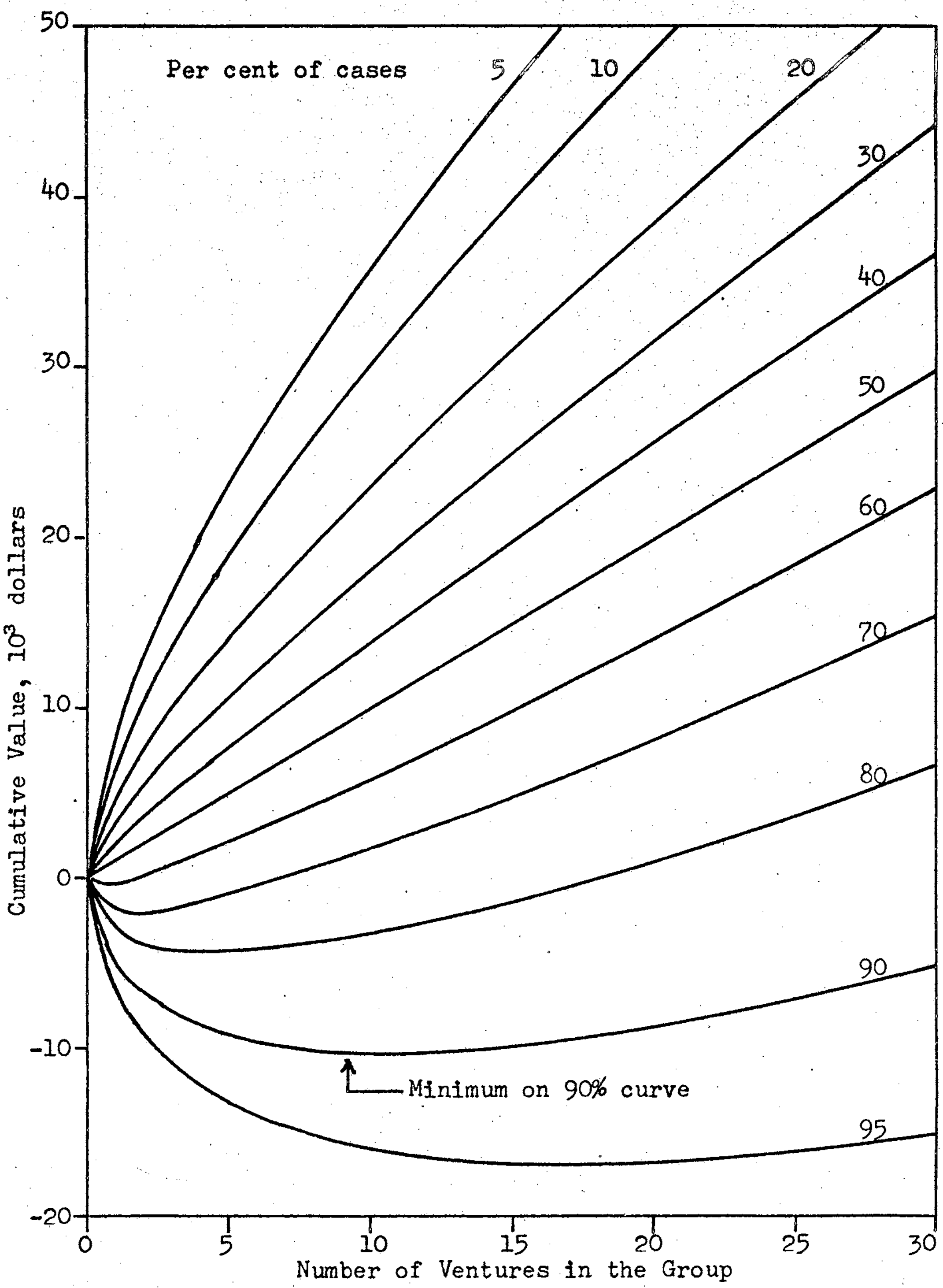

Figure 13. Probability Distribution of the Cumulative Values of a Group of Ventures

statistical interpretation of successive events is much like the interpretation of concurrent events.

The second sense for interpretating relations between cumulative present value and group size is as a group of successive ventures. An example shows how these two usages can be combined. Consider the case of a company which has maintained a number of dispersed interests and has achieved very satisfactory over-all results. But eight attempts have been made to find oil in the Sand in Major County and the results here have been disappointing. The following questions must be asked at this point: Have the probability estimates of the exploration department been proven wrong in this instance? Is it reasonable to try further? These questions might be translated into asking if the point has been passed where there would be a 90 per cent probability of an increase in fortunes, based on the original estimates of the exploration department. In this sense it becomes interesting to be able to calculate the numbers of ventures at which occur the minimum points.

The number of ventures where the minimum occurs can be calculated by setting the derivative of Equation (3) equal to zero, as follows:

$$0 = \frac{d(y - y_o)}{dn} = \frac{d}{dn}\,(n\,X_E + z\,\sigma\,\sqrt{n})$$

whence

$$\sqrt{n} = -\,\frac{z\sigma}{2\,X_E}$$

$$n_{min} = \left(\frac{z\,\sigma}{2\ X_E}\right)^2 . \tag{5}$$

Replacing n in Equation (3) with this expression and simplifying gives the following:

$$y_o - y_{min} = \frac{(z\,\sigma)^2}{4\ X_E}. \tag{6}$$

For the 90 per cent line on Figure 13, the following values apply:

$z = -1.282$

$\sigma = \$5000$, the standard deviation

$X_E = \$1000$, the expectancy.

Using these values in Equation (5) gives the following results:

$$n_{min} = \left(\frac{1.282 \times \$5000}{2 \times \$1000}\right)^2 = 10.3.$$

Using these values again in Equation (6) follows:

$$y_o - y_{min} = \frac{(1.282 \times \$5000)^2}{(4)\ (\$1000)} = \$10,280.$$

These results can be verified by inspection of Figure 13. It can be observed that the lowest point on the 90 per cent line occurs at about 10 ventures. At this point the total decrease in fortunes amounted to $10,280. There is a 10 per cent risk that the losses would be even more than this amount. Beyond the tenth venture, there is a 90 per cent probability that the total fortune will improve. This would be the case if the originally assumed probability distribution were correct.

In the relations which have been developed in this chapter, it has been tacitly assumed that the cost or initial investment was a fixed quantity. This assumption permits a complete development of the probability spectrum on a plot of cumulative present value for a group of ventures versus the number in the group. However, the cost is not always predetermined. It is necessary to direct attention toward solutions which will give the maximum amount which a particular economic unit can safely invest in a specific economic endeavor.

CHAPTER V

DEVELOPMENT OF APPRAISAL EQUATIONS

Foundations for Appraisals

The price which an economic unit can safely pay for an investment must be consistent with the objectives of the unit. Survival and profit within a chosen environment are herein assumed to be the controlling objectives which guide economic decisions. Capital employed by industrial enterprises returns products and services as well as dividends and sufficient return cash flow for development and expansion. These things are ultimately returned to the environment. In final analysis it must be the environment and interaction in the environment which provide the standards whereby an economic venture will be judged. Under present conditions the larger oil producing companies are generating profits which for different companies range from approximately 8 to 18 per cent. Maintaining operations at this level sets a standard for economic behavioral patterns.

It has been pointed out by Barnard that it is a false assumption that the profit motive controls business enterprises.[1] Most assuredly, the hope of profit is the mainspring of industrial enterprise. Profit is necessary for wages, for the necessary increases in capital, for incentives; but, it is the fear of loss which dominates the business complex. The fear of loss dominates also in non-profit organizations, such as governmental departments, hospitals, foundations, and trusts. This principle can be traced back to the concept that the measure of efficiency of an organization is its continued survival. In many situations, there will be present other factors besides survival and profit. Nevertheless, where these two are considered, investments must be limited and distributed so as to assure survival through a series of ventures, and then, secondly, the prospective investments may be further scanned so as to choose those which will maximize profit.

Standards for survival, in a field of economic endeavors wherein most individual ventures actually fail, can hardly be based upon the outcome of any single venture. Neither can survival be based solely upon the expected values, for while one-half of the outcomes are better, one-half are worse. Probabilistic theories referring to gambler's ruin warn against taking this course. Establishing the precise amount of an expected value would be difficult in most cases of exploring for resources, and in general it would not have much significance where results

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[1] C. I. Barnard, Organization and Management (Cambridge, Mass., 1962), p. 16.

spread widely in either direction. It must be noted that in an area of familiarity where the probabilistic pattern of results remains the same, it is possible to establish an expected rate of return value which will compensate for the failures. This method lacks flexibility, and values established in one area of uncertainty could not be transferred to another area of uncertainty without study of the other factors.

Statistical decision methods can be applied to help in decisions affecting this problem of survival, as will be illustrated by the following developments. These methods are widely used in many areas of physical and biological research. For example, an analysis might show that one can be 90 per cent certain that a given treatment is effective in producing a result of interest. In operations research decisions of this type guide the control of inventory levels and logistic activities. In general, a predetermined risk is set as a standard for guidance. An inventory is managed so that there is a one per cent risk of not having a particular part. A store might accept a 10 per cent risk of not being able to supply customers continuously with a certain brand item. As far as drilling exploratory oil wells is concerned, it is conceivable that a very long sequence of dry holes could be drilled. Without accepting some risk, no exploratory wells could be drilled. What cannot be avoided should be managed. The acceptance of a 5 per cent risk has been assumed for the examples in the following developments. Other levels of risk could be used in exactly the same way.

Statistical decision methods can be applied to the distributions of cumulative present values of a group of ventures as considered with respect to the number of ventures in the group. Plots of probability distributions of cumulative present value versus number in the group have been shown on Figures 3, 5, 11, and 13. On these graphs, the lowest curved line shown has usually been marked 95 per cent, which indicates that 95 per cent of the cases will be better on the average than the position of that particular line. There is, then, a 5 per cent risk that the cumulative outcome of the group of ventures will be less than the position of the line, and solutions based on the position of this line will contain a 5 per cent risk of undesirable results.

The relationships of immediate interest are illustrated on Figure 14, which shows parts of the probability distribution which are used in formulating a statistical decision. The solution for the number of ventures in the group which will insure 95 per cent confidence of success occurs where the line representing the required minimum return intersects the 95 per cent line. If there are more ventures in the group, then the chances of success will be greater; less than this number involves a risk greater than 5 per cent.

A mathematical solution which is based on normal

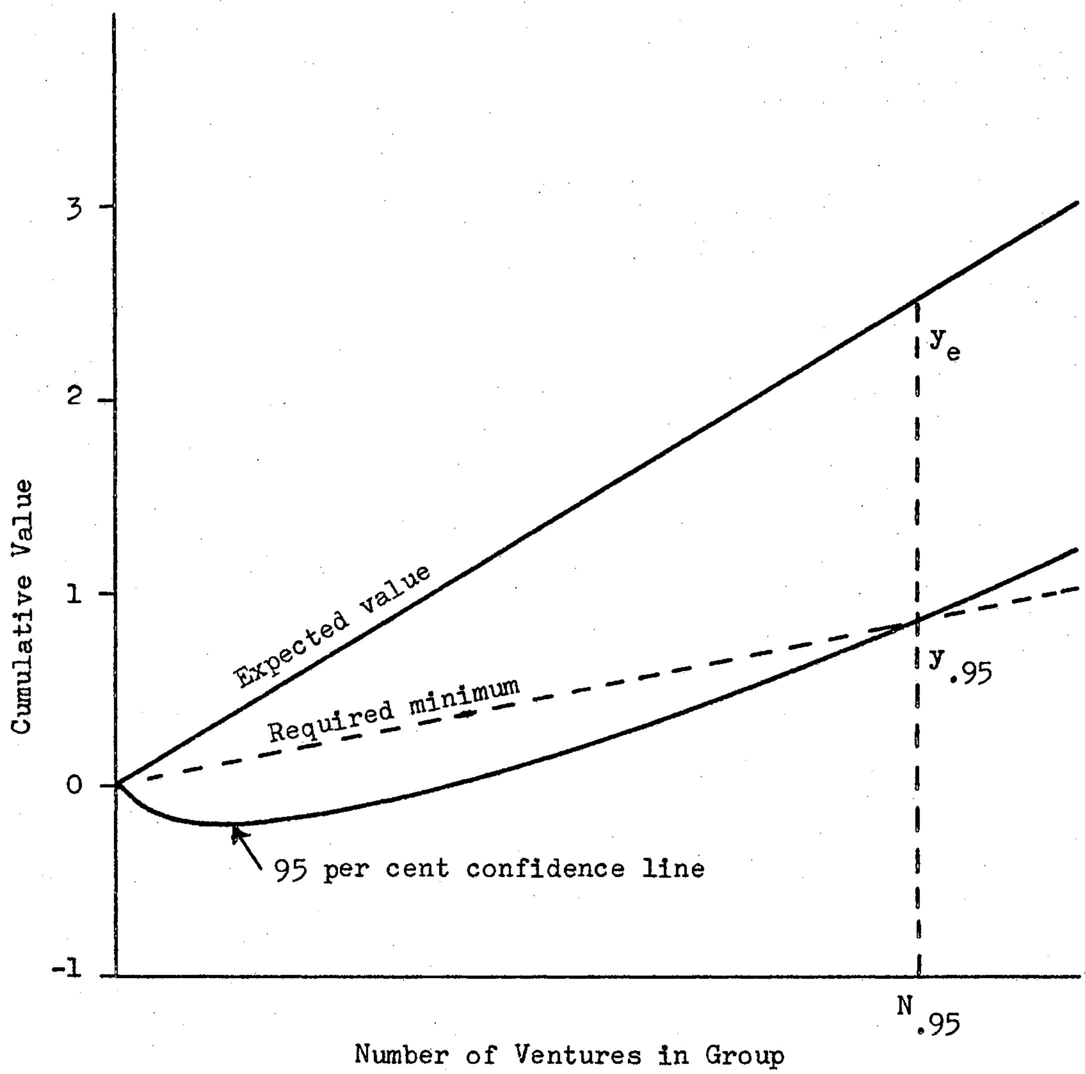

Figure 14. Relations Between the Number of Ventures, the Required Return and the Confidence Limit

relationships can be developed as follows, Equation (2) is rewritten to express the dispersion at the point of solution,

$$y_E - y_{.95} = - \sigma z_{.95} \sqrt{n_{.95}}. \quad (7)$$

The negative sign appears on the right hand side of Equation (7) because z is itself negative.

The value of the total expectancy at $n_{.95}$ is given by:

$$y_E - y_o = n_{.95} X_E. \quad (8)$$

Where the required minimum return per venture is expressed as a fraction w of the initial investment C, the cumulative required minimum value is given by:

$$y_{.95} - y_o = n_{.95} w C. \quad (9)$$

Eliminating y_o from these two relations gives,

$$y_E - y_{.95} = n_{.95}(X_E - wC). \quad (10)$$

Combining Equations (7) and (10) and solving for the number of ventures produces the following relation:

$$\sqrt{n_{.95}} = - \frac{\sigma z_{.95}}{X_E - w C}. \quad (11)$$

Where it is desirable to express the expectancy per venture as a fraction v of the initial investment, it may be indicated as:

$$X_E = v\ C. \tag{12}$$

Using this substitution gives the solution in the form:

$$\sqrt{n_{.95}} = - \frac{\sigma\ z_{.95}}{C\ (v - w)}. \tag{13}$$

More useful or more readily interpreted forms of these relations are developed in the following sections. In general, the value of n in these basic equations is replaced in terms of initial investments and financial resources. Attention is next focused on the development of appraisal equations.

Solution for Maximum Degree of Participation in a Relatively Large Venture

In the early 1950's a rather striking phenomenon occurred among the major oil producing companies in the United States. They formed alliances and banded together in groups of three, four, or five in order to conduct exploration and producing activities in the off-shore areas of the Gulf of Mexico. In other areas they continued to compete as usual, but where the large amounts of risk capital were required they found comfort with each other. This manner of operation, wherein one individual owns only a fractional share of the venture, and puts up only a fraction of the cost, has long been practiced among the small and independent oil operators, who have found the practice necessary for survival. Although the method of sharing the risk is widely practiced, the only formal solution which has been offered for setting participation limits is the indirect method of Grayson, discussed in Chapter II, which is based upon cardinal utility theory. A more direct method is developed in the following paragraphs.

It has been shown in previous sections that the only way of being sufficiently confident of success when engaging in highly uncertain ventures is to participate in a number of them. The amount of money invested in any one enterprise multiplied by the number of ventures required for assurance of success should not exceed the total risk capital available. This concept is expressed in the following equation, where $n_{.95}$ is the number of ventures, f is the fractional participation in the venture, C is the initial investment and M is the total risk capital available.

$$n_{.95} \, f \, C \leq M. \tag{14}$$

Equation (13) is used to eliminate n and gives the following relation:

$$f \leq MC \left(\frac{v - w}{\sigma \, z_{.95}} \right)^2 . \tag{15}$$

Eliminating v in favor of X_E gives:

$$f \leq \frac{M}{C} \left(\frac{X_E - w \, C}{\sigma \, z_{.95}} \right)^2 . \tag{16}$$

Calculations with this equation will be illustrated with the data used in constructing Figure 10, and will be based upon a minimum acceptable rate of return of 8 per cent. Accordingly, in the 8 per cent probability distribution, w = 0. The standard deviation must also apply to this distribution. It is assumed that there is a total of one million venture capital.

Data:

$$C = \$60{,}000$$

$$M = \$1{,}000{,}000$$

$$X_E = (0.748)(116{,}000) - 60{,}000 = \$26{,}770$$

$$\sigma = (0.748)(196{,}280) = \$146{,}820$$

$$z_{.95} = -1.645$$

$$f = \frac{1{,}000{,}000}{60{,}000}\left[\frac{26{,}770 - 0}{(146{,}820)(1.645)}\right]^2$$

= 0.205, or 20.5% interest, the maximum advisable participation.

Solution for Maximum Appraisal Value

Being able to estimate how much one can pay for a particular investment is a requirement for survival. It is a fairly common presumption that the large company can afford to pay more for a certain investment than can the small company or individual, and it will be shown why this is true. A new manner of approach must be used, however, for developing the appraisal equation.

In all previous solutions, it was assumed that the cost, the initial investment, was known. Where success is highly uncertain it is obvious that loss of the initial investment is the most common result and certainly it must be included to give a true picture of the financial outcomes. The cost was one of the elements used in calculating the standard deviation of the parent distribution. The means for circumventing this difficulty have been illustrated on Figure 10. On this graph, consider the second diagram from the top, which is the undeferred net operating income. Next, consider the third diagram from the top, where the cost has been incorporated into the previous probability distribution. Inclusion of the cost, a certainty, merely shifted the entire probability diagram without changing its shape. This geometric interpretation, which is born out by calculations, permits the following statement. The standard deviation calculated with the cost neglected will be the same as that calculated when the cost is included.

Again, Equation (2) is used for developing the appraisal relationship, and it is written as follows:

$$y_E - y_{.95} = -\sigma_A \, z_{.95} \sqrt{n_{.95}}. \qquad (17)$$

In this case, σ_A denotes the fact that the standard deviation must be obtained from the probability distribution of net operating incomes. The quantity σ_A will be taken as the standard deviation of the undeferred net operating income. The practices of the economy indicate that it would be conventional to consider present values rather than undeferred values. Referring again to Figure 10, consider how the lower three diagrams would appear if the cost were not included. There would be the specified probability of zero income, and the amounts of the positive incomes would be proportional to the present value factor. This is to say that the dispersions, and the standard deviations, of the present value probability diagrams are exactly proportional to the present value factor in each instance.[2] Therefore, where D is a present value factor corresponding to the return cash flow and based upon a specified interest rate, Equation (17) is replaced with the more general relation,

$$y_E - y_{.95} = -D\, \sigma_A\, z_{.95} \sqrt{n_{.95}}. \qquad (18)$$

The value of the expectancy at the point of solution must now be expressed in terms of the expected net operating income, as given by the following relation:

$$y_E - y_o = n_{.95}\, (D\, \overline{X}_A - C) \qquad (19)$$

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[2]In some cases the deferment factor, D, will not be a constant throughout the range of possible net operating incomes. An average deferment factor, D_{AV}, obviously can be calculated by dividing the distribution into small increments, reducing these to present value, and calculating the standard deviation of the resulting present value range, which by definition must equal $D_{AV}\, \sigma_A$. In making these calculations, the increments should be weighted according to their respective probabilities.

Here $\overline{X}_A$ designates the undeferred expectancy of the net operating income. The minimum acceptable value is again given by the expression:

$$y_{.95} - y_o = n_{.95}\ w\ C. \tag{9}$$

Eliminating y_o gives:

$$y_E - y_{.95} = n_{.95}(D\ \overline{X}_A - C) - n_{.95}\ w\ C$$

and this can be combined with Equation (18) to produce the following appraisal equation:

$$C \leq \left(\frac{D}{1 + w}\right)\left(\overline{X}_A + \frac{\sigma_A\ z_{.95}}{\sqrt{n_{.95}}}\right). \tag{20}$$

It should be recalled that z is a negative quantity here, so that if the absolute value of z is used the equation should be written as:

$$C \leq \left(\frac{D}{1 + w}\right)\left(\overline{X}_A - \frac{\sigma_A\ z_{.95}}{\sqrt{n_{.95}}}\right). \tag{20a}$$

Equations (20) and (20a) could prove useful where an appraisal value depended upon a predetermined number of ventures. The general solution, however, requires that the number of ventures be eliminated in favor of the financial resources and the cost of the venture. Equation (14) is again used for this purpose. Making this substitution and solving for the cost gives the following appraisal equation.

$$C \leq \left[B_1 + B_2\right]\left[1 - \sqrt{1 - \left(\frac{B_1}{B_1 + B_2}\right)^2}\right] \tag{21}$$

where

C = the appraisal value for 100% ownership

fC = the appraisal value for f fractional ownership

$$B_1 = \frac{D\ \overline{X}_A}{1 + w} \tag{22}$$

$$B_2 = \frac{f}{2M}\left(\frac{D\ z_{.95}\ \sigma_A}{1 + w}\right)^2 . \tag{23}$$

A solution can be formulated in terms of an expected rate of return and a lowest acceptable rate of return. Let the deferment factor D in Equations (22) and (23) be calculated using i, the lowest acceptable interest rate. Then simply set $w = 0$. Let D_R be a deferment factor calculated using r as an interest rate, where r is the minimum expected rate of return. The following indicates the desired relationship:

$$C \leq D_R\ \overline{X}_A . \tag{24}$$

For an example calculation, the data of Figure 10 is again used. However, it will be presumed that the cost has not been firmly set, and the calculation will be made for a maximum appraisal price for the estimated reserves, under the estimated probability conditions, converted to a return cash flow of net operating profits. A minimum acceptable rate of return of 8 per cent is assumed, which sets w = 0 and the present value factor D = 0.748.

$$\bar{X}_A = (0.7)(0) + 0.3\left(\frac{60,000 + 300,000 + 800,000}{3}\right)$$

$$= (0.7)(0) + (0.3)(386,667)$$

$$= \$116,000$$

$$p_1 = 0.7$$

$$p_2 = (1 - p_1)\left(\frac{X_M - X_L}{X_H - X_L}\right) = (1 - 0.7)\left(\frac{300,000 - 60,000}{800,000 - 60,000}\right)$$

$$= 0.0973$$

$$p_3 = (1 - p_1)\left(\frac{X_H - X_M}{X_H - X_L}\right) = (1 - 0.7)\left(\frac{800,000 - 300,000}{800,000 - 60,000}\right)$$

$$= 0.2027$$

$$d = X_H - X_L = 800,000 - 60,000$$

$$= 740,000$$

X_1 = 0, position of the spike

$$\sigma_A^2 = \left(\frac{p_1}{1 - p_1}\right)\left(\bar{X}_A - X_1\right)^2 + \frac{d^2}{18}\left[1 - p_1 - \frac{p_2 p_3}{1-p_1}\right]$$

$$= \left(\frac{0.7}{0.3}\right)(116,000 - 0)^2 + \frac{(740,000)^2}{18}$$

$$\left[0.3 - \frac{(0.973)(0.2027)}{0.3}\right]$$

$$= 38,524 \times 10^6$$

$$\sigma_A = 196,280$$

D = 0.748, based on 8% interest

w = 0

f = 1

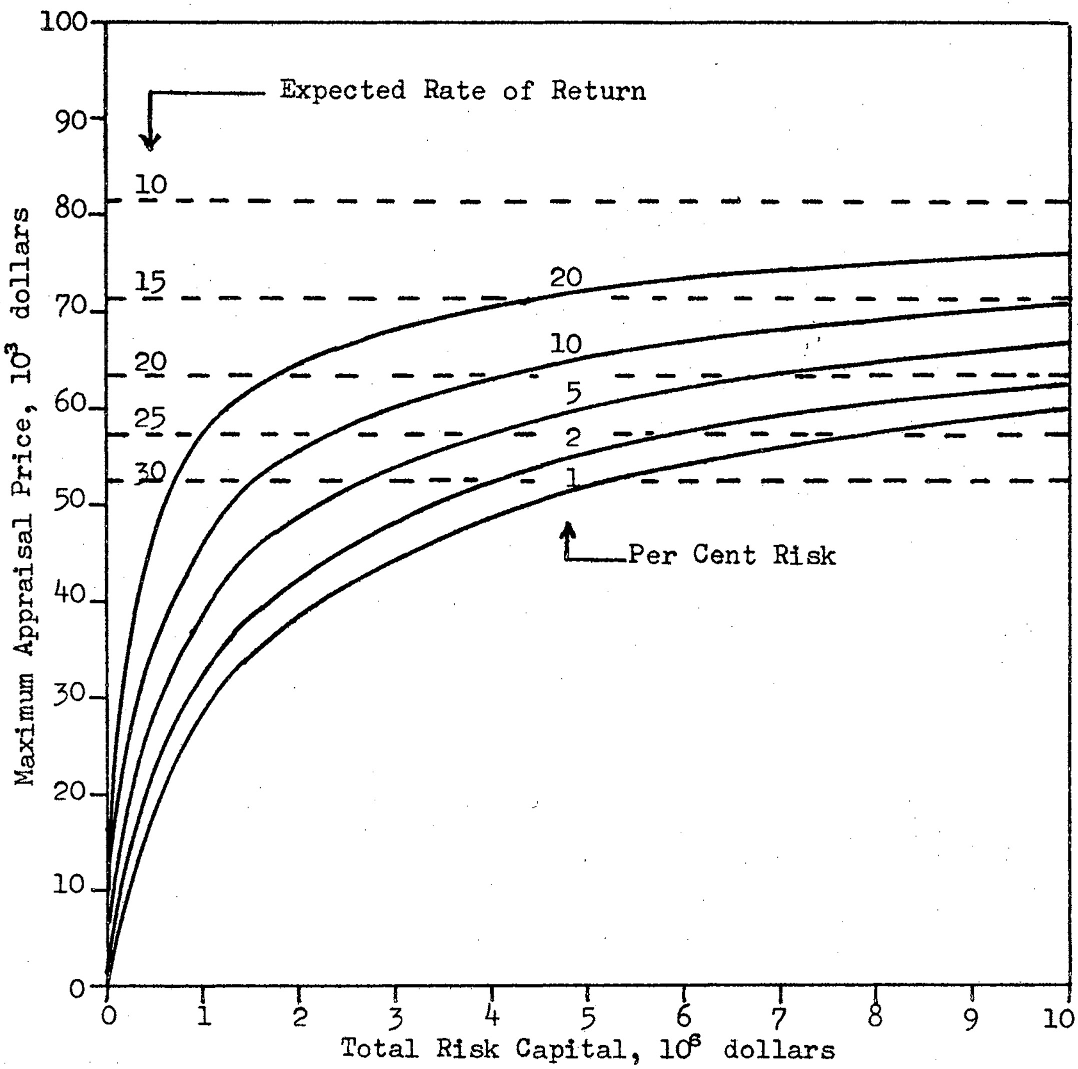

Figure 15. Appraisal Price Versus Total Risk Capital (Probability Distribution is Shown on Figure 10)

$$M = \$1{,}000{,}000$$

$$B_1 = \frac{D\ \overline{X}_A}{1 + w} = \frac{(0.748)(116{,}000)}{1 + 0}$$

$$= 86{,}770$$

$$B_2 = \frac{f}{2M}\left(\frac{D\ z_{.95}\ \sigma_A}{1 + w}\right)^2$$

$$= \frac{1}{(2)\ (1 \times 10^6)}\left[\frac{(0.748)(1.645)(196{,}280)}{1.0}\right]^2$$

$$= 29{,}165$$

$$C \leq [B_1 + B_2]\left[1 - \sqrt{1 - \left(\frac{B_1}{B_1 + B_2}\right)^2}\right]$$

$$\leq \left[86{,}770 + 29{,}165\right]\left[1 - \sqrt{1 - \left(\frac{86{,}770}{86{,}770 + 29{,}165}\right)^2}\right]$$

$$\leq \$39{,}050.$$

The appraisal price which has been calculated is for the case where the total risk capital employed is \$1,000,000. The presumption is that it would all be invested in various ventures where the risk was not greater than 5 per cent, and the expectancy would have to be attractive in order to submit to such risk. For this same identical prospect, the appraisal price is a function of the total risk capital of the individual economic unit.

An economic analysis of the drilling prospect has been developed, by using Equations (19) and (24), and the results are plotted on Figure 15. This shows the maximum appraisal price plotted versus the total risk capital of the investor. The expected rate of return is, of course, independent of financial resources and depends only on the expectation, its time pattern, and the price paid for the venture. The horizontal lines show the price which could be paid in order to anticipate expected rates of return of 10, 15, 20, 25, and 30 per cent. However, the risk is a function of financial resources. As an example, the case where there is a total of 2 million dollars risk capital available will be considered. If the investor were satisfied with an expected rate of return of 15 per cent, he might pay a total of $71,000 in the venture. This amount, however, is above the 20 per cent risk line, which means that if he persists in paying this much for ventures containing so much risk, in more than 1 time out of 5 the total 2 million would be dissipated. If he is willing to accept a 5 per cent risk, which means that in only 1 time in 20 would he end up in trouble, the investor with 2 million could pay $49,000 for the venture. But an investor with 3 million total resources could pay $54,000 on the same basis of 5 per cent risk. However, the investor with 3 million could pay only $60,000 without risking disaster in 1 time out of 10 commitments of his resources. In this case, the expected rate of return is about 23 per cent, but rather obviously the expected rate of return cannot serve as the only criterion for judging the quality of an investment.

Value of Information Which Increases Probability of Success

Additional geophysical efforts, either by more intensified usage of presently employed methods or by incorporating new methods into an exploration program, can sometimes result in reducing the region percentage of failures among wildcat wells. The maximum value of information which decreases the probability of failure can be estimated by the following equation:

$$V_I = C_{NEW} - C_{OLD} \qquad (25)$$

where V_I is the value of the information and C_{NEW} is the appraisal value based on new information and C_{OLD} is based on the original information.

The same data which was used in the previous section will again be used for a calculated example. The added condition arises because of the claim that certain additional geophysical work can reduce the number of dry wells by 20 per cent. The new probabilities are calculated as follows, on the basis of 10 original wells.

	Original		New Conditions	
	Wells	Probability	Wells	Probability
Dry	7	0.7	5.6	0.651
Producers	3	0.3	3.	0.349
	10	1.0	8.6	1.000

It is taken that the probable reserves discovered per well remains the same. On this basis, the new probabilities are:

$$p_1 = 0.651$$

$$p_2 = (1 - p_1)\left(\frac{X_M - X_L}{X_H - X_L}\right) = (1 - 0.651)\left(\frac{300,000 - 60,000}{800,000 - 60,000}\right)$$

$$= 0.1132$$

$$p_3 = (1 - p_1)\left(\frac{X_H - X_M}{X_H - X_L}\right) = (1 - 0.651)\left(\frac{800,000 - 300,000}{800,000 - 60,000}\right)$$

$$= 0.2358$$

$$\overline{X}_A = (0.651)(0) + (0.349)\left(\frac{60,000 + 300,000 + 800,000}{3}\right)$$

$$= 134,950.$$

By calculations identical with those in the previous section, one obtains $\sigma_A = 205,570$. Again using a total risk capital of $1,000,000, the following is obtained:

$$B_1 = \frac{D\ X_A}{1 + w} = \frac{(0.748)(134,950)}{1.0}$$

$$= 100,943$$

$$B_2 = \frac{f}{2M}\left(\frac{D\ z_{.95}\ \sigma_A}{1 + w}\right)^2$$

$$= \frac{1}{2 \times 10^6}\left[\frac{(0.748)(1.645)(205,570)}{1.0}\right]^2$$

$$= 31,991.$$

Using these values in Equation (21) gives:

$$C \leq 51,240.$$

For the case where the total risk capital is $1,000,000, the value of the information, with respect to one well as described, is

$$V_I = 51,240 - 39,050$$
$$= 12,190.$$

Where the risk capital is very large, the value of the information can be calculated from the differences in the expectancies reduced by the acceptable expected rate of return, which in this case is taken as 20 per cent. The following calculation gives the results:

$$V_I = D_R(\overline{X}_{A_{NEW}} - \overline{X}_{A_{OLD}}) \tag{26}$$

$$= 0.549\ (134,950 - 116,000)$$

$$= 10,400.$$

The calculated values happen to be about the same amount in these two cases. On a percentage basis, the information was worth about 16% to the very large company and about 31% to the smaller company.

CHAPTER VI

A PETROLEUM EXPLORATION DECISION MODEL

The Economic Model

Appraisals and economic evaluations of drilling ventures have been treated in previous chapters as though they were completely unattached to a continuously functioning organization. For many cases those methods are adequate, and the circumstances requiring decisions will come when they will come. But other cases can be served best by extending and modifying the methods. In a large number of cases a definite economic commitment must be made months and perhaps years before the discovery of oil by the drill. It is towards an understanding of such problems that the present chapter is directed.

The search for a petroleum deposit starts within a staff function, the research either of an individual or of the geologic department of a large company. Preferably a study is made of the regional geology which should point out the more favorable areas. This might increase the chances of success from one in a hundred to say one in ten. Before detailed work is done within one of the indicated favorable areas, it should be ascertained that leases to drill and produce oil and gas can be obtained in that particular area. In the larger companies the obtaining of the leases from the land owners is done by the land department. There is also legal work required. All of these preliminary functions result in overhead costs which must be born by the individual or by the company. Thus, the successful discovery must pay for such overhead as well as for the actual costs associated with the particular oil deposit.

Oil and gas leases specify that a certain amount, usually one-eighth, of the gross proceeds from the sales of the minerals will be given to the land owner. The operator has the working interest which must bear the costs of drilling and production operations. Most working interests are seven-eighths of the gross production, but one individual may transfer a lease to another while retaining an economic interest in the future production. The immediate focus here is on the obtaining of the leases from the landowners. A payment which is called a bonus is usually made for the lease. For essentially unproven, but attractive, land the bonus might be $40 per acre up to $2000 or more per acre when the property is proven to be very profitable by drilling on surrounding tracts of land. Another device which is used is to buy an option on a lease. For example, a company may feel justified in conducting a seismic exploration program in a certain area. The area of potential promise, for example, may have been narrowed down to 8000 acres, about 12.5 square miles. The company may then obtain an option for $10 per acre before proceeding with the seismic survey. The chances of failure are about 75 per cent; that is, there is about a 75 per cent chance that no oil or gas will be found, in the example case. If the seismic exploration proves to be favorable, then the desired acreage can be leased and a wildcast well drilled to find out if oil is present. There is still the possibility that within those cases that are actually drilled, the size of the discovery will be disappointingly small. Such contingency can be handled by assigning a suitable probability distribution to the expected sizes of oil discoveries. The model as portrayed in Figure 16 was discussed by Louis F. Davis before a meeting of the Society of Petroleum Engineers.[1]

Resource Expectations

The most trusted basis for petroleum exploration ventures is experience gained in previous efforts. The potentiality of any new prospective area is judged by the history of older areas having similar geologic conditions. The original decisions to explore off-shore Louisiana were based on the fact that the sediments there are a continuation of the sediments which start in Arkansas and extend

[1]Louis F. Davis, Distinguished Lecturer of the Society of Petroleum Engineers, Lecture given at May 1967 meeting, Oklahoma City, Okla.

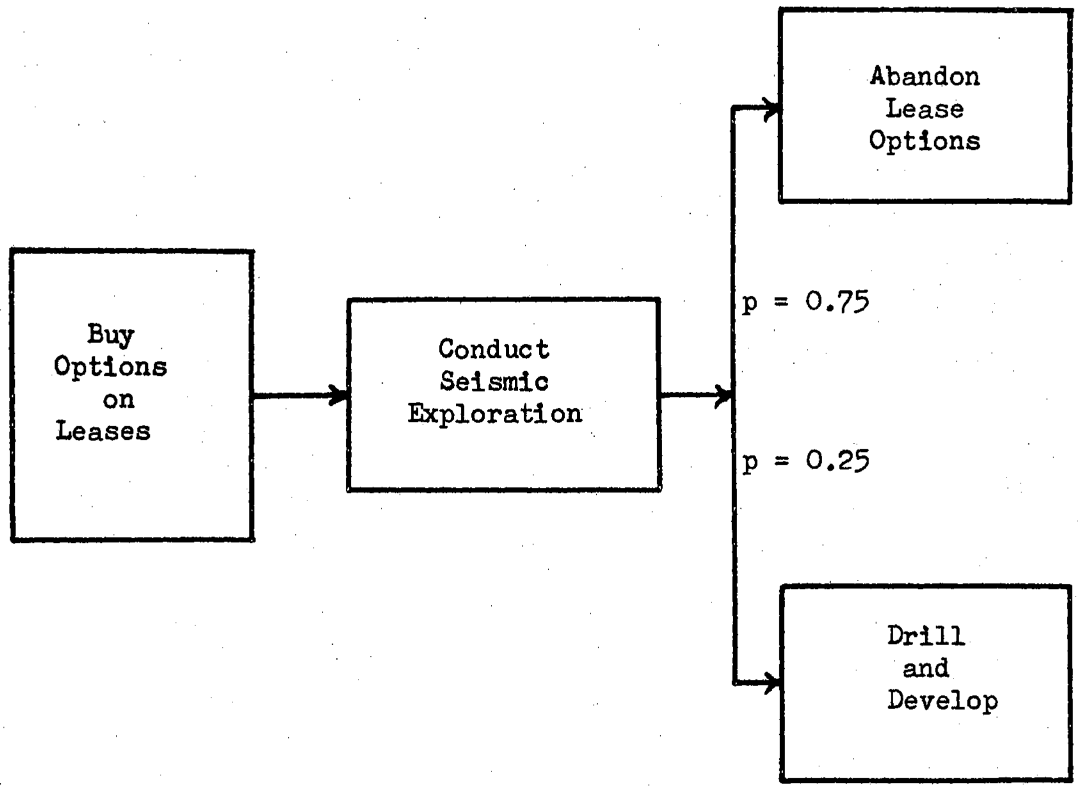

Figure 16. Economic Model for Oil Exploration in the Mid-Continent Region of the United States

southward to the present coastline. Thus, the manner of occurrence and the sizes of the fields to be discovered off-shore could be expected to follow the patterns which had been learned from explorations which had been conducted on the land adjacent to the shore. According to Owen, the following statistics were available for making estimates about the exploration program:[2]

a. The proportion of productive acreage in the over-all area.

b. The number of oil fields and their ultimate production.

c. The ratio between the number of major oil fields and the number of tested piercement domes, deep-seated domes, and other structures, respectively.

d. The average gross reserves for each structural type.

e. The average number of tests required to establish production on each type of structure.

f. The success ratio experienced for the various categories of seismic prospects.

g. The role of additional geological and engineering data which would be disclosed

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[2]Edgar W. Owen, "Petroleum Exploration - Gambling Game or Business Venture?", in Economics of Petroleum Exploration, Development, and Property Evaluation (Englewood Cliffs, New Jersey, 1961).

as drilling progressed.

The degree of the ultimate success of exploratory efforts could be forecast with sufficient accuracy. However, the ultimate success of any one economic unit depended upon its having sufficient capital to bear the expenses of off-shore exploration in a sufficient number of ventures to insure success.

In regions which are already producing, knowledge gained in the explored parts of the region can be extended to the unexplored parts. The characteristics of a particular formation as observed in known areas permit an extrapolation into the unknown areas, such, for example, as setting upper, lower, and most probable values.

The use of the lognormal distribution for describing the size distribution of oil fields was reported by Arps[3] and it was later used by Kaufman[4] in conjunction with expected utility decision theory. The random process underlying the normal distribution is essentially additive in nature, that underlying the lognormal is essentially multiplicative in nature. The logarithms of the sizes of the individual elements are normally distributed. Some of the properties of the lognormal distribution are given by

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[3]J. J. Arps, "The Profitability of Exploratory Ventures," Economics of Petroleum Exploration Development, and Property Evaluation (Englewood Cliffs, N. J., 1961).

[4]Gordon M. Kaufman, Statistical Decision and Related Techniques in Oil and Gas Exploration (Englewood Cliffs, 1963).

Parzen.[5] A more thorough description is given by Aitchison and Brown.[6]

Some results of plotting the size distributions of the ultimate productions of the oil fields in Oklahoma, West Texas, North Louisiana, South Louisiana and off-shore Louisiana are shown in Figures 17 to 21. The data include only those fields which have an ultimate production of at least one million barrels of oil.[7] These are cumulative plots. For example, on Figure 17 it may be observed that 50 per cent of the Oklahoma fields have an ultimate production of less than 35 million barrels and 90 per cent of the Oklahoma fields have an ultimate production of less than 200 million barrels. If the actual size distribution were exactly lognormal, the data would fall in a straight line on the graph.

Deviations from any theoretical distribution are to be expected. The data concern geologic formations and the extractive processes applied to fluid minerals trapped within favorable locations within such formations. The data must be estimated and they inevitably contain errors. Not all of the data are present, for more deposits remain

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[5]Emanuel Parzen, Modern Probability Theory and Its Applications (New York, 1960).

[6]J. Aitchison and J. A. C. Brown, The Lognormal Distribution (Cambridge, 1957).

[7]"Oil and Gas Journal Forecast and Review," Oil and Gas Journal, Vol. 65, No. 5 (Tulsa, Okla., Jan. 30, 1967).

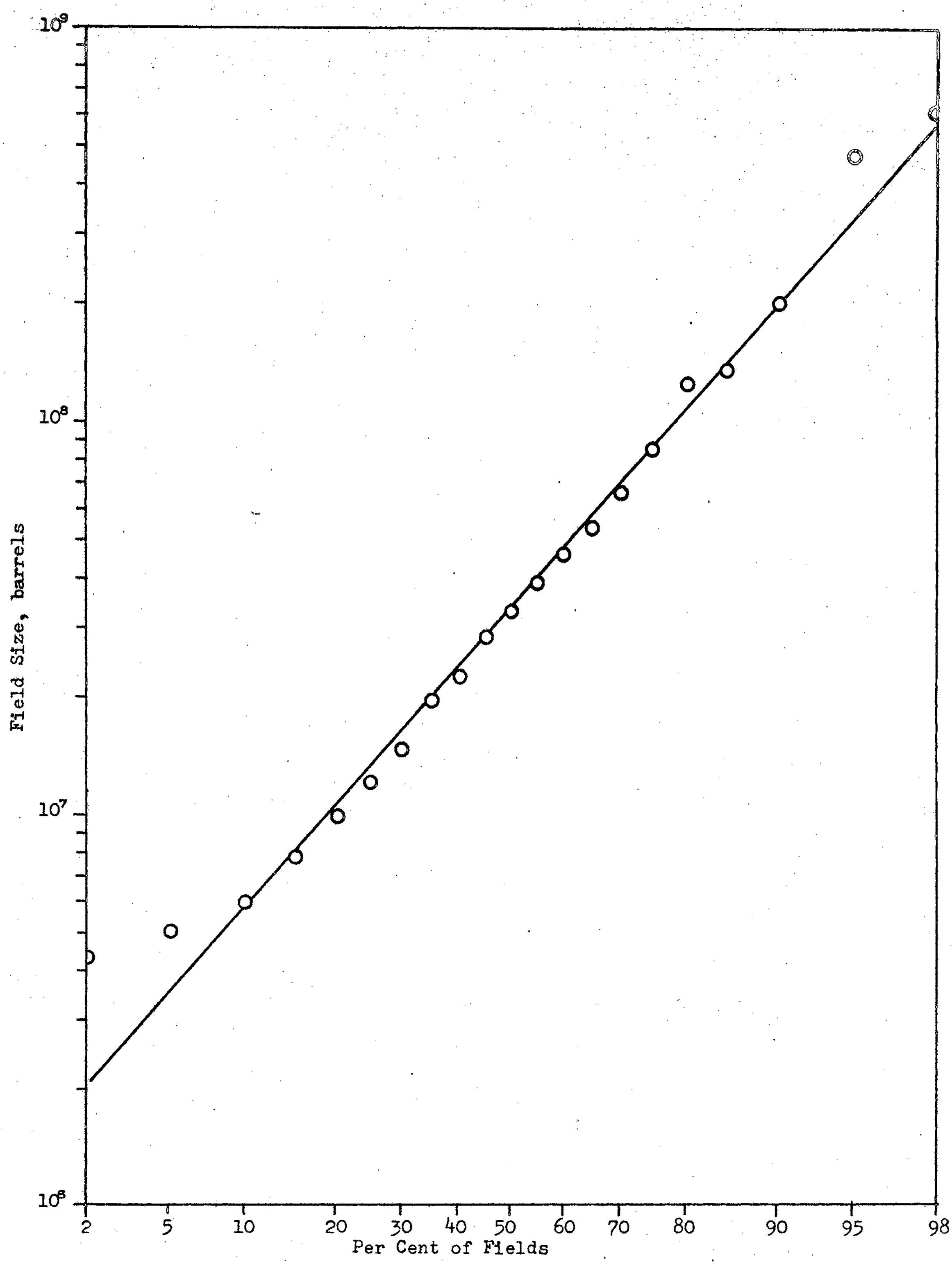

Figure 17. Total Ultimate Proven Reserves in Oklahoma Fields of 10^6 bbl. or More as of January 1967

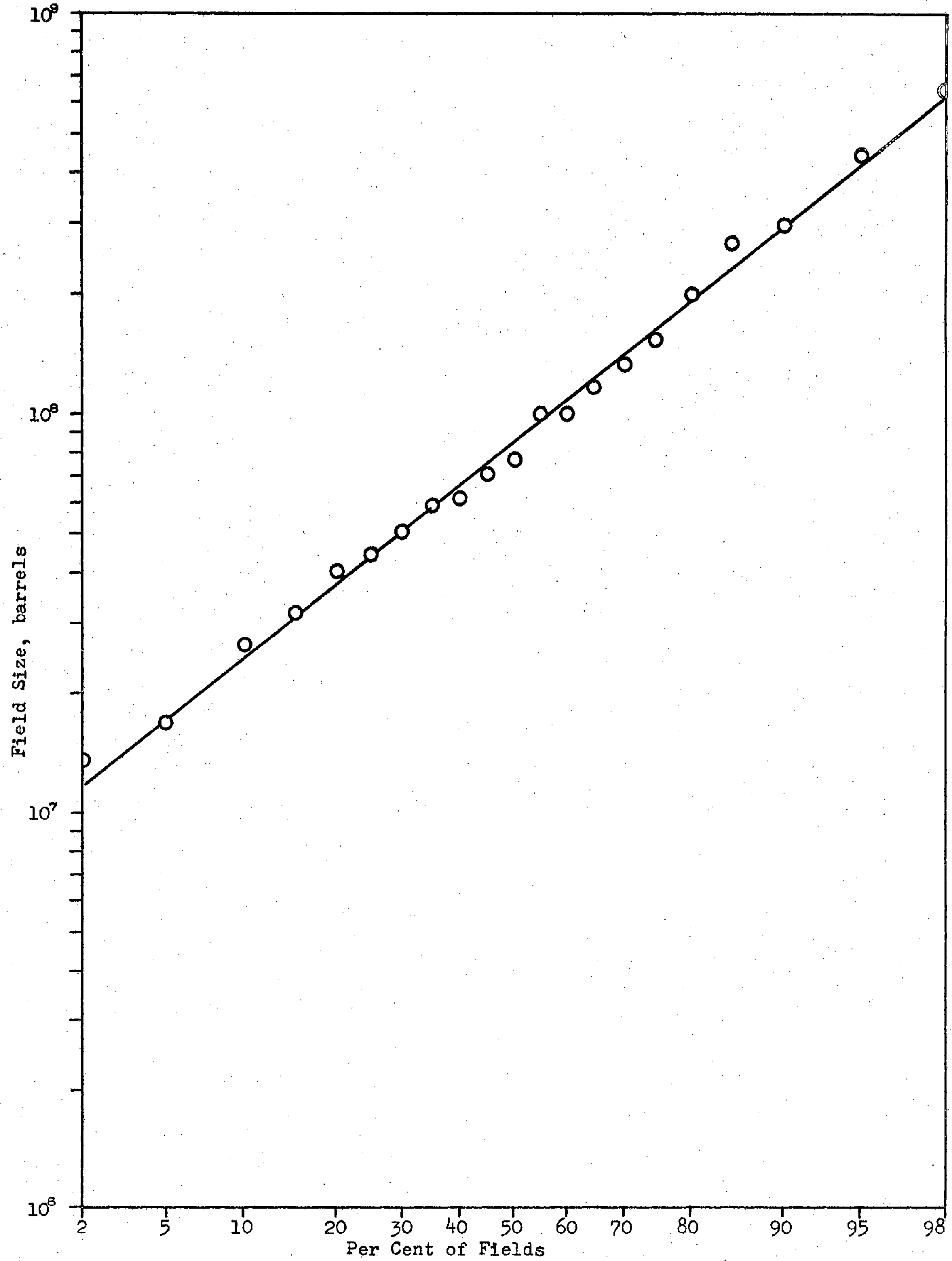

Figure 18. Total Ultimate Proven Reserves in West Texas Fields of 10^6 bbl. or More as of January 1967

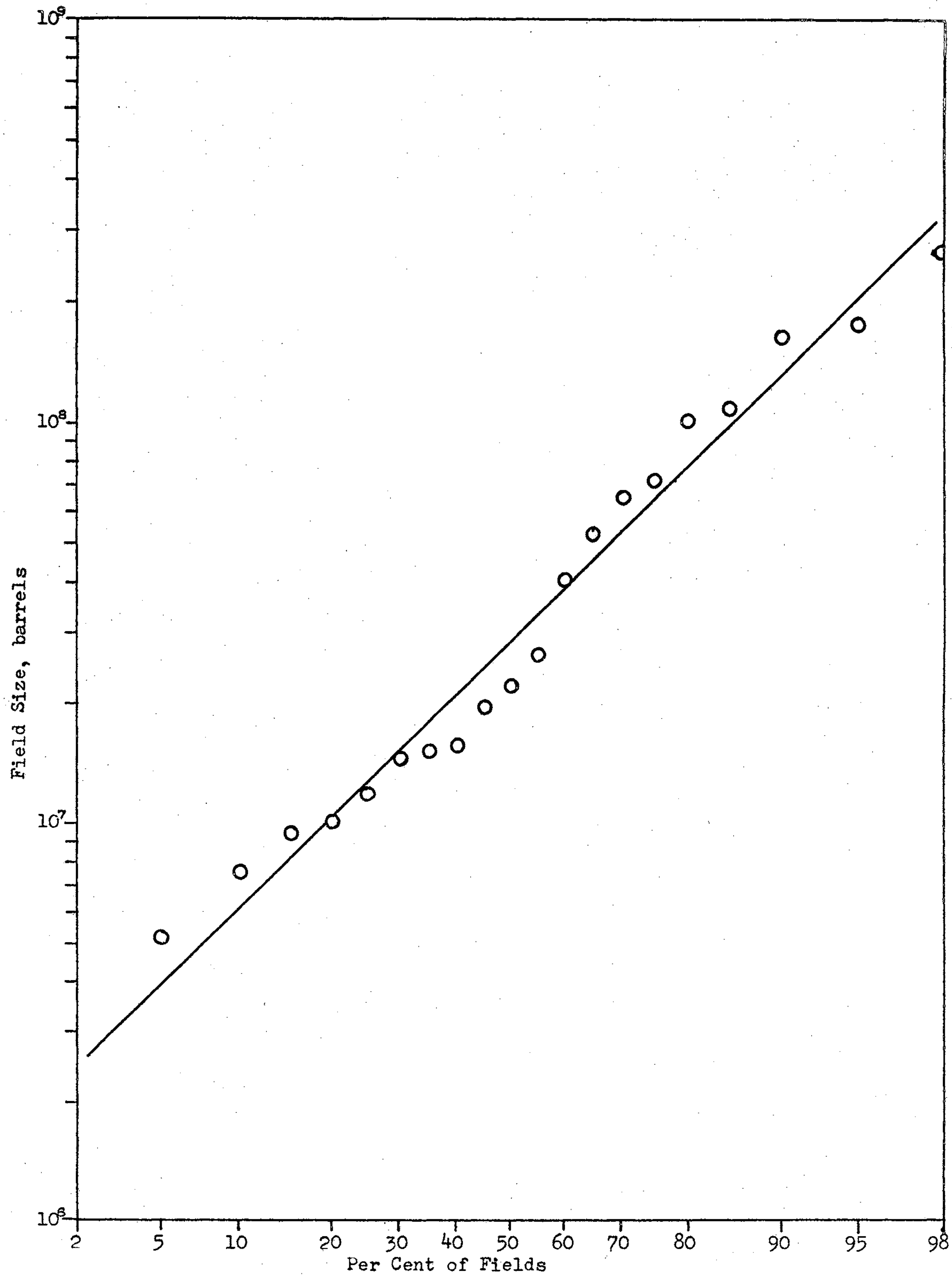

Figure 19. Total Ultimate Proven Reserves in North Louisiana Fields of 10^6 bbl. or More as of January 1967

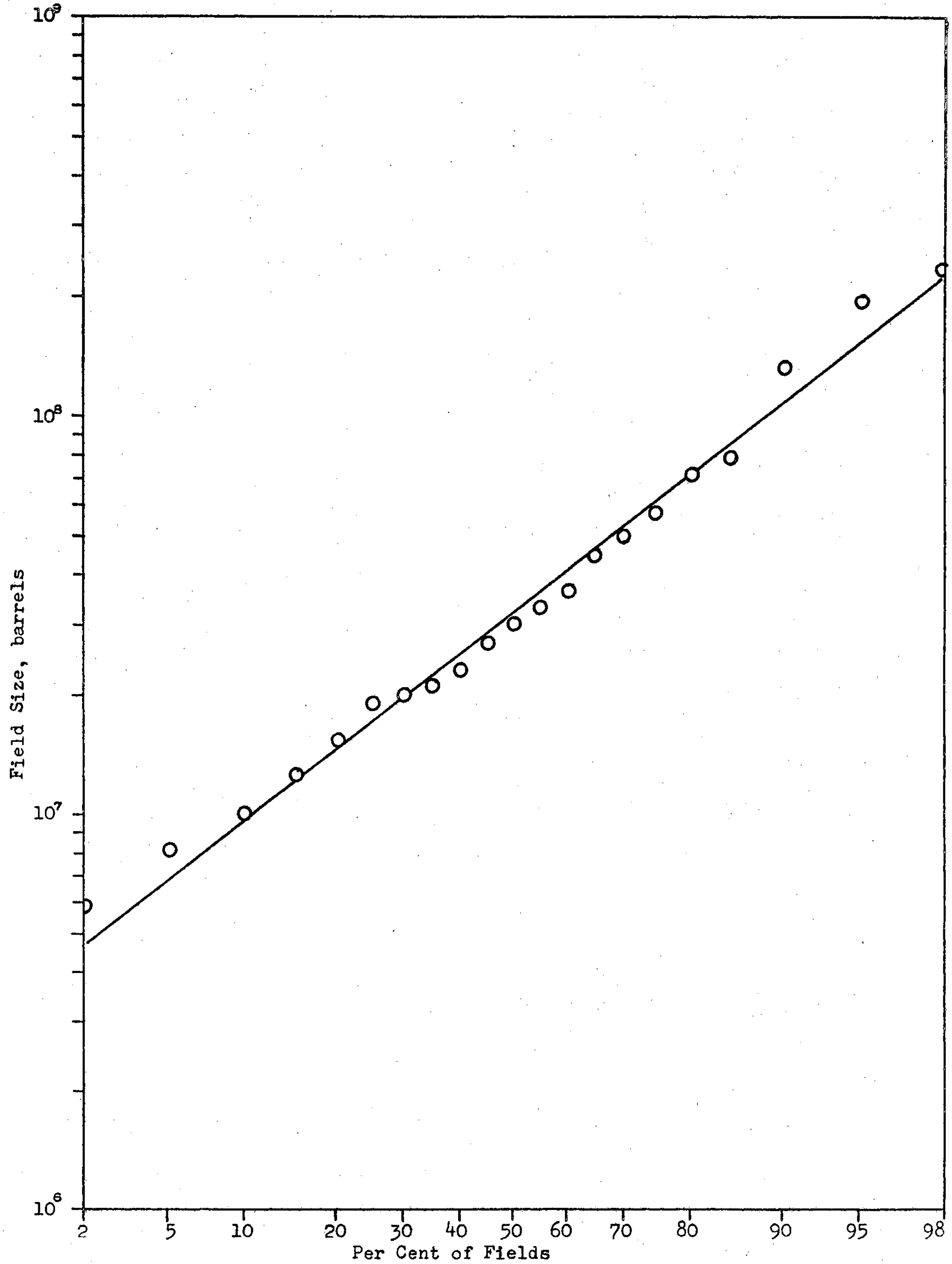

Figure 20. Total Ultimate Proven Reserves in South Louisiana Fields of 10^6 bbl. or More as of January 1967

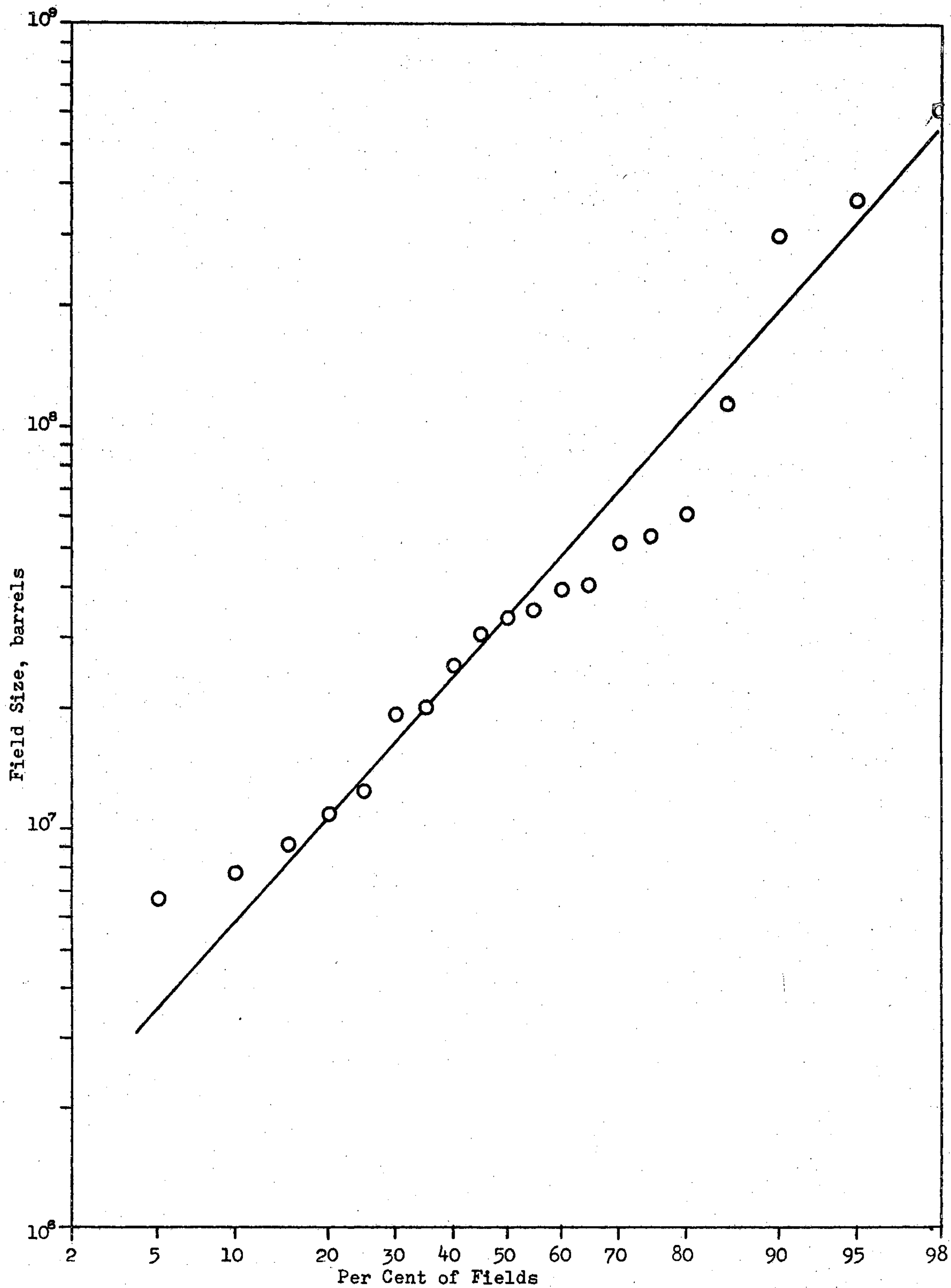

Figure 21. Total Ultimate Proven Reserves in Louisiana Off-Shore Fields of 10^6 bbl. or More as of January 1967

to be discovered. And improved recovery processes will alter the data within the present analysis. Also, the data will be refined by time as additional information is obtained about the various deposits.

Some of the data pertaining to these plots of field sizes in the several regions are shown in Table IV. The standard deviation σ of the lognormal distribution was obtained by dividing the difference between the natural logarithms of the field sizes at the 95 and 5 per cent points by 3.290, which is the number of standard deviations between these percentage points. The mean m was calculated as the natural log of the average field size less one-half of the square of the standard deviation. In order to enhance an appreciation of the physical significance of the distributions, the associated ratio between sizes of fields was investigated. The quantity which was calculated was the average ratio between sizes of fields within adjacent 10 percentile groups, averaged between the 5 and 95 per cent points. For example, if one considers the average size of the Oklahoma fields within the interval between the 10 and 20 per cent points of the number of fields, the average size of the fields within the 20 to 30 per cent group will be about 1.6 times the average size of field within the former group.

The degree of success which was obtained in simulating the lognormal distribution is shown in Figure 22. The mean and standard deviation which were obtained from the

TABLE IV

DATA ON SIZE DISTRIBUTIONS OF OIL FIELDS

Producing Area	Number of Fields	Avg. Size, 10^6 bbl.	Avg. 10% size ratio	Lognormal parameters m	Lognormal parameters σ
Oklahoma	92	90.070	1.655	10.45399	1.38158
West Texas	95	138.556	1.427	11.36360	0.97516
North Louisiana	39	59.946	1.554	10.28429	1.19744
South Louisiana	167	52.330	1.414	10.41713	0.94680
Off-Shore La.	49	85.422	1.655	10.39900	1.38158

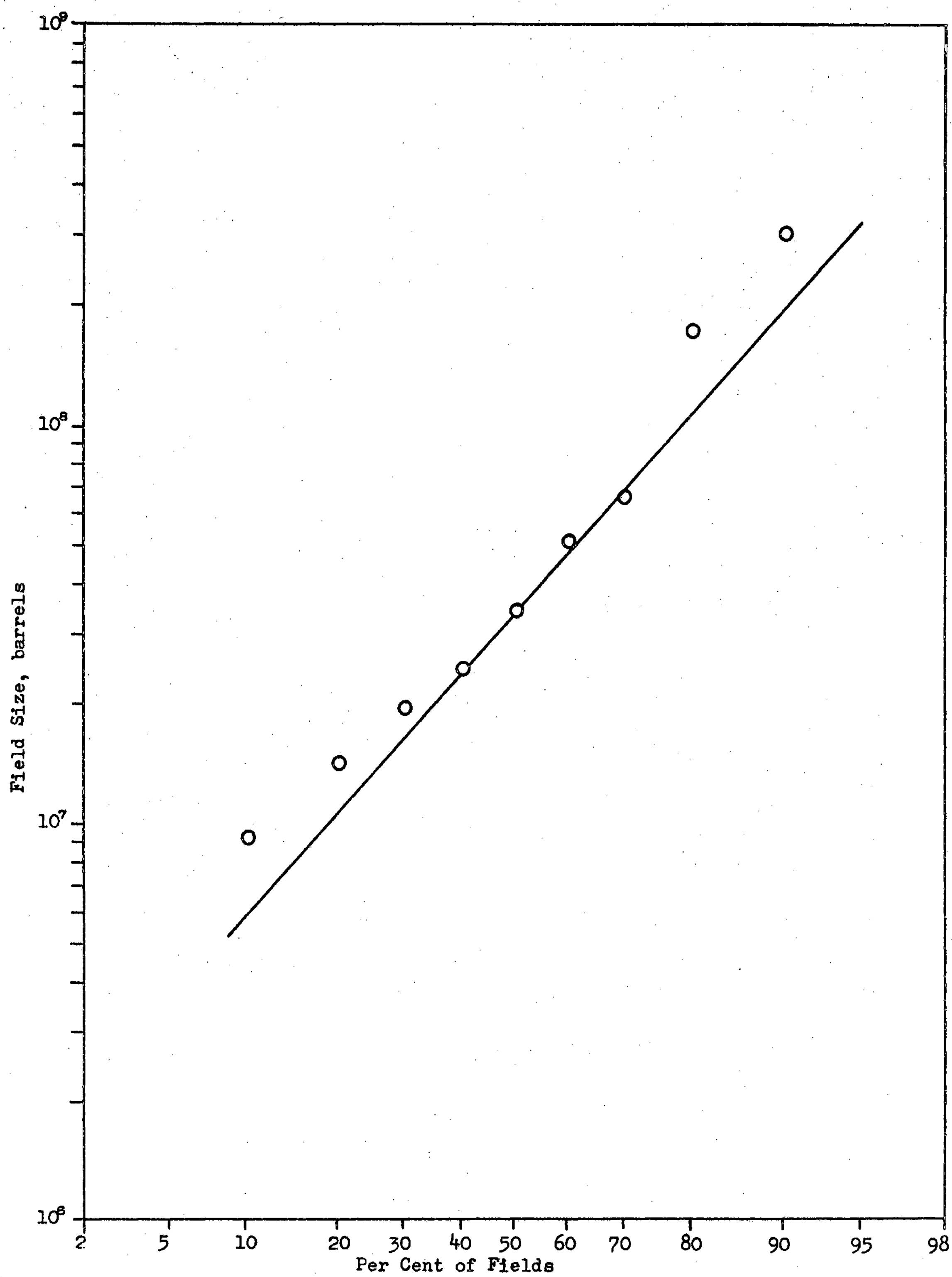

Figure 22. Results of Monte Carlo Simulation of Oil Field Sizes Using Parameters From Oklahoma Fields

Oklahoma oil fields were entered as parameters in the Monte Carlo simulation. Thus, the ultimate results of the simulation process should be a reproduction of data which would lie exactly on the line drawn through the Oklahoma data as shown on Figure 17. This would require the simulation of a large number of individual fields. In the trial run, the number of fields reproduced was limited to a total of 50 in order to obtain some measure of the efficiency of the process. The simulated data deviates by only 0.2 per cent in the value of the mean m, and by 8.2 per cent in the value of σ, the standard deviation.

The use of state-wide or province-wide averages for exploration parameters has been criticized on occasions too numerous to document. It may be seen on Figure 17 that about 75 per cent of the fields in Oklahoma are smaller than the average size of about 90 million barrels ultimate production. The lognormal distribution is highly skewed, and its apparent simplicity as plotted on such graphs as Figure 17 is misleading. The actual nature of the data, i.e., the straight line on Figure 17, is plotted on Figure 23, which shows the true character of the probability distribution. On this latter figure, the most probable field size, the mode, may be seen to be about five million barrels of ultimate production.

The mean and the mode can be calculated from the relations of the lognormal distribution. From Figure 17, it may be seen that the field sizes at the 5 and 95 per cent points are 3.50×10^6 barrels and 3.30×10^8 barrels, respectively. The corresponding standard deviations on the normal distribution are -1.645 and 1.645. The standard deviation of the lognormal is

$$\sigma = \frac{\ln (3.30 \times 10^8) - \ln (3.50 \times 10^6)}{(2)(1.645)}$$

$$= 1.38.$$

The midpoint on the lognormal, expressed as a natural logarithm, is

$$m = \ln (3.40 \times 10^7)$$

$$= 17.362.$$

The mean of the actual distributions, as plotted on Figure 23, can be calculated as follows:

$$X_{mean} = e^{m+\frac{1}{2}\sigma^2}$$

$$= \exp(17.362 + (\tfrac{1}{2})(1.382)^2)$$

$$= 90.0 \times 10^6 \text{ bbl.}$$

This figure can also be obtained as the average of the field sizes which were originally plotted on Figure 17.

The mode of the actual distribution, as plotted on Figure 23, can be calculated as follows:

$$X_{mode} = e^{m - \sigma^2}$$

$$= \exp (17.362 - (1.382)^2)$$

$$= 5.05 \times 10^6 \text{ bbl.}$$

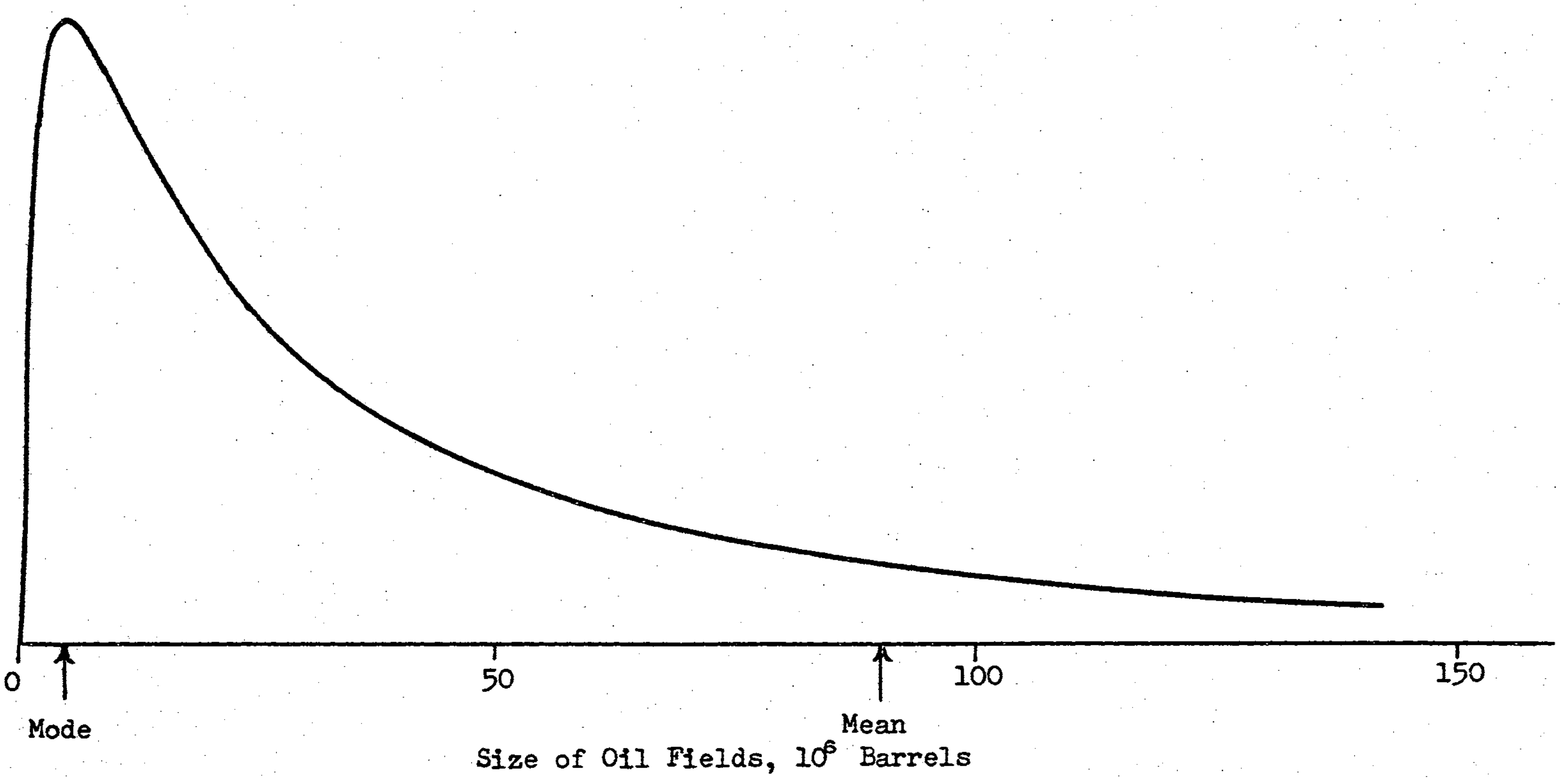

Figure 23. Probability Distribution of Sizes of Oklahoma Oil Fields

In the case of an actual exploration venture, the question arises concerning what size of discovery might be expected. By definition, the most likely size of field is the mode. The median is at the 50 per cent point. For the Oklahoma fields, these data apply.

Mode	5.05×10^6 bbl.
Median	$34. \times 10^6$ bbl.
Mean	$90. \times 10^6$ bbl.

Additional considerations involve the acreage which must be leased and the associated costs. In addition to the cases which are complete failures, it must be recognized that in most successful cases acreage will be leased that eventually does not prove to be productive. And it is inevitable, unless a concession on a very large territory were involved, that in many cases part of a field would extend over onto unleased land. This contingency can be minimized by leasing more acreage than could reasonably be expected to be productive, which would involve additional expense. In any event, the size of the discovery, insofar as the economic unit is concerned, is limited by that portion of the total discovery which is under land leased by the economic unit.

In simulating or otherwise analyzing the probability distribution of expected outcomes of a discovery venture, it would appear to be important to have the mode of the assumed probability distribution coincide with the mode of the actual field sizes. But, in most cases at least, the probability distribution of possible field sizes must be truncated in accordance with the land holdings of the economic unit. For purposes of analysis, distributions other than the lognormal, such as the discrete, the histogram, the triangular, the exponential, and combinations of these should give more realistic results. This is to say that the lognormal is convenient for locating such points as the mean and the mode, but is less convenient for further analysis.

Development of Economic Equations

The exploration process revolves around three major decisions. First, there must be the decision to devote time and energy to the process. This results in the creation of a staff with the attendant overhead charges. The costs associated with this decision tend to amount to about one-third of the total exploration charges.[8] Second, after the staff has assembled data and made recommendations, there must be the decision to obtain leases or options in a certain area. In the example developed here, it is assumed that, if the leases were obtained, seismic geophysical work would then follow. Third, analysis of the geophysical work results in the decision to drill or the decision not to drill. If the exploratory well is drilled,

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[8] Robert Birch, Exploration Department, Mobile Oil Co., Oklahoma City, Okla. Personal communication.

the size of the field, whether large or disappointingly small, will begin to become evident through information obtained from the exploratory well and from subsequent development wells.

For analysis, an economic unit has been assumed with a staff of six geologists, engineers, and landmen. It has further been assumed that they will come up with two deals per year where options will be purchased and seismic work will be undertaken. The basis of analysis is one such deal. In the following calculation, the expenses per man have been presumed sufficient to cover the attendant necessary travel and clerical help. At the time of purchasing options on 8000 acres at the assumed price, the following expenses would be committed:

Option on 8000 acres at $10/acre	$ 80,000
(6 tech. men)($40,000/yr)(½ yr.)	120,000
Seismic exploration at $15/acre	120,000
	$320,000.

The model followed assumes that three times out of four these deals will go no further. The seismic results will be unfavorable and the particular options will not be taken up and that deal will be dropped. In the simulation process, this was provided for by assigning one-fourth of the band of uniformly distributed random numbers to the event which includes drilling the exploratory well.

The income to the working interest per barrel was computed by assuming the current price of $3.00 per barrel, reduced first by 5 per cent state tax and then by the usual one-eighth land-owners royalty, which gave a gross of $2.49 per barrel. An average operating cost of $0.60 was assumed, which left a net of $1.89 per barrel. An allowable of 25 barrels per well was assumed, and 40 acre spacing, which at 8 per cent per year gave a present value factor of 0.585, which reduced the value of the oil to $1.11 present value per barrel. Thus, the present value of the recoverable oil became 1.11 S, where the symbol S designates the size of the discovery in barrels of oil.

There remained to be considered the leases which must be bought and the wells which must be drilled, the amounts of both of which depended upon the total amount of the reserves discovered. On the assumption of 20 feet of productive formation and 150 barrels per acre-foot, there would be a recovery of 3000 barrels per acre. By this reckoning the number of acres leased would be S/3000, and with $30 per acre left to be paid as bonus, the cost of the leasing would be (S/3000)(30) or 0.01 S.

The cost of drilling in the assumed area is taken as $150,000 for the original exploratory well and $80,000 for each additional well. The statistics on development wells show that not all of these are successful, and for each four producers there will average about one dry well. Assuming 40 acre spacing results in 120,000 barrels reserves per well. The number of development wells would be (1.25)(S-120,000)/120,000. The cost of drilling the development wells would be (80,000)(1.25)(S-120,000)/ 120,000. Including the exploratory well, the total cost of drilling the wells was expressed by 50,000 + 0.833 S.

The net effect of actions which follow the decision to drill the exploratory well can be found by subtracting leasing and drilling costs from the present value of the operating income.

$$\text{Net effect} = 1.11S - 0.01S - 50{,}000 - 0.833S$$
$$= 0.267S - 50{,}000.$$

Including the overhead expenses, the cost of the options, and the cost of the seismic exploration, the total change in financial position, for the case where an exploratory well is drilled, become

$$\text{Total change} = 0.267S - 370{,}000.$$

As indicated previously, for the case where an exploratory well is not drilled, the total change in financial position was

$$\text{Total change} = -320{,}000.$$

Results Using a Modified Triangular Distribution of Reserves

A portrayal of the results which might reasonably be expected from the economic model developed in previous sections was arrived at as follows. It was assumed that the most probable size of an oil discovery would correspond to 20 feet thickness of pay formation underlying one-fourth of the total acreage of 8000 acres with an ultimate recovery of 150 barrels per acre-foot, or 6,000,000 barrels. It may be noted that this corresponds favorably to the mode of the Oklahoma fields, which is about 5,000,000 barrels. The highest possible discovery, for the economic unit holding an option on 8000 acres was taken as 24,000,000 barrels, which would correspond to an average of 20 feet of pay underlying the total acreage under option. There would be some probability of an actual field exceeding this extent, and in this example that probability was taken as five per cent, but for the economic unit this additional probability was lumped at the maximum of 24,000,000 barrels. And, of course, it was assumed that a large per cent of the ventures would prove unattractive; a figure of 75 per cent was used in this example. This probability distribution is shown on Figure 24.

The next step was to convert the probability distribution to dollars. This is shown on the lower part of Figure 24. The values were taken as follows:

X_C = -\$320,000, the outcome where no exploratory well is drilled

X_L = -\$370,000, the outcome where a dry exploratory well is drilled

X_E = (6,000,000 bbl.)(\$0.267) - \$370,000
= \$1,232,000, the most probable outcome if an oil field is discovered

X_H = (24,000,000 bbl.)(\$0.267) - \$370,000

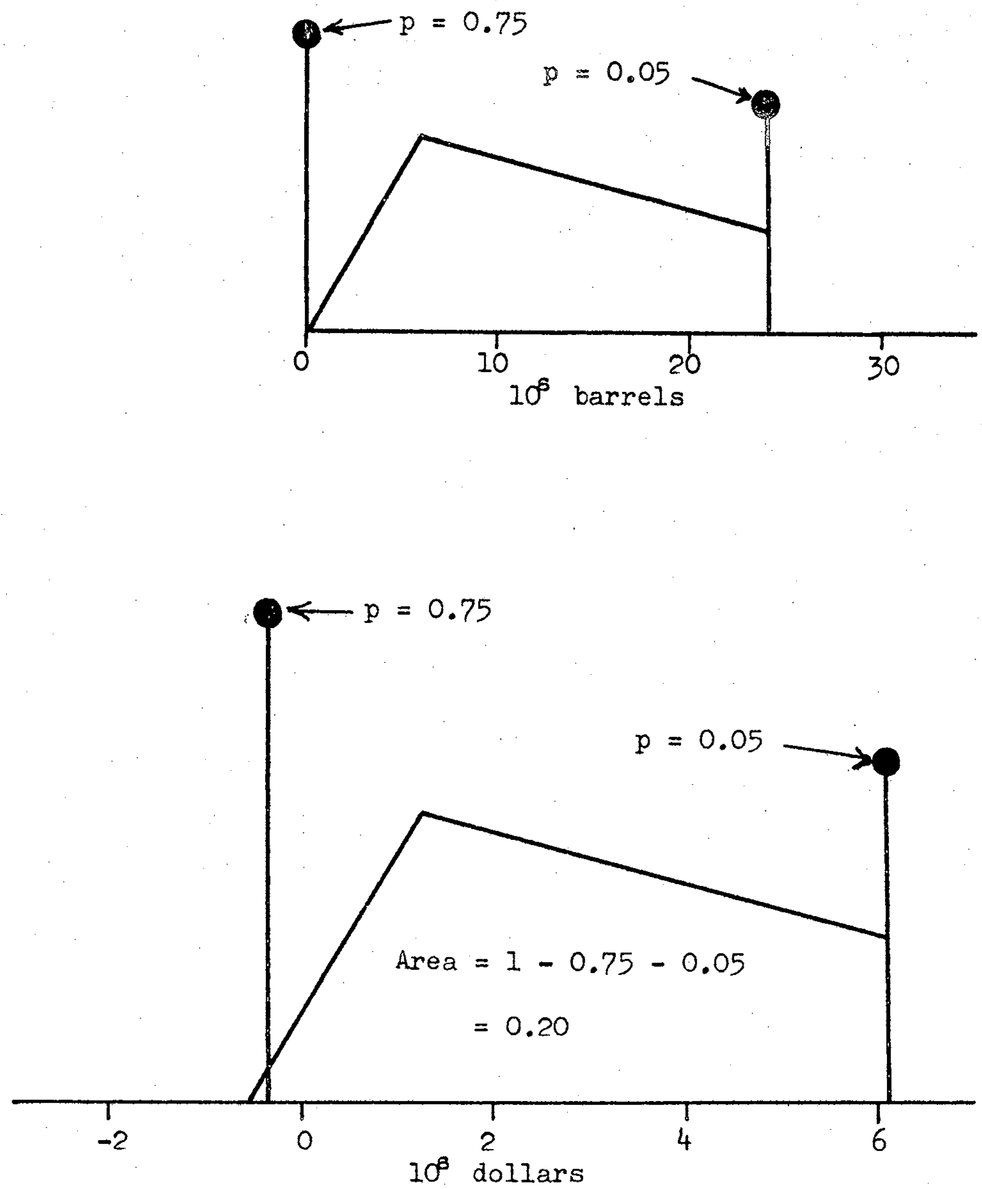

Figure 24. Probability Distribution of the Outcome Values of a Single Exploratory Venture

= \$6,038,000, the upper limit of the favorable outcomes.

The probability distribution of the financial outcomes of successive ventures was simulated by the Monte Carlo method. The results are shown on Figure 25. The expectancy is fairly high, 1.99 times the cost C. Nevertheless, Figure 25 shows that a company should be prepared to invest in 25 such ventures in order to be 95 per cent confident of at least breaking even.

The normalized probability curve, as generated by the simulation of 200 trials of 30 successive ventures in each trial, is shown on Figure 26. It may be seen that the distribution is close to normal in this case. On Figure 27 are plotted the values of the number of standard deviations as a function of the number of ventures. If the square root relation, $\sigma \propto \sqrt{n}$, were followed exactly, these lines would be straight horizontal lines.

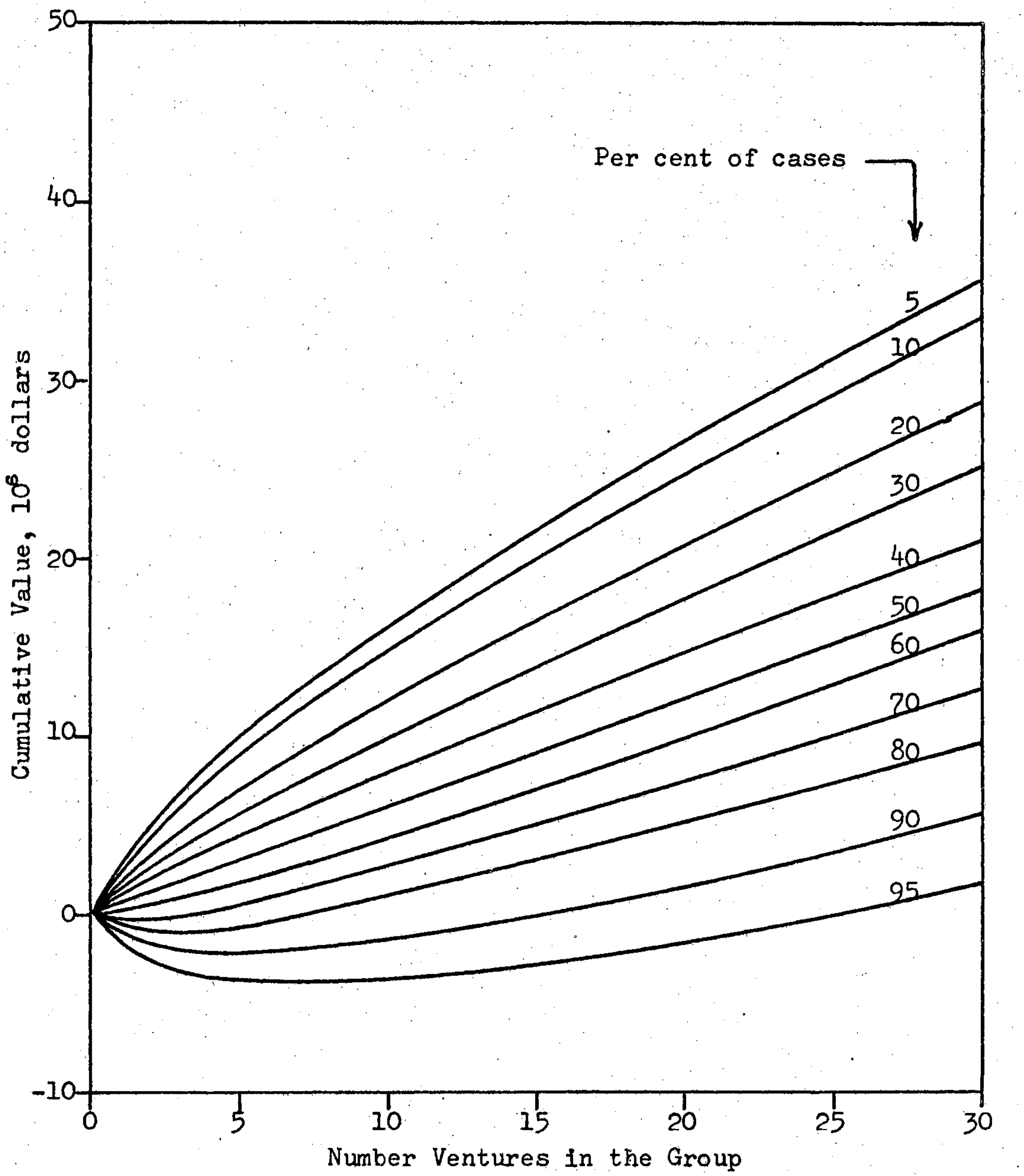

Figure 25. Probability Distribution of the Cumulative Value of Groups of the Individual Venture Shown on Figure 24

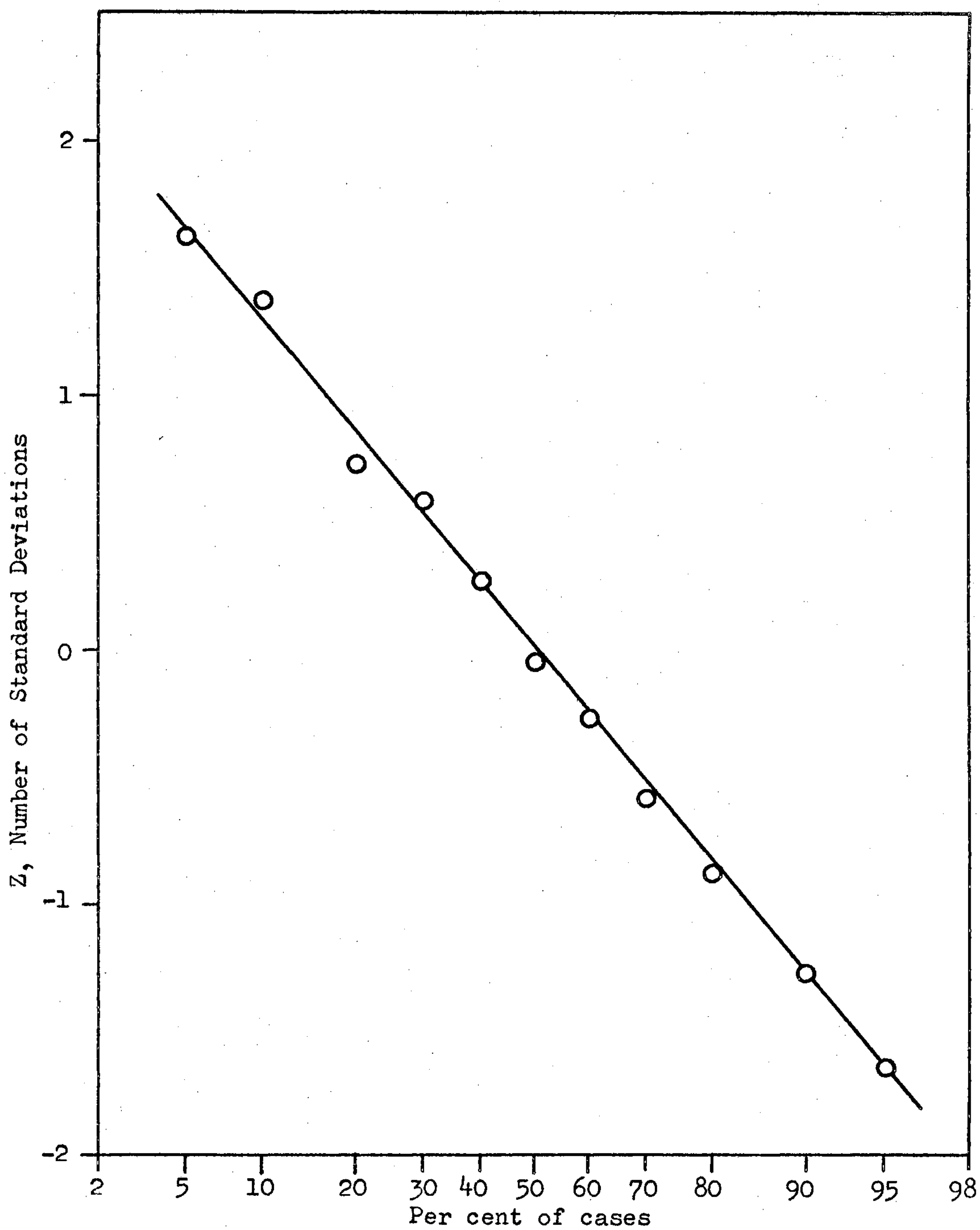

Figure 26. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 24

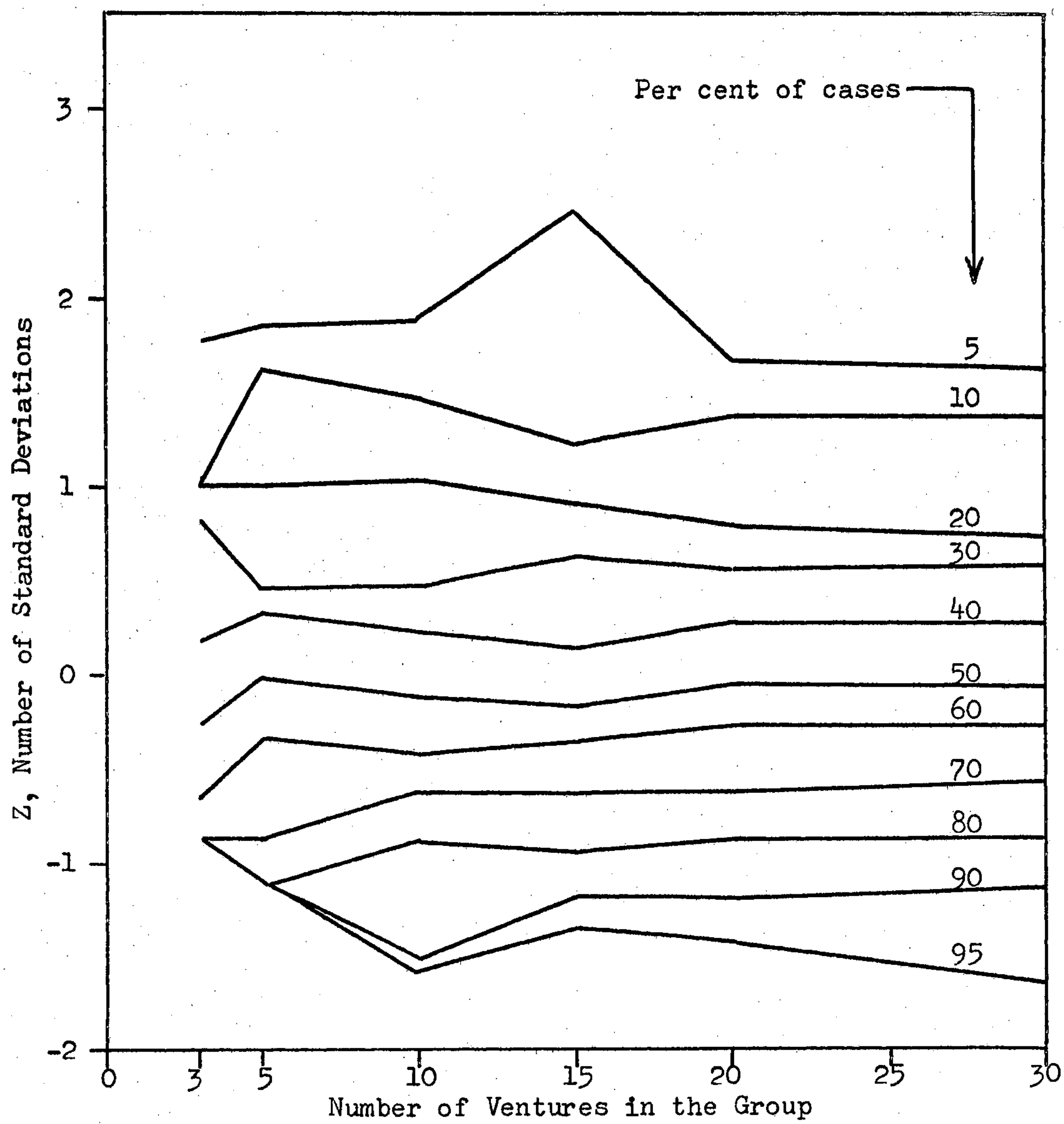

Figure 27. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 24

CHAPTER VII

COMPARISONS TO PREVIOUS SOLUTIONS

Comparisons to Method Based on Expected Values

The features of appraisal method which are based upon expectancies have been discussed in the first two chapters. These include methods proposed by Whitworth and adaptations of his work by Hayward and Pirson. The decision trees of Magee must also be included in this group since they also are dedicated toward formulating an expected value. Direct comparisons between those methods and the methods which have been developed during this investigation are not very meaningful. The methods developed here are dedicated toward portraying the full range of probable outcome values and managing investments so that combined risks will remain small. The former methods attempt to present a single acceptable answer. "A problem involving indeterminate variables does not have a single deterministic solution. To present a single solution to such a problem is incorrect."[1]

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[1] J. E. Walstrom, T. D. Muller, and R. C. McFarlane, "Evaluating Uncertainty in Engineering Calculations" presented at Annual Fall Meeting of SPE, Houston, Texas, Oct. 4, 1967.

Comparison to Proposed Statistical Methods

Statistical methods which attempt to give a complete picture of the probability distribution of possible outcomes of an economic venture are patterned in general after the work of Hertz, which was described in the first two chapters. The original application was for evaluating a proposed manufacturing plant expansion. Walstrom et al., have extended the method to evaluating partially developed oil reservoirs. These methods have been confined to evaluating a single economic venture in which the probability of some degree of success is assured but needs better description. The method developed in this investigation is concerned with combinations of highly uncertain ventures and appraisal methods based upon combining such ventures. Any method of evaluating the probability spectrum of a single venture can be incorporated into the methods which have been developed in this research.

Comparison to Results From Expected Utility Value Method

The expected utility method was developed and applied by Grayson for evaluating exactly the same exploratory or wildcat drilling prospects as have been studied in this investigation. He also used probabilistic descriptions to portray a range of possible different monetary returns. Tables I and II in Chapter II give the results of a numerical example offered by Grayson. Obviously, a comparison of results by the two methods should be possible.

The data of possible financial outcomes in Table I is quite obviously already reduced to present value and corrected for operating expenses. The following quantities can be calculated from the data:

$$X_E = 75{,}000, \text{ expectancy}$$

$$\sigma^2 = p(X_i - X_E)^2$$

$$= 49.375 \times 10^9$$

$$\sigma = 222{,}200 \text{ standard deviation}$$

$$C = 50{,}000 \text{ initial investment}$$

Some necessary information is not given directly but must be estimated. One such quantity is the total risk capital. The basis for calculating Grayson's examples was the utility curve shown on Figure 1. Inspection of this figure shows that a loss of $70,000 would cause the utility value to approach negative infinity, which would indicate an insurmountable loss, so it has been assumed that the risk capital available is $70,000. Since the possible returns have already been reduced to a present value, the value of w is zero.

The solution in Table II indicates that the optimum degree of participation in the venture, for the particular individual, would be about 0.5. In order to test this result in the light of statistical decision methods, Equation (16) is solved for z, the number of standard deviations associated with this solution. Following are the calculations:

$$z = -\left(\frac{X_E - w\,C}{\sigma}\right)\sqrt{\frac{M}{fC}}$$

$$= -\left(\frac{75,000 - 0}{222,200}\right)\sqrt{\frac{70,000}{(0.5)(50,000)}}$$

$$= -0.56.$$

Assuming that the distribution of outcome results is roughly normal, this corresponds to about 70 per cent confidence in ultimately achieving success by this type of decision. In other words, if all of the venture capital were invested in this manner, there is a 30 per cent probability that the capital would be dissipated. Reviewing the original data discloses the reason. About one well out of ten makes the profit. Therefore, investing one-third of the speculative capital into one such venture would not represent a very sound decision.

The distribution of the financial outcomes of successive ventures for this example, as given in Table I, were simulated by the Monte Carlo method. The probability distribution for a single venture is represented as a discrete distribution in Figure 28. The distribution of the outcomes of successive ventures is shown on Figure 29. On Figure 30 is plotted the distribution of outcomes, assuming that the standard deviation is proportional to the square root of the number of ventures. These lines should be straight horizontal lines if this condition held exactly throughout the Monte Carlo process. The degree of

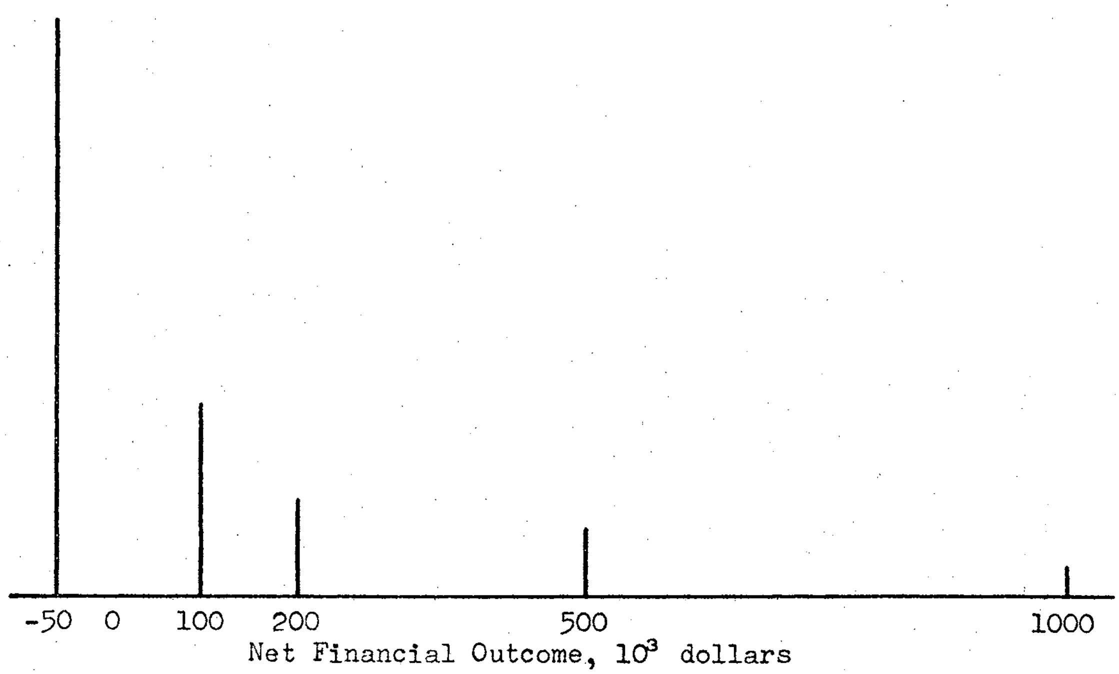

Figure 28. Probablity Distribution for a Single Venture (Grayson's Example)

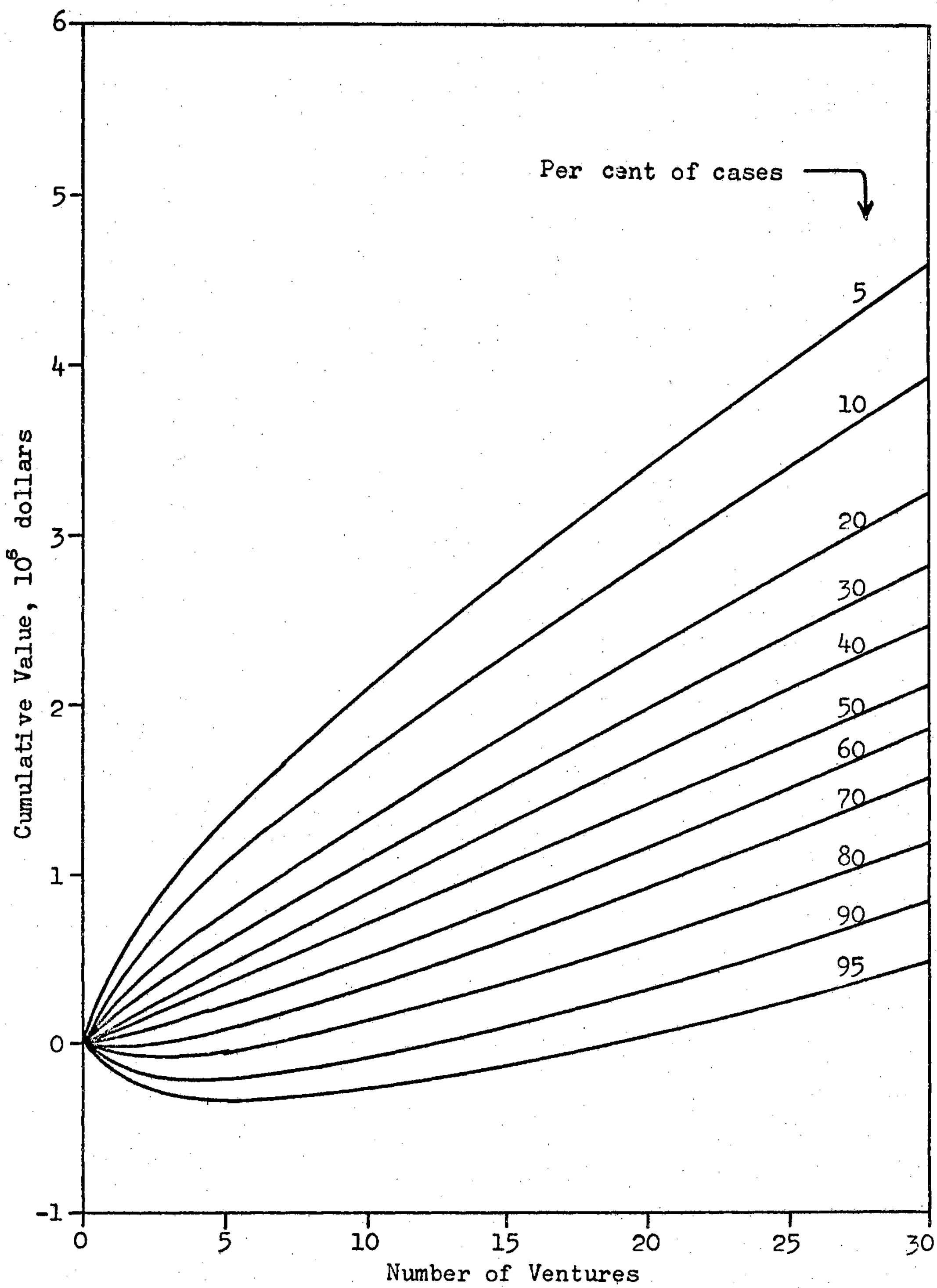

Figure 29. Probability Distribution of the Cumulative Value of a Group of Ventures Shown on Figure 28 (Analysis of Grayson's Example)

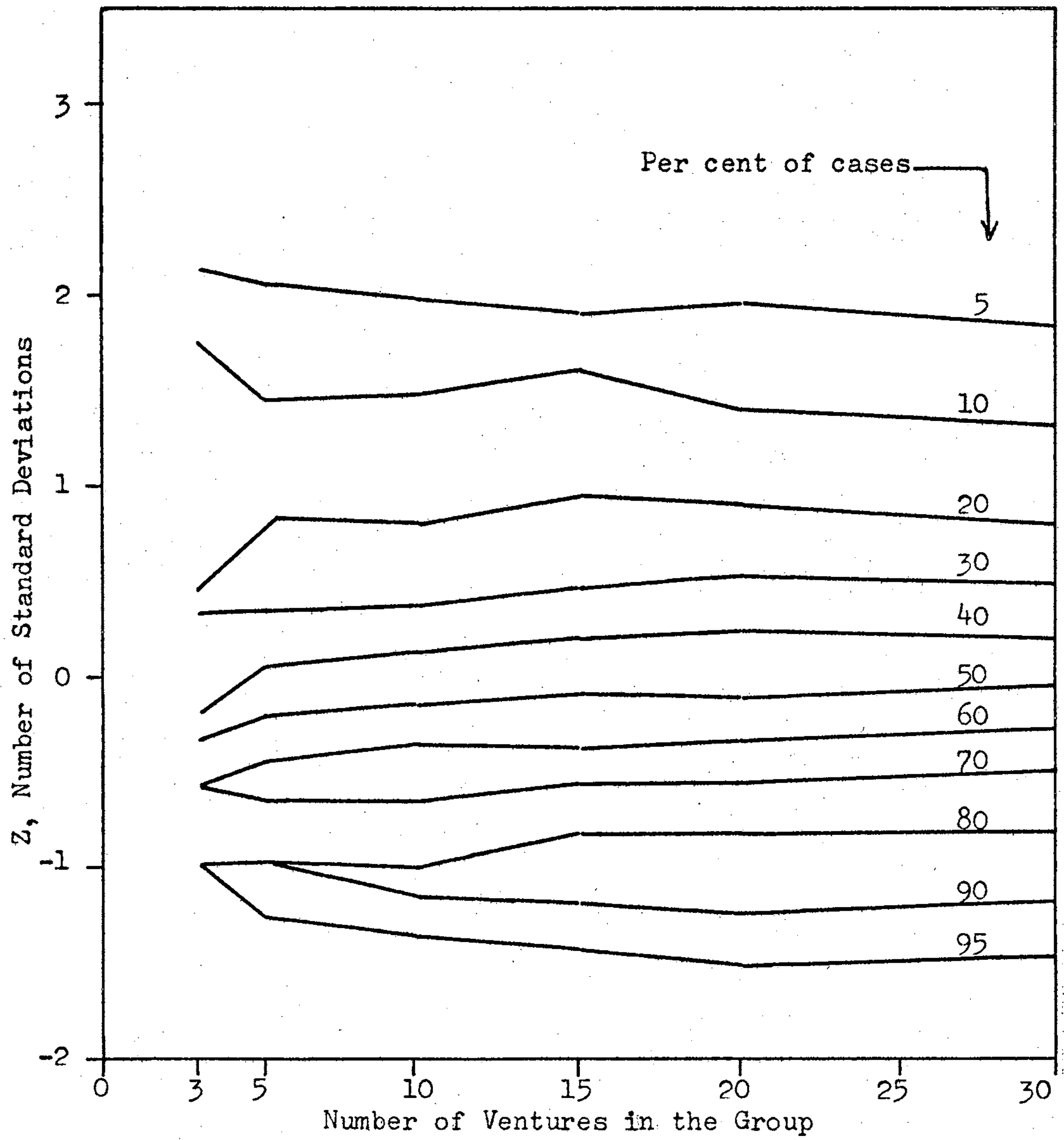

Figure 30. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 28

compliance with this condition is held to be justification for the assumption used in developing Equations (2), (3), and (4). On Figure 31 it may be seen that the distribution is close to normal, although there is definite curvature in the results.

The simulation results as plotted on Figure 29 can be used to verify the solutions which have been obtained from the derived equations. First, if the particular entrepreneur invested $25,000 in this venture, the use of all of his venture capital would provide him with $70,000/$25,000 or 2.8 such ventures. On Figure 29, it may be seen that the 70 per cent line passes up through zero value when the number of ventures is about three. This verifies that, on the average, 30 per cent of such cases have unfavorable results. Not enough ventures could be participated in to assure a reasonably good chance of not going broke.

A better solution is found where the 95 per cent confidence line passes through zero. This occurs at a value of 19 ventures. On this basis, the maximum investment which this investor should make in this investment would be $70,000/19 or $3700. This amount is about one-seventh of the ideal investment as determined by the expected utility value method. If the particular investor were content with a 90 per cent confidence of success, then his recommended maximum investment would be $70,000/12 or $5800. In both cases, the answers are significantly different from that obtained by the expected utility value

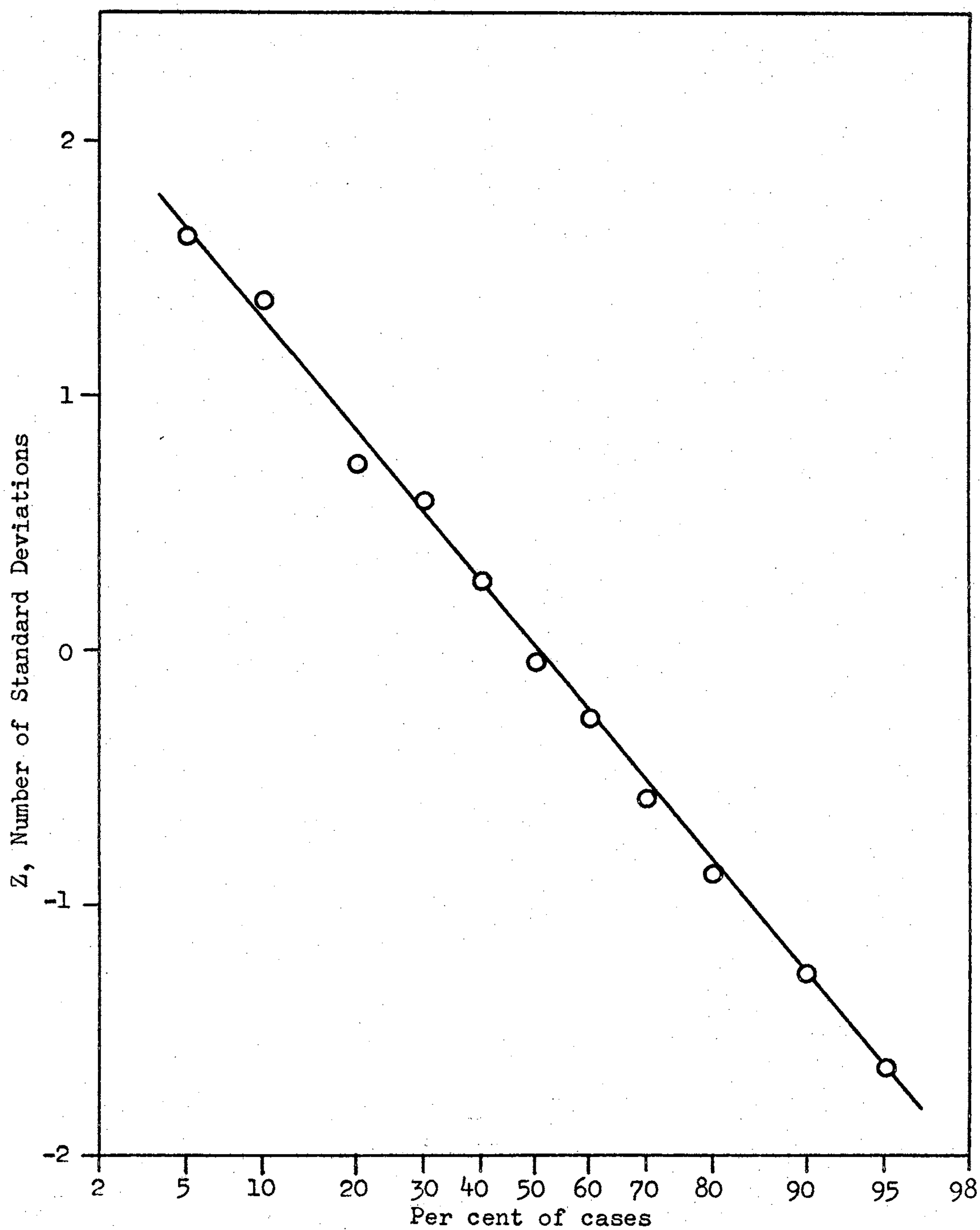

Figure 26. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 24

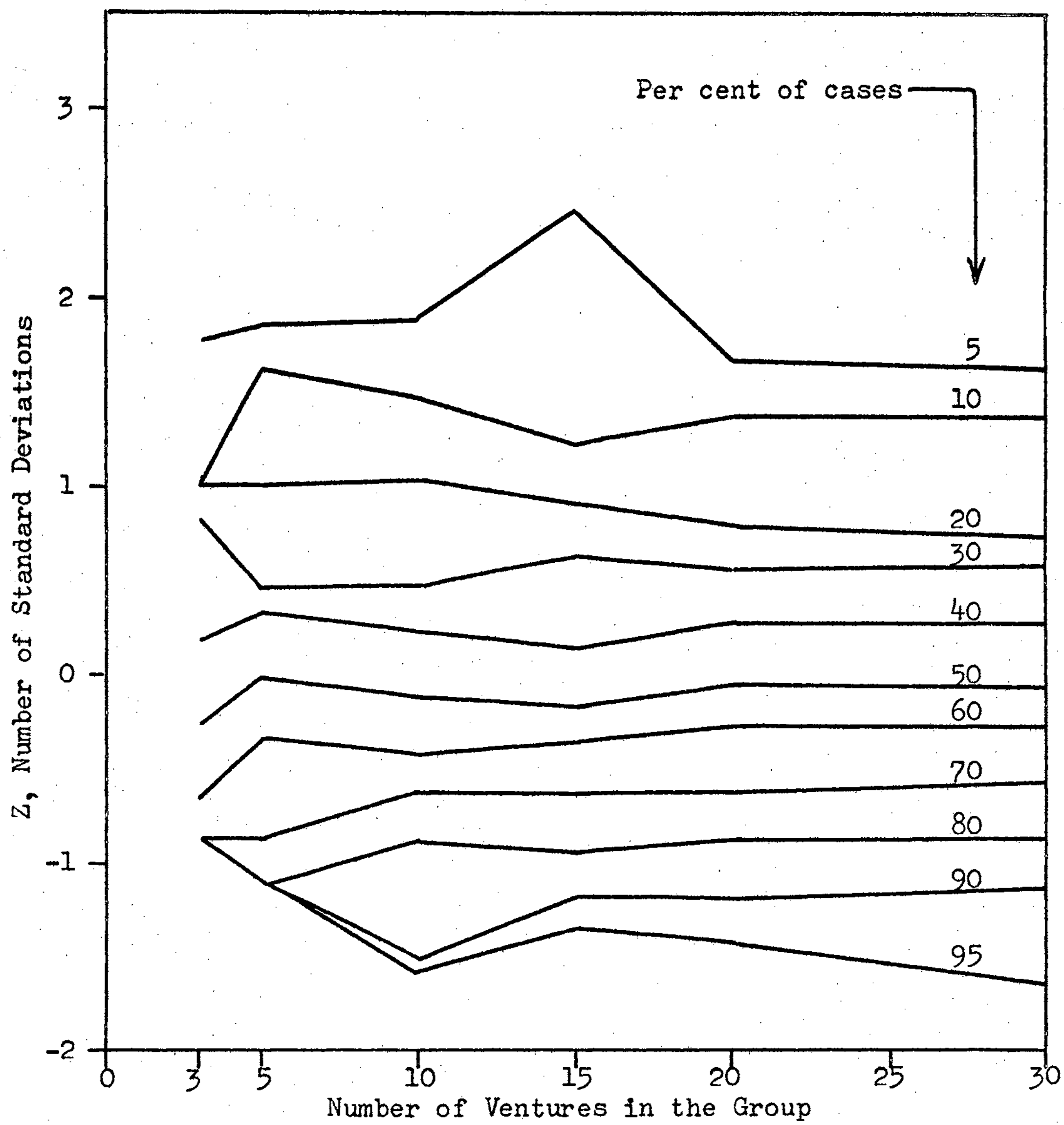

Figure 27. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 24

CHAPTER VII

COMPARISONS TO PREVIOUS SOLUTIONS

Comparisons to Method Based on Expected Values

The features of appraisal method which are based upon expectancies have been discussed in the first two chapters. These include methods proposed by Whitworth and adaptations of his work by Hayward and Pirson. The decision trees of Magee must also be included in this group since they also are dedicated toward formulating an expected value. Direct comparisons between those methods and the methods which have been developed during this investigation are not very meaningful. The methods developed here are dedicated toward portraying the full range of probable outcome values and managing investments so that combined risks will remain small. The former methods attempt to present a single acceptable answer. "A problem involving indeterminate variables does not have a single deterministic solution. To present a single solution to such a problem is incorrect."[1]

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[1]J. E. Walstrom, T. D. Muller, and R. C. McFarlane, "Evaluating Uncertainty in Engineering Calculations" presented at Annual Fall Meeting of SPE, Houston, Texas, Oct. 4, 1967.

Comparison to Proposed Statistical Methods

Statistical methods which attempt to give a complete picture of the probability distribution of possible outcomes of an economic venture are patterned in general after the work of Hertz, which was described in the first two chapters. The original application was for evaluating a proposed manufacturing plant expansion. Walstrom et al., have extended the method to evaluating partially developed oil reservoirs. These methods have been confined to evaluating a single economic venture in which the probability of some degree of success is assured but needs better description. The method developed in this investigation is concerned with combinations of highly uncertain ventures and appraisal methods based upon combining such ventures. Any method of evaluating the probability spectrum of a single venture can be incorporated into the methods which have been developed in this research.

Comparison to Results From Expected Utility Value Method

The expected utility method was developed and applied by Grayson for evaluating exactly the same exploratory or wildcat drilling prospects as have been studied in this investigation. He also used probabilistic descriptions to portray a range of possible different monetary returns. Tables I and II in Chapter II give the results of a numerical example offered by Grayson. Obviously, a comparison of results by the two methods should be possible.

The data of possible financial outcomes in Table I is quite obviously already reduced to present value and corrected for operating expenses. The following quantities can be calculated from the data:

$$X_E = 75{,}000\text{, expectancy}$$

$$\sigma^2 = p(X_i - X_E)^2$$

$$= 49.375 \times 10^9$$

$$\sigma = 222{,}200 \text{ standard deviation}$$

$$C = 50{,}000 \text{ initial investment}$$

Some necessary information is not given directly but must be estimated. One such quantity is the total risk capital. The basis for calculating Grayson's examples was the utility curve shown on Figure 1. Inspection of this figure shows that a loss of $70,000 would cause the utility value to approach negative infinity, which would indicate an insurmountable loss, so it has been assumed that the risk capital available is $70,000. Since the possible returns have already been reduced to a present value, the value of w is zero.

The solution in Table II indicates that the optimum degree of participation in the venture, for the particular individual, would be about 0.5. In order to test this result in the light of statistical decision methods, Equation (16) is solved for z, the number of standard deviations associated with this solution. Following are the calculations:

$$z = -\left(\frac{X_E - w\,C}{\sigma}\right)\sqrt{\frac{M}{fC}}$$

$$= -\left(\frac{75{,}000 - 0}{222{,}200}\right)\sqrt{\frac{70{,}000}{(0.5)(50{,}000)}}$$

$$= -0.56.$$

Assuming that the distribution of outcome results is roughly normal, this corresponds to about 70 per cent confidence in ultimately achieving success by this type of decision. In other words, if all of the venture capital were invested in this manner, there is a 30 per cent probability that the capital would be dissipated. Reviewing the original data discloses the reason. About one well out of ten makes the profit. Therefore, investing one-third of the speculative capital into one such venture would not represent a very sound decision.

The distribution of the financial outcomes of successive ventures for this example, as given in Table I, were simulated by the Monte Carlo method. The probability distribution for a single venture is represented as a discrete distribution in Figure 28. The distribution of the outcomes of successive ventures is shown on Figure 29. On Figure 30 is plotted the distribution of outcomes, assuming that the standard deviation is proportional to the square root of the number of ventures. These lines should be straight horizontal lines if this condition held exactly throughout the Monte Carlo process. The degree of

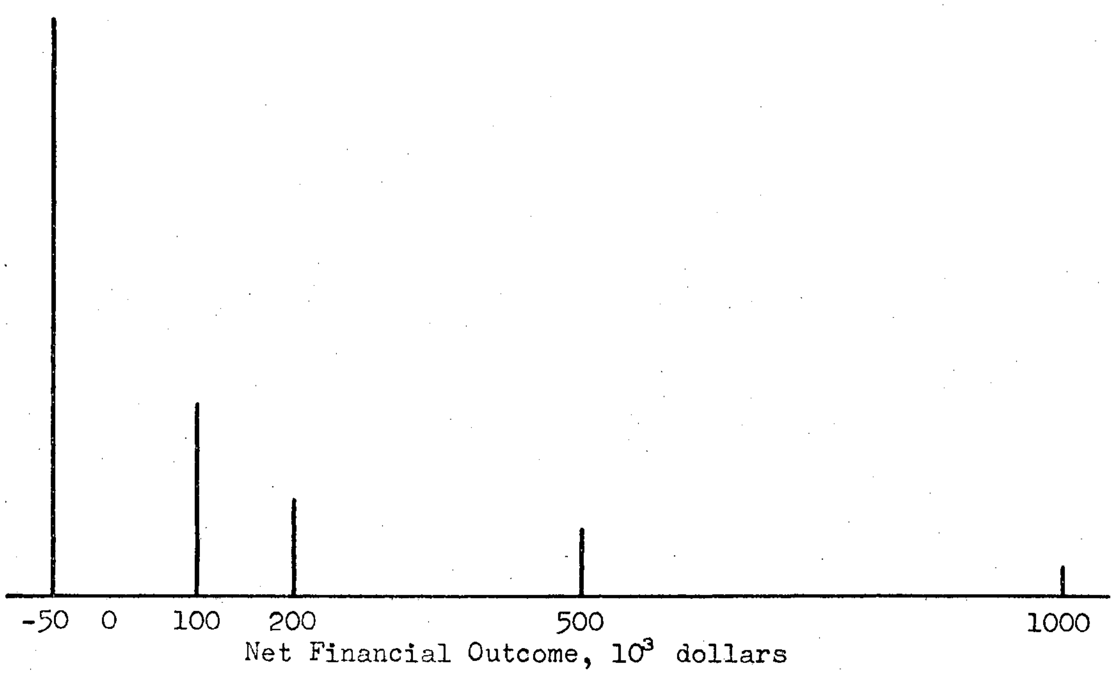

Figure 28. Probablity Distribution for a Single Venture (Grayson's Example)

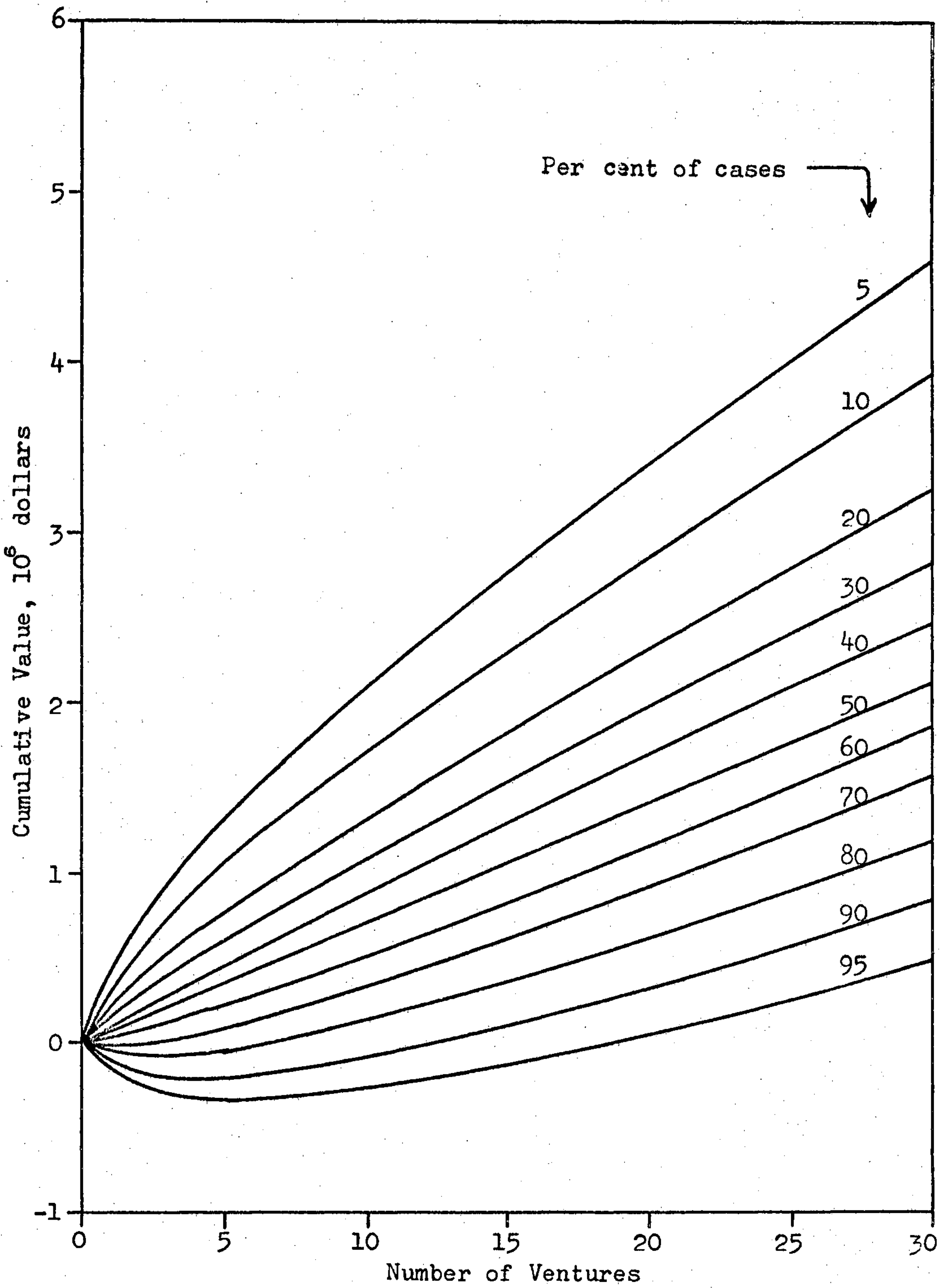

Figure 29. Probability Distribution of the Cumulative Value of a Group of Ventures Shown on Figure 28 (Analysis of Grayson's Example)

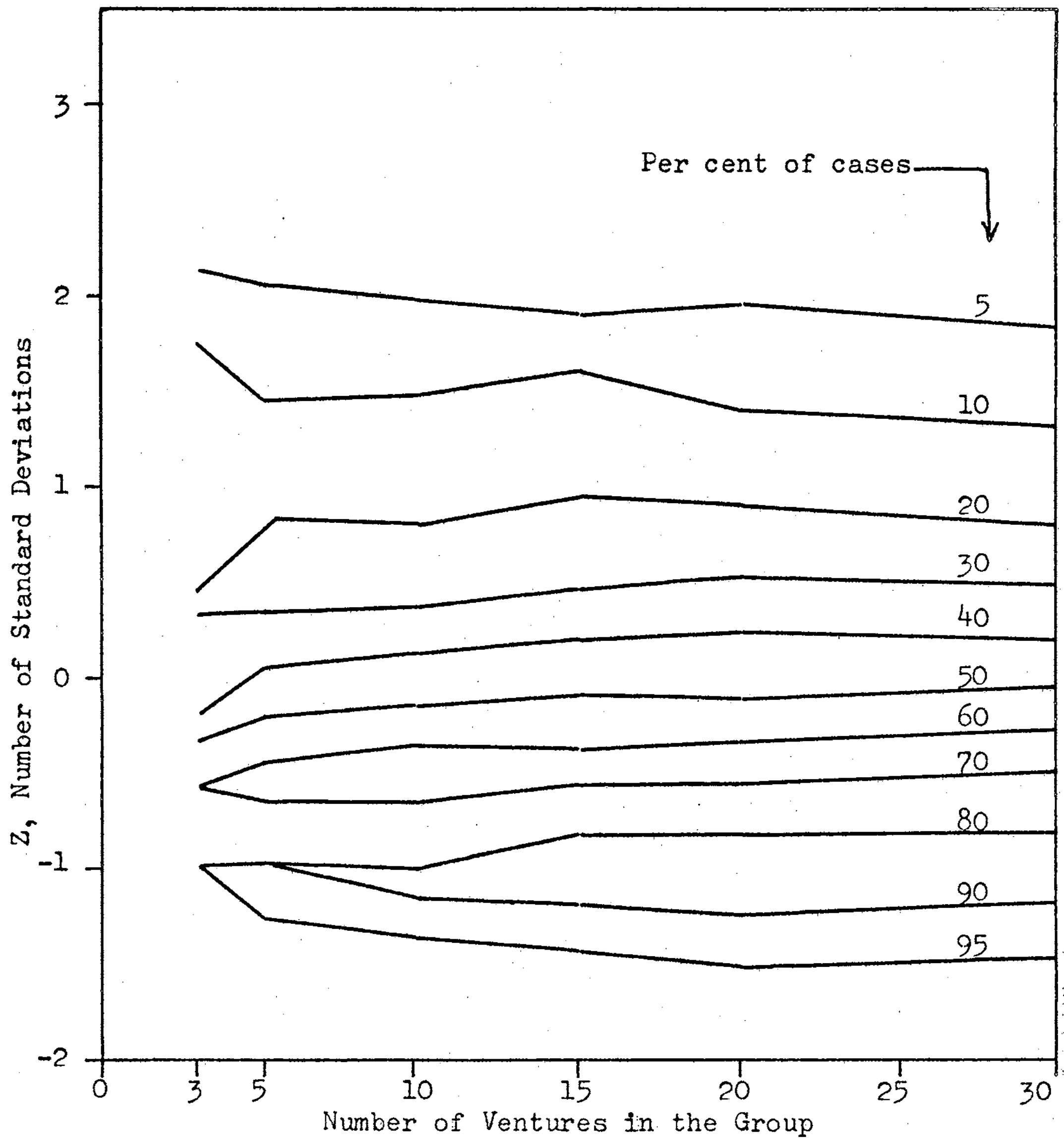

Figure 30. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 28

compliance with this condition is held to be justification for the assumption used in developing Equations (2), (3), and (4). On Figure 31 it may be seen that the distribution is close to normal, although there is definite curvature in the results.

The simulation results as plotted on Figure 29 can be used to verify the solutions which have been obtained from the derived equations. First, if the particular entrepreneur invested $25,000 in this venture, the use of all of his venture capital would provide him with $70,000/$25,000 or 2.8 such ventures. On Figure 29, it may be seen that the 70 per cent line passes up through zero value when the number of ventures is about three. This verifies that, on the average, 30 per cent of such cases have unfavorable results. Not enough ventures could be participated in to assure a reasonably good chance of not going broke.

A better solution is found where the 95 per cent confidence line passes through zero. This occurs at a value of 19 ventures. On this basis, the maximum investment which this investor should make in this investment would be $70,000/19 or $3700. This amount is about one-seventh of the ideal investment as determined by the expected utility value method. If the particular investor were content with a 90 per cent confidence of success, then his recommended maximum investment would be $70,000/12 or $5800. In both cases, the answers are significantly different from that obtained by the expected utility value

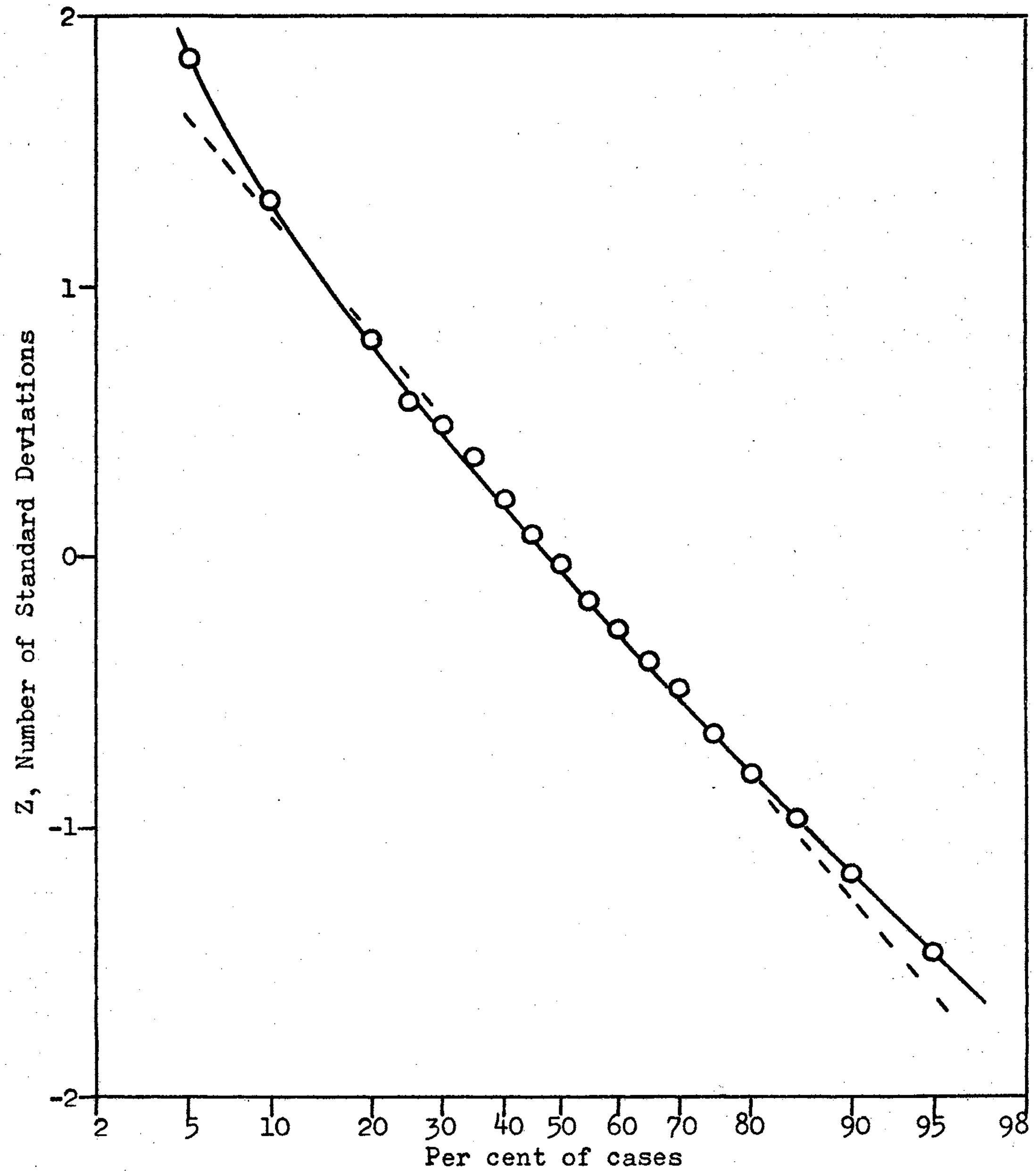

Figure 31. Analysis of Monte Carlo Simulation of the Probability Distribution Shown on Figure 28 (Dotted line shows position of normal distribution)

method. It is quite possible that the two methods might give about the same answer in some cases, depending largely upon the utility function drawn from the answers given by the subject investor to hypothetical investment proposals. But it is believed that the proposed statistical decision model will set forth in each case the pattern which will be generated by investments in that particular type of venture. Because it is responsive to the variations in the results from the venture, as well as to the financial position and asperations of the investor, it should prove to be a useful model in analyzing investments.

CHAPTER VIII

CONCLUSIONS

Assessment of Hypothesis

A hypothesis was presented in Chapter I with the statement that the best basis for making choices for participation in uncertain economic ventures would be found by combining utility theory with statistical decision theory. Utility theory must provide guidance in formulating an over-all policy. Statistical decision methods should be used for implementing the policy. The question must be considered regarding the degree to which the validity of the hypothesis has been demonstrated.

In general it has been accepted throughout this dissertation that both the commitment to engage in a particular industry and the conditions necessary for participation must be decided by taking account of factors both tangible and intangible of which part are outside of that industry. Usually, alternatives exist which are more or less attractive and choice is made by comparison of materialistic analyses and personal beliefs and desires. It is granted that much of this is outside of the scope of statistical decisions and must be handled by other methods. It is believed that such decisions, including the setting of risk factors for statistical decisions, are best made under the general guidance of methods incorporated in utility theory.

Evaluation methods have been developed during this investigation which can be applied to the problems of survival and profit in an economic area characterized by highly uncertain individual ventures. The various solutions have in common the basic characteristic of statistical solutions wherein a predetermined risk is accepted for a modus operandi. Direct solutions have been developed for guiding participation and for the appraisal of individual ventures so that risk is limited. These solutions are dependent upon a knowledge of the probabilistic distribution of the possible different financial outcomes of a single venture. Some means of estimating such distributions have been discussed and examples have been cited in the literature wherein resource expectations are being estimated by simulation methods. In sum this means that methods for supplying the basic data are in an advanced state of development. Such basic data can be handled by the mathematical methods developed in this investigation and it can be interpreted for guiding decisions related to participation in uncertain economic ventures.

It is submitted that the validity and the usefulness of the hypothesis have been demonstrated.

Conclusions

The following conclusions summarize the results of this investigation:

1. The financial aspect of drilling exploratory wells for petroleum is probabilistic in nature. The set of all possible drilling locations is customarily narrowed down by geological methods so that the remaining set of recommended drilling locations results in a much greater success ratio. A number of factors are involved, and the size of the deposits is variable, so that the entire process is probabilistic.
2. The lognormal distribution was assumed to be suitable for describing the size distributions of oil fields in the large oil producing areas, such as states or large areas within states. The most valid use of this relationship consists of determining the mode, the most common size of oil discoveries.
3. The background and training of most men permit them to use simple probability models to describe the outcomes of an infinite number of trials of a probabilistic event. The concepts formed in the mind relate to the array of possible results of a single trial and to the average of such trials.

4. The information handling capacities of the mind are limited and the expected value has generally been used for making decisions.
5. Decisions involving a single venture can be based upon a study of the full probability spectrum, which can be simulated by methods described in recent literature. Any such probability models can be incorporated into the methods developed in this investigation.
6. Methods have been developed during this investigation for interpretations in areas of highly uncertain ventures which are based upon participation in a sufficient number of ventures to insure survival. Statistical decision techniques can be used for obtaining various solutions. Rules have been formulated which provide for great confidence of survival through a series of uncertain ventures.
7. The methods developed during this investigation permit expressing the character of a financial venture in terms of a confidence limit of success, or conversely a risk, as well as in terms of an expectancy. Both of these concepts are easy to understand, and their combination incorporates a more complete probabilistic interpretation into a

decision.

8. Methods developed in this investigation provide a means of appraising the character of a group of concurrent uncertain ventures. They also provide a means of judging the performance of a subset of similar consecutive ventures.

9. Methods of appraising individual ventures were extended to a petroleum exploration model which involved several consecutive decision steps.

10. Appraisal equations were derived for the following situations:

    (1) The maximum advisable fractional participation in a relatively large uncertain venture.

    (2) The maximum appraisal price which can be paid for an uncertain venture, consistent with the financial resources of the investor.

    (3) The maximum value of information which increases the chances of success.

11. The maximum appraisal price is set by the minimum acceptable expected rate of return when financial resources are large. The maximum appraisal price should be set by risk when financial resources

are not large. This effect was observed for resources up to 70 times the expectancy for a typical drilling situation.

12. The groups of ventures are sampling distributions. They were observed to be normally distributed, to a fair approximation when there were five in a group and to a good approximation when there were ten, twenty, and thirty in a group. When the outcomes were expressed as a present value, they were normally distributed about the expectancy. The standard deviation was equal to the standard deviation of the parent distribution times the square root of the number of ventures in the group. The outcomes, when expressed as rates of return, were observed in the range from -30 per cent to +70 per cent to be normally distributed about the expected rate of return. In the latter case, the distribution converged with increasing number of ventures in the group. The parent distributions which were simulated included the rectangular, a triangular distribution with a spike and a discrete distribution.

13. Where the possible financial outcomes of a prospective economic venture are expressed

both in terms of undeferred values and in terms of present values reckoned according to different interest rates, the standard deviations are proportional to the present value factor. The standard deviations are independent of the cost of the venture.

14. Where a minimum acceptable rate of return is used as a basis for calculating an appraisal price, a second rate of return figure should also be used as the minimum acceptable rate at a specified confidence limit. The use of the second and lower rate permits an evaluation which takes account of the risk in an uncertain venture.

Suggestions for Further Work

In many areas, the petroleum producing industry has not developed a capacity for describing its anticipated discoveries in a probabilistic manner. Similar conditions may exist in many other industries. It is suggested that this may be due to the fact that in the past there has not been available a way of translating such probability distributions into meaningful economic information which could serve as a basis for decisions. Expectancies have been used, of course, but the relations between probability distributions and the survival of economic units had not been adequately described in a mathematical relationship.

It is suggested that further work could be done in developing methods of estimating probabilities, types of distributions and their parameters. Presumably both theoretical and numerical methods could be adapted to this work. A related effort would consist of developing methods of correlating and adjusting the distributions, parameters and probabilities as new data became available during an economic development.

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APPENDIX A

PROPERTIES OF SOME PROBABILITY DISTRIBUTIONS

Uniform (Rectangular) Distribution

$$\text{Mean} = X_E = \frac{X_L + X_H}{2}$$

$$\text{Variance} = \frac{(X_H - X_L)^2}{12}$$

L and H are the lowest and highest values, at the limits of the distribution.

Triangular Distribution

$$\text{Mean} = X_E = \frac{X_L + X_M + X_H}{3}$$

$$\text{Variance} = \frac{(X_H - X_L)^2}{18} (1 - p_2 p_3)$$

L and H are the lowest and highest values, at the limits of the distribution. M is the mode, most likely value, at the apex of the triangle.

$$p_2 = \frac{X_M - X_L}{X_H - X_L}$$

$$p_3 = \frac{X_H - X_M}{X_H - X_L}$$

Triangular Distribution With Spike

$$\text{Mean} = X_E = p_1 X_1 + (1 - p_1)\left(\frac{X_L + X_M + X_H}{3}\right)$$

$$\text{Variance} = p_1 (X_1 - X_E)^2 + (1 - p_1)\Big[\left(\frac{X_L + X_M + X_H}{3} - X_E\right)^2$$

$$+ \frac{(X_H - X_L)^2}{18}\left(1 - \frac{p_2 p_3}{(1 - p_1)^2}\right)\Big]$$

$$= \left(\frac{p_1}{1 - p_1}\right)(X_1 - X_E)^2 + \frac{(X_H - X_L)^2}{18}\left[1 - p_1 - \frac{p_2 p_3}{1 - p_1}\right]$$

X_1 is the discrete value designated by the spite

p_1 is the probability associated with X_1

X_L, X_M, X_H are the lowest, the mode and the highest values of the triangular distribution:

$$p_2 = (1 - p_1)\left(\frac{X_M - X_L}{X_H - X_L}\right)$$

$$p_3 = (1 - p_1)\left(\frac{X_H - X_M}{X_H - X_L}\right)]$$

Discrete Distribution

$$\text{Mean} = X_E = \sum_1^n P_i X_i$$

$$\text{Variance} = \sum_1^n P_i (X_i - X_M)^2$$

n is the number of individual terms

P_i is the probability associated with each X_i term.

APPENDIX B

PRESENT VALUE FACTORS FOR DECLINING PRODUCTION

Exponential Decline Relations

The relation between the initial and final instantaneous production rate is

$$q_F = q_o \; R^T .$$

The total oil produced over the decline is obtained as the sum of a geometric progression

$$N_{PD} = Y_1 \left(\frac{1 - R^T}{1 - R} \right) .$$

The total present value is obtained as the sum of a geometric progression. All income received throughout a year is considered to be received as a lump sum received at the mid-point of each year.

Present Value = (D)(Undeferred Value)

$$D = (1 + i)^{0.5} \left(\frac{1 - R}{1 - R^T} \right) \left[\frac{1 - \left(\frac{R}{1+i} \right)^T}{1 + i - R} \right]$$

where D = present value factor

i = yearly interest rate

N_{PD} = total oil produced over the declining production period

q_o = initial instantaneous production rate

q_F = final instantaneous production rate

R = ratio, less than one, between successive yearly productions

T = number of years in the decline period

Y_1 = production of the first year on decline

VITA

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