A QUANTITATIVE ANALYSIS OF FACTORS AFFECTING

ELEMENTARY AND SECONDARY SCHOOLING QUALITY
IN OKLAHOMA, WITH ECONOMIC APPLICATION
FOR RURAL AREAS

By

FREDDIE CAD WHITE
Bachelor of Science Texas Technological University

Lubbock, Texas 1967

Master of Science Oklahoma State University Stillwater, Oklahoma 1969

Submitted to the Faculty of the Graduate College of the Oklahoma State University in partial fulfillment of the requirements
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## Thesis Approved:



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## CHAPTER I

## INTRODUCTION

The Problem

Pressures of tight budgets and resistance of taxpayers to higher taxes on one hand and, on the other hand, pressures of parents for better schooling for their children have placed school administrators in a difficult position. Whereas, administrators once may have been able to improve schooling quality by obtaining more funds, emphasis now is turning to improving quality with given resources - hence increasing efficiency. In some instances, school boards have contracted with private consultants to take over school operations and improve student performance. Use of private consultants and performance contracts highlights public concern with the jobs schools are doing. Such drastic action will not be necessary if current administrators make needed adjustments to improve the efficiency of schools. This study shows opportunities to improve efficiency.

Quality schooling is essential to the well being of youth who must compete for jobs or for a university degree. The close interaction between rural and urban areas provided by migration requires that the quality of education offered in rural areas not lag behind that of urban areas. However, major differences in educational opportunities between urban and rural areas exist. A task force on rural development and family living reported:


#### Abstract

Vocational education in rural areas too often stresses agricultural production where relatively little employment opportunity exists. Other aspects of rural education that still lag behind urban standards are illustrated by: lack of preschool and kindergarten experience; inadequate curriculum offerings and vocational guidance; dearth of special teaching aids, laboratories, libraries, and other educational facilities deemed necessary in modern schools. 1

These inequalities in educational opportunities are of special concern since many rural residents are faced with the prospect of migration. It is widely believed that the mass migration of poorly educated rural residents into urban areas has contributed to urban problems. This migration has also weakened the economic base of many rural communities and raised dependence rates, making it difficult to adequately finance schooling and maintain the quality of education offered. The need for efficient use of available resources is clear.


## Economics of Education

Economic analysis is applied to education in a manner similar to any other activity which uses scarce resources. This analysis encompasses the allocation of a community's scarce resources so as to satisfy wants as fully as possible. It requires adequate information on (1) educational objectives, (2) performance of various educational methods or inputs in attaining these objectives, and (3) costs associated with these educational methods or inputs.

Economic analysis of education can be divided into two separate but related components, external and internal economics. External economics of education deals with the economic value of education. Encompassing issues of economic payoff, equity and financing of education, the external economics of education helps guide public policy. Internal economics of education, concerned with the efficient operation
of a school district or system, applies theory of the firm on a schooldistrict basis. It can be used to determine (1) how to apply given funds most efficiently, and (2) what level of funds are needed to attain a given level of output.

## External Economics of Education

The bulk of previous economic analyses on education have focused on concepts of human capital, social benefit-cost analysis, and educational policy. Although research on external economics of education had begun earlier, T. W. Schultz revived this branch of economics emphasizing the idea of human capital formation. ${ }^{2}$ The amount of educational capital embodied in the population helps to $\operatorname{explain}$ past economic growth and to prescribe needed changes for future growth.

The benefits from investing in education, however, must be weighed against its costs. Beginning with Hansen's ${ }^{3}$ and Weisbrod's ${ }^{4}$ work, almost all benefit-cost studies have shown a favorable rate of return on investment in education and have documented the importance of education's contribution to economic growth. Favorable rates of return on investment indicated a capacity for the educational system to absorb greater investments profitably.

These measurements of the benefits and costs of education provide a basis for educational planning on a state or national level. This approach guides investment in education among rural-urban-suburban sectors and among regions. The optimum level of investment may be based on ability to invest, equity, or profitability of investment. Equity deals with issues of who pays for and who benefits from schooling and examines the disassociation of schooling costs and benefits. An
analysis of schooling investments among regions within the United States showed investment in formal schooling yielded a favorable but unequal rate of return on investment among regions. ${ }^{5}$ Another study showed the rate of return on investment in elementary and secondary education differs among rural and urban residents. ${ }^{6}$ These differences in rates of return among rural-urban sectors and among regions indicate an imbalance in resource allocation.

Initial analyses of the economic value of education concentrated on those aspects of education which were most easily quantifiable. Measuring outputs in dollar values was a logical starting point. Economic analysis of education focused on state, regional, and national schooling problems since data on lifetime earnings, as a measure of educational output, are readily available at this level of aggregation. Lack of output data helps explain the relative neglect by economists of internal economics of schooling.

## Internal Economics of Education

Income data to measure schooling output are usually not available on a school-district basis. Even if they were available, earnings may only partially measure the productivity of education since all of the output of the educational system does not enter the market. Moreover, an analysis emphasizing economic benefits fails to address many relevant questions for a school system. A school system is concerned with internal efficiency - the allocation of its resources in an efficient manner to reduce costs of achieving schooling objectives.

Analysis to improve educational quality requires substantial knowledge of the relationship between educational inputs and educational
output, as well as the relationship between educational inputs and educational costs. An educational input is defined as a resource (e.g., labor, capital, time) which is used in providing educational services. The output of a school's program is measured by the number of students in the program and the quality of schooling. Quality of the schooling program is ultimately measured by the behavior it produces in the students who pass through it. Since the many dimensions of lifetime behavior are difficult to assess, variables such as achievement test scores, dropout rates, and absentee rates (which are known to be related to subsequent behavior) are used to measure school quality. These measures of quality are subsequently used as measures of output for a given number of students educated in the program. Internal economics of education focuses on combining schooling inputs in a least-cost manner to achieve a given level of output. The review of literature in the next chapter reveals that economists have given little attention to this important field of human economic development.

## Objectives

This study is concerned with the internal economics of education in Oklahoma elementary and secondary public schools. The major objective is to determine optimum resource allocations and school district size for rural areas. Specific objectives are to:
(1) Quantify the effect that the various factor inputs have on the output of elementary and secondary schooling in Oklahoma.
(2) Measure the costs of factor inputs.
(3) Determine the allocation of schooling inputs that minimizes the cost of a given quality of education for a given
population density and other characteristics of a geographic area.
(4) Measure the economic payoff from changes in use of inputs in achieving a given level of output.

Organization of the Study


#### Abstract

Published studies relating to the internal economics of education are reviewed in the next chapter. Chapter III identifies relevant educational outputs and inputs and provides a theoretical framework applicable to the internal economics of education. Data sources and characteristics are presented in Chapter IV. The educational production functions and cost relationships are presented in Chapters V and.VI. respectively. The procedure for optimizing the resource mix is presented in Chapter VII. The major empirical chapter, Chapter VIII, describes the long-run optimum allocation of educational resources for a given school district. Summary, conclusions and implications are presented in Chapter IX.


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## CHAPTER II

## REVIEW OF LITERATURE

This chapter examines past research dealing with inputs and outputs of the educational system. Research reveals that numerous variables conceivably affect both the performance and cost of educational systems. Since controlled experiments to establish the interrelationships of a large number of variables in education are costly, research surveys have been used to ascertain the contribution of inputs to output. These surveys measure inputs such as pupil-teacher ratios and instructional materials that influence educational output and that are under the control of school administrators. However, administrators should also take into account other variables or inputs such as student background which must be considered as given in the short run. A few studies have recognized this relationship between instrumental and background variables.

Educational Output and Its Measurement

Although economists have long been concerned with output and its measurement, only recently have they tried to define and measure the output of an educational system. The many quality dimensions of educational output make its measurement particularly difficult, because there is no system of weighting outputs to get a single measure of output.

Lacking measures of output, researchers frequently use inputs such as expenditures per pupil to measure schooling output. Readily observable physical inputs were first used to approximate educational quality. Using the Metropolitan St. Louis survey, Hirsch quantified inputs which were assumed to indirectly measure educational output. ${ }^{1}$ For comparison, the school districts were ranked according to each input and ranks were summed by school districts on all factors to form a composite rank. Those districts which were generally recognized as "being among the bestl' in the area had the highest index scores.

Hirsch's index of educational inputs consisted mainly of factors used in educating students. Schmandt and Stephens thought schooling subfunctions or activities were more closely related to schooling quality. ${ }^{2}$ Activities were measured in terms of the number of school services offered. While these items influence output, they are the ingredients and not the measure of production.

Dollar inputs are frequently used to measure educational quality. Zimmer used current operating expenditures per student in average daily attendance to measure educational quality. ${ }^{3}$ Total expenditures make no distinction between those expenditures which directly affect educational output and other expenditures such as transportation. Instructional costs, on the other hand, would be more closely correlated with educational output. However, an expenditures approach is justifiable only under conditions of linear homogeneity in production and equal efficiency among schools in resource allocation. ${ }^{4}$ If there are economies or diseconomies of size, school systems of different size have different outputs for the same expenditure per pupil.

Standardized achievement tests have been used as one measure of educational output. These tests have shortcomings such as middle-class orientation but avoid some of the limitations inherent in the use of educational inputs as measures of educational output. Standardized tests of achievement have been used by two national research surveys to assess over-all performance in subject areas as a measure of educational output. Project TALENT involved over $4,000,000$ high school students in a representative sample of the country's high schools. 5 Data were obtained on student achievement scores, as well as outcomes such as college attendance, dropout, absentee, and delinquency rates. The Equal Educational Opportunity Survey, as presented by the Coleman Report, examined data from 5,000 schools on student achievement and its relationship to the school. 6 Both studies obtained measures of the student's socioeconomic background which were held constant so that realistic comparisons of schools operating under different conditions could be made.

## Educational Production Function

The production function, a conceptual tool which can be used to analyze efficiency problems in the education industry, expresses the physical relationship between inputs and outputs. The production function for education describes a multitude of choices open to school administrators. ${ }^{7}$ It shows the output that various levels and combinations of inputs will produce for a given state of technology and environmental conditions. Knowledge of these production possibilities allows economic principles to be applied to factor substitution.

The estimation of production functions has been limited by the availability of quantifiable outputs and meaningful measures of inputs. To estimate input-output relationships within school systems, it is essential to control for the effect of family and community on educational outputs. 8 Economic analyses must be different from benefit-cost studies of general education which have not separated in-school from out-of-school influence on educational outputs.

The Project TALENT data indicated that the most important factors associated with school outcomes - achievement, going to college and staying in school - were teacher salaries, teacher experience, number of books in school library, and per-pupil expenditure. ${ }^{9}$ These four factors remained important even after region, rural-urban status, and such socioeconomic factors as median family income and quality of family housing were held constant. Many other factors appeared unlikely to have much influence on school outcomes considered in this study. Among these factors were school size, average size of classes, age of building, and suburban location.

The Coleman Report regressed student performance on student-body composition, facilities, curriculums, and teacher characteristics. Student achievement was related to factors within the school, as well as family background and student attitudes. Background variables parent's education, living pattern, and interest in students schoolwork - were highly correlated with student achievement. The schools showed little statistical effect on student achievement when the socioeconomic background of the students was taken into account. Teacher characteristics showed a stronger relationship to achievement than other school
factors. Little variation in student achievement was accounted for by variation in school facilities and curriculum.

## Size-Performance Relationships

Student performance may vary systematically with the size of
school. A broader curriculum and teacher specialization made possible through larger enrollments are major factors in improving performance. The introduction of ability groupings and specialized inputs such as guidance and reading specialists generally associated with larger enrollments are also designed to increase student performance. One possible source of diseconomies in education may result from crowding and from less contact by teachers with students on an individual basis. Jackson used schooling input data on all southern high schools to study the relationship between high school size and program quality. ${ }^{10}$ The adequacy of the instructional program was influenced by factors such as training and certification of teachers, teaching load, and scope of the curriculum. The relative effectiveness of schools of various sizes were evaluated by separating the schools into categories of size. These results showed that small schools were unable to provide a satisfactory educational program. Course offerings were particularly limited in small schools. Teacher qualifications - certification and advanced degrees - improved with increases in enrollment. Although informative, this study measured the relationship between school size and schooling inputs rather than outputs.

Street, Powell, and Hamblen examined the relationship between performance of elementary students on standardized achievement tests and school size in eastern Kentucky. ${ }^{11}$ Within a district, large schools had
significantly higher mean achievement scores than small schools. Further, they found larger schools tend to have more facilities, attract better-prepared teachers, and be located in areas where there are more educational opportunities for students. Although these factors were not controlled for, it was felt that they were actually responsible for the relationship of school size to achievement.

Kiesling's study of school districts in New York used data from the Quality Measurement Project. ${ }^{12}$ This analysis related student performance to school district size while correcting for the influences of pupil intelligence and socioeconomic background. After school districts were stratified by socioeconomic background, average achievement scores were regressed against size, expenditures, and pupil intelligence. Holding socioeconomic background and pupil intelligence constant, achievement decreased at a decreasing rate with increases in district size.

Data from the Project TALENT Sample were used to establish a relationship between school size and student performance. ${ }^{13}$ In general, the size-performance relationship is not very well defined until the effects of other factors are controlled. With the introduction of intelligence, socioeconomic index, and high school expenditures into the multiple regression equation, however, the size-performance relationship becomes consistently negative. School size per se is somehow detrimental to better school performance. Uncorrected student performance seems to attain a maximum between 1200 and 1600 pupils in $A D A$ and then to decline. After the three control variables have been introduced, student performance declines with each successive increase in size.

## Expenditure-Performance Relationships

Communities hope that increased expenditures will improve the quality of their school systems. A number of studies have analyzed the output of schools with different expenditure levels. In these early studies, all inputs were usually aggregated under the heading of "total expenditures per pupil."

Kiesling's Project TALENT study found a positive relationship between achievement scores and expenditures levels. ${ }^{14}$ The relationship was quite strong when the intelligence, size, and socioeconomic variables were not in the equation. The effect of expenditures on performance net of the three other variables, however, was small and in some cases not significant. This approach makes no distinction between rural and city school districts which may have the same expenditures but different programs.

To determine which measures of expenditures were related to educational quality, Stinson and Krahmer analyzed the correlation between 24 separate expenditure variables and student achievement tests in 80 per cent of North Dakota's school districts. ${ }^{15}$ A statistically significant correlation existed between some measures of per-teacher expenditure and achievement test scores. The three most useful expenditure measures were instructional cost, operating cost less transportation, and total cost less transportation. This correlation appeared consistent among grades and among different high school sizes. No statistically significant relationships existed between any measure of per-pupil expenditure and achievement test scores.

## Educational Costs and Cost Functions

This section examines the cost concepts and empirical studies of educational costs. The large variation in educational programs of different schools requires that quality of the educational program be held constant in making valid comparisons among schools.

Much of the interest in school size is focused on per-unit costs associated with schools of different sizes. For a given program, large schools may have cost advantages over smaller ones. These decreases in unit costs associated with increases in size are called economies of size. This cost reduction arises in education mainly through declining average fixed costs achieved by better utilization of administration, teachers, plant and equipment.

Using school-input variables, Katzman derived two short-run inputcost relationships - current costs per ADM (average daily membership) and instructional costs per ADM. ${ }^{16}$ The percentage of teachers with one to ten years experience had little effect on costs. Increases in median class size, crowding, and student-staff ratios tended to reduce expenditures per student. Both instructional costs and current costs showed significant diseconomies of size at all levels of utilization in the Boston area.

Wright and Pine analyzed factors affecting costs per pupil for 108 rural high school districts in central Kansas. ${ }^{17}$ Although measures of the quality of education are important, they were not included. The teacher-pupil ratio explained more than 90 percent of all the variations in instructional costs per pupil. Other factors which significantly affected costs were curriculum, district assessed valuation, value of plant and equipment, and average daily attendance. School size
accounted for most of the variations in plant operation costs per pupil. Significant cost reductions could have been achieved by enlarging the district size. Consolidation depends in part on time and cost of transporting pupils; however, transportation was not included in this study.

In Hanson's study of 577 districts located in nine states, the unit costs corrected for population characteristics consistently declined as district size increased from 1,500 to 20,000 pupils. 18 He also found substantial but less consistent support for diseconomies of size beyond the optimum size. However, three states did not reveal any tendency for costs to rise in extremely large school districts.

Although early analyses of economies of size in school operation used a school district as the unit of observation, Riew utilized data on individual public high schools located in Wisconsin. 19 Variations in educational programs and qualities among schools were reduced by considering only accredited schools with typical teacher salaries. More than one-half of the variation in average operating expenditures among the 109 high schools was accounted for by six independent variables of which average teacher's salary, enrollment and changes in enrollment were statistically significant. Almost 20 percent of the variation in per-pupil operating expenditures was explained by variation in enrollment. An increase in enrollment with other variables held constant decreased average per-pupil operating expenditures until enrollment reached 1,675 .

Cohn evaluated the per-pupil cost function for a sample of 377 Iowa high school districts. ${ }^{20}$ To examine economies of size in high school operations, he estimated the longørun average cost curve. A
long-run period is a time interval in which the school district size can be changed. The long-run average cost curve is a function of schooling outputs and inputs. Incremental test scores were used as a proxy for educational quality. The study of per-pupil costs indicated the existence of significant economies of size for Iowa high schools.

## Cost-Effectiveness Analysis

In one of the few studies to deal with efficiency problems, Levin applied cost-effectiveness analytic techniques to decisions on teacher recruitment. 21 Teacher characteristics were related to student achievement and average costs. Recruiting teachers with verbal scores was five to ten times as effective per dollar in raising achievement scores as obtaining teachers with more experience. His analysis was only designed to be illustrative. School administrators need this type of information on many more educational inputs.

Summary

Useful measures of educational inputs and outputs have evolved from previous studies on educational production functions. These studies have narrowed the number of variables which conceivably affect educational output. By quantifying the effect various inputs have on educational output, they provide some insight into resource allocation. Policy guidelines as a result of these studies must of necessity be limited because they were not related to efficiency problems.

Most educational cost studies relied entirely on regression techniques, which made little allowance for changes in factor proportions. Also, these studies made little attempt to control for educational
quality. In many cases, factors affecting a district's ability to support education were found to affect the cost of an educational program.

No study has successfully combined production functions and cost functions to provide insights on efficient resource allocation in an educational system. Even the discussion of a theoretical framework for such analysis has been limited. The present study provides a theoretical discussion and an empirical application of the theory of the firm to the problem of efficient resource organization in an educational system.

Economists offered few guidelines to help direct the school district consolidation that has been occurring. Even those studies that were applied in rural areas have ignored transportation costs, an important component of educational costs in sparsely populated areas. These previous studies have provided many misconceptions for policy purposes on school district organization by not taking student density into consideration. The present study relates minimum size of school district to student density and other characteristics of the geographic area.

This study will estimate production functions for achievement scores, absentee and dropout rates. Cost relationships which allow for factor substitution and control for quality will also be developed. The production functions and cost functions will be combined to find the most efficient method of producing a given output. The study will simultaneously determine the optimum organization of resources within a school district, as well as the optimum school district size.

## FOOTNOTES

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\section*{CHAPTER III}

\section*{INTERNAL ECONOMICS OF EDUCATION}

An efficient educational system is defined here as one which combines inputs to produce a given quality of schooling at minimum cost. \({ }^{1}\) A school district can be visualized as a firm utilizing capital and labor to produce its output. It must compete with other firms in purchasing its inputs. It is broadly concerned with efficient use of resources to produce its product. However, there is no well-defined product nor a price that can be attached to its product, and most of its product is not directly sold. Users (students) exert little pressure to increase efficiency, not only because they lack mature judgment and are unaware of input-output relationships (often inputs as well as outputs are ill-defined and uncertain) but also because efficiency gains are not passed to users. Furthermore, factors outside the school system influence output throughout the long production process. Even if the price mechanism does not work, economic principles can still be used in education to allocate resources efficiently. This chapter identifies educational inputs and outputs and relates their use to relevant economic principles which can be followed to improve a school system's efficiency of operation.

Educational Production Functions

Input-output relationships can be used to determine the
consequences of changing input combinations. Output variables ideally should represent what the school system is attempting to produce. Input variables ideally should include all factors which influence output of the school system. However, the variables included in the subsequent model are restricted to those having a measurable and significant effect on the performance of the system. Performance is not measurably sensitive to a great number of variables.

The characteristics of a production function are: (1) the law of diminishing returns and (2) economies of size. The law of diminishing returns states that successive increments of equal size in the use of one resource per unit of time, holding other resources constant, will produce smaller increases in output beyond some point. \({ }^{2}\) This law may be illustrated in education by the fact that doubling the number of teachers per class is not likely to result in a doubling of the class achievement scores. A statistical estimate of the production function may not exhibit diminishing returns because diminishing returns may not have been observed in the sample used in the estimation procedure. However, one must be aware that diminishing returns do exist somewhere beyond the range of input levels observed in the sample.

Scale factors are not readily observable in education because the quality of output is not constant as the size of the school increases. \({ }^{3}\) Since questions of school size and efficiency are important, however, size of school should be included in the model explaining educational output. Physical economies of size may result from division and specialization of labor, while diseconomies may result from crowding, lack of supervision, and other sources.

Educational Output

While the quality of a school's program is ultimately measured by the behavior it produces in the students who pass through it, this study measured output by achievement test scores, dropout rates, and absentee rates. Omission of less easily quantified outputs, such as personality and citizenship, is in no way meant to slight their importance but rather stems from the fact that inadequate data preclude their use in the analysis. Even though the variables used to measure output have imperfections, and other measures of output exist, these variables are believed to be important. Schools characterized by high achievement scores, low absentee rates, and low dropout rates are assumed to have a higher "output" than schools with the opposite characteristics.

\section*{Educational Input}

Three general factors determine the output of an educational system - student input, educational process, and environment. Student input variables are defined as the characteristics and level of attainment of students at the beginning of an educational program. Educational process variables are all the activities in a school that are designed to raise students' level of attainment. Environmental variables are circumstances in the community and the home that facilitate or impede the educational process.

Many more factors affect output in elementary and secondary education than can be included in a statistical model. The specific variables included in the study were thought to have a causal relationship with educational output. To preserve structural validity, the model explaining educational output was specified as a hierarchy of admissible
hypotheses (variables). There are two echelons of inputs into the educational system similar to Tinbergen's conceptualization of factors affecting economic policies. \({ }^{4}\) First, the school board has at its disposal an array of policy instruments or instrumental variables which it can use to reach certain objectives. There are, however, a large number of factors whose values are determined outside the school system; these factors are control variables. All the input variables are listed in Table I.

Instrumental Variables. The educational process variables are the focal point of the allocative analysis and are placed in the first echelon. The importance of the ranking will be apparent in subsequent statistical analysis. These educational process variables include program offerings, teaching innovations, instructional materials, and teacher characteristics.

Teacher characteristics which affect student performance include workloads, qualifications, and salaries. Average teacher experience, percentage of teachers by years of experience, standard salary \({ }^{5}\), and percentage of teachers with a master's degree were the variables used to reflect teacher qualifications. Pupil-teacher ratios and percentage of teachers with a planning period were used to measure teacher workload.

The scope of an educational program is defined as the number and type of course offerings and special services provided by the school. The existence of an adequate kindergarten may improve student performance in elementary education. Team teaching, ability groupings, special education classes, remedial classes, and accelerated classes reflect teaching innovations and special efforts of the school to meet individual student needs.

\section*{OUTPUT VARIABLES}

Achievement test scores
Absentee rates
Dropout rates

\section*{INPUT VARIABLES}

\section*{Educational Process Variables}
```

    Use of team teaching
    Use of ability groupings
    Percentage of students in special education classes
    Percentage of students in accelerated classes
    Availability of adequate kindergarten
    Availability of adequate science laboratory
    Availability of adequate language laboratory
    Availability of adequate industrial shop
    Number of printed volumes per pupil
    Number of periodicals
    Value of audiovisual material
    Classroom teachers' education
    Classroom teachers' experience
    Average district classroom teacher salary
    Standard district classroom teacher salary
    Pupil-teacher ratio
    Percentage of teachers with a planning period
    Number of nonvocational units offered
    Number of vocational units offered
    ```
Student Input Variables

Race
Student scores on Otis-Lennon Mental Ability Test
Time students spent studying outside of school
Type of community in which students have spent most of their lives
Student workloads outside of school
Post high school educational plans
Number of books read during the last summer
Number of times students changed schools
Environmental Variables

Parents \({ }^{\wedge}\) education
Parents \({ }^{\triangleright}\) occupation
Parents \({ }^{\circ}\) net income
Plans for children's education
Number of times parents talk with children about school work

Capital services - plant, grounds, equipment, and instructional materials - are an important factor in an educational system. The volume of capital services actually used is difficult to measure, but the inclusion of approximate measures of capital services in the model appears desirable. Some measures of facilities such as current value of plant and grounds \({ }^{6}\) and crowding \({ }^{7}\) show little impact on educational output. More appropriate measures may be critical facilities: (a) science laboratory, (b) language laboratory, and (c) industrial, vocational, or technical shop. These measures are expected to correlate positively with other measures of the over-all adequacy of facilities.

The number of periodicals, printed volumes per pupil, and value of audiovisual material per pupil measure an important part of the capital stock of a school. Again, the existence of these items does not insure that they are used extensively in the educational program. However, their existence and use are positively related. Since the number and value of these items has no quality dimensions, their usefulness in statistical analysis is further reduced. In any case, an attempt will be made to measure the impact of these capital items on educational output.

Control Variables. The second echelon of inputs into the educational system include those control variables - student input and environmental variables - which at least in the short run must be considered fixed. The student input and environmental variables are interrelated, making it difficult to distinguish their separate effects on output. For this reason, the two types of variables are grouped under one category. The school board and administration must take into account the effect of these control variables on educational output when determining the levels of instrumental variables.

The output of the educational program depends heavily on the quality of students that enter the program. Student aptitude, measured in the study by scores on the Otis-Lennon Mental Ability Test, indicates the student's potential for academic performance. Given the academic potential, motivation is important to realize high levels of achievement, but motivation is difficult to assess. It is hypothesized that there is a direct relationship between a student's commitment to education and his performance. Variables used to measure this commitment are (1) time spent studying outside of school, (2) the number of books the student read the previous summer, and (3) his post high school educational plans. Admittedly, the school has an influence on homework through assignments and post high school plans through counseling.

Time allocated to work decreases time which would otherwise be available for studying and other educational activities. Therefore, it is hypothesized that time spent working outside of school is inversely related to student performance.

The quality of a school's program is influenced by the continuity of students in the school system. Thus, another student input variable considered is residence patterns - type of community in which a student has spent most of his life and number of times he has changed schools. Student performance may suffer from changing schools several times. Home environment is expected to influence student performance. Environmental variables included in the analysis are parents' education, occupation, income, and interest in their children's education. It is hypothesized that a direct relationship exists between student performance and family status variables - parents' education, occupation, and income. Parents' education serves as a proxy for informal education
provided in the home, and average income reflects the family's ability to support the student's education. The interest that parents show in their children's education may influence student performance. Variables used to reflect this interest are the number of times each week that a student talks to his parents about schoolwork and parents' plans for their children's future education.

\section*{Educational Cost Functions}

The costs of production incurred by a school district were broken into (1) costs of education (instruction and attendant activities) and (2) transportation. Educational costs include expenditures on teachers, textbooks, supplies, administration, plant operation and maintenance, buildings, and equipment. The cost function expresses the functional relationship between output and cost.

The cost function depends on the production function which underlies it. This analysis contains school district's cost of production at various output levels. Cost of each output level is based upon (1) resource price and (2) efficiency with which school districts use the resources. \({ }^{8}\) The resource prices are assumed to be fixed for each individual school district. For example, the school district can hire additional teachers as needed at its current salary schedule.

Changes in cost of production associated with changes in output depend on the period of time under consideration. The short run is a time interval long enough to permit changes in output without altering the scale of plant. \({ }^{9}\) The long run, to which most estimates in this study apply, is a period of sufficient duration to allow a school to vary output by changing either the resource combination, district size,
or the scale of plant.
The long-run average cost curve shows the minimum cost of producing various outputs. In other words, a school district can operate on the long-run average cost curve only if resources are combined efficiently for each level of output. \({ }^{10}\) Much concern over school district size relates to per-unit costs associated with educating students in school districts of different sizes. The long-run average cost curve for school districts is thought to exhibit both economies and diseconomies of size.

Internal economies of large-scale production are primarily a longrun phenomenon, dependent upon appropriate adjustment of plant scale to each successive output. \({ }^{11}\) When the size of school district is large, greater teacher specialization may occur with each teacher instructing those classes in which he is most qualified. An illustration of technical internal economies would be the savings in teacher requirements per student made possible by a larger scale of operations. Figure 1 depicts economies of size in educational costs.

On the other hand, increases in the size of school district can involve less efficient operation and, consequently, higher unit costs. These diseconomies are not likely to be very important in elementary and secondary education. The major diseconomies accrue from transportation costs. School districts must provide transportation for all students in the district who live more than 1.5 miles from school. Such costs are particularly prominent in sparsely populated areas. The average cost of transportation for a given population density is also shown in Figure 1.

An examination of a school district's long-run cost curve provides


Figure 1. Average Cost of Education and Transportation Per ADA
some insight into how large the school district should be. The optimum size of school district is defined as that which has minimum long-run average costs with resources combined in a least-cost manner. \({ }^{12}\)

\section*{Least-Cost Combination}

The school district can combine factor inputs in varying proportions to produce its output. The problem facing the school district is to use factor inputs in such a way that, whatever the output produced, the cost will be a minimum. In mathematical terms, the problem is one of constrained cost minimization in which the restrictions on output are the constraints. The economic principles derived are also applicable to maximizing the level of output for a given cost outlay.

Let the school district's cost function and production function be represented, respectively, by:
\[
\begin{aligned}
& C=f\left(X_{1}, X_{2}, \ldots, X_{n}\right) \\
& Q=g\left(X_{1}, X_{2}, \ldots, X_{n}\right)
\end{aligned}
\]
where \(X_{i}\) is the quantity of the \(i^{\text {th }}\) input, \(C\) is cost, and \(Q\) is output. The school district will try to produce a given output \(Q^{*}\) at lowest possible cost. The Lagrangian expression for this constrained minimization problem is:
\[
V=f\left(X_{1}, X_{2}, \ldots, X_{n}\right)+\lambda\left(Q^{*}-g\left(X_{1}, X_{2} \ldots, X_{n}\right)\right)
\]

This expression is minimized by setting each of the partial derivatives equal to zero as follows:
\[
\frac{\partial V}{\partial X_{1}}=\frac{\partial f}{\partial X_{1}}-\lambda \frac{\partial g}{\partial X_{1}}=0
\]
\[
\begin{gathered}
\frac{\partial V}{\partial x_{2}}=\frac{\partial f}{\partial x_{2}}-\lambda \frac{\partial g}{\partial x_{2}}=0 \\
\cdot \\
\cdot \\
\frac{\partial V}{\partial x_{n}}=\frac{\partial f}{\partial x_{n}}-\lambda \frac{\partial g}{\partial x_{n}}=0 \\
\frac{\partial V}{\partial \lambda}=Q^{*}-g\left(x_{1}, x_{2}, \ldots, x_{n}\right)=0
\end{gathered}
\]

The marginal physical product of input \(X_{1}\) and the marginal cost of input \(X_{1}\) are represented by \(\partial g / \partial X_{1}\) and \(\partial f / \partial X_{1}\), respectively. These simultaneous equations can be solved for the optimum values of the \(n\) inputs and \(\lambda\).

\section*{Factor-Product Conditions}

If a dollar value can be assigned to output, then the optimum schooling plan calls for expansion of factor use until marginal cost is just equal to the marginal value of output. Even though educators are unwilling to assign dollar values to output in determining its level, information on the cost of an additional unit of output may be useful to compare with other marginal costs. The simultaneous equations can be solved for \(\lambda\) so that:
\[
\lambda=\frac{\partial f / \partial x_{1}}{\partial g / \partial x_{1}}
\]

This ratio of marginal cost of input \(X_{1}\) to marginal physical product of input \(X_{1}\) represents the change in cost brought about by a one-unit change in output, and is the marginal cost of another unit of output.

\section*{Factor-Factor Conditions}

Given the desired level of output, the problem faced by the school administrators is how to allocate all resources in producing that level of output. Solving the above simultaneous equations yield:
\[
\frac{\partial g / \partial x_{1}}{\partial f / \partial X_{1}}=\frac{\partial g / \partial x_{2}}{\partial f / \partial x_{2}}=\cdots \frac{\partial g / \partial x_{n}}{\partial f / \partial x_{n}} .
\]

The cost of production is minimized by combining factor inputs in such a way that the marginal physical product of a dollar's worth of one input is equal to the marginal physical product of a dollar's worth of every other input used.

\section*{Product-Product Conditions}

The educational production process yields more than one output. The case of joint products, exists whenever the quantities of two or more outputs, such as achievement scores, absentee rates, and dropout rates, are technically interrelated. Restrictions on the levels of these additional outputs can be incorporated into the above model by imposing additional constraints. The added equimarginal principles that must be met to minimize costs are:
\[
\frac{\partial g_{1} / \partial x_{1}}{\partial f / \partial x_{1}}=\frac{\partial g_{2} / \partial x_{1}}{\partial f / \partial x_{1}}=\ldots \frac{\partial g_{n} / \partial x_{1}}{\partial f / \partial x_{1}}
\]

The input \(X_{1}\) should be used in the production of each of the \(m\) outputs to the point that the marginal physical product of \(X_{1}\) in the production of output \(Q_{j}\) per dollar's worth of input \(X_{1}\) is the same for all outputs.

\section*{Theoretical Applications}

This study will apply the economic principles derived above to select the least-cost combination of inputs for a given quality of education. The possible range of substitution among outputs will also be estimated. Assuming that the number of students is variable, the minimum point on the long-run average cost curve, such as point \(A_{1}\) in Figure 2, shows the optimum size of school district. Each long-run average cost curve in Figure 2 is based on a different student density. The less densely populated areas have greater transportation costs associated with busing a given number of students over large distances. Thus, the three curves are long-run average cost curves for the same quality of education, but different student densities. The curve showing the optimum school district size for the different student densities, which is presented in Figure 3, can be derived from Figure 2. Before applying these principles, sources and profiles of data used in the analysis are presented in the following chapter.


Figure 2. Long-run Average Cost Curves for Alternative Student Densities ( \(\mathrm{SD}_{\mathrm{i}}\) )


Figure 3. Optimum Size of School District for Alternative Student Densities
\(1_{\text {Output }}\) is also a dimension of efficiency. Although this study did not determine efficient levels of output, it did examine the impact of changes in educational output.
\({ }^{2}\) Richard H. Leftwich, The Price System and Resource Allocation, 4th ed. (Hinsdale, Illinois, 1970), p. 117.
\({ }^{3}\) Jesse Burkhead, Thomas G. Fox, and John W. Holland, Input and Output in Large-City High Schools (New York, 1967), p. 20.
\({ }^{4}\) Jan Tinbergen, On the Theory of Economic Policy (Amsterdam, 1952), pp. 6-7.
\({ }^{5}\) Salaries of teachers were standardized to the same experience and degree level.
\(6_{\text {Burkhead, Fox, and Holland, pp. 39-76. }}\)
\({ }^{7}\) Martin T. Katzman, "Distribution and Production in a Big City Elementary School System," Yale Economic Essays, Vol. 8, No. 1 (Spring, 1968) , pp. 218-220.
\({ }^{8}\) Leftwich, p. 146.
\(9^{9}\) Jacob Viner, "Cost Curves and Supply Curves," Readings in Price Theory, ed. George J. Stigler and Kenneth E. Boulding, American Economics Association (Homewood, Illinois, 1952), p. 201.
\({ }^{10}\) Leftwich, pp. 160-162.
\({ }^{11}\) Viner, p. 213.
\({ }^{12}\) George J. Stigler, The Theory of Price, 3rd ed. (New York, 1966), p. 159.

\section*{CHAPTER IV}

\section*{DATA}

The data used herein were obtained by the Oklahoma State Department of Education which evaluated student needs in Oklahoma schools. The statewide study assessed school programs, student backgrounds, and student attainment. The data were collected to determine the relationship of student performance to educational process, student input, and environmental variables. Broad categories of variables were included in the study to assess general effectiveness of educational programs irrespective of individual school district goals.

\section*{Sampling Procedure}

To capture the wide differences in characteristics that are known to exist in Oklahoma's public elementary and secondary schools, a stratified random sample of the state's school districts was used. The population of school districts was first divided into 16 populations or strata according to geographic location: the Tulsa area and Oklahoma City area, northwestern, northeastern, southwestern, and southeastern Oklahoma. Figure 4 shows these geographic regions. Each region, except the Oklahoma City and Tulsa areas, was then divided into three strata according to school district sizes: under 500 students, between 500 and 2,000 students, and over 2,000 students. The Oklahoma City and Tulsa areas were divided into two strata: regular districts and Title I


Figure 4. The Geographic Regions From Which the Sample Was Drawn
school districts.

The sample was designed to be proportional - sampling all students in each district until the number of districts included five percent of all students in each stratum. As shown in Table II, the actual sampling rate varied slightly among strata because of the "lumpiness" of school districts. For example, in the stratum containing large schools, adding or subtracting one school would change the sampling rate considerably. A sample of districts was drawn independently and randomly in each stratum.

\section*{Data Gathering}

After the schools were randomly drawn, counselors from the Oklahoma State Department of Education visited the schools in March, 1970, to supervise the administering of tests and questionnaires. Students in the eleventh grade of sampled high schools, and the students in the fourth and eighth grades of schools that "feed" into these high schools completed standardized achievement and IQ instruments and filled out a questionnaire. The questions, as shown in Appendix A, included socioeconomic background, activities, educational preparation, and post high school plans. The numbers of students surveyed in the fourth, eighth, and eleventh grades were \(2,255,1,993\), and 1,903 , respectively. The parents of the students also filled out a questionnaire on their education, occupation, income, and residence. This questionnaire is shown in Appendix B. In addition, data were acquired from school administrators concerning finances, program, organization, facilities, equipment, teacher qualifications, and community characteristics. The administrators' questionnaire is shown in Appendix \(C\).

\section*{DESCRIPTION OF POPULATION AND SAMPLE BY STRATA}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{Strata} & \multirow[b]{2}{*}{ADA in Strata} & \multirow[b]{2}{*}{ADA of Districts Sampled} & \multirow[t]{2}{*}{ADA of Districts Sampled as \% of Stratum ADA} & \multicolumn{3}{|l|}{Percentage of Students Sampled in the} \\
\hline & & & & Fourth Grade & Eighth Grade & Eleventh
Grade \\
\hline Metropolitan Areas & 136,897 & --- & --- & 3.102 & 1.261 & 2.181 \\
\hline \multicolumn{7}{|l|}{Southeast} \\
\hline Greater than 2000 students & 18,312 & 4,739 & 25.879 & 16.943 & 20.844 & 22.874 \\
\hline 501-2000 students & 40,431 & 2,664 & 6.589 & 5.339 & 5.926 & 6.144 \\
\hline O-500 students & 41,042 & 1,801 & 4.388 & 3.895 & 4.206 & 3.896 \\
\hline \multicolumn{7}{|l|}{Southwest} \\
\hline Greater than 2000 students & 49,695 & 3,569 & 7.182 & 6.457 & 4.975 & 5.557 \\
\hline 501 - 2000 students & 31,677 & 1,386 & 4.375 & 3.978 & 3.782 & 3.838 \\
\hline O-500 students & 24,428 & 1,034 & 4.233 & 3.094 & 3.462 & 4.003 \\
\hline \multicolumn{7}{|l|}{Northeast} \\
\hline Greater than 2000 students & 42,089 & 2,634 & 6.258 & 4.713 & 4.799 & 6.310 \\
\hline 501-2000 students & 40, 155 & 1,881 & 4.684 & 3.894 & 4.183 & 4.574 \\
\hline o-500 students & 25,392 & 1,237 & 4.872 & 3.676 & 4.857 & 4.371 \\
\hline \multicolumn{7}{|l|}{Northwest} \\
\hline Greater than 2000 students & 68,257 & 2,857 & 4.186 & 3.487 & 3.028 & 2.419 \\
\hline 501-2000 students & 13,006 & 1,003 & 7.712 & 7.092 & 5.420 & 5.793 \\
\hline O-500 students & 21,739 & 593 & 2.728 & 2.308 & 1.837 & 4.255 \\
\hline
\end{tabular}

Additional information on transportation, course enrollment, and costs by functions for the sampled school districts was obtained from the State Department of Education's files. Most of this data were from school district annual reports.

Complete data were acquired from 27 independent Oklahoma school districts. The size of the school districts sampled ranged from under 100 pupils to more than 70,000 pupils. The stratification insured that every geographic region in the state and school district size was represented in the sample.

\section*{Biases}

Even though 91.8 percent of the students who took achievement tests returned both parent and student questionnaires, there may have been some biases introduced into the data from missing observations associated with the unreturned questionnaires. It is useful to examine how the results may be affected by missing information. The first issue is whether real differences exist between the performance of those students with complete information and those with missing information. Secondly, it is important to decide if any observed differences are likely to bias the results.

The mean composite score of students with complete information was compared to the mean composite scores of students with incomplete student questionnaires and incomplete parent questionnaires. The summary information is shown in Table III. In all three grades, students with complete information had higher average composite achievement scores than students with missing questionnaires. The means of these groups were significantly different at the . 01 level in every instance. The

\section*{TABLE III}

\section*{SELECTED INFORMATION RELATED TO DATA \\ BTAS}
\begin{tabular}{ccc}
\hline & & Average \\
Subpopulations & Number of & Composite \\
Observations & Score & Deviation of \\
\hline
\end{tabular}

\section*{Fourth Grade}
\begin{tabular}{lrrr} 
Complete Information & 2019 & 157.506 & 46.629 \\
Missing Student Questionnaire & 6 & 97.833 & 47.998 \\
Missing Parent Questionnaire & 230 & 140.917 & 46.041 \\
Eighth Grade & & & 72.152 \\
\hline Complete Information & 1854 & 175.553 & 50.907 \\
Missing Student Questionnaire & 10 & 133.200 & 81.186
\end{tabular}

\section*{Eleventh Grade}
\begin{tabular}{lrrr} 
Complete Information & 1776 & 80.676 & 24.280 \\
Missing Student Questionnaire & 9 & 64.778 & 25.464 \\
Missing Parent Questionnaire & 118 & 72.373 & 22.124
\end{tabular}
greatest difference was between students with complete information and students with missing student questionnaires. However, there were only 25 such students in the entire sample.

Without complete information on these students, it is difficult to know what characteristics are responsible for the lower achievement scores. It seems likely that the results will be slightly biased. However, the number of incomplete questionnaires is so small that the conclusions of the study are not likely to be significantly affected.

\section*{Profile of Sample}

The educational output variables used in this study are each district's average absentee rate, dropout rate, and average achievement scores. The absentee and dropout rates are averages for the schools included in the sample rather than for the students sampled. The achievement scores are average school district scores made on Science Research Associates' standardized achievement tests. The tests were administered to students in the fourth, eighth, and eleventh grades. The modern math and language arts scores in all three grades are district averages. The fourth and eighth grade composite scores are made up of test scores on social studies, science, language arts, math, and reading. The eleventh grade composite score is the average district score on social concepts, natural science, expression, quantitative thinking, and reading.

Even though observations were made at the individual student level, the school district is used as the unit of observation to provide guidelines for school district resource organization. Since this study is concerned with school characteristics, using the school district as the unit of observation has a major advantage - that of weighting all
districts equally so that a school district with many pupils cannot bias the results in comparison to using the pupil as the unit of observation. This procedure also has a disadvantage, however, in that a great deal of information is lost through aggregation. For example, the effect of IQ is very important at the individual level, but the averages "wash out" individual student differences and decrease its importance at the district level.

It is useful to compare results of regression equations with the pupil and the school district as units of observation. In the former case, individual pupil achievement scores were regressed on individual pupil characteristics. The resulting relationship provides information to predict each pupil's achievement score given his characteristics of background and schooling. With the school district as the unit of observation, average school district scores were regressed on district averages for educational process and background variables. The equation can be used to predict a given school district's average achievement score, making use of information on its schooling variables and average student characteristics. Almost all of the explained variation in the first equation was attributable to student IQ. All other variables included in this equation explained a very small percentage of the variation in achievement scores. With the school district as the unit of observation, the estimated equation indicated schooling inputs had a much greater impact on average achievement scores.

Tables IV, V, and VI present data which describe the educational outputs by strata for each of the three grades. In general, absentee and dropout rates are higher in the smaller school districts. Northwestern Oklahoma and the Tulsa and Oklahoma City areas consistently had

TABLE IV

FOURTH GRADE EDUCATIONAL OUTPUTS BY SAMPLING STRATA
\begin{tabular}{|c|c|c|c|c|c|}
\hline Strata & \begin{tabular}{l}
Number of \\
Students \\
Sampled
\end{tabular} & \begin{tabular}{l}
School \\
Average \\
Absentee \\
Rate
\end{tabular} & \begin{tabular}{l}
Sample \\
Average \\
Language Arts Score
\end{tabular} & \begin{tabular}{l}
Sample \\
Average \\
Modern \\
Math Score
\end{tabular} & \begin{tabular}{l}
Sample \\
Average \\
Composite Score
\end{tabular} \\
\hline Metropolitan Areas & 364 & 5.00 & 283.11 & 284.74 & 157.91 \\
\hline \multicolumn{6}{|l|}{Southeast} \\
\hline 0-500 students & 266 & 3.96 & 277.93 & 279.51 & 158.38 \\
\hline 501-2000 students & 185 & 1.20 & 282.65 & 292.30 & 164.51 \\
\hline Greater than 2000 students & 137 & 2.39 & 252.85 & 270.28 & 133.52 \\
\hline \multicolumn{6}{|l|}{Southwest} \\
\hline 0-500 students & 275 & 4.10 & 281.77 & 291.32 & 161.09 \\
\hline 501-2000 students & 108 & 3.63 & 279.44 & 289.06 & 153.75 \\
\hline Greater than 2000 students & 84 & 4.63 & 268.60 & 275.71 & 146.20 \\
\hline \multicolumn{6}{|l|}{Northeast} \\
\hline 0-500 students & 170 & 5.00 & 274.72 & 288.48 & 159.63 \\
\hline 501-2000 students & 134 & 5.71 & 281.74 & 296.99 & 167.31 \\
\hline Greater than 2000 students & 80 & \(0.00{ }^{\text {a }}\) & 264.96 & 278.60 & 142.39 \\
\hline \multicolumn{6}{|l|}{Northwest} \\
\hline 0-500 students & 204 & 4.37 & 271.36 & 280.32 & 152.18 \\
\hline 501-2000 students & 79 & 3.00 & 273.73 & 299.76 & 158.58 \\
\hline Greater than 2000 students & 43 & 1.89 & 280.58 & 304.14 & 167.37 \\
\hline
\end{tabular}
\({ }^{\text {absentee rates for }}\) schools were recorded to the nearest percent.

TABLE V
EIGHTH GRADE EDUCATIONAL OUTPUTS BY SAMPLING STRATA
\begin{tabular}{|c|c|c|c|c|c|}
\hline Strata & \begin{tabular}{l}
Number of \\
Students \\
Sampled
\end{tabular} & \begin{tabular}{l}
School \\
Average \\
Absentee \\
Rate
\end{tabular} & \begin{tabular}{l}
Samp1e \\
Average \\
Language Arts Score
\end{tabular} & \begin{tabular}{l}
Sample \\
Average \\
Modern \\
Math Score
\end{tabular} & \begin{tabular}{l}
Sample \\
Average \\
Composite Score
\end{tabular} \\
\hline Metropolitan Areas & 147 & 5.00 & 365.68 & 472.76 & 203.39 \\
\hline \multicolumn{6}{|l|}{Southeast} \\
\hline 0-500 students & 325 & 3.00 & 356.45 & 443.44 & 192.52 \\
\hline 501-2000 students & 204 & 2.26 & 353.67 & 438.73 & 184.50 \\
\hline Greater than 2000 students & 147 & 2.22 & 326.41 & 417.80 & 163.19 \\
\hline \multicolumn{6}{|l|}{Southwest} \\
\hline 0-500 students & 215 & 4.00 & 359.76 & 463.66 & 200.71 \\
\hline 501-2000 students & 102 & 4.25 & 360.70 & 439.86 & 190.86 \\
\hline Greater than 2000 students & 72 & 4.74 & 348.93 & 429.50 & 175.50 \\
\hline \multicolumn{6}{|l|}{Northeast} \\
\hline 0-500 students & 172 & 5.00 & 356.69 & 450.53 & 191.00 \\
\hline 501-2000 students & 143 & 5.67 & 378.26 & 467.27 & 208.47 \\
\hline Greater than 2000 students & 105 & 0.70 & 321.53 & 427.09 & 163.37 \\
\hline \multicolumn{6}{|l|}{Northwest} \\
\hline 0-500 students & 176 & 5.00 & 366.04 & 458.82 & 201.30 \\
\hline 501-2000 students & 60 & 0.00 & 380.90 & 460.45 & 190.02 \\
\hline Greater than 2000 students & 34 & 1.89 & 365.62 & 446.91 & 198.21 \\
\hline
\end{tabular}

TABLE VI

ELEVENTH GRADE EDUCATIONAL OUTPUTS BY SAMPLING STRATA
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Strata & Number of Students Sampled & \begin{tabular}{l}
School \\
Average Absentee Rate
\end{tabular} & \begin{tabular}{l}
School \\
Average \\
Dropout \\
Rate
\end{tabular} & \begin{tabular}{l}
Sample \\
Average \\
Language Arts Score
\end{tabular} & \begin{tabular}{l}
Sample \\
Average \\
Modern \\
Math Score
\end{tabular} & \begin{tabular}{l}
Sample \\
Average Composite Score
\end{tabular} \\
\hline Metropolitan Areas & 226 & 5.97 & 5.13 & 428.26 & 442.33 & 87.60 \\
\hline \multicolumn{7}{|l|}{Southeast} \\
\hline 0-500 students & 317 & 5.00 & 4.00 & 428.10 & 414.52 & 80.95 \\
\hline 501-2000 students & 188 & 2.64 & 6.27 & 414.95 & 416.22 & 80.41 \\
\hline Greater than 2000 students & 121 & 3.01 & 3.54 & 383.72 & 386.15 & 62.58 \\
\hline \multicolumn{7}{|l|}{Southwest} \\
\hline 0-500 students & 209 & 5.00 & 5.00 & 421.40 & 407.80 & 79.24 \\
\hline 501-2000 students & 92 & 2.80 & 4.44 & 428.83 & 427.69 & 81.92 \\
\hline Greater than 2000 students & 74 & 3.71 & 4.54 & 402.29 & 392.73 & 72.62 \\
\hline \multicolumn{7}{|l|}{Northeast} \\
\hline 0-500 students & 201 & 5.00 & 5.00 & 419.84 & 419.09 & 78.57 \\
\hline 501-2000 students & 139 & 5.16 & 4.23 & 436.00 & 431.74 & 87.89 \\
\hline Greater than 2000 students & 84 & 1.07 & 1.13 & 406.26 & 407.38 & 73.14 \\
\hline \multicolumn{7}{|l|}{Northwest} \\
\hline 0-500 students & 125 & 2.00 & 3.00 & 417.45 & 413.38 & 77.21 \\
\hline 501-2000 students & 57 & 0.00 & 3.00 & 429.23 & 432.37 & 85.11 \\
\hline Greater than 2000 students & 70 & 0.75 & 0.00 & 441.33 & 427.64 & 87.57 \\
\hline
\end{tabular}
the highest achievement test scores. However, it is difficult to detect any substantial trends among school district sizes, geographic regions, or grades. The achievement scores are not standardized among grades, and cannot be used directly to compare progress among grades 4, 8, and 11.

Table VII presents simple correlation coefficients between educational outputs. The first number in the second column shows that the correlation between fourth grade absentee rate and language arts score is . 096. In general, the correlations between absentee rates and achievement scores are low in absolute value but statistically significant. The correlations show that composite scores in all grades are closely related. In the eleventh grade, dropout rates are negatively related to all achievement scores.

Simple correlation analysis gives some insight into which variables are associated with higher educational output. A list of selected variables with their means and standard deviations are shown in Table XXXVII of Appendix D. Tables XXXVIII and XXXIX show the correlation coefficients associated with these variables. However, correlation analysis does not take into account the interrelationships among all the variables. Subsequent chapters will address this problem.

TABLE VII
CORRELATION COEFFICIENTS OF EDUCATIONAL OUTPUTS
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline Variable & Absentee Rate \(_{4}\) & Language Arts \(_{4}\) & \begin{tabular}{l}
Modern \\
Math \(_{4}\)
\end{tabular} & Composite Score \(_{4}\) & Absentee Rate \(_{8}\) & Language Arts \(_{8}\) & \begin{tabular}{l}
Modern \\
Math \(_{8}\)
\end{tabular} \\
\hline Absentee Rate \({ }_{4}\) & 1.000 & 0.096 & 0.043 & 0.163 & 0.859 & 0.607 & 0.566 \\
\hline Language \(\mathrm{Arts}_{4}\) & & 1.000 & 0.595 & 0.865 & 0.089 & 0.461 & 0.580 \\
\hline Modern Math \({ }_{4}\) & & & 1.000 & 0.798 & -0.053 & 0.471 & 0.509 \\
\hline Composite \(\mathrm{Score}_{4}\) & & & & 1.000 & 0.099 & 0.588 & 0.707 \\
\hline Absentee Rate \({ }_{8}\) & & & & & 1.000 & 0.465 & 0.439 \\
\hline Language \(\mathrm{Arts}_{8}\) & & & & & & 1.000 & 0.812 \\
\hline Modern Math 8 & & & & & & & 1.000 \\
\hline
\end{tabular}

TABLE VII (Continued)
\begin{tabular}{lllll}
\hline Variable & \begin{tabular}{c} 
Composite \\
Score \(_{8}\)
\end{tabular} & \begin{tabular}{c} 
Absentee \\
Rate \(_{11}\)
\end{tabular} & \begin{tabular}{c} 
Dropout \\
Rate \(_{11}\)
\end{tabular} & \begin{tabular}{c} 
Language \\
Arts \\
11
\end{tabular}
\end{tabular}

\section*{CHAPTER V}

EDUCATIONAL PRODUCTION FUNCTIONS

The main objective of this chapter is to quantify the effect of various inputs on the quality of elementary and secondary education. The educational output variables - absentee rates, dropout rates, and achievement scores - were regressed on the educational process, student input, and environmental variables. This procedure yields the net effect of the educational process variables on educational output, while holding the control variables constant.

\section*{Hierarchical Regression}

Simple correlation analysis fails to account for the interrelationships of educational process variables and student input and environmental variables. Regression analysis is one technique that can be used to estimate the net relationship between educational process variables and educational output while holding the control variables constant.

In the regression analysis, the achievement scores, absentee rates, and dropout rates which measure the quality of educational output are termed the dependent variables. The educational process variables, student input variables, and environmental variables are the independent or explanatory variables. The regression equation is essentially a formula for predicting the value of the dependent variable.

Since there are more admissible hypotheses (variables) explaining
any dependent variable than can be included in the regression equation, stepwise regression is used to select a subset of variables for the final equation. The stepwise linear regression procedure computes successive multiple linear regression equations. At each successive step, the variable which makes the greatest reduction in the error sum of squares is added to the regression equation. The variable added consequently has the highest partial correlation with the dependent variable partialled on the variables which have already been included. Although this procedure does not insure the "best" regression equation, it is considered to be the best of the variable selection procedures. \({ }^{1}\)

Just as the model of the inputs' relationship to output was specified as a hierarchy of admissible hypotheses, the regression procedure was formulated to take this hierarchy into account. Interest centered on the educational process variables, and these comprised the first echelon. Thus, if either educational process or control variables can account for the variation in educational output, the former variables are favored and selected to be included in the equation. This procedure is similar to the one used by Heady and Tweeten. \({ }^{2}\) The most significant variables of the higher echelon (educational process variables) were the first variables allowed to enter the equation by the stepwise regression procedure. Then, the significant variables of the second echelon (control variables) were allowed to enter the equation. As additional variables were introduced, educational process variables were removed from the equation if they became insignificant. This regression procedure may slightly bias the t-test of significance.

\section*{Fourth Grade}

The equations showing the effect of various factors on fourth grade absentee rates, language arts, modern math, and composite achievement scores are shown in Tables VIII, IX, X, and XI, respectively. The coefficients of all the variables included in these equations were significant at the . 10 level. These equations explained from 74 to 82 percent of the variation in the sample data on educational output.

\section*{Educational Process Variables}

The influence of teachers on student performance was especially apparent in the fourth grade. Lighter workloads and higher salaries were associated with improved student performance. Every one percent increase in the number of teachers with a planning period was associated with . 10 unit increase in the composite achievement score and .02 percent reduction in the absentee rate. Large pupil-teacher ratios were detrimental to absentee rates and modern math scores. The absentee rate increased by . 25 percent and the average modern math score declined by 1.38 units for every additional student per teacher. Those school districts offering higher standard salaries attained higher achievement scores and lower absentee rates by attracting better teachers.

Educational output was also linked to teacher experience. Each year of average teacher experience was associated with a reduction of . 30 percent in the absentee rate and an increase of 1.82 units in average language arts score, and .96 units in average composite score. Students tended to have lower absentee rates and higher language arts scores if a greater percentage of their teachers had between three and nine years of experience. On the other hand, students had higher modern

TABLE VIII
EQUATION EXPLAINING FOURTH GRADE ABSENTEE RATE
\begin{tabular}{|c|c|c|}
\hline Variable & Regression Coefficient & Standard Error \\
\hline Constant & 22.8199 & \\
\hline \multicolumn{3}{|l|}{Educational Process Variables} \\
\hline Percentage of teachers with three to nine years of experience & -0.0394 & 0.0228 \\
\hline Percentage of teachers with a planning period & -0.0190 & 0.0061 \\
\hline Pupil-teacher ratio & 0.2460 & 0.0511 \\
\hline Average teacher experience & -0.2972 & 0.0690 \\
\hline Salary in thousand dollars for beginning teachers with bachelor's degree & -3.2050 & 0.9250 \\
\hline \multicolumn{3}{|l|}{Student-Input Variables} \\
\hline Percentage of students who study three or more hours a day & -0. 1903 & 0.0482 \\
\hline Percentage of students who have spent most of their lives in rural areas & -0.0285 & 0.0105 \\
\hline \[
\mathrm{R}^{2}
\] & 0.8056 & \\
\hline
\end{tabular}

TABLE IX

EQUATION EXPLAINING FOURTH GRADE
LANGUAGE ARTS SCORES
\begin{tabular}{|c|c|c|}
\hline Variable & Regression Coefficient & Standard Error \\
\hline Constant & 80.1583 & \\
\hline \multicolumn{3}{|l|}{Educational Process Variables} \\
\hline Percentage of teachers with five to nine years of experience & 0.6479 & 0.2718 \\
\hline Adequate kindergarten & 6.6886 & 4.0334 \\
\hline Number of different periodicals & 0.3476 & 0.2126 \\
\hline Value of audiovisual material in hundred dollars per student & 2.0176 & 0.7003 \\
\hline Average teacher experience & 1.8205 & 0.4238 \\
\hline Salary in thousand dollars for beginning teachers with bachelor's degree & 10.1777 & 5.8997 \\
\hline SQRT (Average daily attendance in thousands) & -2. 1119 & 1.5367 \\
\hline \multicolumn{3}{|l|}{Student-Input Variable} \\
\hline IQ & 0.7516 & 0.4819 \\
\hline Environmental Variable & & \\
\hline Percentage of students whose parents' net income is greater than \(\$ 10,000\)
\[
\mathrm{R}^{2}
\] & 0.5813
0.8289 & 0.2788 \\
\hline
\end{tabular}

TABLE X

\section*{EQUATION EXPLAINING FOURTH GRADE \\ MODERN MATH SCORES}
\begin{tabular}{lll}
\hline Variable & \begin{tabular}{l} 
Regression \\
Coefficient
\end{tabular} & \begin{tabular}{c} 
Standard \\
Error
\end{tabular} \\
\hline Constant & 249.6048
\end{tabular}

Educational Process Variables
\begin{tabular}{lll}
\begin{tabular}{c} 
Percentage of teachers with ten or \\
more years of experience
\end{tabular} & 0.2242 & 0.1104 \\
Number of different periodicals \\
Pupil-teacher ratio & 0.5861 & 0.2290 \\
\begin{tabular}{l} 
Value of audiovisual material in \\
hundred dollars per student
\end{tabular} & -1.3820 & 0.5610 \\
SQRT (Average daily attendance in \\
thousands)
\end{tabular}

Environmental Variables
\begin{tabular}{ccc}
\begin{tabular}{l} 
Percentage of students whose mother's \\
occupation is professional or \\
executive
\end{tabular} & 1.1563 & 0.6066 \\
Parents' average net income \\
\(\mathrm{R}^{2}\) & 4.7960 & 1.2873
\end{tabular}

TABLE XI

\section*{EQUATION EXPLAINING FOURTH GRADE COMPOSITE SCORES}
\begin{tabular}{|c|c|c|}
\hline Variable & Regression Coefficient & Standard Error \\
\hline Constant & \(-25.9263\) & \\
\hline \multicolumn{3}{|l|}{Educational Process Variables} \\
\hline Percentage of teachers who have a planning period & 0.0977 & 0.0488 \\
\hline Value of audiovisual material in hundred dollars per student & 0.9503 & 0.5953 \\
\hline Average teacher experience & 0.9591 & 0.3571 \\
\hline \multicolumn{3}{|l|}{Student-Input Variables} \\
\hline IQ & 1.5406 & 0.4052 \\
\hline Percentage of students who have spent most of their lives in rural areas & -0.1648 & 0.0810 \\
\hline \multicolumn{3}{|l|}{Environmental Variable} \\
\hline Percentage of students whose mother's occupation is professional or executive & 1. 1944 & 0.4400 \\
\hline \(\mathrm{R}^{2}\) & 0.7630 & \\
\hline
\end{tabular}
math scores if a greater percentage of their teachers had ten or more years of experience.

Instructional materials also had an important effect on student performance. Value of audiovisual material per pupil was positively associated with all three achievement scores. The number of periodicals available in a school was positively related to language arts and modern math scores. These measures may actually reflect the impact of a full complement of instructional materials. Student performance was significantly influenced by two additional educational process variables - the availability of an adequate kindergarten and the size of the school district. Fourth grade students had higher language arts scores if the district had an adequate kindergarten. Evidently, kindergarten had a positive effect on learning and helped to build a foundation for elementary education. Those school districts with larger enrollments had lower language arts and modern math scores, other things equal.

\section*{Control Variables}

As expected, student characteristics were also important in explaining student performance. Each unit of \(I Q\) was associated with an increase in the language arts score of .75 units and in the composite achievement score of 1.54 units. Schools had lower absentee rates if a greater percentage of students studied three or more hours a day, other things equal. Time spent studying probably revealed students' interest in education. Those schools with a greater percentage of students who have spent most of their lives in rural areas had lower absentee rates but also lower composite achievement scores. Every one percent of students who have spent most of their lives in rural areas reduced
absentee rates by .03 percent and composite achievement scores by .16 units.

Parents' income and occupation accounted for a large portion of the variation in student achievement. Every one percent of the mothers whose occupation is professional or executive was associated with an increase of 1.16 units in the modern math score and 1.19 units in the composite achievement score. Each thousand dollar increase in parents' average net income was associated with an increase of 4.80 units in the modern math score. Each additional percentage point gain in the proportion of parents with a net income of more than \(\$ 10,000\) was associated with an increase of .58 units in the language arts score.

\section*{Nonsignificant Variables}

Identification of variables which did not have a significant effect on student attainment can also be an important part of evaluation. Ability groupings, special education, and accelerated classes appeared to have little effect on fourth grade student performance. Teacher education showed no significant effect on student attainment. Of course, it is possible that some of these omitted variables may have been important but did not show significance because of limited data in the sample or because they are closely correlated with other variables which were included in the equation.

\section*{Eighth Grade}

Variables affecting eighth grade student performance are shown in Tables XII, XIII, XIV, and XV. These equations reveal as did those for grade 4 that educational process, student input, and environmental

TABLE XII

EQUATION EXPLAINING EIGHTH GRADE ABSENTEE RATE
\begin{tabular}{|c|c|c|}
\hline Variable & Regression Coefficient & Standard Error \\
\hline Constant & 23.0438 & \\
\hline \multicolumn{3}{|l|}{Educational Process Variables} \\
\hline Percentage of teachers with a planning period & -0.0137 & 0.0061 \\
\hline Pupil-teacher ratio & 0.1535 & 0.0472 \\
\hline Average teacher experience & -0. 1993 & 0.0532 \\
\hline Salary in thousand dollars for beginning teachers with bachelor's degree & -2.9833 & 1.0619 \\
\hline \multicolumn{3}{|l|}{Student-Input Variables} \\
\hline Percentage of students who study one or more hours a day & -0.0258 & 0.0188 \\
\hline Percentage of students who spent most of their lives in rural areas & -0.0634 & 0.0141 \\
\hline \(\mathrm{R}^{2}\) & 0.7569 & \\
\hline
\end{tabular}

TABLE XIII

EQUATION EXPLAINING EIGHTH GRADE LANGUAGE ARTS SCORES
\begin{tabular}{|c|c|c|}
\hline Variable & Regression Coefficient & Standard Error \\
\hline Constant & 86.5020 & \\
\hline \multicolumn{3}{|l|}{Educational Process Variables} \\
\hline Percentage of teachers with five to nine years of experience & 0.3016 & 0.2275 \\
\hline Pupil-teacher ratio & -0.6994 & 0.4528 \\
\hline SQRT (Average daily attendance in thousands) & \(-7.3332\) & 1.8858 \\
\hline \multicolumn{3}{|l|}{Student-Input Variables} \\
\hline IQ & 2.1791 & 0.5472 \\
\hline Average number of hours students studied & 15.2785 & 9.7660 \\
\hline Average number of books students read during last summer & 5.9697 & 2.3514 \\
\hline \multicolumn{3}{|l|}{Environmental Variables} \\
\hline Percentage of students whose fathers attended college & 0.5568 & 0.2520 \\
\hline Percentage of students whose parents' net income is greater than \(\$ 10,000\) & 0.6263 & 0.2600 \\
\hline \(\mathrm{R}^{2}\) & 0.8531 & \\
\hline
\end{tabular}

TABLE XIV
EQUATION EXPLAINING EIǴHTH GRADE MODERN
MATH SCORES
\begin{tabular}{lll}
\hline Variable & \begin{tabular}{l} 
Regression \\
Coefficient
\end{tabular} & \begin{tabular}{c} 
Standard \\
Error
\end{tabular} \\
\hline Constant & 403.9570 &
\end{tabular}

Educational Process Variables
\begin{tabular}{lll}
\begin{tabular}{c} 
Percentage of teachers with three \\
to nine years of experience
\end{tabular} & 0.3391 & 0.1040 \\
Number of printed volumes per pupil & 0.6193 & 0.2906 \\
\begin{tabular}{l} 
Pupil-teacher ratio
\end{tabular} & -0.6425 & 0.3017 \\
\begin{tabular}{c} 
SQRT (Average daily attendance in \\
thousands)
\end{tabular} & -7.9208 & 1.6008
\end{tabular}

Environmental Variables
\begin{tabular}{ccc}
\begin{tabular}{c} 
Percentage of students whose fathers \\
attended college
\end{tabular} & 0.5494 & 0.1706 \\
\begin{tabular}{c} 
Percentage of students whose father's \\
occupation is professional or \\
executive
\end{tabular} & 0.7084 & 0.2651 \\
\begin{tabular}{c} 
Percentage of students whose parents' \\
net income is greater than \(\$ 10,000\)
\end{tabular} & 0.9200 & 0.1628 \\
\(\mathrm{R}^{2}\) & 0.9044 &
\end{tabular}

TABLE XV

EQUATION EXPLAINING EIGHTH GRADE COMPOSITE SCORES
\begin{tabular}{|c|c|c|}
\hline Variable & Regression Coefficient & Standard Error \\
\hline Constant & 41.9040 & \\
\hline \multicolumn{3}{|l|}{Educational Process Variables} \\
\hline Percentage of teachers with ten or more years of experience & -0.1019 & 0.0739 \\
\hline Number of printed volumes per pupil & 0.5505 & 0.2358 \\
\hline Number of different periodicals & 0.1860 & 0.0894 \\
\hline SQRT (Average daily attendance in thousands) & -3.9975 & 1. 1644 \\
\hline \multicolumn{3}{|l|}{Student-Input Variables} \\
\hline Fourth grade composite score & 0.2508 & 0.1160 \\
\hline IQ & 0.5070 & 0.3749 \\
\hline Average number of hours students studied & 12.2533 & 5.1364 \\
\hline \multicolumn{3}{|l|}{Environmental Variables} \\
\hline Percentage of students whose fathers attended college & 0.5440 & 0.1388 \\
\hline Percentage of students whose parents plan for their children to attend college & 0.4419 & 0.1727 \\
\hline \(\mathrm{R}^{2}\) & 0.9212 & \\
\hline
\end{tabular}

\begin{abstract}
variables account for a sizable proportion of the variation among districts in student performance. The school that seeks to change educational process variables to improve student performance must recognize constraints imposed by student background.
\end{abstract}

\section*{Educational Process Variables}

Major educational process variables affecting eighth grade student performance include instructional materials, school district size, and factors associated with classroom teachers. Every printed volume per pupil was associated with increased modern math and composite achievement scores of .62 and .55 units, respectively. The number of periodicals available also increased composite achievement scores. Larger school districts had lower achievement scores, other things equal.

Significant factors associated with teachers include their workload, salary, and experience. Giving teachers planning periods improved their performance and, consequently, reduced absentee rates. Increasing pupil-teacher ratios reduced the effectiveness of teachers. Every oneunit increase in the pupil-teacher ratio was associated with .15 percent increase in the absentee rate, .70 unit decline in the language arts score, and .64 unit decline in the modern math score. Those districts with the highest standard teacher salaries had the lowest absentee rates, other things equal. Each thousand dollar increment in salary for beginning teachers with bachelor's degree was associated with a reduction in the absentee rate of 2.98 percent.

Again, teacher experience played a major role in determining student performance. The absentee rate was reduced .20 units on the average for each year of teacher experience. Percentage of teachers with

\begin{abstract}
three to nine years of experience was positively related to language arts and modern math scores. A high percentage of teachers with ten or more years experience was detrimental to composite achievement scores. Thus, teachers with several years of experience are very important in some aspects of education, such as motivating students to stay in school. On the other hand, the skills that older teachers obtained in college may have been of lower quality than currently obtained.
\end{abstract}

\section*{Control Variables}

Aptitudes and educational efforts are the most important student characteristics affecting student performance. Time spent studying was positively related to language arts and composite achievement scores. Also, percentage of students who study one or more hours a day was negatively related to the absentee rate. Each unit increase in the average number of books read was associated with an increase in language arts score of 5.97 units. Student \(I Q\) was positively related to language arts and composite achievement scores. Average fourth grade achievement scores were used as a characteristic of students entering the junior high educational program. Each additional unit of fourth grade composite achievement score was associated with an increase in eighth grade composite achievement score of .25 units, other things equal. Those schools with a greater percentage of students who spent most of their lives in a rural area had a lower absentee rate.

Parents' education, occupation, income, and interest in their children's education were very strongly related to eighth grade student performance. The percentage of fathers who attended college was positively related to all achievement scores. Language arts and modern math
scores improved with an increase in the percentage of students whose parents' net income is greater than \(\$ 10,000\). The percentage of students whose father's occupation is professional or executive was positively related to modern math scores. A greater percentage of parents who plan for their children to attend college was associated with a higher average composite achievement score.

\section*{Nonsignificant Variables}

A pattern of omitted variables in explaining student performance was not as evident for the eighth grade as for the fourth grade. Facilities did not have a significant effect on student performance, and again the teacher's education did not appear significant. All other major categories of educational process variables were important in explaining some aspect of student performance.

\section*{Eleventh Grade}

The equations explaining eleventh grade student performance are shown in Tables XVI, XVII, XVIII, XIX, and XX. Besides absentee rates, language arts, modern math, and composite achievement scores, student performance is expanded in the eleventh grade to include dropout rates. Additional educational process and student input variables which reflect high school education and high school students were included for the eleventh grade. The last part of this section examines the tradeoff between educational outputs.

Educational Process Variables

Factors associated with teachers were again the most important

TABLE XVI

EQUATION EXPLAINING ELEVENTH GRADE DROPOUT RATE
\begin{tabular}{lll}
\hline Variable & \begin{tabular}{l} 
Regression \\
Coefficient
\end{tabular} & \begin{tabular}{c} 
Standard \\
Error
\end{tabular} \\
\hline Constant & 20.5431 &
\end{tabular}

Educational Process Variables
\begin{tabular}{ccc}
\begin{tabular}{c} 
Percentage of teachers with five to \\
nine years of experience
\end{tabular} & -0.0588 & 0.0194 \\
\begin{tabular}{c} 
Percentage of teachers with ten or \\
more years of experience
\end{tabular} & -0.0893 & 0.0198 \\
\begin{tabular}{c} 
Value of audiovisual material in \\
hundred dollars per student
\end{tabular} & -0.3764 & 0.0746 \\
\begin{tabular}{c} 
Salary in thousand dollars for \\
beginning teachers with \\
bachelor's degree
\end{tabular} & -1.8264 & 1.0377 \\
SQRT (Average daily attendance in \\
thousands)
\end{tabular}

Environmental Variable
Percentage of students whose parents -0.0531 0.0299
net income is greater than \$10,000
\(R^{2}\)
0.7856
table XVII
Equation explaining eleventh grade ABSENTEE RATE
\begin{tabular}{lll}
\hline Variable & \begin{tabular}{c} 
Regression \\
Coefficient
\end{tabular} & \begin{tabular}{c} 
Standard \\
Error
\end{tabular} \\
\hline Constant \\
Educational Process Variables & 23.0678 & \\
\hline \begin{tabular}{c} 
Percentage of teachers with five to \\
nine years of experience
\end{tabular} & -0.1008 & 0.0220 \\
Value of audiovisual material in \\
hundred dollars per student
\end{tabular}

TABLE XVIII
EQUATION EXPLAINING ELEVENTH GRADE LANGUAGE
ARTS SCORES
\begin{tabular}{lcc}
\hline Variable & \begin{tabular}{c} 
Regression \\
Coefficient
\end{tabular} & \begin{tabular}{c} 
Standard \\
Error
\end{tabular} \\
Constant \\
Educational Process Variables \\
Percentage of teachers with three to \\
nine years of experience
\end{tabular}

TABLE XIX

\section*{EQUATION EXPLAINING ELEVENTH GRADE MODERN MATH SCORES}
\begin{tabular}{lcc}
\hline Variable & \begin{tabular}{l} 
Regression \\
Coefficient
\end{tabular} & \begin{tabular}{c} 
Standard \\
Error
\end{tabular} \\
\hline Constant & 77.0076
\end{tabular}

Educational Process Variables
\begin{tabular}{lll}
\begin{tabular}{c} 
Percentage of teachers with ten or \\
more years of experience
\end{tabular} & -0.2191 & 0.1483 \\
\begin{tabular}{c} 
Salary in thousand dollars for beginning \\
teachers with a bachelor's degree
\end{tabular} & 12.8106 & 7.9707 \\
\begin{tabular}{l} 
Adequate industrial, vocational or \\
technical shop
\end{tabular} & 7.4269 & 4.5102 \\
Student-Input Variables
\end{tabular}
\begin{tabular}{lll} 
Eighth grade modern math score & 0.5652 & 0.1442 \\
\begin{tabular}{c} 
Average number of hours students \\
studied
\end{tabular} & 14.8942 & 8.2821 \\
\begin{tabular}{c} 
Average number of books students read \\
during last summer
\end{tabular} & 4.7528 & 2.3086 \\
\begin{tabular}{c} 
Average number of hours worked per week \\
outside of school \\
\(R^{2}\)
\end{tabular} & -2.7796 & 1.3291 \\
\hline
\end{tabular}

EQUATION EXPLAINING ELEVENTH GRADE COMPOSITE SCORES
\begin{tabular}{|c|c|c|}
\hline Variable & Regression Coefficient & Standard Error \\
\hline Constant & 28.0791 & \\
\hline \multicolumn{3}{|l|}{Educational Process Variables} \\
\hline Percentage of teachers with master's degree & 0.2058 & 0.1018 \\
\hline Percentage of teachers with ten or more years of experience & -0.2546 & 0.1019 \\
\hline SQRT (number of nonvocational units offered) & 4.1382 & 1.9831 \\
\hline Adequate industrial, vocational or technical shop & 7.8974 & 2.2111 \\
\hline SQRT (Average daily attendance in thousands) & -3.8311 & 1. 1549 \\
\hline \multicolumn{3}{|l|}{Student-Input Variables} \\
\hline Eighth grade composite score & 0.0872 & 0.0304 \\
\hline Percentage of students who have changed schools three or more times & -0.1345 & 0.0966 \\
\hline Percentage of students who plan to continue going to school after graduation & 0.1415 & 0.0769 \\
\hline \multicolumn{3}{|l|}{Environmental Variables} \\
\hline Percentage of students whose fathers attended college & 0.2281 & 0.1473 \\
\hline \(\mathrm{R}^{2}\) & 0.8474 & \\
\hline
\end{tabular}
educational process variables. Standard salary had its major impact in the eleventh grade. Those districts offering higher standard salaries had lower dropout and absentee rates and higher modern math scores. Percentage of teachers with a master's degree was positively related to composite achievement scores. These two variables indicate that highly qualified teachers are especially important in high school education. Teacher experience was important in determining every dimension of student performance. Having high percentage of teachers with five to nine years of experience reduced both absentee and dropout rates and increased language arts scores. However, the percentage of teachers with ten or more years of experience had a greater impact on reducing dropout rates. Teachers with this much experience were detrimental to modern math and composite achievement scores.

Other educational process variables which affected student performance were audiovisual materials, program offerings, facilities, and district size. Absentee and dropout rates declined with increases in the value of audiovisual material available. Those schools with a greater number of nonvocational units had higher composite achievement scores. Schools with an adequate industrial, vocational or technical shop, a proxy for facilities in general, had higher achievement scores. Schools with an adequate language laboratory had lower absentee rates. School district size per se again was negatively related to student performance.

\section*{Control Variables}

Student effort was apparent in performance. The average number of hours studied was positively related to modern math scores. An increase
in the average number of books read last summer was associated with an increase in the modern math scores. Those schools whose students scored higher on eighth grade modern math and composite achievement tests had higher eleventh grade modern math and composite achievement scores. Other significant student input variables include work outside of school, post high school plans, number of times students have changed schools, and residence patterns. Modern math scores were lower in those schools which had many students working outside of school. Students who have changed schools three or more times had higher absentee rates and lower composite achievement scores. Percentage of students who plan to continue going to school after graduation was positively related to language arts and composite achievement scores. Schools with a greater percentage of students who had spent most of their lives in rural areas or towns with less than 2,500 population had lower absentee rates. Environmental variables were not directly as important in determining eleventh grade student performance as they were in earlier grades. Parents' income affected dropout rates and language arts scores. Students were less likely to drop out if their parents had high incomes (over \(\$ 10,000\) ) and more likely to have low language arts scores if their parents had very low incomes (under \(\$ 3,000\) ). Those students whose fathers attended college had higher composite achievement scores. Also, those students whose father's occupation is professional or executive had higher language arts scores.

Nonsignificant Variables

One of the most striking results of the eleventh grade equations was the absence of variables related to teaching workload. Although the

\begin{abstract}
coefficients of these variables (pupil-teacher ratio and percentage of teachers with a planning period) were statistically significant in earlier grades, they were not in the eleventh grade. Teacher qualifications appeared to be just as important as in earlier grades, but instructional materials appeared to be less important. After correcting for parents' education, occupation, and income, race appeared to have no significant effect on student performance in any grade.
\end{abstract}

\section*{Simultaneous Equations}

A system of equations was specified in the eleventh grade to measure the possible substitution effect between dropouts and achievement test scores. Each of the three eleventh grade achievement scores language arts, modern math, and composite achievement scores - were specified as a function of the dropout rate and the same independent variables as presented in Tables XVIII, XIX, and XX, respectively. Eleventh grade dropout rate was specified as a function of eleventh grade composite achievement score and the same independent variables as presented in Table XVI. Two-stage least squares was used to estimate the four equations, which were all over identified. The results of these regressions are shown in Tables XL, XLI, XLII, and XLIII of Appendix E.

In all cases, the regression coefficients in the simultaneous equations were very close to the ones in the previous single equation models. Each one-unit increase in eleventh grade composite achievement score is associated with .015 percent increase in the dropout rate. Since composite achievement score has a mean of 77 and a standard deviation of 9.9 , it has little impact on the dropout rate. In the
achievement score simultaneous equations, the dropout rate regression coefficient is -2.38 for language arts, -1.10 for modern math, and 0.11 for composite achievement score. Average dropout rate is 3.4 percent with a standard deviation of 2.3. In absolute terms, dropout rate has little effect on achievement scores. These equations show little improvement over the single equation models. Moreover, school programs for reducing dropout rates had no apparent effect on achievement test scores.

\section*{Summary and Conclusions}

The major task of this chapter has been to quantify the relationship between educational inputs and outputs as measured by achievement scores, absentee rates, and dropout rates. Among the educational process variables which affect achievement, those factors that are associated with teacher performance appeared particularly important. Teacher performance as measured by students' achievement tended to reach a peak between three and ten years of experience. Teachers with more than ten years of experience appeared to be more successful than other teachers in reducing absentee and dropout rates. A smaller pupil-teacher ratio was more important in elementary than in secondary education. Student performance increased in elementary grades when teachers had a lighter workload. Students in secondary schools attained higher levels of performance if they had better trained and better paid teachers.

Other educational process variables that significantly affected student performance include instructional materials, school district size, facilities, and the number of nonvocational units offered. Printed volumes per pupil, value of audiovisual material, and the number
of periodicals positively affected student performance. Those schools offering a higher number of nonvocational units had higher secondary composite scores. Other things equal, student performance declined with increases in school district size.

The types of educational input-output relationships presented in this chapter can be used to improve the quality of educational output by changing factor inputs. However, policy decisions should not be based solely on the significance of a factor's effect on educational output, but should also be based on its impact on output in relation to its cost relative to other factors. Subsequent analysis will determine efficient ways to improve the quality of elementary and secondary education based on the input-output coefficients estimated in this chapter. The simultaneous equations in Appendix E suggest that the educational outputs considered in this study can be viewed as independent of each other. This finding greatly simplified the subsequent analysis of an efficient schooling organization to meet specific output targets.

\footnotetext{
\({ }^{1}\) N. R. Draper and H. Smith, Applied Regression Analysis (New York, 1966), pp. 171-172.
\({ }^{2}\) Earl O. Heady and Luther G. Tweeten, Resource Demand and Structure of the Agricultural Industry (Ames, 1963), pp. 405-417.
}

\section*{CHAPTER VI}

\section*{EDUCATIONAL COSTS}

This chapter shows costs of elementary (grades 1-8) and secondary (grades 9-12) education applicable to rural areas. A thorough analysis of a school district's operation requires cost data on transportation as well as education. The problem of providing high quality education at a reasonable cost in rural areas is intensified by the high cost of transportation. Educational costs can be divided into a fixed component, embodied in plant and equipment, and a variable component. Major variable costs include administration, instruction, and operation and maintenance of plant.

In planning for educational services, it is useful to know how costs of education vary with the number of students in a district. This study is concerned with a long-run period, a time interval long enough to allow a school district to vary in size. The long-run average cost curve shows the minimum cost per student of educating various numbers of students. The purpose of this chapter is to estimate these cost curves.

The large variation among schools in the quality of educational programs must be accounted for in estimating the size-cost relationship. Measures used herein of quality are average eleventh grade composite achievement scores (the school district's final product), and the number and type of credit units offered.

\section*{Administration}

Administration consists of those activities which regulate and control the affairs of the school district. Three major items were included in administrative expenditures - salaries, contractual services, and other expenses. The variables which were considered to affect administrative costs include average pupil-teacher ratio, average daily attendance (ADA), and average achievement score. An examination of the data showed that, with increases in ADA, administrative costs per ADA declined rapidly at first and then remained relatively constant over a large range of school district sizes. Also, administrative costs declined at a decreasing rate with increases in the pupil-teacher ratio.

A long-run average cost curve was estimated for administrative services using multiple regression. Standard errors are shown in parentheses below the regression coefficients. Each regression coefficient is significant at least at the . 05 level.
\[
\begin{gather*}
\mathrm{AD}=311.743+\underset{(.312)}{. .528} \mathrm{ACH}-\underset{(4.730)}{26.391} \mathrm{PTR}+\underset{(.097)}{.513}(\mathrm{PTR})^{2}+\underset{(2.679)}{6.694}(1 / \mathrm{ADA}) \\
\mathrm{R}^{2}=.937 . \tag{1}
\end{gather*}
\]

The variables are as follows:

AD is administrative costs per student in average daily attendance,

ACH is average eleventh grade composite achievement score,

PTR is pupil-teacher ratio, and

ADA is average daily attendance in 1,000 units.
The long-run average administrative cost curve for a given level of
achievement is derived by finding the minimum cost with respect to the pupil-teacher ratio for each level of ADA. The average cost curve for achievement held at the Oklahoma average is shown in Figure 5 by the curve LR. Under actual conditions, a school district may operate on a short-term cost curve above the level that is achieved under a full long-term adjustment. The short-run curves are based on a given size plant. In the case illustrated, short-run curves \({ }^{\prime} \mathrm{SR}_{1}\) and \(\mathrm{SR}_{2}\) are derived by holding the number of teachers constant at 50 and 100 , respectively. These curves show that average administrative costs per pupil in ADA can be high with too many teachers.

\section*{School Plant Operation and Maintenance}

Accounts for operation and maintenance of plant for the school system record all current expenditures for keeping the grounds, buildings, and equipment in good condition. Operation of plant includes cleaning, lighting, heating, communications, power, and other such activities required on a regular basis. Maintenance consists of repairs or replacements needed to keep the physical plant in good condition.

Cost of operation and maintenance of plant are related to student achievement, pupil-teacher ratio and ADA in Equation (2). The regression coefficients are significant at the . 05 level.

\[
\begin{equation*}
\mathrm{R}^{2}=.852 \tag{2}
\end{equation*}
\]

POM is average cost of plant operation and maintenance per pupil in ADA.


Results indicate initial economies and then slight diseconomies as school district size (ADA) is increased. Initial economies accrue from more intensive utilization of plant, while diseconomies accrue from servicing a large number of school plants within a given district. POM declines at a decreasing rate with increases in the class size.

Figure 6 shows the average cost per pupil of plant operation and maintenance when achievement is held constant at the Oklahoma average. Long-term adjustments in ADA produce significant economies of size up to \(2,000 \mathrm{ADA}\) as shown by LR. The short-run cost curves \(\mathrm{SR}_{1}\) and \(\mathrm{SR}_{2}\) were derived by holding the number of teachers constant, which causes more than the optimum number of classes to be in operation at smaller enrollments. These short-run curves illustrate that only minor deviations in plant operation may cause major deviations from minimum attainable costs.

\section*{Instruction}

Instruction consists of those activities dealing directly with the teaching of students. These are the activities of teachers, principals, and guidance personnel. Attendant costs include salaries of secretarial and clerical assistants, textbooks, school libraries, audiovisual materials, and teaching supplies.

Instructional costs vary because of differences in the quality of program offerings. To make a valid comparison of costs among schools of different sizes, the program quality must be held constant. For this analysis, the high school course offerings were standardized according to number and type of vocational and nonvocational courses offered.

Three high school curriculums - the first designed to be minimum

for accreditation, the second designed to be adequate, and the third designed to be desirable - are presented in Tables XXI, XXII, and XXIII. The most frequently offered courses within each subject area are also presented in the tables. The first program, typical for many small, rural high schools, offers only nonvocational courses. However, this type of program may not meet the needs of all students. Percentages of students by subject area were computed from actual enrollments in the sampled districts. The "adequate" program was developed from the courses offered by the districts in the sample which provided (1) both vocational and nonvocational courses and (2) enough units to be accredited. This program was designed as a minimum standard of broad exposure to the general subject areas, including vocational education. The "desirable" program offered the same courses as the other two programs combined. Percentages of students enrolled in each subject area for the adequate and desirable programs were computed from those districts providing both vocational and nonvocational courses. The course offerings may be expanded as the initial classes of a subject area reach optimum size and new classes are added. High school instructional costs were calculated by incrementing \(A D A\) and distributing high school students among courses as prescribed by the program under consideration.

The number of high school teachers was calculated from the number of classes required for the given program. The number of elementary and junior high school teachers was based on ADA and the optimum studentteacher ratio. Cost calculations were based on the generous assumption that two courses could be combined in a single classroom under one teacher until the number of students in combined courses exceed the maximum class size.

TABLE XXI

MINIMUM PROGRAM OF COURSE OFFERINGS
\begin{tabular}{|c|c|c|c|c|}
\hline Courses & \begin{tabular}{l}
Credit \\
Units
\end{tabular} & Credit Units By Subject Area & Maximum No. Of Students Per Class & Percentage Enrolled in Subject Area \\
\hline Language Arts & & 8 & 35 & 25.305 \\
\hline English I-IV & 4 & & & \\
\hline Speech & 1 & & & \\
\hline Library Science & 1 & & & \\
\hline Creative Writing & 1 & & & \\
\hline Yearbook & 1 & & & \\
\hline Mathematics & & 5 & 28 & 10.668 \\
\hline General Math & 1 & & & \\
\hline Algebra I-II & 2 & & & \\
\hline Plane Geometry & 1/2 & & & \\
\hline Trigonometry & 1/2 & & & \\
\hline Math Analysis & 1 & & & \\
\hline Social Studies & & , & 35 & 16.726 \\
\hline American History & 1 & & & \\
\hline World History & 1 & & & \\
\hline Government & 1/2 & & & \\
\hline Okl ahoma History & 1/2 & & & \\
\hline Sociology & 1 & & & \\
\hline Psychology & 1 & & & \\
\hline Science & & 5 & 22 & 11.373 \\
\hline Biology & 1 & & & \\
\hline Chemistry & 1 & & & \\
\hline General Science & 1 & & & \\
\hline Physics & 1 & & & \\
\hline Physical Science & 1 & & & \\
\hline Business Education & & 5 & 30 & 10.397 \\
\hline Accounting & 1 & & & \\
\hline Business Law & 1 & & & \\
\hline General Business & 1 & & & \\
\hline Shorthand & 1 & & & \\
\hline Typing & 1 & & & \\
\hline Fine Arts & & 4 & 32 & 10.335 \\
\hline Arts & 1 & & & \\
\hline Crafts & 1 & & & \\
\hline Vocal Music & 1 & & & \\
\hline Instrumental Music & 1 & & & \\
\hline Health and Safety & & 3 & 35 & 11.460 \\
\hline Driver Education & 1/2 & & & \\
\hline Health Education & 1 & & & \\
\hline Physical Education & 1 & & & \\
\hline Safety Education & 1/2 & & & \\
\hline
\end{tabular}

TABLE XXI (Continued)
\begin{tabular}{lcccc}
\hline & \begin{tabular}{c} 
Credit \\
Units
\end{tabular} & \begin{tabular}{c} 
Credit Units \\
By Subject \\
Area
\end{tabular} & \begin{tabular}{c} 
Maximum No. \\
Of Students \\
Per Class
\end{tabular} & \begin{tabular}{c} 
Percentage \\
Enrolled in \\
Subject Area
\end{tabular} \\
\hline \begin{tabular}{l} 
Foreign Language \\
French \\
Spanish \\
Latin
\end{tabular} & 1 & 3 & 30 & 3.733 \\
TOTAL CREDIT UNITS & 1 & & & \\
\hline
\end{tabular}

TABLE XXII
ADEQUATE PROGRAM OF COURSE OFFERINGS
\begin{tabular}{|c|c|c|c|c|}
\hline Courses & Credit Units & Credit Units By Subject Area & Maximum No. Of Students Per Class & \begin{tabular}{l}
Percentage \\
Enrolled in \\
Subject Area
\end{tabular} \\
\hline Language Arts & & 6 & 35 & 31.428 \\
\hline Enlgish I-IV & 4 & & & \\
\hline Speech & 1 & & & \\
\hline Library Science & 1 & & & \\
\hline Mathematics & & 4 & 28 & 14.663 \\
\hline General Math & 1 & & & \\
\hline Algebra I-II & 2 & & & \\
\hline Plane Geometry & 1/2 & & & \\
\hline Trigonometry & 1/2 & & & \\
\hline Social Studies & & 4 & 35 & 5.986 \\
\hline American History & 1 & & & \\
\hline World History & 1 & & & \\
\hline Government & 1/2 & & & \\
\hline Oklahoma History & 1/2 & & & \\
\hline Sociology & 1/2 & & & \\
\hline Psychology & 1/2 & & & \\
\hline Science & & 4 & 22 & 10.767 \\
\hline Biology & 1 & & & \\
\hline Chemistry & 1 & & & \\
\hline General Science & 1 & & & \\
\hline Physics & 1 & & & \\
\hline Business Education & & 5 & 30 & 7.916 \\
\hline Accounting & 1 & & & \\
\hline Business Law & 1 & & & \\
\hline General Business & 1 & & & \\
\hline Shorthand & 1 & & & \\
\hline Typing & 1 & & & \\
\hline Fine Arts & & 3 & 32 & 12.336 \\
\hline Arts & 1 & & & \\
\hline Crafts & 1 & & & \\
\hline Music & 1 & & & \\
\hline Health and Safety & & 2 & 35 & 7.792 \\
\hline Driver Education & 1/2 & & & \\
\hline Heal th Education & 1/2 & & & \\
\hline Physical Education & 1 & & & \\
\hline Foreign Language & & 2 & 30 & 2. 108 \\
\hline French & 1 & & & \\
\hline Latin & 1 & & & \\
\hline
\end{tabular}

TABLE XXII (Continued)
\begin{tabular}{lcccc}
\hline & \begin{tabular}{c} 
Credit \\
Units
\end{tabular} & \begin{tabular}{c} 
Credit Units \\
By Subject \\
Area
\end{tabular} & \begin{tabular}{c} 
Maximum No. \\
Of Students \\
Per Class
\end{tabular} & \begin{tabular}{l} 
Percentage \\
Enrolled In \\
Subject Area
\end{tabular} \\
\hline Courses & & 8 & 22 & 16.631 \\
\begin{tabular}{l} 
Vocational Education \\
Vocational Agric.
\end{tabular} & 2 & & & \\
\begin{tabular}{l} 
Home Economics \\
Auto Mechanics
\end{tabular} & 2 & 2 & & \\
\begin{tabular}{l} 
Cosmetology \\
Drafting
\end{tabular} & 1 & & & \\
TOTAL CREDIT UNITS & 1 & & & \\
\hline
\end{tabular}

TABLE XXIII

DESIRABLE PROGRAM OF COURSE OFFERINGS
\begin{tabular}{lcccc}
\hline & & & \begin{tabular}{c} 
Credit Units \\
By Subject \\
Credit \\
Urea
\end{tabular} & \begin{tabular}{c} 
Maximum No. \\
Of Students \\
Per Class
\end{tabular}
\end{tabular} \begin{tabular}{c}
\begin{tabular}{c} 
Percentage \\
Enrolled In \\
Subject Area
\end{tabular} \\
Courses
\end{tabular}

\section*{TABLE XXIII (Continued)}
\begin{tabular}{|c|c|c|c|c|}
\hline Courses & \begin{tabular}{l}
Credit \\
Units
\end{tabular} & Credit Units By Subject Area & Maximum No. Of Students Per Class & Percentage Enrolled In Subject Area \\
\hline Foreign Language & & 3 & 30 & 2.108 \\
\hline French & 1 & & & \\
\hline Spanish & 1 & & & \\
\hline Latin & 1 & & & \\
\hline Vocational Education & & 8 & 22 & 16.631 \\
\hline Vocational Agric. & 2 & & & \\
\hline Home Economics & 2 & & & \\
\hline Auto Mechanics & 2 & & & \\
\hline Cosmetology & 1 & & & \\
\hline Drafting & 1 & & & \\
\hline TOTAL CREDIT UNITS & & 46 & & \\
\hline
\end{tabular}

The State's average salaries were \(\$ 9,028\) for principals, \(\$ 8,657\) for guidance counselors, and \(\$ 4,000\) for secretaries. The Oklahoma full-time equivalent principal-to-ADA ratio is 3:1000. This ratio includes both principals and assistant principals. Adequate full-time equivalent guidance counselors and secretarial personnel-to-ADA ratios are 4:1000 and 3:1000, respectively.

\section*{Buildings}

Construction costs for buildings consist of all expenditures for general construction; installation of plumbing, heating, lighting, architectural services; paint; and any other costs connected with the planning of buildings. Data on recently built schools, reported in School Management, give the general cost of construction by size of school district and state. \({ }^{1}\) Equations (3) and (4) derived from these data give a representative picture of the size-cost relationship in building construction in Oklahoma.
\[
\begin{align*}
& \operatorname{CON}_{E}=1,428.944-.057\left(\mathrm{ADA}_{E}\right)+22,488.895\left(1 / \mathrm{ADA}_{E}\right)  \tag{3}\\
& \operatorname{CON}_{S}=1,909.770-.234\left(\mathrm{ADA}_{S}\right)+1,845.033\left(1 / \mathrm{ADA}_{S}\right) \tag{4}
\end{align*}
\]

The variables are as follows:

CON is construction cost per ADA,
E is elementary school, and
\(S\) is secondary school.

Average costs of building construction for high schools and elementary schools are shown in Figures 7 and 8. In computing annual investment and depreciation, it was estimated that a school building loses


Figure 7. Average Cost Per ADA of Buildings for Elementary School


Figure 8. Average Cost Per ADA of Buildings for Secondary School
only 80 percent of its value in 50 years. The average cost of construction was put on an annual basis by considering depreciation, insurance, and interest on investment.

Equipment

Cost of equipment includes expenditures for items of furniture, furnishings, and machinery that are not integral parts of the building. Some examples of equipment include desks, chairs, tables, bookcases, musical instruments, shop machinery and tools, and typewriters. Some economies of size in providing equipment are realized, given constant quality of equipment. Equations (5) and (6) depict the average cost of equipment for elementary and secondary education, excluding vocational equipment. \({ }^{2}\) These equations were adjusted to \(1969-70\) price levels by the Bureau of Labor Statistics commercial furniture price index.
\[
\begin{align*}
& \mathrm{EQP}_{\mathrm{E}}=238.520+4404.357\left(1 / \mathrm{ADA}_{\mathrm{E}}\right)  \tag{5}\\
& \mathrm{EQP}_{\mathrm{S}}=406.735+7919.032\left(1 / \mathrm{ADA}_{\mathrm{S}}\right) \tag{6}
\end{align*}
\]

EQP is equipment cost per ADA. The average cost curves of equipment for elementary and secondary schools (excluding vocational equipment) are shown in Figures 9 and 10.

The cost of vocational equipment varies by the curriculum offered, as shown in Table XXIV. Equipment for an auto mechanics course with 30 students would cost \(\$ 25,040\) or \([1 \times 18,270+(30 / 3) \times 500+(30 / 10) \times\) 590]. These programs show significant economies associated with large classes. For example, the cost per student in an auto mechanics class is \(\$ 208.60\) for only 10 students compared to \(\$ 83.47\) for 30 students.

The cost of equipment per ADA was based on the number of elementary


Figure 9. Average Cost Per ADA of Equipment for Elementary Education


Figure 10. Average Cost Per ADA of Equipment for Secondary Education

TABLE XXIV

\section*{COST OF EQUIPMENT AND SUPPLIES BY} VOCATIONAL PROGRAM
\begin{tabular}{|c|c|c|c|}
\hline Course & Required for Every & Cost of Equipment & Cost of Supplies Per Person \\
\hline \multirow[t]{4}{*}{Auto Mechanics} & & & 30.09 \\
\hline & Class & \$18, 270 & \\
\hline & 3 Persons & 500 & \\
\hline & 10 Persons & 590 & \\
\hline \multirow[t]{4}{*}{Cosmetology} & & & 24.08 \\
\hline & Class & 2,830 & \\
\hline & 3 Persons & 1,200 & \\
\hline & 8 Persons & 200 & \\
\hline \multirow[t]{3}{*}{Drafting} & & & 8.98 \\
\hline & Class & 7,930 & \\
\hline & 1 Person & 490 & \\
\hline \multirow[t]{4}{*}{Air-Conditioning and Refrigeration} & & & . 04 \\
\hline & Class & 6,185 & \\
\hline & 1 Person & 160 & \\
\hline & 3 Persons & 140 & \\
\hline \multirow[t]{4}{*}{Welding} & & & 35.66 \\
\hline & Class & 15,288 & \\
\hline & 5 Persons & 860 & \\
\hline & 8 Persons & 2,410 & \\
\hline \multirow[t]{4}{*}{Home Economics} & & & 15.00 \\
\hline & Class & 1,716 & \\
\hline & 2 Persons & 300 & \\
\hline & 6 Persons & 1,215 & \\
\hline \multirow[t]{3}{*}{Vocational Agriculture} & & & 30.00 \\
\hline & Class & 5,000 & \\
\hline & 6 Persons & 1,500 & \\
\hline
\end{tabular}

\begin{abstract}
students, secondary students, and secondary vocational students by courses. These costs were converted to annual costs by taking into consideration depreciation, insurance, and interest on investment. The equipment's operating life was assumed to be 25 years for nonvocational equipment and 10 years for vocational equipment.
\end{abstract}

\section*{Transportation}

In Oklahoma school districts, public transportation generally is provided for students whose residence is not within 1.5 miles of the school attended. Thus, the school districts are not responsible for transporting the students who live near their schools. Insofar as possible, buses are approved to go at least within one-half mile of each student's home. Transportation costs in rural areas are particularly important because students frequently are dispersed over many square miles. This section examines costs of transporting students.

\section*{Costs Per Bus}

In many districts, the major transportation expense is the drivers' salaries. The average salary per mile of bus route is 75.94 dollars with a standard deviation of 38.139 dollars. There is no stable relationship in the state between bus drivers' salaries and either total miles or total miles adjusted for the number of students. District salary schedules are typically based on a school district's wealth and local wage rates rather than a uniform state salary schedule. In computing transportation cost, the state's average salary was used.

The least expensive combination of body and chassis was selected for each size of bus. The list prices of buses by size are presented in

Table XXV. Depreciation costs were computed from these list prices and a seven-year operating life, the state average. The approximate cost per mile traveled was derived from the cost per mile of bus route using the number of days traveled (180) and the number of times per day the bus route is covered.

Combined cost of fuel and lubricants is one of the largest items of transportation costs, ranking third among all items. With the many starts and stops that school buses must make, their gas mileage is particularly low. A 30-passenger bus in rural Oklahoma typically gets 6.5 miles per gallon. The typical fuel consumption is . 000937 gallons per mile per horsepower. Since the engines for those bus sizes considered in Table XXV only ranged from 165 to 235 horsepower, it was assumed that the above coefficient of fuel consumption per mile per horsepower was applicable in all cases. The cost per gallon of gasoline was assumed to be 26 cents since public schools are exempted from gasoline taxes. The cost of lubricants - oil and grease - was assumed to be 15 percent of the fuel cost. The combined cost of fuel and lubricants are also presented in Table XXV.

Maintenance, the fourth largest bus-operating cost, includes cost of materials and labor for repairs, overhauls, and equipment replacement such as tires. Equation (7), an engineering relationship for calculating the cost of school bus maintenance, was adopted from Bowers' research on truck replacements. \({ }^{3}\) This equation gives the cumulative maintenance cost. An average school bus was assumed to operate 2520 hours in its seven year life. Average maintenance cost per year is found by dividing the cumulative maintenance cost for its life by the number of years of useful life, seven years. This annual cost is

TABLE XXV
FIXED AND VARIABLE COST OF OPERATING SCHOOL BUSES, BY SIZE OF BUS
\(\left.\begin{array}{ccccccc}\hline \begin{array}{l}\text { Seating } \\ \text { Capacity }\end{array} & \begin{array}{c}\text { List } \\ \text { Price }\end{array} & \begin{array}{c}\text { Depreciation } \\ \text { Per Mile of } \\ \text { Bus Route }\end{array} & \begin{array}{c}\text { Interest on } \\ \text { Average Annual } \\ \text { Investment }\end{array} & \begin{array}{c}\text { Cost of Fuel } \\ \text { and Lubricants } \\ \text { Per Mile of } \\ \text { Bus Route }\end{array} & \begin{array}{c}\text { Cost of } \\ \text { Maintenance } \\ \text { Per Mile of } \\ \text { Bus Route }\end{array} & \begin{array}{c}\text { Annual Cost } \\ \text { of }\end{array} \\ \hline 30 & \$ 6,557 & \$ 36.4323 & \$ 216.38 & \$ 16.6422 & \$ 12.4409 & \$ 218.95 \\ \text { Insurance }\end{array}\right]\)
converted to maintenance cost per mile by dividing by the average length of bus route.
\[
\begin{equation*}
M C_{i}=\frac{0.585}{(4,000)^{1.4}} \times\left(\mathrm{TH}_{1}\right)^{1.4} \times \mathrm{LP} \tag{7}
\end{equation*}
\]

The variables are as follows:
\(M C_{1}\) is maintenance costs through year \(i\),
\(\mathrm{TH}_{1}\) is total hours used through year \(i\), and
LP is list price.
Since fuel, lubricant, and maintenance costs have been derived from engineers, the accuracy of these costs must be verified before confidence can be placed in their use. Combined fuel, lubricant, and maintenance expenditures, available from the sampled school districts, were compared with the derived costs shown in Table XXV. The observed number of students per bus was assumed to be the bus size in use. Using derived costs as predictions of actual costs, over 79 percent of the variation in actual costs was explained. For the sampled school districts, average cost of fuel, lubricants, and maintenance per mile of bus route was \(\$ 39\) compared to \(\$ 35\) for average predicted cost. Actual costs, however, included mileage other than transporting students between home and school.

Insurance and interest on investment are the last components of transportation costs. The charge for interest was computed from average annual investment and a six percent interest rate. The cost of fullcoverage insurance was based on a typical package of comprehensive, collision, and liability insurance. The liability coverage had 100-30025 in thousand dollars of individual bodily injury, total bodily injury, and property damage, respectively.

Organization of District Transportation

\begin{abstract}
The cost of transporting students between home and school varies by size of school district and student density. Analysis of the relationship of cost to size and density requires accurate estimates of the number and size of buses used, as well as the distance traveled. The following discussion outlines the procedure used to design an efficient bus route and presents the program used to calculate the number and size of buses needed to service this route. The bus routes in actual use may be slightly shorter than the ones derived here because of clustering of students. For this reason, an "efficient" bus route used here probably underestimates actual mileage typically used to reach a given population but underestimates mileage saved by clustering of students. These biases may be offsetting.
\end{abstract}

Designing a Bus Route. In general, bus routes should be designed to minimize the total distance covered by all buses in transporting students from given locations to school. Achieving this objective insures that the minimum number of buses will be used. The problem is to determine the exact sequence in which stops will be made. In most cases, it is impossible to examine every possible route. The lockset method is one procedure for establishing a route of relatively low cost. \({ }^{4}\) Although this method does not insure an optimum route, it provides an organized and efficient procedure for reducing distance covered. Solutions derived from this method have been tested against results of dispatching methods used by many firms. The actual routes in use were never shorter than the routes prescribed by the lockset method.

Typically, the new route was eight to twelve percent shorter than the actual route.

An example for one route illustrates the steps used in the procedure. A bus begins its route at the school, makes stops at four locations, and then returns to the school. The problem is to sequence the stops at the various student locations so that the distance traveled is as short as possible. The student locations are given and only the sequence of stops remains to be determined.

The first step in minimizing the distance traveled is to obtain a distance matrix. This matrix shows the number of miles between every possible stop, including the school. The distance matrix for a bus route with four stops and the school is shown in Table XXVI. The second number in the first row shows that there are ten miles between the school \(S_{\Theta}\) and student location \(S_{1}\). There are seven miles between student location \(S_{1}\) and student location \(S_{2}\).

TABLE XXVI

DISTANCE MATRIX EXAMPLE
\begin{tabular}{crcccc}
\hline \multirow{3}{*}{ Origin } & \multicolumn{5}{c}{ Destination } \\
\cline { 2 - 7 } & \(S_{0}\) & \(S_{1}\) & \(S_{2}\) & \(S_{3}\) & \(S_{4}\) \\
\hline\(S_{0}\) & 0 & 10 & 8 & 7 & 3 \\
\(S_{1}\) & 10 & 0 & 7 & 4 & 7 \\
\(S_{2}\) & 8 & 7 & 0 & 8 & 6 \\
\(S_{3}\) & 7 & 4 & 8 & 0 & 4 \\
\(S_{4}\) & 3 & 7 & 6 & 4 & 0 \\
\hline
\end{tabular}

The initial basis assumes that every student location is on a onestop route. This basis is shown in Part A of Figure 11. The coefficients depict the maximum distance that can be traveled to transport these students. The distance is reduced by successive modifications of the route. The first step involves every pair of student locations as shown in Table XXVII. Information shown in this table gives the distance between each pair of stops \(\left(S_{1} S_{j}\right)\), as well as each stop and the school \(\left(S_{0} S_{1}\right.\) and \(\left.S_{0} S_{j}\right)\).

TABLE XXVII

PAIRING LIST AND DISTANCE-SAVED COEFFICIENT EXAMPLE
\begin{tabular}{lcccr}
\hline \multirow{2}{*}{ Pairing } & \multicolumn{4}{c}{ Distance-Saved Coefficients } \\
\(S_{0} S_{1}\) & \(S_{0} S_{j}\) & \(S_{1} S_{j}\) & DSC \\
\hline\(S_{1}\) with \(S_{2}\) & 10 & 8 & 7 & 11 \\
\(S_{1}\) with \(S_{3}\) & 10 & 7 & 4 & 13 \\
\(S_{1}\) with \(S_{4}\) & 10 & 3 & 7 & 6 \\
\(S_{2}\) with \(S_{3}\) & 8 & 7 & 8 & 7 \\
\(S_{2}\) with \(S_{4}\) & 8 & 3 & 6 & 5 \\
\(S_{3}\) with \(S_{4}\) & 7 & 3 & 4 & 6 \\
\hline
\end{tabular}

The second step determines how many miles can be saved by combining \(S_{1}\) and \(S_{j}\) on the same route. This savings, termed the distance-saved coefficient (DSC), is computed as follows:
\[
D S C=S_{0} S_{1}+S_{0} S_{1}-S_{11}
\]

A. Initial Solution

B. Pairing \(\mathrm{S}_{1} \mathrm{~S}_{3}\)

D. Pairing \(\mathrm{S}_{3} \mathrm{~S}_{4}\)

Figure 11. Successive Pairing in Selecting
Route by Lockset Method
where:
\(S_{0}\) is the school,
\(S_{1}\) is student location \(i\),
\(S_{j}\) is student location \(j\), and
\(S_{1} S_{j}\) represents the distance between \(S_{1}\) and \(S_{1}\).
The distance-saved coefficients are presented in Table XXVII.

The third step combines the pair of student locations that result in the largest DSC. This pair is \(S_{1}\) and \(S_{3}\) which has a DSC of 13 miles. Part \(B\) of Figure 11 shows the route, \(S_{0} S_{1} S_{3} S_{0}\), which combines this pair. Before accepting this pairing, two restrictions must be met:
(a) Each stop must have at least one leg connected with the origin.
(b) Each stop has not previously been selected.

The pair \(S_{1}\) and \(S_{3}\) can be joined because the pairing meets both restrictions.

Now consider pairing the two student locations with the next largest DSC. Student locations \(S_{1}\) and \(S_{2}\) have the second largest DSC, 11 miles. This pairing meets the two restrictions above, so it becomes part of the revised route, \(S_{0} S_{2} S_{1} S_{3} S_{0}\), as shown in Part \(C\) of Figure 11. This procedure is repeated until all stops are included in the route. The pair with the next largest \(D S C\) is \(S_{2}\) and \(S_{3}\). These two stops are already included in the route. Pairs \(S_{1} S_{4}\) and \(S_{3} S_{4}\) each have a DSC of six miles. However, \(S_{l}\) does not have a leg connected with the origin. The pairing of \(S_{3}\) and \(S_{4}\) meets both restrictions so it can be incorporated into the route. As shown in Part D of Figure 11, all stops have been incorporated into the \(f\) inal route, \(S_{0} S_{2} S_{1} S_{3} S_{4} S_{0}\).

To apply this method to an actual school district, two additional
restrictions must be met in considering possible pairings:
(c) The bus must be of sufficient size.
(d) The bus must be capable of traveling the distance in the allotted time.

The procedure incorporating these additional restrictions follows the description used in the preceding example. As each pair is accepted or rejected, the remaining pair with the largest DSC on the same route under consideration is tested to see if it meets all four restrictions. After all pairs have been considered, the final solution of the lockset method is identified.

In calculating transportation costs for a given school district, the bus routes had to be specified. The routes used in the following analysis conformed to the lockset method to insure an efficient and feasible routing.

Determining Distance Traveled, Size of Buses, and Number of Buses. A computer program was written to determine the size and number of buses used, as well as the distance traveled in transporting students from their homes to school. This program considers both the size of district and density of transported students.

Student locations and size of district are assumed to be known. District size is based on the concept of area served - the area through which buses must travel to transfer students. Actual district size can differ from the area served if there are no students in an outer portion of the district. Following Oklahoma laws, the bus routes are designed to go at least within one-half mile of each student's home. For a generalized discussion of student transportation costs, it is assumed that the students are evenly dispersed. Even if there is some clustering,
the results will not vary markedly since the bus has to travel the road to pick up any lone student anyway. Also, the roads are assumed to be built in a grid one mile apart. This type of road system is common in rural Oklahoma counties.

Based on the size of district, the program makes a preliminary estimate of distance to be covered by bus routes. The estimated distance and student density are used to compute the bus size needed. Since buses in Oklahoma are stationed at the beginning of their respective routes, the route outlined in Figure 12 in general represents the minimum distance route. This route covers one-half the district and can be doubled to get information for the entire district.

To illustrate the procedure used in determining the distance covered, suppose a bus has already traveled from A to D. Before the bus leaves Point D, the program tests to see if the bus
(a) has the capacity to pick up the students along road segment DE , and
(b) can pick up all the students along road segment \(D E\) and still get to school in the allotted time.

If both restrictions are met, the route of the current bus extends through this segment, in this case segment \(D E\). If either of these restrictions is not met, the bus will pick up only as many students as it can and still meet the restrictions. Then a new bus starts its route where the last route ended. The entire procedure is outlined by the block diagram of Figure 13.

Determining Average Transportation Costs by Student Density. All costs of owning and operating school buses presented in Table XXV were combined with the information on routing to determine the average cost


Figure 12. Generalized Routes For One Size District and Successive Expansions of District Size


Figure 13. Diagram Showing How to Determine Total Distance
Traveled in Transporting Students
of transporting students by student density and district size. Average cost curves for different student densities are presented in Figure 14. The curves for the low density districts show that it is economically infeasible to transport large numbers of students in these sparsely populated areas.


\section*{FOOTNOTES}
\(1^{1 " C o s t ~ o f ~ C o n s t r u c t i o n, " ~ S c h o o l ~ M a n a g e m e n t, ~ V o l . ~} 14\) (July, 1970), pp. 15-35.
\({ }^{2}\) Walter Isard and Robert E. Couglin, Municipal Costs and Revenues Resulting from Community Growth (Wellesley, Mass., 1957), pp. 71-74.
\(3_{\text {Wendell Bowers, Modern Concepts of Farm Machinery Management }, ~}^{\text {Con }}\), Oklahoma State University (Champaign, Ill., 1970), pp. 30-37.
\({ }^{4}\) Leonard W. Schruben and Ruth E. Clifton, "The Lockset Method of Sequential Programming Applied to Routing Delivery and Pickup Trucks," American Journal of Agricultural Economics, Vol. 50, No. 4 (November, 1968), pp. 854-867.

\section*{CHAPTER VII}

\section*{OPTIMIZATION PROCEDURE}

The major objective of this study is to determine the optimum resource mix and size of school district. To meet this objective, the average cost curve is minimized subject to constraints on schooling quality and characteristics of the geographic area. This chapter describes the mathematical programming procedure used to optimize the resource mix.

Linear programming, a mathematical programming technique which allows for the substitution of inputs, is used here to select the optimum alternative. It optimizes a linear function subject to linear constraints. The general linear programming problem can be stated as:
\[
\begin{array}{r}
\text { optimize } Q=C ' X \\
\text { subject to } A X \geq B \\
X \geq 0
\end{array}
\]
where \(X\) is a ( \(N \times 1\) ) vector of activities, \(A\) is a (M \(X\) ) matrix of input-output coefficients, \(B\) is a \((M \times 1)\) vector of constraints, \(C\) is a \((N \times 1)\) vector of costs per unit of each activity and \(Q\) is the value to be optimized.

Even though linear programming can minimize average cost subject to the constraints, the model to be used in this study must also consider the economies and diseconomies of size which may exist in education.

Modified linear programming procedures can be used in the case of nonlinear, convex objective functions. Solutions to these problems are based on the assumption of a convex feasible solution, which can arise from increasing returns to size or decreasing average costs.

Consider the problem of minimizing the average cost of transportation and instruction similar to the average cost curve shown in Figure 15. The average cost of transportation and instruction is convex within the range of values being considered. Normal linear programming procedures cannot be used to solve the model which includes both economies and diseconomies of size. Separable programming may be used to solve the nonlinear problem if the nonlinear function is a function of a single variable. In the present study, the nonlinear function is determined by the number of students transported and educated.

The method for solving these problems containing nonlinear functions was developed by Miller. \({ }^{1}\) The method assumes each nonlinear function can be represented by linear equations coupled with logical restrictions. The simplex method has been modified to enforce these logical restrictions.

Suppose that a variable of interest is in the functional form \(f(Z)\) as represented in Figure 15. This function can be replaced by a piecewise linear approximation based on a finite number of points. Let the coordinates of the points be \(\left(a_{1}, b_{1}\right)\), and certain special variables \(S_{1}, S_{2}, \ldots, S_{n}\) be defined so that:
and
\[
\begin{align*}
& S_{1}+S_{2}+\ldots+S_{n}=1  \tag{1}\\
& a_{1} S_{1}+\ldots+a_{n} S_{n}=z  \tag{2}\\
& b_{1} S_{1}+\ldots+b_{n} S_{n}=f(Z) \tag{3}
\end{align*}
\]


ADA
Figure 15. Average Cost of Instruction and Transportation and Its
Representation by Piecewise Linear Segments

If \(S_{1}=1, S_{z}=\ldots=S_{n}=0\), then \(Z=a_{1}\) and \(f(Z)=b_{p}\) In the separable programming problem, the special variables enter the analysis in sequence. For instance, \(S_{z}\) is the second variable to enter the analysis but can do so only after \(S_{1}\) has reached its upper limit.

For purposes of illustration, a simplified tableau for a separable programming model is presented in Table XXVIII. This matrix contains many relationships that will be used in later analysis. Variables listed are:
\(P T R_{j}^{1}=\) pupil-teacher ratio where \(i\) is the class size and \(j\) is the educational program (1 = elementary and \(2=\) secondary),
\(\operatorname{EXP}_{\mathrm{g}}=\) average teacher experience,
\(\mathrm{TR}=\) accounting transfer column,
\(\mathrm{ADA}=\) number of pupils in average daily attendance,
\(\operatorname{COST}=\) cost associated with transporting students,
\(M_{1 j}, N_{1 j}, P_{1 j}, Q_{1 j}, R_{1 j}\), and \(S_{i j}=\) regression coefficients
\(S_{1}=\) special variables,
\(a_{1}\) and \(b_{1}=\) coordinates of special variables,
\(\mathrm{ACH}_{\mathrm{j}}=\) specified achievement level, and
\(A B S_{j}=\) specified absentee rate.

Row (1) shows the costs associated with each variable. The cost per ADA associated with teacher experience varies by pupil-teacher ratio. For example, \(C_{l_{2}}\), the cost of an average year of experience when the pupil-teacher ratio is 25 , is greater than \(C_{14}\), the cost of an average year of experience when the pupil-teacher ratio is 30. In elementary education, the relationship of experience to pupil-teacher ratio is insured by rows (7), (8), and (9). Row (7) insures that only one pupilteacher ratio exists at a time. Rows (8) and (9) link the experience

\section*{TABLE XXVIII}

MATRIX FOR THE SEPARABLE MODEL: ELEMENTARY AND SECONDARY EDUCATION WITH CONSTRAINTS ON ACHIEVEMENT SCORES AND ABSENTEE RATES
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & \[
\mathrm{PTR}_{1}^{25}
\] & \(\operatorname{EXP}_{1}\) & \(\mathrm{PTR}_{1}{ }^{30}\) & \(\operatorname{EXP}_{1}\) & TR & \[
\mathrm{PTR}_{2}^{25}
\] & \(\mathrm{EXP}_{2}\) & \[
\mathrm{PTR}_{2}^{30}
\] & \(\operatorname{EXP}_{2}\) & ADA & Cost & \[
\frac{\text { Specia }}{S_{1}}
\] & \[
\frac{\text { al Varlables }}{S_{2} \cdots S_{n}}
\] & Type of Restriction & \[
\begin{aligned}
& \text { Right Hand } \\
& \text { Side }
\end{aligned}
\] \\
\hline 1 & \(\mathrm{C}_{11}\) & \(\mathrm{C}_{12}\) & \({ }^{\text {c }} 13\) & \(\mathrm{C}_{14}\) & & \(\mathrm{C}_{21}\) & \(\mathrm{c}_{22}\) & \(\mathrm{c}_{23}\) & \(\mathrm{C}_{24}\) & & 1 & & & N & OBJ \\
\hline 2 & \(\mathrm{M}_{11}\) & \(\mathrm{M}_{12}\) & \(\mathrm{M}_{13}\) & \(\mathrm{M}_{14}\) & -1 & & & & & & & & & \(=\) & 0 \\
\hline 3 & . & & & & 1 & & & & & & & & & \(\geq\) & \(\mathrm{ACH}_{1}\) \\
\hline 4 & \(\mathrm{N}_{11}\) & \(\mathrm{N}_{12}\) & \(\mathrm{N}_{13}\) & \({ }^{\mathrm{N}} 14\) & & & & & & & & & & \(\leq\) & \({ }_{A B S}{ }_{1}\) \\
\hline 5 & & & & & \(\mathrm{P}_{20}\) & & \(\mathrm{P}_{22}\) & & \(\mathrm{P}_{24}\) & & & & & \(\geq\) & \(\mathrm{ACH}_{2}\) \\
\hline 6 & & & & & & \(Q_{21}\) & Q22 & \(Q_{23}\) & \(\mathrm{Q}_{24}\) & \(Q_{25}\) & & & & \(\leq\) & \(\mathrm{ABS}_{2}\) \\
\hline 7 & 1 & & 1 & & & & & & & & & & & = & 1 \\
\hline 8 & \(\mathrm{R}_{11}\) & -1 & & & & & & & & & & & & \(\geq\) & 0 \\
\hline 9 & & & \(\mathrm{R}_{13}\) & -1 & & & & & & & & & & \(\geq\) & 0 \\
\hline 10 & & & & & & 1 & & 1 & & & & & & = & 1 \\
\hline 11 & & & & & & \(\mathrm{s}_{21}\) & -1 & & & & & & & \(\geq\) & 0 \\
\hline 12 & & & & & & & & \(S_{23}\) & -1 & & & & & \(\geq\) & 0 \\
\hline 13 & & & & & & & & & & -1 & & \(\mathrm{a}_{1}\) & \(a_{2} \ldots a_{n}\) & \(=\) & 0 \\
\hline 14 & & & & & & & & & & & -1 & \(\mathrm{b}_{1}\) & \(\mathrm{b}_{2} \cdots \mathrm{~b}_{n}\) & \(=\) & 0 \\
\hline 15 & & & & & & & & & & & & - & - ...- & Lower & Bound \\
\hline 16 & & & & & & & & & & & & 1 & 1 ... 1 & \(\mathrm{U}_{\text {Pper }}\) & Bound \\
\hline
\end{tabular}
variables to the appropriate pupil-teacher ratio.

Rows (3) and (4) are constraints on elementary achievement scores and absentee rates. Each year of average teacher experience adds \(M_{1}\) units to the average achievement score. Row (4) insures a given level of elementary achievement. The elementary achievement score is transferred by column TR to the secondary achievement constraint row. Each unit of elementary achievement adds \(P_{z o}\) units to the average secondary achievement score. Each unit of \(A D A\) adds \(Q_{25}\) units to the secondary absentee rate. The absence of a coefficient in Row (5) under the secondary pupil-teacher ratio variable indicates that all variables do not have to affect all constraints.

\section*{FOOTNOTES}
\({ }^{1}\) Clair E. Miller, "The Simplex Method for Local Separable Programming," Recent Advances in Mathematical Programming, ed. R. L. Graves (New York, 1963), pp. 80-100.

\section*{CHAPTER VIII}

\section*{ANALYSIS}

An understanding of a school district's optimum resource combination to meet specified objectives is essential for efficient operation. The school district must take into account those factors outside its control, such as student background and student density, which affect either productivity or cost. Optimum school district organization is evaluated herein for various alternatives - curriculum, level of educational output, salary schedule, student background, and student density. Several levels of each alternative are presented so that trends in resource adjustments for that alternative can be identified.

\section*{Model Formulation}

The models of this chapter are formulated to determine the minimum average cost of educating and transporting students within a school district to meet specified levels of educational output. The model is specified in a separable programming format in which the objective function average cost is minimized subject to the production constraints.

Production estimates for elementary, junior high, and high school education are based on the production functions discussed in Chapter \(V\). Minimum output levels are specified for each of the production functions. Student input and environmental variables are held constant, while the educational process variables are allowed to adjust to meet
these restrictions at minimum cost. These educational process variables include teacher experience, percentage of teachers with a master's degree, percentage of teachers with a planning period, pupil-teacher ratio, value of audiovisual material, printed volumes per pupil, school district size, and other variables.

These educational process variables have costs associated with them, which must be included in the cost function. Cost estimates used herein were discussed in Chapter VI. There are certain costs such as current expenditures for buildings, equipment, administration, guidance counselors, secretaries, instructional supplies, and transportation that are directly connected to ADA. Once the optimum district size or ADA is determined, these costs are also determined.

\section*{Resource Situation}

The basic model was made up of sample averages, and the high school was assumed to offer the standard adequate program. Educational outputs for all three grades - composite scores, modern math scores, language arts scores, absentee rates, and dropout rates - were specified at the sample averages. All student background variables were set at their averages. The salary used for a beginning teacher with a bachelor's degree was \(\$ 5,750.70\). The school district was assumed to have 265 nontransported \(A D A\) and a student density of 1.8 transported ADA per square mile.

Among other results, the basic separable programming problem solution shows the change in average cost associated with a one-unit change in each educational output. The cost of changing the level of these educational output variables is of interest since it indicates the cost
of alternative strategies used in improving schooling quality. These costs are presented in Table XXIX. The change in average cost of lowering the fourth grade absentee rate one percent below its level in equilibrium was \(\$ 4.29\). In general, the costs associated with a one percent decrease in absentee and dropout rates were high, because one percentage unit change of these rates was more than one-fourth of their mean values. The largest change in average cost for increasing achievement scores was eighth grade modern math.

\section*{Cost-Effectiveness Evaluation}

A cost-effectiveness model was used to evaluate the effect of educational output, salary, student background, student density, and high school curriculum on average cost and optimum resource combination. While any one situation was varying, all other characteristics were held constant at their respective averages. These situations include the main kinds of operations facing school districts. However, a mixture of these pure cases would occur in any given school district. In effect, they represent patterns which can improve judgments about the consequences of a given school district's organization.

\section*{Educational Output}

Educational output measures used in the model are composite achievement scores, modern math scores, language arts scores, absentee rates, and dropout rates. Oklahoma school district averages for these outputs are shown in Table XXX. Three levels of output - low, average, and high - were included in the analysis to represent differences in goals among school districts. Achievement scores were lowered five

\section*{TABLE XXIX}

CHANGES IN AVERAGE COST PER ADA ASSOCIATED WITH ONE UNIT IMPROVEMENT IN EDUCATIONAL OUTPUT FROM EQUILIBRIUM
\begin{tabular}{|c|c|c|c|c|c|}
\hline Variable & Equilibrium Value & Increase in Average Cost For One Unit Increase & Variable & Equilibrium Value & Increase in Average Cost For One Unit Decrease \\
\hline Fourth Grade & & & Fourth Grade & & \\
\hline Language Arts & 281.57 & \$1.19 & Absentee Rate & 2.39 & \$4.29 \\
\hline Modern Math & 285.15 & 3.16 & & & \\
\hline Composite & 152.01 & 2.36 & & & \\
\hline Eighth Grade & & & Eighth Grade & & \\
\hline Language Arts & 351.96 & 0.07 & Absentee Rate & 2.93 & 2.92 \\
\hline Modern Math & 440.26 & 0.03 & & & \\
\hline Composite & 183.92 & 0. 14 & & & \\
\hline Eleventh Grade & & & Eleventh Grade & & \\
\hline Language Arts & 438.03 & 0.96 & Absentee Rate & 1.30 & 5.92 \\
\hline Modern Math & 424.90 & 1.12 & Dropout Rate & 3.44 & 4.26 \\
\hline Composite & 90.84 & 0.29 & & & \\
\hline
\end{tabular}
percent and absentee and dropout rates were raised five percent to develop a measure of "low" educational output. On the other hand, the measures of output were improved five percent to represent "high" educational output. Five percent of the achievement scores represents about one standard deviation. This was the largest uniform variation that could be made and still get an optimum solution with average characteristics of student background.

TABLE XXX

AVERAGES OF SELECTED MEASURES OF EDUCATIONAL OUTPUTS
\begin{tabular}{lcrr}
\hline & \begin{tabular}{c} 
Fourth \\
Grade
\end{tabular} & \begin{tabular}{c} 
Eighth \\
Grade
\end{tabular} & \begin{tabular}{c} 
Eleventh \\
Grade
\end{tabular} \\
\hline Composite Achievement Score & 151.48 & 18.3 .50 & 77.03 \\
Modern Math Score & 285.15 & 440.26 & 412.12 \\
Language Arts Score & 271.50 & 350.09 & 414.67 \\
Absentee Rate & 2.83 & 2.93 & 2.98 \\
Dropout Rate & - & -- & 3.44 \\
\hline
\end{tabular}

Table XXXI presents results of the separable programming model with these three levels of educational output used as constraints. Average cost per ADA ranged from \(\$ 661.67\) for the low level of output to \(\$ 767.62\) for the high level of output. To attain the high level of output, the school district size was reduced from 675 to 377 ADA. Under the low output situation, all pupil-teacher ratios were at their maximum

\section*{TABLE XXXI}

OPTIMUM RESOURCE COMBINATION BY EDUCATIONAL OUTPUT
\begin{tabular}{|c|c|c|c|}
\hline & Low Output & \begin{tabular}{l}
Average \\
Output
\end{tabular} & High Output \\
\hline Average cost per ADA (dollars) & 661.67 & 691.85 & 767.62 \\
\hline ADA & 675.00 & 675.00 & 377.00 \\
\hline \multicolumn{4}{|l|}{Pupil-teacher ratio} \\
\hline High School & 28.00 & 28.00 & 28.00 \\
\hline Junior High & 30.00 & 30.00 & 22.00 \\
\hline Elementary & 30.00 & 26.00 & 22.00 \\
\hline \multicolumn{4}{|l|}{Percentage of teachers with three to nine years experience} \\
\hline High School & 67.03 & 65.48 & 63.86 \\
\hline Junior High & 25.25 & 33.10 & 75.00 \\
\hline Elementary & 50.24 & 20.56 & 21.35 \\
\hline \multicolumn{4}{|l|}{Percentage of teachers with ten or more years experience} \\
\hline High School & 32.97 & 34.51 & 36.14 \\
\hline Junior High & 74.75 & 66.89 & 25.00 \\
\hline Elementary & 49.76 & 79.44 & 78.65 \\
\hline \multicolumn{4}{|l|}{Average teacher experience} \\
\hline High School & 9.16 & 9.51 & 9.87 \\
\hline Junior High. & 18.58 & 16.81 & 7.36 \\
\hline Elementary & 12.95 & 19.64 & 19.46 \\
\hline \multicolumn{4}{|l|}{Percentage of teachers with a planning period} \\
\hline Junior High & -0- & 25.88 & 93.63 \\
\hline Elementary & 64.12 & -0- & -0- \\
\hline \multicolumn{4}{|l|}{Value of audiovisual material per ADA \(8.7 \mathrm{a}^{\text {a }} 8\)} \\
\hline High School & \(\frac{8.70}{8.70}\) & \(\frac{8.70}{8.70}\) & \(\frac{8.70}{8.70}\) \\
\hline Junior High
Elementary & \(\frac{8.70}{8.70}\) & \(\frac{8.70}{99.90}\) & \(24 \frac{8.70}{1.57}\) \\
\hline
\end{tabular}

TABLE XXXI (Continued)
\begin{tabular}{lll}
\hline & \begin{tabular}{c} 
Low \\
Output
\end{tabular} & \begin{tabular}{c} 
Average \\
Output
\end{tabular} \\
\hline \begin{tabular}{l} 
Printed volumes per ADA \\
High School \\
Junior High \\
Elementary
\end{tabular} & \(\underline{5.30}\) & \(\underline{5.30}\) \\
\hline
\end{tabular}
\(\mathrm{a}_{\text {All underlined values have entered the solution }}\) at their lower limit.
allowable levels (30:1 for elementary and junior high and 28:1 for high school). Pupil-teacher ratios were 22:1 in elementary and junior high to achieve high output.

Definite trends in adjustment of teacher experience to higher levels of output are apparent. At low output levels, the percentage of teachers with three to nine years of experience was greater in elementary and high school and smaller in junior high. Greater teacher experience appears to be more important in elementary and high school at high output levels.

Since the percentage of teachers with a planning period had no effect on eleventh grade output, it never entered the solution in high school. A greater percentage of junior high teachers with a planning period was required in order to attain the high output level. Percentage of elementary teachers with a planning period appeared to be redundant at the high output level since it was zero at this level but positive at the low output level.

Other factors included in the analysis were value of audiovisual material and printed volumes per pupil. In most cases these variables entered the solution at their lowest allowable level, as represented by the line under the value in the table. Elementary education required increases in the value of audiovisual material in order to achieve higher levels of output. High school education required an increase in the printed volumes per pupil to attain the high level of output.

Student Background

In this instance student background encompasses home influences such as parents' education and occupation, as well as students' study
habits and attitudes toward education. The three student background levels included in this analysis are ten percent below average, average, and ten percent above average. For example, 78 percent of the fourth graders talked to their parents about schoolwork at least once a week. This figure ( 78 percent) was used in the average student background model. Ninety percent of this figure was used in the low student background model. All other variables relating to student background were similarly adjusted for the different models.

A given level of output with a lower student background is more costly to attain. The results of these three models are presented in Table XXXII. For the students with low backgrounds to attain the average level of educational output, district size was 370 ADA. Average cost of attaining the average level of output was \(\$ 794.93\) for a district with students from a low background versus \(\$ 672.60\) for a district with students from a high background.

Since eleventh grade output was never affected by the pupil-teacher ratio, the high school pupil-teacher ratio always entered the solution at 28:1. On the other hand, the pupil-teacher ratio was 22:1 in both elementary and junior high with the low student background. These ratios were higher with both average and high student backgrounds.

With increases in student background, teacher experience tended to go down in high school and up in junior high. There was no change in fourth grade teacher experience associated with these levels of student background. Percentage of teachers with a planning period declined in both elementary and junior high as student background improved. Elementary education required a much greater investment in audiovisual material to achieve average output as student background declined.

TABLE XXXII

OPTIMUM RESOURCE COMBINATION BY STUDENT BACKGROUND
\(\left.\begin{array}{lrl}\hline & \begin{array}{c}\text { Low } \\ \text { Student } \\ \text { Background }\end{array} & \begin{array}{c}\text { Average } \\ \text { Student } \\ \text { Background }\end{array} \\ \text { Student } \\ \text { Background }\end{array}\right]\)

TABLE XXXIT (Continued)
\begin{tabular}{lccc}
\hline & \begin{tabular}{c} 
Low \\
Student \\
Background
\end{tabular} & \begin{tabular}{c} 
Average \\
Student \\
Background
\end{tabular} & \begin{tabular}{c} 
High \\
Student \\
Background
\end{tabular} \\
\hline \begin{tabular}{l} 
Printed volumes per ADA \\
High School \\
Junior High \\
Elementary
\end{tabular} & \(\underline{5.30}\) & & \\
\hline
\end{tabular}

\section*{High School Curriculum}

The three high school curriculums, which were discussed previously, are as follows:
(1) minimum program (38 academic units and no vocational units),
(2) adequate program (30 academic units and 8 vocational units), and
(3) desirable program (38 academic units and 8 vocational units). These programs are listed in ascending order with respect to their ability to meet student needs.

Table XXXIII shows that it is much more expensive to provide an extensive curriculum which includes vocational courses. Average costs for the three programs ranged from \(\$ 665.48\) without vocational courses to \(\$ 700\) for the desirable program. To achieve minimum average cost, the school district size increased from 550 ADA for the minimum program to 900 ADA for the desirable program.

Among the three programs, there were only minor differences in other policy variables. The high school pupil-teacher ratio with the minimum program was 30:1. Since vocational courses require smaller classes, the adequate and desirable programs had smaller high school pupil-teacher ratios; both were approximately 28:1. There were slight changes in teacher experience in high school and junior high to adjust for potential declines in performance associated with increased school district size. Likewise, 55.42 percent of high school teachers had to have a master's degree to maintain output in the larger district required by the desirable program.

\section*{TABLE XXXIII}

OPTIMUM RESOURCE COMBINATION BY HIGH SCHOOL CURRICULUM
\(\left.\begin{array}{lcc}\hline & \begin{array}{c}\text { Minimum } \\ \text { Program }\end{array} & \begin{array}{c}\text { Adequate } \\ \text { Program }\end{array} \\ \hline \text { Program }\end{array}\right]\)

TABLE XXXIII (Continued)
\begin{tabular}{|c|c|c|c|}
\hline & Minimum Program & Adequate Program & Desirable Program \\
\hline \multicolumn{4}{|l|}{Printed volumes per ADA} \\
\hline High School & 5.30 & 5.30 & 5.30 \\
\hline Junior High & 5.30 & 5.30 & 5.30 \\
\hline Elementary & 5.30 & 5.30 & 5.30 \\
\hline Percentage of high school teachers with a masters degree & -0- & -0- & 55.42 \\
\hline
\end{tabular}

Teacher Salary

The average salary for a beginning teacher with a bachelor's degree was \(\$ 5,750.70\). The three salary levels considered were ten percent below average, average, and ten percent above average. In all three cases, teachers were assumed to receive \(\$ 100\) for each additional year of experience. The adjustments, particularly in this section, are assumed to be long-run adjustments, since the influence of salary on teacher performance takes several years to be fully realized. In other words, increasing the salary level approximately \(\$ 500\) a year will not materially improve present teacher's performance immediately; but in the long run, it will encourage teachers to upgrade their skills through continuing higher education and at the same time attract betterqualified teachers.

Even though the range in teacher salary was greater than \(\$ 1,000\), the range in average cost per ADA was less than \(\$ 20\) - as shown in Table XXXIV. Under the low-salary situation, teacher performance was assumed to have deteriorated. Only by increasing expenditures on other educational process variables could the same level of educational output be attained. With the low salary level, the pupil-teacher ratio had to be lowered in the elementary grades. As the cost of teachers increased, the elementary pupil-teacher ratio increased to substitute less expensive inputs for teachers.

In general, teacher experience had to be increased under the low salary situation to maintain the average level of educational output. All eighth grade teachers and more than one-half of the elementary teachers were given planning periods under the low-salary situation.

TABLE XXXIV
OPTIMUM RESOURCE COMBINATION BY TEACHER SALARY LEVELS
\begin{tabular}{|c|c|c|c|}
\hline & \[
\begin{aligned}
& \text { Low } \\
& \text { Salary }
\end{aligned}
\] & Average Salary & \[
\begin{aligned}
& \text { High } \\
& \text { Salary }
\end{aligned}
\] \\
\hline Average cost per ADA (dollars) & 690.38 & 691.85 & 709.93 \\
\hline ADA & 675.00 & 675.00 & 675.00 \\
\hline \multicolumn{4}{|l|}{Pupil-teacher ratio} \\
\hline High School & 28.00 & 28.00 & 28.00 \\
\hline Junior High & 30.00 & 30.00 & 30.00 \\
\hline Elementary & 24.00 & 26.00 & 28.00 \\
\hline \multicolumn{4}{|l|}{Percentage of teachers with three to nine years experience} \\
\hline High School & 50.63 & 65.48 & 62.42 \\
\hline Junior High & 26.76 & 33.10 & 60.15 \\
\hline Elementary & 23.63 & 20.56 & 20.56 \\
\hline \multicolumn{4}{|l|}{Percentage of teachers with ten or more years experience} \\
\hline High School & 49.37 & 34.51 & 25.00 \\
\hline Junior High & 73.23 & 66.89 & 39.85 \\
\hline Elementary & 76.37 & 79.44 & 79.44 \\
\hline \multicolumn{4}{|l|}{Average teacher experience} \\
\hline High School & 12.86 & 9.51 & 7.14 \\
\hline Junior High & 18.24 & 16.81 & 10.71 \\
\hline Elementary & 18.95 & 19.64 & 19.64 \\
\hline \multicolumn{4}{|l|}{Percentage of teachers with a planning period} \\
\hline Junior High & 100.00 & 25.88 & -0- \\
\hline Elementary & 52.19 & -0- & -0- \\
\hline \multicolumn{4}{|l|}{Value of audiovisual material per ADA} \\
\hline High School & 101.43 & 8.70 & 8.70 \\
\hline Junior High & 8.70 & 8.70 & 8.70 \\
\hline Elementary & 8.70 & 99.90 & 221.39 \\
\hline
\end{tabular}

TABLE XXXIV (Continued)
\begin{tabular}{lccc}
\hline & \begin{tabular}{c} 
Low \\
Salary
\end{tabular} & \begin{tabular}{c} 
Average \\
Salary
\end{tabular} & \begin{tabular}{c} 
High \\
Salary
\end{tabular} \\
\hline \begin{tabular}{l} 
Printed volumes per ADA \\
High School \\
Junior High \\
Elementary
\end{tabular} & \(\underline{5.30}\) & & \\
\hline
\end{tabular}

Also, expenditures on audiovisual material in high school and printed volumes in junior high had to be increased with lower salaries.

Student Density

The high level of transportation cost in rural areas makes its influence on school district organization especially important. The average transportation cost which is based on student density is a major determinant of school district size. Student performance is assumed to deteriorate slightly with increases in district size, causing minor adjustments in other educational process variables to maintain the level of educational output.

On the other hand, differences in student densities cause significant differences in optimum school district size and average cost as shown in Table XXXV. Average cost with optimum organization was \(\$ 744.27\) for a student density of 0.6 transported ADA per square mile and \(\$ 660.68\) for a 3.0 student density. Figure 16 shows that, in the heavily populated areas, school districts can operate anywhere within a wide range of ADA without significant differences in per-unit costs. School districts operating outside this low cost range, particularly in sparsely populated areas, face substantially higher per-unit costs. The optimum school district size ranged from 300 ADA with the light density to 1,075 with the heavy density. The relationship between student density and optimum school district size is shown in Figure 17. This figure shows a positive relationship between student density and optimum school district size; i.e., the optimum school district size is smaller in sparsely populated rural areas.

TABLE XXXV
OPTIMUM RESOURCE COMBINATION BY STUDENT DENSITY


TABLE XXXV (Continued)




\section*{Other Cases}

Since the modern math and composite scores of the lower grades were linked statistically with achievement in the upper grades, they were also linked in the separable programming model. By removing the restrictions on these scores in the lower grades, it was possible to determine where expenditures should be made in the different phases of education to obtain the highest level of eleventh grade achievement. The two alternatives were to increase expenditures in the lower grades or in the upper grades. The results showed that without these restrictions, there was more flexibility in the lower grades with respect to resource combination. Less effort was directed into elementary and junior high education and more into high school education to attain the highest level of high school achievement.

In the basic model, the cost of a year's experience was assumed to be \(\$ 100\). Lowering this cost 20 percent resulted in no change in the optimum resource combination. With a 20 percent increase in the cost of experience, other educational process variables, such as percentage of teachers with a planning period, were substituted to reduce the use of the higher cost experience.

The number of nontransported students can affect the optimum school district size. The 265 nontransported students in the basic model resulted in an optimum school district size of 675 ADA. An increase of 25 nontransported ADA increased optimum school district size 15 ADA. In general, a larger number of nontransported ADA resulted in a larger optimum school district size but smaller number of transported ADA.

Costs considered so far have included charges for buildings and equipment. However, most information available and, consequently, most

\begin{abstract}
previous research included only current costs. Since there are some economies of size associated with providing buildings and equipment, research considering only current cost understates optimum school district size. In this analysis, minimum average current cost of \(\$ 556.75\) per student occurred at 550 ADA compared to minimum average cost of \(\$ 691.85\) per student at 675 ADA.
\end{abstract}

\section*{Sensitivity Analysis}

In evaluating the results of a model, it is important to know how sensitive the results are to changes in the basic parameters of the model. The eleventh grade dropout rate, eighth grade absentee rate and average modern math score, and fourth grade average modern math score were all at their limits in the equilibrium model. The dropout rate could have varied three percentage points with no change in the optimum set of activities. This change is relatively large, since the average dropout rate was 3.4 percent. In the eighth grade, the absentee rate could have varied .6 percentage points and the modern math score could have varied 12 units with no change in activities. Fourth grade modern math score could have varied four units with no change in activities. Looking at the regression coefficients of the production functions, coefficients of the pupil-teacher ratio could have varied five percent in the fourth grade and ten percent in the eighth grade with no change in the optimum pupil-teacher ratio.

\section*{Benefit-Cost Analysis}

Any policy alternative, such as a change in the pupil-teacher ratio, may affect more than one dimension of educational output.

Therefore, it is desirable to have a meaningful aggregation of the output variables. One such aggregation can be accomplished by converting the achievement test scores to yearly equivalent changes in test scores. A comparison of benefits to costs for a particular policy can then be made.

\section*{Benefits}

To measure the benefits of a particular policy alternative, changes in average achievement test scores were estimated from the production functions of Chapter \(V\). The change in yearly equivalents of language arts, modern math, and composite achievement scores were averaged together to obtain an over-all measure of change in yearly equivalents. Assuming that the yearly equivalent change in average test scores is sustained and can be used as a measure of the change in years of schooling, dollar values can be assigned to the benefits. The results will be biased to the extent that improved achievement is lost over time.

Redfern estimated the discounted value of the extra earnings that an individual would expect to receive by obtaining extra schooling. \({ }^{1}\) Adjusting his figures for sex and race of students according to the sampled school districts, the average present value for an additional year of schooling was \(\$ 4,478.66\) in \(1969-70\) for students finishing their twelfth year of schooling. Similar to Becker's work, the average income was assumed to increase two percent a year. \({ }^{2}\) Although lower classmen could expect a greater income upon completion of their schooling, its present value would be smaller since it had to be discounted to the year under consideration. Using a six percent discount rate, the
present value of a year's schooling was \(\$ 2,809.96\) for fourth graders, \(\$ 3,547.51\) for eighth graders, and \(\$ 4,306.40\) for eleventh graders.

Costs

To make a valid comparison between costs and benefits, costs had to be accumulated by compounding (or discounting) to the same period in which the benefits were being considered. Since fourth grade achievement was considered in elementary education, the costs of a policy alternative (e.g., increasing teacher experience) for the first four grades were associated with the benefits of that alternative. Likewise, costs of policy alternatives for the seventh and eighth grades were associated with eighth grade achievement, and costs of policy alternatives for the ninth through eleventh grades were associated with eleventh grade achievement.

Benefit-Cost Ratio

The equilibrium values of the educational process variables from the basic model were used to compute benefit-cost ratios as shown in Table XXXVI. The benefit-cost ratio for average teacher experience ranges from 6:1 in the fourth grade to 16:1 in the eleventh grade. As expected, pupil-teacher ratios, audiovisual material, and printed volumes had a positive benefit in the lower grades, and teacher education had a positive benefit in the eleventh grade.

Surprisingly, the benefit-cost ratio of the pupil-teacher ratio in the fourth grade was very low - .28:1. Other important variables may have failed to show a positive benefit because all schools had a sufficient level of these variables. For example, just because printed

TABLE XXXVI

\section*{AVERAGE BENEFITS AND COSTS ASSOCIATED WITH VARIOUS EDUCATIONAL PROCESS VARIABLES}
\begin{tabular}{|c|c|c|c|c|c|}
\hline Variable & Increment & Change in Yearly Equivalent of Mean Test Score & \begin{tabular}{l}
Present \\
Value of Benefits
\end{tabular} & \begin{tabular}{l}
Present \\
Value of Costs
\end{tabular} & \begin{tabular}{l}
Benefit- \\
Cost \\
Ratio
\end{tabular} \\
\hline Fourth Grade & & & (\$) & (\$) & \\
\hline Percentage of teachers with three to nine years of experience & 10 percent & . 0883 & 248.12 & 10.05 & 24.69 \\
\hline Percentage of teachers with ten or more years of experience & 10 percent & . 0687 & 193.04 & 23.56 & 8.19 \\
\hline Average teacher experience & 1 year & . 0379 & 106.50 & 16.82 & 6.33 \\
\hline Percentage of teachers with a planning period & 10 percent & . 0074 & 20.79 & 17.93 & 1. 16 \\
\hline Pupil-teacher ratio & from 26:1 to 24:1 & . 0079 & 22.20 & 80.63 & . 28 \\
\hline Value of audiovisual material per pupil & 10 dollars & . 0044 & 12.36 & 5.60 & 2.21 \\
\hline Eighth Grade & & & & & \\
\hline Percentage of teachers with three to nine years of experience & 10 percent & . 0645 & 228.81 & 4.12 & 55.54 \\
\hline Percentage of teachers with ten or more years of experience & 10 percent & . 0136 & 48.25 & 9.61 & 5.02 \\
\hline Average teacher experience & 1 year & . 0168 & 59.60 & 6.87 & 8.68 \\
\hline Pupil-teacher ratio & from 30:1 to 28:1 & . 0294 & 104.30 & 28.21 & 3.70 \\
\hline Printed volumes per pupil & 1 volume & . 0125 & 44.34 & . 90 & 49.27 \\
\hline Eleventh Grade & & & & & \\
\hline Percentage of teachers with three to nine years of experience & 10 percent & . 0338 & 145.56 & 6.82 & 21.34 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline Variable & Increment & Change in Yearly Equivalent of Mean Test Score & Present Value of Benefits & \begin{tabular}{l}
Present \\
Value of Costs
\end{tabular} & \begin{tabular}{l}
Benefit- \\
Cost \\
Ratio
\end{tabular} \\
\hline Percentage of teachers with ten or more years of experience & 10 percent & . 0746 & 321.26 & 15.92 & 20.18 \\
\hline Average teacher experience & 1 year & . 0427 & 183.88 & 11.37 & 16.17 \\
\hline Percentage of teachers with a master's degree & 10 percent & . 0046 & 19.81 & 2.27 & 8.73 \\
\hline
\end{tabular}
volumes in the eleventh grade showed no positive benefit, it is still necessary to provide an adequate library.

Although these benefits and costs were based on averages of the optimum solution of the basic model, they are not necessarily appropriate for directing additional expenditures. The tradeoffs between policy alternatives, such as percentage of teachers with three to nine years of experience and ten or more years of experience, indicate that increases in the levels of some educational process variables cause deteriorations in some dimensions of output. For example, the benefit of an additional year of average teacher experience in elementary education is -.0555 yearly equivalents when all variables are allowed to adjust to equilibrium.

Benefits from an increase in an educational process variable may differ from benefits derived above as the level of other educational process variables adjust. Also, costs may increase as additional expenditures on other educational process variables are needed to maintain a given level of educational output. Allowing variables to adjust, the benefit-cost ratio of eighth grade pupil-teacher ratio is \(4.72: 1\) compared to 3.70:1 when other variables are held constant. If a school district wishes to make small adjustments in its resource organization, it should be aware of these possible tradeoffs in ascertaining benefits and costs.

\section*{FOOTNOTES}
\({ }^{1}\) James Martin Redfern, "Social and Private Returns to Investment in Schooling, By Race-Sex Groups and Urban-Rural Residence" (unpub. dissertation, Oklahoma State University, 1970), pp. 74-78.
\({ }^{2}\) Gary S. Becker, Human Capital (New York, 1964), p. 139.

\section*{CHAPTER IX}

\section*{SUMMARY AND CONCLUSIONS}

The problems concerning inequalities of educational opportunities and high cost of providing educational services, particularly in rural areas, are amenable to economic analysis. Through efficient resource allocation and school district organization, the cost of providing a given quality of schooling can be lowered. This chapter summarizes how economic analysis was used to address these problems. The first section gives a summary of objectives, procedures, and major empirical results. The second section discusses implications and limitations of this study, as well as the need for further research.

\section*{Summary}

This analysis applied economic theory to the problems of school district organization in Oklahoma elementary and secondary education. A major objective was to develop and apply a model to efficiently organize a school district and allocate its scarce resources. This model required information on schooling inputs, output, costs, and, to a lesser extent, returns.

Past studies have measured the impact of inputs on educational output, but stopped short of a comprehensive analysis of an efficient allocation of schooling inputs. Previous studies helped identify relevant input-output variables. The present research used this knowledge
of relevant variables to measure the effect of various inputs on selected educational outputs in Oklahoma. Data from a random sample of Oklahoma school districts were used to estimate functional relationships.

Measures of educational output included achievement scores, dropout rates, and absentee rates. Three broad categories of educational inputs - educational process, student input, and environmental variables - affected these measures of output. The educational process variables, which are under the control of the school board, can be manipulated to raise the level of student attainment. The student input and environmental variables, which must be considered as given in the short run, must be taken into account in any decisions relating to changes in students' level of attainment.

Major control variables (student input and environmental variables) relate to students \({ }^{\prime}\) ability and educational effort and family background. IQ and achievement test scores in earlier grades, representing students \({ }^{\prime}\) ability, significantly contributed to educational output in elementary, junior high, and high school. Time spent studying and numm ber of books read outside of school, as measures of educational effort, were also important. Parents education, occupation, income, and interest in their children \({ }^{\circ}\) s education were directly related to student attainment.

Educational process variables affecting educational output included teacher qualifications and workloads, teacher salaries, instructional materials, program, and school district size. Of the educational process variables, teacher performance had the greatest impact on educational output. Workloads, measured by pupil-teacher ratios and
percentage of teachers with a planning period, were especially important in the lower grades. Teacher education (percentage of teachers with a master's degree) was more important in high school. Teacher salary, standardized for experience and education, affected all dimensions of educational output. The level of teacher experience was also important. Experience was more significant in lower grades than in high school and was more important in reducing absentee and dropout rates than in raising achievement.

As measures of the program, availability of kindergarten improved elementary achievement and number of nonvocational units were positively related to high school achievement. Instructional materials - periodicals, printed volumes, and audiovisual material - had major impact in lower grades. Availability of facilities affected high school achievement; school district size was negatively related to educational output, other variables held constant.

However, increasing school district size permitted internal changes that raised student attainment for a given level of expenditures. Existence of economies of size or cost reductions associated with increases in school district size allowed these savings to be reallocated to raise educational output. This apparent contradiction highlights the importance of knowing the physical relationship between inputs and outputs. Costs must also be taken into consideration, and it is necessary to know each input's contribution to output relative to its cost.

The cost of schooling was divided into transportation and educational costs. Educational costs included expenditures for instruction, administration, and plant operation and maintenance, as well as annual charges for buildings and equipment. All educational costs showed
significant economies of size. Transportation costs, on the other hand, showed slight diseconomies in heavily populated areas and significant diseconomies in sparsely populated areas. Combining transportation and educational costs gives insight into school district organization that largely has been ignored in previous studies.

Using a separable programming model, this study related costs to production functions to find the minimum cost of a given quality of education. Activities affecting both cost and production included teacher qualifications and workloads and instructional materials. Costs of administration, buildings, and equipment were linked directly to ADA or school district size. This model provides information on efficient resource allocation and school district organization for given student and community characteristics, as well as school district goals.

The basic model was initially specified to reflect average levels of output, student background, salary, and student density. In turn, each of these situations was varied to identify its impact on the optimum resource combination.

\section*{Conclusions}

\section*{Implications}

It is apparent that the optimum resource organization in education depends on educational objectives, student backgrounds, high school curriculum, teacher salary, and student density.
1. For a given level of student background, higher levels of educational output were associated with substantial increases in per-unit costs. The most efficient way to attain higher levels of educational output requires a reduction in elementary
and junior high teacher workloads, increased teacher experience and increased expenditures on instructional materials.
2. Students with below average abilities and socioeconomic backgrounds place formidable constraints on attainable educational output. Their performance can be improved, however, with properly allocated higher expenditures.
3. Allowing for long-run adjustments to changes in teacher salaries, student performance was positively related to salary. Thus increases in teacher salaries can be accompanied by reductions in expenditures on other educational process variables.
4. A more extensive curriculum requires larger school districts to efficiently utilize the program. The optimum school district size for a school offering only a minimum program is substantially less than one offering a more comprehensive program.
5. Optimum school district size was also affected by student density. In sparsely populated areas, school districts could not expand in size to take full advantage of economies in instruction because transportation diseconomies were an overriding factor. Transportation costs are not nearly as important in heavily populated districts. In these areas, the optimum school district size is much greater, and the average cost curves for heavily populated districts are nearly flat over a very wide range. School districts can operate anywhere within this range without significant differences in per-unit costs. On the other hand, sparsely populated districts have
a very small range in which they can operate at minimum cost. Only minor deviations in this optimum size result in substantially higher costs.

Since this research has shown that it may be more important to know how funds are spent than the total level of funds, it follows that a general increase in expenditures will not necessarily result in any measurable increase in student performance. If funds are to be employed in a measure to strengthen education, careful attention must be given to productive investment opportunities.

Problems in financing local education stem partly from inappropriate school district organization. Since the structure of school district organization was established generations ago, many rural school districts are too small to meet the needs of their students. Reorganization and consolidation for these school districts offer much greater potential for cost reductions than efficiently organizing resources within the school districts. Reorganizations should be based on a minimum school district size to provide a specified quality of education at the least cost.

The major portion of this study examined costs for reaching some prescribed schooling output, but, for some purposes, measures of the dollar value of output need to be compared with costs. Dollar values were assigned to the yearly equivalent test scores to measure the present value of benefits. The present value of benefits and costs for various policy alternatives were then compared. Surprisingly, the benefit-cost ratio for reducing the pupil-teacher ratio was one of the lowest for any alternative considered. Several other changes in the
allocation of schooling resources had very favorable benefit-cost ratios. The results were crude, however, and they must be interpreted cautiously.

\section*{Limitations}

Conclusions relating to such things as pupil-teacher ratios and instructional materials may give insight into changes required to improve efficiency but may be inappropriate for some schools. To determine appropriate changes for any given school, it is necessary to measure input-output relationships for that school and make adjustments accordingly. Then these relationships may be used in cost-effectiveness analysis to determine optimum resource organization. The approach could be similar to the one used in this research in which broad categories of variables were used.

All costs of education such as discomfort of bus rides or loss of sports due to reorganization are nonquantifiable and are not included in this analysis. The analysis reported herein shows opportunity costs of misallocating resources. Such information on the cost of a school that is too small or inefficiently organized can be balanced against the nonquantifiable benefits of allowing these inefficiencies to persist.

A sample containing a larger number of schools would have been more informative. Insufficient variation among schools sampled may have veiled real influences of some factors on educational output. For example, the failure of printed volumes per student to show significant relationships to eleventh grade output may have been because all schools sampled had an adequate number of printed volumes. While a larger
sample would allow more statistically significant variables to be detected, the results would not have revealed any variables with a large impact on the educational output variables.

Another problem relates to school district objectives. These objectives may vary by socioeconomic and other characteristics of the community. Many school districts may place heavy emphasis on scholarly achievement while others emphasize vocational, citizenship, and even sports activities. Lumping school districts with different objectives together is hazardous. However, the output variables selected in this research were considered to be sufficiently broad and basic to apply to most school districts and do not necessarily conflict with pursuit of local objectives such as a good sports program.

Some variables were not as completely specified as would be desired. The only measure of teacher education was percentage of teachers with a master's degree. Other measures of education could also be included. Also, measures of teacher qualifications obtained by testing of teachers in verbal and other skills would have been useful. The pupil-teacher ratio was the ratio of enrollment to total teachers. This variable might be improved by using average class size. A further modification would be to select only certain classes, ignoring classes which are unlikely to affect the dimension of quality currently under consideration.

Availability of science laboratory, language laboratory, and industrial, vocational, or technical shop were the measures of facilities considered in the analysis. While these measures are undoubtedly correlated with the quality of the entire physical plant, it is recognized that the measures are not fully adequate.

Much criticism has been directed at the use of linear regression input-output analysis. Linear equations imply that the next unit of an input has the same effect on student performance as did the last unit of that input and that the effect is independent of other inputs. However, interaction terms could have been used to show that the input's effect on output depends on the level of other inputs. While the linear and squared variables were used to express the pupil-teacher ratio, a larger sample would have permitted use of more squared variables.

The regression equations explained from 70 to 90 percent of variation in the educational output variables. Since all the variation was not explained, there may be some discrepancies between expected and actual values of an output. A1so, the analysis was restricted to conventional teaching methods. These limitations must be considered in interpreting the results of the mathematical programming model.

\section*{Further Research}

Continuation of this line of research using data gathered for more schools and students and for more years will provide further insight into schooling efficiency. More refinement is needed on estimates of the returns to be expected from variations in the inputs used in the educational system. The schools being considered could be categorized by similar characteristics. Although the school district appeared to be the obvious unit of analysis in rural areas, similar analysis could be applied to just elementary, junior high or high schools.

More dimensions of educational output need to be included in the analysis. One such possibility is job-oriented tests. Estimates of how well measures of student performance are maintained over time should
be developed. Job-oriented tests should be related to post high school employment, and achievement tests should be related to success in higher education.

Further research linking school programs to individual students is needed. Average class size that an individual student actually attends and the qualifications of teachers that he is actually exposed to are the relevant variables in determining his performance. Since characteristics of schools in which students attended in earlier grades may be dissimilar to current school characteristics, longitudinal studies are necessary to link these characteristics in earlier grades to current student performance.

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APPENDIX A

SAMPLE QUESTIONNAIRE - STUDENT INFORMATION

\section*{STUDENT INFORMATION}

(Check appropriate blank.)
1. Sex: (1) Male
(2) Female \(\qquad\)
2. Race: (1) Indian \(\qquad\) (2) B1 ack \(\qquad\) (3) White \(\qquad\)
(4) Mexican American \(\qquad\) (5) Oriental \(\qquad\)
(6) Other \(\qquad\)
3. Did you go to kindergarten before you started the first grade?
(1) Yes \(\qquad\) (2) No \(\qquad\)
4. Did you go to Headstart or nursery school before you went to kindergarten?
(1) Yes \(\qquad\) (2) No \(\qquad\)
(Circle the correct answer.)
5. About how many times have you changed schools since you started the first grade? (Not counting promotions from one school to another.)
1. Never
2. Once
3. Twice
4. Three times
5. Four times or more
6. On an average school day, how much time do you spend studying outside of school?
1. None or almost none
2. About \(1 / 2\) hour a day
3. About 1 hour a day
4. About 1 \(1 / 2\) hours a day
5. About 2 hours a day
6. About 3 hours a day
7. About 4 or more hours a day
7. How often do you and your parents or guardians talk about your schoolwork?
1. Just about every day
2. Once or twice a week
3. Once or twice a month
4. Never or hardly ever
8. Did you read any books during the last summer?
1. No
2. Yes, 1 or 2
3. Yes, about 5
4. Yes, about 10
5. Yes, more than 10
9. In what type of community have you spent most of your life? (Give your best estimate if you are not sure.)
1. Rural (farm or ranch)
2. Town or city (0-500)
3. Town or city (500-2,500)
4. Town or city ( \(2,500-5,000\) )
5. Town or city (5,000-10,000)
6. Town or city ( \(10,000-30,000\) )
7. Town or city (30,000-above)
10. Do you feel that you can get to see a guidance counselor when you want to or need to?
1. Yes
2. No
3. We have no guidance counselor.
11. How many times did you talk to a guidance counselor in an individual or group situation last year?
1. Never
2. Once
3. Two or three times
4. Four or five times
5. Six or more times
6. We had no guidance counselor.
12. With whom do you live?
1. Both parents
2. Father
3. Mother
4. Guardian
5. Other

THE FOLLOWING QUESTIONS SHOULD BE ANSWERED BY EIGHTH (8th) AND ELEVENTH (11th) GRADERS ONLY.
13. Circle the activities below in which you participate.
1. Athletic 5. Social clubs
2. Band
6. Honor Society
3. Chorus
7. Others
4. Subject-related clubs
14. Do you work after school (other than household duties)?
1. No
2. Less than 10 hours a week
3. About 10 to 20 hours a week
4. More than 20 hours a week

THE FOLLOWING QUESTIONS SHOULD BE ANSWERED BY ELEVENTH (11th) GRADERS ONLY.
15. I plan to make the following occupation my career:
1. Office work (cashier, clerk, bookkeeper, etc.)
2. Professional (doctor, teacher, lawyer, minister, nurse, etc.)
3. Executive (manage large business, industrial firm)
4. Semi-skilled work (janitor, farm hand, plumber's helper, waiter, truck driver, etc.)
5. Salesman (insurance, real estate, auto, store, etc.)
6. Skilled work (secretary, mechanic, welder, appliance service man, etc.)
7. Own, rent, manage small business (store, station, cafe, etc.)
8. Military
9. Own, rent, manage farm or ranch
10. Housewife
16. After graduation from high school, I plan:
1. To continue going to school
2. To get a job
3. To become an apprentice
4. To go into military service
5. To work at my home
6. I have no definite plans
7. Other \(\qquad\)

\section*{APPENDIX B}

SAMPLE QUESTIONNAIRE - PARENT INFORMATION

\section*{SAMPLE QUESTIONNAIRE}

\section*{PARENT INFORMATION}

(Circle correct answer.)
1. Father's education
1. Less than high school
2. Attended high school
3. Graduated from high school
4. Attended trade or business school
5. Attended college
6. Graduated from college
7. Has a master's or doctor's degree
2. Father's occupation
1. Office work (cashier, clerk, bookkeeper, etc.)
2. Professional (doctor, teacher, lawyer, minister, etc.)
3. Executive (manages large business, industrial firm)
4. Semi-skilled work (janitor, farm hand, plumber's helper, waiter, truck driver, etc.)
5. Salesman (insurance, real estate, auto, store, etc.)
6. Skilled work (mechanic, welder, appliance service man, etc.)
7. Owns, rents, manages small business (store, station, cafe, etc.)
8. Military
9. Owns, rents, manages farm or ranch
10. Disabled, retired
11. Deceased
3. Mother's education
1. Less than high school
2. Attended high school
3. Graduated from high school
4. Attended trade or business school
5. Attended college
6. Graduated from college
7. Has master's or doctor's degree
4. Mother's occupation
1. Housewife (does not work outside of home)
2. Professional (teacher, doctor, lawyer, etc.)
3. Executive (manages large business)
4. Semi-skilled work (waitress, cleaning woman, etc.)
5. Saleslady
6. Skilled worker (secretary, lab technician, etc.)
7. Owns, rents, manages small business
8. Office work (clerk, filing, receptionist)
9. Disabled, retired
10. Deceased
5. Residence
1. Rural (farm or ranch)
2. Town or city ( \(0-500\) )
3. Town or city (500-2;500)
4. Town or city ( \(2,500-5,000\) )
5. Town or city (5,000-10,000)
6. Town or city ( \(10,000-30,000)\)
7. Town or city ( 30,000 -above)
6. Number of miles from school
1. \(0-1\) mile
2. 1-3 miles
3. 3-10 miles
4. 10 or more miles
7. Parent's or guardian's net income
1. Less than \(\$ 1,500\)
2. \(\$ 1,500\) to \(\$ 3,000\)
3. \(\$ 3,000\) to \(\$ 5,000\)
4. \(\$ 5,000\) to \(\$ 10,000\)
5. \(\$ 10,000\) to \(\$ 20,000\)
6. \(\$ 20,000\) and above
8. Our plan for our children's education
1. Attend high school
2. Complete high school
3. Attend business college (secretarial, barber, cosmetology, etc.)
4. Vocational-technical school
5. Junior college
6. Four-year college
7. University

APPENDIX C

\section*{SAMPLE QUESTIONNAIRE}

\section*{ADMINISTRATORS INFORMATION}


\section*{PROGRAM}
1. Specific grade(s), at this school site, covered by this questionnaire.

THE FOLLOWING INFORMATION SHOULD APPLY ONLY TO THE GRADES AT THIS SCHOOL SITE, BUT SEPARATE QUESTIONNAIRES SHOULD BE FILLED OUT FOR EACH OF THE SCHOOL GROUPINGS AT THIS SITE AS INDICATED IN QUESTION NUMBER ONE。
2. Is team teaching utilized to any extent?
3. Are students grouped by ability to any extent?
4. Total number of students in remedial classes.
5. Total number of special education students.
6. Total number of students in accelerated classes.
7. Is your elementary school instructional program
(a) Self-contained classrooms
(b) Departmentalized
(c) Combination of the above
(a)Yes \(\qquad\) (b) No
(a)Yes (b) No
\(\qquad\)
\(\qquad\)
\(\qquad\)

Instructional staff, fall 1969 (Total number of filled positions, full-time equivalent.)
\begin{tabular}{|l|l|c|}
\hline Type of Personnel & Elementary & Secondary \\
\hline Principals & 12 & 13 \\
\hline Classroom teachers & 14 & 15 \\
\hline \begin{tabular}{c} 
Guidance and counseling \\
personnel
\end{tabular} & 16 & 17 \\
\hline
\end{tabular}

Characteristics of Classroom Teachers by degree status and type of certificate, fall 1969.

Degrees. In the following table, report the number of teachers in each level of school organization by highest degree obtained and type of teaching certificate. Count each teacher once in each of the two sections of the table.
\begin{tabular}{|c|c|c|c|}
\hline \multirow[b]{3}{*}{Organizational Level} & \multicolumn{3}{|c|}{Number of Teachers} \\
\hline & \multicolumn{3}{|c|}{Highest Degree Obtained} \\
\hline & Bachelor's & Master's or higher & No Degree \\
\hline Elementary & 18 & 19 & 20 \\
\hline Secondary (including junior high) & 21 & 22 & 23 \\
\hline Total number of teachers & 24 & 25 & 26 \\
\hline
\end{tabular}

Salary. Full-time classroom teachers by salary groups, fall 1969.
Include as salary all adjustments and increments such as additional sums for dependents, and for services as coach, librarian, etc.
\begin{tabular}{|c|c|c|}
\hline & Sal ary Groups & Number of Teachers \\
\hline 27. & \$5,000 to \$5,499 & \\
\hline 28. & \$5,500 to \$5,999 & \\
\hline 29. & \$6,000 to \$6,999. & \\
\hline 30. & \$7,000 to \$7,999 & \\
\hline 31. & \$8,000 to \$8,999 & \\
\hline 32. & \$9,000 to \$9,999 & \\
\hline 33. & \$10,000 and over & \\
\hline 34. & Total number of teachers & \\
\hline
\end{tabular}
35. \(\qquad\) None
36. \(\qquad\) 1 or 2
37. \(\quad 3\) or 4
38. \(\qquad\) 5 to 9
39. \(\qquad\) 10 to 14
40. \(\qquad\) 15 to 19
41. \(\qquad\) 20 to 29
42. \(\qquad\) 30 or longer
43. Number of instructional periods per day at this school site.
(a) 5 \(\qquad\) (b) 6 \(\qquad\) (c) 7 \(\qquad\) (d) Modular
(e) Other
\(\qquad\)
44. Number of teachers who have planning period.
45. Number of teachers who do not have planning period.

\section*{FACILITIES}

If instructional rooms are needed to eliminate inadequate facilities, what type of special facilities are needed, if any? (Check as many as appropriate.)
46. Science Laboratory
(a)Yes
(b) No
47. Language Laboratory
(a)Yes (b) No
48. Industrial, Vocational or Technical Shop
49. Kindergarten
(a)Yes (b) No
(a)Yes (b) No
50. What is the total number of printed volumes per pupil
in this school? Count the total number of books, exclusive of textbooks, whether in a central media center or elsewhere in the building, and divide by the total number of students in membership.
51. What is the total number of different periodicals which are available regularly in this school?

Below is a partial list of audiovisual equipment which is frequently used in classroom instruction. List how many of each item of equipment you have in this school building.
52. 16 mm projectors
53. 8 mm projectors
54. Filmstrip projectors
55. Slide projectors
56. Record players
57. Tape recorders
58. Overhead projectors
59. Television receivers

In the chart below include all audiovisual materials available for use at this school site.
\begin{tabular}{|l|l|l|}
\hline & \multicolumn{2}{|c|}{\begin{tabular}{c} 
Number of Titles \\
Available \\
Types of Materials
\end{tabular}} \\
\hline Motion pictures & Elementary & 61 \\
\hline Filmstrips & 60 & 63 \\
\hline Slides & 62 & 65 \\
\hline Disc recordings & 64 & 67 \\
\hline Tape recordings & 66 & 69 \\
\hline Picture sets & 70 & 71 \\
\hline Maps & 72 & 75 \\
\hline Globes & 74 & 77 \\
\hline Charts & 76 & 79 \\
\hline Transparencies & 78 & 83 \\
\hline Other types (list) & 80 & 7 \\
\hline Totals & & \\
\hline
\end{tabular}

\section*{ORGANIZATION - ADMINISTRATION - MANAGEMENT}
84. Total number of grades offered at this school site.
85. What is the pupil-teacher ratio at this school site? (classroom teachers only)
86. What is the retention rate at this school site?
87. What is the absentee rate (percent) of students at this school site for first semester 1969-70?
\[
\frac{\text { Total days absent }}{\text { Total days on roll }} \times 100
\]
88. What percent of the student population comes from ranches, farms, or open country at this school site?
89. Total ADA of this school site at the end of 1968-69 school year.

ALL QUESTIONS APPLY TO THE ENTIRE SCHOOL DISTRICT

\section*{FINANCES}
90. What is the assessed valuation per ADA?
91. Expenditures for instructional purposes per ADA?
92. Expenditures for school library purposes per ADA?
93. Expenditures for transportation purposes per ADA?
94. Expenditures for instructional supplies per ADA?
95. Expenditures for all functions per ADA? (Do not include capital outlay and building fund.)

SCHOOL DISTRICT INFORMATION
96. What is the student density figure (enrollment per square mile) for the school district?
\[
\frac{\text { Total Enrollment }}{\text { Total Sq. Miles }}
\]
97. Total number of high school graduates.
98. Total ADA of this school district.
99. Typical salary for this school site.
(a) Beginning teacher with bachelor's degree
(b) Teacher with 10 years experience, bachelor's degree
(c) Teacher with 10 years experience, master's degree

APPENDIX D

\section*{MEANS, STANDARD DEVIATIONS, AND CORRELATION COEFFICIENTS OF SELECTED ELEVENTH GRADE VARIABLES}

\section*{TABLE XXXVII}

\section*{ELEVENTH GRADE VARIABLES}
\begin{tabular}{|c|c|c|c|}
\hline Variable & Description & Estimated Mean & Estimated Standard Deviation \\
\hline & \multicolumn{3}{|l|}{Educational Outputs} \\
\hline \(\mathrm{X}_{1}\) & Absentee rate & 2.982 & 2.389 \\
\hline \(\mathrm{X}_{2}\) & Dropout rate & 3.444 & 2.313 \\
\hline \(\mathrm{X}_{3}\) & Average language arts score & 414.666 & 22.153 \\
\hline \(\mathrm{X}_{4}\) & Average modern math score & 412.122 & 21.065 \\
\hline \(\mathrm{X}_{5}\) & Average composite achievement score & 77.031 & 9.943 \\
\hline & \multicolumn{3}{|l|}{Instrumental Variables} \\
\hline \(\mathrm{Y}_{1}\) & Percentage of teachers with 10 or more years of experience & 47.293 & 16.351 \\
\hline \(\mathrm{Y}_{2}\) & Percentage of teachers with a planning period & 51.041 & 47.175 \\
\hline \(\mathrm{Y}_{3}\) & Adequate science laboratory & 0.555 & 0.506 \\
\hline \(\mathrm{Y}_{4}\) & Printed volumes per pupil & 12.549 & 7.642 \\
\hline \(\mathrm{Y}_{5}\) & Pupil-teacher ratio & 20.845 & 5.719 \\
\hline \(\mathrm{Y}_{6}\) & Percentage of teachers with a master's degree & 34.068 & 15.470 \\
\hline \(\mathrm{Y}_{7}\) & Value of audiovisual material per pupil (\$100) & 2.105 & 3.319 \\
\hline \(\mathrm{Y}_{8}\) & Average teacher salary (\$1,000) & 6.963 & 0.448 \\
\hline \(\mathrm{Y}_{9}\) & Average teacher experience & 11.617 & 3.420 \\
\hline
\end{tabular}

\section*{TABLE XXXVII (Continued)}
\begin{tabular}{|c|c|c|c|}
\hline Variable & Description & Estimated Mean & \begin{tabular}{l}
Estimated \\
Standard \\
Deviation
\end{tabular} \\
\hline \(\mathrm{Y}_{10}\) & Salary for beginning teachers with bachelor's degree (\$1000) & 5.750 & 0.295 \\
\hline \(\mathrm{Y}_{11}\) & Number of academic units offered & 44.537 & 19.253 \\
\hline \(\mathrm{Y}_{12}\) & Number of vocational units offered & 13.685 & 12.242 \\
\hline \(\mathrm{Y}_{13}\) & Adequate industrial, vocational or technical shop & 0.518 & 0.509 \\
\hline \(\mathrm{Y}_{14}\) & Expenditures for all functions except transportation per ADA & 526.703 & 129.122 \\
\hline \(\mathrm{Y}_{15}\) & Average daily attendance (1,000 units) & 5.934 & 18.064 \\
\hline & Control Variables & & \\
\hline \(\mathrm{Z}_{1}\) & Average IQ & 95.962 & 8.652 \\
\hline \(\mathrm{Z}_{2}\) & Percentage of students whose fathers attended college & 22.798 & 15.728 \\
\hline \(\mathrm{Z}_{3}\) & Percentage of students whose fathers' occupation is professional or executive & 12.538 & 11.509 \\
\hline \(\mathrm{Z}_{4}\) & Parents' average net income (\$1,000) & 8.672 & 2.634 \\
\hline \(\mathrm{Z}_{5}\) & Percentage of students whose parents plan for their children to attend college & 62.713 & 14.178 \\
\hline \(\mathrm{Z}_{6}\) & Percentage of students who are Indian & 8.961 & 14.904 \\
\hline \(\mathrm{Z}_{7}\) & Percentage of students who are black & 5.908 & 12.314 \\
\hline \(\mathrm{Z}_{8}\) & Average number of times student has changed schools & 1.732 & 0.425 \\
\hline
\end{tabular}

TABLE XXXVII (Continued)
\begin{tabular}{|c|c|c|c|}
\hline Variable & Description & Estimated Mean & Estimated Standard Deviation \\
\hline \(\mathrm{Z}_{9}\) & Average number of hours studied & 0.895 & 0.285 \\
\hline \(\mathrm{Z}_{10}\) & Percentage of students who talk to parents about schoolwork at least once a week & 72.316 & 11.017 \\
\hline \(\mathrm{Z}_{11}\) & Average number of books students read during the last summer & 3.874 & 0.954 \\
\hline \(\mathrm{Z}_{12}\) & Percentage of students who have spent most of their lives in rural areas & 40. 110 & 25.410 \\
\hline Z 13 & Percentage of students who have spent most of their lives in a town or city with 0-2500 population & 15.865 & 16.868 \\
\hline \(\mathrm{Z}_{14}\) & Average number of hours worked outside of school per week & 4.547 & 1.690 \\
\hline \(\mathrm{Z}_{15}\) & Percentage of students who plan to go to college & 61.014 & 13.456 \\
\hline
\end{tabular}

TABLE XXXVIII
CORRELATION COEFFICIENTS OF EDUCATIONAL OUTPUTS AND SELECTED EDUCATIONAL PROCESS VARIABLES FOR THE ELEVENTH GRADE
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline Variable & \(\mathrm{X}_{1}\) & \(\mathrm{X}_{2}\) & \(\mathrm{x}_{3}\) & \(\mathrm{x}_{4}\) & \(\mathrm{x}_{5}\) & \(Y_{1}\) & \(\mathrm{Y}_{2}\) & \(Y_{3}\) & \(Y_{4}\) & \(\mathrm{I}_{5}\) \\
\hline \(\mathrm{X}_{1}\) & 1.000 & 0.592 & 0.057 & 0.076 & 0.156 & 0.002 & 0.115 & 0.089 & -0.317 & 0.567 \\
\hline \(\mathrm{x}_{2}\) & & 1.000 & -0.217 & -0.073 & -0.004 & -0.294 & 0.213 & 0.044 & -0.240 & 0.572 \\
\hline \(\mathrm{x}_{3}\) & & & 1.000 & 0.881 & 0.920 & -0.324 & 0.262 & 0.053 & 0.174 & -0.207 \\
\hline \(\mathrm{X}_{4}\) & & & & 1.000 & 0.939 & -0.274 & 0.358 & 0.049 & 0.230 & 0.030 \\
\hline \(\mathrm{x}_{5}\) & & & & & 1.000 & -0.358 & 0.367 & 0.139 & 0.253 & 0.047 \\
\hline \(\mathrm{Y}_{1}\) & & & & & & 1.000 & 0.037 & -0.100 & -0.169 & -0.148 \\
\hline \(\mathrm{Y}_{2}\) & & & & & & & 1.000 & -0.060 & -0.196 & -0.107 \\
\hline \(\mathrm{Y}_{3}\) & & & & & & & & 1.000 & 0.244 & 0.188 \\
\hline \(Y_{4}\) & & & & & & & & & 1.000 & 0.027 \\
\hline \(\mathrm{Y}_{5}\) & & & & & & & & & & 1.000 \\
\hline
\end{tabular}

TABLE XXXVIII (Continued)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline Variable & \(Y_{6}\) & \(\mathrm{Y}_{7}\) & \(\mathrm{Y}_{8}\) & \(\mathrm{Y}_{9}\) & \(\mathrm{Y}_{10}\) & \(\mathrm{Y}_{11}\) & \(\mathrm{Y}_{12}\) & \(\mathrm{Y}_{13}\) & \(\mathrm{Y}_{14}\) & \({ }^{1} 15\) \\
\hline \(\mathrm{X}_{1}\) & 0.017 & -0.374 & -0.266 & -0.032 & -0.194 & 0.472 & 0.433 & -0.007 & -0.454 & 0.350 \\
\hline \(\mathrm{x}_{2}\) & -0.123 & -0.555 & -0.385 & -0.198 & -0.321 & 0.291 & 0.347 & -0.203 & -0.678 & 0.209 \\
\hline \(\mathrm{x}_{3}\) & -0.009 & 0.230 & 0.104 & -0.329 & 0.450 & 0.291 & 0.204 & 0.414 & 0.521 & 0.189 \\
\hline \(\mathrm{x}_{4}\) & 0.136 & 0.039 & 0.217 & -0.242 & 0.476 & 0.493 & 0.204 & 0.418 & 0.302 & 0.423 \\
\hline \(\mathrm{X}_{5}\) & 0.019 & 0.078 & 0.137 & -0.328 & 0.433 & 0.404 & 0.229 & 0.438 & 0.340 & 0.319 \\
\hline \(Y_{1}\) & 0.571 & -0.154 & 0.506 & 0.881 & -0.025 & -0.131 & -0.030 & 0.190 & -0.058 & -0.225 \\
\hline \(\mathrm{Y}_{2}\) & 0.159 & -0.075 & 0.201 & 0.087 & 0.053 & 0.411 & 0.243 & 0.553 & -0.009 & 0.308 \\
\hline \(\mathrm{Y}_{3}\) & -0.400 & 0.308 & 0.165 & -0.050 & 0.054 & 0.162 & -0.085 & 0.331 & 0.175 & 0.250 \\
\hline \(\mathrm{Y}_{4}\) & -0.124 & 0.197 & 0.417 & 0.051 & 0.243 & -0.181 & -0.150 & 0.041 & 0.244 & -0.100 \\
\hline \(Y_{5}\) & -0.011 & -0.538 & -0.168 & -0.009 & -0.016 & 0.351 & 0.310 & -0.238 & -0.716 & 0.310 \\
\hline \(\mathrm{Y}_{6}\) & 1.000 & -0.478 & 0.452 & 0.546 & 0.371 & 0.162 & 0.251 & 0.029 & -0.102 & 0.047 \\
\hline \(\mathrm{Y}_{7}\) & & 1.000 & 0.003 & -0.221 & 0.004 & -0.153 & -0.252 & 0.232 & 0.681 & -0.021 \\
\hline \(\mathrm{Y}_{8}\) & & & 1.000 & 0.566 & 0.281 & 0.011 & 0.018 & 0.381 & 0.237 & 0.066 \\
\hline \(\mathrm{Y}_{9}\) & & & & 1.000 & 0.019 & -0.120 & 0.015 & 0.122 & -0.177 & -0.202 \\
\hline \(\mathrm{Y}_{10}\) & & & & & 1.000 & 0.293 & 0.247 & 0.138 & 0.293 & 0.270 \\
\hline \(\mathrm{P}_{11}\) & & & & & & 1.000 & 0.449 & 0.375 & -0.145 & 0.867 \\
\hline \(\mathrm{Y}_{12}\) & & & & & & & 1.000 & 0.006 & -0.240 & 0.298 \\
\hline \(\mathrm{Y}_{13}\) & & & & & & & & 1.000 & 0.458 & 0.274 \\
\hline \(\mathrm{Y}_{14}\) & & & & & & & & & 1.000 & -0.045 \\
\hline \(\mathrm{Y}_{15}\) & & & & & & & & & & 1.000 \\
\hline
\end{tabular}

CORRELATION COEFFICTENTS OF EDUCATIONAL OUTPUTS AND SELECTED STUDENT INPUT AND ENVIRONMENTAL VARIABLES FOR THE ELEVENTH GRADE
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline Variable & \(\mathrm{x}_{1}\) & \(\mathrm{x}_{2}\) & \(\mathrm{x}_{3}\) & \(\mathrm{x}_{4}\) & \(\mathrm{X}_{5}\) & \(\mathrm{z}_{1}\) & \(\mathrm{z}_{2}\) & \(\mathrm{z}_{3}\) & \(\mathrm{Z}_{4}\) & \(\mathrm{z}_{5}\) \\
\hline \(\mathrm{X}_{1}\) & 1.000 & 0.592 & 0.057 & 0.076 & 0.156 & -0.004 & 0.306 & 0.449 & 0.127 & 0.178 \\
\hline \(\mathrm{X}_{2}\) & & 1.000 & -0.217 & -0.073 & -0.004 & -0.015 & 0.190 & 0.349 & 0.042 & 0.197 \\
\hline \(\mathrm{X}_{3}\) & & & 1.000 & 0.881 & 0.920 & 0.518 & 0.532 & 0.404 & 0.691 & 0.636 \\
\hline \(\mathrm{X}_{4}\) & & & & 1.000 & 0.939 & 0.503 & 0.689 & 0.590 & 0.750 & 0.687 \\
\hline \(\mathrm{X}_{5}\) & & & & & 1.000 & 0.595 & 0.672 & 0.548 & 0.744 & 0.757 \\
\hline \(\mathrm{z}_{1}\) & & & & & & 1.000 & 0.344 & 0.146 & 0.256 & 0.447 \\
\hline \(\mathrm{z}_{2}\) & & & & & & & 1.000 & 0.916 & 0.795 & 0.675 \\
\hline \(\mathrm{z}_{3}\) & & & & & & & & 1.000 & 0.739 & 0.527 \\
\hline \(\mathrm{Z}_{4}\) & & & & & & & & & 1.000 & 0.660 \\
\hline \(\mathrm{z}_{5}\) & & & & & & & & & & 1.000 \\
\hline
\end{tabular}

TABLE XXXIX (Continued)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline Variable & \(z_{6}\) & \(\mathrm{z}_{7}\) & \(\mathrm{z}_{8}\) & \(z_{9}\) & \(z_{10}\) & \(\mathrm{z}_{11}\) & \(\mathrm{Z}_{12}\) & \(\mathrm{z}_{13}\) & \(\mathrm{z}_{14}\) & \(\mathrm{Z}_{15}\) \\
\hline \(\mathrm{X}_{1}\) & 0.035 & 0.078 & 0.366 & 0.319 & 0.511 & -0.079 & -0.354 & -0.060 & 0.051 & 0.049 \\
\hline \(\mathrm{x}_{2}\) & -0.222 & 0.073 & 0.387 & 0.260 & 0.636 & -0.246 & -0.422 & 0.228 & 0.060 & 0.038 \\
\hline \(\mathrm{x}_{3}\) & -0.503 & -0.368 & -0.534 & 0.246 & 0.067 & 0.479 & -0.464 & 0.022 & 0.024 & 0.769 \\
\hline \(\mathrm{x}_{4}\) & -0.555 & -0.326 & -0.310 & 0.415 & 0.182 & 0.436 & -0.519 & 0.078 & 0.021 & 0.821 \\
\hline \(\mathrm{X}_{5}\) & -0.601 & -0.439 & -0.367 & 0.360 & 0.205 & 0.436 & -0.565 & 0.106 & 0.146 & 0.837 \\
\hline \(\mathrm{z}_{1}\) & -0.365 & -0.366 & -0.296 & 0.250 & 0.131 & 0.328 & -0.209 & 0.231 & 0.294 & 0.417 \\
\hline \(\mathrm{z}_{2}\) & -0.489 & -0.154 & 0.137 & 0.457 & 0.159 & 0.228 & -0.658 & 0.048 & 0.342 & 0.687 \\
\hline \(\mathrm{z}_{3}\) & -0.338 & -0.037 & 0.312 & 0.522 & 0.292 & 0.031 & -0.673 & 0.054 & 0.260 & 0.540 \\
\hline \(\mathrm{z}_{4}\) & -0.630 & -0.358 & -0.041 & 0.231 & 0.111 & 0.321 & -0.639 & -0.066 & 0.280 & 0.640 \\
\hline \(z_{5}\) & -0.692 & -0.253 & -0.077 & 0.223 & 0.378 & 0.503 & -0.602 & 0.115 & 0.286 & 0.846 \\
\hline \(\mathrm{z}_{6}\) & 1.000 & 0.242 & 0.262 & -0.185 & -0.341 & -0.370 & 0.494 & -0.135 & -0.277 & -0.606 \\
\hline \(\mathrm{Z}_{7}\) & & 1.000 & 0.182 & 0.103 & 0.090 & -0.525 & 0.094 & 0.020 & -0.302 & -0.279 \\
\hline \(\mathrm{z}_{8}\) & & & 1.000 & 0.187 & 0.419 & -0.151 & -0.096 & -0.089 & 0.132 & -0.286 \\
\hline \(\mathrm{Z}_{9}\) & & & & 1.000 & 0.431 & 0.004 & -0.435 & -0.101 & -0.170 & 0.297 \\
\hline \(\mathrm{z}_{10}\) & & & & & 1.000 & 0.204 & -0.368 & 0.015 & 0.013 & 0.229 \\
\hline \(\mathrm{z}_{11}\) & & & & & & 1.000 & -0.109 & -0.045 & 0.051 & 0.436 \\
\hline \({ }^{2} 12\) & & & & & & & 1.000 & -0.217 & -0.112 & -0.467 \\
\hline \(\mathrm{z}_{13}\) & & & & & & & & 1.000 & -0.119 & -0.039 \\
\hline \({ }_{2}\) & & & & & & & & & 1.000 & 0.216 \\
\hline \(\mathrm{z}_{15}\) & & & & & & & & & & 1.000 \\
\hline
\end{tabular}

APPENDIX E

SIMULTANEOUS EQUATIONS EXPLAINING

ELEVENTH GRADE EDUCATIONAL

OUTPUT

TABLE XL

\section*{SIMULTANEOUS EQUATION EXPLAINING ELEVENTH GRADE DROPOUT RATE}
\begin{tabular}{|c|c|c|}
\hline Variable & Regression Coefficient & Standard Error \\
\hline Constant & 19.6590 & \\
\hline \multicolumn{3}{|l|}{Endogenous Variable in Equation} \\
\hline Eleventh Grade Composite Scores & 0.0151 & 0.0003 \\
\hline \multicolumn{3}{|l|}{Educational Process Variables} \\
\hline Percentage of teachers with five to nine years experience & -0.0556 & 0.0002 \\
\hline Percentage of teachers with ten or more years experience & -0.0890 & 0.0002 \\
\hline Value of audiovisual material in hundred dollars per student & -0.3757 & 0.0006 \\
\hline Salary in thousand dollars for beginning teachers with bachelor's degree & -1.8503 & 0.0081 \\
\hline SQRT (Average daily attendance in thousands) & 0.5888 & 0.0016 \\
\hline \multicolumn{3}{|l|}{Environmental Variable} \\
\hline Percentage of students whose parents' net income is greater than \(\$ 10,000\) & -0.0617 & 0.0003 \\
\hline \(\mathrm{R}^{2}\) & 0.7874 & \\
\hline
\end{tabular}

TABLE XLI

SIMULTANEOUS EQUATION EXPLAINING ELEVENTH GRADE LANGUAGE ARTS SCORES
\begin{tabular}{|c|c|c|}
\hline Variable & Regression Coefficient & Standard Error \\
\hline Constant & 373.7004 & \\
\hline \multicolumn{3}{|l|}{Endogenous Variable in Equation} \\
\hline Eleventh Grade Dropout Rate & -2.3787 & 0.0083 \\
\hline \multicolumn{3}{|l|}{Educational Process Variables} \\
\hline Percentage of teachers with three to nine years of experience & 0.3469 & 0.0014 \\
\hline Adequate industrial, vocational or technical shop & 16.8045 & 0.0370 \\
\hline SQRT (Average daily attendance in thousands) & -9.4460 & 0.0210 \\
\hline \multicolumn{3}{|l|}{Student Input Variable} \\
\hline Percentage of students who plan to continue going to school after graduation & 0.4843 & 0.0013 \\
\hline \multicolumn{3}{|l|}{Environmental Variables} \\
\hline Percentage of students whose father's occupation is professional or executive & 1.8637 & 0.0042 \\
\hline Percentage of students whose parents' net income is less than \(\$ 3,000\) & -0.2299 & 0.0011 \\
\hline \(\mathrm{R}^{2}\) & 0.6697 & \\
\hline
\end{tabular}

TABLE XLII
SIMULTANEOUS EQUATION EXPLAINING ELEVENTH
GRADE MODERN MATH SCORES
\begin{tabular}{|c|c|c|}
\hline Variable & Regression Coefficient & Standard Error \\
\hline Constant & 93.6721 & \\
\hline \multicolumn{3}{|l|}{Endogenous Variable in Equation} \\
\hline Eleventh Grade Dropout Rate & -1.0966 & 0.0081 \\
\hline \multicolumn{3}{|l|}{Educational Process Variables} \\
\hline Percentage of teachers with ten or more years experience & -0.2621 & 0.0012 \\
\hline Salary in thousand dollars for beginning teachers with a bachelor's degree & 10.1036 & 0.0662 \\
\hline Adequate industrial, vocational, or technical shop & 6.8520 & 0.0359 \\
\hline \multicolumn{3}{|l|}{Student Input Variables} \\
\hline Eighth grade modern math score & 0.5716 & 0.0011 \\
\hline Average number of hours students studied & 17.5971 & 0.0685 \\
\hline Average number of books students read during last summer & 4.5126 & 0.0184 \\
\hline Average number of hours worked per week outside of school & -2.6208 & 0.0106 \\
\hline \(\mathrm{R}^{2}\) & 0.8471 & \\
\hline
\end{tabular}

TABLE XLIII

SIMULTANEOUS EQUATION EXPLAINING ELEVENTH
GRADE COMPOSITE SCORES
\(\left.\begin{array}{lcc}\hline \text { Variable } & \begin{array}{c}\text { Regression } \\ \text { Coefficient }\end{array} & \begin{array}{c}\text { Standard } \\ \text { Error }\end{array} \\ \hline \text { Constant } & 31.6542 & \\ \hline \text { Endogenous Variable in Equation }\end{array}\right] 0.1055\) Dropout Rate \(\quad 0.0040\)

Student Input Variables
\begin{tabular}{lll} 
Eighth grade composite score & 0.0912 & 0.0002 \\
\begin{tabular}{l} 
Percentage of students who have \\
changed schools three or \\
more times
\end{tabular} & -0.0650 & 0.0016 \\
\begin{tabular}{l} 
Percentage of students who plan to \\
continue going to school after \\
graduation
\end{tabular} & 0.1403
\end{tabular}

Environmental Variable
\begin{tabular}{ccc}
\begin{tabular}{c} 
Percentage of students whose fathers \\
attended college
\end{tabular} & 0.3185 & 0.0011 \\
\(\mathrm{R}^{2}\) & 0.8317 \\
\hline
\end{tabular}

\title{
2 \\ VITA \\ Freddie Cad White \\ Candidate for the Degree of \\ Doctor of Philosophy
}

Thesis: A QUANTITATIVE ANALYSIS OF FACTORS AFFECTING ELEMENTARY AND SECONDARY SCHOOLING QUALITY IN OKLAHOMA, WITH ECONOMIC APPLICATION FOR RURAL AREAS

Major Field: Agricultural Economics
Biographical:
Personal Data: Born in Big Spring, Texas, November 21, 1944, the son of Mr. and Mrs. F. W. White.

Education: Graduated from Big Spring High School, Big Spring, Texas, in May, 1963; received Bachelor of Science degree in Agricultural Economics from Texas Technological University in 1967; received Master of Science degree in Agricultural Economics from Oklahoma State University in 1969; completed requirements for the Doctor of Philosophy degree at Oklahoma State University in May, 1972.

Professional Experience: Research Assistant, Oklahoma State University, Stillwater, Oklahoma, 1970-71; NDEA Fellow, Oklahoma State University, Stillwater, Oklahoma, 1967-70.```

