Spatial Price Equilibrium Analyses of the Livestock Economy

I. Methodological Development and Annual Spatial Analyses of the Beef Marketing Sector

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PREFACE

This study is the first of a series reporting the results of research concerning the application of spatial price equilibrium models to the livestock marketing sector of the economy. As such, the research deals primarily with estimating the equilibrium geographical prices, consumption and flows for livestock products under alternative conditions and assumptions.

This initial study is concerned with developing an operational model for spatial price equilibrium analysis and applying it to the beef marketing sector of the economy. For purposes of presentation, the study is divided into two parts: (1) statement of the general problem and methodological considerations underlying the development of an operational spatial price equilibrium model and (2) spatial price equilibrium solutions for beef under alternative assumptions and conditions and general economic implications relating to this type of analysis. Those readers interested in only the results of the spatial price equilibrium models for beef should go directly to Section III, "The Data."

SPATIAL PRICE EQUILIBRIUM ANALYSES

OF THE LIVESTOCK ECONOMY

The three studies reported in this series are:

1. Methodological Development and Annual Spatial Analyses of the Beef Marketing Economy

2. Applications of Spatial Analysis to Quarterly Models and Particular Problems within the Beef Marketing System

3. Spatial Price Equilibrium Models of the Pork Marketing Sector

SPATIAL PRICE EQUILIBRIUM ANALYSES OF THE LIVESTOCK ECONOMY

1. Methodological Development and Annual Spatial Analyses Of the Beef Marketing Sector

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I Introduction

Economists have been concerned over a long period of time with the development of a general theory of location and space economy. General equilibrium theory as elaborated by Walras (28), Pareto (21) and Wicksell (29), and its modern counterpart as exemplified by the writings of Hicks (19), and Mosak (18), Lange (15) and Samuelson (22), treat an economy in which all factors and producers, products and consumers are, in effect, located at one point (transportation costs are taken as zero). Productive efforts by such men as Weber (7), Englander (4), Ohlin (19), Palander (20), Hoover (11), Losch (17) and others have extended the general theory in the direction of embracing the total spatial array of economic activities. Contributions by Leontief (16) in developing the input-output technique for general equilibrium analysis and Koopmans (13, pp. 33-97), Dantzig (13, pp. 19-32) and others in developing the activity analysis model of production and allocation are of consequence here since they permit an attack on a specific set of significant problems which logically fall within the province of a general theory of location and space economy.

With the objective of specifying a conceptual framework and the design of the corresponding operational model, Enke (5), Samuelson (23), Beckmann (2) and Baumol (1) set out the problem of interconnected competitive markets in a new form, and at the same time, opened a new approach to spatial pricing systems and competitive locational equilibrium. This new operational formulation permits space to be treated explicitly and presents the rationale whereby a purely descriptive problem in nonnormative economics can be converted into an extremum problem in which linear programming can be employed as a tool of analysis. By converting the spatial equilibrium system into an extremum problem, insights can be obtained relative to the geographic location of production, spatial equilibrium prices and the optimum geographical flows consistent with a given set of data.

In addition to being operational in the sense that computations involved are manageable, solutions to spatial equilibrium models of the Enke-Samuelson type generate information that is basic to choice by decision makers at the government and firm levels. For example, using the method of "comparative statics," the spatial equilibrium models can be used to ascertain the consequences of changes in: (1) transport costs, (2) the level and regional distribution of production, (3) level and distribution of consumer income, and (4) the level and regional distribution of population, on production, consumption, geographical prices and flows. Given the setting, this study is concerned with an interregional analysis of the beef sector of the U. S. economy within the conceptual framework defined by the postulates of the Enke-Samuelson formulation of spatial price equilibrium analysis.

A. The General Problem

The general problem of equilibrium among spatially separated markets has been stated in one of its simplest forms by Enke. His formulation proceeds as follows:

> "There are three (or more) regions trading a homogenous good. Each region constitutes a single and distinct market. The regions of each possible pair of regions are separated - but not isolated - by a transportation cost per physical unit which is independent of volume. There are no legal restrictions to limit the actions of the profit-seeking traders in each region. For each region, the functions which relate local production and local use to local price are known and, consequently, the magnitude of the difference which will be exported or imported at each local price is also known. Given these trade functions and transportation costs, we wish to ascertain: (1) the net price in each region; (2) the quantity of exports or imports for each region; (3) which regions import, export, or do neither; (4) the aggregate trade in each commodity; (5) the volume and direction of trade between each possible pair of regions...." (5, p. 41).

Enke then suggests how a solution to this problem may be obtained by electric analogue.

Samuelson has demonstrated how the Enke problem contains within it the following Koopmans-Hitchcock minimum transportation cost problem:

> "A specified number of (empty or ballast) ships is to be sent out from each of a number of ports. They are to be allocated among a number of other receiving ports, with the total sent in to each such port being specified. If we are given the unit costs of shipments between every two ports, how can we minimize the total cost of the program?" (23, p. 284). [1/]

1/ For discussions relating to this problem, see (13, pp. 222-259) and (14).

It should be noted that the Koopmans-Hitchcock problem implicitly assumes that the scale of output and the demand in each given region are known.

Samuelson has investigated the many-region location problem posed by Enke, and has suggested how it can be couched mathematically into a maximum problem which can be solved by trial and error or by a systematic procedure of varying shipments in the direction of increasing the social pay-off, (23, p. 292). Baumol (1) has presented a solution similar to that of Samuelson. Beckmann (2) has extended the formulation and has considered the case of continuous geographical intensity distributions of production where every infinitesimally small area in an economy both produces and consumes a given commodity. Under these formulations, theoretically, both the geographic distribution of production and consumption and the optimum geographical pattern of interregional flows would have been derived simultaneously. These formulations ignore a number of basic locational forces, but if a finite number of production and consumption points or regions are specified, the Enke-Samuelson-Beckmann models offer an efficient approach to spatial pricing systems and the determination of the resulting geographical flows of commodities.

From an empirical standpoint, the following studies should be mentioned. Fox (6) and Judge (12), employing the Enke-Samuelson-Beckmann formulations, developed spatial price equilibrium models of the feed-livestock and egg sectors of the economy. Snodgrass and French (24) and Henry and Bishop (9) employed the transportation model as formulated by Koopmans to derive optimum geographical flows for the dairy and broiler sectors of the economy. These studies give an operational content to the theoretical formulations discussed above and suggest the practicability of research relating to spatial pricing systems and competitive locational equilibrium.

B. The Particular Problem Investigated

Given the general problem and the Enke-Samuelson-Beckmann formulation, this study is concerned with developing a spatial price equilibrium model for the beef sector of the economy. The particular problem follows that of Enke, except that, in this study the regional supply of beef, population and income are considered as predetermined variables, i.e. the optimum regional level and location of beef production is not considered in the problem and is taken as given for any point in time. Therefore, given the regional demand relationships and transport costs along with the existing values of the predetermined variables, the problem becomes one of ascertaining:

1. A set of spatial equilibrium prices of beef and the quantities consumed in each region,

2. The quantity of beef exported and imported from each region under the equilibrium conditions,

3. The aggregate net trade and the corresponding total transport cost, and

4. The volume and direction of trade between each possible pair of regions that will maximize net returns to each source and permit the geographical distribution of beef at a minimum transport cost.

The spatial price equilibrium solution reflected by the unique set of basic data for a particular year will be considered first. The problem will then be broadened to obtain the changes in the basic data, such as:

1. An increase or decrease in unit transport costs between regions i and j,

2. An increase or decrease in the total supply of beef,

3. A change in the geographical distribution of beef production,

4. A change in the level and geographical distribution of population and income,

5. The assumption that all beef is slaughtered where it is produced, and,

6. A change in the price or supply of pork available.

Each of these questions will be analyzed to determine the direction and magnitude of the changes on equilibrium regional prices, regional consumption and the pattern of geographical flows of beef.

II The Model

A. Assumptions

In an attempt to reduce the model to a simplified version of reality, the following restrictive and expository assumptions are made. Perfect competition assumptions dictate the requirements for the regional pattern of prices and flows of the commodity. Therefore, each firm is assumed to have the objective of maximizing profits and thus will make shipment decisions which yield the greatest per unit net return. The supply source and market for each geographical area is assumed to be represented by a fixed point. It is assumed that regional demands can be represented by known linear demand functions, and regional supplies are taken as predetermined for the given time period. All regions are connected by transport costs that are independent of the direction and volume of trade, and flows of beef among regions are assumed unhampered by governmental or other interference. It is assumed that consumers are indifferent as to source of supply and that the product is homogeneous. The observed values of factors affecting regional consumption over and above the price of the commodity are taken as predetermined or given. Imports and exports of beef outside the continental United States are taken as negligible and it is assumed for any time period t that total production and total consumption of beef are equal.

Obviously, both production and consumption of beef can take place in all regions and beef consumed out of local production does not require transporting since each region is represented by a point. Another obvious assumption, although required explicitly in a mathematical sense, is that there can be no negative shipments. It may also be observed that as a result of the profit maximization postulate there can be no cross-hauling of the product, deficit regions cannot export, and surplus regions can only export to deficit regions.

B. The Formal Model

Given the postulates and the objectives, the problem can be divided for simplicity into the following three parts: (1) general solution for determining equilibrium beef prices, consumption and surpluses or deficits for all regions, (2)deriving minimum cost flows of beef among regions, and (3) estimating objective regional price differentials and determining the final spatial price equilibrium solution. It should be understood that division of the problem into three parts is motivated by hopes of expository simplification and that the determination of all unknowns is, of course, a mutually dependent process.

(1) Determination of regional prices, consumption and surpluses and deficits: The two-regional competitive case is presented since it can be easily diagrammed and provides an insight for the logic underlying the solution for the general case. Generalization to n regions is not straightforward but should prove simpler once the theoretical solution is established for the trivial example. First, consider the geometrical representation presented in Figure 2.1. 2/

S_1 and S_2 represent fixed supplies for regions 1 and 2, respectively, and D_1 and D_2 depict linear demand schedules for the two regions. Assuming that none of the product will be shipped, equilibrium price would be at P_1' and consumption would be established at A for region 1, while price and the quantity consumed in region 2 would be P_2' and B.

Now assuming that shipment between regions can occur at a unit transport cost of C, a joint equilibrium would be established at price P_0 for region 1 (determined by the intersection of ES_1 and ES_2, the

2/ See Judge (12, p. 9) and Samuelson (23, p. 286).

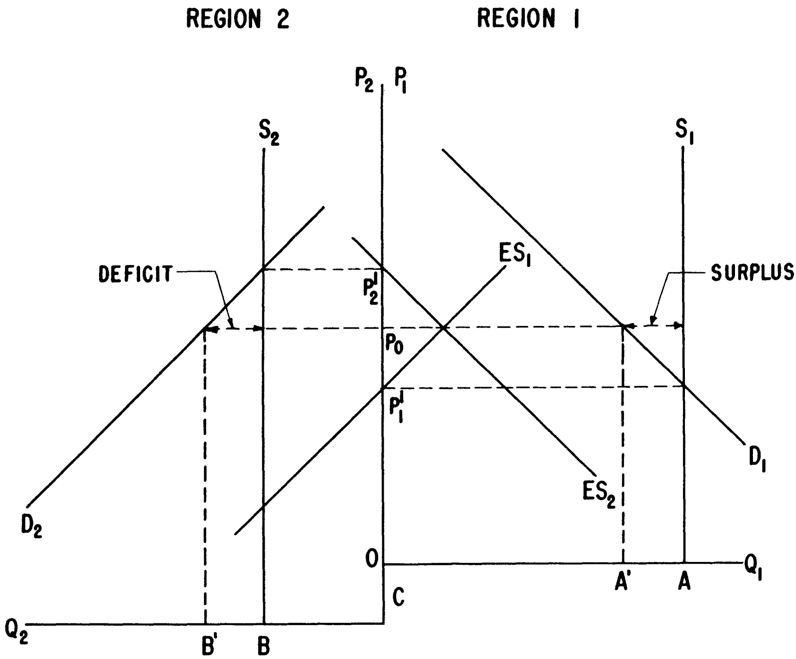

Fig. 2.1 Determination of Equilibrium Prices, Consumption and Surplus and Deficit Quantities for the Two-Region Case

excess supply curves for the two regions). The price for region 2 would be $P_0 + C$, and region 1 would consume the amount A' (A' $<$ A) while region 2 would increase consumption to B' (B' $>$ B). Obviously, A' + B' = A + B and thus A - A' = B' - B. Thus, knowing the demand schedules for regions 1 and 2, the supplies for both regions and the transport cost between the two regions permit the determination of joint equilibrium prices and consumption for both regions, surplus and deficit production and the amount of the product that will be transported to satisfy the equilibrium condition. Total transport costs may then be estimated by multiplying C times either B' - B or A - A'.

The algebraic analogue of the geometric model (Fig. 1) may be depicted as follows: let the two demand schedules be represented as,

$$D_1: \quad A' = \beta P_o + k, \tag{2.1}$$

$$D_2: \quad B' = \beta(P_o + C) + k, \tag{2.2}$$

and

$$A' + B' = A + B, \tag{2.3}$$

where β is the known slope and k the known intercept of the linear demand function (assuming that both regions have the same demand function). [3] A + B is the total production or total consumption for both regions and is known. C represents the cost of transporting a unit of product between the two regions and is assumed fixed and known. The unknowns include A', B' and P_o, and equation (2.3) provides enough information so that equations (2.1) and (2.2) may be solved uniquely for these unknowns. Adding equations (2.1) and (2.2) yields:

$$A' + B' = 2\beta P_o + \beta C + 2k. \qquad (2.4)$$

And from (2.3):

$$A + B = 2\beta P_o + \beta C + 2k. \qquad (2.5)$$

The solution for P_o, the equilibrium price in the surplus region, yields:

$$P_o = \frac{1}{2\beta}(A + B - \beta C - 2k). \qquad (2.6)$$

Given the unique P_o, equilibrium price in the deficit region is, then, $P_o + C$, and these values may be substituted into (2.1) and (2.2) for unique solution for A' and B'.

Before proceeding to the case of N regions, it should be pointed out that if the price differential, $(P_1' - P_2')$, assuming no shipments, is less than the cost of shipping between regions there would be no movement of the product between the two regions. Also, if the unlikely case of the price differential being equal to the unit transport cost occurred, either region would be indifferent to shipping a unit of product. [4]

In generalizing the two-region case to N regions, consideration is given to other regional factors that may be associated with the determination of the level of regional consumption. A typical equation for one region in the N region case may be written as:

$$Y_{1i} = \beta_i(Y_{20} + d_i) + \sum_{j=1}^{k} \alpha_{ij} Z_{ij} + \alpha_{i0}, \qquad i=1,\ldots,N \qquad (2.7)$$ [5]

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[3] This assumption is, of course, not necessary for a unique solution of the problem.

[4] See Samuelson (23) for a rigorous treatment of the various alternatives in question.

[5] Here the assumption that the demand function is homogeneous for all regions is omitted.

where Y_{1i} is consumption in the i^{th} region, β_i is the known coefficient connecting consumption and price in the i^{th} region, Y_{20} is price in the base region, d_i is the price differential between region i and the base and may be zero, positive or negative, Z_{ij}, $j = 1, \ldots, k$, represent all other factors that may affect regional consumption such as population, income, prices of complementary and competing products, etc., the α_{ij} are the coefficients of the Z_{ij} and α_{i0} is the equation constant. It is assumed that the β_i, d_i, α_{ij}, Z_{ij} and α_{i0} are known. Summing equation (2.7) over, the N regions yield:

$$\sum_{i=1}^{N} Y_{1i} = Y_{20} \sum_{i=1}^{N} \beta_i + \sum_{i=1}^{N} \beta_i d_i + \sum_{i=1}^{N} \sum_{j=1}^{k} \alpha_{ij} Z_{ij} + \sum_{i=1}^{N} \alpha_{i0}. \qquad (2.8)$$

Since total consumption $\left(\sum_{i=1}^{N} Y_{1i}\right)$ for the N regions is assumed equal to total production, which is known, equation (2.8) can be solved for Y_{20} (The price in the base region).

$$Y_{20} = \frac{1}{\sum_{i=1}^{N} \beta_i} \left[\sum_{i=1}^{N} Y_{1i} - \sum_{i=1}^{N} \beta_i d_i - \sum_{i=1}^{N} \sum_{j=1}^{k} \alpha_{ij} Z_{ij} - \sum_{i=1}^{N} \alpha_{i0} \right] \qquad (2.9)$$

With Y_{20} determined and assuming the d_i, $i=1, \ldots, N$, are known, equilibrium regional consumption can be established for all regions by substituting $Y_{20} + d_i$ in the regional demand equation (2.7). Unfortunately, the d_i are not ordinarily known for the N case since in considering more than two regions, the price advantage is no longer strictly equal to the transport cost between the base and the i^{th} regions. The problem relating to the determination of the d_i will be discussed in the next section.

Assuming for the moment that the d_i, $i=1, \ldots, N$, are known for all regions, the amount of surplus or deficit product for each region can be calculated since production or available supply is assumed predetermined for each region.

(2) Minimum cost flows of beef among regions: Given the surplus and deficit regions and the quantities of the commodity involved in each case, the problem of determining minimum cost flows may be treated as a linear programming problem. To make the problem explicit, the following tableau is employed (Table 2.1).

With the determination of the a_i and b_j, $i=1, \ldots, n$; $j=1, \ldots, m$; $n + m = N$, the problem is one of satisfying all demands out of total supplies in such a way as to minimize transport costs. The problem may be stated algebraically as finding a set of X_{ij} such that:

$$\sum_{j=1}^{m} \sum_{i=1}^{n} X_{ij}C_{ij} = \text{minimum}, \tag{2.10}$$

subject to

$$\sum_{j=1}^{m} X_{ij} = a_i; \quad i=1,\ldots, n, \tag{2.11}$$

$$\sum_{i=1}^{n} X_{ij} = b_j; \quad j=1,\ldots, m, \tag{2.12}$$

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \tag{2.13}$$

and

$$X_{ij} \geq 0 \text{ for all } i, j, \tag{2.14}$$

Table 2.1 The General Transportation Problem Tableau

	Destinations (Deficit Regions)										Totals a_i
	i \ j	(1)	(2)	.	.	.	.	.	.	(m)	
Origins (Surplus Regions)	(1)	C_{11}	C_{12}							C_{1m}	a_1
	(2)	C_{21}	C_{22}							C_{2m}	a_2
	.										.
	.				X_{ij}						.
	.										.
	.										.
	.										.
	.										.
	(n)	C_{n1}	C_{n2}							C_{nm}	a_n
Totals	b_j	b_1	b_2	.	.	.	.	.	.	b_m	$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j$

where X_{ij} represents the amount of product shipped from the i^{th} surplus region to the j^{th} deficit region; a_i represents the amount of product available for export from the i^{th} surplus region; b_j is the amount required by the j^{th} deficit region and C_{ij} is the per unit cost of shipping from the i^{th} surplus to the j^{th} deficit region. There are many solutions to (2.11) and (2.12) subject to (2.13) and (2.14), and given any feasible solution of $n + m - 1$ or $N - 1$ shipments, an iterative procedure known as the simplex method provides a means of converging to the optimum program (the one that satisfies 2.10).[6]

Therefore, given the a_i's and b_j's (regional excess supplies and demands), the linear programming transportation model may be used to determine the optimum distribution system. After setting forth and carrying through the problem in this form, it can be shown that the resulting minimum cost set of flows is the one that would be determined under the conditions of perfect competition. This conclusion follows since the equilibrium prices are tied together by a specific set of transport costs and the relevant transport costs, used in obtaining the optimum set of flows, are less than for every possible alternative delivery which is not made. The solution obtained will be unique except for the case when two or more sources find two markets equally profitable. In this case, of course, more than one optimum shipment plan exists.

(3) Determining regional price differentials (d_i): If the regional price differentials are known, then one can proceed in a straightforward manner to derive the spatial equilibrium prices and flows. However, in the real world these geographical price differentials constitute one of the unknowns necessary for a solution as they condition regional prices and the level of regional consumption. Therefore, in some cases, the magnitude of the price differentials may determine whether a region is surplus or deficit and by how much.

Since for practical problems the number of regions employed are finite, it is possible to obtain an initial approximate set of price differentials by the following procedure. For any given time period, regional supplies and population are known and average total per capita consumption and per capita supply of each region are also known. This information can be used to initially partition each region as being surplus or deficit. Given information relative to the classification of each region, an approximate set of price differentials can be generated by employing the following rules: (1) if one region ships to another, the prices must differ by the known unit transportation cost and (2) if two surplus regions ship to the same deficit region, the difference between prices in the surplus regions will be equal to the difference between their unit transport costs to the deficit region. Thus, the system involves a structure of regional prices bound together by specific transport costs. By selecting the i^{th} region as the base, the approximate regional price differential may then be used to determine the equilibrium

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6/ See Dantzig (13, pp. 359-73). For example of the use of the simplex technique in obtaining a solution for the transportation problem, see Appendix A.

regional prices. These prices can then be used to generate the data necessary for the transportation problem and thus the optimum geographical commodity flows under the assumed price differentials.

Up to this point, the analysis has proceeded on the basis that the initial vector of price differentials is the "correct" one. Also, the programming problem, as such, has been solved without recourse to price differentials, the prime economic allocators of regional distribution. However, as with any linear programming problem, the solution implicitly places values on the various inputs and outputs involved. Therefore, with the aid of the duality theorem of linear programming, a unique set of price differentials may be derived which correspond to the equilibrium set of flows and which may be used to check the approximate differentials of the initial formulation. Thus, given a minimum cost transportation solution, the dual problem is concerned with deriving the vector of regional price differentials consistent with this solution.

In developing the dual solution, let V_j be associated with the destinations and U_i be associated origins or supply points. The problem may then be set forth in the following equations as that of maximizing:[7/]

$$S = \sum_{j=1}^{M} b_j V_j - \sum_{i=1}^{N} a_i U_i \tag{2.15}$$

subject to the restrictions:

$$V_j - U_i \leq C_{ij}, \tag{2.16}$$

$$U_i,\ V_j > 0, \tag{2.17}$$

Since equation (2. 15) is equal to S, the total cost of transportation derived in the minimum formulation, the maximum problem may be thought of as finding the values of the U_i and V_j that will maximize the total gain in value of amounts shipped subject to non-positive profits on each shipment. Within this framework, it is then possible to interpret the U_i as the value of the product at supply origin i and the V_j as the value of the product delivered at destination j. By rewriting equation (2. 17) as:

$$V_j \leq U_i + C_{ij}, \tag{2.18}$$

7/ This development of the dual follows that given by R. Dorfman, P. A. Samuelson and R. M. Solow (3, pp. 122-127).

this relationship then states that for any supply-destination pair, the value at the destination must be no greater than the value of the input at the supply point plus transportation cost. For routes in the basis, destination value equals supply point value plus transportation costs. For those routes not in the basis, destination value is equal to or less than the supply point value plus transport costs. For any given problem, once the supply-destination pairs are known, then for that set of pairs, equation (2.18) may be rewritten as:

$$V_j - U_i = C_{ij}. \tag{2.19}$$

This then defines a set of linear equations involving $n + m$ unknown values of the U_i and V_j. Since there are only $n + m - 1$ observed unit transport costs in a basic solution, a unique solution to the set of equations requires assigning an arbitrary value to either one U_i or V_j. By choosing the value at the i^{th} supply point as equal to zero, a set of price differentials is generated subject to this choice or base. By then making use of the U_i and V_j estimates of the dual, objective estimates of the d_i, the regional price differentials, are obtained. 8/

In addition to providing objective estimates of the regional price differentials, the U_i and V_j also contain two types of useful economic information: (1) the values of U_i measure the comparative advantages of the surplus regions and (2) the values of the V_j give the delivered price differentials that correspond to the most economical allocation of the supply from the viewpoint of minimum aggregate transportation costs.

The next step is to ascertain whether or not the set of price differentials generated by the dual solution agree with those developed in the initial approximate formulation. If the new set of price differentials differ from the initial formulation, the process of determining equilibrium prices and flows is repeated using the new estimates of the d_i. Thus, an iterative process is employed to determine the price differentials and the final stage is reached when the d_i, $i = 1, \ldots, N$, generated by the last optimum shipment program agree with the differentials used in determining equilibrium price, consumption, etc. The results of this iterative process are similar to those of basing-point prices in that the prices at each supply point are influenced by the transportation costs pertaining to the other supply points. Therefore, the price at supply point 1 may be greater than those at supply point 2 because supply point 2 has a locational disadvantage and could not otherwise dispose of its product without loss. The results obtained in this manner are the competitive equilibrium solution that would result from the efforts of the supply points to dispose of their supplies at the maximum possible prices and the solution to the value and flow problem is simultaneous and interdependent. An example of this iterative procedure is given in the empirical results section.

8/ It should be mentioned that only the regional price differences are determinate by this formulation. The regional equilibrium prices or P_0 plus or minus the price differential relative to the base region is determined by using equation 2.9.

III The Data

In order to convert the conceptual framework to an operational model, real world counterparts must be defined and the data used to reflect the variables must be specified. In converting the formal model to a reflection of a real world situation, the area under study must first be demarcated into meaningful geographical units or regions. Given the regional demarcation, the model then specifies a need for the following data: (1) regional market demand relationships for beef, (2) observed values of the predetermined variables - regional supply of beef, population, disposable income and the price of pork - in time t, and (3) a structure of transportation rates between all possible pairs of regions. The data basic to the first analysis, which involves the 1955 time period, is presented in this section. Basic data for alternative time periods will be discussed when the particular analysis is introduced in the empirical results section.

A. The Regional Demarcation

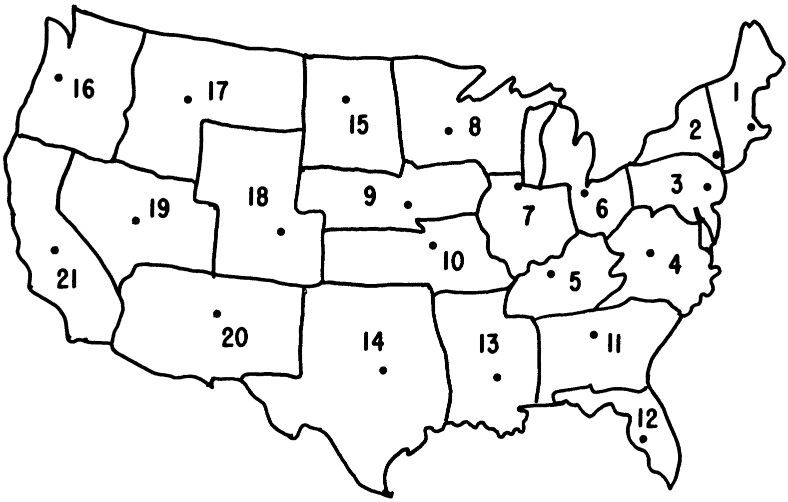
Although the particular problem conditions to a large extent the division of an economic territory into geographically contiguous units, the final demarcation also involves subjective considerations along with restrictions relative to the availability of data. In arriving at the final demarcation, the investigator usually seeks a regional model that will be both manageable and reasonably realistic or meaningful.

Employing these criteria, the United States was partitioned into 21 geographically contiguous regions (Fig. 3.1). States are the smallest geographical units for which data for the predetermined variables are available and, thus, each region is composed of one or more states. Each regional market or source of supply is represented by a point that is identified with a certain city near the geographical center of each area.

B. Market Demand Relationship

Converting the formal model to an operational form requires the specification of regional market demand relationships for beef. Since there are no adequate data on beef consumption by state or regions, the regional demand relationships were derived from an econometric demand analysis of the United States as a whole by Wallace and Judge (27). The aggregated sector model underlying this analysis specified behavior relationships that were logarithmic in functional form, and generated parameter estimates of beef price and income elasticity of demand and pork price elasticity of substitution of -0.86, +0.59 and +0.32, respectively. Using these parameter estimates, the market demand function was converted to a functional form linear in natural units by employing the following identity:

Figure 3.1. The Regional Demarcation, Demand and Supply Points, Spatial Beef Model



Regions	States	Demand and Supply Point
1	Vermont, New Hampshire, Maine, Massachusetts, Connecticut and Rhode Island	Boston
2	New York	New York
3	Maryland, Delaware and Washington, D. C., Pennsylvania and New Jersey	Philadelphia
4	West Virginia, Virginia, North Carolina	Roanoke
5	Kentucky and Tennessee	Bowling Green
6	Michigan and Ohio	Toledo
7	Illinois and Indiana	Chicago
8	Minnesota and Wisconsin	St. Paul
9	Nebraska and Iowa	Omaha
10	Kansas and Missouri	Kansas City
11	Alabama, Georgia and South Carolina	Atlanta
12	Florida	Tampa
13	Arkansas, Mississippi and Louisiana	Vicksburg
14	Oklahoma and Texas	Ft. Worth
15	North Dakota and South Dakota	Bismarck
16	Washington and Oregon	Portland
17	Montana and Idaho	Butte
18	Wyoming and Colorado	Denver
19	Utah and Nevada	Ely
20	Arizona and New Mexico	Gallup
21	California	Fresno

$$B_i = \frac{y_i dy_1}{y_1 dy_i}, \qquad (3.1)$$

where B_i is the relevant elasticity estimate; y_1, per capita consumption of beef and y_i can take on definitions of retail price of beef, per capita disposable income and retail price of pork. The formulation dy_1/dy_i represents the partial derivative of y_1 with respect to y_i. Data applying to the 1955 observed values of the variables were then employed along with the elasticity estimates to derive the desired linear coefficients. This transformation resulted in the following linear in natural units market demand relationship which was postulated for each of the twenty-one regions:

$$Y_{1i} = -1.0529Y_{2i} + 0.6509Y_{3i} + 0.0303Z_{1i} + 78.3543 \qquad (3.2)$$

where Y_{1i} is the per capita consumption of beef in the i^{th} region; Y_{2i}, the retail price of beef in the i^{th} region; Y_{3i} is price of pork in the i^{th} region and Z_{1i} represents per capita disposable income for the i^{th} region. In applying equation (3.2), the price of pork was excluded from the 1955 market demand relationship since available data were not sufficient to establish regional pork prices. However, in this case, the average impact of the price of pork is included in the constant term. The price of pork is included in demand relationships reflecting other time periods for which these data are available.

C. Regional Values of the Predetermined Variables

The postulated model specifies the need for regional data relating to beef supplies, population and disposable income.[9] The model specified observed values of these series as predetermined variables and the data relating to the regional values in 1955 were obtained from records published by the Departments of Agriculture and Commerce. The observed values of these predetermined variables are assumed to be free of errors of measurement and are accepted as accurate portrayals of the variables they are supposed to reflect. The 1955 regional values of the predetermined variables are given in Table 3.1. The sources and derivation of the variables are given in the footnotes accompanying the Table.

D. Transportation Rates

The market and supply sources as formulated in the model are assumed to be designated by a single point in each region. Since the structure of transport rates for beef are basic to the spatial solution, it is necessary to obtain estimates of the costs between the points that represent each pair of regions.

9/ Beef supplies are defined as the total annual regional availability of commercial and farm slaughter beef in carcass weight form. The regional data, therefore, relate not to farm production but rather to the availability of slaughtered beef.

Table 3.1 - Data Pertaining to the Total and Per Capita Supply of Beef Available for Consumption, Per Capita Disposable Income and Population, by Regions, 1955

Regions	Total Beef Supply[1/] (1000 pounds)	Per Capita Beef Supply (pounds)	Per Capita Disposable Income ($)[2/]	Population[3/] (thousands)
1	148,710	15.5	1,817	9,619
2	383,783	24.0	1,970	16,021
3	830,980	41.1	1,785	20,213
4	198,091	20.0	1,180	9,907
5	342,418	53.3	1,087	6,425
6	1,066,866	65.6	1,823	16,271
7	1,499,690	110.0	1,865	13,630
8	1,324,079	192.1	1,512	6,892
9	2,109,061	518.8	1,362	4,065
10	1,167,530	186.5	1,523	6,261
11	333,962	36.8	1,066	9,080
12	166,320	46.5	1,441	3,580
13	211,793	30.8	994	6,869
14	947,597	86.5	1,386	10,958
15	271,697	204.9	1,137	1,326
16	387,537	90.3	1,677	4,292
17	120,353	97.0	1,442	1,241
18	477,962	257.1	1,534	1,859
19	108,058	104.7	1,528	1,032
20	81,210	45.1	1,317	1,800
21	1,318,935	101.8	1,978	12,961
U. S.	13,496,632	82.1	1,608	164,302

[1/] Agricultural Marketing Service, "The Livestock and Meat Situation," U. S. Department of Agriculture, August 1948 and March 1956; Agricultural Marketing Service, "Livestock Slaughter," U. S. Department of Agriculture, May 1956, p. 4; and Agricultural Marketing Service, "Meat Animal, Farm Production, Disposition and Income, By States," U. S. Department of Agriculture, Bulletin 184, 1956, pp. 6-10. Estimates by state of beef production (including farm slaughter) are directly available for 1947 and 1954. For 1955, commercial slaughter in liveweight was added to farm slaughter and the total divided by the appropriate ratio to obtain carcass weight supplied by states.

[2/] U. S. Bureau of Foreign and Domestic Commerces, "Supplement to Survey of Current Business," U. S. Department of Commerce, 1956, p. 141. Per capita disposable income is not available for 1955 on a state basis, so it was necessary to adjust these data on the basis of state personal income payment to obtain estimates of this series.

[3/] U. S. Bureau of Foreign and Domestic Commerce, "Supplement to Survey of Current Business," U. S. Department of Commerce, 1956, p. 145. These data apply to population estimates as of July 1.

A model to reflect rail rates between market and supply source points was postulated as:

$$C^*_{ij} = \beta_1 M^*_{ij} + \beta_2 M^{*\frac{1}{2}}_{ij} + \epsilon, \qquad (3.3)$$

where C^*_{ij} represents the cost in cents of shipping a pound of carcass beef from point i to point j by rail, M^*_{ij} is the rail milage between i and j, β_2 and β_1 are parameters to be estimated and ϵ is an unobservable random error.

The above functional form was postulated in the belief that rail rates are an increasing function of milage but should increase at a decreasing rate. For obvious reasons, the function was postulated as having a zero intercept.

Since beef is shipped by both truck and rail, an additional model was constructed to represent truck rates.

$$C^{**}_{ij} = \alpha_1 M^{**}_{ij} + \alpha_2 M^{**\frac{1}{2}}_{ij} + \epsilon' , \qquad (3.4)$$

where C^{**}_{ij} is the per pound cost by truck in shipping carcass beef from point i to point j, M^{**}_{ij} is the highway mileage between region i and j, the α_i are unknown parameters to be estimated and ϵ' is an unobservable random error.

A sample of data was secured to represent the observable variables, and the least squares procedure using moments about zero was used to estimate the unknown parameters.10/ The results were:

$$C^{*}_{ij} = .0008M^{*}_{ij} + .0464M^{*\frac{1}{2}}_{ij} \qquad R^2 = .970 \qquad \text{(Rail)} \qquad (3.5)$$

and

$$C^{**}_{ij} = .0015M^{**}_{ij} + .0226M^{**\frac{1}{2}}_{ij} \qquad R^2 = .969 \qquad \text{(Truck)} \qquad (3.6)$$

If rail mileages were the same as highway mileages between cities, the minimum rate matrix could have been established by solving equations (3.5) and (3.6) simultaneously and using equation (3.6) to generate costs up to the intersection of the two functions. Then equation (3.5) could have been used to estimate costs for mileages greater than the intersection point. However, since distance for the two types of transportation are not equal, both rail and highway mileage data were secured between all basing points. Both functions were then used to generate truck and rail costs and the minima of those alternative rates were chosen as representative.11/

The decision to generate transport costs by smooth functions rather than using actual rates as determined by the Interstate Commerce Commission was influenced by several factors. The basing point

10/ Rail rates for fresh beef (n=66) were obtained from the Commodity Stabilization Service, Transportation and Storage Services, USDA; corresponding rail mileages were taken from "Rand McNally Commercial Atlas and Marketing Guide," 86th edition, Rand, McNally and Co. (1955). Truck rates for fresh beef (n=25) were obtained from Wilson and Co., Oklahoma City Branch and Armour and Co., Chicago Branch. Corresponding highway mileages were obtained from "Standard Highway Mileage Guide," Rand, McNally and Co., Chicago, 1955.

11/ As a point of interest, under the assumption that highway mileage is the same as rail mileage between i and j, the simultaneous solution for M for (3.5) and (3.6) was approximately 1089 miles. Thus, under 1089 miles, truck rates were less than rail and over 1089 miles, the reverse was true.

cities were chosen in many cases because of their proximity to the center of the geographical regions. Thus, the I. C. C. rate between, say, Ely, Nevada, and Bowling Green, Kentucky, could be quite distorted, if available at all, since it is possible that not enough beef has been shipped between the two to establish a representative rate. Another reason for using functions was that this provides a method for estimating transport costs between any two cities in the United States with a minimum of resources. It was felt, too, that under pure competitive assumptions, transport costs should approximate a continuous function of the form specified. The estimated transport cost data are presented in Table 3.2.

IV The Empirical Results

Being provided a method by the formal model and a means of relating the model to the real world by the basic data, the objectives motivating this research can now be realized to the extent that the simplifying assumptions are valid. In presenting the results, the method of analysis is discussed in detail for the year 1955. For all other, similar analyses, only the final results are given and certain of the more important implications are developed.

A. Regional Price and Consumption for 1955 Under Alternative Assumptions

Before proceeding to the major objectives of the study, several alternative assumptions such as absence of interregional trade, equal regional per capita incomes, etc., were posed to provide descriptive indications of the importance of the various factors on the regional price and consumption of beef. These results (see Table 1) were obtained by employing the logarithmic counterpart of the demand equation (3.2) presented in Chapter III and the relevant regional data for 1955.[12/] In the first analysis, regional consumption was assumed equal to the regional supplies of beef, i.e. no interregional movement of beef.

The results of this analysis make apparent the importance of interregional trade on the geographical prices of beef (Table 4.1, Column 1). Under the assumption of no interregional movements, the estimated price of beef becomes quite high in New England, New York and other high income, low producing areas, while beef is almost a valueless good in Iowa and Nebraska (Region 9). If such a catastrophic breakdown in transportation did actually occur, it could be expected that inter-industry adjustments would partially alleviate the large price differentials. However, these estimates serve to point out the degree of geographical specialization in beef slaughter and the impact of competition and geographical flows on prices.

12/ The choice of the logarithmic functional form was based on the objective of portraying more accurately the extreme quantities involved.

Table 3.2 - Estimates of Transport Rates for Fresh Beef between Specified Points, by Regions, United States[1/]

Region	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	(Cents per Pound or Dollars per 100 lbs.)																				
1	0	.67	.87	1.63	2.41	1.77	2.21	2.85	2.98	2.92	2.38	2.86	3.04	3.49	3.44	5.14	4.30	3.69	4.69	4.29	5.36
2		0	.35	1.19	2.00	1.46	1.90	2.68	2.76	2.62	2.00	2.53	2.72	3.18	3.29	5.01	4.16	3.55	4.55	4.11	5.23
3			0	1.00	1.85	1.31	1.76	2.54	2.62	2.48	1.83	2.40	2.59	3.06	3.17	4.90	4.09	3.43	4.43	4.00	5.12
4				0	1.21	1.24	1.58	2.38	2.39	2.19	1.16	1.81	1.92	2.60	3.14	4.75	3.92	3.18	4.30	3.62	4.76
5					0	1.13	1.06	1.84	1.74	1.34	.89	1.83	1.23	1.81	2.67	4.36	3.49	2.52	3.68	3.04	4.42
6						0	.70	1.56	1.67	1.67	1.61	2.50	2.03	2.53	2.35	4.19	3.30	2.69	3.67	3.26	4.40
7							0	1.10	1.22	1.27	1.68	2.57	1.79	2.22	1.98	3.89	2.99	2.28	3.40	2.96	4.12
8								0	1.00	1.18	2.42	3.21	2.34	2.22	1.16	3.38	2.39	1.97	3.16	2.88	3.95
9									0	.63	2.29	3.01	1.92	1.65	1.44	3.33	2.44	1.35	2.62	2.30	3.50
10										0	1.92	2.75	1.53	1.34	1.83	3.59	2.67	1.50	2.90	2.14	3.33
11											0	1.20	1.19	1.98	3.06	4.64	3.84	3.00	4.14	3.26	4.29
12												0	1.73	2.50	3.73	5.25	4.42	3.60	4.50	3.82	4.69
13													0	1.07	3.01	4.38	3.75	2.30	3.70	2.52	3.55
14														0	2.56	3.92	3.24	1.78	3.00	1.76	3.15
15															0	2.84	1.61	1.70	2.63	2.66	3.54
16																0	1.67	2.79	1.96	2.85	1.81
17																	0	1.88	1.53	2.08	2.48
18																		0	1.76	1.29	2.72
19																			0	1.60	1.47
20																				0	1.89
21																					0

[1/] These rates were estimated by using equations (3.5) and (3.6).

Table 4.1 - Estimated Regional Price and Consumption of Beef Under Alternative Assumed Conditions (1955)

Region	(1) Regional price assuming no interregional trade	(2) Regional price assuming equal per capita regional consumption	(3) Regional per capita consumption assuming equal prices in all regions	(4) Regional price assuming a 20% increase in regional pork price[1/]	(5) Regional price assuming a 20% decrease in regional pork price[1/]
	dollars/pound	cents/pound	pounds	cents/pound	cents/pound
1	5.17	74.8	89.8	75.5	64.9
2	3.30	79.0	94.2	75.0	64.5
3	1.65	73.9	88.9	75.2	64.6
4	2.86	55.7	69.6	74.9	64.4
5	.87	52.6	66.3	71.4	61.4
6	.98	74.9	89.9	74.1	63.7
7	.55	76.1	91.2	72.5	62.3
8	.25	66.0	80.6	70.9	60.9
9	.07	61.4	75.8	71.6	61.6
10	.26	66.3	81.0	71.5	61.5
11	1.32	51.9	65.5	72.4	62.3
12	1.24	63.8	78.3	76.3	65.6
13	1.54	49.6	62.9	71.8	61.7
14	.59	62.1	76.6	72.4	62.3
15	.19	54.3	68.1	69.9	60.1
16	.64	70.8	85.7	69.9	60.1
17	.53	63.9	78.3	70.9	60.9
18	.18	66.6	81.9	69.7	59.9
19	.50	66.4	81.7	69.9	60.1
20	1.20	60.0	74.2	71.0	61.0
21	.62	79.2	94.5	70.7	60.7

[1/]See Table 4.5, Column 1, for the estimated regional equilibrium prices of beef under the 1955 actual pork price.

Alternatively, the estimates given in columns two and three of Table 1 provide some indication of the net effect of regional incomes on regional price and consumption of beef. For example, under these assumptions, California (Region 21) has the highest estimated consumption and price since it had the largest per capita disposable income for the year considered. The estimated regional prices suggest a differential range of 29.6 cents between regions and the estimated regional consumptions suggest a differential range of 31.6 per capita pounds of beef between regions, which may be attributed to variation in the level of the income variable. From a practical standpoint, substitution within grades probably serves to dampen the effect of regional income differences in terms of total beef consumed per region. The estimates presented in Column 2 could be interpreted as equilibrium regional prices under a rationing scheme similar to that employed for beef during World War II. In the event of such a program, this type of procedure could be employed to develop equitable regional controls if price was used as the allocator. Alternatively, the estimated regional price and consumption differences give an indication of the extent of distortion in equilibrium values when either regional price or consumption or both are specified under a controlled situation.

The final two columns of Table 1 were generated by using regional equilibrium consumption estimates for 1955 (see the next Section) along with a uniform 20 per cent increase and decrease in the 1955 pork price. The effect of assuming a uniform 20 per cent decrease in pork prices, certeris paribus, was to decrease regional beef prices. Correspondingly, a uniform increase in regional pork prices resulted in generally higher regional prices of beef. A 20 per cent increase or decrease in the regional price of pork generated, on the average, a change of 5 cents in the price of beef. These results make explicit the degree of competition between pork and beef as consumer goods.

B. Equilibrium Geographical Prices and Flows for Beef, 1955

(1) Determination of regional price, consumption, surpluses and deficits - (First Approximation): Given observations relating to disposable income, population, production and market demand relationships by regions, the first task is to determine a structure of geographical prices that are tied together by transport costs. The procedure followed was to choose a base region and then approximate a set of price differentials relative to that base. Region 9 (Iowa and Nebraska) was chosen as the base because of its proximity to the center of the United States and because it is obviously a surplus producing and processing region. The per capita production (slaughter) observations of Table 3.1 provide some indication as to which regions are surplus and which are deficit, and thus give some clues as to the probable pattern of shipment. In approximating the differentials for 1955, it was decided by observing the level of regional per capita supplies in Table 3.1, that regions 1, 2, 3, 4, 5, 6, 11, 12, 13, and 20 would be surplus; regions 7, 8, 9, 10, 15, 17, 18, 19 and 21 would be deficit and regions 14 and 16 would go either way depending on their final equilibrium prices. Proceeding on the basis of this subjective analysis and using the relevant transport costs, along with the method outlined in Chapter II, an approximate set of price differentials was obtained (Table 4.2). As an example of this procedure, assume that the base region (Region 9) ships to region 2, the price differential would be equal to the unit transport cost between the two regions or 2.76 cents. Assuming that both regions 9 and 10 ship to region 5, the difference between equilibrium prices in the surplus regions must be equal to the differences between their transport costs to the deficit region. Since, at this stage in the analysis, it is not definitely known which surplus regions ship to each deficit region, (neither is it known in all cases whether a region will be clearly surplus or deficit), this manner of determining differentials is unsatisfactory as it stands. However, it does provide a useful approximation, and given the resulting equilibrium flow solution, it can then be shown that the linear programming simplex tableau provides an objective means of determining the "correct" price differentials.

Provided a set of regional price differentials, the problem is now one of solving for the equilibrium price in the base region (Y_{20}).

Following the procedure outlined in the model section, consider the following equation.[13/]

$$Y_{1i} = -1.0529(Y_{20} + d_i) + .0303Z_{1i} + 104.9777 \qquad (4.1)^{14/}$$

where Y_{1i} represents per capita consumption of beef for the i^{th} region, Y_{20} is price in the base region (9), d_i is the estimated price differential between the base and i^{th} regions and Z_{1i} is the per capita disposable income for the i^{th} region, and summing over all regions yields:

$$\sum_{i=1}^{21} P_i Y_{1i} = -1.0529 Y_{20} \sum_{i=1}^{21} P_i - 1.0529 \sum_{i=1}^{21} P_i d_i + .0303 \sum_{i=1}^{21} Z_{1i} P_i + 104.9777 \sum_{i=1}^{21} P_i \qquad (4.2)$$

Since $\sum_{i=1}^{21} P_i Y_{1i}$ is the total consumption of beef in the United States and is assumed equal to the known total supply, $\sum_{i=1}^{21} P_i$ is total population, $\sum_{i=1}^{21} Z_{1i} P_i$ is the total disposable income for the United States for 1955 and the P_i and d_i are known, the solution for Y_{20} is straightforward. Inserting the observed regional values of the variables in equation (4.2) and carrying through the analysis, the base price, Y_{20}, was found to be 66.26 cents per pound. Price in the i^{th} region was determined by using the relationships, $Y_{2i} = Y_{20} + d_i$, where Y_{2i} is the price in question. The price differential, d_i, can be positive, negative or zero.

With the determination of equilibrium prices, the linear demand relationship (4.1) was used to estimate per capita consumption for each region. The per capita estimates of consumption were then multiplied by regional population and total regional consumption estimates were subtracted from regional supply data for estimates of surpluses and deficits. The following Table (Table 4.2) is a presentation of the estimates of regional price, consumption, etc. employing the initial approximate d_i.

(2) Determination of optimum flows for beef (First Approximation): Given estimates of the regional excess supplies and demands, the problem becomes one of determining the geographical flow pattern for beef such that the total cost of transportation is minimized.

13/ Since the variables are linear in natural units, an alternate iterative procedure could be employed for arriving at the equilibrium set of geographical prices by choosing a base region price, estimating regional consumption and then making the adjustment in price necessary to equate supplies with demands.

14/ This linear demand equation is the same as equation (3.2) of the preceeding Chapter, except that the price of pork is taken as an average for all regions and included in the constant terms.

Table 4.2 - Regional Equilibrium Price Differentials, Prices, Consumption and Surpluses and Defecits, (1955), First Approximation

Region	$\hat{d}_i$, Price Differential	Price	Per Capita Consumption	Total Consumption	Surplus and Deficit[1/]
	cents	cents	lbs.	1,000 lbs.	1,000 lbs.
1	2.98	69.24	87.1	832,952	-684,242
2	2.76	69.02	92.0	1,465,172	-1,081,389
3	2.62	68.88	86.5	1,738,485	-907,325
4	2.39	68.65	68.4	672,961	-474,870
5	1.54	67.80	66.5	424,038	- 81,620
6	1.67	67.93	88.7	1,434,591	-367,725
7	0.86	67.12	90.8	1,230,624	269,066
8	0.08	66.34	80.9	554,074	770,005
9	0.00	66.26	76.5	308,730	1,800,331
10	0.20	66.46	81.1	504,689	662,841
11	2.12	68.38	65.3	587,966	-254,004
12	2.64	68.90	76.1	270,487	-104,167
13	1.21	67.47	64.0	436,322	-224,529
14	0.14	66.40	77.0	838,633	108,964
15	-0.53	65.73	70.2	92,406	179,291
16	-2.25	64.01	88.4	377,111	10,426
17	-1.40	64.86	80.4	99,104	21,249
18	-0.79	65.47	82.5	152,422	325,504
19	-1.79	64.47	83.4	85,501	22,557
20	-0.28	65.98	75.4	134,782	-53,572
21	-2.17	64.09	97.4	1,255,762	63,173

[1/] A minus preceding an observation indicates that the region in question occupies a deficit position and the related number indicates the magnitude of the deficit.

As a basis for this determination, the following table (4.3) presents transport costs between all surplus and deficit regions, the total amounts of beef available for distribution by the surplus regions and the total amounts demanded by the deficit regions.

Using this data, the linear programming transportation model was used to derive an optimum geographical shipment pattern for beef.[15/] The optimum tableau, given the transport cost and rim quantity requirements, is presented in Table 4.4.

[15/] For discussions of this method, see Dantzig (13, pp. 359-373) and Henderson and Schlaefer (8) and for an example of the logic of the simplex method as applied to the transportation problem, see Appendix A. To establish a first basic solution that would be a closer approximation to this optimum than some arbitrary procedure, a method known as "Vogel's Approximation" was used. See Appendix B.

Table 4.3 - Transport Costs (Cents per Pound) and Rim Requirements (1955) First Approximation

Origins i \ Destinations j	1	2	3	4	5	6	11v	12	13	20	Total Supplies (1,000 lb.)
7	2.21	1.90	1.76	1.58	1.06	.70	1.68	2.57	1.79	2.96	269,066
8	2.85	2.68	2.54	2.38	1.84	1.56	2.42	3.21	2.34	2.88	770,005
9	2.98	2.76	2.62	2.39	1.74	1.67	2.29	3.01	1.92	2.30	1,800,331
10	2.92	2.62	2.48	2.19	1.34	1.67	1.92	2.75	1.53	2.14	662,841
14	3.49	3.18	3.06	2.60	1.81	2.53	1.98	2.50	1.07	1.76	108,964
15	3.44	3.29	3.17	3.14	2.67	2.35	3.06	3.73	3.01	2.66	179,291
16	5.14	5.01	4.90	4.75	4.36	4.19	4.64	5.25	4.38	2.85	10,426
17	4.30	4.16	4.09	3.92	3.49	3.30	3.84	4.42	3.75	2.08	21,249
18	3.69	3.55	3.43	3.18	2.52	2.69	3.00	3.60	2.30	1.29	325,504
19	4.69	4.55	4.43	4.30	3.68	3.67	4.14	4.50	3.70	1.60	22,557
21	5.36	5.23	5.12	4.76	4.42	4.40	4.29	4.69	3.55	1.89	63,173
Total Demands (1,000 lbs.)	684,242	1,081,389	907,325	474,870	81,620	367,725	254,004	104,167	224,529	53,572	4,233,407 1/

1/ Totals may not be additive due to rounding.

Table 4.4 - Optimum Shipment Pattern for Beef (First Approximation, 1955)
(1,000 lbs.)

Origins	Destinations 1	2	3	4	5	6	11	12	13	20	Totals	U_i
7	.22	.08	.08	.13	.46	269,066	.50	.67	1.19	3.86	269,066	0.94
8	335,310	336,036	0	.07	.38	98,659	.38	.45	.88	2.92	770,005	0.08
9	.05	745,353	907,325	147,653	.20	.03	.17	.17	.38	2.26	1,800,331	0.00
10	.19	.06	.06	327,217	81,620	.23	254,004	.11	.19	2.30	662,841	0.20
14	1.03	.89	.91	.68	.74	1.36	.33	.13	108,964	2.19	108,964	0.47
15	179,291	.02	.04	.24	.62	.20	.43	.38	.96	2.11	179,291	-0.51
16	10,426	.04	.07	.15	.61	.34	.31	.20	.63	.60	10,426	-2.21
17	21,249	.03	.10	.16	.58	.29	.35	.21	.84	.67	21,249	-1.37
18	137,930	.03	.05	.03	.22	.29	.12	72,009	115,565	.49	325,504	-0.76
19	.20	.23	.25	.35	.58	.47	.46	.10	.60	22,557	22,557	-1.56
21	.58	.62	.65	.52	1.03	.91	.32	32,158	.16	31,015	63,173	-1.85
Totals	684,242	1,081,389	907,325	474,870	81,620	367,725	254,004	104,167	224,529	53,572	4,233,407[1]	
V_j	2.93	2.76	2.62	2.39	1.54	1.64	2.12	2.84	1.54	.04		

[1] Totals may not be additive due to rounding. Total costs = $\sum_{i}^{n} \sum_{j}^{m} C'_{ij} x_{ij}$ = \$104,572,214.

The cells of Table 4.4 in which the bold face type appear represent the activities which appear in the final basic solution, and the corresponding number represents the quantities of beef involved in the optimum flow solution. For example, the optimum flow solution indicates that region 8 would supply 98,659 thousand pounds of beef to region 6, region 21 would supply 32,158 thousand pounds of beef to region 12, etc. The plain type numbers appearing in the cells of Table 4.4 may be interpreted as the difference between direct and indirect transportation costs. The indirect costs refer to the opportunity cost of not including a particular activity (supply and destination combination) in the basic solution or, alternatively, the consequences of including it in the basic solution. The general theory of the simplex solution states that an optimum is reached when all direct minus indirect costs are positive or zero. This solution indicates that a change of activities appearing in the basis would result in equal or added total costs. For example, region 7 could ship a unit of product to region 1 only at a 0.22 cent loss or by increasing total transport cost by 0.22 cents per unit shipped. Alternatively, it could be inferred that the transport cost between region 1 and 7 would have to decrease at least 0.22 cents per unit before any product would be shipped in that direction, given the alternative shipping possibilities and costs.

The dummy variables U_i' and V_j are determined from the following set of linear equations for the minimum problem:

$$U_i' + V_j = C_{ij}'; \qquad i = 1, \ldots, n: \qquad j = 1, \ldots, m, \qquad (4.3)$$

where C_{ij}' is the transport cost of shipping a unit of product from region i to region j for the supply-destination pairs that appear in the basic solution. Since there are n + m, U_i' and V_j to be determined and only n + m - 1 transport costs in a basic solution, it is necessary to assign an arbitrary value to either one U_i' of V_j. Once that value is assigned, the other values are unique. For example, in Table 4.4, the U_i opposite region 9, the base region, is assigned the value, zero. Given this assigned value, and using equation (4.3), the V_j opposite deficit region 4 is determined from equation (4.3) as 2.39, the transport cost of shipping from region 9 to region 4. Note that in this case an alternative optimum feasible program exists since there is a zero difference in direct and indirect costs for the activity of shipping a unit of product from region 8 to region 3. Given solutions for U_i' and V_j for any basic solution, the simplex procedure for converging to the optimum solution stems from the idea that for all activities not in the basic solution there are two costs - the direct cost of shipping a unit of product from region i to region j and the indirect cost ($U_i' + V_j$) that indicates the opportunity cost of not having that activity in the basic solution. Thus, the plain type numbers in Table 4.4 are a result of differencing the relevant C_{ij} and $U_i' + V_j$, and the optimum is reached when all direct costs minus indirect costs are positive or zero. This indicates that to change from the optimum basic solution would result in equal or added costs.

Given the optimum set of activities from the minimum solution, the duality theorem may then be used to obtain estimates of the d_i, the regional price differentials.16/ Since the minimum problem has been solved, the solution to the dual is trivial. In this case, the dual may be generated by multiplying the vector of U_i' by the scalor (-1) and defining it as U_i. Or alternatively, given the optimum set of activities, the price differentials relative to a given base can be derived by solving the set of linear equations:

$$V_j - U_i = C'_{ij}, \tag{4.4}$$

By then assigning a value of zero to the U_i opposite the base region, the economic interpretation of the other U_i is that they represent locational price differentials of the surplus or shipping regions relative to the base region (in this case, region 9). The V_j represent locational price differentials of the deficit regions relative to the base region. A change of base could easily be accomplished by assigning the value zero to the U_i or V_j opposite any other region and expressing all other U_i and V_j in relation to that value. It should be noted that given the minimum solution, the dual does not need to be formulated and solved since the activities (supply and destination pairs) are identical under the minimum and maximum formulations.

Solving the linear equations given in equation 4.4 results in the vector of price differentials (U_i and V_j) given in Table 4.4. Comparing these price differential with the initial specification of column 1, Table 4.2, reveals certain differences between the two price differential vectors. For example, region 1, under the new formulation, has a price differential of 2.93 cents relative to region 9 instead of 2.98 cents under the initial approximate specification. Similar differences obtain for several of the other importing and exporting regions.

(3) Equilibrium price pattern, regional consumption and surpluses and deficits, 1955 (Final Solution): The process outlined previously for determining regional equilibrium prices, consumption and regional surpluses and deficits was repeated using regional price differentials provided by the U_i and V_j of Table 4.4. The results of this analysis are presented in Table 4.5.

The resulting regional equilibrium prices varied from a low of 64.46 in region 16 (Washington and Oregon) to a high of 69.60 in region 1 (New England). Per capita regional consumption varied from a low of 63 lbs. in region 13 (Arkansas, Louisiana and Mississippi) to a high of 97 lbs. in region 21 (California). The equilibrium solution resulted in eleven surplus and ten deficit regions.

In regard to the classification of the regions, it is rather surprising that California (Region 21) was a surplus producing area for 1955,

16/ See Section B in the chapter relating to the model employed.

Table 4.5 - Regional Equilibrium Prices, Consumption and Surplus and Deficits (1955) Final Solution

Region	Price	Per Capita Consumption	Total Consumption	Surplus and Deficit
	(cents/lb.)	(1,000 lbs.)	(1,000 lbs.)	(1,000 lbs.)
1	69.60	86.7	834,188	-685,478
2	69.43	91.5	1,466,503	-1,082,720
3	69.29	86.1	1,739,937	-908,957
4	69.06	68.0	673,644	-475,553
5	68.21	66.1	424,531	- 82,113
6	68.31	88.3	1,436,127	-369,261
7	67.61	90.3	1,230,411	269,279
8	66.75	80.5	554,705	769,374
9	66.67	76.0	309,046	1,800,015
10	66.87	80.7	505,213	662,317
11	68.79	64.8	588,638	-254,676
12	69.51	75.4	270,035	-103,715
13	68.21	63.3	434,518	-222,725
14	67.14	76.3	835,637	111,960
15	66.16	69.8	92,486	179,211
16	64.46	87.9	377,243	10,294
17	65.30	79.9	99,146	21,207
18	65.91	82.0	152,506	325,456
19	65.11	82.7	85,343	22,715
20	66.71	74.6	134,318	-53,108
21	64.82	96.6	1,252,456	66,479

since they had a higher per capita disposable income and a larger population relative to most other regions. A possible explanation of this is offered by the Western Livestock Marketing Research Technical Committee (26) in that although California is a deficit producing area, live inshipments, from other contigious regions, for slaughter generate an excess supply situation. As might be expected, the heavily surplus producing regions included the corn belt states and states adjacent on the north and south of the corn belt areas. The heavily deficit areas included New England, the Atlantic States and the Southeastern States.

(4) Optimum shipment program for beef, 1955 (Final Solution): The changes in magnitude of the surpluses and deficits determined by the optimum price differentials were insufficient to change the direction of any shipments from the "first approximate" spatial solution. Table 4.6 presents the new solution for flows of beef for 1955.

Since the U_i and V_j for this spatial equilibrium solution are the same as the d_i (price differentials) used to obtain estimates of regio-

Table 4.6 - Final Optimum Flows of Beef in the United States (1955)
(1,000 lbs.)

Origins	Destinations										Totals	U_i
	1	2	3	4	5	6	11	12	13	20		
7	.22	.08	.08	.13	.46	269,279	.50	.67	1.19	3.86	269,279	0.94
8	327,705	341,687	0	.07	.38	99,982	.38	.45	.88	2.92	769,374	0.08
9	.05	741,033	908,957	150,025	.20	.03	.17	.17	.38	2.26	1,800,015	0.00
10	.19	.06	.06	325,528	82,113	.23	254,676	.11	.19	2.30	662,317	0.20
14	1.03	.89	.91	.68	.74	1.36	.33	.13	111,960	2.19	111,960	0.47
15	179,211	.02	.04	.24	.62	.20	.43	.38	.96	2.11	179,211	-0.51
16	10,294	.04	.07	.15	.61	.34	.31	.20	.63	.60	10,294	-2.21
17	21,207	.03	.10	.16	.58	.29	.35	.21	.84	.67	21,207	-1.37
18	147,061	.03	.05	.03	.22	.29	.12	67,629	110,766	.49	325,456	-0.76
19	.20	.23	.25	.35	.58	.47	.46	.10	.60	22,715	22,715	-1.56
21	.58	.62	.65	.52	1.03	.91	.32	36,086	.16	30,393	66,479	-1.85
Totals	685,478	1,082,720	908,957	475,553	82,113	369,261	254,676	103,715	222,725	53,108	4,238,307[1/]	
V_j	2.93	2.76	2.62	2.39	1.54	1.64	2.12	2.84	1.54	.04		

[1/] Totals may not be additive due to rounding. TC = $104,756,372.

nal price, consumption, surpluses and deficits, the sequential analysis is complete. It is possible that a third stage would be necessary if the rim totals had changed so that the new optimum would involve different combinations and directions of shipment. The resulting U_i and V_j of the final optimum solution contain two types of useful economic information. First, the U_i measures the comparative location advantage of the supply points relative to region 9 (Iowa and Nebraska). For example, beef is worth 0.47 cents per lb. more at region 14 (Oklahoma and Texas) than at region 9 because of its proximity to the consuming centers of the Southeast. Alternatively, beef is worth 0.76 cents per lb. less in region 18 (Wyoming and Colorado) than at region 9 because of its distance from the major consuming centers. Second, the values of the V_j give the delivered price differentials relative to region 9 for the deficit regions. For example, the price of beef is 2.93 cents per lb. higher in region 1 (New England) than in region 9. The resulting price differentials obtained are the competitive equilibrium price differentials (or prices) that would result from the eleven surplus regions attempting to sell their excess supplies to the ten deficit regions at the maximum possible gain. Therefore, the optimum values and the direction of flows are simultaneous and interdependent.

Observing that regions 16, 17 and 18 ship to region 1 points up the fact that "closest" markets are not necessarily "optimum" in the general equilibrium sense. Although there are deficit areas closer to these regions than New England, total transport costs would be increased by any such program. Another way of looking at this is that though there are closer markets for regions 16, 17, and 18, the most economically attractive market is region 1.

C. The Effect of Changes in Transport Costs on Regional Equilibrium Price Patterns and Flows of Beef (1955)

In order to assess the elasticity of demand for transportation services to the beef industry and the corresponding equilibrium prices and flows, the conditions (1) a 20 per cent increase and (2) a 20 per cent decrease in transport costs, were postulated for the year 1955. Although for these and all subsequent analyses the sequential process was employed to converge to optimum regional prices and flows for beef, only the final solutions will be presented.

(1) <u>The effect of a 20 per cent increase</u>: Table C.1 of Appendix C contains the regional equilibrium prices, consumption, surpluses and deficits and the optimum flows, assuming all transport costs connecting the surplus and deficit producing regions were increased by 20 per cent over 1955 rates. With the increase of 20 per cent in transportation rates between regions, total transport expenditures increased by 19.08 per cent, while total geographical flows decreased by only 0.49 per cent - thus, indicating a very inelastic demand for beef transport services within the range considered. The classification of regions and the direction of flows remained the same as the initial 1955 analysis, the only change being a slight decrease in amounts exported

and imported. The effect of the increase in transport costs on regional prices was to lower the base price by 0.64 cents while accentuating the price differentials between regions. In general, the effect on the magnitude of consumption was a decrease in the deficit regions and an increase in the surplus regions.

(2) The effect of a 20 per cent decrease in transport costs: Table 2 of Appendix C contains the results of the alternative assumption - a 20 per cent decrease in transport costs between regions for 1955. Under the assumption that transportation costs decrease by 20 per cent, total shipments of beef increase .50 per cent while total transport expenditures for beef decreased by 19.37 per cent. The reason for the highly inelastic demand for transportation service is the relative insignificance of transport costs to the selling price of beef in the range considered. That the intercept for the demand curve for transportation services for beef is far above the origin is reflected in Table 4.1. For example, under the assumption of no interregional trade, the price differential between Region 9 and 1 was estimated at $5.10 per pound. Thus, the transport cost between these two regions could be $5.09 per pound and still not preclude some shipment of beef.

The effect of the decrease of transport costs on price resulted in a higher base price and depressed price differentials. Consumption tended to be lower in the surplus areas and higher in the deficit areas due to the decreased transport costs. As in the case of increased costs, regional consumption was not sufficiently changed to allow any changes in direction of optimum flows from the 1955 program.

D. Equilibrium Price Pattern and Optimum Flows for 1955 Assuming that Regional Farm Supplies of Beef were Slaughtered Locally

To obtain some indication of the impact of changes in the geographical supply of beef on the equilibrium solution and the tendency of slaughter plants to be market rather than production oriented, a price and spatial analysis was carried out for 1955 under the assumption that beef was slaughtered in the producing region. To obtain the data necessary for the analysis, estimates of farm production of beef by regions for 1955 were multiplied by the ratio of total slaughter divided by total farm production.[17] These liveweight estimates were then converted to a carcass weight equivalent basis. The results of the analysis carried out under this assumption are presented in Table 4.7.

Relative to the initial 1955 solution, the alternative regional supply specification yields many interesting changes in the classification of regions, magnitude and sign of price differentials, size of surplus and deficits and the total geographical flows of beef. Under the assumption that processing plants are production oriented, regions 13 (Ark-

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17/ These data were obtained from "Meat Animals, Farm Production and Income, by States," AMS, USDA, Washington, D. C. (February, 1956).

Table 4.7 - Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows (1955) Assuming Regional Production of Beef for Slaughter is Slaughtered Where Produced

Region	Equil. Price cents/pound	Equil. Cons. 1,000 pounds	Surplus and Deficit 1,000 pounds	Origins and Quantities of Shipments (1,000 lbs.) 8	9	10	13	14	15	16	17	18	19	20	V_j
1	69.45	835,815	-754,920	.05	.07	.15	.48	.16	313,615	1.51	104,558	336,747	1.40	.58	2.91
2	69.30	1,468,875	-1,274,702	.03	0	438,609	.31	372,880	463,213	1.53	.01	.01	1.41	.55	2.76
3	69.16	1,742,929	-1,496,053	.03	1,104,958	391,095	.32	.02	.02	1.56	.08	.03	1.43	.58	2.62
4	68.72	677,302	-385,903	.31	.21	.15	.09	385,903	.43	1.85	.35	.22	1.74	.64	2.18
5	67.93	426,496	-21,231	.56	.35	.09	.19	21,231	.75	2.25	.71	.35	1.91	.85	1.39
6	68.20	1,438,193	-939,852	307,401	632,451	.14	.71	.44	.15	1.80	.24	.24	1.62	.79	1.67
7	67.75	1,228,553	-238,082	238,082	.01	.20	.93	.59	.24	1.96	.39	.29	1.81	.95	1.21
8	66.65	555,508	545,482												
9	66.54	309,648	1,737,409												
10	66.68	506,536	829,704												
11	68.08	595,528	-182,473	.99	.75	.52	182,473	.02	.99	2.38	.91	.68	2.22	.92	1.54
12	68.62	273,429	-121,123	1.24	.93	.81	33,620	87,503	1.12	2.45	.95	.74	2.04	.94	2.08
13	66.89	444,142	216,093												
14	66.12	847,528	867,517												
15	66.01	92,709	776,828												
16	66.82	366,626	304												
17	65.15	99,355	496,373												
18	65.76	152,820	336,747												
19	66.16	84,213	100,603												
20	65.74	136,177	151,561												
21	67.63	1,214,252	-644,283	2.97	2.41	2.38	2.81	1.64	1.92	304	391,815	.85	100,603	151,561	1.09
			U_i	.11	0	.14	.35	-.42	-.53	-.72	-1.39	-.78	-.38	-.80	

Total shipments (1,000 lbs.) = 6,058,622; Total costs = $152,348,777.

ansas, Louisiana and Mississippi) and 20 (Arizona and New Mexico) which were deficit areas in the initial analysis, became surplus. Also, regions 17 (Idaho and Montana), 19 (Utah and Nevada), 14 (Oklahoma and Texas) and 15 (North and South Dakota), were considerably more surplus than under the previous formulation. California (Region 21) and Illinois and Indiana (Region 7), initially surplus regions, became deficit under the assumption that beef is slaughtered where produced. This analysis makes explicit the magnitude of the live movement of beef into California for processing. This live movement of beef to California results in an excess supply for this region and, therefore, generates a certain amount of cross-hauling, i.e. shipped live in to California and shipped back out as processed meat. The scale effects of large processing plants or locational advantages in terms of labor, capital and/or natural resources may serve to explain this apparent uneconomic flow pattern. However, that a region such as Arizona and New Mexico is highly surplus in farm production and deficit in slaughtered beef, raises a strong possibility of resource misallocation, and suggests several problem areas for research. This analysis may also be used to suggest how far East that California must now reach to satisfy its demands and thus generates an East-West line of self-sufficiency.

The total costs for this alternative program were 45 per cent higher than for the initial 1955 analysis, i.e. assuming that processing plants were production oriented in 1955, the total cost for shipping processed beef would have been approximately 45 per cent higher than under the locational matrix of processing plants and supplies that did exist. The increase in transport costs results from the fact that processing plants are to some extent consumer oriented and the cost of live movements of beef is not included in the initial analysis. In regard to total product movement, the total geographical beef flows increased by 43 per cent over the initial 1955 program. Comparing the total beef flows for this program (6 billion lbs.) with the total shipped under the first 1955 program (4.2 billion lbs.), indicates an estimated live interregional shipment of approximately 1.8 billion pounds of beef.[18/]

From the standpoint of value, the equilibrium price in the base region (9) under the alternative specifications are approximately the same. However, changes in the classification of regions generates a new set of regional price differentials. In particular, the price differentials in regions 11 and 12 are decreased although their deficit classification does not change. Since regions 13 and 20 change from a deficit to a surplus classification, their relative equilibrium prices are decreased. The largest change in price differential occurs in region 21 (California). In the initial analysis, the price differential for California was -1.85, while in this analysis the price differential becomes + 1.09. This value analysis makes explicit the impact of the location of production and processing firms on the resulting comparative advantage or disadvantage of a particular region.

18/ This is in terms of carcass weight and these estimates are based on the regional demarcation as postulated.

E. Equilibrium Prices and Flows, 1947 and 1952

Having investigated the equilibrium solutions for one period of time, it may now be instructive to consider alternative formulations which involve changes in the geographical distribution and level of income, population and supplies. Therefore, a spatial analysis was carried out for 1947 and 1952 conditions in the hope that any trends in regional beef production, consumption, etc. would become discernable when compared with the 1955 analysis. The year 1947, like 1955, represents a high production period for beef. Alternatively, 1952 represents a period of high prices and relatively low production for the beef industry. The results of the analysis for 1947 are presented in Table 4.8.

In this analysis, the price differentials and production of beef were such that regions 16 and 17 became self-sufficient and thus there were no economic incentive for flows either in or out.[19] Therefore, an equilibrium price for these regions was obtained by equating each region's market demand with its supply. These regions were then dropped from the resulting equilibrium flow solution. In the equilibrium analysis, all other regions were unchanged from the initial 1955 solution in terms of being surplus and deficit. Regional prices for 1947 were somewhat lower than for 1955, due primarily to the considerably lower regional incomes in 1947.

Under the regional breakdown as postulated, 30.8 per cent of total beef production was shipped interregionally for 1947 compared to 31.4 per cent for 1955. The average cost of shipping for 1947 was $2.39 per hundred pounds of beef shipped and for 1955, the average cost of shipping was $2.47 per hundred pounds. This difference in the per unit flow costs between time periods indicates some decentralization of processing plants since 1947 as the average length of haul is somewhat larger for 1955. Shifts in population tend to minimize the impact of decentralization on per unit flow costs, and the impact of changes in liveweight shipments has not been estimated.

The Southern states, regions 11, 12, and 13, had an approximate 44 per cent increase in per capita farm production of beef over the years 1947 to 1955, but their per capita deficit position, in the equilibrium sense, is approximately the same (about 27 lbs. per capita) for processed beef. This indicates that processing is not keeping pace with the increased farm production in these states.

The analysis for 1952 is presented in Table 4.9.

19/ The reader will recall that in the theoretical solution for the two-region case the price differential between the surplus and deficit regions must be greater than the transport cost between the two or there would be no incentive to ship. An analagous case occurred in the 21 region model in that if regions 16 and 17 were treated as surplus, the price differential forced the computed consumption to be less than production and when treated as deficit, the resulting differentials forced computed consumption to be greater than production.

Table 4.8 - Regional Equilibrium Prices, Consumption, Surplus and Deficits and Optimum Flows for Beef (1947)

Region	Equil. Price cents/pound	Equil. Cons. 1,000 pounds	Surplus and Deficit 1,000 pounds	Origins and Quantities of Shipments (1,000 lbs.) 7	8	9	10	14	15	18	19	21	V_j
1	62.01	679,854	-481,646	.14	378,443	.05	.19	1.14	103,205	.11	.21	.69	2.93
2	61.84	1,185,122	-857,557	265,816	0	591,741	.06	1.00	.02	.14	.24	.73	2.76
3	61.70	1,372,875	-679,147	0	210,602	468,545	.06	1.02	.04	.16	.26	.76	2.62
4	61.47	535,757	-405,357	.05	.07	136,930	268,427	.79	.24	.14	.36	.63	2.39
5	60.62	340,184	-138,846	.38	.38	.20	138,846	.85	.62	.33	.59	1.14	1.54
6	60.64	1,048,517	-126,328	126,328	.08	.11	.31	1.55	.28	.48	.56	1.10	1.50
7	59.94	962,972	392,144										
8	59.16	437,464	589,045										
9	59.08	263,264	1,197,216										
10	59.28	402,068	688,076										
11	61.20	458,434	-228,930	.42	.38	.17	228,930	.44	.43	.23	.47	.43	2.12
12	62.03	166,232	-84,862	.48	.34	.06	51,873	.13	.27	9,949	7,597	15,443	2.95
13	60.73	356,252	-192,469	1.00	.77	.27	.08	60,356	.85	132,113	.50	.16	1.65
14	59.66	630,242	60,356										
15	58.57	87,701	103,205										
16	57.33	297,639	-										
17	57.67	81,464	-										
18	58.43	112,479	142,062										
19	57.53	58,705	26,837										
20	59.13	81,832	-19,240	3.77	2.91	2.25	2.29	2.29	2.10	.59	19,240	.10	.05
21	57.34	873,363	15,443										
			U_i	.86	.08	0	.20	.58	-.51	-.65	-1.55	-1.74	

Total shipments (1,000 lbs.) = 3,214,382; Total transport costs = $76,478,131.

Table 4.9 - Regional Equilibrium **Prices**, Consumption, **Surpluses** and **Deficits**, and Optimum Flows for Beef (1952)

Region	Equil. Price	Equil. Cons.	Surplus and Deficit	Origins and Quantities of Shipments (1,000 lbs.)										V_j
	cents/ pound	1,000 pounds	1,000 pounds	7	8	9	10	14	15	17	18	19	21	
1	91.66	600,646	-464,722	.14	273,183	.05	.19	1.03	124,306	1,915	65,318	.04	.42	2.93
2	91.49	1,042,997	-724,710	101,018	0	623,692	.06	.89	.02	.03	.03	.07	.46	2.76
3	91.35	1,259,916	-672,886	0	264,390	408,496	.06	.91	.04	.10	.05	.09	.49	2.62
4	91.12	504,229	-376,471	.05	.07	172,694	203,777	.68	.24	.16	.03	.19	.36	2.39
5	90.27	307,987	-104,073	.38	.38	.20	104,073	.74	.62	.58	.22	.42	.87	1.54
6	90.29	982,143	-166,699	166,699	.08	.11	.31	1.44	.28	.37	.37	.39	.83	1.56
7	89.59	874,590	267,717											
8	88.81	392,386	537,573											
9	88.73	238,505	1,204,882											
10	88.93	362,360	534,359											
11	90.85	427,663	-226,512	.42	.38	.17	226,512	.33	.43	.35	.12	.30	.16	2.12
12	91.41	174,049	-100,949	.75	.61	.33	.27	.29	.54	.37	.16	.10	100,949	2.68
13	90.27	327,344	-236,982	1.11	.88	.38	.19	68,170	.96	.84	150,630	.44	18,182	1.54
14	89.20	606,896	68,170											
15	88.22	66,447	124,306											
16	87.08	-	-											
17	87.36	74,006	1,915											
18	87.97	105,621	215,948											
19	87.01	56,692	10,138											
20	88.61	91,475	-30,232	3.94	3.08	2.42	2.46	2.35	2.27	.83	.65	10,138	20,094	-.12
21	86.72	854,276	139,225											
			U_i	.86	.08	0	.20	.47	-.51	-1.37	-.76	-1.72	-2.01	

Total shipments (1,000 lbs.) = 3,104,236; Total transport costs = $76,722,230.

Region 16 was again dropped from the analysis as a self-sufficient area. In general, 1952 is characterized by relatively high regional prices (from 86.7 to 91.7 cents per lb.) and low output of beef (on the average, only 62 lbs. of beef was available per capita). In the equilibrium solution, the classification of surplus and deficit regions was the same as for 1947 and 1955. Per capita shipment for 1952, under the regional breakdown of the model, was approximately 20 pounds. Per capita shipment for 1955 was 26 pounds. This analysis points up explicity the impact of short supplies of beef on regional prices and suggests the revenue effects of changes in the level of production.

In order to facilitate comparisons between 1947, 1952 and 1955, transport costs were assumed equal for all years. Therefore, changes in the equilibrium values of the prices and flows were assessed independently of changes in interregional transport costs. It is estimated by the United States Department of Agriculture that average transport costs for meat increased by 60 per cent over the period 1947-1956 (25). Therefore, using 1955 transport costs overestimates the actual regional price differentials for the years 1947 and 1952, and this should be kept in mind when drawing inferences for these two observation periods.

F. Equilibrium Prices and Flows, Extrapolated for 1963

Spatial models are also useful tools for estimating the equilibrium conditions that may exist in some future time period. These resulting predictions are, or course, conditioned by the "goodness" of the projected future values of the predetermined variables. In this connection, a spatial analysis was generated for 1963 by extrapolating the basic regional data of production, population and disposable income. Population for 1963 was estimated by multiplying 1955 population by the regional percentage change of 1955 compared to 1947. Slaughter (regional supplies) of beef for 1963 was estimated in the same manner. Disposable income was estimated for 1963 by adding to 1955 income the linear increment of 1955 less 1947 income for each region. Given the 1963 value estimates of these variables, separate analyses were made under the alternative assumptions that (1) population, production and income all increased from 1955 to 1963 as postulated above, (2) population and income increased as described above, but total regional production remain unchanged from 1955 levels, (3) population and income increased in the manner postulated, but per capita regional production is the same as for 1955 and (4) population and production increase as described, but per capita incomes remain the same as for 1955.

(1) Spatial and price analysis, 1963, assuming population, production and incomes all increase: In order to assess the effect of changes in the basic data, assuming that all relevant factors increase at rates similar to the 1947-1955 time period, it was postulated that regional production of beef, regional population and regional dispos-

able incomes all increase from 1955 to 1963 as described above. It was further assumed that 1955 transport costs would prevail for this future period. The solution for regional equilibrium prices of beef, consumption and optimum flows was derived and the results are presented in Table 4.10.

The essential interpretation of all estimates shown in Table 4.10 is that if the basic factors change from 1955 to 1963, as they did from 1947 to 1955, these estimates will result. The projections indicate an average per capita availability of beef of 95 pounds. In this analysis, region 5 (Kentucky and Tennessee) became a surplus producing area. This result obtained because of the relatively large increase in production between 1947 and 1955, i.e., supply of beef increased relatively more than population. Relative to the initial 1955 analysis, the total flows of beef increased by 34 per cent and regional prices remained approximately the same.

(2) Spatial and price analysis, 1963, assuming population and income increase while total production remains fixed at the 1955 level: Recognizing that beef production is subject to periodic fluctuations, this analysis attempts to assess the impact of a low production year if population and income keep climbing at rates equivalent to the 1947-1955 rates of increase. The results of this analysis are presented in Table 3 of Appendix C.

This set of assumptions generated an average per capita availability of beef of 72 pounds. As expected, the effect of constant total supplies and increased population and income is reflected by high regional prices (92.4 to 96.7). California, region 21, appears as a deficit region under these assumptions, due to the relatively high rates of increase in population and income in the period 1947-1955. To satisfy this deficit, California imported the entire surpluses of regions 16 and 19 along with most of the surplus of region 17. This change in classification, of course, generated changes in the regional price differentials and thus the regional comparative advantages and disadvantages.

(3) Spatial and price analysis, 1963, assuming that population and income increase while per capita production remains the same as for 1955: Since 1955 represents a large production period for beef (approximately 81.2 lbs. per capita), it is possible that an increase in population and income may occur in the next few years with little or no increase in the regional per capita supplies of beef. Therefore, an analysis was made to assess the effect of constant regional per capita supplies and is presented in Table 4 of Appendix C.

Again, under this assumption, regional prices are affected upward from the initial equilibrium solution for 1955. Although the surpluses and deficits are different in magnitude, there were no changes in the pattern of shipment from the initial 1955 optimum shipment program.

Table 4.10 - Regional Equilibirum Prices, Consumption, Surpluses and Deficits and Optimum Flows (1963) Assuming Production, Population and Income All Increase

Region	Equil. Price cents/pound	Equil. Cons. 1,000 pounds	Surplus and Deficit 1,000 pounds	Origins and Quantities of Shipments (1,000 lbs.)												V_j
				5	7	8	9	10	14	15	16	17	18	19	21	
1	70.08	1,034,216	-922,643	.71	.22	102,007	.05	.19	1.03	281,073	1,186	49,202	489,175	.69	1.07	2.93
2	69.91	1,892,059	-1,442,410	.47	.08	198,168	1,244,242	.06	.89	.02	.04	.03	.03	.72	1.11	2.76
3	69.77	2,239,240	-1,243,853	.46	.08	0	1,243,853	.06	.91	.04	.07	.10	.05	.74	1.14	2.62
4	69.54	864,455	-563,534	.05	.13	.07	183,354	380,180	.68	.24	.15	.16	.03	.84	1.01	2.39
5	68.38	534,447	47,907													
6	68.79	1,985,512	-751,271	.72	65,629	685,642	.03	.23	1.36	.20	.34	.29	.29	.96	1.40	1.64
7	68.09	1,594,059	65,629													
8	67.23	722,093	985,817													
9	67.15	374,220	2,671,449													
10	67.35	640,957	609,452													
11	69.27	763,139	-277,175	47,907	.50	.38	.17	229,272	.33	.43	.31	.35	.12	.95	.81	2.12
12	69.99	443,976	-104,018	.22	.67	.45	.17	.11	.13	.38	.20	.21	104,018	.59	.49	2.84
13	68.69	537,086	-263,210	.92	1.19	.88	.38	.19	180,141	.96	.63	.84	83,069	1.09	.65	1.54
14	67.62	1,120,095	180,141													
15	66.64	105,606	281,073													
16	64.94	503,956	1,186													
17	65.78	128,810	49,202													
18	66.39	213,191	684,298													
19	66.08	127,564	8,937													
20	67.68	224,992	-119,626	3.74	3.37	2.43	1.77	1.81	1.70	1.62	.11	.18	8,036	8,937	102,653	.53
21	65.79	1,854,568	102,653													
			U_i	1.23	.94	.08	0	.20	.47	-.51	-2.21	-1.37	-.76	-1.07	-1.36	

Total shipments (1,000 lbs.) = 5,687,740; Total transport costs = $143,505,702.

(4) Spatial and price analysis, 1963, assuming that population and production increase while per capita incomes remain at 1955 levels: As an antithesis to the latter two "sellers' market" assumptions, the following analysis was carried out under the assumption that regional incomes, per capita, will remain at the same levels as for 1955 while population and production increase at the rates they did from 1947 to 1955. The results of this analysis are presented in Table 5 of Appendix C.

Assuming no change in per capita incomes between the two periods, the 1963 analysis resulted in generally lower prices than those prevailing in 1955. Region 5 became a surplus region under those assumptions and, therefore, the shipping pattern was rearranged. Obviously, many other admissable alternative formulations could have been specified and analyzed. However, the sole purpose of these projected analyses was not to make pragmatic predictions relative to a future time period, but rather to suggest how spatial price analyses may be employed and the type of possible "if-then" statements that may be made when alternative conditions are specified.

V General Implications of the Postulated Models

The specification and solution of spatial models and their dual system may be used to obtain insights to many theoretical problems involving economic choice. By employing this new type of analysis, the efficiency and competitive structure of individual sectors may be investigated and knowledge may be obtained relative to problems of industrial structure and comparative-statics when the consequences of changes or actions are desired.

From a methodological point of view, analyses of this type yield implications for many areas of economic research. The requirements of specifying spatial market demand relationships generates a need and use for parameter estimates such as price and income elasticity which are the prime objective of sector models. Furthermore, by using these parameter estimates, it is possible to obtain estimates of spatial consumption patterns under alternative price, income and population situations. By using these market demand relationships and employing the linear programming dual, it is possible to obtain regional price differentials that are consistent with a perfectly competitive market formulation. This type of spatial model is valuable because it is operational research-wise and the computations required are manageable. By employing principles which seek to narrow down the number of geographical market and supply points, the spatial formulation alluded to in the preceding sections treats the space factor explicitly and offers an efficient approach to the determination of regional prices and the resulting geographical flows of a commodity. The model has general application and can be applied to any industry or sector which satisfy the underlying assumptions with a reasonable degree of accuracy.

From an economic policy point of view, the interdependent nature or our economy necessitates an analytical model, depicting the joint determination of sector variables, if the consequences and repercussions of certain policy actions are to be isolated. In this connection, spatial models offer an operational tool to the policy maker since they can be used to answer questions of a comparative static nature and thus indicate the changes in the optimum values of geographical prices, flows and differentials brought about by a specified change in the data or action. In particular, the model provides information basic to determining the consequences under given conditions of changes in: (1) transport costs and (2) geographical distribution of population, income and product supply, on the level of geographical prices and flows. The comparative static method of analysis may be best illustrated by an example. How would the optimum solutions for 1955 change if the transport costs between each pair of points were increased by 20 per cent? To answer this question, we could merely change the existing cost structure by 20 per cent, recompute the new optimum solutions and compare these with the initial result (Section B, Chapter III). The number of admissable alternative applications of this type are, or course, very large and depend on the "if-then" questions to be asked. Knowledge generated by application of the model permits a ready application to such practical problems as production - shifting legislative policies or the level and distribution of population and income. By employing this type of analysis, policy makers may ascertain in advance of a specified change both the direction and magnitude of the disturbance on the system.

As a basis for policy action, the perfect market concept used in formulating the spatial equilibrium models provides a standard of comparison whereby the pricing and distribution of a commodity can be judged as efficient or inefficient relative to this base. The impact or value of this normative approach to economic policy rests, of course, on the relationship of the conceptual model to general welfare. The normative model, in this case the competitive model, depicts what could happen under given ends and assumptions and thus may not indicate what will obtain in the real world. However, from an economic policy standpoint, when "what could be" and "what is" differ, the divergence does indicate possible areas for choice or action. For example, what are the consequences of slaughtering beef in the region where produced rather than shipping it live to consumer-oriented processing plants? Whether or not action will be taken will, of course, depend on the social goals to be pursued.

Spatial equilibrium analysis provides information for decision making by both producing and processing firms. In regard to the beef producing firm, these analyses should suggest how changes in transportation, and the geographical distribution and level, population, income and supply might alter beef prices. These expected prices could then be used as a basis for resource adjustments. By introducing the time dimension, insights into the changing character of the beef industry may be discerned and the long-run competitive position of one region relative to another can be analyzed.

From the standpoint of the processing and distributing firm, the information provided by the analyses should make possible a rational choice among alternative geographical destinations of product shipments. By employing the duality theorem of linear programming, a unique set of regional price differentials can be derived which correspond to the optimum set of product flows. These results may then be used by the processing firm in ascertaining the comparative or location advantage of one region relative to others. Through this analysis, some insights into the optimum geographical location of beef processing firms may be obtained which are a function of both location and the scale of the firm.

The results of the spatial analysis have suggested several possibilities for future research in the beef sector of the economy. The more important research possibilities implied are: (1) consideration of a less aggregative model so that market and supply points will be more descriptive of a region, (2) recognition of seasonal variation in beef supplies and thus possibly considering equilibrium models on a quarterly rather than an annual basis, (3) consideration of the quality of beef produced and demanded in each region and (4) an activity analysis model of the livestock-feed economy. These and other restrictive assumptions underlying the analysis such as uncertain demand and product carryover will be explored in future research.

VI Summary

Recent developments in the field of linear programming have made possible a new type of economic analysis and have made feasible an operational model of general equilibrium analysis under which the space factor may be treated explicitly. The general method of linear programming has in this study been applied to the problem of developing a spatial price equilibrium model for beef. Specifically, the study has attempted to derive the geographical equilibrium prices and flows of beef under alternative sets of conditions for the controlled variables. Alternative models were employed to ascertain the economic consequences of specified disturbances in the initial set of data. Finally, the implications of the results for decision making and action by the government and firm were reviewed and suggestions for possible future research were enumerated.

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Appendix A

An Example of the Simplex Technique in Solving A Transportation Problem

Most material published on the simplex procedure for solving the transportation problem required some knowledge of vector and matrix theory to appreciate its logic. This appendix represents an attempt to give the reader an understanding of the logic of the technique even though he has little mathematical training. Most of the material presented here parallels that of Dantzig (13, pp. 359-73). The authors make no claim for originality but only hope that the number of research people who may use linear programming with some understanding may be increased.

As a means of presenting the simplex procedure, an example involving three origins (supplies) and five destinations (demands) is used. Generalization to any sized matrix is immediate. The program requirements and transport costs for the hypothetical problem are assumed to be as in Table A. 1.

Table A. 1 - Program Requirements and Transport Costs

		Destinations					Totals
	i \ j	(1)	(2)	(3)	(4)	(5)	a_i
Origins	(1)	1	3	4	6	3	50
	(2)	0	5	6	9	2	80
	(3)	8	5	3	2	9	120
Totals	b_j	90	25	35	40	60	250

In words, the destinations require 90, 25, 35, 40 and 60 units of a homogeneous product (b_j) while the origins have available for shipment 50, 80 and 120 units of the product (a_i). The assumed transport costs appear in the cells of the body of Table A. 1. For example, the cost is 4 for transporting a unit of the product from origin 1 to destination 3; shipping from origin 3 to destination 5 entails a cost of 9, etc. There is no restriction on the magnitude or sign of the costs, therefore a constant may be subtracted from all original costs. That this can be done is emphasized by the zero cost between origin 2 and destination 1.

The table provides the following set of equations:

$$(1) \qquad x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = a_1 = 50$$

$$(2) \qquad x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = a_2 = 80$$

(3) $x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = a_3 = 120$

(4) $x_{11} + x_{21} + x_{31} = b_1 = 90$

(5) $x_{12} + x_{22} + x_{32} = b_2 = 25$

(6) $x_{13} + x_{23} + x_{33} = b_3 = 35$

(7) $x_{14} + x_{24} + x_{34} = b_4 = 40$

(8) $x_{15} + x_{25} + x_{35} = b_5 = 60$

In total there are 8 equations in 15 unknowns, but since:

(9) $$\sum_{j}^{5} \sum_{i}^{3} x_{ij} = 250 = \sum_{i=1}^{3} a_i = \sum_{j=1}^{5} b_j$$

there are only 7 independent equations. Therefore, many independent solutions are feasible for the system as given. Any feasible solution is referred to as a basis. At least one solution will minimize total costs. i.e.,

(10) $$\sum_{j=1}^{5} \sum_{i=1} C_{ij}x_{ij} = \text{minimum.}$$

This optimum or optimum basic solution is, then, to be sought, subject to the restriction that all x_{ij} must be positive or zero.

First Feasible Solution and Check for Optimality

Keeping in mind that only 7 of the x_{ij} can be non-zero, a trial solution may be obtained by starting with x_{11} and continuing across and down the tableau until all origins are exhausted and all destinations are satisfied. Thus, a first feasible basis could be $x_{11} = 50$, $x_{21} =$

40, $x_{22} = 25$, $x_{23} = 15$, $x_{33} = 20$, $x_{34} = 40$ and $x_{35} = 60$.

The total cost for this solution is:

$$C_{11}x_{11} + C_{21}x_{21} + C_{22}x_{22} + C_{23}x_{23} + C_{33}x_{33} + C_{34}x_{34} + C_{35}x_{35} = 945.$$

Given a feasible basis, the problem is one of determining whether costs can be reduced further. The fundamental approach of the simplex technique is to express all activities in the system in terms of the basic set that constitute a feasible solution. An activity of shipping a unit of product from i to j can be denoted as A_{ij}, where x_{ij} represents the level of A_{ij}. To sustain a unit level of any activity, one unit of product at the i^{th} origin is required as input and one unit is made available at the j^{th} destination. Following this formulation, any cost C_{ij} in the basic solution may be expressed as a sum of an input cost, U_i', and an output cost, V_j. For the first feasible basis, this formulation yields the following set of equations:

(11) $U_1' + V_1 = C_{11} = 1$

(12) $U_2' + V_1 = C_{21} = 0$

(13) $U_2' + V_2 = C_{22} = 5$

(14) $U_2' + V_3 = C_{23} = 6$

(15) $U_3' + V_3 = C_{33} = 3$

(16) $U_3' + V_4 = C_{34} = 2$

(17) $U_3' + V_5 = C_{35} = 9$

By assigning one of the U_i' of V_j some arbitrary value, each of the remaining ones are uniquely determined.

Table A. 2 presents the first feasible basis, the direct costs and the components of the indirect costs, the U_i' and V_j. Using equations (11) through (17) and assigning U_1 a value of zero permitted determining the U_i' and V_j as given in Table A. 2.

Table A. 2 - First Feasible Basis

	i \ j	(1)	(2)	(3)	(4)	(5)	Totals	U_i'
Origins	(1)	1 / 50					50	0
	(2)	0 / 40	5 / 25	6 / 15			80	-1
	(3)			3 / 20	2 / 40	9 / 60	120	-4
Totals		90	25	35	40	60	250	
V_j		1	6	7	6	13		

The determination of whether the solution of table A. 2 is optimum stems from the idea that each cost for those activities not in the basic solution can be expressed as either the direct (transport) cost or as indirect costs - i. e., $\bar{C}_{ij} = U_i' + V_j$, where $\bar{C}_{ij}$ is the cost of not having some activity A_{ij} in the basic solution. If

(18) $C_{ij} < \bar{C}_{ij}$ or $C_{ij} < U_i' + V_j$,

then it costs more not having A_{ij} in the basic solution than the direct or transportation cost would be if it were in the solution. Therefore, the difference $C_{ij} - \bar{C}_{ij}$ is examined for all activities not in the basic solution. If the differences are all positive, the optimum program has been reached - if some are negative, the program must be re-arranged.

Using the costs versus indirect cost criterion, it can be seen from Table A. 3 that activities A_{12}, A_{13}, A_{15}, and A_{25} are admissable candidates for entering a new solution. Since A_{15} and A_{25} are causing the largest unfavorable costs by not being in the basis, it seems logical to bring in either of these activities in a second feasible basis. (The iterative process allows only one change per program - after bringing in one new activity and dropping an old one, the program must be re-evaluated.)

Table A. 3 - $C_{ij} - \overline{C}_{ij}$ First Feasible Basis

		Destinations				
	i \ j	(1)	(2)	(3)	(4)	(5)
Origins	(1)	0	-3	-3	0	-10
	(2)	0	0	0	4	-10
	(3)	11	3	0	0	0

Program Adjustment

It has been established that costs can be reduced by bringing in the activity A_{15} at some level. The problem is now one of determining at what level this activity can be introduced.

In Table A. 4, the activity A_{15} has been introduced at a level α. In order that the rim requirements are not violated, this requires subtracting α from x_{11} which requires α being added to x_{21}, etc. until all rows and columns still sum to their respective totals.

Table A. 4 - Program Readjustment

		Destinations					Totals
	i \ j	(1)	(2)	(3)	(4)	(5)	
Origins	(1)	$50 - \alpha$				α	50
	(2)	$40 + \alpha$	25	$15 - \alpha$			80
	(3)			$20 + \alpha$	40	$60 - \alpha$	120
Totals		90	25	35	40	60	250

Since there can be no more than 7 activities $(n + m - 1)$ and all x_{ij} must be positive or zero, the obvious choice for α is 15. If α were larger than 15, x_{23} would enter as negative and α were smaller than 15, there would be 8 activities in the second basis. As a generalization, the equation $x_{ij} - \alpha = 0$ can be considered and α chosen as the smallest positive solution.

This completes the full cycle of the iterative process. Table A.5 contains the second feasible basis which results from choosing $\alpha = 15$.

To converge to a minimum cost program, the steps just presented are repeated until $C_{ij} - \overline{C}_{ij}$ is positive or zero for all i, j. Total costs are

$$z_0 + \alpha(C_{15} - \overline{C}_{15}) \text{ or } 945 - 15(10) = 795.$$

Table A.5 - Second Feasible Basis

		Destinations					Totals
	i \ j	(1)	(2)	(3)	(4)	(5)	
Origins	(1)	35				15	50
	(2)	55	25				80
	(3)			35	40	45	120
Totals		90	25	35	40	60	250

Logic of Iterative Technique

The mathematical logic enabling one to express the "so-called" indirect costs is that given a linearly independent set of vectors that span a vector space (a basis) all other vectors in the space can be expressed as a linear combination of the basic set. Thus, the "cost" of some activity not in a basic solution can be expressed as a linear combination of the costs that appear in the basic solution. An intuitive grasp of the indirect cost formulation can be gained, however, by tracing through a unit program change. First, let A_{15} (Table A.4) be sustained at a unit level. In order that the row and column sums remain invariant, x_{11} must be decreased by 1, x_{21} must be increased by 1, x_{23} is decreased by 1, x_{33} is increased by 1 and x_{35} is decreased by 1. Each of the activities in the basic solution have a direct cost, therefore everywhere that a unit of product has been subtracted costs have been decreased by the associated unit costs and where the unit of product has been added, total costs have been increased by the unit costs. Letting a minus represent cost reduction and a plus represent addition to costs and summing gives the result:

$$-C_{11} + C_{21} - C_{23} + C_{33} - C_{35} = -1 + 0 - 6 + 3 - 9 = -13$$

Bringing in a unit level of activity in A_{15} only entails a direct cost of 3, therefore, total costs can be reduced by 10 units by the program readjustment considered. This is the same inference that obtains by considering $C_{15} - \overline{C}_{15}$ in the iterative procedure where,

$\bar{C}_{15} = U_1' + V_5.$

Since in the iterative process, A_{15} is brought in at $\alpha = 15$, total costs are reduced by 150. This corresponds to the formula given for determining total costs for the second feasible basis.

By introducing slack or storage vectors, the simplex procedure can be used to solve a transportation problem where supplies are assumed greater than demands. Alternatively, the problem where demands are greater than supplies can be solved using this same approach. So-called slack vectors are merely unit vectors that transform inequalities into equalities. Degeneracy is ignored here, but can easily be resolved if it appears at any stage in the iterative process. (See 13, pp. 365-67).

Appendix B

The "Vogel Approximation" Method for Determining A First Feasible Basis [1]

Artificial and poorly motivated methods of obtaining an initial feasible solution in the Simplex Procedure have been subjected to presistent objections. In this connection, the "Vogel Approximation" method provides a means of establishing initial solutions to transportation problems that have been found to reduce the requisite number of Simplex iterations. In fact, for small matrices, this method often provides the optimum program without resorting to any other type of iterative process.

In order to present the Vogel method, the hypothetical problem posed in Appendix A is again used.

(See following page for Table B. 1)

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[1] Presented at a Quality Control Industrial Engineering Conference held at Milwaukee, Wisconsin, in 1954.

Table B.1 - Vogel's Approximation Method for Establishing a Feasible Basis[1/]

Origins	i \ j	Destinations (1)	(2)	(3)	(4)	(5)	Totals	Row Cost Differences (1)	(2)	(3)	(4)	(5)	(6)	(7)
	(1)	1 10	3	4	6	3 40	50	2	2	2	0	x		
	(2)	0 80	5	6	9	2	80	2	2	x				
	(3)	8	5 25	3 35	2 40	9 20	120	1	2	2	2	2	2	2
Totals		90	25	35	40	60	250							

Cost Differences Column		(1)	(2)	(3)	(4)	(5)
	(1)	1	2	1	4	1
	(2)	1	2	1	x	1
	(3)	7	2	1		6
	(4)	x	2	1		6
	(5)		5	3		9
	(6)		5	3		x
	(7)		x	3		
	(8)			x		

Steps:

1) Construct the tableau with costs in the upper right hand boxes.
2) Enter the difference of the two lowest row costs for each row in column (1) of the row cost difference table. Do the same for column costs.
3) Choose the maximum value of row and column (1) of the cost difference tables. Circle this value. (4 in row 1 of column cost differences.)
4) Examine the row or column in the tableau that corresponds to the circled value and make the maximum possible assignment to the minimum cost appearing in that row of columns. In the example, this satisfies all demands for destination (4); therefore, it is eliminated from further consideration. Thus, an x is entered in the corresponding cost difference column.
5) Repeat the process except that the costs in column (4) of the tableau are not considered in finding the new row cost differences.

[1/] This method was called to the attention of the authors by Mr. Robert Ashworth of the Industrial Engineering Department, Oklahoma State University. The method was presented before an Industrial Engineering Quality Control Conference held at Milwaukee, Wisconsin, 1954. For this particular example, the Vogel method provides an optimum tableau. (The reader may satisfy himself of this by using the simplex procedure of Appendix A to find $C_{ij} - \overline{C}_{ij}$.)

Appendix C

Tables Relative to Equilibrium Solutions for Alternative Levels of Transport Costs and Projected 1963 Conditions

Table C.1 - Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows Assuming a 20% Increase in Transport Costs (1955)

Region	Equil. Price cents/ pound	Equil. Cons. 1,000 pounds	Surplus and Deficit 1,000 pounds	Origins and Quantities of Shipments (1,000 lbs.) 7	8	9	10	14	15	16	17	18	19	21	V_j
1	69.54	831,360	-682,650	.26	339,715	.07	.23	1.24	178,718	7,161	20,523	136,533	.24	.69	3.51
2	69.34	1,461,328	-1,077,545	.09	328,393	749,152	.07	1.07	.02	.04	.03	.03	.27	.74	3.31
3	69.17	1,734,047	-903,067	.09	0	903,067	.08	1.09	.04	.08	.12	.06	.30	.77	3.14
4	68.90	671,175	-473.084	.15	.08	146,755	326,329	.81	.28	.17	.18	.03	.41	.61	2.87
5	67.88	424,079	-81,661	.54	.45	.24	81,661	.88	.73	.72	.69	.25	.69	1.22	1.85
6	67.99	1,434,812	-367,946	268,372	99,574	.04	.28	1.64	.24	.41	.35	.35	.56	1.09	1.96
7	67.15	1,231,318	268,372												
8	66.12	556,397	767,682												
9	66.03	310,087	1,798,974												
10	66.27	506,553	660,977												
11	68.57	586,949	-252,987	.60	.45	.21	252,987	.40	.51	.37	.42	.14	.55	.38	2.54
12	69.43	268,841	-102,521	.80	.54	.21	.14	.16	.46	.24	.25	76,011	.12	26,510	3.40
13	67.87	434,108	-222,315	1.43	1.06	.46	.24	110,192	1.15	.76	1.01	112,123	.72	.19	1.84
14	66.59	837,405	110,192												
15	65.41	92,979	178,718												
16	63.37	380,376	7,161												
17	64.38	99,830	20,523												
18	65.11	153,295	324,667												
19	64.15	85,956	22,102												
20	66.07	134,778	-53,568	4.63	3.51	2.72	2.77	2.63	2.53	.72	.81	.59	22,102	31,464	.04
21	63.80	1,260,961	57,974												
			U_i	1.12	.09	0	.24	.56	-.62	-2.66	-1.65	-.92	-1.88	-2.23	

Total shipments (1,000 lbs.) = 4,217,345; total transport costs = $124,739,983.

Table C.2 - Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows Assuming a 20% Decrease in Transport Costs (1955)

Region	Equil. Price	Equil. Cons.	Surplus and Deficit	Origins and Quantities of Shipments (1,000 lbs.)											v_j
	cents/ pound	1,000 pounds	1,000 pounds	7	8	9	10	14	15	16	17	18	19	21	
1	68.87	837,565	-688,855	.18	316,076	.03	.15	.82	179,709	13,442	21,896	157,731	.16	.47	2.35
2	68.73	1,471,622	-1,087,839	.07	354,733	733,106	.05	.71	.02	.04	.03	.03	.19	.50	2.21
3	68.62	1,745,756	-914,776	.07	0	914,776	.04	.73	.04	.06	.08	.04	.20	.53	2.10
4	68.43	676,080	-477,989	.11	.06	153,188	324,801	.55	.20	.13	.14	.03	.29	.43	1.91
5	67.75	424,960	-82,542	.38	.31	.16	82,542	.60	.51	.50	.47	.19	.47	.84	1.23
6	67.84	1,437,384	-370,518	270,236	100,282	.02	.18	1.08	.16	.27	.23	.23	.38	.73	1.32
7	67.28	1,229,454	270,236												
8	66.59	552,988	771,091												
9	66.52	307,991	1,801,070												
10	66.68	503,852	663,678												
11	68.22	590,297	-256,335	.40	.31	.13	256,335	.26	.35	.25	.28	.10	.37	.26	1.70
12	68.80	271,216	-104,896	.54	.36	.13	.08	.10	.30	.16	.17	59,176	.08	45,720	2.28
13	67.76	434,905	-223,112	.95	.70	.30	.14	113,767	.77	.50	.67	109,345	.48	.13	1.24
14	66.90	833,830	113,767												
15	66.12	91,988	179,709												
16	64.76	374,095	13,442												
17	65.43	98,457	21,896												
18	65.92	151,710	326,252												
19	65.28	84,728	23,330												
20	66.56	133,850	-52,640	3.09	2.33	1.80	1.83	1.75	1.69	.48	.53	.39	23,330	29,310	.04
21	65.05	1,243,905	75,030												
			U_i	.76	.07	0	.16	.38	-.40	-1.76	-1.09	-.60	-1.24	-1.47	

Total shipments (1,000 lbs.) = 4,259,501; Total costs = $84,468,340.

Table C.3 - Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows, 1963, Assuming Total Production Remains at the 1955 Level while Population and Income Increase

Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows, 1963, Assuming Total Production Remains at the 1955 Level while Population and Income Increase

Region	Equil. Price	Equil. Cons.	Surplus and Deficit	Origins and Quantities of Shipments (1,000 lbs.)										V_j
	cents/ pound	1,000 pounds	1,000 pounds	7	8	9	10	14	15	16	17	18	19	
1	96.69	785,954	-637,244	.22	435,401	.05	.19	1.14	192,474	1.51	9,369	.11	1.40	2.93
2	96.52	1,436,866	-1,053,083	.08	206,061	847,022	.06	1.00	.02	1.55	.03	.14	1.43	2.76
3	96.38	1,700,101	-869,121	.08	0	869,121	.06	1.02	.04	1.58	.10	.16	1.45	2.62
4	96.15	657,173	-459,082	.13	.07	111,382	347,700	.79	.24	1.66	.16	.14	1.55	2.39
5	95.30	402,772	-60,354	.46	.38	.20	60,354	.85	.62	2.12	.58	.33	1.76	1.54
6	95.40	1,502,322	-435,456	296,326	139,129	.03	.23	1.47	.20	1.85	.29	.40	1.67	1.64
7	94.70	1,203,364	296,326											
8	93.84	543,488	780,591											
9	93.76	281,536	1,827,525											
10	93.96	482,711	684,819											
11	95.88	579,419	-245,457	.50	.38	.17	245,457	.44	.43	1.82	.35	.23	1.66	2.12
12	96.71	337,333	-171,013	.56	.34	.06	31,308	.13	.27	1.60	.10	139,705	1.19	2.95
13	95.41	406,268	-194,475	1.08	.77	.27	.08	104,242	.85	2.03	.73	90,233	1.69	1.65
14	94.34	843,355	104,242											
15	93.25	79,223	192,474											
16	93.06	371,313	16,224											
17	92.39	96,402	23,951											
18	93.11	159,798	318,164											
19	93.40	94,992	13,066											
20	94.40	169,436	-88,226	3.26	2.32	1.66	1.70	1.70	1.51	1.51	.07	88,226	.60	.64
21	94.87	1,362,807	-43,872	3.95	2.92	2.39	2.42	2.62	1.92	16,224	14,582	.96	13,066	1.11
			U_i	.94	.08	0	.20	.58	-.51	-.70	-1.37	-.65	-.36	

Total shipments (1,000 lbs.) = 4,257,383; Total transport costs = $102,213,557.

Table C.4 - Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows, 1963, Assuming Per Capita Supplies Remain Fixed at 1955 Levels

Region	Equil. Price	Equil. Cons.	Surplus and Deficit	Origins and Quantities of Shipments (1,000 lbs.)											v_j
	cents/ pound	1,000 pounds	1,000 pounds	7	8	9	10	14	15	16	17	18	19	21	
1	88.24	899,317	-741,414	.22	260,705	.05	.19	1.03	215,009	30,911	31,266	203,523	.20	.56	2.93
2	88.07	1,644,452	-1,205,139	.08	470,978	734,161	.06	.89	.02	.04	.03	.03	.23	.62	2.76
3	87.93	1,945,747	-1,026,117	.08	0	1,026,117	.06	.91	.04	.07	.10	.05	.25	.65	2.62
4	87.70	751,355	-531,463	.13	.07	187,915	343,548	.68	.24	.15	.16	.03	.35	.52	2.39
5	86.85	461,044	-92,561	.46	.38	.20	92,561	.74	.62	.61	.58	.22	.58	1.03	1.54
6	86.95	1,720,963	-461,212	306,823	154,389	.03	.23	1.36	.20	.34	.29	.29	.47	.91	1.64
7	86.25	1,379,313	306,823												
8	85.39	623,296	886,072												
9	85.31	322,871	1,948,193												
10	85.51	553,544	729,595												
11	87.43	662,793	-293,486	.50	.38	.17	293,486	.33	.43	.31	.35	.12	.46	.32	2.12
12	88.15	386,223	-151,702	.67	.45	.17	.11	.13	.38	.20	.21	89,187	.10	62,515	2.84
13	86.85	465,582	-242,544	1.19	.88	.38	.19	123,316	.96	.63	.84	119,228	.60	.16	1.54
14	85.78	967,819	123,316												
15	84.80	90,823	215,009												
16	83.10	432,520	30,911												
17	83.94	110,709	31,266												
18	84.55	183,598	411,938												
19	83.75	110,323	31,739												
20	85.35	195,481	-77,013	3.86	2.92	2.26	2.30	2.19	2.11	.60	.67	.49	31,739	45,274	.04
21	82.47	1,616,828	107,789												
			U_i	.94	.08	0	.20	.47	-.51	-2.21	-1.37	-.76	-1.56	-1.85	

Total shipments (1,000 lbs.) = 4,822,651; Total transport costs = $120,053,244.

Table C.5 - Regional Equilibrium Prices, Consumption, Surpluses and Deficits and Optimum Flows, 1963, Assuming Per Capita Incomes Remain at the 1955 Level

Region	Equil. Price	Equil. Cons.	Surplus and Deficit	Origins and Quantities of Shipments (1,000 lbs.)											V_j
	cents/ pound	1,000 pounds	1,000 pounds	5	7	8	9	10	14	15	17	18	19	21	
1	60.37	1,017,932	-906,359	.71	.22	123,498	.05	.19	1.03	266,095	42,177	474,589	.69	1.07	2.93
2	60.20	1,930,301	-1,480,652	.47	.08	198,936	1,281,716	.06	.89	.02	.03	.03	.72	1.11	2.76
3	60.06	2,214,154	-1,218,767	.46	.08	0	1,218,767	.06	.91	.04	.10	.05	.74	1.14	2.62
4	59.83	859,858	-558,937	.05	.13	.07	160,459	398,478	.68	.24	.16	.03	.84	1.01	2.39
5	58.67	529,591	52,763												
6	59.08	1,953,400	-719,159	.72	64,141	655,018	.03	.23	1.36	.20	.29	.29	.96	1.40	1.64
7	58.38	1,595,547	64,141												
8	57.52	730,458	977,452												
9	57.44	384,727	2,660,942												
10	57.64	641,526	608,883												
11	59.56	749,132	-263,168	52,763	.50	.38	.17	210,405	.33	.43	.35	.12	.95	.81	2.12
12	60.28	437,537	-97,579	.22	.67	.45	.17	.11	.13	.38	.21	97,579	.59	.49	2.84
13	58.98	527,422	-253,546	.92	1.19	.88	.38	.19	188,978	.96	.84	64,568	1.09	.65	1.54
14	57.91	1,111,256	188,980												
15	56.93	120,584	266,095												
16	57.74	505,402	- 260	5.29	4.53	3.16	3.03	3.49	4.09	2.03	260	1.73	.59	.15	.30
17	56.07	135,575	42,437												
18	56.68	219,926	677,563												
19	56.37	129,165	7,336												
20	57.97	224,749	-119,383	3.74	3.37	2.43	1.77	1.81	1.70	1.62	.18	40,827	7,336	71,220	.53
21	56.08	1,886,001	71,220												
			U_i	1.23	.94	.08	0	.20	.47	-.51	-1.37	-.76	-1.07	-1.36	

Total shipments (1,000 **lbs.**) = 5,617,810; **Total transport costs** = **$141,390,537.**