A Stochastic Daily Weather Simulation Model for El Reno, Oklahoma



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#### <u>ABSTRACT</u>

A stochastic daily weather simulation model was implemented following the guidelines from Larsen and Pense 1981 and 1982 for agronomic models. Twenty years of daily weather were used to estimate precipitation and temperature parameters. A first order Markovian relationship was used to determine the sequence of dry-dry, dry-wet, wet-dry and wet-wet days. Amounts of daily precipitation were simulated with two parameter gamma distributions for each month conditioned to previous day rainfall status. Daily maximum and minimum temperatures conditioned to wet or dry day were simulated with a nonlinear model coupled with bi-variate normal distributions of daily temperatures for each month. Nine years were used to estimate solar radiation parameters. Solar radiation was simulated with gamma distributions for dry days while beta distributions were used for wet days. Variability in daily precipitation, measured with the coefficient of variability, is around 100% with no seasonal pattern throughout the year. In contrast, the variability of maximum and minimum daily temperatures is higher in the winter (100% or more) than in the summer (35% or less). Similar to the variability in temperatures, coefficients of variability of solar radiation are larger in winter (up to 87%) than summer (up to 36%). These results suggest that in general, weather in El Reno is more variable in winter than summer.

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#### INTRODUCTION

The weather simulation model presented here was implemented as part of a project to ascertain the production and marketing risks in wheat grazing systems. This model needed to be sensitive to daily rainfall, temperatures and solar radiation and it is used by a wheat grazing system model under rainfed conditions (Rodríguez et al. 1988). Further, the weather and wheat grazing systems models will be linked to a price generator and management models to evaluate the effects of weather and price risk in wheat grain and beef production decision making.

The level of accuracy required of different weather models depends on the objectives of the risk analysis. For example, if we are interested in determining the probabilities of drought in a region comprising several states in a period of one hundred years, or if we are interested in determining the probabilities of having a sequence of dry weeks during a specific month, or the probability of having rainfall events at dawn in a given month, etc., the accuracy required for a weather simulator may be different. Rainfed wheat grazing systems are highly variable with regard to weather variations and resultant forage response (Christiansen et al. 1988). Most of the wheat process models, including a modified CERES-wheat used by Rodríguez et al. (1988), operate in a daily time step, thus, a day to day weather model seems adequate.

Several risk analysis studies that use bioeconomic models rely on historical weather data and often fail to generate more than one or two dozen "states" of nature (see Harris and Mapp 1986; Boggess et al. 1983; Parsch and Loewer 1987, Hamilton 1986, among others). One advantage of using simulation models is that a large number of years can be simulated and the simulated outcomes reflect the possible events that characterize the weather conditions of one site (see Rodríguez and Bartlett, 1988 for an application of stochastic rainfall to cattle growth variability). The climatological characterization of one site is relatively complex and laborious because of the large number of statistical parameter estimations and their tests for validity. However, daily weather characterization permits a better understanding of the changing and not necessarily normally distributed inputs of agricultural systems. Non-normality of agricultural crop yield variability has been discussed by Day (1965), Anderson (1974), Antle (1983), and Taylor (1984), among others; and it is the result of the interaction between uncontrollable inputs (i.e., weather) and sequential decision making processes within growing seasons. Thus, better management practices can be expected when intraseasonal variability in inputs and outputs are considered. Intraseasonal expectations can be known by producers and provide orientation to assess production risks.

#### MODEL DEVELOPMENT

Two data sets were used to implement the weather model. One from the Oklahoma Climatological survey (see McDonald et al. 1983) and the other one from the United States Department of Agriculture - Agricultural Research Service (USDA - ARS). The first data set consisted of daily precipitation, and daily maximum and minimum temperature from 1966 to 1985 at El Reno, OK. These weather variables were used to estimate a) monthly sets of two parameter gamma distributions conditioned to previous day precipitation and b)

monthly bi-variate normal distributions for maximum and minimum temperatures conditioned to current day precipitation.

The second data set consisted of daily solar radiation and precipitation from 1978 to 1986 at the Forage and Livestock Research Laboratory near El Reno, OK. These data was used to estimate gamma and beta distributions of solar radiation conditioned to dry and wet days, respectively.

#### Precipitation

A first order Markov chain (Bond 1979, Larsen and Pense 1982) was used to determine the probability of a wet or dry day depending upon the state of the previous day (wet or dry). A wet day was defined as any day with more than or equal to .24 mm of precipitation. This limit was set to avoid rainfall events between 0 and the lower boundary of climatological data (a hundred of an inch) in the data set<sup>1</sup>. Complementarily, a dry day was defined as a day with 0 to less than .24 mm of precipitation. The probability of a wet day i given that the previous day i-1 was wet (Pi (W|W)) is expressed as:

(1) 
$$P_i(W|W) = 1 - P_i(D|W)$$

where  $P_i$  (D|W) is the probability of day i being dry given that the day i-1 was wet. Similarly, the probability of a wet day i given that the previous day i-1 was dry (Pi (W|D)) is:

(2) 
$$P_i(W|D) = 1 - P_i(D|D)$$

where P<sub>i</sub> (D|D) is the probability of a dry day i given that the previous day was also dry. The modeled sequence of dry and wet days is fully described by the probabilities P<sub>i</sub> (W|W), P<sub>i</sub> (W|D) and the presence of a wet or dry day in the previous day (Richardson 1981). For every month two gamma distributions (k=1 or 2) with two parameters were estimated to assign rainfall intensities on wet days. The two parameter gamma distribution  $f_k(X)$  is defined as:

(3) 
$$f_{k}(X) = \frac{1}{B(k)^{G(k)} \lceil G(k)} X^{G(k)-1} e^{-X/B(k)};$$
   
  $B > 0$   
 $G > 0$   
 $k=1,2$ 

where X is the random variable, B(k) is the scale parameter on X, G(k) is the shape parameter,  $\lceil$  is the usual gamma function<sup>2</sup>, and f<sub>k</sub>(X)=0 for X<0. The index k=1 implies that a wet day was observed the previous day while k=2

<sup>&</sup>lt;sup>1</sup> For those cases where a precipitation "trace" is provided in the weather data set, see Larsen and Pense (1982) for the appropriate treatment within a simulation procedure.

 $<sup>{}^{2}\</sup>Gamma(n) = \int_{0}^{\alpha} u^{(n-1)} e^{-u} du$  for any n>0, e.g.,  $\Gamma(1/2) = \sqrt{\pi}$ . For any positive integer r,  $\Gamma r = (r-1)!$ , e.g.,  $\Gamma(1) = 1$  (see Larson 1982, p. 199).

implies that a dry day occurred the previous day. According to Thom (1958) this is a positively skewed distribution depending inversely on the shape factor G(k).

#### <u>Temperature</u>

Daily maximum or minimum temperatures (T) were used to estimate the parameters of the following equation:

(4)  $T = \alpha SIN((Julian day-\beta)2\pi/365)+\delta$ 

where  $\beta$  determines the placement of the sine wave with respect to Julian Day,  $\alpha$  determines the amplitude of the sine wave and  $\delta$  represents the yearly average maximum and minimum. As suggested by Larsen and Pense (1982), non-linear least squares (NLLS) with the Marquardt method (SAS 1985) were used to estimate  $\alpha$ ,  $\beta$  and  $\delta$  for maximum and minimum temperatures. Differences between a) actual minimum or maximum daily temperature conditioned to rainfall status and b) the corresponding non-linear predictions were calculated. Similar calculations were done for actual previous day maximum temperature and current day maximum temperature predicted by the non-linear model. Thus, a set of three monthly means, variances and correlation coefficients were estimated for dry or wet days.

"Daily temperatures were generated using one bivariate normal to simulate either current maximum or current minimum temperature from previous day maximum temperature. Which current temperature was simulated depended on the higher of the two correlations (i.e. the serial correlation between previous and current maximum temperature and the lag cross-correlation between previous maximum and current minimum temperature). The second bivariate normal was used to simulate the remaining current temperature generated by the first bivariate normal. This procedure takes advantage of the highest correlations." (Larsen and Pense 1982, p. 511)

The general formula of the bivariate normal distribution to generate maximum or minimum temperature differences in a given month is:

(5) 
$$y_2 = \hat{y}_{2j} + \hat{\rho}_{12j} \hat{s}_{2j} (y_1 - \hat{y}_{1j})/s_{1j} + \hat{s}_{2j} (1 - \hat{\rho}^2_{12j})^{1/2} Z$$

where

- y<sub>1</sub> = difference for either previous day maximum temperature or current temperature,
- y<sub>2</sub> = current temperature difference,
- Z = standard normal random variate (zero mean and variance of one),

- j = current day is a wet or dry day (1 or 2, respectively),
- s = standard deviation,
- $\rho$  = correlation coefficient between y<sub>1</sub> and y<sub>2</sub>,
- ^ denotes an estimated parameter.

The final simulated temperatures resulted from adding the difference  $y_2$  to the prediction T of the non-linear model.

## Solar Radiation

For dry days, B and G parameters of the gamma functions were estimated for transformed solar radiation differences by month (see Larsen and Pense 1982). For wet days, solar radiation differences on the interval [0,1] were estimated using the transformation suggested by Larsen and Pense (1982). These differences were used to estimate monthly p and q parameters for the standard beta distribution. Beta random variates were simulated using the following relationship:

(6) W (p,q) = 
$$\frac{\int (p,1)}{\int (p,1) + \int (q,1)}$$

This relationship generates a random variate using two gamma random variates (Mihra 1972). The beta distribution can look similar to a normal or posses a skew in either direction Larsen and Pense (1981). For both wet and dry days the transformations were reversed to original scales according to Larsen and Pense (1982) using the software provided by Larsen (personal communication) as a guideline.

## RESULTS

## Precipitation. Temperature and Solar Radiation Parameters

The parameters in Table 1 describe the first order Markov chain that is required to generate rainfall events and the gamma distribution conditioned to previous day precipitation status (dry or wet). Parameters of the non-linear least squares (NLLS) models for estimating maximum and minimum daily temperatures are shown in Table 2. These estimated daily temperatures were used to obtain differences with actual temperatures for every month conditioned to whether a given day was dry or wet (Tables 3 and 4). Correlation coefficients of temperature differences for a) previous day maximum temperature and current day maximum temperature, b) previous day maximum temperature and current day minimum temperature conditioned to current day wet or dry are shown in Table 5. Table 6 presents the parameters of the gamma distribution used to simulate daily solar radiation for dry days. Table 7 presents the parameters p and q of the beta distribution used to simulate daily solar radiation for wet days.

#### MODEL PERFORMANCE AND VALIDATION

Larsen and Pense (1982) validated their model for five different locations in the U.S. with good results. One way to validate the model is to compare a set of simulated results with a non-used set for the estimated parameters. Unfortunately, the data set available does not allow this comparison (only twenty years). In spite of this limitation comparisons between simulated output with twenty years of historical data are presented here. A similar situation prevailed for daily radiation where only nine years were used for validation.

#### **Precipitation**

Descriptive statistics of historical and simulated daily precipitation in a month for wet days given that the previous day was wet or dry are presented in Tables 8 and 9, respectively. In the same tables, results are presented for testing equality of means, variances, and cumulative distribution functions (CDF's) with T,F, and two sample Kolmogorov-Smirnoff (K-S) statistics, respectively.

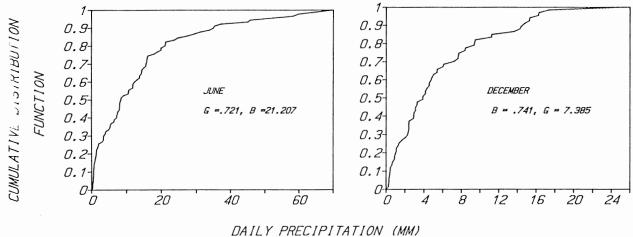
Chi-square tests were done to compare the frequencies of historical and simulated rainy days. No significant differences were found between historical and simulated frequencies of number of rainy days for both previous day wet and previous day dry.

Table 8 shows that daily rainfall by month given that previous day was wet have a bi-modal distribution, with peaks in May and September (13.14 and 15.54 mm day<sup>-1</sup>, respectively) and the lowest mean precipitation in January (2.96 mm day<sup>-1</sup>). In general, the coefficients of variability of daily precipitation by month are slightly larger than 100%. Table 9 shows that the means of daily rainfall by month given that previous day was dry have another bi-modal distribution. The peaks occur in May and August (15.72 and 13.98 mm day<sup>-1</sup>, respectively) and the lowest mean daily precipitation in December (5.33 mm day<sup>-1</sup>). In general, the coefficients of variability of daily precipitation by month are larger than 100%.

No differences were found between precipitation means of simulated and historical data for both previous day wet and dry (Tables 8 and 9, respectively). When the previous day was wet (Table 8), precipitation variances were different in January, March, June, September, October, November, and December while the means and the CDF's were equal between historical and simulated precipitations. When the previous day was dry (Table 9), precipitation variances were different in April and December, and the CDF's were different in January, February, and April.

The CDF's of the simulated daily precipitation given that the previous day was dry for the months of June and December are presented in Fig.1. In June, the frequency of days with rainfall events larger than the mean (14 mm) is about 33%, while in December the frequency of days with rainfall events larger than the mean (5 mm) is about 38%. Each month has a different CDF; with different curvatures G and different scales B (see Table 1). Each CDF yields different statistics that are summarized in Table 9.

The number of variances that were different is within the range of significantly different variances found by Larsen and Pense (1981) in five different locations. The significant differences in CDF's given that the previous



Cumulative distribution functions of simulated daily precipitation given that the previous day was dry at El Reno, Oklahoma. Figure 1.

day was dry during January, February, and April do not have corresponding significant differences in means or variances. Thus, in general the model was accurate to simulate rainfall.

#### <u>Temperature</u>

Historical data of maximum and minimum temperatures by month conditioned to current day precipitation from historical data and simulated data were compared with simulated results. Similar to precipitation, means, variances and CDF's were compared with T, F, and K-S statistics.

Maximum average daily temperatures by month show the highest temperatures during July and the lowest temperatures in January for both current rainy day or current dry day (Tables 10 and 11, respectively). Those days with rainfall are cooler (have lower maximum temperatures) than dry days. The differences between maximum temperatures of current day dry and current day wet are smallest in summer (June) and largest in winter (January).

Comparisons between maximum temperatures conditioned to current wet days (Table 10) showed that none of the means, variances, or CDF's were different. Similar comparisons conditioned to current dry day (Table 11) showed differences between means in February, July, September and December. Variances were different in August and October. Differences between CDF's were found in February, July, September and December.

Minimum average temperatures by month showed the highest temperatures during July and the lowest temperatures in January for both current rainy days and current dry days (Tables 12 and 13, respectively). From May to October the lowest temperatures occured in wet days and from November to January the lowest temperatures occured in dry days. This implies that during summer rainfall decreases temperature but the opposite does not occur in the winter.

Comparisons between minimum temperatures conditioned to current wet days are shown in Table 12. Only the variances in June were different. However, comparisons between minimum temperatures conditioned to current dry days (Table 13) showed differences in means during July and December, and differences in variances in August. Differences in CDF's were present in March, April, July, August, November and December.

Coefficients of variability of maximum and minimum daily temperatures by month are larger or equal to 100% during the winter and 35% or less during the summer for both current day dry and current day wet.

## Solar Radiation

Daily solar radiation (Langleys) in wet days was lowest in December and highest in August, with 72 and 383 Langleys, respectively (Table 14). Similar to temperature, the coefficients of variability were larger in winter than summer, ranging from 87% in December to 36% in June. Simulated daily solar radiation in July, September and December were underestimated while they were overestimated in the other months. No significant differences in means, variances and cumulative distributions were found between historical and simulated daily solar radiation in a given month except the variance of December. Daily solar radiation in dry days was lowest in December and highest in June, with 181 and 539 Langleys, respectively (Table 15). Coefficients of variability were larger in winter than in summer, ranging from 34% in December to 19% in June. Simulated daily solar radiation was underestimated in all months but June; these underestimations were larger in winter than summer. The t tests showed that the means of historical and simulated daily solar radiation were different in January, February, September, October and November (Table 15). Only the variances of January and February, April, September, October and November had significant differences.

Daily solar radiation in wet days was lower than daily solar radiation in dry days; however, the variability was higher in wet days than dry days. These results are in agreement with those from Larsen and Pense (1981) and Richardson (1983).

The CDF's of the simulated daily solar radiation in wet days for the months of January and May are shown in Fig. 2. In January, the frequency of days with solar radiation above the mean (127 Langleys) is about 32%, while in May the frequency of days with solar radiation above the mean (358 Langleys) is about 48%. Each month is characterized with a different set of parameters of the beta distribution, as shown in Table 7, and these parameters yield the summarized statistics in Table 14.

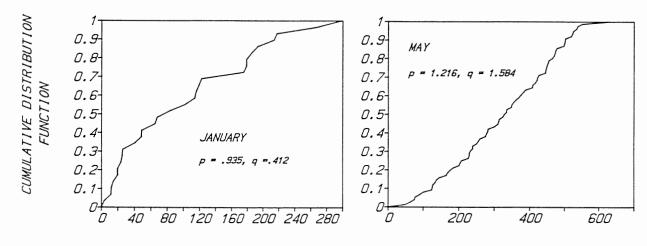
The relatively large number of significant differences in means, variances and cumulative distributions found in daily solar radiation in dry days was concentrated in fall and winter. This suggests that gamma distributions do not characterize appropriately the historical solar radiation during these months<sup>3</sup>. Results provided in Larsen and Pense (1981) showed significant differences in variances of solar radiation for dry days in both Alburquerque, New Mexico and Columbia, Missouri (six variances in each location). However, those differences were scattered throughout the year.

#### DISCUSSION

Daily precipitation was predicted reasonably well with the two parameter gamma distribution. Seven variances of daily precipitation out of twelve were different given that the previous day was wet. Two variances of daily precipitation out of twelve, and three CDF's of daily precipitation out of twelve were different given that the previous day was dry. Maximum temperatures were predicted accurately on wet days while the accuracy decreased in February, July, October, and December for dry days, with tendency to simulate colder days. A similar situation occured for daily minimum temperatures; accurate predictions on wet days while a few differences occurred in July, August, November, and December with a tendency to simulate colder days.

These differences in means, variances, and CDF's need to be considered in light of the precision of the biological model that is fed with daily simulated weather. In our case, using CERES-wheat (Ritchie and Otter 1985) with this weather simulator seems appropriate. More precision could be accomplished by reducing the monthly parameters to weekly parameters but at the expense of computer costs. An approach similar to Richardson (1981)

<sup>&</sup>lt;sup>3</sup> Parameters of beta distributions were estimated for generating daily solar radiation on dry days. However, the resulting statistical differences in means, variances and cumulative distribution functions were larger than those found when using gamma distributions.



DAILY SOLAR RADIATION (LANGLEYS)

Figure 2. Cumulative distribution functions of simulated daily solar radiation for wet days at El Reno, Oklahoma.

could be followed as long as the Fourier coefficients do not smooth out significantly the observed weather variability. The Fourier coefficients approach has the advantage of providing continuous parameters rather than discrete parameters as determined here. Further, this method could eliminate the laborious task of determining differences in temperatures and solar radiation. Larsen (1981) stated that most of the distributional problems with the temperature simulator were a result of skewness in the observed data. Skewness could be accommodated by replacing the bi-variate normals with bi-variate betas.

Variability in daily precipitations, measured with the coefficient of variability, is around 100% with no seasonal pattern throughout the year. In contrast, the variability of maximum and minimum daily temperatures is higher in the winter (100% or more) than in the summer (35% or less), suggesting that, in general, weather in El Reno is less variable in the summer than in the winter. Native vegetation takes advantage of this climatic characteristic, growing during April to October. Small cereals planted in the fall present vegetative growth during the time of the year when temperature is highly variable (late fall, winter and early spring). A model sensitive to daily temperature changes during the winter months is an appropriate tool for simulating growth and development of winter small grain cereals.

The results obtained for El Reno showed an average underestimation of 10% in daily solar radiation for dry days in January, February, September, October and November. More solar radiation data could enhance the parameter estimation of the gamma distributions (our estimations were based on less than one half of the sample size used by Larsen and Pense 1981). The 10% underestimation of daily solar radiation for the fall and winter dry days seems important for modeling growth and development of "winter grown" small grain cereals. This is an important input that cannot be neglected; however, it needs to be weighed with other sources of inaccuracy, such as: approximations in the model equations to reflect biological processes and plant measurement errors in the data for model validation. It is known where the model is relatively innacurate; therefore, corrective measurements in the modeling process are relatively simple.

#### CONCLUDING COMMENTS

The model developed by Larsen and Pense (1982) was implemented for El Reno, Oklahoma. Simulated number of rainy days, precipitation amounts per day, and minimum and maximum temperatures were accurately simulated. Simulated daily solar radiation in wet days was accurate; however, simulated solar radiation in dry days was underestimated 10% during five months in the fall and winter.

In light of our objective to provide a reliable stochastic weather simulator to feed a wheat-grazing systems model, this study accomplished its purpose. Applications for El Reno can be done with precise knowledge of the statistical characteristics of daily weather. Further, the model can be used to analyze weather events larger than one day in order to fulfill other modeling requirements or experimental designs.

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	No.			k <sup>4</sup> =	1	k =	= 2
	Observations	P <sub>i</sub> (W D) <sup>2</sup>	P <sub>i</sub> (W W) <sup>3</sup>	G	В	G	В
January	620	.0952	.2877	.873	4.157	.542	15.856
February	565	.1404	.3627	.746	6.446	.705	9.430
March	620	.1656	.3817	.822	10.176	.732	12.870
April	600	.1938	.4238	.740	15.617	.685	12.184
Мау	620	.2494	.4398	.653	21.543	.742	20.237
June	600	.2022	.4194	.690	13.485	.721	21.207
July	620	.1333	.3818	.889	11.240	.789	15.802
August	620	.1488	.3534	.743	15.698	.692	16.464
Septembe	er 600	.1485	.4180	.667	19.431	.702	17.306
October	620	.1500	.3750	.638	21.110	.707	14.440
Novembe	r 600	.1024	.5000	.780	12.530	.899	7.016
Decembe	r 620	.1221	.3333	.623	8.325	.741	7.385

Monthly Probabilities of Rainfall Occurrences and Parameters of Two Parameter Gamma Distributions for Daily Rainfall Events at El Reno, OK<sup>1</sup>

TABLE 1

 $1_{f_k(x)} = \frac{1}{B(k)G(k)G(k)} X^{G(k)-1} e^{-X/B(k)}$ 

Daily precipitation (1966-1985) was used for these parameter estimations.

<sup>2</sup>Probability of a wet day given a wet previous day.

<sup>3</sup>Probability of a wet day given a dry previous day.

 $^{4}\mbox{The}$  index k defines the specific distribution for those wet days with previous day wet

or dry (k = 1 or 2, respectively).

# TABLE 2 Estimated Parameters For Maximum and Minimum Daily Temperature at EI Reno, OK.<sup>1</sup>

T	α	β	δ	Residual Mean Square
Maximum Daily	12.942	107.893	22.205	30.259
Temperature (C)	(0.090)	(0.408)	(0.064)	
Minimum Daily	12.682	108.711	8.628	23.435
Temperature (C)	(0.080)	(0.367)	(0.056)	

 $^{1}\text{T} = \alpha~SIN((JULIAN~DAY - \beta)~2~\pi/365) + \delta$  Daily observations from 1966 to 1985 were used for the non-linear least squares (NLLS) estimation with the Marquardt method (SAS 1985). Asymptotic standard errors are in parentheses.

## Means and Standard Deviations of Differences of Temperatures Between NLLS Estimations and Actual Temperatures (Table 2) By Month on a Wet Day at El Reno, OK<sup>1</sup>

		MEAN (C)		ST	STANDARD DEVIATION			
	LTMAX <sup>2</sup>	TMAX <sup>3</sup>	<u>TMIN</u> <sup>4</sup>	LT	MAX	<u>TMAX</u>	TMIN	
January	3.48	6.24	0.91	7.	.34	7.55	6.18	
February	1.81	3.76	0.14	7.	.12	6.69	6.05	
March	-0.39	1.34	-1.89	6.	.26	6.22	5.33	
April	0.46	1.32	-0.82	5.	.03	4.83	4.74	
May	2.21	2.92	1.42	3.	.92	3.95	3.05	
June	2.54	3.12	2.06	3.	.35	3.29	2.79	
July	1.52	2.71	1.16	3.	.23	3.30	2.34	
August	0.38	1.45	0.79	3	.97	3.62	2.01	
September	1.12	3.00	0.65	4.	.98	4.90	3.51	
October	0.24	2.33	-0.61	4	.96	4.85	4.45	
November	2.52	4.40	0.32	6	.24	5.12	4.14	
December	3.27	4.46	0.20	6	.53	6.71	5.63	

<sup>1</sup>Daily data from 1966 to 1985 were used for these parameter estimations.

<sup>2</sup>LTMAX = Previous day maximum temperature.

<sup>3</sup>TMAX = Current day maximum temperature.

<sup>4</sup>TMIN = Current day minimum temperature.

Note: A positive bias occurs for the average differences (LTMAX, TMAX and TMIN) between NLLS estimations and actual temperatures. In a wet day actual temperatures are below the sine functions in Table 2, these sine functions were estimated with both rainy and dry days, the bias is compensated when the average differences include dry days (see Table 4).

# Means and Standard Deviations of Differences of Temperatures Between NLLS Estimations and Actual Temperatures (Table 2) By Month on a Dry Day at El Reno, OK<sup>1</sup>

TABLE 4

		MEAN(C)		STANDARD DEVIATION(C			
	<u>LTMAX<sup>2</sup></u>	<u>TMAX</u> 3	<u>TMIN</u> <sup>4</sup>	LT	MAX	<u>TMAX</u>	TMIN
January	1.36	0.92	0.93	-	7.56	7.35	5.94
February	-0.78	-1.42	-0.57	7	7.39	7.15	5.57
March	-2.46	-2.86	-1.32	6	6.52	6.33	5.91
April	-1.63	-1.79	-0.52	2	1.98	4.81	5.56
May	0.87	0.59	1.13	3	3.62	3.45	4.50
June	1.25	0.97	0.43	2	2.82	2.61	3.51
July	-0.10	-0.34	-0.19	3	3.02	2.87	3.14
August	-0.94	-1.26	-0.76	3	3.20	3.11	3.14
September	-1.03	-1.52	-0.99	4	4.45	4.20	4.44
October	-1.47	-1.92	-0.42	4	4.68	4.40	5.11
November	-0.04	-0.37	0.16	Ę	5.82	5.83	5.09
December	0.33	0.20	0.88	e	6.62	6.52	5.39

<sup>1</sup>Daily data from 1966 to 1985 were used for these parameter estimations.

<sup>2</sup>LTMAX = Previous day maximum temperature.

<sup>3</sup>TMAX = Current day maximum temperature.

<sup>4</sup>TMIN = Current day minimum temperature.

## Correlation coefficients of temperature differences for a) previous day maximum temperature and current day maximum temperature, b) previous day maximum temperature and current day minimum temperature, and c) current day maximum temperature and current day minimum temperature conditioned to rainfall status.

		WET			DRY	
	LTMAX <sup>2</sup> -TMAX <sup>3</sup>	LTMAX-TMIN <sup>4</sup>	TMAX-TMIN	LTMAX-TMAX	LTMAX-TMIN	TMAX-TMIN
January	.66	.51	.80	.71	.64	.72
February	.74	.59	.72	.75	.69	.70
March	.64	.55	.69	.66	.60	.68
April	.64	.55	.63	.60	.60	.65
May	.45	.39	.54	.59	.56	.61
June	.63	.37	.47	.66	.52	.56
July	.59	.34	.58	.80	.58	.63
August	.57	.19	.38	.79	.58	.62
September	.62	.36	.54	.79	.64	.67
October	.52	.34	.60	.62	.49	.55
November	.62	.35	.56	.66	.55	.62
December	.66	.46	.74	.67	.62	.67

<sup>1</sup>Daily data from 1966 to 1985 were used for these parameter estimations.

<sup>2</sup>LTMAX = Previous day maximum temperature

<sup>3</sup>TMAX = Current day maximum temperature

<sup>4</sup>TMIN = Current day minimum temperature

	Number of Observations	Shape parameter G	Scale parameter B
January	189	2.360	47.10
February	168	1.740	86.99
March	161	1.894	87.14
April	180	1.574	87.43
Мау	171	1.527	94.59
June	172	1.374	97.87
July	220	1.244	108.71
August	165	1.866	83.67
September	201	2.082	77.16
October	196	2.427	64.22
November	196	2.178	55.21
December	204	3.157	35.56

Monthly parameters of Two Parameter Gamma Distributions used to simulate daily solar radiation (Langleys) on dry days at El Reno, Ok.<sup>1</sup>

$$f(x) = \frac{1}{B^{G-G}} X^{G-1} e^{-X/B}$$

	Number of Observations	р	q
January	18	0.935	0.412
February	29	0.655	0.421
March	49	1.105	0.620
April	58	0.767	0.869
May	76	1.216	1.584
June	67	1.577	2.266
July	28	1.268	1.481
August	38	0.872	1.434
September	38	0.718	0.817
October	52	1.027	0.734
November	35	0.983	0.463
December	31	1.290	0.578

 TABLE 7

 Monthly parameters of beta distributions<sup>1</sup> used to simulate daily solar radiation (Langleys) on wet days at El Reno, OK.

 $1 W (p,q) = \frac{\left\lceil (p,1) \right\rceil}{\left\lceil (p,1) + \left\lceil (q,1) \right\rceil}$ 

Descriptive Statistics of Historical (H) and Simulated (S) Daily Precipitation
given Previous Day Wet in a Month At El Reno, OK. Tests for Equality In Means
(T), Variances (F), and Cumulative Distribution Functions (K-S) Between
H and S

			Pre	ecipation	n (mm)				
		Number of bservations <sup>1</sup>	Mean	Std	Min	Max	т	F	K-S <sup>2</sup>
January	н	22	3.62	4.45	0.25	18.54	0.53	2.63*	.31
	s	23	2.96	2.74	0.25	9.64			
February	н	37	4.81	8.27	0.25	47.49	-0.37	1.81	.58
	S	38	4.17	6.15	0.24	34.94			
March	н	50	8.36	9.57	0.25	43.18	-1.21	2.51**	.84
	S	47	11.52	15.16	0.28	69.24			
April	н	64	11.54	14.59	0.25	58.16	-0.45	1.40	.11
	s	73	12.60	12.35	0.24	55.30			
Мау	н	84	12.87	14.97	0.25	68.32	-0.11	1.02	.82
	S	72	13.14	14.82	0.25	60.40			
June	н	65	9.30	12.23	0.25	44.95	1.00	2.13**	.80
	s	80	7.51	8.37	0.25	46.57			
July	н	42	9.98	10.41	0.25	50.03	0.31	1.53	.22
	S	43	9.33	8.40	0.30	33.20			
August	н	41	11.66	15.16	0.25	58.42	-0.25	1.37	.13
	s	48	12.43	12.94	0.25	63.30			
September	н	51	12.96	19.01	0.25	99.31	-0.77	1.83*	.16
	s	49	15.54	14.06	0.24	48.86			
October	н	45	11.25	12.84	0.25	50.03	-1.01	2.12*	.83
	S	49	14.57	18.68	0.24	86.45			
November	н	51	9.77	12.12	0.25	61.72	.58	1.98*	.68
	S	61	8.58	8.61	0.25	40.96			
December	н	31	5.19	7.52	0.25	33.52	.31	2.38*	.70
	S	30	4.69	4.87	0.24	16.54			

<sup>1</sup> Chi square test.

<sup>2</sup> Two sample Kolmogorov - Smirnoff test.

\* Significantly different at P<.05.

\*\* Significanlty different at P<.01.

Descriptive Statistics of Historical (H) and Simulated (S) Daily Precipitation given Previous Day Dry in a Month At El Reno, OK. Tests for Equality In Means (T), Variances (F), and Cumulative Distribution Functions (K-S) Between H and S.

		N	P	recipitatio	n (mm)				
		Number of bservations <sup>1</sup>	Mean	Std	Min	Max	т	F	K-S <sup>2</sup>
January	н	52	8.59	13.86	0.25	65.53	-0.78	1.31	.44*
	S	51	10.88	15.85	0.25	98.14			
February	н	65	6.64	9.13	0.25	43.68	-0.54	1.64	.38*
	s	62	7.43	7.13	0.24	30.03			
March	н	81	9.42	12.36	0.25	51.81	-0.17	1.14	0.46
	s	74	9.79	13.22	0.24	63.65			
April	н	87	8.34	11.29	0.25	53.34	0.09	1.66*	.57*
	s	89	8.20	8.75	0.25	39.59			
Мау	н	107	15.01	17.35	0.25	86.86	0.09	1.03	0.81
	s	112	15.72	17.09	0.24	78.73			
June	н	90	15.29	18.67	0.25	76.45	0.42	1.39	0.60
	s	87	14.19	15.83	0.29	69.45			
July	н	68	10.99	12.55	0.25	59.43	-0.49	1.24	0.88
	s	64	12.05	11.27	0.24	58.25			
August	н	75	11.39	16.07	0.25	87.63	-0.98	1.10	0.99
	S	84	13.98	16.86	0.24	80.92			
September	н	71	12.15	14.72	0.25	62.48	0.41	1.08	0.78
	S	68	11.14	14.18	0.25	81.90			
October	н	75	10.21	13.00	0.25	58.67	-0.44	1.01	1.14
	s	75	11.16	13.05	0.24	84.56			
November	н	51	6.30	7.25	0.25	32.25	-0.34	1.22	0.73
	s	52	6.77	6.57	0.24	28.30			
December	н	65	5.47	8.47	0.25	41.14	0.11	2.47**	1.01
	S	71	5.33	5.39	0.24	24.75			

<sup>1</sup>Chi-square test.

<sup>2</sup>Two sample Kolmogorov - Smirnoff test.

\*Significantly different at P<.05.

\*\*Significantly different at P<.01.

# Descriptive Statistics of Historical (H) and Simulated<sup>1</sup> (S) Daily Maximum Temperature for Wet Days in a Month at El Reno, OK. Tests of Equality in Means (T), Variances (F), And Cumulative Distribution Functions (K-S) Between H and S.

		Niumber of	Te	mperat	<u>ure (C)</u>				
		Number of bservations	Mean	Std	Min	Max	т	F	K-S <sup>2</sup>
January	н	74	3.19	7.54	-10.00	18.88	-1.81	1.05	1.23
	S	74	5.42	7.36	-13.94	18.91			
February	н	102	7.18	6.95	-8.88	20.55	-0.16	1.07	0.53
	S	100	7.02	7.19	-7.13	23.52			
March	н	131	14.28	6.03	1.11	27.22	-0.45	1.13	0.63
	s	121	14.64	6.41	-0.55	32.08			
April	н	151	20.73	5.11	5.55	32.77	-0.69	1.11	0.59
	s	162	21.15	5.38	8.18	41.38			
May	н	191	25.48	4.44	7.77	35.00	0.20	1.18	1.13
	s	184	25.38	4.81	12.19	35.85			
June	н	155	29.75	3.61	18.33	36.11	-1.35	1.13	0.86
	s	167	30.29	3.39	20.30	39.27			
July	н	110	32.26	3.29	23.88	41.11	-0.52	1.10	0.86
	S	107	32.51	3.45	24.90	41.33			
August	н	116	31.96	3.75	20.55	38.33	-1.85	1.11	1.10
	S	132	32.83	3.56	20.23	40.93			
September	н	122	25.97	5.44	13.88	37.22	-1.42	1.23	0.87
	S	117	26.92	4.90	13.05	37.34			
October	н	120	20.00	5.52	6.66	31.11	-1.02	1.34	0.87
	s	124	20.67	4.77	9.61	34.56			
November	н	102	11.79	5.59	1.11	25.00	-0.42	1.02	0.89
	s	113	12.11	5.64	-0.06	28.40			
December	н	96	6.62	6.57	-8.33	21.66	-0.01	1.05	0.59
	S	101	6.61	6.73	-13.54	23.46			

<sup>1</sup>Using parameters in Tables 1-5.

<sup>2</sup>Two sample Kolmogorov - Smirnoff test.

# Descriptive Statistics of Historical (H) and Simulated<sup>1</sup> (S) Daily Maximum Temperature for Dry Days in a Month at El Reno, OK. Tests of Equality in Means (T),Variances (F), And Cumulative Distribution Functions (K-S) Between H and S.

		NI 1	Te	mperat	ure (C)				
		Number of bservations	Mean	Std	Min	Max	т	F	K-S <sup>2</sup>
January	н	546	8.48	7.35	-12.22	26.66	-0.51	1.04	1.02
	s	546	8.71	7.19	-12.91	29.75			
February	Н	463	12.38	7.41	-10.00	27.77	2.25*	1.08	1.74**
	S	465	11.30	7.13	-6.56	30.26			
March	н	489	18.18	6.54	-1.66	33.88	-1.09	1.02	0.88
	S	499	18.64	6.48	1.62	45.84			
April	Н	449	23.41	4.77	7.77	39.44	-0.62	1.10	1.00
	s	438	23.62	5.00	8.44	38.00			
Мау	н	429	27.52	3.79	15.00	40.55	-0.72	1.06	1.10
	s	436	27.70	3.90	14.79	38.57			
June	н	445	32.15	2.94	22.22	42.22	-1.12	1.01	1.07
	s	433	32.37	2.93	23.75	40.08			
July	Н	510	35.33	2.89	25.55	43.33	2.88**	1.13	2.48**
	s	513	34.79	3.07	27.31	44.53			
August	Н	504	34.68	3.07	24.44	42.22	0.27	1.21*	0.91
	s	488	34.63	3.38	18.19	42.54			
September	н	478	30.32	4.61	12.22	39.44	-1.99*	1.03	1.47*
	s	483	30.91	4.53	17.83	43.40			
October	н	500	24.40	5.00	7.22	36.66	1.22	1.28**	2.22**
	s	496	24.04	4.41	11.83	35.35			
November	н	498	16.34	6.19	-0.55	29.44	-0.91	1.00	0.81
	s	487	16.70	6.19	-1.65	31.90			
December	н	524	11.06	6.60	-13.88	26.11	2.84**	1.02	2.13**
	S	519	9.90	6.53	-15.31	27.53			

<sup>1</sup>Using parameters in Tables 1-5.

<sup>2</sup>Two sample Kolmogorov - Smirnoff test.

\*Significantly different at P<.05.

\*\*Significantly different at P<.01.

Descriptive Statistics of Historical (H) and Simulated <sup>1</sup> (S) Daily Minimum							
Temperature for Wet Days in a Month at El Reno, OK. Tests of Equality in Means							
(T), Variances (F), And Cumulative Distribution							
Functions (K-S) Between H and S.							

		Niumah ay af	<u>Te</u>	mperat	<u>ure (C)</u>	<u>Temperature (C)</u>			
		Number of bservations	Mean	Std	Min	Max	т	F	K-S <sup>2</sup>
January	н	74	-4.78	6.16	-18.33	7.77	-1.71	1.61	0.82
	s	74	-3.14	6.19	-20.74	11.39			
February	н	102	-2.91	6.25	-17.77	10.55	0.33	1.01	0.92
	s	100	-2.87	6.24	-17.65	12.62			
March	н	131	3.93	5.15	-11.66	15.55	0.71	1.09	0.85
	s	121	3.42	5.39	-10.49	18.77			
April	н	151	9.13	4.98	-3.88	23.33	0.46	1.18	1.15
	s	162	8.86	5.41	-5.90	22.95			
Мау	н	191	13.12	3.48	5.00	19.44	0.44	1.11	1.17
	s	184	12.95	3.66	4.35	23.24			
June	н	155	16.92	3.12	9.44	25.00	-0.92	1.40*	1.07
	s	167	17.22	2.64	10.95	24.59			
July	н	110	19.97	2.35	11.11	26.11	-0.49	1.01	0.68
	s	107	20.13	2.35	12.82	26.16			
August	н	116	18.91	2.15	12.77	23.88	-0.40	1.08	1.01
	S	132	19.02	2.24	12.94	24.55			
September	н	122	14.75	4.01	4.44	21.66	-1.05	1.06	0.74
	s	117	15.31	4.13	6.70	28.30			
October	н	120	9.54	4.81	-1.11	20.00	-0.94	1.02	0.64
	s	124	10.13	4.86	-0.28	23.45			
November	н	102	2.57	4.72	-6.66	13.88	-0.16	1.02	0.56
	s	113	2.68	4.73	-7.49	17.14			
December	н	96	-2.38	5.52	-17.22	12.77	0.08	1.03	0.88
	s	101	-2.47	5.45	-15.54	10.39			

<sup>1</sup>Using parameters in Tables 1-5.

<sup>2</sup>Two sample Kolmogorov - Smirnoff test.

\*Significantly different at P<.05.

Descriptive Statistics of Historical (H) and Simulated<sup>1</sup> (S) Daily Minimum Temperature for Dry Days in a Month at El Reno, OK. Tests of Equality in Means (T), Variances (F), And Cumulative Distribution Functions (K-S) Between H and S.

		Marchard	Te	mperat	ure (C)				
		Number of bservations	Mean	Std	Min	Max	т	F	K-S <sup>2</sup>
January	н	546	-4.84	5.93	-23.33	14.44	-0.54	1.02	0.75
	s	546	-4.64	5.97	-21.05	14.27			
February	н	463	-1.91	5.82	-17.22	16.66	0.79	1.02	0.84
	s	465	-2.21	5.88	-22.11	16.44			
March	н	489	3.06	6.07	-15.55	19.44	-0.64	1.07	1.37*
	s	499	3.31	6.28	-13.52	30.83			
April	н	449	8.40	5.80	-7.77	22.77	-0.80	1.05	1.44*
	s	438	8.71	5.65	-9.66	26.05			
May	н	429	13.12	4.83	2.22	25.55	-0.53	1.11	1.23
	s	436	13.31	4.58	1.41	27.20			
June	н	445	18.80	3.70	7.22	29.44	-0.37	1.16	1.20
	s	433	18.90	3.44	6.47	28.75			
July	н	510	21.33	3.15	10.55	30.55	2.35*	1.02	2.77**
	S	513	20.87	3.12	12.18	32.16			
August	н	504	20.47	3.12	10.00	29.44	-0.17	1.22*	1.48*
	s	488	20.50	3.45	4.23	29.77			
September	н	478	16.23	4.83	1.11	30.00	-0.87	1.17	1.20
	s	483	16.50	4.47	2.46	30.27			
October	н	500	9.50	5.43	-5.00	22.22	1.47	1.12	1.05
	S	496	9.00	5.13	-4.11	21.51			
November	н	498	2.51	5.42	-11.66	17.77	-1.02	1.01	1.65**
	s	487	2.86	5.45	-15.45	18.40			
December	н	524	-2.88	5.52	-21.11	18.33	2.26*	1.06	1.80**
	S	519	-3.66	5.70	-21.10	12.61			

<sup>1</sup>Using parameters in Tables 1-5.

<sup>2</sup>Two sample Kolmogorov-Smirnoff test.

\*Significantly different at P<.05.

\*\*Significantly different at P<.01.

Descriptive Statistics of Historical (H) and Simulated<sup>1</sup> (S) Daily Solar Radiation for Wet days in a month at El Reno, Ok. Tests of equality in Means (T), Variances (F) and Cummulative Distribution Functions (K-S) between H and S.

<u> </u>			Solar	Radiatio	n (Langl	evs)			
		mber of ervations		Std	Min	Max	т	F	K-S
January	H S	18 29	86.8 126.6	79.4 88.9	6.0 3.4	270.0 298.2	-1.59	1.25	1.17
February	H S	29 38	140.4 175.7	111.6 116.9	11.5 0.8	341.1 404.4	-1.25	1.10	1.10
March	H S	49 56	187.6 203.1	147.8 132.6	4.8 7.5	513.4 471.2	-0.55	1.24	0.74
April	H S	58 60	299.8 310.6	167.2 180.2	10.7 15.4	554.9 598.5	-0.33	1.16	0.84
May	H S	76 76	365.1 358.5	161.3 135.1	6.2 48.0	640.6 630.7	0.27	1.43	0.73
June	H S	67 68	377.4 379.1	138.1 159.3	13.1 53.3	631.1 625.1	-0.06	1.33	1.01
July	H S	28 28	380.8 376.8	154.9 149.9	46.2 115.1	667.4 628.2	0.09	1.14	0.53
August	H S	38 49	382.6 387.2	128.1 132.0	83.9 145.3	564.2 575.7	-0.16	1.06	0.67
September	H S	38 38	281.9 274.9	139.1 129.1	45.6 9.7	489.4 470.7	0.22	1.16	0.57
October	H S	52 52	161.0 171.4	111.5 99.9	4.3 20.0	380.3 406.9	-0.50	1.25	0.69
November	H S	35 43	93.7 97.4	76.2 71.7	11.9 2.9	267.5 266.8	-0.22	1.13	0.81
December	H S	31 38	71.9 59.5	62.0 43.7	1.5 0.2	228.7 156.9	0.93	2.01*	0.57

<sup>1</sup>Using parameters in Tables 1-7.

Descriptive Statistics of Historical (H) and Simulated<sup>1</sup> (S) Daily Solar Radiation for dry days in a month at El Reno, Ok. Tests of equality in Means (T), Variances (F) and Cummulative Distribution Functions (K-S) between H and S.

<b>-</b>			Solar	Radiati	on (Lang	leys)			
		umber of servations	Mean	Std	Min	Max	т	F	K-S
January	H S	189 219	207.2 190.3	69.5 59.4	17.2 25.7	315.4 288.6	2.66**	1.37*	2.31**
February	H S	168 188	260.2 237.6	105.4 90.6	6.4 20.6	408.5 423.4	2.17*	1.35*	2.07**
March	H S	161 192	371.1 360.3	114.5 119.5	17.0 27.4	533.2 565.9	0.87	1.09	1.07
April	H S	180 180	475.9 456.3	111.8 107.2	53.4 20.5	610.6 621.1	1.70	1.09	1.37*
Мау	H S	171 172	513.8 507.2	116.0 117.2	117.8 54.0	655.2 649.7	0.52	1.02	0.69
June	H S	172 172	539.3 553.1	101.2 99.5	241.7 122.1	670.8 675.0	-1.27	1.03	1.24
July	H S	220 220	537.4 527.5	117.4 114.6	1.5 29.7	669.6 666.8	0.89	1.05	1.00
August	H S	165 199	458.4 443.8	113.5 119.4	37.7 29.6	611.5 607.1	1.18	1.11	0.87
September	H S	201 202	395.7 353.4	106.2 109.7	45.9 1.5	553.4 526.7	3.9**	1.07	2.15**
October	H S	196 196	299.2 251.8	94.8 87.8	35.3 6.4	452.1 428.6	5.1**	1.16	2.88**
November	H S	196 196	211.1 194.3	77.2 73.6	15.7 8.2	328.3 325.8	2.2*	1.10	1.87**
December	H S	204 210	180.9 173.6	62.6 61.7	21.7 11.9	290.1 279.3	1.19	1.03	1.09

<sup>1</sup>Using parameters in Tables 1-7.

## APPENDIX Listing of the Program Weather

```
PROGRAM NEWPROG
С
      FEBRUARY 17, 1988. REVISED SEPT 1, 1988.
С
      THIS IS THE PROGRAM THAT IS USED TO READ THE NECESSARY
С
      PARAMETERS FOR RUNNING THE WEATHER SUBROUTINE USING THE
С
      IMPLEMENTED MODEL OF LARSEN AND PENSE (1982) FOR EL RENO, OKLAHOMA
      THIS PROGRAM IS USED WITHIN THE WHEAT GRAZING SYSTEMS MODEL
С
С
      DEVELOPED BY RODRIGUEZ ET AL. (1988). DEPT. AGRIC. ECONOMICS,
С
      OKLA. STATE UNIV.
      DIMENSION MONTH(12,2)
      REAL MAXTMP, MINTMP, CORCOF (12, 2, 3), MEAN (12, 2, 3), STD (12, 2, 3)
      DATA MONTH/31,28,31,30,31,30,31,31,30,31,30,31,
     1
                  31,29,31,30,31,30,31,31,30,31,30,31/
      DATA IX/-7/
      OPEN(8, FILE='ERSTAT.DAT', STATUS='OLD')
      OPEN (7, FILE='WEAOUT.PRN', STATUS='UNKNOWN')
      DO 512 MON=1,12
         READ (8,113) CORCOF (MON, 2, 1), CORCOF (MON, 2, 2), CORCOF (MON, 2, 3),
     1
                       CORCOF (MON, 1, 1), CORCOF (MON, 1, 2)
         READ (8,213) CORCOF (MON, 1, 3), MEAN (MON, 2, 1), MEAN (MON, 2, 2),
                       MEAN (MON, 2, 3), STD (MON, 2, 1), STD (MON, 2, 2)
     1
         READ (8,213) STD (MON, 2, 3), MEAN (MON, 1, 1), MEAN (MON, 1, 2),
     1
                       MEAN (MON, 1, 3), STD (MON, 1, 1), STD (MON, 1, 2)
         READ (8,213) STD (MON, 1, 3)
  512 CONTINUE
  113 FORMAT (13X,5(F11.8,1X))
  213 FORMAT (6(1X,F11.8))
       The model assumes that the day previous to the beginning of
С
       simulation was dry and that average maximum temperature was 9.0.
С
С
       These conditions can be changed to start in any day of any month
С
       of any year changing indices for do 100.
       PPT=0.0
       MAXTMP=9.0
       DO 100 IYR=1,50
         JDATE=1
         LY=1
         IF (MOD(IYR, 4).EQ.0) THEN LY=2
         DO 100 MON=1,12
           DO 100 IDAY=1, MONTH (MON, LY)
             CALL WEATH (MON, JDATE, MAXTMP, MINTMP,
      1
                PPT, SOLRAD, CORCOF, MEAN, STD, IX)
             if (maxtmp.le..5.and. ppt.ge..24) goto 399
             go to 401
       Conditional statement for ice/snow conditions.
C
             write (7,400) MON, IDAY, IYR, PPT, MAXTMP, MINTMP, SOLRAD
  399
   400
             FORMAT (1X, I2, 1X, I2, 1X, I2, 1X, 4 (F5.1, 1X))
   401
              jdate=jdate+1
  100 CONTINUE
       STOP
       END
С
С
С
       SUBROUTINE WEATH (MON, JDATE, MAXTMP, MINTMP,
      1 PPT, SOLRAD, CORCOF, MEAN, STD, IX)
C THIS SUBROUTINE SIMULATES THE PRECIPITATION, MAX AND MIN TEMPERATURES,
```

```
С
      AND SOLAR RADIATION OF EL RENO, OK ON A DAILY BASIS.
С
      FEBRUARY 17, 1988.
С
      ONLY MON, JDATE, MAXTMP AND PPT FROM THE PREVIOUS DAY
С
С
      WILL BE PASSED FROM THE MAIN PROGRAM TO THE SUBROUTINE WHEN CALLED
С
      AND THE RETURNED VALUES WILL BE MAXTMP, MINTMP, PPT AND SOLRAD
С
      FOR THE CURRENT DAY.
С
      MAKE SURE THAT PPT AND MAXT ARE SET PROPERLY FOR THE INITIAL
С
      SIMULATION DAY
С
С
      MON=MONTH (1-12)
С
      JDATE=JULIAN DAY
С
      MAXTMP=MAXIMUM TEMPERATURE (C)
С
      MINTMP=MINIMUM TEMPERATURE (C)
С
      PPT=PRECIPITATION (MM)
С
      SOLRAD=SOLAR RADIATION (LANGLEYS)
С
      CORCOF=CORRELATION COEFFICIENT BETWEEN A) LMAXTEMP-MAXTEMP,
      B) LMAXTEMP-MINTEMP AND C) MAXTEMP-MINTEMP MEAN= MEAN FOR A, B AND C ABOVE
С
С
      STD= STANDARD DEVIATION FOR A, B AND C ABOVE
С
С
      IX=SEED USED BY THE RANDOM NUMBER MIN THE WEATHER SIMULATION
С
      REAL MAXTMP, MINTMP, CORCOF (12,2,3), MEAN (12,2,3), STD (12,2,3)
С
      CALL PRECIP(PPT, MON, IX)
      CALL TEMP (CORCOF, MEAN, STD, MAXTMP, MINTMP, PPT, MON, JDATE, IX)
      CALL SOLAR (MON, JDATE, SOLRAD, PPT, IX)
С
      RETURN
      END
C
C
      SUBROUTINE PRECIP(PPT, MON, IX)
      DIMENSION ALPHA(12,2), BETA(12,2), RAIN(12,2)
C ALPHA (CURVATURE/SLOPE)
      DATA ALPHA/.542192,.704913,.732444,.684617,.742160,.721039,.788783
     1,.692401,.702219,.707361,.898631,.741098,.872851,.746553,.821739,
     2.739518, .652823, .689956, .888316, .743108, .667450, .638242, .779817,
     3.623528/
C BETA (SCALING FACTOR)
      DATA BETA/15.8559,9.4295,12.8695,12.1836,20.2372,21.2066,15.8024,
     1 16.4638,17.3061,14.4399,7.0164,7.3849,4.1571,6.4460,10.1756,
     2 15.6170,21.5430,13.4852,11.2399,15.6981,19.4306,21.1100,12.5305,
     3 8.3254/
C PROBABILITY OF PRECIPITATION
      DATA RAIN/.095238,.140389,.165644,.193764,.249417,.202247,
     1 .133333,.148810,.148536,.150000,.102410,.122137,
     2 .287671,.362745,.381679,.423841,.439791,.419355,
     3 .381818, .353448, .418033, .375000, .500000, .333333/
      IF (PPT .LT. .24) J=1
      IF (PPT .GE. .24) J=2
      U=RAN3(IX)
      IF (U .LE. RAIN(MON, J)) GO TO 300
      PPT=0.
      RETURN
  300 Z=0.0
      K=ALPHA (MON, J)
      F=K
```

```
IF (K) 303,303,301
  301 PROD=1.0
      DO 302 L=1,K
      U=RAN3(IX)
      PROD=PROD*U
302 CONTINUE
      Z=-ALOG (PROD)
  303 D=ALPHA(MON, J)-F
      IF (D) 308,308,304
  304 A=1.0/D
      B=1.0/(1.0-D)
      L=1
  305 U=RAN3(IX)
      X=U**A
      U=RAN3(IX)
      Y = (U * *B) + X
      IF (Y-1.0) 307,307,306
  306 L=L+2
      GO TO 305
  307 W=X/Y
      U=RAN3(IX)
      Y=-ALOG(U)
      PPT=(Z+W*Y)*BETA(MON, J)+.24
      GO TO 309
  308 PPT=Z*BETA(MON, J)+.24
  309 PPT=(AINT(PPT*100.))/100.
      IF (PPT .LT. .24) PPT=0.00
      RETURN
      END
С
С
      SUBROUTINE TEMP (CORCOF, MEAN, STD, MAXTMP, MINTMP, PPT, MON, JDATE, IX)
      REAL MAXTMP, MINTMP, CORCOF (12, 2, 3), MEAN (12, 2, 3), STD (12, 2, 3)
      INTEGER A, B, C, D, E, F, DRY
      DATA EX/0.0/
      DATA STDX/1.0/
      DATA AMAX/107.8931375/
      DATA BMAX/12.9423650/
      DATA CMAX/22.2056646/
      DATA AMIN/108.7114750/
      DATA BMIN/12.6824934/
      DATA CMIN/8.6287619/
      DATE=JDATE
      COMTMX=SIN((DATE-AMAX)*.017214)*BMAX+CMAX
      COMTMN=SIN((DATE-AMIN)*.017214)*BMIN+CMIN
      DPTMP1=COMTMX-MAXTMP
      DPTMP2=COMTMX-MAXTMP
      DRY=2
      IF (PPT.EQ.0.0) DRY=1
      A=2
      B=1
      D=3
      E=1
      IF ((CORCOF(MON, DRY, 1).GE.CORCOF(MON, DRY, 2)).AND.
          (CORCOF (MON, DRY, 2).LT.CORCOF (MON, DRY, 3))) E=2
     1
      IF ((CORCOF(MON, DRY, 1).LT.CORCOF(MON, DRY, 2)).AND.
     1
          (CORCOF (MON, DRY, 1) .LT.CORCOF (MON, DRY, 3))) B=3
      C=A+B-2
```

```
F=D+E-2
      CALL NORMAL (EX, STDX, Z, IX)
С
      MAXTMP=COMTMX- (MEAN (MON, DRY, A) +CORCOF (MON, DRY, C) *STD (MON, DRY
     1, A) * (DPTMP1-MEAN (MON, DRY, B) ) / STD (MON, DRY, B) + STD (MON, DRY, A) *
     2SQRT (1-CORCOF (MON, DRY, C) *CORCOF (MON, DRY, C)) *Z)
      CALL NORMAL (EX, STDX, Z, IX)
      IF (E.EQ.2) DPTMP2=COMTMX-MAXTMP
      MINTMP=COMTMN-(MEAN(MON, DRY, D)+CORCOF(MON, DRY, F)*STD(MON, DRY
     1,D) * (DPTMP2-MEAN (MON, DRY, E)) / STD (MON, DRY, E) + STD (MON, DRY, D) *
     2SQRT (1-CORCOF (MON, DRY, F) *CORCOF (MON, DRY, F))*Z)
С
      IF (B.EQ.3) THEN
        CALL NORMAL(EX, STDX, Z, IX)
        DPTMP1=COMTMN-MINTMP
        MAXTMP=COMTMX-(MEAN (MON, DRY, A) +CORCOF (MON, DRY, C) *STD (MON,
       DRY, A) * (DPTMP1-MEAN (MON, DRY, B)) / STD (MON, DRY, B) + STD (MON, DRY
       , A) *SQRT (1-CORCOF (MON, DRY, C) *CORCOF (MON, DRY, C) ) *Z)
     2
      ENDIF
С
            IF MAXTEMP<MINTEMP
      IF (MAXTMP.LT.MINTMP) MINTMP=MAXTMP
С
      RETURN
      END
С
    1 SUBROUTINE NORMAL(EX, STDX, X, IX)
    2 SUM=0.0
    3 DO 5 I=1,12
    4 R=RAN3(IX)
    5 SUM=SUM+R
    6 X=STDX*(SUM-6.0)+EX
    7 RETURN
      END
      SUBROUTINE SOLAR (MO, JDATE, SOLRAD, PRECIP, IX)
      REAL LAT
      DIMENSION ALPHAR(12,2), BETAR(12,2), AMXDFR(12,2), AMNDFR(12,2)
      DIMENSION DUMMY(12,2)
      DATA ALPHAR/2.35977,1.73952,1.89422,1.57437,1.52689,1.37410,
     1 1.24369, 1.86584, 2.08193, 2.42681, 2.17762, 3.15666,
     2 .93493,.65516,1.10519,.76670,1.21589,1.57732,1.26820,.87245,
     3.71780,1.02736,.98304,1.28967/
     DATA BETAR/47.097,86.994,87.146,87.428,94.593,97.873,
     1 108.708,83.671,77.163,64.225,55.212,35.565,
     2.41211,.42085,.62028,.86884,1.58369,2.26590,1.48079,
     3 1.43430,.81731,.73398,.46317,.57846/
      DATA AMXDFR/322.174,443.239,494.035,625.810,597.599,495.384,
     1 721.570,636.049,515.607,433.516,367.809,287.920,
     2 355.558,464.122,587.685,641.052,703.001,722.877,
     3 680.708,618.535,574.365,505.953,367.613,301.511/
```

C

С

С C

```
DATA AMNDFR/46.9780,57.9558,29.9406,83.4325,79.1611,64.3380,
   1 63.5500,90.9249,84.6561,72.7867,59.6731,19.8606,
   2 64.401,67.070,92.983,91.988,93.781,105.976,
   3 63.905,122.050,130.120,108.116,73.411,75.236/
    LAT=35.5
    S1=SIN(LAT*.017453)
    C1=COS (LAT*.017453)
    DEC=.4092*SIN(.017202*(JDATE-82.2))
    RADV=1.0001+(.016723*SIN(.017202*(JDATE-94.5)))
    S2=S1*SIN(DEC)
    C2=C1*COS (DEC)
    HAS=ACOS (-S2/C2)
    A2=106.-LAT
    A1=(90.93*(S2+C2)/(RADV**2))-A2
    AMXRAD=(2.*(A1*HAS+A2*SIN(HAS)))/.2618
    IF (PRECIP.GT.0.0) GO TO 300
    JT=1
    GO TO 350
300 JT=2
320 G1=GAMMAD (ALPHAR, DUMMY, MO, JT, IX, U)
    G=GAMMAD (BETAR, DUMMY, MO, JT, IX, U)
    SDIFFB=(G1*(AMXDFR(MO,JT)-AMNDFR(MO,JT))/(G1+G))+AMNDFR(MO,JT)
    SOLRAD=(AINT(((AMXRAD-SDIFFB)+.05)*10.))/10.
    IF (SOLRAD.LE.0.0) GO TO 320
    RETURN
350 SDIFFR=GAMMAD (ALPHAR, BETAR, MO, JT, IX, U) +AMNDFR (MO, JT) -3.
    SOLRAD=(AINT(((AMXRAD-SDIFFR)+.05)*10.))/10.
    IF (SOLRAD.LE.0.0) GO TO 350
    RETURN
    END
    FUNCTION GAMMAD (ALPHA, BETA, MO, J, IX, U)
    DIMENSION ALPHA(12,2), BETA(12,2)
    INTEGER IX
    REAL U
    Z=0.0
    K=ALPHA (MO, J)
    F=K
    IF (K) 303,303,301
301 PROD=1.0
    DO 302 L=1,K
      U=RAN3(IX)
302 PROD=PROD*U
    Z=-ALOG (PROD)
303 D=ALPHA (MO, J) -F
    IF (D) 308,308,304
304 A=1.0/D
    B=1.0/(1.0-D)
    L=1
305 U=RAN3(IX)
    THE CONSTANT -50 IN 'UA' AND 'UB' BUST BE CHANGED TO -30 IF 'DOS'
   OPERATING SYSTEM IS USED. A MAINFRAME COMPUTER COULD HANDLE
    LARGER NEGATIVE NUMBERS (ie., A CRAY MX 48 CAN HANDLE A -150. THE
    IMPROPER USE OF THIS CONSTANT CAUSES OVERFLOWS.
    UA=-30/ALOG10(U)
    X=0.
    IF (A.LT.UA) X=U**A
```

C C

C C

С

С

```
U=RAN3(IX)
      UB=-30/ALOG10(U)
      Y=X
      IF (B.LT.UB) Y=U**B+X
IF (Y-1.0) 307,307,306
  306 L=L+2
      GO TO 305
  307 W=X/Y
      U=RAN3(IX)
      Y=-ALOG (U)
      GAMMAD = (Z + W * Y) * BETA (MO, J)
      RETURN
  308 GAMMAD=Z*BETA(MO,J)
      RETURN
      END
С
С
C SUBROUTINE FROM PRESS ET AL. 1986
С
      "Numerical Recipes", Cambridge Univ. Press.
С
      This subroutine returns a random deviate between 0 and 1. Set the
С
      IDUM value to a negative value to initialize or reinitialize the
С
      sequence.
FUNCTION RAN3 (IDUM)
          IMPLICIT REAL*4(M)
          PARAMETER (MBIG=4000000., MSEED=1618033., MZ=0., FAC=2.5E-7)
      PARAMETER (MBIG=100000000, MSEED=161803398, MZ=0, FAC=1.E-9)
С
      DIMENSION MA(55)
      DATA IFF /0/
      IF (IDUM.LT.0.OR.IFF.EQ.0) THEN
        TFF=1
        MJ=MSEED-IABS (IDUM)
        MJ=MOD (MJ, MBIG)
        MA(55)=MJ
        MK=1
        DO 11 I=1,54
          II=MOD(21*I,55)
          MA(II)=MK
          MK=MJ-MK
          IF (MK.LT.MZ) MK=MK+MBIG
          MJ=MA(II)
        CONTINUE
11
        DO 13 K=1,4
          DO 12 I=1,55
            MA(I) = MA(I) - MA(1 + MOD(I + 30, 55))
            IF (MA(I).LT.MZ) MA(I) = MA(I) + MBIG
12
          CONTINUE
13
        CONTINUE
        INEXT=0
        INEXTP=31
        IDUM=1
      ENDIF
      INEXT=INEXT+1
      IF (INEXT.EQ.56) INEXT=1
      INEXTP=INEXTP+1
      IF (INEXTP.EQ.56) INEXTP=1
      MJ=MA(INEXT)-MA(INEXTP)
      IF (MJ.LT.MZ) MJ=MJ+MBIG
```

MA(INEXT)=MJ RAN3=MJ\*FAC RETURN END

# File ERSTAT.DAT

1.00000000 0.72464059	0.66343066 3.48777983	0.51873279	0.80010154	0.71342544 7.34204750	0.64122941 7.55412512
6.18237292 5.94044062	1.36617594	0.92443094	0.93812623	7.56681025	7.35799499
2.00000000	0.74022898	0.59062247 3.76801596	0.72990529 0.14290630	0.75084543 7.12140607	0.69202968
6.05989048		-1.42868197		7.39764732	7.15770703
5.57696925 3.00000000	0.64817064	0.55581728	0.69600255	0.66243955	0.60930658
5.33937934	-0.39148038 -2.46033884	1.34395315	-1.89937838 -1.32056641	6.26443106 6.52008436	6.22848726 6.33677017
5.91091085 4.00000000	0.64101627	0.55379128	0.63983308	0.60401859	0.60000953
0.65383760 4.74871389	0.46719072 -1.63745787	1.32025034 -1.79480285	-0.82990022 -0.52659009	5.03657338 4.98284314	4.83831859 4.81670073
5.56094539 5.00000000	0.45550315	0.39014292	0.54894137	0.59934396	0.56273409
0.61604068 3.05412153	2.21568931 0.87463092	2.92355352 0.59311446	1.42932319 1.13550767	3.92585913 3.62395572	3.95529575 3.45935665
4.50091408 6.00000000	0.63578886	0.37782203	0.47280285	0.66588448	0.52529318
0.56011233 2.79201306	2.54792597 1.25882623	3.12258565 0.97457784	2.06798474 0.43096561	3.35943197 2.82218314	3.29101092 2.61259528
3.51597447 7.00000000	0.59254291	0.34369332	0.58004909	0.80451157	0.58130709
0.63036568 2.34518476	1.52698286 -0.10222247	2.71107632 -0.34549864	1.16908135 -0.19054153	3.23358574 3.02783583	3.30111831 2.87307085
3.14159855 8.00000000	0.57661752	0.19249457	0.38327599	0.79210681	0.58472000
0.62127917 2.01321901	0.38369202 -0.94798212	1.45793598 -1.26594179	0.79486273 -0.76695516	3.97705699 3.20024925	3.62791807 3.11669446
3.14716839 9.00000000	0.62546872	0.36017085	0.54881015	0.79138865	0.64377843
0.67526742 3.51708070	1.12832628 -1.03244356	3.00059889 -1.52475459	0.65293400 -0.99226172	4.98039699 4.45526681	4.90437243 4.20055626
4.44862274 10.0000000	0.52888783	0.34256695	0.60741166	0.62920441	0.49206224
0.55086098	0.24511153 -1.47717331	2.33067930 -1.92058572	-0.61407801 -0.42109401	4.96842929 4.68415446	4.85904807 4.40134173
5.11946803 11.00000000	0.62077541	0.35200388	0.56315755	0.66682853	0.55782866
0.62833767	2.52827683 -0.04260788	4.40825848 -0.37321128	0.32401703 0.16201314	6.24472221 5.82404837	5.12957529 5.83571909
5.09455476	0.66576303	0.46006045	0.74798588	0.67166734	0.62794513
0.67992914 5.63709183 5.39604132	3.27313377 0.33788958	4.46503693 0.20622878	0.20416466 0.88450705	6.53038968 6.62016278	6.71983712 6.52270325