An Application of Optimal Control Techniques to Agricultural Policy Analysis

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AN APPLICATION OF OPTIMAL CONTROL TECHNIQUES TO AGRICULTURAL POLICY ANALYSIS

James W. Richardson and Daryll E. Ray*

Legislation to support farm prices and incomes was first enacted in 1933. Although the names and procedural details of farm support legislation have changed through time, the basic support devices have remained the same. Each act has included a mix of supply controls, price supports through nonrecourse loans and direct payments. The legislation provides guidelines and boundaries but usually authorizes the Secretary of Agriculture to set the exact levels of the various support devices.

The Secretary is faced with conflicting interest among farmers, consumers and taxpayers with regard to farm programs. Farmers want high incomes with minimal governmental interference. Consumers want a stable supply of food at low prices. Taxpayers want low treasury costs. The overall economic policy of the administration also comes into play since farm programs affect the rate of general inflation, the size of the federal budget and the balance of payments.

Program specification by the Secretary or Administration may be viewed as a constrained maximization problem. In this setting, the Secretary would generate the greatest or desired additional farm receipts given enabling legislation and expected market conditions. The constraints might be administration targets for food and general price inflation and the level of government expenditures. Obviously, expectations about future domestic supply, export demand and other market conditions may not materialize.

This stochastic nature of agricultural markets may influence the selection of the program mix. Political acceptability and other intangables may also be part of program deliberations. Even though the process of program specification includes subjective components, it has many features of an optimal control problem. This study focuses on the application of control theory as a conceptual aid in farm policy development and analysis.

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Purpose of the Study

The overall purpose of this study is to investigate the potential using optimal control procedures as an analytical technique for analyzing farm policy. Optimal control theory has an unique nomenclature and methodology for addressing a problem. One objective is to describe how an agricultural policy problem can be cast into the optimal control methodology. A second objective is to apply an optimal control technique to select values for loan rates, target prices and other policy variables under assumed conditions including a hypothetical prespecified criterion for measuring attainment of policy goals.

Nomenclature of Optimal Control Theory

Optical control theory is a mathematical technique to determine values for decision variables that cause a particular system to satisfy a given set of constraints and optimize a given performance measure [Jacobs, Kirk, Sage]. Control might be applied to a real system itself (e.g., rocket deployment) or to a model of a system (e.g., an agricultural sector).

As defined, optimal control theory encompasses standard optimization techniques such as multiperiod programming models and even static linear programming as special cases. Control theory generalizes the optimization framework to include positivistic or normative behavioral rep sentatives of systems with continuous or discrete timing of data. While static optimization may fall within the broad definition of control theory, the theoretical evolution and typical application of control theory have been with dynamic models.[1]

The mathematical representation of the system is not altered by the application of control. Once the mathematical structure of the system is formulated, the optimal control procedure uses the behavioral relationships to search for feasible levels of key decision (control) variables that will best achieve the specified objective.

Formulation of an optimal control problem requires:

1) A mathematical model of the system to be controlled,
2) Designation of boundary constraints or statements of feasible ranges for the decision (or control) variables,
3) A statement of the performance measure, that is, a mathematical relationship for measuring the performance of the system and
4) Selection of a particular numerical optimal control procedure [Kirk].

[1] Numerous economists have applied optimal control theory to dynamic discrete-time models including: Cooper and Fischer; Taylor and Talpaz; Frohberg and Taylor; Rausser and Freebairn; Theil; Rindyck; Kendrick and Taylor; Chow; Bray; Pindyck and Roberts; Livesey; Tintner; Rauler-son and Longhorn; Rausser and Howitt; Trapp; and Arzac and Wilkinson.

In control theory, the endogenous variables in the behavioral model are referred to as state variables. The subset of state variables used in the performance measure are called output variables. Exogenous variables that can not be influenced by the decision maker are denoted as uncontrollable exogenous variables. Exogenous variables that can be manipulated by the decision maker are called controls. Controls are a function of time and/or the state variables. Values for the control variables over the period analyzed constitute the control path (or history) and the values for the state variables form the state trajectory [Kirk]. When the controls are a function of only time, the system is an open-loop control problem. If controls are a function of state variables, the system is in a closed-loop form and allows system feedback in problem solution.

Boundary constraints are usually specified to limit the control and selected state variables within boundaries established by the user in light of physical, economic, social and political limits. Hence, the boundaries establish the feasible set of control values (and acceptable associated values for key state variables) that can be spanned in the search for an optimal control path.

The performance measure assigns an unique real number for each state trajectory (set of state variable values) associated with an admissible control path. ("Admissible" means the control path satisfies all constraints.) The performance measure is defined by a mathematical equation that sums weighted functions of the output variables for each state trajectory generated by the system being controlled. The functional form of the performance measure varies somewhat depending upon the type of problem being analyzed (e.g., tracking of a specified trajectory through time, reaching specified targets in the final year, optimizing the annual weighted values of output variables, etc.). The optimal control procedure solves for the set of control values (or control path) that causes the performance measure to be optimized.

Theoretical descriptions of control problems generally utilize first-order differential equations. However, numerical techniques are available that do not require conversion of model equations to this form. Numerical solution of a control problem is usually accomplished with the use of direct-search methods. Kirk and Swann describe several alternative direct-search methods for solving constrained optimization problems.[2] One such method is the Complex Procedure developed by Box. The procedure can handle a number of controls, allows for closed-loop feedback and does not require the equations in the system to be in first difference form.

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[2] For detailed discussion of direct-solution techniques see: Chow; Kirk; Sage; or Intrilligator. For a discussion of numerical optimization techniques see: Box; Fair; Goldfeld et al.; Holbrook; Kuester and Mize; Kendrick and Taylor; and Swann.

The Complex Procedure maximizes the performance measure (F) subject to the boundary constraints:

$$\text{Maximize: } F(X_1, X_2, X_3, \ldots, X_n)$$
$$\text{Subject to: } G_j \leq U_j \leq H_j,\ j = 1, 2, \ldots, m$$

where $X_1, \ldots, X_n$ are output variables; $U_1, \ldots, U_m$ are control inputs; G_j and H_j are lower and upper boundary constraints respectively for the j^{th} control.

Values for admissible controls are used as input in the mathematical model of the system to obtain predicted values for the state variables. The output variables, a subset of the state variables, are used to compute the value of the performance measure. The control mechanism systematically searches the surface of the performance measure for its global maximum by iteratively selecting control paths that increase the value of the performance measure. The solution is at a maximum when a change in any control variable results in reducing the value of the performance measure. The control path associated with the maximum value of the performance measure is the optimum set of values for the control variables.[3]

To insure that the final solution is at the global maximum for the given performance measure, the problem should be run several times. Each time a different set of starting values for the control variables should be used so that the procedure is forced to search a different set of control paths. If the procedure returns the same answer each time, the analyst can be fairly certain of having found the global maximum.[4]

Applying Optimal Control Theory to Farm Policy Analysis

As indicated earlier, determination of policy instrument values for farm programs can be cast into a constrained maximization problem. Viewed in this way, the objective could be to maximize farm revenue subject to constraints on food price inflation, government program costs and other economic as well as political considerations. The remainder of this section discusses the specific methodology used to address a farm policy optimization problem within the context of optimal control theory. The discussion is organized around the procedural steps given earlier for formulating a control problem (model specification, selection of boundary constraints for the relevant controls, specification of a performance measure and selection of an optimization routine).

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[3] Theoretical descriptions of optimal control theory, using advanced mathematical procedures are available in the following references: Chow; Pindyck; Dorfman; Kirk; Arrow; Sage; and Intrilligator. A detailed description of optimal control theory, using simple mathematical procedures is available in Richardson, Ray and Trapp.

[4] Each farm program evaluated in this study was run three times to ensure that a global maximum had been reached.

Model Specification

The model used in this study is the National Agricultural Policy Simulator (POLYSIM), a partially disaggregated macro model of the U. S. agricultural economy with the following crop and livestock categories: feed grains, wheat, cotton, soybeans, cattle and calves, hogs, sheep and lambs, chickens, turkeys, eggs and milk. The impacts on these commodities and on total farm incomes, consumer expenditures for food and government costs can be simulated with the model. This is done with specified levels of the following agricultural policy instruments: target prices and resulting deficiency payments, loan rates, alternative CCC buy-and-sell criteria, allotments, voluntary or mandatory set-aside acreages, per acre payment schedules for voluntary set-aside, program participation rates and acreage or production quotas.

The policy instruments selected as control variables for this analysis are loan rates, target prices and set-aside acreages for the crops. Since a detailed description of the mathematical relationship in POLYSIM is available elsewhere [Ray and Richardson], the model equations are not repeated here.

Boundary Constraints

Where possible, the Food and Agriculture Act of 1977 was used to develop boundary constraints for the control variables. The lower boundary constraints for wheat and corn loan rates are set at the legal minimums specified in the Act. The upper loan rate boundaries are the estimated target prices authorized by the Act (Table I). The target price estimates, based on the provisions of the 1977 Act, also are used as the lower boundaries for the target price variables in the analysis.

The 1978 upper boundaries on target prices are set at about 150 percent of the 1978 lower boundaries. The upper 1979-1981 target price con-

Table I. Upper and Lower Boundary Constraints for Loan Rates for Wheat, Corn and Cotton, 1978-81.

	Wheat		Corn		Cotton	
	Lower	Upper	Lower	Upper	Lower	Upper
Year	$/bu.		$/bu.		$/lb.	
1978	2.00	3.00	1.75	2.10	.37	.52
1979	2.00	3.16	1.75	2.21	.37	.55
1980	2.00	3.34	1.75	2.34	.37	.58
1981	2.00	3.52	1.75	2.47	.37	.61

Source: Lower boundaries for wheat and corn 1978-81 are given in the Food and Agriculture Act of 1977. The proportion of minimum to announced 1977 loan rates for wheat and corn (88%) was assumed in setting the minimum loan rate for cotton. Upper boundaries are estimated target prices based on the provisions of the 1977 Act.

Table II. Upper and Lower Boundary Constraints for Target Prices for Wheat, Corn and Cotton, 1978-81.

	Wheat		Corn		Cotton	
	Lower	Upper	Lower	Upper	Lower	Upper
Year	$/bu.		$/bu.		$/lb.	
1978	3.00	4.00	2.10	3.10	.52	.75
1979	3.16	4.21	2.21	3.26	.55	.79
1980	3.34	4.45	2.34	3.45	.58	.84
1981	3.52	4.69	2.47	3.64	.61	.88

Source: Lower boundaries for 1978 are from the 1977 Act. The upper boundaries for 1978 for set at about 150 percent of the 1978 lower boundaries. The 1979, 1980 and 1981 values were computed using the escalation clause of the 1977 Act assuming annual percentage increases in variable production costs of 5.4, 5.7 and 5.4 percent during the three-year period.

Table III. Upper and Lower Boundary Constraints for Acreage Set-Aside Levels for Wheat, Feed Grains and Cotton, 1978-81.

	Wheat		Feed Grains		Cotton	
	Lower	Upper	Lower	Upper	Lower	Upper
Year	m. ac.					
1978	0	24.8	0	37.7	0	3.2
1979	0	24.8	0	37.7	0	3.2
1980	0	24.8	0	37.7	0	3.2
1981	0	24.8	0	37.7	0	3.2

Source: The Agricultural Act of 1977 specifies that the maximum acreage set-aside cotton can not exceed 28% of planted acreage in the previous year. For cotton, planted and harvested acreages are nearly equal to harvested acreage. For this analysis, the maximum set-aside for cotton is 28% of harvested acreage in 1977. For feed grains and wheat, planted acreage is often much larger than harvested acreage so the maximum set-aside for these crops is set at 35% of harvested acreage in 1977.

straints were estimated using the escalation clause of the Act (Table II). The maximum set-aside acreages were set at 35 percent of 1977 for wheat and feed grains. The maximum set-aside was set at 28 percent for cotton (Table III). The cotton set-aside limit is specified in the Act in terms of planted acreage. Since harvested and planted acreages differ substantially for grains, a higher set-aside proportion of harvested feed grains and wheat acreages were specified.

Performance Measure

Rausser and Freebairn [1974b] outline a three stage approach for identifying a performance measure for use in an optimal control problem. The first stage is to determine the relevant variables to include in the performance measure. In this study farm producers, consumers and taxpayers have been identified as three broad political interest groups in the area of farm policy. Farmers want farm income to be as high as possible hile taxpayers want to hold down the level of payments to farmers and ..e administration costs of farm programs. Consumers are interested in a stable and abundant supply of food at reasonable prices. With this in mind, output variables used in the performance measure for this study include farm income, farm program related government expenditures to farmers and the cost of food for consumers. Other output variables of particular importance for a specific policy objective can be added as needed.

The second stage of the Rausser-Freebairn approach is to determine the mathematical form of the performance measure. One widely-used functional form is Theil's quadratic preference function. With this performance criterion, the control path is selected which minimizes the squared deviations of target and realized output values. This functional form requires the user to provide single valued target levels for each output variable for each time period of the analysis. A similar functional form is used for the performance measure in this study. However, the scalar target for each output variable in the j^{th} period is replaced with an acceptable range of values and, again in contrast to Theil's function, different penalties are allowed for positive and negative deviations from the designated range.

This more general mathematical description for the performance easure, as applied here, can be expressed as:

If lower bound of specified range is violated—

$$JL_{ij} = H_{ij} \,|\, Y_{ij} - LB_{ij}|$$

If upper bound of range is violated—

$$JU_{ij} = I_{ij} \,|Y_{ij} - UB_{ij}|$$

$$\text{Maximize:} \quad J = \sum_{j=1}^{4} \left(NFY_j + NLY_j - \sum_{i=3}^{n} (JL_{ij} + JU_{ij}) \right)$$

where H_{ij} is the weight for output variable Y_i violating its lower boundary LB_i in period j; I_{ij} is the weight for output variable Y_i violating its upper boundary UB_i in period j; NFY_j is the level of realized net farm income in period j; and NLY_j is the level of net income for livestock producers in period j, defined here as livestock income after deducting variable production expenses.[5] No ranges are specified for the farm income variables since the performance measure is designed to maximize farm income subject to the other variables in the function.

The third step in specifying the performance measure is to estimate the parameter weights for the output variables (H_{ij} and I_{ij}). Bray suggests

[5] Livestock income (about $18 billion per year as defined) is included in the performance measure to prevent the control mechanism from increasing net income for the crop sector without regard to the impacts on the livestock sector. Farm programs that support feed prices at high levels result in high costs of feed stuffs to livestock producers which immediately reduce the net incomes for livestock producers and cause cut-backs in livestock production in the following years.

that the parameter weights may be determined through interviews with decision makers and government planners. Rausser and Freebairn [1976b] include Bray's method in their direct approach and add two other approaches: the indirect approach which involves study of past decisions and the arbitrary approach in which the analyst assigns arbitrary values for the parameter weights based upon his understanding of the system being controlled.

With the performance function used here, the analyst also must identify ranges of acceptable values for the output variables. While using range-approach is not as demanding on the policy maker or analyst as providing single valued targets for each output variable for the analysis period, it still requires information that is not readily available.

Hypothetical ranges and parameter weights for the output variables used in this study are presented in Table IV. The ranges and parameter weights only illustrate plausable values that facilitate application of optimal control techniques for agricultural policy analysis.

The cost of food to consumers in the U.S. or consumers' total expenditures for food is one of the primary output variables in the performance measure. The baseline values for consumers' total expenditures for food by commodity analysts in CED, USDA are presented in Table IV.[6] These values are used as lower boundaries, LB_{3j}, for the acceptable ranges for food costs.

The upper boundaries for the total food cost ranges, UB_{3j}, are set 101 percent of the baseline values. The baseline values assume a per capita increase in food costs of three to four percent per year over the study period. So the one percent increase, used to obtain the upper boundaries of the acceptable ranges, puts the annual increase in food costs at about four to five percent.

Values of total food costs are free to move within the designated ranges ($188,300 million and $190,183 million in year 1) without changing the performance measure. Values outside these ranges, of course, do cause changes in the performance measure. The magnitude of the changes is determined by the parameter weights identified for the output variables as well as the extent to which the variable values miss the acceptable ranges.

Two sets of parameter weights are presented in Table IV for several of the output variables. The parameter weights for consumers' total expenditures for food values that are below and above the acceptable ranges are +2.0 and —2.0 for set 1 and +4.0 and —4.0 for set 2, respectively.

Parameter weight +2.0 (H_{3j} in set 1) implies that each dollar decrease in total food costs below the lower boundary of the specified range

[6] USDA baseline data for 1978-81 used in this analysis were developed in July, 1977 based on information available at that time.

able IV. Upper and Lower Boundary Levels and Their pective Weights for an Aggregate Agricultural Perf ance Measure Used with POLYSIM

Output Variables$_i$ for years$_j$	Projected Values[1] by CED Commodity Analysts (Baseline Values)	Acceptable Range LB_{ij}	Acceptable Range UB_{ij}	Parameter Weights Set 1 Lower Boundary (H_{ij})	Parameter Weights Set 1 Upper Boundary (I_{ij})	Parameter Weights Set 2 Lower Boundary (H_{ij})	Parameter Weights Set 2 Upper Boundary (I_{ij})
Consumers Total Expenditures for Food (m. $) (i=3)[2]							
Year 1	188,300.0	188,300.0	190,183.0	+2.0	—2.0	+4.0	—4.0
Year 2	192,500.0	192,500.0	194,425.0	+2.0	—2.0	+4.0	—4.0
Year 3	202,600.0	202,600.0	204,626.0	+2.0	—2.0	+4.0	—4.0
Year 4	211,400.0	211,400.0	213,514.0	+2.0	—2.0	+4.0	—4.0
Total Government Payments to Farmers (m. $) (i=4)							
Year 1	2,054.5	850.0	3,700.0	0.0	—1.5	0.0	—4.0
Year 2	3,025.8	850.0	3,700.0	0.0	—1.5	0.0	—4.0
Year 3	3,362.9	850.0	3,700.0	0.0	—1.5	0.0	—4.0
Year 4	3,654.5	850.0	3,700.0	0.0	—1.5	0.0	—4.0
Total CCC Interest and Storage Costs (m. $) (i=5)							
Year 1	310.4	0.0	600.0	0.0	—1.5	0.0	—4.0
Year 2	452.0	0.0	600.0	0.0	—1.5	0.0	—4.0
Year 3	598.8	0.0	600.0	0.0	—1.5	0.0	—4.0
Year 4	739.2	0.0	600.0	0.0	—1.5	0.0	—4.0
Ending Year Stocks of Feed Grains (m. t.) (i=6)							
Year 1	70.4	30.0	60.0	—104.0	—82.5	—104.0	—82.5
Year 2	82.6	30.0	60.0	—104.0	—82.5	—104.0	—82.5
Year 3	87.5	30.0	60.0	—104.0	—82.5	—104.0	—82.5
Year 4	89.3	30.0	60.0	—104.0	—82.5	—104.0	—82.5

Table IV. Continued

Output Variables$_i$ for years$_j$	Projected Values[1] by CED Commodity Analysts (Baseline Values)	Acceptable Range		Parameter Weights Set 1		Parameter Weights Set 2	
		LB_{ij}	UB_{ij}	Lower Boundary (H_{ij})	Upper Boundary (I_{ij})	Lower Boundary (H_{ij})	Upper Boundary (I_{ij})
Ending Year Stocks of Wheat (m. bu.) (i=7)							
Year 1	1,539.0	600.0	1,200.0	—3.4	—2.8	—3.4	—2.8
Year 2	1,827.0	600.0	1,200.0	—3.4	—2.8	—3.4	—2.8
Year 3	2,112.0	600.0	1,200.0	—3.4	—2.8	—3.4	—2.8
Year 4	2,374.0	600.0	1,200.0	—3.4	—2.8	—3.4	—2.8
Ending Year Stocks of Cotton (m. bales) (i=8)							
Year 1	4.3	2.0	4.0	—284.8	—268.6	—284.8	—268.6
Year 2	4.2	2.0	4.0	—284.8	—268.6	—284.8	—268.6
Year 3	4.5	2.0	4.0	—284.8	—268.6	—284.8	—268.6
Year 4	4.1	2.0	4.0	—284.8	—268.6	—284.8	—268.6

[1] Baseline values were developed by the USDA based on information available July 1977.

[2] Realized net farm income and income to livestock producers after variable production expenses are output variables 1 and 2, respectively. Since incomes are to be conditionally maximized, no ranges are specified for these output variables.

increases the value of the performance measure by two units. So if total food costs in the U. S. decreased below the boundary level by an average of $1 per consumer, the performance measure would be increased by about 440 million units with a population estimate of 220 million people. The parameter weight —2.0 for values that exceed a designated range (I_{3j} for set 1) implies that each $1 increase in total food costs above the range reduces the value of the performance measure by two units.

The second set of parameter weights for food costs reflect a trade-off of 1:—4 or twice the impact of the first set. The first set of parameters are used for all farm program structures analyzed in the study. One of the farm programs is also analyzed using the second set of weights to evaluate the sensitivity of the optimal solution to changes in the performance measure.

Government expenditures for the U. S. farm program are separated into two categories, direct payments to farmers and Commodity Credit Corporation (CCC) storage and interest costs. The annual baseline values for government payments and CCC costs over the study period are presented in Table IV. The lower boundaries for acceptable ranges of direct payments to farmers, LB_{4j}, are set at the baseline estimates of government payments for miscellaneous farm programs (basically excludes deficiency and diversion payments for wheat, feed grains and cotton). The upper boundaries, UB_{4j}, are set at $3,700 million, which approximates the baseline estimate of total payments during the last year of the study, 1981.

The designated range for CCC storage and interest costs is zero to $600 million. The upper value is the estimated storage and interest cost for holding a reserve of 30 million tons of feed grains and 600 million bushels of wheat.[7]

Parameter weights for total government payments to farmers and CCC storage and interest costs are assumed to be equal for this study since both variables represent costs to the taxpayer for having farm programs. The lower and upper boundary parameter weights (H_{ij} and I_{ij}, $i = 4, 5$) for government expenditures are 0.0 and —1.5 for set 1 and 0.0 and —4.0 for set 2, respectively (Table IV).

The upper boundary parameter weights —1.5 and —4.0 indicate that a dollar of government expenditures above the designated ranges (UB_{4j} and UB_{5j}) causes the performance measure to be decreased by more than a dollar or $1.50 and $4, respectively. The true weights would represent the marginal disutility that taxpayers receive for each unit of expense above the range boundaries of $3,700 million and $600 million, respectively. A weight of —1.5 assumes a disutility of $1.50 for each additional dollar of government expenditure.

[7] CCC reserve levels are based on Waugh and Tweeten, Kalbfleisch and Lu. Waugh recommends a feed grain reserve of 30-40 million tons and a wheat reserve of 550 to 650 million bushels. Tweeten et al. recommends a wheat reserve of 600 million bushels.

Total ending year carryover of feed grains, wheat and cotton a[illegible] included in the performance measure (J) to penalize the value of the performance measure when shortages or surpluses of these crops are encountered. The lower boundaries for acceptable carryover ranges are set at 30 million tons of feed grains, 600 million bushels of wheat and two million bales of cotton (Table IV). The upper range boundaries are set at about twice the lower boundary levels for all three crops.

The parameter weights for penalizing the performance measure when ending year carryovers of feed grains, wheat and cotton exceed their ranges are stated in terms of the costs to society associated with a "surplus carryover." (A "surplus carryover" is defined as an ending year carryover in excess of the relevant upper boundary level.)

The costs to society of a surplus carryover is the sum of direct costs (interest and storage charges) and opportunity costs for the resources used to produce the surplus carryover. Annual storage costs are about \$7.20 per ton for feed grains and \$0.22 per bushel for wheat. Annual storage cost for cotton is about \$5.40 per bale. (Storage costs used here are based on values used by commodity analysts in CED, USDA to estimate total CCC storage costs.) Interest charges for CCC storage are 7.5 percent of the value of the stock, where value is based on the average loan rate.

The opportunity cost to society for surplus carryovers is the value [illegible] resources used in producing the surplus stocks. Theoretically, the opportunity cost (marginal cost of production) for producing excess carryovers is measured as the area under the supply curve, associated with the quantity of excess production [Tweeten]. It is assumed in this study that the average per unit marginal (variable) cost of production for excess carryover is the particular crop's loan rate.

Using the announced loan rates in the Agricultural Act of 1977 (\$2.00/bushel for corn, \$2.35/bushel for wheat and \$0.51/lb. for cotton) the marginal (variable) costs of production for surplus carryovers are \$70 per ton of feed grains, \$2.35 per bushel of wheat and \$244.80 per bale of cotton. Thus, for feed grains the upper boundary parameter weight (I_{ij}) is —82.5 per ton or the sum of storage costs (\$7.20/ton), interest costs (\$5.30/ton) and the marginal (variable) cost of production (\$70/ton). So for every ton of feed grains in ending year carryover above 60 million tons, the performance measure is reduced 82.5 units.

The parameter weight for exceeding the range of acceptable ending year stocks of wheat is —2.8 or for each bushel of wheat above 1,200 million bushels the performance measure is reduced by 2.8 units. The parameter weight for ending year stocks of cotton exceeding the upper boundary is —268.6 (Table IV).

The parameter weights for ending year carryovers of feed grains, wheat and cotton that fall below their respective ranges should be larger, in absolute terms, than the respective parameter weights for exceeding the designated carryover ranges. Higher parameter weights are justified because as ending year stocks get smaller the greater the possibility that they will be needed to meet domestic demands. The lower boundary parameter weights (H_{ij}) for feed grain, wheat and cotton ending year carryovers are —104.0, —3.4, and —284.8, respectively (Table IV). The lower boundary parameter weights used in the study reflect higher storage costs than the upper boundary weights to account for society's increased marginal value of holding a reserve in the face of relatively tight supplies.

Modifications Made to POLYSIM

To incorporate Box's Complex Procedure, an iterative optional control procedure, in the POLYSIM model required several changes that affect data input and the order in which model components are executed. The original data cards required for POLYSIM are described in Richardson and Ray [1975a] and coding instructions for additional data cards required for the Control Theory Option are presented in Appendix B of Richardson [1978].

The optimal control version of POLYSIM begins with a call to the complex algorithm to select a control path or a time scripted set of admissible values for the policy variables (Figure 1). The policy variable values are inputed to the agricultural model which estimates corresponding values for the endogenous variables of the simulation model, that is, the corresponding state trajectory. Estimated values for the output variables are used in the performance measure to obtain a unique real number for evaluating the particular control path.

The process of selecting control paths, executing the POLYSIM subroutines and evaluating the performance measure is repeated until the optimal control path is found. When the optimal control path is determined, the program re-enters POLYSIM, executes the subroutines in the model and prints the results in the normal output tables (Figure 1).

Farm Programs Selected for Analysis

Four farm programs are analyzed using the Control Theory Option in POLYSIM to demonstrate the use of the technique for selecting values of particular farm policy variables. The farm programs analyzed are the following: I—a price and income support program, II—a price and income support program with voluntary acreage diversion, III— a price and income support program with voluntary acreage diversion and in-

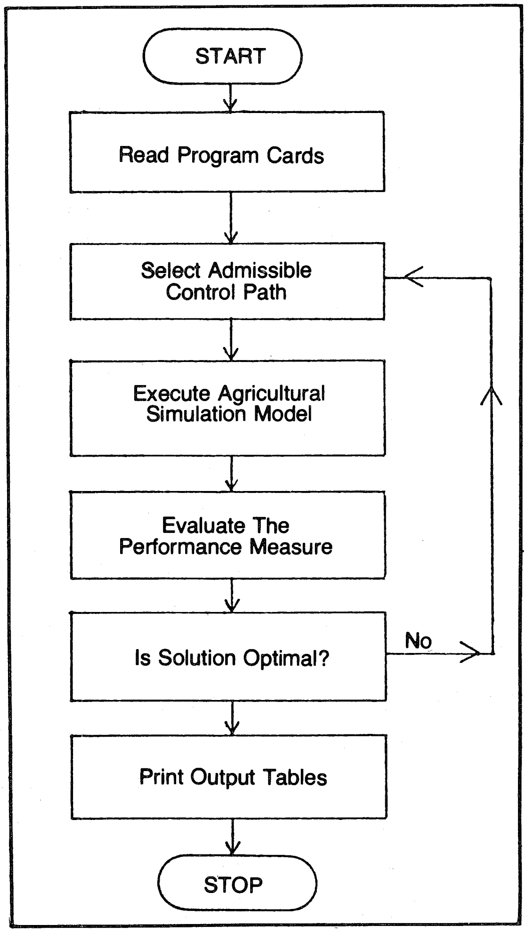


Figure 1. Simplified Flowchart of POLYSIM with Control Theory Option.

creased export demands for feed grains, wheat and cotton during the first year simulated. Each farm program is analyzed for the four year period of 1978-81.

Farm program I, a price and income support program, guarantees feed grain, wheat and cotton farmers a minimum price and cash receipts for their eligible production. The minimum prices to participating farmers are the loan rates for the respective crops. If production is sufficiently large to cause average prices to fall below the loan rate, the Commodity Credit Corporation (CCC) is authorized to make loans to farmers using their crops, valued at the loan rate, as collateral. In the event that prices continue to be low, the farmer can turn the collateral over to the CCC and the loan is considered paid.

Additional income support may be provided to farmers in the form of a deficiency payment to make up the difference between the price participating farmers receive for their crops in the market and the corresponding crop target price. To summarize for program I, the control mechanism selects loan rate and target price values for feed grains, wheat and cotton that maximize the performance measure in Table IV. The loan rates and target prices must be within the upper and lower boundary constraints for these controls (Tables I and II).

Farm programs II and IV have the price and income support provisions found in program I and in addition have an acreage diversion provision. Acreage diversion programs usually require that participating farmers divert a percent of their land to soil conserving uses. Farmers complying with the voluntary acreage diversion requirements are then eligible for price and income supports as well as a payment for diverting the land. A non-zero payment rate for diverted acreage is used to insure participation in the programs.

The control mechanism selects acreage diversion levels and loan rates for feed grains, wheat and cotton that maximize the performance measure in Table IV. The loan rates and acreage set-aside levels must be within the upper and lower boundary constraints for these control variables (Tables I and III). Target price levels from the 1977 Agricultural Act are used as the target prices in programs II and IV for the income support provision.

Based on the projections of commodity analysts in the USDA, the ending year carryovers of feed grains, wheat and cotton assuming no set-aside are expected to increase annually over the next four years from their relatively high levels in 1976 and 1977. Given this prospect, the diverted acreage control variables for farm program II will most likely be set by the control mechanisms at relatively high levels in an attempt to reduce ending year carryovers to the upper boundary levels specified in the performance measure (Table IV). The situation is made slightly more complicated for program IV by assuming higher 1978 exports than is assumed in running program II and thereby reducing the excess supply in the first year simulated.

A grain reserve program III is analyzed to demonstrate how control theory can be used to select loan rates and acreage set-aside levels for feed grains, wheat and cotton that would allow the CCC to maintain a relatively fixed reserve of grains. A grain reserve of 20 million tons of feed grains and 500 million bushels of wheat is assumed to be established in 1977 by the CCC, and the performance measure is modified slightly to encourage the CCC to hold the stocks over the four-year period simulated, 1978-1981. The control variables, loan rates and acreage diversion levels for the three crops, are constrained to the upper and lower boundary constraints for these variables (Tables I and III).

For the control mechanism to maximize the performance measure used in this study, it must select values for the control variables (loan rates, target prices and acreage set-aside levels for feed grains, wheat and cotton in 1978-81) based on their estimated impacts on the state variables in POLYSIM and the output variables in the performance measure. The control mechanism tries to raise net farm income as high as possible over the four-year period while at the same time minimizing penalties that accrue when other output variables go outside their acceptable ranges. The control mechanism considers one unit of added income (or penalty) in 1978 equal to one unit of added income (or penalty) in 1979 or 1981 since the parameter weights are not discounted for time.

The complexity of the solution process becomes evident by tracing the model interactions that occur in the selection of only one control or instrumental variable value, say, the 1978 loan rate for wheat. To determine the so-called "optimal value" for the wheat loan rate in 1978, the control mechanism must consider the immediate impacts in 1978 but also the longer run impacts in 1979-81 on the endogenous variables in the model and particularly the impacts on the output variables. The immediate impacts on the following endogenous variables must be considered: the market price of wheat, quantity of domestic and export demands and wheat cash receipts as well as impacts on the output variables in the performance measure.

The indirect 1979-81 impacts that must be considered—because of their linkages to the output variables in the performance function—include harvested acreage and supply of wheat, feed grains, cotton and soybeans, market prices of wheat, feed grains cotton and soybeans; the quantity of domestic and export demands for the four model crops; livestock production and prices (resulting from changes in grain prices) and income for all crops and livestock. Changes in these variables affect the level of ending year inventories, CCC costs, income to livestock producers, total realized net farm income and food costs—all of which are variables considered in the performance measure.

Only the selection of the 1978 wheat loan rate is considered above. Actually, the control mechanism simultaneously selects values for the loan rates of corn, wheat and cotton in 1978, 1979, 1980 and 1981, after considering the impacts of the loan rates on the output variables in the performance measure. The immediate and longer run interrelationships described for 1978, thus, become confounded with the immediate and longer run impacts because of selecting loan rates in each of the remaining years. The complexity mushrooms with the introduction of additional controls such as target prices for farm program I and diverted acreage for the other programs.

Data Requirements

The POLYSIM model requires a reference baseline of forecasted data. The baseline must include projections of commodity supplies, prices and utilization as well as aggregate values for receipts and costs. Commodity specialists in ERS develop the five-year projections used in POLYSIM using formal and informal forecasting models tempered with their own experienced judgments. The projections contain explicit assumptions concerning the rates of change in population, per capita incomes, consumer preferences, export demand, technology (including crop yields and livestock gains) and other supply and demand shifters. These projections also assume a specific value for government farm program instruments.

POLYSIM estimates the impact on the agricultural economy of changes in the levels of farm program instruments [Ray and Richardson]. The baseline used for this study is the July 1977 baseline and assumes continuation of the 1973 Agriculture and Consumer Protection Act through 1982 with modifications in loan rate and target price levels consistent with the 1977 Food and Agriculture Act.[8]

For the farm programs analyzed in this study, it was necessary to provide program participation rates, acreage diversion payment rates, slippage rates, farm program acreages and yields. Values for these policy variables used in this study are presented in Table V.

Acreage diversion payment rates can be determined several ways: by sealed bids from individual farmers, by using the total fixed costs of owning the land and machinery to operate it and by using the total fixed charges for the land.

In this study, the acreage diversion payment rate for each crop is determined as the sum of total general overhead costs, total machinery ownership costs, and 20 percent of the interest and tax charges on the land. Using the average cost of production data provided by the congressional agricultural committee and the formula above, the per acre diversion payment rates for feed grains, wheat and cotton would have been about $52.38, $31.59 and $76.73, respectively, in 1977. (The acreage diversion payment rate for feed grains is a weighted average of the payment rate for corn, sorghum and barley.) The per acre diversion pay-

[8] The study was completed prior to the final passage of the Food and Agriculture Act of 1977. Although earlier legislative drafts were drawn upon, the farmer-held grain reserve, the announced feed grain, wheat set-aside and acreage diversion programs as well as recent market developments were not included in the baseline data.

Table V. Assumed Values for Program Participation Rates, Diversion Payment Rates, Slippage Rates and Farm Program Acreage and Yields Used For Four Farm Programs

Variable and Crop	Farm Programs: I Price and Income Support	II & IV Price and Income Support 'with Acreage Diversion	III Price Support & Acreage Diversion with a Grain Reserve Program
Program Participation Rate[1]			
Feed Grains	0.95	0.65	0.80
Wheat	0.95	0.80	0.80
Cotton	0.95	0.80	0.80
Diversion Payment Rates[2]			
Feed Grains		53.43	53.43
Wheat		32.23	32.23
Cotton		78.26	78.26
Slippage Rate[1]			
Feed Grains		0.40	0.40
Wheat		0.40	0.40
Cotton		0.40	0.40
Farm Program Acreages[1]			
Feed Grains	76.0	76.0	
Wheat	56.9	56.9	
Cotton	10.0	10.0	
Farm Program Yields[3]			
Feed Grains	2.06	2.06	
Wheat	31.00	31.00	
Cotton	480.00	480.00	

[1] Values for 1978 through 1981.
[2] Acreage diversion payment rates are for 1978; values for 1979, 1980 and 1981 are obtained by inflating the 1978 value by two percent per year.
[3] Farm program yields are for 1978 for feed grains and wheat. Feed grain farm program yields are increased by 0.03 ton per acre each year to obtain values for 1979, 1980 and 1981. Wheat farm program yields are annually increased 0.5 bushels per acre. No change is assumed for cotton farm program yields in 1978 through 1981.

ment rates for 1978, reported in Table V, are obtained by increasing the 1977 values by two percent.

Slippage is the portion of each acre of diversion that does not actually result in reducing production because of farmers diverting their least productive land and using variable resources more intensively on remaining land under cultivation. Tweeten reports that prior to 1970 the slippage rate for feed grains was about 0.40 meaning that for each acre of set-aside, production was reduced by only 0.6 acres. Garst and Miller report the slippage rate for wheat at 0.39 during 1960-1970 and being as high as 0.59 between 1971-1974. The slippage rates selected for the acreage set-aside provisions in programs II, III, and IV are 0.40 for feed grains, wheat and cotton (Table V).

Farm program acreages for feed grains, wheat and cotton are used to calculate deficiency payments. The farm program acreages in the July baseline are as follows: 76.0 million acres for feed grains, 56.9 million acres for wheat and 10.0 million acres for cotton.

Results for Farm Program I

For program I, the control mechanism selects the values of loan rates and target prices for feed grains, wheat and cotton that maximize the performance measure using the first set of parameter weights in Table IV. The "optimal values" of the control variables and simulated values for selected state variables in the final solution are presented in Table VI. The solution is optimal for the conceptual performance measure used, the baseline data and simulation model used and the assumptions made concerning the farm program. The optimal wheat loan rates are higher than the baseline values for 1980 and 1981, but lower than the baseline in 1978 and 1979. Corn and cotton loan rates are lower than the baseline values in all four years.

The corn loan rates are set just below the calculated corn price in all four years to prevent the CCC loan actions from raising the price of corn and resulting in higher livestock feed costs and lower net returns for livestock producers. Also, higher corn prices would have caused an increase in consumers' expenditures for food by reducing livestock production and increasing the prices received for livestock. On the other hand, loan rates for wheat are used to support the average price of wheat in 1980 and 1981 since wheat prices have only minor influence on the consumer's expenditures for food, and raising the price of wheat tends to increase net farm income.

The optimal values of target prices for corn, wheat and cotton are set at their respective lower boundary constraints in 1980 and 1981 (Table VI). Also, the optimal target prices for cotton in 1978 and 1979 are equal to their lower boundary constraints. The optimal target prices for the three crops are set to their lower boundaries in an effort to reduce deficiency payments, since total government payments exceed the upper boundary in the performance measure ($3,700 million) in 1979, 1980 and 1981 by $218 million, $1,162 million and $2,032 million, respectively (Table VI).

The control mechanism can select control paths that result in the output variables exceeding their range boundaries if the additional unit increase in the output variable increases net farm income by more than the added penalty decreases the value of the performance measure. In this particular case, the control mechanism could only reduce government

Table VI. Optimal Values of Control Variables and the Simulated Values of Selected State Variables for Farm Program I.

Item	Unit	Baseline Values				Simulated Values			
		1978	1979	1980	1981	1978	1979	1980	1981
CONTROL VARIABLES									
Price Support Levels									
Corn	$/bu.	2.00	2.00	2.00	2.00	1.83	1.89	1.91	1.91
Wheat	$/bu.	2.35	2.35	2.35	2.35	2.01	2.21	2.43	2.68
Cotton	$/lb.	.51	.51	.51	.51	.42	.44	.48	.50
Income Support Levels									
Corn	$/bu.	2.10	2.21	2.34	2.47	2.26	2.27	2.34*	2.47*
Wheat	$/bu.	3.00	3.16	3.34	3.52	3.17	3.19	3.34*	3.52*
Cotton	$/lb.	.52	.55	.58	.61	.52*	.55*	.58*	.61*
Diverted Acres									
Feed grains	m. ac.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Wheat	m. ac.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Cotton	m. ac.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
STATE VARIABLES									
Harvested Acreage									
Feed grains	m. ac.	107.7	107.7	107.4	107.2	107.7	109.3	109.3	108.6
Wheat	m. ac.	70.7	71.1	71.1	71.1	70.7	71.3	71.7	72.7
Cotton	m. ac.	11.6	11.4	11.7	11.2	11.6	11.7	11.9	11.4
Yield									
Feed grains	t./ac.	2.06	2.09	2.12	2.15	2.06	2.09	2.12	2.14
Wheat	bu./ac.	31.00	31.50	32.00	32.49	31.00	31.48	32.04	32.74
Cotton	lb./ac.	480.00	480.00	480.00	480.00	480.00	480.00	478.77	477.34
Export Levels									
Feed grains	m. t.	50.4	52.2	53.7	55.4	50.4	52.5	54.6	56.6
Wheat	m. bu.	1025.0	1070.0	1110.0	1160.0	1033.2	1068.2	1090.9	1077.2
Cotton	m. bales	4.5	4.5	4.4	4.4	4.5	4.5	4.5	4.5
Total Utilization									
Feed grains	m. t.	206.2	213.3	233.0	228.6	206.2	211.5	223.2	230.5
Wheat	m. bu.	1925.0	1953.0	1991.0	2049.0	1935.2	1950.8	1966.7	1945.8
Cotton	m. bales	11.6	11.7	11.6	11.8	11.6	11.7	11.7	

Table VI. Continued.

Item	Unit	Baseline Values 1978	1979	1980	1981	Simulated Values 1978	1979	1980	1981
Ending Year Carryovers									
Feed grains	m. t.	70.4	82.6	87.5	98.3	70.4	87.7	96.1	98.1
Wheat	m. bu.	1539.0	1827.0	2112.0	2374.0	1528.8	1824.2	2156.8	2590.9
Cotton	m. bales	4.3	4.2	4.5	4.1	4.3	4.4	4.9	4.5
CCC Inventory and Outstanding Loans									
Feed grains	m. t.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Wheat	m. bu.	776.0	1130.0	1497.0	1848.0	28.9	28.9	162.5	693.5
Cotton	m. bales	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Commodity Prices									
Corn	$/bu.	2.00	2.00	2.00	2.00	2.00	1.98	1.94	1.93
Wheat	$/bu.	2.35	2.35	2.35	2.35	2.31	2.37	2.43	2.66
Soybeans	$/bu.	5.60	5.60	5.70	5.80	4.24	4.68	4.92	4.92
Cotton	$/lb.	.54	.55	.52	.55	.54	.54	.51	.53
Cattle and Calves	$/lb.	.42	.45	.49	.50	.42	.45	.49	.50
Hogs	$/lb.	.35	.41	.40	.37	.35	.41	.40	.37
Total Government Payments	B. $	2.019	3.549	4.712	5.850	3.650	3.918	4.862	5.732
Total CCC Storage and Interest Costs	B. $	0.150	0.310	0.452	0.599	0.150	0.012	0.012	0.065
Consumers' Food Expenditures	B. $	188.3	196.8	205.0	214.0	188.3	196.8	204.7	213.4
Livestock Producer's Income Over Variable Costs	B. $	17.312	18.844	19.967	21.289	17.667	18.955	20.005	21.068
Realized Net Farm Income	B. $	18.118	18.949	18.812	19.550	19.186	18.547	18.283	18.621
Performance Measure									111,999.0

[1] Optimal control variables that equal their lower boundary constraints are denoted by superscript "*" and those that equal their upper boundary constraints are denoted by superscript "†".

[2] The performance measure for the optimal solution presented here uses the first set of parameter weights.

payments by increasing the loan rate thus reducing the deficiency payment rates. However, such action would have raised market prices for the crops and reduced livestock production, increased livestock prices and raised consumer expenditures for food. Commodity Credit Corporation and total grain ending year carryovers would also increase. These factors would have penalized the value of the performance measure by more than the high level of government payments.

Harvested acreage for feed grains, wheat and cotton are slightly higher under program I than the baseline because of the supply response from the slight increase in wheat prices, and the decrease in soybean prices being relatively greater than decreases in either corn or cotton prices (Table VI). The increase in harvested acreage of wheat and the decrease in the exports of wheat causes ending year stocks of wheat to increase 3.2 percent over the baseline for the four-year period. (Wheat exports are less than the baseline because of higher wheat prices in the last three years.) Ending year carryovers for feed grains and cotton increase 6.8 percent and 5.4 percent, respectively, over the baseline values because of the increases in production being slightly greater than increases in demand for these crops.

Simulated values of realized net farm income for farm program I are less than those for the reference baseline in 1979, 1980 and 1981 (Table VI). Over the four-year period, simulated values for realized net farm income are less than the baseline by about one percent. The reason for the lower farm income even though the performance measure seeks to maximize net farm income is that the program also took into consideration the control path's impacts on consumers' food expenditures, the levels of ending year carryovers for the crops, total CCC storage and interest costs and the levels of total government payments.

Results for Farm Program II

Farm program II is a price and income support program with a voluntary acreage diversion provision to enable the government to reduce harvested acreages for feed grains, wheat and cotton. The control mechanism selects the optimal loan rates and acreage diversion levels for feed grains, wheat and cotton in 1978, 1979, 1980 and 1981.

The optimal solution of the performance measure (first set of parameter weights) in Table IV for farm program II is presented in Table VII. The value of the performance measure for the optimal solution is 127,968. The optimal loan rates for corn, wheat and cotton are less than the average prices of the respective crops. Therefore, the loan rates for the three crops do not influence the average prices received by farmers and could be set to their respective lower boundary constraints without appreciably changing the solution.

For this analysis, target prices for corn, wheat and cotton are predetermined at their respective levels established in the 1977 Agricultural Act. By fixing the target prices, allotted acreage and farm program yields, the deficiency payments are completely determined once loan rates and market prices are known. When the market price is greater than the loan rate, as in Table VII, the deficiency payment rate is the target price minus the market price. As the market price is brought closer to the target price, the deficiency payment approaches zero. This relationship partially explains why diverted acreage is used to raise the average market price for corn, wheat and cotton to relatively high levels (Table VII).

Optimal acreage diversion levels for wheat are equal to the upper boundary constraints for wheat (about 24.8 million acres) in all four years (Table VII). The optimal acreage diversion levels for cotton are equal to their upper boundary constraints (about 3.2 million acres) in the first two years, 1978 and 1979. The acreage diversion levels for feed grains (about 20 million acres) are about one half as large as their upper limits of 37.7 million acres. High levels of diversions for wheat are used in the solution because they reduce wheat production causing the average market price of wheat to increase which reduces the deficiency payments for wheat. Another reason for the high levels of diverted acreage for wheat is to decrease the ending year wheat carryovers to within the upper boundary of the range specified in the performance measure (1,200 million bushels).

Acreage diversion levels for feed grains are sufficiently high each year to reduce the ending year carryovers of feed grains to about 60 million tons, the upper boundary in the performance measure for this output variable. To achieve this goal, the optimal quantity of feed grain acreage diversion changes from year to year are 10 million acres in 1978, 20 million in 1979 and 1980 and 18 million in 1981 (Table VII).

Higher levels of feed grain diversions are not used since they do not improve the value of the performance measure after once reducing carryovers to 60 million tons. Also, higher levels of feed grain diversions would reduce the value of the performance measure by increasing corn prices which result in decreases in net incomes for livestock producers and in the following year result in increases in consumer expenditures for food. Acreage diversion levels for cotton cause the ending year carryovers of cotton to be reduced to the acceptable range of 2.0 to 4.0 million bales in the performance measure. The resulting prices of cotton are greater than the target price for cotton, thus, eliminating deficiency payments for cotton (Table VII).

Government payments for farm program II are less than the $3,700 million limit imposed on the performance measure in all but the last year simulated when government payments are $4,266 million (Table

Table VII. Optimal Values of Control Variables and the Simulated Values of Selected State Variables for Farm Program II.

Item	Unit	Baseline Values				Simulated Values			
		1978	1979	1980	1981	1978	1979	1980	1981
CONTROL VARIABLES									
Price Support Levels									
Corn	$/bu.	2.00	2.00	2.00	2.00	1.80	1.82	1.82	1.84
Wheat	$/bu.	2.35	2.35	2.35	2.35	2.18	2.21	2.39	2.39
Cotton	$/lb.	.51	.51	.51	.51	.47	.49	.51	.53
Income Support Levels									
Corn	$/bu.	2.10	2.21	2.34	2.47	2.10	2.21	2.34	2.47
Wheat	$/bu.	3.00	3.16	3.34	3.52	3.00	3.16	3.34	3.52
Cotton	$/lb.	.52	.55	.58	.61	.52	.55	.58	.61
Diverted Acres									
Feed grains	m. ac.	0.0	0.0	0.0	0.0	9.9	19.9	19.6	18.1
Wheat	m. ac.	0.0	0.0	0.0	0.0	24.7†	24.8†	24.8†	24.6
Cotton	m. ac.	0.0	0.0	0.0	0.0	3.2†	3.3†	3.0	2.5
STATE VARIABLES									
Harvested Acreage									
Feed grains	m. ac.	107.7	107.7	107.4	107.2	101.8	97.3	97.8	97.9
Wheat	m. ac.	70.7	71.1	71.1	71.1	55.9	56.7	57.6	58.2
Cotton	m. ac.	11.6	11.4	11.7	11.2	9.7	10.0	10.8	10.5
Yield									
Feed grains	t./ac.	2.06	2.09	2.12	2.15	2.06	2.10	2.15	2.19
Wheat	bu./ac.	31.00	31.50	32.00	32.49	31.00	31.61	32.42	33.15
Cotton	lb./ac.	480.00	480.00	480.00	480.00	480.00	488.69	497.08	500.56
Export Levels									
Feed grains	m. t.	50.4	52.2	53.7	55.4	49.3	49.1	49.2	50.1
Wheat	m. bu.	1025.0	1070.0	1110.0	1160.0	991.0	933.1	910.9	926.6
Cotton	m. bales	4.5	4.5	4.4	4.4	4.2	4.0	3.9	3.9
Total Utilization									
Feed grains	m. t.	206.2	213.3	223.0	228.6	204.6	204.6	209.8	214.8
Wheat	m. bu.	1925.0	1953.0	[illegible].0	2049.0	1884.0	1786.9	1747.5	176[illegible]
Cotton	m. bales	11.6	11.7	11.6	11.8	11.2	11.0	10.8	11.1

Table VII. Continued.

Item	Unit	Baseline Values				Simulated Values			
		1978	1979	1980	1981	1978	1979	1980	1981
Ending Year Carryovers									
Feed grains	m. t.	70.4	82.6	87.5	89.3	59.9	59.9	60.3	60.1
Wheat	m. bu.	1539.0	1827.0	2112.0	2374.0	1119.9	1125.4	1247.5	1417.3
Cotton	m. bales	4.3	4.2	4.5	4.1	2.7	2.1	2.6	2.6
CCC Inventory and Outstanding Loans									
Feed grains	m. t.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Wheat	m. bu.	776.0	1130.0	1497.0	1848.0	0.8	0.0	0.0	0.0
Cotton	m. bales	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Commodity Prices									
Corn	$/bu.	2.00	2.00	2.00	2.00	2.09	2.21	2.26	2.27
Wheat	$/bu.	2.35	2.35	2.35	2.35	2.51	2.90	3.00	3.03
Soybeans	$/bu.	5.60	5.60	5.70	5.80	4.32	4.83	6.00	6.28
Cotton	$/lb.	.54	.55	.52	.55	.61	.65	.61	.63
Cattle and Calves	$/lb.	.42	.45	.49	.50	.43	.46	.51	.52
Hogs	$/lb.	.35	.41	.40	.37	.35	.42	.43	.40
Total Government Payments	B. $	2.019	3.549	4.712	5.850	3.180	3.460	3.680	4.266
Total CCC Storage and Interest Costs	B. $	0.150	0.310	0.452	0.599	0.150	0.017	0.0	0.0
Consumers' Food Expenditures	B. $	188.3	196.8	205.0	214.0	188.3	197.7	207.1	216.5
Livestock Producer's Income Over Variable Costs	B. $	17.312	18.844	19.967	21.289	17.345	18.848	20.668	22.420
Realized Net Farm Income	B. $	18.118	18.949	18.812	19.550	18.874	18.913	19.926	21.589
Performance Measure									127,968.0

[1] Optimal control variables that equal their lower boundary constraints are denoted by superscript "*" and those that equal their upper boundary constraints are denoted by superscript "†".

[2] The performance measure for the optimal solution presented here uses the first set of parameter weights in Table IV.

VII). Total government payments could not be decreased to the $3,700 million limit in the last year simulated. This is because higher wheat acreage diversion to reduce wheat deficiency payments is not possible. Also higher cotton diversion only increases cotton payments for acreage withdrawal since no deficiency payments are paid for cotton. Furthermore, higher levels of feed grain diversions result in higher corn prices which cause an immediate reduction of net income to livestock producers and, thus, reduces the value of the performance measure.

In the aggregate, farm program II tends to increase both net incomes for livestock producers and total realized net farm income over the values in the baseline (Table VII). Realized net farm income is increased 10.4 percent in 1982 over its baseline value and the average increase over the four-year period is about five percent. The optimal levels of acreage diversion for feed grains, wheat and cotton in farm program II result in moderate increases in consumers' food expenditures over the baseline. Over the four-year period, total consumers' food costs are estimated to increase about 0.7 percent over the baseline values.

Results for Farm Program III

Farm program III is a price support and acreage diversion program with a grain reserve provision. The control variables for the farm program are loan rates and acreage diversion levels for feed grains, wheat and cotton in 1978, 1979, 1980 and 1981. The optimal values for the control variables in farm program III are presented in Table VIII.

Farm program III includes a grain reserve provision that encourages the CCC to hold 20 million tons of feed grains and 500 million bushels of wheat. The CCC reserve of feed grains and wheat is assumed to be acquired in 1977. The objective is to determine loan rates and acreage diversion levels that maximize the performance measure (first set of parameter weights) in Table IV, subject to the added constraint of maintaining the initial level of grain reserves from 1978 through 1981. The CCC release rule used for this farm program is to release CCC held reserves if the average market price exceeds the loan rate by 50 percent and to release only the amount of stocks needed to lower the average market price to 150 percent of the loan rate.

The control mechanism tries not to use loan rates to support the market price in this particular farm program since the support action results in the CCC acquiring control of additional stocks. So acreage diversion is the predominate control variable for farm program III. The optimal acreage diversion levels for wheat and cotton are equal to the crop's respective upper boundary constraints in each of the four years (Table VIII). Optimal acreage diversion levels for feed grains range from 12 million acres to 32 million acres over the period simulated (Table

VIII). Hence, the feed grain average diversion levels are less than the upper boundary constraints (about 37 million acres), but are larger than the diversion levels for feed grains in farm program II (Table VII and VIII).

The high levels of acreage diversion for feed grains, wheat and cotton cause the average market prices for these crops to be greater than the respective market prices in the baseline for each of the years simulated (Table VIII). The corn loan rate is increased from year to year but is never greater than the market price but neither does the market price exceed the loan rate by 150 percent. So the CCC acquisition and release rules for corn are never activated. A similar situation exists for wheat.

The total government payments for miscellaneous farm programs and acreage diversion is less than the $3,700 million upper boundary of the range imposed on the performance measure (Table VIII). The upper boundary is almost passed in 1981 with total government payments of $3,696 million. Additional diverted acreage of feed grains is possible in 1981; however, higher levels of diversion would increase total government payments over the upper limit and penalize the performance measure. Realized net farm income for farm program III is higher than the baseline values in each year simulated, and over the four years, simulated net farm income is nine percent greater than the baseline.

Results for Farm Program IV

The optimal levels of acreage diversion and loan rates for farm program IV (program II with increased export demands in 1978) are presented in Table IX. The quantity of exports in 1978 for feed grains, wheat and cotton is predetermined at a relatively high level which reduces the ending year carryovers of these crops (Table IX). The predetermined value of exports equals the baseline export value in 1978, plus the percentage increase in exports between 1971 and 1972 (86 percent for feed grains, 58 percent for wheat and 57 percent for cotton). Target prices for farm program IV are fixed at the baseline levels. The value of the performance measure for the optimal solution of program IV is 131,946.0 as compared to 127,968.0 for farm program II.

The high level of exports in 1978 reduces the ending year carryovers of feed grains, wheat and cotton, thus reducing the need for diverted acreage in 1978 for these crops (Table IX). Optimal acreage diversion levels for feed grains and cotton are less than 1.0 million acres in 1978. The resulting ending year carryovers for feed grains and cotton are approximately equal to the lower boundaries of the designated ranges for these state variables in the performance measure, 30 million tons and 2 million bales, respectively (Table IX).

Table VIII. Optimal Values of Control Variables and the Simulated Values of Selected State Variables for Farm Program III.

Item	Unit	Baseline Values				Simulated Values			
		1978	1979	1980	1981	1978	1979	1980	1981
CONTROL VARIABLES									
Price Support Levels									
Corn	$/bu.	2.00	2.00	2.00	2.00	1.80	1.94	2.10	2.18
Wheat	$/bu.	2.35	2.35	2.35	2.35	2.23	2.26	2.44	2.46
Cotton	$/lb.	.51	.51	.51	.51	.38	.38	.42	.46
Income Support Levels									
Corn	$/bu.	2.10	2.21	2.34	2.47	0.00	0.00	0.00	0.00
Wheat	$/bu.	3.00	3.16	3.34	3.52	0.00	0.00	0.00	0.00
Cotton	$/lb.	.52	.55	.58	.61	0.00	0.00	0.00	0.00
Diverted Acres									
Feed grains	m. ac.	0.0	0.0	0.0	0.0	11.8	20.9	27.8	31.9
Wheat	m. ac.	0.0	0.0	0.0	0.0	24.6†	24.8†	24.8†	24.8†
Cotton	m. ac.	0.0	0.0	0.0	0.0	3.2†	3.3†	3.1†	3.2†
STATE VARIABLES									
Harvested Acreage									
Feed grains	m. ac.	107.7	107.7	107.4	107.2	100.6	96.3	92.9	90.3
Wheat	m. ac.	70.7	71.1	71.1	71.1	55.9	57.7	58.5	58.6
Cotton	m. ac.	11.6	11.4	11.7	11.2	9.7	9.9	10.7	10.0
Yield									
Feed grains	t./ac.	2.06	2.09	2.12	2.15	2.06	2.10	2.16	2.22
Wheat	bu./ac	31.00	31.50	32.00	32.49	31.00	31.89	32.71	33.37
Cotton	lb./ac.	480.00	480.00	480.00	480.00	480.00	480.66	497.21	501.24
Export Levels									
Feed grains	m. t.	50.4	52.2	53.7	55.4	49.1	46.9	45.9	46.0
Wheat	m. bu.	1025.0	1070.0	1110.0	1160.0	900.4	854.6	857.8	894.0
Cotton	m. bales	4.5	4.5	4.4	4.4	4.2	4.0	3.9	3.8
Total Utilization									
Feed grains	m. t.	206.2	213.3	223.0	228.6	228.6	201.0	201.2	[illegible]
Wheat	m. bu.	1925.0	1953.0	[illegible]	2049.0	1770.6	1688.6	1681.6	[illegible]
Cotton	m. bales	11.6	11.7	11.6	11.8	11.2	11.0	10.8	10.9

Table VIII. Continued.

Item	Unit	Baseline Values				Simulated Values			
		1978	1979	1980	1981	1978	1979	1980	1981
Ending Year Carryovers									
Feed grains	m. t.	70.4	82.6	87.5	89.3	57.8	59.5	59.7	55.8
Wheat	m. bu.	1539.0	1827.0	2112.0	2374.0	1235.2	1385.9	1620.0	1852.6
Cotton	m. bales	4.3	4.2	4.5	4.1	2.7	2.0	2.5	2.1
CCC Inventory and Outstanding Loans									
Feed grains	m. t.	0.0	0.0	0.0	0.0	20.0	20.0	20.0	20.0
Wheat	m. bu.	776.0	1130.0	1497.0	1848.0	500.0	500.0	500.0	500.0
Cotton	m. bales	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Commodity Prices									
Corn	$/bu.	2.00	2.00	2.00	2.00	2.37	2.37	2.45	2.49
Wheat	$/bu.	2.35	2.35	2.35	2.35	2.92	3.11	3.11	3.09
Soybeans	$/bu.	5.60	5.60	5.70	5.80	4.32	4.88	6.16	6.49
Cotton	$/lb.	.54	.55	.52	.55	.60	.65	.61	.65
Cattle and Calves	$/lb.	.42	.45	.49	.50	.42	.46	.52	.53
Hogs	$/lb.	.35	.41	.40	.37	.35	.42	.45	.41
Total Government Payments	B. $	2.019	3.549	4.712	5.850	2.544	3.144	3.359	3.696
Total CCC Storage and Interest Costs	B. $	0.150	0.310	0.452	0.599	0.150	0.253	0.253	0.253
Consumers' Food Expenditures	B. $	188.3	196.8	205.0	214.0	188.3	197.9	208.7	218.1
Livestock Producer's Income Over Variable Costs	B. $	17.312	18.844	19.967	21.289	17.169	18.425	21.549	23.232
Realized Net Farm Income	B. $	18.118	18.949	18.812	19.550	18.641	18.995	21.634	22.911
Performance Measure									123,162.0

[1] Optimal control variables that equal their lower boundary constraints are denoted by superscript "*" and those that equal their upper boundary constraints are denoted by superscript "†".

[2] The performance measure for the optimal solution presented here uses the first set of parameter weights in Table IV.

Acreage diversion for wheat in 1978 is 24.3 million acres, just slightly less than the upper boundary for wheat (24.7 million). High levels of wheat diversion are used in 1978 in an effort to reduce the large carryover of wheat from 1977 (1,270 million bushels) in anticipation of the large carryovers in 1981. The acreage diversion levels of wheat in 1979 and 1980 (24.4 and 24.8 million acres, respectively) are also about equal to the upper boundary constraint for these control variables (Table IX). The diversion levels for wheat are high in 1979 and 1980 in an effort to hold carryovers for 1981 as close as possible to the 1,200 million bushel boundary in the performance measure.

Acreage diversion levels for feed grains in 1979 and 1980 are slightly higher for farm program IV than for program II; but acreage diversions are well below the maximums established by the upper boundary constraints in Table III. The reason for the increase in diverted acreage is to maintain ending year carryover of feed grains at about 60 million tons, additional acreage set-aside is needed to remove the effect of increases in feed grain harvested acreage and yields that result from increases in feed grain price (corn).

Optimal acreage diversion levels for cotton are equal to their upper boundary constraints in 1979 and 1980 (3.3 and 3.1 million acres, respectively) in an effort to reduce the carryover in 1980 to four million bales, the upper boundary in the performance measure (Table IX). The complexities of farm program IV demonstrate the dynamic properties of control theory, i.e., the optimal values selected for the acreage diversion control variables in 1979 and 1980 are selected due not only to their immediate but also their longer run impacts on the performance measure.

The average market prices received for the model crops are considerably higher under farm program IV than their respective values in the baseline and in program II (Table IX). The higher prices are due in part to the increase in export demands for feed grains, wheat and cotton and to the high levels of acreage diversions selected by the control mechanism. Also, loan rates for wheat and cotton are used to support the average market prices in 1980 and 1981. The selection of the loan rates for cotton are interesting in that the 1978 and 1979 values are the lowest possible values that permit a loan rate of $0.55 per pound in 1980 (Table IX). A constraint on the annual increase in loan rates prohibits increases of more than 10 percent a year and this is the rate of increase between 1978 and 1979 and between 1979 and 1980.

Prices of beef cattle and hogs increase 6 percent and 11 percent, respectively, over the baseline values for 1978-1981 for farm program IV (Table IX). The increases in livestock prices are because of lower livestock supplies in response to increases in feed costs. The higher livestock prices are passed on to the consumer in the form of higher food costs

(Table IX). Over the four year period, total expenditures for food increase 1.6 percent over the baseline values and for the last three years, food expenditures increase about three percent. The impacts on feed costs because of increases in the feed costs in 1978 primarily accrue in 1979 and 1980.

Realized net farm income for farm program IV increases about 23 percent over the baseline between 1978 and 1981 (Table IX). The primary increase in net farm income is in 1979 when net farm income increases from $18.9 billion to $32.1 billion. This 70 percent increase is due primarily to higher livestock prices as a result of negative livestock production response to the higher feed costs in 1978.

Sensitivity of the Optimal Solution to Changes in the Performance Measure

The final solutions for farm programs I, II, III and IV are "optimal" using the first set of parameter weights for the performance measure in Table IV. A change in the weights used in the performance measure would likely change the optimal solutions for the farm programs. A sensitivity analysis could be done for each farm program to determine the sensitivity of the optimal solution to changes in parameter weights and acceptable ranges for the critical variables in the performance measure. To demonstrate the type of information sensitivity analysis can provide, farm program I is solved a second time using the second set of parameter weights for the performance measure (Table IV).

The second set of parameter weights has higher weights or penalties (in absolute terms) for values outside the designated acceptable ranges for consumers' food expenditures, total government payments and total CCC and interest costs (Table IV). For example, when the upper boundary for the acceptable range of total government payments is exceeded, the parameter weight becomes —4.0 rather than —1.5, implying that for each unit of expense over the upper boundary level ($3,700 million) the disutility is $4 or 2.66 times more than when a weight of —1.5 is used. The increased penalty (disutility) associated with government payments exceeding its acceptable range should cause the control mechanism to select values for the control variables that tend to hold government payments closer to the acceptable range. The optimal solutions for farm program I, using both performance measures in Table IV, are presented in Table X.

As hypothesized above, total government payments for farm program I are less for the optimal solution using the second set of weights than for the optimal solution using the lower first set of weights (Table X). The increase in total food costs because of shifting from the low to the high set of weights for government payment is about $0.3 billion over

Table IX. Optimal Values of Control Variables and the Simulated Values of Selected State Variables for Farm Program IV.

Item	Unit	Baseline Values				Simulated Values			
		1978	1979	1980	1981	1978	1979	1980	1981
CONTROL VARIABLES									
Price Support Levels									
Corn	$/bu.	2.00	2.00	2.00	2.00	2.10†	2.19	2.20	2.20
Wheat	$/bu.	2.35	2.35	2.35	2.35	3.00†	3.14	3.30	3.48
Cotton	$/lb.	.51	.51	.51	.51	.46	.51	.55	.58
Income Support Levels									
Corn	$/bu.	2.10	2.21	2.34	2.47	2.10	2.21	2.34	2.47
Wheat	$/bu.	3.00	3.16	3.34	3.52	3.00	3.16	3.34	3.52
Cotton	$/lb.	.52	.55	.58	.61	.52	.55	.58	.61
Diverted Acres									
Feed grains	m. ac.	0.0	0.0	0.0	0.0	0.6	21.1	22.0	16.7
Wheat	m. ac.	0.0	0.0	0.0	0.0	24.3	24.4	24.8†	24.7†
Cotton	m. ac.	0.0	0.0	0.0	0.0	0.8	3.3†	3.1†	2.8
STATE VARIABLES									
Harvested Acreage									
Feed grains	m. ac.	107.7	107.7	107.4	107.2	106.8	99.9	97.2	98.0
Wheat	m. ac.	70.7	71.1	71.1	71.1	57.9	58.7	59.3	60.1
Cotton	m. ac.	11.6	11.4	11.7	11.2	11.0	12.1	11.0	10.1
Yield									
Feed grains	T./ac.	2.06	2.09	2.12	2.15	2.07	2.21	2.21	2.22
Wheat	bu./ac.	31.00	31.50	32.00	32.49	31.52	32.50	33.14	33.84
Cotton	lb./ac.	480.00	480.00	480.00	480.00	480.00	551.09	515.42	505.57
Export Levels									
Feed grains	m. t.	50.4	52.2	53.7	55.4	93.8	64.0	54.2	52.1
Wheat	m. bu.	1025.0	1070.0	1110.0	1160.0	1617.3	1069.2	884.9	819.1
Cotton	m. bales	4.5	4.5	4.4	4.4	7.1	5.4	4.6	4.3
Total Utilization									
Feed grains	m. t.	206.2	213.3	223.0	228.6	243.1	194.2	214.8	218.[illegible]
Wheat	m. bu.	1925.0	1953.0	[illegible].0	2049.0	2477.4	1890.9	1689.0	161[illegible]
Cotton	m. bales	11.6	11.7	11.6	11.8	13.4	12.1	11.4	11.4

Table IX. Continued.

Item	Unit	Baseline Values 1978	1979	1980	1981	Simulated Values 1978	1979	1980	1981
Ending Year Carryovers									
Feed grains	m. t.	70.4	82.6	87.5	89.3	32.7	59.8	60.0	59.2
Wheat	m. bu.	1539.0	1827.0	2112.0	2374.0	619.4	637.8	915.4	1333.6
Cotton	m. bales	4.3	4.2	4.5	4.1	1.9	3.8	4.4	3.9
CCC Inventory and Outstanding Loans									
Feed grains	m. t.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Wheat	m. bu.	776.0	1130.0	1497.0	1848.0	122.2	122.2	129.8	314.7
Cotton	m. bales	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.5
Commodity Prices									
Corn	$/bu.	2.00	2.00	2.00	2.00	3.12	2.28	2.23	2.25
Wheat	$/bu.	2.35	2.35	2.35	2.35	3.45	3.22	3.30	3.43
Soybeans	$/bu.	5.60	5.60	5.70	5.80	4.46	6.02	6.50	6.01
Cotton	$/lb.	.54	.55	.52	.55	1.07	.55	.54	.59
Cattle and Calves	$/lb.	.42	.45	.49	.50	.42	.54	.49	.52
Hogs	$/lb.	.35	.41	.40	.37	.35	.56	.40	.40
Total Government Payments	B. $	2.019	3.549	4.712	5.850	1.755	3.137	3.600	3.707
Total CCC Storage and Interest Costs	B. $	0.150	0.310	0.452	0.599	0.150	0.049	0.049	0.054
Consumers' Food Expenditures	B. $	188.3	196.8	205.0	214.0	188.3	206.6	205.4	216.7
Livestock Producer's Income over Variable Costs	B. $	17.312	18.844	19.967	21.289	13.501	27.449	19.564	22.728
Realized Net Farm Income	B. $	18.118	18.949	18.812	19.550	18.800	32.055	20.272	21.861
Performance Measure									131,946.0

1 Optimal control variables that equal their lower boundary constraints are denoted by superscript "*" and those that equal their upper boundary constraints are denoted by superscript "†".

2 The performance measure for the optimal solution presented here uses the first set of parameter weights in Table IV.

Table X. Optimal Values for Control Variables and the Simulated Values of Selected State Variables for Farm Program I Using the First and Second Set of Parameter Weights in the Performance Measure.

Item	Unit	Simulated Values Using First Set				Simulated Values Using Second Set			
		1978	1979	1980	1981	1978	1979	1980	1981
CONTROL VARIABLES									
Price Support Levels									
Corn	$/bu.	1.83	1.89	1.91	1.91	1.75*	1.78	1.90	2.03
Wheat	$/bu.	2.01	2.21	2.43	2.68	2.39	2.62	2.89	3.17
Cotton	$/lb.	.42	.44	.48	.50	.51	.54	.58	.61
Income Support Levels									
Corn	$/bu.	2.26	2.27	2.34*	2.47*	2.10*	2.21*	2.34*	2.47*
Wheat	$/bu.	3.17	3.19	3.34*	3.52*	3.47	3.47	3.47	3.52*
Cotton	$/lb.	.52*	.55*	.58*	.61*	.56	.57	.58*	.61*
Diverted Acres									
Feed grains	m. ac.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Wheat	m. ac.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Cotton	m. ac.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
STATE VARIABLES									
Harvested Acreage									
Feed grains	m. ac.	107.7	109.3	109.3	108.6	107.5	108.7	108.3	107.7
Wheat	m. ac.	70.7	71.3	71.7	72.7	71.2	72.5	73.6	74.4
Cotton	m. ac.	11.6	11.7	11.9	11.4	11.6	11.7	12.2	12.1
Yield									
Feed grains	t./ac.	2.06	2.09	2.12	2.14	2.06	2.09	2.12	2.15
Wheat	bu./ac.	31.00	31.48	32.04	32.74	31.10	31.70	32.50	33.30
Cotton	lb/ac.	480.00	480.00	478.77	477.34	480.00	480.03	483.72	494.27
Export Levels									
Feed grains	m. t.	50.4	52.5	54.6	56.6	50.4	52.4	54.2	55.2
Wheat	m. bu.	1033.2	1068.2	1090.9	1077.2	972.6	991.7	962.4	914.4
Cotton	m. bales	4.5	4.5	4.5	4.5	4.5	4.5	4.2	4.1
Total Utilization									
Feed grains	m. t.	206.2	211.5	223.2	230.5	206.1	211.2	222.6	227.9
Wheat	m. bu.	1935.2	1950.8	[illegible]66.7	1945.8	1859.2	1853.6	1805.2	18[illegible]
Cotton	m. bales	11.6	11.7	11.7	11.9	11.6	11.7	11.3	11.4

Table X. Continued.

Item	Unit	Simulated Values Using First Set 1978	1979	1980	1981	Simulated Values Using Second Set 1978	1979	1980	1981
Ending Year Carryovers									
Feed grains	m. t.	70.4	87.7	96.1	98.1	70.0	86.5	96.6	97.6
Wheat	m. bu.	1528.8	1824.2	2156.8	2590.9	1625.6	2073.3	2660.0	3396.2
Cotton	m. bales	4.3	4.4	4.9	4.5	4.3	4.4	5.5	6.9
CCC Inventory and Outstanding Loans									
Feed grains	m. t.	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.3
Wheat	m. bu.	28.9	28.9	162.5	693.5	375.0	554.7	1201.1	2017.6
Cotton	m. bales	0.0	0.0	0.0	0.0	0.0	0.0	2.7	4.0
Commodity Prices									
Corn	$/bu.	2.00	1.98	1.94	1.93	2.00	1.99	1.96	2.03
Wheat	$/bu.	2.31	2.37	2.43	2.66	2.59	2.62	2.86	3.15
Soybeans	$/bu.	4.24	4.68	4.92	4.92	4.24	4.71	4.92	4.92
Cotton	$/lb.	.54	.54	.51	.53	.54	.54	.57	.61
Cattle and Calves	$/lb.	.42	.45	.49	.50	.42	.45	.49	.50
Hogs	$/lb.	.35	.41	.40	.37	.35	.41	.40	.37
Total Government Payments	B. $	3.650	3.918	4.862	5.732	2.969	3.724	3.824	3.834
Total CCC Storage and Interest Costs	B. $	0.150	0.012	0.012	0.065	0.150	0.150	0.277	0.480
Consumers' Food Expenditures	B. $	188.3	196.8	204.7	213.4	188.3	196.8	204.8	213.6
Livestock Producer's Income over Variable Costs	B. $	17.667	18.955	20.005	21.068	17.609	18.915	19.983	20.858
Realized Farm Income	B. $	19.186	18.547	18.283	18.621	18.862	18.916	18.425	18.213
Performance Measure					111,999.0				93,715.0

[1] Optimal control variables that equal their lower boundary constraints are denoted by superscript "*" and those that equal their upper boundary constraints are denoted by superscript "†."

the four-year period. Total CCC costs are higher for the second set of weights; however, the costs do not exceed the $600 million upper boundary of the established range in the performance measure.

The two solutions for farm program I indicate that changes in the parameter weights in the performance measure can result in significant changes in the optimal values of the control variables and the state variables. The sensitivity of the optimal values for the control variables to changes in the parameter weights for the performance measure is a critical factor in using control theory and requires careful attention. In application, analysts may be able to obtain estimates of the parameter weights directly from the decision makers, and as Rausser and Freebairn [1974a] indicate, a range of parameter weights may be used in the analysis rather than using single-valued estimates of the parameter weights.

Summary and Conclusions

The purpose of this study was to investigate the potential of using optimal control procedures as an analytical tool for analyzing farm policy. The specific objective was to indicate the type of results one can obtain from using control theory techniques to select values for farm policy variables, such as loan rates, target prices and acreage diversion levels.

The optimal levels of the control variables for farm program I are significantly different for the two performance measures. Loan rates for wheat with the second set weights are $2.39, $2.62, $2.89 and $3.17 for 1978 through 1981, compared to $2.01, $2.21, $2.43 and $2.68 for 1978 through 1981, with the first set of weights (Table X). The higher loan rates for wheat are used to support the average market price of wheat to higher levels and thus result in larger accumulation of stocks by the CCC (2,017.6 million bushels versus 693.5 million bushels in 1981).

An explanation for the control mechanism using the wheat loan rate to support the price of wheat is the higher penalty of total government expenditures makes the use of large deficiency payments to raise net farm income less desirable. Since higher market prices reduce the deficiency payment rate for a given target price and increase net farm income, the control mechanism uses the loan rate to increase the price. The same explanation can be used to explain the high support prices of cotton in 1980 and 1981 that cause the CCC to acquire stocks of cotton (2.7 million bales in 1980 and 1.3 million bales in 1981).

The larger parameter weight on government payments above the designated range leads the control mechanism to select values for the target prices that are closer to the market prices for the respective crops than the target prices for the lower range of weights (Table X). The reason for this action is to reduce the deficiency payment rates.

A control theory procedure was adapted to the National Agricultural Policy Simulator (POLYSIM), a computerized model developed by the authors at Oklahoma State University [Ray and Richardson, 1978]. Control theory is a mathematical technique that can be used to determine the levels of control variables that cause a particular system to satisfy a given set of boundary constraints and at the same time optimize a given performance measure. In application the control mechanism selects values for the control variables, determines their impacts on the system's output variables and evaluates the performance measure based on the values of the relevant output variables. This process is usually repeated in a systematic fashion until any change in the control variables results in a reduction in the value of the performance measure.

The system being controlled in this study is the agricultural economic system in the United States. The control variables in the system are the farm policy variables—loan rates, target prices and diverted acreage for feed grains, wheat and cotton. The state variables in the system are commodity supplies, prices and utilization as well as aggregate values for production expenses, government expenditures and cash receipts.

The performance measure for control theory is similar to the objective function for programming models. For farm policy analysis, the performance measure is a mathematical statement of the trade-offs, both explicit and implicit, between the primary interest groups—farmers, consumers and taxpayers. The variables included in the performance measure are analyzed net farm income, income for livestock producers above variable costs, total consumers' expenditures for food, Commodity Credit Corporation (CCC) expenditures for storage and interest charges and ending year carryovers for feed grains, wheat and cotton.

The functional form of the performance measure used in the study is a generalization of the quadratic preference function introduced by Theil. The functional form allows the user to target output variables within acceptable ranges and provides a weighting procedure that differentiates between positive and negative deviations from the acceptable range. Parameter weights and acceptable ranges for the output variables in the performance measure are synthesized from various sources to demonstrate the use of control theory for analyzing farm policy.

Four different farm programs are analyzed using the Control Theory Option in POLYSIM to demonstrate the uses of the technique for selecting values of particular farm policy variables. The farm programs analyzed are the following: I—a price and income support program; II—a price and income support program with voluntary acreage diversion; III—a price support and acreage diversion program with a grain reserve provision; and IV—a price and income support program with voluntary acreage diversion and increased export demands for feed grains, wheat and cotton during the first year simulated. Each farm program is analyzed for the four-year period of 1978-81.

The results suggest that optimal control provides a convenient means of explicitly recognizing multiple goals in policy formulation. The control mechanism sought to balance the impacts of raising farm prices and/or government payments (by various means) on farm income and on other goal considerations including food costs, government outlays and levels of ending year inventories.

Since the mix and levels of policy instruments were determined after considering non-farm goals, net farm incomes were not always increased. In fact in farm program I, which used only support rates and deficiency payments, net farm income decreased somewhat in the latter years of the four year period. Lower corn loan rates and prices compared to the baseline allowed increased production and lower prices for livestock products. Part of the impact on feed grain producers was offset with higher target prices during part of the period. The resulting lower consumer expenditures for food more than compensated for the lower farm incomes.

Farm programs that included land diversion as a control variable increased net farm income especially in the latter part of the analysis period even though consumer expenditures for food were increased as a result. The reduced ending carryover for grains and smaller government payment levels increased the performance measure value by more than it was diminished by higher food costs.

Limitations to the present application of control theory include the following:

- the weights used in the performance measure are not purported to be the true values but illustrative values that demonstrate the technique,
- the parameter weights in the performance measure are not discounted for time,
- the POLYSIM response parameters (elasticities) may not be the true values,
- and the July 1977 CED, USDA baseline may not be correct with respect to the projections of supply and utilizations for the commodities in the model.

In view of these limitations, the values of the control variables reported in the study should be viewed as examples of the type of information that can be obtained by applying control theory techniques to farm policy analysis.

In general, the primary limitation of using optimal control techniques for farm policy analysis is the need for a performance measure that incorporates goal trade-offs among farmers, consumers and taxpayers. Hopefully the functional form for the performance measure and empirical results from this study will provide information that will be useful in developing more meaningful performance measures in the future.

The values for the parameter weights may best be developed from working directly with farm policy decision makers. The policy makers could specify a set of parameter weights, and after evaluating the results of the analysis modify the parameter weights and repeat the analysis. The process could be repeated several times until the policy maker fully understand the meaning of the parameter weights and becomes comfortable with the chosen values for the weights.

Also, the control theory model could be made stochastic by specifying random errors for all or certain relationships in the model. This linkage of optimal control to the stochastic version of POLYSIM is technically feasible but beyond the scope and budget of this study.

REFERENCES

Arrow, K. W. 1968. "Applications of Control Theory to Economic Growth." American Math. Soc., *Mathematics of the Decision Sciences, Part 2.*

Arzac, Enrique R. and Maurice Wilkinson. 1977. *Optimal Stabilization of a Quarterly Econometric Model of the U. S. Livestock and Feed Grain Markets: Some Preliminary Results.* Presented at the Sixth Annual NBER Conference on Economics and Control, Yale University, May 25-27.

Box, M. J. 1965. "A New Method of Constrained Optimization and a Comparison with Other Methods." *Computer J.* 8:42-52.

Bray, Jeremy. 1974. "Predictive Control of a Stochastic Model of the U. K. Economy Simulating Present Policy Making Practice by the U. K. Government." *Ann. Econ. and Soc. Meas.* 3:239-55.

Chow, Gregory. 1975. *Analysis and Control of Dynamic Economic Systems.* New York: John Wiley and Sons.

_____________. 1976. "An Approach to the Feedback Control of Nonlinear Econometric Systems." *Ann. Econ. and Soc. Meas.* 5:297-309.

Cooper, J. Phillip and Stanley Fischer. 1974. "A Method for Stochastic Control of Nonlinear Econometric Models and An Application: Abstract." *Ann. Econ. and Soc. Meas.* 3:205-06.

Dorfman, Robert. 1969. "An Economic Interpretation of Optimal Control Theory." *Amer. Econ. Rev.* 59:817-31.

Ericksen, Milton H., Daryll E. Ray and James W. Richardson. 1976. "Farm Programs and Grain Reserves—Simulated Results." *Analyses of Grain Reserves, A Proceedings.* Compiled by David J. Eaton and W. Scott Steele, ERS-USDA, in cooperation with the National Science Foundation, ERS Report No. 634.

Fair, Ray C. 1974. "Methods for Computing Optimal Control Solutions on the Solution of Optimal Control Problems as Maximization Problems." *Ann. Econ. and Soc. Meas.* 3:135-53.

Frohberg, Klaus K. and C. Robert Taylor. 1977. "Optical Agricultural Erosion-Sedimentation Control Considering Both Soil Conservation and Water Quality." Technical Article No. 13833 of the Texas Agri. Exp. Sta.

Garst, Gail D. and Thomas A. Miller. 1975. "Impact of the Set-Aside Program on the U. S. Wheat Acreage." *Agri. Econ. Res.* 27:30-37.

Goldfeld, Stephen M., Richard E. Quandt, and Hale F. Trotter. 196[illegible] "Maximization of Quadratic Hill-Climbing." *Econometrica.* 34:541-51.

Heady, Earl O. 1962. *Agricultural Policy Under Economic Development.* Ames: Iowa State University Press.

Henderson, James M. and Richard E. Quandt. 1958. *Microeconomic Theory: A Mathematical Approach.* New York: McGraw-Hill Book Company.

Holbrook, Robert S. 1974. "A Practical Method for Controlling a Large Non-linear Stochastic System." *Ann. Econ. and Soc. Meas.* 3:155-175.

Intriligator, Michael D. 1971. *Mathematical Optimization and Economic Theory.* Englewood Cliffs: Prentice-Hall, Inc.

Jacobs, O. L. R. 1974. *Introduction to Control Theory.* London: Oxford University Press.

Kendrick, Davis and Lance Taylor. 1970. "Numerical Solution on Nonlinear Planning Models." *Econometrica.* 38:453-67.

Kirk, Donald E. 1970. *Optimal Control Theory: An Introduction.* Englewood Cliffs: Prentice-Hall, Inc.

Kuester, James L. and Joe H. Mize. 1973. *Optimization Techniques with Fortran.* New York: McGraw-Hill Book Co.

Livesey, D. A. 1971. "Optimizing Short-Term Economic Policy." *The Econ. J.* 81-525-46.

Maxwell, J. C. 1868. "On Governors." *Proc. Royal Society.* 16:270-83.

Pindyck, Robert S. 1973. *Optimal Planning for Economic Stabilization.* London: North-Holland Publishing Company.

Pindyck, Robert S. and Steven M. Roberts. 1974. "Optimal Policies for Monetary Control." *Ann. Econ. and Soc. Meas.* 3:207-37.

Raulerson, Richard C. and Max R. Langham. 1970. "Evaluating Supply Control Policies for Frozen Concentrate Orange Juice With an Industrial Dynamics Model." *Amer. J. Agr. Econ.* 52:197-208.

Rausser, Gordon C. and John W. Freebairn. 1974a. "Approximate Adaptive Control Solutions to U. S. Beef Trade Policy." *Ann. Econ. and Soc. Meas.* 3:177-203.

____________. 1974b. "Estimation of Policy Preference Functions: An Application to U. S. Beef Import Quotas." *Rev. Econ. and Stat.* 56: 437-49.

Rausser, Gordon C. and Richard Howitt. 1975. "Stochastic Control of Environment Externalities." *Ann. Econ. and Soc. Meas.* 4:271-92.

Ray, Daryll E. and James W. Richardson. 1978. *Technical Description of POLYSIM.* Okla. Agri. Exp. Sta. Tech. Bul. T-151.

Richardson, James W. 1978. *An Application of Optimal Control Theory to Agricultural Policy Analysis.* Ph.D. thesis, Oklahoma State University.

Richardson, James W. and Daryll E. Ray. 1975a. *User's Manual for the National Agricultural Policy Simulator (POLYSIM).* Okla. Agr. Exp. Sta. Res. Rep. P-727.

_______________. 1975b. *User's Manual for UPDATE: An Auxiliary Program for POLYSIM*. Okla. Agr. Exp. Sta. Res. Rep. P-726.

Richardson, James W., Daryll E. Ray, and James N. Trapp. 1979. "Illustrative Application of Optimal Control Theory Techniques to Problems in Agricultural Economics." Okla. Agr. Exp. Sta. Bul., B-739.

Ryan, D. M. 1974. "Penalty and Barrier Functions." ed. P. E. Gill and W. Murray, pp. 175-90. *Numerical Methods for Constrained Optimization*. London: Academic Press.

Sage, Andres P. 1968. *Optimum Systems Control*. Englewood Cliffs: Prentice-Hall, Inc.

Swann, W. H. 1974. "Constrained Optimization by Direct Search." ed. P. E. Gill and W. Murray, pp. 191-217. *Numerical Methods for Constrained Optimization*. London: Academic Press.

Taylor, Robert C. and Hovav Talpaz. 1977. "Approximately Optimal Carryover Levels for Wheat in the United States." Amer. J. Agr. Econ., 61:32-40.

Theil, Henri. 1965. "Linear Decision Rules for Macrodynamic Policy Problems." *Quantitative Planning of Economic Policy*. ed. B. G. Hickman, pp. 18-37, Brookings.

Tintner, Gerhard. 1969. "What Does Control Theory Have to Offer?" *J. Farm Econ*. 51:383-98.

Trapp, James N. 1977. "Peanut Market Structure: An Analysis of Its Influence Upon Peanut Policy Trade-Offs." Mimeo, Department of Agricultural Economics, Oklahoma State University.

Tweeten, Luther. 1970. *Foundations of Farm Policy*. Lincoln: University of Nebraska Press.

Tweeten, Luther, Dale Kalbfleisch, and Y. C. Lu. 1971. *An Economic Analysis of Carryover Policies for the United States Wheat Industry*. Okla. Agr. Exp. Sta. Tech. Bul. T-132.

U. S. Congress. Senate. 1977. *Food and Agriculture Act of 1977*. Conference Report No. 95-418, 95th Cong., 1st session, September 9.

_______________. Committee on Agriculture and Forestry. 1977. *Costs of Producing Selected Crops in the United States 1975, 1976 and Projections for 1977*. Prepared by ERS-USDA, 95th Cong., 1st session, January 21.

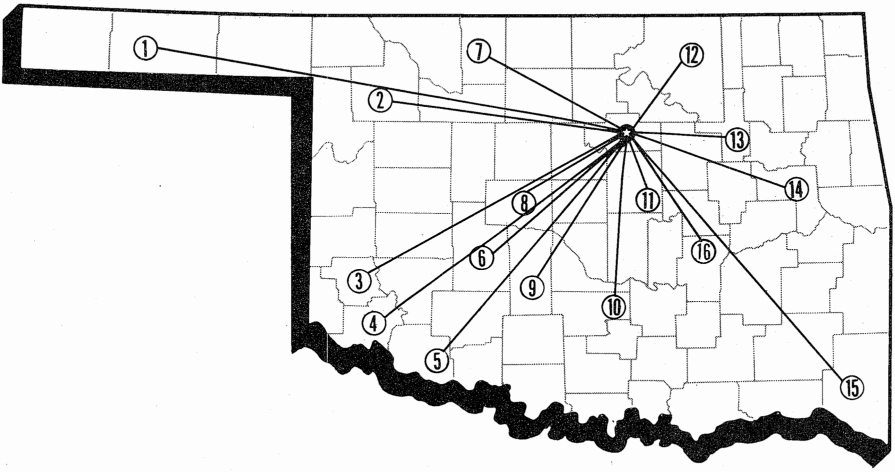
_______________. 1973. *Agriculture and Consumer Protection Act of 1973*. Public Law 93-86, Senate Bill 1888, 93rd Cong., 1st session, August 10.

Waugh, Frederick V. 1967. "Reserve Stocks of Farm Products." *Agricultural Policy: A Review of Programs and Needs*. Vol. V. National Advisory Commission on Feed and Fiber, Washington, D. C. pp. 3-52.

OKLAHOMA

Agricultural Experiment Station

System Covers the State



Main Station — ***Stillwater, Perkins and Lake Carl Blackwell***

1. **Panhandle Research Station** — ***Goodwell***
2. **Southern Great Plains Field Station** — ***Woodward***
3. **Sandyland Research Station** — ***Mangum***
4. **Irrigation Research Station** — ***Altus***
5. **Southwest Agronomy Research Station** — ***Tipton***
6. **Caddo Research Station** — ***Ft. Cobb***
7. **North Central Research Station** — ***Lahoma***
8. **Southwestern Livestock and Forage Research Station** — ***El Reno***
9. **South Central Research Station** — ***Chickasha***
10. **Agronomy Research Station** — ***Stratford***
11. **Pecan Research Station** — ***Sparks***
12. **Veterinary Research Station** — ***Pawhuska***
13. **Vegetable Research Station** — ***Bixby***
14. **Eastern Research Station** — ***Haskell***
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