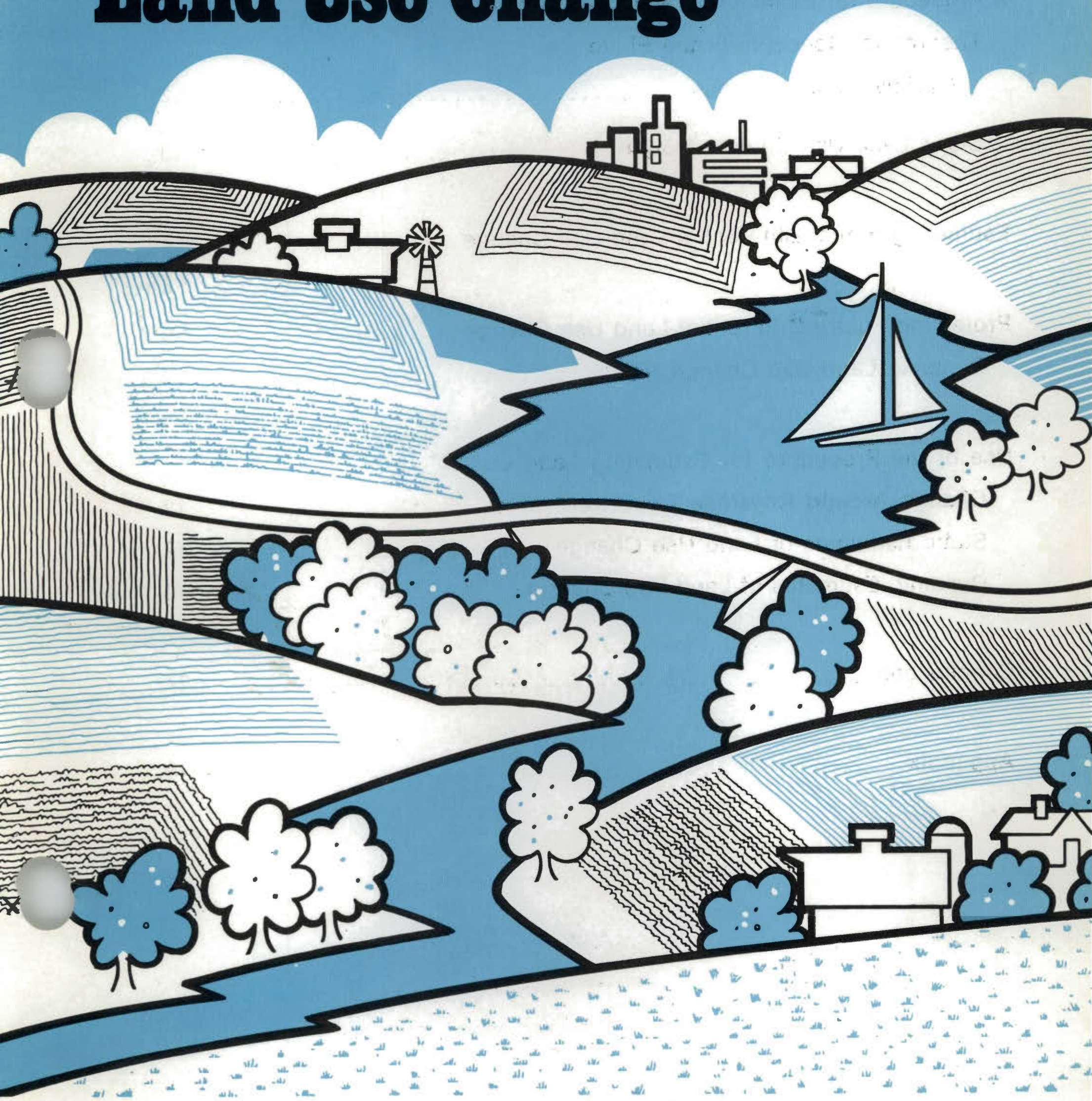
The Use of Markov Processes in Estimating Land Use Change



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The Use of Markov Processes in Estimating Land Use Change*

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Large scale public investments by the Corps of Engineers and similar agencies invariably affect the land use pattern in the immediate vicinity of the project. Changes that occur in land use patterns frequently produce economic, environmental and aesthetic externalities. Consequently, the land use impact of such projects is an important dimension in the overall evaluation of the project.

The ex post evaluation of the net impact of large scale public investments on land use patterns is a complex problem. The researcher must compare the land use pattern that is observed to exist following the project with an estimate of what that land use pattern would have been if the public investment had not been made, ceteris paribus. The Markov process procedure described herein is an appropriate and effective technique for handling this problem. Following a complete discussion of the methodology and several variants thereof, an example of its use in analyzing land use changes around Keystone Reservoir in Oklahoma will be presented.

A PROCEDURE FOR ESTIMATING LAND USE CHANGE

Economists are frequently interested in measuring the change in economic variables through time and in estimating what paths these variables may take in future periods of time. The Markov process (or chain) is a statistical procedure which may be used to generate such information. Although the basic concepts of Markov chains were introduced in 1907, their use by economists is a relatively recent phenomenon.

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The Markov process has been used by several authors to project farm numbers. Krenz in 1964 used the process to project farm numbers in North Dakota for the years 1975 and 2000 [9]. He made use of several different base periods for each projection and concluded that Markov chains have important advantages over traditional procedures when used to project farm numbers: (1.) projections can be made more conveniently for each size category of farms; and, (2.) the method provides additional information which is not readily obtainable with traditional techniques.

Hallberg employed the technique to analyze the size distribution of plants manufacturing frozen milk products in Pennsylvania during the period 1944-1963 [3]. He suggested a method based on multiple regression techniques of replacing the constant transition probabilities with probabilities which are a function of various factors including structural characteristics in the industry. More recently, Burnham, has used the Markovian framework to project future land use patterns in the Southern Mississippi Alluvial Valley [1]. He concludes that the process can be adapted to project the future implications of observed land use trends provided appropriately specified data are available. In addition, the model provides a framework for analyzing alternative institutional policies designed to attain specific land use futures.

Theoretical Concepts of the Finite Markov Chain Process

A stochastic process may be described as a sequence of experiments or events in which the outcome of each individual experiment in the sequence depends on some probability, P. A finite stochastic process exists when the range of possible outcomes is finite. If the probability, P, does not depend on the history of the system prior to the previous time period, a special type of stochastic process called a Markov process exists. According to Kemeny:

> **A Markov chain process is determined by specifying the following information: There is given a set of states ($S_1, S_2, \ldots S_r$). The process can be in one and only one of these states at a given time and it moves successively from one state to another. Each move is called a step. The probability that the process moves from S_i to S_j depends only on the state S_i that it occupied before the step. The transition probability P_{ij}, which gives the probability that the process will move from S_i to S_j is given for every ordered pair of states. Also an initial starting state is specified at which the process is assumed to begin [6, p. 148].**

Assume the variable of interest is land use. The finite Markov chain process requires that r different land use categories be defined and that movements between these land use categories over time be summarized in a land use flow matrix. Land use transitions must be regarded as a stochastic process. Once the land use flow matrix is estimated, the probability (p_{ij}) of moving from one land use category (S_j) to another land use category (S_j) is computed as:

$$P_{ij} = \frac{S_{ij}}{\sum_{i} S_{ij}} \tag{1}$$

Each P_{ij} represents the fraction of land that started in land use category S_i in period t and moved to land use category S_j in the following period. Therefore, p_{11} represents the proportion of land that started in S_1 in time t and continued in S_1 in time t + 1. These transition probabilities may be expressed in the form of a matrix such as:

$$P = \begin{array}{c|cccc} & S_1 & S_2 & \cdots & S_r \\ S_1 & p_{11} & p_{12} & \cdots & p_{1r} \\ S_2 & p_{21} & p_{22} & \cdots & p_{2r} \\ & \cdots & \cdots & \cdots & \cdots \\ S_r & p_{r1} & p_{r2} & \cdots & p_{rr} \end{array} \tag{2}$$

Where P is a transition probability matrix.

An important kind of Markov process is the regular Markov chain process. A Markov chain process is regular if the p_{ij} elements of each row sum to unity and are non-negative. These two assumptions are appropriate for projecting land uses since they imply land is neither created nor destroyed during the land use transition process.

A Markov chain process may be either stationary or dynamic. Stationarity in a Markov chain process means that the transition probabilities in P do not change over time. In a land use analysis, this means that factors influencing land use change over the time period in which the transition matrix is constructed remain the same throughout future time periods. A dynamic Markov process is one in which the transition probabilities are assumed to change with time in some sort of regularly described pattern. Both stationary and dynamic probability estimates are considered in this report.

Static Land Use Change Model

The transition matrix given in (2) and an initial vector of land uses completely defines the Markov chain process. With these data it is possible to project land uses in the n^{th} time period or step. If Q_0 represents the initial land use vector (of length r), then the following procedure may be used to project land use patterns in each future time period:

$$Q_oP = Q_1$$
$$Q_1P = Q_2$$
$$\vdots \quad \vdots$$
$$Q_{n-1}\,P = Q_n$$

or Q_n may be written as:

$$Q_n = Q_o[P]^n$$

The static Markov chain process may also be used to project equilibrium land use distributions. If a Markov chain process is regular, then as the transition matrix is raised to successively higher powers, all rows converge to a unique row vector termed the equilibrium vector. The equilibrium vector represents the unique organization of land uses in which net movements from one land use category to another is zero, i.e., land use movements out of each state are exactly equal to movements into that state. More specifically, if P is a regular transition matrix, there exists a matrix T, consisting of identical rows, to which P^n will converge as n approaches infinity. Each row of T is the same vector t, and all elements of t are non-negative.

One method for calculating the equilibrium vector is to multiply the P matrix times itself a large number of times until some power of P reaches the equilibrium configuration; however, this would be a tedious process. Judge and Swanson [5] propose another method for calculating the equilibrium vector. They note that in equilibrium the distribution vector must be invariant, so $tP = t$, or

$$t(P - I) = 0 \tag{3}$$

where I is an identity matrix. The system in (3) contains n — 1 linearly independent equations and n unknowns. Since t is a probability vector,

$$\sum_j t_j = 1 \tag{4}$$

When combined, (3) and (4) form a system of n linearly independent equations and n unknowns from which it is possible to solve for the unique values of t.

Estimating Actual Differential Land Use Change

Estimates of future land use patterns are determined by the transition probability matrix and the original state, or original distribution of the land among use categories. The initial state is designated as vector Q_a of length r, and the land use pattern at the end of the time period (i.e., the period over which the r by r transition probability matrix ${}_{ab}P$ is computed) is Q_b. Then it follows that:[1]

$$Q_b = Q_a \cdot {}_{ab}P \tag{5}$$

Assuming that land use change is a stochastic process in which any future movement is independent of past movements and that ${}_{ab}P$ is both regular and stationary, then (5) can be generalized to predict land use patterns in n, where $n > b$ and $n = 0$ in a:

$${}_{ab}Q_n = Q_a \cdot {}_{ab}P^n \tag{6}$$

${}_{ab}Q_n$ denotes an estimated land use vector in time period n based on a transition probability matrix constructed over the time period a,b. The land use prediction model in (6) is valid only if the stability of P is assumed between b and n. With this ceteris paribus requirement, it must be assumed that the rate of change of economic and other factors influencing land use change patterns remains constant over the projection period. This assumption is maintained throughout the remainder of this study, unless explicitly stated otherwise.

Suppose that a large scale public investment such as the construction of a reservoir occurred in the study area in time period m_1 to m_2 where $b > m_1 > m_2 > n$. Then the land use pattern *predicted* by (6) for time period n (${}_{ab}Q_n$) may deviate from the actual land use pattern *observed* in n (Q_n). The difference between the predicted land use pattern that would have existed in n without the lake, and the actual observed land use pattern in n with the lake is the differential land use change caused by development of the lake. Thus the differential land use impact (D_n) of the reservoir in time period n may be computed as:

$$D_n = Q_n - {}_{ab}Q_n = Q_n - Q_a\,[{}_{ab}P]^n \tag{7}$$

Vector D_n in (7) provides a more accurate estimate of the differential land use impact of reservoir construction than "with and without" techniques frequently used in project analysis. This is because the pattern of land use change in the pre-investment time period a, b is projected to time n, thereby accounting for land use changes that would have occurred, ceteris paribus, if the reservoir had never been constructed.

The computation in (7) may be represented graphically. Actual differential land use change over time for a single land use category i is

[1] In the notational conventions used in this study, all subscripts refer to either points in time or time periods. A left subscript is the time period (base period) over which the variable is estimated or measured, while the right subscript is the time at which the variable is estimated or measured. Land use vectors (Q) for which there is no left subscript are observed. Those with a left subscript are estimated by the Markov model. A superscript is the power to which the variable is to be raised.

illustrated in Figure 1. The actual quantity of land use i is shown by the solid line over time while the projected land use had the reservoir not been constructed follows the broken line. Actual differential land use change associated with reservoir construction at any time from m_1 to n is the vertical distance between these two lines. Figure 1 is a two dimensional representation of differential land use change for a single land use. Estimates generated by a Markov model are multi-dimensional. Each land use category is estimated simultaneously with the restriction that the sum of all changes must be equal to zero.

Projecting Future Differential Land Use Change

The above model may be extended to project the future impacts of land use change associated with reservoir construction. Projected differential land use change impacts of reservoir construction are differential land use changes at some future time period when it is not possible to

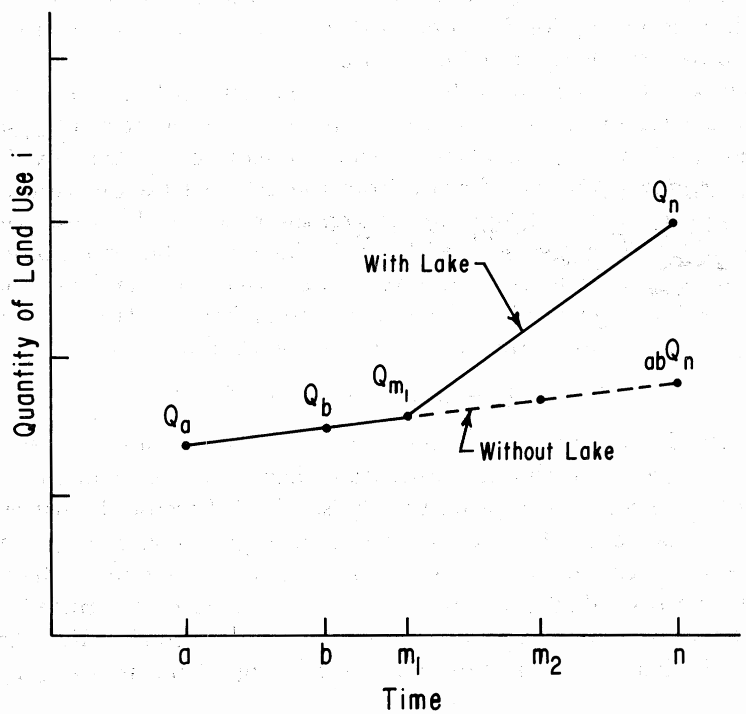

Figure 1. Illustration of Actual Differential Change in Use i Associated with Reservoir Construction

observe actual land use patterns. In this case actual observations of Q_n in (7) are replaced by Markovian estimates of future land use patterns based on a post-investment (a time period following reservoir construction) matrix of transition probabilities. The difference between estimates of land use patterns at time n based on pre-investment and post-investment transition probabilities is a measure of the projected differential impact of the investment at time n.

More specifically, let ${}_{ab}P$ (where $a < b < m_1$) be the transition matrix reflecting the land use transition patterns before the lake was initiated and ${}_{cd}P$ (where $m_2 < c < d$) be the transition probabilities derived over a time period following completion of the project. If the presence of the lake affects the land use transition process, then ${}_{ab}P = {}_{ed}P$.

The estimated land use pattern in n (where $n > d$) that would have occurred if the investment had not been made is estimated using pre-investment transition probabilities.

$$ {}_{ab}Q_n = Q_a \left[{}_{ab}P\right]^n \tag{8} $$

The land use pattern that is projected to exist in n with the reservoir development is estimated using post-investment transition probabiliies and a post-investment original state (Q_c):

$$ {}_{cd}Q_n = Q_c \left[{}_{cd}P\right]^{n-c} \tag{9} $$

The difference between the estimated land use patterns in (9) and (8) is the projected differential land use impact (D_n) of the investment at time (n).

$$ D_n = {}_{cd}Q_n - {}_{ab}Q_n = Q_c \left[{}_{cd}P\right]^{n-c} - Q_a \left[{}_{ab}P\right]^n \tag{10} $$

The computation of the projected differential land use change for one land use is illustrated in Figure 2. The actual quantity of land in use i is shown by the solid line while the estimated land in use i had the reservoir not been constructed follows the broken line. Projected differential land use change for land use i associated with the reservoir construction at time n is the vertical distance between ${}_{cd}Q_n$ and ${}_{ab}Q_n$.

Since ${}_{cd}P$ and ${}_{ab}P$ are regular transition matrices, (10) may be estimated for any $n > d$ including n at infinity. As n approaches infinity, ${}_{ab}P$ and ${}_{cd}P$ approach equilibrium states in which net land use transitions in each will be zero. The equilibrium projected differential land use change provides an estimate of the eventual, total land use impact of the reservoir development in which all land use adjustments attributable to the lake are considered. These estimates should be of special interest in analyzing and evaluating the long-term impacts of reservoir construction and are comparable to estimates of lifetime benefits usually computed in benefit-cost analyses.

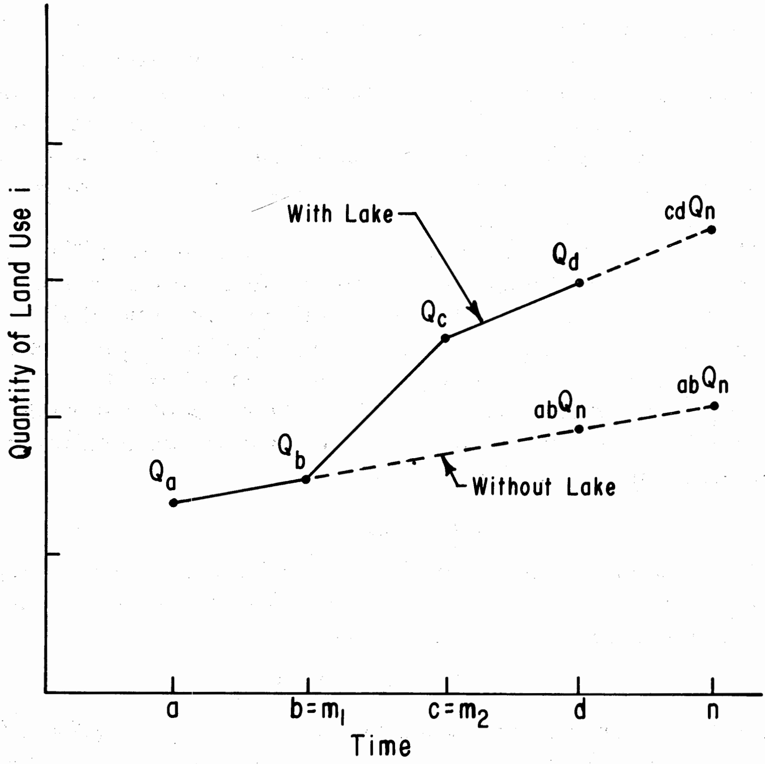

Figure 2. Illustration of Projected Differential Change in Land Use i Associated with Reservoir Construction

Dynamic Land Use Change Model

In the previous section a differential land use change model was developed in which land use change is taken as the difference between two estimates of land use, each estimate being derived by a static Markov model. As mentioned previously, a static Markov change model is one in which all of the transition probabilities are assumed to be constant. In a dynamic Markov model the transition probabilities are assumed to change over time. Since the land use changes caused by the construction of a water resource development project may initially be quite great but diminish over time, dynamic transition probabilities are conceptually attractive. It may well be that the pattern of land use change after the initial water resource development project impacts have dissipated will be no different than that which existed prior to the project.

The essential difference between a static transition probability matrix such as (2), and a dynamic transition probability matrix is that in a dynamic matrix each element p_{ij} is a function of time[2]. Each of the elements in a dynamic transition probability matrix must satisfy the basic conditions for a Markovian transition probability matrix: each p_{ij} must be greater than or equal to 0; and the sum of the p_{ij} in each row must be exactly equal to 1. Within a dynamic framework the second assumption becomes very crucial since each p_{ij} changes over time. This particular assumption requires that the sum of several independent functional relationships must be equal to 1 in each time period. With a static model this assumption is not critical because the values of the p_{ij} do not change. However, within the dynamic framework the sum of the elements of each row must be equal to one *in each time period.*

Previous studies utilizing dynamic Markov change probabilities have followed either one of two techniques for generating dynamic transition probabilities. The first technique was introduced by Hallberg in 1969 [3]. This technique calls for a linear regression of each element in each row with time as an exogenous variable. The regression procedure used ensures that the sum of the elements of each row is equal to 1. The disadvantages of this procedure are twofold. In the first place it must be assumed that the rate of change of the estimated transition probabilities in response to changes in the exogenous variables is constant over time. Thus, even though estimates are dynamic, they are relatively inflexible. The second difficulty is that even though the procedure forces the sum of all elements of each row to be equal to 1, it does not prevent the possibility of a given element in a row falling below zero or above 1. In such cases it is necessary to adjust all estimates within the row by arbitrarily setting problem elements equal to zero or one and adjusting all other elements such that the row elements sum to 1.

The second approach to estimating non-linear transition probabilities is a geometric adjustment model developed by Salkin, Just, and Cleveland [10]. In this approach it is assumed that each element of the transition matrix adjusts to the previous year's change by a fixed proportion as shown in equation (11).

$$p_{ij,\,t+1} = p_{ij,\,t} + \theta_i\,(p_{ij,\,t} - p_{ij,\,t-1}) \tag{11}$$

Each p_{ij} in each time period $t + 1$ is equal to the previous year's transition matrix element ($p_{ij,\,t}$) plus a certain proportion of the difference between the previous year's and the next previous year's transition matrix elements. The proportion of adjustment in (11) is θ_i. This θ_i is the proportion of the previous years adjustment which occurs in the current year. For instance, if θ_i is equal to 50%, then in each year 50% of the previous

[2] Non-stationary transition probabilities may be a function of time, or any other variable. The dynamic transition probabilities discussed herein are related to time alone.

year's adjustment occurs. This geometric adjustment procedure causes the transition probabilities to adjust rapidly in the first years following the initial change and then to taper off as time increases. The dynamic adjustment model in (11) may be solved by converting (11) into a structural equation as shown in (12).

$$\begin{aligned}
&\text{let: } a_{ij} = p_{ij,\,-1} \\
&\text{and } B_{ij} = p_{ij,\,o} - p_{ij,\,-1} \\
&\text{then } p_{ij,\,o} = a_{ij} + B_{ij} \\
&\quad p_{ij,\,1} = p_{ij,\,o} + \theta_1 B_{ij} = a_{ij} + B_{ij} + \theta_i B_{ij} \\
&\quad p_{ij,\,2} = p_{ij,\,1} + \theta_i^2 B_{ij} = a_{ij} + B_{ij} + \theta_1 B_{ij} + \theta_i^2 B_{ij} \\
&\qquad \cdot \\
&\qquad \cdot \\
&\qquad \cdot \\
&\quad p_{ij,\,t} = a_{ij} + B_{ij} + B_{ij} \sum_{n=o}^{t} \theta_i^n
\end{aligned} \tag{12}$$

The general structural equation may be expressed as

$$p_{ij,\,t} = a_{ij} + B_{ij} \frac{1 - \theta_i^t}{1 - \theta_i} \tag{13}$$

in which the value of p_{ij} in any time period is a function of a_{ij}, B_{ij}, θ_i and time. Since a_{ij}, B_{ij}, and θ_i do not vary with time, there is a non-linear relationship between the value of $p_{ij,\,t}$, and t which is determined by the value of these three parameters. By means of a maximum likelihood estimation procedure [4] it is possible to estimate the values of the parameters, a_{ij}, B_{ij}, and θ_i based on observed values of the transition probabilities and time. The estimated values of the parameters may then be used to generate a system of dynamic transition probability matrices for each future time period. Since θ_i in (11) is assumed constant for each row of the transition probability matrix, it is possible to estimate the p_{ij} such that the sum of all of the estimates is always equal to 1 using the constrained least squares technique mentioned above. As before there is a problem that sometimes the estimated p_{ij} values will be less than zero or greater than one. In these cases it is necessary to adjust all other elements such that the total of the elements be equal to 1.

The use of dynamic transition probabilities for estimating land use change parallels that described previously for static transition probability matrices. The principle difference is that with dynamic transition probability matrices there will be a unique transition probability matrix for each and every year.

$$P_t = \begin{matrix} p_{11,\,t} & \cdots & p_{ri,\,t} \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot & \cdot \\ p_{ri,\,t} & \cdots & p_{rr,\,t} \end{matrix} \qquad (14)$$

Matrix (14) may be compared to (2) in which all of the transition probabilities were assumed static. The land use pattern for each year may be computed using the dynamic transition probability matrix P_n:

$$D_n = {}_dQ_n = Q_o[P_n] \qquad (15)$$

where ${}_dQ_n$ is used to identify the vector of estimated land uses in n generated by dynamic transition matrices. Equation (15) should be compared with (6) in which a static transition probability matrix was used to estimate land use change. In the static case the transition probability matrix P is raised to the n^{th} power and then multiplied times the original land use vector in order to obtain a land use projection for time period n. In the dynamic case each element of P_n is directly estimated for the n^{th} time period by (13).

The differential land use change in any time period **n** is the difference between the dynamic estimates for time period n, and the static estimate based on the pre-lake transition probability matrix (${}_{ab}p^n$):

$${}_dD_n = {}_dQ_n - {}_{ab}Q_n = Q_oP_n - Q_o\ {}_{ab}P^n \qquad (16)$$

Differential land use change estimated by the dynamic transition probability matrix is nothing more than the difference between the two matrices, the dynamic and the initial static matrix. Equation (16) should be compared to (10). Pre-lake land use vectors are estimated with static transition probability matrices in both (10) and (16). In (10), the post lake land use vector is estimated with a static transition probability matrix while in (16) this vector is estimated by a dynamic transition probability matrix.

A similar procedure may also be used to estimate actual land use changes associated with the construction of a water resource development project. To do this one simply needs to replace the second term of (16) with observed land use change is time period n. When dynamic transition probability matrices are used it usually is not possible to estimate an equilibrium land use vector.

USE OF THE PROCEDURE FOR ESTIMATING LAND USE CHANGE AROUND KEYSTONE RESERVOIR

Keystone Reservoir is a large Corps of Engineers project which was completed in 1965. Due to its location near Tulsa, Oklahoma, the lake has attracted substantial residential and commercial development. The extent of the differential impact of the project on land use patterns around the lake may be computed using the procedures described above.[3]

Static Estimates of Land Use Change

Land uses in the study area for each of three use categories were measured in two pre-project years (1948 and 1958) and two post-project years (1964 and 1970). The land use flow matrices based on these data are shown in Tables 1 and 2. To estimate the land use pattern that would have existed in future time periods had the lake not been constructed, transition probabilities for the pre-project time period which are derived from the flow data in Table 1 are multiplied times the original land use pattern of 1948. The estimated patterns for selected years are shown

[3] For a complete description of the Keystone area and the data collection procedures, see Vandeveer [11] and Drummond [12].

Table 1.—Pattern of Land Use Change in Pre-Project Time Period: Keystone Lake

	1958 Uses—Acres			
1948 Uses	**Agricultural**	**Residential**	**All Other**	**Total**
Agricultural	87,889	283	766	88,938
Residential	212	602	14	828
All Other	494	15	1,396	1,905
Total	88,596	899	2,175	91,670

Table 2.—Pattern of Land Use Change in Post-Project Time Period: Keystone Lake

	1970 Uses—Acres			
1964 Uses	**Agricultural**	**Residential**	**All Other**	**Total**
Agricultural	86,789	347	528	87,665
Residential	128	1,083	29	1,240
All Other	518	23	2,224	2,766
Total	87,434	1,454	2,781	91,670

in Table 3.[4] The equilibrium estimate refers to the land use pattern that will exist when the net rate of change of each class is zero.

The actual 1970 differential land use change associated with the Keystone project is shown in Table 4. By 1970, the amount of land in agricultural uses was 876 acres less than what it is estimated it would have been if the project had not been constructed. Residential and all other uses experienced differential increases of comparable magnitudes, but the relative increase in residential was much greater.

The differential change in land use patterns which can be expected in future time periods is estimated by (10). The projected impact of Keystone is equal to the difference between: a) future land use patterns estimated by pre-project transition probabilities (Table 3); and, (2) future land use patterns estimated by post-project transition probabilities (Table 5). Any differences between the two estimates may be due to different elements in the transition probabilities matrices and/or different initial land use vectors. Projected differential land use change estimates for 2000 and in equilibrium are shown in Tables 6 and 7.

[4] Since the transition probabilities matrix was estimated over a 10 year period (1948-58), future estimates are obtained for 1968, 1978, etc. The data presented in Table 3 are extrapolated from such estimates.

Table 3.—Estimated Land Use: Keystone (Based on Static 1948-8 Transition Matrix)

Year	Agricultural	Residential	All Other
Observed Land Use	——acres——	——acres——	——acres——
1948	88,938	828	1,905
1958	88,596	899	2,175
Estimated Land Use			
1964	88,446	930	2,291
1970	88,310	959	2,400
1980	88,138	996	2,535
2000	87,921	1,044	2,705
Equilibrium	87,674	1,102	2,894

Table 4.—Actual Differential Land Use Change: Keystone, 1970

Land Use	1970 Acreage Estimated by 1948-58 Static Transition Matrix	1970 Acreage Observed	1970 Differential Land Use Change
Agricultural	88,310	87,434	− 876
Residential	959	1,454	+ 495
All Other	2,400	2,781	+ 381

Table 5.—Estimated Land Use: Keystone (Based on 1964-70 Static Transition Matrix)

Year	Agricultural	Residential	All Other
Observed Land Use	——acres——	——acres——	——acres——
1964	87,664	1,240	2,766
1970	87,434	1,454	2,781
Estimated Land Use			
1980	87,112	1,748	2,809
2000	86,635	2,175	2,859
Equilibrium	85,787	2,898	2,985

Table 6.—Projected Differential Land Use Change: Keystone, 2000

Land Use	2000 Acreage Estimated by Pre-WRDP Static Transition Matrix	2000 Acreage Estimated by Post-WRDP Static Transition Matrix	2000 Differential Land Use Change
Agricultural	87,921	86,635	— 1,286
Residential	1,044	2,175	+ 1,131
All Other	2,705	2,859	+ 154

Table 7.—Projected Differential Land Use Change: Keystone, Equilibrium

Land Use	Equilibrium Acreage Estimated by Pre-WRDP Static Transition Matrix	Equilibrium Acreage Estimated by Post-WRDP Static Transition Matrix	Equilibrium Differential Land Use Change
Agricultural	87,674	85,787	− 1,887
Residential	1,102	2,898	+ 1,796
All Other	2,894	2,985	+ 91

Two interesting trends are evident in the projected differential land use changes. First, the differential impact on residential uses continues to grow beyond 1970, finally reaching nearly 1800 acres by equilibrium. Second, the differential impact on all other uses diminishes over time. Since most of these latter uses are infrastructural (transportation, schools, commercial, etc.), this finding is not unexpected since immediately after completion of the project all infrastructure deemed necessary for future demands was installed. Initially, this represented more infrastructure that would have been necessary with pre-project growth patterns. However, over time the pre-project growth in the all other uses category "catches up" with the somewhat slower post-project growth rate. Such a pattern of land use change could be shown graphically by a figure similar to Figure 2 but in which the growth patterns between time periods d and n converge rather than diverge.

Dynamic Estimates of Land Use Change

Land use patterns in the Keystone study area may also be projected using a system of dynamic transition probabilities described above. Future land use patterns for the Keystone study area estimated by a system of dynamic transition probabilities are shown in Table 8. These data were obtained from (15) where the transition probabilities are estimated by (13). The estimation procedure used is more fully described in the appendix.

As with static estimates of land use patterns, the dynamic estimates indicate that agricultural uses will decline while residential and all other uses increase. With dynamic transition probabilities it is impossible to derive an equilibrium vector, therefore, these data are not shown in Table 8. Note however that by 1980 residential land use has already achieved an apparent steady state since there was little land use change between 1980 and 2000. Estimates for further time periods not shown here indicate that the pattern estimated for 2000 is near an eventual equilibrium.

Actual 1970 and projected differential land use changes estimated by dynamic transition probabilities are shown in Table 9. The actual differential land use change is simply the difference between the 1970 land use pattern estimated by the dynamic transition matrix and the land use pattern observed in 1970. The actual change in 1970 estimated by dynamic transition matrices is less than the static differential land use change.

Projected differential land use change is the difference between estimated land use based on dynamic transition probabilities shown in Table 8 and land use patterns shown in Table 3 which were estimated

Table 8.—Estimated Land Use: Keystone (Based on dynamic transition matrix)

Year	Agricultural	Residential	All Other
Observed Land Use	*—acres—*	*—acres—*	*—acres—*
1948	88,938	828	1,905
Estimated Land Use			
1958	88,210	1,135	2,324
1964	88,101	1,186	2,382
1970	88,003	1,232	2,435
1980	87,895	1,263	2,511
2000	87,776	1,263	2,630

Table 9.—Dynamic Differential Land Use Change: Keystone

Year	Agricultural	Residential	All Other
Actual Change	*—acres—*	*—acres—*	*—acres—*
1970	—569	+222	+346
Projected Change			
1980	—243	+267	— 24
2000	—145	+219	— 75

by the pre-project static transition probabilities. The differential land use change estimated in this manner is shown for 1980 and 2000 in Table 9. These results should be compared to those in Tables 6 and 7. Several differences are evident.

Perhaps the most interesting characteristic of the dynamic land use change estimates is that the amount of estimated change is much less than that estimated by the static transiion probabiliy marices. This is probably a consequence of the geometric adjustment which is used in estimating the dynamic transition probabilities. As a consequence of this adjustment mechanism the transition probabilities in the dynamic model tend towards an equilibrium transition probability matrix not unlike that of the pre-project time period. As a consequence, most of the land use change estimated to occur within the dynamic model occurs soon after the completion of the project. By contrast, in the static estimates the same rate of change is assumed to occur during all future time periods.

Another interesting finding in Table 9 is that the all other land use category is estimated to have a negative differential land use change in 1980 and 2000. This means that as a consequence of the construction of the Keystone project the amount of land used in the study area in the all other category is less than what would have been expected had the project not been built. This finding is contrary to apriori expectations and is difficult to sustain based on observed patterns of land use change near Keystone and other water resource development projects. Although the direction of change estimated for all other land use is negative, the magnitude of the change is quite small.

Finally, the results for residential land use in the dynamic model suggest that the initial impact of the Keystone project on residential land use declines over time. The estimated increase in residential land use in 1970 is 273 acres but by 2000 it has declined to 219 acres. In other words, the rate of increase of residential land use estimated by the dynamic model in future time periods is less rapid than that estimated by the 1948-58 static transition probabilities. Consequently, the magnitude of the differential increase in residential land use tends to decline over time.

CONCLUSIONS

The methodology used in this project and the illustrative results reported herein are primarily of an ex post nature. The estimated changes in land use patterns based on pre-project transition matrices are deemed to be consistent with a priori expectations. Since these estimates and projections are not subject to validation, it is impossible to test the accuracy of the estimates or the methodology. Nonetheless, the nature of the results obtained suggests that the methodology is sensitive to the particular characteristics of the study area.

The potential for using the methodology in an ex ante manner is considered quite limited. If a priori expectations of future land use changes associated with any contemplated project are obvious, then the methodology may be feasible. However, the difficulty of quantifying ex ante transition probability matrices would probably nullify any beneficial attitudes the methodology may possess.

Attempts to use dynamic transition probability matrices to estimate or project land use patterns were generally unsatisfactory. The problems associated with this methodology are many. In the first place the data requirements are much greater than for the estimation of a static transition probability matrix. In the second place the geometric adjustment model used in this study requires the estimation of an equation with nonlinear parameters. The techniques available to perform such estimates are somewhat limited and appear to give results that are very sensitive to changes in the data. Finally, the dynamic transition probability estimation procedure averages changes over a number of time periods such that the estimated pattern of change is not characteristic of any time period but instead is characteristic of an average of all time periods. As is always the case, an average often tends to obscure more than it reveals.

Ex post estimates of land use change using static transition probabilities have been shown to be useful tools in evaluating the impact of water projects. Most previous impact studies have focused on the changes in economic patterns in the impacted area. The results in Vandeveer [11] suggest that these analyses have failed to identify and evaluate the economic changes associated with land use changes which have occurred. Knight and Drummond [8] have demonstrated the efficacy of the methodology used in this study as a tool in impact analysis by evaluating the impact of the Keystone project on the property tax base and the demand for public services within the study area. A variety of other uses of the ex post estimates of land use change may be envisioned.

APPENDIX

The elements of the matrix of dynamic transition probabilities were estimated by equation (13) within a nonlinear ordinary least squares framework that forced the sum of the elements of each row to be equal to one. The regression procedure is taken from Hallberg [3].

Equation (13) may be rewritten in the form used for estimating each row i of a 3 x 3 dynamic transition probability matrix based on three static probability matrices:

$$\text{(A1)} \qquad p_{ijt} = b_1D_1 + b_2D_2 + b_3D_3 + b_4D_1 \frac{(1+\theta_i)^t}{1+\theta_i} + b_5D_2 \frac{(1+\theta_i)^t}{1+\theta_i} + b_6D_3 \frac{(1+\theta_i)^t}{1+\theta_i}$$

for: $j = 1, \ldots, 3$
$t = 1, \ldots, 3$

where: p_{ijt} is the static transition probability of land use i in time $t - 1$ moving into use j by time t;

D_k is a dummy variable equal to 1.0 when $j = k$ and 0 in all other cases;

θ_i geometric adjustment factor for row i, assumed to be equal for all j elements of that row;

t time, measured in number of periods covered by data.

Equation (A1) is estimated without an intercept term. The estimated coefficients (and t-values[5]) of (A1) for each of the three rows in the dynamic transition probabilities matrix are shown in Table A1.

In all cases, the R^2 statistics were very high but the t-values on some coefficients were well below standard levels of acceptance. The value of the geometric adjustment paramenter (θ) is near 1.0 for the first two rows (all other and residential land uses) suggesting that the adjustment process for these land uses will be extended over a long period of time.

The justification for the dummy variable estimation procedure is demonstrated by the values of the estimated coefficients. Note that for each row, $\sum_{i=1}^{3} b_i = 1.0$ and $\sum_{k=4}^{6} b_k = 0.0$. Since each of the first three coefficients is multiplied times 1.0 once per row, the row total is equal

[5] The statistical properties of maximum likelihood estimates of nonlinear models is not well stablished. The t-ratios presented are generated by the statistical package used to estimate (A1) [4].

to 1.0; and, since each of the last three coefficients is multiplied once times a common (to each row and each time) variable, the sum of the last three terms is zero. Hence, the sum of the six terms is always equal to 1.0 for any given θ_i and for all t.

The value of the first element in row i is given by:

$$(A2) \qquad P_{i1t} = b_1 + b_4 \frac{(1 + \theta_i)^t}{1 + \theta_i}$$

Since $D_1 = 1$ and $D_2 = D_3 = 0$, each element of the transition probability matrix for each year $t = 1, \ldots, n$ is computed as in equation (A2). Unfortunately, the estimation procedure does not preclude the possibility of any $p_{ijt} > 1.0$ or $p_{ijt} < 0.0$. In the first case, all other row elements must equal 0.0 and the p_{ijt} is fixed at 1.0. If $p_{ijt} < 0.0$, then the estimated value is arbitarily set 0.0 and all other row elements are increased proportionately such that the row total of all elements equals 1.0.

Table A1.—Estimated Coefficients of Geometric Adjustment Model of Dynamic Transition Probabilities

Coefficient	Row 1	Row 2	Row 3
b_1	0.685347	0.016927	0.166880
	(22.07)	(0.48)	(0.007)
b_2	0.007986	0.638396	0.093407
	(0.26)	(18.32)	(0.004)
b_3	0.307182	0.344676	1.357265
	(9.89)	(9.89)	(0.06)
b_4	0.037072	0.003355	3.027211
	(4.02)	(0.32)	(0.22)
b_5	0.000368	0.076293	7.449627
	(0.04)	(7.36)	(0.54)
b_6	—0.037544	—0.079648	—10.68164
	(—4.07)	(—7.69)	(—0.77)
θ	0.984615	0.984615	0.015385
R^2	0.99	0.99	0.99

Note: t-values are shown in parentheses directly below the coefficient estimates.

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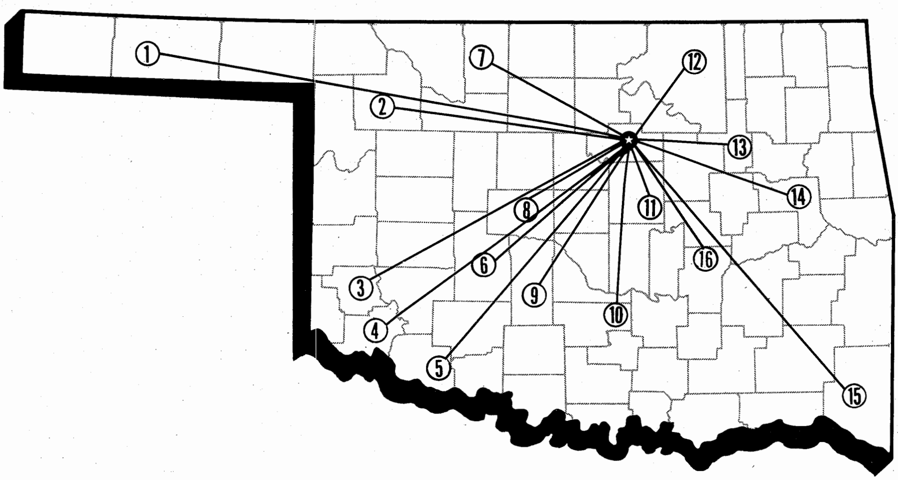

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