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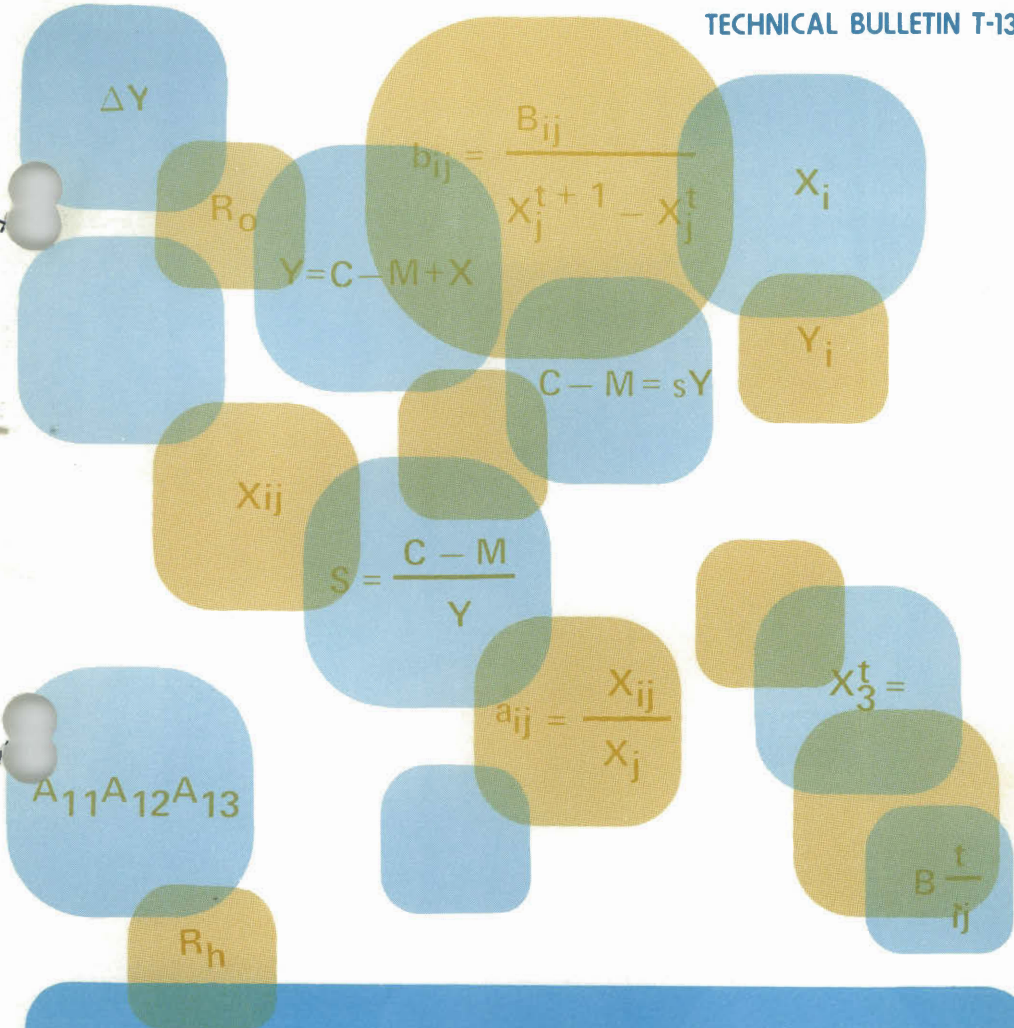

Interindustry Models for Rural Development Research

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CONTENTS

Interindustry Models For Rural Development Research*

by

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The objective of this paper is to present four models used in rural development research. The four models include input-output, from-to, dynamic input-output, and simulation. Input-output and from-to analysis are static models and are used to measure interindustry effects in the short-run. Dynamic input-output is introduced to measure interindustry effects over time. Simulation is a model which can be used for short and long-run analysis.

The format of the paper will be to present each model in four sections. (1) The basic components of each model will be discussed. (2) The assumptions of each model will be evaluated. (3) An application of each model will be presented to illustrate its empirical uses. (4) A mathematical presentation of each model will be outlined in an appendix.

The bulletin is intended to present the models in a non-mathematical manner for researchers and students who have not had previous experience with the models. The mathematical appendices are presented for those who are more mathematically inclined and for those who desire deeper knowledge of the underlying theory.

PART I

THE INPUT-OUTPUT MODEL

The most popular interindustry model is the input-output model. The model as used today is based mainly upon work completed by Professor Wassily Leontief [9]. Leontief completed input-output studies of the U.S. economy for the years 1919, 1929, and 1939 and is now working

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on a world input-output model. A more detailed input-output study was completed in 1947 by Duane W. Evans and Marvin Hoffenberg [7].

The Bureau of Economic Analysis (USDC) has subsequently initiated a policy to complete a U.S. input-output study every five years. Thus far, the Bureau has published studies for 1958, 1963, and 1967. Polenske et.al. [12, p. 1] have developed a consistent set of multiregional input-output tables for each state for 1963. In addition to these national and state studies, many regional input-output studies have been completed [1].

Basic Components

The input-output model[1] consists of three basic components: a transaction or flow table, a set of direct coefficients, and direct and indirect coefficients. The flow table is the base of the model as the direct, and direct and indirect coefficients are derived from it. A five sector model of the Oklahoma economy will be used to illustrate the model. Each sector consists of a set of relatively homogeneous industries aggregated according to a pre-determined classification.

The Flow Table

To illustrate the flow table, consider a model having four producing sectors and a single final demand sector. Each producing sector has a certain amount of output, which is used within the sector, purchased by other sectors, or purchased for final demand by the consumer. The flow table presented in Table 1.1 may help to explain the model. The upper left-hand part of the table is known as the processing section and contains these sectors producing goods and services from the other sectors. In most empirical input-output studies, this portion of the flow table is greatly expanded and often contains numerous sectors.

The final demand sectors consist of activities of those who purchase goods and services from the producing sectors for final disposition. Sectors generally found in this section are: households, governments, exports, inventory change and capital formation. The primary input section consists mainly of imports, households, governments, and depreciation. The figures in this row indicate the amount of primary inputs purchased by the sectors in the processing and final demand sections. The small model used here for illustration purposes has two primary input sectors and one final demand sector. The sectors within each were aggregated for simplicity.

Reading down a column indicates the amount purchased by that sector from the industries represented by the row. For example, the agri-

[1] For a complete mathematical presentation of the model, see Appendix 1.1.

Table 1.1. Flow Table for Oklahoma, 1963 (Millions of Dollars).

Producing Sector	Purchasing Sector: Agricultural Production	Agricultural Processing	Manufacturing and Mining	Service	Final Demand	Total Output
Agricultural Production	183	191	8	27	378	787
Agricultural Processing	46	70	3	18	358	495
Manufacturing and Mining	33	24	921	528	1,677	3,183
Service	112	71	544	1,103	3,252	5,082
Households	246	71	927	2,274	1,596	5,114
Other Primary Inputs	167	68	780	1,132	2,628	4,775
Total Inputs	787	495	3,183	5,082	9,889	19,436

Source: [5].

cultural production sector purchases goods and services valued at $183 million from firms within the sector, $46 million from the agricultural processing sector, $33 million from the manufacturing and mining sector, $112 million from the service sector, $246 million from households and $167 million worth of goods and services from the other primary inputs sector. The total amount of purchases by the agricultural production sector was $787 million.

The amount of sales from one industry to the other industries is obtained by reading across the row. For example, the agricultural production sector sold goods and services valued at $183 million to the firms in the sector, $191 million to agricultural processing, $8 million to manufacturing and mining, $27 million to the service sector and $378 million to final demand. Total sales of the agricultural production sector was $787 million. The amount sold equals the amount purchased, which is true for all producing sectors, but not necessarily for the final demand column and primary input row.

Direct Coefficients

The direct coefficients indicate the input requirements per dollar of output for a given sector. Direct coefficients (sometimes called technical coefficients) are relevant only for the processing sectors; therefore, technical coefficients are computed only for the columns of the purchasing sectors. Calculation of the coefficients consists of dividing all the entries in each industry's column by total output for that sector. The direct coefficients for the Oklahoma model are presented in Table 1.2.

Considering a particular column, say column one, the direct coefficients are interpreted as follows: For each dollars worth of output of the agricultural production sector, the sector requires direct purchases of 23 cents from firms in the sector, 6 cents from agricultural processing, 4 cents from manufacturing and mining, 14 cents from the service sector, 31 cents from households and 2 cents from the other primary inputs sector. These coefficients show the direct effects in all sectors due to one dollar change in output of the agricultural production sector.

Direct and Indirect Coefficients

The direct and indirect coefficients (sometimes called interdependence coefficients) indicate the total change in input requirements as a result of a one dollar change in final demand. The direct coefficients indicate the first round effect of change in final demand in a given sector. The direct and indirect coefficients yield the total effect as a result of first, second, third, etc. rounds. For example, if final demand of the agricultural processing sector increases by one dollar and output by intra-industry transactions increases by \$.14 (column 2 and row 2 of Table 1.2), then output of the agricultural processing sector will increase by at least \$1.14. In addition, firms in the agricultural processing industry will purchase more from the other industries.

The agricultural processing sector will purchase 44 cents (1.14 x .39) from the agricultural production sector as a result of the increased activities in the agricultural processing sector. This effect occurs in all sectors. However, these are not all the indirect effects. When the agricultural production sector expands its production to meet the new demands of

Table 1.2. Direct Coefficients for Oklahoma, 1963.

	Agricultural Production	Agricultural Processing	Manufacturing and Mining	Services
Agricultural Production	.23	.39	.00	.01
Agricultural Processing	.06	.14	.00	.00
Manufacturing and Mining	.04	.05	.29	.10
Service	.14	.14	.17	.22
Households	.31	.14	.29	.45
Other Primary Inputs	.22	.14	.25	.22
Total	1.00	1.00	1.00	1.00

the agricultural processing sector, the agricultural production sector buys more inputs and all sectors selling to it experience an increase in sales. If all the indirect effects from the one dollar change in the agricultural processing were added, the end result would yield the direct and indirect coefficients as presented in Table 2.3.

The direct and indirect coefficients for the Oklahoma model are shown in Table 1.3. They are obtained by manipulating the set of direct coefficients in Table 1.2.[2] As an example, a one dollar increase in final demand from the agricultural production sector will change output by one dollar and 35 cents in the agricultural production sector, 9 cents in agricultural processing, 13 cents in manufacturing and mining and 29 cents in the service sector. These changes are due to the interaction among sectors as output changes to meet the increased demand.

Assumptions of the Input-Output Model

The input-output model is based upon two fundamental assumptions. The most restrictive assumption is that the direct coefficients are fixed. The assumption of fixed coefficients implies that technology remains constant, no external economies or diseconomies exist, and no substitution occurs to changes in relative prices or availability of new materials.

The fixed coefficient assumption places limits on the use of the input-output model as a long-range forecasting technique. Cameron and Chenery conducted research to check on the reasonableness of this assumption. Cameron [2] found that the model yielded a reasonable short-term approximation of the actual Australian economy. Chenery [4] con-

[2] For a mathematical description and procedure for computing the direct and indirect coefficients, see Appendix 1.1.

Table 1.3. Direct and Indirect Coefficients for Oklahoma, 1963.

	Agricultural Production	Agricultural Processing	Manufacturing and Mining	Services
Agricultural Production	1.35	.61	.01	.01
Agricultural Processing	.09	1.21	.00	.01
Manufacturing and Mining	.13	.17	1.45	.20
Services	.29	.37	.32	1.32
Output Multipliers	1.86	2.36	1.78	1.54

cluded that the fixed coefficient assumption is realistic for the short run; however, continued technological change causes the actual relationships to change over time. Therefore, periodical adjustments of the coefficients[3] or the construction of a new table is suggested.

The other assumption of the basic input-output model is that there are no errors of aggregation in combining industries into sectors. Industries within a sector are homogeneous and different from industries in other sectors. This implies that a given product is supplied by only one sector and there are no joint products. So the coefficients for a sector are representative of all the industries within that sector. Conclusions drawn from the analysis indicate the average conditions of the industries within the sector. The more sectors included in the model, the less chance that errors of aggregation will arise.

Application of the Input-Output Model

The input-output model coefficients describe the "structural interdependence" of an economy. From these coefficients various predictive devices can be computed which can be useful in analyzing economic changes in a region. Multipliers indicate the relationships between some observed change in the economy and the total change in economic activity created throughout the economy. Output requirements necessary for meeting a given final demand are helpful in predicting production patterns of a region.

The Output Multiplier

The output multiplier for a sector measures the change in total output from all sectors resulting from a one dollar change in final demand for the products of that sector. It is computed directly from the direct and indirect coefficients (Table 1.3) by adding down the column of a purchasing sector.[4] For example, by adding down the column for agricultural production in Table 1.3 the sector output multiplier is 1.86. This indicates that a one dollar change in final demand for agricultural production will cause a change in output of all sectors by 1.86. The output multiplier for each sector is listed in Table 1.3 under the respective columns. In addition to agricultural production, the output multipliers are 2.36 for agricultural processing, 1.78 for manufacturing and mining, and 1.54 for the services sector.

[3] Miernyk [11, pp. 117-125] has employed a technique called "best practice" to update technical coefficients.

[4] For the mathematical procedure used to calculate output multipliers, see Appendix 1.2.

The Income Multiplier

The income multiplier measures the total change in income throughout the economy resulting from a one dollar change in income in a sector.[5] The concept of the input-output income multiplier was developed by Hirsch [8].

The underlying basis of the income multiplier is that a certain amount of income is generated with each change in output. Direct and direct and indirect income effects are used to compute the income multipliers.[6] The direct income effect is the amount of each dollar of output which goes to households in the form of income either as wages and salaries, proprietor income, rents or profits. The direct income effects for the aggregate model are taken from the household row in Table 1.2 and listed in column (1) of Table 1.4.

Direct and indirect income effects are the total changes in income as a result of a one dollar change in final demand. This effect is measured by considering how output in each sector changes as a result of an initial one dollar change in final demand and how the output change affects income. For example, from Table 1.3, it can be seen that a dollar change in final demand for agricultural products will change output in that sector by $1.35. Households receive as income 31 cents of every dollar change in output; therefore, an initial change will cause household income to change by 42 cents (1.35 x .31).

The initial change in final demand for agricultural production will cause a direct and indirect change in output of 9 cents in the agricultural processing sector. From the direct effect, 14 cents of every dollar change in output in the agricultural processing sector goes to households. Thus, households income changes by 1 cent as the result of the one dollar change in the agricultural processing sector. Using the same procedure, the change in income going to households in the manufacturing and

[5] The income multiplier definition measures the direct and indirect effects. It does not include induced effects. For a discussion of induced effects see [11, pp. 42-50].
[6] For the mathematical procedure used to calculate income multipliers, see Appendix 1.3.

Table 1.4. Direct Income Effects, Direct and Indirect Income Effects, and Income Multipliers for the Oklahoma Model, 1963.

	Direct Income Effects (1)	Direct & Indirect Income Effects (2)	Income Multipliers (3)
Agricultural Production	.31	.60	1.93
Agricultural Processing	.14	.58	4.14
Manufacturing and Mining	.29	.59	2.03
Services	.45	.69	1.53

mining sector equals 4 cents and for the service sector equals 13 cents.

The sum of these income changes will give the total amount of direct and indirect income generated as a result of the initial one dollar change in final demand for that sector (.42 +.01 + .04 + .13 = .60). The same procedure is used for each sector to compute the amount of the direct and indirect effects which are listed in column (2) of Table 1.4.

Income multipliers for all sectors are listed in column (3) of Table 1.4. They are computed by dividing the direct and indirect income effect by the direct income effect (column 2 ÷ column 1). Each multiplier indicates the total amount of income generated by the increase of one dollar of income in that sector.

The Employment Multiplier

The employment multiplier as computed from the input-output model is defined as the total change in employment due to a one unit change in the employed labor force of a particular sector. The concept of input-output employment multipliers was developed by Moore and Peterson [10]. The basic assumption in computing employment multipliers is that there is a linear relationship between employment and output in a sector.

The computational procedure[7] for the employment multiplier is again related to a change in output. The change in output creates direct and indirect employment effects. The direct employment effect indicates the number of men employed per year per unit of output. These direct effects are listed in column (1) of Table 1.5. The direct employment effect of the agricultural processing sector indicates that 32.22 additional man-years of employment will be needed if output for that sector increases by one million dollars.

[7] For the mathematical procedure used to calculate employment multipliers, see Appendix 1.4.

Table 1.5. Direct Employment Effect, Direct and Indirect Employment Effect and Employment Multipliers for the Oklahoma Model, 1963.

	Direct Employment Effect (1)	Direct & Indirect Employment Effect (2)	Employment Multiplier (3)
	(Man-Years per million dollars output)		
Agricultural Production	18.24	52.18	2.86
Agricultural Processing	32.22	81.73	2.53
Manufacturing & Mining	37.23	76.03	2.04
Services	68.34	98.16	1.44

The direct and indirect employment effects are computed by considering the repercussions on employment in all sectors as a result of the initial change in final demand in a sector. For example, a one million dollar increase in final demand for agricultural production will increase output directly and indirectly within the agricultural production sector by 1.35 million dollars (Table 1.3). This increase will require 24.62 man-years of additional employment. In addition, as a result of this initial increase in final demand, the direct and indirect effect on output of the agricultural processing sector will be .09 million dollars. Since this sector requires 32.22 man-years of employment per million dollars worth of output, 2.90 additional jobs are created. Direct and indirect employment effects in the manufacturing and mining sector and services sector are 4.84 and 19.82 man-years respectively. Adding the employment effects created in each sector will yield the total direct and indirect employment effect of 52.18 ($24.62 + 2.90 + 4.84 + 19.82 = 52.18$).

The employment multipliers are derived by dividing the direct effect into the direct and indirect effect (column 2 $\div$ column 1). These multipliers are presented in column (3) of Table 1.5. If employment increases by one unit in the agricultural production sector, the total employment change throughout the economy is 2.86.

Prediction of Future Output

The input-output model can be used to predict the change in output of each sector necessary to meet some specified change in final demand. First, final demand for each sector must be estimated. For the four processing sectors in the input-output model, final demand for Oklahoma in 1980 was estimated as follows[8]

Agricultural Production	$ 598,828,000
Agricultural Processing	728,437,000
Manufacturing and Mining	3,155,694,000
Services	5,960,567,000

The output requirements for a sector necessary to meet the projected final demand is found by multiplying the total estimated final demand for each sector times the direct and indirect coefficients.[9] The output requirements for each sector to meet the final demands estimated for 1980 are :

Agricultural Production	$1,343,927,000
Agricultural Processing	994,909,000
Manufacturing and Mining	5,969,551,000
Services	9,320,952,000

[8] In constant 1963 dollars and from [5].
[9] For the calculation procedure of the output requirements, see Appendix 1.5.

The output requirements are considerably larger than the initial projected final demand. The magnitude of these figures indicate the type of pattern needed to meet the projected final demands for 1980.

Input-Output References

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Appendix 1.1

Mathematical Presentation of the Input-Output Model[10]

The Flow Table

Transactions of the four-sector economy as discussed in the body of this paper can be presented in a system of equations.

$$\begin{aligned}
X_1 &= x_{11} + x_{12} + x_{13} + x_{14} + Y_1 \\
X_2 &= x_{21} + x_{22} + x_{23} + x_{24} + Y_2 \\
X_3 &= x_{31} + x_{32} + x_{33} + x_{34} + Y_3 \\
X_4 &= x_{41} + x_{42} + x_{43} + x_{44} + Y_4 \\
R_h &= y_{h1} + y_{h2} + y_{h3} + y_{h4} + Y_h \\
R_o &= y_{o1} + y_{o2} + y_{o3} + y_{o4} + Y_o
\end{aligned}$$

X_i = gross output of the i^{th} sector
R_h = household input
R_o = other primary input
x_{ij} = purchases of the j^{th} sector from the i^{th} sector needed to produce X_j
y_{hj} = purchases from households by the j^{th} sector needed to produce X_j
y_{oj} = purchases of other primary inputs by the j^{th} sector needed to produce X_j
Y_i = final or consumer demand for products of sector i
Y_h = final or consumer demand for household inputs
Y_o = final or consumer demand for other primary inputs

An outlined form of the flow table may help to explain the system of equations. The equations inserted in the outlined form are presented in Figure 1.

[10] For a more rigorous mathematical presentation, see [13 and 16].

		Purchasing Sectors				Final	Total
		(1)	(2)	(3)	(4)	Demand	Output
Producing Sectors	(1)	x_{11}	x_{12}	x_{13}	x_{14}	Y_1	X_1
	(2)	x_{21}	x_{22}	x_{23}	x_{24}	Y_2	X_2
	(3)	x_{31}	x_{32}	x_{33}	x_{34}	Y_3	X_3
	(4)	x_{41}	x_{42}	x_{43}	x_{44}	Y_4	X_4
Primary Inputs	(1) Households	y_{h1}	y_{h2}	y_{h3}	y_{h4}	Y_h	R_h
	(2) Other Primary Inputs	y_{o1}	y_{o2}	y_{o3}	y_{o4}	Y_o	R_o
Total		X_1	X_2	X_3	X_4	Y	

Figure 1. Flow Table with Mathematical Notation Inserted

The Direct Coefficients

The direct or technical coefficients are derived from the flow table by assuming that the relationship between the purchases of a sector and the level of output of that sector is linear [3]. This relationship can be expressed in the following form:

$$x_{ij} = a_{ij} X_j \qquad j = 1, 2, 3, 4; \quad i = 1, 2, 3, 4.$$

The a_{ij}'s (purchases of the j^{th} sector from the i^{th} sector irrespective of the level of output of the j^{th} sector) are parameters in the expression. The technical coefficient (a_{ij}) is the ratio of the purchases of output from industry i by industry j over the gross output of industry j. Mathematically, this is presented as:

$$(2) \quad a_{ij} = \frac{x_{ij}}{X_j}$$

Each a_{ij} indicates the direct dependence on industry i per dollar of output of industry j.

Direct and Indirect Coefficients

The calculation of the direct and indirect coefficients begins by subtracting the matrix of technical coefficients from an identity matrix. An identity matrix has the diagonal elements equal to 1 and the remaining elements zero. Then the inverse of the resulting matrix provides the set of direct and indirect coefficients. The mathematical procedure is as follows: First, the $a_{ij}X_j$'s are substituted for the x_{ij}'s in the set of equations listed in (1). The equations are then solved for Y_i.

$$
(3)\qquad
\begin{aligned}
Y_1 &= X_1 - a_{11}X_1 - a_{12}X_2 - a_{13}X_3 - a_{14}X_4\\
Y_2 &= X_2 - a_{21}X_1 - a_{22}X_2 - a_{23}X_3 - a_{24}X_4\\
Y_3 &= X_3 - a_{31}X_1 - a_{32}X_2 - a_{33}X_3 - {}_{34}X_4\\
Y_4 &= X_4 - a_{41}X_1 - a_{42}X_2 - a_{43}X_3 - a_{44}X_4
\end{aligned}
$$

Rewriting equation (3),

$$
(4)\qquad
\begin{bmatrix} 1-a_{11} & -a_{12} & -a_{13} & -a_{14}\\ -a_{21} & 1-a_{22} & -a_{23} & -a_{24}\\ -a_{31} & -a_{32} & 1-a_{33} & -a_{34}\\ -a_{41} & -a_{42} & -a_{43} & 1-a_{44} \end{bmatrix}
\bullet
\begin{bmatrix} X_1\\ X_2\\ X_3\\ X_4 \end{bmatrix}
=
\begin{bmatrix} Y_1\\ Y_2\\ Y_3\\ Y_4 \end{bmatrix}
$$

In matrix notation,[11] it would read as:

(4a) $(\underline{I} - \underline{A})\ \underline{X} = \underline{Y}$

where $\underline{I} = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $\underline{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$

The matrix (I-A) is the direct and indirect coefficient matrix (also known as the "Leontief Matrix") and has the special properties that diagonal elements are positive, while all remaining elements are negative or zero. The solution of the set of equations in (4) is simply obtained by finding the inverse of the Leontief Matrix. This solution is as follows:

$$
(5)\qquad
\begin{bmatrix} X_1\\ X_2\\ X_3\\ X_4 \end{bmatrix}
=
\begin{bmatrix} A^{11}A^{12}A^{13}A^{14}\\ A_{21}A_{22}A_{23}A_{24}\\ A_{31}A_{32}A_{33}A_{34}\\ A_{41}A_{42}A_{43}A_{44} \end{bmatrix}
\bullet
\begin{bmatrix} Y_1\\ Y_2\\ Y_3\\ Y_4 \end{bmatrix}
$$

In matrix notation the equation is:

(5a) $\underline{X} = (\underline{I} - \underline{A})^{-1}\ \underline{Y}$

Each A_{ij} which is an element of the $(I - A)^{-1}$ matrix, and indicates the amount of direct and indirect production from sector i necessary to sustain a final demand of one unit in sector j.

Appendix 1.2
Output Multipliers

The matrix of direct and indirect coefficients is multiplied by a one unit change in final demand which takes place in a particular sector. For example, if X_1 increases by one unit, the output multiplier for sector one would be:

$$
(6)\qquad
\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14}\\ A_{21} & A_{22} & A_{23} & A_{24}\\ A_{31} & A_{32} & A_{33} & A_{34}\\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}
\bullet
\begin{bmatrix} 1\\ 0\\ 0\\ 0 \end{bmatrix}
= A_{11}+A_{21}+A_{31}+A_{41} = \sum_{i=1}^{4} A_{i1}
$$

[11] The underscore is used to denote a matrix.

This indicates that a one unit change in final demand in the first sector will cause a total change in all sectors equal to $\sum_{i=1}^{4} A_{i1}$. Similarly, the effects can be measured of a one unit change in any other sector of the economy.

Appendix 1.3
Income Multipliers

The income multipliers are computed by completing three steps. First, the direct income effects (h_j) are calculated. These are computed from the flow table as follows:

$$(7) \quad h_j = \frac{y_{hj}}{X_j}$$

Second, the direct and indirect income effects are calculated by transposing the direct and indirect coefficients matrix and multiplying by the column of direct income effects. In mathematical notation, it is as follows:

$$(8) \quad \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}' \bullet \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{14} \end{bmatrix}$$

where h_{Ii} is the direct and indirect income effect for sector i.

The third step in computing the income multipliers is to divide the direct income effects into the direct and indirect income effects.

$$(9) \quad \begin{bmatrix} \frac{h_{11}}{h_1} \\ \frac{h_{12}}{h_2} \\ \frac{h_{13}}{h_3} \\ \frac{h_{14}}{h_4} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

Where I_j is the income multiplier for sector j. This multiplier shows the direct and indirect income generated for each dollar of direct income in sector j.

Appendix 1.4
Employment Multipliers

The employment multiplier is computed in a similar manner to the income multiplier. First, the employment output ratios e_i must be determined for each sector. These are calculated by dividing the output per year of a sector into the number of man-years employed in that sector. Second, the direct and indirect employment effects are calculated by transposing the direct and indirect coefficient matrix and multiplying by the column of direct employment effects (employment output ratios). In mathematical notation, it is as follows:

$$(10) \quad \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}' \bullet \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \\ e_{14} \end{bmatrix}$$

where e_{Ij} is the direct and indirect employment effect for sector j.

The final step in computing the employment multipliers is to divide the direct employment effects into the direct and indirect employment effects.

$$(11) \quad \begin{bmatrix} \frac{e_{11}}{e_1} \\ \frac{e_{12}}{e_2} \\ \frac{e_{13}}{e_3} \\ \frac{e_{14}}{e_4} \end{bmatrix} \quad \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix}$$

where E_j is the employment multiplier for sector j. E_j is the total direct and indirect employment associated with each man-year of direct employment in sector j.

Appendix 1.5
Output Requirements Needed to Meet an Estimated Final Demand

Th matrix of direct and indirect coefficients is multiplied by the estimated final demand. If Y_i^* represents estimated output requirements to meet the estimated demand, the mathmatical equation can be written as follows:

$$(12) \quad \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \bullet \begin{bmatrix} Y_1^* \\ Y_2^* \\ Y_3^* \\ Y_4^* \end{bmatrix} = \begin{bmatrix} X_1^* \\ X_2^* \\ X_3^* \\ X_4^* \end{bmatrix}$$

In matrix notation the equation is:

$$(12a) \quad (\underline{I} - \underline{A})^{-1} \bullet \underline{Y}^* = \underline{X}^*.$$

PART II
FROM-TO ANALYSIS

An interindustry model which is receiving increasing attention is the from-to model. From-to analysis is simply a variation of the input-output model. The variation has less restrictive data requirements and, for relatively small regions, no less realistic assumptions compared to the input-output model. The concept was first introduced by Charles L. Leven [5]. Examples of empirical applications include a study by Hansen and Tiebout [1], a study by Kalter [3] and one by Muncrief [6].

Basic Components

The from-to model[1] consists of four basic components: a distribution table, a transaction table, a direct requirements table, and a total direct and indirect requirements table. The distribution table is the base of the model with the transaction table derived from it. In turn, the direct requirements and direct and indirect requirements are derived from the transaction table.

The Distribution Table

To illustrate the distribution table, consider a model for South Central Oklahoma [6] with three producing sectors and three final demand sectors. The distribution table for the aggregated South Central Oklahoma study [6] is presented in Table 2.1. The producing sectors in-

[1] For a complete mathematical presentation of this model, see Appendix 2.1.

Table 2.1. Distribution of Industry Sales (Percent) Planning Region Nine, South Central, Oklahoma, 1970.

Sector To: From:	Agriculture Mining and Manu-facturing (1)	Con-struction, Transpor-tation Utilities, Finance and Services (2)	Retail and Wholesale Trade (3)	Household Con-sumption (4)	Exports (5)	Other Final Demand (6)	Total (7)
1. Agriculture, Mining and Manufacturing	23.5	2.5	5.6	4.0	60.3	4.1	100
2. Construction, Transportation, Utilities, Finance, and Services	7.5	8.3	7.9	44.4	15.2	16.7	100
3. Retail and Wholesale Trade	8.1	3.6	7.0	57.0	8.2	16.1	100

Source: Adopted from [7].

clude (1) agriculture, mining, and manufacturing; (2) construction, transportation, utilities, finance and services; (3) and retail and wholesale trade. Each sector sells its product to industries within the region or to final demand sectors. Final demand sectors include household consumption, exports and other final demand. In most models, the number of producing and final demand sectors would be greatly expanded.

Regional trade relationships are obtained from the distribution table. Reading across a row indicates the percent of sales from the sector represented by that row to the sector represented by that column. For example, the agricultural, mining and manufacturing sector sells 23.5 percent of its total sales to industries within that sector, 2.5 percent to the construction, transportation, utilities, finance, and service sector, 5.6 percent to the wholesale and retail trade sector, 4.0 percent to households, 60.3 percent to exports, and 4.1 percent to the other final demand sectors. Reading down a column in the distribution table indicates the relative magnitude of each row sector's output serving as inputs to the column industry. For example, the agriculture, mining, and manufacturing sector required 23.5 percent of the same sectors output as intraindustry sales; 7.5 percent of all output from the construction, transportation, utilities, finance and service sector; and 8.1 percent of the output from retail and wholesale trade.

The Transaction Table

The transaction table indicates employment flows from each producing sector to each purchasing sector and to the final demand sectors.[2] To

[2] Flow variables other than employment may be used if data are available such as sales or value-added.

calculate the transaction table, two steps were necessary. First, the employment in each sector was estimated from secondary data. Second, the employment totals are multiplied by the respective rows in the distribution table. The transaction table for the aggregate South Central Oklahoma model is presented in Table 2.2.

The interpretation of the transaction table is similar to that of the distribution table only now in terms of employment. For example, agriculture, mining and manufacturing serves industries within that sector equal to output from 5,181 annual average employment;[3] construction, transportation, utilities, finance, and services with 547 employment; re--tail and wholesale trade with 1,240 man-years; the household sector with 883 employment; the export sector with 13,303 employment; and the other final demand sector with 914 man-years.

Direct Coefficients

The direct coefficients indicate the direct employment in any one sector per employee in a specified sector. Each coefficient is calculated by dividing each entry in the transaction table by the row total of the sector named at the top of the column. For example, the first entry (.23) in Table 2.3 is obtained by dividing 5,181 by 22,068; the second (.06) by divided 1,367 by 22,068; and the third (.05) by dividing 1,173 by 22,068.

The column figures show, for each producing sector, the employment needs related directly to the output of the sector names at the head of the column. For example, for every person employed in the agriculture, mining, and manufacturing sector due to its output, .23 employment is needed from other firms in that sector, .06 from the construction, transportation, utilities, finance, and service sector, and .05 from retail and wholesale trade due to its inputs.

Direct and Indirect Requirements

The direct and indirect requirements table is calculated from the direct requirements table[4]. The direct and indirect coefficients for the aggregate South Central Oklahoma model are presented in Table 2.4. Each entry in the columns of this table shows the employment required both directly and indirectly from the industry at the left of the row, for each unit of employment used to produce for final demand by the industry names at the head of the column. For example, consider the wholesale and retail trade sector of the South Central Oklahoma model. For every

[3] Employment control totals are from the Oklahoma Employment Security Commission. Employment is determined from an established survey and covers the week which includes the 12th of each month. Consequently, *each job* is counted and recorded by *place of employment*.

[4] For the mathematical procedure to calculate the direct and indirect requirements see Appendix 2.1.

Table 2.2. Employment Transaction Flow Table (Persons Employed), Planning Region Nine*, South Central Oklahoma, 1970.

Sector To: From:	Agriculture Mining and Manufacturing (1)	Construction, Transportation Utilities, Finance and Services (2)	Retail and Wholesale Trade (3)	Household Consumption (4)	Exports (5)	Other Final Demand (6)	Total (7)
1. Agriculture, Mining and Manufacturing	5,181	547	1,240	883	13,303	914	22,068
2. Construction, Transportation, Utilities, Finance, and Services	1,367	1,506	1,438	8,057	2,759	3,036	18,163
3. Retail and Wholesale Trade	1,173	522	1,011	8,240	1,183	2,334	14,463

*Source: Adopted from [6].

Table 2.3. Direct Requirements Per Employee. Planning Region Nine, South Central Oklahoma, 1970.

Sector	Agriculture, Mining and Manufacturing (1)	Construction, Transportation, Utilities, Finance, and Services (2)	Retail and Wholesale Trade (3)
1. Agriculture, Mining, and Manufacturing	.23	.03	.09
2. Construction, Transportation, Utilities, Finance and Services	.06	.08	.10
3. Retail and Wholesale Trade	.05	.03	.07

Table 2.4. Direct and Indirect Requirements from South Central Oklahoma Per Unit of Employment Generated from Sales to Final Demand, 1970.

Sector	Agriculture, Mining and Manufacturing (1)	Construction, Transportation, Utilities, Finance, and Services (2)	Retail and Wholesale Trade (3)
1. Agriculture, Mining, and Manufacturing	1.31	.05	.13
2. Construction, Transportation, Utilities, Finance, and Services	.09	1.09	.13
3. Retail and Wholesale Trade	.07	.04	1.09

person employed to produce final demand output by the wholesale and retail trade sector, .13 units of employment are needed directly and indirectly from the agriculture, mining, and manufacturing sector; .13 from the construction, transportation, utilities, finance, and service sector, and 1.09 from other firms within the same sector.

From-to Analysis and Input-Output

Kalter [2] and Leven [5] have made a comparison of from-to analysis and input-output analysis. A summary of their results is presented here. The from-to model makes no change from normal input-output analysis with respect to output flows. Unlike the input-output model the from-to model ignores data on imports. The transaction table for the from-to analysis is similar to the input-output model except that it does not include a primary import section. This creates at least one major advantage and one minor disadvantage.

The advantage is that data requirements for from-to analysis are greatly reduced. The analyst only has to ask firms to whom they sold their product and what proportion. With few intermediate buyers and a relatively uncomplicated interdependent structure for most rural and non-metropolitan areas, firms are generally aware of whom they sold their products; thus this data are generally easily obtained. The transaction table shows input flows from area industries to other industries of the same area or to final demand sales. The final demand columns show the proportion going to local consumers and to exports. Thus, the model gives a representation of regional flows. The model thus permits the researcher to ignore imported inputs. This is of substantial importance since data on imports are frequently difficult to obtain.

A disadvantage of from-to analysis is that there is no cross checking mechanism as found in the input-output model. In the input-output model, column and row totals are equal, whereas in from-to analysis, data are available for rows only.

It is difficult to say which technique is more appropriate or gives more accurate results. The model to use will depend on the questions asked, data available, and regional structure [5, p. 170]. Selecting the appropriate model for a given region, in part depends upon whether the pattern of domestic supply of intermediate commodities depends primarily on changes in technical production functions or on the spatial distribution of production. While, the supply of intermediate commodities depends on both, Leven has arrived at a tentative judgement that the limit where from-to analysis might be preferable "would probably be somewhere near the point where a region became big enough so that trade with the rest of the world accounted for less than half of its production and/or final domestic demand," [5, p. 171] With this criterion, the input-state regions, and national economics, whereas the from-to model would be preferred for counties, multi-counties, and even small SMSA's.

Assumptions of the From-to Model

From-to analysis, in addition to the same assumptions which apply to input-output, carries with it the assumption of constant trade coefficients. This implies that the supply pattern of intermediate inputs of a sector is fixed by the spatial distribution of production. Thus, from-to analysis assumes constant production coefficients and constant trade coefficients.

Kalter [4, pp. 14-15] discussed two reasons why constant trade and production coefficients are not as restrictive as first appearance when applying from-to analysis at the local or small region level. First, the greater the amount of imported inputs, the less critical are the assumptions. A change in production technology in this case would not greatly affect the values in the direct and indirect coefficient table. Changes in trading patterns normally occur when new firms move into a region. If such new firms do not produce inputs for other firms in the region, trading coefficients will change very little.

Second, Kalter states that if a model is classified as to disaggregate the retail and service sectors, the coefficients will be relatively stable. This results from the fact that the biggest share of the product sold by the retail sector are imported and the trade patterns change very slowly.

Application of From-to Analysis

The from-to model can be used to estimate structural parameters of local economies and to make empirical projections similar to the input-output model. Thus, employment, sales, and income multipliers as well as final demand multipliers can be estimated and projections made of future output.

Projecting Future Output[5]

For projection purposes, estimates must be made of final demand for some future time period. For the South Central Oklahoma model, assume employment used to produce in final demand for 1975 for each of the three sectors is as follows:

Agriculture, Mining and Manufacturing:	16,610
Construction, Transportation, Utilities, Finance and Services; and	15,929
Retail and Wholesale Trade	15,284

Total sector employment is obtained by multiplying the estimated projected final demand employment for each sector times the direct and indirect coefficients. The employment requirements for each sector to meet the final demand employment estimated for 1975 are:

Agriculture, Mining and Manufacturing:	24,542
Construction, Transportation, Utilities, Finance and Services; and	20,845
Retail and Wholesale Trade	18,459

[5] The mathematical procedure for projecting future output is similar to that used with the input-output model. The procedure is discussed in Appendix 1.5.

Employment, Value-added and Sales Multipliers[6]

Industry multipliers can be obtained from the direct and indirect coefficient table. The industry output multipliers are calculated by adding the columns of the direct and indirect coefficient table. If the transaction table is in terms of sales, the procedure leads to sector sales multipliers. For employment and value-added multipliers, the transaction table must be in terms of employment and value-added flows, respectively. For the South Central Oklahoma model, which is in terms of employment, the sector employment multipliers are as follows:

Agriculture, Mining and Manufacturing:	1.47
Construction, Transportation, Utilities, Finance and Services; and	1.18
Retail and Wholesale Trade	1.35

Each multiplier shows the total number of jobs created directly and indirectly from the original job serving final demand. For example the multiplier for agriculture, mining and manufacturing indicates that for each person directly employed by that sector to serve the final demand markets, a total of 1.47 jobs are needed throughout the economy. (This includes the original final demand job).

Final Demand Multipliers [7]

Final demand multipliers are frequently calculated in from-to analysis. Such multipliers are useful to show the general effect on local or regional economies of a change in the export base on the federal government sector. Sales, value added, and employment of the producing sectors are tied directly and indirectly to final demand sectors. By tracing the output flows of the producing sectors to final demand sectors, sales, employment or value-added can be divided into a direct or indirect category. For the South Central Oklahoma model, the direct and indirect employment flows are presented in Table 2.5. The table shows employment for each sector which is assigned directly to the final demand categories. The percentage of direct to total employment for the household consumption sector is 78, for the export sector is 70, and for the other final demand sector is 78.

Linkages of the producing sectors can also be obtained from the data. The percent of direct to total employment for agriculture, mining and manufacturing is 68; for construction, transportation, utilities, finance, and services is 76; and for retail and wholesale trade is 81.

[6] For the mathematical procedure to calculate these multipliers see Appendix 2.2.

[7] For a mathematical procedure to calculate these multipliers see Appendix 2.3 and for a discussion of the procedure see [1].

Table 2.5. Direct and Indirect Employment Transaction Flow to Final Markets, South Central Oklahoma, 1970.

Sector		Household Consumption (1)	Exports (2)	Other Final Demands (3)	Total (4)	Direct and Indirect Percentages (5)
			(number employed)			
Agriculture, Mining	Direct	883	13,303	914	15,100	68
and Manufacturing	Indirect	1,704	4,541	723	6,968	32
	Total	2,587	17,844	1,637	22,068	100
Construction, Trans-	Direct	8,057	2,759	3,036	13,852	76
portation, Utilities,	Indirect	1,924	1,713	674	4,311	24
Finance and Services	Total	9,981	4,472	3,710	18,163	100
Retail and Wholesale	Direct	8,240	1,183	2,334	11,757	81
Trade	Indirect	1,075	1,232	399	2,706	19
	Total	9,315	2,415	2,733	14,463	100
Industry Total	Direct	17,180	17,245	6,284	40,709	74
	Indirect	4,703	7,486	1,796	13,985	26
	Total	21,883	24,731	8,080	54,694	100
Percentages	Direct	78	70	78	74	
	Indirect	22	30	22	26	
	Total	100	100	100	100	
Government						
State and Local				8,097		
Federal				6,396		
Total Employment		21,883	24,731	22,573	69,187	

Final demand interindustry multipliers can be calculated for each sector in the model in the short-run. The final demand interindustry multiplier for the export sector is $\frac{24,731}{17,245} = 1.43$, and for the other final demand sector is $\frac{8,080}{6,284} = 1.29$. Each multiplier indicates the total change in employment, direct and indirect, as a result of one-unit change in direct employment in that final demand sector.

In the short-run, it is assumed that the household sector will have constant demand [7]. Household expenditures will increase due to added employment in exports or other final demands and give rise to an "induced effect". This effect is the result of increased household spending which in turn will generate additional economic activity. The induced effect is obtained by the following procedure:

$$\frac{\text{Total Employment in Household Consumption Sector}}{\text{Total Employment} - \text{Total Employment in Household Consumption}}$$

$$\frac{21,883}{47,304} = .46$$

This may be expressed as a market employment multiplier by adding one to the above ratio. This means that for each person needed to produce goods for the export and other final demand sector, 0.46 employment is needed to serve local consumption needs.

The total multiplier resulting from a change in export employment includes the interindustry impact as well as the household induced effect. This multiplier is calculated for the export sector as:

$$\text{Export Multiplier} = 1.46 \text{ x } 1.43 = 2.09$$

This indicates that for each additional job for export demand, a total of 2.09 jobs arise due to interindustry and induced household effects.

From-To References

[1] Hansen, W. L. and Charles M. Tiebout, "An Intersectional Flows Analysis of the California Economy", *The Review of Economics and Statistics,* 45: 409-418, Nov. 1963.

[2] Kalter, Robert J., "A Model to Estimate the Economic Effects of Water-Based Recreation Projects on Local Political Subdivisions," Ph.D. Thesis, University of Wisconsin, 1966.

[3] Kalter, Robert J., *An Interindustry Analysis of the Central New York Region.* Cornell University, Agricultural Experiment Station Bulletin 1025, May 1969.

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[5] Leven, Charles L., "Regional Income and Product Accounts: Construction and Applications." Chapter 6 in *Design of Regional Accounts,* Resources for the Future. Baltimore, The Johns Hopkins Press, 1961, pp. 148-195.

[6] Muncrief, George E., *Development of Interindustry Analysis of Use Within A Public Service Planning Framework to Estimate Generation of Solid Wastes in South Central Oklahoma.* Unpublished M. S. thesis, Oklahoma State University, 1972.

[7] Schreiner, Dean F. and Dave Flood, "Area Analysis by Means of an Economic Base Study", Article in Proceedings of Rural Development Workshop, July, 1971. Compiled by Department of Agricultural Economics, Oklahoma State University.

[8] Tiebout, Charles M., *The Community Economic Base Study,* Committee for Economic Development, Supp. Paper No. 16, Dec. 1962.

Appendix 2.1
Mathematical Presentation of the From-To Model

The Sales Distribution Table

Distribution of industry sales to intermediate producing sectors and to final demand sectors represents the results of the estimation procedure of the from-to model. A questionnaire was administered to a sample of randomly drawn firms in the South Central Oklahoma study to estimate the distribution of industry sales. Employment of individual firms was used to construct a weighted index of industry or sector level distribution of sales.

The Transaction Table

The transaction or flow table of the from-to model is derived by applying control totals of regional output proxy variables such as employment, sales or value-added to the distribution table as estimated in the preceding section. Transactions of the from-to model can be presented as a system of equations

$$
\begin{aligned}
& X_1 = X_{11} + X_{12} + X_{13} + Y_{11} + Y_{12} + Y_{13} \\
(1)\quad & X_2 = X_{21} + X_{22} + X_{23} + Y_{21} + Y_{22} + Y_{23} \\
& X_3 = X_{31} + X_{32} + X_{33} + Y_{31} + Y_{32} + Y_{33}
\end{aligned}
$$

where

X_i = gross output (in terms of sales, employment or value-added) of the i^{th} sector

X_{ij} = flows of output from the i^{th} producing sector to the j^{th} producing sector

Y_{ij} = flows of output from the i^{th} producing sector to the j^{th} final demand sector.

An outlined form of the transaction table is presented in Figure 2.

		To						
		Intermediate Sectors			Final Demand Sectors			Total Output
From		(1)	(2)	(3)	(1)	(2)	(3)	
	(1)	X_{11}	X_{12}	X_{13}	Y_{11}	Y_{12}	Y_{13}	X_1
Producing Sectors	(2)	X_{21}	X_{22}	X_{23}	Y_{21}	Y_{22}	Y_{23}	X_2
	(3)	X_{31}	X_{32}	X_{33}	Y_{31}	Y_{32}	Y_{33}	X_3

Figure 2. Representation of the From-To Transaction Table.

Direct Coefficients

The direct coefficients are derived from the transaction table by assuming that the relationship between the sales to a sector and the level of output of that sector is linear. The direct coefficients are calculated as follows:

$$a_{ij} = \frac{X_{ij}}{X_j} \tag{2}$$

These coefficients are defined as the amount of output of the i^{th} industry purchased from firms within the region per unit of output of the j^{th} industry. Thus, the coefficient reflects production and trading patterns of the region.

Direct and Indirect Coefficients

Calculation of the direct and indirect coefficients is identical to the procedure used for the input-output model. See Appendix 1.1 for that procedure. Each coefficient in the direct and indirect matrix $(I-A)^{-1}$ indicates the amount of output (sales, employment, or income) from sector i necessary to sustain a final demand of one unit in sector j.

Appendix 2.2
Industry Multipliers

Depending upon the units employed in the analysis. (Sales, employment, or value-added) multipliers can be obtained from the direct and indirect coefficients. If the matrix of direct and indirect coefficients is in terms of employment, the employment multiplier for sector 1 can be calculated as follows:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = A_{11} + A_{21} + A_{31} = \sum_{i=1}^{3} A_{i1}$$

where the A_{ij}'s are the direct and indirect coefficients. This indicates that for a one unit change in final demand employment for sector 1, the total change in employment in all sectors equals $\sum_{i=1}^{3} A_{i1}$. Effects on the total economy of a one unit change in final demand of the other sectors can be measured in a similar manner.

If the direct and indirect coefficients are expressed in terms of sales or value-added, the identical procedure is used to calculate the industry sales and value-added multipliers.

Appendix 2.3
Final Demand Multipliers

Final demand multipliers are of two categories: an interindustry effect and a household or induced effect.

Interindustry Effect

The interindustry effect measures the indirect effects created in other sectors as a result of a direct change in one final demand sector. For example, if there is a direct increase in export demand for sector 1, there will be indirect effects on employment in all sectors. The interindustry effect for a final demand sector is calculated by dividing the direct employment (for that sector) by the direct and indirect employment for that sector. For example, the interindustry effect for exports is $\frac{24,731}{17,245} = 1.43$ (Table 2.5). This multiplier states that for every increase of one job in direct private export employment a total increase of 1.43 jobs will result throughout the economy.

Household Effect

The household effect measures the induced effect resulting from increased household spending due to a change in a given final demand sector. The effect is derived from the following simple model [1].

$$Y = C - M + X \qquad\qquad C - M = sY$$

where

Y = net regional product,
C = consumption,
s = marginal propensity to spend locally,
X = exports,
M = imports.

It is assumed that employment is proportional to local income and hence can be used as a proxy variable for regional income. It is also assumed that average propensity to spend equals marginal propensity to spend, hence s can be calculated as follows: $s = \dfrac{C-M}{Y}$

In the example, $C - M = 21{,}883$ and $Y = 69{,}187$ (Table 2.5). Thus, $s = \dfrac{21{,}883}{69{,}137} = .316$. The household effect is as follows: $\Delta Y = \dfrac{1}{1\text{-}s}\,\Delta X$.

In the example, this equals $\dfrac{1}{1-.316} = 1.46$. This multiplier measures the induced change in employment due to a change in household spending from a change in export base employment.

Total Multiplier

The total multiplier includes the interindustry effect and the household effect. It is calculated using the following formula:

$$\Delta Y = \frac{1}{1-s} \bullet (d + id)\,\Delta X.$$

where

$\dfrac{1}{1-s}$ = household or induced effect
$(d + id)$ = direct and indirect effect
ΔY = total change in employment resulting from a one-unit employment change in exports, and
ΔX = change in exports.

The total multiplier for the export sector is calculated as follows:

$$\Delta Y = \frac{1}{1-s} \bullet (d + id)\ \Delta X \qquad \Delta Y = 1.46 \bullet \Delta X = 2.09$$

The multiplier indicates that for a one-unit change in export employment, a total of 2.09 employment units will be generated throughout the economy due to interindustry effects and household (induced) effects. Multipliers for other final demand sectors can be calculated in a similar fashion.

PART III
DYNAMIC INPUT-OUTPUT ANALYSIS

The static input-output model measures interdependence by the flows of goods and services among sectors in the economy and is acceptable for structural economic analysis in the short-run. The dynamic input-output model in contrast to static input-output models includes effects of capital formation over time on current and future production. Analysis or projections for the long-run should include the capital formation effect. In which case a dynamic input-output model or similar models can be applied.

Application of dynamic input-output has proceeded more slowly than static input-output and from-to analysis. Slow adaptation of the dynamic input-output model is due largely to its extensive data requirements. A few notable dynamic input-output studies include Clopper Almon's [1] 10 sector dynamic model of the American Economy, Wassily Leontief's [4] 52 sector model for the U.S., and Alan Manne's [5] dynamic multisectoral model for India.

Basic Components[1]

The dynamic input-output model[2] is an extension of the static model in that it accounts for the expansion of capital stock to meet new levels of final demand. Basic components of the dynamic model include the flow table, direct requirement coefficients, capital coefficients, and the dynamic inverse.

[1] For a complete mathematical presentation of the model, see Appendix 3.1.
[2] There are several versions of the dynamic input-output model. For a summary of these, see [2, pp. 24-29].

The Flow Table and Direct Requirements Coefficients

To illustrate the dynamic flow table, consider the same four sector model as presented in the static input-output section. The flow table remains the same except that the private capital formation column of the static model is now represented by the actual sector capital flows. The dynamic flow table is presented in Table 3.1. Reading down a capital flow column slows the amount purchased by that sector from the industries represented by that row for capital formation. For example, the Agricultural production sector purchases capital goods valued at $63 million from the manufacturing and mining sector, $138 million from the service sector, and $114 from the other primary input sector (imports of capital equipment).

Reading across a row, the sales from one industry to the other industries for capital formation is obtained. Consider the manufacturing and mining sector; it sells capital goods worth $63 million to the Agricultural production sector, $9 million to the Agricultural processing, $156 million to industries within that sector, and $249 million to the service sector.

The direct coefficients remain the same for the dynamic model. Direct coefficients for the Oklahoma model are presented in Table 1.2.

Capital Coefficients[3]

Capital output coefficients describe the amount of investment in buildings, machinery, spare parts, and other supplies needed to expand sector output by one unit. The capital unit coefficients indicate the amount of capital goods produced by the row sector and purchased by the column sector for each additional unit of output capacity of the column sector. Calculation of the capital-output coefficients consists of dividing the capital flow data from Table 3.1 by the unit change in sector output.[4] For the example given, capital-output coefficients were derived from other studies as reported in [3].

The capital unit coefficients are presented in Table 3.2. For example, in the service sector, the capital unit coefficients indicate that for each additional unit of output capacity for that sector, 15 cents worth of capital goods are required from the manufacturing sector, 39 cents from other industries in the service sector and none from the Agricultural production and Agricultural processing sectors [3].

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[3] For a complete presentation and discussion of the capital data for Oklahoma, see [3].

[4] For a mathematical description and computational procedure for the capital coefficients, see Appendix 3.1.

Table 3.1. Dynamic Flow Table for Oklahoma, 1963, (Millions of Dollars).

					Capital Flows					
	Agricul-tural Pro-duction	Agricul-tural Pro-cessing	Manu-facturing and Mining	Services	Agricultural Production	Agricultural Processing	Manu-facturing and Mining	Services	Final Demand	Total Outputs
Agricultural Production	183	191	8	27	0	0	0	0	378	787
Agricultural Processing	46	70	3	18	0	0	0	0	358	495
Manufacturing and Mining	33	24	921	528	63	9	156	249	1,200	3,183
Service	112	71	544	1,103	138	29	323	635	2,127	5,082
Households	246	71	927	2,274	0	0	0	0	1,596	5,114
Other Primary Inputs	167	68	780	1,132	114	19	277	473	1,745	4,775
Total Inputs	787	495	3,183	5,082	315	57	756	1,357	7,404	

Source: [3] and Table 1.1.

Table 3.2. Capital Unit Coefficients, Oklahoma, 1963.

Sector	Agricultural Production	Agricultural Processing	Manufacturing and Mining	Service
Agricultural Production	.00	.00	.00	.00
Agricultural Processing	.00	.00	.00	.00
Manufacturing and Mining	.27	.05	.21	.15
Service	.60	.16	.44	.39

Source: [3].

The Dynamic Inverse

The dynamic inverse is the counterpart in the dynamic system to the direct and indirect coefficients of the static system. The dynamic inverse is more difficult to construct and becomes very large as the numbers of sectors and years increase. For illustration purposes, the dynamic inverse for the Oklahoma four sector model is presented in Table 3.3 for a three year period. Each element describes the direct and indirect input requirements generated by the delivery to final demand of one unit of the product of that sector for year t. These requirements are distributed backward over time. The last four rows (lower right corner of Table 3) show the input requirements that must be made in year 3. This is the same year that the final deliveries are made. In this illustration the matrix is identical to the direct and indirect coefficient matrix in the static model.[5] The middle four rows illustrate input needs for year 2 to meet a specified final demand. These needs include input requirements of year 2 and increased capital formation to meet increased output levels in year 3. The first four rows illustrate input requirements to meet final demand in year 1 and capital formation needs to meet the increased output levels for year 2.

A number of negative coefficients appear in the dynamic inverse. This is the result of the well-known effect called the accelerator principle.[7] As long as the total sum of the positive output requirements exceed the total sum of the negative input requirements, sector output will increase.[6]

[5] In the above illustration, the direct and capital unit coefficients are assumed the same for each year. Technological change is introduced into the model by allowing these coefficients to change over time.

[6] For a complete mathematical presentation of the convergence properties of the dynamic inverse and an explanation of the properties, see [4].

[7] For a complete discussion of the accelerator principle, see [6, pp. 261-262].

Table 3.3. The Dynamic Inverse, Oklahoma, 1963.

Time Period	Sector	Agri. Production	Agri. Processing	Manu. and Mining	Services	Agri. Production	Processing	Manu. and Mining	Services	Agri. Production	Agri. Processing	Manu. and Mining	Services
1	Agricultural Production	1.3432	.6039	.0018	.0077	.0043	.0035	.0033	.0027	.0042	.0032	.0033	.0025
1	Agricultural Processing	.0892	1.2058	.0002	.0041	.0021	.0017	.0016	.0013	.0021	.0016	.0016	.0012
1	Manufacturing and Mining	—.3129	—.1419	1.0921	—.0443	.2357	.1500	.1990	.1182	.2067	.1564	.1634	.1207
1	Servises	—.4541	—.2227	—.2515	.8594	.3760	.3114	.2802	.2456	.3700	.2822	.2911	.2183
2	Agricultural Production					1.3432	.6039	.0018	.0077	.0085	.0066	.0066	.0052
2	Agricultural Processing					.0892	1.2058	.0002	.0041	.0042	.0033	.0032	.0025
2	Manufacturing and Mining					—.3126	—.1375	1.0920	—.0443	.4423	.3156	.3625	.2389
2	Services					—.4542	—.2237	—.2515	.8594	.7459	.5925	.5713	.4638
3	Agricultural Production									1.3517	.6106	.0084	.0129
3	Agricultural Processing									.0934	1.2091	.0034	.0066
3	Manufacturing and Mining									.1294	.1737	1.4545	.1947
3	Services									.2918	.3698	.3198	1.3232

Assumptions of the Dynamic Input-Output Model

The dynamic model permits the direct input-output coefficients to change whereas these coefficients were assumed constant in the static input-output system. Thus, technological change, external economies and diseconomies, and substitution possibilities are reflected in the changing direct input-output coefficients and capital unit coefficients. The other major assumption under the static model remains with the dynamic model. This assumption is that there are no errors of aggregation in combining industries into sectors.

Ability of the dynamic model to introduce technological change leads to limited use of the model. To implement the dynamic model, the researcher needs a vast amount of data. Data requirements for the dynamic model are much larger than for the static input-output model.

Application of the Model

Application of dynamic input-output models has proceeded much slower than the theoretical development. The slow progress in application is due to the large data requirements of the dynamic input-output model. To illustrate how the dynamic input-output model can be used, the Oklahoma model will be used to predict output needs to meet a given final demand. The most useful property of the open input-output system is the linear additivity of their solutions with respect to any changes in final demand [4, p. 21].

Prediction of Future Output

The dynamic input-output model can be used to predict the change in output of each sector necessary to meet some specified final demand. Using the data as derived in the preceeding sections, sector output estimates are made for three final demand situations. These situations are depicted in Table 3.4. The output requirements for a sector necessary to meet the projected final demand is found by multiplying the total estimated final demand for each sector for each time period times the dynamic inverse coefficients.[8] The output estimates for each final demand situation are presented in Table 3.5.

To check on the internal consistency of the model, the first situation assumes final demand remains the same for the three periods. In this case, capital investment to expand the capacity of a sector is not required and sector output requirements are the same for each period.

[8] The calculation procedure is the same illustrated for the static input-output model, except the dynamic inverse is used instead of the direct and indirect coefficient matrix. See appendix 1.5, for the static input-output calculation procedure.

Table 3.4. Alternative Final Demand Estimates for Oklahoma for Three Time Periods.

Period	Sector	Alternative One	Alternative Two (Millions of Dollars)	Alternative Three
1	Agricultural Production	378	378	378
1	Agricultural Processing	358	358	358
1	Manufacturing and Mining	1,200	1,200	1,200
1	Services	2,126	2,126	2,126
2	Agricultural Production	378	470	470
2	Agricultural Processing	358	516	516
2	Manufacturing and Mining	1,200	2,310	2,310
2	Services	2,126	4,360	4,360
3	Agricultural Production	378	470	599
3	Agricultural Processing	358	516	728
3	Manufacturing and Mining	1,200	2,310	3,156
3	Services	2,126	4,360	5,961

Table 3.5. Output Estimates to Meet Alternative Final Demands.

Period	Sector	Alternative One	Alternative Two (Millions of Dollars)	Alternative Three
1	Agricultural Production	767	788	796
1	Agricultural Processing	486	496	500
1	Manufacturing & Mining	2,271	3,298	3,688
1	Services	3,440	5,273	5,976
2	Agricultural Production	767	1,026	1,042
2	Agricultural Processing	486	705	713
2	Manufacturing & Mining	2,271	4,360	5,175
2	Services	3,440	6,835	8,283
3	Agricultural Production	767	1,026	1,358
3	Agricultural Processing	486	705	987
3	Manufacturing & Mining	2,271	4,360	5,955
3	Services	3,440	6,835	9,341

The second situation assumes that final demand increases in the second period and remains at that level in the third period. In this case, sector output requirements increase in the first period as output capacity has to be constructed in that period in order that sufficient capacity exists in period two to meet the final demand of period two. Sector output needs for period two and three are identical as final demand remains the same for these periods. The third situation assumes that final demand increases in each time period, thus during each period sector output must meet present final demand requirements as well as produce enough capital goods to insure adequate sector capacity for the next period.

Dynamic Input-Output References

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[2] Bargur, Jona, *A Dynamic Interregional Input-Output Programming Model of the California and Western States Water Economy*. Contribution No. 128, Water Resources Center, Berkeley, June, 1969.

[3] Doeksen, Gerald A. and Dean F. Schreiner, *An Analysis of the Capital Structure by Private Sectors in Oklahoma*. Oklahoma Agricultural Experiment Station Bulletin, B-694, July, 1971.

[4] Leontief, Wassily, "The Dynamic Inverse" published in *Contributions to Input-Output Analysis*, edited by A. P. Carter and A. Brady, North-Holland Publishing Company, Amsterdam, 1970. pp. 17-46.

[5] Manne, Alan S. and Thomas E. Weisshope, "A Dynamic Multisectoral Model for India, 1967-75" in *Applications of Input-Output Analysis*, edited by A. P. Carter and A. Brady, North-Holland Publishing Company, Amsterdam, 1970, pp. 70-103.

[6] Samuelson, Paul A. *Economics, An Introductory Analysis*, Sixth edition, New York: McGraw-Hill Book Company, 1964.

Appendix 3.1
Mathematical Presentation of the Dynamic Input-Output Model

The Flow Table

Transactions of the four-sector economy as discussed in the static input-output section and dynamic section can be presented as a system of equations.

(1)

$$X_1^t = x_{11}^t + x_{12}^t + x_{13}^t + x_{14}^t + B_{11}^t + B_{12}^t + B_{13}^t + B_{14}^t + Y_1^t$$
$$X_2^t = x_{21}^t + x_{22}^t + x_{23}^t + x_{24}^t + B_{21}^t + B_{22}^t + B_{23}^t + B_{24}^t + Y_2^t$$
$$X_3^t = x_{31}^t + x_{32}^t + x_{33}^t + x_{34}^t + B_{31}^t + B_{32}^t + B_{33}^t + B_{34}^t + Y_3^t$$
$$X_4^t = x_{41}^t + x_{42}^t + x_{43}^t + x_{44}^t + B_{41}^t + B_{42}^t + B_{43}^t + B_{44}^t + Y_4^t$$

X_i^t = gross output of the i^{th} sector in year t.

x_{ij}^t = purchases of the j^{th} sector from the i^{th} sector in year t.

B_{ij}^t = purchases of the j^{th} sector from the i^{th} sector needed to expand capacity output in sector X_j in year t+1.

Y_i^t = final or consumer demand for products of sector in year t.

Direct and Capital Unit Coefficients

The direct requirements coefficients are calculated in a similar manner as in the static model. The technical coefficient (a_{ij}) is the ratio of the purchases of output of industry i by industry j over the gross output of sector j. Mathematically, this is presented as:

$$(2) \quad a_{ij} = \frac{x_{ij}}{X_j}$$

The capital coefficients (b_{ij}) indicate the capital goods required by sector j from sector i to increase its output capacity by one unit. Mathematically, these are derived as follows:

$$(3) \quad b_{ij} = \frac{B_{ij}}{X_j^{t+1} - X_j^{t}}$$

This relationship assumes that capital goods produced in year t are installed and put into operation in year t+1.

Substituting the direct requirement coefficients and capital coefficients from (2) and (3) into (1) results in the following system:

$$(4) \quad \begin{aligned}
X_1^t &= a_{11}X_1^t + a_{12}X_2^t + a_{13}X_3^t + a_{14}X_4^t + b_{11}(X_1^{t+1} - X_1^t) + b_{12}(X_2^{t+1} - X_2^t) + b_{13}(X_3^{t+1} - X_3^t) + b_{14}(X_4^{t+1} - X_4^t) + Y_1^t \\
X_2^t &= a_{21}X_1^t + a_{22}X_2^t + a_{23}X_3^t + a_{24}X_4^t + b_{21}(X_1^{t+1} - X_1^t) + b_{22}(X_2^{t+1} - X_2^t) + b_{23}(X_3^{t+1} - X_3^t) + b_{24}(X_4^{t+1} - X_4^t) + Y_2^t \\
X_3^t &= a_{31}X_1^t + a_{32}X_2^t + a_{33}X_3^t + a_{34}X_4^t + b_{31}(X_1^{t+1} - X_1^t) + b_{32}(X_2^{t+1} - X_2^t) + b_{33}(X_3^{t+1} - X_3^t) + b_{34}(X_4^{t+1} - X_4^t) + Y_3^t \\
X_4^t &= a_{41}X_1^t + a_{42}X_2^t + a_{43}X_3^t + a_{44}X_4^t + b_{41}(X_1^{t+1} - X_1^t) + b_{42}(X_2^{t+1} - X_2^t) + b_{43}(X_3^{t+1} - X_3^t) + b_{44}(X_4^{t+1} - X_4^t) + Y_4^t
\end{aligned}$$

Elements 1-4 on the right hand side represent the current input requirements of all sectors in year t. Elements 5-8 indicate investment requirements needed to expand capacity output from year t to year t+1.

The Dynamic Inverse

The dynamic inverse was developed by Professor Leontief [2]. Solution to the system of equations in (4) can be obtained by finding the dynamic inverse. To illustrate, assume the system is analyzed for two years. Rewriting (4) for two years and solving for Y the following results:

$$(5) \quad (1 - a_{11} + b_{11})X_1^1 - (a_{12} - b_{12})X_2^1 - (a_{13} - b_{13})X_3^1 - (a_{14} - b_{14}) X_4^1 - b_{11}X_1^2 - b_{12}X_2^2 - b_{13}X_3 - b_{14}X_4^2 = Y_1^1$$

$-(a_{21}-b_{21})X_1^1+(1-a_{22}+b_{22})X_2^1-(a_{23}-b_{23})X_3^1-(a_{24}-b_{24})$
$X_4^1-b_{21}X_1^2-b_{22}X_2^2-b_{23}X_3-b_{24}X_4^2=Y_2^1$
$-(a_{31}-b_{31})X_1^1+(-a_{32}+B_{32})X_2^1+(1-a_{33}+b_{33})X_3^1-(a_{34}-b_{34})$
$X_4^1-b_{31}X_1^2-b_{32}X_2^2-b_{33}X_3-b_{34}X_4^2=Y_3^1$
$-(a_{41}-b_{41})X_1^1+(-a_{42}+b_{42})X_2^1-(a_{43}-b_{43})X_3^1+(1-a_{44}+b_{44})$
$X_4^1-b_{41}X_1^2-b_{42}X_2^2-b_{43}X_3-b_{44}X_4^2=Y_4^1$

$$(1-a_{11})X_1^2-a_{12}X_2^2-a_{13}X_3^2-a_{14}X_4^2=Y_1^2$$
$$-a_{21}X_1^2+(1-a_{22})X_2^2-a_{23}X_3^2-a_{24}X_4^2=Y_2^2$$
$$-a_{31}X_1^2-a_{32}X_2^2+(1-a_{33})X_3^2-a_{24}X_4^2=Y_3^2$$
$$-a_{41}X_1^2-a_{42}X_2^2-a_{43}X_3^2+(1-a_{44})X_4^2=Y_4^2$$

Rewriting (5) in matrix notion and solving for the X_j^t's results in the system of equations on the following page.

The square matrix on the right hand side of system (7) is the inverse of the structural matrix in (6). It is called the dynamic inverse. Solution of the system determines the time sequence of sector outputs that would permit the economy to yield projected levels of final demand for time periods t=1 and t=2.

The dynamic inverse describes the direct and indirect input requirements generated by the delivery of final demand of one unit of output of any industry in year t as well as the direct and indirect input requirements for capital formation in year t to meet projected increased capacity for any industry in years t+1, t+2. . . . t+n. The lower right portion of the dynamic matrix (4x4) shows the input requirements that must be met in year t and is the same as the inverse of the static input-output model.

The dynamic input-output model allows technology to change by having a new a_{ij} and b_{ij} matrix for each year. Thus, the dynamic model allows for technological changes which the static model does not.

Solving system (6) for X, the set equations in (7) result

(6)

$$
\begin{bmatrix}
1-a_{11}+b_{11} & -a_{12}+b_{12} & -a_{13}+b_{13} & -a_{14}+b_{14} & -b_{11} & -b_{12} & -b_{13} & -b_{14} \\
-a_{21}+b_{21} & 1-a_{22}+b_{22} & -a_{23}+b_{23} & -a_{24}+b_{24} & -b_{21} & -b_{22} & -b_{23} & -b_{24} \\
-a_{31}+b_{31} & -a_{32}+b_{32} & 1-a_{33}+b_{33} & -a_{34}+b_{34} & -b_{31} & -b_{32} & -b_{33} & -b_{34} \\
-a_{41}+b_{41} & -a_{42}+b_{42} & -a_{43}+b_{43} & 1-a_{44}+b_{44} & -b_{41} & -b_{42} & -b_{43} & -b_{44} \\
 & & & & 1-a_{11} & -a_{12} & -a_{13} & -a_{14} \\
 & & & & -a_{21} & -a_{22} & -a_{23} & -a_{24} \\
 & & & & -a_{31} & -a_{32} & 1-a_{33} & -a_{34} \\
 & & & & -a_{41} & -a_{42} & -a_{43} & 1-a_{44}
\end{bmatrix}
\bullet
\begin{bmatrix} X_1^1 \\ X_2^1 \\ X_3^1 \\ X_4^1 \\ X_1^2 \\ X_2^2 \\ X_3^2 \\ X_4^2 \end{bmatrix}
=
\begin{bmatrix} Y_1^1 \\ Y_2^1 \\ Y_3^1 \\ Y_4^1 \\ Y_1^2 \\ Y_2^2 \\ Y_3^2 \\ Y_4^2 \end{bmatrix}
$$

(7)

$$
\begin{bmatrix} X_1^1 \\ X_2^1 \\ X_3^1 \\ X_4^1 \\ X_1^2 \\ X_2^2 \\ X_3^2 \\ X_4^2 \end{bmatrix}
=
\begin{bmatrix}
1-a_{11}+b_{11} & -a_{12}+b_{12} & -a_{13}+b_{13} & -a_{14}+b_{14} & -b_{11} & -b_{12} & -b_{13} & -b_{14} \\
-a_{21}+b_{21} & 1-a_{22}+b_{22} & -a_{23}+b_{23} & -a_{24}+b_{24} & -b_{21} & -b_{22} & -b_{23} & -b_{24} \\
-a_{31}+b_{31} & -a_{32}+b_{32} & 1-a_{33}+b_{33} & -a_{34}+b_{34} & -b_{31} & -b_{32} & -b_{33} & -b_{34} \\
-a_{41}+b_{41} & -a_{42}+b_{42} & -a_{43}+b_{43} & 1-a_{44}+b_{44} & -b_{41} & -b_{42} & -b_{43} & -b_{44} \\
 & & & & 1-a_{11} & -a_{12} & -a_{13} & -a_{14} \\
 & & & & -a_{21} & 1-a_{22} & -a_{23} & -a_{24} \\
 & & & & -a_{31} & -a_{32} & 1-a_{33} & -a_{34} \\
 & & & & -a_{41} & -a_{42} & -a_{43} & 1-a_{44}
\end{bmatrix}
\bullet
\begin{bmatrix} Y_1^1 \\ Y_2^1 \\ Y_3^1 \\ Y_4^1 \\ Y_1^2 \\ Y_2^2 \\ Y_3^2 \\ Y_4^2 \end{bmatrix}
$$

PART IV
SIMULATION

Simulation is increasingly becoming a useful analytical tool for the study of regions. The adoption of simulation to regional studies began in the mid 50's. Since then, a number of studies have been completed. Simulation is defined as the use of a model to represent, over time, essential characteristics of a system or process under study [8, p. 2]. In setting up the simulation framework, the system is given the initial conditions, parameters, and variables. The simulation model then generates values of certain preselected variables. These values, in turn, are used for the next time span or decision period and the model is rerun. Simulation allows the introduction of many relationships which conventional optimizing models do not. In this sense, simulation is a flexible technique for testing and evaluating a proposed system in a laboratory environment. Due to the complexity and interdependencies of the many relationships in a system, it takes an analytical tool such as simulation to identify and quantify the effects of changes in the levels of variables in a system.

Basic Components and Assumptions

Simulation, unlike the previous tools, does not have certain basic components and assumptions. The flexibility of simulation permits each model to have its own basic components and unique assumptions. To illustrate flexibility two simulation models will be briefly presented. The discussion will include the basic framework of the model, its major assumptions, and empirical uses. The two regional models selected for discussion, Hamilton et. al. [5] and Doeksen and Schreiner [4].[3]

Hamilton et. al. Simulation Model

The model[2] was constructed principally to analyze the economy of the Susquehanna River Basin and to define the role the basin's water resources would or could play in the future development of the economy. The Susquehanna River Basin was divided into 8 subregions and each region was modeled separately but in a similar fashion. The model is composed of three major components that describe the demographic, employment, and water supply characteristics. The demographic and employment components are tied together by a feedback loop. The water component is a technical part of the model and could be replaced by another component if the model is adopted to another region.

[1] An extensive survey of the simulation studies have been completed by Johnson and Rausser [6]. These men have classified the simulation studies completed in agricultural economics by the following schemes: firm and process models; market and industry models; aggregate models; development models; and resource models.

[2] For a complete description of the model and results of the study, see [4].

The Demographic Component. Three major factors—births, deaths, and migration—are involved in the demographic component of the model. Each factor directly influences population change in a region. To determine the effect of these factors, the regional population was classified into six age classes. Births are directly affected by the number of women in the child bearing age. Deaths are influenced by the number of people in the upper age groups. The rate of migration (inward or outward) is related to the difference between the subregional unemployment rate and an assumed, long-run national rate. Migration is the most dynamic element in the model which causes population to change in a region. Also, an important feedback is created between the demographic and employment components of the model.

The Employment Component. Economic activity is expressed in terms of employment in the model. Employment is categorized into three groups. First, a certain number of industries are viewed as selling their output outside the subregion. Workers for these industries are classified as export base employment. Second, workers supplying goods and services to other businesses in the subregion are classified as business serving employment. Third, employment providing goods and service for households are referred to as household serving employment.

The driving force of the employment component of the model is market area demand operating through export industry demand. Growth in employment in export industries is determined by the subregion's locational attractiveness (determined through transportation and labor costs) and market demand.

The Water Component. This component of the model could be replaced by another component suited for the objective of another study. Thus, the component will not be discussed in detail other than indicating the purpose of the component for the model. The water component was included in the model to simulate conditions in the river related to both water quality and quantity. The important point is that if one is more interested in some other objective for regional analysis, the model is constructed such that the objective could replace the water component or could be added in addition to the water component.

Model Output. The simulation model has hundreds of variables which could be plotted and analyzed. However, certain variables were selected for illustration purposes. Specific experiments were run to evaluate a number of situations. For example, runs were completed to show the relationship of skills and education to regional growth patterns; runs were completed to determine the sensitivity of migration parameters; and runs were made to determine how wages adjusted to unemployment.[3]

[3] Many other simulation runs were completed and for a discussion of these see [5].

One way of illustrating the model results is to look at a particular region. Consider subregion E and look at migration, population and unemployment from 1950 to 2010. The variables are presented in Figure 4.1.

The economy of subregion E in 1960 has a considerable amount of unemployment. As the model simulates the economy, two effects operate to change conditions. The high level of unemployment in 1960 is associated with a low wage rate. The possibility of cheap labor attracts new firms to locate in the region. The high unemployment rate also leads to a high rate of outmigration. The direct effect of outmigration reduces the population of the region in the shortrun. Also, population is indirectly reduced as births decrease as the younger couples often migrate. After the period of outmigration, unemployment falls and population eventually begins to increase.

Doeksen and Schreiner Model

The Doeksen and Schreiner simulation model is formulated around the basic Leontief input-output system. An overview of the model is presented in three sections. First, the Oklahoma social accounting system is discussed. Second, the simulation model is outlined. Third, the model is summarized.

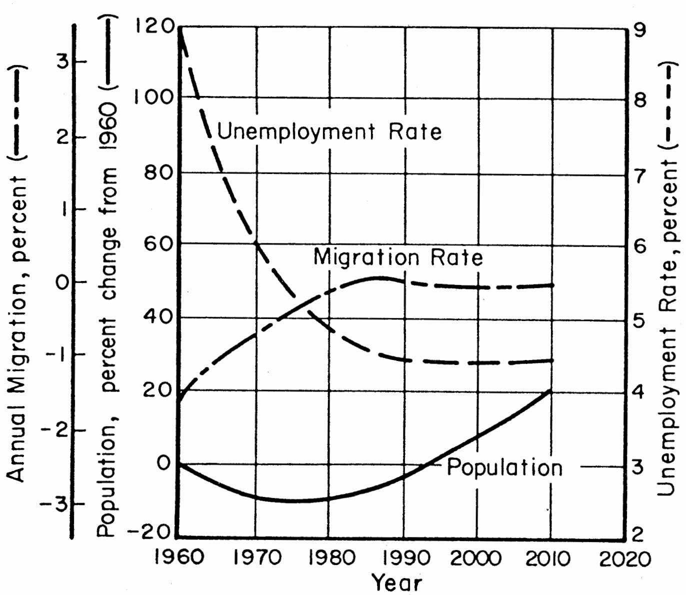

Figure 4.1. Projections of Unemployment, Migration and Population from 1960 to 2010, Subregion E.

Source: [5, p 236]

The Oklahoma Social Accounting System. The Oklahoma social accounting system is outlined in Figure 4.2. The system is divided into three main accounts: (1) a capital account; (2) an interindustry account; and (3) a human resource account. The interindustry account forms the core of the complete system with interrelated capital and human resource accounts.

The Oklahoma economy was divided into twelve endogenous sectors and five exogenous sectors. The endogenous sectors included major divisions of agriculture, manufacturing, services, and mining. Agricultural activities were divided into two sectors: crops and livestock and livestock products.

Manufacturing activities were divided into four sectors based on the economic activity in the state. These include petroleum processing, agricultural processing, machinery and other manufacturing. Service-type activities were aggregated into five sectors: transportation, communication, and public utilities; finance, insurance, and real estate; services; wholesale and retail trade; and construction. Also, a separate sector for mining was included. Five exogenous or final demand sectors were included in the model: federal government, state and local governments, households, private capital formation, and exports.

As noted in Figure 4.2 the interindustry section of the Oklahoma social accounting system consists of three basic parts: a transaction table or flow table, a direct coefficient table, and a direct and indirect coefficient table.[4] The capital coefficient matrix forms the base of the Oklahoma capital analysis.[5] Each capital coefficient indicates the amount of capital goods required from each sector per dollar's worth of capital expenditures in that sector. Capital-output ratios were computed for the twelve endogenous sectors and defined as the ratio of total cost of plant and equipment to output at capacity. Estimates of capacity-operating levels for each sector were obtained from employment data. Peak employment was assumed equal to 100 percent capacity operation.

A capital unit matrix is derived from the capital-output ratios and the capital coefficient matrix. Each coefficient in this matrix indicates the capital goods required from each sector to produce one unit of output capacity for that sector. The capital unit coefficients are computed by multiplying the capital coefficients of a sector times the capital-output ratio of that sector. A capital stock matrix can be derived from the capital-output ratios, an output estimate, and the capital coefficient matrix. The capital-output ratio times the estimated output at capacity yields the amount of capital in each sector. The capital stock matrix can be determined by multiplying total sector capital stock by the capital

[4] The basic parts were discussed in the input-output sector of this bulletin.
[5] For a complete discussion of each matrix associated with the capital accounts, see [2].

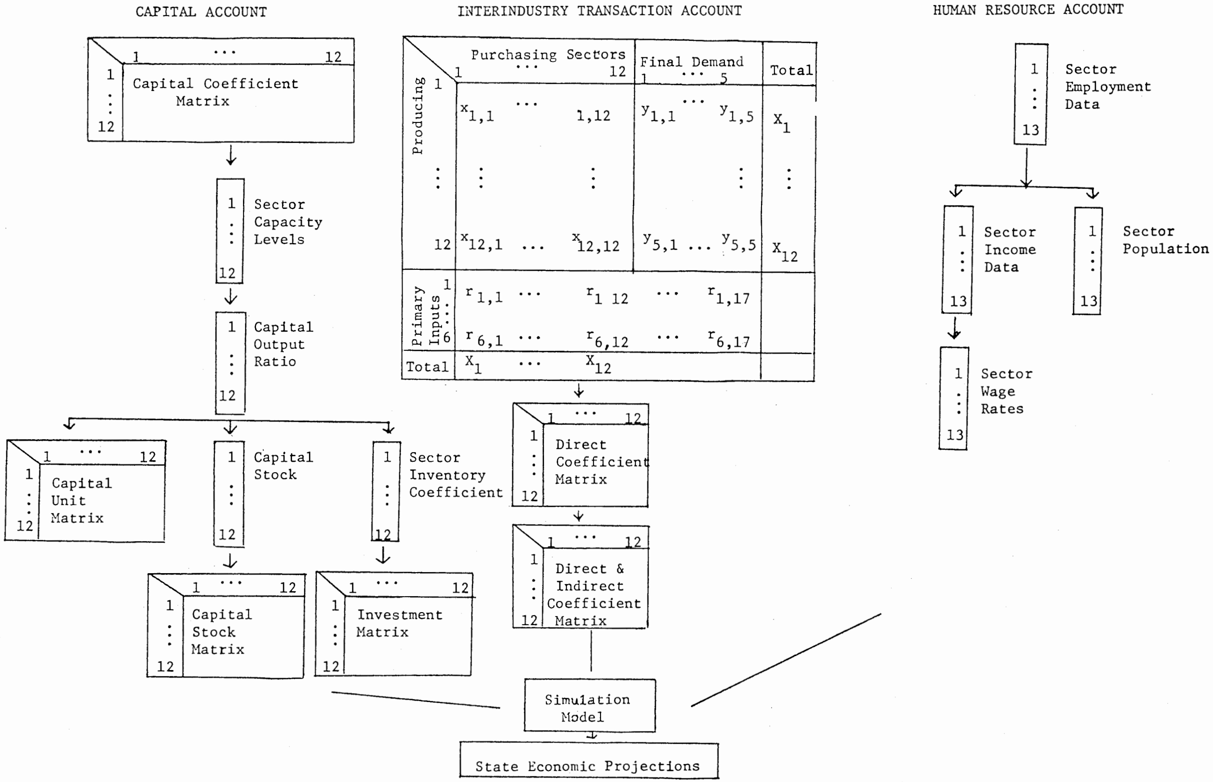

Figure 4.2: The Oklahoma Social Accounting System.

coefficient matrix. Each element in the capital stock matrix represents the total value of capital goods produced by the sector represented by that row and invested in the sector represented by that column.[5]

Inventory coefficients were derived that indicate the amount of inventory needed per unit of output.[5] By adding the capital unit coefficient and the inventory coefficient for a sector, the total amount of capital required per unit of output expansion is estimated. This addition yields a combined capital and inventory matrix from which the investment matrix is calculated. Each coefficient is obtained by the total of all entries for that column. Investment coefficients are defined as the value of output of the producing sector (row sector) needed by the purchasing sector (column sector) per unit of investment in the column sector. To complete the capital structure analysis, depreciation coefficients were estimated. Depreciation rates were estimated as the ratio of depreciation to depreciable assets. The human resource account provides data on employment, income and population of the state. Included are estimates of wage and salary and proprietor employment by sector. Labor-output ratios are developed using employment data from the human resource account and the output data from the transaction table. Income rates for wage and salary workers as well as proprietors are calculated.

The Simulation Model. The simulation model is formulated around the basic Leontief input-output system.[6] First, equations were derived which estimates changes in final demand. Second, these equations were used to stimulate the economy from year to year. The model incorporates growth and development through changes in capital investment (capital-output ratios and changes in capital-output ratios), human resource productivity (labor-output ratios, changes in labor-output ratios, and changes in wage rates), and current activity (changes in population, government expenditures, and exports).

The multiple-sector recursive model consists of 51 major equations. Many of the major equations are disaggregated into subequations representing each endogenous sector in the Oklahoma economy. Thus, the entire system includes over 300 equations. The model was formulated in Fortran and can be run on the computer at relatively low cost, thus enabling the researcher to experiment with the model by changing variables and measuring their impact.

Summary of Model Output. Empirical results from the model include projections of key variables and impact parameters. Data from the Oklahoma social accounts were used to simulate levels of state economic activity from 1963 to 1980. The simulated results obtained from the model are compared with published data from 1963 to 1970. Wage and

[6] The model is broadly outlined in Appendix 4.1.

salary employment by sector, proprietor employment, wage and salary payments by sector, proprietor income, disposable income, per capita income, gross state product, federal government revenue, and state and local government revenue are some of the economic variables which were projected. Figure 4.3 contains an example of projections obtained from the model.[7]

Figure 4.3 contains estimates of total employment proprietor employment, and wage and salary employment. The solid line indicates values derived from the simulation model. The broken line shows the actual estimates as published by the Oklahoma Employment Security Commission (OESC). Total employment projected to increase from 874,700 in 1963 to 1,347,645 in 1980. The simulated data from 1964 to 1970 are slightly higher than the OESC estimates. Wage and salary employment is projected to increase from 638,400 in 1963 to 1,094,841 by 1980. The projections are above the OESC estimates for 1964 and 1965 and slightly below for 1968 and 1970. Proprietor employment according to the simulation model is projected to increase only slightly from 236,300 in 1963 to 252,804 in 1980. The simulation results are higher than the OESC estimates. The reason proprietor employment changes very little is that the decreasing number of farmers is offset by a slight increase in proprietor employment for the service sectors.

[7] For a complete presentation of the income and employment results see [3].

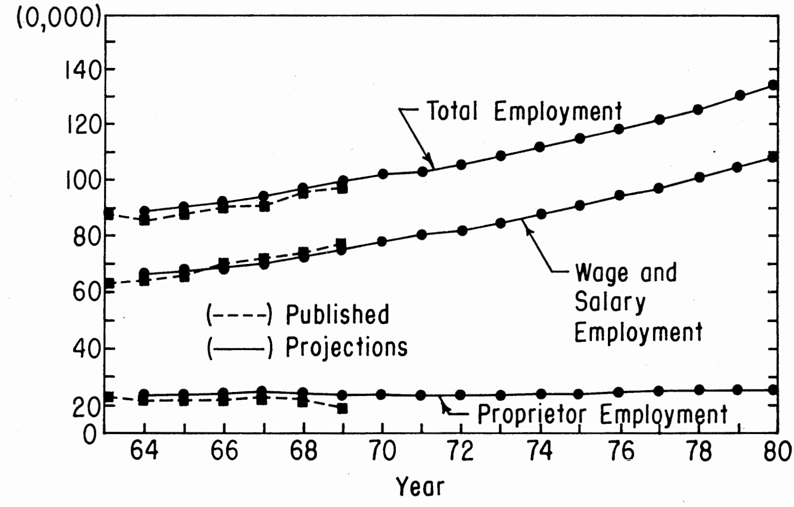

Figure 4.3: Total Employment, Proprietor Employment and Wage and Salary Employment, Oklahoma.

Impact parameters or multipliers derived from a simulation analysis can be used to determine the total regional effects resulting from induced changes occurring in the region.

Employment Multipliers. To trace short and long run employment benefits from private investment, the procedure was to assume a one million dollar investment by sector in 1970. Simulation runs were made for each endogenous sector.[8] The impact was measured in terms of new employment created in 1970 and through 1980. Listed in columns (1) and (2) of Table 4.1 are employment multipliers.

The short run employment multipliers are listed in column (1). Each multiplier indicates the change in direct and indirect employment throughout the Oklahoma economy by a one-unit change in production employment in the specified sector. The petroleum sector has the largest employment multipliers at 7.25. The magnitude results from the sector's large interaction with other sectors, particularly mining and manufacturing. Agricultural processing has the second largest employment multiplier at 6.29. This multiplier is interpreted to mean that for each man-year directly employed in processing agricultural products for delivery to

[8] For a presentation of the methodology involved in the procedure, see [4]. The short run multipliers include the direct and indirect effects, whereas the long run multipliers measure the direct, indirect, and induced effects.

Table 4.1. Short and Long Run Employment Multipliers and Investment Cost Per Hundred Jobs Created in Oklahoma, 1970.

	Short Run Employment Multiplier (1)	Long Run Employment Multiplier (2)	Cost Per 100 Jobs Directly Created in the Short Run (3)	Cost Per 100 Jobs Directly and Indirectly Created Short Run (4)	Cost Per 100 Jobs Created Directly, Indirectly and Induced in the Long Run (5)
Livestock and Livestock Products	2.37	2.05	1,695	714	826
Crops	1.24	.72	901	724	1,250
Agricultural Processing	6.29	6.25	1,282	204	205
Petroleum & Coal Products	7.25	6.25	8,333	1,149	1,333
Machinery, Except Electrical	2.02	2.58	1,316	649	510
Other Manufacturing	1.87	3.13	1,219	654	389
Mining	2.12	2.10	3,125	1,471	1,492
Transportation, Communication & Public Utilities	1.54	1.65	4,167	2,703	2,500
Real Estate, Finance & Insurance	1.52	1.59	1,250	803	787
Services	1.30	1.62	452	347	279
Wholesale & Retail	1.29	1.56	443	344	283
Construction	2.36	2.57	658	279	256

final demand, a total of 5.29 additional man-years are generated throughout the economy. Long run employment multipliers are listed in column (2). Each multiplier indicates the total employment generated (directly, indirectly and induced) in 1980 resulting from the one man-year production employment in 1970. Petroleum, agricultural processing, and other manufacturing have the largest long run employment multipliers at 6.25, 6.25, and 3.13, respectively.

Investment Cost per job created. To determine the number of jobs created per unit of capital, investment cost per 100 jobs created were derived for each sector. These costs are presented in columns (3), (4), and (5) of Table 4.1. The cost to directly employ 100 men is presented in column (3). For example, to directly employ 100 men in the agricultural processing sector, $1,282,000 (1963 prices) must be invested in that sector. The wholesale and retail trade sector has the lowest short run investment requirements per 100 jobs. Following in second order is the service sector.

Investment costs per 100 jobs created directly and indirectly in the short run by each sector are presented in column (4) of Table 4.1. These costs indicate the direct investment needed in a particular sector to create 100 jobs. Jobs are directly created in the sector receiving the investment; however, employment created by the interaction of sectors is also included. Thus, all sectors may witness an increase in employment. For example, if $204,000 were invested in agricultural processing, 100 jobs would be created throughout the economy in the short run. The agricultural processing sector has the lowest short run direct investment requirements per 100 men employed. Next in order of magnitude are the construction, wholesale and retail trade and service sectors.

Investment costs per 100 jobs created in the long run are presented in column (5) of Table 4.1. In the long run, employment is increased directly, indirectly, and induced. Employment created by additional consumer spending is measured as the induced effect. Each figure in column (5) indicated the amount of direct investment required in 1970 to increase employment throughout the economy by 100 jobs in 1980. The agricultural processing sector requires $205,000 of direct investment in 1970 to create 100 jobs in 1980. Following this sector in order of increased investment costs are construction, services, and wholesale and retail trade.

Other uses. Projections and impact measures were presented above to illustrate how the simulation model could be used. Additional uses of the simulation model include measuring the impact of government spending, measuring the impact of revenue and expenditure of different taxes, evaluating various government programs, measuring the impact of labor needs of industrial expansion, and estimating future needs for public services.

The regional simulation model is only a beginning illustration. Its major limitations are that it includes a small number of sectors, assumes constant production coefficients, and includes a limited number of equations. Additional research is needed to study, evaluate and improve the model, thus making it more realistic and sensitive. In brief, there is still much to learn about the construction and use of regional simulation models.

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Appendix 4.1
Mathematical Summary of Doesksen-Schreiner Model[9]

Several equations will be presented to give an overview of the Doeksen-Schreiner model. This includes estimating final demand, determining sector output, and projecting economic variables.

Estimating Final Demand

Final demand is the sum of demands from households, federal government, state and local government, exports, and capital formation. It is expressed as follows:

(1) $Z_t = (CA)_t + F_t + (SL)_t + (H_t)_t + (E_t)_t$

where

Z_t = final demand vector,

$(CA)_t$ = column vector of composition of capital investment in year t,

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$(SL)_t$ = column vector of state and local government purchases in year t.

$(H_t)_t$ = column vector of household purchases of durable goods, non-durable goods and services in year t.

$(E_t)_t$ = column vector of export demand of durables and nondurables in year t, and

(F_t) = column vector of federal government purchases in year t.

Final demand elements are determined individually. For example, capital investment is explained by the accelerator principle and household purchases are a function of population and income.

Determining Output Estimates

Once final demand estimates are made for year t, sector output necessary to produce the final demand can be made as follows:

(2) $X_t^d = A_1 Z_t$

where

X_t^d = output vector required to produce final demand, and

A_1 = matrix of direct and indirect coefficients.

Maximum output per sector that can be produced due to labor availability is as follows:

(3) $X_t^L = (A_2)_{t-1} A_3 L_t$

where

X_t^L = column vector of maximum output level due to labor restriction in year t,

[9] For the complete model see [1].

$(A_2)_{t-1}$ = diagonal matrix of output-labor ratios in year t—1,

A_3 = diagonal matrix of one plus annual rate of growth in output labor ratios, and

L_t = column vector of labor available by sector in year t.

Maximum output per sector that can be produced due to capital availability is:

(4) $X_t^c = (X^c)_{t-1} + (I_n)_t/[A_4)_t \bullet A_5]$

where

$(X^c)_{t-1}$ = column vector of maximum production due to capital restriction for year t—1,

$(I_n)_t$ = column vector of new plant and equipment investment in year t,

$(A_4)_t$ = diagonal matrix of capital output ratios defined at capacity levels in year t,

A_5 = diagonal matrix of one plus change in capital-output ratio, and

X_t^c = column vector of maximum production due to capital restriction in year t.

Realized sector output X_t^R is the minimum constrained by final demand, plant capacity, or labor force.[10] It is expressed as follows:

(5) $X_t^R = \min [(X_t^d) (X_t^L) (X_t^L)]$

Projecting Economic Variables

Once output is estimated by above method, the simulation model projects employment (wage and salary workers, and proprietors), income (wage and salary, proprietor, property, and transfer payments), value added, state and local taxes, federal taxes, and disposable income. Equations (6) and (7) are presented to illustrate how two of these variables (wage and salary employment, and wage and salary income) were obtained.[11] Wage and salary employment is obtained as follows:

(6) $(L^w)_t = (A_6)_{t-1} A_7 (L^e)_t$

where

$(L^w)_t$ = column sector of wage and salary employment in year t,

$(A_6)_{t-1}$ = ratio of wage and salary employment to total employment in year t—1,

A_7 = one plus growth rate of ratio in A_6, and

$(L^e)_t$ = column vector of state employment by sector in year t.

Wage and salary income is expressed as:

(7) $(WS)_t = (A_8)_{t-1} A_9 (L^w)_t$

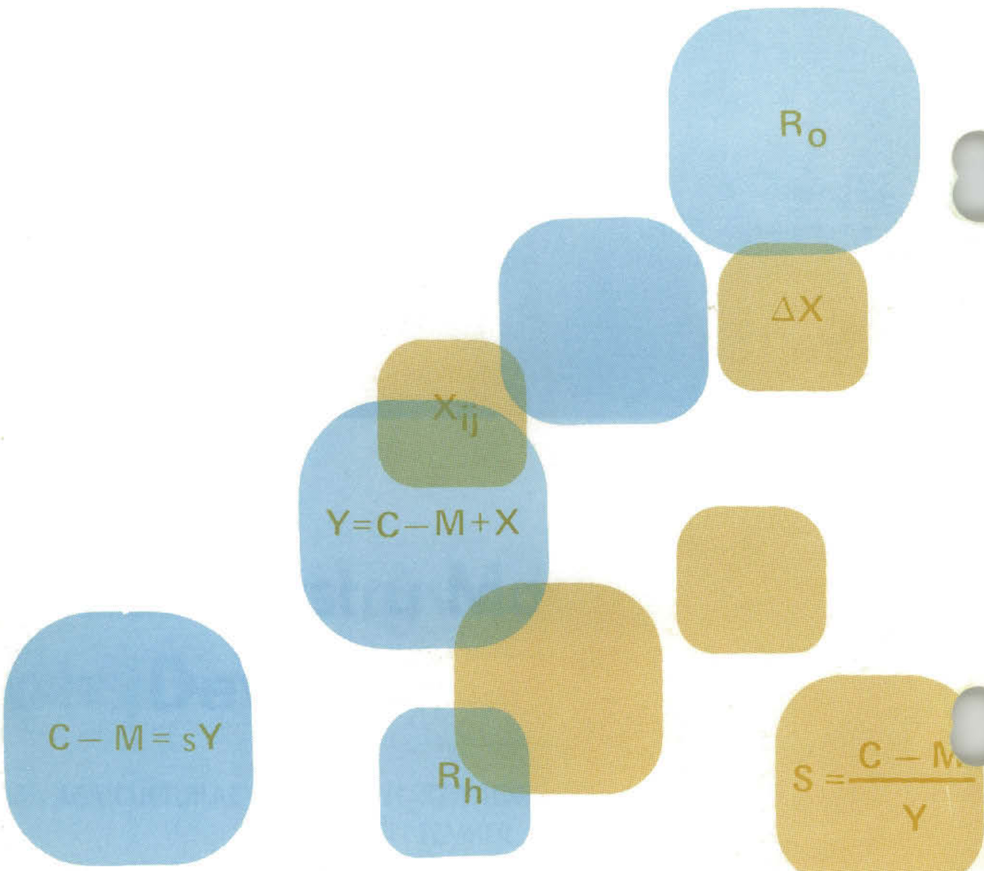
[10] This procedure follows earlier methodology established by Maki, Sutton, and Barnard [7].
[11] For a complete presentation of how each variable was derived, see ([1].

where

$(WS)_t$ = column vector of wage and salary income in year t,

$(A_8)_{t-1}$ = wage rates by sector in year t−1, and

(A_9) = annual growth rate of wages by sector.



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