Measurement Errors in Nuclear Counters Calibrated By the Ratio Method

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Introduction

Instruments which employ nuclear radiation to measure properties of materials, e.g., uniformity of density, composition, water content, and the like, are commonly calibrated to indicate directly the desired characteristics of the material under measurement. In making this determination, a reading will be based on a radiation count including background effects as well as the effects directly under measurement. Background count is often contained within the calibration. The calibration can be stabilized by periodically making an adjustment based on a reading on a reference material or by reading a source of radioactivity, if such is not already a part of the instrument.

Such stabilization tends to preserve the calibration from any effects of count drift such as radioactive decay, and transient drift such as might be caused by the counting instrument itself. Still, any indication of the instrument is subject to counting error due to the random nature of nuclear radiation counting.

Errors resulting from counting random events and error from instrument characteristics, dead time and background are well understood. However, the stabilized indication of such direct calibration instruments has the error characteristics of a ratio of the count made on the material to the count made on the stabilization system. The error of this measurement, which may contain a background count, is more devious and is considered in this paper. The magnitude of the stabilization count can be selected to cancel or effectively reduce the effects of background uncertainty and dead time on the ratio.

Analysis of Problem

If a set of radioactive counts is stabilized by division by a count on a stabilization system, each ratio can be represented

$$R_m/R_s = (R'_m + R_o) \,/\, (R'_s + R_o) \qquad \mathbf{1}$$

where R_s is the recorded count rate on the stabilization system, R_m

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is the recorded count rate on the material under measurement, R'_m is the portion of the count due to the reading on the material, R'_s is the portion of the count due to the stabilization system and R_o is the background count rate. Of concern here is the fractional error of this ratio as affected by uncertainty of the measured activity, dead time of the counter system, and uncertainty of the background.

The basis for evaluation of random counting error will be an equation (1)[1] relating the fractional standard error f associated with a count rate on a material R_m including background count rate R_o to counting time t, R_m and R_o:

$$f = (tR_o)^{-1/2} (R_m/R_o + 1)^{1/2} (R_m/R_o - 1)^{-1} \qquad \mathbf{2}$$

A second kind of error involves the relationship between R'_s and R_o. If R_o changes in the same proportion that R'_s changes, equation 1 indicates that no error is incurred in the calibrated reading for a given condition of the material of measurement. This second error then is concerned with non-proportional behavior of R_o, which would introduce error, as seen by inspection of equation 1.

A third kind of error is concerned with dead time. A counting system for radioactive emanations translates electrical information through use of a rate meter, scaler, or the like. The minimum time interval between successive events in the detector which will produce recorded information is herein called dead time. In practice, radioactive emanations traversing the detector in this dead time are either completely ignored by the system or increase the dead time interval. Herein assume that they are ignored. A recorded count is corrected for dead time by deducting the total "off time" which occurred during the count interval. This is accomplished by the following equation (2):

$$R = {}^*R/(1 + {}^*R\tau) \qquad \mathbf{3}$$

Where R is the recorded count rate, $^*R$ is the actual count rate, and τ is the dead time.

In addition to its effect on the counting interval, dead time also disturbs the estimate of random counting error, for example, equation 2. Cohn (1) has analyzed this problem for both types of dead time. Cohn's work can be utilized in judging whether one must make correction for dead time in employing equation 2. Consider errors at 1 and 10 percent levels for example. At the 1 percent level, a $\tau = 5\mu$ sec counter could be operated in a 2000 cps flux; a $\tau = 100\mu$ sec, up to 2000 cps. Note that these errors are errors in estimating the f and not the count. Consequently, it seems safe to assume the effect analyzed by Cohn can be safely

[1] Numbers in parenthesis refer to Literature Cited.

neglected for purposes herein. Consequently, no allowance for effect of dead time on the estimation of counting error will be made, and attention will be confined to the main effect of dead time on counting.

Derivation of Error Equations.

Consider first the error introduced by the uncertainty of the background count and its effect on the stabilized ratio. The fractional standard error f_r due to the nature of random counting which includes a background count for the ratio in equation 1 will be the square root of the sum of the squares of two terms like that in equation 2, one involving R_m and one involving R_s:

$$f_r = (tR_o)^{-1/2} [(R_m/R_o + 1)(R_m/R_o - 1)^{-2} + (R_s/R_o + 1)(R_s/R_o - 1)^{-2}]^{1/2} \quad 4$$

which can be rewritten

$$f^2_r = (tR_s)^{-1} \left[\left(\frac{R_m}{R_s} + \frac{R_o}{R_s}\right)\left(\frac{R_m}{R_s} - \frac{R_o}{R_s}\right)^{-2} + \left(1 + \frac{R_o}{R_s}\right)\left(1 - \frac{R_o}{R_s}\right)^{-2}\right] \quad 5$$

Figure 1 shows a plot of f_r for ranges of R_m/R_s and R_o/R_s for $R_m > R_o$. The line of lower limit is the point where $R_o = 0$. Here the fractional error is due entirely to R_m and R_s. This figure can be used to determine f_r for an existing system or aid in the selection of a suitable stabilization system for instrument design. The calculations of figure 1 are based on the standard error so about one-third of the observed errors will be larger than predicted by the graph.

Consider next the error arising from inclusion of a background R_o which varies in a manner not proportional to the stabilization reading R_s. As considered earlier, the ratio R'_m/R'_s (equation 1) would more properly be considered constant over a range of instrument conditions.

To make provision for nonproportional change in R_o, that is, R_o/R_s not constant, consider the ratio in equation 1 for a count reading on a particular material. This can be written

$$R_m/R_s = (k + R_o/R'_s)(1 + R_o/R'_s)^{-1} \quad 6$$

since for a given condition of the medium of measurement R'_m/R'_s can be considered constant, represented k.

The rate of change in the ratio with respect to change in the stabilization reading is $d(R_m/R_s)/dR'_s$. Performing this operation on

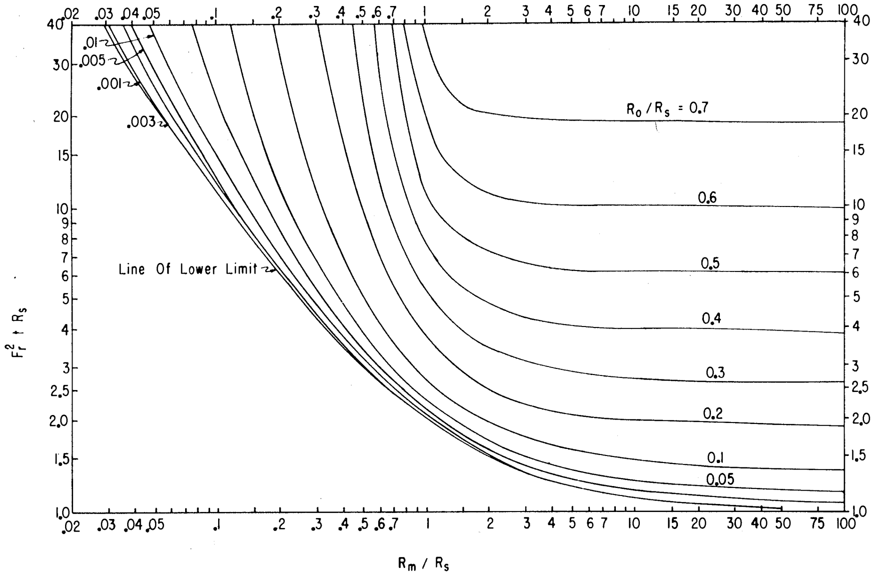

Figure 1. Graphic presentation of equation 5.

equation 6 yields.

$$\frac{d(R_m/R_s)}{dR'_s} = \frac{(k - 1)\,(R_o/R'_s - dR_o/dR'_s)}{(1 + R_o/R'_s)^2 R'_s} \qquad \textbf{7}$$

Fractional error on the ratio due to nonproportional drift effect f_n would be $\frac{\Delta(R_m/R_s)}{R_m/R_s}$ where the incremental change is that due equation 7. Applying this incremental expression to equation 7 yields

$$f_n = \frac{(k - 1)\,(R_o/R'_s - dR_o/dR'_s)}{(k + R_o/R'_s)\,(1 + R_o/R'_s)}\,\frac{\Delta R'_s}{R'_s} \qquad \textbf{8}$$

If $k = 1$, the error is zero. This is the point in the ratio set where the $R_m = R_s$, provided R_s is selected such that this occurs. Note that the greater R_o/R'_s, the smaller the error. The more R_o deviates from proportionality to R'_s, the greater the error, and the equation also indicates zero error when R_o is proportional, that is, when $dR_o/dR'_s, = R_o/R'_s$, as required from earlier consideration of equation 1.

Now consider the effect of system dead time. For this discussion assume background to be zero or at least to vary in proportion to R'_s. Suppose that the initial calibration condition of the instrument is represented by a set of R_{m_1} and also the stabilization reading stays close to R_{s_1}. These observed readings would contain some error due to dead time as described by equation 3. Assume that at some later time R_{s_1} changes by some substantial amount to R_{s_2} (condition 2) by virtue of the true stabilization reading changing from $*R'_s$ to $b*R'_s$ where b is a constant of change. A calibrated point will now be correctly represented

$$\frac{R_{m_2}}{R_{s_2}} = \frac{b*R_m' \left(1 + b*R'_m \tau\right)^{-1}}{b*R_s' \left(1 + b*R'_s \tau\right)^{-1}} \qquad \mathbf{9}$$

Note that the ratio R_{m_1}/R_{s_1} is represented by equation 9 when $b = 1$. The points of interest in this discussion is what is the fractional error caused by change b when dead time is significant but neglected. This fractional error is

$$f_d = \frac{\dfrac{R_{m_2}}{R_{s_2}} - \dfrac{R_{m_1}}{R_{s_1}}}{\dfrac{R_{m_1}}{R_{s_1}}} = \frac{R_{m_2} R_{s_1}}{R_{s_2} R_{m_1}} - 1. \qquad \mathbf{10}$$

Use of equation 9 for representing the ratio for conditions 1 and 2 and substituting these in equation 10 gives

$$f_d = \frac{\tau(*R'_s - *R'_m)\,(b-1)}{(1 + b*R'_m \tau)\,(1 + *R'_s \tau)} \qquad \mathbf{11}$$

Here the error is seen to approach zero as dead time approaches zero, as $*R'_s$ approaches $*R'_m$, and if b approaches 1 ($b = 1$ if the $*R'_s$ does not change in the first place).

Discussion

If error expressed in equation 5 is larger than can be tolerated, several possibilities are open for error reduction. These possibilities involve manipulation of the independent variables, R_m, R_s, R_o, and t. Presumably, R_m and R_o are determined by the system so error reduction is obtained through manipulation of R_s and t. However, if such manipulation does reduce error suitably the system should be redesigned for more suitable R_m and R_o. One can reduce fractional error by making R_s larger and making t larger (longer time) for counting the stabilization system than for R_m, although equipment is generally considered more practical when t can be decreased. Note that by changing R_s, the total fractional error cannot be made smaller than the part due to the R_m and R_o.

If a longer counting time is used for evaluating R_s, the part of the error due to R_s will be reduced by $n^{-1/2}$ for each integral multiple n of unit counting time. One reason for making readings on the stabilization system is to check on the operation of the system. In instrumentation where counts are recorded and translated to calibration data, it is likely that several stabilization readings are made during the course of obtaining a series of R_m and these are available for reducing the error on the determination of R_s.

If error due to nonproportional R_o (equation 8) imposes limitations on operation, this error can be reduced by selecting the stabilization system such that R_m equals R_s at some point on the instrument calibration. Presumably at a point where the fractional errors are least tolerable.

As was true for error due to nonproportional background, the fractional error due to the dead time is minimized at the point on the intsrument calibration where true indication on the material of measurement equals the true indication on the stabilization system, as seen in equation 11.

Example:

Consider a hypothetical instrument as an example of handling the error equations 5, 8, and 11. Assume a calibrated instrument where $R = 150$ cpm, $R_s = 5{,}000$ cpm, $1{,}000 < R_m < 10{,}000$ cpm. Assume $\tau = 1.5 \times 10^{-6}$ min. and $dR_o/dR'_o = 0.1$ when $k = 2$. Assume that R'_s increases 200 cpm from initial calibration so $\Delta R'_s = 200$ cpm. Note that $b = 1.04$. Figure 1 indicates that fractional error f_r on the ratio R_m/R_s due to random counting, including background, ranges from 0.042 to 0.018 as R_m/R_s ranges from 0.2 to 2.

Equation 8 shows that the fractional error f_r on the ratio R_m/R_s due to the nonproportional characteristics of the background, following the change in reading the stabilization system is $f_m = 0.003$.

Equation 11 indicates that the fractional error f_d on the ratio R_m/R_s due to dead time, following the change in reading on the stabilization system is $f_d = 0.0003$. The latter two errors are determinate and thus additive to the random counting error so the range of total fractional error f where the $R_m/R_s = 2$ is $(+0.003 - 0.0003) \pm .018$ to give a range of $-.0153$ to $.0207$. Whether or not this is significant depends upon the tolerance in the calibrated measurement on the material at this ratio.

Summary

Nuclear counters are often calibrated by comparing a count in the medium under study to a count in a standard system. Errors due to the well known uncertainties of radioactive counting are thereby often masked in the final computation. Desirable features of the reference material are specified and methods of handling and computing counting errors in the calibrated reading are suggested. This can also serve as an aid to improving the use of instruments employing ratio calibration.

Literature Cited

1. Cohn, C. E. 1966. The effect of dead time on counting errors. Nuclear Inst. and Meth. 41:338-340.
2. Halliday, D. 1950. Introductory Nuclear Physics. John Wiley and Sons, New York. p. 207 & 208.

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