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PRECAST CONCRETE HYPERBOLIC PARABOLOID SHELLS

for Agricultural Light Industrial and Commercial Uses

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PRECAST CONCRETE HYPERBOLIC PARABOLOID SHELLS For Agricultural, Light Industrial And Commercial Uses

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The hyperbolic paraboloid (h-p) shell develops unusually high strength from its doubly curved surface. Because of this capability, the h-p shell can span large distances without floor obstructions making it well suited for many commercial, agricultural and light industrial structures.

The research reported in this publication was made to develop methods of prefabricating the structural elements with subsequent assembling of the shell rapidly at a selected site. Thus, shell elements in several modular sizes could be precast and stockpiled under factory conditions by construction crews during slack construction periods. Construction crews could work more efficiently by eliminating on-the-job forming and curing.

Objectives of the Study

The objectives of this study were: (1) to design precast structural elements which could be joined to form a h-p shell structure, (2) to develop a support system which could be moved to job sites, adjusted to various shell sizes, and rigidly stabilize the structure during erection, (3) to develop a standard procedure for assembling precast shell components, and (4) to evaluate construction costs.

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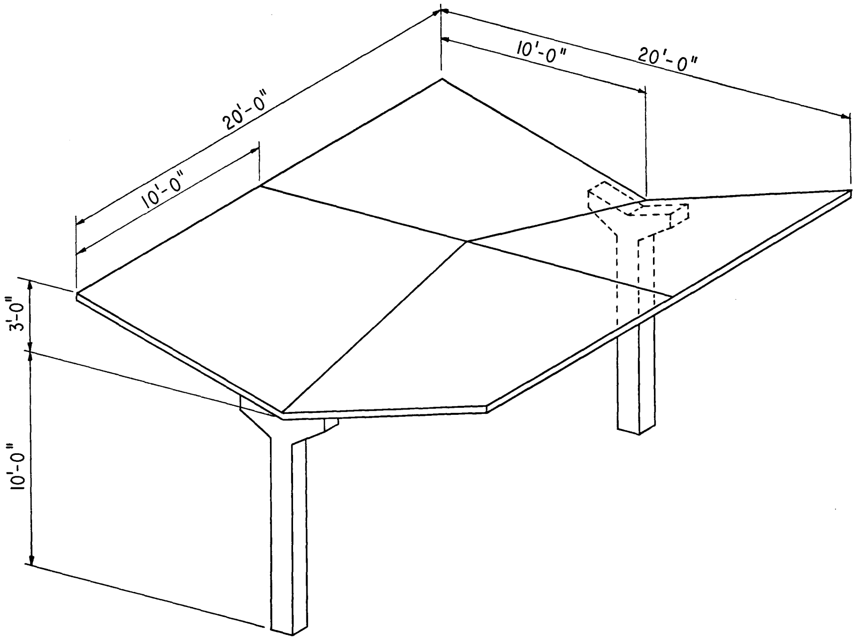


Figure 1 Hyperbolic paraboloid shell principal dimensions and configuration for precasting studies.

Structural Specifications

A two-column, 20-foot square shell was selected for this study (Figure 1). The shell design was made using h-p shell structural equations listed by Portland Cement Association (1). A combined dead plus live load of 63 pounds per square foot was used to study stress and load characteristics of the structure. A detailed design and structural analysis is presented by Noyes (2).

After evaluation of alternative construction joints that appeared feasible, welding exposed steel members cast into the structural elements was selected as a means of developing rigid connections between columns, shell quadrants, the roof center tie bar and the column tie rod. The shell structure was divided into design groups as follows: shell, column, footing and tie bar. A general discussion of these elements is presented in the following paragraphs.

Design of Shell Quadrants

The shell quadrants were designed with a constant top surface slope and 3½ inch exterior edge beams thickened on the lower surface.

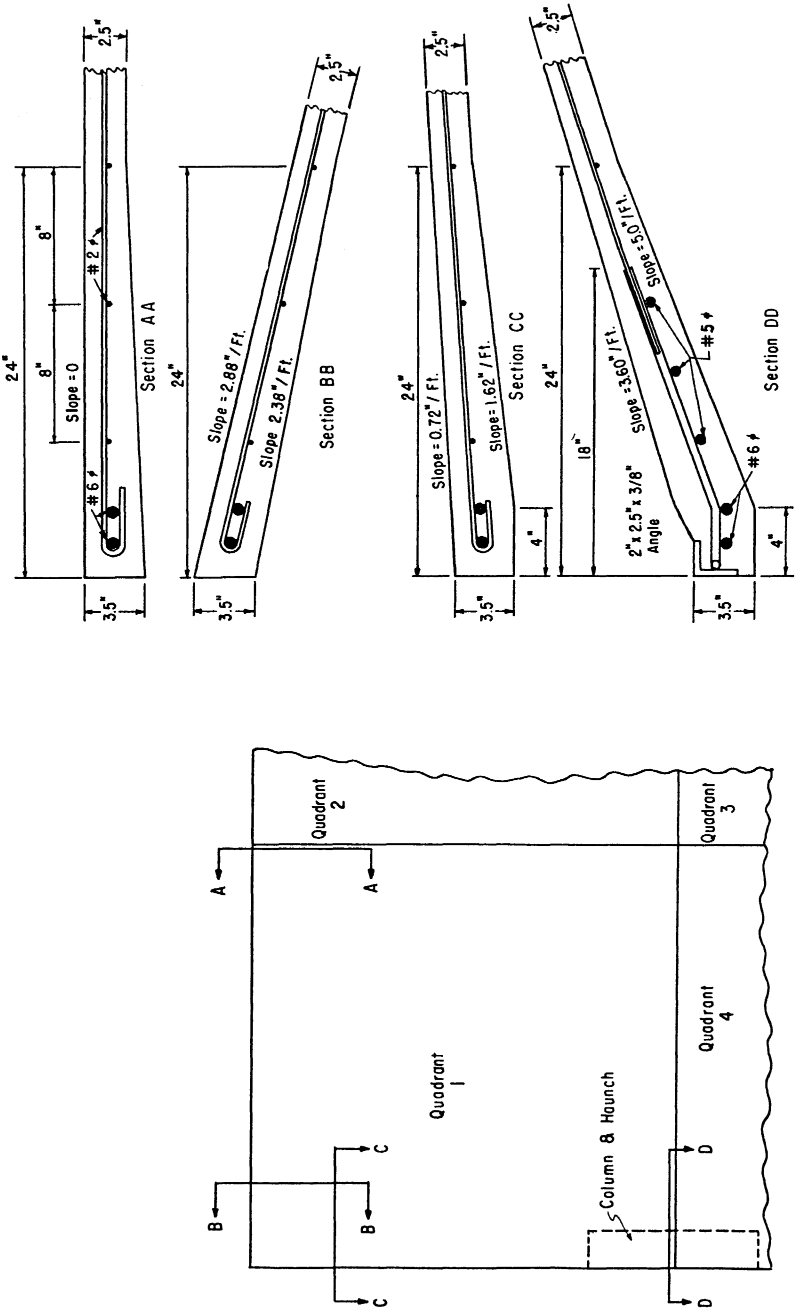


Figure 2 Edge beam and reinforcement details for experimental shell structure.

Four exterior edge beam sections are shown at various surface slopes, Figure 2.

Construction joints between quadrants were formed by welding a flat bar to steel angles, precast into interior quadrant edges, forming the interior edge beams. Figure 3 illustrates the quadrant reinforcement design, ready for casting. Eight-inch dowells on eight-inch centers overlapped the No. 2 shell steel. Two No. 6 bars provided the exterior edge-beam design strength.

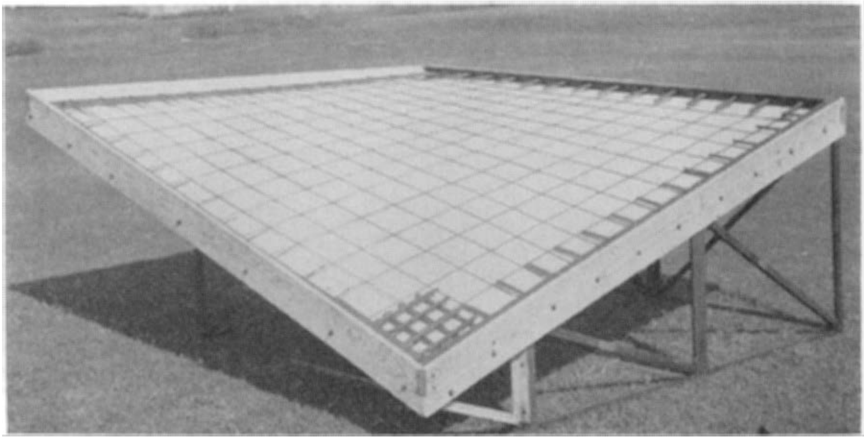


Figure 3 Installation of reinforcement for one quadrant of precast shell.

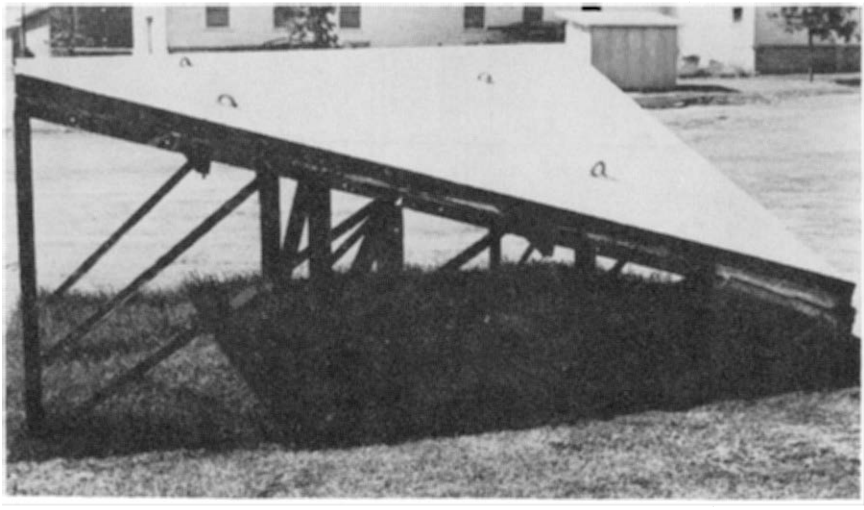


Figure 4 Shell quadrant as precast on casting platform.

A reinforcing mat was connected to the edgebeam steel at the column to reduce localized bending, Figures 2 and 3. The mat was connected to a steel angle, cast into the exterior edgebeam to provide a welded construction joint with column haunch arms to resist overturning.

Lift rings, placed under shell steel at quarter points of the shell surface were used for lifting, Figure 4. The rings were cut off flush with the surface and grouted over after the erection.

Roof quadrants were precast of lightweight aggregate concrete which reduced the roof dead load by 20 percent.

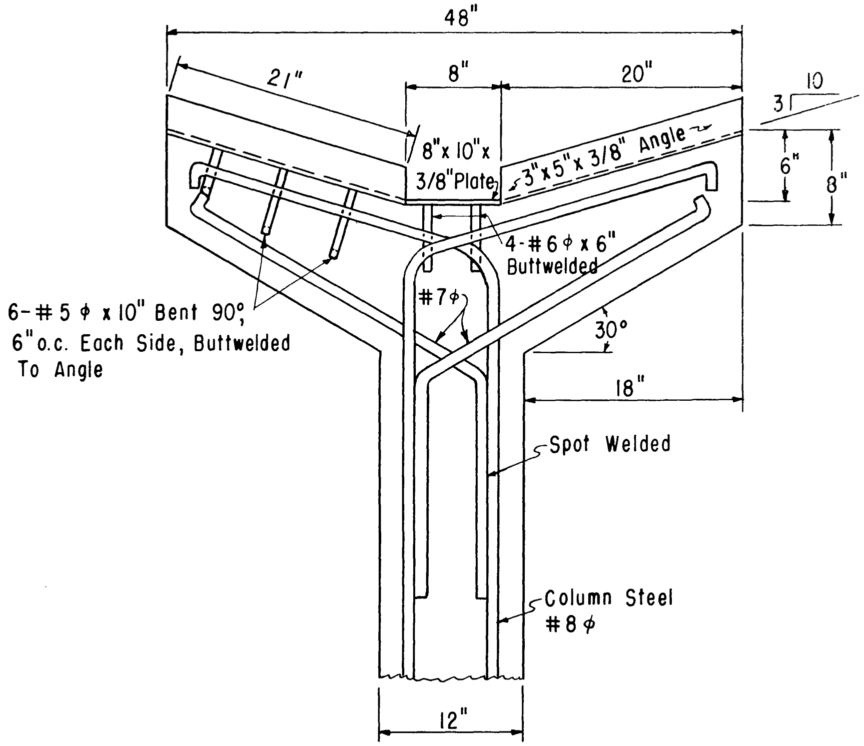


Figure 5 Details of haunch reinforcement at top of column.

Column Design

The columns were designed by ultimate strength methods (3) to support the roof under unbalanced load conditions without external supports. (For multiple shell units, or single shells with wall support, the haunches could be partially or entirely eliminated.) The design required a 10 by 12 inch section with six No. 8 bars spaced three per side. Eighteen-inch haunch arms at the top of the column were designed to resist overturning moments caused by unbalanced roof and wind loads, Figure 5. The haunches were used as lift arms during the column erection and provided a working platform for quadrants during assembly.

An inverted steel "T" section, used as a column-tie rod weld joint, was welded to a steel cap cast into the column cap between haunch arms, Figure 6.

Figure 6 Shell support column precast with attachment for welding to horizontal tie-rod.

Tie Bar

The 1.05 square inch tie bar was designed to absorb the full side thrust developed by the roof loads so that the columns were not subjected to heavy bending.

Footings

The pole building footing "depth of set" equation (4) was used for the footing design. This equation is:

$$D = \frac{2.37P + \sqrt{(2.37P)^2 + 10.56\ PHS_1B^1}}{2\ S_1B}$$

Where:

$D =$ required embedment depth, ft.

$P =$ horizontal thrust, lbs.

$H =$ height above ground line of horizontal thrust, ft.

$S_1 =$ average soil pressure (resistance) about point of rotation, lb. per sq. ft.

$B =$ average diameter of embedded portion of pole, ft.

Wing walls provided resistance to overturning, Figure 7. The combined bearing surface of the footing and wing wall was used to provide the soil bearing area. A reinforcement steel cage was prefabricated to be placed in the wing wall excavations before the columns were set. Columns were lowered through the cage and plumbed before the footings were cast.

Forming Precast Elements

Forming precast elements included the design and construction of shell and column formwork (including form support framework, braces and casting surfaces), steel and concrete placement and concrete finishing.

Shell and Column Formwork

The shell forms, Figure 3, were constructed with a steel framework for strength and rigidity to withstand transportation and handling stresses. Plywood surfaces were fastened to the framework with elevator bolts. Forms were constructed with about 6 inches ground clearance at the low corner and 4 feet along the top edge to provide a ground-level working height.

Column forms were constructed from 2 by 12-inch lumber to obtain rigid support during casting.

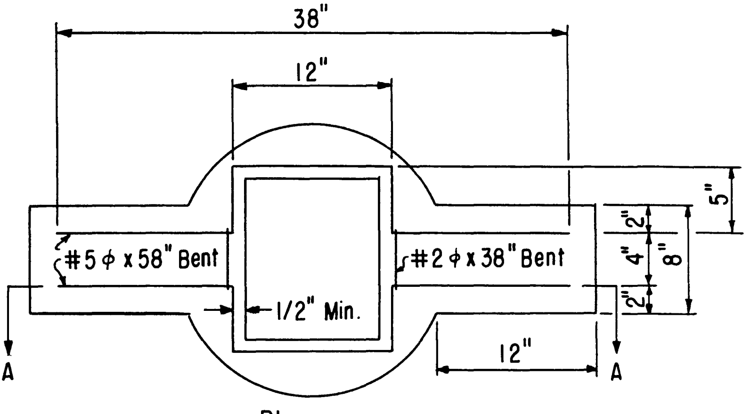
Steel Forming and Placement

Interior edge beam cross-section area was provided by two 2 x 2½ x ⅜ inch angles, Figure 3. One end of each member was cut at 45° and bent down slightly to form the 90° interior edge beam corner. Ten-inch No. 6 dowels were bent 90°, two inches from one end, and were fillet welded to the edge beams on 8-inch centers.

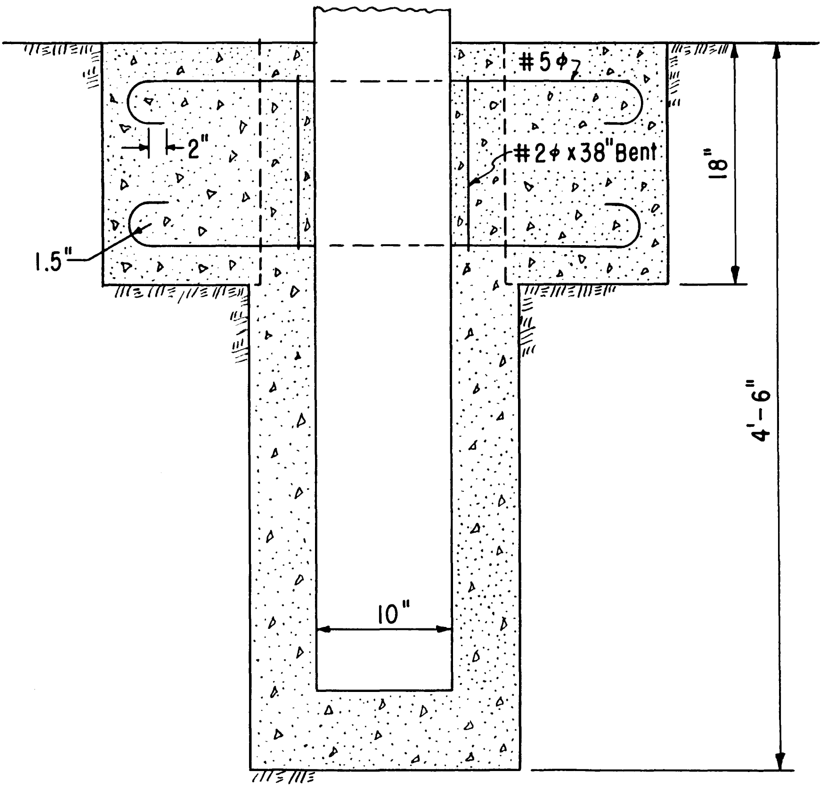
Two No. 6 bars were bent to form the exterior edge beam reinforcement, Figures 2 and 3. Both bars were butt welded to the interior edge beam angles for efficient stress transfer and ease in shell casting.

The 18-inch square reinforcing mat in the lower corner of each quadrant was constructed of No. 5 bars. The bars were welded on 4-inch centers to the steel angles cast into the edge beams, section DD, Figure 2.

Shell steel was placed in the forms immediately following the appli-



Plan



Section AA

Figure 7 Column wing-wall and footing details.

cation of bond breaker. No. 2 bars on 8-inch centers were hooked around the No. 6 exterior edge beam steel; the No. 2 bars were pulled tight and tack-welded to edge beam dowels for direct transfer of shear forces from shell to edge beams.

The column steel cage was formed by tack-welded No. 2 ties to the column steel, Figure 5, to save time during concrete placing.

Shell and Column Casting and Curing

A five-man crew cast the shell quadrants. Two men placed and raked concrete on the forms while three men worked the concrete around the steel. Stiff (3750 psi) concrete had to be rodded, vibrated and handworked under the edge beam steel angles and shell reinforcement to prevent honeycombing.

After concrete was placed in the first form, one man worked the concrete onto the second form while two men screeded the first shell surface. One man worked concrete under the edge beam angles ahead of the screeding while the fifth man followed the screeding with a "wood float" operation. Each man moved to the next form as soon as he completed work on the previous form. While the fourth shell was being finished, the first shell concrete had set enough to be prepared for curing.

All shell surfaces were cured by covering with two layers of wet burlap material and a four milli-inch thick plastic sheet, weighted down securely. The burlap was soaked at 12-hour intervals for the first 4 days, then once each morning for 4 more days. Two weeks after casting, cover materials and form sides were removed.

Column forms were blocked up and sprayed with a bond breaker, the steel cages were lowered into the forms and blocked up. A three-man crew using an electric vibrator placed the column concrete in 1½ hours. Column curing included two layers of burlap soaked continuously for 8 days, no soaking for the next six days, then removal of the burlap after 14 days.

Erection Apparatus

Several pieces of auxiliary equipment were designed and constructed for use in assembling precast structural elements. These included a shell assembly support framework, frame for lifting precast shell quadrants and supports for plumbing the columns while casting the footings.

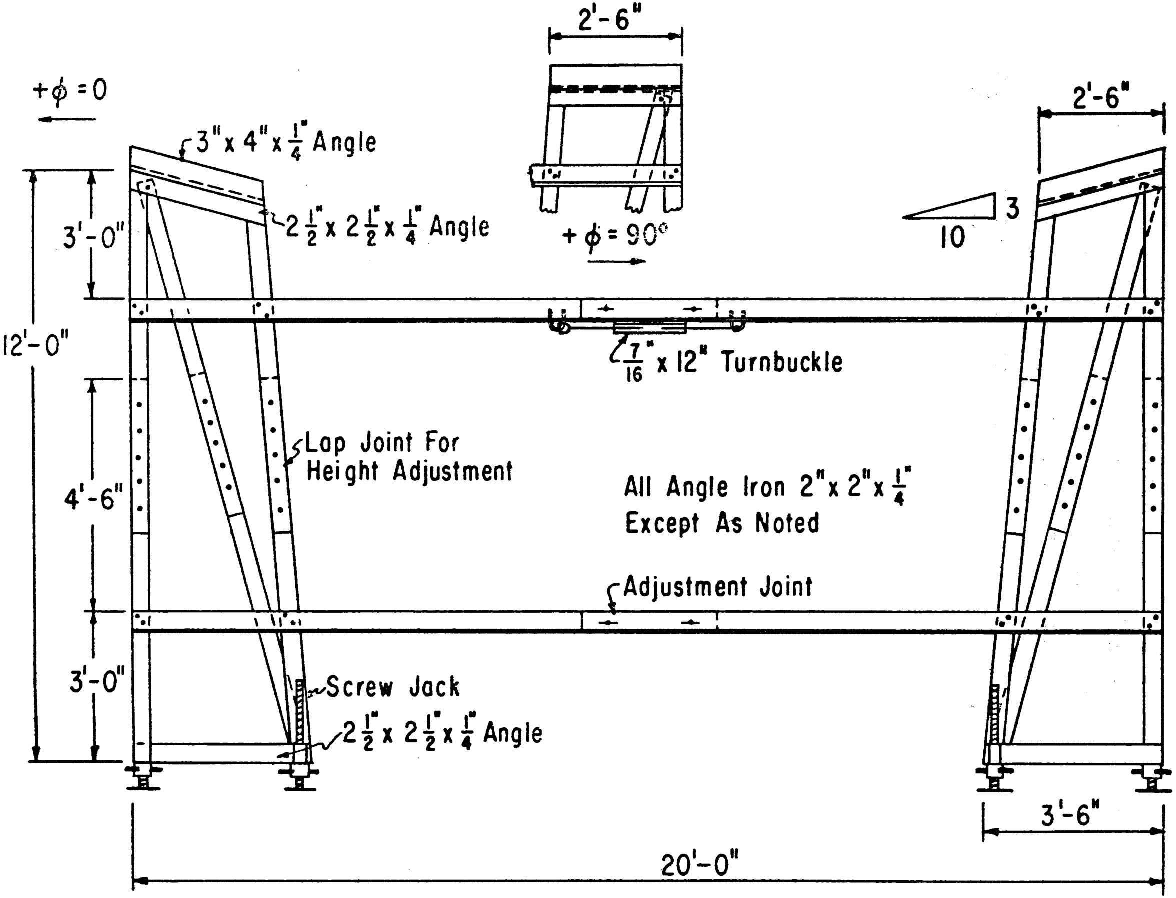


Figure 8 Steel frame for temporary support during erection of precast shell.

Support System

A rigid steel assembly frame, Figure 8, was developed to support the the quadrant corners and provide precise vertical and horizontal stability of shell quadrants during shell erection.

A wooden support frame, Figure 9, provided a platform to support the four quadrants at the center during the roof assembly. The frame

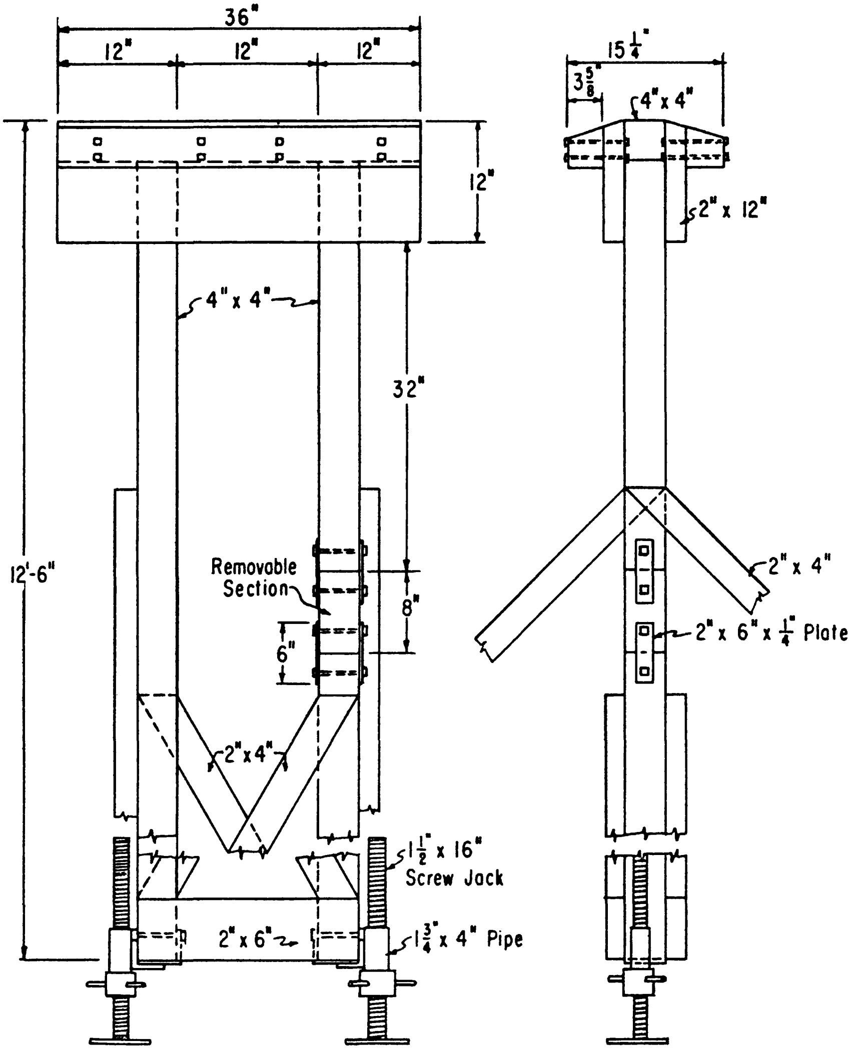


Figure 9 Wooden support frame for temporary support of center of precast shell.

had screw jacks in both legs for vertical adjustment and a removable leg section for assembly around the tie bar. Two wooden towers supported the horizontal roof edges.

A system of 4- by 4-inch wood cribbing clamps supported each column vertically while footings were cast. Wood braces nailed to ground stakes and to boards clamped at the top of each column were used to stabilize the columns while casting footings.

Lift Frame

A square-shaped steel lifting frame was designed to spread the lift chains. The chains were bolted to the corners so that the frame could not shift. Only vertical forces acted on each lift ring during lifting operations.

Site Layout

Site preparation for the erection of the prefabricated structure required two concurrent construction phases. These were: construction site preparation, Figure 10, and material layout for erection, Figure 11.

After the building site was leveled and staked, a rotary drilling truck was used to drill two footing excavations. Wing wall excavations were completed by hand.

The concrete columns and shell quadrants were hauled to the construction site on a flat bed equipment trailer. Wooden spacer blocks were used between quadrants; a wooden frame supported the high corner of the shell during hauling. The columns and roof quadrants were laid out according to erection sequence so that erection equipment could maneuver easily, Figure 11.

Erection Procedure

The erection process was conducted using an untrained crew, with an Agricultural Engineering Department staff member acting as "general contractor." He supervised precasting of concrete elements, site preparation, and erection of the structure.

The research erection sequence consisted of (1) erecting the columns, Figure 12; (2) casting the footings; (3) welding the tie bar to the columns; (4) assembling and aligning the support system; (5) lifting roof quadrants onto columns and support system, Figure 13; (6) welding quadrants to columns, edge beams and the roof center tension bar,

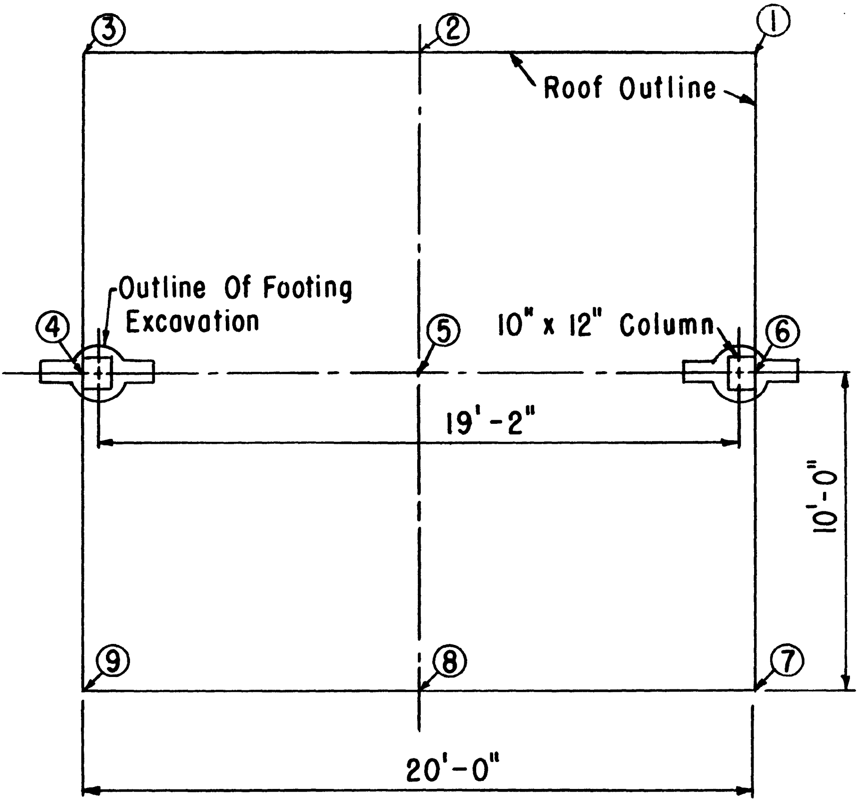


Figure 10 Worksite layout with 9 construction control points.

Figure 14; and (7) lowering supports. The erected structure is shown after supports were lowered in Figure 15.

A transit and carpenter's level were used to check vertical alignment; the transit was placed over a corner control point, Figure 12, to adjust the horizontal position of each column.

Recommended Shell Construction Procedure

Concrete elements for precast h-p shells could be produced at local concrete plants and transported to the construction site, or precast and stockpiled at the construction site. Based on these procedures, the following erection sequence was developed (Note: sub-letters denote concurrent operations):

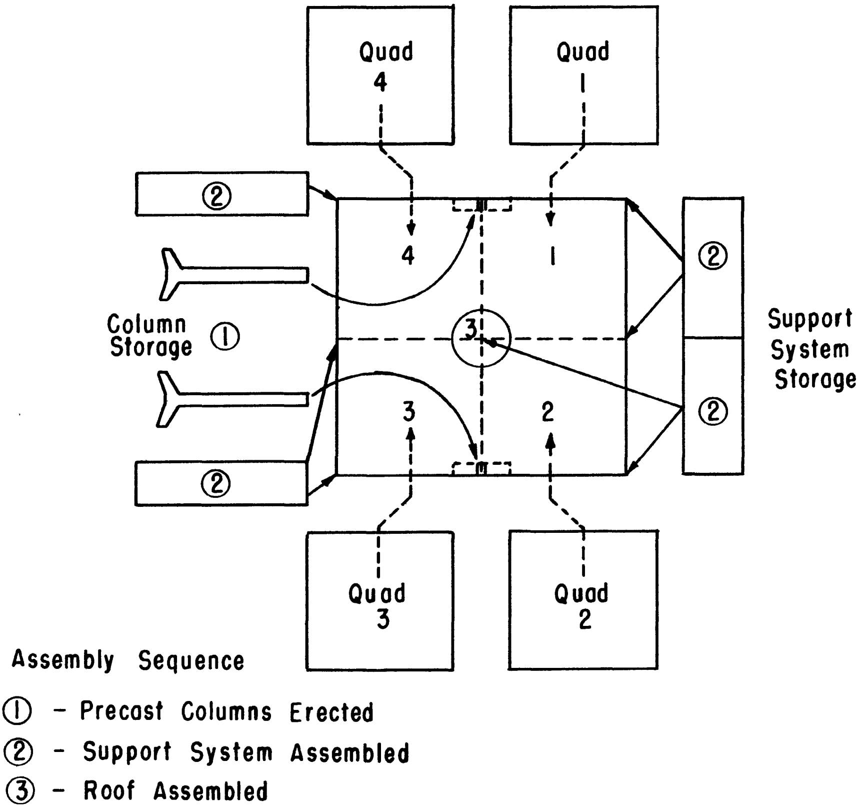


Figure 11 Placement of materials for precast shell erection.

1. Prepare construction site by leveling, staking and excavating footing holes (4 to 6 inches deeper than column base).

2a. Cast a concrete footing pad of early-high-strength, quick setting concrete in the bottom of each excavation at the proper elevation for the base of the columns.

2b. Transport precast elements and erection apparatus to site and place in construction sequence.

3a. Lower the first column onto the footing pad through the steel cage framework (or place cage around column), align, plumb and brace, and cast (quick setting, early-high-strength) concrete footing around column.

3b. Place second column in footing excavation while first footing

Figure 12 Precast column with temporary support.

is being cast. Space the second column the proper top and base distances from the first column and plumb, brace, and cast the footing.

4a. Erect and align support system while column footings are curing.

4b. Weld tie bar to first column; support tie bar at midpoint to prevent sag while bar is welded to the second column.

5. Assemble shell quadrants on the support system and pull quadrants together using a chain jack between lift rings and horizontal adjustment of assembly frame cross bars.

6. Weld shell edge beams to inverted "T" section on column cap; weld tension plates at center of horizontal interior edge beam.

7. Remove support system.

8. Complete edge beam welding. Weld along the lengths of flat

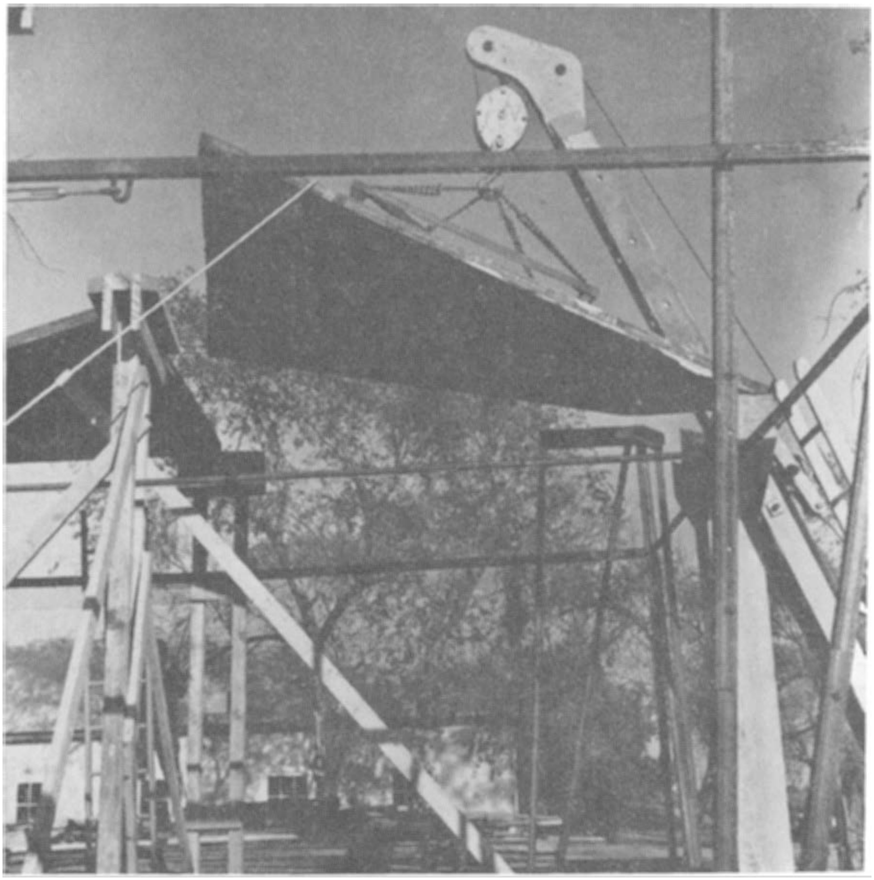


Figure 13 Placement of second quadrant onto temporary support system.

steel laid on edge beam angles at periodic intervals, 2-3 inches of weld per 12 inches of length.

9. Grout between shell and column.
10. Water proof steel edge beams and other exposed steel at joints. Use expandable grout over interior edge beam angles and lift ring steel.

Construction Cost Analysis

The construction cost analysis was subdivided into three major sections: (1) labor, (2) equipment, and (3) material costs. The hourly wage values used in computing labor costs were taken from *Estimating Construction Costs,* Table 1-2, "Union Wage Scale in the United States, in Dollars," [6]. The average rate used for unskilled laborers was $2.18

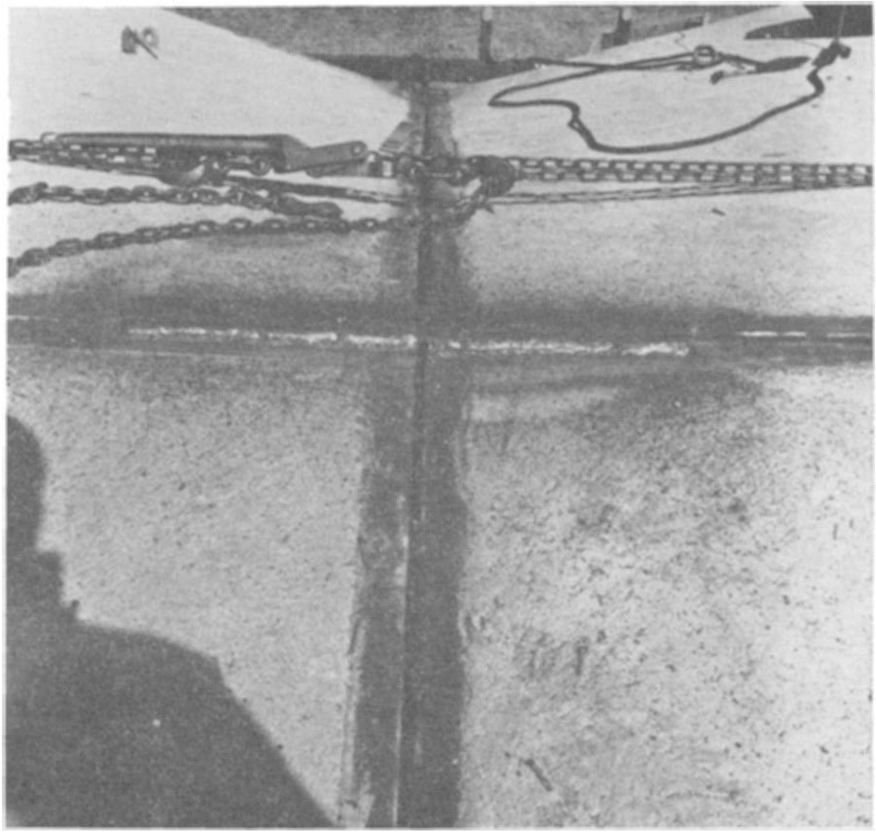


Figure 14 Chain jack system to pull quadrants together at center of shell.

per hour. The average rate for carpenters, $3.13 per hour, was used for skilled labor or supervision. Equipment costs were actual or average local values. Material costs were actual costs incurred on the project.

Labor Costs

Original labor costs for the prototype shell are unrealistic as they were obtained under research conditions. From observations and experience gained during the study, original labor cost data were adjusted by skill or experience factors to expected labor values for future shells cast. Figure 16 shows expected trends in labor as a percent of the first shell labor (man hours) requirements for additional shells constructed.

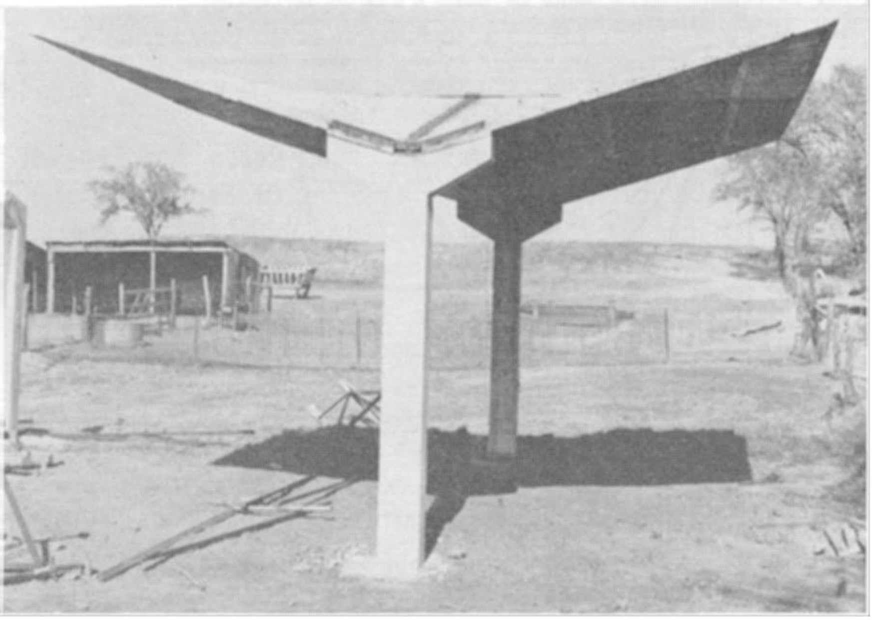


Figure 15 Completed assembly of precast shell after removal of temporary supports.

TABLE I: First Unit Labor Adjusted by Skill Factors

Operation	Skilled Labor Factor	Unskilled Labor Factor	Adjusted Skilled Labor	Adjusted Unskilled Labor
			— Man	Hours —
Column Construction	.500	.775	5.5	82.2
Shell Construction	.400	.800	12.4	159.2
Support System Construction	.765	.736	30.6	22.1
Lift Frame Construction	.700	----	4.2	----
Site Preparation	.625	.600	2.5	7.8
Site Layout	.545	.665	4.9	11.3
Column Erection	.600	.761	6.0	38.1
Support System Erection	.500	.500	3.5	10.5
Shell Erection	.482	.605	10.6	21.2
FINAL TOTAL (Man-Hours)			80.2	352.7

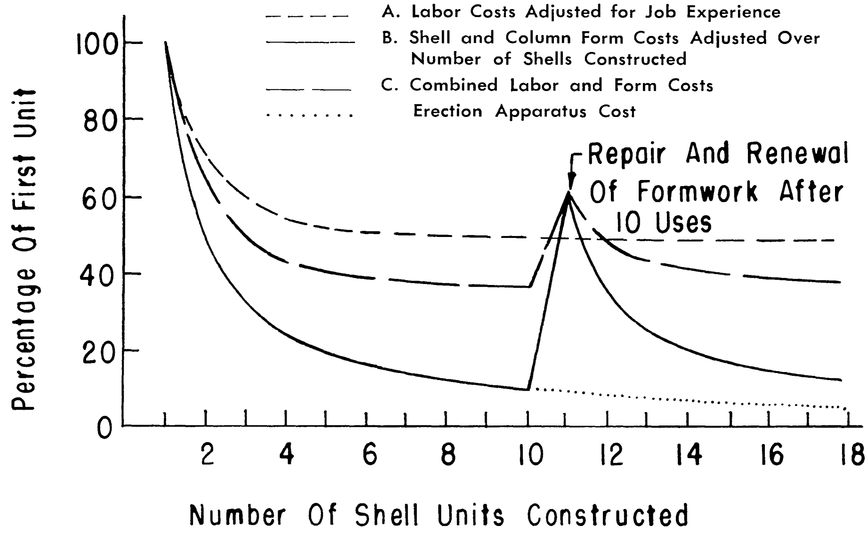


Figure 16 Effect of number of shell units constructed on labor and erection apparatus cost per unit.

The original labor total was 140 man-hours for skilled and 472 man-hours for unskilled labor. These totals included constructing forms, erection apparatus, and structural elements, plus the shell erection. The total labor cost was $1467.16. Table I lists the expected labor operations for construction of the second 20-foot square prototype unit; the first unit labor totals were adjusted by skill factors based on expected work efficiency during construction of the second h-p shell.

Table I reflects data which would not be reproduced, or would be reproduced periodically, such as resurfacing forms and constructing the support system, lifting frame, and column cribbing. By removing these items, the revised total man-hours for supervision was 37.4 man-hours; the total for unskilled labor was 202.0 man-hours. Figure 16 shows the expected trends of the percentage of first unit labor and erection apparatus costs adjusted for skill. Curve A shows labor for column and shell casting, on-site assembly for the support system, and column and shell erection, adjusted for increasing skill. Curve B shows form costs adjusted for the number of units constructed; form surfaces are expected to last for 10 units. With re-use of erection apparatus, the percent of first unit costs, Curve C, would gradually taper off after the first 20 units.

TABLE II: Equipment Costs for First and Second Shells

Item	Hours		Cost	
	First Unit	Second Unit	First Unit	Second Unit
Acetylene Welder, $3.00/hr.	28.0	15.2	$84.00	$45.60
Electric Welder, $2.00 to 3.00/hr.	75.0	26.7	172.00	65.70
Tractor & Equipment Trailer, $2.50/hr.	9.0	6.0	22.50	15.00
Tractor with drawbar hoist, $2.50/hr.	2.0	2.0	4.00	4.00
Fork Lift Truck, 10 Ton, $3.00/hr.	2.0	2.0	6.00	6.00
Craine, 10 Ton (w/operator) $6.00/hr.	8.0	4.6	48.00	27.60
Tractor Dozer for Site Leveling (w/operator) $6.00/hr.	2.0	1.8	12.00	10.80
Rotary Drill Truck (w/operator) $12.50/hr.	1.0	1.0	12.50	12.50
Power Hack Saw $2.00/hr.	9.0	3.5	18.00	7.50
TOTAL EQUIPMENT COSTS			$379.00	$191.20

Equipment Costs

Table II shows the equipment costs for the first shell and the expected equipment costs for the second shell based on labor adjustments by skill factors; the equipment use varies with labor use. The first unit costs *included* the cost of constructing the erection apparatus and forms whereas the second unit costs *exclude* them; thus, a savings of about $187.00 could have been expected between the first and second unit equipment costs.

Material Costs

The cost of materials was separated from labor and equipment costs to provide a clear outline of the expenditures charged to each

TABLE III: Material Costs for First and Second Shells

Item	First Unit Cost	Second Unit Cost
Welding Materials	$ 26.35	$ 17.56
Concrete	108.12	108.12
Steel Material	521.96	199.84
Lumber & Miscellaneous	225.16	0
Final Material Cost TOTAL	$881.59	$325.52

cost area. Table III lists the material costs for the first and second units; the variation in material costs is due to the one-time-construction of erection apparatus and forms. The material cost *difference* between the first and second units was $556.07.

The difference in direct costs between the first and second units was $1191.01. The values does not reflect prorated form and support frame costs over both shells. Additional cost reductions would be mainly from labor and equipment time saved.

Projection to Multiple Shells

The procedure developed in this study can be used to construct one shell or can be modified for use in precasting and erecting multiple shell units. If multiple shells are constructed to form a continuous roof in two directions, the interior columns should be redesigned to provide drainage for the inverted umbrella sections. A storm drain system would be required. An additional modification would be required for the sloped and horizontal exterior edge beams on each *interior* shell unit. The horizontal interior edge beams might be connected by a sealing material to act as an expansion joint.

Crane lifting requirements for multiple shells roof structures will be greater than for a single shell. The crane may be required to lift one or two quadrants into position (depending on maneuverability of the crane) across one or more complete quadrant widths.

Material layout patterns and erection sequences can be developed so the columns and roof quadrants for each shell are placed in the general vicinity of the shell unit. Also, elements could be stockpiled and moved to the crane on trailers, trucks, or moving frames, as needed.

Foundation hole drilling could be scheduled to coincide with the column and roof erection sequence. Primary footing requirements on multiple shell columns will be to provide adequate bearing area.

Summary

The study on a 20 x 20 foot square "saddle" shell by precasting column, roof, and footing elements give the following results:

1. A simple procedure was developed for precasting and erecting h-p shell structures.
2. Connecting the quadrants by a welded edge beam is an effective means of developing an efficient construction joint.
3. The steel and wooden assembly frames effectively and safely stabilized the shell during erection.
4. The h-p shell roof quadrants were assembled on the support system in one hour and 20 minutes.

Conclusions

The following conclusions were derived from the study:

1. Precasting reduces the amount of form work, forming time, and materials compared to conventional h-p shells cast in place.
2. Better working conditions on h-p shells are obtained at ground level.
3. Forms which are maintained at one casting location should last longer than those moved to construction sites for each casting.
4. The cribbing system used for the column erection was too laborious. Columns can be erected faster if (1) a pad is cast in the bottom of each hole at the proper elevation to eliminate vertical adjustments. (2) A concrete footing could be cast which contained four leveling bolts protruding above the footing for leveling the column. Pipes cast into the base of the column would sit on leveling nuts over the leveling bolts. Pipe and bolt connections would be welded after being leveled and the joint could be grouted with expandable grout.
5. Continuous welding of interior edge beams is not required, approximately 1″ per 6-8″ of edgebeam would be sufficient.
6. After ten uses of forms and erection apparatus, the total cost of an erected 20 x 20 foot square precast h-p shell can be reduced to about 37 percent of the first unit *Total Cost.*
7. Steel angles used for edgebeams require a high degree of accuracy in alignment for casting so that quadrants will match up when erected. A maximum allowable error of ½ degree is recommended for a 10-foot edgebeam (20-foot shell unit). A ½ degree variation in 10 feet will result in a 1.05 inch gap.

8. Steel angles tend to warp when dowels are welded to them. Warping increases difficulty of alignment of edge beam steel. Clamping edge beam angles to quadrant forms which are well aligned and braced until shell concrete has set will aid in keeping them aligned.
9. The steel assembly frame and shell forms can be used for cast-in-place construction with minor adaptions.
10. The lifting frame in conjunction with the lift rings provide a satisfactory means of moving the quadrants.

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