

STUDIES ON DROUGHT FLOW DISTRIBUTION AND
USE OF ZONE-TREATMENT PRINCIPLE IN
WATER QUALITY MANAGEMENT

By

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CHAPTER I

INTRODUCTION

The stream assimilative capacity is one of the major concerns in a water pollution control program. As to this aspect the distribution of the drought flow and the program of managing the basinwide water quality are two important problems. The study of drought flow distribution can provide a justified basis for the determination of the design flow. The design flow is the expected stream flow under a specific probability of the drought severity. It is used to determine the dilution capacity of the flow to the waste strength. It also affects the flow velocity and therefore affects the rate of reaeration. The development of a water quality management program is a vital problem in regard to the utilization of the stream assimilative capacity and its allocation to the waste dischargers. Effectiveness and equitability are two key points a program should provide to ensure its success in water quality management.

The drought flow and the flood flow are the two extremes of the stream runoff. But not like the extensive studies made on the flood flow, the number of previous studies on the drought flow is limited. Among them the use of the Type III asymptotic distribution for smallest values to express the drought flow distribution by Gumbel (5) is the most well known application. The distribution of drought flow was considered as only being bounded at the lower end when the Type III

distribution was used. To assume a non-upper limit distribution for the drought flow is not as logical as that for the flood flow. Therefore, in this study the Johnson S_B distribution, a probability function considering the limits at both ends of the distribution, was used to express the drought flow distribution. It was its first application in the low flow study. The goodness-of-fit of the Johnson S_B distribution to the drought flows was compared with that of the Type III asymptotic distribution for smallest values. Theoretical descriptions of these two distributions are in Chapter III.

Annual minimum flows with various numbers of consecutive days were the flow data used in the study of the drought flow distribution. They were generated from the daily flows of each recorded year. The flow records of the three stations located in the Arkansas River basin were used. These stations include a small flow station, a moderate flow station, and a large flow station.

The design flow could be objectively determined from the drought flow distribution. The magnitude of the design flow varies as the number of consecutive days and/or the specified probability of occurrence changes. These aspects were also investigated in this study.

In the achievement of the basinwide water quality objectives, effective and equitable use of the waste assimilative capacity of the stream is an essential concern. The purpose of this part of the study was to extend the use of the zone-treatment principle in various ways to manage the quality of river water. The idea of the zone-treatment was first used in the water quality study of the Delaware River Basin (4) (19). In that study treatment zones were grouped according to their geographical locations. The treatment requirements for various sources

of wastes discharged within each zone were uniform. However, the treatment level could be different for different treatment zones.

The feasibility of three different zoning criteria in grouping the treatment zones were investigated in this study. These three criteria are weight of influent BOD, sub-basin, and BOD-flow ratio. With the first criterion, treatment plants are grouped into treatment zones based on their daily pounds of influent BOD. Among the treatment zones, the one having more pounds of influent BOD should not have less percent BOD removal than others. With the second criterion, a river basin is divided into several sub-basins according to the distribution of its stream system. Treatment plants which discharge their wastes into the receiving waters in the same sub-basin are considered belonging to the same treatment zone. Finally, with the third criterion, treatment plants are zoned according to their ratios of influent BOD and flow. The flow in this case is the summation of influent waste water and the design stream flow at the discharging location of the waste water. Similarly, between each zone, the one having larger BOD-flow ratio should not have less percent BOD removal than the other. For all three classifications, the percent BOD removal required should be uniform for all plants within the same treatment zone; however, they could be different between zones.

In order to make a comprehensive evaluation of each zone-treatment management program, the minimum treatment program and the uniform treatment program were also investigated. The first program allows different percents of BOD removal among treatment plants in the river basin. The second program requires a uniform percent BOD removal throughout the whole river basin.

Bio-degradable organic matter was the type of waste considered in this study. Therefore, the dissolved oxygen (D.O.) in the stream was adopted as the quality measured parameter. The dissolved oxygen is not only essential to the aquatic life, but also a good indicator of the over-all quality of the natural waters and the degree of the presence of pollutants which utilize oxygen.

A regional water quality management model involving the inter-relationship of the organic waste discharged, the stream assimilative capacity, and the dissolved oxygen in the stream was formulated into a linear structure with an application of the Streeter-Phelps equation (21). The technique of linear programming was used to obtain the optimal solution for each management program. Because of much complexity involved in minimizing the summation of non-linear cost functions from all treatment plants in the basin, the cost function was not taken as the objective function. Instead, the stream assimilative capacity in terms of pounds of BOD per day discharged into the streams was selected as the optimization objective.

Though the flow data of three gage stations in the Arkansas River basin were used in the study of drought flow distribution, the water quality data in this basin in many cases were not available. Therefore, a large scale hypothetical drainage basin was used in the study of regional water quality management. Data of design flows, influent BOD, influent D.O., and rates of deoxygenation and reaeration were assumed. It was also assumed that each or a number of waste sources were collected and treated at treatment plants, either municipal treatment plants or industrial treatment plants, before being discharged into the receiving streams.

CHAPTER II

REVIEW OF LITERATURE

A. Drought Flow Distribution and Its Concern in Pollution Control

Contrary to the studies of flood distribution, there are only limited numbers of statistical studies previously done on the drought flow distribution. In 1950 Velz (40) applied Type I asymptotic distribution for largest values, a distribution function first applied by Gumbel (6) on the flood frequency analysis, to express the drought flow distribution. The probability density function of Type I distribution is shown in Appendix A. The droughts were simply arranged in the order of severity and plotted on probability paper. Later on, in 1954 Gumbel (5) applied the Type III asymptotic distribution for smallest values on the drought frequency analysis. Type III distribution is sometimes known as Weibull distribution since Weibull (42) (43) first applied it to the description of the strength of brittle materials. In his application, Gumbel took into account the lower limit of the drought flow distribution. The lower limit was assumed to be zero, or a small positive value. Another comparative study on the drought flow distribution was done by Matalas (13). The goodness-of-fits of four probability distributions to low flow data were investigated. These distributions were Type III distribution for smallest values, log-normal distribution, gamma (Pearson Type III) distribution, and Pearson Type V distribution.

Matalas concluded that Type III and gamma distributions fitted flow data equally well and were more representative of the distribution of low flows than either log-normal or Pearson Type V distribution. The probability density functions of log-normal distribution, gamma distribution, and Pearson Type V distribution are listed in Appendix A, while the probability density function of Type III distribution for smallest values is shown in Chapter III.

The analysis of drought flow distribution has direct applicability to pollution control problems. From it the assimilative capacities of a stream and the degrees of waste treatment required to meet various values of low flow may be determined. In one such instance, Velz (41) estimated the cost of waste treatment in meeting various magnitudes of drought flow. He concluded that for meeting the low flow quality requirements, in certain circumstance, reduction in industrial production might be justifiable rather than costly treatment. In another instance of applying low flow analysis to pollution abatement problem, Camp (2) pointed out that the feasibility and cost of low flow augmentation should be considered as a means of reducing the treatment cost.

There are two basic viewpoints in establishing the standard for determining statewide or basinwide design flow from the analysis of drought flows. One of them is the adoption of a uniform standard to all streams regardless of water uses, intrastate or interstate status, and regulated or unregulated flow condition. The other one is the application of a flexible standard to different groups of receiving waters. As the uniform standard is concerned, there are two categories of design flow prevailingly adopted by water quality regulatory agencies. One of them is the minimum 7-day flow once in 10 years. It is the minimum

average flow for a period of seven consecutive days expected to recur once in ten years. The Delaware River Basin (24), Ohio River Valley (35), and states of Arkansas, Massachusetts, Ohio, and Virginia are among those adopting this kind of design flow (23) (27) (34) (37).

Another category of the uniform design flow is the minimum 7-day flow in the most recent 10 years. Wisconsin is a state adopting this design flow (38). It is obvious that this standard underlines the flow condition of the recent years which could be significantly different from the condition in the previous period due to regulations from man-made or natural causes. In fact, with the application of this design flow, the determination of flow magnitude is not based upon the analysis of low flow distribution. This design flow in a sense is not the flow with a 10-year return frequency, unless the period of flow records is exactly ten years. On the contrary, the minimum 7-day flow once in 10 years is a flow determined from the analysis of drought flow distribution. When regulation does exist and it results in variations in flow significantly different from the natural pattern of variation, this minimum 7-day flow once in 10 years may reflect the effects due to regulation (24).

Where a flexible standard is applied, different design flow criteria are applied to various groups of receiving waters in a state or a drainage basin. Water uses, intrastate or interstate streams, and unregulated or regulated flow condition are three primary aspects generally considered in the selection of design flows. In Minnesota (29) (30), four kinds of design flows have been specified to various classes of receiving waters with fisheries and recreation uses. Streams with these water uses have been classified into three classes: Class A,

Class B, and Class C. For the intrastate waters, the minimum 30-day average flows once in 25 years, 20 years, and 10 years are the design flows for streams of Classes A, B, and C, respectively. For the interstate waters, the minimum 7-day flow once in 10 years is the design flow for each class of stream with fisheries and recreation purposes. The selection of a design flow based upon whether the receiving streams are unregulated or regulated have also been adopted in the states of Georgia and North Carolina (26) (33). For the unregulated streams these two states have also adopted the minimum 7-day flow once in 10 years as the design flow. For the regulated waters, the state of Georgia specifies that specific criteria or standards set for the various water quality parameters apply to all flows. The State of North Carolina sets that the governing flow for quality standards and for the design of water treatment facilities shall be the instantaneous minimum flow. In fact, these two states adopt the same design flow to the regulated streams. It is an instantaneous flow minimum at any time to come and has no way to be determined from the analysis of the recorded low flow data.

B. Zone-Treatment Principle, Effluent BOD,
and Stream Dissolved Oxygen Standards in
the Management of Bio-degradable Wastes

The application of the zone-treatment principle in basinwide water quality management was first seen in the Delaware Estuary Comprehensive Study (4) (19). This study was undertaken by the Federal Water Quality Administration in cooperation with the Delaware River Basin Commission (DRBC), the New Jersey Department of Health, the Pennsylvania Department of Health, and the City of Philadelphia Water Department.

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In 1967 DRBC, a joint federal-state administrative agency established in 1961, adopted this zone approach in the development of water quality standards of the Delaware River Basin. These standards in turn were incorporated into basin regulations of water quality adopted March 7, 1968 (15) (16) (24).

In the Delaware Estuary Comprehensive Study, the estuary was divided into 30 sections from upstream to downstream. The physical, hydrological, and biochemical characteristics were considered uniform within each section. Five sets of water quality objectives were specified. They ranged from current existing water quality to the maximum feasible enhancement of the river quality. Each objective set consisted of twelve water quality parameters, of which dissolved oxygen was the most important. The method of zone treatment was one of the three approaches applied in the cost allocation to achieve the various water quality objectives. The estuary was classified into four treatment zones according to their geographical locations. The same treatment level was required for all waste dischargers within a given zone; however, the treatment level requirements could be different for different zones. The treatment level in each zone was determined so as to achieve the regional water quality goal at minimum overall cost. The other two cost allocation approaches applied were the uniform treatment and the cost-minimization programs. The uniform treatment program required that an identical percentage of the bio-degradable raw waste from each source be removed before discharge. The cost-minimization program allowed different treatment levels at each waste source so as to obtain water quality goals of the region at a minimum cost. Although it was realized that the relationship between cost and percent BOD

removal was generally a concave curve above the primary treatment, the total cost of the cost-minimization program was still estimated through the application of linear programming techniques in an approximation manner. The total cost of either the zone-treatment and/or the uniform treatment program was estimated less accurately by the use of a search technique, because the linear programming technique was not applicable for these two cases. It was shown that for any given quality objective set, the cost-minimization program yielded the smallest cost and the uniform treatment program yielded the largest. The cost of zone-treatment program was in between the other two.

When concerned with the pollution control of bio-degradable wastes, many state regulatory agencies have compiled the allowable concentrations of effluent BOD, or the treatment requirements, and/or stream dissolved oxygen standards into their water quality regulations. In general, three types of quality standards prevail at the present. The first type is the sole adoption of stream dissolved oxygen standards for various water uses. With this selection any discharge of bio-degradable waste into the receiving waters is subjected to the restriction of not causing dissolved oxygen depletion below the level specified. The quality regulations of Massachusetts, Michigan, New York, and Wisconsin (27) (28) (32) (38) belong to this category. As a typical instance, the dissolved oxygen standards of Michigan intrastate rivers are shown in Table I. The second type is the combined uses of the treatment requirement and the stream D.O. standard. A great number of states, Alabama, Florida, Georgia, North Carolina, Pennsylvania (22) (25) (26) (33) (36), and others, have adopted this type of standard. The Delaware River Basin is a large drainage basin that also has adopted

TABLE I
DISSOLVED OXYGEN STANDARDS OF
THE MICHIGAN INTRASTATE RIVERS

Water Use	Dissolved Oxygen
Domestic and industrial water supplies	Present at all times in sufficient quantities to prevent nuisance
Recreation (total and partial body contact)	Present at all times in sufficient quantities to prevent nuisance
Fish, wildlife, and other aquatic life:	
1. Intolerant fish, cold-water species such as salmon, trout, whitefish	Not less than 6 mg/l at any time
2. Intolerant fish, warm-water species such as bass, pike, walleye, and panfish	Average daily dissolved oxygen not less than 5 mg/l, nor shall any single value be less than 4 mg/l
3. Tolerant fish, warm-water species such as carp and bullheads	Average daily dissolved oxygen not less than 4 mg/l, nor shall any single value be less than 3 mg/l
4. Anadromous salmonid migrations in warm-water rivers that serve as principle anadromous fish migration routes	Maintain more than 5 mg/l of dissolved oxygen during times of migration
Agricultural use	Not less than 3 mg/l at any time
Commercial and other uses	Average daily dissolved oxygen not less than 2.5 mg/l, nor any single value less than 2 mg/l

this standard. Table II summarizes the treatment requirements and the stream dissolved oxygen standards of the Alabama Water Quality Criteria. Ways of specifying the minimum BOD treatment requirements are slightly different for different states. Some are rigid in substance, such as those for Alabama (75% BOD removal), Delaware River Basin (85% BOD removal), Georgia (secondary or equivalent treatment), Florida (90% BOD removal, effective not later than Jan. 1973), and Pennsylvania (85% BOD removal from May 1 to Oct. 31, and 75% BOD removal for the remainder of year). Others are flexible. The minimum treatment requirement in the water quality standards of North Carolina is a typical example. The North Carolina regulations state that in the interest of maintaining and enhancing water quality, secondary treatment or equally effective treatment and control shall be considered the minimum treatment for all significant waste sources, unless it can be demonstrated that the quality of the receiving waters will be maintained and enhanced by a lesser degree of treatment or control. The regulations also state that advanced waste treatment processes shall be required insofar as practicable in the instances where a higher degree of treatment is required to maintain the assigned water quality standards. The third type is the combined uses of the effluent BOD concentration and the stream dissolved oxygen to regulate the discharge of bio-degradable wastes. This is specified in the water quality standards of Minnesota (29) (30) (31). Table III shows the summaries of the effluent BOD standards and the stream dissolved oxygen standards for intrastate and interstate streams of Minnesota.

TABLE II
BOD TREATMENT REQUIREMENTS AND STREAM
DISSOLVED OXYGEN STANDARDS IN ALABAMA

<u>Waste</u>	<u>Min. % BOD Removal</u>
Sewage	Secondary treatment*
Industrial waste	Secondary treatment or its equivalent**
<u>Water Use</u>	<u>Min. Dissolved Oxygen, mg/l</u>
Water supply for drinking or food-processing purposes	4.0
Shellfish propagation and harvesting	4.0
Fish and wildlife	4.0
Agricultural and industrial water supply	2.0
Navigation	Sufficient to prevent the development of an offensive condition

*75 ≤ % BOD removal ≤ 95 (for Alabama)

**The equivalent of secondary treatment means control and restriction, generally through in-plant measure or storage and regulation of discharge, of waste constituents capable of producing pollution effects to a degree comparable to that obtained through applicable secondary treatment process.

TABLE III
STANDARDS OF EFFLUENT BOD AND STEAM
DISSOLVED OXYGEN IN MINNESOTA

Water Use	Max. Effluent 5-day BOD*, mg/l		Min. Dissolved Oxygen, mg/l	
	Intrastate	Interstate	Intrastate	Interstate
Domestic consumption	25	25	Trace	—
Fisheries and recreation**				
Class A	25	25	7 (Octo. 1 to May 31) 5 (at other times)	7 (Oct. 1 to May 31) 5 (at other times)
Class B	25	25	6 (April 1 to May 31) 5 (at other times)	6 (April 1 to May 31) 5 (at other times)
Class C	25	25	5 (April 1 to May 31) 3 (at other times)	5 (April 1 to May 31) 4 (at other times)
Industrial consumption***				
Class A	25	25	Trace	—
Class B	50	25	Trace	—
Class C	50	25	Trace	—
Agriculture and wild life	50	25	Trace	—
Navigation and waste disposal	50	25	Trace	—

*For the cases without adequate dilution the max. effluent 5-day BOD is 20 mg/l for intrastate and interstate waters.

**Class A. The quality of this class shall be such as to permit the propagation and maintenance of warm or cold water sport or commercial fishes and be suitable for aquatic recreation of all kinds, including bathing, for which waters may be usable.

Class B. The quality of this class shall be such as to permit the propagation and maintenance of sport or commercial fishes and be suitable for aquatic recreation of all kinds, including bathing, for which waters may be usable.

Class C. The quality of this class shall be such as to permit the propagation and maintenance of fish of species commonly inhabiting waters of the vicinity under natural conditions, and be suitable for boating and other forms of aquatic recreation not involving prolonged intimate contact.

***Class A. The quality of this class shall be such as to permit uses without chemical treatment, except softening for ground water, for most industrial purposes, except food processing and related uses, for which a high quality of water is required.

Class B. The quality of this class shall be such as to permit uses for general industrial purposes, except food processing, with only a moderate degree of treatment.

Class C. The quality of this class shall be such as to permit uses for industrial cooling and materials transport without a high degree of treatment being necessary to avoid severe fouling, corrosion, scaling, or other unsatisfactory conditions.

CHAPTER III

DATA, METHODS, AND THEORETICAL CONSIDERATIONS

A. Application of Probability Distribution in Drought Flow Analysis

1. Generation of Drought Flow Data

In this study the minimum discharges, rather than the average minimum discharges, for various numbers of consecutive days within a year were used as drought flow data. In order to cover the driest period as a whole, a year was defined as beginning on April 1 and ending on March 31. The average minimum flow is equal to the minimum flow divided by its corresponding number of consecutive days, e.g., average minimum 7-day flow = (minimum 7-day flow)/7. The determination of a design flow by using the distribution of either the minimum flows or the average minimum flows is essentially the same. However, for a small flow station, the use of minimum flows could provide more accurate results.

Three stations in the Oklahoma part of the Arkansas River basin were investigated. They are Stations 1775, 1645, and 1945. Station 1775, located at Bird Creek near Sperry, Oklahoma, has small flows. There have been a significant number of zero daily flow records at this station. Stations 1645 and 1945, located respectively on the Arkansas River near Tulsa and Muskogee have moderate to large magnitudes

of flows. Daily flows recorded at these two stations are all greater than zero.

Annual minimum flows for various numbers of consecutive days at the three stations are shown in Appendix B. These flow data were generated from a computer analysis of daily flow records. Numbers of consecutive days taken into consideration included 1, 3, 7, 14, 30, 60, and 90 days. Flows in Appendix B are listed in a non-decreasing order of magnitudes. Each flow datum has a corresponding observed cumulative probability. The observed cumulative probability of the M^{th} order flow is given by $M/(N+1)$, the Weibull formula (3), where N is the total years of records, M is the ranking number of flows in a non-decreasing order.

2. Johnson Distributions

Johnson distributions proposed by N. L. Johnson in 1949 (7) (10), are empirical distributions which transform a random variable to a standard normal variate. Such a transformation has made the standard normal distribution applicable in many cases. Although the Johnson distributions have been known for many years, no previous applications of them have been found in the field of water resources. The general form of the transformation is

$$z = \gamma + \eta \tau(x; \epsilon, \lambda), \quad \eta > 0, \quad -\infty < \gamma < \infty, \\ \lambda > 0, \quad -\infty < \epsilon < \infty, \quad (3.1)$$

where z is a standard normal variate, x is the random variable in the Johnson distributions, γ , η , ϵ , and λ are four parameters, and τ is an arbitrary function. Three forms of function τ were proposed by Johnson as follows:

$$\tau_1(x; \epsilon, \lambda) = \ln \left(\frac{x - \epsilon}{\lambda} \right), \quad x \geq \epsilon, \quad (3.2)$$

$$\tau_2(x; \epsilon, \lambda) = \ln \left(\frac{x - \epsilon}{\lambda + \epsilon - x} \right), \quad \epsilon \leq x \leq \epsilon + \lambda, \quad (3.3)$$

$$\text{and } \tau_3(x; \epsilon, \lambda) = \sinh^{-1} \left(\frac{x - \epsilon}{\lambda} \right), \quad -\infty < x < \infty. \quad (3.4)$$

The distribution of variable x defined by (3.1) and (3.2) is known as the Johnson S_L distribution. Its probability density function (p.d.f.) is

$$f_1(x) = \frac{\eta}{\sqrt{2\pi} (x - \epsilon)} \exp \left\{ -\frac{1}{2} \left[\gamma + \eta \ln \left(\frac{x - \epsilon}{\lambda} \right) \right]^2 \right\},$$

$$x \geq \epsilon, \quad \eta > 0, \quad -\infty < \gamma < \infty, \quad \lambda > 0, \quad -\infty < \epsilon < \infty. \quad (3.5)$$

In substance, this is a three-parameter log-normal distribution with

$$\eta = \frac{1}{\sigma} \text{ and } \gamma - \eta \ln \lambda = -\frac{\mu}{\sigma},$$

where μ and σ are mean and standard deviation of a normal distribution.

The distribution of variable x defined by (3.1) and (3.3) is called Johnson S_B distribution. And its p.d.f. is given by

$$f_2(x) = \frac{\eta}{\sqrt{2\pi}} \frac{\lambda}{(x - \epsilon)(\lambda - x + \epsilon)} \exp \left\{ -\frac{1}{2} \left[\gamma + \eta \ln \left(\frac{x - \epsilon}{\lambda - x + \epsilon} \right) \right]^2 \right\},$$

$$\epsilon \leq x \leq \epsilon + \lambda, \quad \eta > 0, \quad -\infty < \gamma < \infty, \quad \lambda > 0,$$

$$-\infty < \epsilon < \infty. \quad (3.6)$$

Finally, for the variable x defined by (3.1) and (3.4), its distribution is named Johnson S_U distribution. The p.f.d. is

$$f_3(x) = \frac{\eta}{\sqrt{2\pi}} \frac{1}{\sqrt{(x - \epsilon)^2 + \lambda^2}} \exp \left[-\frac{1}{2} \left(\gamma + \eta \ln \left\{ \left(\frac{x - \epsilon}{\lambda} \right)^2 + \left[\left(\frac{x - \epsilon}{\lambda} \right)^2 + 1 \right]^{\frac{1}{2}} \right\} \right)^2 \right],$$

$$\begin{aligned}
 -\infty < x < \infty, \quad \eta > 0, \quad -\infty < \gamma < \infty, \quad \lambda > 0, \\
 -\infty < \epsilon < \infty.
 \end{aligned}
 \tag{3.7}$$

The cumulative distributions of the variate z from each of Johnson distribution families are in the form of the standard cumulative normal distribution. It is given by

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-\frac{1}{2}z^2} dz. \tag{3.8}$$

A method of calculating the approximate cumulative probability (9) was used in this study. This method is particularly convenient for the use of the electronic computer. The approximation is:

$$F(z) = 1 - g(z) \sum_{i=1}^5 a_i \omega^i, \tag{3.9}$$

where

$$\omega = 1/(1 + pz)$$

$$g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$p = 0.2316419$$

$$a_1 = 0.3193815$$

$$a_2 = -0.3565638$$

$$a_3 = 1.781478$$

$$a_4 = -1.821256$$

$$a_5 = 1.330274$$

The maximum error of this approximation is 7×10^{-7} .

To determine which of these three Johnson families is applicable for a given set of data, the first procedure is to estimate the relative measures of skewness and kurtosis, $\sqrt{\beta_1}$ and β_2 , respectively, of the

distribution (7). The estimates of $\sqrt{\beta_1}$ and β_2 , denoted respectively as $\sqrt{b_1}$ and b_2 , are given by

$$\sqrt{b_1} = \frac{m_3}{(m_2)^{1.5}}, \quad (3.10)$$

and

$$b_2 = \frac{m_4}{(m_2)^2} \quad (3.11)$$

where m_k is an estimate from the data of the k^{th} central moment. It is expressed by

$$m_k = \frac{\sum_{i=1}^n (x_i - \bar{x})^k}{n}, \quad k = 2, 3, 4. \quad (3.12)$$

An observation is denoted as x_i , \bar{x} is the data mean, and n is the sample size.

The second procedure is entering the estimates of β_1 , and β_2 into the chart of $\beta_1 - \beta_2$ relationship to determine the appropriate form of Johnson distribution. Figure 1 shows the regions in the (β_1, β_2) plane. The curve for the Johnson S_L distribution in Figure 1 is given by the parametric equations (10)

$$\beta_1 = (\omega - 1)(\omega + 2)^2, \quad (3.13)$$

and

$$\beta_2 = \omega^4 + 2\omega^3 + 3\omega^2 - 3, \quad \omega = 1, 2, \dots \quad (3.14)$$

If the (β_1, β_2) point falls close to the S_L curve, the distribution of flow may be represented by the log-normal distribution. If (β_1, β_2) point falls between the S_L curve and the line $\beta_1 - \beta_2 - 1 = 0$, the

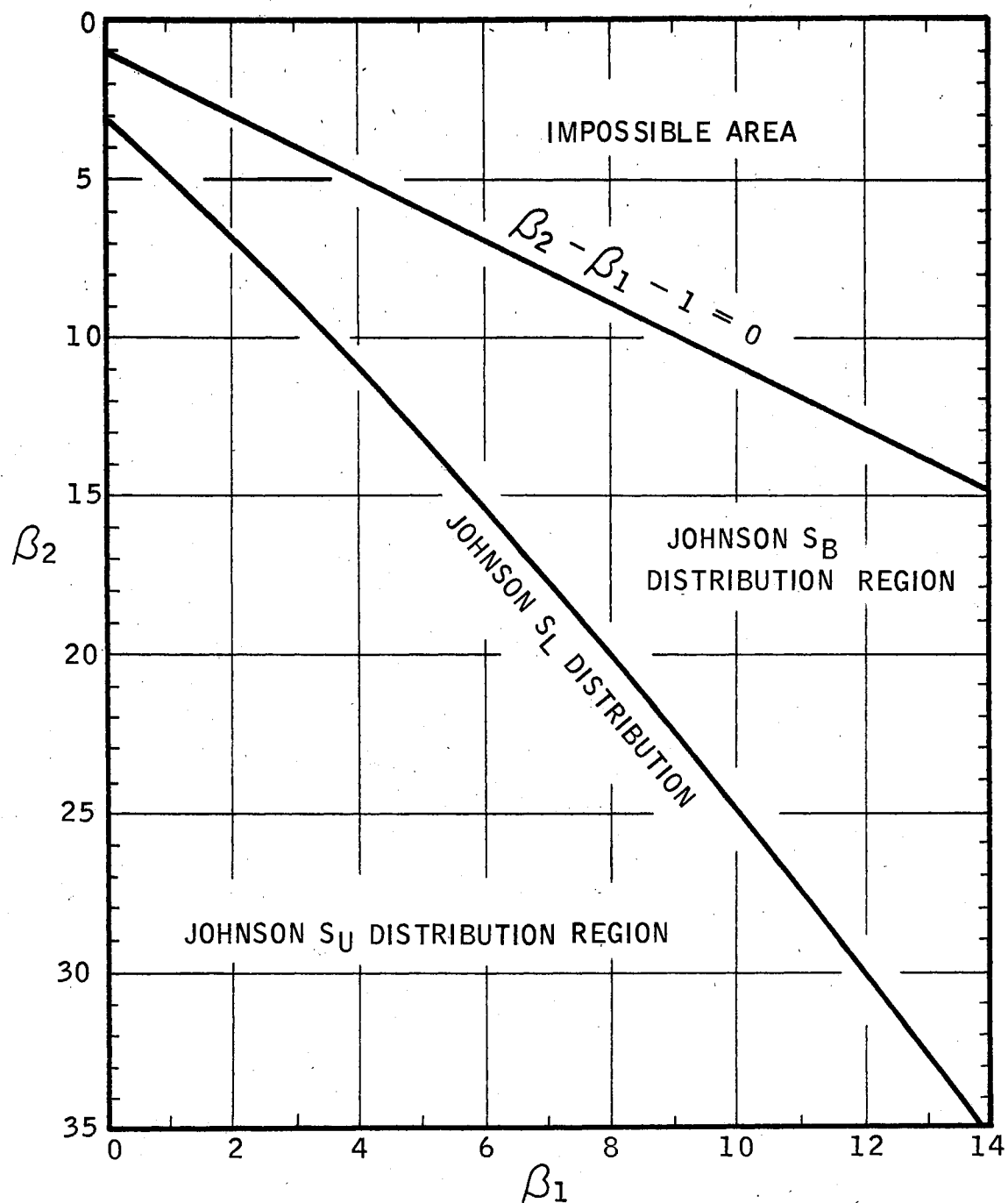


Figure 1. Chart for Determining Appropriate Johnson Distribution Approximation

distribution of flow may be represented by the Johnson S_B distribution. If the (β_1, β_2) point falls below the Johnson S_L distribution line, the distribution of flow may be represented by the Johnson S_U distribution.

3. Type III Asymptotic Distribution for Smallest Values

This is a distribution of minimum values from various initial distributions bounded at the left. It was first used by Weibull to represent the distribution of the breaking strength of materials (42) (43), and it has become well-known in the low flow study since Gumbel applied it in the drought flow analysis (5) (13).

The probability density function of this distribution is

$$f(x) = \begin{cases} \frac{\eta}{\sigma - \epsilon} \left(\frac{x - \epsilon}{\sigma - \epsilon} \right)^{\eta-1} \exp \left[-\left(\frac{x - \epsilon}{\sigma - \epsilon} \right)^{\eta} \right], & x \geq \epsilon \geq 0, \sigma \geq 0, \eta > 0, \\ 0 & \text{elsewhere} \end{cases} \quad (3.15)$$

where x is the variable, σ and ϵ are location parameters, and η is the shape parameter.

The cumulative probability function of this distribution is

$$F(x) = \begin{cases} 1 - \exp \left[-\left(\frac{x - \epsilon}{\sigma - \epsilon} \right)^{\eta} \right], & x \geq \epsilon \geq 0, \sigma \geq 0, \eta > 0, \\ 0 & \text{elsewhere,} \end{cases} \quad (3.16)$$

which defines the integration of $f(x)$ from ϵ to x .

4. Criterion for the Comparison of Goodness-of-fits

The maximum deviation, a test statistic used in Kolmogorov-Smirnov one-sample test (18), was the measure used in this investigation to compare the goodness-of-fits of the distribution functions applied. It is defined as the largest deviation between the theoretical

cumulative probability distribution and the observed cumulative probability distribution. In the mathematical expression, it is given by

$$D = \text{Max.} \left| F_O(X) - S_N(X) \right|, \quad (3.17)$$

where $F_O(X)$ is the theoretical cumulative distribution function, and $S_N(X)$ is the observed cumulative probability distribution of a sample with N observations. It is a measure of the degree of agreement between the distribution observed and the theoretical distribution assumed. In its application to this drought flow study, $S_N(X) = M/(N + 1)$, where M denotes the number of flows equal to or less than a flow X , and N is the total number of observations.

5. Determination of Design Flow

For the drought flows expressed by the Johnson S_B distribution, with the use of equations (3.1) and (3.3), the design flow is given by

$$Q_n = \frac{1}{n} \frac{\left[\exp \left(\frac{z-\gamma}{\eta} \right) (\epsilon + \lambda) + \epsilon \right]}{\left[1 + \exp \left(\frac{z-\gamma}{\eta} \right) \right]}, \quad (3.18)$$

where

Q_n : the design flow determined from the distribution of minimum n -day flows,

n : number of consecutive days,

$\gamma, \eta, \lambda, \epsilon$: parameters of distribution function,

and

z : standard normal variate associated with a specific probability of occurrence.

For the low flows expressed by the Type III distribution, with the use of equation (3.16), the design flow is given by

$$Q_n = \frac{1}{n} \left[\left\{ \frac{1}{\eta} \ln \ln \left[\frac{1}{1-P} \right] + \ln (\sigma - \epsilon) \right\} + \epsilon \right], \quad (3.19)$$

where Q_n : the design flow determined from the distribution of minimum n-day flows,

n : number of consecutive days,

η, σ, ϵ : parameters of distribution function,

and

P : a specific probability of occurrence.

B. Study of Regional Water Quality Management

1. Methods for the Classifications of Treatment Zones

In this extensive application of the zone-treatment principle to manage the basinwide water quality, some premises were specified. First, bio-degradable organic matter was the type of waste considered. The polluttional strength of the bio-degradable organic matter was expressed as the biochemical oxygen demand (BOD), either in the unit of mg/l or of pounds/day. It was also assumed that each or a number of waste sources were collected and treated at a treatment plant, either municipal or industrial, before being discharged to the receiving streams. Secondly, the dissolved oxygen (D.O.) content was adopted as the measure of water quality. Dissolved oxygen is not only a significant regulator of aquatic life, but also an indicator of water quality conditions. It is the most sensitive quality parameter in measuring the degree of pollution from the organic wastes. The

saturated content of the dissolved oxygen in a natural stream at 20°C is approximately 9.2 mg/l (14).

Three kinds of criteria for grouping treatment zones were investigated in this study. They are described as follows:

(1) Classifications of Sub-basins: Sub-basins were classified according to their geographical locations in a river basin. Each sub-basin was then considered as a treatment zone. The requirements of percent BOD removal of treatment plants discharging their wastes into the receiving streams of the same sub-basin were uniform. However, the percent BOD removal could be different for treatment plants discharging wastes into different sub-basins. The grouping of treatment plants depended on where their wastes were discharged, not on where they were located. The stream dissolved oxygen standards of the sub-basins could be either different or identical.

(2) Weight of Influent BOD: Treatment plants in the basin were grouped according to their weights of influent BOD, e.g., pounds of influent BOD per day. Each group consisted of one or more treatment plants. Treatment plants having similar weights of influent BOD were classified into the same group. It required that the treatment plant having larger weight of influent BOD should not have less degree of treatment than others. The weight of influent BOD per day for each plant could be determined from the analysis of the daily influent BOD records obtained over a period of time.

(3) BOD-flow Ratio: The BOD-flow ratio was the weight ratio between the influent BOD and flow. The flow in this classification was the summation of the influent waste flow and the design stream flow at the point of discharge. As the design flow was selected, the BOD-flow

ratio at each treatment plant could be determined from the analysis of daily influent BOD records. The number of treatment zones from the application of this criterion depended upon the number of treatment plants in the basin and the distribution of the BOD-flow ratios of treatment plants. Similarly, it required that the treatment plant having larger BOD-flow ratio should not have less degree of treatment than others.

For making a comprehensive evaluation of the zone-treatment programs, the minimum treatment program and the uniform treatment program were also considered. The former allowed different degrees of BOD removal among treatment plants. The latter required a uniform degree of BOD removal at all treatment plants throughout the basin.

2. Optimization of Basinwide Stream Assimilative Capacity

To apply the foregoing zone-treatment programs in managing the quality of a regional water the primary aspects involved are the selection of the quality objective, the formulation of a water quality system, the specifications of the lower and the upper limit of percent BOD removal, and the choice of the objective function to be optimized. The application of a mathematical programming approach is needed in order to optimize the objective function and simultaneously determine the percent BOD removal required at each treatment plant. Results from the use of this optimization approach can then be used to compare and evaluate the methods of zone-treatment applied.

As mentioned earlier, the water quality objective was expressed in terms of dissolved oxygen concentration in the stream. The quality objective could be determined from the consideration of the specific

beneficial use or of the mutual beneficial uses of stream water. In order to make a clear evaluation of each zone-treatment method proposed, the uniform minimum dissolved oxygen in the streams was adopted. The three levels of uniform quality objective used were 4.0, 4.5, and 5.0 mg/l of minimum dissolved oxygen concentration.

Two levels of minimum treatment requirement were used. They were 30% and 75%. The 30% BOD removal is the efficiency the primary treatment can achieve. Primary treatment of the waste source before its discharge to the receiving waters had been accepted as the minimum requirement for many years. In the recent establishments of stream quality standards, the minimum efficiency achieved by secondary treatment has been adopted in many cases as the minimum treatment requirement. The quality standards adopted in Alabama, Delaware River Basin, Georgia, and Pennsylvania are some examples (22) (24) (26) (36). The minimum efficiencies of secondary treatment specified in different standards are slightly different. The maximum efficiency that secondary treatment could achieve was the upper limit of percent BOD removal used in this study. The 95% BOD removal is generally accepted as the maximum efficiency, obtained by the activated sludge process or by the trickling filter process (14).

An application of optimization technique is necessary in order to determine the percent BOD removal required at each treatment plant for maintaining the D.O. objective in the streams. Many workers treated this problem either with dynamic programming, such as the work by Liebman and Lynn (12), or with linear programming, such as those by Thomann (39), Sobel (20), and ReVelle et al. (17). In general, two aspects should be considered in the application of mathematical

programming to the water quality management problem. They are the selection of the objective function to be optimized and the formulation of the water quality system. The treatment cost has been selected by many investigators as the objective function (12) (17) (20) (39).

In these instances the relationship between the cost and the percent BOD removal for a treatment plant had been assumed as a linear function when the BOD removal varies from 35% up to 85%, or even extended to 90%. With this assumption and the requirement of primary treatment as a mandatory minimum, the optimization problem was treated to minimize the total cost of maintaining the D.O. standards at all points on the stream. However, the lineality of the cost-efficiency relationship is doubtful. An investigation of the cost function was made in a research project of water resources closely related to this study (11). Using the data of treatment cost from the Robert A. Taft Water Research Center and the design criteria of the activated sludge process, the relationship between the annual treatment cost and the percent BOD removal in the range between primary and secondary treatment was expressed by the exponential function $y = a + br^x$, where y and x were treatment cost and percent BOD removal, respectively, and a , b , and r were coefficients determined from an asymptotic regression analysis. This non-linear function is far from being possibly assumed as a linear function in a meaningful way. It was also realized that there would have been too much complexity involved in the minimization of the total cost from all treatment plants in a basin if the non-linear cost function were taken as the objective function in the application of mathematical programming. Therefore, instead of choosing the treatment cost, the stream assimilative capacity was taken as the objective to be optimized by the use

of linear programming. The percent BOD removal required at each treatment plant was determined through the maximization of the stream assimilative capacity without violating the minimum D.O. standards. The stream assimilative capacity was defined as the total pounds of BOD discharged from the whole basin that streams can assimilate.

The Streeter-Phelps equation (21) was used to formulate in linear form the regional water quality system. The equation expresses the D.O. content in the stream as the function of the deoxygenation rate due to organic waste discharged, the reaeration rate, the flowing time of streamflow from one location to another, and the initial levels of BOD and D.O. deficit. It is given by the equation

$$D_t = \frac{k_1 L_i}{k_2 - k_1} (e^{-k_1 t} - e^{-k_2 t}) + D_i e^{-k_2 t}, \quad (3.20)$$

where

D_i, D_t : D.O. deficits at initial time and time t , respectively,
mg/l,

L_i : initial BOD concentration in the stream, mg/l,

k_1 : deoxygenation rate, day^{-1} ,

k_2 : reaeration rate, day^{-1} ,

t : flowing time, day.

Associated with equation (3.20) an equation expressing the change of BOD with time is given by

$$L_t = L_i e^{-k_1 t}, \quad (3.21)$$

where L_t = BOD concentration in the stream at time t , mg/l.

Let

$$\alpha = e^{-k_1 t},$$

$$\beta = e^{-k_2 t},$$

$$\text{and } \gamma = \frac{k_1}{k_2 - k_1} (e^{-k_1 t} - e^{-k_2 t}),$$

then equations (3.20) and (3.21) become respectively

$$D_t = \gamma L_i + \beta D_i, \quad (3.22)$$

and

$$L_t = \alpha L_i. \quad (3.23)$$

Furthermore, let

O_t = D.O. in the stream at time t ,

O_s = saturated D.O. in the stream,

and O_i = initial D.O. in the stream,

then equation (3.22) becomes

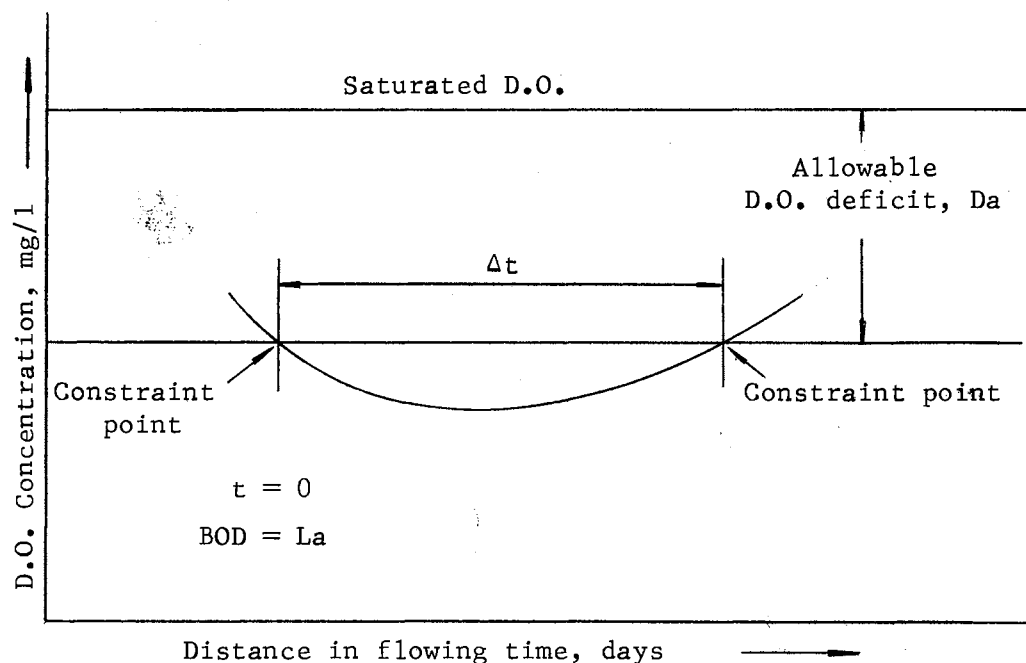
$$\beta O_i - O_t - \gamma L_i = O_s (\beta - 1). \quad (3.24)$$

Therefore, when values of k_1 's, k_2 's, and t 's are known, a basinwide water quality system can be formulated into a linear structure with the use of equations (3.23) and (3.24). Combining this linear system with the quality objectives of the receiving streams, the application of the principle of zone-treatment, and with other constraints, the desired treatment efficiencies of plants can be determined optimally through the use of linear programming techniques.

3. Spacing of D.O. Constraint Points

Linear programming can be used to manage the water quality system in a manner to satisfy the minimum D.O. objective at discrete points;

however it is still possible to have violation of the D.O. standard between two adjacent points. The following sketch shows a D.O. violation in the lower portion of an oxygen sag curve between two quality constraint points.



It is obvious that the smaller the spacing between two adjacent point, the less the chance of a D.O. violation. However, if too many constraints points are located, the calculation work would become unbearable. Therefore, it is necessary to determine an appropriate spacing so that the violation of D.O. standard would become insignificant.

The expression of maximum D.O. violation in terms of the spacing between two adjacent constraint points can be derived as follows:

The oxygen sag curve in the above sketch shows that the D.O. deficit equals D_a at $t = 0$ and again it equals D_a at $t = \Delta t$,

where

D_a = allowable D.O. deficit

Δt = interval of the flowing time between crossings of D_a .

Therefore, using the Streeter-Phelps equation, D_a becomes

$$D_a = \frac{k_1 La}{k_2 - k_1} (e^{-k_1 \Delta t} - e^{-k_2 \Delta t}) + D_a e^{-k_2 \Delta t}, \quad (3.25)$$

in which the only unknown is La , the BOD concentration in the stream at $t = 0$.

The critical D.O. deficit (21), D_c , becomes

$$D_c = \frac{k_1 La}{k_2} \exp \left\{ \frac{-k_1}{k_2 - k_1} \ln \left[\frac{k_2}{k_1} \left(1 - \frac{(k_2 - k_1) D_a}{k_1 La} \right) \right] \right\}. \quad (3.26)$$

From equations (3.25) and (3.26), the critical D.O. deficit becomes

$$D_c = \frac{(k_2 - k_1) D_a (1 - e^{-k_2 \Delta t})}{k_2 (e^{-k_1 \Delta t} - e^{-k_2 \Delta t})} \exp \left\{ \frac{-k_1}{k_2 - k_1} \ln \left[\frac{k_2}{k_1} \left(1 - \frac{(e^{-k_1 \Delta t} - e^{-k_2 \Delta t})}{(1 - e^{-k_2 \Delta t})} \right) \right] \right\} \quad (3.27)$$

The maximum violation of the D.O. standard, V , is the difference between

D_c and D_a , i.e.,

$$V = D_c - D_a \quad (3.28)$$

The relationship between V and Δt can then be established from the use of equations (3.27) and (3.28). Therefore, when a maximum allowable D.O. violation, V , is specified, the required spacing between two adjacent points, Δt , can be determined. With the value of Δt determined, additional control points can then be added in the management of the water quality system.

CHAPTER IV

RESULTS

A. Drought Flow Distributions and Design Flows

1. The Appropriate Johnson Distribution—Johnson S_B Distribution

Through the use of equations (3.10), (3.11), and (3.12) in Chapter III, the values of b_1 and b_2 of the drought flow data for various numbers of consecutive days were calculated. Results for the three stations investigated are shown in Table IV. With the use of Figure 1, these values indicate that among the three families of Johnson distribution the S_B distribution is the most appropriate one to express the distribution of drought flows. With the application of Johnson S_B distribution the distribution of drought flows is confined in a range between a lower limit, ϵ , and an upper limit, $\epsilon + \lambda$. Since the magnitude of streamflow can never be less than zero, ϵ was considered equal to or greater than zero in this application.

2. Drought Flow Distributions in Forms of Johnson S_B Distribution and Type III Distribution

The Johnson S_B distribution and the Type III distribution were used respectively to fit each set of drought flow data listed in Appendix B. For the use of the Johnson S_B distribution in fitting the flow data, the equation $z = \gamma + \eta \ln \left(\frac{x - \epsilon}{\lambda + \epsilon - x} \right)$ was used. This

TABLE IV

APPROPRIATE JOHNSON DISTRIBUTIONS FOR MINIMUM FLOWS
WITH VARIOUS NUMBERS OF CONSECUTIVE DAYS

Station 1775

	1-day	3-day	7-day	14-day	30-day	60-day	90-day
b_1	3.9628	3.7611	3.7347	4.5198	10.4529	4.6897	3.3262
b_2	6.6880	6.3422	6.1789	7.0526	13.0210	7.1918	6.3966
Distribution	S_B	S_B	S_B	S_B	S_B	S_B	S_B

Station 1645

	1-day	3-day	7-day	14-day	30-day	60-day	90-day
b_1	0.4663	0.7457	0.8154	0.7785	0.9324	1.2668	0.5415
b_2	2.8626	3.0624	3.1292	3.1976	3.4870	4.1825	2.8385
Distribution	S_B	S_B	S_B	S_B	S_B	S_B	S_B

Station 1945

	1-day	3-day	7-day	14-day	30-day	60-day	90-day
b_1	0.6703	0.8811	0.8498	1.6322	2.1417	2.3525	0.8444
b_2	3.2385	3.5142	3.2532	4.3787	5.0114	5.3478	2.9354
Distribution	S_B	S_B	S_B	S_B	S_B	S_B	S_B

equation is a combination of equations (3.1) and (3.3), in which z is the standard normal variate corresponding to the observed cumulative probability of the drought flow. For the use of the Type III distribution in fitting the flow data, equation (3.16) was used. A computer program for non-linear regression (1) was used to estimate the parameters of the two distribution functions. Values of estimated parameters in the Johnson S_B distribution and the Type III distribution are summarized in Tables V and VI, respectively. Table V shows that for most cases, ϵ 's, the lower limits of the drought flow distributions obtained from the use of the S_B distribution, were equal to zero. Among twenty one minimum flow distributions investigated, only two cases, one with 60 consecutive days at Station 1645 and another one with 90 consecutive days at Station 1945, had lower limits greater than zero. Also indicated in Table V is that each upper limit of the drought flow distributions from the use of the S_B distribution, $\epsilon + \lambda$, was in the near upper side of the largest observation of minimum flows. Table VI also indicates that most of the estimated lower limits of the drought flow distributions from the use of the Type III distribution were zero.

The theoretical cumulative probability distributions of drought flows from the use of each distribution function are shown in Figures 2 through 22. The observed cumulative probability of each drought flow are also shown in each figure. These figures express the curve fitting of drought flows on the normal probability paper from the use of Johnson S_B distribution and the Type III distribution based on the least squares principle. The normal probability paper is the type of paper frequently used in engineering hydrology for showing the cumulative probability distribution of stream flow. Each figure represents a fitting of the

TABLE V

ESTIMATED VALUES OF PARAMETERS IN
JOHNSON S_B DISTRIBUTION

Station	Parameter	Minimum Flows						
		1-day	3-day	7-day	14-day	30-day	60-day	90-Day
1775	γ	0.89156	0.83905	0.86115	1.10408	1.73426	1.46252	1.05479
	η	0.28644	0.27505	0.29906	0.38274	0.46931	0.45524	0.38166
	ϵ	0	0	0	0	0	0	0
	λ	22.16	66.54	164.76	499.76	4437.84	11529.60	25536.60
1645	γ	0.88846	1.26156	1.01037	0.68905	0.81994	0.88796	0.61588
	η	0.70837	0.75587	0.72258	0.64851	0.66632	0.67673	0.67717
	ϵ	0	0	0	0	0	751.86	0
	λ	2930.84	13123.80	28557.97	51701.00	149191.00	417449.13	606431.00
1945	γ	0.64003	0.72585	0.76350	0.90892	0.96851	1.08188	0.86016
	η	0.72690	0.71770	0.69388	0.70794	0.69731	0.70792	0.64153
	ϵ	0	0	0	0	0	0	7095.14
	λ	5650.29	20901.00	58931.39	152231.00	415411.00	1169301.00	1941364.00

TABLE VI
ESTIMATED VALUES OF PARAMETERS IN
TYPE III DISTRIBUTION

Station	Parameter	Minimum Flows						
		1-Day	3-Day	7-Day	14-Day	30-Day	60-Day	90-Day
1775	σ	2.67	8.57	22.50	58.47	198.50	912.97	3318.02
	η	0.40131	0.40082	0.42026	0.48641	0.46048	0.55741	0.49772
	ϵ	0	0	0	0	0	0	0
1645	σ	908.18	2991.70	8026.34	18586.70	47968.94	127759.31	228641.94
	η	1.05705	1.01353	1.08954	1.06610	1.05114	1.05322	1.19576
	ϵ	20.40	86.43	0	0	0	0	0
1945	σ	2195.53	7570.33	20299.35	46405.32	118923.50	301149.31	588474.56
	η	1.25398	1.18890	1.13301	1.10823	1.06756	1.02665	0.99200
	ϵ	0	0	0	0	0	4292.27	10927.57

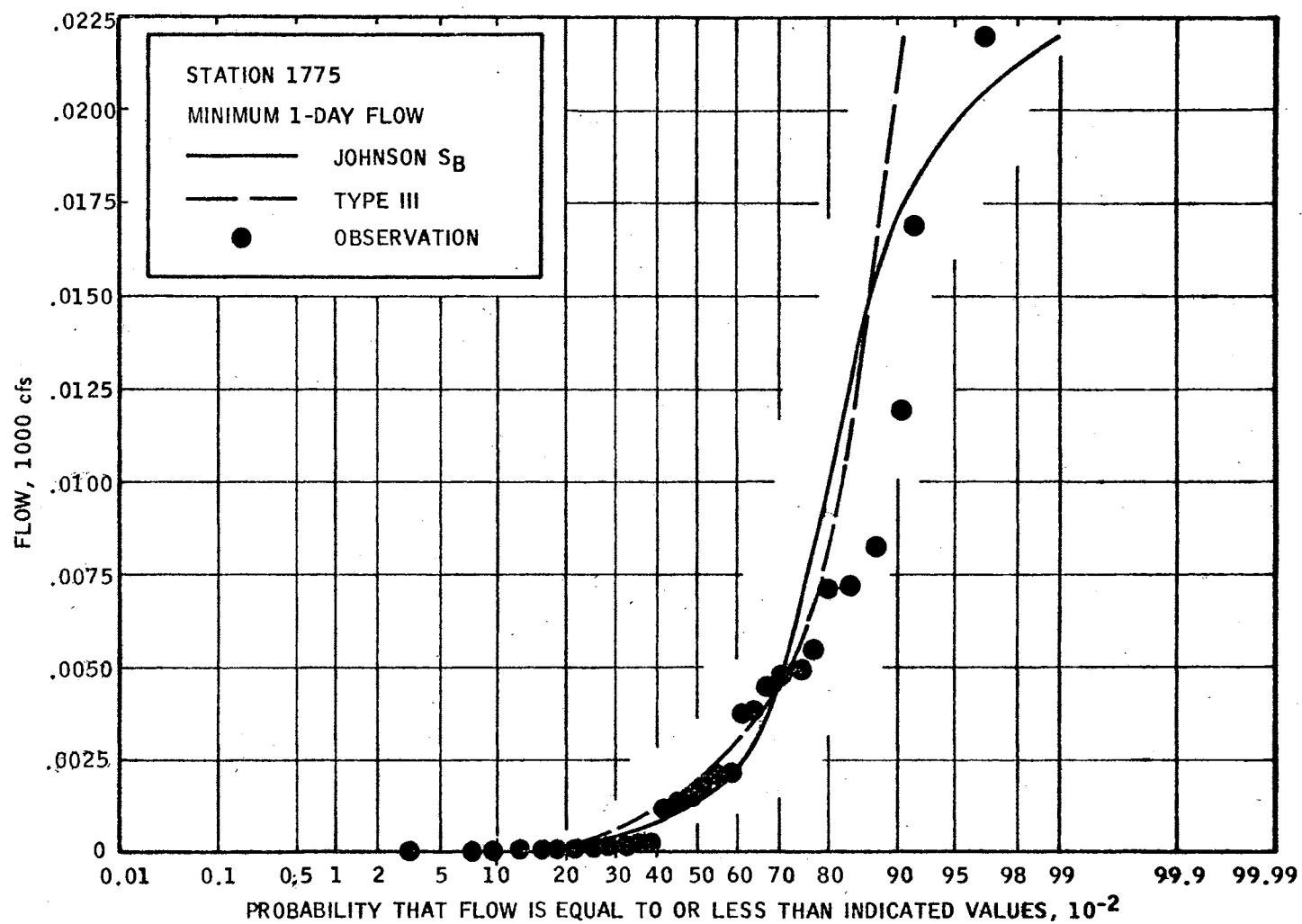


Figure 2. Cumulative Probability Distribution of Annual Minimum 1-Day Consecutive Flow at Station 1775

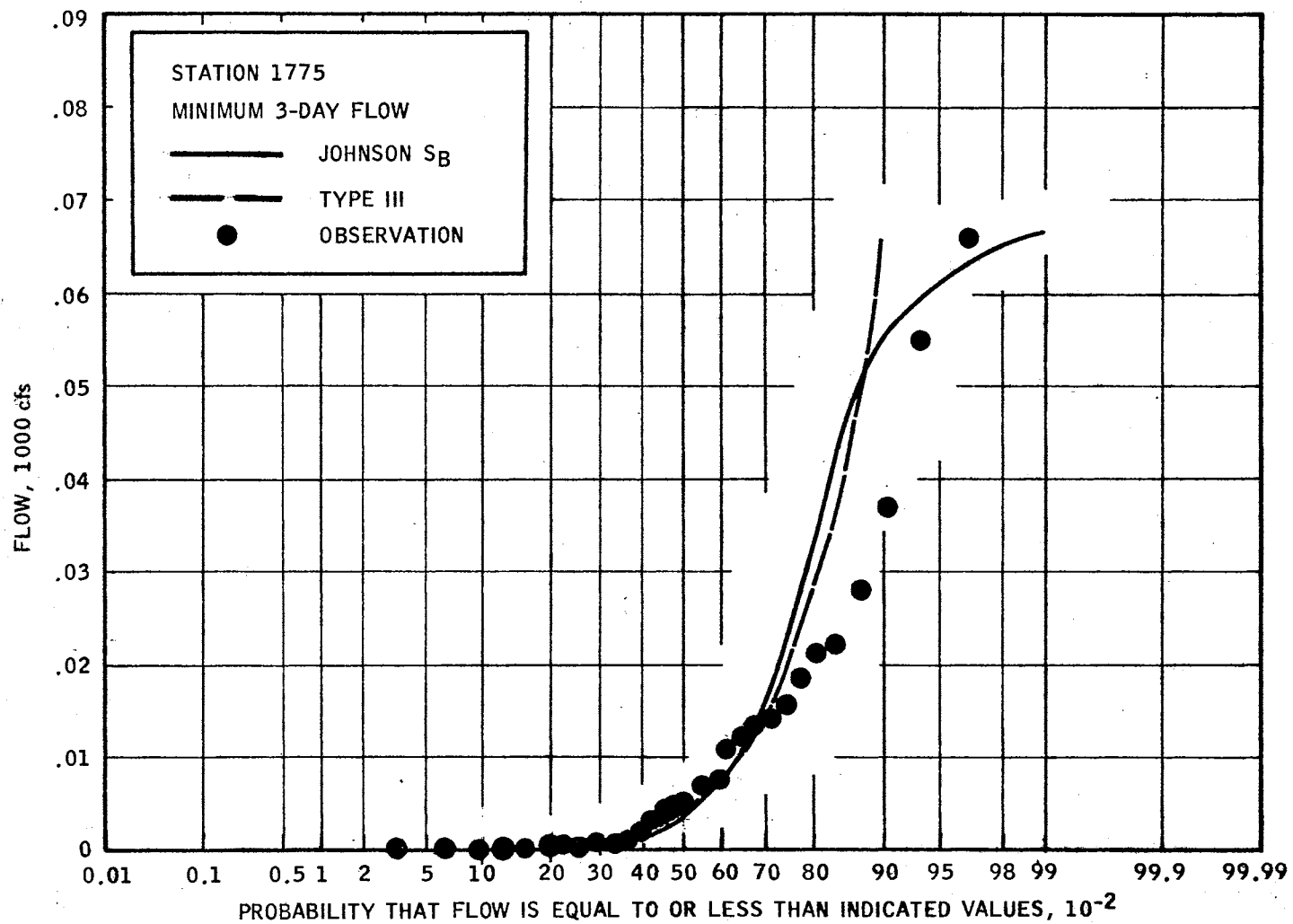


Figure 3. Cumulative Probability Distribution of Annual Minimum 3-Day Consecutive Flow at Station 1775

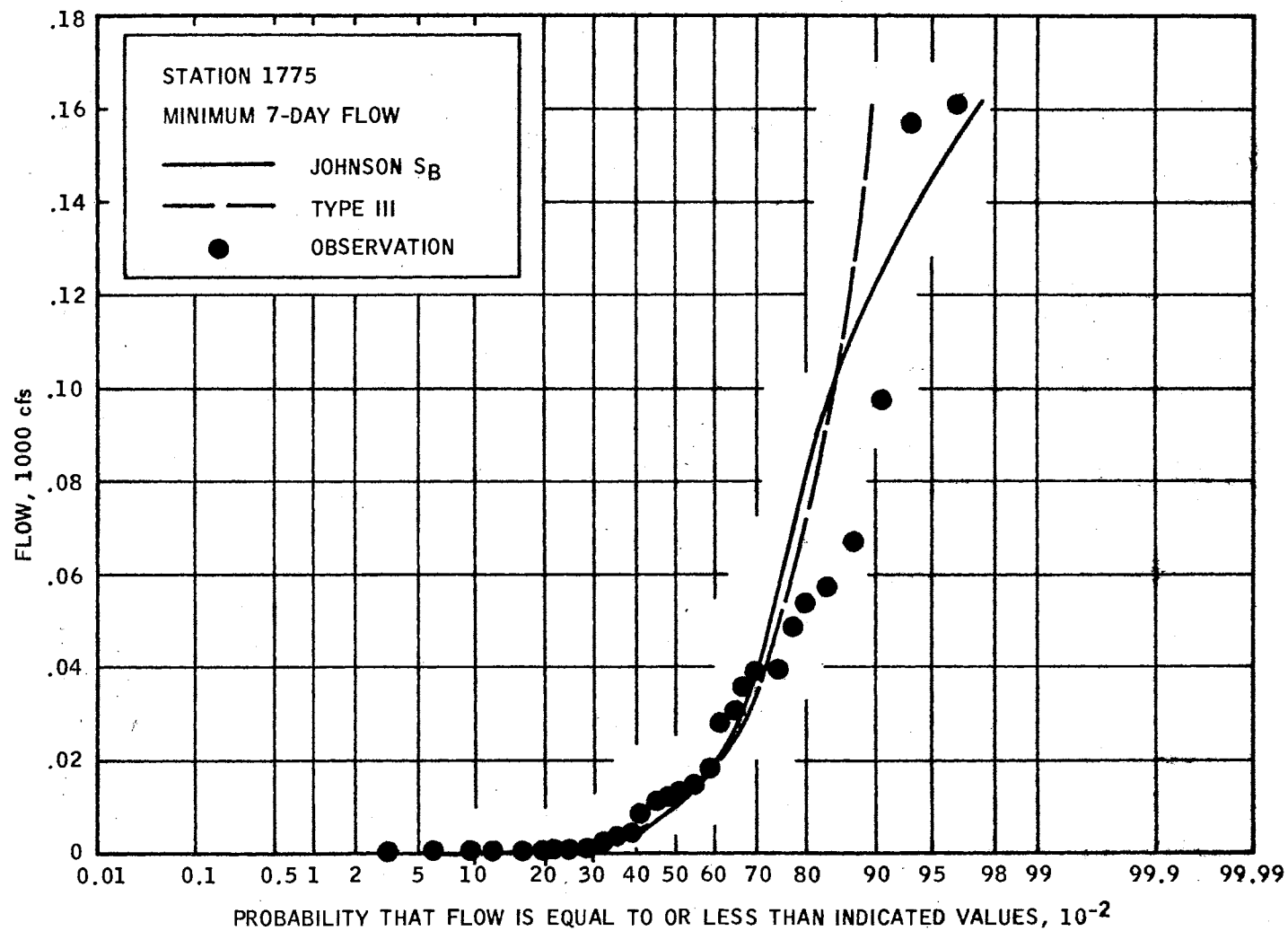


Figure 4. Cumulative Probability Distribution of Annual Minimum 7-Day Consecutive Flow at Station 1775

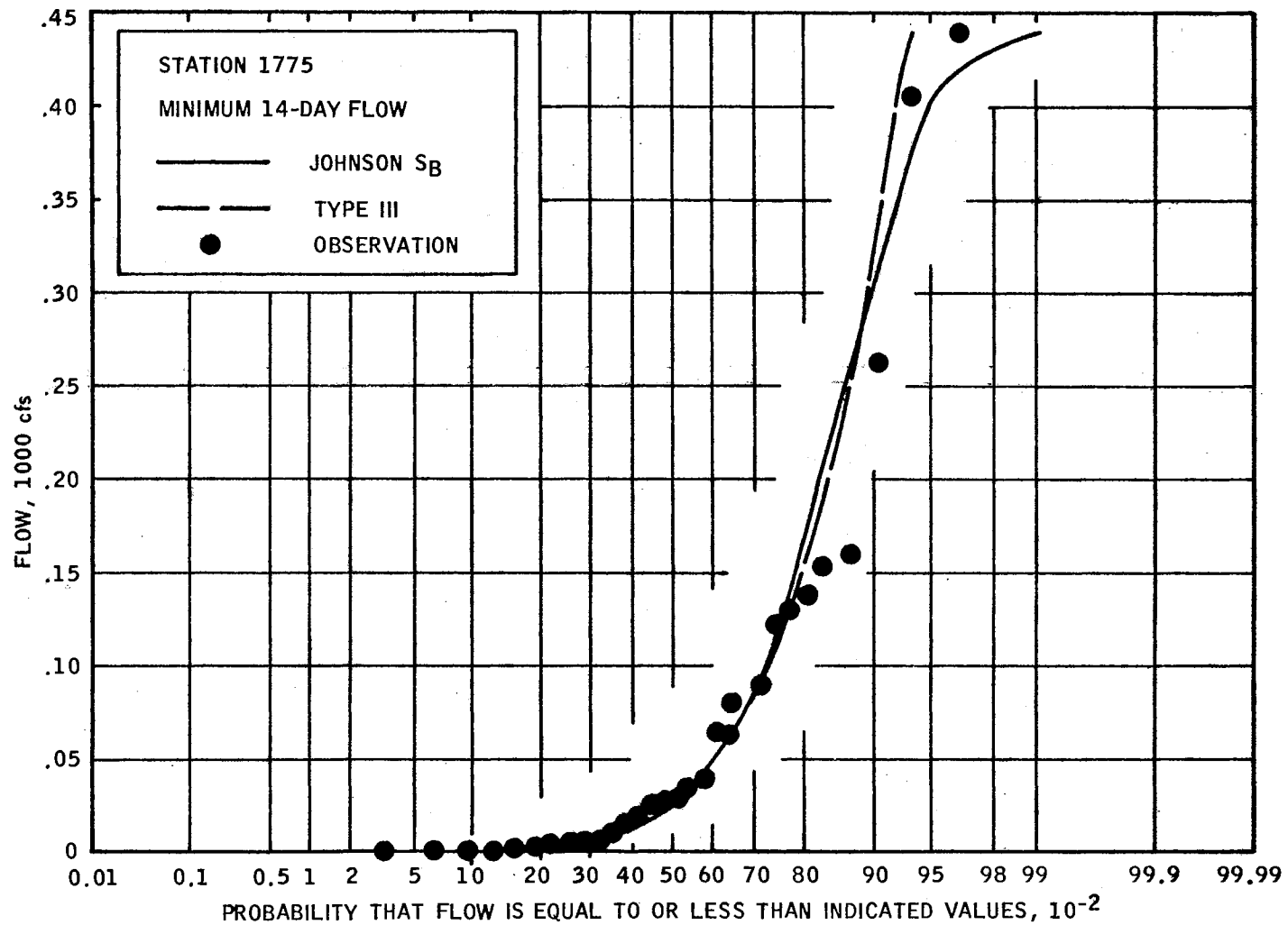


Figure 5. Cumulative Probability Distribution of Annual Minimum 14-Day Consecutive Flow at Station 1775

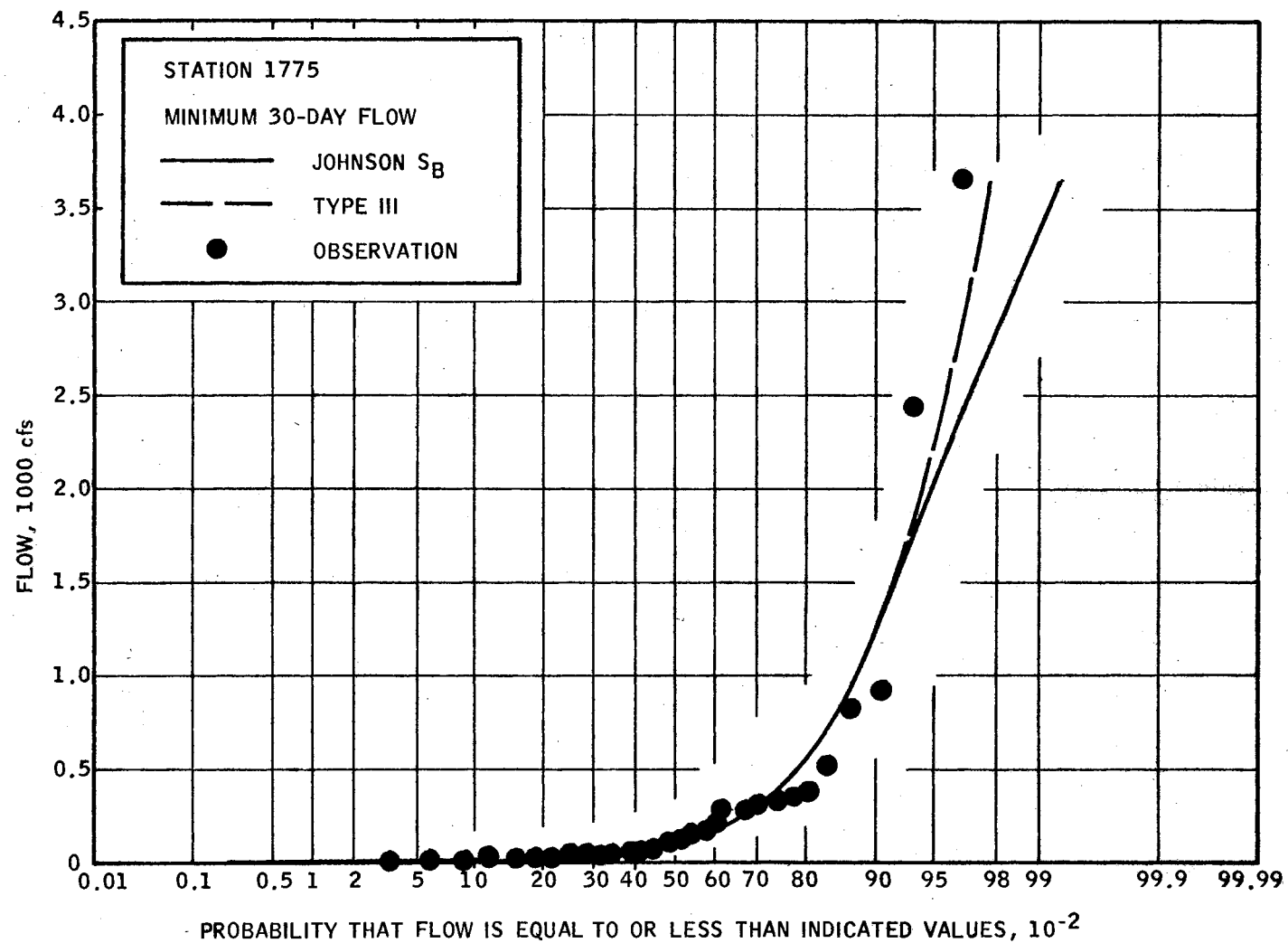


Figure 6. Cumulative Probability Distribution of Annual Minimum 30-Day Consecutive Flow at Station 1775

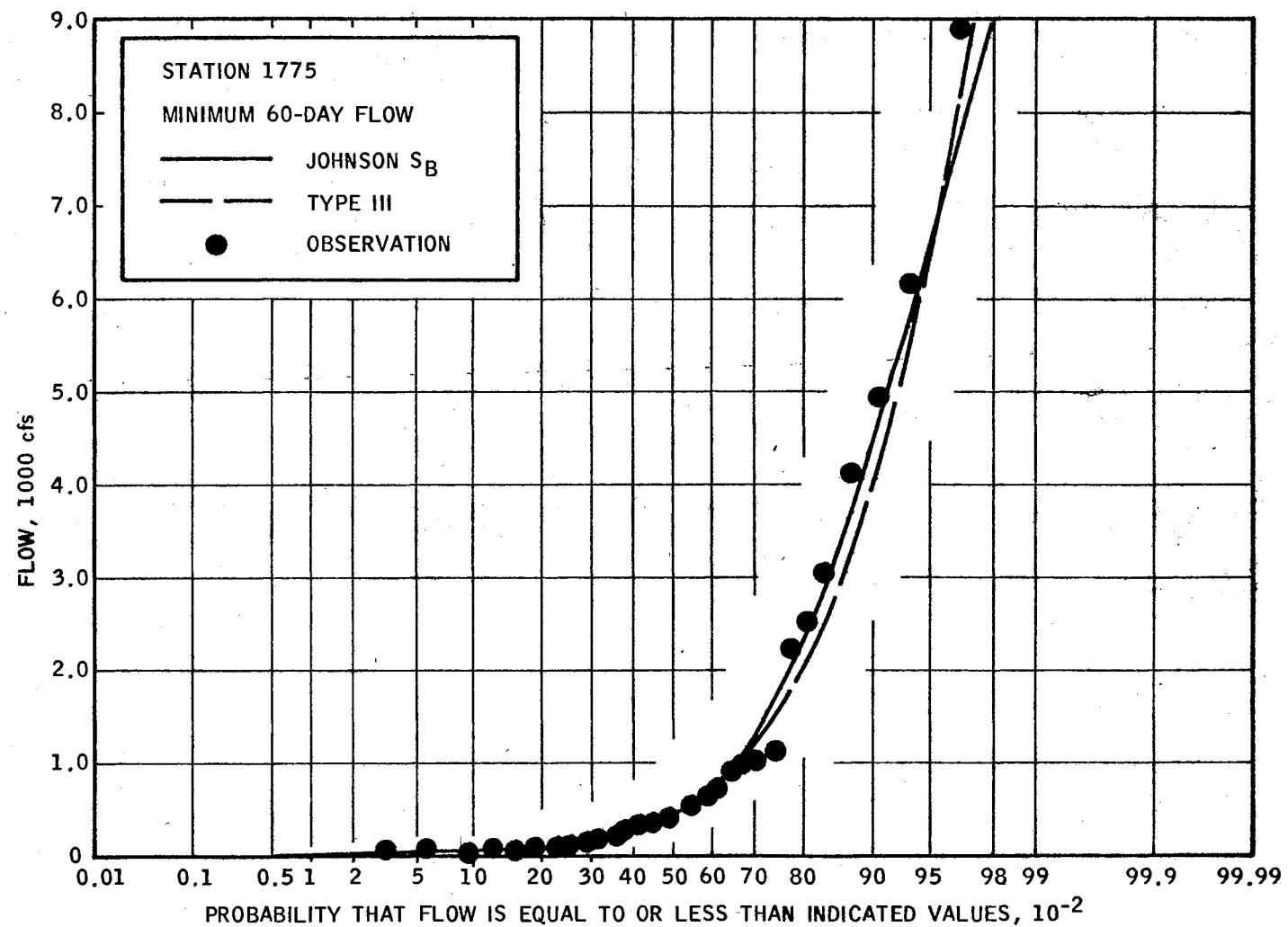


Figure 7. Cumulative Probability Distribution of Annual Minimum 60-Day Consecutive Flow at Station 1775

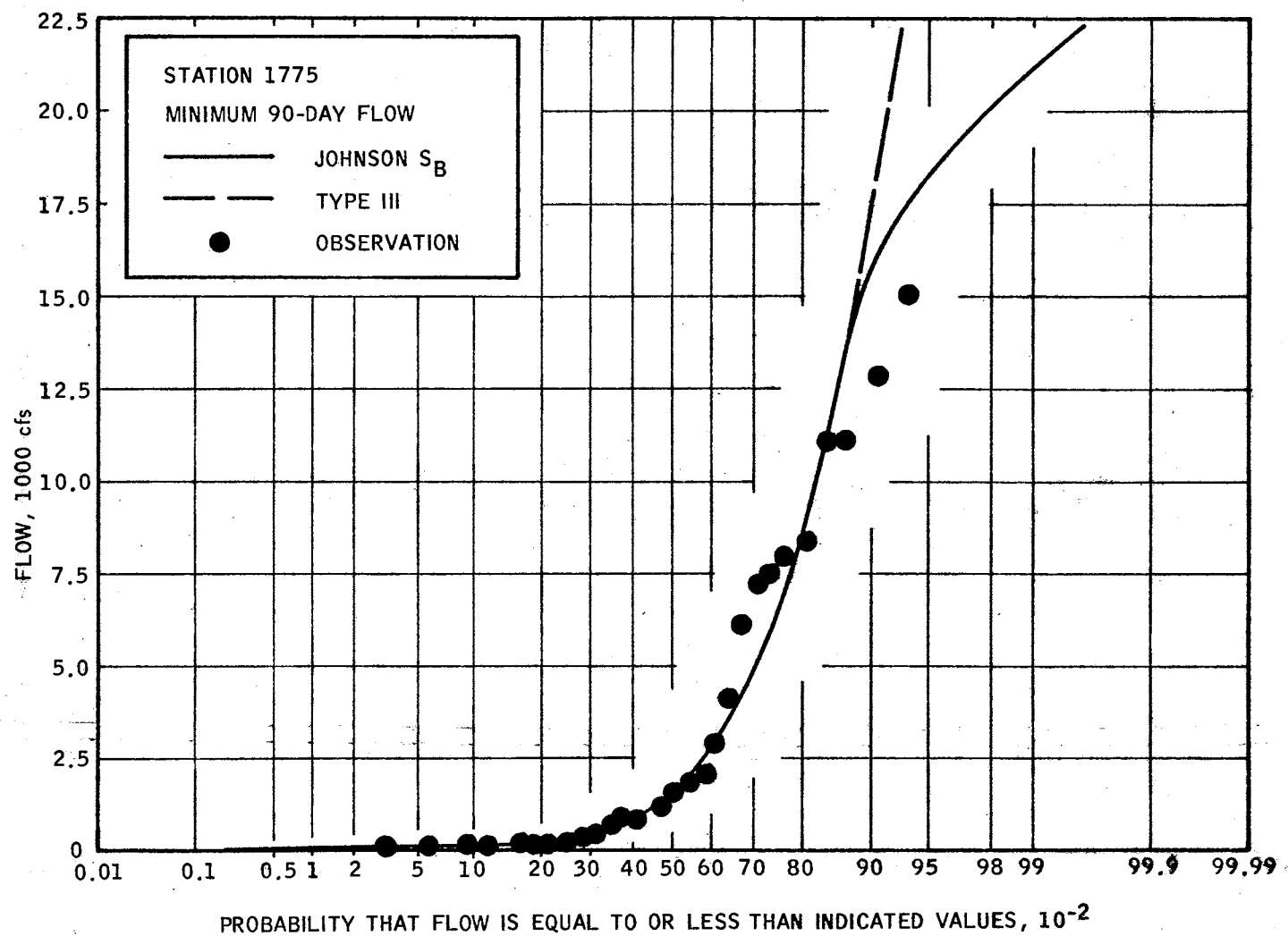


Figure 8. Cumulative Probability Distribution of Annual Minimum 90-Day Consecutive Flow at Station 1775

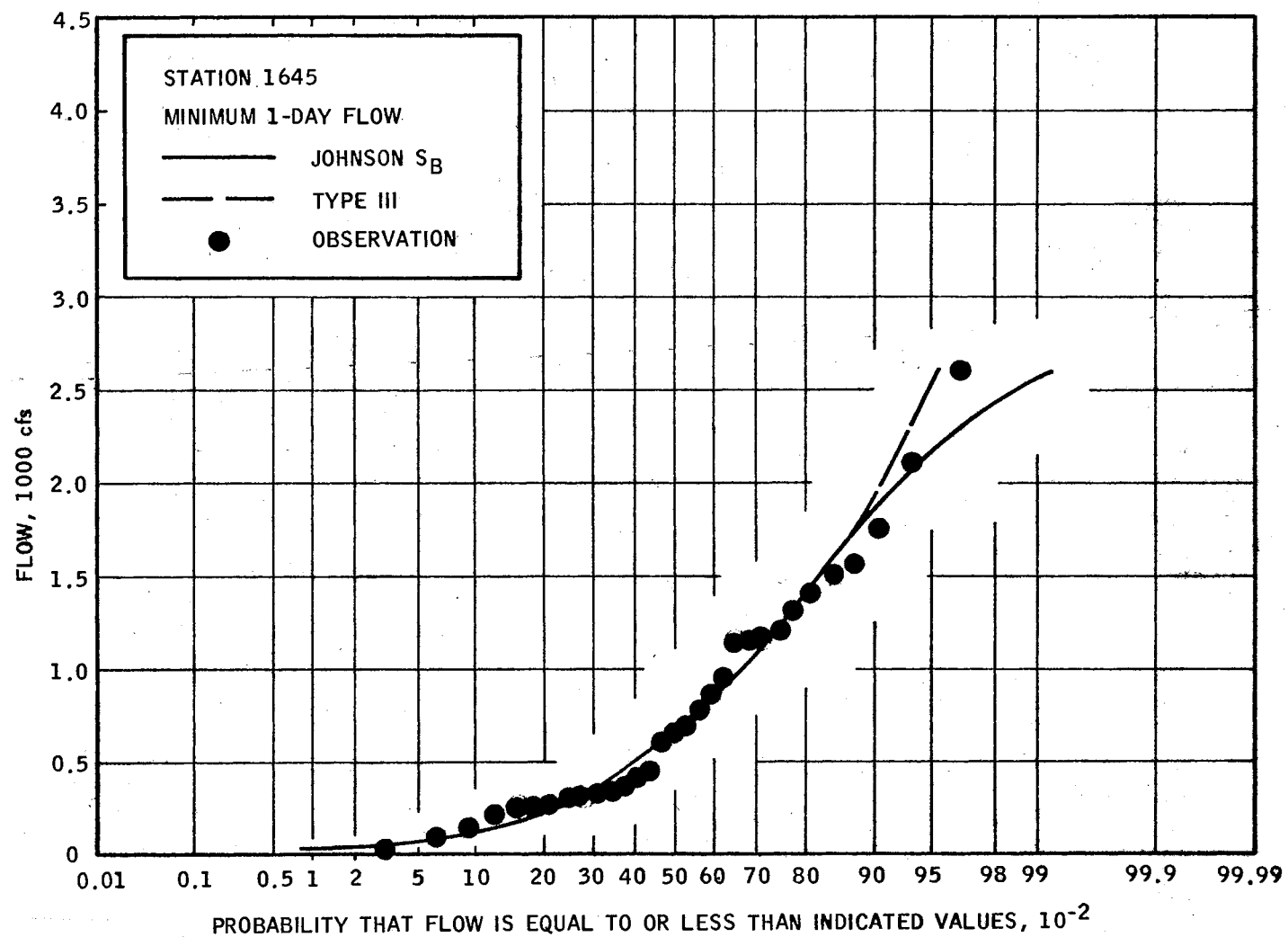


Figure 9. Cumulative Probability Distribution of Annual Minimum 1-Day Consecutive Flow at Station 1645

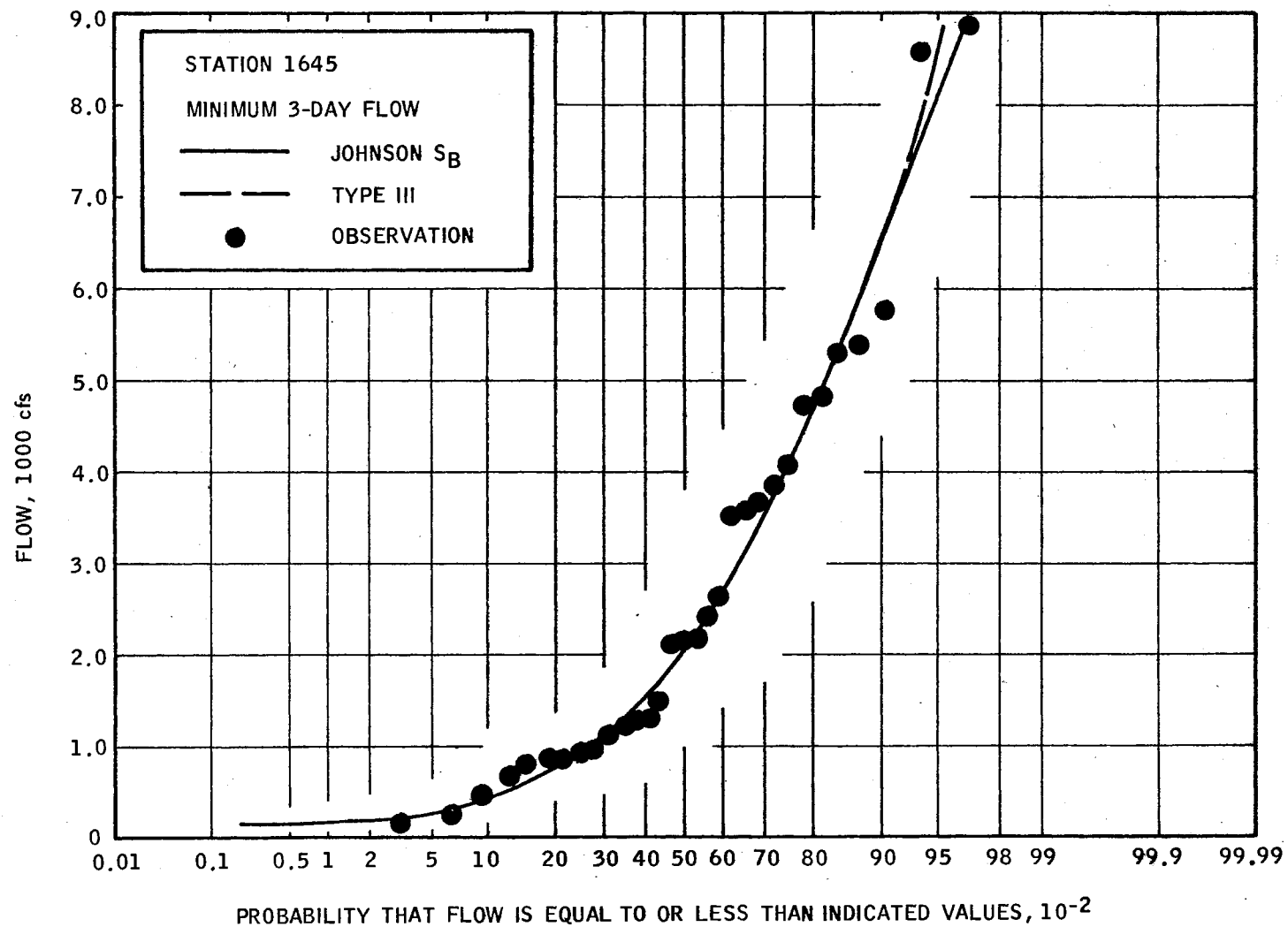


Figure 10. Cumulative Probability Distribution of Annual Minimum 3-Day Consecutive Flow at Station 1645

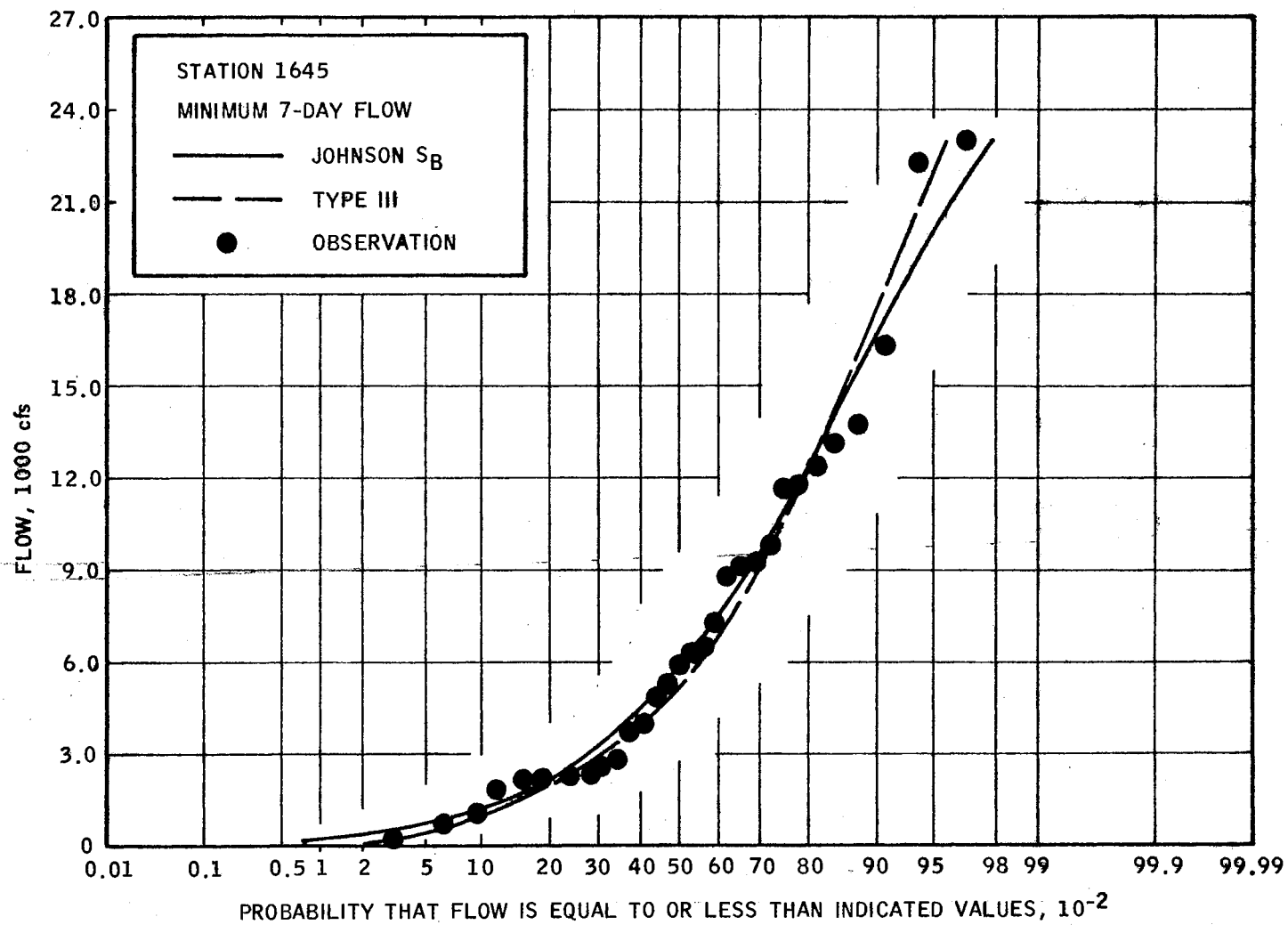


Figure 11. Cumulative Probability Distribution of Annual Minimum 7-Day Consecutive Flow at Station 1645

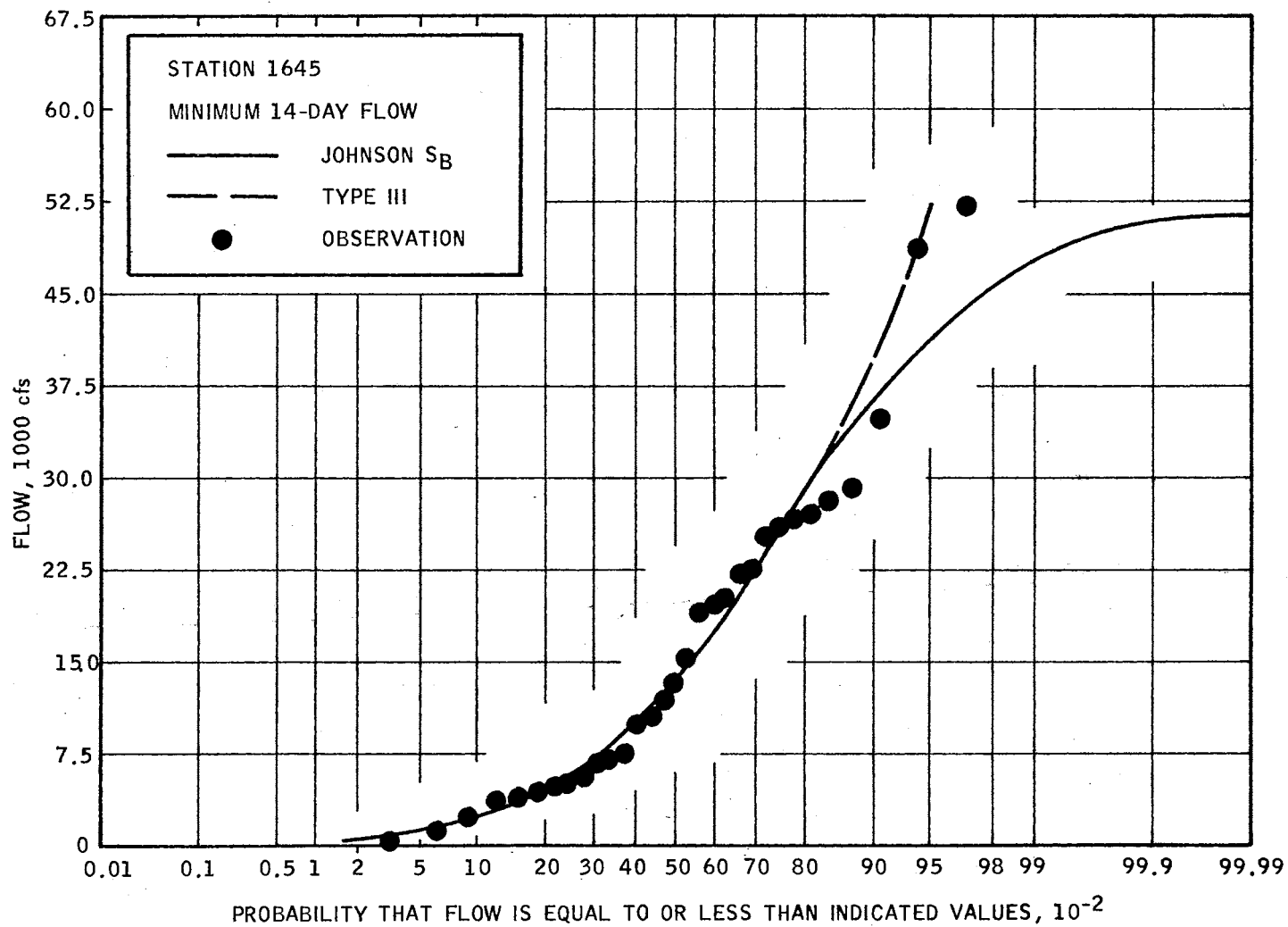


Figure 12. Cumulative Probability Distribution of Annual Minimum 14-Day Consecutive Flow at Station 1645

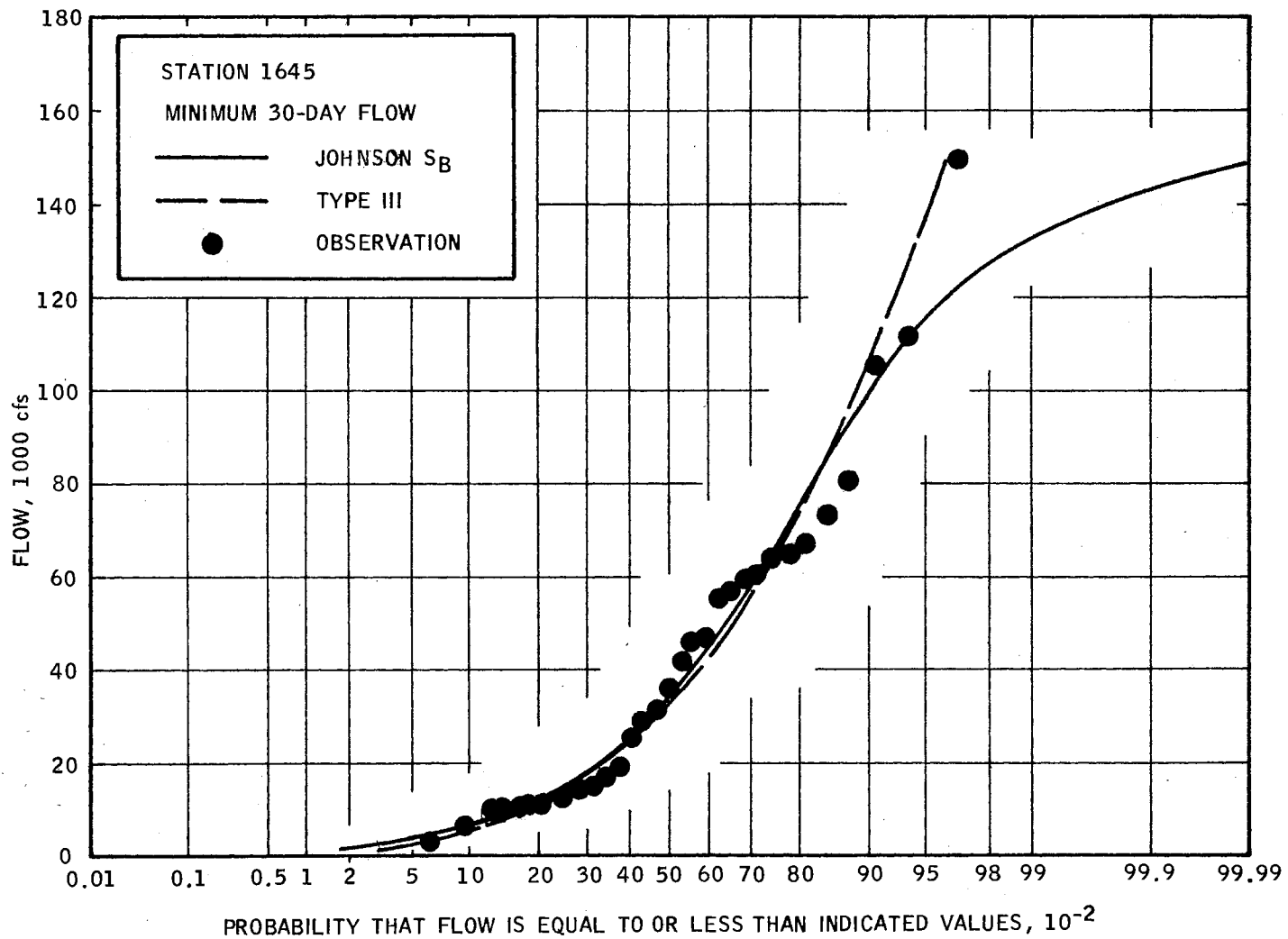


Figure 13. Cumulative Probability Distribution of Annual Minimum 30-Day Consecutive Flow at Station 1645

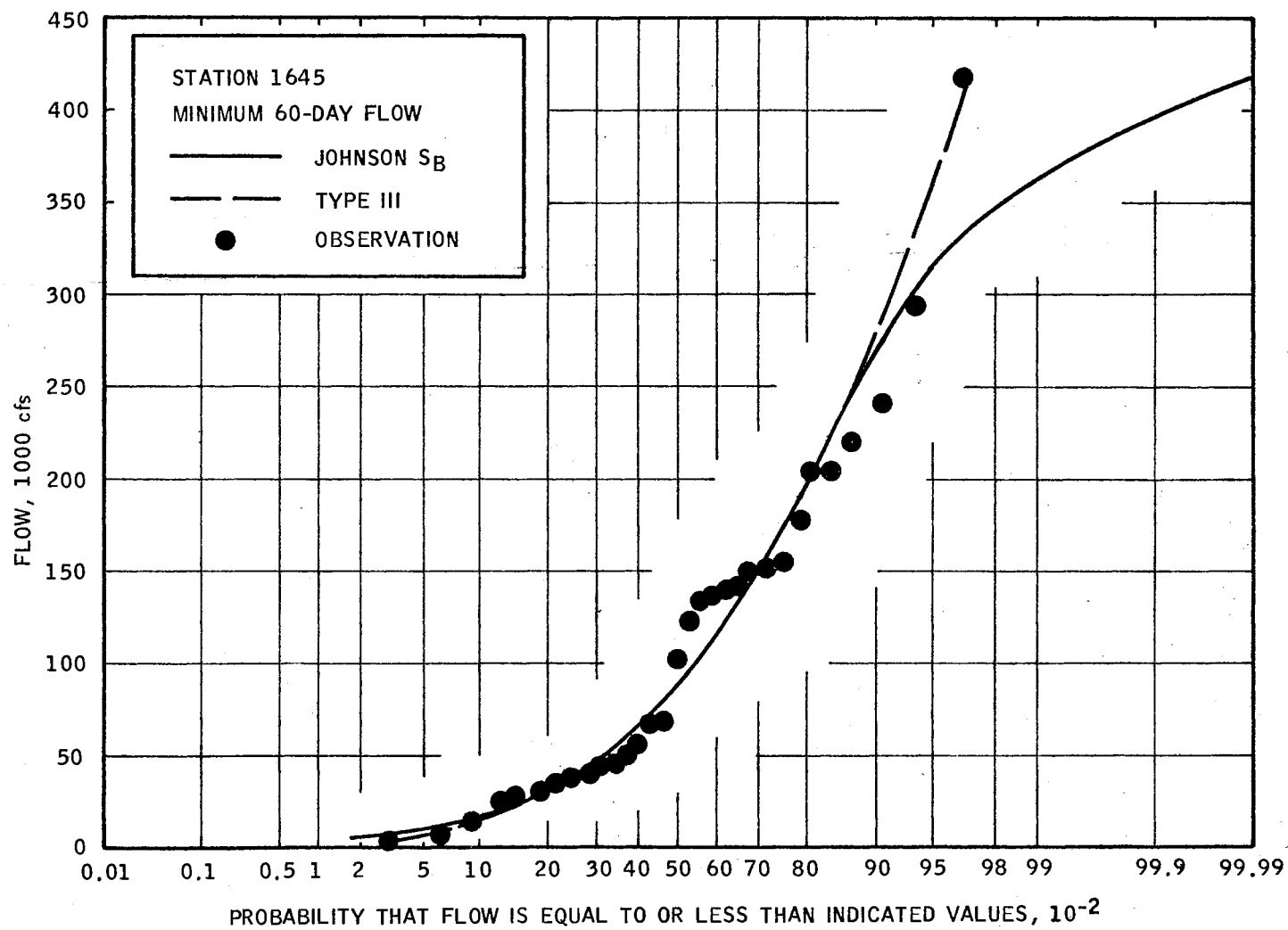


Figure 14. Cumulative Probability Distribution of Annual Minimum 60-Day Consecutive Flow at Station 1645

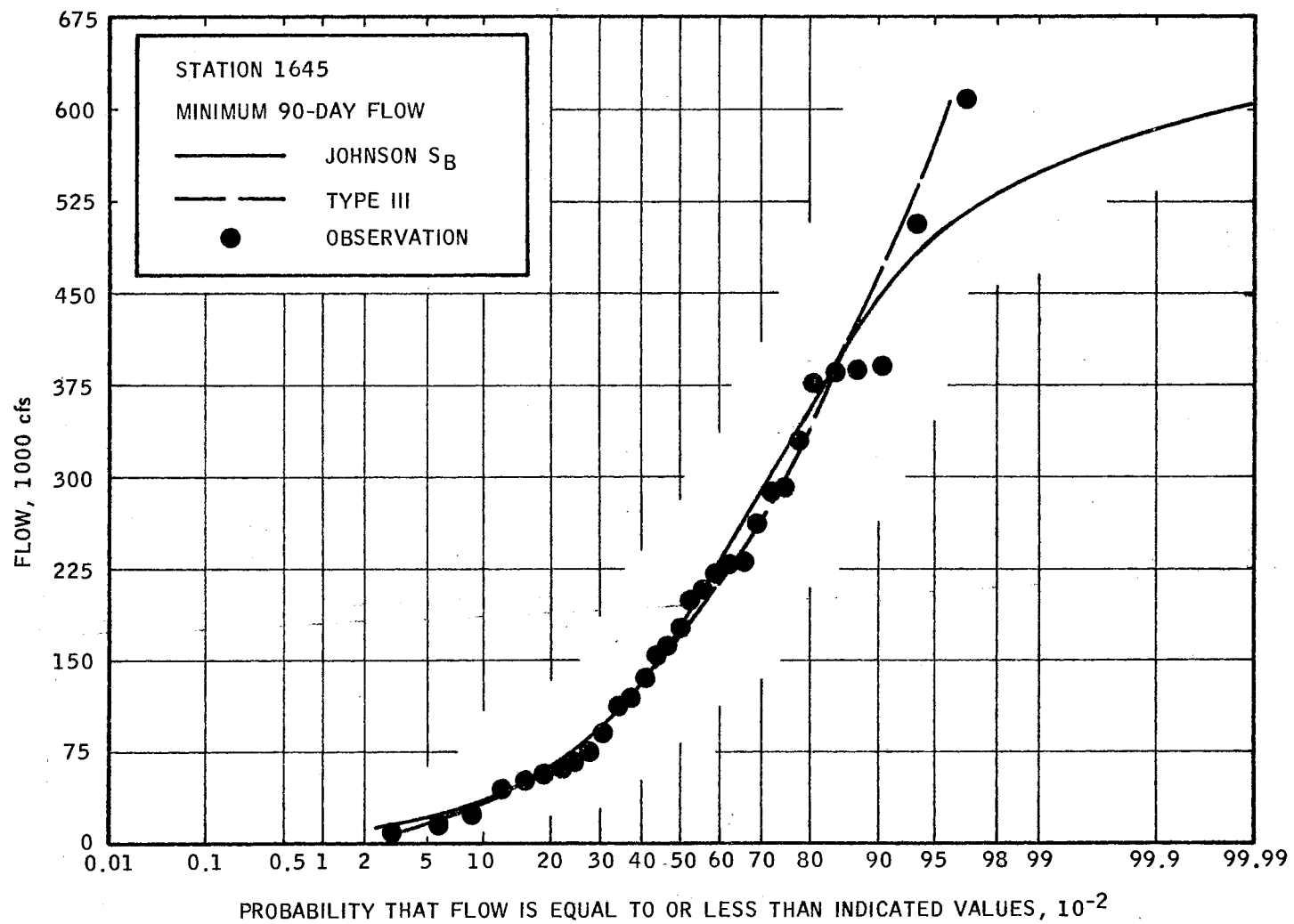


Figure 15. Cumulative Probability Distribution of Annual Minimum 90-Day Consecutive Flow at Station 1645

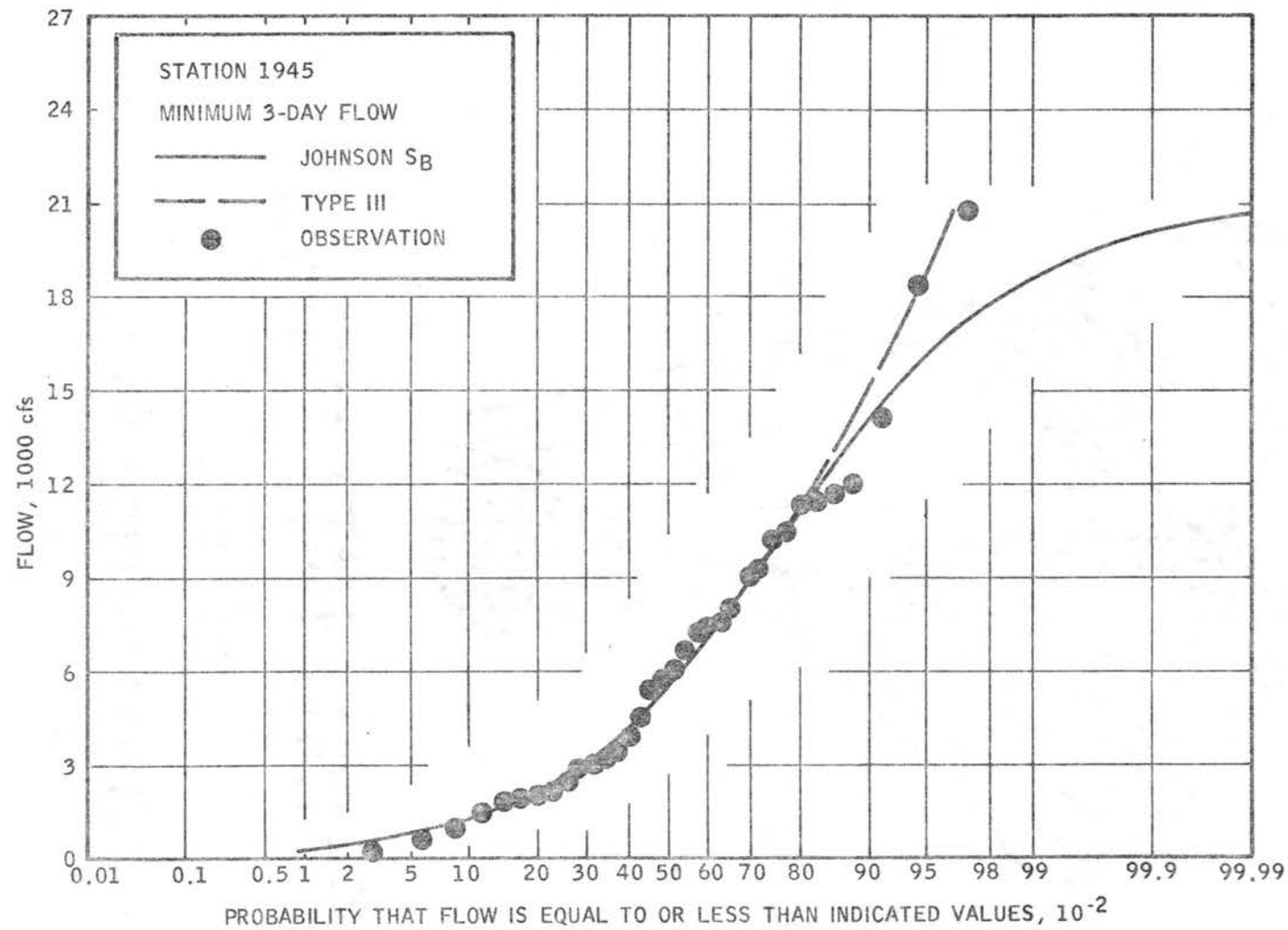


Figure 17. Cumulative Probability Distribution of Annual Minimum 3-Day Consecutive Flow at Station 1945

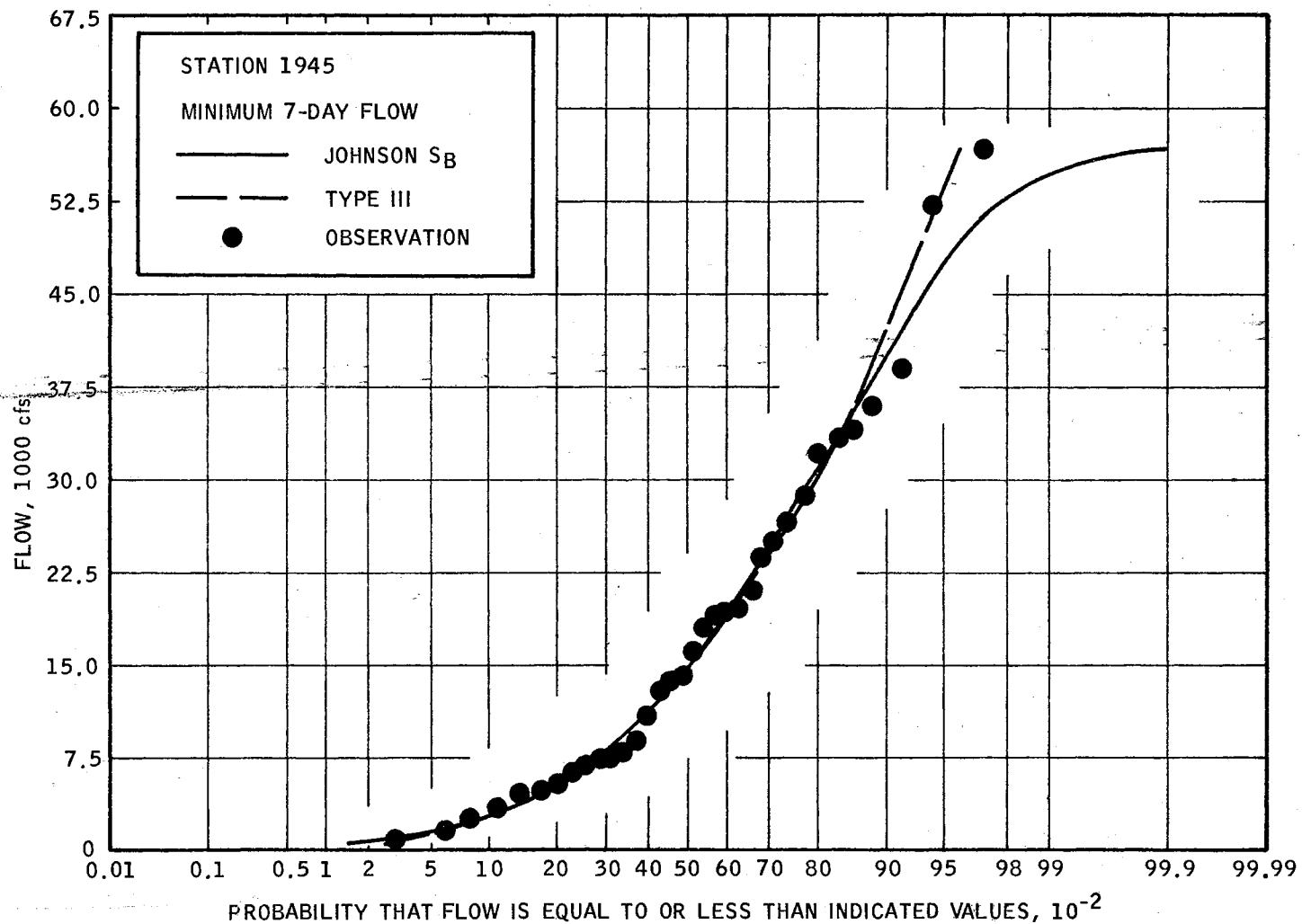


Figure 18. Cumulative Probability Distribution of Annual Minimum 7-Day Consecutive Flow at Station 1945

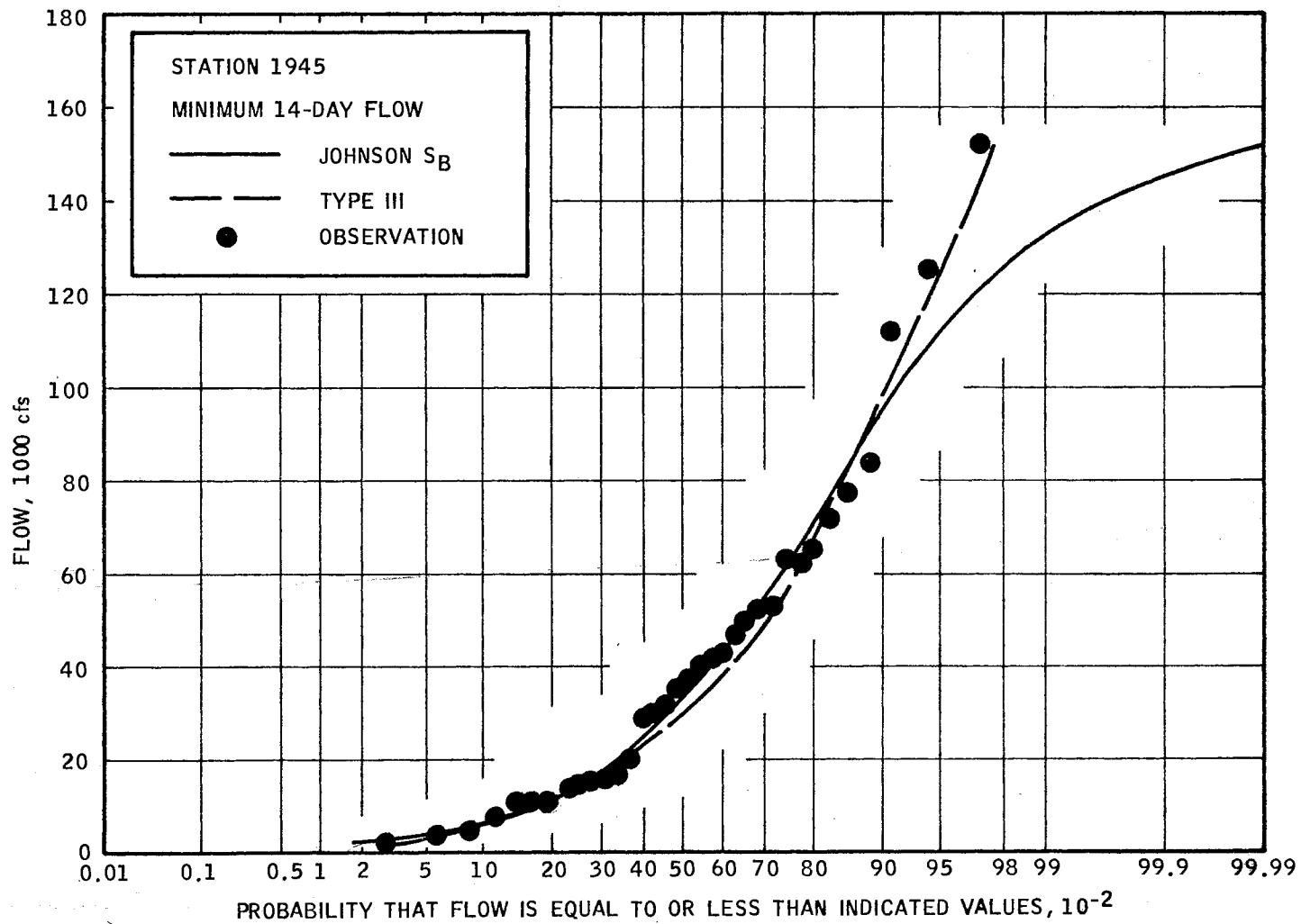


Figure 19. Cumulative Probability Distribution of Annual Minimum 14-Day Consecutive Flow at Station 1945

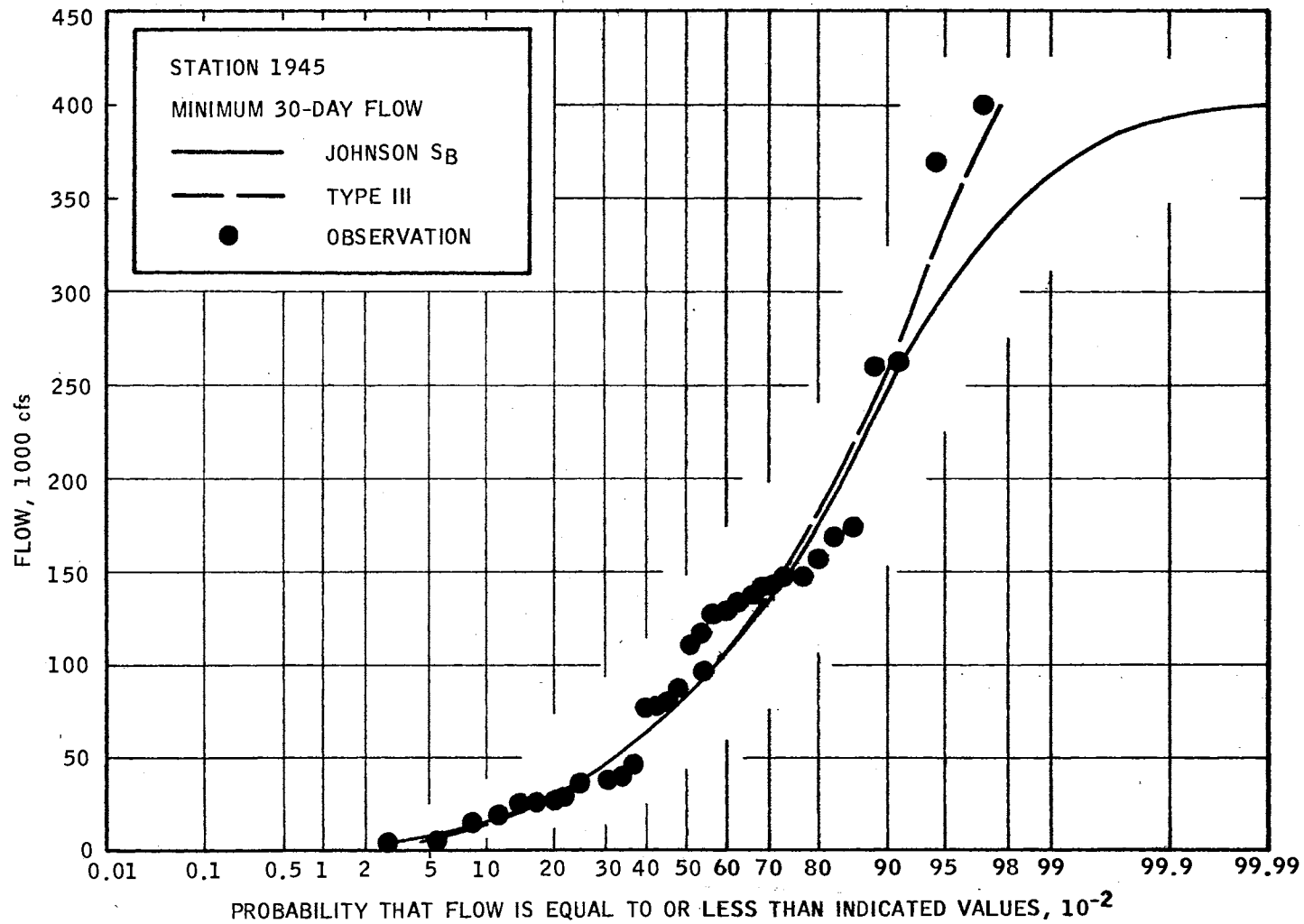
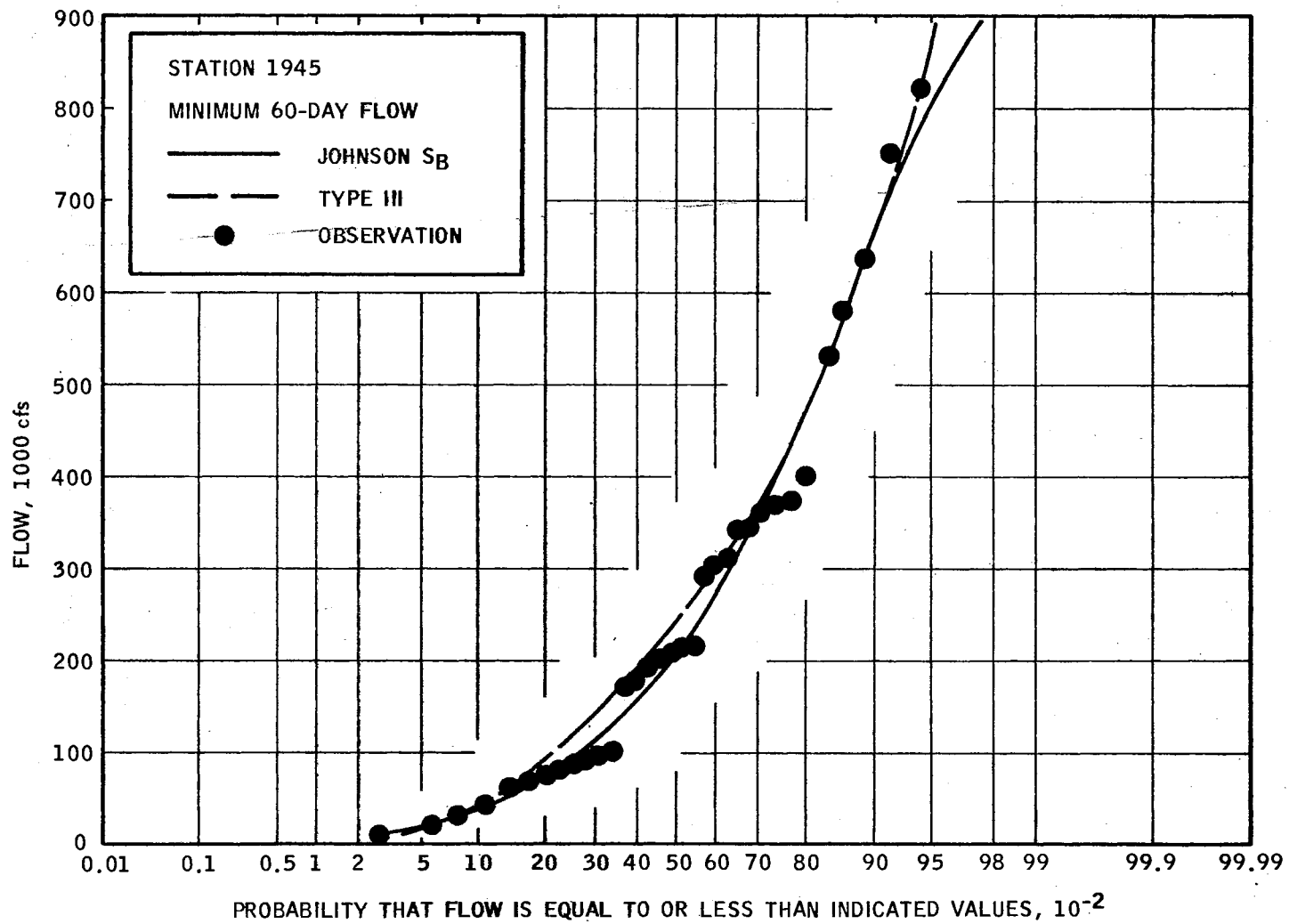


Figure 20. Cumulative Probability Distribution of Annual Minimum 30-Day Consecutive Flow at Station 1945



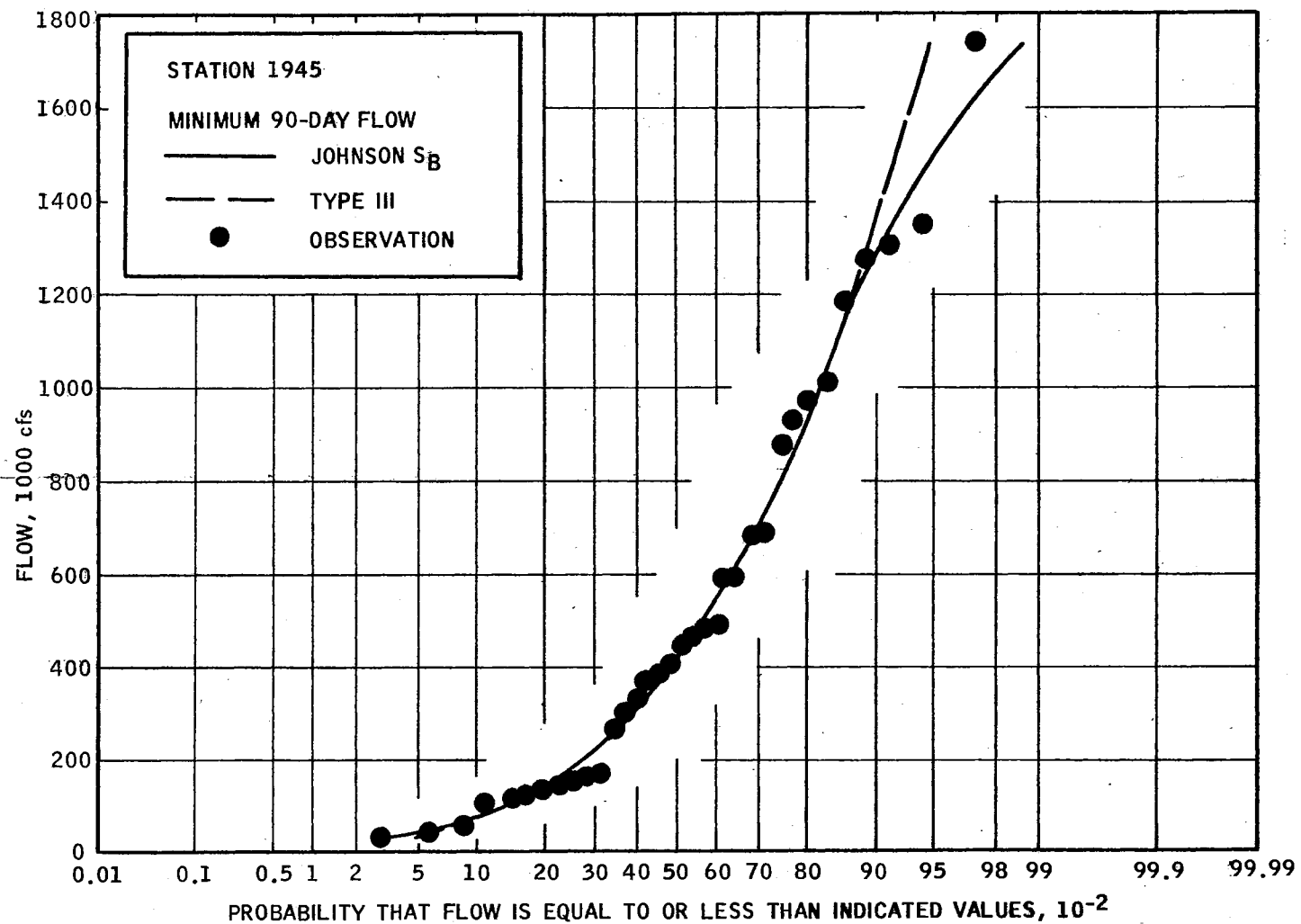


Figure 22. Cumulative Probability Distribution of Annual Minimum 90-Day Consecutive Flow at Station 1945

minimum flows with a specific number of consecutive days at each of the three stations investigated. Flows with 1, 3, 7, 14, 30, 60, and 90 consecutive days were included. The shapes of the two curves in each figure are different in their upper parts. Starting from the place where the cumulative probability is about 0.85 or more, the S_B curve is increasingly departing to the right from the Type III curve. The difference in the shapes of the upper parts of the two curves is attributed to the combined effect of two factors. First, the scale of the normal probability paper gradually increases from the middle to both ends of the paper. The scale in either the lower portion or the upper portion is much larger than that in the middle portion of the paper. Secondly, the upper limit of the Johnson S_B probability density function is at the near upper side of the largest observation of the drought flows while that of the Type III probability density function is infinite. The shapes of the Johnson S_B and the Type III curves are similar when they are plotted on an arithmetic paper. As typical examples, the curves in Figures 6, 13, and 20 for the minimum 30-day flows respectively at Stations 1775, 1645, and 1945 were also plotted on arithmetic paper as shown in Appendix C.

The maximum deviations from the use of Johnson S_B distribution and Type III distribution in fitting the drought flows are shown in Table VII. These maximum deviations served as the criteria in comparing the goodness-of-fits of the two distribution functions applied. Table VII shows that for Stations 1645 and 1945, the two stations having stream flow throughout the year, the S_B distribution provided better goodness-of-fit to the minimum flows than the Type III distribution. For Station 1775, a small flow station having a significant number of zero

TABLE VII

MAXIMUM DEVIATIONS FROM THE USE OF JOHNSON S_B
DISTRIBUTION AND TYPE III DISTRIBUTION
IN THE FITTING OF MINIMUM FLOWS

Station	Distribution	1-Day	3-Day	7-Day	14-Day	30-Day	60-Day	90-Day
1775	S_B	0.1351	0.1690	0.1730	0.1457	0.1262	0.1001	0.0651
	Type III	0.1251	0.1612	0.1553	0.1172	0.1026	0.0640	0.0638
1645	S_B	0.0757	0.0695	0.0699	0.0774	0.0655	0.0865	0.0632
	Type III	0.0763	0.0717	0.0721	0.0790	0.0726	0.0894	0.0581
1945	S_B	0.0550	0.0636	0.0487	0.0725	0.0857	0.0661	0.0740
	Type III	0.0651	0.0668	0.0558	0.0797	0.0863	0.0707	0.0811

daily flows each year, the Type III distribution had better curve fitting. All maximum deviations for Station 1945 and six out of seven maximum deviations for Station 1645 from the use of S_B distribution were smaller than those from the use of Type III distribution. On the contrary, for Station 1775, all maximum deviations from the S_B distribution were greater than those from the Type III distribution.

3. Design Flows from the Drought Flow Distributions

The design flows determined from the application of the Johnson S_B and the Type III distributions are shown in Table VIII and IX, respectively. Each value of design flow associated with a number of consecutive days and a probability of occurrence was calculated by equation (3.18) or (3.19). Estimated parameters of distribution functions in Tables V and VI were used. Numbers of consecutive days included in the determinations of design flows were 1, 3, 7, 14, 30, 60, and 90 days. The associated probabilities of occurrence included 0.01, 0.05, 0.1, 0.15, 0.2, and 0.25.

The design flows in Tables VIII and IX are shown in Figures 23 to 25. They are respectively the design flows at Stations 1775, 1645, and 1945. Each figure shows the magnitude of design flow versus the number of consecutive days of drought flow for various probabilities of occurrence, P 's. The number of consecutive days is shown on the logarithmic scale in the horizontal axis while the design flow is shown on the arithmetic scale in the vertical axis. Except for the cases where $P = 0.01$, the three figures show that for a specific probability of occurrence the magnitude of design flow increased as the number of consecutive days increased. The design flows determined from the S_B

TABLE VIII
DESIGN FLOWS FROM JOHNSON S_B
CUMULATIVE DISTRIBUTION*

Station 1775

Prob.	1-Day	3-Day	7-Day	14-Day	30-Day	60-Day	90-Day
0.01	0.00	0.00	0.00	0.00	0.03	0.05	0.04
0.05	0.00	0.00	0.01	0.03	0.11	0.21	0.24
0.10	0.01	0.01	0.02	0.07	0.24	0.46	0.62
0.15	0.03	0.02	0.04	0.13	0.40	0.79	1.18
0.20	0.05	0.05	0.08	0.22	0.61	1.21	1.96
0.25	0.09	0.09	0.14	0.34	0.87	1.74	3.03

Station 1645

Prob.	1-Day	3-Day	7-Day	14-Day	30-Day	60-Day	90-Day
0.01	31.02	37.66	39.91	35.00	43.88	72.25	86.34
0.05	79.76	91.57	100.88	98.31	120.06	173.50	230.89
0.10	130.76	146.13	164.06	168.69	203.46	283.32	385.29
0.15	181.69	199.78	226.90	241.42	289.04	395.49	540.52
0.20	234.35	254.83	291.79	318.35	379.27	513.48	701.17
0.25	290.85	313.71	361.40	402.23	477.59	641.87	873.03

Station 1945

Prob.	1-Day	3-Day	7-Day	14-Day	30-Day	60-Day	90-Day
0.01	93.91	97.76	96.94	111.53	121.80	158.80	228.09
0.05	233.62	247.01	253.80	287.09	318.79	409.18	504.72
0.10	374.90	400.28	419.55	471.08	528.22	672.98	817.69
0.15	512.20	550.98	585.65	655.02	739.74	938.35	1145.91
0.20	650.83	704.69	757.58	845.47	960.56	1214.93	1497.91
0.25	796.21	867.41	941.89	1050.05	1199.57	1514.20	1887.11

*The amount of each design flow in the body of table was determined by equation (3.18). The unit of flow is cfs.

TABLE IX

DESIGN FLOWS FROM TYPE III
CUMULATIVE DISTRIBUTION*Station 1775

Prob.	1-Day	3-Day	7-Day	14-Day	30-Day	60-Day	90-Day
0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.05	0.00	0.00	0.00	0.01	0.01	0.07	0.09
0.10	0.01	0.01	0.02	0.04	0.05	0.27	0.40
0.15	0.03	0.03	0.04	0.10	0.13	0.58	0.96
0.20	0.06	0.07	0.09	0.19	0.25	1.03	1.81
0.25	0.12	0.13	0.17	0.32	0.44	1.63	3.02

Station 1645

Prob.	1-Day	3-Day	7-Day	14-Day	30-Day	60-Day	90-Day
0.01	31.83	39.16	16.82	17.75	20.10	27.00	54.22
0.05	73.85	80.49	75.07	81.87	94.77	126.91	211.91
0.10	126.01	133.96	145.35	160.82	187.96	251.36	386.89
0.15	179.55	190.06	216.36	241.49	283.88	379.33	555.90
0.20	235.21	249.28	289.43	325.12	383.81	512.56	724.67
0.25	293.56	312.08	365.43	412.60	488.74	652.37	896.21

Station 1945

Prob.	1-Day	3-Day	7-Day	14-Day	30-Day	60-Day	90-Day
0.01	56.02	52.67	50.01	52.20	53.30	127.57	183.55
0.05	205.52	207.49	210.80	227.23	245.38	345.66	442.77
0.10	364.89	380.15	397.92	435.07	481.58	624.18	785.36
0.15	515.54	547.36	583.34	643.29	722.75	914.45	1149.15
0.20	663.83	714.62	771.69	856.33	972.65	1219.40	1536.14
0.25	812.91	884.87	965.64	1076.95	1233.96	1541.66	1949.06

*The amount of each design flow in the body of table was determined by equation (3.19). The unit of flow is cfs.

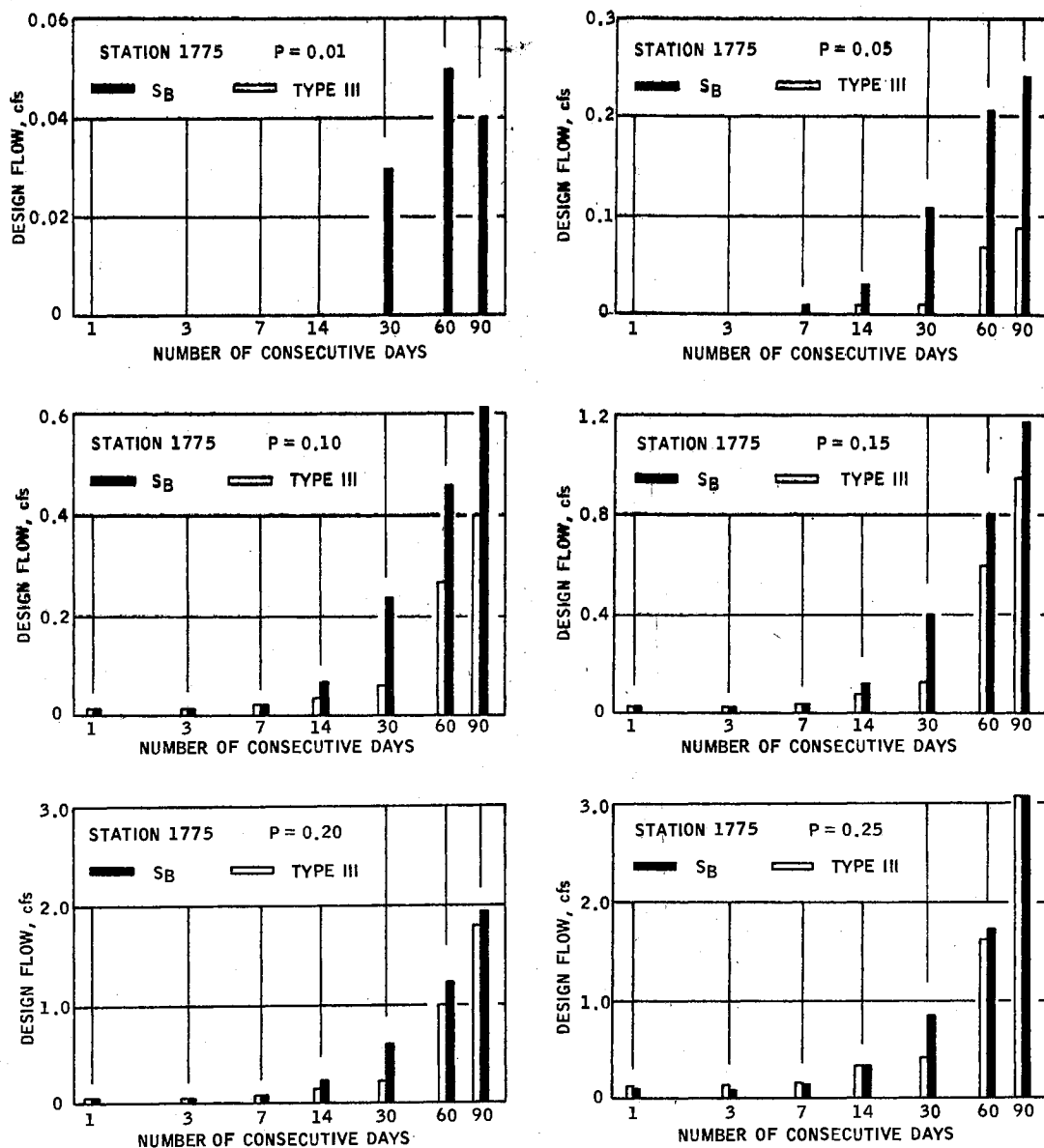


Figure 23. Design Flow vs. Number of Consecutive Days of Minimum Flow for Various Probabilities of Occurrence at Station 1775

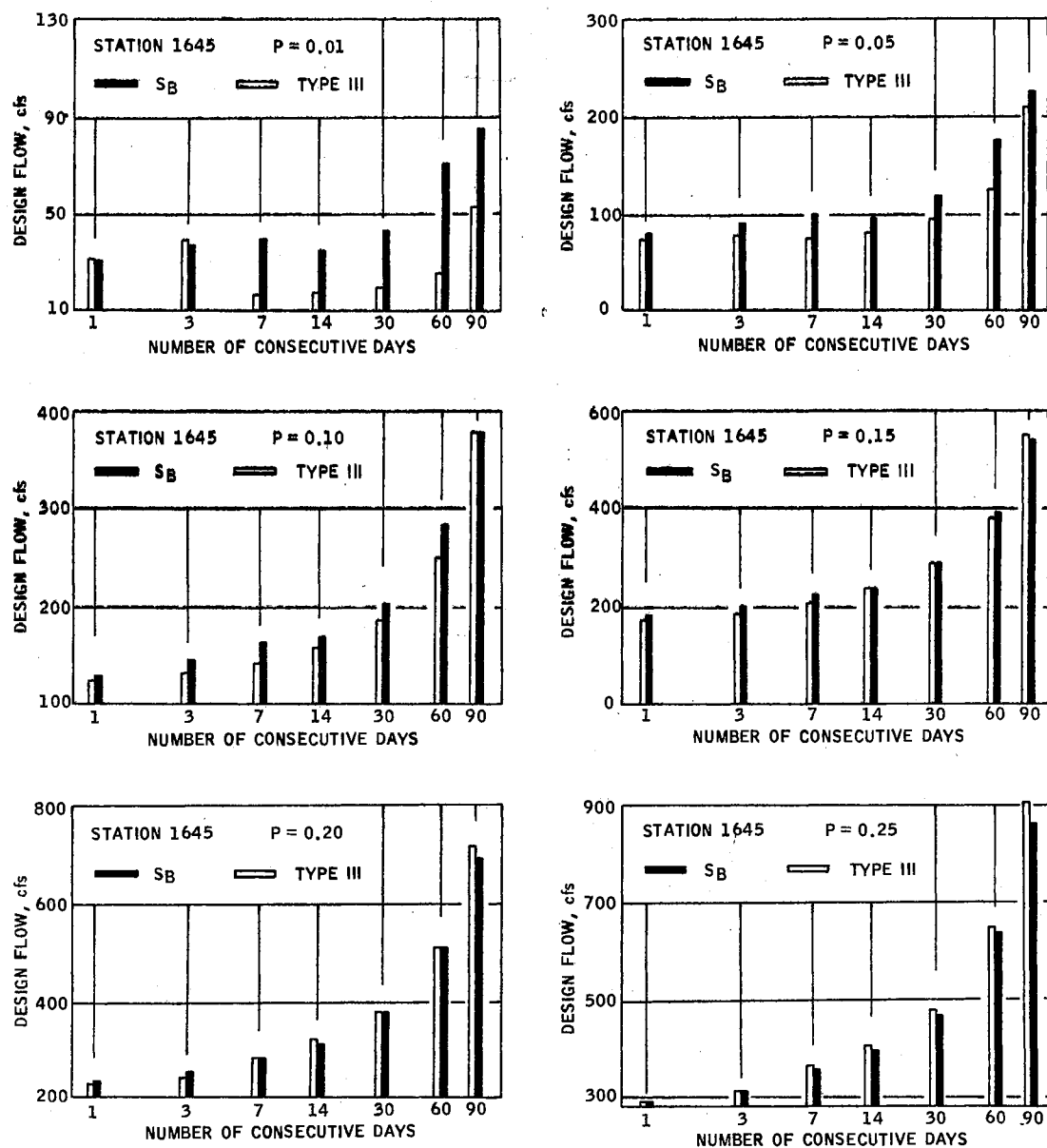


Figure 24. Design Flow vs. Number of Consecutive Days of Minimum Flow for Various Probabilities of Occurrence at Station 1645

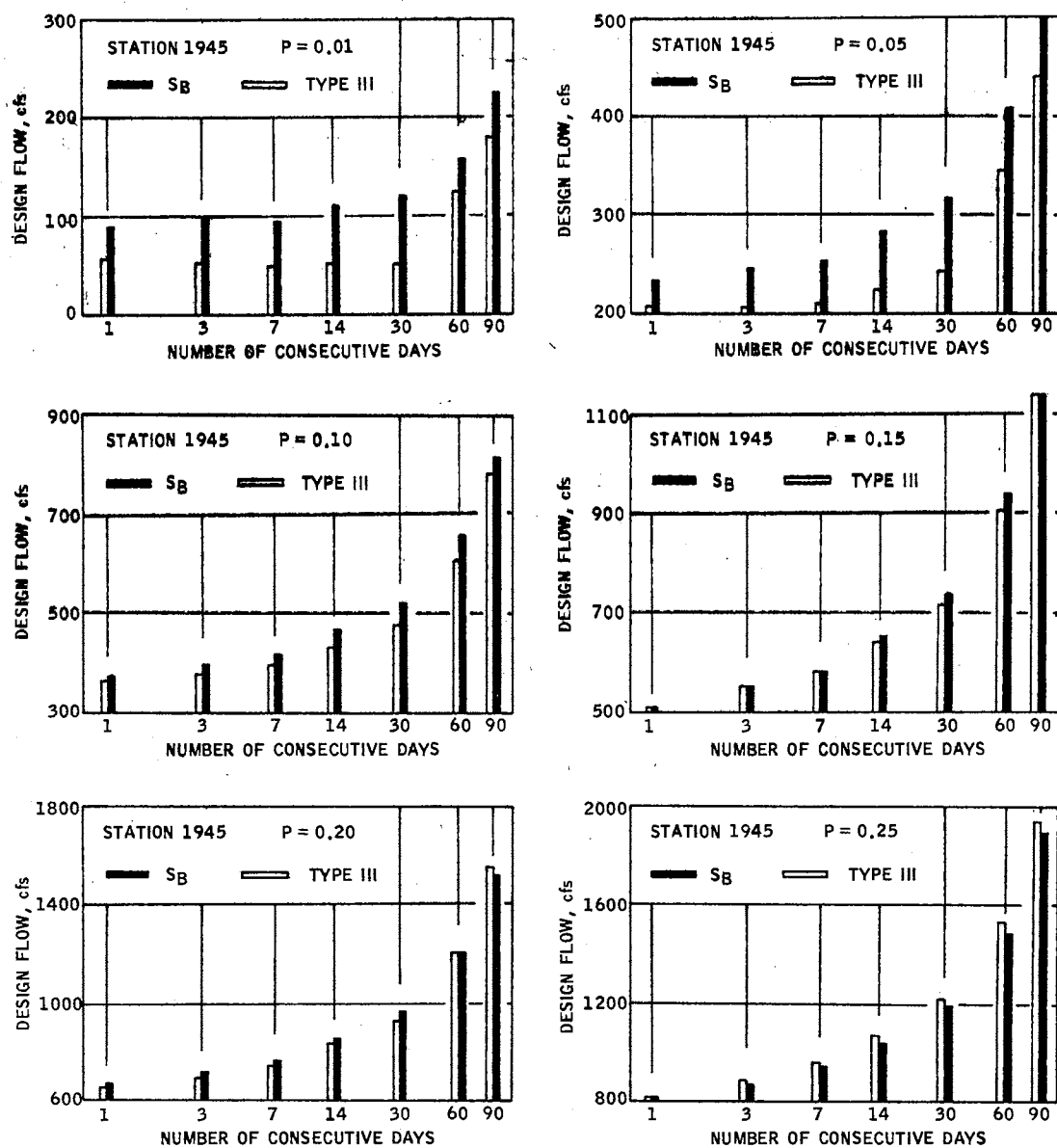


Figure 25. Design Flow vs. Number of Consecutive Days of Minimum Flow for Various Probabilities of Occurrence at Station 1945

distribution were more consistent in this aspect than those from the Type III distribution. This relation was particularly pronounced when the probability of occurrence was equal to or greater than 0.1.

B. A Hypothetical Study of Water Quality

Management

1. General Description and Spacing of D.O. Constraint

In order to evaluate the applicability of the zone-treatment methods described in Chapter III, a hypothetical river basin was assumed as shown in Figure 26. The river system consists of a main river and six major tributaries. Six sub-basins were defined according to their geographical locations. Discharge locations of seven municipal treatment plants, and seven industrial treatment plants are also indicated in the figure. Design stream flows, waste flows, influent BOD concentration, rates of deoxygenation and reaeration, and other related data are listed in Table X. A D.O. concentration of 4.0 mg/l was assumed in the effluents of the waste treatment plants. A uniform stream saturated D.O. concentration was also assumed equal to 9.0 mg/l.

A sketch of the river system is shown in Figure 27. Either the discharge locations of the wastes or the stream intersections were considered as the D.O. constraint points. At each constraint point the D.O. level should not be less than the stream D.O. standard. Distance in terms of flowing time between constraint points are also indicated in the figure. Each river section between two constraint points was counted as a reach. Within each reach, the design stream flow, the deoxygenation rate, and the reaeration rate were assumed constant. There are seventeen reaches in the river basin as shown in Figure 27.

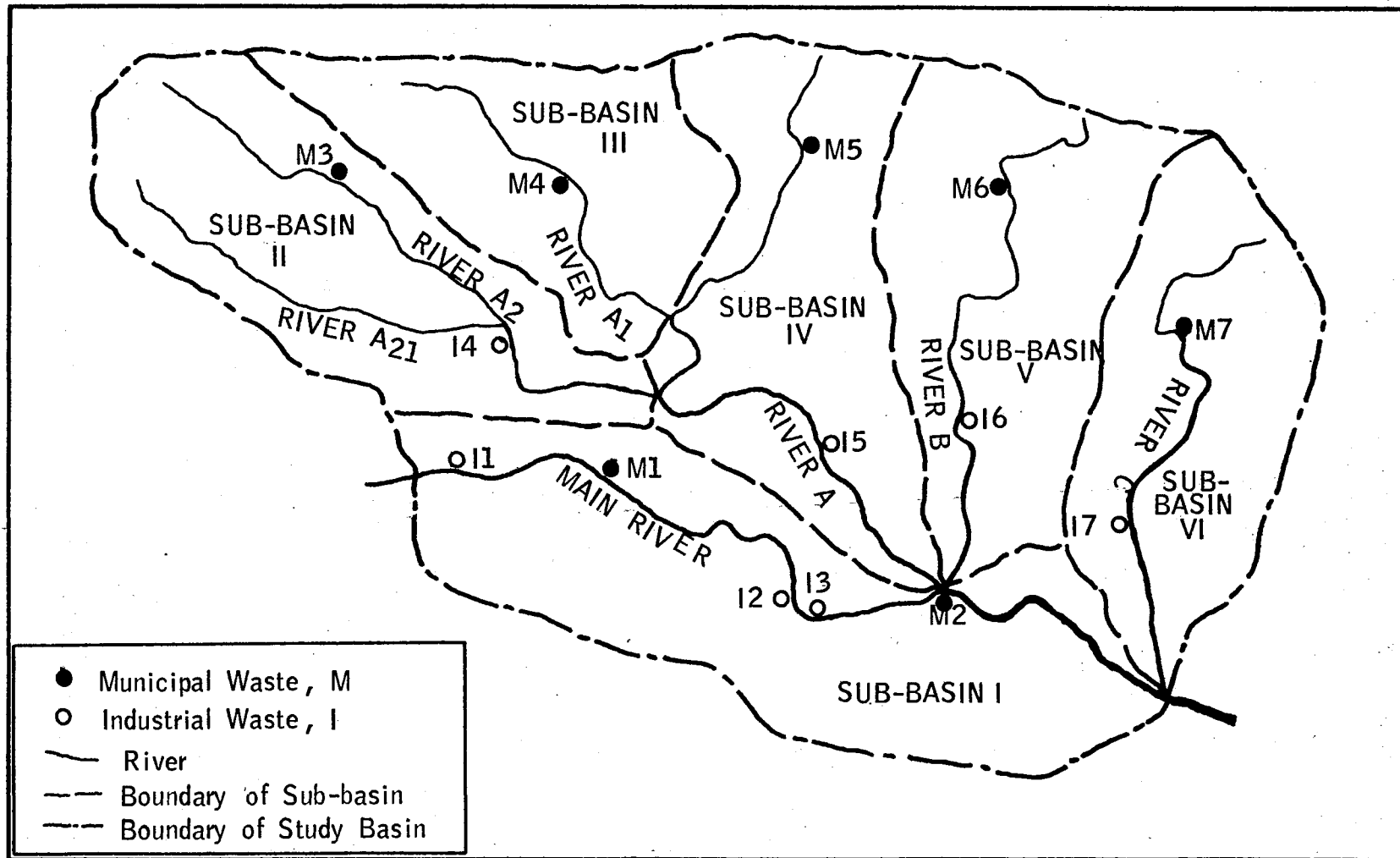


Figure 26. A Hypothetical River Basin

TABLE X
WATER QUALITY DATA OF HYPOTHETICAL RIVER BASIN

River	Items Location	Q, cfs		D.O., mg/l		BOD, mg/l		k_1 , day ⁻¹	k_2 , day ⁻¹
		Design Flow	Effluent	Stream	Effluent	Stream	Untreated Flow		
Main River	A - + (I1)	250 "	5	7.4 *	4.0	1.6 *	500	0.23 0.30	0.45 "
	B - + (M1)	" 260	35	* *	4.0	* *	380	" 0.28	" "
	C - + (I2)	" 275	6	* *	4.0	* *	650	" 0.32	" "
	D - + (I3)	" 280	3	* *	4.0	* *	975	" 0.34	" "
	E - + (M2)	" 450	12	* *	4.0	* *	240	" 0.31	" 0.48
	" + (River A)	"		* *		* *		" "	" "
	" + (River B)	"		* *		* *		" "	" "
	F - + (River C)	" 500		* *		* *		" 0.30	" "
River A2	G - + (M3)	10 "	8	7.5 *	4.0	1.0 *	220	0.24 0.26	0.35 "
	H - + (River A21)	" 20	5	* *	8.0	* *	1.0	" 0.23	" "
	I - + (I4)	" "	2	* *	4.0	* *	550	" 0.29	" "
	M -	"		*		*		"	"
River A1	J - + (M4)	25 "	7	7.6 *	4.0	0.9 *	270	0.20 0.25	0.36 "
	L -	30		*		*		"	"
River A	K - + (M5)	28 "	10	8.0 *	4.0	1.2 *	300	0.24 0.25	0.40 "
	L - + (River A1)	" 60		* *		* *		" "	" "
	M - + (River A2)	" 80		* *		* *		" 0.27	" 0.45
	N - + (I5)	" 84	4.2	* *	4.0	* *	1040	" 0.31	" "
	E -	"		*		*		"	"
River B	O - + (M6)	34 "	7.5	7.8 *	4.0	2.1 *	350	0.20 0.22	0.50 "
	P - + (I6)	" 50	3.6	* *	4.0	* *	700	" 0.27	" "
	E -	"		*		*		"	"
River C	R - + (M7)	32 "	7.5	7.7 *	4.0	1.5 *	275	0.21 0.23	0.46 "
	S - + (I7)	" 46	2.5	* *	4.0	* *	865	" 0.31	" "
	F -	"		*		*		"	"

- : IMMEDIATE UPSTREAM SIDE
+ : POINT OF INFLOW

* : VALUE NEEDED TO BE DETERMINED
" : SAME AS THE ABOVE VALUE

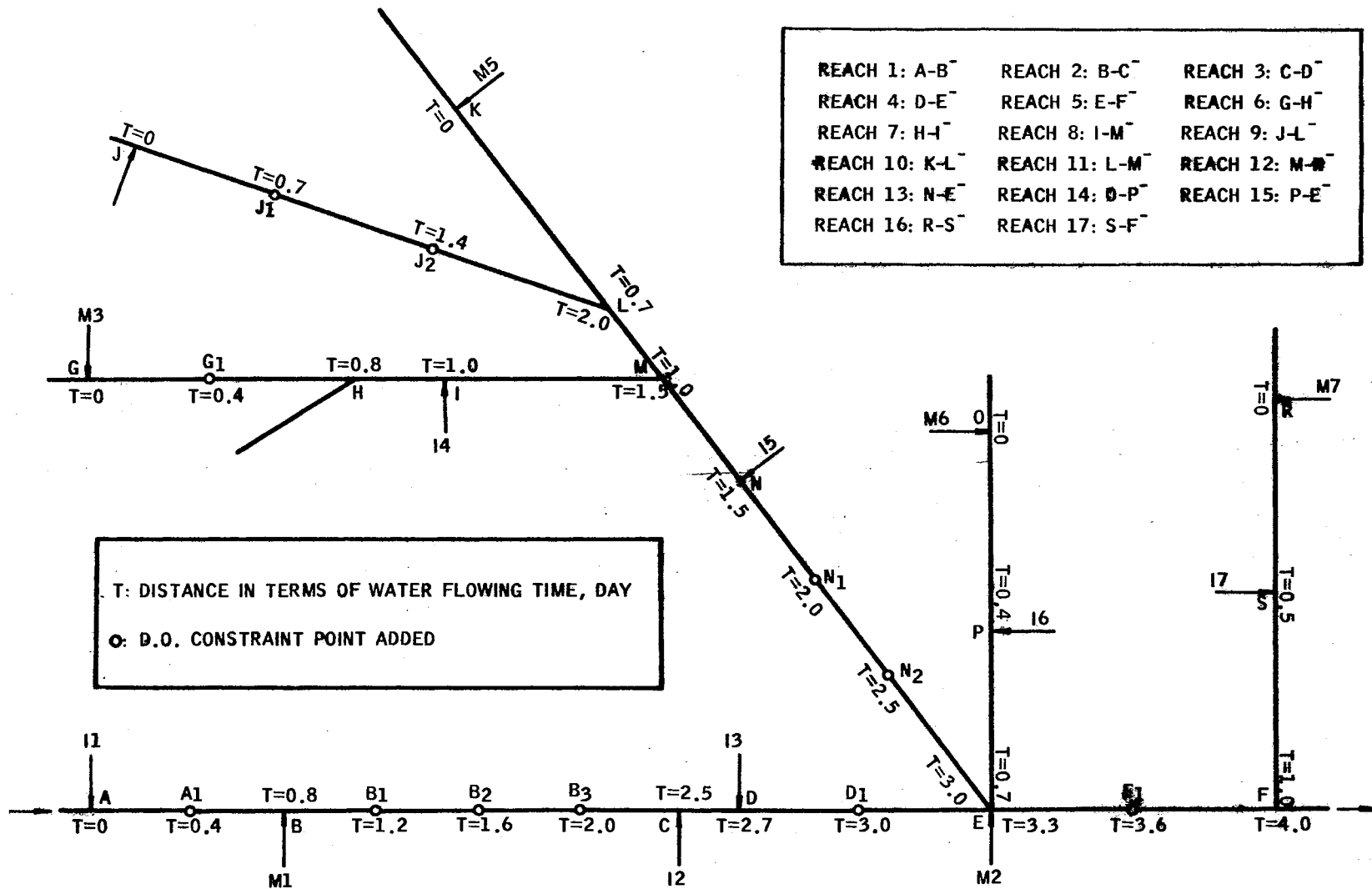


Figure 27. Layout of Stream System

In order to examine the possibility of the violation of the D.O. standards between two adjacent constraint points, the spacings of D.O. constraint points and their corresponding maximum D.O. violations in each river reach were calculated by the use of equations (3.27) and (3.28) in Chapter III. The maximum possible violations of D.O. standards and their corresponding spacings of D.O. constraints are listed in Table XI. The maximum allowable violation of D.O. standard was specified as 0.025 mg/l for each reach. One or more of the D.O. constraint points were added in each reach where it was necessary. The additional constraint points are also shown in Figure 27.

2. Optimal Solutions of Water Quality Management Programs

(1) General. From the data in Table X a basinwide water quality system in terms of the relationship between BOD and D.O. was formulated into a linear structure by the use of equations (3.23) and (3.24). Details of the system formulation are listed in Appendix D.

As described in Chapter III, the total stream assimilative capacity was the objective to be optimized in the management program of water quality. For the convenience of using the computer program of IBM/360 (8) to solve the linear programming problem, this objective function was arranged into a form as the total pounds of BOD removed per day from all treatment plants in the basin. The minimization of the total pounds of BOD removal is equivalent to the maximization of the total stream assimilative capacity. From the data in Table X the objective function is then given by

TABLE XI
MINIMUM D.O. VIOLATIONS VS. SPACINGS OF D.O. CONSTRAINTS FOR EACH RIVER REACH

<u>REACH NO. 1 (A - B)</u>		<u>REACH NO. 6 (G - H)</u>		<u>REACH NO. 11 (L - M)</u>	
SPACING, DAY	D.O. VIOLATED, MG/L	SPACING, DAY	D.O. VIOLATED, MG/L	SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0008	0.10	0.0005	0.10	0.0004
0.20	0.0034	0.20	0.0021	0.20	0.0018
0.30	0.0076	0.30	0.0046	0.30	0.0039
0.40	0.0135	0.40	0.0082		
0.40	0.0211	0.50	0.0128	<u>REACH NO. 12 (M - N)</u>	
0.60	0.0305	0.60	0.0185	SPACING, DAY	D.O. VIOLATED, MG/L
0.70	0.0415	0.70	0.0251	0.10	0.0005
		0.80	0.0329	0.20	0.0021
<u>REACH NO. 2 (B - C)</u>		<u>REACH NO. 7 (H - I)</u>		0.30	0.0048
SPACING, DAY	D.O. VIOLATED, MG/L	SPACING, DAY	D.O. VIOLATED, MG/L	0.40	0.0085
0.10	0.0008	0.10	0.0005	0.50	0.0133
0.20	0.0032	0.20	0.0018	<u>REACH NO. 13 (N - E)</u>	
0.30	0.0071			SPACING, DAY	D.O. VIOLATED, MG/L
0.40	0.0126	<u>REACH NO. 8 (I - M)</u>		0.10	0.0006
0.50	0.0197	SPACING, DAY	D.O. VIOLATED, MG/L	0.20	0.0024
0.60	0.0284	0.10	0.0006	0.30	0.0055
0.70	0.0387	0.20	0.0023	0.40	0.0098
0.80	0.0506	0.30	0.0052	0.50	0.0153
0.90	0.0641	0.40	0.0092	0.60	0.0220
1.00	0.0792	0.50	0.0143	0.70	0.0300
1.10	0.0960			0.80	0.0392
1.20	0.1143	<u>REACH NO. 9 (J - L)</u>		0.90	0.0497
1.30	0.1344	SPACING, DAY	D.O. VIOLATED, MG/L	1.00	0.0614
1.40	0.1561	0.10	0.0005	1.10	0.0744
1.50	0.1795	0.20	0.0020	1.20	0.0887
1.60	0.2046	0.30	0.0046	1.30	0.1043
1.70	0.2314	0.40	0.0081	1.40	0.1212
1.80	0.2599	0.50	0.0127	1.50	0.1393
<u>REACH NO. 3 (C - D)</u>		0.60	0.0183	<u>REACH NO. 14 (O - P)</u>	
SPACING, DAY	D.O. VIOLATED, MG/L	0.70	0.0249	SPACING, DAY	D.O. VIOLATED, MG/L
0.10	0.0009	0.80	0.0325	0.10	0.0006
0.20	0.0036	0.90	0.0411	0.20	0.0022
<u>REACH NO. 4 (D - E)</u>		1.00	0.0508	0.30	0.0050
SPACING, DAY	D.O. VIOLATED, MG/L	1.10	0.0616	0.40	0.0088
0.10	0.0010	1.20	0.0733	<u>REACH NO. 15 (P - E)</u>	
0.20	0.0038	1.30	0.0862	SPACING, DAY	D.O. VIOLATED, MG/L
0.30	0.0086	1.40	0.1001	0.10	0.0007
0.40	0.0153	1.50	0.1150	0.20	0.0027
0.50	0.0240	1.60	0.1310	0.30	0.0061
0.60	0.0345	1.70	0.1481	<u>REACH NO. 16 (R - S)</u>	
<u>REACH NO. 5 (E - F)</u>		1.80	0.1663	SPACING, DAY	D.O. VIOLATED, MG/L
SPACING, DAY	D.O. VIOLATED, MG/L	1.90	0.1856	0.10	0.0006
0.10	0.0009	2.00	0.2059	0.20	0.0024
0.20	0.0037	<u>REACH NO. 10 (K - L)</u>		0.30	0.0054
0.30	0.0084	SPACING, DAY	D.O. VIOLATED, MG/L	0.40	0.0095
0.40	0.0149	0.10	0.0004	0.50	0.0149
0.50	0.0233	0.20	0.0018	<u>REACH NO. 17 (S - F)</u>	
0.60	0.0336	0.30	0.0039	SPACING, DAY	D.O. VIOLATED, MG/L
0.70	0.0457	0.40	0.0070	0.10	0.0008
		0.50	0.0110	0.20	0.0032
		0.60	0.0158	0.30	0.0072
		0.70	0.0215	0.40	0.0128
				0.50	0.0201

$$\begin{aligned}
P &= \Sigma (\text{waste flow} \times \text{influent BOD concentration} \times \% \text{ BOD removal}) \\
&= (5 \times 500 E_{i1} + 6 \times 650 E_{i2} + 3 \times 975 E_{i3} + 2 \times 550 E_{i4} \\
&\quad + 4.2 \times 1040 E_{i5} + 3.6 \times 700 E_{i6} + 2.5 \times 865 E_{i7} \\
&\quad + 35 \times 380 E_{m1} + 12 \times 240 E_{m2} + 8 \times 220 E_{m3} + 2 \times 270 E_{m4} \\
&\quad + 10 \times 300 E_{m5} + 7.5 \times 350 E_{m6} + 7.5 \times 275 E_{m7}) C/100,
\end{aligned}$$

where

P = total pounds of BOD removed per day by the treatment plants
in the river basin,

E_{ir} = % BOD removal at the r^{th} industrial treatment plant,
 $r = 1, 2, \dots, 7,$

E_{ms} = % BOD removal at the s^{th} municipal treatment plant,
 $s = 1, 2, \dots, 7,$

and

C = a factor to convert a mg/l of BOD concentration in a cfs of
flow to pounds of BOD per day as follows:

1 cfs = 0.646 million gallons/day, and

1 mg/l = 8.345 pounds/million gallons, therefore

$$C = (0.646) (8.345) = 5.39.$$

The units of waste flow and influent BOD concentration are cfs and
mg/l, respectively.

The objective function then becomes

$$\begin{aligned}
P &= 134.75 E_{i1} + 212.905 E_{i2} + 157.657 E_{i3} + 59.29 E_{i4} \\
&\quad + 235.435 E_{i5} + 135.828 E_{i6} + 116.558 E_{i7} + 716.87 E_{m1} \\
&\quad + 155.232 E_{m2} + 94.864 E_{m3} + 101.871 E_{m4} + 161.7 E_{m5} \\
&\quad + 141.487 E_{m6} + 111.168 E_{m7}.
\end{aligned} \tag{4.1}$$

(2) Minimum Treatment. In the use of the minimum treatment program to manage the water quality of the river basin, the percent BOD removal required at each treatment plant was determined from the maximization of assimilative capacity utilization. Different percent BOD removals at each treatment plant were allowed. The linear programming formulation of this management program is listed as follows:

Minimize

P in equation (4.1)

subject to

constraints 1 to 92 in Appendix D, and stream D.O. standards.

The D.O. standards included 4.0, 4.5, and 5.0 mg/l. Two ranges of allowable percent BOD removal were between 30 and 95%, and between 75 and 95%. The results from the optimal solution of this linear programming problem are listed in Table XII. Several points are shown in this table. First of all, the differences of treatment requirement among treatment plants were large. The largest one was the difference between the lower and the upper limit of the permissible range of BOD removal. These differences decreased as the range of BOD removal became smaller. Furthermore, Table XII shows that when the lower BOD limit was fixed, the treatment requirement of each plant became greater, or at least unchanged, as the D.O. standard became higher. But when the D.O. standard was fixed, some high treatment requirements, such as those of I1, I2, M1, M6, and M7, became smaller as the lower limit of BOD removal increased from 30 to 75%; while the low treatment requirements, such as those of I3, I6, I7, and M2, increased to the level of 75% BOD removal. And as what could be anticipated, this table also indicates that the maximum utilizable stream assimilative capacity

TABLE XII

OPTIMAL WATER QUALITY MANAGEMENT FROM THE
MINIMUM TREATMENT PROGRAM

Range of BOD Removal, %	Min. D.O. Standard, mg/l	Max. Stream Assimilative Capacity, lbs BOD/day	Min. Requirement of % BOD Removal													
			I1	I2	I3	I4	I5	I6	I7	M1	M2	M3	M4	M5	M6	M7
30-95	4.0	51,450	95.0	"	37.58	95.0	"	30.0	49.06	95.0	30.0	83.91	95.0	73.31	88.74	95.0
	4.5	45,675	95.0	"	55.53	95.0	"	30.40	57.83	95.0	30.0	86.87	95.0	77.65	95.0	"
	5.0	39,332	95.0	"	70.35	95.0	"	45.13	66.59	95.0	30.0	89.93	95.0	81.94	95.0	"
75-95	4.0	44,698	75.0	"	"	95.0	"	75.0	"	88.48	75.0	83.91	92.71	75.0	"	79.36
	4.5	39,728	75.0	"	"	95.0	"	75.0	"	93.18	75.0	86.87	95.0	77.65	75.53	84.64
	5.0	34,601	75.0	81.28	75.0	95.0	"	75.0	"	95.0	75.0	89.93	95.0	81.94	81.96	89.93

" : Same as the left value.

decreased as the lower limit of BOD removal and/or the D.O. standard became higher.

(3) Uniform Treatment. The use of the uniform treatment program to achieve the D.O. objective requires a uniform percent BOD removal at each treatment plant. The linear programming formation of this management program is the following:

Minimize

P in equation (4.1)

subject to

constraints 1 to 92 in Appendix D, and

$$E_{i1} = E_{i2} = \dots = E_{i7} = E_{m1} = E_{m2} = \dots = E_{m7},$$

and stream D.O. standards.

The D.O. standards and the ranges of allowable BOD removal used in this program are the same as those used in the minimum treatment program. The results of the optimal solution of this program are shown in Table XIII. Besides the result that the maximum utilizable stream assimilative capacity decreased as the D.O. standard increased, two points are obvious in this table. First, the increase of the uniform treatment requirements due to the increase of the D.O. standard were small. The treatment requirements for each treatment plant were 88.38, 89.78, and 91.22% for 4.0, 4.5, and 5.0 mg/l of D.O. standards, respectively. Secondly, the treatment requirement was insensitive to the lower limit level of the range of BOD removal. For each D.O. standard, the treatment requirements were the same for 30 and 75% lower limits of BOD removal.

(4) Weight of Influent BOD. The pounds of influent BOD per day at each treatment plant were computed to form the management program.

TABLE XIII

OPTIMAL WATER QUALITY MANAGEMENT FROM THE
UNIFORM TREATMENT PROGRAM

Range of BOD Removal, %	Min. D.O. Standard, mg/l	Max. Stream Assimilative Capacity, lbs BOD/day	Min. Requirement of % BOD Removal													
			I1	I2	I3	I4	I5	I6	I7	M1	M2	M3	M4	M5	M6	M7
30 - 95 or 75 - 95	4.0	29,475	88.38	"	"	"	"	"	"	"	"	"	"	"	"	"
	4.5	25,906	89.78	"	"	"	"	"	"	"	"	"	"	"	"	"
	5.0	22,260	91.22	"	"	"	"	"	"	"	"	"	"	"	"	"

" : Same as the left value.

These calculations are listed in an increasing order as follows:

Influent BOD, lbs/day	Treatment Plant
5933	I4
9493	M3
10193	M4
11124	M7
11663	I7
13484	I1
13592	I6
14158	M6
15533	M2
15776	I3
16181	M5
21035	I2
23559	I5
71734	M1

The distribution of the weights of influent BOD did not provide a clear-cut picture for grouping the treatment zones. One way to group the treatment zones for this case was to take each treatment plant as a treatment zone. It then required that the treatment plant having a larger weight of influent BOD should not have less degree of treatment than the others. The linear programming formation of this management program is as follows:

Minimize

P in equation (4.1)

subject to

constraints 1 to 92 in Appendix D, and

$$E_{i4} \leq E_{m3} \leq E_{m4} \leq E_{m7} \leq E_{i7} \leq E_{i1} \leq E_{i6} \leq E_{m6} \leq E_{m2} \leq E_{i3} \leq E_{m5} \leq E_{i2} \leq E_{i5} \leq E_{m1}, \text{ and stream D.O. standards.}$$

The results from the optimal solution of this management program are in Table XIV. This table shows that through the optimization procedure the number of treatment zones had been automatically reduced from fourteen down to two for both 4.0 and 4.5 mg/l of D.O. standards and

TABLE XIV

OPTIMAL WATER QUALITY MANAGEMENT FROM THE
 ZONE-TREATMENT PROGRAM BASED ON THE
 WEIGHT OF INFLUENT BOD
 (INITIAL NUMBER OF
 TREATMENT ZONES=14)

Range of BOD Removal, %	Min. D.O. Standard, mg/l	Max. Stream Assimilative Capacity, lbs BOD/day	Min. Requirement of % BOD Removal													
			Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7	Zone 8	Zone 9	Zone 10	Zone 11	Zone 12	Zone 13	Zone 14
			I4	M3	M4	M7	I7	I1	I6	M6	M2	I3	M5	I2	I5	M1
30 - 95 or 75 - 95	4.0	29,977	86.88	"	"	"	"	"	"	"	"	"	89.36	"	"	"
	4.5	26,152	89.05	"	"	"	"	"	"	"	"	"	90.26	"	"	"
	5.0	22,260	91.22	"	"	"	"	"	"	"	"	"	"	"	"	"

" : same as the left value.

down to one for 5.0 mg/l of D.O. standard. It also shows that as the D.O. standard becomes higher the difference in treatment requirements between zones becomes smaller. For each treatment plant the increase in treatment requirement was also small when the D.O. standard increased. Similar to that in Table XIII, this table shows that the treatment requirement was insensitive to the range of BOD removal. Under a D.O. standard the treatment requirement of a plant was unchanged as the range limits of BOD removal increased from that between 30 and 95% to that between 75 and 95%.

Another way of grouping the treatment zones according to the weight of influent BOD was based upon the similarity of the weights. Five treatment zones were grouped from the review of the weights of influent BOD at those fourteen treatment plants. They are listed as follows:

Zone No.	Influent BOD, lbs/day	Treatment Plant
1	5933	I4
2	9493	M3
2	10193	M4
2	11124	M7
2	11663	I7
3	13484	I1
3	13592	I6
3	14158	M6
3	15533	M2
3	15776	I3
3	16181	M5
4	21035	I2
4	23559	I5
5	71734	M1

Its linear programming formation is shown as follows:

Minimize

P in equation (4.1)

subject to

constraints 1 to 92 in Appendix D, and

$$E_{14} \leq E_{m3} = E_{m4} = E_{m7} = E_{17} \leq E_{11} = E_{16} = E_{m6} = E_{m2} = E_{13} \\ = E_{m5} \leq E_{12} = E_{15} \leq E_{m1}, \text{ and stream D.O. Standards.}$$

The results from the optimal solution of this management program are shown in Table XV. This table shows that through the solution procedure the five treatment zones initially assigned had been automatically reduced to one zone only. These results are identical to those from the uniform treatment program in Table XIII. They demonstrated that to group the treatment zones based on the similarity of weight of influent BOD could not ensure a better management outcome than what the uniform treatment program could provide.

(4) Sub-basin. From the use of a sub-basin as the treatment zone in the water quality management, the treatment plants in the river basin were grouped into six treatment zones according to their discharge locations. They are

Zone No.	Sub-basin	Treatment Plant
1	I	I1, I2, I3, M1, M2
2	II	I4, M3
3	III	M4
4	IV	I5, M5
5	V	I6, M6
6	VI	I7, M7

The treatment plants in the same treatment zone were subjected to a uniform treatment requirement. The linear programming formation of this management program is listed as follows:

Minimize

P in equation (4.1)

TABLE XV

OPTIMAL WATER QUALITY MANAGEMENT FROM THE
 ZONE-TREATMENT PROGRAM BASED ON THE
 WEIGHT OF INFLUENT BOD
 (INITIAL NUMBER OF
 TREATMENT ZONES=5)

Range of BOD Removal, %	Min. D.O. Standard, mg/l	Max. Stream Assimilative Capacity, lbs BOD/day	Min. Requirement of % BOD Removal				
			Zone 1	Zone 2	Zone 3	Zone 4	Zone 5
			I4	M3, M4, M7, I7	I1, I6, M6, M2, I3, M5	I2, I5	M1
30 - 95 or 75 - 95	4.0	29,474	88.38	"	"	"	"
	4.5	25,906	89.78	"	"	"	"
	5.0	22,260	91.22	"	"	"	"

" : same as the left value.

subject to

constraints 1 to 92 in Appendix D, and

$$E_{i1} = E_{i2} = E_{i3} = E_{m1} = E_{m2},$$

$$E_{i4} = E_{m3},$$

$$E_{i5} = E_{m5},$$

$$E_{i6} = E_{m6},$$

$$E_{i7} = E_{m7}, \text{ and stream D.O. standards.}$$

The results from the optimal solution of this zone-treatment management program are shown in Table XVI. In this table three points have been shown. First of all, under each of the three D.O. standards, the difference of treatment requirement between each treatment zone was not considered to be large. Secondly, for each range of BOD removal, the increase in minimum treatment requirement of a zone was small when the D.O. standard became higher. Finally, this table shows that either the treatment requirement or the utilizable stream assimilative capacity was insensitive to the lower limit of the range of BOD removal. When the lower limit increased from 30 to 75%, the treatment requirements of zones 1 and 5 only and the stream assimilative capacity under 4.0 mg/l of D.O. standard were affected. The treatment requirement of zone 5 increased from 70.89 to 75% while that of zone 1 decreased from 83.25 to 82.58%, and the stream assimilative capacity showed a small decrease from 43,747 to 43,524 pounds of BOD per day.

(5) BOD-flow Ratio. In the use of the BOD-flow ratio as the criterion to group the treatment zones, the BOD-flow ratio of each treatment plant was computed to be as follows.

TABLE XVI

OPTIMAL WATER QUALITY MANAGEMENT FROM THE
ZONE-TREATMENT PROGRAM BASED ON
SUB-BASIN CLASSIFICATION

Range of BOD Removal, %	Min. D.O. Standard, mg/l	Max. Stream Assimilative Capacity, lbs BOD/day	Min. Requirement of % BOD Removal					
			Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
			I1, I2, I3, M1, M2	I4, M3	M4	I5, M5	I6, M6	I7, M7
30 - 95	4.0	43,747	83.25	86.88	76.72	92.13	70.89	77.72
	4.5	38,191	85.16	89.05	79.83	92.82	75.37	81.02
	5.0	32,634	87.07	91.22	82.94	93.53	79.84	84.31
75 - 95	4.0	43,524	82.58	86.88	76.72	92.13	75.00	77.72
	4.5	38,191	85.16	89.05	79.83	92.82	75.37	81.02
	5.0	32,634	87.07	91.22	82.94	93.53	79.84	84.31

BOD-flow ratio $\times 10^{-6}$	Treatment plant
6.24	M2
9.81	I1
10.34	I3
13.88	I2
44.60	I7
45.10	M1
47.03	I6
49.54	I5
50.02	I4
52.24	M7
59.09	M4
63.28	M6
78.98	M5
97.82	M3

This distribution of BOD-flow ratios did provide a clear-cut picture for the classification of the treatment zones. Treatment plants having similar ratios were grouped into a treatment zone. Five treatment zones were obtained as follows:

Zone 1: M2, I1, I3, I2

Zone 2: I7, M1, I6, I5, I4, M7

Zone 3: M4, M6

Zone 4: M5

Zone 5: M3

The linear programming formation of this zone-treatment management program is the following:

Minimize

P in equation (4.1)

subject to

constraints 1 to 92 in Appendix D and

$$E_{m2} = E_{i1} = E_{i3} = E_{i2} \leq E_{i7} = E_{m1} = E_{i6} = E_{i5} = E_{i4} = E_{m7} \leq$$

$$E_{m4} = E_{m6} \leq E_{m5} \leq E_{m3}, \text{ and stream D.O. standards.}$$

The results from the optimal solution of this management program are shown in Table XVII. This table shows that through the optimization procedure the number of treatment zones had been automatically reduced from five to three. These three treatment zones are

Zone 1: M2, I1, I3, I2

Zone 2: I7, M1, I6, I5, I4, M7, and

Zone 3: M4, M6, M5, M3.

When the D.O. standard was 4.0 mg/l, the treatment requirements of Zones 3, 4, and 5 were equal to the upper limit of the range of BOD removal, while the treatment requirements of Zones 1 and 2 were close to each other. As the D.O. standard increased, the treatment requirement of Zone 2 had increased at a greater rate than that of Zone 1, while the treatment requirements of Zones 3, 4, and 5 remained unchanged. It is also noted from Table XVII that the treatment requirement was insensitive to the lower limit of the range of BOD removal. Under each D.O. standard, the treatment requirement of each zone was unchanged when the lower limit increased from 30 to 75%. This table also shows that for each D.O. standard the difference of treatment requirement between each treatment zone was not large.

TABLE XVII

OPTIMAL WATER QUALITY MANAGEMENT FROM THE
 ZONE-TREATMENT PROGRAM BASED ON THE
 CLASSIFICATION OF BOD-FLOW RATIOS

Range of BOD Removal, %	Min. D.O. Standard, mg/l	Max. Stream Assimilative Capacity, lbs BOD/day	Min. Requirement of % BOD Removal				
			Zone 1	Zone 2	Zone 3	Zone 4	Zone 5
			M2, I1, I3, I2	I7, M1, I6, I5, I4, M7	M4, M6	M5	M3
30 - 95 or 75 - 95	4.0	41,816	80.39	80.83	95.0	"	"
	4.5	36,633	81.97	83.94	95.0	"	"
	5.0	31,450	83.55	86.85	95.0	"	"

" : same as the left value.

CHAPTER V

DISCUSSION

A. Assessment of Distribution Functions for Drought Flows and Design Flow Criteria

The comparison of the maximum deviations in Chapter IV indicated that for Stations 1645 and 1945 the Johnson S_B distribution had better goodness-of-fit in the characterization of drought flow distribution than the Type III distribution. On the contrary, the Type III distribution provided better curve fitting to the drought flows for Station 1775. Stations 1645 and 1945 had moderate to large magnitude of stream flows. Daily flows recorded at these two stations were all greater than zero. Station 1775 was a small flow station. There were a significant number of zero daily flows recorded at this station. Only those streams having flow throughout the year can be considered as having an assimilative capacity. It is obvious that to determine a design flow for either station 1645 or 1945 has more significance than that for Station 1775. Therefore, the better goodness-of-fit of Johnson S_B distribution to drought flows at Stations 1645 and 1945 is much more meaningful than that of Type III distribution to flows at Station 1775. Besides, the Type III distribution is bounded only by the lower limit, while the Johnson S_B distribution has both lower and upper limits. Intuitively, to assume that the distribution of drought flows are bounded at both ends, rather than bounded at only the lower end, is more reasonable.

With these considerations, the use of the Johnson S_B distribution is sounder than the use of the Type III distribution.

When the S_B distribution is applied to characterize the drought flows, the lower limit and the upper limit defining the range of flow distribution are of interest. Table V in Chapter IV indicates that for the drought flow distributions at Stations 1645 and 1945, most of the lower limits were equal to zero. The minimum 60-day consecutive flow at Station 1645 and the minimum 90-day consecutive flow at Station 1945 were the only two cases having a lower limit greater than zero. From this result it is reasonable to assume a zero lower limit in the S_B distribution when it is applied to express the distribution of a minimum flow with 30 or less consecutive days. As the upper limit of the flow distribution is concerned, Table V also indicates that it was in the near upper side of the largest observation of the drought flows. Therefore, it is not difficult to make a reasonable initial estimate of the upper limit of the drought flow distribution when the Johnson S_B distribution is used.

The amount of design flow obtained from the analysis of drought flow distribution depends upon the level of probability of recurrence selected and the type of minimum consecutive flow analyzed. It is definitely that the amount of design flow increases as the level of probability of recurrence increases. In regard to the effect of the type of minimum flow analyzed, Figures 23 to 25 in Chapter IV indicate that the design flow became larger when the number of consecutive days of flow increased. This is probably due to the fact that the fluctuation of the low flow become less as the number of consecutive days become larger.

The design flow is not only an important factor in evaluating the

stream assimilative capacity, but is also a basis for evaluating the quality of stream water, such as pH value, and concentrations of D.O., BOD, total dissolved solids, coliform organisms, and inorganic salts. A criterion for determining the design flow always specifies a probability of recurrence and a type of minimum flow with a specific number of consecutive days, such as the minimum 7-day consecutive flow once in ten years. As reviewed in Chapter II, there are two basic viewpoints in establishing the design flow criterion (or criteria) in a state or a drainage basin. One is to apply a uniform design flow criterion to all streams regardless of water uses, intrastate or interstate status, and regulated or unregulated status. Another viewpoint is to apply different criteria to determine the design flows for different groups of receiving streams. As mentioned earlier, selections of different probability of recurrence and/or different types of minimum flow with various numbers of consecutive days would result in significant difference in the amounts of design flow, therefore, to apply different design flow criteria in a state or a drainage basin would in essence allow different water quality standards in different streams and inevitably would result in inequitable responsibility of pollution control among the waste dischargers. Therefore, it is understandable that the uniform design flow criterion is more effective and equitable in managing a statewide or a basinwide water quality management program.

B. Evaluation of Water Quality

Management Programs

The minimum treatment program, the uniform treatment program, and the three zone-treatment programs respectively based on the weight of

influent BOD, the sub-basin, and the BOD-flow ratio were the five management programs investigated in this study. The summarized utilizations of the stream assimilative capacity achieved from each program are shown in Table XVIII on percentage basis. In Table XVIII the assimilative capacity utilization from the minimum treatment program under each specific D.O. standard and each range of BOD removal was taken as 100%. The assimilative capacity utilizations from the other four programs were expressed in percent of the utilization from the minimum treatment program. This table shows several points concerning the utilization of stream assimilative capacity related to the management program, the stream D.O. standard, and the allowable range of BOD removal.

First of all, though the minimum treatment program could achieve the greatest utilization of the assimilative capacity among the five programs investigated, the inequity in treatment requirements resulted from its use are obvious. This program created large differences of treatment requirement among dischargers, even among those located close to each other. The degree of inequity was particularly tremendous when the allowable range of BOD removal was large. As an example, the treatment requirements of plants I2 and I3 were extremely inequitable. They were located at opposite banks of the Main River and were close to each other. They could be expected to cause a similar degree of damage to the stream water quality. But from the use of this program the treatment requirements were 95 and 37.58% for plants I2 and I3, respectively. The degree of treatment inequity became much less as the range of percent BOD removal decreased from that between 30 and 95% to that between 75 and 95%. Table XII also shows that at each treatment

TABLE XVIII

PERCENTAGE COMPARISON OF THE UTILIZATIONS OF STREAM
ASSIMILATIVE CAPACITY FROM VARIOUS PROGRAMS
OF WATER QUALITY MANAGEMENT

Range of BOD Removal, %	Minimum D.O. Standard, mg/l	Management Program				
		Minimum Treatment	Uniform Treatment	Weight of Influent BOD*	Sub-basin	BOD-Flow Ratio
30 - 95	4.0	100	57.28	58.26	85.02	81.27
	4.5	100	56.71	57.25	83.61	80.20
	5.0	100	56.59	56.59	82.97	79.96
75 - 95	4.0	100	65.94	67.06	97.37	93.55
	4.5	100	65.20	65.82	96.13	92.20
	5.0	100	64.33	64.33	94.31	90.89

*The results in Table XIV were used.

plant the variation of treatment requirement due to the change of the allowable range of BOD removal was more sensitive than that due to the change of the stream D.O. standards.

The uniform treatment program provides a uniform treatment requirement to each waste treatment plant; therefore, its administrative simplicity is obvious. However, it is inefficient in the utilization of the stream assimilative capacity. As indicated in Table XVIII this program resulted in the least utilization of the assimilative capacity among the five management programs. Two other points can also be noted from Table XVIII. First, when considering the utilization of the assimilative capacity from the minimum treatment program as 100%, the percent utilization of assimilative capacity achieved from this program decreased insignificantly as the stream D.O. standard increased. For the range of BOD removal between 30 and 95%, the percent utilizations of the assimilative capacity were 57.28, 56.71, and 56.59 for 4.0, 4.5, and 5.0 mg/l of D.O. standards, respectively. The range of BOD removal between 75 and 95%, showed a similar trend, they were 65.94, 65.20, and 64.33 for the three D.O. standards, respectively. Secondly, the percent utilization of the assimilative capacity increased significantly as the lower limit of the allowable BOD removal increased from 30 to 75% while the upper limit was fixed at 95%. Therefore, as the minimum requirement of BOD removal gets high, such as 80% or more, the uniform treatment program could possibly become a feasible management program. It can be seen from Table XIII that the uniform percent BOD removal increased accordingly only in a small amount as the D.O. standard increased. The percent BOD removals were 88.38, 89.78, and 91.22 for 4.0, 4.5, and 5.0 mg/l of D.O. standards, respectively. Furthermore,

the requirement of BOD removal was insensitive to the range of BOD removal. Under each D.O. standard, the treatment requirement was the same for the range of BOD removal either that between 30 and 95% or that between 75 and 95%.

The fair aspect of the zone-treatment management program based on the weight of influent BOD was that it required the treatment plant with a larger weight of influent BOD should not have less degree of treatment than others. However, because it had not taken into account the stream flow available at the discharge location, this program did not show significant improvement in the utilization of the stream assimilative capacity over the uniform treatment program. Table XVIII shows that under 4.0 and 4.5 mg/l of D.O. standards there were respectively only 0.98 and 0.54% more of assimilative capacity utilization gained from this program than those from the uniform treatment program as the minimum requirement of BOD removal was 30%. Similarly, there were respectively only 1.12 and 0.62% more utilization gained from this program under 4.0 and 4.5 mg/l of D.O. standard as the minimum requirement of BOD removal was 75%. Furthermore, under the 5.0 mg/l of D.O. standard this program resulted in a uniform treatment requirement to each treatment plant for both ranges of BOD removal. This result was identical with what resulted from the uniform treatment program.

The management program taking the sub-basins as the treatment zones resulted in a high degree of assimilative capacity utilization. Table XVIII shows that under 4.0, 4.5, and 5.0 mg/l of D.O. standards, in the range of allowable BOD removal between 30 and 95%, the utilizations of assimilative capacity achieved from this program were

respectively 85.02, 83.61, 82.97% of those from the minimum treatment program. They were respectively up to 97.37, 96.13, and 94.3% when the range of allowable BOD removal became between 75 and 95%. It is obvious that the percent utilization of assimilative capacity improved significantly as the minimum treatment requirement got higher. Also indicated in Table XVIII is that the utilization of assimilative capacity decreased slightly as the stream D.O. standard increased. The treatment requirements of treatment plants in Table XVI show that there was no tremendous treatment difference among the six treatment zones even when the minimum treatment requirement was low. As the minimum treatment requirement was 30%, the largest treatment differences were 22.24, 17.45, and 13.69% for 4.0, 4.5, and 5.0 mg/l of D.O. standards, respectively. As the minimum treatment requirement went up to 75%, the largest treatment differences were 17.13, 17.45, and 13.69% for the three different D.O. standards, respectively. This management program is nearly as easy to implement as the uniform treatment program since it requires only locating the discharging points of waste sources within treatment zones. It tends to reduce the objections of individual dischargers regarding their treatment requirements as compared to those discharging their wastes in the same sub-basin. The determination of the treatment requirement for each sub-basin was completely based upon the optimization of the overall stream assimilative capacity. Among the treatment zones no specification concerning the order of the treatment degree was made in advance. For the particular case that each sub-basin consisted of only one waste discharger this program would then become identical to the minimum treatment program. It could be imagined that the amount of treatment difference depends on the number

of treatment zones classified. The more the number of treatment zones, the larger the treatment difference would be, and accordingly, the higher the utilization of the stream assimilative capacity. However, to classify the river basin into many treatment zones could diminish the virtue of this program.

The management program based on the use of the BOD-flow ratio to group the treatment zones looks more logical than the others. It takes into account both the weight of influent BOD and the dilution capacity of the design flow at the discharge location. From the viewpoint of administration this program is easy to implement, since it requires only the determination of the BOD-flow ratios of the treatment plants. It requires uniform treatment to the plant having similar BOD-flow ratios in the same treatment zone. It also requires no less degree of treatment to the treatment zones with larger BOD-flow ratio than others. Therefore, this program could reduce the objections of individual dischargers regarding their situations as compared to others. In addition, it could reduce the possibility of discharging waste into the small flow stream. The utilization of stream assimilative capacity achieved from this program were reasonably high. They were 81.27, 80.2, and 79.96% of those from the minimum treatment program respectively under 4.0, 4.5, and 5.0 mg/l of D.O. standards as the range of BOD removal was that between 30 and 95%. Similarly, they were 93.55, 92.2, and 90.89% respectively for the three D.O. standards as the range of BOD removal was between 75 and 95%. As indicated by other management programs, these results show that the percent utilizations of assimilative capacity associated with high minimum treatment requirement was much larger than those associated with the low minimum treatment

requirement. Another point shown in Table XVIII is that the utilization of assimilative capacity decreased insignificantly as the stream D.O. standard increased. Table XVII also indicates that from the use of this program the treatment differences among the treatment zones were in a reasonable range even when the minimum treatment requirement was low. For both ranges of BOD removal the largest differences were 14.61, 13.03, and 11.45% under 4.0, 4.5, and 5.0 mg/l of D.O. standards, respectively. The treatment difference decreased as the stream D.O. standard increased.

CHAPTER VI

CONCLUSIONS

From the studies of the drought flow distribution and the management programs of the basinwide water quality, the following conclusions can be drawn:

1. The Johnson S_B distribution is a good probability distribution function in the drought flow analysis. It specifies the lower limit and the upper limit of the drought flow distribution. For the stations having daily stream flow throughout the year, this distribution did show better goodness-of-fit than the Type III distribution for the smallest values. The use of S_B distribution could provide an objective determination of the design flow for the evaluation of the stream assimilative capacity.

2. When the number of consecutive days of flow is equal to or less than 30 days, it is reasonable to assume that the lower limit of S_B distribution is zero. The upper limit of this distribution is usually located in the near upper side of the largest observation of the minimum flows and therefore could be estimated without much difficulty.

3. For a fixed probability, the magnitude of the design flow increases as the number of consecutive days of flow increases.

4. The program of zone-treatment management with the use of BOD-flow ratio has shown its virtue of effectiveness and equitability in the utilization and allocation of the stream assimilative capacity.

It takes both the weight of influent BOD and the dilution capacity of design flow into account in the classifications of the treatment zones. Therefore, it could achieve a high degree of assimilative capacity utilization. And because it requires that the treatment zone having larger BOD-flow ratio should have no less treatment requirement than others, it could reduce the objections of individual dischargers regarding their situation as compared to that of others. It could also reduce the possibility of discharging waste into the small flow stream.

5. The zone-treatment program based on the use of a sub-basin as the treatment zone could effectively achieve a high utilization of the stream assimilative capacity. It requires a uniform treatment to the dischargers in the same treatment zone. Therefore, it is able to prevent large treatment difference among the dischargers located nearby to each other. But this program has no basis to ensure an equitable treatment requirements among the sub-basins. The treatment requirement in each treatment zone is completely determined through the optimization of the stream assimilative capacity. The use of this program therefore would possibly still have objections from the treatment zones concerning their treatment requirements.

6. The zone-treatment program based on the weight of influent BOD is in essence equitable in the allocation of the stream assimilative capacity. But because it does not take the dilution capacity of the stream flow into account in the classification of the treatment zones, the utilization of assimilative capacity from this program is insignificant in its improvement over that from the uniform treatment program.

7. Although the minimum treatment program can achieve the highest

degree of the utilization of stream assimilative capacity, it is extremely inequitable in the allocation of the assimilative capacity among the dischargers.

8. The uniform treatment program is inefficient in the utilization of the stream assimilative capacity though it is simple to administer in management of water quality. However, as the minimum treatment requirement is high, e.g., 80% or more of BOD removal, it could possibly become a feasible management program.

9. Considering the utilization of the assimilative capacity achieved from the minimum treatment program as 100%, the percent utilization of the assimilative capacity achieved from either the zone-treatment programs or the uniform treatment program gets slightly smaller as the stream D.O. standard increases, but it gets larger as the minimum treatment requirement increases.

CHAPTER VII

SUGGESTIONS FOR FUTURE WORK

Several suggestions for further studies related to this investigation are outlined as follows:

1. Study the effect of the selection of uniform design flow criterion to the water quality management in a real drainage basin. The management results could be compared in terms of utilizable stream assimilative capacity and treatment degrees of waste dischargers. The selection of design flow criterion is based on the specifications of the probability of occurrence and the number of consecutive day of drought flow, such as the minimum 7-day flow once in 10 years and the minimum 30-day flow once in 20 years.
2. Use of other water quality parameters, such as effluent BOD concentration or BOD concentration in the stream, to evaluate the zone-treatment management program.
3. Sensitivity analysis of the D.O. standard, and the lower and upper limit of BOD removal to the treatment degrees of waste dischargers and to the overall utilization of stream assimilative capacity.

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APPENDIX A

PROBABILITY DENSITY FUNCTIONS OF TYPE I ASYMPTOTIC
DISTRIBUTION FOR LARGEST VALUES, LOG-NORMAL
DISTRIBUTION, GAMMA DISTRIBUTION, AND
PEARSON TYPE V DISTRIBUTION

1. Type I Asymptotic Distribution for Largest Values (Gumbel Distribution)

$$f(x; \mu, \sigma) = \frac{1}{\sigma} \exp \left[-\frac{1}{\sigma} (x - \mu) - e^{-(1/\sigma)(x-\mu)} \right],$$

$$-\infty < x < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0.$$

2. Log-normal Distribution

$$f(x; \mu, \sigma) = \frac{1}{\sigma x \sqrt{2\pi}} \exp \left[-\frac{1}{2\sigma^2} (\ln x - \mu)^2 \right],$$

$$x > 0, \quad -\infty < \mu < \infty, \quad \sigma > 0.$$

3. Gamma (Pearson Type III) Distribution

$$f(x; \eta, \lambda) = \frac{\lambda^\eta}{\Gamma(\eta)} x^{\eta-1} e^{-\lambda x},$$

$$x \geq 0, \quad \lambda > 0, \quad \eta > 0.$$

$$\text{where } \Gamma(\eta) = \int_0^\infty x^{\eta-1} e^{-x} dx.$$

4. Pearson Type V Distribution

$$f(x; \gamma, p) = \frac{\gamma^{p-1}}{\Gamma(p-1)} x^{-p} e^{-\gamma/x},$$

$$0 \leq x \leq \infty.$$

APPENDIX B

ANNUAL MINIMUM FLOWS AT STATIONS

1645, 1775 AND 1945

ANNUAL MINIMUM FLOWS FOR VARIOUS NUMBERS OF CONSECUTIVE DAYS AT STATION 1645

M	M/(N+1)	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
1	0.0313	30.0	92.0	240.0	653.0	1834.0	5593.0	12427.0
2	0.0625	85.0	263.0	624.0	1350.0	3210.0	8986.0	16007.0
3	0.0938	147.0	447.0	1071.0	2440.0	6151.0	15685.0	24598.0
4	0.1250	202.0	692.0	1996.0	4323.0	10266.0	26962.0	49495.0
5	0.1563	270.0	852.0	2160.0	4400.0	11229.0	28140.0	51318.0
6	0.1875	280.0	880.0	2209.0	4501.0	11284.0	31910.0	61844.0
7	0.2188	290.0	880.0	2209.0	4700.0	12595.0	37301.0	63046.0
8	0.2500	300.0	920.0	2210.0	4730.0	12865.0	39817.0	67259.0
9	0.2813	326.0	989.0	2361.0	5474.0	13576.0	42319.0	74883.0
10	0.3125	327.0	1116.0	2698.0	6899.0	15873.0	45610.0	88535.0
11	0.3438	342.0	1193.0	3070.0	7058.0	17236.0	48002.0	112252.0
12	0.3750	372.0	1218.0	3731.0	7683.0	19896.0	51414.0	120559.0
13	0.4063	394.0	1307.0	4250.0	9982.0	25484.0	57286.0	133623.0
14	0.4375	436.0	1494.0	5047.0	10500.0	28440.0	68012.0	155117.0
15	0.4688	615.0	2116.0	5306.0	11933.0	29939.0	69021.0	160678.0
16	0.500	688.0	2134.0	5973.0	13217.0	36667.0	103918.0	174297.0
17	0.5313	694.0	2170.0	6377.0	15498.0	42070.0	124057.0	201910.0
18	0.5625	790.0	2403.0	6458.0	18800.0	47100.0	134120.0	208360.0
19	0.5938	850.0	2638.0	7354.0	19850.0	47848.0	136730.0	220550.0
20	0.6250	944.0	3540.0	8850.0	19954.0	56880.0	141910.0	228580.0
21	0.6563	1150.0	3550.0	9180.0	22210.0	57980.0	142100.0	233200.0
22	0.6875	1200.0	3640.0	9240.0	22530.0	59550.0	149754.0	261520.0
23	0.7188	1220.0	3880.0	9840.0	25000.0	60710.0	151420.0	288520.0
24	0.7500	1260.0	4070.0	11750.0	25860.0	62450.0	154920.0	288780.0
25	0.7813	1310.0	4770.0	11890.0	27060.0	64360.0	177120.0	328920.0
26	0.8125	1400.0	4820.0	12430.0	27190.0	66290.0	207900.0	377600.0
27	0.8438	1500.0	5320.0	13220.0	27960.0	72870.0	207910.0	380890.0
28	0.8750	1540.0	5400.0	13830.0	28710.0	80220.0	218970.0	386440.0
29	0.9063	1750.0	5790.0	16360.0	34820.0	105390.0	241390.0	388510.0
30	0.9375	2100.0	8600.0	22310.0	48770.0	111790.0	294520.0	508610.0
31	0.9688	2620.0	8880.0	23000.0	51700.0	149190.0	418200.0	606430.0

REMARKS:

1. PERIOD OF RECORDS: APRIL 1, 1938 - MARCH 31, 1969
2. M: RANKING NUMBER OF FLOWS IN A NON-DECREASING ORDER
3. N: TOTAL NUMBER OF YEARS
4. M/(N+1): OBSERVED CUMULATIVE PROBABILITY
5. UNIT OF FLOW: CFS

ANNUAL MINIMUM FLOWS FOR VARIOUS NUMBERS OF CONSECUTIVE DAYS AT STATION 1775

M	M/(N+1)	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
1	0.0323	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0645	0.0	0.0	0.0	0.0	0.0	0.1	0.1
3	0.0968	0.0	0.0	0.0	0.0	0.0	6.8	24.8
4	0.1290	0.0	0.0	0.0	0.0	0.3	7.2	29.8
5	0.1613	0.0	0.0	0.0	0.1	11.4	40.1	112.1
6	0.1935	0.0	0.0	0.0	1.8	15.3	52.6	116.5
7	0.2258	0.0	0.0	0.6	3.2	17.2	83.3	237.7
8	0.2581	0.1	0.3	0.7	5.2	19.8	126.6	382.8
9	0.2903	0.1	0.5	1.8	5.5	43.9	194.2	429.4
10	0.3226	0.2	0.8	2.2	6.9	46.1	249.0	466.7
11	0.3548	0.3	0.9	2.4	12.6	46.7	263.2	725.6
12	0.3871	0.3	1.7	4.5	15.3	50.4	318.2	925.4
13	0.4194	1.2	3.6	9.0	20.7	84.7	349.3	934.1
14	0.4516	1.3	4.3	11.5	27.8	85.0	352.1	1203.7
15	0.4839	1.5	5.4	13.5	29.4	107.0	407.5	1217.4
16	0.5161	1.7	5.5	14.7	29.6	135.4	424.8	1507.8
17	0.5484	2.1	6.3	14.7	35.7	176.6	592.7	1858.2
18	0.5806	2.2	6.6	18.4	43.6	177.8	666.4	2043.2
19	0.6129	3.8	11.4	28.2	66.0	254.0	685.8	2935.3
20	0.6452	3.8	12.2	30.8	66.8	307.8	901.2	4130.6
21	0.6774	4.5	13.5	36.0	80.8	308.8	925.0	6080.9
22	0.7097	4.8	14.4	39.7	89.7	315.2	1052.3	7258.7
23	0.7419	5.0	15.8	39.7	124.3	345.3	1158.3	7482.3
24	0.7742	5.6	18.4	48.8	134.4	369.1	2274.0	7935.1
25	0.8065	7.2	21.6	54.8	137.3	376.4	2503.2	8369.0
26	0.8387	7.2	22.3	57.9	159.5	521.3	3069.0	11036.5
27	0.8710	8.6	28.0	67.9	161.4	832.4	4163.1	11316.0
28	0.9032	12.0	37.0	98.0	264.0	923.0	4939.8	12768.6
29	0.9355	17.0	55.0	157.0	405.0	2437.0	6194.0	15034.0
30	0.9677	22.0	66.0	161.0	482.0	3665.0	8973.0	25535.6

REMARKS:

1. PERIOD OF RECORDS: APRIL 1, 1939 - MARCH 31, 1969
2. M: RANKING NUMBER OF FLOWS IN A NON-DECREASING ORDER
3. N: TOTAL NUMBER OF YEARS
4. M/(N+1): OBSERVED CUMULATIVE PROBABILITY
5. UNIT OF FLOW: CFS

ANNUAL MINIMUM FLOWS FOR VARIOUS NUMBERS OF CONSECUTIVE DAYS AT STATION 1945

M	M/(N+1)	1-DAY	3-DAY	7-DAY	14-DAY	30-DAY	60-DAY	90-DAY
1	0.0286	76.0	276.0	843.0	2060.0	6969.0	17379.0	36425.0
2	0.0571	200.0	605.0	1445.0	3284.0	8276.0	24204.0	54024.0
3	0.0857	255.0	1020.0	2460.0	5115.0	14015.0	37733.0	59913.0
4	0.1143	500.0	1500.0	3500.0	7460.0	17917.0	38725.0	111559.0
5	0.1429	566.0	1956.0	4823.0	10837.0	25831.0	66089.0	120878.0
6	0.1714	585.0	2045.0	4958.0	10857.0	29514.0	75640.0	128605.0
7	0.2000	638.0	2054.0	5213.0	10953.0	30373.0	75868.0	132510.0
8	0.2286	650.0	2177.0	6272.0	14552.0	32780.0	83351.0	141192.0
9	0.2571	678.0	2366.0	7190.0	14990.0	35331.0	85342.0	149230.0
10	0.2857	910.0	2980.0	7400.0	15810.0	35842.0	90820.0	157350.0
11	0.3143	983.0	3049.0	7506.0	15909.0	37030.0	91110.0	162670.0
12	0.3429	1040.0	3120.0	7799.0	15910.0	38629.0	101440.0	265740.0
13	0.3714	1040.0	3430.0	8830.0	19940.0	47380.0	170140.0	299780.0
14	0.4000	1110.0	4040.0	10800.0	29360.0	76890.0	172620.0	329880.0
15	0.4286	1470.0	4630.0	12870.0	29430.0	77180.0	194541.0	371390.0
16	0.4571	1680.0	5400.0	13970.0	31720.0	79080.0	202240.0	380670.0
17	0.4857	1810.0	5790.0	14120.0	35120.0	87150.0	212460.0	395390.0
18	0.5143	1850.0	6050.0	15910.0	37340.0	93840.0	217430.0	451421.0
19	0.5429	1890.0	6750.0	17930.0	40880.0	96580.0	217550.0	464560.0
20	0.5714	1960.0	7250.0	19350.0	42890.0	126930.0	291020.0	482670.0
21	0.6000	2190.0	7650.0	19500.0	42970.0	129070.0	303380.0	496740.0
22	0.6286	2240.0	7780.0	19740.0	47670.0	131580.0	308910.0	595450.0
23	0.6571	2340.0	8050.0	20940.0	50050.0	137430.0	341810.0	615680.0
24	0.6857	2530.0	9060.0	23410.0	52430.0	139700.0	343930.0	680130.0
25	0.7143	2640.0	9200.0	25220.0	53000.0	139830.0	360260.0	684220.0
26	0.7429	2790.0	10300.0	27660.0	62720.0	143380.0	366000.0	877090.0
27	0.7714	3020.0	10530.0	28170.0	63290.0	148200.0	374240.0	936940.0
28	0.8000	3090.0	11340.0	32240.0	65580.0	157710.0	402530.0	969290.0
29	0.8286	3380.0	11390.0	33520.0	70950.0	169580.0	535280.0	1010460.0
30	0.8571	3490.0	11440.0	34030.0	76930.0	174510.0	581470.0	1181510.0
31	0.8857	3510.0	11880.0	35740.0	83610.0	261040.0	633030.0	1275460.0
32	0.9143	3650.0	14170.0	39130.0	111870.0	262280.0	754560.0	1309740.0
33	0.9429	5100.0	18450.0	50120.0	125310.0	367930.0	822730.0	1352030.0
34	0.9714	5650.0	20900.0	56750.0	152230.0	415410.0	1169300.0	1741100.0

REMARKS:

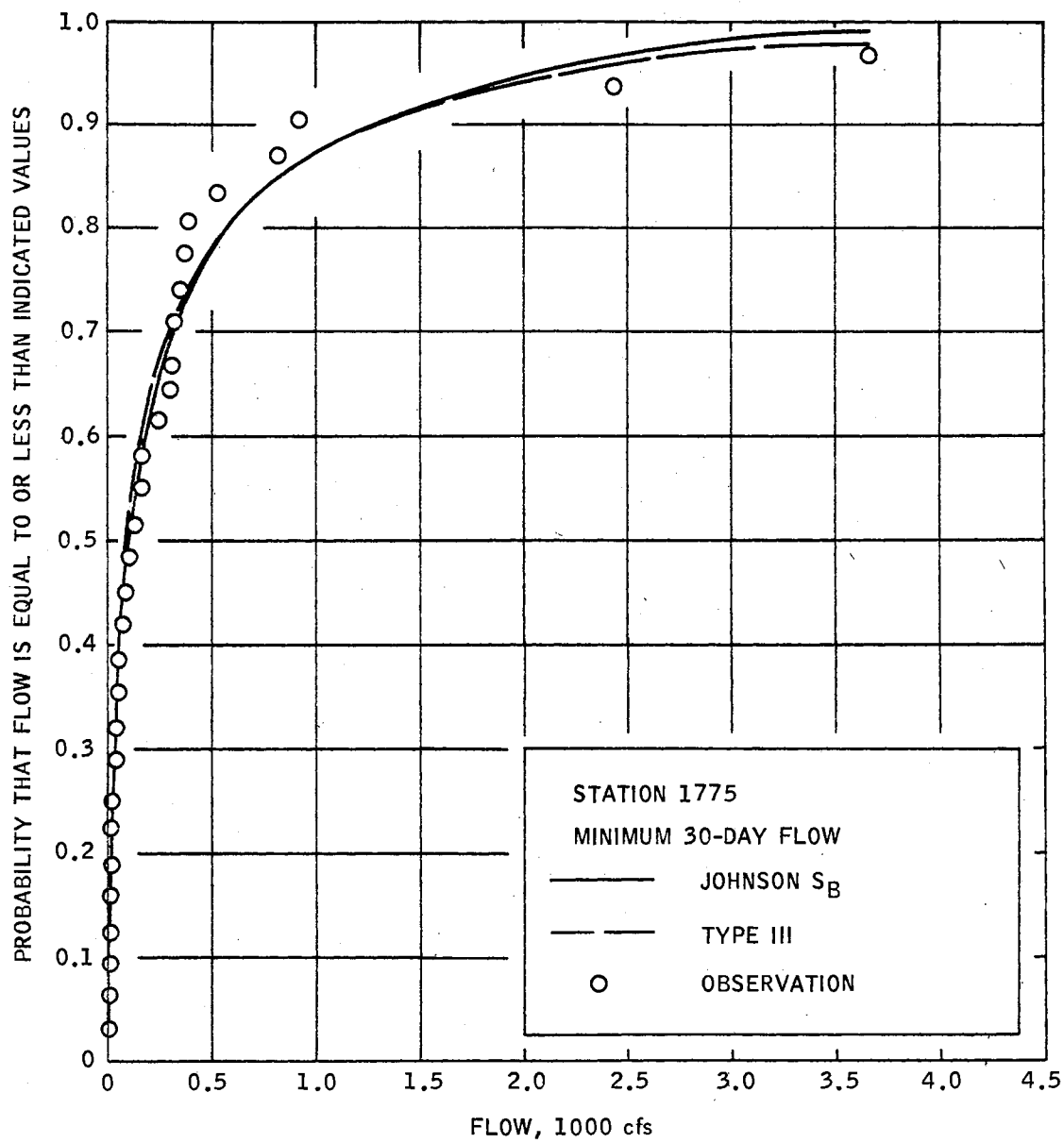
1. PERIOD OF RECORDS: APRIL 1, 1935 - MARCH 31, 1969
2. M: RANKING NUMBER OF FLOWS IN A NON-DECREASING ORDER
3. N: TOTAL NUMBER OF YEARS
4. M/(N+1): OBSERVED CUMULATIVE PROBABILITY
5. UNIT OF FLOW: CFS

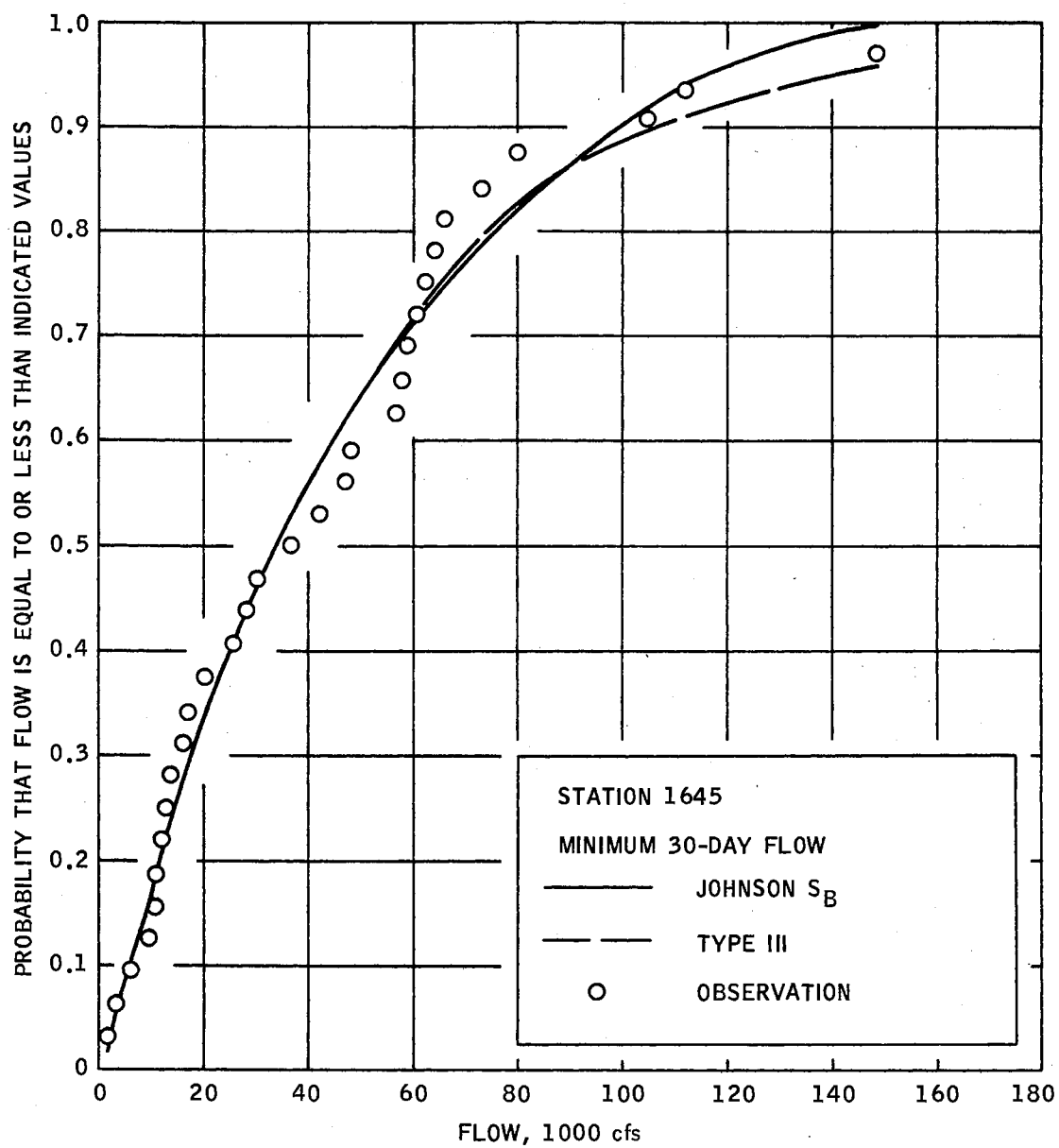
APPENDIX C

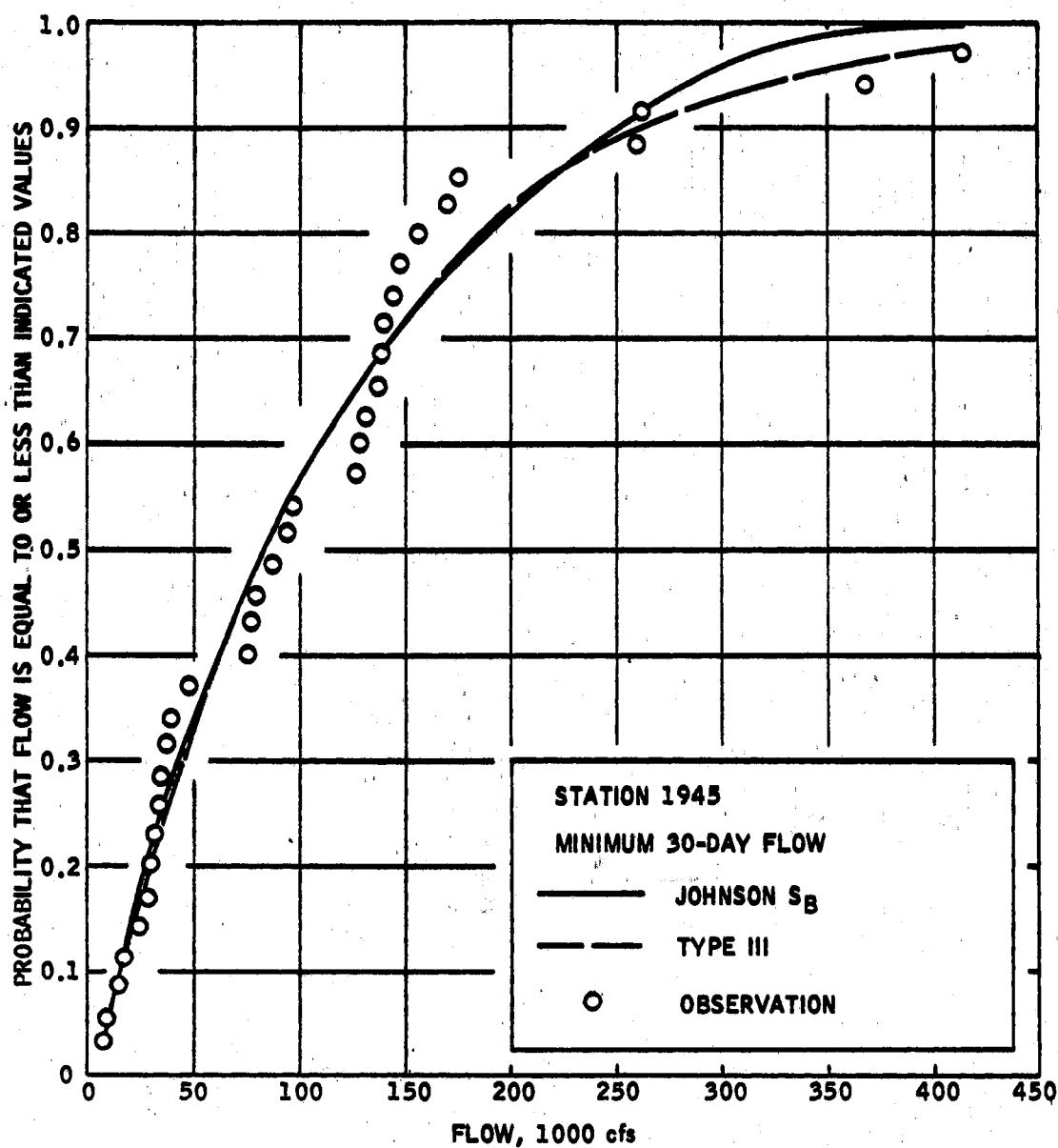
PROBABILITY CUMULATIVE DISTRIBUTIONS OF MINIMUM

30-DAY FLOWS AT STATIONS 1775, 1645,

AND 1945 ON THE ARITHMETIC PAPER







APPENDIX D

INVENTORY OF BOD AND D.O. CONSTRAINTS

Using equations (3.23) and (3.24), and the data in Table X, the constraints of BOD and D.O. for the regional quality system were developed as follows.

[Reach 1]

At A

$$DO_{1a} = (250 \times 7.4 + 5 \times 4)/(250 + 5)$$

$$DO_{1a} = 7.33 \quad (1)$$

$$BOD_{1a} = [250 \times 1.6 + 5 \times 500 \times (100 - E_{i1})/100]/(250 + 5)$$

$$255 BOD_{1a} + 25 E_{i1} = 2900 \quad (2)$$

At A₁

$$0.83527 DO_{1a} - FO_{1a1} - 0.1033 BOD_{1a} = -1.48257 \quad (3)$$

$$BOD_{1a1} - 0.88692 BOD_{1a} = 0 \quad (4)$$

At B⁻

$$0.72979 DO_{1a} - DO_{1b} - 0.16159 BOD_{1a} = -2.43189 \quad (5)$$

$$BOD_{1b} - 0.81058 BOD_{1a} = 0 \quad (6)$$

[Reach 2]

At B

$$DO_{2b} = (260 DO_{1b} + 35 \times 4)/(260 + 35)$$

$$295 DO_{2b} - 260 DO_{1b} = 140 \quad (7)$$

$$BOD_{2b} = [260 BOD_{1b} + 35 \times 380 \times (100 - E_{m1})/100]/(260 + 35)$$

$$295 BOD_{2b} - 260 BOD_{1b} + 133 E_{m1} = 13300 \quad (8)$$

At B₁

$$0.79852 DO_{2b} - DO_{2b1} - 0.11668 BOD_{2b} = -1.81332 \quad (9)$$

$$BOD_{2b1} - 0.86936 BOD_{2b} = 0 \quad (10)$$

At B₂

$$0.66698 \text{ DO}_{2b} - \text{DO}_{2b2} - 0.18162 \text{ BOD}_{2b} = -2.99718 \quad (11)$$

$$\text{BOD}_{2b2} - 0.77724 \text{ BOD}_{2b} = 0 \quad (12)$$

At B₃

$$0.55711 \text{ DO}_{2b} - \text{DO}_{2b3} - 0.22694 \text{ BOD}_{2b} = -3.98601 \quad (13)$$

$$\text{BOD}_{2b3} - 0.69489 \text{ BOD}_{2b} = 0 \quad (14)$$

At C⁻

$$0.44486 \text{ DO}_{2b} - \text{DO}_{2c} - 0.2623 \text{ BOD}_{2b} = -4.99626 \quad (15)$$

$$\text{BOD}_{2c} - 0.60411 \text{ BOD}_{2b} = 0 \quad (16)$$

[Reach 3]

At C

$$\text{DO}_{3c} = (275 \text{ DO}_{2c} + 6 \times 4) / (275 + 6)$$

$$281 \text{ DO}_{3c} - 275 \text{ DO}_{2c} = 24 \quad (17)$$

$$\text{BOD}_{3c} = [275 \text{ BOD}_{2c} + 6 \times 650 \times (100 - E_{i2}) / 100] / (275 + 6)$$

$$281 \text{ BOD}_{3c} - 275 \text{ BOD}_{2c} + 39.5 E_{i2} = 3950 \quad (18)$$

At D⁻

$$0.91393 \text{ DO}_{3c} - \text{DO}_{3d} - 0.05926 \text{ BOD}_{3c} = -0.77463 \quad (19)$$

$$\text{BOD}_{3d} - 0.93801 \text{ BOD}_{3c} = 0 \quad (20)$$

[Reach 4]

At D

$$\text{DO}_{4d} = (280 \text{ DO}_{3d} + 3 \times 4) / (280 + 3)$$

$$283 \text{ DO}_{4d} - 280 \text{ DO}_{3d} = 12 \quad (21)$$

$$\text{BOD}_{4d} = [280 \text{ BOD}_{3d} + 3 \times 975 \times (100 - E_{i3}) / 100] / (280 + 3)$$

$$283 \text{ BOD}_{4d} - 280 \text{ BOD}_{3d} + 20.25 E_{i3} = 2925 \quad (22)$$

At D₁

$$0.87372 \text{ DO}_{4d} - \text{DO}_{4d1} - 0.09061 \text{ BOD}_{4d} = -1.13652 \quad (23)$$

$$\text{BOD}_{4d1} - 0.90303 \text{ BOD}_{4d} = 0 \quad (24)$$

At E⁻

$$0.76338 \text{ DO}_{4d} - \text{DO}_{4e} - 0.16098 \text{ BOD}_{4d} = -2.12958 \quad (25)$$

$$\text{BOD}_{4e} - 0.81546 \text{ BOD}_{4d} = 0 \quad (26)$$

[Reach 6]

At G

$$\text{DO}_{6g} = (10 \times 7.5 + 8 \times 4)/(10 + 8)$$

$$\text{DO}_{6g} = 5.95 \quad (27)$$

$$\text{BOD}_{6g} = [10 \times 1 + 8 \times 220 \times (100 - E_{m3})/100]/(10 + 8)$$

$$18 \text{ BOD}_{6g} + 17.6 E_{m3} = 1770 \quad (28)$$

At G₁

$$0.86936 \text{ DO}_{6g} - \text{DO}_{6g1} - 0.09206 \text{ BOD}_{6g} = -1.17576 \quad (29)$$

$$\text{BOD}_{6g1} - 0.9123 \text{ BOD}_{6g} = 0 \quad (30)$$

At H⁻

$$0.75578 \text{ DO}_{6g} - \text{DO}_{6h} - 0.163 \text{ BOD}_{6g} = -2.19798 \quad (31)$$

$$\text{BOD}_{6h} - 0.81221 \text{ BOD}_{6g} = 0 \quad (32)$$

[Reach 7]

At H

$$\text{DO}_{7h} = (15 \text{ DO}_{6h} + 5 \times 8)/20$$

$$20 \text{ DO}_{7h} - 15 \text{ DO}_{6h} = 40 \quad (33)$$

$$\text{BOD}_{7h} = (15 \text{ BOD}_{6h} + 5 \times 1)/20$$

$$20 \text{ BOD}_{7h} - 15 \text{ BOD}_{6h} = 5 \quad (34)$$

At I

$$0.93239 \text{ DO}_{7h} - \text{DO}_{7i} - 0.04341 \text{ BOD}_{7h} = -0.60849 \quad (35)$$

$$\text{BOD}_{7i} - 0.95504 \text{ BOD}_{7h} = 0 \quad (36)$$

[Reach 8]

At I

$$\begin{aligned} \text{DO}_{8i} &= (20 \text{ DO}_{7i} + 2 \times 4)/(20 + 2) \\ 22 \text{ DO}_{8i} - 20 \text{ DO}_{7i} &= 8 \end{aligned} \quad (37)$$

$$\begin{aligned} \text{BOD}_{8i} &= [20 \text{ BOD}_{7i} + 2 \times 550 \times (100 - E_{i4})/100]/(20 + 2) \\ 22 \text{ BOD}_{8i} - 20 \text{ BOD}_{7i} + 11 E_{i4} &= 1100 \end{aligned} \quad (38)$$

At M

$$0.83946 \text{ DO}_{8i} - \text{DO}_{8m} - 0.12357 \text{ BOD}_{8i} = -1.44486 \quad (39)$$

$$\text{BOD}_{8m} - 0.86502 \text{ BOD}_{8i} = 0 \quad (40)$$

[Reach 9]

At J

$$\begin{aligned} \text{DO}_{9j} &= (25 \times 7.6 + 7 \times 4)/(25 + 7) \\ \text{DO}_{9j} &= 6.81 \end{aligned} \quad (41)$$

$$\begin{aligned} \text{BOD}_{9j} &= [25 \times 0.9 + 7 \times 270 \times (100 - E_{m4})/100]/(25 + 7) \\ 32 \text{ BOD}_{9j} + 18.9 E_{m4} &= 1912.5 \end{aligned} \quad (42)$$

At J₁

$$0.77724 \text{ DO}_{9j} - \text{DO}_{9j1} - 0.14139 \text{ BOD}_{9j} = -2.00484 \quad (43)$$

$$\text{BOD}_{9j1} - 0.83946 \text{ BOD}_{9j} = 0 \quad (44)$$

At J₂

$$0.60411 \text{ DO}_{9j} - \text{DO}_{9j2} - 0.22859 \text{ BOD}_{9j} = -3.56301 \quad (45)$$

$$\text{BOD}_{9j2} - 0.70469 \text{ BOD}_{9j} = 0 \quad (46)$$

At L

$$0.48675 \text{ DO}_{9j} - \text{DO}_{9l} - 0.27222 \text{ BOD}_{9j} = -4.61925 \quad (47)$$

$$\text{BOD}_{9l} - 0.60653 \text{ BOD}_{9j} = 0 \quad (48)$$

[Reach 10]

At K

$$\text{DO}_{10k} = (28 \times 8 + 10 \times 4)/(28 + 10)$$

$$\text{DO}_{10k} = 6.95 \quad (49)$$

$$\text{BOD}_{10k} = [28 \times 1.2 + 10 \times 300 \times (100 - E_{m5})/100]/(28 + 10)$$

$$38 \text{ BOD}_{10k} + 30 E_{m5} = 3033.6 \quad (50)$$

At L

$$0.75578 \text{ DO}_{10k} - \text{DO}_{10l} - 0.13946 \text{ BOD}_{10k} = -2.19798 \quad (51)$$

$$\text{BOD}_{10l} - 0.83946 \text{ BOD}_{10k} = 0 \quad (52)$$

[Reach 11]

At L

$$\text{DO}_{11l} = (30 \text{ DO}_{9l} + 30 \text{ DO}_{10l})/60$$

$$60 \text{ DO}_{11l} - 30 \text{ DO}_{9l} - 30 \text{ DO}_{10l} = 0 \quad (53)$$

$$\text{BOD}_{11l} = (30 \text{ BOD}_{9l} + 30 \text{ BOD}_{10l})/60$$

$$60 \text{ BOD}_{11l} - 30 \text{ BOD}_{9l} - 30 \text{ BOD}_{10l} = 0 \quad (54)$$

At M

$$0.88692 \text{ DO}_{11l} - \text{DO}_{11m} - 0.06804 \text{ BOD}_{11l} = -1.01772 \quad (55)$$

$$\text{BOD}_{11m} - 0.92772 \text{ BOD}_{11l} = 0 \quad (56)$$

[Reach 12]

At M

$$\text{DO}_{12m} = (60 \text{ DO}_{11m} + 20 \text{ DO}_{8m})/80$$

$$80 \text{ DO}_{12m} - 60 \text{ DO}_{11m} - 20 \text{ DO}_{8m} = 0 \quad (57)$$

$$\text{BOD}_{12m} = (60 \text{ BOD}_{11m} + 20 \text{ BOD}_{8m})/80$$

$$80 \text{ BOD}_{12m} - 60 \text{ BOD}_{11m} - 20 \text{ BOD}_{8m} = 0 \quad (58)$$

At N⁻

$$0.79852 \text{ DO}_{12m} - \text{DO}_{12n} - 0.1128 \text{ BOD}_{12m} = -1.81332 \quad (59)$$

$$\text{BOD}_{12n} - 0.87372 \text{ BOD}_{12m} = 0 \quad (60)$$

[Reach 13]

At N

$$\text{DO}_{13n} = (84 \text{ DO}_{12n} + 4.2 \times 4)/(84 + 4.2)$$

$$88.2 \text{ DO}_{13n} - 84 \text{ DO}_{12n} = 16.8 \quad (61)$$

$$\text{BOD}_{13n} = [84 \text{ BOD}_{12n} + 4.2 \times 1040 \times (100 - E_{i5})/100]/(84 + 4.2)$$

$$88.2 \text{ BOD}_{13n} - 84 \text{ BOD}_{12n} + 43.68 E_{i5} = 4368 \quad (62)$$

At N₁

$$0.79852 \text{ DO}_{13n} - \text{DO}_{13n1} - 0.1282 \text{ BOD}_{13n} = -1.81332 \quad (63)$$

$$\text{BOD}_{13n1} - 0.85642 \text{ BOD}_{13n} = 0 \quad (64)$$

At N₂

$$0.63763 \text{ DO}_{13n} - \text{DO}_{13n2} - 0.21217 \text{ BOD}_{13n} = -3.26133 \quad (65)$$

$$\text{BOD}_{13n2} - 0.73345 \text{ BOD}_{13n} = 0 \quad (66)$$

At E⁻

$$0.50916 \text{ DO}_{13n} - \text{DO}_{13e} - 0.26345 \text{ BOD}_{13n} = -4.41756 \quad (67)$$

$$\text{BOD}_{13e} - 0.62814 \text{ BOD}_{13n} = 0 \quad (68)$$

[Reach 14]

At O

$$\text{DO}_{14o} = (34 \times 7.8 + 7.5 \times 4)/(34 + 7.5)$$

$$\text{DO}_{14o} = 7.1 \quad (69)$$

$$\text{BOD}_{14o} = [34 \times 2.1 + 7.5 \times 350 \times (100 - E_{m6})/100]/(34 + 7.5)$$

$$41.5 \text{ BOD}_{14o} + 26.25 E_{m6} = 2696.4 \quad (70)$$

At P

$$0.81873 \text{ DO}_{14o} - \text{DO}_{14p} - 0.07624 \text{ BOD}_{14o} = -1.63143 \quad (71)$$

$$\text{BOD}_{14p} - 0.91576 \text{ BOD}_{14o} = 0 \quad (72)$$

[Reach 15]

At P

$$\begin{aligned} \text{DO}_{15p} &= (50 \text{ DO}_{14p} + 3.6 \times 4) / (50 + 3.6) \\ 53.6 \text{ DO}_{15p} - 50 \text{ DO}_{14p} &= 14.4 \end{aligned} \quad (73)$$

$$\begin{aligned} \text{BOD}_{15p} &= [50 \text{ BOD}_{14p} + 3.6 \times 700 \times (100 - E_{i6}) / 100] / (50 + 3.6) \\ 53.6 \text{ BOD}_{15p} - 50 \text{ BOD}_{14p} + 25.2 E_{i6} &= 2520 \end{aligned} \quad (74)$$

At E

$$0.86071 \text{ DO}_{15p} - \text{DO}_{15e} - 0.07218 \text{ BOD}_{15p} = -1.25361 \quad (75)$$

$$\text{BOD}_{15e} - 0.92219 \text{ BOD}_{15p} = 0 \quad (76)$$

[Reach 5]

At E

$$\begin{aligned} \text{DO}_{5e} &= [(450 - 84 - 50) \text{ DO}_{4e} + 84 \text{ DO}_{13e} + 50 \text{ DO}_{15e} + 12 \times 4] \\ &\quad / (450 + 12) \\ 462 \text{ DO}_{5e} - 316 \text{ DO}_{4e} - 84 \text{ DO}_{13e} - 50 \text{ DO}_{15e} &= 48 \end{aligned} \quad (77)$$

$$\begin{aligned} \text{BOD}_{5e} &= [(450 - 84 - 50) \text{ BOD}_{4e} + 84 \text{ BOD}_{13e} + 50 \text{ BOD}_{15e} + 12 \\ &\quad \times 240 \times (100 - E_{m2}) / 100] / (450 + 12) \\ 462 \text{ BOD}_{5e} - 316 \text{ BOD}_{4e} - 84 \text{ BOD}_{13e} - 50 \text{ BOD}_{15e} + 28.8 E_{m2} &= 2880 \end{aligned} \quad (78)$$

At E₁

$$0.86589 \text{ DO}_{5e} - \text{DO}_{5e1} - 0.08262 \text{ BOD}_{5e} = -1.20699 \quad (79)$$

$$\text{BOD}_{5e1} - 0.91119 \text{ BOD}_{5e} = 0 \quad (80)$$

At F⁻

$$0.71462 \text{ DO}_{5e} - \text{DO}_{5f} - 0.16468 \text{ BOD}_{5e} = -2.56842 \quad (81)$$

$$\text{BOD}_{5f} - 0.80493 \text{ BOD}_{5e} = 0 \quad (82)$$

[Reach 16]

At R

$$\text{DO}_{16r} = (32 \times 7.7 + 7.5 \times 4)/(32 + 7.5)$$

$$\text{DO}_{16r} = 7.0 \quad (83)$$

$$\text{BOD}_{16r} = [32 \times 1.5 + 7.5 \times 275 \times (100 - E_{m7})/100]/(32 + 7.5)$$

$$39.5 \text{ BOD}_{16r} + 20.625 E_{m7} = 2110.5 \quad (84)$$

At S⁻

$$0.79453 \text{ DO}_{16r} - \text{DO}_{16s} - 0.09683 \text{ BOD}_{16r} = -1.84923 \quad (85)$$

$$\text{BOD}_{16s} - 0.89137 \text{ BOD}_{16r} = 0 \quad (86)$$

[Reach 17]

At S

$$\text{DO}_{17s} = (46 \text{ DO}_{16s} + 2.5 \times 4)/(46 + 2.5)$$

$$48.5 \text{ DO}_{17s} - 46 \text{ DO}_{16s} = 10 \quad (87)$$

$$\text{BOD}_{17s} = [46 \text{ BOD}_{16s} + 2.5 \times 865 \times (100 - E_{17})/100]/(46 + 2.5)$$

$$48.5 \text{ BOD}_{17s} - 46 \text{ BOD}_{16s} + 21.625 E_{17} = 2162.5 \quad (88)$$

At F⁻

$$0.79453 \text{ DO}_{17s} - \text{DO}_{17f} - 0.12789 \text{ BOD}_{17s} = -1.84923 \quad (89)$$

$$\text{BOD}_{17f} - 0.85642 \text{ BOD}_{17s} = 0 \quad (90)$$

At F

$$\text{DO}_{17ff} = [(500 - 46) \text{ DO}_{5f} + 46 \text{ DO}_{17f}]/500$$

$$500 \text{ DO}_{17ff} - 454 \text{ DO}_{5f} - 46 \text{ DO}_{17f} = 0 \quad (91)$$

$$\text{BOD}_{17ff} = [(500 - 46) \text{ BOD}_{5f} + 46 \text{ BOD}_{17f}]/500$$

$$500 \text{ BOD}_{17ff} - 454 \text{ BOD}_{5f} - 46 \text{ BOD}_{17f} = 0 \quad (92)$$

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