Name: E. Leon Hibbs Date of Degree: May 26, 1957

Institution: Oklahoma A. and M. College

Location: Stillwater, Oklahoma

Title of Study: A SEMINAR REPORT ON A STUDY OF MEASURING AND SURVEYING INSTRUMENTS SUITABLE FOR A GENERAL MATHEMATICS LABORATORY

Pages in Study: 35

Candidate for Degree of Master of Science

Major Field: Natural Science

Nature of Study: This report deals with the measuring and surveying instruments suitable for use in a general mathematics laboratory. It deals specifically with the appearance and description, mathematical principles involved, cost, use, construction, and places to borrow twenty-five of the some one hundred instruments studied.

Findings and Conclusion: A general mathematics laboratory can be a reality in nearly any school system.

Use of Study: As a reference when teaching general mathematics in a laboratory approach method.

ADVISER'S APPROVAL James H. Zant

A SEMINAR REPORT ON A STUDY OF MEASURING AND SURVEYING INSTRUMENTS SUITABLE FOR A GENERAL MATHEMATICS LABORATORY

By

E. Leon Hibbs

Bachelor of Science
Northwestern State College
Alva, Oklahoma
1952

Master of Education
University of Oklahoma
Norman, Oklahoma
1956

Submitted to the faculty of the Graduate School of
the Oklahoma Agricultural and Mechanical College
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
May, 1957

A SEMINAR REPORT ON A STUDY OF MEASURING AND SURVEYING INSTRUMENTS SUITABLE FOR A GENERAL MATHEMATICS LABORATORY

Seminar Report Approved:

____
Seminar Report Adviser

____

____
Dean of the Graduate School

PREFACE

Many teachers of high school mathematics contend that general mathematics should never be included in the curriculum. Others say that general mathematics is one of the most valuable courses offered in our high schools.

The purpose of this report is not to prove or disprove the value of general mathematics, but we must assume that there are classes in general mathematics in our high schools before this report could claim validity.

In fact, general mathematics has largely replaced arithmetic in grades seven and eight and is nearly always one leg of the double track program in ninth grade mathematics curricula.

If one takes the old axiom that the whole is greater than any of its parts and applies it to the field of mathematics, he can easily see that general mathematics could be considered as the whole while the sequential courses such as algebra, geometry, and trigonometry could be considered as the parts.

Some teachers consider the interrelations between the parts of more significance than the parts themselves. I consider a knowledge of the interrelationship of the parts as being a valuable asset as well as a detailed study of the parts as being of cultural and educational value for both the non-college and college bound pupil. In other words, we need both.

We profess that the bulk of the general mathematics curriculum deals with the material which every well-informed citizen should have at his command, but many of our teachers of general mathematics do not possess this information, not to mention the background necessary for teaching it to boys and girls. I felt that I was one of these teachers. With this knowledge of my inadequacy of training and background, I began a study of the whole field of general mathematics.

I first tried to determine the area of general mathematics in which I felt most inadequate. In this endeavor, I always arrived at the fact that I was inadequate in showing the practical applications of mathematics to my pupils. In short, I was one of the students described by Zant[1] when he said:

> Many students who have a thorough knowledge of sequential mathematics do not know the more practical things such as consumer mathematics, simple statistics, the ability to use tables and the like. Hence, we must see that students who are interested in sequential courses in mathematics are also required to make themselves competent in that part of mathematics which is needed by everyone.

---

[1] James H. Zant, "The Improvement of High School Mathematics Courses as Recommended by the Commission on Post-War Plans", The Mathematics Teacher, Vol. 39, (1946), p. 273.

After having established this weakness in my training and background, I then began a study of how to remove this weakness. Realizing that I must become acquainted with as many applications as possible in a short length of time, I obtained as much material as I could on mathematics laboratories.

> Strictly speaking, mathematics is not an experimental science, yet it has an almost unlimited number of applications to economic, commercial, engineering, and industrial situation, and to the natural sciences. These applications are probably more interesting to most students than the abstract material of textbooks. If applications are selected and studied in such a way as to emphasize the mathematical principles which underlie them, they are valuable from a mathematical standpoint.[2]

I was surprised to find that the mathematics laboratory is not just a mere phrase, but that many teachers are having great success in teaching general mathematics in this manner. I was also surprised to find that so much material has been written on this subject.

It must be emphasized that the whole course of general mathematics does not lend itself to laboratory or field work, but many units do have wonderful practical applications. It must also be emphasized that the laboratory method of instruction does not take the place of any part of general mathematics, but only tends to strengthen certain units throughout the course.

---

[2] C. H. Butler and F. L. Wren, <u>The Teaching of Secondary Mathematics</u>, (New York: McGraw-Hill Book Company, Inc., 1951), pp. 120-1.

Several leaders in the field of mathematics teaching feel that the laboratory method of instruction lends itself well to certain phases of mathematics. For example, Mejdak[3] says: "If the teacher wishes to make the mathematics period interesting and worthwhile he should transform it, whenever possible, into a sort of laboratory period [..]" Olds[4] says: "[..] applications should be used in order to increase interest, to aid understanding, to provide practice in applying knowledge, and to give the students and the community an appreciation of the value of mathematical education." Gorman[5] made this statement:

> One of the fundamental elements of the program of instruction in elementary and high school mathematics is that of assisting the pupils to develop clear objective concepts of the useful units of measure and the instruments of business. Moreover, it must provide him with basic experience in using them. The underlying question is: How is the school properly to assist the child to develop these concepts and give him the necessary basic experience? Certainly one essential is the observation and use of the actual instruments by the pupils. Instruction in this element of mathematics therefore in large part should be of the work-shop or laboratory type.

---

[3]Raymond J. Mejdak, "Classroom Practice in the Teaching of Everyday Mathematics," The Mathematics Teacher, Vol. 34, (1941), p. 369.

[4]E. G. Olds, "The Use of Applications for Instructional Purposes," The Mathematics Teacher, Vol. 34, (1941), p. 83.

[5]F. H. Gorman, "What Laboratory Equipment for Elementary and High School Mathematics?" School Science and Mathematics, Vol. 43, (1943), p. 335.

After I became convinced that a laboratory approach to the teaching of a part of general mathematics was a logical and adequate approach, I began an investigation as to which phases of the laboratory material suggested by leaders in the field would be of most value for me to study. After careful investigation, I found that I was weakest in preparation in mathematics of finance and the use of and mathematical principles involved in measuring and surveying instruments. My enrollment in a course entitled "Mathematics of Finance" seemed to be a logical way to correct the first weakness. But what of the measuring and surveying instruments and their applications to general high school mathematics? There was no course available. So, being a persistent individual, I proceeded to find the answer for myself.

During the remainder of this paper, I shall report a part of my findings on "Measuring and Surveying Instruments Suitable for a General Mathematics Laboratory." It is my wish that the bringing together of this material will not only be a great addition to my training as a teacher of general mathematics, but that the reporting of this information will make ready reference material available to myself in my teaching position and for any of the other members of the National Science Foundation teachers who care to make use of this paper.

In order to simplify the report and to make it easier to use for future reference, I shall limit my report to only about twenty-five of the some one hundred instruments studied. These twenty-five were chosen because of either the mathematical principle involved in their uses, ease with which they can be obtained or made, or interesting applications in which they are used.

For further simplification, I shall divide the remainder of the report into six sections in the following manner:

1. The name of the instruments or pieces of equipment, their general appearance, and a source of available classroom-size photographs, where applicable.

2. The mathematical principle involved in each of the instruments, where applicable.

3. The current retail price of each of the instruments. This price will be for the lowest priced instrument of good grade as found in current catalogs of scientific supply houses.

4. Some uses and applications of the instruments adaptable to the mathematics laboratory.

5. The instruments that can be homemade and procedure, where deemed necessary or applicable.

6. The instruments that can be borrowed from other institutions or agencies and where each of these could be borrowed.

ACKNOWLEDGEMENTS

The writer wishes to express his sincere appreciation and gratitude to Dr. James H. Zant, Professor of Mathematics, who has contributed much of his time in listening to ideas on the preparation of this report, and to Maxine Hibbs, wife, who did the typing.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER I

INSTRUMENTS AND THEIR APPEARANCE

Following is a list of instruments chosen for the basis of this report.

(1) Angle mirror
(2) Jacob's staff
(3) Plane table
(4) Alidade
(5) Proportional dividers
(6) Clinometer
(7) Groma or surveyor's cross
(8) Hypsometer
(9) Planimeter
(10) Level
(11) Compass
(12) Transit and rod
(13) Sextant
(14) Vernier
(15) Pantograph
(16) Quadrant
(17) Plumb line and bob
(18) Odometer
(19) Chartometer
(20) Theodolite
(21) Ruler, tape, and meter stick
(22) Micrometer
(23) Balances horn pan type specifically
(24) Carpenter's square
(25) Nephoscope

The angle mirror is used in laying out angles or circles when given a known diameter. It consists of two mirrors on a frame, the mirrors meeting at an angle.

The Jacob's or cross staff has the appearance of a cross with one of the bars shorter than the other. The shorter bar is made so that it will slide along the longer bar in four directions, to the right, left, forward, or backward.

A plane table is nothing more than a drawing board mounted on a tripod. The plane table usually has a compass along one edge. It is free to rotate on the tripod, but fitted with a locking device so that it can be held in one position.

An alidade is nothing more than a ruler with a sighting device attached. Some alidades have sights with vertical slits at each end and others have telescopes attached.

Proportional dividers or compasses consist of two equal bars joined by an adjustable screw in which the distance from the screw to one end of one bar is equal to the distance from the screw to one end of the second bar. This also makes the distances from the screw to the other ends of the bars equal to one another. This is the same as having two sets of dividers with different lengths of legs but open at the same angle.

"The clinometer is a sighting tube with cross wires placed at the outer end. A quadrant scale graduated in degrees from 0 to 90 is mounted on the side."[1]

"The Surveyor's Cross is a modern adapation of the groma of Roman times. The former consists of a double

---

[1]B. C. Haynes, Techniques of Observing the Weather, (New York: John Wiley and Sons, Inc., 1947), pp. 55-56.

double alidade with the two parts rigidly set at right angles on a tripod."[2]

A hypsometer consists of a rectangle that has been divided into small squares and a plumb line and bob attached to one corner of the rectangle.

A planimeter is an instrument used to measure areas of irregularly shaped figures. It appears to be two rods connected by a hinge. The other end of one of the rods is arranged so that it will not slide while the free end of the second rod is on a roller with the roller being graduated.

Nearly everyone has seen a carpenter's level. The surveyor's level is very similiar except that it has a telescope attached and the level is attached to a tripod when in use.

The compass is a direction finder and appears in many forms. Most people have seen one at an early age in life. The compass is simply a magnetized needle free to turn on an axis.

The transit is simply a telescope mounted on a vertical protractor and these two parts can be turned on a horizontal protractor. The leveling rod is used in conjunction with the transit and level and is just a long rod marked off in equal lengths similiar to a ruler.

---

[2] A. Leonhardy, M. Joseph, and R. D. McLeary, New Trend Geometry, (New York: Charles E. Merrill Co., 1940), p. 341.

The sextant consists of a sight piece, two mirrors and a graduated arc arranged so that angles in any plane can be measured.

In 1631, Pierre Vernier invented the vernier. It consists of a small sliding scale parallel to a large one. Verniers are found on sextants, mercurial barometers, and other precision instruments.

The pantograph consists of at least four rods joined to form a parallelogram. It is adjustable to form other parallelograms having sides of different ratios.

A quadrant is a quarter of a circle. In making use of this instrument, we attach a plumb line and bob at the right angle [or center of the circle] and graduate the arc in degrees.

A plumb line and bob is simply a string with a weight attached to one end.

The odometer invented by Leonardo Da Vinci is a wheel with an attached counter that counts the number of revolutions the wheel makes when it is rolled along the ground.

The chartometer is very much like the odometer only much smaller. The chartometer is used with paper on a desk while the odometer is used in the field.

The modern theodolite looks much like the modern transit but differs in one respect. Instead of having the telescope directly lined with the eyepiece as in the transit, the theodolite has the tube of the telescope perpendicular to the eyepiece with a mirror to reflect the light. It will be shown later why there is a difference in construction.

The micrometer caliper is U shaped with a bar that can be extended from the inside of one end of the open U to the other.

A balance is a bar with half of the bars weight extending on either side of a knife edge on which the bar is resting. The horn pan type of balance has pans of equal weight attached to the ends of the bar which allows objects to be placed in the pans and compared as to weight.

A carpenter's square is two bars meeting one another at one end of each bar at right angles. One bar is usually larger than the other.

A nephoscope looks very much like a television antenna and mast except that the crosspieces are the same length and all the crosspieces are equidistant apart.

Classification of Mathematical Instruments,[3] With Sources*

*Abbreviations used:
B. & S. - - Browne and Sharpe Co., Providence, R. I.
Cenco Cat - Catalog of the Central Scientific Co., Chicago, Illinois.

---

[3]The National Council of Teachers of Mathematics, Multi-Sensory Aids in the Teaching of Mathematics, Eighteenth Yearbook, (New York: Teachers College, Columbia University, 1945), pp. 198-200.

Comp. - Used in "From Comp" to mean picture supplied by the manufacturer.
K & E - Keuffel and Esser Co., New York City.
K W - - Catalog, Kelvin White Co., Boston, Massachusetts.

In the following classification will be found the name of the instrument, followed by a few selected sources of pictures large enough to be of use in the classroom.

Section I - MEASUREMENT INSTRUMENTS
Part A: Laboratory Instruments

....

2. Length
Calipers - Cenco Cat.; B. & S. Cat.
Micrometer screw - Cenco Cat.; B. & S. Cat.

3. Mass and Weight
....
Analytic Balances - Cenco Cat., From Comp.
....

Part B: Applied Instruments

1. Surveying
Groma - So. Kensington Museum
Quadrant - Third Yearbook National Council of Teachers of Mathematics
Plane Table - K & E Catalog, From Comp.
Theodolite - So. Kensington Museum, From Comp.
Transit - K & E Cat., From Comp.
Level - K & E Cat., From Comp.
Leveling Rod - K & E Cat.
Surveying Compass - K & E Cat.

2. Navigating
....
Sextant - K & E Cat., From Comp.
Nautical Compass - K W Cat.
....

Section II - CALCULATION INSTRUMENTS
Part A: Graphical Methods

1. Drawing Instruments
Proportional dividers - K & E Cat.

Pantograph - K & E Cat.

....

2. Nomographs
   Vernier Scale - Picture of sextant
3. Lengths of curves
   Curve Measurer - A. Galle, _Mathematische Instrumente_, Leipzig, 1912, p. 65.
4. Areas
   Planimeter - K & E Cat., From Comp.

....

Part C: Solving Equations

1. With balances - L. Jacob, _Calcul Mecanique_, Paris, 1911, p. 132.

....

CHAPTER II

THE MATHEMATICAL PRINCIPLES INVOLVED

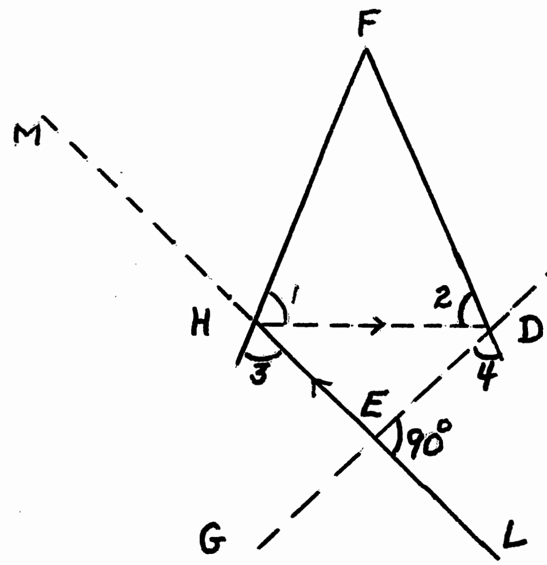

Figure 1. Principles of angle mirror

If a perpendicular is to be constructed to line KE at E, sight from G through a window above the mirror at D to stake K at a convenient point in line KE. Then have a second stake L set up by an assistant so that its image is reflected twice, as indicated by the arrows in the diagram, and appears in the mirror at D, directly under and in line with the stake K seen through the window. Then the line EL is perpendicular to KE as desired.[1]

The mathematical principle is that angle KEL will always be twice angle HFD.

---

[1]Leonhardy, Joseph, and McLeary, p. 147.

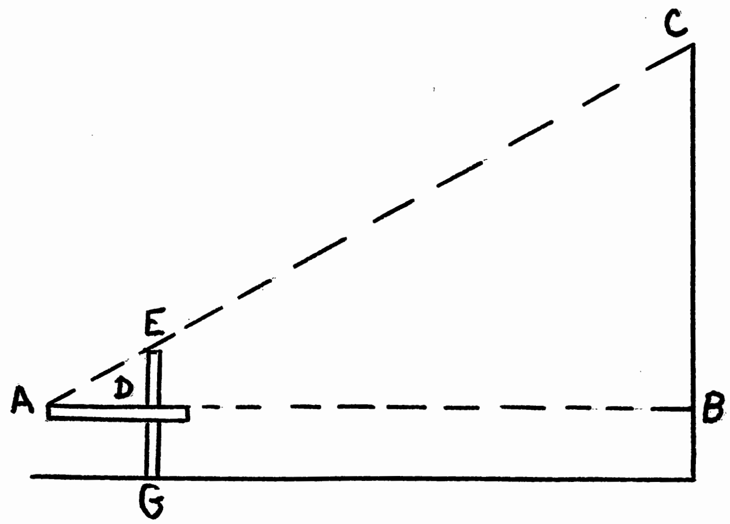

Figure 2. Principle of measurement of vertical distance with Jacob's staff.

$$BC = \frac{AB \times DE}{AD}$$

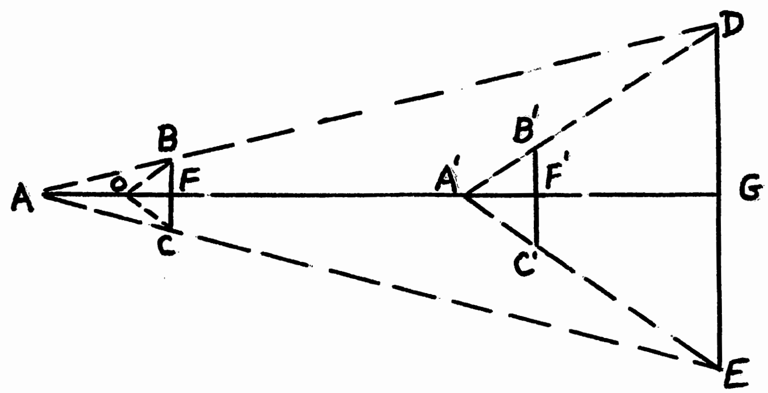

Figure 3. Mathematical Principle of measuring horizontal inaccessible distance with a Jacob's staff.

$$DE = \frac{AA \times BC}{AO}$$

The fundamental operation of the Jacob's staff is that of measuring the angle of sight of a given object. This is done by placing at a determinable distance from the eye a suitable object such as a short rod, so that its ends coincide with the lines of sight from the ends of the object to be measured.

> The natural means of carrying out such an operation are the outstretched arm and the fingers of the hand held close together and at right angles to the arm. At some period of man's history some unknown individual must have noticed the relationship between the size of the angle of sight and the number of fingers held together.[2]

The method used in the simplest form of a Jacob's staff is to change the length of the arm which varies the angle instead of varying the length determined by the closed fingers.

One finds the desired distances by solving for parts of similiar triangles as shown in Figures 1 and 2.

The plane table and alidade are used as a team in elementary mapping and surveying. Hence, they are discussed as a team rather than separate instruments.

They use one of two principles, either the principle of radiation from a point or intersections of lines of sight taken from both ends of a given, or established, base line. In either case the alidade is the sighting instrument and the plane table is the drawing board, level, and point of reference.

Proportional dividers are used for enlarging or reducing maps and such. This instrument is simply based on the principle of proportionality of corresponding parts of similiar triangles.

---

[2]The National Council of Teachers of Mathematics, Surveying Instruments, Nineteenth Yearbook, (New York: Teachers College, Columbia University, 1947), p. 83.

The clinometer illustrates the principle of indirect measurement. It is used to find the angle of elevation or depression. One can then measure one side of the right triangle and solve for any of the other parts by use of trigonometric functions.

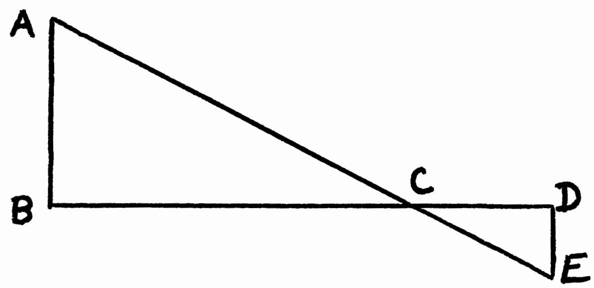

Figure 4. Method of indirect measurement with a groma.

> The Surveyor's Cross is a modern adaptation of the groma of Roman times. The former consists of a double alidade with the two parts rigidly set at right angles on a tripod. The purpose of this instrument is to construct right angles, and it is used particularly in laying out streets and highways.[3]

We shall see in Chapter IV how the groma may be used to measure indirectly the line AB in Figure 4.

The hypsometer is a modern form of the geometric square. Direct measurements can be made with it when one side of a similar triangle is intentionally made equal to scale to one side of the triangle to be solved. This makes it possible to read the dimensions of the other two sides directly from the instrument.

---

[3]The National Council of Teachers of Mathematics, A Source Book of Mathematical Application, Seventeenth Yearbook, (New York: Teachers College, Columbia University, 1942), p. 205.

The tracer of a planimeter is passed round the periphery of an area to be measured. This moves the flange wheel, which either turns or slides on the paper according to the direction the tracer is moving.

The planimeter is actually an integrater. The instrument simply measures the ordinate of the integral curve and we multiply by the constant of the machine to get the area of the figure.

The basic principle involved in the level which is used in surveying is to establish planes in which the lines of sight will be horizontal.

The principle involved in the compass is that no matter where a person is located on the northern hemisphere, one end of the compass needle nearly always points to the same point.

> The compass needle does not point to the true north pole but to the magnetic pole. The angle between true north-south meridian and the magnetic meridian is called the magnetic declination. Imaginary lines connecting places having the same declination are called isogonic lines. These lines are very irregular. The declination in the United States ranges from 22 degrees west to 24 degrees east.[4]

The transit is an instrument used to measure angles as well as to level. It measures angles in both vertical and horizontal planes. "It thus provides a combined

---

[4]The National Council of Teachers of Mathematics, Selected Topics in the Teaching of Mathematics, Third Yearbook, (New York: Teachers College, Columbia University, 1928), p. 215.

means for securing all data needed in any field problems in surveying or mapping or in the indirect measurement of distances."[5]

The leveling rod is required for all leveling work and for some of the work with the transit. It is simply an oversized ruler.

> A convenient hand instrument used for measuring angles in any plane and an indispensable aid in determining the latitude of a ship at sea is a sextant. The sextant's name is derived from the fact that it contains a sixty degree arc of a circle. In Figure 5, DD' is a sixty degree arc of a circle whose center is M; this arc is graduated from zero degrees to 120 degrees. MC is a movable arm, which has a mirror at M in line with MC and perpendicular to the plane of the sextant. Fixed at M' is another mirror, the horizon glass, also perpendicular to the plane of the sextant and parallel to MD'. E is a small telescope fixed across MD' and pointed toward the mirror M'. Since only the lower half of this mirror is silvered, if the observer wishes to find the angle of elevation of the sun, he first sights the horizon through the unsilvered portion. Then the arm MC is moved so that the image of the sun is reflected from mirror M to mirror M' and thence through the telescope to the eye. The observation consists in bringing the horizon and the image of the sun into coincidence. The angle of elevation of the sun is read on the graduated arc.
>
> Because it is often difficult to see the horizon, a bubble sextant is usually used. A small spirit level is attached to the tube of the telescope, and when the bubble is centered, the effect is the same as that which is produced when the horizon is sighted.[6]

---

[5]Butler and Wren, p. 115.

[6]Leonhardy, Joseph, and McLeary, pp. 231-233.

Angle MFM' is equal to twice angle D'MC. Hence, arc DD' must be graduated from zero degrees to 120 degrees. This comes from the fact that an exterior angle of a triangle equals the sum of the opposite interior angles.

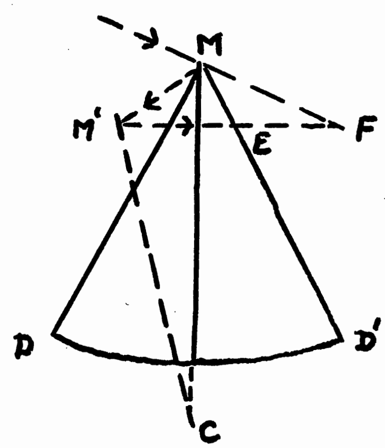

Figure 5. Principles of sextant

A vernier is sometimes attached to a sextant or other instruments when accurate measurements are wanted.

The fact that two scales side by side one of which has all primary units subdivided into ten equal parts and the other has nine of these subdivisions divided into ten equal parts which makes each division of this scale one-tenth of a primary subdivision shorter than a primary subdivision enables us to measure accurately to one-one hundredth of the main unit. An instrument such as this is called a vernier, after its inventor Pierre Vernier.

The pantograph, an instrument used in enlarging and reducing maps and pictures is based on this principle; if two polygons are radically situated, they are similar.

The quadrant, as the name implies, is a quarter arc of a circle and the principle involved in its use is the same as in the clinometer.

The plumb-bob when suspended from a line will point the line to the center of the earth on the bob end and toward the zenith on the suspended end.

The odometer and chartometer use the principle that the circumference of a circle is two times pi times the radius of the circle. Thus by counting the number of times the wheel rotates one can obtain the distance traveled by measuring the radius of the wheel and computing the circumference.

The theodolite uses exactly the same mathematical principles as the transit.

The ruler, tape, and meter stick are used by the principle of finding a certain distance in terms of a smaller known distance by counting the number of smaller distances within the larger distance.

The micrometer uses the principle of the vernier in measuring distances very accurately.

The horn pan balance uses the principle of equality. Two objects equal in mass, one in each pan, will make the bar of the balances assume a horizontal position.

The carpenters square uses the geometry of the right triangle as its basis.

The nephoscope uses the principle of similar triangles and parallel lines to obtain the direction of movement and velocity of clouds.

CHAPTER III

COST OF INSTRUMENTS

TABLE I

APPROXIMATE COST OF INSTRUMENTS

Name of Instrument	Approximate Cost Each
Angle Mirror	$ 20.00
Jacob's Staff	5.25
Plane Table and Tripod	80.00
Alidade without telescope	20.00
Proportional dividers	9.25
Clinometer	35.00
Surveyor's cross	[not listed in catalogs]
Hypsometer	[not listed in catalogs]
Planimeter	31.00
Level surveyor's	38.00
Compass	1.50
Transit and Rod	70.00
Sextant student type	4.95
Vernier on calipers	6.00
Pantograph	5.00
Quadrant	[not listed in catalogs]
Plumb line and bob	2.00
Odometer	12.00
Chartometer	5.00
Theodolite	600.00
Ruler	.50
Tape	2.00
Meter stick	.50
Micrometer	9.00
Horn pan balance	3.15
Carpenter's square	4.00
Nephoscope	[not listed in catalogs]

CHAPTER IV

USE OF INSTRUMENTS IN CLASS

Figure 6. One use of angle mirror

Figure 6 shows an exercise that can be performed with an angle mirror, tape, and pole. A person wants to sketch a map of the bank of a creek or some other irregular line.

> Lay off a straight line AB some distance from the creek and mark every ten feet or some other interval. Then place at pole at B and have one student at A with an angle mirror while another student stands at the bank at C with another pole. The student at A has the one at C move until both poles line up. Angle C,AB is then ninety degrees. The distance C,A is measured and noted. As soon as all the offsets are measured it is easy to plot the map by connecting C_1, C_2, and so on. The area between the line AB and the creek may be found by using the trapezoidal rule: Multiply the distance between the offsets by half the sum of the first and last offsets plus the sum of the other offsets.[1]

---

[1]The National Council of Teachers of Mathematics, Third Yearbook, p. 205.

Other projects that can be solved by use of the angle mirror are the laying out of a football field or tennis court.

A Jacob's staff can be used to measure vertical or horizontal distances when one or both ends of the line one wishes to measure is inaccessible. Some examples are the height of a flagpole or the distance between two points on the other side of a river [there isn't a bridge and the operator can't swim].

The plane table and alidade can be used to map an area bounded by straight lines and also to find the horizontal distance between two points, each inaccessible from the other and from any third point. The Jacob's staff can also be used in this case of inaccessibility.

Proportional dividers are extremely fascinating to students and can be used to enlarge or draw to scale almost any picture, map, or drawing.

The clinometer is used to find the angle of elevation of any structure such as a building or flagpole. The tangent of the angle found in this manner times the distance from the structure will give the height of the structure.

The chief use of the groma is to lay out right angles. It can be used to make indirect measurements such as AB in Figure 4 of this paper. Angles B and D are made right angles by use of the groma, if BCD and

ACE are straight lines, and if the distance BC, CD, and DE are known or can be measured then the length of AB can be computed.

The hypsometer, a simple form of height-measuring instrument, is often used by forest rangers and lumbermen to determine the height of trees. Its uses are very similiar to those of a clinometer only a different principle of mathematics is used in the hypsometer as described in Chapter II.

The only practical use of the planimeter is to measure area of regular or irregular figures drawn on a piece of paper. However, this area on the paper may be a map and in proportion to a larger area such as a field or pasture thereby making it possible to determine the actual area of a field by this method.

To level means to find how much higher or lower one point on the earth's surface is than another. There are several factors such as difference in elevation and the nature of the terrain between the points that affect the method of leveling. The modern bubble tube level isn't necessary in carrying out these projects but is certainly one of the instruments which make the job much easier. However, some of the simple instruments mentioned earlier may be used if the modern level is not available.

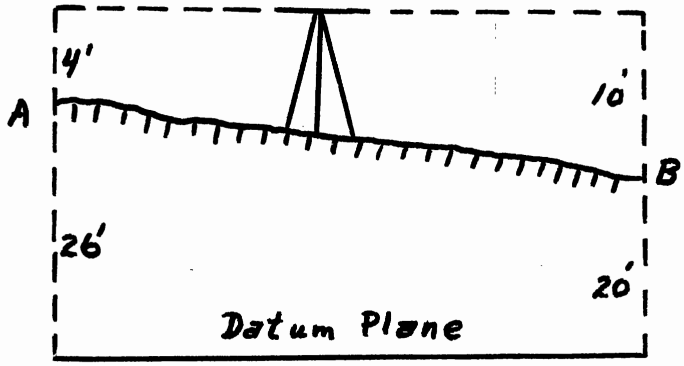

Figure 7. Principle of leveling

> The fundamental principle of ordinary leveling is made clear in Figure 7. The height of a point A above an arbitrarly chosen horizontal plane the datum plane is assumed. A level instrument is set up in such a position that sights can be taken on a rod held at A and B. All the lines of sight from the instrument at any fixed height determine a horizontal plane. Suppose that the sight on A reads 10 ft., while that on B reads 4 ft. Then the horizontal plane of sight is 10 ft. above A or 30 ft. above the datum plane; therefore B, which is 4 ft. below the plane of sight must be 26 ft. above the datum plane, and the difference in elevation between A and B is 6 ft.[...][2]

If the points A and B are so far apart that they both cannot be seen from one point, then the elevation of points between A and B must be found and the leveling instrument moved to these points until the difference in elevation of A and B can be calculated.

A compass can be used in the classroom to show the students how to determine directions if they are lost.

---

[2]The National Council of Teachers of Mathematics, Nineteenth Yearbook, p. 354.

It doesn't lend itself well to repeated exercise, but every person should know how to use a compass.

The transit can be used to solve any of the problems which can be solved by the use of the level, hypsometer, groma, clinometer, Jacob's staff, or angle mirror previously described. It can also be used to solve problems in which the quadrant or theodolite are used which will be described presently.

The sextant can be used to determine latitude and the height of objects. March twenty-first and September twenty-third are the best dates to determine latitude. On these two dates there need not be correction for declination.

Most sextants have a vernier attached to their arc. This makes it possible to teach the use and principles of the vernier right along with teaching the use and principles of the sextant. This offers another opportunity of teaching two principles at one time.

The pantograph can be used to enlarge or reduce any plane figure of reasonable size.

The quadrant can be used to measure the angle of elevation of any point above the horizon. This is due to the fact that angles with the same complement are equal.

The quadrant can also be used in leveling which was described earlier in conjunction with the level.

It is interesting to note that the first attempts to measure the earth's circumference were made by the use of the quadrant.

The plumb line and bob have no use in their own right other than to find the vertical. However, they are employed as a part of certain instruments such as the quadrant and hypsometer.

The odometer can be used to measure distances such as the dimensions of a football field or even curved distances such as the length of a terrace line or curved sidewalk. This fascinates boys and girls because they can quickly measure long distances with no more of a measuring device other than a short ruler to determine the radius of the wheel used on the odometer.

The chartometer can be used to measure the perimeter of any plane figure or the circumference of solid figures such as the cylinder. A good exercise is to measure the miles of shoreline of a certain lake on a map which has been drawn to the desired scale.

Wind direction and velocity aloft can be determined by the use of a pilot baloon and a theodolite. The theodolite is a transit with the eyepiece making right angles to the main tube of the telescope instead of directly in line with it. The pilot baloon filled with a certain amount of helium will ascend at a known rate. Therefore it can be followed with the theodolite and the baloon's elevation and azimuth determined at certain time intervals in which the baloon should be at a known altitude. This makes it possible to calculate the wind direction and velocity at that altitude.

A transit could be used for this work and sometimes is. The only difficulty involved is that the transit is always pointing near the vertical and the observer soon tires at his job when he has to look up for any length of time. The theodolite with a mirror arrangement allows the observer to look in a horizontal direction all of the time.

Nearly everyone knows of many uses of the tape, ruler, and meter stick. The chief purpose of having a meter stick in a mathematics program is to introduce the metric system. Some examples would be for the boys and girls to determine the length of their feet in centimeters. Many young people do not actually understand the construction or use of the ruler. The first step, then, in the effective teaching of linear measurement is the study of the measuring instrument. The teacher should know that the pupils understand the construction and principles of the instrument before asking for specific exercises to be done by the pupils.

An interesting demonstration with a tape is easily performed. A right angle can be laid out with a one hundred foot tape as shown in Figure 8 with the ends of the tape at A and B and the right angle at B.

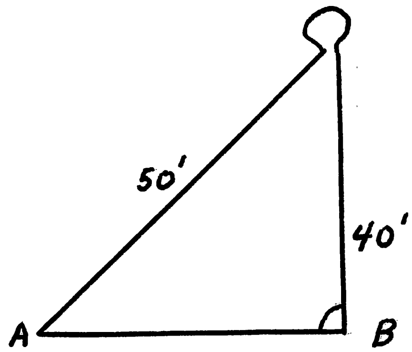

Figure 8. Laying out a right angle

A micrometer can be used to measure the thickness of a sheet of paper and other small dimensions that are nearly impossible to measure by any other ordinary means.

The horn pan balances are sometimes used to show that two objects are equal in mass thereby emphasizing the principle of equality.

The carpenter's square is a versatile instrument for carpenters as well as mathematics laboratories. Students use it to find the horizontal distance between two points, one being accessible, to determine the height of an object, to bisect an angle, to lay out parallel lines, or to solve nearly any problem involving proportion.

The use of the carpenter's square constitutes a five semester hour course in several universities. This statement is made so the reader might realize the many applications to which one might put the carpenter's square.

The only use of the nephoscope found was to find the direction and velocity of some object in the sky such as a cloud, bird, or airplane when the object's height is known. This last statement "height is known" renders this instrument almost useless to the general mathematics student. Therefore, a discussion of this instrument to any great extent other than to say that it is an extremely interesting application of the principles of similiar triangles would be somewhat of a waste of time.

CHAPTER V

HOW CAN ONE MAKE THESE INSTRUMENTS?

Homemade instruments will not usually give as precise measurements as commercial instruments, but for classroom use they are sufficiently accurate. Homemade instruments have two advantages over the commercially produced. First, students take more pride in equipment made by themselves. This helps in motivating them to use the instruments. Second, the mathematical principles upon which the instruments are based are more likely to be understood by the children if they make their own instruments.

The angle mirror is easily constructed by using a miter box to cut two grooves in a short piece of one by four lumber at forty-five degree angles. These grooves should overlap one another near the edge of the board and be just wide enough so that cheap mirrors will fit snugly into the grooves meeting at the point of intersection of the grooves. When the mirrors are inserted the instrument is completed.

A Jacob's staff can be constructed from two yard sticks and a stove bolt with a wing-tip nut. Cut one foot off one of the yard sticks, saving the two foot piece for the instrument, then take both sticks and remove a center portion about one-fourth inch wide extending to within one inch of the ends of the sticks. Then place the shorter stick on the longer one in the form of a cross, insert the bolt through both sticks, and tighten the nut.

Butler and Wren[1] have given complete directions on how to make the plane table, alidade, and clinometer.

Proportional dividers can be made by a procedure much the same as in the making of a Jacob's staff. The chief differences are that the sticks should be of equal length and shorter [the length used is entirely dependent upon the wishes of the craftsman] than in the former procedure and both sticks should be pointed on both ends. A good way to make the point is to start about two inches from an end of the stick from the inside of the angle formed and cut diagonally to the very tip of the stick to the outside of the angle.

A surveyor's cross can be made by taking two rulers and fastening them rigidly together with their flat sides toward each other. Pins can be inserted into the rulers equidistant from the edges to be used as sights. It is very important that the rulers cross each other at right angles and at points midway between their ends.

---

[1]Butler and Wren, pp. 119-120.

A hypsometer is easily constructed by pasting a sheet of ordinary graph paper on a rectangular board and attaching a plumb bob and sights. Shellac will help to protect the paper. The plumb bob should be attached directly under the far sight and so that the plumb line will superimpose one of the dark division lines of the graph paper when the hypsometer is sighted in a horizontal position.

The planimeter, theodolite, and micrometer are much too difficult to construct to be attempted by an amateur. It seems best to leave their construction to the professional instrument makers.

The Eighteenth Yearbook[2] of The National Council of Teachers of Mathematics gives a complete description of how to make a transit and their Nineteenth Yearbook[3] shows how to make a level.

A leveling rod may be made by taking a ten foot one by four board and marking it off in feet and inches or tenths of feet.

A compass is made by magnetizing a needle by rubbing it on a magnet, inserting the needle through a cork with

---

[2]The National Council of Teachers of Mathematics, Multi-Sensory Aids to the Teaching of Mathematics, Eighteenth Yearbook, (New York: Teachers College, Columbia University, 1945), pp. 189-191.

[3]The National Council of Teachers of Mathematics, Nineteenth Yearbook, pp. 129-141.

each end sticking out an equal distance, and floating this apparatus in a container of water.

Since a student sextant can be obtained at a nominal price it is not recommended that time be spent in trying to construct one.

The pantograph could be constructed fairly easily, but time spent in constructing one could be better used, since the principle of the pantograph can be demonstrated by even a toy pantograph which is available in local department stores at a lower price than enumerated in Chapter III.

> Construct a simple scale with all primary units subdivided into ten equal parts along one edge of a piece of cardboard in the making of a vernier. On another cardboard draw a line equal in length to nine of these subdivisions and subdivide this again into ten equal parts. Each division of this scale is one-tenth of a primary subdivision shorter than a primary subdivision. Therefore, when the zero of the secondary scale coincides with the zero on the main scale, the first division of the secondary scale is one-one hundredth of the main unit shorter than the first primary subdivision of the main scale. This value is known as the least count of the vernier [...]
>
> In general the division of the vernier which coincides with a division mark on the main scale gives the reading of the second decimal point on such a scale.[4]

A quadrant is constructed by cutting out one-fourth of a circle from a piece of lumber on a jig-saw, marking

---

[4]The National Council of Teachers of Mathematics, Nineteenth Yearbook, p. 342.

this arc off in degrees, and attaching a plumb bob and line at the point of the ninety degree angle which is a radius distance from the arc.

A plumb bob and line is made by taking a small weight such as a fisherman's weight and attaching it to a piece of string.

An easy way to improvise an odometer is to take the front wheel and fork from a bicycle and paint a stripe across the tire. This enables the operator to count the number of turns the wheel makes when pushed over the line to be measured.

A typist's eraser in the form of a small wheel should be marked so that the number of turns can be counted when it is rolled over a line to be measured and one will have a perfectly good chartometer.

Rulers, tapes, and meter sticks can be made by carefully copying from a commercially manufactured specimen.

Horn pan balances need not be purchased if they are not available. Simply attach small Easter egg baskets or something similar to a meter stick or yard stick that has been balanced on a knife edge. Care should be taken that the balance is not disturbed when the "pans" are attached.

Since carpenter's squares are so readily available, time should not be spent in trying to construct one.

A nephoscope is not of much value unless the height of the object observed is known. Therefore, unless a nephoscope is readily available, a class in mathematics should not be concerned too much in the actual use of this instrument, but the study of the mathematical principle involved and the showing of a picture of a nephoscope certainly seem to be worthwhile.

CHAPTER VI

WHERE CAN ONE BORROW THEM?

It is possible to borrow the following instruments from the county engineer, soil conservation district, or county agricultural stabilization committee. Some of them are sometimes borrowed from the parents of boys and girls in the class where they will be used: angle mirror, plane table, alidade, proportional dividers, clinometer, planimeter, surveyor's level, transit and rod, odometer, chartometer, and pantograph.

A school nearly always owns a carpenter's square, horn pan balances, rulers, tapes, and meter sticks.

There are not many of the following instruments available in many communities so it would be very difficult to borrow them: nephoscope, theodolite, quadrant, sextant, hypsometer, surveyor's cross, and Jacob's staff.

However, most of these are easily constructed so they are readily available to any teacher who wants to go to a little trouble to have them made.

Usually some student in the class will have a compass, and is glad to make it available.

Verniers are usually attached to some of the instruments mentioned above.

The micrometer can be borrowed from machine shops in the community.

A WORD OF CAUTION

Some of these instruments are quite expensive as has been shown and some are also very delicate. Therefore, if an instrument is borrowed, the teacher should assume full responsibility in seeing that the instrument is returned as soon as possible and in as good condition as it was when borrowed. It is better to make or buy an instrument if possible because of the fact that ill-feeling is sometimes created from borrowing.

It has been said that the pure mathematician is never as happy as when he does not know what he is talking about - W. F. G. Swann.

When you can measure what you are speaking about and express it in numbers you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind. Lord Kelvin

BIBLIOGRAPHY

Butler, C. H., and Wren, F. L. _The Teaching of Secondary Mathematics_. New York: McGraw-Hill Book Co., 1951

Gorman, F. H. "What Laboratory Equipment for Elementary and High School Mathematics?" _School Science and Mathematics_, Vol. 43, 1943.

Haynes, B. C. _Techniques of Observing the Weather_. New York: John Wiley and Sons, Inc., 1947.

Leonhardy, A., Joseph, M., and McLeary, R. D. _New Trend Geometry_. New York: Charles E. Merrill Co., 1940.

Mejdak, Raymond J. "Classroom Practice in the Teaching of Everyday Mathematics." _The Mathematics Teacher_, Vol. 34, 1941.

The National Council of Teachers of Mathematics. _A Source Book of Mathematical Application_. Seventeenth Yearbook, New York: Teachers College, Columbia University, 1942.

________. _Multi-Sensory Aids in the Teaching of Mathemathics_. Eighteenth Yearbook, New York: Teachers College, Columbia University, 1945.

________. _Selected Topics in the Teaching of Mathematics_. Third Yearbook, New York: Teachers College, Columbia University, 1928

________. _Surveying Instruments_. Nineteenth Yearbook, New York: Teachers College, Columbia University, 1947.

Olds, E. G. "The Use of Applications for Instructional Purposes." _The Mathematics Teacher_. Vol. 34, 1941.

Zant, James H. "The Improvement of High School Mathematics Courses as Recommended by the Commission on Post-War Plans." _The Mathematics Teacher_. Vol. 39, 1946.

VITA

Elvin Leon Hibbs

Candidate for the Degree of

Master of Science

Seminar Report: A SEMINAR REPORT ON A STUDY OF MEASURING AND SURVEYING INSTRUMENTS SUITABLE FOR A GENERAL MATHEMATICS LABORATORY

Major Field: Natural Science

Biographical:

Personal data: Born near Balko, Oklahoma, October 15, 1930, the son of P. O. and Louella Hibbs.

Education: Attended elementary school in Balko and Beaver, Oklahoma; graduated from Beaver High School in 1948; received the Bachelor of Science degree from Northwestern State College, Alva, Oklahoma, with a major in Mathematics, in May, 1952; received the Master of Education degree from the University of Oklahoma, with a major in Administration, in August, 1956; completed requirements for the Master of Science degree in May, 1957.

Professional experience: Entered teaching profession in 1952 as an elementary and junior high school teacher at Greenough Consolidated School, Liberal, Kansas, also as Principal of this same school until May, 1956.

Professional organizations: Beaver County Schoolmaster's, Beaver County Teachers, Oklahoma Education Association, National Education Association, National Council of Teachers of Mathematics, National Association of Secondary School Principals, Kappa Delta Pi, Who's Who in American Colleges and Universities, 1951.