INVESTIGATION OF FEED POINT IMPEDANCE OF THE THREE ELEMENT PARASITIC ANTENNA ARRAY

By

DANIEL P. HEARN

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Oklahoma Agricultural and Mechanical College

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THESIS AND ABSTRACT APPROVED:

Harold Fristoe
Thesis Adviser

A. Naeter
Faculty Representative

D. C. McIntosh
Dean of the Graduate School

PREFACE

The author gratefully acknowledges the assistance of Professor H.T. Fristoe, for his guidance during the experimental phases of this problem and for his many helpful suggestions during the preparation of the paper. He also wishes to thank Professor A.L. Betts and Professor D.L. Johnson for their constructive criticism and for proof reading the manuscript.

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INVESTIGATION OF FEED POINT IMPEDANCE OF THE THREE ELEMENT PARASITIC ANTENNA ARRAY.

I. INTRODUCTION.

It is a well known fact that the parasitic type of close spaced antenna array will provide more directional gain per unit space than any other type of antenna. Due to the large number of variables in the device however, its development has been slow and many of its essential characteristics are as yet unknown. It has been only about one year since the first satisfactory mathematical analysis of the simpler two element array was published. [1] No mathematical analysis has as yet been published for the three element type of array.

Due to the complexity of the mutual impedances existing between the elements of the three element parasitic array a thorough analysis would be exceedingly difficult. Up to the present time a small amount of experimental data have been taken on this type of antenna but the functioning of the mutual impedances is still somewhat in doubt, particularly for element spacings which are not multiples of 0.05 wavelength.

In his original work on the two element array Brown

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[1] Ronald King, "The Field of a Dipole with a Tuned Parasite at Constant Power," Proceedings of the I.R.E., (July, 1948)

found that the maximum forward gain for the two element array occurred at a spacing of 0.10 wavelength for the parasitic element when used as a director and at 0.15 wavelength when used as a reflector. [2]

Attempts have been made to extend these results to the arrays using more than two elements by spacing all reflectors 0.15 wavelength and all directors 0.10 wavelength in the belief that these spacings would give maximum gain. Such was not the case. It soon became evident that spacings other than these gave higher gain. Much confusion still exists on the subject of element spacing for arrays using more than two elements.

Since it is very difficult, even though remotely possible, to analyze the parasitic array mathematically, the only other approach is to measure experimentally the characteristics of the array under all possible combinations of spacings. The two characteristics of most interest are the feed point impedance and the gain of the antenna.

II. EXPERIMENTAL ANALYSIS.

It was decided that a reasonable approach to an experimental analysis of the three element parasitic array would be to vary the spacing of the director and reflector in small increments and at each spacing adjust the element

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[2] G. R. Brown, "Directional Antennas," <u>Proceedings of the I.R.E.</u>, (January, 1937)

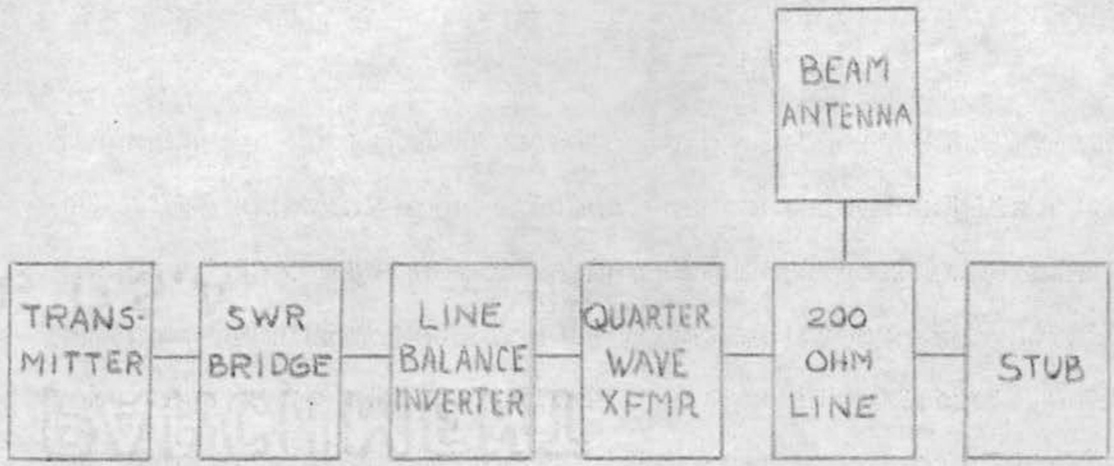
lengths for maximum forward gain as indicated by greatest field strength. Once this adjustment was established, the feed point impedance and gain were measured. The relative gain measurement was made by first removing the parasitic elements and measuring the field strength of the dipole. The power applied to the line was noted and established as the reference throughout the succeeding measurements. The parasitic elements were then replaced and adjusted for maximum forward field strength. At the same time the beam was matched to the transmission line, keeping the power applied to the line at the established reference value. The gain of the beam relative to the dipole, in decibels, was calculated according to the relation:

$$G = 20 \log_{10} \frac{Eb}{Ed}$$

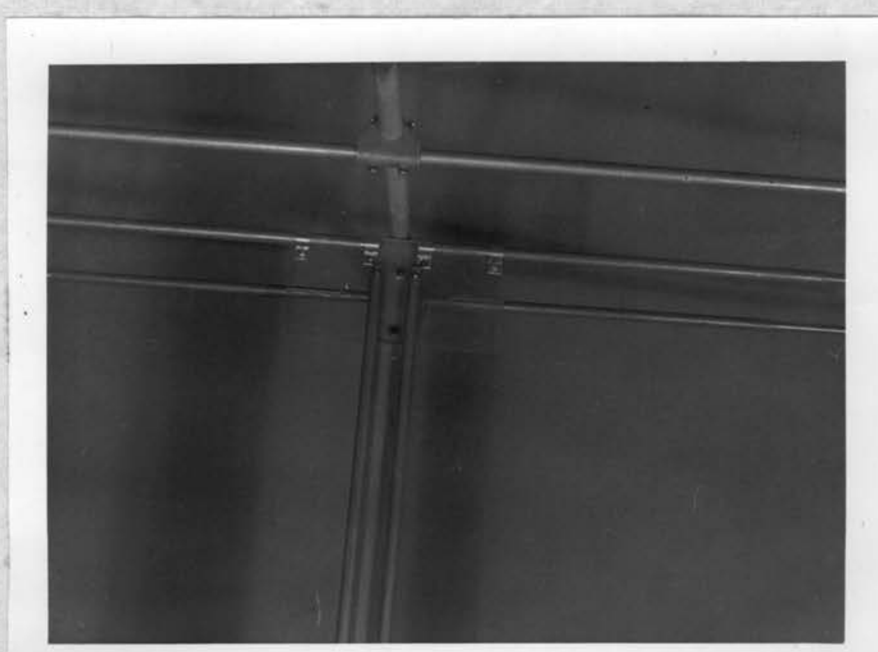
G = Relative gain in decibels of beam over dipole.
Eb = Field strength of beam.
Ed = Field strength of dipole.

The apparatus was set up on the grounds of the college golf course. The beam height was 5 feet above the ground surface and the field strength dipole was supported on a telephone pole at a height of 30 feet.

The distance between the beam and the dipole was approximately 100 feet. The component arrangement was as shown below:

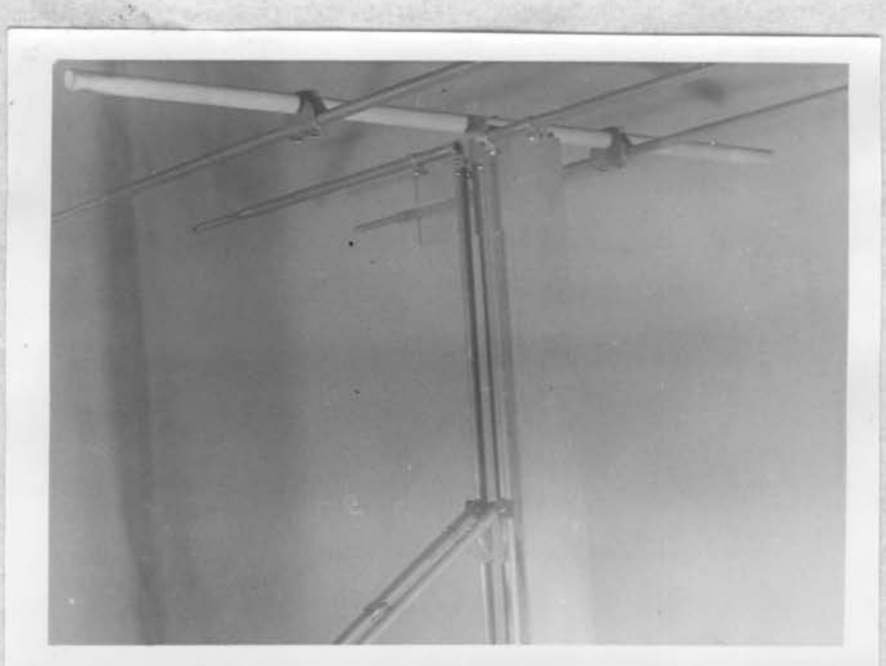


A. The Antenna. Each element of the antenna was made up of a center section thirty inches long of ½ inch O.D. aluminum tubing. Into each end was telescoped a ten inch length of 3/8 inch O.D. aluminum tubing. Thus, the overall length could be varied from thirty inches to approximately fifty inches. The driven element was cut in the center and mounted on a piece of polystyrene so that it could be driven from a 200 ohm line. The two parasitic



elements were mounted in the same plane horizontally and parallel to the driven element, one on each side in such a manner that they could be moved toward and away from the driven element. The support for the elements was a three foot length of 3/4 inch diameter maple dowel rod.

B. Matching Stub. Probably the most versatile device for radio frequency impedance matching is the shorted stub. This device was chosen since it is capable of matching a transmission line to a wide variety of impedances of a resistive or complex nature. A surge impedance of 200 ohms



was chosen for the stub and associated line used in this problem since that value gives a convenient spacing of half inch conductors. The design is as follows:

Diameter tubing available = $\frac{1}{2}$ inch O.D.
Required surge impedance = 200 ohms.

$$Z_o = 120 \cosh^{-1} \frac{d}{2r}$$

$$d = 2r \cosh \frac{Z_o}{120}$$

$$r = .25 \text{ inch.}$$

$$d = .5 \cosh \frac{200}{120}$$

$$= .5 \cosh 1.67$$

$$= .5 \times 2.75$$

$$= 1.375 \text{ inches spacing center to center.}$$

$$\text{Length of quarter wave} = \frac{246}{f}$$

$$= \frac{246}{146}$$

$$= 1.67 \text{ feet.}$$

The transmission line and stub were both constructed for a surge impedance of 200 ohms. The line length was made slightly greater than one half wavelength in order to match impedances smaller than 200 ohms. The stub was made one quarter wavelength and provided with an adjustable shorting bar.

C. Quarter Wave Transformer. Since it was convenient to use 52 ohm coaxial cable to carry power from the transmitter to the antenna, some method of matching the 52 ohm coaxial cable to the 200 ohm tubing line was necessary. The quarter wave transformer was selected for this task. The design is as follows:

$$Z_t = \sqrt{Z_1 x\, Z_2}$$

Z_t = Required surge impedance of quarter wave transformer.

Z_1 = Impedance of line connected to one end of transformer.

Z_2 = Impedance of line connected to other end of transformer.

$$Z_t = \sqrt{52 x\ 200}$$

$$= 102 \text{ ohms required } Z_o$$

$$Z_o = 120 \text{ Cosh}^{-1} \frac{d}{2r}$$

r = .58 inches for tubing on hand.

$$d = 2r \text{ Cosh } \frac{Zo}{120}$$

$$= 1.16 \text{ Cosh } \frac{102}{120}$$

$$= 1.16 \text{ Cosh } .85$$

$$= 1.16 \text{ x } (1.384)$$

$= 1.60$ inches spacing center to center.

$$\text{Length} = \frac{246}{f}$$

$$= \frac{246}{146}$$

$= 1.67$ feet

D. Line Balance Inverter. It was desirable to ground the outer conductor of the coaxial cable in order to minimize the effects of line radiation on the results. This meant that some sort of line balance inverter was necessary in

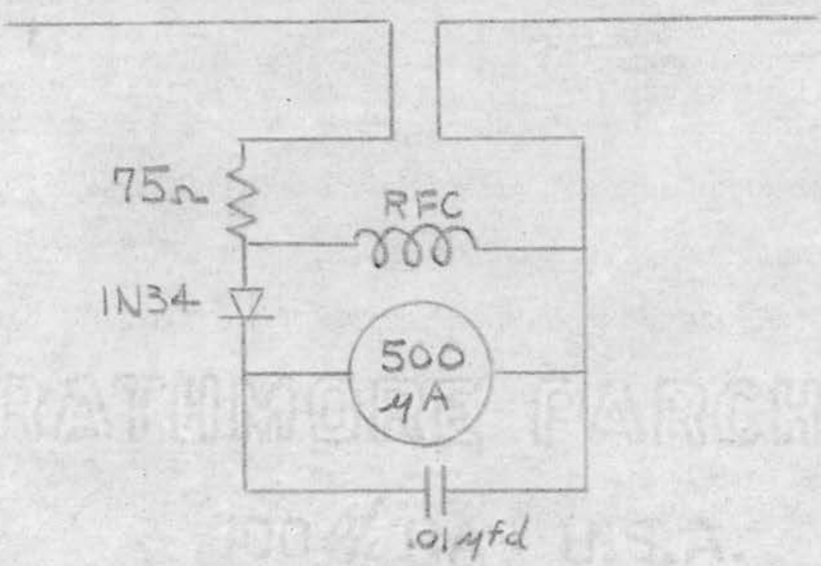
order to transfer the coaxial cable from a balanced to an unbalanced condition so that the outer shield could be grounded. The device selected is the so called "Bazooka" type detuning sleeve. The length was made one quarter wavelength as calculated previously. The diameter of the tubing sleeve is not critical. It should be about 4 times the center conductor diameter.

E. Standing Wave Ratio Bridge. The standing wave ratio bridge chosen was the "Micro Match", manufactured by M. C. Jones Electronics Company of Bristol Connecticut. The specifications of the model M M 252 as given by the manufacturer are as follows:

Frequency Range	3 to 162 megacycles
Transmission Line Impedance	52 ohms
Wattmeter Scales	0 to 10,100, and 1000
Range of Power Measurement	1/10 to 1000 watts
Power Range for SWR Measurement	2 to 1000 watts
Reflection Coefficient	Less than ½ db.
Power Loss in Instrument	Less than 3/10 db.
Connectors	Amphenol 82-24

F. Transmitter. The source of power for the experiments performed was the transmitter portion of a surplus army radio set SCR522. This transmitter uses crystals in the 8 megacycle range and has power output in the vicinity of 144 megacycles. The maximum output, with the 350 volt power supply used, is slightly over 5 watts. A variable voltage electronic regulated power supply was used in order that the power input to the line could be kept constant with different conditions of loading and tuning.

G. Field Strength Meter. The field strength meter used consisted of a dipole antenna with 75 ohm line connecting the dipole to an r.f. voltmeter.



A curve of voltage against rectified current reading was experimentally determined and it was found that the rectifier was reasonably linear over the small range of current used (500 micro-amps) as shown in Graph 1.

III. MEASUREMENT PROCEDURE.

The order of adjustments for each reading taken was as follows:

1. Set reflector and director spacings to any value desired. Adjust reflector to approximately 5% greater length than ½ wavelength. Adjust director to approximately 5% shorter than ½ wavelength.

2. Adjust stub position and length simultaneously for lowest standing wave ratio as shown on MicroMatch.

3. Adjust director and reflector lengths for maximum field strength as shown on the field strength meter.

4. Readjust matching stub for lower standing wave ratio.

5. Reverse MicroMatch and set power input into transmission line to the reference value.

6. Adjust MicroMatch for full scale reading.

7. Reverse MicroMatch and read standing wave ratio.

8. Record field strength meter reading.

9. Record distance from antenna to point where stub connects onto line.

10. Record stub length.

IV. METHOD OF CALCULATION.

The gain of the parasitic array over a reference dipole is calculated from the ratio of the two field strengthsas described previously.

The feed point impedance of the array may be determined from the length of matching stub and the point of connection which gives unity standing wave ratio.

Probably the most simple method of calculating the antenna impedance from the stub dimensions and point of connection is to use a rectangular transmission line circle diagram or a polar transmission line diagram or "Smith Chart". Both are capable of quite good accuracy for line terminations which are reasonably near the value of the line surge impedance. In the present case the terminations are in some cases less than 10% of the line surge impedance. This results in some inaccuracies in the calculations. The solutions on the polar diagram, in the latter case, fall at a point where there is crowding of lines and the accuracy is poor. If an attempt to use the rectangular form of chart is made for the latter case, the answers fall off the ordinary chart and it is necessary to construct a larger one. The rectangular chart gives readings which tend to magnify inaccuracies, since solutions on this diagram depend upon subtracting two large numbers of experimental accuracy in order to obtain a difference very near zero. An error of a few percent in either of the large numbers results in badly distorted values for the antenna impedance.

It was decided that the rectangular chart was best suited to the problem at hand so an enlarged version of the chart, about three feet by four feet, was constructed and used for all calculations.

The explanation of the rectangular impedance chart is as follows:

The rectangular units to left and right on graph 3 represent real components of admittance. All work is carried out on a basis of "admittance per unit characteristic admittance of transmission line", or "admittance per unit Y_o". The final result is divided into Z_o to give the correct values of impedance.

When the circle diagram is used in determining values of Z_r from the geometry of the stub matching system required for SWR of one, the procedure is as follows:

1. Convert distance from antenna to stub into electrical degrees, 360 degrees being considered equal to one wavelength at the frequency in use.

2. Convert length of closed stub into electrical degrees by the same method.

3. Locate the point on the diagram corresponding to the stub admittance. This will be somewhere along the left side of the diagram since the stub is considered to be a pure susceptance with no resistance.

4. With a straight edge draw a line through this point and the point (.5,0). An extension of this line will intersect the unit imaginary axis at some one point. This point represents a susceptance which is the inverse of the stub susceptance.

5. The circles about point (1,0) are plots of a function $\frac{2r}{1-r^2}$ where r is the reflection coefficient of the load. Each of the circles is a plot of the function with some value of r substituted. Since the reflection coefficient r is related to Z_r, it is from these circles that Z_r is indirectly determined. The previously described intersection point is located and the angle number of the half

circle passing through that point is noted. From this angle is subtracted the electrical angle representing the distance from load to stub. The circle of constant r is followed counter clockwise until it intersects the half circle calibrated with the previously noted difference angle. The intersection point is the "per unit Y_o" admittance of the load. From this value, of course, the correct Z_r may be determined in complex form.
Movement along the circles of constant r may be visualized as traveling along the transmission line, each point representing the "per unit Y_o" admittance of the line at that point. By moving along the circle a distance equal to the distance from stub to load, the actual load admittance may be found. Travel along the circle counter clockwise may be thought of as travel down the line toward the load. Travel in a clockwise direction may be thought of as travel toward the generator.

V. SAMPLE CALCULATION.

A sample calculation for the case of the dipole alone was as follows:

l_1 = 32.25 inches = Distance from antenna to point where stub connects.

l_2 = 10.5 inches = length of stub.

f = 146 megacycles

$Z_o = 200 + j0$

Bl_1 = length in electrical degrees of l_1

$= \frac{32.25}{80.9} \times 360 = 140$ degrees or from graph 2

Bl_2 = length of l_2 in electrical degrees.

= 46.7 degrees or from graph 2

Locate the point where the 46.7 degree line crosses the left margin of the diagram. This is point P_1.

Draw a line through point P_1 and the point R= 0.50, X= 0 on the chart, this line crosses the R= 1 line at point P_2. The half circle passing through P_2 is noted to be 147 degrees. This angle is Ø. The value of Ø minus Bl_1 is 147 degrees minus 143 degrees. It is noted that the complete circle passing through P_2 is marked 82%. This circle is traveled counter clockwise to the point where it crosses the 4 degree line, point P_3. Point P_3 is the unit antenna admittance. This value is read as 2.45 minus j.35. This is the same as the admittance $2.47\underline{/-8.13}$. The impedance of the antenna is $\frac{200}{2.47\underline{/8.13}}$ or $81\underline{/8.13}$ or 80 + j1.14 ohms. The calculations for the beam impedance measurements may be carried out similarly except that values of $\frac{Z_o}{Z_r}$ greater than 10 will be off the chart.

VI. DISCUSSION OF ERROR DUE TO LINE ATTENUATION.

The measurement for antenna impedance in this project is dependent upon very accurate determination of standing wave ratio. (hereafter designated as SWR). The directional coupler or MicroMatch used for SWR measurement obtains its values for SWR from the ratio of incident power to reflected power. Any amount of attenuation present between the MicroMatch and load or antenna in this case will show up as an SWR reading which is lower than the actual SWR at the load. The reason for this is that the reflected power is reduced due to the line attenuation. The normal attenuation of a line is expressed as a certain number of db loss per wavelength at a certain frequency as shown in graph 4 for RG8U Coaxial cable at 146 megacycles. This figure refers to a unidirectional traveling wave. In other words, it refers to the attenuation of the line per unit length when the line length is infinite or else terminated in its characteristic impedance, and no standing waves exist on the line.

The presence of reflection, as shown by the existence of standing waves, increases the percentage of total power dissipated in the line and thereby decreases the efficiency of the line as shown in graph 5 for RG8U Coaxial cable at 146 megacycles. The reasoning for this is as follows:

The presence of standing waves indicates reflection. This means that there is both an incident wave component and a reflected wave component on the line. The standing wave is the resultant of the two traveling waves. The power actually delivered to the load is equal to the difference between the energy contained in the incident wave and the energy contained in the reflected wave. If the reflection coefficient is high, there is a considerable amount of energy flowing both ways, yet only a comparatively small amount of power is dissipated at the load. This increases the ratio of power dissipated in the line and reduces the overall line efficiency.

The effects of line loss were very noticable in this problem. In the initial tests the line was both open and short circuited at the antenna end. In no case would the standing wave ratio rise to a value higher than 10. If the output terminals of the MicroMatch were open or short circuited, the standing wave ratio would rise to a very high value. This shows definitely that the section of line between the MicroMatch and the antenna has sufficient attenuation to hold the standing wave ratio to a maximum value of 10.

VII. TEST DATA AND GRAPHS

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.06	0.20	410	1.95	38.0	3.0	169	13.4	169	0	21+j0	21.0/0	9.5	7.1
0.06	0.18	480	2.29	38.6	3.0	172	13.4	169	-3	10+j12	15.6/50	12.8	7.1
0.06	0.16	480	2.29	38.6	3.0	172	13.4	169	-3	10+j12	15.6/50	12.8	7.3
0.06	0.14	490	2.33	39.0	3.0	174	13.4	169	-5	5+j9	10.3/61	19.4	7.3
0.06	0.12	490	2.33	39.2	3.0	175	13.4	169	-6	4.4+j8	9.1/61	22.0	7.3
0.06	0.10	500	2.38	38.8	3.5	173	15.6	167	-6	4.4+j8	9.1/61	22.0	7.5
0.06	0.08	490	2.33	39.0	3.5	174	15.6	167	-7	3.6+j7.5	8.2/64	24.4	7.3
0.06	0.06	480	2.29	38.5	5.0	172	22.3	163	-9	3.4+j4.6	5.8/54	34.4	7.1

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_0}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ø From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.07	0.20	450	2.14	37.5	3.6	167	16.0	167	0	18.0+j0	18.0/0	11.1	6.5
0.07	0.18	480	2.28	38.0	3.6	169	16.0	167	-2	13.5+j9	18.2/42	11.0	7.1
0.07	0.16	500	2.38	38.0	3.6	169	16.0	167	-2	13.5+j9	18.2/42	11.0	7.4
0.07	0.14	500	2.38	39.2	3.6	175	16.0	167	-8	2.6+j6.2	6.25/65	32.0	7.4
0.07	0.12	500	2.38	38.7	3.6	173	16.0	167	-6	4.2+j8.1	9.2/63	21.8	7.4
0.07	0.10	500	2.38	38.8	3.6	173	16.0	167	-6	4.2+j8.1	9.2/63	21.8	7.4
0.07	0.08	490	2.33	38.7	3.6	173	16.0	167	-6	4.2+j8.1	9.2/63	21.8	7.2
0.07	0.06	490	2.33	38.6	4.1	172	18.3	165	-7	4.0+j6.2	7.3/57	27.4	7.2

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.08	0.20	460	2.19	37.6	3.8	168	16.9	167	-1	17+j4.5	17.5/15	11.4	6.7
0.08	0.18	450	2.14	37.5	3.7	167	16.8	167	0	18+j0	18.0/0	11.1	6.5
0.08	0.16	490	2.34	38.0	3.7	169	16.8	167	-2	13.5+j9	16.3/34	12.3	7.3
0.08	0.14	500	2.38	38.2	3.7	170	16.8	167	-3	10+j11	15.0/48	13.3	7.4
0.08	0.12	490	2.34	38.2	3.7	170	16.8	167	-3	10+j11	15.0/48	13.3	7.3
0.08	0.10	500	2.38	38.5	3.7	171	16.8	167	-4	7+j8.5	11.1/51	18.1	7.4
0.08	0.08	500	2.38	39.0	3.7	174	16.8	167	-7	3.6+j6.9	7.7/62	26.0	7.4
0.08	0.06	500	2.38	39.0	5.0	174	22.3	163	-11	2.4+j4.2	4.8/60	41.7	7.4

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.09	0.20	500	2.38	38.4	4.0	171	17.8	166	-5	5.3+j7.4	9.0/54	22.2	7.4
0.09	0.18	500	2.38	38.5	4.0	171	17.8	166	-5	5.3+j7.4	9.0/54	22.2	7.4
0.09	0.16	490	2.33	38.3	4.0	171	17.8	166	-5	5.3+j7.4	9.0/54	22.2	7.3
0.09	0.14	440	2.10	37.5	4.0	167	17.8	166	-1	15.3+j3.5	15.7/13	12.8	6.4
0.09	0.12	420	2.00	37.4	4.0	167	17.8	166	-1	15.3+j3.5	15.7/13	12.8	6.1
0.09	0.10	430	2.05	37.3	4.0	166	17.8	166	0	16.2+j0	16.2/0	12.3	6.2
0.09	0.08	490	2.33	38.0	4.0	169	17.8	166	-3	10.5+j8	13.2/37	15.1	7.3
0.09	0.06	500	2.38	38.2	4.0	170	17.8	166	-4	7+j7	9.9/45	20.2	7.4

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E2}{E1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.10	0.20	460	2.19	37.5	3.8	167	17.0	166	-1	15.3+J3.5	15.7/13	12.8	6.7
0.10	0.18	500	2.38	38.0	3.8	169	17.0	166	-3	10.5+J8.0	13.2/37	15.1	7.4
0.10	0.16	490	2.33	38.0	3.8	169	17.0	166	-3	10.5+J8.0	13.2/37	15.1	7.3
0.10	0.14	490	2.33	38.0	3.8	169	17.0	166	-3	10.5+J8.0	13.2/37	15.1	7.3
0.10	0.12	500	2.38	38.3	3.8	171	17.0	166	-5	5.3+J7.4	9.0/54	22.2	7.4
0.10	0.10	490	2.33	38.5	3.8	172	17.0	166	-6	4.8+J7.2	8.6/56	23.2	7.3
0.10	0.08	500	2.38	38.8	3.8	173	17.0	166	-7	3.4+J6.5	7.2/62	27.8	7.4
0.10	0.06	500	2.38	39.0	4.8	174	17.0	163	-11	2.4+J5.3	5.9/66	34.0	7.4

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.11	0.20	410	1.95	37.4	3.8	167	17.0	166	-1	15.3+j3.5	15.7/13	12.8	5.7
0.11	0.18	420	2.00	37.3	4.0	166	17.8	166	0	16.2+j0	16.2/0	12.3	6.1
0.11	0.16	420	2.00	37.5	4.0	167	17.8	166	-1	15.3+j3.5	15.7/13	12.8	6.1
0.11	0.14	480	2.28	38.0	4.0	169	17.8	166	-3	10.5+j8.0	13.7/37	15.1	7.0
0.11	0.12	490	2.33	38.3	4.0	171	17.8	166	-5	5.3+j7.4	9.0/54	22.2	7.2
0.11	0.10	490	2.33	38.5	4.0	172	17.8	166	-6	4.8+j7.2	8.6/56	23.2	7.2
0.11	0.08	500	2.38	39.0	4.0	174	17.8	166	-8	3.0+j5.4	6.2/61	32.2	7.4
0.11	0.06	450	2.14	39.0	4.0	174	17.8	166	-8	3.0+j5.4	6.2/61	32.3	6.5

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ø From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.12	0.20	460	2.19	37.0	4.0	164	17.8	166	2	12.5-j6.8	$14.3\angle{-29}$	14.0	6.2
0.12	0.18	480	2.28	37.3	4.0	165	17.8	166	1	15.3-j3.5	$15.7\angle{-13}$	12.8	7.1
0.12	0.16	490	2.33	37.5	4.0	166	17.8	166	0	16.2-j0	$16.2\angle{0}$	12.3	7.2
0.12	0.14	470	2.24	37.8	4.0	167	17.8	166	-1	15.3+j3.5	$15.7\angle{13}$	12.8	6.9
0.12	0.12	400	1.91	38.0	4.0	168	17.8	166	-2	12.5+j6.8	$14.3\angle{29}$	14.0	5.6
0.12	0.10	390	1.86	38.5	4.0	169	17.8	166	-3	10.5+j8.0	$13.2\angle{37}$	15.1	5.3
0.12	0.08	490	2.28	38.5	4.0	169	17.8	166	-3	10.5+j8.0	$13.2\angle{37}$	15.1	7.1
0.12	0.06	500	2.38	39.0	4.0	174	17.8	166	-8	3.0+j5.4	$6.2\angle{61}$	32.3	7.4

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.13	0.20	460	2.19	37.0	4.0	165	17.8	166	1	15.3-j3.5	$15.7\angle{-13}$	12.8	6.2
0.13	0.18	490	2.33	37.4	4.0	167	17.8	166	-1	15.3+j3.5	$15.7\angle{13}$	12.8	7.2
0.13	0.16	450	2.14	37.5	4.0	167	17.8	166	-1	15.3+j3.5	$15.7\angle{13}$	12.8	6.5
0.13	0.14	480	2.28	38.3	4.0	171	17.8	166	-5	5.3+j7.4	$9.0\angle{54}$	22.2	7.1
0.13	0.12	490	2.33	38.5	4.0	172	17.8	166	-6	4.8+j7.2	$8.6\angle{56}$	23.1	7.2
0.13	0.10	420	2.00	38.5	4.0	172	17.8	166	-6	4.8+j7.2	$8.6\angle{56}$	23.1	6.1
0.13	0.08	460	2.19	39.0	4.0	174	17.8	166	-8	3.0+j5.4	$6.2\angle{61}$	32.3	6.7
0.13	0.06	470	2.24	39.5	4.0	176	17.8	166	-10	1.6+j5.2	$5.5\angle{73}$	36.4	6.4

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.14	0.20	410	1.95	36.6	4.0	163	17.8	166	3	11.0-j7.5	$13.3\angle{-34}$	15.1	5.7
0.14	0.18	410	1.95	37.4	4.0	167	17.8	166	1	15.3-j3.5	$15.7\angle{-13}$	12.8	5.7
0.14	0.16	400	1.91	37.5	4.0	167	17.8	166	1	15.3-j3.5	$15.7\angle{-13}$	12.8	5.6
0.14	0.14	400	1.91	38.5	4.0	172	17.8	166	-6	4.8+j7.2	$8.6\angle{56}$	23.2	5.6
0.14	0.12	410	1.95	38.5	4.0	172	17.8	166	-6	4.8+j7.2	$8.6\angle{56}$	23.2	5.7
0.14	0.10	430	2.05	38.8	4.0	173	17.8	166	-7	3.4+j6.5	$7.2\angle{62}$	27.8	6.2
0.14	0.08	420	2.00	39.0	4.0	174	17.8	166	-8	3.0+j5.4	$6.2\angle{61}$	32.3	6.1
0.14	0.06	400	1.91	39.5	4.0	176	17.8	166	-10	1.6+j5.2	$5.5\angle{73}$	36.4	5.6

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance Y_r	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.15	0.20	490	2.33	36.4	4.6	162	20.5	163	1	7.8-j1.8	8.0/-13	25.0	7.2
0.15	0.18	490	2.33	36.7	5.0	163	22.3	163	0	10.6-j0	10.6/0	18.8	7.2
0.15	0.16	450	2.14	36.6	5.0	163	22.3	163	0	10.6-j0	10.6/0	18.8	6.5
0.15	0.14	450	2.14	37.0	5.0	165	22.3	165	-2	9.6+j3.2	10.1/18	19.8	6.5
0.15	0.12	480	2.29	37.5	5.0	167	22.3	163	-4	6.4+j5.2	8.2/39	24.4	7.1
0.15	0.10	440	2.10	38.3	5.2	171	23.2	162	-9	3.2+j4.2	5.3/53	37.8	6.4
0.15	0.08	450	2.14	38.3	5.2	171	23.2	162	-9	3.2+j4.2	5.3/53	37.8	6.5
0.15	0.06	400	1.91	39.2	4.2	174	18.7	166	-8	3.0+j5.5	6.2/61	32.2	5.6

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.16	0.20	420	2.00	38.0	3.0	169	13.4	167	-2	14+j8	$16.2\underline{/30}$	12.3	6.1
0.16	0.18	400	1.91	38.5	3.2	172	13.4	167	-5	5.2+j8	$9.5\underline{/57}$	21.0	5.6
0.16	0.16	430	2.05	38.5	3.2	172	13.4	167	-5	5.2+j8	$9.5\underline{/57}$	21.0	6.2
0.16	0.14	410	1.96	38.7	3.2	173	13.4	167	-6	4.8+j7.8	$9.1\underline{/58}$	22.0	5.8
0.16	0.12	320	1.53	38.8	4.0	173	17.6	164	-9	3.0+j5.1	$6.0\underline{/60}$	33.3	3.4
0.16	0.10	420	2.00	39.0	4.0	174	17.6	164	-10	1.8+j5.0	$5.3\underline{/70}$	37.7	6.6
0.16	0.08	400	1.91	39.5	4.0	176	17.6	164	-12	1.8+j4.2	$4.6\underline{/67}$	43.5	5.6
0.16	0.06	430	2.05	40.0	4.2	178	18.7	164	-14	1.4+j4.0	$4.3\underline{/71}$	46.5	6.2

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.17	0.20	400	1.91	35.4	3.6	158	16.0	166	8	2.0-j6.0	6.3/-71.6	31.6	5.6
0.17	0.18	420	2.00	35.6	3.8	159	16.0	165	6	5.0-j6.5	7.8/-50	25.6	6.1
0.17	0.16	430	2.05	36.1	3.8	161	16.9	165	4	6.0-j6.5	8.8/-47	22.7	6.2
0.17	0.14	430	2.05	36.5	3.5	162	15.6	166	4	7.0-j7.0	9.9/-45	20.1	6.2
0.17	0.12	400	1.91	37.2	3.5	166	15.6	166	0	14.0-j0	14.0/0	13.9	5.6
0.17	0.10	420	2.00	37.9	3.5	169	15.6	166	-3	9.4+j6.5	11.4/35	17.5	6.1
0.17	0.08	410	1.96	38.3	3.5	171	15.6	166	-5	5.5+j7.0	8.9/52	22.4	5.8
0.17	0.06	400	1.91	38.5	3.5	172	15.6	166	-6	5.0+j6.6	7.9/51	25.3	5.6

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.18	0.20	490	2.33	35.2	3.6	157	16.0	166	-9	2.2+j5.3	5.75/67	34.8	7.2
0.18	0.18	490	2.33	38.5	3.6	172	16.0	166	6	10.0-j7.0	12.2/-55	16.5	7.2
0.18	0.16	500	2.38	36.4	3.3	162	14.7	167	-5	5.5+j7.5	9.3/54	21.5	7.3
0.18	0.14	450	2.14	36.8	3.3	164	14.7	167	-3	10.0+j8.0	16.0/51	12.5	6.5
0.18	0.12	440	2.10	37.3	3.3	166	14.7	167	-1	15.3+j3.5	15.7/13	12.8	6.4
0.18	0.10	450	2.14	38.0	3.3	169	14.7	167	2	12.0-j7.0	13.9/-30	14.4	6.5
0.18	0.08	480	2.29	38.5	3.2	171	14.3	167	4	7.0-j9.0	11.4/-52	17.5	7.1
0.18	0.06	480	2.29	38.5	3.7	171	16.5	164	7	4.1-j5.5	6.8/-53	29.1	7.1

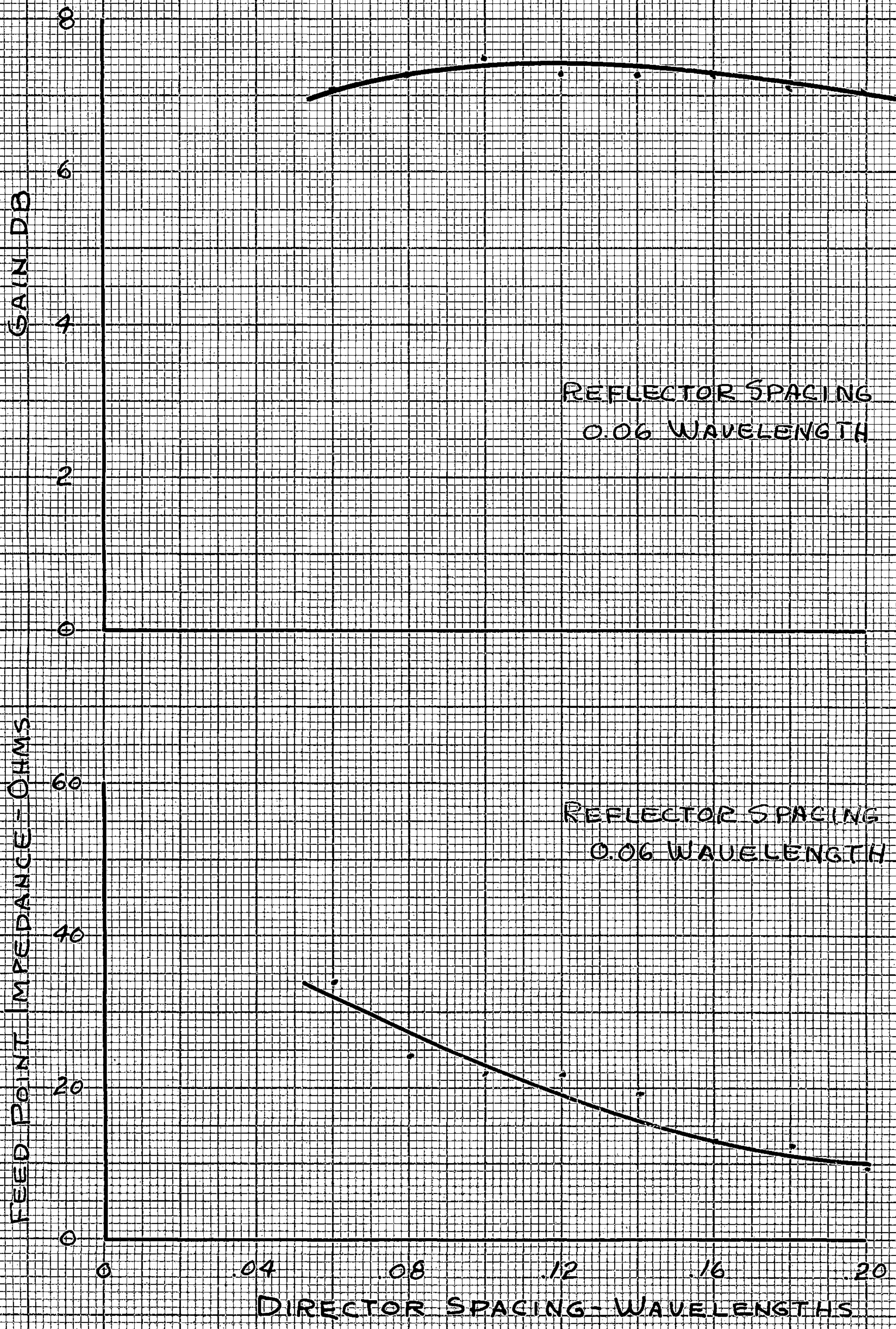
Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.19	0.20	400	1.91	35.5	4.5	156	20.0	162	6	5.1-j4.4	6.8/-41	29.4	5.6
0.19	0.18	410	1.96	35.5	4.4	158	19.5	163	5	5.6-j5.3	7.7/-43	26.0	5.8
0.19	0.16	400	1.91	36.0	4.0	160	17.8	164	4	6.6-j6.2	9.0/-43	22.2	5.6
0.19	0.14	400	1.91	36.5	3.5	162	15.6	166	4	7.2-j7.0	10.0/-44	20.0	5.6
0.19	0.12	410	1.96	37.4	3.5	166	15.6	166	0	14.5-j0	14.5/0	13.8	5.8
0.19	0.10	430	2.05	38.0	3.6	169	16.0	167	-2	13.0+j9.0	15.8/35	12.7	6.2
0.19	0.08	430	2.05	38.3	3.7	171	16.5	167	-4	6.5+j9.5	11.6/56	17.2	6.2
0.19	0.06	430	2.05	38.5	3.7	172	16.5	167	-5	5.0+j7.0	8.7/55	22.0	6.2

Experimental and Calculated Data

Reflector Spacing Wavelengths	Director Spacing Wavelengths	Field Strength Micro-amps	Voltage Ratio $\frac{E_2}{E_1}$	Distance to Stub-Inches	Length of Stub-Inches	Distance to Stub-Degrees	Length of Stub-Degrees	Phase Angle ϕ From Chart	Difference Angle	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Unit Antenna Admittance $\frac{Y_r}{Y_o}$	Antenna Impedance Magnitude	Antenna Gain Over Dipole db
0.20	0.20	80	1.6	34.5	6.2	154	27.0	156	2	4.9-j0.7	$4.95\angle{-8.1}$	40.4	4.1
0.20	0.18	105	2.1	35.5	5.0	159	22.0	162	3	7.0-j3.2	$8.6\angle{-24}$	23.2	6.4
0.20	0.16	120	2.4	34.2	7.0	153	31.0	153	0	4.6-j0	$4.6\angle{0}$	43.5	7.6
0.20	0.14	110	2.2	35.5	6.0	159	26.0	158	-1	6.2+j0.3	$6.2\angle{2.8}$	32.3	6.8
0.20	0.12	120	2.4	36.1	5.1	162	23.0	160	-2	7.4+j2.0	$7.7\angle{15.2}$	26.0	7.6
0.20	0.10	115	2.3	37.0	4.3	166	19.0	163	-3	7.5+j3.3	$8.2\angle{24}$	24.4	7.5
0.20	0.08	120	2.4	37.5	4.4	169	20.0	163	-6	5.2+j4.5	$7.0\angle{42}$	28.6	7.6
0.20	0.06	100	2.0	38.5	2.5	173	11.0	168	-5	6.0+j10	$11.7\angle{59}$	17.1	6.0

GAIN AND IMPEDANCE CHARACTERISTICS



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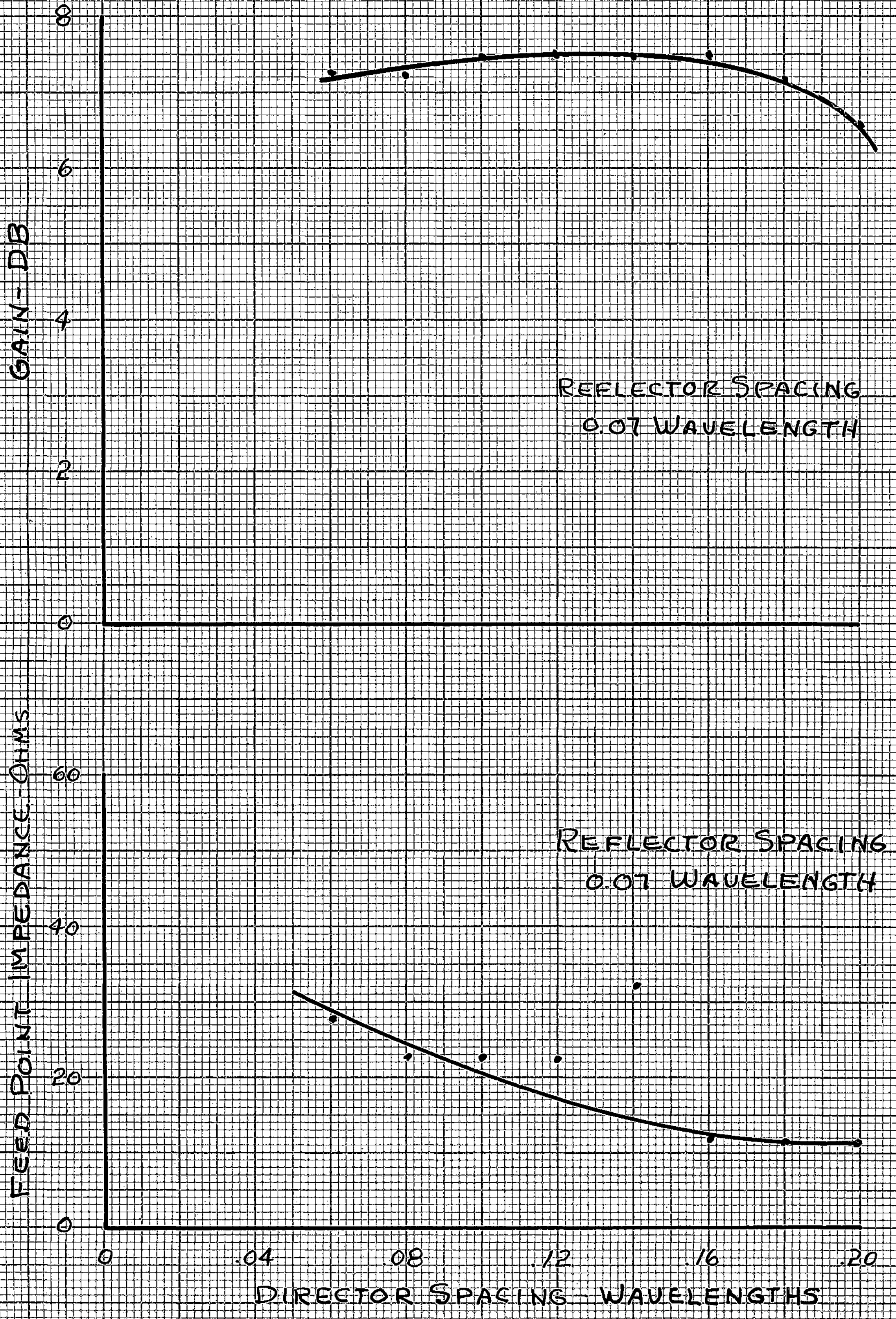

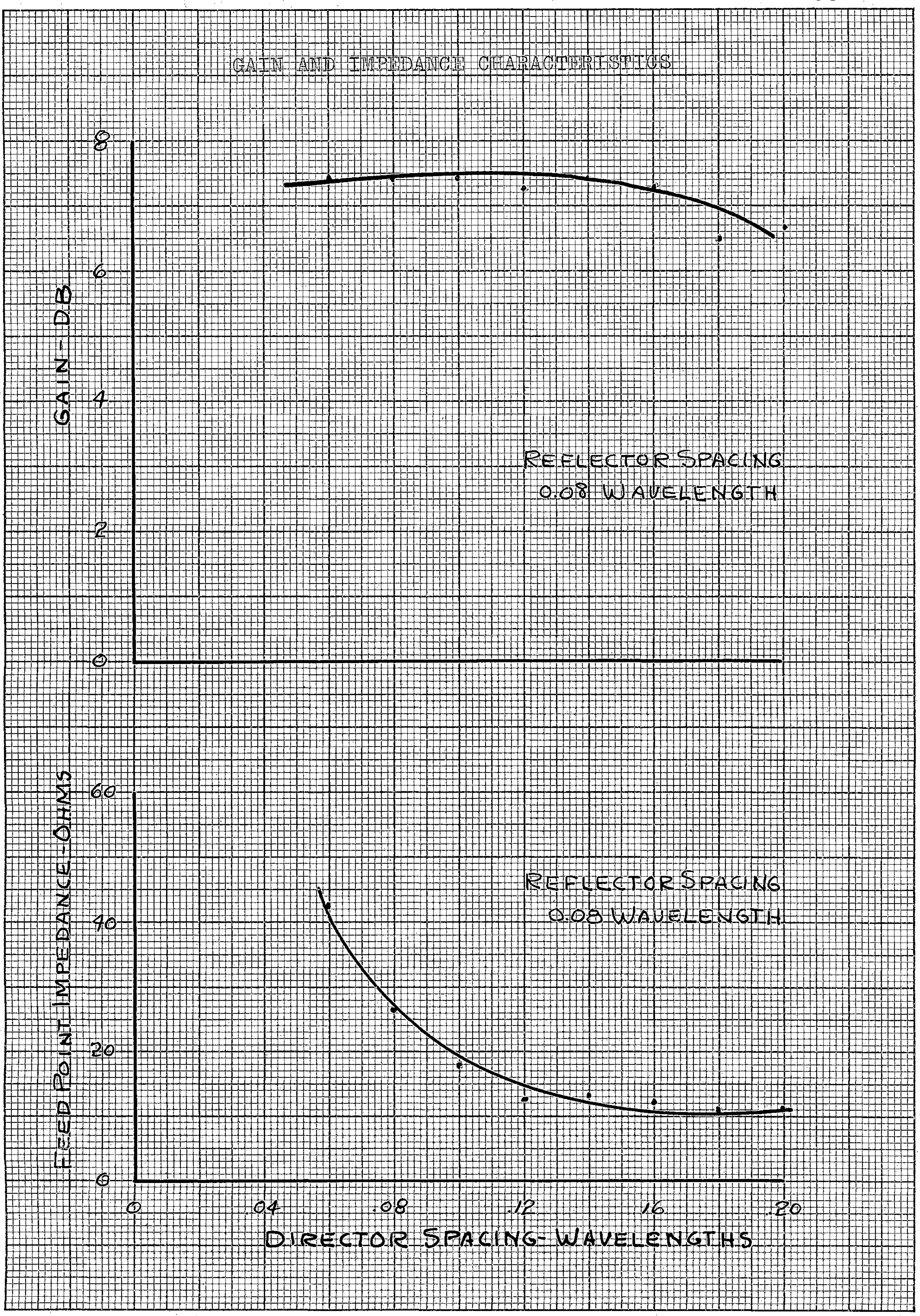
GAIN AND IMPEDANCE CHARACTERISTICS
GAIN - DB
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4
2
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REFLECTOR SPACING
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FEED POINT IMPEDANCE-OHMS
60
40
20
0
REFLECTOR SPACING
0.08 WAVELENGTH
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.08
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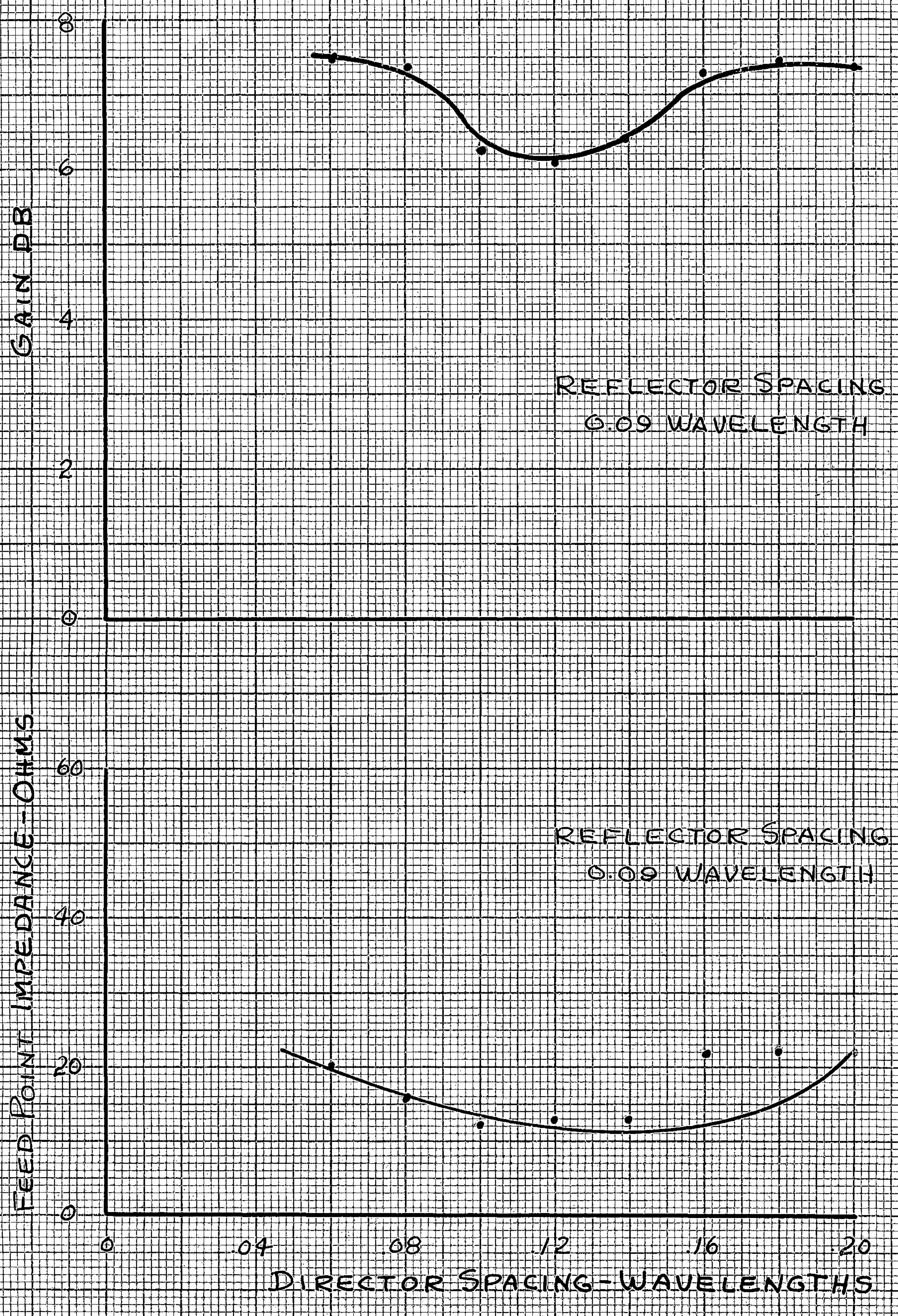
.20
DIRECTOR SPACING-WAVELENGTHS

GAIN AND IMPEDANCE CHARACTERISTICS



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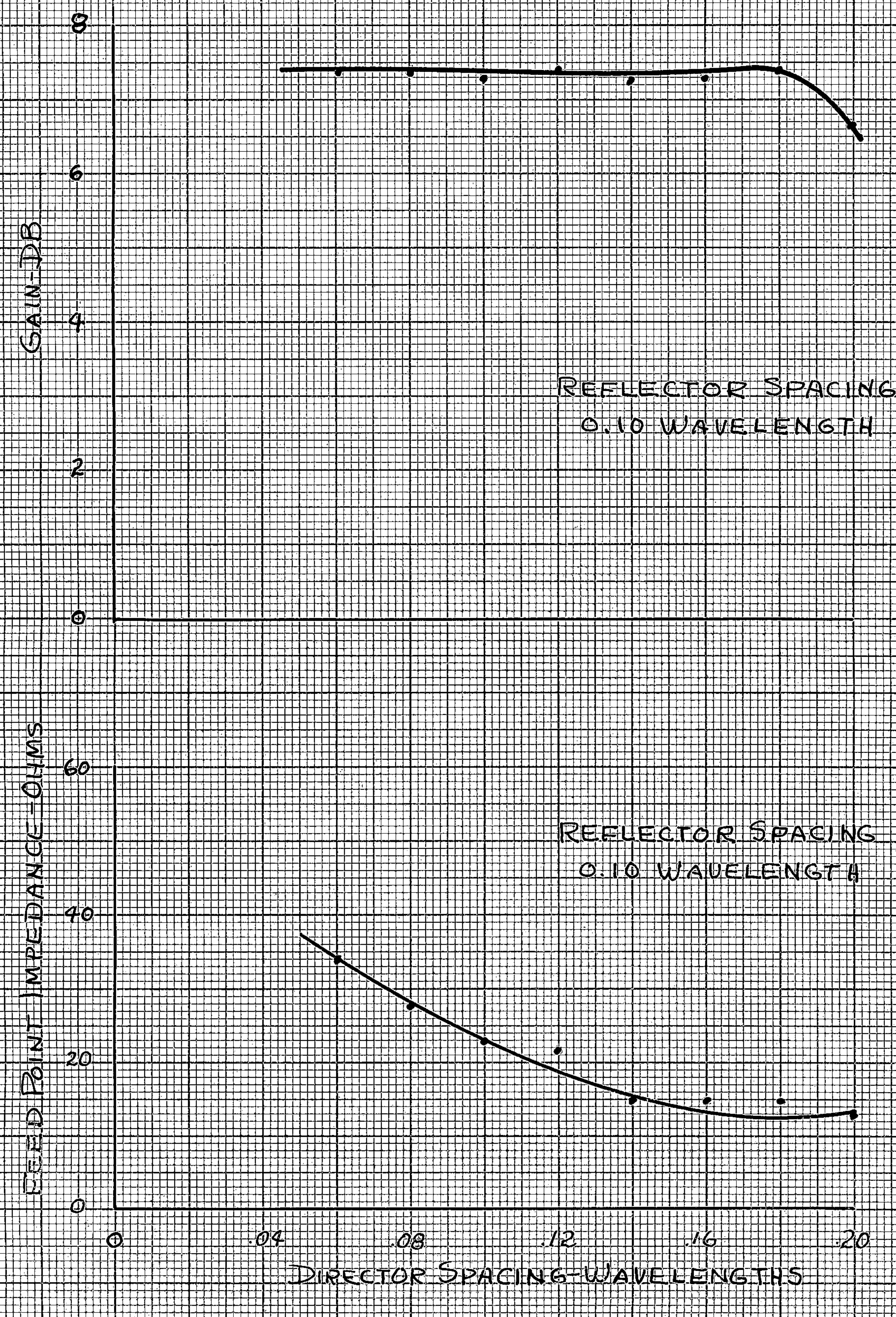

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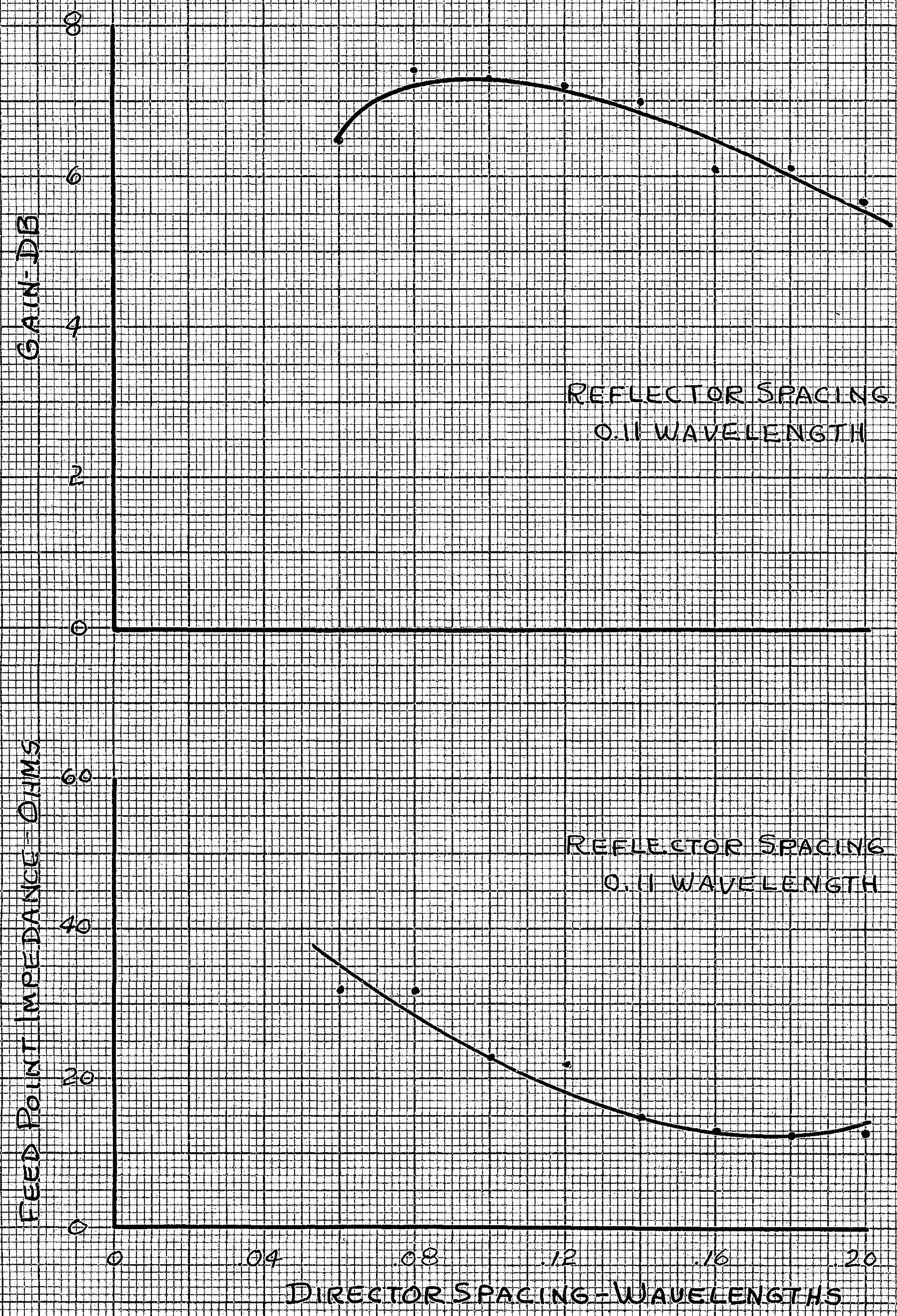

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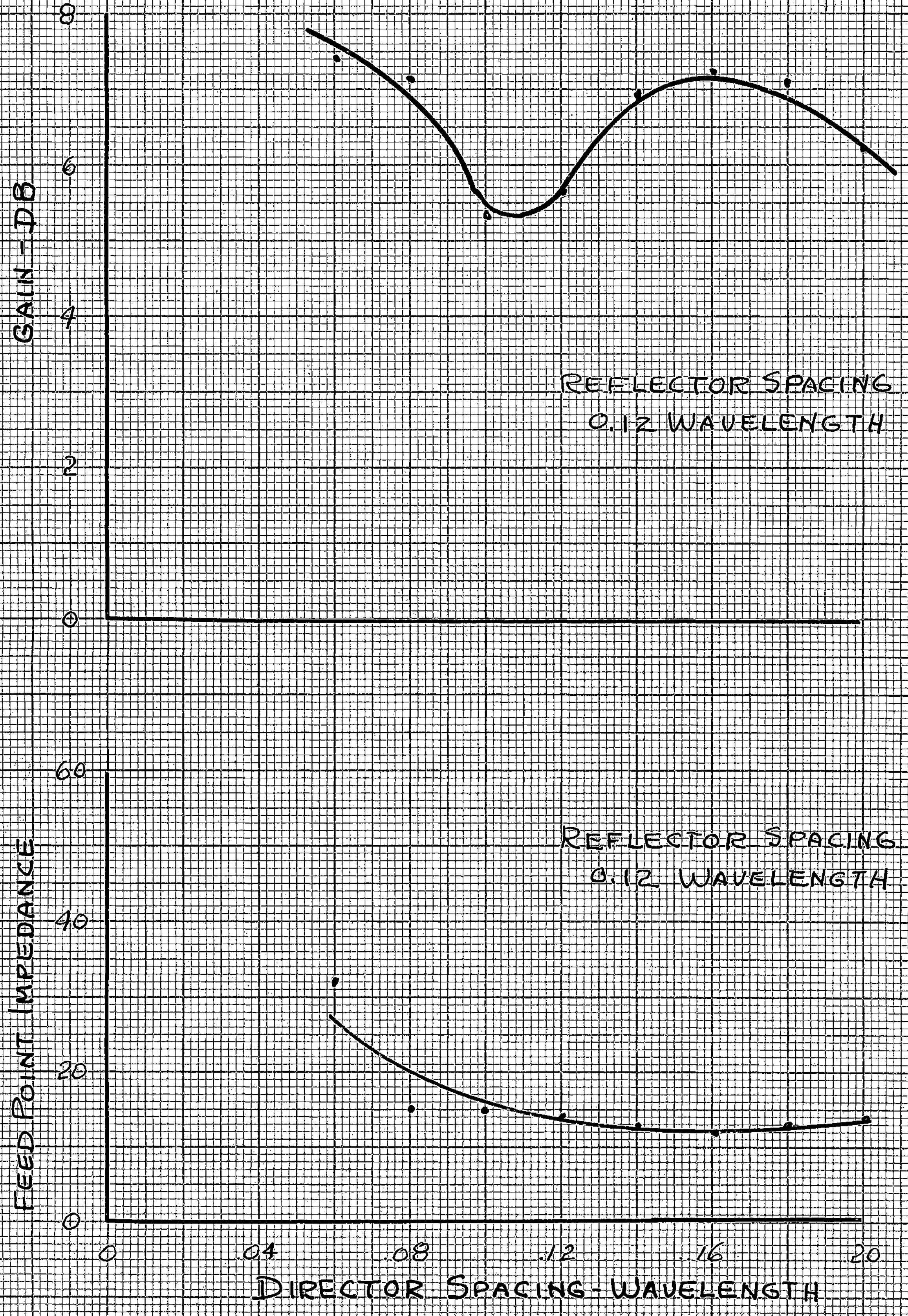

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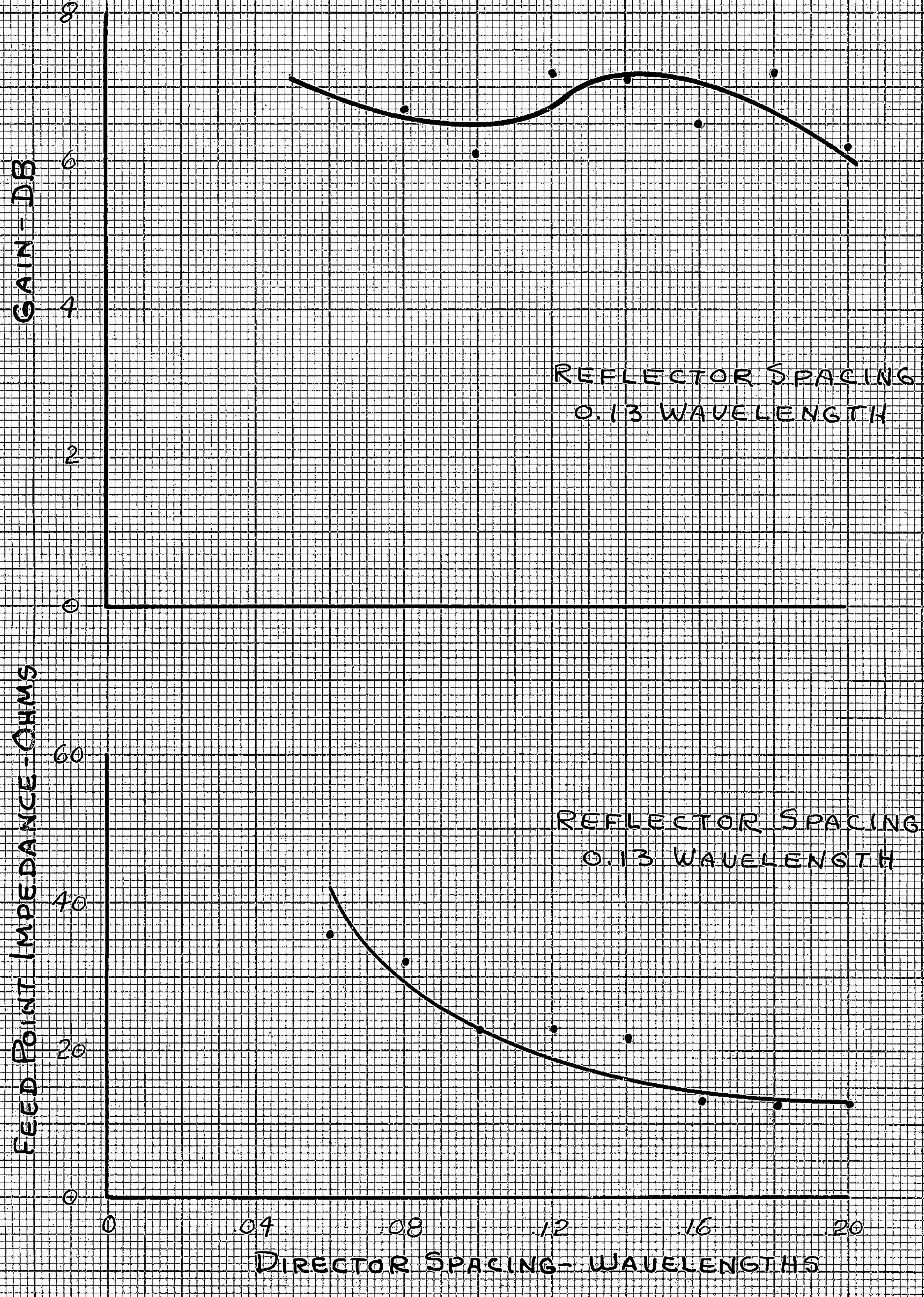

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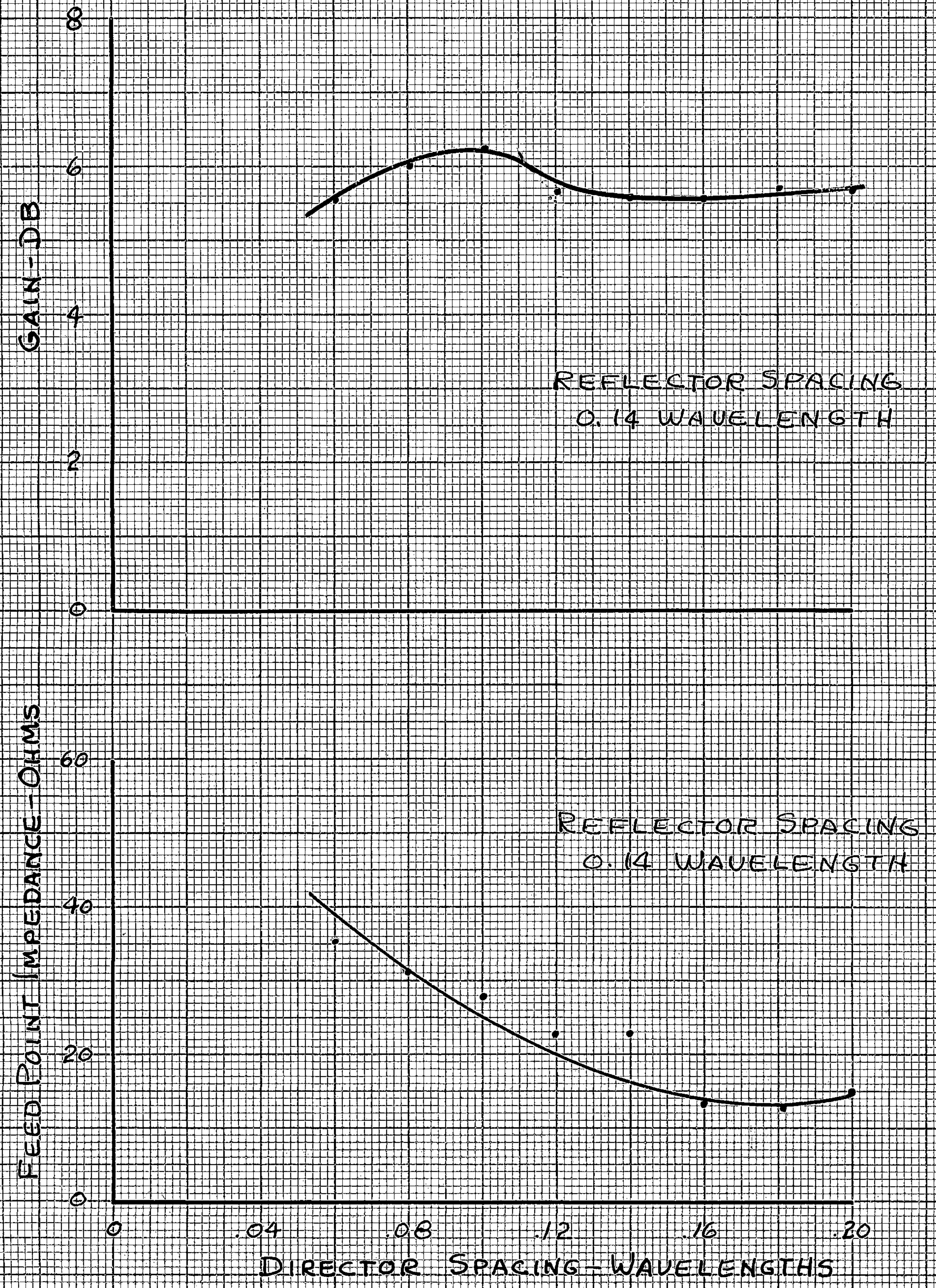

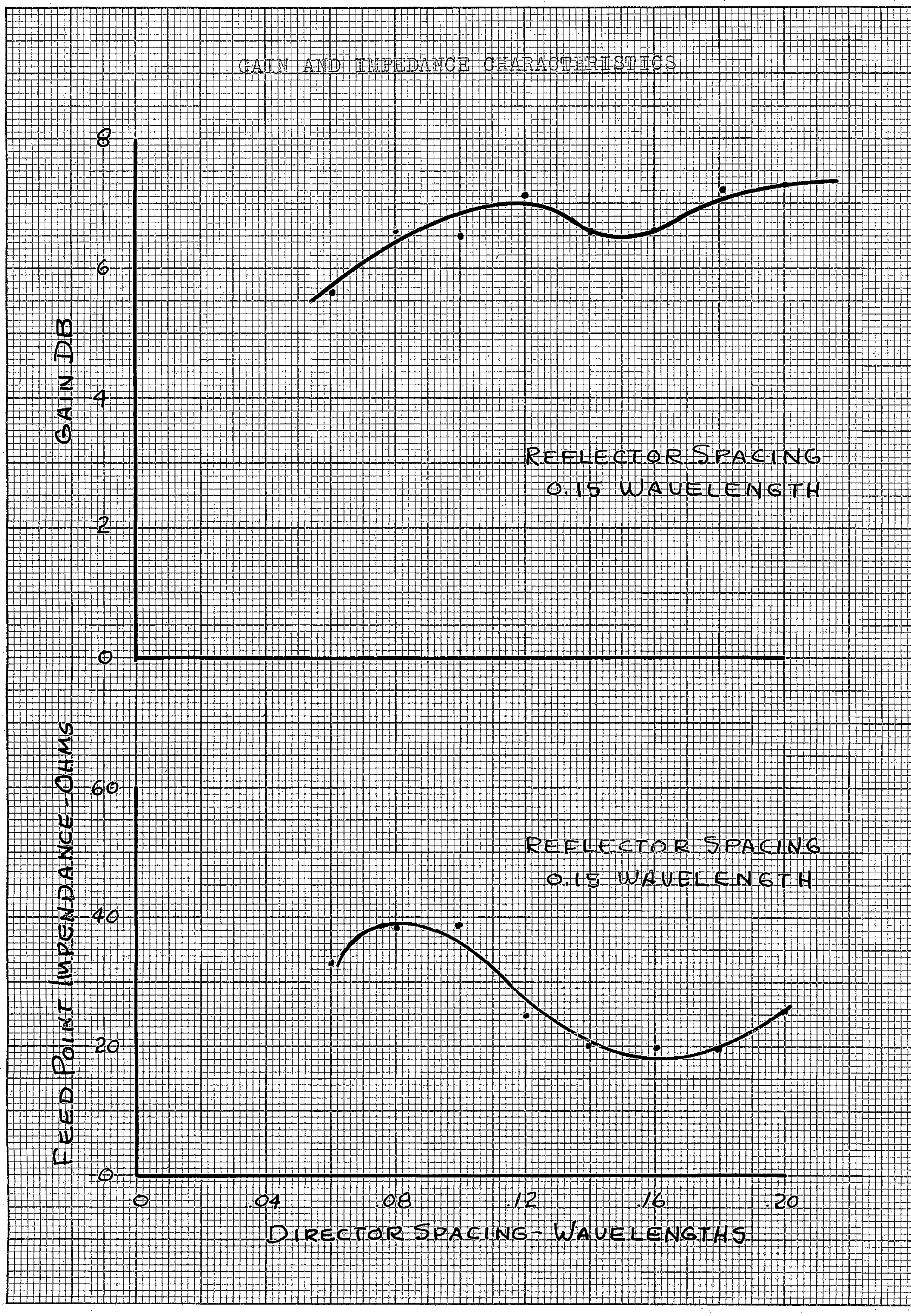
GAIN AND IMPEDANCE CHARACTERISTICS
GAIN DB
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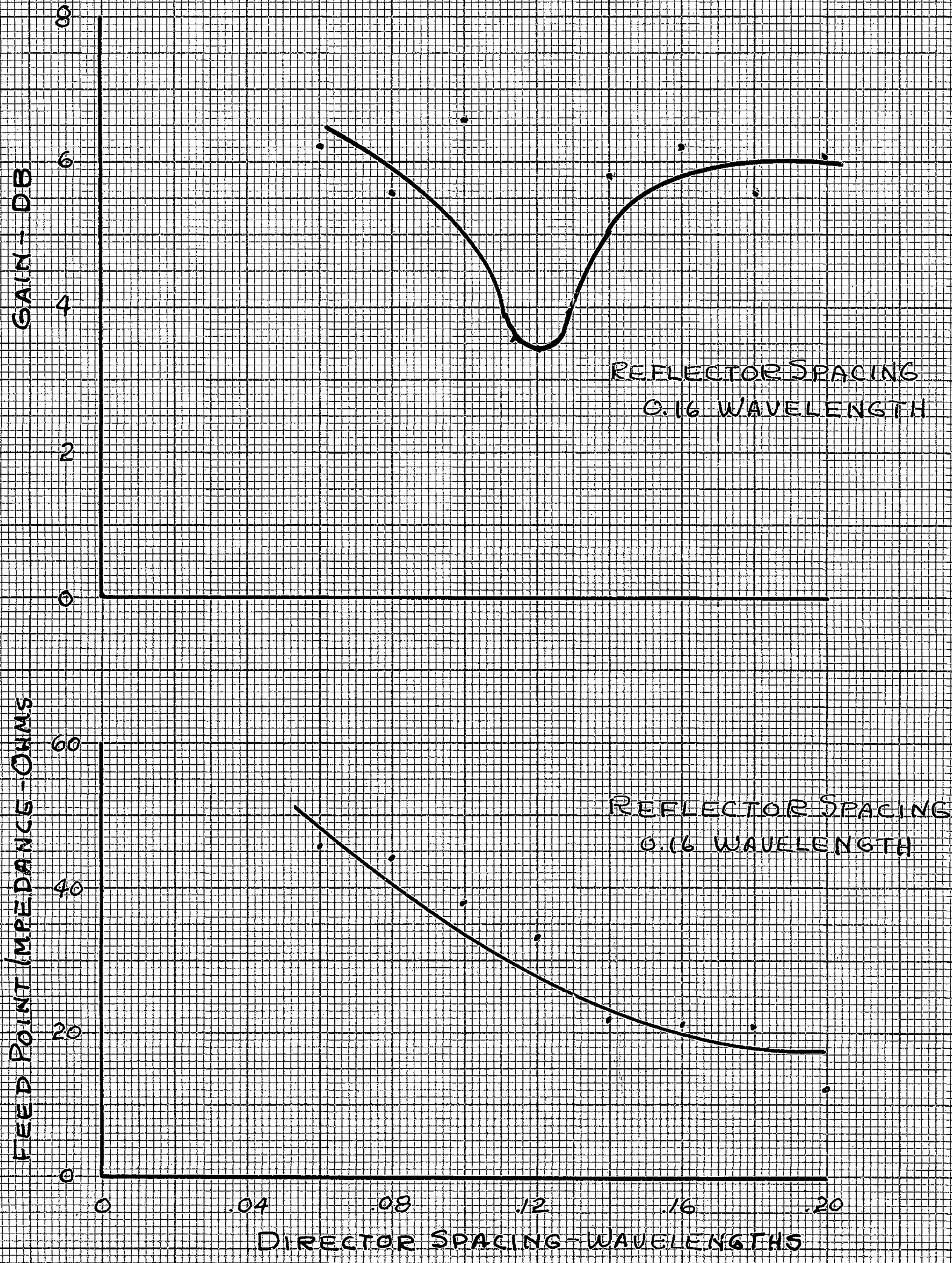
6
4
2
0
REFLECTOR SPACING
0.15 WAVELENGTH
FEED POINT IMPENDANCE-OHMS
60
40
20
0
REFLECTOR SPACING
0.15 WAVELENGTH
0
.04
.08
.12
.16
.20
DIRECTOR SPACING- WAVELENGTHS

GAIN AND IMPEDANCE CHARACTERISTICS



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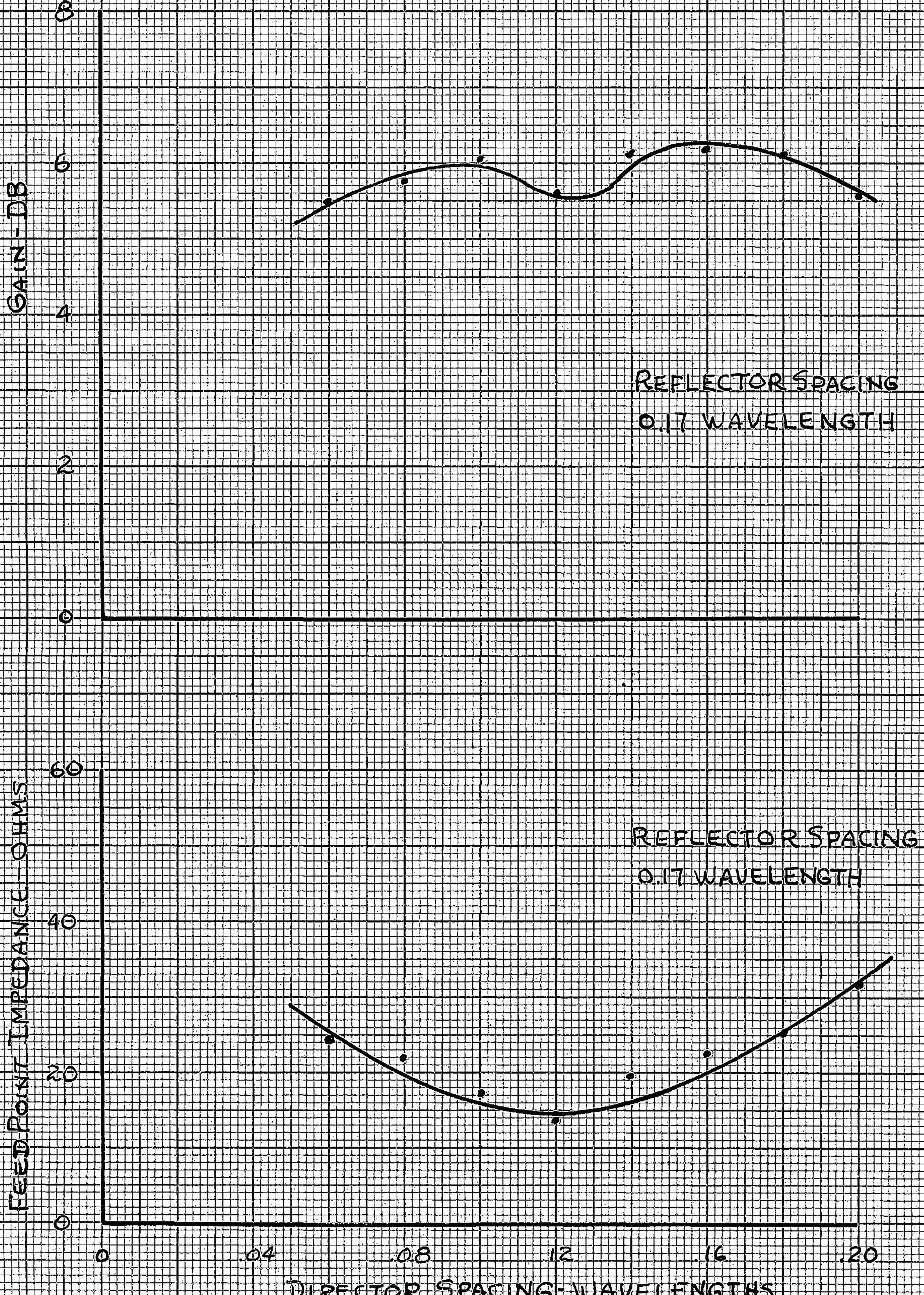

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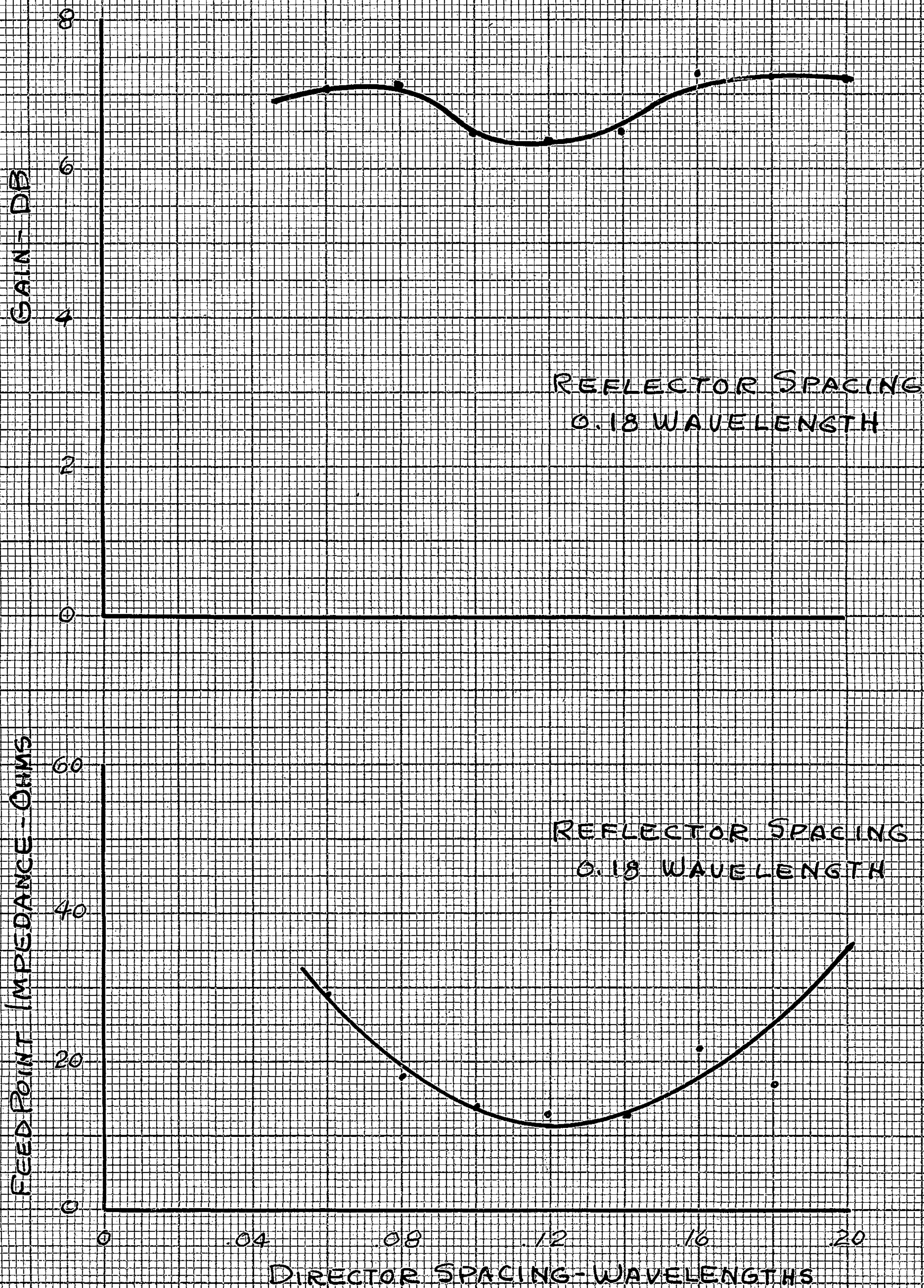

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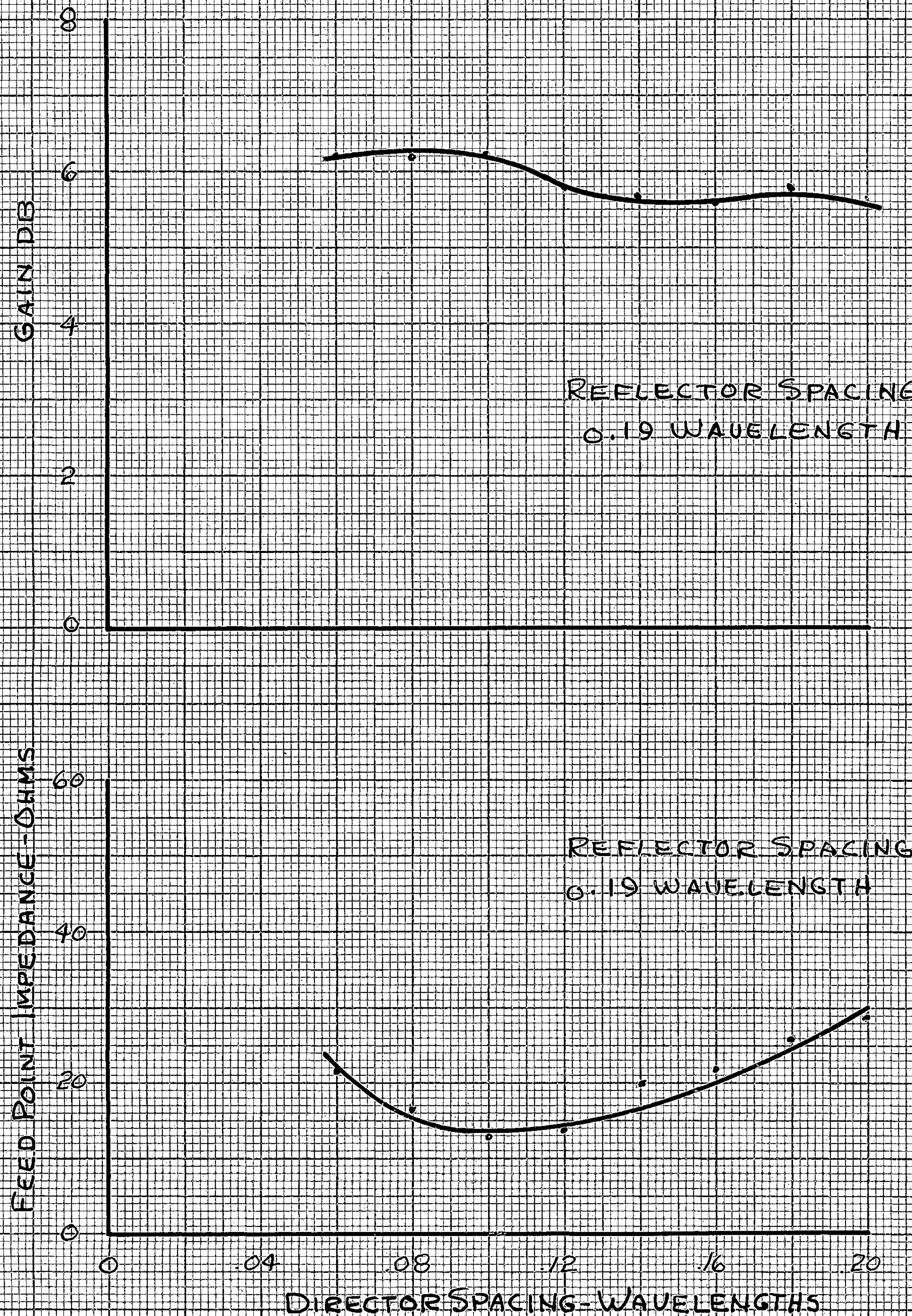

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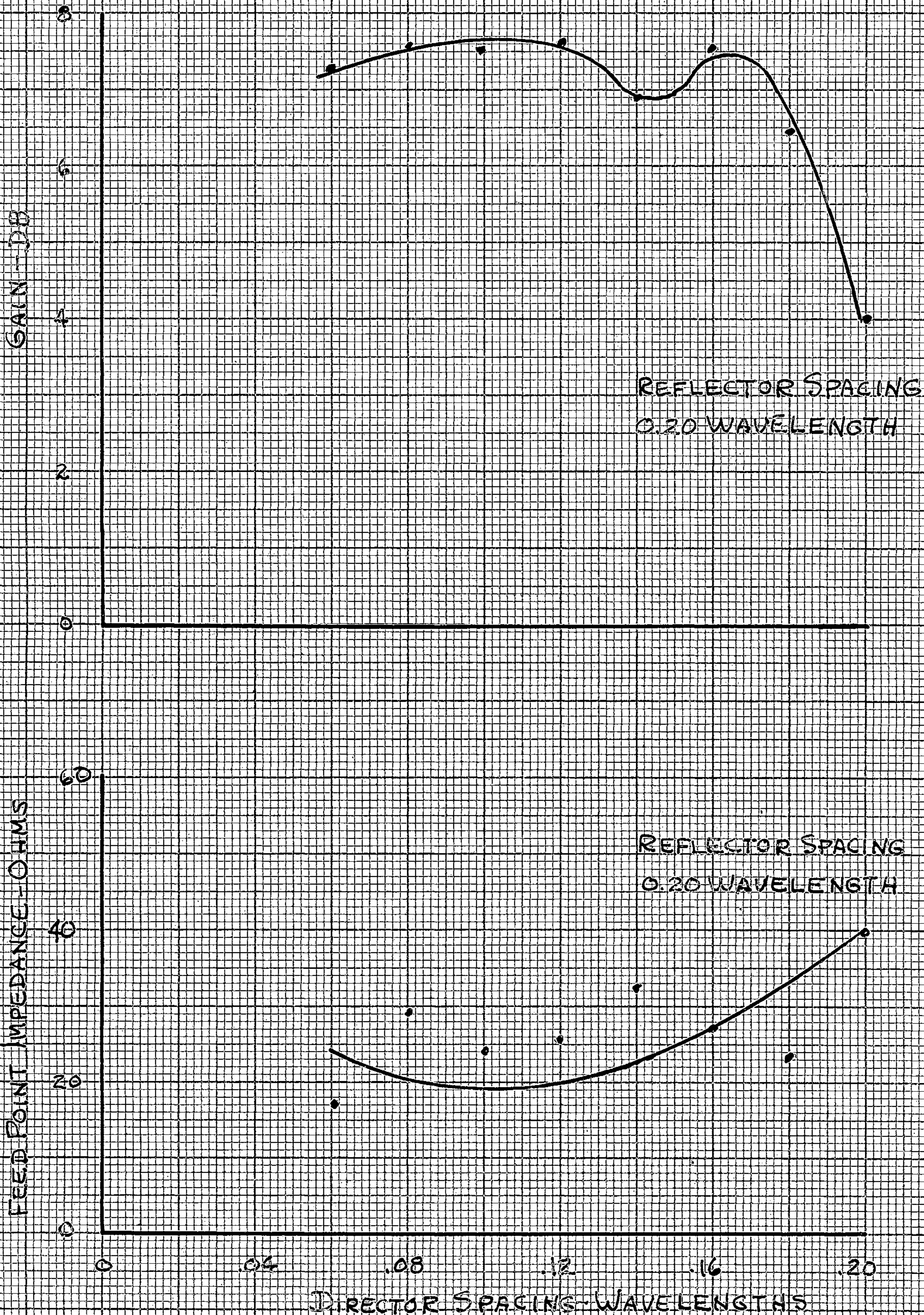
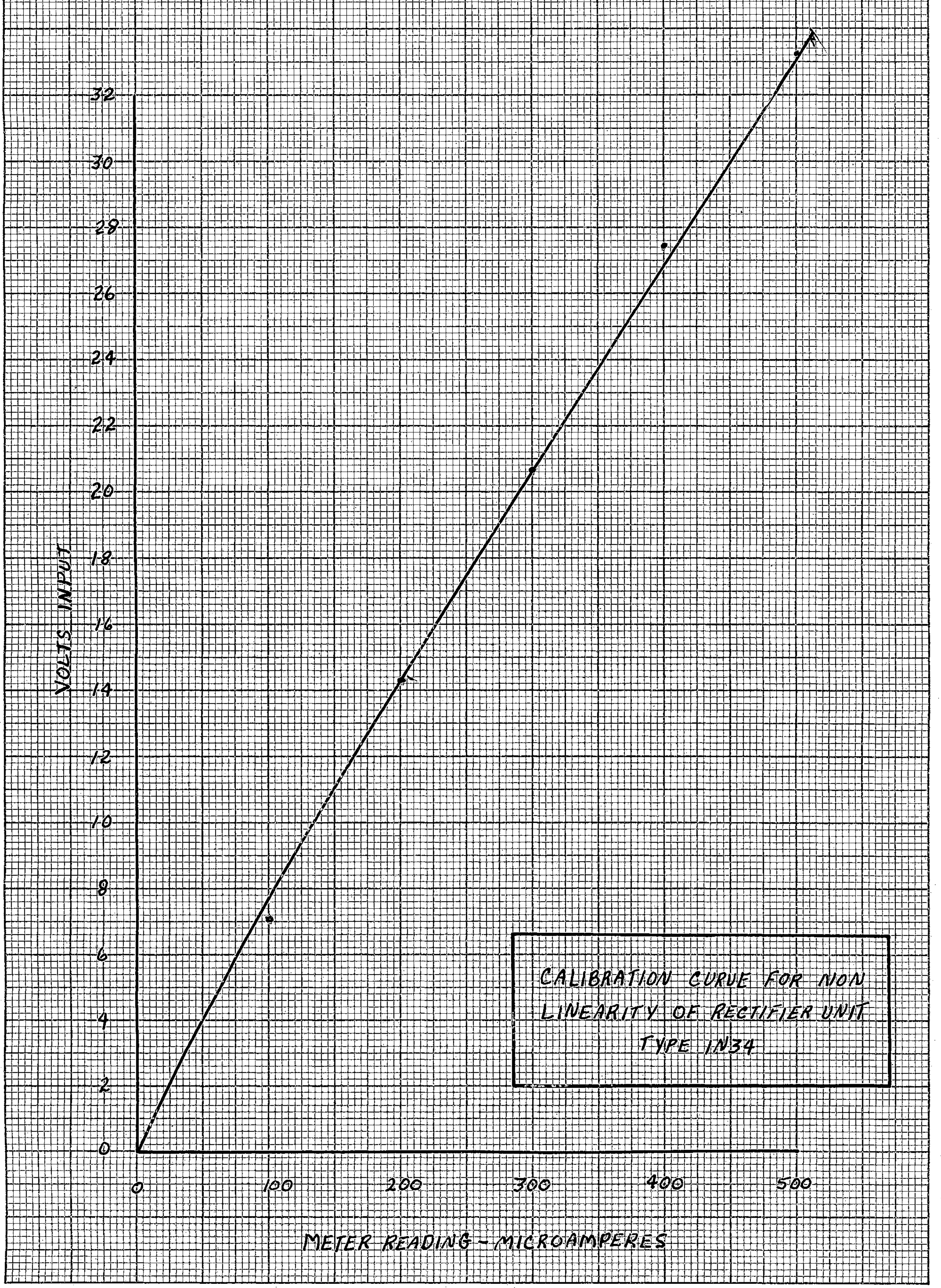


GRAPH I.


VOLTS INPUT
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8
10
12
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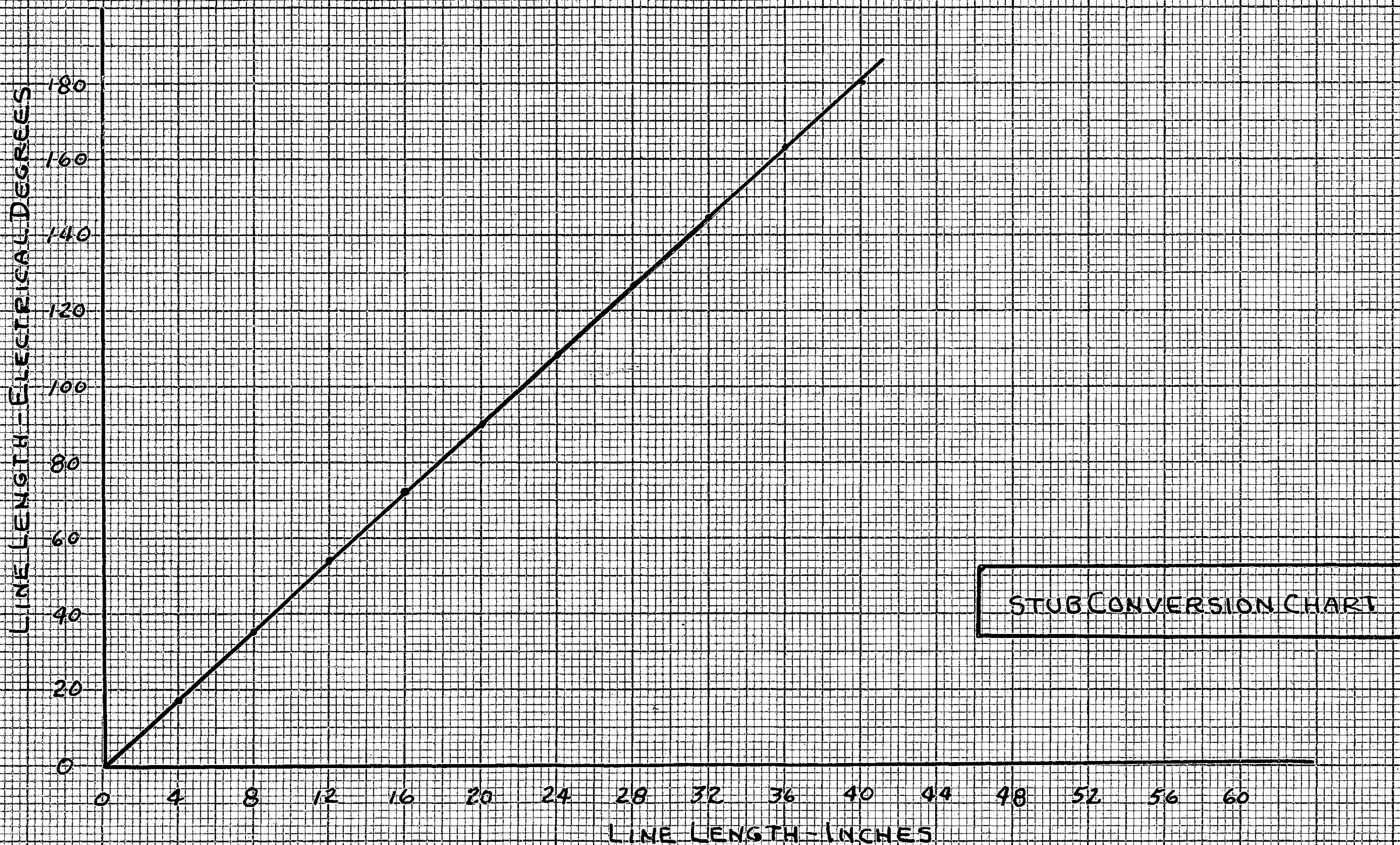
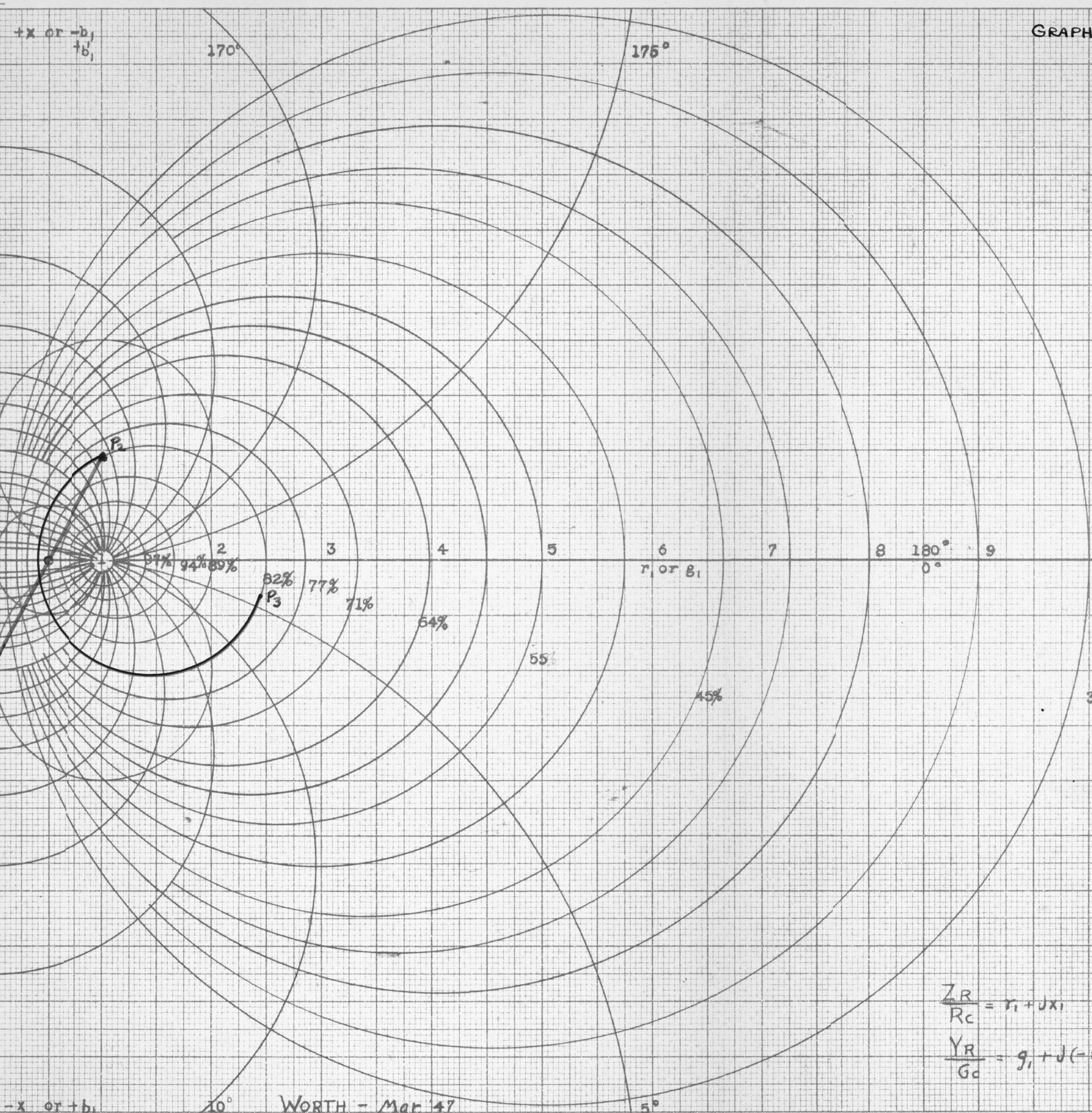
18
20
22
24
26
28
30
32
0
100
200
300
400
500
METER READING - MICROAMPERES
CALIBRATION CURVE FOR NON LINEARITY OF RECTIFIER UNIT TYPE IN34



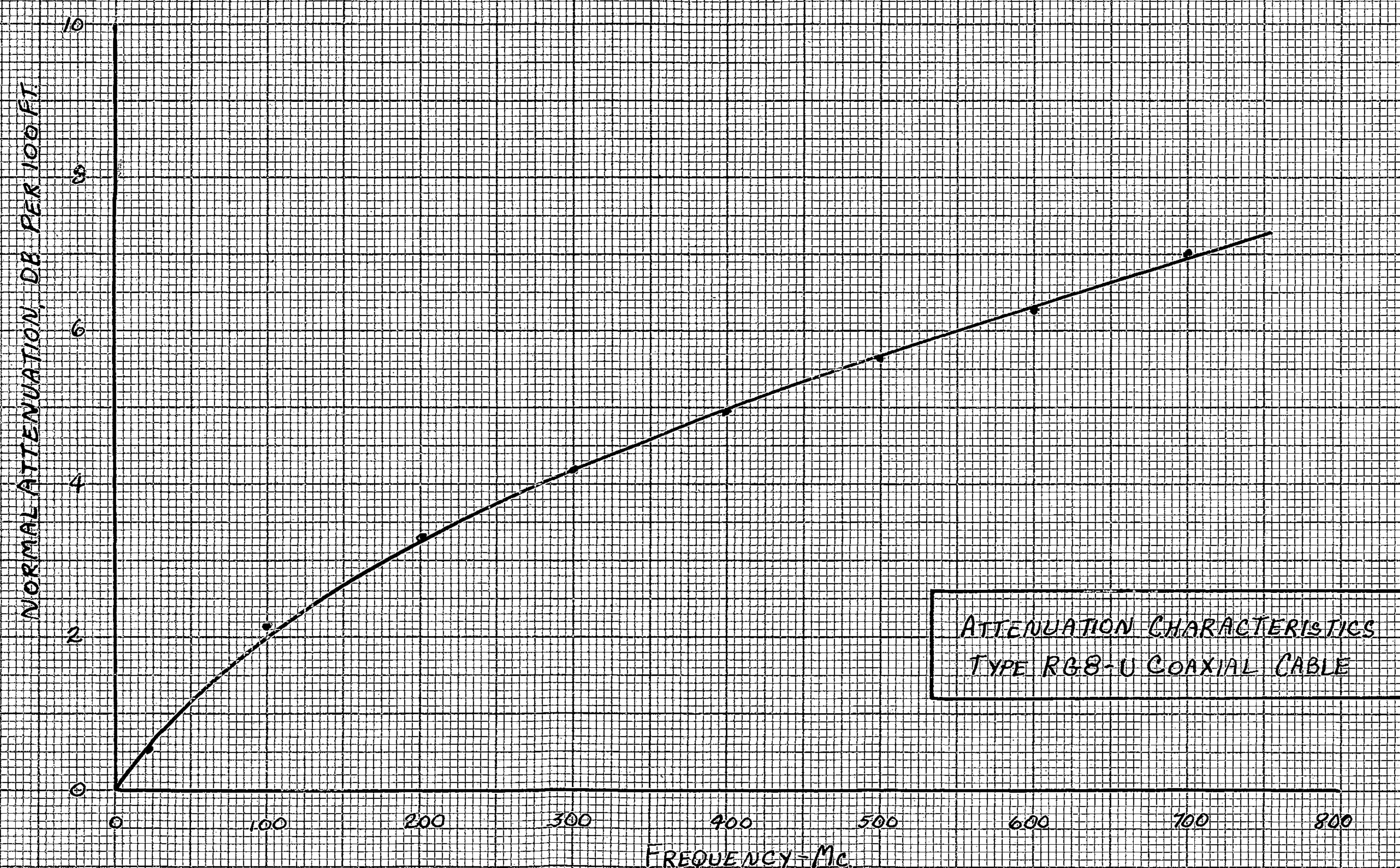
LINE LENGTH - ELECTRICAL DEGREES
180
160
140
120
100
80
60
40
20
0
0
4
8
12
16
20
24
28
32
36
40
44
48
52
56
60
LINE LENGTH - INCHES
STUB CONVERSION CHART

GRAPH 2

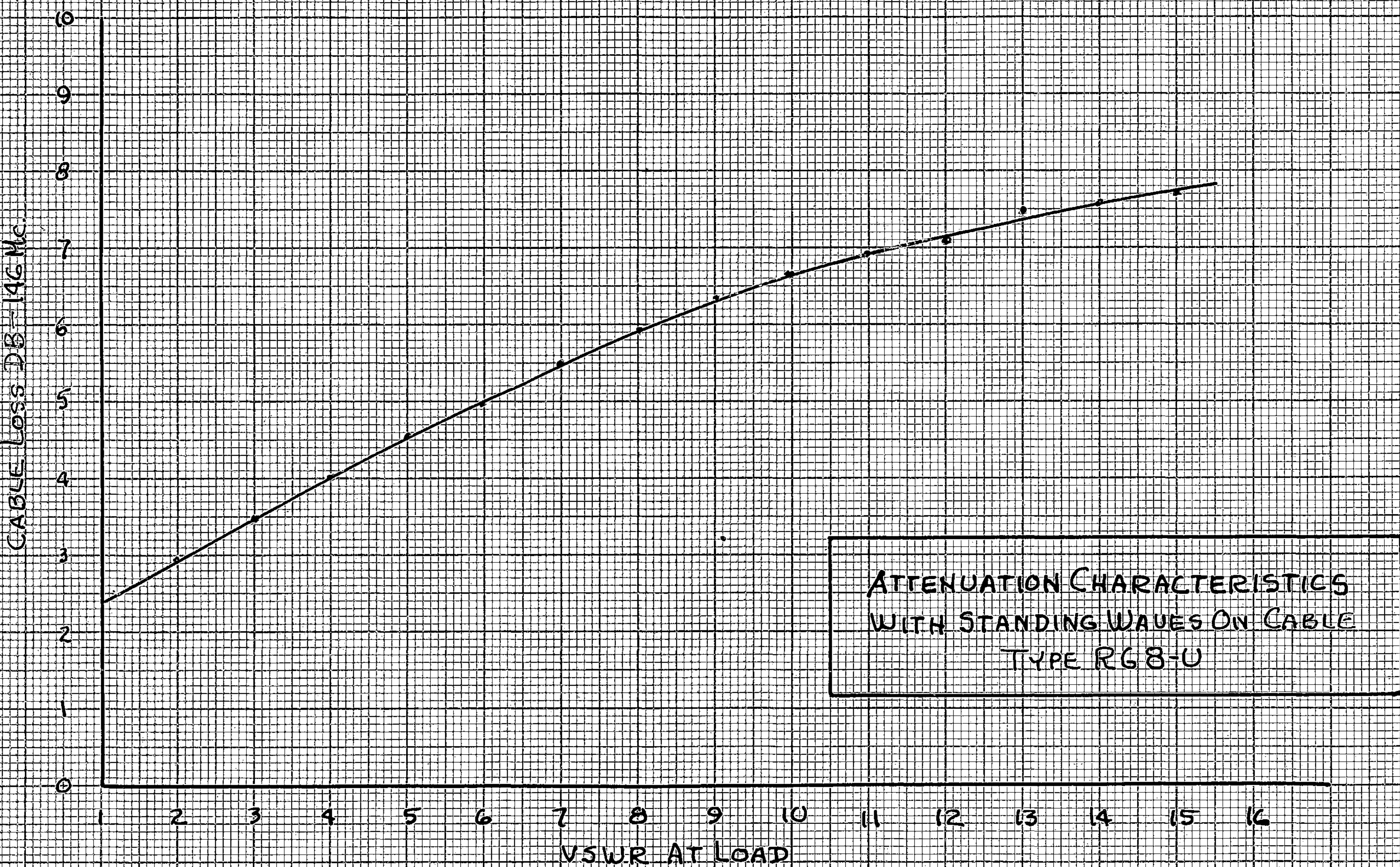


GRAPH
+x or -b1
170°
175°
P2
P3
2
3
4
5
6
7
8
9
180°
0°
r1 or g1
97%
94%
89%
82%
77%
71%
64%
55
45%
ZR/Rc = r1 + jx1
YR/Gc = g1 + j(-
-x or +b1
10°
5°
WORTH - Mar '47

MADE IN U. S. A.



GRAPH 4.


ATTENUATION CHARACTERISTICS
WITH STANDING WAVES ON CABLE
TYPE RG8-U
CABLE LOSS DB-146 Mc.
0
1
2
3
4
5
6
7
8
9
10
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
VSWR AT LOAD

GRAPH 5.

VIII. DISCUSSION OF RESULTS

The importance of the parasitic array is increasing daily due to its widespread use in television reception and the rapid expansion of UHF and VHF applications. Available literature contains no mathematical or empirical derivations and only limited experimental data for the design and prediction of the three element parasitic array. The need certainly exists for more accurate design information.

While it was realized that a complete analysis leading to a solution of this problem would not be possible immediately, it was believed that a preliminary investigation would initiate an effort leading to an ultimate solution of this problem.

The scope of the problem may be seen when the number and nature of the inter-related variables involved are considered.

For example, the feed point impedance of the three element parasitic array is a function of six distinct variables, each of which in turn is a function of other variables. The feed point impedance is dependent upon the following:

a. The self impedance of the driven element.

b. The self impedance of the reflector.

c. The self impedance of the director.

d. The mutual impedance between the driven element and the reflector.

e. The mutual impedance between the driven element and the director.

f. The mutual impedance between the director and reflector.

Each of these is dependent on the length and diameter of the several elements and the space relations of the elements. These variables all combine to determine the feed point impedance, therefore an investigation of feed point impedance as affected by the variation of relative element spacing and length was instituted.

Since the forward gain of the antenna is a prime consideration, the antenna was adjusted for maximum forward gain, and all measurements of impedance were made under this condition.

The equipment was installed on the grounds of the college golf course and impedance measurements were made in accordance with the above conditions. In order to observe the effects of element spacing on gain and impedance, data were obtained for reflector spacing increments of 0.01 wavelength and for director spacing increments of 0.02 wavelength. The limits of the spacing variations were 0.20 wavelength and 0.06 wavelength. Spacing greater than 0.20 wavelength were impossible due to the limitations of the element supporting structure. Spacing less than 0.06 wavelength were likewise not possible due to the method of support which obstructed the elements at close spacings.

The measured data and calculations were summarized and presented on the family of curves.

An inspection of the curves for feed point impedance will show that the general shape for all is the same. The major difference between the individual curves is that the point of lowest impedance occurs at a different director spacing for each value of reflector spacing used. The minimum value is rather constant and is between 10 and 15 ohms in all cases except the very wide spacings. The curve for 0.20 wavelength reflector spacing shows a minimum impedance at a director spacing of about 0.08 wavelength. At 0.19 wavelength reflector spacing the minimum point comes at a director spacing of 0.085 wavelength. At 0.18 wavelength reflector spacing the minimum point continues to move to the right (increasing director spacing) and occurs at 0.12 wavelength director spacing. At 0.17 wavelength reflector spacing the minimum has moved over to about 0.13 wavelength director spacing. At 0.16 wavelength reflector spacing the point of minimum impedance has moved still farther to the right to about 0.14 wavelength director spacing. The curve for 0.15 wavelength reflector spacing shows that the minimum has moved to about 0.16 wavelength director spacing. For a reflector spacing of 0.14 wavelength the minimum occurs at 0.17 wavelength director spacing. For a reflector spacing of 0.14 wavelength the minimum occurs at 0.17 wavelength director spacing. The last curve showing this trend was the reflector spacing of 0.13 wavelength with a minimum value of feed point impedance at 0.18 wavelength.

At spacings of less than 0.13 wavelength for the reflector the variation is more rapid and seems to repeat in the opposite direction (toward closer director spacing). The point of lowest impedance moves to the left rather than to the right and reverses at a reflector spacing of 0.09 wavelength. It then travels back to the right again and at 0.16 wavelength spacing it occurs at a director spacing greater than 0.20 wavelength. The minimum value in these instances is between 10 and 15 ohms.

The curves for gain fall into two general types. One type is essentially a straight line while the other type is reasonably straight but has a well defined dip in it. The curves with the sharp dip occur in conjunction with the impedance curves which have a spacing such that a reversal of direction of impedance minimum occurs. The gain is lowest at the spacing which gives lowest feed point impedance.

The results, while inconclusive, are indicative of the general trends of gain and impedance variations at different element spacings. Due to the limited time available the increments of spacing used were necessarily large. Additional data should be secured at intermediate element spacings in order to more completely define the behavior of the feed point impedance. This investigation has raised many interesting questions which were impossible to solve in the time alloted. Investigation of the following factors would be profitable: a study of the complete radiation pattern at all horizontal and vertical angles under different tuning conditions; the mutual impedance between parasitically excited elements; the effects of mounting the parasitic elements in other horizontal planes than that of the driven element. A further study of the parameters investigated in this problem would be of value.

It is the authors hope that this paper may serve as a guide and incentive for those interested in antennas and that it may have a small place in the development of the large concepts which ultimately may lead to a more complete understanding of the characteristics and more reliable predictions of performance of the three element parasitic antenna array.

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Typist: Blanche Hawkins