

HIDDEN SYMMETRIES AND
THEIR IMPLICATIONS FOR PARTICLE PHYSICS

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2000

Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
MASTER OF SCIENCE
July, 2004

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ACKNOWLEDGMENTS

There is usually one person who played the pivotal role as a mentor in one's graduate career. To me, this person has been Prof. K.S. Babu. As such an outstanding model building phenomenologist, he has a distinguished view of connection between experiments and theories. I wish to express my deepest gratitude to him for his constructive guidance, constant encouragement, kindness and great patience. Without his guidance and collaboration, I would not have been able to finish this work. Many thanks are also due to Prof. Mingxing Luo, who has led me into theoretical high energy physics many years ago; Ilia Gogoladze, our former postdoctoral research associate, who has also been a good teacher and a wonderful collaborator for years, and our current postdoctoral research associate, Gerhart Seidl, whose encouragement and suggestion has been invaluable in completing this work.

I would also like to express my sincere appreciation to Prof. Paul Westhaus for his assistance during my study and my other committee member Prof. John Mintmire. My appreciation extends to Prof. Satya Nandi, Prof. Jacques H.H. Perk, Prof. Aihua Xie, Prof. Xincheng Xie, Gary Don Thacker, Susan Cantrell, Cindi Raymond, Karen S. Hopkins, and all our faculty members and staff in the department. Moreover, I want to thank many of my physics colleagues for their support and friendship, especially, Ye Xiong, Zhenyue Zhu, Brian Timmons, Jianning Wang, Beining Nie, Cyril Anoka, Abdelghafour Bachri, Tsendenbaljir Enkhbat and Corneliu Marius Rujoiu.

I would like to thank the physics department at Oklahoma State University for years of support during my study and the U.S. Department of Energy for the summer financial support in part by DOE Grant # DE-FG03-98ER-41076, DOE Grant # DE-FG02-01ER-45684.

I wish to thank my parents for their constant support and encouragement. Finally, I want to thank Qinan for her support and patiently listening while I talked about my work. She has made graduate school which should have seemed interminable, instead sweet and happy. I look forward to many more years of the same.

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CHAPTER 1

INTRODUCTION

1.1 The Standard Model and Beyond

The Standard Model (SM) of elementary particle physics is a chiral gauge theory that gives a successful description of strong, weak and electromagnetic interactions [1]. It has been highly successful in explaining all experimental observations in the energy regime up to $M_{\text{EW}} \sim \mathcal{O}(10^2 \text{ GeV})$. The theory is invariant under the gauge group $G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$. The $SU(3)_C$ Quantum Chromodynamics (QCD) describes the strong interaction which is supported by evidence from deep inelastic collision experiments. The $SU(2)_L \times U(1)_Y$ gauge symmetry corresponds to the Weinberg-Salam model of electroweak interaction which has been verified by a host of experiments, including the UA1/UA2 [2] and LEP [3].

In this thesis, we use the conventional notations for the SM matter fields. They are shown in Table 1.1.

	Q	u^c	d^c	ℓ	e^c	H
$SU(3)_C$	3	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	1	1	1
$SU(2)_L$	2	1	1	2	1	2
$U(1)_Y$	1/6	-2/3	1/3	-1/2	1	1/2

TABLE 1.1. Transformation properties of the SM fields under G_{SM}

As a chiral theory, the left-handed and right-handed fermions have different transformation properties with respect to G_{SM} . Under $SU(2)_L$, the left-handed particles transform as the doublets

$$Q = \begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}_L \quad \text{and} \quad \ell = \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{pmatrix}_L,$$

while right-handed particles are $SU(2)_L$ singlets:

$$(u^c, c^c, t^c), \quad (d^c, s^c, b^c), \quad (e^c, \mu^c, \tau^c).$$

The electroweak gauge symmetry is spontaneously broken via the Higgs mechanism by a scalar $SU(2)$ doublet [4]

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}.$$

The masses of the quarks and leptons arise from Yukawa couplings from the lagrangian:

$$\mathcal{L}_{\text{SM}} = Y_u Q u^c H + Y_d Q d^c \bar{H} + Y_e \ell e^c \bar{H} + \text{h.c.}, \quad (1.1)$$

where \bar{H} is defined by $\bar{H} = i\sigma_2 H^\dagger$ and Y_u , Y_d and Y_e are dimensionless coupling constants known as Yukawa couplings. Note that the generation and color indices are contracted here.

Quantum correction to the Higgs boson mass induces the only quadratic divergence in the theory. For example, at the one-loop-level, the top quark Yukawa couplings induces a quadratic divergence given by

$$\Delta m_H^2 = \frac{\lambda_t^2}{8\pi^2} \Lambda^2,$$

where the cutoff scale Λ can be as large as M_{Pl} of order $\mathcal{O}(10^{19} \text{ GeV})$ [5]. If so, the entire Higgs mechanism explanation of electroweak symmetry breaking would fail or require fine tuning of parameters. In order to address this so-called gauge hierarchy problem [5], one would have to introduce new physics at the TeV scale. The most elegant solution is known as supersymmetry (SUSY) [6], where to each particle, there exists a SUSY partner with different spin. For instance, the superpartner of the

matter fermion top quark is a scalar known as *stop* \tilde{t} . The quadratic divergence in the Higgs mass is now removed via the cancellation between top loop and stop loop. At the one-loop-level, it is

$$\Delta m_H^2 = \frac{\lambda_t^2}{8\pi^2} \Lambda^2 + \frac{\lambda_{\tilde{t}}^2}{8\pi^2} (m_{\tilde{t}}^2 - \Lambda^2) \quad (1.2)$$

This cancellation is valid up to all loop corrections and is thus technically natural.

The minimal SUSY version of the SM is called the Minimal Supersymmetric Standard Model (MSSM) [6, 7] which is described by the superpotential

$$\mathcal{W}_{\text{MSSM}} = Y_u Q u^c H_u + Y_d Q d^c H_d + Y_e l e^c H_d + \mu H_u H_d, \quad (1.3)$$

where all the matter fields are now chiral superfields and two Higgs doublets

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \quad \text{and} \quad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$$

are introduced since the superpotential must be holomorphic, i.e., the MSSM is a two-Higgs model [7]. It is interesting enough to see the extra Higgs boson also playing an important role in cancelling the Higgsino contribution to the $SU(2)_L \times U(1)_Y$ mixed anomaly, $U(1)_Y^3$ anomaly, gravitational trace anomaly, and also to cancel the global $SU(2)_L$ Witten anomaly [8].

The SM or MSSM has been an extremely successful theory with exception of the puzzles, such as flavor hierarchy, neutrino masses, the μ -term problem, R -parity, the strong CP problem, etc.

In this thesis, our goal is to apply a new model building tool — discrete gauge symmetries [9] to solve the problems or puzzles mentioned above [10–13].

1.2 Global Symmetries in the SM

The SM provides one highly successful description the particle physics up to M_{EW} . However, it is believed to be only an effective field theory valid up to a cut-off scale Λ . In the low-energy effective theory, corrections from new physics beyond SM arise as non-renormalizable operators which are invariant under G_{SM} . Unlike the

renormalizable couplings, the coupling constants of these non-renormalizable operators are expected to be suppressed by appropriate powers of $1/\Lambda$ and have thus a negative dimension of mass [14, 15].

In the SM, there is a unique unbroken $U(1)$ gauge symmetry which is known as the $U(1)_Y$ hypercharge symmetry. The hypercharge assignment except of its normalization is determined by requiring the theory to be free from triangle gauge anomalies [16]. The gauge anomalies are violations of conservative laws due to loop corrections. They are generated via the triangle diagrams. For example, the $[SU(3)_C]^2 \times U(1)$ mixed anomaly arise from the following diagram:

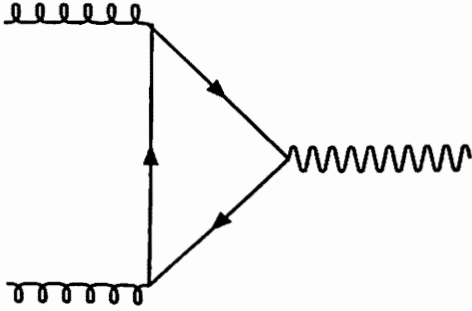


Figure 1.1. The diagram that generates $[SU(3)_C]^2 \times U(1)$ mixed anomaly.

where the internal lines are fermions, quarks in this case.

Being free from triangle gauge anomalies is a required condition for any gauge theory to make essential sense, namely the renormalizability. The anomalous Ward identity must be avoided. Anomaly matching condition should be satisfied. In order to make the discussion more concrete, let us look at the explicit example of computing anomaly coefficients invoking the hypercharge symmetry. Suppose under the $U(1)_Y$, q , u , d , l , e and h are corresponding charge for Q , u^c , d^c , l , e^c and H . $U(1)_Y$ invariance of the SM Yukawa couplings shown in 1.1 requires

$$q + u + h = 0, \quad q + d - h = 0, \quad l + e - h = 0. \quad (1.4)$$

The mixed anomaly coefficients should all vanish.

$$\begin{aligned}
A_{[SU(3)_C]^2 \times U(1)_Y} &= \frac{N_g}{2}(2q + u + d) = 0 \\
A_{[SU(2)_L]^2 \times U(1)_Y} &= \frac{N_g}{2}(3q + l) = 0 \\
\text{Tr}U(1)_Y &= N_g(6q + 3u + 3d + 2l + e) = 0 \\
A_{[U(1)_Y]^3} &= N_g(6q^3 + 3u^3 + 3d^3 + 2l^3 + e^3) = 0, \tag{1.5}
\end{aligned}$$

where the trace is the gravitational anomaly. In this particular case, cubic anomaly condition is equivalent to the gravitational anomaly condition. One can then solve the set of 6 independent equations and obtain the hypercharge assignment without its overall normalization. The hypercharge normalization can be determined when imposing conditions from physics beyond SM, e.g., GUTs.

When the anomaly cancellation constraints are relaxed, the extra degrees of freedom correspond to the following global symmetries:

- Baryon number B
- Lepton number L .

They cannot be realized as part of a fundamental gauge symmetry. An ultimate theory, like string theory [17], is believed to contain a theory of gravity which presumably violates all global symmetries and therefore has to be a full gauge theory. It is then unclear where these global symmetries arise from and how they can survive down to low energies. One usually expects that global symmetries can arise as accidental symmetries in the low energy effective theory. However, there is still no fundamental reason for global symmetries to be protected. Both B and L could be violated but their violation has not been directly observed yet. When an additional Higgs doublet is introduced, the new degree of freedom correspond the Peccei-Quinn (PQ) symmetry [18]. The PQ symmetry is broken explicitly near $f_a \sim \mathcal{O}(10^{11}\text{GeV})$ thereby generating an axion to compensate the CP violation in QCD and thus provides a solution to the strong CP problem *.

*A detailed discussion can be found in chapter 6.

	Q	u^c	d^c	ℓ	e^c	ν^c	H_u	H_d
$U(1)_B$	1/3	-1/3	-1/3	0	0	0	0	0
$U(1)_L$	0	0	0	1	-1	-1	0	0
PQ	0	0	-1	0	-1	0	1	0

TABLE 1.2. Global Symmetries in the two-Higgs SM

In Table 1.2, we list the global charges with respect to $U(1)_B$, $U(1)_L$ and the PQ symmetry for the two-Higgs SM which can be naturally embedded into a SUSY version of the SM.

Since neither B nor L is a part of G_{SM} , quantum gravity is believed to violate both B or L via non-renormalizable operators of the type:

$$\mathcal{L}_{\text{NR}} \supset \ell\ell HH/M_{\text{Pl}} + QQQ\ell/M_{\text{Pl}}^2 + \text{h.c.} \quad (1.6)$$

The first term violates L by two units ($\Delta L = 2$) and can give rise to neutrino masses, while the second term violates both B and L by one unit ($\Delta B = 1$, $\Delta L = 1$) which leads to proton decay, for example, via $p \rightarrow e^+\pi^0$. Provided the four-dimensional (4D) quantum gravity scale of M_{Pl} is roughly of order $\mathcal{O}(10^{19} \text{ GeV})$, one obtains a lower bound on the neutrino masses (m_ν) and an upper bound on the proton lifetime (τ_p) of the orders:

$$m_\nu \gtrsim 10^{-5} \text{ eV} \quad \text{and} \quad \tau_p \lesssim 10^{45} \text{ yrs.}$$

The above neutrino mass scale does not agree with the current experimental bound. For several decades, massless neutrinos have played an important role in understanding the chiral character of weak interaction. The SM does not contain massive neutrinos. However, since Super-Kamiokande water Cherenkov Detector discovered the oscillation between different flavor states of neutrinos suggesting that neutrinos are massive, our knowledge about neutrino masses has been remarkably improved by solar [19], atmospheric [20], and reactor [21] neutrino oscillation data. For instance, solar and atmospheric neutrino oscillations imply the neutrino mass squared splittings

$\Delta m_{\odot}^2 = 7.5 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{\text{atm}}^2 = 2.0 \times 10^{-3} \text{ eV}^2$ respectively. These mass squared splittings yield a lower bound on neutrino mass around $\sim 10^{-1} \text{ eV} \gg 10^{-5} \text{ eV}$ which is much greater than the mass possibly induced by quantum gravity effects.

Lepton number does not necessarily have to be violated in order to understand the existence of massive neutrinos. Neutrinos could be Dirac particles, in which case, neutrino masses may arise from the usual Yukawa couplings:

$$\mathcal{L}_{\text{Dirac}} = \ell \nu^c H_u + \text{h.c.} \quad (1.7)$$

where right-handed neutrinos ν^c are the SM singlet. Then, the hierarchy problem in Yukawa coupling constants must be addressed since there exists a 10^{12} order hierarchy in $Y_t/Y_\nu \sim m_t/m_\nu \sim 174 \text{ GeV}/10^{-10} \text{ GeV} \sim 10^{12}$. The hierarchy provides a strong hint that a new physics scale should be much greater than M_{EW} . One natural way to understand this hierarchy, i.e., the smallness of neutrino masses, is provided by the seesaw mechanism [22]. In this framework, the right-handed neutrinos are Majorana particles and the right-handed neutrino scale is $M_R \sim 10^{14} - 10^{15} \text{ GeV}$. The renormalizable lagrangian responsible for neutrino masses is then given by

$$\mathcal{L}_{\text{seesaw}} = \ell \nu^c H_u + M_R \nu^c \nu^c + \text{h.c.} \quad (1.8)$$

Note that the Majorana neutrino mass terms $M_R \nu^c \nu^c$ explicitly break the L . This allows to test the scenario in current and future neutrinoless double beta decay $(\beta\beta)_{0\nu}$ experiments. At low energies, the non-renormalizable L -violating operators generated by the seesaw mechanism can be realized after integrating out the heavy right-handed neutrino as the dimension-five term [15, 23]

$$\mathcal{L}_{\Delta L=2} = \ell \ell H H / \Lambda_L, \quad (1.9)$$

where Λ_L stands for the effective scale of L -violation which is M_R in this case. After integrating out the heavy states ν^c , one arrives at realistic neutrino masses in the range $M_{\text{EW}}^2/M_R \sim 10^{-10} \text{ GeV}$.

The $\tau_p \sim 10^{45}$ yrs limit predicted by quantum gravity corrections from operators of the type $QQQ\ell/M_{\text{Pl}}^2$ is much above the current experimental bounds on the proton

lifetime [24]:

$$\tau_p > 5 \times 10^{33} \text{ yrs for } p \rightarrow e^+ \pi^0 \text{ and } \tau_p > 1.6 \times 10^{33} \text{ yrs for } p \rightarrow \bar{\nu} K^+.$$

These limits indicate that the baryon number violation scale must be $\Lambda_B > 10^{15}$ GeV.

The high energy scales Λ_L and Λ_B find a natural origin in Grand Unified Theories (GUTs) [25]. As an elegant extensions of the SM, GUTs provide a unified picture of the SM gauge interactions $SU(3)_C \times SU(2)_L \times U(1)_Y$ and are consistent with the gauge unification picture which LEP and other experiments tested many years ago [26]. GUTs give a natural explanation of charge quantization as well. As a result of putting baryons and leptons in to the same gauge multiplets, GUTs (with or without SUSY) typically generate $\Delta B = 1$ and $\Delta L = 1$ operators with

$$\Lambda_B \sim 10^{14} - 10^{16} \text{ GeV},$$

which are close to the experimental limits from nucleon decay [27].

Besides the new physics effects discussed above, B can be violated even in the SM via non-perturbative effect such as electroweak instanton [28] and sphaleron processes [29]. These effects, however, are at the $\Delta B = 3 \bmod 3$ level due to the existence of three generations. For instance, the non-perturbative sphaleron interaction in the SM lagrangian can be thought of as state

$$\prod_{i=1}^3 (u_L d_L d_L \nu_L)_i, \quad (1.10)$$

where $i = 1, 2, 3$ stands for generation index. These B and L violating processes play an extremely important role in cosmology, e.g., in the context of baryogenesis or the electroweak phase transition. It is interesting to note that there exists a symmetry known as baryon parity [12, 30] in the SM lagrangian. The physical consequence of this symmetry is also an effective Baryon number at the mod three level ($\Delta B = 3 \bmod 3$). In the next chapter, we present this symmetry and discuss its physical implications.

CHAPTER 2

HIDDEN SYMMETRY IN THE SM

2.1 Discrete Gauge Symmetry and Anomalies

Discrete global symmetries have been widely discussed in particle physics for various phenomenological purposes. As mentioned previously, global symmetries will have to face a potential violation induced by quantum gravitational effects [31]. If those discrete symmetries can be realized as gauge symmetries, such violation can then be avoided. The idea of discrete gauge symmetries was first introduced in the Lattice gauge theory [32]. One can make use of these discrete gauge symmetries for field regularization purpose on the lattices [33]. In the context of string theory, discrete gauge symmetries are also widely discussed as relics, emerging after dimensional reduction, of higher-dimensional general coordinate invariance or spontaneously broken high-dimensional gauge symmetries. Moreover, they turn out to be crucial in orbifold constructions [34]. Discrete gauge symmetries are also introduced in 4D field theories as remnants of a spontaneously broken gauge symmetry [9, 35, 36]. As a new model building tool, discrete gauge symmetries have been widely discussed in various applications [10–13, 30, 37–40].

In order to understand the idea of discrete gauge symmetries, let us consider an explicit realization of a discrete gauge symmetry in a $U(1)$ theory. Assume a 4D $U(1)$ gauge theory containing two scalar fields, the Higgs η with charge N and the scalar ψ with charge -1 under the $U(1)$ symmetry. After the Higgs η develops a vacuum expectation value (VEV) and breaks the $U(1)$, the gauge-invariant term $\eta\psi^N$ restricts

$$\psi \rightarrow e^{-i2\pi/N}\psi. \quad (2.1)$$

However, since the term $\eta\psi^N$ is non-renormalizable if $N \geq 4$, it is not clear whether the symmetry should really be preserved. A renormalizable example in a chiral theory can be given in terms of the SM language, where masses arise from usual Yukawa couplings. For this purpose, we suppose there exists a new $U(1)_X$ symmetry. Thus the total gauge symmetry of the theory is $G_{\text{SM}} \times U(1)_X$. Suppose $U(1)_X$ is broken along the electroweak symmetry via the SM Higgs VEV. The Yukawa coupling invariance then leads to

$$Qu^c H : q + u + h = 0 \quad (2.2)$$

where q , u and h stand for the $U(1)_X$ charges of Q , u^c , and H , respectively. Hence, the fields transform as

$$Q \rightarrow e^{-iq\theta(x)}Q, \quad u^c \rightarrow e^{-iu\theta(x)}u^c, \quad H \rightarrow e^{-iN\theta(x)}H, \quad (2.3)$$

where we also assume all the charges are integers and have set $h = N$. After electroweak symmetry breaking, the lagrangian exhibits a discrete Z_N symmetry

$$Qu^c \rightarrow e^{-i(q+u)2\pi/N}Qu^c = e^{-i(N)2\pi/N}Qu^c = Qu^c, \quad (2.4)$$

under which Q and u^c transform as

$$Q \rightarrow e^{-iq2\pi/N}Q, \quad u^c \rightarrow e^{-iu^c2\pi/N}u^c. \quad (2.5)$$

In the effective theory, the two discrete Z_N symmetries are indistinguishable. However, this indeed provides hints to high energy theory. Our above consideration provides a constraint on the proper charge assignment. In fact, a condition must be satisfied, since spontaneous symmetry breaking does not induce any gauge anomaly. Therefore, if the Z_N is a subgroup of a gauge symmetry, it must be free of gauge anomaly since the original theory is also anomaly-free.

Another puzzle arises as how to define a gauge anomaly in terms of discrete gauge symmetries [9, 35, 36]. At low energies, gauge bosons decouple from the theory and there is no gauge current associated with discrete gauge symmetries. It seems then to be difficult to realize a triangle anomaly [16]. However, as we mentioned earlier, gauge anomalies cannot be induced via spontaneous symmetry breaking (SSB), it

should be possible to realize the anomaly prior to SSB. We can simply take the discrete charges to compute anomaly coefficients in the same way we compute anomalies before SSB. It is clear that the linear conditions will still hold. However, the non-linear conditions like cubic anomalies cannot be simply extended to discrete symmetries.

Besides the above change, the anomaly cancellation condition may be modified due to possible existence of vectorial heavy fermions. Suppose the discrete Z_N gauge symmetry arise from a full $U(1)$. The field that acquires a VEV and breaks $U(1)$ to Z_N can supply large masses at very high scale to a set of heavy fermions which have Yukawa couplings involving this field. Such fields may include Majorana fermions as

$$\mathcal{L} \supset SQQ, \quad (2.6)$$

and Dirac fermions as

$$\mathcal{L} \supset SQ\bar{Q}. \quad (2.7)$$

These heavy fields can carry SM gauge quantum numbers, but they must transform vectorially under the SM. In order that their mass terms be invariant under the unbroken Z_N , it must be that

$$\begin{aligned} 2q_i &= 0 \text{ mod } N \text{ (Majorana fermion)} \\ q_i + \bar{q}_i &= 0 \text{ mod } N \text{ (Dirac fermion)} \end{aligned} \quad (2.8)$$

where q_i are the $U(1)$ charges of these heavy fermions. The index i is a flavor index corresponding to different heavy fields. These heavy fermions, being chiral under the $U(1)_A$, contribute to gauge anomalies. Their contribution to the $[SU(3)_C]^2 \times U(1)$ gauge anomaly is given by $A_3 = \sum_i q_i m_i = (N/2) \sum_i p_i m_i$ (Majorana fermion) or $A_3 = \sum_i (q_i + \bar{q}_i) m_i = (N) \sum_i p_i m_i$ (Dirac fermion) where ℓ_i is the quadratic index of the relevant fermion under $SU(3)_C$ and the p_i are integers. We shall adopt the usual normalization of $m = 1/2$ for the fundamental of $SU(N)$. Then, for the case of a heavy Dirac fermion, one has $A_3 = p(N/2)$ where p is an integer, as the index of the lowest dimensional (fundamental) representations is $1/2$ and those of all other representations are integer multiples of $1/2$. The same conclusion follows for the case of Majorana fermions for a slightly different reason. All real representations of

$SU(3)_C$ (such as an octet) have integer values of m , so that $\sum_i p_i m_i$ is an integer. Analogous conclusions follow for the $[SU(2)_L]^2 \times U(1)$ anomaly coefficient.

2.2 Baryon Parity

In this section, we show that the SM lagrangian with the seesaw mechanism for small neutrino masses has a discrete Z_6 gauge symmetry which forbids all $\Delta B = 1$ and $\Delta B = 2$ baryon violating effective operators *. This can be seen as follows. The SM Yukawa couplings incorporating the seesaw mechanism to generate small neutrino masses is

$$\mathcal{L}_{\text{mass}} = Qu^c H + Qd^c \bar{H} + le^c \bar{H} + l\nu^c H + M_R \nu^c \nu^c + \text{h.c.} \quad (2.9)$$

Here we have used the standard (left-handed) notation for the fermion fields and have not displayed the Yukawa couplings or the generation indices. This lagrangian respects a discrete Z_6 symmetry with the charge assignment as shown in Table 2.2. Also shown in Table 2.2 are the charge assignments under the Z_3 and Z_2 subgroups of Z_6 . The Z_3 assignment is identical to that in Ref. [41]

	Q	u^c	d^c	ℓ	e^c	ν^c	H
Z_6	6	5	1	2	5	3	1
Z_3	3	2	1	2	2	3	1
Z_2	2	1	1	2	1	1	1

TABLE 2.1. Family-independent Z_6 charge assignment to the SM fields along with the charges under the Z_3 and Z_2 subgroups.

*Since there exists an unbroken $U(1)_Y$ symmetry, one can always take the hypercharge subgroup to redefine the discrete symmetry as

$$H \rightarrow e^{-i2\pi/3 \times (1)} \cdot e^{-i2\pi\alpha/N \times (3)} H.$$

For instance, under all the symmetries we discuss here, Higgs fields transform non-trivially which may lead to potential domain wall problem. But one can always rotate it away by shifting a combination of hypercharge. We would then instead obtain a Z_9 symmetry.

From Table 3.1 it is easy to calculate the Z_6 crossed anomaly coefficients with the SM gauge groups. We find the $SU(3)_C$ or $SU(2)_L$ anomalies to be

$$\begin{aligned} A_{[SU(3)_C]^2 \times Z_6} &= 3N_g \\ A_{[SU(2)_L]^2 \times Z_6} &= N_g \end{aligned} \quad (2.10)$$

where N_g is the number of generations. The condition for a Z_N discrete group to be anomaly-free is

$$A_i = \frac{N}{2} \pmod{N} \quad (2.11)$$

where i stands for $SU(3)_C$ and $SU(2)_L$. For Z_6 , this condition reduces to $A_i = 3 \pmod{6}$, so when $N_g = 3$, Z_6 is anomaly-free. Obviously, the Z_3 and Z_2 subgroups are also anomaly-free. The significance of this result is that unknown quantum gravitational effects will respect this Z_6 . It is this feature that we utilize to stabilize the nucleon. Absence of anomalies also suggests that the Z_6 may have a simple gauge origin.

To see how the Z_6 forbids $\Delta B = 1$ and $\Delta B = 2$ processes, we note that it is a subgroup of $U(1)_{2Y-B+3L}$ where Y is SM hypercharge [42]. We list in Table 2.2 the charges under the three $U(1)$ symmetries. It is clear that the Z_6 can be a subgroup

	Q	u^c	d^c	ℓ	e^c	ν^c	H
$U(1)_{2Y-B+3L}$	0	-1	1	2	-1	-3	1

TABLE 2.2. Charge assignment under $U(1)_{2Y-B+3L}$ which contains the Z_6 .

of $U(1)_{2Y-B+3L}$. Any Z_6 invariant effective operator must then satisfy

$$2\Delta Y - \Delta B + 3\Delta L = 0 \pmod{6}. \quad (2.12)$$

Invariance under $U(1)_Y$ implies $\Delta Y = 0$. Consider $\Delta B = 1$ effective operators which must then obey (from Eq. (2.12)) $3\Delta L = 1 \pmod{6}$. This has no solution, since $3\Delta L = 0 \pmod{3}$ from Table 2.2. Similarly, $\Delta B = 2$ operators must obey $3\Delta L = 2 \pmod{6}$ which also has no solution. $\Delta B = 3$ operators, which corresponds to $3\Delta L = 0 \pmod{6}$, are allowed by this Z_6 . Such operators have dimension 15 or

higher and have suppression factors of at least Λ^{-11} . These will lead to “triple nucleon decay” processes where three nucleons in a heavy nucleus undergo collective decays leading to processes such as $pnn \rightarrow e^+\pi^0$. We estimate the rates for such decay in Section 2.3 and find that Λ can be as low as 10^2 GeV.

2.3 Triple Nucleon Decays

The existence of baryon parity ensures the absence of $\Delta B = 1$ and $\Delta B = 2$ effective operators. We now list the lowest dimensional (d=15) $\Delta B = 3$ effective operators which are consistent with the baryon parity. Imposing gauge invariance and Lorentz invariance, we find them to be:

$$\begin{aligned} & \bar{u}^c \bar{d}^c \bar{e}^c, \quad \bar{u}^c \bar{d}^c \bar{e}^c, \quad Q \bar{u}^c \bar{d}^c \bar{e}^c, \quad Q \bar{u}^c \bar{d}^c \bar{e}^c, \quad Q^2 \bar{u}^c \bar{d}^c \bar{e}^c, \\ & Q^2 \bar{u}^c \bar{d}^c \bar{e}^c, \quad Q^3 \bar{u}^c \bar{d}^c \bar{e}^c, \quad Q^3 \bar{u}^c \bar{d}^c \bar{e}^c, \quad Q^4 \bar{u}^c \bar{d}^c \bar{e}^c, \quad Q^4 \bar{u}^c \bar{d}^c \bar{e}^c, \\ & Q^4 \bar{d}^c \bar{e}^c, \quad Q^5 \bar{u}^c \bar{d}^c \bar{e}^c, \quad Q^5 \bar{d}^c \bar{e}^c, \quad Q^6 \bar{u}^c \bar{d}^c \bar{e}^c, \quad Q^7 \bar{d}^c \bar{e}^c, \quad Q^8 \bar{d}^c \bar{e}^c. \end{aligned} \quad (2.13)$$

Here Lorentz, gauge and flavor indices are suppressed. These operators can lead to “triple nucleon decay”. The dominant processes are

$$\begin{aligned} ppp & \rightarrow e^+ + \pi^+ + \pi^+ \\ ppn & \rightarrow e^+ + \pi^+ \\ pnn & \rightarrow e^+ + \pi^0 \\ nnn & \rightarrow \bar{\nu} + \pi^0. \end{aligned} \quad (2.14)$$

Tritium (3H) and Helium-3 (3He) are examples of three-nucleon systems in nature. These nuclei are unstable and undergo β -decay with relatively short lifetime. In the presence of operators of Eq. (2.13), ${}^3H \rightarrow e^+ + \pi^0$ and ${}^3He \rightarrow e^+ + \pi^+$ decays can occur. However, there is no stringent experimental limit arising from these nuclei. So we focus on triple-nucleon decay in the Oxygen nucleus where there are experimental constraints from water detectors. To estimate the decay lifetime we need first to convert the nine-quark operators of Eq. (2.13) into three-nucleon operators and subsequently into the Oxygen nucleus.

We choose a specific operator $Q^5 \bar{d}^c \bar{\ell} / \Lambda^{11}$ as an example to study the process $pnn \rightarrow e^+ + \pi^0$ triple nucleon decay process. This induces the effective three-nucleon operator in the Oxygen nucleus

$$\frac{Q^5 \bar{d}^c \bar{\ell}}{\Lambda^{11}} \sim \frac{\beta^3(1+D+F)}{\sqrt{2}f_\pi\Lambda^{11}}(\pi n n p e), \quad (2.15)$$

where $\beta \simeq 0.014 \text{ GeV}^3$ is the matrix element to convert three quarks into a nucleon [43]. $F \simeq 0.47$, $D \simeq 0.80$ are chiral lagrangian factors, and $f_\pi = 139 \text{ MeV}$ is the pion decay constant.

We now estimate the wave-function overlap factor for three nucleons inside Oxygen nucleus to find each other. This is based on a crude free Fermi gas model where the nucleons are treated as free particles inside an infinite potential well. A single nucleon wave function is given by $\psi_m(x) = \sqrt{2/r} \sin(m\pi x/r)$, where r is the size of the nucleus and m is the energy level. Incorporating isospin and Pauli exclusion principle, the highest energy level which corresponds to $m = 4$ is found to have 2 protons and 2 neutrons. We assume the highest level has the highest probability to form a Tritium-like “bound state” of three nucleons. The probability for three nucleons in the Oxygen nucleus to overlap in a range of the size of the Tritium nucleus is

$$P \sim \frac{4}{3} \int_0^{\sqrt[3]{\frac{3}{16}}} d\left(\frac{x_1}{r}\right) d\left(\frac{x_2}{r}\right) d\left(\frac{x_3}{r}\right) \left(\sin\left(\frac{4\pi x_1}{r}\right) \sin\left(\frac{4\pi x_2}{r}\right) \sin\left(\frac{4\pi x_3}{r}\right) \right)^2 \sim 0.0253, \quad (2.16)$$

where $\sqrt[3]{\frac{3}{16}}$ is the ratio between the radii of the Tritium and the Oxygen nucleus, since $R \propto A^{1/3}$ (A is the atomic number). So the effective baryon number violating operator of Eq. (2.15) becomes

$$\frac{P\beta^3}{\sqrt{2}f_\pi\Lambda^{11}R^3}({}^3H\pi e). \quad (2.17)$$

The triple nucleon decay lifetime can then be estimated to be

$$\tau \sim \frac{16\pi f_\pi^2 \Lambda^{22} R^6}{P^2 \beta^6 M_{3H}}. \quad (2.18)$$

By putting the current limit on proton lifetime of 3×10^{33} yrs, we obtain:

$$\Lambda \sim 10^2 \text{ GeV}. \quad (2.19)$$

Thus we see the Z_6 symmetry ensures the stability of the nucleon. To test our crude model of nuclear transition, we have also evaluated the double nucleon decay rate within the same approach and found our results to be consistent with other more detailed evaluations [44].

2.4 Gauging Baryon Parity

It is interesting to see if the Z_6 symmetry of Table 3.1 can be realized as an unbroken subgroup of a gauged $U(1)$ symmetry. Although the Z_6 is a subgroup of the $U(1)_{(2Y-B+3L)}$, this $U(1)$ would be anomalous without enlarging the particle content. We have found a simple and economic embedding of Z_6 into a $U(1)$ gauge symmetry associated with $I_R^3 + L_i + L_j - 2L_k$. Here L_i is the i th family lepton number and $i \neq j \neq k$. No new particles are needed to cancel gauge anomalies. With the inclusion of right-handed neutrinos, $I_R^3 = Y - (B - L)/2$ is an anomaly-free symmetry. $L_i + L_j - 2L_k$, which corresponds to the λ_8 generator acting in the leptonic $SU(3)$ family space, is also anomaly-free.

The charges of the SM particles under this $U(1)$ are

$$\begin{aligned} Q_i &= (0, 0, 0), & u_i^c &= (-1, -1, -1), & d_i^c &= (1, 1, 1), \\ \ell_i &= (-4, 2, 2), & e_i^c &= (5, -1, -1), & \nu_i^c &= (3, -3, -3), & H &= 1. \end{aligned}$$

This charge assignment allows all quark masses and mixings as well as charged lepton masses. When the $U(1)$ symmetry breaks spontaneously down to Z_6 by the vacuum expectation value of a SM singlet scalar field ϕ with a charge of 6, realistic neutrino masses and mixings are also induced. The relevant lagrangian for the right-handed neutrino Majorana masses is

$$\mathcal{L}_{\text{neutrino mass}} = M_{12}\nu_1^c\nu_2^c + M_{13}\nu_1^c\nu_3^c + \nu_3^c\nu_3^c\phi + \nu_2^c\nu_2^c\phi + \nu_1^c\nu_1^c\phi^* + \nu_3^c\nu_2^c\phi. \quad (2.20)$$

After integrating out the heavy right-handed neutrinos we obtain the following $\Delta L = 2$ effective operators:

$$\mathcal{L}_{\Delta L=2} = \frac{1}{\Lambda}(\ell_1\ell_2HH + \ell_1\ell_3HH + \ell_1\ell_1HH\epsilon + \ell_2\ell_2HH\epsilon^* + \ell_2\ell_3HH\epsilon^* + \ell_3\ell_3HH\epsilon^*). \quad (2.21)$$

Here $\Lambda \sim M_{12} \sim M_{13}$ is the scale of L -violation and we have defined $\epsilon \equiv \langle \phi \rangle / \Lambda$. For $\epsilon \ll 1$, this lagrangian leads to the inverted mass hierarchy pattern for the neutrinos which is well consistent with the current neutrino oscillation data. This neutrino mass mixing pattern is analogous to the one obtained from $L_e - L_\mu - L_\tau$ symmetry [45]. However, here the $U(1)$ is a true gauge symmetry.

We have also investigated other possible $U(1)$ origins of the Z_6 symmetry and found the $I_R^3 + L_i + L_j - 2L_k$ combination to be essentially unique. To see this, let us assign a general $U(1)_X$ charge for i th generation of the SM fermions consistent with the Z_6 symmetry as

$$\{Q_i, u_i^c, d_i^c, \ell_i, e_i^c, \nu_i^c\} = \{6m_1^{(i)}, 5 + 6m_4^{(i)}, 1 + 6m_3^{(i)}, 2 + 6m_2^{(i)}, 5 + 6m_5^{(i)}, 3 + 6m_6^{(i)}\}$$

where $m_j^{(i)}$ are all integers. The Higgs field has a charge $H = 1 + 6m_0$. If we impose the invariance of the Yukawa couplings of the charged fermions and Dirac neutrinos for each generation, the anomaly coefficients from the i th generation become

$$\begin{aligned} A_{[SU(3)_C]^2 \times U(1)_X}^{(i)} &= 0 \\ A_{[SU(2)_L]^2 \times U(1)_X}^{(i)} &= 1 + 9m_1^{(i)} + 3m_2^{(i)} \\ A_{[U(1)_Y]^2 \times U(1)_X}^{(i)} &= -(1 + 9m_1^{(i)} + 3m_2^{(i)}) \\ A_{[U(1)_X]^2 \times U(1)_Y}^{(i)} &= [5 + m_0] A_{[SU(2)_L]^2 \times U(1)_X}^{(i)} \\ A_{[U(1)_X]^3}^{(i)} &= [5 + m_0]^2 A_{[SU(2)_L]^2 \times U(1)_X}^{(i)}. \end{aligned} \quad (2.22)$$

The coefficient for the mixed gravitational anomaly for each generation is zero. From Eq. (2.22), it follows that $A_2 = \sum_i A_{[SU(2)_L]^2 \times U(1)_X}^{(i)} = \sum_i (1 + 9m_1^{(i)} + 3m_2^{(i)}) = 0$ can be satisfied only when all three generation contributions are included. Once $A_2 = 0$ is satisfied, all other anomaly coefficients will automatically vanish. $A_2 = 0$ can be rewritten in a familiar form as $3 \sum_i Q_i + \sum_i \ell_i = 0$. Thus we see that any $U(1)$ symmetry satisfying this condition and consistent with the Z_6 charge assignment can be a possible source of Z_6 . If the Q_i are different for different generations, quark mixings cannot be generated without additional particles. By making a shift proportional to hypercharge, we can set $Q_i = 0$ for all i . Two obvious solutions to $\sum_i \ell_i = 0$ are $\ell_i = (1, 1, -2)$ and $\ell_i = (1, -1, 0)$. The latter one does not reproduce

the Z_6 charge assignment while the former one does, which is our solution when I_R^3 is added to it.

A related $B - 3L_\tau$ has been discussed in Ref. [46]. This is the same as $B - L$ plus $L_e + L_\mu - 2L_\tau$. In Ref [46], only one right-handed neutrino ν_τ^c is introduced so the seesaw mechanism applies only for one light neutrino. The other two neutrinos receive small masses from radiative corrections. In our model, since there are three right-handed neutrinos, all the neutrino masses arise from the conventional seesaw mechanism.

The 4D simple GUT will explicitly break the Z_3 baryon parity as it predicts the $D = 6$ operator which violates the Baryon number at $\Delta B = 1$. One can also see that the embedding of Z_6 into $U(1)$ of $I_R^3 + L_i + L_j - 2L_k$ is not a consistent picture of the simple GUTs. The baryon parity provides a strong hint to GUTs type physics beyond SM. It is interesting that the anomaly-free fact of Z_6 is a result of existence of three generations and consistent with Baryon number violation due to the electric instanton or SM sphaleron processes.

CHAPTER 3

GAUGED R -PARITY AND $B - L$ SYMMETRY

3.1 MSSM and Gauged R -parity

The following couplings

$$\mathcal{W}_{\mathcal{R}} = u^c d^c d^c + Q l d^c + l l e^c + l H_u, \quad (3.1)$$

are G_{SM} gauge invariant but absent in the non-supersymmetric SM, since they violate Lorentz invariance. However, when the theory is extended to MSSM, this constraint no longer exists and the couplings will appear in the superpotential. These couplings essentially violate B or L at the renormalizable level, which are presumably global symmetries in the SM. The L violating couplings can give rise to the neutrino masses via one-loop effects but the B violating terms lead to a rapid proton decay. The strong experimental bound on the proton lifetime therefore requires these couplings to be sufficiently suppressed, such that the MSSM can become an acceptable theory. For this purpose, one usually assumes a discrete global Z_2 symmetry, under which, the SM particles are taken to be even while their superpartners are odd. This symmetry is known as R -parity. The assumption of R -parity has profound implications for supersymmetric particle search at colliders as well as for cosmology. Due to R -parity, e.g., SUSY particles would be produced at colliders only in pairs. Moreover, R -parity implies that the lightest SUSY particle (LSP), for instance a neutralino in the mSUGRA scenario, will be stable. This stable LSP is then a leading candidate for cosmological cold dark matter.

Since the global R -parity is not part of the MSSM gauge symmetry, it is potentially violated by quantum gravitational effects. These effects (associated with worm holes, black holes, etc.) are believed to violate all global symmetries [31]. Gauge

symmetries, however, are protected from such violations. As noted in Chapter 2, when a gauge symmetry breaks spontaneously, often a discrete subgroup is left intact. Such discrete symmetries, called discrete gauge symmetries [9], are also immune to quantum gravitational effects. Not all discrete symmetries can however be gauge symmetries. For instance, since the original continuous gauge symmetry was free from anomalies, its unbroken discrete subgroup should be free from discrete gauge anomalies [47, 48]. This imposes a non-trivial constraint on the surviving discrete symmetry and/or on the low energy particle content [9, 30, 37, 46–49]. It will be of great interest to see if R -parity of MSSM can be realized as a discrete gauge symmetry, so that one can rest assured that it won't be subject to unknown quantum gravitational violations [50, 51].

After a systematic analysis in [10], we can conclude the simplest exact R -parity is family-independent Z_2 subgroup of $U(1) I_R^3$ gauge symmetry. Notice together with

	Q	u^c	d^c	ℓ	e^c	ν^c	H_u	H_d	α
Z_2	2	1	1	2	1	1	1	1	2

TABLE 3.1. Gauged Z_2 R -parity.

the Z_3 Baryon Parity, they form a discrete Z_6 R -parity [10].

At the $U(1)$ level, I_R^3 can be realized as linear combination of $U(1)_{B-L}$ and $U(1)_Y$. Therefore, one can always take a hypercharge subgroup to do a redefinition of an exact Z_2 R -parity. Hence, the exact R -parity can always be realized in terms of subgroups of $B - L$ [50].

Proton decay violates both B and L . However, $B - L$ is still conserved. The $U(1)_{B-L}$ is known as a global symmetry in the SM. By introducing a right-handed neutrino for each generation, it becomes a full gauge symmetry free of anomalies. An interesting example of a discrete gauge symmetry in the SM with seesaw neutrino masses is the Z_6 subgroup of $B - L$. The introduction of the right-handed neutrino for generating small neutrino masses makes $B - L$ a true gauge symmetry. When the ν^c fields acquire super-large Majorana masses, $U(1)_{B-L}$ breaks down to a discrete Z_6

subgroup. It is worth mentioning that the Z_6 symmetry has a Z_2 and a Z_3 subgroup as well.

Field	Q	u^c	d^c	ℓ	e^c	ν^c	H_u	H_d
$U(1)_{B-L}$	1/3	-1/3	-1/3	-1	1	1	0	0
Z_6	1	5	5	3	3	3	0	0

TABLE 3.2. The $B-L$ charges of the SM fields along with the unbroken Z_6 subgroup after the seesaw mechanism.

3.2 Gauged $B-L$ without ν_R

One physical consequence of the existence of the Z_3 Baryon Parity in the theory is that the new physics cut-off scale can be lowered to TeV even without violating current proton decay limits. If the threshold of new physics is as low as a few TeV, the induced neutrino mass via $\ell\ell H_u H_u/\Lambda$ will be too large. Here we show a mechanism by which such operators can be suppressed by making use of a discrete Z_N symmetry (with N odd) surviving to low energy. This Z_N has a natural embedding in the $B-L$ gauge symmetry. The question arises here is essentially how to gauge $U(1)_{B-L}$ without a right-handed neutrino.

Consider the following effective operators in the low energy lagrangian:

$$\mathcal{L} \supset \ell\ell H H \frac{S^6}{\Lambda^7} + \frac{S^{2N}}{\Lambda^{2N-4}}. \quad (3.2)$$

Here S is a scalar singlet field which has charge $(1, 3)$ under $Z_N \times Z_6$ while ℓ has charge $(-3, 2)$. The first term in Eq. (3.2) respects a $U(1)$ symmetry while the second term reduces this to $Z_6 \times Z_N$. If S develops a VEV of order 10^2 GeV, realistic neutrino masses can arise even when Λ is low. For example, if $\Lambda = 10$ TeV and $S = 10^2$ GeV, the neutrino mass is of order $v^2 \langle S \rangle^6 / \Lambda^7 \sim 0.4$ eV, which is consistent with the mass scale suggested by the atmospheric neutrino oscillation data.

Two explicit examples of the Z_N symmetry with $N = 5$ and 7 are shown in Table 3.2. These Z_N symmetries are free from gauge anomalies. In the Z_5 example, the

crossed anomaly coefficients for $SU(3)_C$ and $SU(2)_L$ are $5N_g$ and $5N_g/2$ respectively showing that Z_5 is indeed anomaly-free. For Z_7 , these coefficients are $7N_g$ and $7N_g/2$, so it is also anomaly-free.

Field	Q	u^c	d^c	ℓ	e^c	H	S
Z_5	1	4	4	2	3	0	1
Z_7	1	6	6	4	3	0	1

TABLE 3.3. Z_N charge assignment for $N = 5$ and 7 .

It is interesting to ask if the Z_N can be embedded into a gauged $U(1)$ symmetry. A simple possibility we have found is to embed this Z_N into the anomalous $U(1)_A$ symmetry of string origin with the anomalies cancelled by the Green-Schwarz mechanism [52]. Consider $U(1)_{B-L}$ without the right-handed neutrinos but with the inclusion of vector-like fermions which have the quantum numbers of $5(3)$ and $\bar{5}(2)$ under $SU(5) \times U(1)_A$. This $U(1)_A$ is anomaly-free by virtue of the Green-Schwarz mechanism. When this $U(1)_A$ breaks down to Z_5 , the extra particles get heavy masses and are removed from the low energy theory which is the $Z_6 \times Z_5$ model.

Without the second term in Eq. (3.2), the phase of the S field will be massless upon spontaneous symmetry breaking. This Majoron field [53] would however acquire a mass from the second term of Eq. (3.2). In the $Z_6 \times Z_5$ model, the mass of the Majoron is of order $\langle S \rangle^7 / \Lambda^6 \sim 100$ keV. In the $Z_6 \times Z_7$ model, the Majoron mass is of order $\langle S \rangle^{11} / \Lambda^{10} \sim 10$ eV. Such a Majoron with a mass of either 100 keV or 10 eV is fully consistent with constraints from early universe cosmology [54]. The interaction term $\ell\ell HHS^6 / \Lambda^7$ induces the Majoron decay $S \rightarrow \nu\nu$ with a Yukawa coupling $Y_{S \rightarrow \nu\nu} = 6m_\nu / \langle S \rangle \sim 10^{-11}$. The decay rate of the Majoron can be estimated to be

$$\Gamma = \frac{Y_{S \rightarrow \nu\nu}^2 m_S}{8\pi} \sim 10^{-23} m_S. \quad (3.3)$$

This corresponds to a Majoron lifetime of $\tau \sim 10$ sec for the $Z_6 \times Z_5$ model and $\tau \sim 10^5$ sec for the $Z_6 \times Z_7$ model. Such a Majoron can modify the big-bang nucleosynthesis processes. However the modification is not significant since the Majoron will decouple

before the electro-weak phase transition. Its contribution to the expansion rate is equivalent to that of $0.047 \times 4/7 \sim 0.027$ light neutrino species [55]. This extra contribution is well within observational uncertainties.

CHAPTER 4

THE μ -PROBLEM: A SYMMETRY APPROACH

The μ problem has been an intriguing puzzle of MSSM. One of the main goals of SUSY is to solve the gauge hierarchy problem while the arising μ -problem brings the hierarchy problem back to the theory [7, 56]. μ and B are the Higgs mass parameters in the MSSM, where μ appears in the superpotential

$$W_{\text{MSSM}} \supset \mu H_u H_d \quad (4.1)$$

and B is in the soft breaking sector

$$\mathcal{L}_{\text{MSSM soft}} \supset \mu B \tilde{H}_u \tilde{H}_d . \quad (4.2)$$

The phase of B is also a main source of CP violation in MSSM, usually known as the SUSY CP problem which is strictly constrained by electric dipole moment (EDM) experiments. To allow electroweak symmetry breaking, μ has to be of order the soft SUSY breaking mass scale M_{SUSY} ($\sim M_{\text{EW}}$), while one would usually expect μ to be of order the Planck scale M_{Pl} , the cut-off scale in the 4D theory, since it is a priori not protected by any (gauge) symmetry. μ cannot vanish either, otherwise there would be massless charged fermions (charged Higgsino).

Generally, understanding of the μ and B parameters is usually tied up with the SUSY breaking mechanism [6, 7]. In some scenario, it is directly related to the generation of gaugino masses [56, 57].

4.1 Peccei-Quinn Symmetry

In order to explain the μ and B parameters from physics beyond MSSM, one must introduce a new symmetry to ensure the absence of the bare μ term in the superpotential and it is reasonable to assume this new symmetry to be flavor independent.

However, the fact that under the new symmetry, H_u and H_d are not vectorial,

$$h_u + h_d \neq 2\alpha, \quad (4.3)$$

where h_u and h_d are the corresponding charges of the MSSM two Higgs doublets and α is the charge of gaugino. This eventually leads to a global PQ symmetry. Suppose the new symmetry is an Abelian symmetry G . After imposing the Yukawa couplings condition, the mixed QCD anomaly is given as

$$\begin{aligned} A_{[SU(3)]^2 \times G} &= 3\alpha + \frac{3}{2}(2(q - \alpha) + (u - \alpha) + (d - \alpha)) \\ &= 3\alpha - \frac{3}{2}(h_u + h_d), \end{aligned} \quad (4.4)$$

where q, u, d, h_u and h_d are the corresponding charge under G for the SSM superfields Q, u^c, d^c, H_u and H_d . α stands for the gaugino charge.

It is clear that this additional symmetry which forbids the bare μ term in the superpotential carries mixed QCD anomaly so it can be identified as the PQ symmetry [18].

A naive extension of the MSSM is to introduce a SM singlet S with trilinear coupling $H_u H_d S$ [58]. The quartic-coupling can arise radiatively. After the PQ symmetry is broken, S develops a VEV of order of M_{EW} . Hence, μ arises via

$$\mu \sim \langle S \rangle \sim M_{EW}. \quad (4.5)$$

Global PQ symmetry is explicitly broken by its mixed QCD anomaly giving rise to a pseudo-Goldstone particle known as the axion. The axion mass and PQ symmetry breaking scale could both lead to phenomenological inconsistency*.

Based on the different ways to address this PQ symmetry problem, the solutions to the μ problem can be classified into the following categories:

- Explicit breaking of PQ symmetry [58]
- Gauging the PQ symmetry by adding exotic quarks [59]
- Addition of a discrete global R-symmetry [60]

*Details of axion physics are discussed in Chapter 6.

- Realization as a subgroup of Anomalous $U(1)_A$ gauge symmetry [10, 57]
- Supersymmetric invisible axion solution [11, 13, 61, 62]

The first solution is realized as Next-to-Minimal model (NMSSM), which is defined by

$$\mathcal{W}_{\text{NMSSM}} \supset H_u H_d S + S^3 \quad (4.6)$$

where S^3 breaks explicitly the $U(1)_{\text{PQ}}$. However, at the same time, a new Z_3 discrete global symmetry has to be introduced, where S transform non-trivially under the discrete Z_3 symmetry. As a consequence, a domain wall is formed at the scale M_{SUSY} which is much lower than the inflation scale. So this poses a serious cosmological problem.

Following the second approach, a string motivated $U(1)'$ gauge symmetry has been proposed [59]. The μ -term solution here is quite similar to the NMSSM but without discrete Z_3 symmetry and involves the superpotential terms

$$\mathcal{W}_{U(1)'} = h H_u H_d S + \lambda S_1 S_2 S_3. \quad (4.7)$$

S gets a VEV near M_{SUSY} from soft SUSY breaking sector. It arises from a string originated E_6 symmetry with symmetry breaking pattern

$$E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi,$$

so it is a full gauge symmetry. But the theory has many exotic matter particles decoupled near one TeV. The $U(1)'$ also predicts an extra Z' boson which may contribute to precision electroweak tests.

In the following three sections, we will present here the last two solutions.

4.2 Giudice-Masiero Mechanism

One attractive scenario which achieves a μ -term solution in the SUGRA mediated SUSY breaking mechanism is the Giudice-Masiero mechanism [57] where a bare μ -term in the superpotential is forbidden by some symmetry, either discrete or continuous. μ is induced in the lagrangian via a non-renormalizable term

$$\mathcal{L} = \int d^4\theta \frac{H_u H_d Z^*}{M_{\text{Pl}}} \quad (4.8)$$

where Z is a spurion field which parameterizes SUSY breaking via $\langle F_Z \rangle \neq 0$, with $\langle F_Z \rangle / M_{\text{Pl}} \sim M_{\text{SUSY}} \sim 10^2 \text{ GeV}$. For instance, the gaugino masses are generated from

$$\mathcal{L}_{\text{soft}} \supset \int d^2\theta W_\alpha W^\alpha \frac{Z}{M_{\text{Pl}}} \quad (4.9)$$

For this mechanism to work, there must exist a symmetry that forbids a bare μ term in the superpotential. Such a symmetry cannot be a continuous symmetry, consistent with the requirement of non-zero gaugino masses, and therefore must be discrete. * It would be desirable to realize this as a discrete gauge symmetry so that the symmetry will be protected even at M_{Pl} . However, as mentioned previously, one must avoid the PQ symmetry problem and one of the ways is to gauge the PQ symmetry through the Green-Schwarz (GS) mechanism and realize the discrete symmetry as a subgroup of the anomalous $U(1)_A$ gauge symmetry [52].

4.3 Green-Schwarz Anomaly Cancellation Mechanism

String theory, when compactified to 4D, generically contains an ‘‘anomalous $U(1)_A$ ’’ gauge symmetry. A subset of the gauge anomalies in the axial vector $U(1)_A$ current can be cancelled via the Green-Schwarz (GS) mechanism in the following way [52]. In 4D, the Lagrangian for the gauge boson kinetic energy contains the terms

$$\mathcal{L}_{\text{kinetic}} = \varphi(x) \sum_i k_i F_i^2 + i\eta(x) \sum_i k_i F_i \tilde{F}_i, \quad (4.10)$$

where $\varphi(x)$ denotes the string dilaton field and $\eta(x)$ its axionic partner. The sum i runs over the different gauge groups in the model, including $U(1)_A$. k_i are the Kac-Moody levels for the different gauge groups, which must be positive integers for the non-Abelian groups, but may be non-integers for Abelian groups. The GS mechanism makes use of the transformation of the string axion field $\eta(x)$ under a $U(1)_A$ gauge variation,

$$V_A^\mu \rightarrow V_A^\mu + \partial_\mu \theta(x), \quad \eta(x) \rightarrow \eta(x) - \theta(x) \delta_{\text{GS}} \quad (4.11)$$

*Without the μ -term and the gaugino mass term, the MSSM Lagrangian has two $U(1)$ symmetries, a PQ symmetry and a $U(1)_R$ symmetry. The μ -term breaks the PQ symmetry and the gaugino mass term breaks the $U(1)_R$ symmetry down to a discrete subgroup.

where δ_{GS} is a constant. If the anomaly coefficients involving the $U(1)_A$ gauge boson and any other pair of gauge bosons are in the ratio

$$\frac{A_1}{k_1} = \frac{A_2}{k_2} = \frac{A_3}{k_3} = \dots = \frac{A_{\text{gravity}}}{24} = \delta_{\text{GS}} , \quad (4.12)$$

these anomalies will be cancelled by gauge variations of the $U(1)_A$ field arising from the second term of Eq. 4.11. δ_{GS} is known as Green-Schwarz constant which is defined in term of the mixed gravitational anomaly. All other crossed anomaly coefficients should vanish, since they cannot be removed by the shift in the string axion field.

Consider the case when the 4D gauge symmetry just below the string scale is $G_{\text{SM}} \times U(1)_A$. Let A_3 and A_2 denote the anomalies associated with $[SU(3)_C]^2 \times U(1)_A$ and $[SU(2)_L]^2 \times U(1)_A$ respectively. Then if $A_3/k_3 = A_2/k_2 = \delta_{\text{GS}}$ is satisfied, from Eq. (4), it follows that these mixed anomalies will be cancelled. The anomaly in $[U(1)_Y]^2 \times U(1)_A$ can also be cancelled in a similar way if $A_1/k_1 = \delta_{\text{GS}}$. However, in practice, this last condition is less useful, since k_1 is not constrained to be an integer as the overall normalization of the hypercharge is arbitrary. If the full high energy theory is specified, there can be constraints on A_1 as well. For example, if hypercharge is embedded into a simple group such as $SU(5)$ or $SO(10)$, $k_1 = 5/3$ is fixed since hypercharge is now quantized. $A_1/k_1 = \delta_{\text{GS}}$ will provide a useful constraint in this case. We shall remark on this possibility in our discussions. Note also that cross anomalies such as $[SU(3)] \times [U(1)_A]^2$ are automatically zero in the SM, since the trace of $SU(N)$ generators is zero. Anomalies of the type $[U(1)_Y] \times [U(1)_A]^2$ also suffer from the same arbitrariness from the Abelian levels k_1 and k_A . Finally, the $[U(1)_A]^3$ anomaly can be cancelled by the GS mechanism, or by contributions from fields that are complete singlets of the SM gauge group.

As discussed in Section 2.1, discrete version of anomaly cancellation will need to be modified due to the possible existence of vectorial fermion pairs. If the Z_N symmetry that survives to low energies was part of $U(1)_A$, the Z_N charges of the fermions in the low energy theory must satisfy a non-trivial condition: the anomaly coefficients A_i for the full theory is given by A_i from the low energy sector plus an integer multiple of $N/2$. These anomalies should obey GS mechanism, leading to the

discrete version of the Green–Schwarz anomaly cancellation mechanism:

$$\frac{A_3 + \frac{p_1 N}{2}}{k_3} = \frac{A_2 + \frac{p_2 N}{2}}{k_2} = \delta_{\text{GS}}, \quad (4.13)$$

with p_1, p_2 being integers. Since δ_{GS} is an unknown constant (from the effective low energy point of view), the discrete anomaly cancellation conditions are less stringent than those arising from conventional anomaly cancellations. If $\delta_{\text{GS}} = 0$, the anomaly is cancelled without assistance from the Green–Schwarz mechanism. We shall not explicitly use the condition that $\delta_{\text{GS}} \neq 0$, so our solutions will contain those obtained by demanding $\delta_{\text{GS}} = 0$, viz., $A_3 = -p_1(N/2)$, $A_2 = -p_2(N/2)$ with p_1, p_2 being integers.

The anomalous $U(1)_A$ symmetry is expected to be broken just below the string scale. This occurs when the Fayet–Iliopoulos term associated with the $U(1)_A$ symmetry is cancelled, so that SUSY remains unbroken near the string scale, by shifting the matter superfields that carry $U(1)_A$ charges [60]. Although the $U(1)_A$ symmetry is broken, a Z_N subgroup of $U(1)_A$ can remain intact. Suppose that we choose a normalization wherein the $U(1)_A$ charges of all fields are integers. (This can be done so long as all the charges are relatively rational numbers.) Suppose that the scalar field which acquires a vacuum expectation value (VEV) and breaks the $U(1)_A$ symmetry has a charge N under $U(1)_A$ in this normalization. A Z_N subgroup is then left unbroken down to low energies.

In our analysis we shall not explicitly make use of the condition $A_1/k_1 = A_2/k_2$, since, as mentioned earlier, the overall normalization of hypercharge is arbitrary. However, once a solution to the various Z_N charges is obtained, we can check for the allowed values k_1 , and in particular, if $k_1 = 5/3$ is part of the allowed solutions. This will be an interesting case for two reasons. If hypercharge is embedded in a simple grand unification group such as $SU(5)$, one would expect $k_1 = 5/3$. Even without a GUT embedding $k_1 = 5/3$ is interesting. We recall that unification of gauge couplings is a necessary phenomenon in string theory. Specifically, at tree level, the gauge couplings of the different gauge groups are related to the string coupling constant g_{st}

which is determined by the VEV of the dilaton field as [61]

$$k_3 g_3^2 = k_2 g_2^2 = k_1 g_1^2 = 2g_{st}^2 \quad (4.14)$$

where k_i are the levels of the corresponding Kac–Moody algebra. In particular, if $k_1 : k_2 : k_3 = 5/3 : 1 : 1$, we would have $\sin^2 \theta_W = 3/8$ at the string scale, a scenario identical to that of conventional gauge coupling unification with simple group such as $SU(5)$. For these reasons, we shall pay special attention to the case $k_1 = 5/3$.

4.4 Discrete Z_4 Gauge Symmetry from $U(1)_A$

As discussed in the above two sections, the symmetry which is consistent with the Giudice-Masiero mechanism must be discrete but carries a mixed QCD anomaly [57]. So the only realization of discrete gauge symmetries must arise from the anomalous $U(1)_A$ gauge symmetry. We have done a systematic analysis in [10].

Here, one of examples of Z_4 subgroup of the anomalous $U(1)_A$ is given in Table 4.4.

q	u	d	l	e	n	h	\bar{h}	α
1	1	1	1	1	1	0	0	1

TABLE 4.1. Z_4 subgroup of the Anomalous $U(1)_A$

The mixed anomaly coefficients are

$$A_3 = 3 \pmod{4}, \quad A_2 = 1 \pmod{4} \quad (4.15)$$

which satisfies the discrete version of Green-Schwarz anomaly cancellation condition. The charge assignment shown in Table 4.4 is clearly compatible with grand unification. The Kac Moody level associated with hypercharge will be $k_1 = 5/3$ with a GUT embedding. Gauge coupling unification is then predicted, since $\sin^2 \theta_W = 3/8$ near the string scale. This is true even if there were no covering GUT symmetry. It also acts as a exact R -parity. The anomalous $U(1)_A$ is broken to

$$U(1)_A \rightarrow Z_4 \rightarrow Z_2,$$

where after the SUSY breaking, Z_4 is broken into the Z_2 subgroup of the I_R^3 as in chapter 3.

4.5 QCD Axion Solution to the μ Problem

As mentioned previously, the PQ symmetry implies the presence of an axion. It is interesting to note that an axion is required to solve the Strong CP problem. All the acceptable axion solutions must be “invisible”. Here, we present a model making use of the real QCD axion to address the μ -term problem. This is a natural solution in terms of the PQ symmetry while the QCD axion is an elegant solution to the Strong CP problem and at the same time, it provides candidate for cold dark matter.

In all the above approaches, the μ -term solutions eventually make use of SUSY breaking and directly relate the M_{SUSY} to μ . Imposing a new physics scale M_{PQ} ($f_a = (10^{10} - 10^{12})$ GeV), the axion models provide another approach to the μ -term problem[11, 13, 62, 63] by relating

$$\mu \sim \frac{M_{\text{PQ}}^2}{M_{\text{Pl}}}. \quad (4.16)$$

In the case of the Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) axion model [64], a μ -term automatically arises after PQ symmetry breaking.

The question is now how to naturally understand the origin of M_{PQ} from a higher energy theory. It is interesting that in the SUGRA mediated SUSY breaking models, one also has to impose a new physics scale of order $\mathcal{O}(10^{11})$ GeV). In these models, this intermediate scale can be generated dynamically. Practically, this intermediate scale can then be identified as M_{PQ} . Here we propose a model involving SUSY breaking [45]. Having made use of M_{SUSY} , this approach certainly requires that the SUSY breaking mediation scale is greater than M_{PQ} . A simple realization of this idea is the SUGRA model. The superpotential of the model contains

$$\mathcal{W} \supset \lambda_1 H_u H_d S^2 / M_{\text{Pl}} + \lambda_2 (S\tilde{S})^2 / M_{\text{Pl}} + S^{22} / M_{\text{Pl}}^{19} \quad (4.17)$$

which is also consistent with the Z_{22} symmetry in the previous section. By minimizing the leading-order potential including SUSY breaking effects,

$$V = (\lambda_2 C (S\tilde{S})^2 / M_{\text{Pl}} + h.c) + m_S^2 |S|^2 + m_{\tilde{S}}^2 |\tilde{S}|^2 + 4\lambda_2 |S\tilde{S}|^2 (|S|^2 + |\tilde{S}|^2) / M_{\text{Pl}}^2, \quad (4.18)$$

where m_S and $m_{\tilde{S}}$ are soft breaking masses of order M_{SUSY} , one obtains

$$f_a^2 = C \pm \sqrt{C^2 - 12m_S^2 M_{\text{Pl}} / 12\lambda_2}. \quad (4.19)$$

So

$$f_a \sim \sqrt{M_{\text{Pl}} M_{\text{SUSY}}} \sim 10^{11} \text{ GeV}. \quad (4.20)$$

Since the F -component of the field S obeys

$$F_S \sim M_{\text{PQ}} M_{\text{SUSY}}, \quad (4.21)$$

the dominant contribution for the B parameter which appears in the soft bilinear SUSY breaking term

$$\mathcal{L}_{\text{soft}} \supset B\mu H_u H_d \quad (4.22)$$

arises from the superpotential $H_u H_d S^2 / M_{\text{Pl}}$ as

$$B\mu = \langle S \rangle \langle F_S \rangle / M_{\text{Pl}} \sim M_{\text{SUSY}}^2. \quad (4.23)$$

So it is difficult to distinguish it from the usual MSSM via electroweak physics. However, as the axion can be a cold dark matter candidate, one can still distinguish the model in cosmology. In this model, the two PQ Higgs bosons have masses of order M_{SUSY} but their mixings with the doublet Higgs are highly suppressed. The orthogonal combination to the axion acquires a mass of order M_{SUSY} . The axino and saxino masses are both around M_{SUSY} . The axino can mix with the Higgsino with a tiny mixing angle of order $(M_{\text{SUSY}} / M_{\text{Pl}})^{1/2} \sim 10^{-7}$. Therefore, the axino can decay to a bottom quark and a sbottom squark with a lifetime

$$\tau \sim 10^{-11} \text{ sec}. \quad (4.24)$$

This is a consistent picture with big-bang cosmology since the axino decays occur earlier than the nucleosynthesis era.

CHAPTER 5

DISCRETE FLAVOR GAUGE SYMMETRY

The flavor hierarchy problem has been a very challenging problem in model building for many years [65]. It mainly addresses the following two questions:

- How is the apparent 10^{12} order hierarchy in m_t/m_ν generated?
- What is the origin of the observed mass ratios and mixing angles of the SM quarks and leptons?

As mentioned earlier in the discussion of L violation, neutrino masses provides a hint for a new physics scale when they are understood as emerging in the low-energy theory from operators such as $\ell\ell H_u H_u/\Lambda_L$. In the following, we use the seesaw mechanism, one natural approach as the realization of this new physics:

$$\mathcal{W}_{\text{seesaw}} = \ell\nu^c H_u + M_R \nu^c \nu^c \quad (5.1)$$

where there exists heavy Majorana neutrinos ν^c at $M_R \sim \mathcal{O}(10^{14} \text{ GeV})$ and the small neutrino masses are given by $M_{\text{EW}}^2/M_R \sim 10^{-10} \text{ GeV}$.

In the SM quark and charged lepton sectors, the mixing angles are small. However, there has been a strong evidence for large neutrino mixing from recent solar, atmospheric and reactor neutrino oscillation data. It is then a challenge to understand why there exists such a discrepancy in the mixing angles, especially in the context of GUTs where quarks and leptons are unified in the same GUT multiplets.

In a 4D framework, flavor gauge symmetries have been a leading candidate solution to this problem. And even in the string theory which has achieved family unification in extra spacetime dimensions, flavor gauge symmetries exist as well. Here, the symmetries are usually broke by the boundary conditions explicitly. Such orbifold

models, however, correspond to special points in the moduli space of the Calabi-Yau manifold at which there is an extra gauge symmetry that acts on the flavors. The more generic Calabi-Yau models can then be considered as models in which the flavor gauge symmetries are spontaneously broken.

SUSY is a promising candidate for physics beyond SM. But when SUSY is introduced, a new flavor problem arises in the soft breaking sector known as the SUSY flavor problem. In the SM, one can make use of the GIM mechanism [66] to suppress harmful flavor changing neutral currents (FCNC's) by a suitable unitary transformation between mass and gauge eigenstates. Alternatively: it is, however, not clear why soft sfermion masses sector and the fermion masses sector in SUSY models should transform similarly. The difference between the usual Yukawa and soft breaking sectors may thus lead to flavor violation, which is strictly constrained from $K - \bar{K}$ mixing and lepton flavor violation measurement like $\mu \rightarrow e\gamma$. This issue depends on the understanding of the SUSY breaking mechanism as well as the flavor gauge symmetries. A popular solution is to assume universality in the soft breaking sector. Then universal structure will remain universal after the unitary transformation. For instance, gauge mediated SUSY breaking or string dilaton dominant SUSY breaking both provide a universal soft sector. In the most widely discussed SUGRA type models, people usually assume the universality. However, any flavor gauge symmetry will bring a splitting between different generations back to the theory known as the D -term splitting problem. A discrete flavor symmetry, on the other hand, would avoid this problem as there is no D -term associated with it. In the following sections, we present an explicit example of discrete flavor gauge symmetry approach.

5.1 Froggatt-Nielsen Mechanism and Anomalous $U(1)_A$ Realization

The most straightforward example of a flavor gauge symmetry is the $U(1)_F$ symmetry employed as in the Froggatt-Nielsen mechanism [67]. Here, a SM singlet scalar which couples to SM matter Yukawa terms is introduced, which and transforms under the new $U(1)_F$ symmetry. The quarks and leptons also carry different $U(1)_F$ flavor charges. Some new physics generates the non-renormalizable couplings terms

consistent with $G_{\text{SM}} \times U(1)_F$ invariance. In terms of MSSM, the superpotential is given as

$$\begin{aligned} \mathcal{W} = & \frac{y_{ij}^u}{n_{ij}^u!} Q_i u_j^c H_u \left(\frac{S}{\Lambda_{\text{FN}}} \right)^{n_{ij}^u} + \frac{y_{ij}^d}{n_{ij}^d!} Q_i d_j^c H_d \left(\frac{S}{\Lambda_{\text{FN}}} \right)^{n_{ij}^d} \\ & + \frac{y_{ij}^e}{n_{ij}^e!} L_i e_j^c H_d \left(\frac{S}{\Lambda_{\text{FN}}} \right)^{n_{ij}^e} + \frac{y_{ij}^\nu}{n_{ij}^\nu!} L_i \nu_j^c H_u \left(\frac{S}{\Lambda_{\text{FN}}} \right)^{n_{ij}^\nu} \\ & + M_{Rij} \nu_i^c \nu_j^c \left(\frac{S}{\Lambda_{\text{FN}}} \right)^{n_{ij}^c} + \mu H_u H_d, \end{aligned} \quad (5.2)$$

where $i, j = \{1, 2, 3\}$ are family indices, n_{ij}^u , n_{ij}^d , n_{ij}^e , n_{ij}^ν and n_{ij}^c are positive integers fixed by the choice of $U(1)_F$ charge assignment. The quantities y_{ij}^x , where $x = u, d, e, \nu$, are Yukawa coupling coefficients which are all taken to be of order one. Here, M_R is the right-handed neutrino mass scale.

When the SM singlet S acquires a VEV, the $U(1)_F$ symmetry is spontaneously broken. Hierarchy and mixings thus arise as suppression of different powers of S/Λ_{FN} .

In the Froggatt-Nielsen mechanism, the usual parametrization of the fermion mass matrices requires $S/\Lambda_{\text{FN}}(\epsilon) \sim 1/5$. The anomalous $U(1)_A$ symmetry which we discuss in Section 4.3 [52, 68] is a promising realization here. The anomalous $U(1)_A$ symmetry is broken below the string scale $M_{\text{St}} \sim \mathcal{O}(10^{17} \text{ GeV})$. Hence, a natural realization of ϵ comes as

$$\epsilon \sim \langle S \rangle / M_{\text{St}} \sim .2 \quad (5.3)$$

5.2 A Lopsided Structure and Discrete Flavor Gauge Symmetry

As mentioned earlier, it is a challenge to address flavor hierarchy problem in a GUT framework.

At low energy, the fermion masses are [69]

$$\begin{aligned} m_u(1 \text{ GeV}) = 5.11 \text{ MeV}, \quad m_c(m_c) = 1.27 \text{ GeV}, \quad m_t(m_Z) = 174 \text{ GeV}, \\ m_d(1 \text{ GeV}) = 8.9 \text{ MeV}, \quad m_s(1 \text{ GeV}) = 130 \text{ MeV}, \quad m_b(m_b) = 4.25 \text{ GeV}. \end{aligned} \quad (5.4)$$

The CKM mixing matrix elements are

$$|V_{us}| \sim 0.222, \quad |V_{ub}| \sim 0.0035, \quad |V_{cb}| \sim 0.04. \quad (5.5)$$

In [10], we proposed an $SU(5)$ GUT compatible model. An acceptable flavor texture which gives the correct pattern of fermion masses and mixings as shown in 5.4 and ?? is:

$$\begin{aligned}
 U_{ij} &= \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} H_u, & D_{ij} &= \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^p H_d, \\
 L_{ij} &= \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon \\ \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \epsilon^p H_d, & \nu_{ij}^D &= \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{a_1} H_u,
 \end{aligned} \tag{5.6}$$

where U_{ij} , D_{ij} , L_{ij} and ν_{ij}^D correspond to the up-quark, down quark, charged lepton and Dirac neutrino Yukawa matrices resulting from the appropriate powers of the S field in Eq. (5.2). The integer p can be either 0, 1 or 2, corresponding to large, medium and small $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$ respectively. Notice that the down-type quark mass matrix and the charged lepton mass matrix are transpose of each other as required by an embedding into an $SU(5)$ GUT.

Once the charged lepton sector and Dirac neutrino sector are constructed, we can uniquely define the form of the heavy Majorana neutrino mass matrix. In the present example it is

$$\nu_{ij}^M = M_R \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{a_2}. \tag{5.7}$$

Any $SU(5)$ compatible theory automatically satisfies the GS anomaly cancellation mechanism. This structure can naturally arise from the anomalous $U(1)_A$ type model. However, as indicated earlier, the anomalous $U(1)_A$ is broken, we then identify discrete subgroups of the anomalous $U(1)_A$ symmetry as the discrete flavor gauge symmetry.

Three examples of Z_{14} symmetric models are presented in Table 5.2. We have chosen the charge of S to be 2 and fixed the charge of θ to be 7 in these examples. Discrete anomaly cancellation is enforced via GS mechanism at Kac-Moody level 1. We have also imposed the conditions that the Z_{14} symmetry forbid all R -parity violating couplings.

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	H_u	H_d	θ	S	A_2	A_3
A	0,2,6	1,3,7	3,5,5	4,6,6	13,1,5	5,7,7	1	13	7	2	6	13
B	4,6,10	13,1,5	11,13,13	6,8,8	9,11,1	5,7,7	13	1	7	2	13	13
C	6,8,12	5,7,11	1,3,3	0,2,2	7,9,13	5,7,7	9	5	7	2	13	6

TABLE 5.1. Examples of the flavor-dependent Z_{14} symmetry which forbids all R -parity breaking terms. $i = 1, 2, 3$ is the flavor index and charges are in order of 1-3.

We are considering $p = 2$ and $q = 0$ in Eq. (5.6) which corresponds to medium values of $\tan \beta \sim 10$. We have taken $a_2 = 0$ in Eq. (5.7) for simplicity.

The above discrete gauge symmetries are consistent with realistic structure of fermion masses hierarchy in 5.4 and ???. And at the same time, it gives the large mixing of neutrinos ν_μ and ν_τ . Moreover, as discrete gauge symmetries, the famous D -term splitting problem can be then avoided.

CHAPTER 6

STABILIZATION OF AXION SOLUTIONS

6.1 Strong CP Problem and QCD Axion

CP violation (CPV) can exist in the QCD Lagrangian arising from the instanton induced Chern-Simons type gluon-gluon coupling

$$\mathcal{L} \supset \theta g_s^2 \epsilon^{\mu\nu\sigma\rho} G_{\mu\nu}^\alpha G_{\sigma\rho}^\alpha / 64\pi^2 = \theta g_s^2 G_{\mu\nu}^\alpha \tilde{G}^{\alpha\mu\nu} / 32\pi^2. \quad (6.1)$$

In addition, there is another CPV source from the quark mass matrices. This results in an observable parameter $\bar{\theta}$ defined as

$$\bar{\theta} = \theta + \arg(\det M_U \det M_D). \quad (6.2)$$

Such a $\bar{\theta}$ would lead to a neutron electric dipole moment (EDM) of order $d_n \simeq 5 \times 10^{-16} \bar{\theta}$ ecm, while the current experiment limit is $d_n < 10^{-25}$ ecm. This puts a strong constraint, $\bar{\theta} < 10^{-10}$. The PQ symmetry [18] is an elegant solution to this so-called strong CP problem. It introduces a global $U(1)$ symmetry, broken by the QCD anomaly, which generates a pseudo-Goldstone particle a , the axion. Non-perturbative effects then induce a term in the lagrangian of the form

$$\mathcal{L} \supset (a/f_a) g_s^2 G_{\mu\nu}^\alpha \tilde{G}^{\alpha\mu\nu} / 32\pi^2. \quad (6.3)$$

$\bar{\theta}$ is then promoted to this dynamical field axion as $a(x)/f_a$. Minimizing the axion potential

$$V(a) \propto \Lambda_{\text{QCD}}^4 (1 - \cos(a(x)/f_a)), \quad (6.4)$$

consequently $\bar{\theta} = \langle a \rangle / f_a = 0$.^{*} The strong CP problem is then solved. f_a is the (model dependent) axion decay constant [70] and it is constrained to be $f_a = (10^{10} - 10^{12})$ GeV

^{*}Due to the periodicity of the potential, $\langle a \rangle = 2n\pi f_a$. Some detailed discussion can be found in various review papers listed in Ref. [42].

by the combined limits from laboratory experiments, astrophysics and cosmology. Hence, only the “invisible axion” models, which have appropriate values of f_a , are favored [64, 71]. The couplings of the axion with the SM fields are highly suppressed in these models. Although the axion arise as a pseudo-Goldstone particle when the PQ symmetry is explicitly broken by its QCD anomaly, the axion can acquire a tiny mass through higher order non-perturbative effects. The mass of the axion can be estimated to be

$$m_a \sim \Lambda_{\text{QCD}}^2 / f_a \sim 10^{-4} \text{ eV}. \quad (6.5)$$

6.2 Discrete Gauge Symmetry Stabilizing the Axion

Quantum gravitational effects can potentially violate the global PQ symmetry as they can break all global symmetries while respecting gauge symmetries. In the axion models, a possible quantum gravity generated non-renormalizable term

$$\mathcal{L} \supset S^n / M_{\text{Pl}}^{n-4} \quad (6.6)$$

is in principle allowed. This term would lead to

$$\bar{\theta} \simeq f_a^n / (M_{\text{Pl}}^{n-4} \Lambda_{\text{QCD}}^4). \quad (6.7)$$

Since both $\bar{\theta}$ and f_a are highly constrained, $n \geq 10$ is necessary. To avoid such kind of violations, one solution is to introduce a discrete gauge symmetry [9]. The PQ symmetry arises only as an accidental global symmetry from it.

Conventionally, absence of anomalies complicates the particle spectrum of axion models. However, the Type I and Type IIB string theories provide a new candidate that cancels the anomalies without enlarging the particle content. In the low energy effective theory of such string theories, there exists one anomalous $U(1)_A$ symmetry as mentioned in Section 4.3 [52, 60]. There, GS mechanism is effective in cancelling the anomalies. The anomalous $U(1)_A$ symmetry is broken by a Higgs field spontaneously near the M_{St} . A discrete version of GS mechanism as in the Eq. 4.13 is applied here.

6.3 Stabilization of the DFSZ Axion

The non-SUSY DFSZ axion model [64] introduces two Higgs doublets H_u and H_d and a SM singlet scalar S . The Lagrangian of the model relevant for the discussion of axion physics is

$$\mathcal{L} = Qu^c H_u + Qd^c H_d + Le^c H_d + l\nu^c H_u + \nu^c \nu^c - \lambda(H_u H_d S^2 + \text{h. c.}). \quad (6.8)$$

Here we have used a standard notation that easily generalizes to our SUSY extension as well.

The \mathcal{L} has three $U(1)$ symmetries, as can be inferred by solving the six conditions imposed on nine parameters. These three $U(1)$ symmetries can be identified as the SM hypercharge $U(1)_Y$, baryon number $U(1)_B$ and a PQ symmetry $U(1)_{PQ}$. If we denote the charges of (Q, u^c, d^c) as (q, u, d) , the symmetries can be realized as $B = q - u - d$, $PQ = -d$, $Y/2 = q/6 - 2u/3 + d/3$. The $U(1)$ charges of the various particles under these symmetries are listed in the Table 6.3.

	Q	u^c	d^c	ℓ	e^c	ν^c	H_d	H_u	S
$Y/2$	1/6	-2/3	1/3	-1/2	1	0	-1/2	1/2	0
B	1	-1	-1	0	0	0	0	0	0
PQ	0	0	-1	0	-1	0	1	0	-1/2

TABLE 6.1. $Y/2$, B and PQ symmetries corresponding to hypercharge, baryon number and PQ charge respectively. The charges are assumed to be generation independent.

After H_d , H_u and S fields develop VEVs, the global PQ symmetry is broken and the light spectrum contains a Goldstone boson, the axion. Non-perturbative QCD effects induce an axion mass [72] given by

$$m_a^{DSFZ} \simeq 0.6 \times 10^{-4} \text{ eV} \frac{10^{11} \text{ GeV}}{f_a}, \quad (6.9)$$

where $f_a \sim \langle S \rangle$ is the axion decay constant.

We now apply the GS mechanism for discrete anomaly cancellation to stabilize the axion from quantum gravity corrections. Even though the model under discussion

is non-SUSY, the GS mechanism for anomaly cancellation should still be available, since SUSY breaking in superstring theory need not occur at the weak scale in principle. Since baryon number has no QCD anomaly, any of its subgroup will be insufficient to solve the strong CP problem via the PQ mechanism. On the other hand, the PQ symmetry does have a QCD anomaly, although with the charges listed in Table 1 it has no $SU(2)_L$ anomaly. Since hypercharge Y is anomaly free, we attempt to identify the anomalous $U(1)_A$ symmetry as a linear combination of PQ and B :

$$U(1)_A = PQ + \gamma B. \quad (6.10)$$

According to the Eq. (6.10) and the charge assignment presented in Table 1, we have for the anomaly coefficients for the $U(1)_A$,

$$\begin{aligned} A_3 &\equiv [SU(3)]^2 \times U(1)_A = -\frac{3}{2} \\ A_2 &\equiv [SU(2)]^2 \times U(1)_A = \frac{9}{2} \gamma. \end{aligned} \quad (6.11)$$

If we identify $\gamma = -k_2/(3k_3)$, the anomalies in $U(1)_A$ will be cancelled by GS mechanism. Thus we have

$$U(1)_A = PQ - \frac{1}{3} \frac{k_2}{k_3} B. \quad (6.12)$$

The simplest possibility is $k_2 = k_3 = 1$, corresponding to the levels of Kac-Moody algebra being one. Normalizing the charge of the singlet field S to be an integer, Eq. (6.12) can be rewritten as

$$U(1)_A = 6(PQ) - 2(B). \quad (6.13)$$

The corresponding charge assignment is given in Table 6.2. As discussed earlier, since hypercharge Y is anomaly free, one can add a constant multiple of $Y/2$ to the $U(1)_A$ charges, and still realize GS anomaly cancellation mechanism. The charges listed in Table 6.2 assumes the combination $-\frac{5}{3}(6PQ - 2B + \frac{4}{5}Y)$. As can be seen from Table 2, this choice of charges is compatible with $SU(5)$ grand unification.

Suppose that the $U(1)_A$ symmetry is broken near the string scale by the VEV of a scalar field which has a $U(1)_A$ charge of N in a normalization where all $U(1)_A$ charges have been made integers. A Z_N symmetry will then be left unbroken to low

scales. Two examples of such Z_N symmetries are displayed in Table 2 for $N = 11, 12$. Invariance under these Z_N symmetries will not be spoiled by quantum gravity, it is this property that we use to stabilize the axion.

Potentially dangerous terms that violate the $U(1)_{PQ}$ symmetry are S^n/M_{Pl}^{n-3} , $H_u H_d S^{*m}/M_{\text{Pl}}^{m-2}$ etc, for positive integers n, m . For the induced $\bar{\theta}$ to be less than 10^{-10} , the integers n, m must obey $n \geq 10, m \geq 5$. The choice of $N = 11, 12$ satisfy these constraints. Note that a Z_{10} discrete symmetry would have allowed a term S^2 , which would be inconsistent with the limit on $\bar{\theta}$. Z_N symmetries with N larger than 12 can also provide consistent solutions. Since by construction, the $U(1)_A$ symmetry in Table 1 is anomaly-free by GS mechanism, any of its Z_N subgroup is also anomaly-free by the discrete GS mechanism, as can be checked directly. In the Z_{11} model, for example, we have $A_3 = A_2 = 4$. Consistent with the Z_{11} invariance, terms that violate the $U(1)_{PQ}$ symmetry and give rise to an axion mass are S^{11}/M_{Pl}^7 , $H_u H_d S^{*9}/M_{\text{Pl}}^7$ etc, all of which are quite harmless. We conclude that the DFSZ axion can be stabilized against potentially dangerous non-renormalizable terms arising from quantum gravitational effects in a simple way.

	Q	u^c	d^c	ℓ	e^c	ν^c	H_u	H_d	S
$U(1)_A$	2	2	4	4	2	0	-4	-6	5
Z_{11}	2	2	4	4	2	0	7	5	5
Z_{12}	2	2	4	4	2	0	8	6	5

TABLE 6.2. The anomalous $U(1)$ charge assignment for the DFSZ axion model. Also shown are the charges under two discrete subgroups Z_{11} and Z_{12} which can stabilize the axion.

The discussion can be easily extended to its SUSY version. The superpotential of the DFSZ axion model contains a term $\lambda H_u H_d S^2/M_{\text{Pl}}$. After H_u, H_d and S develop VEVs, the global PQ symmetry is broken and the axion arises as a pseudo-Goldstone particle. Since the superpotential is holomorphic, one cannot write $S^{\dagger 2} S^2$ type term. In addition to the S field, another singlet \tilde{S} is needed so that the axion is invisible

and at the same time, PQ can be broken. The superpotential of the model now is

$$W \supset \lambda_1 H_u H_d S^2 / M_{\text{Pl}} + \lambda_2 S^2 \tilde{S}^2 / M_{\text{Pl}}. \quad (6.14)$$

One explicit example of Z_{22} discrete gauge symmetry is given. The charge assignment under Z_{22} is listed as

$$\{Q = 3, u^c = 19, d^c = 1, \ell = 11, e^c = 15, \nu^c = 11, H_u = 22, H_d = 18, S = 13, \tilde{S} = 20\}. \quad (6.15)$$

The mixed anomalies are $\{A_2 = 6, A_3 = 17\}$. It apparently satisfies the GSM condition. $S^{22}/M_{\text{Pl}}^{19}$ is the leading allowed term in the superpotential due to potential quantum gravity correction, which only induces $\bar{\theta} \lesssim 10^{-130}$.

In this model, the R -parity is not automatic, for instance, $LH_u S \tilde{S}$ is allowed. To get an exact R -parity, one can introduce an additional Z_2 where all the SM matter fields are odd but H_u, H_d, S and \tilde{S} are even. This is the unbroken subgroup of the gauge symmetry $U(1)_{\text{B-L}}$ even with the presence of Majorana neutrino mass term.

6.4 Stabilization of KSVZ Axion

The Kim-Shifman-Vainshtein-Zakharov (KSVZ) Axion model[72], can also be stabilized by discrete gauge symmetries. The scalar sector of the non-SUSY KSVZ axion model [71] contains the SM doublet and a singlet field S . All the SM fermions are assumed to have zero PQ charge under the global $U(1)_{\text{PQ}}$ symmetry. The Yukawa sector involving the SM fermions is thus unchanged. An exotic quark-antiquark pair $\Psi + \bar{\Psi}$ is introduced, which transform vectorially under the SM (so the magnitude of its mass term can be much larger than the electroweak scale), but has chiral transformations under $U(1)_{\text{PQ}}$. The QCD anomaly needed for the axion potential arises from these exotic quarks. The Lagrangian involving the singlet field and these vector quarks is given by

$$\Delta\mathcal{L} = S\Psi\bar{\Psi} + \text{h.c.} \quad (6.16)$$

When S field develops a VEV, the PQ symmetry is spontaneously broken leading to the axion in the light spectrum.

The global PQ $U(1)$ symmetry is susceptible to unknown quantum gravity corrections. We shall attempt to stabilize the KSVZ axion by making use of discrete gauge symmetries with anomaly cancellation by the GS mechanism. The most dangerous non-renormalizable term in the scalar potential that can destabilize the axion is S^n/M_{pl}^{n-4} , as in the case of the DFSZ axion. We seek a discrete gauge symmetry that would forbid such terms.

In order for the GS mechanism for anomaly cancellation to be viable, the anomaly coefficients A_2 and A_3 corresponding to the $[SU(2)_L]^2 \times U(1)_A$ and $[SU(3)_C]^2 \times U(1)_A$ should equal each other at the $U(1)$ level. This would imply that the $\Psi + \bar{\Psi}$ fields can not all be singlets of $SU(2)_L$. The simplest example we have found is the addition of a $5 + \bar{5}$ of $SU(5)$ to the SM spectrum. Such a modification is clearly compatible with grand unification. The 5 contains a $(3, 1)$ and a $(1, 2)$ under $SU(3)_C \times SU(2)_L$. We allow the following Yukawa coupling involving these fields:

$$\mathcal{L} \supset \lambda 5\bar{5}S + \text{h.c.} \quad (6.17)$$

If we denote the PQ charges of 5 and $\bar{5}$ as ϕ and $\bar{\phi}$, invariance under a surviving discrete Z_N symmetry would imply

$$\phi + \bar{\phi} + s = pN \quad (6.18)$$

where p is an integer. In this simple model, all the SM particles are assumed to be trivial under the PQ symmetry. The discrete anomaly coefficients are then $A_3 = A_2 = \frac{3}{2}(\phi + \bar{\phi}) = \frac{3}{2}(pN - s)$. Since $A_2 = A_3$, the gauge anomalies are cancelled by the GS mechanism. As long as $N \geq 10$, all dangerous couplings that would destabilize the axion through non-renormalizable terms will be sufficiently small. We see that the KSVZ axion can be made consistent in a simple way.

We have also examined the possibility of stabilizing the axion by introducing only a single pair of fermions under the SM gauge group, rather than under the grand unified group. Let us consider a class of models with a pair of fermions transforming under $G_{\text{SM}} \times U(1)_A$ as

$$\Psi(\mathbf{3}, n, y, \psi) + \bar{\Psi}(\bar{\mathbf{3}}, \bar{n}, -y, \bar{\psi}) , \quad (6.19)$$

along with a scalar field $S(1, 1, 0, s)$. The Lagrangian of this model contains a term $\Psi \bar{\Psi} S$ and its invariance under an unbroken Z_N symmetry imposes the constraint

$$\psi + \bar{\psi} + s = pN \quad (6.20)$$

where p and N are integers. Since the SM particles all have zero anomalous $U(1)$ charge, the anomaly coefficients arise solely from the $(\Psi + \bar{\Psi})$ fields. They are

$$\begin{aligned} A_3 &= \frac{1}{2}(n\psi + n\bar{\psi}) = \frac{n}{2}(pN - s) \\ A_2 &= \frac{(n-1)n(n+1)}{12}(3\psi + 3\bar{\psi}) \\ &= \frac{(n-1)n(n+1)}{4}(pN - s). \end{aligned} \quad (6.21)$$

The GS discrete anomaly cancellation condition implies

$$s = pN + \frac{2(-m + bm')}{n(b(n^2 - 1) - 2)} \quad (6.22)$$

where $b \equiv k_3/k_2$.

By choosing specific values of the Kac-Moody levels, one can solve for s , the singlet charge. For instance, in the simple case when $k_3 = k_2 \Leftrightarrow b = 1$,

$$s = \frac{2(m' - m)}{n^3 - 3n} N. \quad (6.23)$$

We have normalized all $U(1)_A$ charges to be integers, including s , so the unbroken Z_N symmetry will be transparent.

When $n = 2$, Ψ and $\bar{\Psi}$ are $SU(2)$ doublets. One can calculate the charge of S and determine the allowed discrete symmetries. For $b = 1$, the solution is $s = 0 \pmod{N}$. This solution would imply that S^n terms in the potential are allowed for any n , in conflict with the axion solution. A similar conclusion can be arrived at for $b = 1/2$. For other values of b , the Z_N symmetry typically turns out to be too small to solve the strong CP problem. For example, if $b = (2, 3, 1/3, 3/2)$, the allowed discrete symmetries are (Z_4, Z_7, Z_3, Z_5) . A special case occurs when $b = 2/3$, in which case s is undetermined, since $A_3/k_3 = A_2/k_2$. If one chooses $s \geq 10$, the KSVZ axion can be stabilized in this case.

If the quarks Ψ and $\bar{\Psi}$ are triplets of $SU(2)_L$, stability of the KSVZ axion solution can be guaranteed in a simple way. For $b \equiv k_3/k_2 = (1, 2, 3, 1/2, 1/3, 2/3, 3/2)$, which are the allowed possibilities if we confine to Kac-Moody levels less than 3, we have the unbroken discrete symmetries to be $(Z_9, Z_{21}, Z_{33}, Z_6, Z_3, Z_{15}, Z_{30})$ respectively. For all Z_N with $N \geq 10$, the axion solution will be stable against quantum gravitational corrections.

CHAPTER 7

Conclusions

In this thesis, we study the discrete gauge symmetries in the SM and also as a model building tool to solve various problems in the SM as well as the MSSM.

In the second chapter, we discuss a hidden discrete gauge symmetry in the non-SUSY flavor independent SM at the renormalizable level. A discrete Z_3 symmetry is found in the SM and is embedded into a discrete Z_6 symmetry in the extension of the SM with seesaw mechanism for the small neutrino masses. Both Z_3 and Z_6 are free from mixed G_{SM} anomalies at the discrete level. It is anomaly free as a result of the existence of three generations ($N_g = 3$). The symmetry can effectively act as the baryon number up to the $\Delta B = 3 \bmod 3$ level which is also consistent with the prediction from non-perturbative corrections in the Standard Model, such as electroweak instanton and sphaleron processes.

Quantum mechanically, we estimate the triple nucleon decay rate which is predicted by the existence of this symmetry. It turns out, that as a result of baryon parity, the current bounds on the proton lifetime show that the cutoff scale in 4D can be as low as $\mathcal{O}(10^2 \text{ GeV})$.

We also find a simple $U(1)$ realization from which this baryon parity can naturally emerge. It is a $U(1)$ of $I_R^3 + L_i + L_j - 2L_k$, where I_R^3 is the lepton number.

Effects arising from simple GUTs will explicitly break the baryon parity. Hence, whether there exists a baryon parity puts a strong hint to GUT physics.

In Chapter 3, gauged R -parity is studied. It is shown that a Z_2 subgroup of I_R^3 plays an important role as R -parity. After shifting the charges by a hypercharge rotation, one can realize this from a Z_6 subgroup of the $U(1)_{B-L}$ symmetry.

In the fourth chapter, we study the different approaches to the μ -term problem, one puzzle in supersymmetric model building, via a symmetry classification. Discrete

gauge symmetries from the anomalous $U(1)_A$ symmetry can be applied to solve this problem. One explicit example in terms of a Z_4 symmetry is given, where the μ -term problem is addressed by the Giudice-Masiero mechanism. The SUSY DFSZ QCD axion is also discussed as other realization of the μ -term problem. Here, new physics scale M_{PQ} is imposed and μ arises as $\mu \sim M_{\text{PQ}}^2/M_{\text{Pl}}$.

Discrete flavor gauge symmetries are studied in the following Chapter 5 which can explain the observed hierarchical structure of fermion masses while avoiding the D -term splitting problem in the usual SUSY soft breaking sector. Discrete Z_{14} gauged flavor symmetries are found to be consistent of one Lopsided hierarchical structure of fermion masses.

In the last chapter, we show how to use discrete gauge symmetries to stabilize the “invisible” axion solutions from violation due to quantum gravity. The axion is an elegant solution to the strong CP problem. Both *DFSZ and KSVZ “invisible axion”* models are discussed. The PQ symmetry only arises as an accidental symmetry. Examples of discrete Z_{11} and Z_{12} gauge symmetries are given to stabilize the non-SUSY DFSZ axion. For the SUSY DFSZ case, a discrete Z_{22} gauge symmetry is applied to stabilize the solution.

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