By
ZHI QUAN
Bachelor of Engineering
Beijing University of Posts and Telecommunications
Beijing, China1999
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## PREFACE

The next generation of wired and wireless network architectures will support a variety of applications with very high speed communication. At the network layer, scheduling algorithms play a critical role in provisioning quality-of-service ( QoS ) guarantees for different traffic classes. Therefore, performance evaluation of the proposed scheduling schemes have became a critical issue in the network design and control.

In this thesis, we first consider the problem of provisioning statistical QoS guarantees for real-time traffic in a wireless network node with selective-repeat automatic repeat request (ARQ) error control. We present a novel approach for evaluating the packet loss probabilities in the network and physical layers [1,2]. The results provide a new perspective for the network control and optimization using cross-layer design techniques.

Secondly, we study a high-speed switch deploying the preemptive priority scheduling policy. An analytical approach has been developed to estimate the per-class buffer overflow probabilities [3]. From the results, we find that when considering a certain traffic class, all the traffic with a higher priority can be lumped together and all the lower priority traffic can be ignored.

Finally, we investigate a high-speed network node scheduled by the earliest-deadlinefirst (EDF) policy. We propose a statistical framework $[5,6]$ to analyze the per-class deadline violation probabilities of an EDF scheduler. Based on the theoretical foundation, we derive the admission conditions and call admission control algorithm [7, 8]. In addition, we show that the statistical QoS guarantees that an EDF scheduler actually assures have an asymptotical ordering property [8]. As shown by the numerical evaluation, the theoretical results derived in this thesis provide fairly good approximations to the real metrics.

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## CHAPTER 1

## INTRODUCTION

### 1.1 Motivation

Future high speed packet-switching networks are expected to provide quality of service (QoS) differentiation for a variety of traffic classes. Statistical multiplexing of the traffic generated from sources with different characteristics on a switch may introduce considerable flexibility and potential savings in network resource allocation. Due to the diverse traffic characteristics, different classes of traffic should be treated separately according to their respective QoS requirements. Effective and practical packet scheduling schemes, therefore, are required to provide different QoS guarantees in an integrated network. To achieve this goal, three important scheduling mechanisms, strict priority (SP), generalized processor sharing (GPS), also known as weighted fair queueing (WFQ), and earliest deadline first (EDF) have been proposed for CAC frameworks. They offer considerable flexibility in providing high degrees of service differentiation and extracting statistical multiplexing gains. In particular, two variants of GPS (i.e., class-based WFQ and self-clocked fair queuing) have been implemented into switches and routers, respectively by the Cisco System and IBM.

Motivated by these observations, we investigate the issues of statistical QoS guarantees in communication networks deploying different scheduling algorithms. In particular, we wish to address the following problems in this thesis.

1. Consider a wireless network with automatic repeat request (ARQ) error control and first-in-first-out (FIFO) scheduling. Is it possible to provide statistical QoS guaran-
tees for real-time traffic under various channel conditions?
2. For a high-speed multiplexer deploying SP scheduling, how to obtain the per-class QoS metrics?
3. In a network node with EDF scheduling, how can one analyze the deadline violation probabilities of individual traffic classes? How to design an efficient and effective CAC algorithm for EDF networks? What kind of information can be derived so that the network protocols and applications can adaptively adjust their behaviors (e.g., QoS requirements and flow rates) for resource efficiency?

In order to solve the above problems, we need an appropriate traffic model to characterize the traffic processes. Given the traffic model, we can derive the solutions for the above problems by extending some of the important results on stochastic performance analysis from the literature. The traffic model and stochastic performance analysis will be introduced in chapter 2. The next section summarizes the contributions of this thesis.

### 1.2 Contributions of the Thesis

The contributions of this thesis are summarized as follows.

1. In the context of wireless communications, we study the provisioning of QoS for real-time traffic over a wireless channel with ARQ error control. We derive an analytical model to evaluate the queueing related loss and the wireless channel related loss. In contrast to the previous work, this model quantifies the interaction between the network and physical layers, and then it enables the admission controllers of the wireless network to improve the utilization while satisfying the traffic QoS constraints through cross-layer design techniques. This piece of research has resulted in [1] and [2].
2. Differentiated services (DiffServ) networking technologies are under development
with the objective to support diverse traffic classes that require different QoS guarantees. Recent studies have shown that real network traffic exhibits self-similarity or long-range dependence (LRD) in high-speed communication networks, which has a deteriorating impact on the network performance. To assist the development of admission control mechanisms which can accommodate heterogeneous traffic, including short-range independence and long-range dependence, this thesis proposes a measurement-based approach to estimate the buffer overflow probabilities of each priority queue in a multiplexer deploying a static priority (SP) scheduling discipline. The accuracy and effectiveness of this model have been verified by simulations. The results of this thesis will provide a practical insight into the buffer dimensioning and admission control design of a multiplexer with SP scheduling. Relevant publications regarding this work are [3] and [4].
3. The design of call admission control (CAC) mechanisms has been a critical issue in providing QoS guarantees for heterogeneous traffic flows over integrated service (IntServ) and/or differentiated service (DiffServ) networks. Earliest deadline first (EDF) is an ideal scheduler for real-time services because of its optimal admissible region and delay bound properties. The major difficulty in developing an effective and efficient CAC algorithm for statistical services is the analysis of per-class deadline violation (loss) probabilities with respect to the delay bounds. In this thesis, we provide an analytical approach to evaluate the aggregate and per-class deadline violation probabilities of an EDF scheduler. Based on these theoretical foundations, we derive the admission control conditions and then propose a CAC algorithm for statistical services under EDF scheduling. In addition, we show that the QoS metrics that an EDF scheduler actually guarantees have an asymptotic ordering property, which provides important insights into the design and control of EDF networks. The effectiveness and performance of our proposed algorithm have been validated by trace-driven simulation experiments using MPEG and H. 263 encoded video sources.

These results have been presented in [5], [6], [7], and [8].

### 1.3 Structure of the Thesis

We have introduced the context and motivation of the issues to be addressed. The remainder of this thesis is constructed as follows. In chapter 2, we provide an overview for the issues and challenges in high-speed networks supporting statistical QoS guarantees. In particular, we will review the issues regarding traffic characterization, performance analysis, scheduling algorithms, and admission control. In chapter 3, we present an analytical model to evaluate the packet loss probability in a wireless network with ARQ error control. In chapter 4, we give an approach to estimate per-class QoS metrics in a multiplexer deploying SP scheduling. In chapter 5, an analytical framework is developed to study the per-class deadline violation probabilities of an EDF scheduler. Based on this theoretical foundation, we further derive the admission control conditions and algorithm for networks with EDF scheduling. Chapter 6 summaries this thesis with a discussion on some future research issues.

## CHAPTER 2

# STATISTICAL QUALITY-OF-SERVICE GUARANTEES IN HIGH-SPEED NETWORKS: AN OVERVIEW 

### 2.1 Introduction

In this chapter, we overview the issues and challenges in high-speed networks supporting statistical QoS guarantees.

The issues regarding scheduling and admission control were originally studied in the context of a deterministic setting (see [9], [10], [11], [12], [14], and [15]). These deterministic frameworks are intrinsically conservative, since they have to consider the worst cases that might occur in the schedulers with very low probabilities. As a result, the network utilization achievable will be very low for bursty traffic. Therefore, they are limited to only applications requiring deterministic services (without packet loss), and are not appropriate for adaptive statistical services that can both tolerate and adapt to certain amounts of loss and end-to-end delay. Due to this, the scheduler behaviors under the context of statistical multiplexing have received a lot of attention in various aspects.

Traffic characterization is an very important issue in understanding QoS guarantees. To analyze the statistical behaviors of different traffic schedulers, we first need an elegant and promising traffic model to accurately characterize the traffic arrival processes. In particular, the traffic model is expected to be amenable for queueing analysis and capable of capturing the self-similar feature of real network traffic. This issue is addressed in section 2.2.

In section 2.3, we introduce some important statistical performance bounds developed in the literature. These theoretical foundations serve as the building blocks for the developments of our analytical frameworks presented in this thesis.

The scheduler is an indispensable part in the design of call admission control mechanisms. In section 2.4, we briefly review some representative scheduling disciplines for QoS guarantees and various important aspects regarding call admission control.

### 2.2 Traffic Models

Traffic specification and modeling are very important for network performance evaluation. Without the knowledge of traffic characteristics, it is impossible for networks to provide QoS guarantees. Recent studies [16] [17] of high-quality, high-resolution traffic measurements have convincingly shown that real network traffic exhibits self-similar (or long-range dependent) characteristics in existing packet-switching networks. These studies present a fundamentally different set of problems for the analysis and design of networks, and many of the previous analytical models based on short-range dependent processes (e.g., Poission and Markovian processes) are no longer valid in the presence of self-similarity. In this section, we first review the definitions and properties of self-similarity and then introduce some representative self-similar traffic models that are amenable for queueing analysis.

### 2.2.1 Self-Similarity

Intuitively, an object is said to be self-similar if it looks roughly the same in any scale. Selfsimilarity has important consequences [16] for the design of computer networks, since typical network traffic has self-similar properties. This implies that traditional short-range dependent traffic models are inaccurate and networks designed without taking self-similarity into account are likely to function in unexpected ways. In the following, we introduce the mathematical definition and properties of self-similarity [16].

Consider a stationary stochastic process $X=\left\{X_{t}: t=0,1, \ldots\right\}$ with mean $\mu$, variance $\sigma^{2}$, and autocorrelation function $r(k), k \geq 0$. It is assumed that

$$
\begin{equation*}
r(k) \sim k^{-\beta} L(t), \quad \text { as } k \rightarrow \infty, \tag{2.1}
\end{equation*}
$$

where $0<\beta<1$ and $L$ is slowly varying at infinity, i.e., $\lim _{t \rightarrow \infty} L(t x) / L(t)=1$, for all $x>0$. For ease of presentation, $L$ is assumed to be asymptotically constant.

For each $m=1,2, \ldots$, we let

$$
\begin{equation*}
X_{k}^{(m)}=\frac{1}{m}\left(X_{k m-m+1}+\cdots+X_{k m}\right), \quad k \geq 1 \tag{2.2}
\end{equation*}
$$

Definition 2.2.1 (exactly second-order self-similar) The process is called exactly secondorder self-similar with Hurst parameter $H=1-\beta / 2$, iffor all $m=1,2, \ldots, \operatorname{var}\left(X^{(m)}\right)=$ $\sigma^{2} m^{-\beta}$ and $r^{(m)}(k)=r(k)$.

Definition 2.2.2 (asymptotically second-order self-similar) The process is called asymptotically second-order self-similar with Hurst parameter $H=1-\beta / 2$, if for all $k$ large enough, $r^{(m)}(k) \sim r(k)$, as $m \rightarrow \infty$.

In other words, $X$ is exactly or asymptotically second-order self-similar if the corresponding aggregated processes $X^{(m)}$ are the same as $X$ or become indistinguishable from $X$, at least with respect to their autocorrelation functions.

Mathematically, self-similarity manifests itself in a number of equivalent ways:

1. Slowly Decaying Variance: the variance of the sample mean decreases more slowly than the reciprocal of the sample size, i.e., $\operatorname{var}\left(X^{(m)}\right) \sim \mathrm{cm}^{-\beta}$, as $m \rightarrow \infty$ with $0<\beta<1$. Notice that $c$ is a certain positive constant here and below.
2. Long Range Dependence (LRD): the autocorrelations decay hyperbolically rather than exponentially fast, which implies a non-summable autocorrelation function, i.e., $\sum_{k} r(k)=\infty$.
3. $1 / f$-Noise: the spectral density function $f(\cdot)$ obeys a power-law near the origin, i.e., $f(\lambda) \sim c \lambda^{-\gamma}$, as $\lambda \rightarrow 0$, with $0<\gamma<1$ and $\gamma=1-\beta$.

Intuitively, the most striking feature of second-order self-similar processes is that their aggregated processes $X^{(m)}$ possess a non-degenerate correlation structure as $m \rightarrow \infty$.

### 2.2.2 Self-Similar Traffic Modeling

Traffic modeling is critical in simulating and evaluating communications networks. Regarding self-similar traffic, two formal mathematical models that yield elegant representations of the self-similarity are introduced in [16], i.e., fractional Gaussian noise (FGN) and fractional autoregressive integrated moving-averaging (ARIMA) processes.

In this thesis, we model the cumulative traffic during time interval $[0, t)$ as a fluid Gaussian process $A(t)$ with stationary increments. Generally, $A(t)$ can be rewritten as

$$
\begin{equation*}
A(t)=\mu t+Z(t) \tag{2.3}
\end{equation*}
$$

where $\mu$ is the mean arrival rate and $Z(t)$ is a centered continuous Gaussian process with variance $\sigma^{2}(t)$. In particular, if $A(t)$ is modeled as a fractional Brownian motion (fBm) process, we then have

$$
\begin{equation*}
\sigma^{2}(t)=V t^{2 H} \quad \text { for } t>0 \tag{2.4}
\end{equation*}
$$

where $V$ is a constant.
The advantages for choosing the Gaussian process as the traffic model are as follows. First, the high-speed packet-switching networks are expected to support a large number of heterogeneous applications. According to the central limit theorem, the aggregated traffic can be effectively characterized by a Gaussian process, even when each individual flow is not Gaussian. Second, a Gaussian process can be completely characterized by its mean and covariance, which makes it more appealing for analysis under large aggregation when compared with Markov modulated fluid (MMF) processes. In spite of these appealing features, the assumption of Gaussian input has not been fully accepted in queueing theory because of the positive probability of negative input, which is nonsense in practice. However, the application of the Gaussian process has been justified in [18], [19], [20], and references therein, where significantly accurate results have been obtained proving its effectiveness as a stochastic traffic model.

### 2.3 Performance Evaluation

Before proceeding, we first describe the system of interest in this thesis. We consider a high-speed multiplexing system with a large number of independent traffic sources. Let $b$ and $c$ respectively represent the buffer size and link capacity of each source. This highspeed link then has a shared buffer of size $B=N b$ and link capacity $C=N c$, where $N$ is the scaling size of the system. For source $i(1 \leq i \leq N)$, we denote the amount of arrival traffic during time interval $[\tau, t)$ by $A_{i}(\tau, t)$. The aggregate arrival process during interval $[\tau, t)$ is given by $A(\tau, t)=\sum_{i=1}^{N} A_{i}(\tau, t)$. To ensure the system stability, it should have $\lim _{t \rightarrow \infty} \mathbb{E}[A(-t, 0)] / t<C$.

To evaluate the performance of this system, we can use the large deviation theory and maximum variance asymptotic, which provide us with the most important time and space scales of the queue length distribution. These two approaches are presented as follows.

### 2.3.1 Large Deviation Theory

The large deviation theory has been extensively studied for queueing analysis, either with the large buffer asymptotic or with the many sources asymptotic. The large buffer case considers a single server and a single arrival stream with finite arrival rate, while the many sources asymptotic deals with a queueing system multiplexing a large number of arrival processes. For an actual high-speed multiplexor (usually up to $10^{4}$ streams), the many sources asymptotic works better than the large buffer method. Moreover, most high-speed networks have small or moderate buffer sizes where the large buffer approximation can not be applied.

Using the results from the many sources asymptotic approach based on large deviation techniques, we can estimate the buffer overflow probability for large $N$. The stochastic behavior of a traffic source can be characterized by its effective bandwidth. For source
$A_{i}(0, t)$, its effective bandwidth is defined as [47]

$$
\begin{equation*}
\alpha_{i}(s, t)=\frac{1}{s t} \log \mathbb{E}\left[e^{s A_{i}(0, t)}\right] \quad 0<s, t<\infty, \tag{2.5}
\end{equation*}
$$

where $s$ and $t$ are the system parameters defined by the context of the source, i.e., the characteristics of the multiplexed traffic, their QoS requirements, and the link resources (capacity and buffer). Specifically, the time parameter $t$ (measured in, e.g., msec) corresponds to the most probable duration of the busy period of the buffer prior to a buffer overflow [44] (i.e., the time-to-fill the buffer). The space parameter $s$ (measured in, e.g., kilobits ${ }^{-1}$ ) corresponds to the degree of multiplexing and depends, among others, on the size of the peak rate of the multiplexed sources relative to the link capacity. Effective bandwidths are increasing with respect to $s$ [46]. In particular, for link capacities much larger than the peak rate of the multiplexed sources, $s$ tends to zero and $\alpha_{i}(s, t)$ approaches the mean rate of the source; for link capacity not much larger than the peak rate of the sources, $s$ is large and $\alpha_{i}(s, t)$ approaches the maximum value that the random variable $A_{i}(0, t) / t$ can achieve.

Denote by $Q_{t}$ the queue length at time $t$, which can be expressed as

$$
\begin{equation*}
Q_{t}:=\sup _{\tau \leq t}[A(\tau, t)-C(t-\tau)] . \tag{2.6}
\end{equation*}
$$

For ease of exploration, $A_{i}(\tau, t)(i=1,2, \ldots, N)$ are assumed to be independent and identically distributed (i.i.d.) processes with stationary increments. The asymptotic tail distribution of $Q_{t}$ is given by

$$
\begin{equation*}
\lim _{N \rightarrow \infty} \frac{1}{N} \log \operatorname{Pr}\left\{Q_{t}>B\right\}=I(b) \tag{2.7}
\end{equation*}
$$

where

$$
\begin{align*}
I(b) & =\inf _{t} \sup _{s}\left[s(c t+b)-s t \alpha_{1}(s, t)\right]  \tag{2.8}\\
& =\inf _{t} \sup _{s}\left\{s(c t+b)-\log \mathbb{E}\left[e^{s A_{1}(0, t)}\right]\right\}
\end{align*}
$$

is defined as the asymptotic rate function. The queue length distribution can be written as $\operatorname{Pr}\left\{Q_{t}>B\right\}=e^{-N I(b)+o(N)}$ for large $N$, which leads to the following approximation
using the Bahadur-Rao improvement [47]

$$
\begin{align*}
\operatorname{Pr}\left\{Q_{t}>B\right\} & =\frac{1}{\sigma s^{*} \sqrt{2 \pi N}} e^{-N I(b)}\left(1+O\left(\frac{1}{N}\right)\right)  \tag{2.9}\\
& \approx e^{-N I(b)-\frac{1}{2} \log \left(2 \pi N \sigma^{2} s^{2}\right)}
\end{align*}
$$

The authors of [45] have shown that $\frac{1}{2} \log \left(2 \pi N \sigma^{2} s^{* 2}\right)$ can be approximated by $\frac{1}{2} \log (4 \pi N I)$. Thus, it does not require any more computations with respect to (2.8).

If the arrival processes are not i.i.d., we can use the effective bandwidth of the aggregate traffic for queueing analysis. Let $N I=\inf _{t} \sup _{s}[s(B+C t)-s t \alpha(s, t)]$, where $\alpha(s, t)=$ $\sum_{j=1}^{N} \alpha_{j}(s, t)$ is the aggregate effective bandwidth. Therefore, we have

$$
\begin{equation*}
\operatorname{Pr}\left\{Q_{t}>B\right\} \approx \exp \left[-N I-\frac{1}{2} \log (4 \pi N I)\right] \tag{2.10}
\end{equation*}
$$

The many sources asymptotic is very generally applicable because it applies for periodic sources, fractional Brownian motion, Markovian sources, and policed and shaped sources [46]. Moreover, it is not necessary to assume a particular traffic model.

### 2.3.2 Maximum Variance Asymptotic

Maximum variance asymptotic (MVA) was developed by the authors of [18] for Gaussian traffic. According to the central limit theorem, using a Gaussian process as the traffic model is practical because we consider a high-speed network where a large number of independent sources are multiplexed together.

The MVA approach works as follows. Consider a general multiplexing system with a constant service rate $C$. The amount of input traffic arriving during time interval [ $s, t$ ) is denoted by $A(s, t)$, and thus $X(t)=A(-t, 0)-C t$ is the net amount of fluid input during time interval $[-t, 0)$. The buffer overflow probability in steady-state is given as [20]

$$
\begin{equation*}
\operatorname{Pr}\{Q>x\}=\operatorname{Pr}\left\{\sup _{t \geq 0} X(t)>x\right\} . \tag{2.11}
\end{equation*}
$$

In particular, the overflow probability can be approximated as

$$
\begin{equation*}
\operatorname{Pr}\left\{\sup _{t \geq 0} X(t)>x\right\} \approx \sup _{t \geq 0} \operatorname{Pr}\{X(t)>x\} \tag{2.12}
\end{equation*}
$$

where " $\approx$ " denotes the asymptotic equivalence. This approximation has been proven by the large deviation theory in at least two distinct regimes: large buffer asymptotic (i.e., the buffer level $x \rightarrow \infty$ ) [28] and many source asymptotic (i.e., the number of sources $n \rightarrow \infty$ ) [43]. Under either limiting regime, (2.12) is justified by the fact that the probability of the union of many rare events is dominated by the probability of the most probable of those events, which is generally stated as rare events take place only in the most probable way. Also, this approximation has been shown to be quite accurate using extreme value theories in [18], [19], and [20]. Therefore, provided that the system is stable (i.e., $\mu=$ $E\{A(-t, 0)\} / t<C)$, there must exist a finite value of $t=\hat{t}$, called the dominant time scale (DTS), at which the function $\operatorname{Pr}\{X(t)>x\}$ attains its maximum. It is easy to see that $[-\hat{t}, 0)$ is the most probable duration of a busy period prior to a buffer overflow at time zero [20]. For simplicity, we use $A(t)$ to represent $A(-t, 0)$ throughout the rest of this thesis, and $A_{i}(t)=0$ for $t<0$.

In [18], the authors developed lower and upper bounds for the steady-state buffer overflow probability through the DTS, i.e.,

$$
e^{-\alpha^{2} / 2} \geq \operatorname{Pr}\{Q>x\} \geq \frac{e^{-\alpha^{2} / 2}}{\sqrt{2 \pi} \alpha}
$$

where

$$
\begin{equation*}
\alpha:=\inf _{t \geq 0}\left[\frac{(C-\mu) t+x}{\sigma(t)}\right] . \tag{2.14}
\end{equation*}
$$

It has been shown that if $\sigma^{2}(t)$ is differentiable at $\hat{t}$, the value of $\alpha$ can be found by solving the following equation [31],

$$
\begin{equation*}
\frac{\sigma(\hat{t})}{\sigma^{\prime}(\hat{t})}-\hat{t}=\frac{x}{C-\mu} . \tag{2.15}
\end{equation*}
$$

To ensure that the Gaussian processes with stationary increments are smooth, it is assumed that there exists $\varepsilon<2$ such that $\lim _{t \rightarrow \infty} \sigma^{2}(t) / t^{\epsilon}=0$.

For more details of the MVA analysis, the interested reader is referred to [18], [19], [20], and the references therein.

### 2.4 Scheduling and Call Admission Control

The packet scheduler is a critical network component to provide QoS guarantees. Due to the diverse traffic characteristics and different QoS constraints of service classes, the major function of a packet scheduler is to classify traffic and control bandwidth sharing among different service classes. In recent years, many packet scheduling algorithms have been proposed to support QoS requirements. The most basic scheduling is the first-in-first-out (FIFO) policy, by which packets are processed in order of their arrivals. Some of the most important scheduling algorithms include static priority (SP), generalized processor sharing (GPS), and earliest deadline first (EDF). In this thesis, we will focus on three of the most representative scheduling policies, i.e., FIFO, SP, and EDF. The interested reader is referred to [21] for a detailed survey of other network scheduling algorithms.

Call admission control (CAC) mainly deals with whether or not to accept a connection for a new call request in order to prevent network congestion. A variety of CAC algorithms have been proposed in the literature. According to the admission decision making mechanisms used, CAC algorithms can be categorized into two classes, i.e., parameterbased and measurement-based. The parameter-based CAC mechanisms require the explicit knowledge of traffic parameters while the measurement-based algorithms use on-line measurements to extract the traffic characteristics. Generally, the parameter-based algorithms are not accurate nor effective due to the difficulty of obtaining the explicit traffic characterization, especially for self-similar traffic which exhibits high burstiness over short time scales, e.g., the variable-bit-rate (VBR) compressed video. This drawback greatly limits the use of parameter-based CAC algorithms in high-speed networks with integrated service (IntServ) and/or differentiated service (DiffServ) architectures. In this thesis, we adopt the maximum variance asymptotic (MVA) as a building block for performance analysis, thus the frameworks developed can be easily used for the measurement-based CAC design.

## CHAPTER 3

## ANALYSIS OF PACKET LOSS FOR REAL-TIME TRAFFIC IN WIRELESS MOBILE NETWORKS WITH ARQ FEEDBACK

### 3.1 Introduction

Real-time traffic is expected to account for a large portion of the traffic in future wireless networks. In general, transporting real-time traffic over wireless networks is a very challenging problem. This is due to the time-varying characteristics of wireless channels and the stringent delay constraints of real-time applications (i.e., voice, video, and real-time control signals). Because of stringent time requirements, real-time services need to be received before a certain deadline, or else are useless. Feedback strategies [25], such as ARQ, are very effective in providing reliable communications for real-time services in wireless environments, given that the round-trip time (RTT) is relatively small compared to the allowed delay. In a high capacity wireless channel, the packet duration is usually small such that there might be enough time for several round-trip feedbacks before its deadline.

QoS has been extensively studied in wired networks in terms of delay and loss. The major existing approaches for evaluating the QoS metrics are large deviation technique [27], [28], bufferless fluid flow model [29], and maximum variance asymptotic (MVA) approximation [18] [20]. However, these approaches are not directly applicable for wireless networks due to the time-varying channel characteristics. Hence, to find an efficient and effective method to study the performance of wireless networks in terms of QoS has become an important open issue.

Generally, the actual delay experienced by a packet consists of two components. The first is the queueing delay that happens in the arrival buffer, i.e., the time between the
packet arrival and the instant of its first transmission over the wireless link. The second part is the time between the first transmission and the successful transmission, referred to as transmission delay. Nevertheless, the issue of reliable communications over a fading channel falls mainly within the context of information theory, where the delay is typically ignored completely or only the transmission delay is considered. The queueing delay is usually considered as a network layer problem and separated from physical layer considerations. Thus, it is very difficult to apply physical layer channel models to QoS provisioning mechanisms (e.g., packet scheduling, call admission control, and resource reservation).

Our work is motivated by the fact that next generation wireless communication technologies and services, such as, wireless asynchronous transmission mode (ATM), wireless integrated services digital network (ISDN), IMT-2000, and the future 4th generation wireless systems, are required to support various QoS requirements and traffic characteristics. In this chapter, we focus on networking conditions when the utilization is relatively high (i.e., close to the link capacity). The reason for this is based on the fact that packet loss and delay become most critical under these conditions, but are much less of a concern when the network utilization is low. In addition, newly developed wireless network applications strongly demand high data rate services with improved QoS constraints, thereby requiring the network devices to support services at much higher utilization than before.

This work differs from previous results in the following aspects. First, the MVA approach is used as the building block for performance analysis by assuming that the input traffic can be modeled as a stationary Gaussian process. In queueing theory, the Gaussian process has not been fully accepted as a traffic model due to the positive probability of negative input, nevertheless its effectiveness and significant accuracy as a stochastic traffic model has been justified by extensive studies in [18], [20], and [19]. In particular, the Gaussian traffic model has been extended to analyze the stochastic scheduling systems, such as priority queueing [3], generalized processor sharing (GPS) [31], and earliest deadline first (EDF) [5], and shown to offer fairly good approximations to the real metrics. Second, we
focus on the selective repeat ARQ feedback strategy because of its efficiency ${ }^{1}$. It has been shown in [26] that this ARQ technique obtains a significant gain in energy efficiency for reliable communications under various channel conditions.

This chapter is constructed as follows. Section 3.2 provides a detailed description of the system model. In section 3.3, we present the approaches for evaluating the loss probabilities in the network and physical layers. Section 3.4 investigates the ARQ error control in the time-varying Rayleigh fading channel. In section 3.5, we numerically validate the effectiveness and accuracy of our framework. Section 3.6 concludes this chapter.

### 3.2 System Model Description

Consider a wireless channel model with a service capacity $C(t)$, shown in Fig. 3.1. We characterize the accumulative arrival traffic as a fluid process $A(t)$ with stationary increments and the size of a packet is infinitesimal. The arrival traffic is queued into the arrival buffer before receiving services. Generally, the wireless channel capacity $C(t)$ is variable. For simplicity, we assume that the channel capacity is slowly time-varying or constant, then we have $C(t)=C$ in the following exploration. Let $R(t)$ represent the accumulative process of the retransmitted traffic. To guarantee the stability condition, the arrival process should satisfy $\lim _{t \rightarrow \infty}[A(t)+R(t)] / t<C$.

Suppose the selective-repeat ARQ protocol is deployed in the physical layer, and only the damaged or lost packets are retransmitted. Specifically, if a packet is corrupted, a negative acknowledgement (SREJ) packet is returned and this packet is retransmitted immediately; otherwise a positive acknowledgement packet is sent back and new packets are continuously transmitted. To facilitate the analysis, we create an ARQ buffer to store the retransmitted packets, and let the packets requesting retransmission have strict priority over the packets in the arrival buffer. Therefore, the arrival buffer can only receive service when

[^0]the ARQ buffer is empty. Without loss of generality, we assume that the feedback channel is perfect and each positive or negative acknowledging packet is always received successfully. Denote by $T_{r}$ the measured moving average of the round-trip time, i.e., the time between a transmission/retransmission and the receival of its positive/negative acknowledging message.

For real-time traffic, each packet has a stringent delay constraint and has to be successfully transmitted before a specific deadline; otherwise, it will be dropped. Let $D$ represent the maximum delay that a packet can tolerate, then the packet arriving at time $t$ should be considered lost if it cannot be successfully transmitted before its deadline $t+D$. In this chapter, the loss probability is defined as the fraction of time that a deadline violation occurs. We also assume that the arrival buffer is large enough, thus there is no packet loss due to insufficient buffer capacity.

### 3.3 Packet Loss Probabilities

The actual delay experienced by a packet consists of queueing delay and transmission delay. Accordingly, the losses (deadline violations) caused by queueing delay and transmission delay are referred to as queueing loss and transmission loss, respectively. In the following, we will present the approaches to evaluate these two quantities.

### 3.3.1 Queueing Loss

For the error free channel, the packet loss is caused only by the queueing delay, which can be evaluated by

$$
\begin{equation*}
\operatorname{Pr}\{d>D\} \approx \sup _{\boldsymbol{t}} \operatorname{Pr}\{A(t)>C(t+D)\} \tag{3.1}
\end{equation*}
$$

The time instant $t=\hat{t}$ at which $\operatorname{Pr}\{A(t)>C(t+D)\}$ attains its maximum is referred to as the dominant time scale (DTS) in [18] and [20]. Suppose a packet loss occurs at time zero. It is easy to see that $[-\hat{t}, 0)$ is the most probable duration of a busy period prior to this packet loss, which is consistent with a well-known statement that rare events take place only in the
most probable way. This approximation has been shown to be quite accurate using large deviation techniques in [27] and [28], and extreme value theories in [18] and [20].

In wireless networks, due to the shadowing, fading, and user mobility, some of the packets transmitted will be corrupted and will require retransmission. Thus, the overall system capacity has to be divided into two parts. We define the fraction of capacity dedicated for retransmission as the $A R Q$ capacity, denoted by $C_{A}(t)$, and the fraction of capacity dedicated to the first transmission as the effective capacity, denoted by $C_{E}(t)$. Thus, at each time $t$, we have

$$
\begin{equation*}
C=C_{E}(t)+C_{A}(t) \tag{3.2}
\end{equation*}
$$

Consequently, the queueing loss probability can be approximated by

$$
\begin{equation*}
P_{q} \approx \sup _{t} \operatorname{Pr}\left\{A(t)>\int_{0}^{t+D} C_{E}(\tau) d \tau\right\} \tag{3.3}
\end{equation*}
$$

Here we give the formulae to calculate the ARQ capacity $C_{A}(t)$ and the effective capacity $C_{E}(t)$. Let $\hat{n}$ be the maximum number of retransmissions that the delay bound $D$ allows, i.e.,

$$
\begin{equation*}
\hat{n}=\left\lfloor\frac{D}{T_{r}}\right\rfloor \tag{3.4}
\end{equation*}
$$

where $\lfloor x\rfloor$ denotes the largest integer that is less than or equal to $x$. For an arbitrary packet, we denote the probability of its failure exactly on the $i$ th transmission by $P_{i}(1 \leq i \leq \hat{n}+1)$. During the busy period (i.e., the link is fully loaded), we can obtain the ARQ capacity as follows. Since there is no ARQ traffic in the transmitter during time interval $\left[0, T_{r}\right.$ ), we have $C_{A}(t)=0$. For interval $\left[T_{r}, 2 T_{r}\right)$, we have $C_{A}(t)=C P_{1}$ because the ARQ traffic will be given a strict priority when competing with the new arrival traffic for resource. By induction, we then have the following recursion formula for the ARQ capacity.

Proposition 3.3.1 The amount of $A R Q$ capacity at time $t$ is given by

$$
\begin{equation*}
C_{A}(t)=C_{A, k}, \quad \text { if } k T_{r} \leq t<(k+1) T_{r} \tag{3.5}
\end{equation*}
$$

where

$$
C_{A, k}= \begin{cases}\sum_{i=0}^{k-1}\left(C-C_{A, i}\right) \prod_{j=1}^{k-i} P_{j} & 0 \leq k<\hat{n}  \tag{3.6}\\ \sum_{i=k-\hat{n}}^{k-1}\left(C-C_{A, i}\right) \prod_{j=1}^{k-i} P_{j} & k \geq \hat{n}\end{cases}
$$

and $C_{A, 0}=0$.

Based on Proposition 3.3.1, the effective capacity can be easily derived, i.e.,

$$
\begin{equation*}
C_{E}(t)=C-C_{A}(t), \quad \text { for all } t>0 \tag{3.7}
\end{equation*}
$$

It is easy to show that under constant channel conditions without packet combining (i.e., memoryless ARQ), where $P_{i}=p(1 \leq i \leq \hat{n}+1)$, we have

$$
\begin{equation*}
\lim _{t \rightarrow \infty} C_{A}(t)=\frac{C\left(1-p^{\hat{n}}\right) p}{1-p^{\hat{n}+1}}, \quad \text { and } \lim _{t \rightarrow \infty} C_{E}(t)=\frac{C(1-p)}{1-p^{\hat{n}+1}} \tag{3.8}
\end{equation*}
$$

For the case of memory ARQ deployed in a time-varying fading Rayleigh channel, Fig. 3.2 shows the evolutions of $C_{A}(t)$ and $C_{E}(t)$ over the time scale. Note that the values of $P_{i}(1 \leq i \leq \hat{n}+1)$ are given to ensure that the probability of failure is less than 0.0001 provided that the maximum number of transmissions $\hat{n}+1=5$.

Consider a continuous Gaussian process $A(t)$ with stationary increments. Let $\mu$ and $\sigma^{2}(t)$ respectively represent the mean arrival rate and variance of $A(t)$. Then, the quantity of equation (3.3) can be evaluated by the following proposition ${ }^{2}$.

Proposition 3.3.2 The queueing loss probability in steady-state can be approximated by the following formula

$$
\begin{equation*}
P_{q} \approx L e^{-\alpha^{2} / 2} \tag{3.9}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha=\inf _{t>0}\left(\frac{\int_{0}^{t+D} C_{E}(\tau) d \tau-\mu t}{\sigma(t)}\right) \tag{3.10}
\end{equation*}
$$

and $L$ is the asymptotic constant, which tends to a constant as goes to infinity.

[^1]Proof. The arrival process can be written as $A(t)=\mu t+Z(t)$, where $Z(t)$ is a centered continuous Gaussian process with variance $\sigma^{2}(t)$. Substituting $A(t)$ into (3.3), we get

$$
\begin{aligned}
P_{q} & \approx \sup _{t} \operatorname{Pr}\left\{Z(t)>\int_{0}^{t+D} C_{E}(\tau) d \tau-\mu t\right\} \\
& =\sup _{t} \Psi\left(\frac{\int_{0}^{t+D} C_{E}(\tau) d \tau-\mu t}{\sigma(t)}\right)
\end{aligned}
$$

where $\Psi(z)=1 / \sqrt{2 \pi} \int_{z}^{\infty} e^{-x^{2} / 2} d x$. Using the MVA bounds developed in [18], i.e., $e^{-\alpha^{2} / 2} \geq \Psi(\alpha) \approx \frac{e^{-\alpha^{2} / 2}}{\sqrt{2 \pi} \alpha}$, and $L$ as an approximation [5] [3], the proposition follows.

Remark: In the previous work for wired networks [3] [5] [6], extensive experimental results have shown that setting $L=\sqrt[-1 / 4]{2 \pi \alpha^{2}}$ does provide fairly good approximations to the real probabilities, although there is no exact solution for it. One may also choose an alternate value for special cases. For example, when using the theory of effective bandwidth [30], the asymptotic constant is given by $L=\operatorname{Pr}\{Q>0\}$, which is the probability that the buffer is nonempty at a randomly chosen time instant. As shown by the numerical examples below, the approximation (3.9) is also fairly accurate in wireless networks with selective repeat ARQ.

### 3.3.2 Transmission Loss

Now, we present the approach to evaluate the transmission loss probability. Before proceeding, we first claim an important statement.

Claim 3.3.1 The delay experienced by the retransmitted packet in the ARQ buffer is zero.

Proof. As defined before, the ARQ traffic has strict priority over the new arrival traffic. Consider the worst case, where $P_{i}=1(1 \leq i \leq \hat{n}+1)$, we have $C_{A}(t)=C$ as $t \rightarrow \infty$. Thus, $C$ is the upper bound of the ARQ capacity $C_{A}(t)$, and $C_{A}(t)$ can never be greater than $C$. We can conclude that the ARQ buffer is always empty and the ARQ traffic does not experience any queueing delay.


Figure 3.1: Block diagram of a wireless mobile system.


Figure 3.2: The effective capacity $C_{E}(t)$ and ARQ capacity $C_{A}(t)$ versus time in a time-varying Rayleigh fading channel, where the memory ARQ is deployed and $P_{1}=$ $0.3229, P_{2}=0.0589, P_{3}=0.0074, P_{4}=0.0007, P_{5}=0.0001, D=25 \mathrm{~ms}$, and $T_{r}=$ 5 ms .

To derive the transmission loss, we need to obtain the steady-state probability density function (PDF) of the queueing delay. Denote the queueing delay of a packet in the arrival buffer by $d$, which has tail probability distribution as

$$
\begin{align*}
\operatorname{Pr}\{d>t\} & \approx \sup _{\tau>0} \operatorname{Pr}\left\{A(\tau)>\int_{0}^{\tau+t} C_{E}(\gamma) d \gamma\right\} \\
& \approx L \exp \left(-\frac{\left[\int_{0}^{\hat{t}+t} C_{E}(\gamma) d \gamma-\mu \hat{t}\right]^{2}}{2 \sigma^{2}(\hat{t})}\right), \tag{3.11}
\end{align*}
$$

where $\hat{t}$ is the dominant time scale. Note that the quantities of $\hat{t}, \mu$, and $\sigma^{2}(t)$ can be estimated from the measurement of real network traffic. Thus, the probability density function (PDF) in steady-state is given by

$$
\begin{equation*}
f_{d}(t)=-\frac{d \operatorname{Pr}\{d>t\}}{d t} . \tag{3.12}
\end{equation*}
$$

For a head-of-line (HOL) packet at a randomly chosen time $t$, the delay available for transmission is $D-d$. If a packet can not be transmitted successfully before its deadline, then it is considered lost. Thus, the maximum number of retransmissions allowed is given by

$$
\begin{equation*}
n=\left\lfloor\frac{D-d}{T_{r}}\right\rfloor . \tag{3.13}
\end{equation*}
$$

Denote the allowable maximum number of retransmissions by $M$, which is a random number determined by the queueing delay $d$. Then, the probability distribution function of $M$ is discrete and can be derived as

$$
\begin{align*}
& \operatorname{Pr}\{M=n\} \\
& =\int_{D-(n+1) T_{r}}^{D-n T_{r}} f_{d}(\tau) d \tau  \tag{3.14}\\
& =\operatorname{Pr}\left\{d>D-(n+1) T_{r}\right\}-\operatorname{Pr}\left\{d>D-n T_{r}\right\},
\end{align*}
$$

when $0 \leq n<\hat{n}$, and

$$
\begin{equation*}
\operatorname{Pr}\{M=n\}=\operatorname{Pr}\{d>0\}-\operatorname{Pr}\left\{d>D-\hat{n} T_{r}\right\}, \tag{3.15}
\end{equation*}
$$

when $n=\hat{n}$.

Let $P_{c}(i)$ denote the probability of failure within the first $i(1 \leq i \leq \hat{n}+1)$ transmissions, i.e.,

$$
\begin{equation*}
P_{c}(i)=1-\operatorname{Pr}\{\text { success within } i \text { transmissions }\} . \tag{3.16}
\end{equation*}
$$

Combining equations (3.14) through (3.16), the following proposition gives the approximation to the transmission loss probability.

Proposition 3.3.3 The steady-state loss probability caused by the channel corruption can be estimated by

$$
\begin{equation*}
\mathbb{E}\left(P_{c}\right)=\sum_{i=0}^{\hat{n}} P_{c}(i+1) \operatorname{Pr}\{M=i\} . \tag{3.17}
\end{equation*}
$$

So far, we have present the general approaches to evaluate the queueing loss probability and the transmission loss probability by assuming that $P_{i}(1 \leq i \leq \hat{n}+1)$ are given. In practice, the metrics of $P_{i}$ are determined by the channel characteristics, ARQ error control mechanisms, and energy utilization. In the section below, we will show how to extract the values of $P_{i}$ from the time-varying Rayleigh fading channels when memory and memoryless ARQ protocols are deployed.

### 3.4 ARQ Error Control in Time-Varying Rayleigh Fading Channels

In this section, we consider the model of [25] and [26], where it is assumed that packet combining is carried out on a per symbol basis of a packet for the ARQ samples. The overall packet is presumed to have near perfect error detection and possibly also error correction. This assumption is valid when considering the current existing space-time coding and turbo coding techniques combined with the memory ARQ topologies. With ideal combining [25], assuming a perfect receiver estimate of the signal-to-noise ratio (SNR), fixed noise power but possibly varying received signal power, there is a rather sharp threshold of success probability relative to the total received energy per symbol [32]. If the total received energy over $n$ transmissions of a symbol is denoted by $E_{n}$ and the noise power by
$N$, we can define an equivalent signal-to-noise ratio denoted by $S_{n}$, as

$$
\begin{equation*}
S_{n}=\frac{E_{n}}{N} \tag{3.18}
\end{equation*}
$$

The assumption here is that the success probability will be zero if $S_{n}<S_{t}$, where $S_{t}$ is the threshold, and 1 if $S_{n} \geq S_{t}$. As an example, suppose the channel received signal power is varying. For simplicity, assume the received power remains fixed during a specific packet copy transmission. In the following, a fast-varying fading mobile communication channel where each packet retransmission experiences an independent channel gain is considered. For numerical illustration, a Rayleigh distribution is selected to describe the received power variation [25] [26] [32] of the fast fading channel characteristics. From the Rayleigh amplitude probability density function of $f_{D}(d)=\frac{d}{\sigma^{2}} e^{-d^{2} / 2 \sigma^{2}}$ (for $d \geq 0$ ), the variables are changed into power signal to noise ratios. Let $S$ be the random variable representing the power SNR and $R$ the mean power signal to noise ratio, by setting $S=d^{2} / 2$ and $R=\sigma^{2}$, we then obtain

$$
\begin{equation*}
f_{s}(s)=\frac{1}{R} e^{-s / R} \tag{3.19}
\end{equation*}
$$

In the $i$ th transmission, let the cumulative distribution function (CDF) and the probability density function (PDF) of the cumulating received signal energy be $F_{i}(s)$ and $f_{i}(s)$, respectively. The probability of success within $i$ transmissions is

$$
\begin{equation*}
\operatorname{Pr}\{\text { success within } i \text { transmissions }\}=1-F_{i}\left(S_{t}\right) . \tag{3.20}
\end{equation*}
$$

From this, the probability of succeeding exactly on the $i$ th transmission becomes

$$
\begin{equation*}
\operatorname{Pr}\{\text { success in exactly } i \text { transmissions }\}=F_{i-1}\left(S_{t}\right)-F_{i}\left(S_{t}\right) . \tag{3.21}
\end{equation*}
$$

As a result, the average number of transmissions per success is given by

$$
\begin{equation*}
\mathbb{E}\{\text { number of transmissions per success }\}=1+\sum_{i=1}^{n} F_{i}\left(S_{t}\right) \tag{3.22}
\end{equation*}
$$

The Rayleigh density and distribution for a single transmission are $f_{1}(s)=\frac{1}{R} e^{-s / R}$ and $F_{1}(s)=1-e^{-s / R}$. For $n$ receptions, the CDF of the sum of the independent random
variables is required. The corresponding density function of the sum of the independent random variables will be

$$
\begin{equation*}
f_{n}(s)=\frac{s^{n-1}}{(n-1)!R^{n}} e^{-s / R} \tag{3.23}
\end{equation*}
$$

For a fixed number of transmissions, the threshold $S_{t}$ was set to be $F_{n}\left(S_{t}\right)=P$. Using the integral relationships of transcendental functions, the probability of failure on the $n$th transmission for memory ARQ can be given as follows,

$$
\begin{equation*}
P_{n}=F_{n}\left(S_{t}\right)=e^{-S_{t} / n} \sum_{i=n}^{\infty} \frac{\left(S_{t} / R\right)^{i}}{i!} . \tag{3.24}
\end{equation*}
$$

In addition, for the ARQ strategy with $n=\infty$, the average number of transmissions per success ( $\bar{n}_{A R Q, \infty}$ ) can be represented by [13],

$$
\begin{align*}
\bar{n}_{A R Q, \infty} & =\mathbb{E}\{\text { number of transmissions per success }\} \\
& =1+\frac{S_{t}}{R} \tag{3.25}
\end{align*}
$$

This gives an upper bound on the expected number of transmissions per success. Note that the upper bound (3.22) is rather tight, since more than $n$ copies should rarely be needed.

For a simple memoryless ARQ protocol, $P_{i}$ can be given as

$$
\begin{equation*}
P_{i}=F_{1}^{i}\left(S_{t}\right)=\left[F_{n}\left(S_{t}\right)\right]^{i / n}, \tag{3.26}
\end{equation*}
$$

in which $F_{n}\left(S_{t}\right)$ presents the failure probability bound for a maximum of $n-1$ retransmissions.

### 3.5 Numerical Validation

The appropriateness of a model is ultimately determined by its ability to accurately predict the actual behavior. In this section, we present numerical results from computer simulations to evaluate the performance of our analytical framework. Various channel conditions and ARQ error control mechanisms will be investigated.

We simulate the system depicted in Fig. 3.1 using MPEG-4 video traces from [33]. The input traffic trace is composed of 40 multiplexed video traces. Thus, it has Gaussian
characteristics and a $1 m s$ frame interval. The sample interval $T_{s}$ is set to $100 \mu \mathrm{~s}$ and each simulation runs 360 seconds in all scenarios. Note that the round-trip time $T_{r}$ is fixed at $3 m s$ in the following simulation scenarios.

### 3.5.1 Constant Channel Condition without Packet Combing

In this simulation scenario, we assume that the channel conditions are constant and a memoryless ARQ scheme is deployed. Then, the packet error probabilities of each transmission are the same and independent with each other (i.e., $P_{i}=p, 1 \leq i \leq \hat{n}+1$ ). We compare the numerical results from proposition 3.3.2 and proposition 3.3.3 with the simulations for the cases of $p=0.1, p=0.01$, and $p=0.001$, as shown in Fig. 3.3, Fig. 3.4, and Fig. 3.5 respectively. The curves of queueing loss probabilities and transmission loss probabilities are plotted against the delay bounds. It is easy to see that the metrics predicted by the proposed model are very close to the real probabilities.

### 3.5.2 Time-Varying Rayleigh Fading Channel

We now consider the time-varying Rayleigh fading channels with memory ARQ and memoryless ARQ, respectively. Fig. 3.6 and Fig. 3.7 show the curves of loss probabilities against the delay bounds. We can see that our analysis can still capture the loss behavior in the time-varying fading channel with different ARQ strategies.

From the above simulation results, we observe that the probabilities decrease subexponentially with regard to the delay bounds. This is most likely due to the long-range dependence (the autocorrelation function is not summable) or self-similarity of the variable bit rate (VBR) traffic. Intuitively, long-range dependent traffic is bursty (highly variable) over a wide range of time scales, and the cumulative effect of the correlations for large lags has serious implication for the packet loss. As a result, increasing the delay bound does not help reduce the loss probabilities significantly.


Figure 3.3: The loss probabilities in reference to the delay bound, where $\mu=0.8310 \mathrm{C}$ and $P_{1}=\ldots=P_{\hat{n}+1}=0.1$.


Figure 3.4: The loss probabilities in reference to the delay bound, where $\mu=0.8814 C$ and $P_{1}=\ldots=P_{\hat{n}+1}=0.01$.


Figure 3.5: The loss probabilities in reference to the delay bound, where $\mu=0.8814 C$ and $P_{1}=\ldots=P_{\hat{n}+1}=0.001$.


Figure 3.6: The loss probabilities in reference to the delay bound, where the memory ARQ is delpoyed and $\mu=0.8555 C$.

### 3.6 Conclusion

In this chapter, we present an analytical model to evaluate the packet loss probabilities in wireless networks deploying ARQ error control mechanisms. The basic strategy is to quantify the queueing delay distributions in the network and physical layers individually, via introducing the concepts of $A R Q$ capacity and effective capacity. The equations and results obtained in this chapter can be used as a guideline in developing wireless mobile network systems, as it provides a unique connection of evaluating packet error control and QoS control together. The proposed framework can be used in admission control for wireless networking devices that are controlling and optimizing the network QoS of the traffic streams using the instantaneous channel status information. In particular, we adopt MVA approximations as the building block for performance analysis, and thus the results of this chapter are applicable for on-line measurements directly. It should be emphasized that using the MVA approach does not preclude the choice of other performance evaluation techniques. In heavy-loaded traffic environments, the proposed model has been shown to be able to accurately capture the real packet loss behavior by extensive simulations under a wide range of conditions.


Figure 3.7: The loss probabilities in reference to the delay bound, where memoryless ARQ is deployed and $\mu=0.8555 C$.

## CHAPTER 4

## ANALYSIS OF STATIC PRIORITY QUEUEING SYSTEMS

### 4.1 Introduction

Static priority is commonly used for traffic scheduling because it is less costly than other scheduling policies. A work-conserving SP scheduler works as follows. The packets queued in the buffer are ordered with respect to the priority assignments. within a priority class, the packets are placed in order of arrival. At any time, the scheduler always selects the packet with the highest priority to serve. In other words, a lower priority packet can only be processed if and only if the queues with higher priority assignments are empty.

DiffServ networking technologies are being standardized as the transport mechanism to integrate and support diverse classes of network traffic, such as data, voice, and video in a single integrated network. DiffServ based scheduling and multiplexing of the traffic generated from diverse sources with different characteristics on a high speed link may introduce considerable flexibility and potential savings in the allocation of network resources. The success of the forthcoming network will depend upon being able to provide guaranteed QoS levels to diverse classes of network traffic. In order to meet the different QoS requirements of individual service classes in an integrated network, several service disciplines, such as the head-of-line (HOL) priority and the generalized processor sharing (GPS) [11], also called weighted fair queueing (WFG), have been proposed. Although GPS is considered as an ideal scheduling algorithm in terms of its combined delay and fairness properties, its asymptotic computation complexity increases linearly with the number of sessions serviced by the scheduler, thus making its implementation prohibitively expensive in high-speed networks. In this chapter, we consider a HOL priority queueing system,
which is served according to a strict priority discipline.
This work was motivated by the maximum variance asymptotic (MVA) approach developed in [18], [19], and [20], which is able to accurately estimate the buffer overflow probability in a general multiplexer. This approach, unlike the large deviation principle which is for general traffic and thus would suffer from a loss of accuracy, is specified for Gaussian random processes, hence leading to tighter boundary conditions. A similar model called the most probable path (MPP) has been developed independently in [31]. This work is an extension of the above mentioned work. The objective is to develop a measurementanalytical model to evaluate the buffer overflow probability for each priority queue in HOL priority queueing systems, given that the network is accessed by stationary Gaussian traffic. Simulations and numerical studies have validated the accuracy of this model, and the comparison with other existing models in the literature has also been investigated in this chapter.

### 4.2 Related Work

Priority queueing systems have been studied extensively in recent years. In [37], Berger and Whitt have proposed the empty buffer approximation (EBA) for a two-class priority queue. The idea behind the EBA is that the total queue actually consists almost exclusively of lower class traffic, and thus the total queue length distribution is a good approximation to that of the lower class queue. Its applicability for the Gaussian traffic model has been checked by the examples in [31], and has shown to be a very good principle in most practically interesting priority scenarios. However, this approximation may become worse if the traffic load of the higher class is much larger than that of the lower class. Also, it is not scalable for the case of multiple service classes.

Another model relevant to this chapter is [38]. The authors in [38] have shown that $\operatorname{Pr}\left\{A_{1}(t)+A_{2}(t)>C t+x, A_{1}(t) \leq C t\right\}$ can asymptotically approximate $\operatorname{Pr}\left\{Q_{2}(t)>x\right\}$. Nevertheless, one can easily see that $\left\{A_{1}(t)+A_{2}(t)>C t+x\right\} \cap\left\{A_{1}(t) \leq C t\right\}$ is only
a subset of the event $\left\{Q_{2}(t)>x\right\}$. Therefore, using this approach to evaluate the buffer overflow probabilities of lower class queues may result in overly optimistic results.

Furthermore, one common issue in these existing approaches is that the analysis is too complicated, and somewhat depends too much on a specific type of traffic, thus not applicable for on-line QoS measurements. This chapter is in fact an extension of these previous works, and a performance comparison will be presented in section 4.4.

### 4.3 Queueing Analysis

In this section, we first introduce an asymptotic result for a general multiplexer, and we then show how to extend this result to a two-class priority queue as well as a multi-class case. It is assumed in this chapter that the arrival processes $A_{i}(t)(i=1, \ldots, N)$ are independent continuous Gaussian processes with stationary increments. We denote $\mu_{i}$ as the mean arrival rate of the class $i$ traffic. To guarantee the existence of the DTS, the aggregated input traffic should satisfy the stability condition, i.e., $\sum_{i=1}^{N} \mu_{i}<C$.

### 4.3.1 A General Multiplexer

As mentioned above, the lower and upper bounds for the buffer overflow probability can be obtained using MVA approximations. From our experience, we find that the geometric average of the lower bound and the upper bound given in [18] can provide a fairly good approximation to the exact buffer overflow probability. Thus, it is reasonable to come to the following result.

Proposition 4.3.1 In a general multiplexing system, the buffer overflow probability can be asymptotically approximated by the following formula

$$
\begin{equation*}
\operatorname{Pr}\{Q>x\} \approx L e^{-\frac{a^{2}}{2}} . \tag{4.1}
\end{equation*}
$$

where $L=\sqrt[-1 / 4]{2 \pi \alpha^{2}}$ is the asymptotic constant, $\hat{t}$ is the dominant time scale, and $\alpha$ is
given by (2.14).

Proof. This result can be easily obtained by calculating the geometric average of the lower and upper bounds in (2.13).

As seen from example 1 in section 4.4, this approximation can provide a very accurate estimation to the buffer overflow probability over a large buffer scale.

### 4.3.2 A Two-Class Priority Queue

In this section, we consider a two-class HOL priority queueing discipline, in which the higher priority class has strict priority over the lower. Let class 1 have higher priority over class 2. Accordingly, the backlog in buffer 2 can receive service only when buffer 1 is empty. Thus, the buffer overflow probability of the high priority queue (HPQ) can be accurately estimated using (4.1).

For the lower priority queue (LPQ), the queue length at time zero can be written as follows

$$
\begin{equation*}
\operatorname{Pr}\left\{Q_{2}>x\right\}=\operatorname{Pr}\left\{Q_{2}>x, Q_{1}=0\right\}+\operatorname{Pr}\left\{Q_{2}>x, Q_{1}>0\right\} . \tag{4.2}
\end{equation*}
$$

Let $-\hat{t} \leq 0$ denote the last time before zero when the system is empty, (i.e., both queues are empty). $\hat{t}$ is indeed the DTS of this system when considering that the two classes of traffic are combined together. We now show how to evaluate the two terms in the right hand side of the above equation.

Lemma 4.1 For a two-class priority queue, the probability that the queue length of buffer 2 exceeds $x$ when buffer 1 is empty can be expressed as

$$
\begin{equation*}
\operatorname{Pr}\left\{Q_{2}>x, Q_{1}=0\right\}=\int_{0}^{C \hat{t}} \Phi\left[\beta_{2}(u, \hat{t})\right] f_{A_{1}}(u) d u \tag{4.3}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta_{2}(u, \hat{t}):=\frac{\left(C-\mu_{2}\right) \hat{t}+x-u}{\sigma_{2}(\hat{t})}, \tag{4.4}
\end{equation*}
$$

and

$$
\begin{equation*}
f_{A_{1}}(x)=\frac{1}{\sqrt{2 \pi} \sigma_{1}(\hat{t})} \exp \left[-\frac{\left(x-\mu_{1} \hat{t}\right)^{2}}{2 \sigma_{1}^{2}(\hat{t})}\right] \tag{4.5}
\end{equation*}
$$

is the probability density function (PDF) of the random variable $A_{1}(\hat{t})$.

Proof. First notice that,

$$
\begin{align*}
& \operatorname{Pr}\left\{Q_{2}>x, Q_{1}=0\right\} \\
& =\operatorname{Pr}\left\{Q_{2}>x \mid Q_{1}=0\right\} \times \operatorname{Pr}\left\{Q_{1}=0\right\}  \tag{4.6}\\
& =\operatorname{Pr}\left\{A_{1}(\hat{t})+A_{2}(\hat{t})>C \hat{t}+x \mid A_{1}(\hat{t}) \leq C \hat{t}\right\} \times \operatorname{Pr}\left\{A_{1}(\hat{t}) \leq C \hat{t}\right\}
\end{align*}
$$

By the assumption that $A_{1}(\hat{t})$ and $A_{2}(\hat{t})$ are independent of each other, thus

$$
\begin{equation*}
\operatorname{Pr}\left\{Q_{2}>x, Q_{1}=0\right\}=\int_{0}^{C \hat{t}} \operatorname{Pr}\left\{A_{2}(\hat{t})>C \hat{t}+x-u\right\} f_{A_{1}}(u) d u \tag{4.7}
\end{equation*}
$$

Since $\operatorname{Pr}\left\{A_{2}(\hat{t})>C \hat{t}+x-u\right\}$ can be written in terms of a standard Gaussian tail distribution, i.e., $\Phi\left[\beta_{2}(u, t)\right]^{1}$, we obtain (4.3).

Lemma 4.2 For a two-class priority queue, the probability that the queue length of buffer 2 exceeds $x$ when buffer 1 is backlogged can be estimated as follows

$$
\begin{equation*}
\operatorname{Pr}\left\{Q_{2}>x, Q_{1}>0\right\} \approx \Phi\left[\alpha_{2}(\hat{t})\right] \Phi\left[\alpha_{1}(\hat{t})\right], \tag{4.8}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{1}(\hat{t})=\frac{\left(C-\mu_{1}\right) \hat{t}}{\sigma_{1}(\hat{t})}, \quad \text { and } \quad \alpha_{2}(\hat{t})=\frac{x-\mu_{2} \hat{t}}{\sigma_{2}(\hat{t})} . \tag{4.9}
\end{equation*}
$$

Proof. Using conditional probability, we can have the following relationship,

$$
\begin{align*}
& \operatorname{Pr}\left\{Q_{2}>x, Q_{1}>0\right\} \\
& =\operatorname{Pr}\left\{A_{2}(\hat{t})>x+\varepsilon \mid A_{1}(\hat{t})>C \hat{t}\right\} \operatorname{Pr}\left\{A_{1}(\hat{t})>C \hat{t}\right\}  \tag{4.10}\\
& =\operatorname{Pr}\left\{A_{2}(\hat{t})>x+\varepsilon\right\} \operatorname{Pr}\left\{A_{1}(\hat{t})>C \hat{t}\right\},
\end{align*}
$$

where $\varepsilon \geq 0$ is the possible service that the class 2 traffic might receive during time interval $[-\hat{t}, 0)$. In practical observations $\varepsilon$ is usually negligible when compared with $x$. Therefore,

$$
{ }^{\prime} \Phi(z)=1 / \sqrt{2 \pi} \int_{z}^{\infty} e^{-x^{2} / 2} d x
$$

we can have $\operatorname{Pr}\left\{A_{2}(t)>x+\varepsilon\right\} \approx \operatorname{Pr}\left\{A_{2}(t)>x\right\}$. By the assumption that $A_{1}(t)$ and $A_{2}(t)$ are independent, (4.8) can be obtained using the standard Gaussian tail distribution.

Now combining the results in lemma 4.1 and lemma 4.2, we can easily reach the following asymptotic approximation to the overflow probability for buffer 2 .

Proposition 4.3.2 In a two-class priority queue, the queue length distribution (buffer overflow probability) of the lower priority queue can be asymptotically approximated by

$$
\begin{equation*}
\operatorname{Pr}\left\{Q_{2}>x\right\} \approx \int_{0}^{C \hat{t}} \Phi\left[\beta_{2}(u, \hat{t})\right] f_{A_{1}}(u) d u+\Phi\left[\alpha_{2}(\hat{t})\right] \Phi\left[\alpha_{1}(\hat{t})\right] . \tag{4.11}
\end{equation*}
$$

By identifying the mean and covariance of the input traffic through an on-line measurement, we will be able to employ this result to estimate the buffer overflow probability, and evaluate the performance of the multiplexer deploying strict priority. Its accuracy and comparison with existing results are validated by simulations in section 4.4.

### 4.3.3 A Multi-Class Queueing System

We now consider a multi-class queueing system with strict priorities, and show that the above results can be easily extended to obtain the asymptotic buffer overflow probabilities of lower class queues.

To estimate the buffer overflow probability of the $n$-th queue in this system, we let $M_{n}=\sum_{i=1}^{n-1} \mu_{i}$ and $v_{n}(t)=\sum_{i=1}^{n-1} \sigma_{i}^{2}(t)$ denote the mean and variance of the aggregated traffic, which has a higher priority assignment than class $n$. Note $\hat{t}$ is the DTS of this system when considering $n$ classes combined together. Therefore, we can come to the following result.

Proposition 4.3.3 For a multi-class queueing system deploying strict priority, the buffer overflow probability of the $n$-th queue can be estimated by

$$
\begin{equation*}
\operatorname{Pr}\left\{Q_{n}>x\right\} \approx \int_{0}^{C \hat{t}} \Phi\left[\beta_{n}(u, \hat{t})\right] f_{A_{n-1}}(u) d u+\Phi\left[\alpha_{n}(\hat{t})\right] \Phi\left[\gamma_{n}(\hat{t})\right] \tag{4.12}
\end{equation*}
$$

where

$$
\begin{gather*}
\beta_{n}(u, t):=\frac{\left(C-\mu_{n}\right) t+x-u}{\sigma_{n}(t)},  \tag{4.13}\\
f_{A_{n-1}}(x)=\frac{1}{\sqrt{2 \pi v_{n}(t)}} \exp \left[-\frac{\left(x-M_{n} \hat{t}\right)^{2}}{2 \cdot v_{n}(\hat{t})}\right],  \tag{4.14}\\
\alpha_{n}(\hat{t})=\frac{x-\mu_{n} \hat{t}}{\sigma_{n}(\hat{t})}, \tag{4.15}
\end{gather*}
$$

and

$$
\begin{equation*}
\gamma_{n}(\hat{t})=\frac{\left(C-M_{n}\right) \hat{t}}{\sqrt{v_{n}(\hat{t})}} \tag{4.16}
\end{equation*}
$$

Proof. The result above can be easily proved if we consider all the classes with higher priority assignment than class $n$ as an aggregated one.

Since fractional Brownian motion (FBM) has been considered one of the well accepted models for self-similar traffic [34], we now show how to apply the results above to such a case. A general FBM traffic can be given by the following stochastic process

$$
\begin{equation*}
A_{i}(t)=\mu_{i} t+Z_{i}^{H_{i}}(t),(i=1, \ldots, N) \tag{4.17}
\end{equation*}
$$

where $\mu_{i}$ and $H_{i} \in\left[\frac{1}{2}, 1\right)$ are the mean arrival rate and the Hurst parameter of the input traffic $A_{i}(t)$, respectively. $Z_{i}^{H_{i}}(t)$ is a centered continuous Gaussian process with variance $\sigma_{i}^{2}(t)=V_{i} t^{2 H_{i}}$. The procedure of obtaining the buffer overflow probability of the class $n(1 \leq n \leq N)$ queue is as follows.

Suppose $\sigma_{i}^{2}(t)(i=1, \ldots, N)$ are differentiable at $\hat{t}$. We then can obtain the value of $\hat{t}$ by solving the following equation

$$
\begin{equation*}
\frac{\sum_{i=1}^{n} \sigma_{i}(t)}{\sum_{i=1}^{n} \sigma_{i}^{\prime}(t)}-\hat{t}=\frac{x}{C-\sum_{i=1}^{n} \mu_{i}} \tag{4.18}
\end{equation*}
$$

Substitute $v_{n}(t)$ with $\sum_{i=1}^{n-1} V_{i} i^{2 H_{i}}$. Then $\operatorname{Pr}\left\{Q_{n}>x\right\}$ can be obtained by using equations (4.12) through (4.16).

Nevertheless, one has to be very careful when using the above results for performance evaluation, since they may not hold if the input traffic has an unbounded instantaneous rate, or the input traffic does not have Gaussian characteristics.

### 4.4 Numerical Examples And Simulations

In this section, we evaluate the accuracy of the analytical method presented above, and report the comparative studies of the existing results. In our experiments, we use the classical method of batch mean [39] to calculate $95 \%$ confidence intervals for each probability estimated through simulations.

Example 1: This example investigates the accuracy of formula (4.1) as well as the MVA analysis. We use real MPEG-4 video traffic from [33], which has been shown to exhibit self-similarity, to generate a single trace with 40 multiplexed sources. Therefore, this traffic trace has a Gaussian characterization and 1 ms frame interval. Fig. 4.1 shows the buffer overflow probability in a general multiplexer. As one can see, the boundary conditions obtained using MVA analysis encapsulate the real value within an order of magnitude, and the approximation given by (4.1) can provide a fairly good estimation.

Example 2: In this example, we examine the buffer overflow probability obtained in a two-class queue deploying HOL discipline. Consider a class 1 traffic trace composed of 40 multiplexed sources and a class 2 trace composed of 8 Ethernet sources obtained from [16]. From observations of our experiments, multiplexing a small number of independent sources can still obtain a fairly good Gaussian traffic trace. As shown in Fig. 4.2, the approximations using the formulae in (4.1) and (4.11) are very close to the exact probabilities. Based on the same simulation scenario, Fig. 4.3 gives the comparative results of the existing models. The results from [37] and [38] are labelled as EBA and Delas respectively.

Example 3: In this experiment, we investigate a three-class queueing system deploying HOL priority. In this system, class 1 traffic consists of 20 multiplexed MPEG-4 movies and 4 multiplexed Ethernet sources. Class 2 traffic is also composed of 20 multiplexed MPEG-


Figure 4.1: The buffer overflow probability in a general multiplexer.


Figure 4.2: The buffer overflow probabilities in a two-class priority queueing system, with $\mu_{1}=0.6 C$ and $\mu_{2}=0.2 C$.

4 traffic and 4 multiplexed Ethernet sources, where class 3 is composed of 10 multiplexed MPEG-4 video traffic and 4 multiplexed Ethernet sources. Fig. 4.4 gives the buffer overflow probabilities of the second and third queues, including simulation and analysis. Class 1 is not plotted in this figure since its metric is extremely small.

### 4.5 Conclusion

In this chapter, a HOL priority queueing system has been studied and a measurementanalytical approach has been proposed to estimate the buffer overflow probability for each priority queue. The simulation studies using real traffic have shown the effectiveness and accuracy of the proposed model with comparisons of existing results. Given that the selfsimilar traffic is commonly experienced in high-speed networks, this analytical model will provide a practical insight into the call admission control (CAC) design which is supposed to accommodate heterogeneous QoS requirements. More experiments to validate this approach will be presented in future work.


Figure 4.3: The buffer overflow probability of the lower priority queue (LPQ).


Figure 4.4: The buffer overflow probabilities of the first, second, and third class queues in a three-class priority queueing system, with $\mu_{1}=0.34 C, \mu_{2}=0.32 C$ and $\mu_{3}=0.19 C$.

## CHAPTER 5

# ADMISSION CONTROL FOR STATISTICAL SERVICES WITH EARLIEST DEADLINE FIRST SCHEDULING 

### 5.1 Introduction

Future high speed packet-switching networks are expected to provide heterogenous QoS guarantees for a variety of applications. Call admission control plays a critical role in achieving this purpose, which is an integration of the traffic models, scheduling disciplines, and QoS specifications. Its major task is to decide whether a new connection can be granted while the QoS requirements of this new connection and all the existing connections can still be guaranteed. This becomes much more complicated and challenging for the networks supporting various applications with heterogenous QoS requirements.

The scheduler is an indispensable part in the design of call admission control mechanisms. Among the existing scheduling disciplines in the literature, static priority (SP), generalized processor sharing (GPS) [11], also known as weighted fair queueing (WFQ), and EDF [50] are probably the most promising to provide heterogeneous QoS guarantees. Although GPS has been considered an ideal scheduling discipline in terms of its combined delay and fairness properties, it has been shown to be sub-optimal by the studies in [13]. On the other hand, EDF has been shown in [22], [23], [56], [54], and [60] to be able to offer a substantial performance gain over GPS in terms of the schedulable ${ }^{1}$ region under both deterministic and statistic contexts. In particular, the authors of [60] have proved that GPS inherently requires dynamic weight re-synchronization in order to realize the maximum schedulable region. In this chapter, we will focus on the CAC issues for a network

[^2]node deploying the EDF scheduling discipline.
The scheduler behaviors under the context of statistical multiplexing have been received a lot of attentions in various aspects. In [51], Elwalid et al. analyzed the first-in-first-out (FIFO) scheduling using the effective bandwidth with Chernoff bounds, where the traffic patterns studied were independent and periodic ON-OFF sources. The results were then extended for GPS scheduling by the authors of [52] and [53], and for the EDF discipline in [54]. On the other hand, some researchers have dedicated to extend the deterministic models to the statistical setting. The authors of [55] introduced the concept of effective envelopes and devised the admission control tests for a set of scheduling algorithms (i.e., FIFO, SP, and EDF). Qiu and Knightly [57] proposed using adaptive and measurementbased maximal rate envelopes to characterize the traffic processes for multi-class networks with link sharing. Other results on statistical multiplexing include [37] [3] for SP, [58] [31] for GPS, and [59] for EDF.

Rigorous investigation of statistical performance metrics for EDF scheduling dates back to [60] and [59]. By assuming that all delay bounds are close together (i.e., about the same order of magnitude) and considering traffic models with Markovian sources, the authors of [59] derived the aggregate deadline violation probabilities. However, the assumptions in [60] and [59] limit the extension to various delay requirements of different flows. Also, the long-range dependent or self-similar characteristic [16] of the real network traffic has made the previous analysis of EDF scheduling face a fundamentally different set of problems. To develop an efficient statistical CAC scheme for the EDF scheduler, the analysis of per-flow loss probabilities with respect to the delay bounds becomes necessary. It is well known that the analysis of a stochastic system such as an EDF (or GPS) scheduler, where several flows with different QoS requirements share a server and a queue, is generally very difficult. The reason behind this is that the amount of service received by a flow at any moment depends not only on its arrival process and content in the queue at that moment, but also on the arrival processes of other flows and their contents in the queue [61]. To circumvent
this difficulty, the authors of [60] developed a mechanism to obtain per-flow metrics by exploiting a fair packet discard scheme, which makes the implementation more costly.

Recently, the studies on the statistical analysis of EDF scheduling [5] have present an efficient and accurate framework, within which the asymptotic solutions for the aggregate and per-class metrics have been derived via defining the virtual queueing system and extending the notation of the dominant time scale (DTS) [18] [49] [19] [20]. These theoretical results will enable us to derive the explicit admission condition and to develop the CAC algorithm for the network supporting diverse statistical QoS guarantees under EDF scheduling.

This work differs from the previous results on EDF scheduling in the following aspects. First, in contrary to [54] and [55], the admission conditions are derived from the per-class analysis of EDF scheduling [5]. Second, comparing our work with [59] and [60], one can see that: i) this chapter relaxes the stringent assumptions on delay bound assignments and thus can handle more diverse delay requirements; ii) it will be able to work under a more heterogeneous traffic environment because of the Gaussian traffic models adopted; iii) the analysis does not use any packet discard policy in order to obtain the per-class deadline violation probabilities. Therefore, this chapter extends and/or complements the existing results on the EDF scheduling.

The rest of this chapter is constructed as follows. In section 5.2, we introduce the necessary background for the proposed algorithm and the issues we wish to address in this chapter. Section 5.3 theoretically analyzes the deadline violation probabilities of the EDF scheduler. In section 5.4, we derive the admission control conditions and present our statistical algorithm. In section 5.5, we investigate some interesting properties of the EDF scheduler and their implications for the network design and control. Section 5.6 numerically evaluates the proposed algorithm via trace-driven simulations. Section 5.7 concludes this chapter.

### 5.2 Background and Problem Formulation

In this section, we briefly review the EDF scheduling discipline and state the main issues to be addressed in this chapter.

Before proceeding, we need to introduce the notation of deadline violation probability. In this chapter, the delay bound and deadline violation probability are the two QoS metrics considered for the admission criterion. By violation probability, we mean the fraction of time that a flow experiences deadline violation over the entire time scale, which corresponds to the buffer overflow probability in [18]. It is calculated by the following formula

$$
\begin{equation*}
P_{L}=\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^{N} I(\text { Deadline is violated at time slot } i .), \tag{5.1}
\end{equation*}
$$

where $I(A)$ is an indicator function, i.e.,

$$
I(A)= \begin{cases}1 & \text { if } A \text { is true }  \tag{5.2}\\ 0 & \text { otherwise }\end{cases}
$$

### 5.2.1 EDF Scheduling

We now briefly introduce the EDF scheduling discipline. EDF has long been known as a processor scheduling algorithm, and has been applied to high speed packet-switching networks recently. Former results in [22] and [23] have shown that EDF has the largest schedulable region among all existing scheduling disciplines.

A work-conserving EDF scheduler (see Fig. 5.1) operates as follows: each traffic class $i$ at the scheduler is associated with a local delay bound $d_{i}$; then, a class $i$ packet arriving at the scheduler at time $t$ is stamped with a deadline $t+d_{i}$, and packets in the scheduler are served according to the increasing order of their deadlines. Given $N$ traffic classes, each of which has an arrival process $A_{i}(t)$ and a worst-case delay requirement $d_{i}(i=1,2, \ldots, N)$, the schedulability check is given by

$$
\begin{equation*}
\sum_{i=1}^{N} A_{i}\left(t-d_{i}\right) \leq C t, \forall t>0 \tag{5.3}
\end{equation*}
$$

where the traffic is assumed to be fluid, $C$ denotes the constant link capacity, and $A_{i}(t)=0$ for $t<0$. It is assumed that $d_{1} \leq d_{2} \leq \cdots \leq d_{N}$. Therefore, the aggregate deadline violation probability can be given as

$$
\begin{equation*}
P_{L}=\operatorname{Pr}\left\{\sup _{t \geq 0}\left[\sum_{i=1}^{N} A_{i}\left(t-d_{i}\right)-C t\right]>0\right\} . \tag{5.4}
\end{equation*}
$$

From [24], we know that under some mild assumptions (for example, $A_{i}(t)$ is stationary and ergodic, and the mean of all the inputs is not greater than the link rate $C$ ), the fraction of traffic that does not meet its deadline at the scheduler has a steady-state value; equivalently, the loss probability exists and has a stationary value. Here, we assume that the queue length is large enough so that there is no packet loss due to insufficient buffer capacity.

### 5.2.2 Problem Formulation for Admission Control

We aggregate the application connections with similar QoS requirements into a class. Let $A_{i, j}(t)$ denote the arrival process of the $j$-th connection in class $i$, and $n_{i}$ be the number of connections in class $i$. Then the class $i$ arrival process is given by

$$
\begin{equation*}
A_{i}(t)=\sum_{j=1}^{n_{i}} A_{i, j}(t) . \tag{5.5}
\end{equation*}
$$

The QoS requirements of the class $i$ sources can be characterized by the parameter duplet ( $d_{i}, P_{i}$ ), where $d_{i}$ and $P_{i}$ are the maximum delay and violation probability that the class $i$ traffic can tolerate.

In this chapter, we wish to address the following two problems for the admission control under EDF scheduling, where a set of different traffic classes are scheduled according to their QoS requirements.

1. Given a new connection $A_{i, n_{i}+1}(t)$ with QoS requirements ( $d_{i}, P_{i}$ ), should the network admit this connection, such that all the admitted connections will still be schedulable with respect to their QoS requirements?
2. What kind of information can be derived from the network deploying EDF scheduling disciplines, so that the network protocols and applications can adaptively adjust their behaviors (e.g., QoS requirements and flow rates) for resource efficiency?

The connections admitted can be described by the admissible region, in terms of which the optimality of the CAC schemes is defined. The notation of the admissible region is given below.

Definition 5.1 The admission region is a vector $\mathcal{R} \in \mathbb{R}^{N}$, defined as

$$
\begin{align*}
& \mathcal{R}=\left\{\left(u_{1}, u_{2}, \ldots, u_{N}\right)\right. \mid \text { The QoS requirements of }  \tag{5.6}\\
&\text { all classes are guaranteed }\}
\end{align*}
$$

where $u_{i}\left(0 \leq u_{i} \leq 1\right.$ and $\left.1 \leq i \leq N\right)$ is the utilization ${ }^{2}$ of the class $i$ connections that are admissible.

In an EDF scheduler, to ensure the stability condition, the aggregate load for admissible region $\mathcal{R}$ should satisfy

$$
\begin{equation*}
\lim _{t \rightarrow \infty} \frac{\sum_{i=1}^{N} A_{i}(t)}{t}<C, \quad \text { or } \sum_{i=1}^{N} u_{i}<1 . \tag{5.7}
\end{equation*}
$$

It is clear that the stability is a necessary condition for all admission criteria.

### 5.3 Evaluating Deadline Violation Probabilities

In this section, we asymptotically evaluate the deadline violation probabilities and analyze the resource allocation behavior of the EDF scheduler. Throughout the following presentation, it is assumed that the individual connections are independent of each other. Now we give the approaches to evaluate the aggregate and per-class violation probabilities of an EDF scheduler, which will constitute the theoretical foundation for the development of the admission criterion under EDF scheduling.

[^3]The rest of this section is exploited as follows. Definitions 5.2 and 5.3 define the virtual queueing system and extend the notation of the dominant time scale, which will be used as the tools for analyzing the deadline violation probabilities. Proposition 5.3.1 serves as a starting point by considering a FIFO scheduling link that deals with all the arrival traffic indiscriminatingly. Proposition 5.3 .2 gives the result of the aggregate violation probability of an EDF scheduler, which accommodates a set of different traffic classes. In proposition 5.3.3, we present the approach to evaluate the per-class violation probabilities of an EDF scheduling link with different traffic classes. This result will be used as the foundation for the admission control conditions derived in the next section.

To evaluate the aggregate and individual deadline violation probabilities, we first introduce the virtual queueing system, which has been shown to be able to asymptotically capture the characteristics of the real EDF scheduler. Its notation is given in the following definition.

Definition 5.2 We define a virtual queueing system $A_{V}: \mathbb{R} \rightarrow \mathbb{R}$, which accommodates all the aggregated class traffic that is supposed to receive service before t. Thus, its cumulative arrival process $A_{V}(t)$ is given by

$$
\begin{equation*}
A_{V}(t)=\sum_{i=1}^{N} A_{i}\left(t-d_{i}\right) \tag{5.8}
\end{equation*}
$$

We rewrite $A_{i}(t)$ as follows

$$
\begin{equation*}
A_{i}(t)=\mu_{i} t+Z_{i}(t) \tag{5.9}
\end{equation*}
$$

where $\mu_{i}$ is the mean arrival rate of $A_{i}(t)$, and $Z_{i}(t)$ is a centered continuous Gaussian process. Substituting (5.9) into (5.8) yields

$$
\begin{align*}
A_{V}(t) & =\sum_{i=1}^{N}\left[\mu_{i}\left(t-d_{i}\right)+Z_{i}\left(t-d_{i}\right)\right] \\
& =\sum_{i=1}^{N} \mu_{i} t+\sum_{i=1}^{N} Z_{i}\left(t-d_{i}\right)-\sum_{i=1}^{N} \mu_{i} d_{i}  \tag{5.10}\\
& =\mu_{V} t+Z_{V}(t)-l_{V},
\end{align*}
$$

where $\mu_{V}=\sum_{i=1}^{N} \mu_{i}$ is the mean of the aggregated arrival rate and $l_{V}=\sum_{i=1}^{N} \mu_{i} d_{i}$ is referred to as the equivalent queue length. It is easy to show that $A_{V}(t)$ is also a continuous Gaussian process with stationary increments [5]. Its variance can be computed as $\sigma_{V}^{2}(t)=$ $\sum_{i=1}^{N} \sigma_{i}^{2}\left(t-d_{i}\right)$.

To exploit the MVA approach for the analysis of this virtual system, we need to extend the notation of the dominant time scale [20] as follows.

Definition 5.3 By [20], we know that the function $\operatorname{Pr}\left\{\sum_{i=1}^{N} A_{i}\left(t-d_{i}\right)-C t>0\right\}$ obtains its maximum at $t=\hat{t}, \forall t \in(0, \infty)$. We refer to $\hat{t}$ as the first order DTS of the EDF system, also denoted by $\hat{t}_{1}$. Then one can find an integer $n(1 \leq n \leq N)$, such that the dominant time scale $\hat{t}$ falls into interval $\left(d_{n}, d_{n+1}\right]^{3}$. If $n<N$, one can also find another integer $m(n<m \leq N)$, such that $\operatorname{Pr}\left\{\sum_{i=1}^{N} A_{i}\left(t-d_{i}\right)-C t>0\right\}$ obtains its maximum at $t=\hat{t}_{2}$, $\forall t \in\left(d_{n+1}, \infty\right)$, where $\hat{t}_{2} \in\left(d_{m}, d_{m+1}\right]$. We define $\hat{t}_{2}$ as the second order DTS, or sub-DTS (SDTS). Similarly, if $m<N$ we will have the third order DTS $\hat{t}_{3}$, etc.

For simplicity, we first consider the asymptotic violation probability of an EDF system accessed by homogeneous traffic, where all the connections have the same QoS requirements (i.e., a FIFO scheduler). The following result is a restatement of proposition 4.3.1.

Proposition 5.3.1 Consider an EDF system with homogenous traffic input. If the traffic has delay bound d, then its deadline violation probability can be asymptotically approximated by the following formula

$$
\begin{equation*}
P_{L} \approx L e^{-\frac{\beta^{2}}{2}} \tag{5.11}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta:=\inf _{t \geq 0}\left[\frac{(C-\mu) t+\mu d}{\sigma(t-d)}\right], \tag{5.12}
\end{equation*}
$$

and $L=\sqrt[-1 / 4]{2 \pi \beta^{2}}$ is the asymptotic constant.

[^4]Proof. The proof of this proposition is similar to that of proposition 4.3.1 and thus omitted.

Remark Extensive experimental results have shown that $L$ does provide a fairly good asymptotic approximation to the exact probability, but there is no exact solution for it [3]. One may also choose an alternate value for special cases. For example, when using the large buffer asymptotic the asymptotic constant can be give by $L=\operatorname{Pr}\{Q>0\}$ [61], which is the probability that the buffer is nonempty at a randomly chosen time instant. For the case of the many source asymptotic [40], the Bahadur-Rao improvement is adopted.

In real applications, Internet services can be categorized into two broad classes, rigid and tolerant, according to the stringency of their respective properties and QoS requirements. For the rigid services, the QoS requirements must be strictly fulfilled, regardless of the deterministic or statistical settings. On the other hand, the tolerant applications do not have such stringent QoS requirements as long as the performance runs within a tolerable range. The applications will be able to adapt to minor fluctuations with regard to their own QoS requirements. Consequently, $L$ becomes a useful tunable parameter, which can be configured during the setup stage of the network according to the characteristics of the services.

Denote by $\hat{t}$ the first-order DTS of an EDF scheduler with $N$ different classes of traffic flows. The following proposition gives the asymptotic violation probability of the aggregated traffic.

Proposition 5.3.2 The deadline violation probability $P_{L}$ of an EDF scheduler asymptotically equals the probability that a virtual queueing system with the first $k$ classes experiences buffer overflow in the steady state, where $k$ is given by

$$
\begin{equation*}
k=\max \left\{i: 1 \leq i \leq N, d_{i}<\hat{t}\right\} . \tag{5.13}
\end{equation*}
$$

Thus, we have the following approximation

$$
\begin{equation*}
P_{L} \approx L_{V} e^{-\frac{\beta_{V}^{2}}{2}}, \tag{5.14}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta_{V}:=\frac{\left(C-\mu_{V}\right) \hat{t}+l_{V}}{\sigma_{V}(\hat{t})} \tag{5.15}
\end{equation*}
$$

and $L_{V}=\sqrt[-1 / 1]{2 \pi \beta_{V}^{2}}$, which is subject to adjusting with regard to the service requirement. Note that the subscript $V$ in the above formulae denotes a virtual queueing system with the first $k$ traffic classes.

Proof. Suppose the system encounters a deadline violation at time $0 .[-\hat{t}, 0)$ is the most probable duration of a busy period prior to the deadline violation before time zero. Since $A_{i}(t)=0$ for $t<0,(5.4)$ can be written as

$$
\begin{equation*}
P_{L}=\operatorname{Pr}\left\{\sum_{i=1}^{N} A_{i}\left(\hat{t}-d_{i}\right)>C \hat{t}\right\}=\operatorname{Pr}\left\{\sum_{i=1}^{k} A_{i}\left(\hat{t}-d_{i}\right)>C \hat{t}\right\} \tag{5.16}
\end{equation*}
$$

To evaluate the aggregate violation probability, one just needs to consider the first $k$ traffic classes. By using the first $k$ traffic classes to construct an aggregate flow like (5.10) and deploying the result obtained in proposition 5.3.1, one can obtain proposition 5.3.2.

To efficiently support heterogeneous applications with different QoS requirements, the admission controller needs to predict the performance metrics of each class, so as to make its decision. Fortunately, the following result provides the exact approach to evaluate the violation probabilities of each class with respect to their delay bounds in a general EDF scheduler.

Given an EDF system with a set of admitted traffic classes $\left\{A_{i}(t): 1 \leq i \leq N\right\}$, the dominant time scales $T:=\left\{\hat{t}_{1}, \ldots, \hat{t}_{m}\right\}$ partition the delay bounds associated with each class into $m$ disjoint delay sets, i.e.,

$$
\begin{aligned}
& D_{1}:=\left\{d_{1}, \ldots, d_{k_{1}}\right\}, \\
& D_{2}:=\left\{d_{k_{1}+1}, \ldots, d_{k_{2}}\right\}, \\
& \ldots \\
& D_{m}:=\left\{d_{k_{m-1}+1}, \ldots, d_{k_{m}}\right\},
\end{aligned}
$$

such that

$$
\begin{aligned}
& d_{k_{1}}<\hat{t}_{1}, \\
& \hat{t}_{1} \leq d_{k_{1}+1}, \text { and } d_{k_{2}}<\hat{t}_{2}, \\
& \ldots \\
& \hat{t}_{m-1} \leq d_{k_{m-1}+1}, \text { and } d_{k_{m}}<\hat{t}_{m} .
\end{aligned}
$$

Note that $k_{m}=N$ in the above formulae. The proposition below gives the asymptotic metrics of the individual traffic classes.

## Proposition 5.3.3 Consider an EDF scheduler with $N$ independent traffic classes and an

 $m$-th order DTS. If $d_{i} \in D_{j}(1 \leq i \leq N)$, then the violation probability of class $i$ can be evaluated by$$
\begin{equation*}
P_{L_{i}} \approx P_{D_{j}} \tag{5.17}
\end{equation*}
$$

where

$$
\begin{equation*}
P_{D_{j}} \approx L_{V} \exp \left\{-\frac{\left[\left(C-\mu_{V}\right) \hat{t}_{j}+l_{V}\right]^{2}}{2 \sigma_{V}^{2}\left(\hat{t}_{j}\right)}\right\} \tag{5.18}
\end{equation*}
$$

Proof. The proof is similar to that of proposition 5.3.2 and thus omitted.
It should be noted that $L_{V}, \mu_{V}, l_{V}$, and $\sigma_{V}^{2}(t)$ are the corresponding parameters of a virtual queueing system with the first $k_{j}$ classes. From the results in [5], it can be shown that the traffic classes that fall into a delay set should have the same asymptotic violation probability. The results presented above have been numerically validated by extensive experiments in [5] and the simulation study in section 5.6.

### 5.4 Call Admission Control: Conditions and Algorithms

### 5.4.1 Admission Conditions

In this section, we analyze the resource allocation behavior of the EDF scheduler. From the analysis, we obtain the explicit admission conditions for the network deploying the EDF scheduling discipline. To proceed, we need the following two definitions.

Definition 5.4 In an EDF scheduler, the available service function of the $j$-th connection in class $i, W: \mathbb{R} \rightarrow \mathbb{R}$, over the interval $(0, t)$ is defined as

$$
\begin{equation*}
W_{i, j}(t) \stackrel{d e f}{=} C t-\sum_{k \neq i} A_{k}\left(t-d_{k}\right)-\sum_{k \neq j} A_{i, k}\left(t-d_{i}\right) . \tag{5.19}
\end{equation*}
$$

Suppose the connection $A_{i, j}(t)$ has QoS requirements ( $d_{i}, P_{i}$ ), its available service function should satisfy

$$
\begin{equation*}
\operatorname{Pr}\left\{\sup _{i>d_{i}}\left[A_{i, j}\left(t-d_{i}\right)-W_{i, j}(t)\right]>0\right\} \leq P_{i} \tag{5.20}
\end{equation*}
$$

This condition constitutes a connection-based admission test. The probability on the left can be evaluated using the Gaussian approximations if the arrival process of $A_{i, j}(t)$ is Gaussian. However, this may not be always the case, and thus the evaluation of the deadline violation probability will be very difficult.

Fortunately, it is reasonable to consider all the connections within a class as a whole because they have the same QoS requirements. According to the central limit theorem, when the number of independent connections is large enough ( $\sim 5$ or larger), the aggregate traffic exhibits Gaussian characteristics even though the individual connection is not Gaussian. Before presenting the class-based admission test, we introduce the following notation.

Definition 5.5 In an EDF scheduler, the available service for class i, denoted by $S_{i}: \mathbb{R} \rightarrow$ $\mathbb{R}$, over the time interval $(0, t)$ is defined as

$$
\begin{equation*}
S_{i}(t) \stackrel{\text { def }}{=} C t-\sum_{j \neq i} A_{j}\left(t-d_{j}\right) . \tag{5.21}
\end{equation*}
$$

Accordingly, the violation probability bound places a constraint on the available service of class $i$ traffic, i.e.,

$$
\begin{equation*}
\operatorname{Pr}\left\{\sup _{t>d_{i}}\left[A_{i}\left(t-d_{i}\right)-S_{i}(t)\right]>0\right\} \leq P_{i} \tag{5.22}
\end{equation*}
$$

where

$$
A_{i}(t)=\sum_{j=1}^{n_{i}} A_{i, j}(t)
$$

The following claim is straightforward to show the equivalence of the connection-based and class-based tests.

Claim 5.4.1 The admission test (5.20) is equivalent to (5.22).

Proof. Since we have

$$
\begin{aligned}
& A_{i, j}\left(t-d_{i}\right)-W_{i, j}(t) \\
& =A_{i, j}\left(t-d_{i}\right)-C t+\sum_{k \neq i} A_{k}\left(t-d_{k}\right)+\sum_{k \neq j} A_{i, k}\left(t-d_{i}\right) \\
& =A_{i}\left(t-d_{i}\right)-S_{i}(t)
\end{aligned}
$$

This establishes the equivalence of (5.20) and (5.22).

Remark In most cases, it is preferred to use (5.22) for admission tests because the class-based test has many advantages. First, it can be easily seen that $A_{i}(t)$ is Gaussian with a moderate $n_{i}$ (up to 5 or larger) even though $A_{i, j}(t), j \in\left\{1, \ldots, n_{i}\right\}$, does not have Gaussian characteristics. Therefore, it is more amenable for the analysis using Gaussian approximations. The second advantage of this approach is that the standard deviation of the aggregate process $A_{i}(t)$ parameter is in general significantly smaller than the sum of the standard deviations of the individual connection $A_{i, j}(t)$ parameters. This tells us an important fact that the measurement of the aggregate process is much more reliable than the sum of each connection process measurements, which can be explained by statistical multiplexing of the measurement errors. Third, by aggregating traffic with similar QoS requirements into a class, the network does not maintain any connection-based information across the network, thus eliminating the overhead of maintaining huge state information and the connection setup costs. The class-based admission test is compatible with the emerging DiffServ architecture for providing absolute QoS guarantees.

To evaluate the quantity on the left of (5.22), one can use the result from proposition 5.3.3.

### 5.4.2 Admission Control Algorithms under EDF Scheduling

In this section, we first study a deterministic CAC algorithm, and then present our statistical CAC algorithm using the theoretical results obtained in the previous section.

## The Deterministic CAC Algorithm

In [31], the authors present a criterion, which can be used for admission control in any work-conserving scheduling algorithms. By specifying their approach, we can obtain a criterion for the call admission control under EDF scheduling.

Let $s_{i}(t)$ denote the service rate of class $i$ traffic at time $t$, and $S_{i}(s, t)=\int_{s}^{t} s_{i}(\tau) d \tau$ be the service process of class $i$ during time interval $(s, t]$. Let $S_{i}(s, t)$ denote the available service process, and $D_{i}(s, t)$ be the departure processes respectively. Thus, a useful result follows.

Proposition 5.4.1 Consider an EDF scheduler with $N$ different traffic classes. If $A_{i}(s, t)(1 \leq$ $i \leq N)$ is differentiable, $d A_{i}\left(s, t-d_{i}\right) / d t \leq s_{i}(t)$ in time interval $\left(t_{1}, t_{2}+d_{i}\right.$ ], and no deadline violation occurs at time $t_{1}$, then class $i$ is schedulable for all $t \in\left(t_{1}, t_{2}+d_{i}\right)$.

Proof. By the assumption

$$
\sup _{s \leq t_{1}-d_{i}}\left[A_{i}\left(s, t_{1}-d_{i}\right)-S_{i}\left(s, t_{1}\right)\right] \leq 0,
$$

$\forall \tau \in\left(t_{1}, t_{2}+d_{i}\right]$, we have

$$
A_{i}\left(s, \tau-d_{i}\right)=A_{i}\left(s, t_{1}-d_{i}\right)+\int_{t_{1}}^{\tau} d A_{i}\left(s, u-d_{i}\right)
$$

and

$$
S_{i}(s, \tau)=S_{i}\left(s, t_{1}\right)+\int_{t_{1}}^{\tau} s_{i}(u) d u
$$

Since

$$
\frac{d A_{i}\left(s, u-d_{i}\right)}{d u} \leq s_{i}(u), \text { for all } u \in\left(t_{1}, \tau+d_{i}\right] \subset\left(t_{1}, t_{2}+d_{i}\right],
$$

we have

$$
\begin{aligned}
& \sup _{s \leq \tau-d_{i}}\left[A_{i}\left(s, \tau-d_{i}\right)-S_{i}(s, \tau)\right] \\
& =\sup _{s \leq \tau-d_{i}}\left\{A_{i}\left(s, t_{1}-d_{i}\right)-S_{i}\left(s, t_{1}\right)+\int_{t_{1}}^{\tau}\left[\frac{d A_{i}\left(s, u-d_{i}\right)}{d u}-s_{i}(u)\right] d u\right\} \\
& \leq \sup _{s \leq t_{1}-d_{i}}\left[A_{i}\left(s, t_{1}-d_{i}\right)-S_{i}\left(s, t_{1}\right)\right] \leq 0
\end{aligned}
$$

This completes the proof.

Suppose at time $t_{1}$, a new connection $A_{i, n_{i}+1}\left(t_{1}, t_{2}\right)$ tries to join into the overall class $A_{i}$ until time $t_{2}$. The scheduler then checks whether the new arrival rate of class $i$ will be still less than the available service rate for class $i, s_{i}(t)$, via allocating more bandwidth to $s_{i}(t)$ from the leftover server capacity, or decreasing the service rates of some other flows with lower QoS requirements. If this new connection is not schedulable, it should be rejected.

Nevertheless, this scheme is intrinsically conservative, thus resulting in sub-optimal network resource utilization.

Claim 5.4.2 This scheme is sub-optimal.
Proof. To prove this claim, we just need to show that the criterion rejects connections that may be otherwise admitted under an optimal admission criterion.

Suppose a new connection $A_{i, n_{i}+1}\left(t_{1}, t_{2}\right)$ requests to access the network from time $t_{1}$ until $t_{2}$. The evolutions of $A_{i}(t), A_{i}\left(t-d_{i}\right), S_{i}(t)$ and $D_{i}(t)$ are plotted in Fig. 5.2. $A_{i}(t)$ and $D_{i}(t)$ converge at point $f$, where all the class $i$ traffic has been serviced.

The backlog in the buffer at time $t$ is given by

$$
\begin{equation*}
B_{i}(t)=A_{i}(t)-D_{i}(t) \tag{5.23}
\end{equation*}
$$

and the delay experienced by the class $i$ traffic arriving at $t$ is given by

$$
\begin{equation*}
d_{i}(t)=D_{i}^{-1}\left[A_{i}(t)\right]-t \tag{5.24}
\end{equation*}
$$

Since $d A_{i}\left(t_{1}, t-d_{i}\right) / d t>s_{i}(t)$ for $t \in\left(t_{a}, t_{b}\right)$, this connection should be rejected. However, it can be easily seen that the maximum delay of $A_{i}(t)$ is $d_{i}(t)=t_{f}-t_{e}$, which is
less than $d_{i}$. Thus, this connection should be admitted under an optimal admission scheme.

Although the EDF schedulability condition in (5.3) is expressed simply, the algorithms to perform optimal admission control for deterministic services can be computationally very complex. One reason behind this is that CAC algorithms require an accurate deterministic model to characterize the arrival traffic. The traditional deterministic models, such as, the leaky bucket model and the (peak rate, burst length, average rate) model are not accurate to capture the burstiness of compressed video traces. This issue has been addressed by Knightly and Zhang in [14], where a novel traffic model called deterministic bounding interval-length dependent (D-BIND) was proposed to capture the important multiplexing properties of bursty sources. The accuracy and effectiveness of D-BIND models depend on the number of intervals used to characterize the traffic process. In [15], Firoiu et al. followed up the D-BIND traffic model and presented their CAC algorithms for EDF schedulers.

We would like to emphasize that the CAC algorithms discussed above are not limited to deterministic services, though they were originally developed for this purpose. Recently, Qiu and Knightly [57] have presented a framework to extend the results in [14] to a stochastic setting by using statistical aggregate traffic envelopes. The basic strategy is to quantify the uncertainty of predicted future traffic, and then examine how the extreme values of the aggregate traffic rate result in packet loss. For more details about this approach, the interested reader is referred to [14].

## The Statistical CAC Algorithm Proposed

Next we present the statistical call admission control algorithm based on the admission test given by (5.22). Let $A_{i, n_{i+1}}(t)$ be a new connection that requests to access the network. Suppose the stability condition holds after the new connection is admitted. The deadline violation probabilities of $A_{i}(t)(1 \leq i \leq N)$ with respect to their available service will be


Figure 5.1: The architecture of an EDF scheduling system. The delay bound and loss bound of class $i$ sources are denoted by $d_{i}$ and $P_{i}$, respectively.


Figure 5.2: The evolutions of $A_{i}(t), A_{i}\left(t-d_{i}\right), S_{i}(t)$ and $D_{i}(t)$ over the time scale. Note $A_{i}(t)$ is $\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}\}$, and $D_{i}(t)$ is $\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{f}\}$.
given by

$$
\begin{equation*}
P_{i}^{\prime}=\operatorname{Pr}\left\{\sup _{t>d_{i}}\left[A_{i}\left(t-d_{i}\right)-S_{i}(t)\right]>0\right\} . \tag{5.25}
\end{equation*}
$$

If the QoS requirement $P_{i}$ can place an upper bound on the value of $P_{i}^{\prime}$ after the new connection is admitted, i.e.,

$$
\begin{equation*}
P_{i}^{\prime} \leq P_{i}, \text { for all } i \in\{1, \ldots, N\}, \tag{5.26}
\end{equation*}
$$

then this connection is admissible, otherwise, it should be rejected. To calculate $P_{i}^{\prime}$, we first need to obtain the $j$-th order dominant time scale $\hat{t}_{j}$, and the delay set $D_{j}$ that includes $d_{i}$. Within $D_{j}$, we can find the largest delay bound $d_{k_{j}}$, which is associated with class $k_{j}$. From proposition 5.3.3, we can evaluate $P_{i}^{\prime}$ through a virtual queueing system with the first $k$ classes of traffic.

In the following, we give the detailed procedure to perform the admission condition using (5.25):

Step 1: Calculate the traffic statistics of each class (including the new connection), and obtain $\mu_{i}$ and $\sigma_{i}^{2}(t),(1 \leq i \leq N)$.

Step 2: Define a function $f(t)$ as

$$
\begin{equation*}
f(t)=\frac{\sum_{i=1}^{N} \sigma_{i}^{2}\left(t-d_{i}\right)}{\left[\left(C-\sum_{i=1}^{N} \mu_{i}\right) t+\sum_{i=1}^{N} \mu_{i} d_{i}\right]^{2}}, \tag{5.27}
\end{equation*}
$$

then we can obtain the dominant time scales corresponding to each class by

$$
\begin{equation*}
\hat{t}_{j}=\arg \max _{t>d_{i}}\{f(t)\}, i=1, \ldots, N . \tag{5.28}
\end{equation*}
$$

Suppose the EDF system has $m$ different dominant time scales. Accordingly, we partition the delay bounds into $m$ disjoint delay sets, through which we find the local largest delay bounds $d_{k_{j}}(1 \leq j \leq m)$.

Step 3: Perform the following test

$$
\begin{equation*}
\frac{\left[\left(C-M_{j}\right) \hat{t}_{j}+U_{j}\right]^{2}}{V_{j}\left(\hat{t}_{j}\right)} \geq-2 \log \frac{P_{i}}{L_{i}} \tag{5.29}
\end{equation*}
$$

for all $\left\{i: d_{i} \in D_{j}, j=1, \ldots, m\right\}$, where

$$
\begin{equation*}
M_{j}=\sum_{l=1}^{k_{j}} \mu_{l}, U_{j}=\sum_{l=1}^{k_{j}} \mu_{l} d_{l}, \text { and } V_{j}(t)=\sum_{l=1}^{k_{j}} \sigma_{l}^{2}\left(t-d_{l}\right) . \tag{5.30}
\end{equation*}
$$

If the above test can be guaranteed for all the traffic classes, then this new connection is admissible. Otherwise, it should be rejected.

In general, this algorithm enables the network to accept more connections than that of proposition 5.4.1, even though the CAC algorithm from proposition 5.4.1 is based on a stochastic setting. We believe that the reasons for this are as follows. First, the admission test in proposition 5.4.1 is actually a sufficient condition for all scheduling disciplines. As shown in claim 5.4.2, it rejects some connections that should have been accepted. Second, the EDF is optimized for meeting delay bounds, as has been shown in the context of deterministic setting [22] [23]. The statistical CAC algorithm proposed is derived directly from the EDF schedulability condition (5.3), and the admission test (5.29) is performed according to the violation probability bound with respect to the delay bound for each traffic class. Namely, a new connection can be accepted if and only if the QoS requirements of all traffic classes can be guaranteed after it is admitted. Therefore, (5.29) becomes a necessary and sufficient condition for the admission control. This implies that the proposed CAC algorithm is optimal in terms of its admissible region. Consequently, if a new connection cannot be admitted by this algorithm, neither can it be admitted by other algorithms.

Next, we examine the computational complexity of our admission control algorithm. Using the techniques from [20], the computational complexity for estimating each pair of $\mu_{i}$ and $\sigma_{i}^{2}(t)$ is $\mathcal{O}\left(n^{2}\right)$, given $t=1,2, \ldots, n$. Thus, estimating the traffic statistics in step 1 requires $\mathcal{O}\left(N n^{2}\right)$ computations. In step 2, we need to search through $t=1,2, \ldots, n$ to obtain all the dominant time scales, and this results in $\mathcal{O}(N n)$ complexity. In step 3, to perform the admission test for all the classes, the overall computations required is less than $\mathcal{O}\left(N^{2}\right)$. In general, we have $N \ll n$. Thus, the bottleneck in this procedure is the actual estimation of the traffic statistics, rather than the calculation of dominant time scales
and violation probabilities. In the real implementation, the traffic statistics of all traffic classes can be stored as the network state information in reference to a certain time period, then the network controller just needs to estimate traffic statistics of the class that the new connection belongs to when performing its admission test.

### 5.5 Statistical Properties of QoS Guarantees under EDF Scheduling

We now study the statistical properties of QoS assurances in an EDF scheduler. With standard EDF scheduling, the order of packets to be served is completely determined by their deadline (or delay bounds $d_{i}$ ). Therefore, the violation probability bounds, $P_{i}$, have no influence on the scheduling operation. In the following, we will provide the analysis of how the violation probabilities can be tuned by the delay bound assignments.

### 5.5.1 Trade-Off Property of QoS Constraints

In reality, some applications are more sensitive to one QoS constraint than to others. For example, real-time applications such as voice and video are delay sensitive, but they can tolerate some loss. In contrast, non-real-time data traffic can tolerate some delay, but is very sensitive to transmission error and loss. Therefore, when the network resource (i.e., capacity) is stringent, a reasonable option is to assure some QoS constraints by sacrificing others that are tolerable.

In the next proposition, we present an intuitive result, which is useful for adaptive and/or intelligent congestion control in high-speed networks deploying EDF scheduling.

Proposition 5.5.1 Consider an EDF scheduler with $N$ different classes of admitted connections, the violation probability function of class $i, P_{i}\left(d_{i}\right)$, decreases monotonically as its delay bound $d_{i}$ increases.

Proof. To prove this proposition, we just need to prove $P_{i}\left(d^{\prime}\right)>P_{i}(d)$ if and only if $d^{\prime}<d$. Since the arrival process $A_{i}(t)$ can be considered as a monotonically increasing function of
$t$ and the available service for class $i, S_{i}(t)$, is given, then we obtain

$$
\begin{aligned}
d^{\prime}<d & \Leftrightarrow A_{i}\left(t-d^{\prime}\right)>A_{i}(t-d) \quad \text { for all } t \\
& \Leftrightarrow A_{i}\left(t-d^{\prime}\right)-S_{i}(t)>A_{i}(t-d)-S_{i}(t) \\
& \Leftrightarrow P_{i}\left(d^{\prime}\right)>P_{i}(d)
\end{aligned}
$$

This completes the proof.

Consequently, if the stability condition can be satisfied after a new connection $A_{i, n_{i}+1}(t)$ is admitted (i.e., $\mathbb{E}\left[W_{i, n_{i}+1}(t)-A_{i, n_{i}+1}\left(t-d_{i}\right)\right]>0$, for all $t>0$ ), then an interesting result follows.

Corollary 5.1 Consider an EDF scheduler with a set of admitted connections. For any new connection $A^{\prime}(s, t)$ with a packet deadline violation probability bound $P$, there exists a unique delay $d^{*}$ such that $A^{\prime}(s, t)$ can be admitted if and only if $d \geq d^{*}$.

In an EDF system with $N$ heterogeneous traffic classes, the individual connections compete with each other for the network resource according to their packet deadlines. It is intuitively clear that the connections with tighter delay bounds will be able to receive relatively more resource (i.e., capacity) during a certain time period. Consequently, the possibility of a new connection with a short delay bound to be admitted is relatively small when the available network resources are tight. An interesting issue raised from corollary 5.1 is how to find the minimum $d^{*}$, which will be addressed in the future work.

Similarly, one might also draw a conclusion as follows.

Corollary 5.2 Consider an EDF scheduler with a set of admitted connections. For any new connection $A^{\prime}(s, t)$ with delay bound $d$, there exists a unique violation probability $P^{*}$ such that $A^{\prime}(s, t)$ can be admitted if and only if $P \geq P^{*}$.

However, this result holds only for homogeneous traffic (i.e., the admitted connections having the same QoS requirements or in a FIFO queueing system). The reason behind this
is that the EDF scheduler lacks isolation among different traffic classes. The new connection with a tight delay bound will squeeze much bandwidth from the on-going connections, thus degrading their QoS metrics. Nevertheless, if the EDF scheduler is combined with a fair packet discard policy (see [60] for example), corollary 5.2 is still valid for an EDF system with heterogeneous traffic. In contrast, the GPS schedulers provide isolation among different traffic classes, and then enable one to tune the violation probabilities by changing the weights.

From the discussion above, we know that given the available bandwidth, there is a trade-off between the delay bound $d$ and the violation probability $P$ for a new connection. If both its QoS requirements $(d, P)$ can not be guaranteed simultaneously, the application can adaptively negotiate with the network on its QoS constraints in order to be admitted.

### 5.5.2 Asymptotic Ordering Property

We know from proposition 5.5 .1 that for a certain traffic class, the violation probability is strictly coupled with its delay bound. From the system perspective, we find that the actual QoS guarantees of an EDF scheduler have an asymptotic ordering property.

Proposition 5.5.2 Consider an EDF scheduler with $N$ different classes of admitted connections, the violation probabilities that each class experiences decrease asymptotically with respect to their delay bounds.

Proof. From definition 5.3 and proposition 5.3.3, we know that $\operatorname{Pr}\left\{\sum_{i=1}^{N} A_{i}\left(t-d_{i}\right)-C t>\right.$ $0\}$ obtains its maximum at $t=\hat{t}_{1}$ for all $t \in(0,+\infty)$. For $t \in\left(d_{k_{1}+1},+\infty\right)$, it obtains the maximum at $t=\hat{t}_{2}$. So we have $t=\hat{t}_{3}$ for $t \in\left(d_{k_{2}+1},+\infty\right)$, etc. Consequently, the violation probabilities of each delay set have a strict ordering property, that is, $P_{D_{1}}>$ $P_{D_{2}}>\ldots>P_{D_{m}}$. Since the traffic classes in a delay set have the same asymptotic violation probabilities, the proposition is implied.

From a practical point of view, this ordering property provides serious implications for the design and control of networks deploying EDF scheduling. A simple example is given as follows. Suppose there are three classes of admitted connections accessing an EDF scheduling link and their QoS requirements are ( $10 \mathrm{~ms}, 10^{-3}$ ), ( $80 \mathrm{~ms}, 10^{-6}$ ), and ( $120 \mathrm{~ms}, 10^{-5}$ ), respectively. If all these three pairs of QoS requirements are satisfied, then the actual violation probability bound that the EDF scheduling link assures for class 3 traffic will be less than or equal to $10^{-6}$, instead of $10^{-5}$.

### 5.6 Numerical Evaluation

The appropriateness of a model is ultimately determined by its ability to accurately predict the actual behavior. To evaluate the performance of the call admission control algorithm, we conduct trace-driven simulation experiments using real network traffic (i.e., MPEG-4 and H. 263 encoded video traces from [33] and Ethernet sources from [16]). It is expected that MPEG-4 and H. 263 encoded video will account for a large portion of the traffic in future wired and wireless networks. The MPEG-4 video traces were encoded at the highquality level, having a constant frame rate of 25 frames $/ \mathrm{sec}$ and a variable bit rate (VBR). The H. 263 video traces were encoded at a constant bit rate (CBR) of $256 \mathrm{kbit} / \mathrm{sec}$, while having a variable frame rate. We refer the interested reader to [33] for a more detailed description of the encoding procedure.

In the simulation experiments, we select twenty independent MPEG-4 video traces and twenty independent H. 263 video traces (all from different movies in [33]), and then obtain two sets of traffic traces. By concatenating each set of traces together, we get two long traces. Each source (either MPEG-4 or H.263) can be produced by randomly selecting a starting point over the corresponding long trace. In doing this way, the dependence between the multiplexed sources is very weak ${ }^{4}$. Thus, we can assume that the individual sources are independent of each other. These are the same procedures that have been applied in [18].

[^5]In the following experiments, we first examine how accurate the proposed virtual queueing system can approximate the real EDF scheduler, and calculate the $95 \%$ confidence intervals for each probability estimated through simulations. We take the admissible region as a performance indication of the CAC algorithm, and compare its performance with the simulation results.

### 5.6.1 Example 1

We first consider a single-class EDF scheduler, where the traffic classes have the same QoS requirements. Fig. 5.3 and Fig. 5.4 show the deadline violation probabilities as a function of the delay bound in a single-class EDF system. It can be observed that the approximations can capture the actual deadline violation behavior in most of the range.

Fig. 5.5 and Fig. 5.6 give the admissible region in terms of utilization that a single-class EDF scheduler with a 45 Mbps link rate (a T 3 link) can achieve against the delay bound. From Fig. 5.5 we observe that after the delay bound of 10 ms , increasing the delay bound (or equivalently the buffer capacity) may not improve the network utilization significantly. This is most likely due to the long-range dependence (the autocorrelation function is not summable) or self-similarity of the VBR traffic. This fractional behavior will become more significant as the number of the aggregated VBR flows increases. Intuitively, long-range dependent traffic is bursty (highly variable) over a wide range of time scales. The cumulative effect of the correlation for large lags has a serious implication in the packet deadline violation. As a result, for VBR traffic, increasing the delay bound may not improve the statistical multiplexing gain significantly.

From Fig. 5.6, we can see that the achieved utilization increases significantly with respect to the delay bound, reaching almost $100 \%$ after a certain delay bound. The reason behind this is that for CBR traffic, as the number of the aggregated flows increases, the aggregate traffic is not so bursty. Thus, increasing the delay bound will result in a substantial statistical multiplexing gain. Fig. 5.7 shows the traffic traces of the aggregated MPEG-4


Figure 5.3: A trace composed of 40 multiplexed MPEG-4 video sources.


Figure 5.4: A trace composed of 80 multiplexed H. 263 video sources.
sources and aggregated H. 263 sources, respectively.

### 5.6.2 Example 2

Now we consider a two-class EDF scheduler, where two independent traffic classes with different QoS requirements are multiplexed into a link.

First, we investigate the deadline violation probabilities in the EDF scheduler. Class 1 traffic is a trace composed of 60 multiplexed H .263 video sources and class 2 traffic consists of 40 multiplexed MPEG-4 video sources. We fixed the delay bound of class 1 traffic at 2 $m s$, and change the delay bound of class 2 . Fig. 5.8 shows the aggregate and individual violation probabilities. From this figure, we can see that the analytical framework presented earlier can provide fairly good approximations to the real probabilities. In particular, after the delay bound of class 2 is greater than the first-order DTS, i.e., $d_{2}>\hat{t}_{1}$, the curves of class 1 flatten considerably. This indicates that increasing $d_{2}$ further will decrease the violation probability of class 2 traffic only, while has very little effect on the violation probability of the class 1 traffic.

Next we examine the admissible region that a 45 Mbps EDF link can achieve in the case of two different traffic classes. For class 1, we use H. 263 video traces as the traffic sources; for class 2, we use MPEG-4 video traces. Their detailed QoS requirements are given in table 5.1. We compare the simulated admissible region with the results obtained from the CAC algorithm. Fig. 5.9 shows the maximum amount of traffic that the EDF scheduling link can admit, expressed in terms of average utilization. It shows that our CAC algorithm can approximate the real admissible region quite closely.

### 5.6.3 Example 3

This example investigates an EDF system with 3 traffic classes. Class 1 and class 2 are respectively composed of 20 MPEG-4 traces and 4 Ethernet sources, where class 3 consists of 10 MPEG-4 traces and 4 Ethernet sources. Table 5.2 presents the simulation and


Figure 5.5: The admissible region vs. delay bound in a 45 Mbps EDF link accessed by MPEG-4 video traffic.


Figure 5.6: The admissible region vs. delay bound in a 45 Mbps EDF link accessed by H. 263 video traffic.


Figure 5.7: The amount of traffic arriving during a time interval of 0.01 second.


Figure 5.8: The deadline violation probabilities vs. delay bounds in a two-class EDF scheduler, where $\mu_{1}=0.2861 C$ and $\mu_{2}=0.5639 C$.


Figure 5.9: The admissible region for heterogeneous traffic in a two-class EDF scheduler. The link rate of the EDF system is fixed at 45 Mbps .

| Class |  |  |  |
| :---: | :---: | :---: | :---: |
| Assignment | Delay bound <br> $(\mathrm{msec})$ | Violation Prob. <br> $(\log P)$ | Sources |
| class 1 | 10 | -3 | $H .263$ |
| class 2 | 80 | -5 | $M P E G-4$ |

Table 5.1: The traffic parameters for numerical example 2 of section 5.6.

| Delay Bound | $d_{1}=3(m s), d_{2}=10(m s), d_{3}=30(m s)$ |  |
| :---: | :---: | :---: |
| Assignments | Simulations | Analysis |
| class 1 | $(0.119420 \pm 0.0199)$ | 0.109549 |
| class 2 | $(0.119919 \pm 0.0199)$ | 0.109549 |
| class 3 | $(0.118331 \pm 0.0198)$ | 0.109549 |
| Agg. | $(0.120964 \pm 0.0200)$ | 0.109549 |
| DTS | $\hat{t}_{1}=699(m s)$ | $\hat{t}_{2}=\infty$ |
| Delay Bound | $d_{1}=1(m s), d_{2}=2(m s), d_{3}=500(m s)$ |  |
| Assignments | Simulations | Analysis |
| class 1 | $(0.064734 \pm 0.006561)$ | 0.064097 |
| class 2 | $(0.063198 \pm 0.006544)$ | 0.064097 |
| class 3 | $(0.008311 \pm 0.005991)$ | 0.008415 |
| Agg. | $(0.073832 \pm 0.006674)$ | 0.064097 |
| DTS | $\hat{t}_{1}=5(m s) \hat{t}_{2}=4419(m s)$ | $\hat{t}_{3}=\infty$ |

Table 5.2: The violation probabilities in a three-class EDF system with $\mu_{1}=0.36 C, \mu_{2}=$ $0.35 C$, and $\mu_{3}=0.20 C$.
analytical results. Although our algorithm can be applied for many different traffic classes, it is in general very difficult to graphically present the admission region with more than 3 classes. Therefore, the admission region will not be given in this example.

### 5.7 Conclusion

This chapter has extensively analyzed the aggregate and individual violation probabilities. Based on these statistical results, novel admission control conditions and algorithm have been developed for the EDF scheduling discipline. The extended notation of the dominant time scale and the virtual queueing system developed play a critical role in the proposed framework, because they provide an accurate approach to evaluate the deadline violation
asymptotic of the individual traffic classes. In particular, we found that due to the strict coupling between the violation probability and the delay bound, the QoS metrics that an EDF scheduler actually guarantees have an asymptotic ordering property, which has serious implications in the design and control of EDF networks. The simulation experiments have illustrated the performance of the proposed CAC algorithm.

## CHAPTER 6

## SUMMARY AND FUTURE WORK

### 6.1 Summary

In this section, a summary of the research results of this thesis is presented.
In chapter 2, we investigated the self-similar characteristics of real network traffic, and reviewed some important techniques for performance evaluation, as well as some relevant issues regarding traffic scheduling and admission control.

In chapter 3, we studied the provision of quality-of-service ( QoS ) for real-time traffic over a wireless channel deploying automatic repeat request (ARQ) error control. By introducing the concepts of $A R Q$ capacity and effective capacity, an analytic model has been derived to evaluate the loss probabilities in both the network layer and the physical layer. In contrast to the previous results, this model quantifies the interaction between the network and physical layers. The results enable the call admission controller in wireless networks to control and optimize traffic QoS using instantaneous channel status information.

In chapter 4, we proposed a measurement-based approach to estimate the buffer overflow probability for each priority queue in a multiplexer deploying a static priority scheduling discipline. This approach is based on the maximum variance asymptotic and the notation of the dominant time scale. The result shows that when considering a particular priority queue, all the traffic with higher priority can be lumped together and all the traffic with lower priority can be ignored.

In chapter 5, we extended the notations of the dominant time scale and introduced the notation of the virtual queueing systems. With these tools, we were able to analyze the per-class QoS metric of each traffic class in an EDF scheduler. Based on this theoretical
foundation, we derived the explicit admission control conditions for a network node deploying EDF scheduling. In addition, we have shown that the QoS metrics that an EDF scheduler actually guarantees have an asymptotic ordering property, which provides important implications for the design and control of EDF networks.

### 6.2 Future Research

Overall, this thesis serves as a framework for a number of important extensions. Specifically, two interesting possibilities in this regard are as follows:

1. Because we use the MVA approach as the building block for the performance analysis, the frameworks proposed in this thesis can be readily applied to the measurementbased CAC design. Therefore, a robust control mechanism is necessary to determine the ideal setting of the measurement window. For example, the studies in [20] and [62] may complement this work.
2. From the perspective of real networks, only the end-to-end issue is of interest. The discussions in this thesis only consider a single node. The major difficulties in extending this work to the multiple hop case are: i) traffic from different sources will become correlated and then the assumption of statistical independence of the sources may not hold; ii) the traffic characteristics may be distorted when travelling through the network; iii) the methods to optimally assign the end-to-end delay to individual hops in order to minimize the violation probabilities will require further investigation. Recently, there are some recent works that addressed various aspects of this issue (see [54], [63], [64], and [65] for example). These studies provide a foundation for the continuing investigation of end-to-end statistical QoS guarantee methodologies.

Besides the theoretical issues, many practical issues still need to be resolved before these algorithms can be applied in practice.

## BIBLIOGRAPHY

[1] Z. Quan and J.-M. Chung, "Analysis of packet loss for real-time traffic in wireless mobile networks with ARQ feedback," in Proc. IEEE Wireless Communication and Networking Conference (WCNC'04), Atlanta, GA, USA, Mar. 21-25, 2004.
[2] Z. Quan and J.-M. Chung, "Asymptotic loss of real-time traffic in wireless mobile networks with selective-repeat ARQ," IEEE Commun. Letters, 2004 (accepted for publication).
[3] Z. Quan and J.-M. Chung, "Priority queueing analysis for self-similar traffic in highspeed networks," in Proc. IEEE International Conference on Communications (ICC'03), Anchorage, AK, USA, May 2003, pp. 1606-1610.
[4] Z. Quan and J.-M. Chung, "Queue length analysis of non-preemptive DiffServ networks," Int. J. Electron. Commun. (AEU), vol. 57, no. 5, pp. 338-340, Sept. 2003.
[5] Z. Quan and J.-M. Chung, "A statistical framework for EDF scheduling," IEEE Commun. Letters, vol. 7, no. 10, pp. 493-495, Oct. 2003.
[6] Z. Quan and J.-M. Chung, "An analytical framework for EDF schedulers based on the dominant time scale," in Proc. IEEE Consumer Communications and Networking Conference (CCNC'04), Las Vegas, Nevada, USA, Jan. 5-8, 2004.
[7] Z. Quan and J.-M. Chung, "Admission control for probabilistic services with earliest deadline first scheduling," in 13th IEEE Workshop on Local and Metropolitan Area Networks (IEEE LANMAN'04), San Francisco Bay Area, CA, Apr. 25-28, 2004.
[8] Z. Quan and J.-M. Chung, "Statistical admission control for real-time services under earliest deadline first scheduling," Computer Networks, Elsevier, 2004 (currently under revision).
[9] R. L. Cruz, "A calculus for network delay, part I: Network elements in isolation," IEEE Trans. Info. Theory, vol. 37, no. 1, pp. 114-131, Jan. 1991.
[10] R. L. Cruz, "A calculus for network delay, part II: Network analysis," IEEE Trans. Info. Theory, vol. 37, no. 1, pp. 132-141 Jan. 1991.
[11] A. K. Parekh and R. G. Gallager, "A generalized processor sharing approach to flow control in integrated services networks: the single-node case," IEEE/ACM Trans. Networking, vol. 1, no. 3, pp. 344-357, June 1993.
[12] A. K. Parekh and R. G. Gallager, "A generalized processor sharing approach to flow control in integrated services networks: the multiple node case," IEEE/ACM Trans. Networking, vol. 2, no. 2, pp. 137-150, Apr. 1994.
[13] L. Georgiadis, R. Guerin, V. Peris, and K. N. Sivarajan, "Efficient network QoS provisioning based on per node traffic shaping," IEEE/ACM Trans. Networking, vol. 4, no. 4, pp. 482-501, Aug. 1996.
[14] E. Knightly and H. Zhang, "D-BIND: An accurate traffic model for providing QoS guarantees to VBR traffic," IEEE/ACM Trans. Networking, vol. 5, no. 2, pp. 219-231, Apr. 1997.
[15] V. Firoiu, J. Kurose, and D. Towsley, "Efficient admission control of piecewise linear traffic envelopes at EDF schedulers," IEEE/ACM Trans. Networking, vol. 6, no. 5, pp. 558-570, Oct. 1998.
[16] W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson, "On the self-similar nature of Ethernet traffic (extended version)," IEEE/ACM Trans. Networking, vol. 2, no. 1, pp. 1-15, Feb. 1994.
[17] V. Paxson and S. Floyd, "Wide area traffic: the failure of poisson modeling," IEEE/ACM Trans. Networking, vol. 3, no. 3, pp. 226-244, June 1995.
[18] J. Choe and N. B. Shroff, "A central-limit-theorem based approach for analyzing queue behavior in high-speed networks," IEEE/ACM Trans. Networking, vol. 6, no. 5, pp. 659-671, Oct. 1998.
[19] E. W. Knightly and N. B. Shroff, "Admission control for statistical QoS: Theory and practice," IEEE Network, vol. 13, no. 2, pp. 20-29, 1999.
[20] D.-Y. Eun and N. B. Shroff, "A measurement-analytic approach for QoS estimation in a network based on the dominant time scale," IEEE/ACM Trans. Networking, vol. 11, no. 2, pp. 222-235, Apr. 2003.
[21] H. Zhang, "Service disciplines for guaranteed performance service in packetswitching networks," Proc. of the IEEE, vol. 83, no. 10, pp. 1374-1396, Oct. 1995.
[22] J. Liebeherr, D. E. Wrege, and D. Ferrari, "Exact admission control for networks with a bounded delay service," IEEE/ACM Trans. Networking, vol. 4, no. 6, pp. 885-901, Dec. 1996.
[23] L. Georgiadis, R. Gurin, and A. K. Parekh, "Optimal multiplexing on a single link: Delay and buffer requirements," IEEE Trans. Inform. Theory, vol. 43, no. 5, pp. 15181535, Sept. 1997.
[24] R. M. Loynes, "The stability of a queue with non-independent inter-arrival and service times," Proc. Cambridge Philos. Soc., vol. 58, pp. 497-520, 1962.
[25] J. J. Metzner, Reliable Data Communications. New York: Academic, 1998.
[26] J. J. Metzner and J.-M. Chung, "Efficient energy utilization with time constraint in mobile time varying channels," IEEE Trans. Veh. Technol., vol. 49, no. 4, pp. 11691177, July 2000.
[27] P. W. Glynn and W. Whitt, "Logarithmic asymptotics for steady-state tail probabilities in a single server queue," J. Appl. Prob., vol. 31, pp. 131-155, 1994.
[28] N. G. Duffield and N. O'Connell, "Large deviations and overflow probabilities for the general single server queue, with applications," Proc. Cambridge. Philos. Soc., no. 118, pp. 363-374, 1995.
[29] G. Mao and D. Habibi, "Loss performance analysis for heterogeneous ON-OFF sources with application to connection admission control," IEEE/ACM Trans. Networking, vol. 10, no. 1, pp. 125-138, Feb. 2002.
[30] D. Wu and R. Negi, "Effective capacity: A wireless link model for support of quality of service," IEEE Trans. Wireless Commun., vol. 2, no. 4, pp. 630-643, July 2003.
[31] P. Mannersalo and I. Norros, "GPS schedulers and Gaussian traffic," in Proc. IEEE INFOCOM'02, New York, NY, USA, June 2002.
[32] J. F. Hayes, "Adaptive feedback communications," IEEE Trans. Commun. Technology, vol. 16, pp. 29-34, Feb. 1968.
[33] F. H. P. Fitzek and M. Reisslein, "MPEG-4 and H. 263 video traces for network performance evaluation," IEEE Network, vol. 15, no. 6, pp. 40-54, Nov. 2001.
[34] I. Norros, "On the use of fractional brownian motion in the theory of connectionless networks," IEEE J. Select. Areas Commun., vol. 13, no. 6, pp. 953-962, Aug. 1995.
[35] W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson, "On the self-similar nature of Ethernet traffic (extended version)," IEEE/ACM Trans. Networking, vol. 2, no. 1, pp. 1-15, Feb. 1994.
[36] J. Beran, R. Sherman, M. S. Taqqu, and W. Willinger, "Long-range dependence in variable-bit-rate video traffic," IEEE Trans. Commun., vol. 43, no. 2/3/4, pp. 1566-1579, 1995.
[37] A. W. Berger and W. Whitt, "Effective bandwidths with priorities," IEEE/ACM Trans. Networking, vol. 6, no. 4, pp. 447-460, Aug. 1998.
[38] S. Delas, R. R. Mazumdar, and C. Rosenberg, "Cell loss asymptotics in priority queues accessed by a large number of independent stationary sources," in Proc. IEEE INFOCOM '99, vol. 2, New York, NY, USA, May 1999, pp. 551-558.
[39] P. Bratley, B. Fox, and L. Schrage, A Guide to Simulation, 2nd ed. New York: Springer-Verlag, 1987.
[40] C. Courcoubetis, A. Dimakis, and G. D. Stamoulis, "Traffic equivalence and substitution in a multiplexer with applications to dynamic available capacity estimation," IEEE/ACM Trans. Networking, vol. 10, no. 2, pp. 217-231, Apr. 2002.
[41] N. G. Duffield and S. H. Low, "The cost of quality in networks of aggregate traffic," in Proc. IEEE INFOCOM'98, vol. 2, San Francisco, CA, Apr. 1998, pp. 525-532.
[42] A. Shwartz and A. Weiss, Large Deviations For Performance Analysis: Queues, Communication and Computing. New York: Chapman and Hall, 1995.
[43] N. Likhanov and R. R. Mazumdar, "Cell loss asymptotics for buffers fed with a large number of independent stationary sources," J. Appl. Prob., vol. 36, no. 1, pp. 86-96, 1999.
[44] C. Courcobetis and R. Weber, "Buffer overflow asymptotics for a switch handling many traffic sources," J. Appl. Prob., vol. 33, pp. 886-903, 1996.
[45] M. Montgomery and G. de Veciana, "On the relevance of time scales in performance oriented traffic characterization," in Proc. IEEE INFOCOM'96, San Francisco, CA, USA, Apr. 1996, pp. 513-520.
[46] F. P. Kelly, "Notes on effective bandwidths," in Stochastic Networks: Theory and Applications, F. P. Kelly, S. Zachary, and I. B. Ziedins, Eds. Oxford University Press, 1996, pp. 141-168.
[47] C. Courcoubetis, V. A. Siris, and G. D. Stamoulis, "Application of the many sources asymptotic and effective bandwidths to traffic engineering," Telecommunication Systems, vol. 12, no. 2-3, pp. 167-191, 1999.
[48] B. K. Ryu and A. Elwalid, "The importance of long-range dependence of VBR video traffic in ATM traffic engineering: Myths and realities," in Proc. ACM SIGCOMM'96, Stanford, CA, USA, Aug. 1996, pp. 3-14.
$149]$ J. Choe and N. B. Shroff, "On the supremum distribution of integrated stationary gaussian processes with negative linear drift," Advances in Applied Probability, vol. 31, pp. 134-156, Mar. 1999.
[50] D. Ferrari and D. Verma, "A scheme for real-time channel establishment in wide-area networks," IEEE J. Select. Areas Commun., vol. 8, no. 3, pp. 368-379, Apr. 1990.
[51] A. Elwalid, D. Mitra, and R. H. Wentworth, "A new approach for allocating buffers and bandwidth to heterogeneous, regulated traffic in an ATM node," IEEE J. Select. Areas Commun., vol. 13, no. 6, pp. 1115-1127, Aug. 1995.
[52] A. Elwalid and D. Mitra, "Design of generalized processor sharing schedulers which statistically multiplex heterogeneous QoS classes," in Proc. IEEE INFOCOM'99, New York, USA, March 1999.
[53] K. Kumaran, G. E. Margrave, D. Mitra, and K. R. Stanley, "Novel techniques for the design and control of generalized processor sharing schedulers for multiple QoS classes," in Proc. IEEE INFOCOM'00, Tel-Aviv, Israel, March 2000.
[54] M. Andrews, "Probabilistic end-to-end delay bounds for earliest deadline first scheduling," in Proc. IEEE INFOCOM'00, Tel-Aviv, Israel, March 2000.
[55] R. R. Boorstyn, A. Burchard, J. Liebeherr, and C. Oottamakorn, "Statistical service assurances for traffic scheduling algorithms," IEEE J. Select. Areas Commun., vol. 18, no. 12, pp. 2651-2664, Dec. 2000.
[56] F. M. Chiussi and V. Sivaraman, "Achieving high utilization in guaranteed services networks using earliest-deadline-first scheduling," in Proc. International Workshop on Quality of Service, Napa, CA, USA, May 1998.
[57] J. Qiu and E. Knightly, "Measurement-based admission control with aggregate traffic envelopes." IEEE/ACM Trans. Networking, vol. 9, no. 2, pp. 199-210, Apr. 2001.
[58] Z.-L. Zhang, D. Towsley, and J. Kurose, "Statistical analysis of generalized processor sharing scheduling discipline," IEEE J. Select. Areas Commun., vol. 13, no. 6, pp. 10711080, Aug. 1995.
[59] V. Sivaraman and F. M. Chiussi, "Statistical analysis of delay bound violations at an earliest deadline first (EDF) scheduler," Performance Evaluation, vol. 36-37, no. 1, pp. 457-470, 1999.
[60] V. Sivaraman, F. M. Chiussi, and M. Gerla, "End-to-end statistical delay service under GPS and EDF scheduling: A comparison study," in Proc. IEEE INFOCOM'01, Anchorage, AK, USA, Apr. 2001.
[61] Z.-L. Zhang, "End-to-end support for statistical quality-of-service guarantees in multimedia networks," Ph.D. dissertation, Department of Computer Science, University of Massachusetts, Amherst, Feburary 1997.
[62] M. Grossglauser and D. N. Tse, "A framework for robust measurement-based admission control," IEEE/ACM Trans. Networking, vol. 7, no. 3, pp. 293-309, June 1999.
[63] V. Sivaraman and F. Chiussi, "Providing end-to-end statistical delay guarantees with earliest deadline first scheduling and per-hop traffic shaping," in Proc. IEEE INFOCOM'OO, Tel-Aviv, Israel, March 2000.
[64] J. Liebeherr, S. Patek, and E. Yilmaz, "Tradeoffs in designing networks with end-toend statistical QoS guarantees," in Proc. IEEE/IFIP 8th Int. Workshop Quality of Service (IWQoS'OO), Pittsburgh, PA, USA, June 2000.
[65] C. Li and E. Knightly, "Coordinated multihop scheduling: A framework for end-toend services," IEEE/ACM Trans. Networking, vol. 10, no. 6, pp. 776-789, Dec. 2002.

Zhi Quan
Candidate for the Degree of
Master of Science

## Thesis: Statistical Service Guarantees for Traffic Scheduling in High-Speed Data Networks

Major Field: Electrical Engineering
Biographical:
Personal Data: Born in Liuzhou, P.R.China, on January 14, 1978, son of Zuhua Quan and Yun Su.

Education: Received the B.E. degree from Beijing University of Posts and Telecommunications, Beijing, China, in July 1999, in Communication Engineering; Completed the requirements for the Master of Science degree with a major in Electrical Engineering and with a minor in Applied Mathematics at Oklahoma State University in May 2004.

Experience: Research Assistant at Oklahoma State University from January 2002 to May 2004.

Professional Memberships: Student Member of IEEE and ACM.
Professional Activities: Technical Reviewer for IEEE Communications Letters and IEEE ICC 2004.


[^0]:    'An efficient retransmission scheme is defined as one that only retransmits packets that are erroneous and continuously transmits new packets as long as no error occurs.

[^1]:    ${ }^{2}$ This approximation is actually an upper bound of the steady-state queueing loss probability.

[^2]:    ${ }^{\prime}$ Throughout this chapter, we will use the notations of schedulable and admissible interchangeably.

[^3]:    ${ }^{2}$ The utilization here means the ratio of the average arrival rate against the overall server capacity, i.e, $u_{i}=\lim _{t \rightarrow \infty} \frac{A_{1}(t)}{C t}$.

[^4]:    ${ }^{3}$ We let $d_{N+1}=+\infty$ because class $N+1$ is not a realistic class in the scheduler. For the detailed proof, the reader is referred to lemma 2 in [5].

[^5]:    ${ }^{4}$ They are generally independent if the two sources are from different movie traces.

