STABILITY OF SLOPES IN A TWO-LAYER SYSTEM OF ANISOTROPIC SOILS

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NOMENCLATURE

β	slope angle
H_1	thickness of top layer
H_2	thickness of bottom layer
γ_1	unit weight of soil in top layer
γ_2	unit weight of soil in bottom layer
C_1 and C_2	cohesion along vertical and horizontal planes, respectively, in top layer
C_1' and C_2'	cohesion along vertical and horizontal planes, respectively, in bottom layer
α and λ_1	geometrical parameters characterizing the slip surface in top layer
α' and λ_2	geometrical parameters characterizing the slip surface in bottom layer
θ	geometrical parameter
f	the angle between the failure plane and the plane normal to the direction of the major principal stress
i	angle of major principal stress from vertical, measured clockwise
C_u	undrained strength
p	relative strength index
n	thickness ratio
K and K'	coefficients of anisotropy
m	γ_2/γ_1
n	H_2/H_1
C_1'	$(p + 1)\ C_1$

C'_2	$(p + 1)\, C_2$
$\frac{C_2}{C_1}$	K
$\frac{C'_2}{C'_1}$	K'

CHAPTER I

INTRODUCTION

The stability of earth slopes is a matter of considerable importance in the construction of highways, railways, and earth dams, as well as in connection with landslides. During the last four decades, numerous efforts have been made to deal with the problem of stability of slopes with the aim of computing the factor of safety with respect to the sliding of slopes of cuts and embankments. The properties of the material (soil) in which sliding occurs deviate quite considerably from those of elastic solids. Hence, the laws of strength of materials and theory of elasticity do not hold true precisely for soils.

The absence of any mathematical theory which could be used to deal with real soil materials, coupled with the necessity to solve problems in soils engineering, require that several assumptions be made idealizing this material, the most important among them being homogeneity and isotropy. Since most of the methods for stability analysis are based on these two basic assumptions, they give only a rough estimate of the factor of safety.

In nature, there are two deviations from the ideal homogeneous material. The first is the case in which the subsoil consists of layers of distinctly different soils. The second is the case of a soil deposit which lacks any distinct stratification but whose properties vary from one point to another over a wide range. This type of non-homogeneity makes it difficult to arrive at representative soil properties to be used in calculations.

Again, in nature, most soils are anisotropic because of the mode of deposition, the stress metamorphosis after deposition, or both. There is considerable evidence in literature showing that the stability of earth slopes is influenced by non-homogeneity and anisotropy in strength (Gibson and Morgenstern, 1962; Lo, 1965; Livneh, 1967; and others).

In this thesis, an analytical method is presented for analyzing slope stability problems in a two-layered system of non-homogeneous and anisotropic soils. Chapter II deals with a brief review of published literature regarding the methods available for slope stability analysis. Since the published material in this area of soil mechanics is quite extensive, only that material which is directly related to the present work is reviewed in detail. The reader is referred to the references cited in the bibliography for additional information.

Chapter III deals with the analytical method for solving slope stability problems in a two-layered system of non-homogeneous and anisotropic soils. The working formulae for stability number and factor of safety are derived in detail. In Chapter IV, the results obtained in this analysis are discussed, and the conclusions drawn from this study are listed. Some aspects related to this area which merit further research are suggested. Charts, slope angle versus stability number, thickness ratio versus stability number, and relative strength index versus stability number are presented (Figures 7 through 30).

Two hypothetical problems dealing with slope stability in a layered system are worked out in Appendix A. These demonstrate the usefulness of the charts in analyzing stability problems in layered soils. The computer program used in obtaining numerical results in this study is given in Appendix B.

CHAPTER II

REVIEW OF LITERATURE

The construction of earthen embankments is as old as civilization itself. Many ancient man-made embankments exist in China, India, and the Middle East. Rao (1961) describes 18 existing earth dams constructed in southern India between 1000 A.D. and 1800 A.D.

The dams described above, as well as those built in other countries during that or earlier periods, were designed by men who had only an empirical knowledge of mechanics and material properties. The successful functioning of these structures clearly indicates that embankment design could be carried out based on some experience and intuition. The main drawback of this approach was that it did not permit a quantitative assessment of the safety of structure. Nor could it handle unusual conditions encountered in embankment design. Real progress in this direction was to wait until tools of mathematics and mechanics and scientific knowledge of soil behavior were available.

Probably the earliest contribution dealing with soil behavior was that of Coulomb (1773), who has given the expression for the critical height of vertical clay slope as

$$H_c = \frac{4c}{\gamma} \frac{\cos \varphi}{1 - \sin \varphi}$$

where c = cohesion,

γ = unit weight, and

φ = internal friction of clay.

In 1820, Francais extended Coulomb's analysis to the case of a clay bank sloping at an angle of θ to the horizontal and, on the assumption that the slip surface was a plane passing through the toe of the slope, deduced the following expression for the height of limiting stability:

$$H = \frac{4c}{\gamma} \left\{ \frac{\cos\varphi \sin\theta}{1 - \cos(\theta - \varphi)} \right\} .$$

This equation is generally, but incorrectly, attributed to Culmann, who published it forty-six years later in his book "Die Graphische Statik" (1866).

The earliest pioneering work in stability of clay slopes was done by Collin (1846). He published a memoir which contained careful field observations on approximately fifteen slips in cuttings, embankments, and earth dams, a description of shear box tests on clay samples, and an approximate analysis of stability which was the primitive forerunner of the present-day $\varphi = 0$ analysis. Yet after its publication, the book remained almost unknown for seventy years until the study of the stability of clay slopes was again placed on a sound basis by the work of such men as Resal (1910), Bell (1915), Petterson and Hultin (1916), Fellenius (1916-22), Frontard (1922), and others.

The curious neglect of such an important work and, indeed, the almost stationary position of soil mechanics during this interim period is due to two main causes. Firstly, the mechanical properties of soils and especially clays are considered to be more complex than those of construction materials such as masonry, concrete, and steel. Consequently, research was more profitably directed toward structures and hydraulics. Secondly, the period 1840-1910 was dominated in the field of soil mechanics by the conventionalized theories of bearing capacity, earth pressure and slope stability, due principally to Poncelet (1840) and Rankine (1857 and 1862), which although of some value for sands are in most cases misleading for clays. Cohesion was ignored, the curved surface of rupture was forgotten, and the false "angle of repose" reigned supreme.

Modern developments leading to the present state of art began during the second decade of the century. The revival of interest appears to have stemmed from several serious landslides in Sweden and from massive landslides that took place during the construction of the Panama Canal. There are probably two other causes for the rapid progress that followed this renewal of interest. First, at about this time, improved and more widespread understanding of the soil properties was developing. Second, there appeared on the scene the great guiding genius of Terzaghi, who was to weld the principles of mechanics and the properties of soil into a coherent whole that would lay the foundation for a new science.

The slip of Stigberg quay into the harbor at Gothenberg, Sweden, in 1916 touched off an investigation that had profound consequences in slope stability analysis. Petterson and Hultin (1916) investigated the slide. In addition to analyzing this slide, they also studied earlier case histories, assuming circular rupture surfaces. According to the concepts of the time, clay was treated purely as a frictional material.

Professor Moller studied the problem concerning friction angles and safety factors more carefully than had been done earlier and published a book Erddruch-Tabellen. He brought out a second edition of the book in 1922 with the subtitle "Augmented with New Earth Pressure Investigations." This is the first publication on circular sliding surfaces to be internationally known. Professor Fellenius continued working on circular sliding surfaces and published several papers during 1918-26. As a partly new method of analysis instead of the friction method formerly solely used, he took up the question of bringing the cohesive strength of soil into the calculations. He referred to investigations and experiments made by the Geotechnical Commission of the Swedish State Roads. The report, published in 1922, contained many examples of circular slides in Sweden. By direct mathematical treatment combined with graphical methods, he arrived at a series of tables and diagrams for predicting the critical slip surfaces. The circular arc analysis for slope stability problems has, in time, come to be known as the "Swedish Method."

Professor Terzaghi (1936) compared circular arc analysis with logarithmic spirals and cycloids and decided that the circular arc method is the most convenient, as well as sufficiently accurate, for many engineering problems. Since the development of the Swedish Method, many new procedures have been advanced to solve the slope stability problem. These are tabulated, showing the assumptions involved and the originators, in Table I.

Most of the methods available for performing slope stability analysis may be categorized as limit equilibrium methods. The basic assumption of the limit equilibrium approach is that Coulomb's failure criterion is satisfied along the failure surface. A weakness of the limit equilibrium method is that it neglects the soil's stress-strain relationship. In an attempt to take into account the stress-strain relationship in analyzing the stability of slopes, it has been suggested that the theory of plasticity be applied to the problem (Drucker and

TABLE I

CLASSIFICATION OF SLOPE STABILITY ANALYSIS BY LIMIT EQUILIBRIUM METHOD

(After Fank and Hirst, 1970)

Type of Failure Plane	Name of Method	Type of Solution	Basic Assumptions	References
Straight line	Culmann Method	Analytical	Failure occurs on a plane through the toe of the slope.	Culmann (1866)
	Method of Infinite Slope	Analytical	The slope is constant with unlimited extent.	Resal (1910)
			A vertical column is typical of the entire mass. No cohesion may be depended on within the depth to which tension occurs.	Frontard (1922)
	Wedge	Semigraphi-cal-analyti-cal	Sliding block mechanism is assumed with lateral earth forces.	Culmann (1866), Terzaghi and Peck (1948)
Circular Arc	Slices Method	Semigraphi-cal-numeri-cal	The lateral forces are equal on two sides of each slice.	Fellenius (1939)
	Bishop's Method	Analytical-Numerical	Oblique side forces on each slice are considered.	Bishop (1955)
	Simplified Bishop's	Analytical-Numerical	Vertical component of lateral earth forces are considered to be equal and opposite.	Bishop (1955), Little and Price (1958)

TABLE I (CONTINUED)

Type of Failure Plane	Name of Method	Type of Solution	Basic Assumptions	References
	φ - Circle Method	Analytical-Graphical	Resultant acting on rupture arc is tangential to a concentric circle with radius = R sin φ_d.	Gilboy (1933), Taylor (1937), Casagrande (1934)
	Modified φ - Circle	Analytical-Graphical	The resultant misses tangency to the φ_d circle by a small amount. Radius of φ_d circle = K R sin φ_d.	Taylor (1937)
Logarithmic Spiral	Log-spiral Method	Analytical	No assumptions required to make the problem statically determinate.	Rendulic (1935), Taylor (1937), Spencer (1969)
Irregular	Irregular	Analytical-Numerical	General slip surface. Forces between slices are considered.	Morgenstern and Price (1965)

Prager, 1952). Analytical procedures have been developed to solve homogeneous and isotropic earth slopes using plastic theory (Chen, Giger and Fang, 1969; Fang and Hirst, 1970; Chen and Giger, 1971).

In most of these methods, however, the basic principles have remained the same. The earlier methods were based on the assumptions that the soil is homogeneous and isotropic. Neither of these is true, as may be seen from the brief review of the published literature cited elsewhere in this chapter. In view of this, some recent procedures have utilized more realistic assumptions for the solution of slope stability problems (Morgenstern and Gibson, 1962; Lo, 1965; and others).

It is often necessary to determine the factor of safety of cuts in normally consolidated clays. These clays characteristically exhibit a linear increase of strength with depth, as shown in Figure 1. Close to the surface, the clay is usually overconsolidated due to desiccation; and the strength of this crust is higher than that of the material immediately below it. Morgenstern and Gibson (1962) have analyzed the slope stability problem in homogeneous soils with strength increasing linearly with depth, as shown in Figure 2.

The expression for the factor of safety established by Morgenstern and Gibson implies that the ground water table is at or above the ground surface. Their approach to this problem was extended by Hunter and Schuster (1969) to include an analytical solution for the common case of ground water table below the ground surface. This method permits a better estimate of the factor of safety where the shear strength C is greater than zero at the ground surface but still increases linearly with depth. Hunter and Schuster present graphically the results of

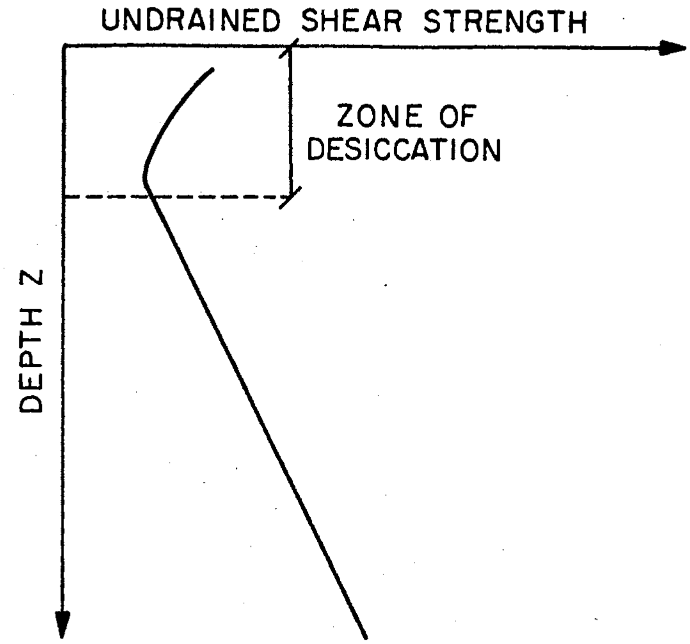


Figure 1. Typical Variation of Undrained Strength with Depth for a Normally Consolidated Clay

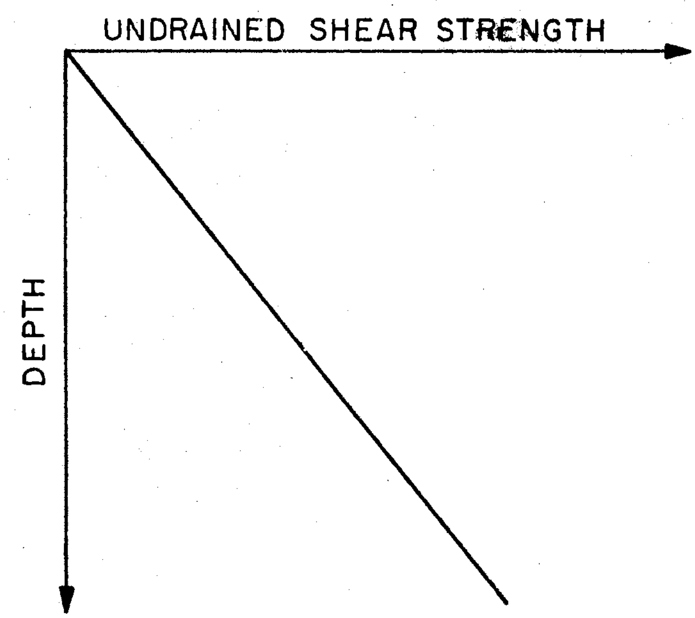


Figure 2. Idealized Strength Depth Relationship

computations of stability numbers for a range of slope inclinations and depths of water table.

In nature, the clay-size particles are nonsymmetrical in shape. They are usually longer in two directions than in the third. Due to this nonsymmetrical shape, the soil deformations and the soil-forming processes produce within the soil fabric an anisotropic structure. This anisotropic structure is reflected by directional variations in the physical properties of soil such as strength, compressibility, and permeability. There is ample evidence in literature confirming the occurrence of anisotropic soil fabric and anisotropic physical properties in natural and remolded cohesive soils.(Mitchell, 1956; Hvorslev, 1960; Duncan and Seed, 1966; and others).

Anisotropy of clays with respect to strength is probably related to the orientation of clay particles. The structure of clay was studied by many investigators (Lambe, 1953; Mitchell, 1956; Pacey, 1956; Martin, 1962; and others). These studies have shown that the clay particles tend to become oriented parallel to the major principal plane during anisotropic consolidation. Mitchell (1956) studied the structure of seven undisturbed marine clays and one lacustrine clay with the aid of a petrographic microscope. From these studies, he concluded that these clays had some degree of parallel orientation. With one exception, these clays were consolidated to nearly 3 kg/sq cm. Quigley and Thompson (1966), studying the relationship between soil fabric and the anisotropic consolidation characteristics of Leda Clay from Ottawa found that anisotropic consolidation of undisturbed samples causes reorientation of the clay platelets into a plane perpendicular to the direction of major principal consolidation pressure. Martin (1962) studied the structure of Kaolinite Clay using X-ray diffraction technique. He compared the peak amplitudes of diffracted X-rays from 002 and 020 planes of the clay and concluded that the clay was approximately "ideally oriented" after one-dimensional consolidation to 197 kg/sq cm., and approximately "ideally random" after isotropic consolidation to 1 kg/sq cm.

It may be concluded from these studies that there is a tendency for the clay-size particles to become oriented parallel to the plane on which the major principal stress acts during consolidation. This parallel particle orientation might be expected to cause anisotropy in physical properties such as strength, compressibility, and permeability. Unconsolidated undrained triaxial tests on samples trimmed in different directions have shown that anisotropically consolidated clays are anisotropic with respect to undrained strength.

For a soil to be perfectly isotropic, the coefficient of earth pressure at rest, K_o, should be 1, as isotropic consolidation requires a hydrostatic state of stress to exist. As early as 1920, Terzaghi reported the value of K_o for a coarse sand to be 0.42. In 1925, he reported a value of 0.7 for a yellow residual clay and a blue marine clay; and experiments by Kjellman with triaxial equipment yielded values ranging from 0.5 to 1.5. K_o is supposed to be a function of stress history of soil. The assumptions that $K_o = 1 - \sin \varphi'$ (Jaky) or $K_o = 0.95 - \sin \varphi'$ (Brooker and Ireland, 1965) indicate that few soils have $K_o = 1$.

In reviewing the published literature regarding anisotropic strength characteristics of clays, the definition for shear strength as shown in Figure 3 is followed.

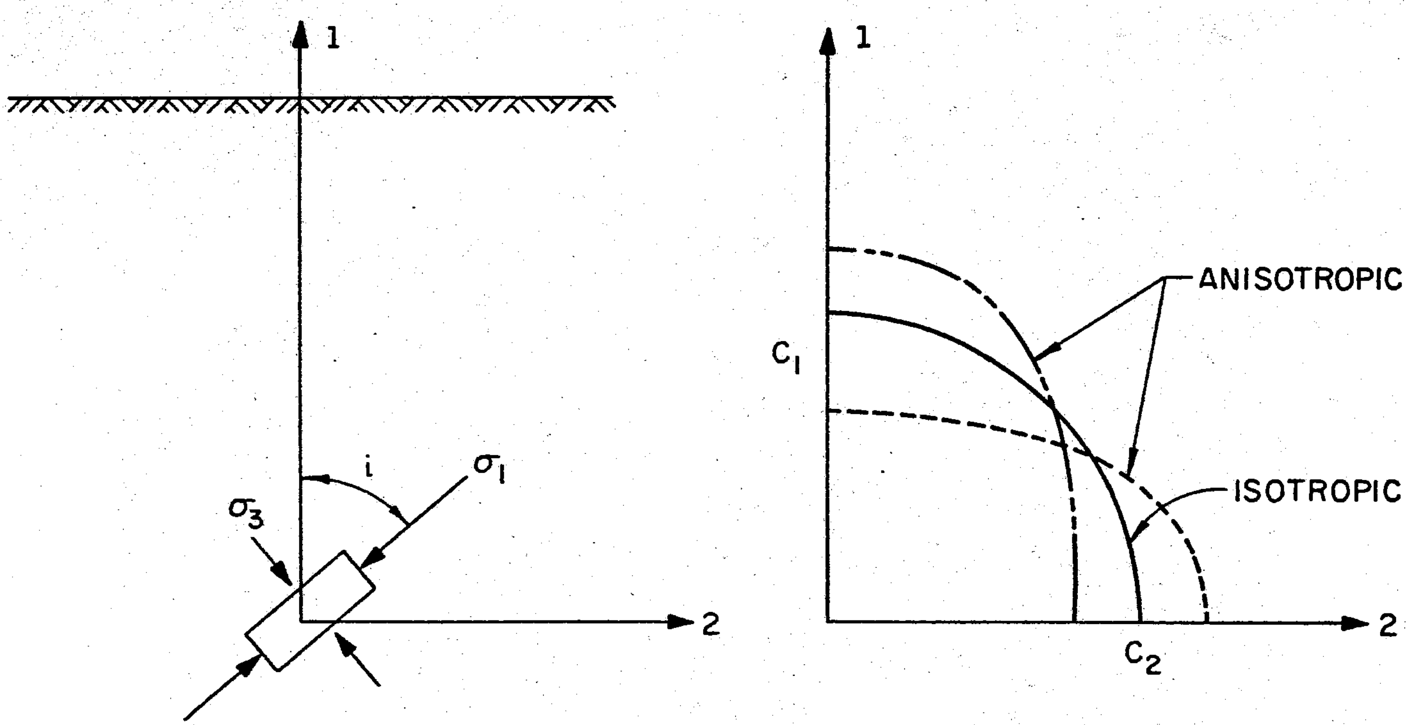


Figure 3. Definition of Shear Strength Variation with Direction (After Lo, 1965)

The physical vertical and horizontal directions, which usually coincide with lines perpendicular and parallel to the bedding planes of soil deposit are the principal directions. If the sample is tested with the major principal stress direction coinciding with the principal directions, the strengths thus determined (C_1 and C_2) are known as principal strengths. When the major principal stress makes an angle, i, with the vertical, the strength then determined is designated as C_i. For a soil with isotropic strength characteristics, the principal strengths C_1 and C_2 and C_i are equal. In other words, the curve traced by C_i in a vertical plane is a circle. However, for a soil having anisotropic strength characteristics, the principal strengths C_1 and C_2 are not equal and the curve traced by C_i is not a circle. The ratio of principal strengths C_2/C_1 is termed as the degree of anisotropy. (Ranganatham and Mathai, 1967, denote it as the Coefficient of Orthotropy.) Depending on the stress history, clay particle orientation, etc., the ratio C_2/C_1 is less than or greater than one. For convenience, the former is designated as M-anisotropy and the latter as C-anisotropy (Lo, 1965). Soil deposits with C_2/C_1 equal to unity are rather rare in nature.

Lo (1965) performed unconfined compression tests on undisturbed samples of clay from Welland, Ontario, Canada. It was reported that the horizontal strengths were less than the vertical strengths, the ratio C_2/C_1 varying between 0.64 and 0.8 (M-anisotropy).

Aas (1965), using vanes of different shapes, performed vane tests on Canadian clays. He reported the ratio of undrained shear strengths acting along horizontal and vertical failure surfaces to be 1.5 to 2.0 (C-anisotropy). Ward, Samuels, and Butler (1959) have reported from their tests on London clay that horizontal strengths were greater than vertical strengths. The ratio C_2/C_1 was established to be 1.3 ± 0.1 (C-anisotropy). The higher strengths exhibited in the horizontal samples may be related to the fact that London clay is heavily overconsolidated and the horizontal stresses in the ground are considerably higher than the vertical stresses (Skempton, 1961). Skempton has shown that the ratio of horizontal-to-vertical stress in the overconsolidated London clay varies from 2.5 at the top to 1.5 at a depth of 100 feet below the surface. Some examples in which C_u, the undrained strength, depends on principal stress directions during shear are furnished in Table II.

From the brief review of published literature, it may be concluded that in nature the rule is anisotropy, isotropy being an exception. There is ample evidence in literature to show that stability of earth masses is affected by strength anisotropy (Lo, 1965; Ranganatham and Mathai, 1967; Livneh, 1967; and others). These investigators have used different strength variations, which are reviewed below, to account for strength anisotropy.

Lo (1965) developed a general method of stability analysis for anisotropic soils. Rigorous solutions were obtained for two cases: (a) the vertical strength was constant with depth, and (b) the vertical strength varied linearly with depth. He assumed the following strength variation, as suggested by Carillo and Casagrande (1942).

$$C_i = C_2 + (C_1 - C_2) \cos^2 i,$$

where

C_i = shear strength when the major principal stress at failure is inclined at an angle, i, to the vertical

C_1 and C_2 = principal strengths in directions of principal stresses.

Charts of slope angle versus stability number were presented.

TABLE II

SOME EXAMPLES IN WHICH C_u DEPENDS ON PRINCIPAL STRESS DIRECTIONS DURING SHEAR (TESTS IN SITU OR ON UNDISTURBED SAMPLES)

1. C_u from field vane lower than C_u from piston samples or block samples. (Vold, 1956; Coates and McRostie, 1963).

2. C_u from field vane for vertical plane lower than for horizontal plane. (Aas, 1965).

3. C_u from block samples with axis horizontal lower than with axis vertical in lightly overconsolidated clay. (Lo, 1965).

4. C_u from block samples with axis horizontal higher than with axis vertical in heavily overconsolidated clay. (Ward, Marsland, and Samuels, 1965).

Livneh (1967) has also studied the effect of strength anisotropy on slope stability. In his analysis, the following variation for strength was assumed (Figure 4).

$$C_\alpha = C_1 \left[1 + (n - 1) \sin^{2K} (\alpha - \psi)\right]$$

where

C_1 = minimum cohesion,

C_2 = maximum cohesion,

n = the anisotropy index defined as the ratio C_2/C_1,

α = the angle between the horizontal axis and an arbitrary axis of reference,

ψ = the angle between the horizontal axis and the minimum cohesion axis, and

K = a positive integer permitting characterization of different cohesion patterns in terms of angle α.

Livneh presented charts giving the slope angle versus stability factor ($\gamma H/C$) for various values of ψ, K, and n. He showed that neglecting the anisotropy factor would lead to results that are either conservative or in error on the unsafe side. Disregarding the anisotropy factor and assuming that the soil is isotropic may lead to a decrease in the computed factor of safety. For example, a 27 percent decrease is obtained for $\beta = 10^\circ$, $n = 2$, $K = 1$, $n_d = 1.5$, and $\psi = 60^\circ$.

Ranganatham and Mathai (1967) have analyzed the effect of strength anisotropy on the stability of earth masses. They have assumed the following variation to account for anisotropy:

$$C = C_h(\cos^2 \theta + n \sin^2 \theta)$$

where

C = cohesion along a plane inclined at an angle to θ to horizontal,

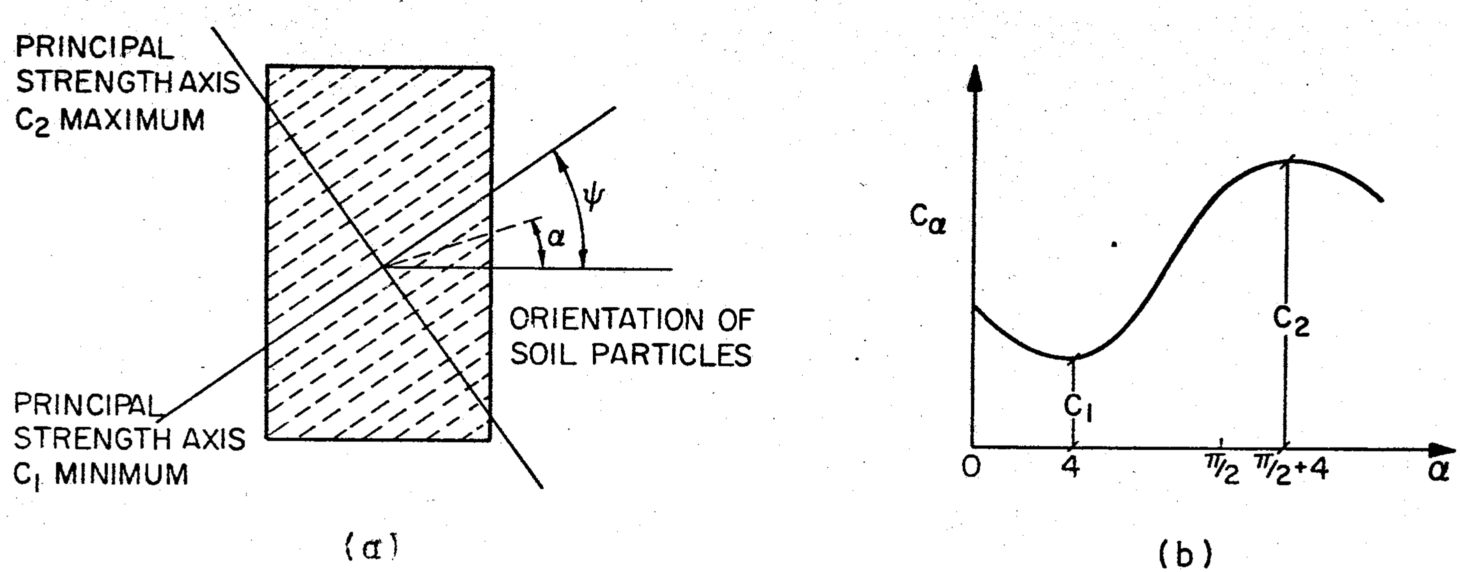


Figure 4. The Definition of Anisotropic Cohesional Medium
(a) The Principal Strength Axes
(b) The Variation of Cohesion with α
(After Livneh, 1967)

C_h = cohesive strength along the horizontal plane,

C_v = cohesive strength along the vertical plane, and

n = C_v/C_h, called the anisotropic strength ratio (also called the coefficient of orthotropy).

The proposed method followed the analysis of Janbu (1954) (based on dimensionless parameters for base failure in purely cohesive soil) except for strength anisotropy. Charts were presented giving the slope angle versus the stability number ($C/\gamma H$) for various values of the coefficient of orthotropy and depth to hard stratum. It was noticed that for vertical cohesive strength ranging from half to double the horizontal, the stability number changed by about +30 percent to -40 percent of the isotropic case. From the numerical results, it was concluded that the influence of anisotropy on stability is much greater than is the depth to hard stratum.

In the above methods, two assumptions are made: (1) Soil mass is homogeneous, and (2) it is purely cohesive ($\varphi = 0$ condition).

When estimating the stability of foundations and slopes, it is often assumed that the soil is homogeneous and isotropic; but it is known that the shear strength increases with depth beyond the zone of desiccation and is also dependent on the direction of the failure surface. While it is difficult to describe the exact functional relationship between the shear strength and depth and direction of failure surface, Ranganatham, Sani, and Sreenivasulu (1969) investigated through carefully-planned experiments the probable variation of strength with depth and direction of failure plane and used these findings in evaluating slope stability. Experimental work was done to obtain the variation in shear strength (1) with direction of failure surface, keeping consolidation pressure constant, and (2) with consolidation pressure, keeping the direction of failure surface constant.

This experimental study lends support to the hypothesis that the undrained strength of an element of soil along a plane other than the horizontal or vertical is equal to the vectorial sum of those acting on the projected areas of the element in the vertical and horizontal planes. Expressed mathematically, the strength C_θ along a plane inclined at an angle θ to the horizontal is given by

$$C_\theta = C_h \cos^2 \theta + C_v \sin^2 \theta$$

where C_h and C_v are strengths in the horizontal and vertical planes, respectively.

The shear strength at any depth Z in relation to the direction of failure surface was defined as follows:

$$C_{hZ} = C_{ho} (1 + \ell_h \frac{Z}{H})$$

$$C_{vZ} = C_{vo} (1 + \ell_v \frac{Z}{H})$$

$$= n_{co} C_{ho} (1 + \ell_v \frac{Z}{H})$$

where

C_{hZ} = strength along the horizontal plane at any depth Z,

C_{vZ} = strength along the vertical plane at any depth Z,

C_{ho} = strength along the horizontal plane at the surface,

C_{vo} = strength along the vertical plane at the surface, and

ℓ_h and ℓ_v = coefficients defining the variation of C_h and C_v over a significant depth H.

On substituting for C_{hZ} and C_{vZ} in the equation for C_θ, the following expression for the undrained shear strength at any depth Z on any failure surface inclined at an angle θ to the horizontal is obtained:

$$C_{\theta Z} = C_{ho}(1 + \ell_h \frac{Z}{H}) \cos^2 \theta + n_{co}(1 + \ell_v \frac{Z}{H}) \sin^2 \theta .$$

Control charts, providing a critical combination of stability number and tangent of friction angle were presented for a given slope ($\beta = \tan^{-1} 1/25$) and various values of the coefficient of orthotropy, depth to hard layer, etc. Numerical results presented demonstrate the influence of strength anisotropy and strength increase with depth on the control charts.

CHAPTER III

ANALYTICAL METHOD FOR STABILITY OF EARTH SLOPES IN NON-HOMOGENEOUS AND ANISOTROPIC SOILS

Introduction

It is evident from the review of published literature (Chapter II) that soil is neither homogeneous nor isotropic. Consequently, when analyzing the stability of earth slopes, this fact should be recognized and accounted for.

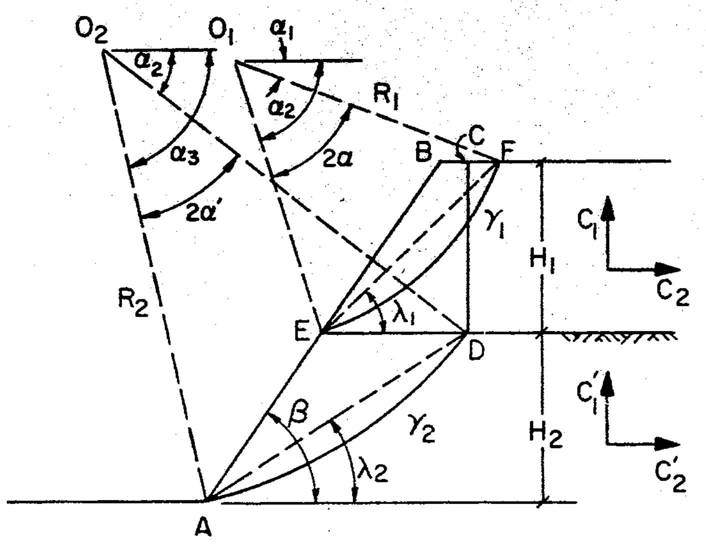
In this chapter, an analytical method is suggested for evaluating the stability of slopes in a two-layered system of anisotropic soils. The basic assumptions made in the analysis are listed below. Following this, the working formulae used in arriving at the factor of safety for the slope are derived in detail.

Basic Assumptions

1. The controlling potential surface of failure is either cylindrical or a combination of planar and cylindrical surfaces, as shown in Figure 5.

2. The soil in each layer is homogeneous with respect to shear strength.

3. The coefficient of anisotropy is the same at all points in the slope.



$n = H_2/H_1$

$m = \gamma_2/\gamma_1$

$K = C_2/C_1 \; ; \; K' = C'_2/C'_1$

$C'_1 = (p+1)C_1$

$C'_2 = (p+1)C_2$

Figure 5. Slip Surfaces for a Two-Layer System of Anisotropic Soils

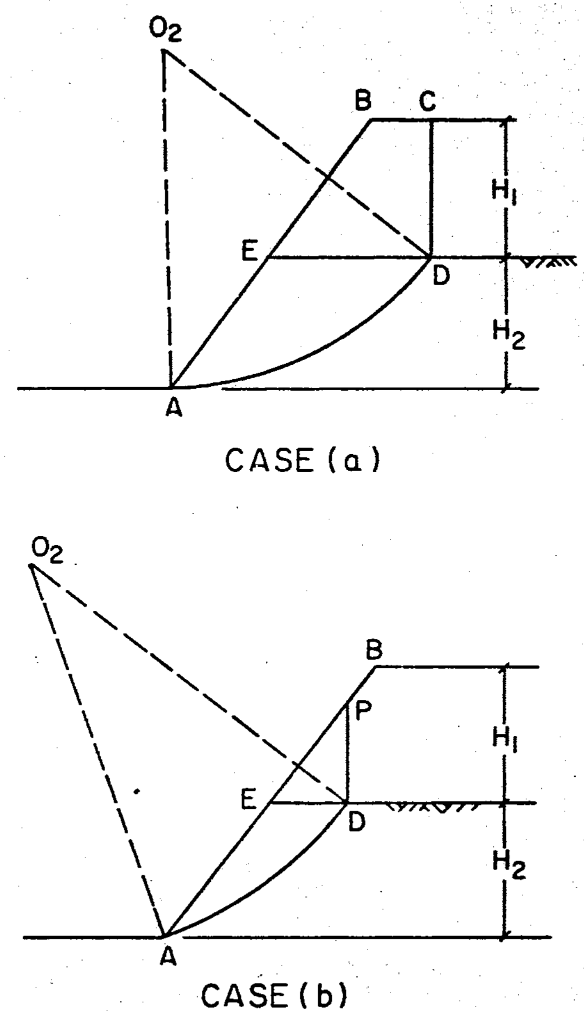


Figure 6. Slip Surfaces for [illegible] Layer 2

4. The anisotropic strength in each layer is characterized by the following equation:

$$C_i = C_h + (C_v - C_h) \cos^2 i$$

where

C_i = shear strength along a plane inclined at an angle, i, to the vertical; and,

C_h and C_v = shear strengths along the horizontal and vertical planes respectively.

5. The stability of the slope is analyzed by considering the stability of the individual layers.

Derivation of Working Formulae

The working formulae for the factor of safety are derived separately for the two layers.

Layer 2

There are two possible types of slip surfaces, as shown in Figure 6. These could be designated as Case (a) and Case (b). For each one of these two cases, the expressions for disturbing moment, resisting moment, and the factor of safety are derived in detail.

Case (a). For limiting equilibrium of the mass above the potential surface of rupture ADC (Figure 6a), the total disturbing moment about O_2 must be equal to the total resisting moment about the same point.

While evaluating the disturbing moment for this case, the mass of soil above the interface ED is taken to be acting as surcharge. Hence, the disturbing moment due to this is considered in addition to that contributed by the soil above the slip surface AD. The expressions for these two moments are derived below.

Let the disturbing moment due to surcharge load (EBCD) be M_{D_1}.

Weight of Soil Mass W_1

$$= \gamma_1 H_1 H_2 (\cot \lambda_2 - \cot \beta) - \frac{H_1^2}{2} \cot \beta.$$

Let ℓ_1 be the lever arm for this mass about O_2.

$$\ell_1 = \gamma_1 H_1 \left[H_2 (\cot \lambda_2 - \cot \beta) - H_1 \cot \beta\right] \left[R_2 \cos \alpha_2\right.$$

$$\left. + \frac{H_1}{2} \cot \beta - \frac{H_2}{2} (\cot \lambda_2 - \cot \beta)\right] + \frac{\gamma_1 H_1^2}{2} \cot \beta$$

$$\times \left[R_2 \cos \alpha_2 - H_2 (\cot \lambda_2 - \cot \beta) + \frac{2}{3} H_1 \cot \beta\right]$$

$$\div \gamma_1 \left[H_1 H_2 (\cot \lambda_2 - \cot \beta) - H_1^2 \cot \beta + \frac{H_1^2}{2} \cot \beta\right].$$

Moment of EBCD about O_2

$$= W_1 \ell_1.$$

$$M_{D_1} = \gamma_1 \left[H_1 \left\{H_2 (\cot \lambda_2 - \cot \beta) - H_1 \cot \beta\right\} \left\{R_2 \cos \alpha_2\right.\right.$$

$$\left. + \frac{H_1}{2} \cot \beta - \frac{H_2}{2} (\cot \lambda_2 - \cot \beta)\right\} + \frac{H_1^2}{2} \cot \beta$$

$$\left. \times \left\{R_2 \cos \alpha_2 - H_2 (\cot \lambda_2 - \cot \beta) + \frac{2}{3} H_1 \cot \beta)\right\} \right].$$

Let the disturbing moment due to mass of soil enclosed in ADE (Figure 6a) be M_{D_2}.

Weight of soil mass (triangle AED)

$$= \frac{\gamma_2 H_2^2}{2} (\cot \lambda_2 - \cot \beta).$$

Weight of soil mass (segment AD)

$$= \frac{\gamma_2 R_2^2}{2} (\alpha' - \frac{1}{2} \sin 2\alpha').$$

Total weight of woil mass (W_2)

$$= \frac{\gamma_2 H_2^2}{2} (\cot \lambda_2 - \cot \beta) + \frac{\gamma_2 R_2^2}{2} (\alpha' - \frac{1}{2} \sin 2\alpha') .$$

Let ℓ_2 be the lever arm for this mass about O_2.

$$\ell_2 = \Big[\Big\{ \frac{H_2^2}{2} (\cot \lambda_2 - \cot \beta) \Big\} \Big\{ R_2 \cos \alpha_2 - \frac{1}{3} (2H_2 \cot \lambda_2 - H_2 \cot \beta) \Big\} + \Big\{ R_2^2 \alpha' - \frac{R_2^2}{2} \sin (\alpha_3 - \alpha_2) \Big\} \times \frac{2}{3} \frac{R_2 \sin^3 \alpha'}{(\alpha' - \sin \alpha' \cos \alpha')} \cos (\alpha_2 + \alpha') \Big] \Big/ \frac{H_2^2}{2} (\cot \lambda_2 - \cot \beta) + R_2^2 \alpha' - \frac{R_2^2}{2} \sin 2\alpha'.$$

Moment of ADE about O_2

$$= W_2 \ell_2.$$

$$M_{D_2} = \gamma_2 \Big[\frac{H_2^2}{2} (\cot \lambda_2 - \cot \beta) \Big\{ R_2 \cos \alpha_2 - \frac{1}{3} (2H_2 \cot \lambda_2 - H_2 \cot \beta) \Big\} + \frac{2R_2^3}{3} \sin^3 \alpha' \cos (\alpha_2 + \alpha') \Big].$$

The total disturbing moment ($M_{D_{II}}$) is the sum of M_{D_1} and M_{D_2}.

$$M_{D_{II}} = \gamma_1 \Big[H_1 \Big\{ H_2 (\cot \lambda_2 - \cot \beta) - H_1 \cot \beta \Big\} \Big\{ R_2 \cos \alpha_2$$

$$+ \frac{H_1}{2} \cot \beta - \frac{H_2}{2} (\cot \lambda_2 - \cot \beta) \Big\} + \frac{H_1^{\ 2}}{2} \cot \beta$$

$$\times \Big\{ R_2 \cos \alpha_2 - H_2 (\cot \lambda_2 - \cot \beta) + \frac{2}{3} H_1 \cot \beta) \Big\} \Big]$$

$$+ \gamma_2 \Big[\frac{H_2^{\ 2}}{2} (\cot \lambda_2 - \cot \beta) \Big\{ R_2 \cos \alpha_2 - \frac{1}{3} (2H_2 \cot \lambda_2$$

$$- H_2 \cot \beta) \Big\} + \frac{2R_2^{\ 3}}{3} \sin^3 \alpha' \cos (\alpha_2 + \alpha') \Big] .$$

From the geometry of the problem,

$$R_2 = \frac{H_2}{2 \sin \alpha' \sin \lambda_2},$$

$$\alpha_2 = 90 - (\lambda_1 + \alpha),$$

$$\frac{H_2}{H_1} = n$$

$$\frac{\gamma_2}{\gamma_1} = m.$$

Substituting these values in the above equation and simplifying,

$$M_{D_{II}} = \gamma_2 H_2^{\ 3} \Big[\frac{1}{2mn} (\cot \lambda_2 - \cot \beta) (\cot \alpha' + \cot \beta)$$

$$+ \frac{1}{4mn^2} \cot \beta (\cot \lambda_2 - \cot \alpha' - 4 \cot \beta + 2 \cot^2 \beta)$$

$$- \frac{1}{6mn^3} \cot^2 \beta + \frac{1}{12} (1 - 2 \cot^2 \beta + 3 \cot \lambda_2 \cot \alpha'$$

$$+ 3 \cot \beta \cot \lambda_2 - 3 \cot \beta \cot \alpha') \Big] .$$

The resisting moment for this case consists of two parts, M_{R_1} and M_{R_2}.

M_{R_1} = Resisting moment due to cohesion along CD,

M_{R_2} = Resisting moment due to cohesion along AD.

$$M_{R_1} = (C_1)(\overline{CD})\, \ell_1$$

where ℓ_1 = lever arm about O_2.

$$M_{R_1} = C_1 H_1 R_2 \cos \alpha_2 .$$

From the geometry of the problem,

$$R_2 = \frac{H_2}{2 \sin \alpha' \sin \lambda_2}$$

$$\alpha_2 = 90 - (\lambda_2 + \alpha').$$

Substituting these values for R_2 and α_2 in the above equation,

$$M_{R_1} = \frac{C_1 H_1 H_2 \sin (\lambda_2 + \alpha')}{2 \sin \alpha' \sin \lambda_2} .$$

Putting $H_2/H_1 = n$, and simplifying,

$$M_{R_1} = \frac{C_1 H_2^{\,2}}{n} (\cot \alpha' + \cot \lambda_2).$$

$$M_{R_2} = R_2 \int_{\alpha_2}^{\alpha_3} C(\theta, Z)\, R_2 d\theta$$

$$= R_2^{\,2} \int_{\alpha_2}^{\alpha_3} \left[C_2' + (C_1' - C_2') \right] \cos^2 i \, d\theta$$

$$= R_2^{\,2} \int_{\alpha_2}^{\alpha_3} C_1' \left[\frac{C_2'}{C_1'} + (1 - \frac{C_2'}{C_1'}) \right] \cos^2 i \, d\theta . \; .$$

On integrating the above expression, and putting $C_2'/C_1' = K'$, the explicit value for M_{R_2} is obtained as

$$M_{R_2} = R_2^2\left[(1 + K')\,C_1'\alpha' - \frac{1}{2}(1 - K')\,C_1' \sin 2\alpha' \cos(2\alpha' - 2\lambda_2)\right].$$

On further simplification, the above expression for M_{R_2} reduces to

$$M_{R_2} = \frac{C_1' H_2^2}{8 \sin^2\alpha' \sin^2\lambda_2}\left[2(1 + K')\alpha' - (1 - K') \sin 2\alpha' \cos(2\alpha' - 2\lambda_2)\right].$$

The total resisting moment ($M_{R_{II}}$) for this case is the sum of M_{R_1} and M_{R_2}.

$$M_{R_{II}} = \frac{C_1 H_2^2}{n}(\cot\alpha' + \cot\lambda_2) + \frac{C_1' H_2^2}{8 \sin^2\alpha' \sin^2\lambda_2}$$

$$\times \left[2\alpha'(1 + K') - (1 - K') \sin^2\alpha' \cos(2\alpha' - 2\lambda_2)\right].$$

Putting $C_1' = (p + 1)\,C_1$,

$$M_{R_{II}} = C_1 H_2^2\left[\frac{1}{n}(\cot\alpha' + \cot\lambda_2) + \frac{(p + 1)}{8 \sin^2\alpha' \sin^2\lambda_2}\right.$$

$$\left.\times \left\{2\alpha'(1 + K') - (1 - K') \sin 2\alpha' \cos(2\alpha' - 2\lambda_2)\right\}\right].$$

The factor of safety, F, is given by

$$F = \frac{\text{Resisting Moment } (M_{R_{II}})}{\text{Disturbing Moment } (M_{D_{II}})}.$$

$$F = C_1 H_2^{\,2} \Big[\frac{1}{n}(\cot \alpha' + \cot \lambda_2) + \frac{(p+1)}{8 \sin^2 \alpha' \sin^2 \lambda_2}$$

$$\times \Big\{ 2\alpha' (1 + K') - (1 - K') \sin 2\alpha' \cos (2\alpha' - 2\lambda_2) \Big\} \Big]$$

$$\div \; \gamma_2 H_2^{\,3} \Big[\frac{1}{2mn} (\cot \lambda_2 - \cot \beta)(\cot \alpha' + \cot \beta)$$

$$+ \frac{1}{4mn^2} \cot \beta \, (\cot \lambda_2 - \cot \alpha' - 4 \cot \beta + 2 \cot^2 \beta)$$

$$- \frac{1}{6mn^3} \cot^2 \beta + \frac{1}{12} (1 - 2 \cot^2 \beta + 3 \cot \lambda_2 \cot \alpha'$$

$$+ 3 \cot \beta \cot \lambda_2 - 3 \cot \beta \cot \alpha') \Big].$$

The above expression for the factor of safety may be conveniently expressed as

$$F = \frac{C_1}{\gamma_2 H_2} N_2$$

where N_2 is termed a stability number.

$$N_2 = \frac{\cot \alpha' + \cot \lambda_2}{2n} + \frac{(p+1)}{8 \sin^2 \alpha' \sin^2 \lambda_2} \Big\{ 2\alpha' (1 + K')$$

$$- (1 - K') \sin 2\alpha' \cos (2\alpha' - 2\lambda_2) \Big\} \Big/ \frac{1}{2mn}$$

$$\times (\cot \lambda_2 - \cot \beta)(\cot \alpha' + \cot \beta) + \frac{1}{4mn^2} \cot \beta$$

$$\times (\cot \lambda_2 - \cot \alpha' - 4 \cot \beta + 2 \cot^2 \beta) - \frac{1}{6mn^3}$$

$$\times \cot^2 \beta + \frac{1}{12} (1 - 2 \cot^2 \beta + 3 \cot \lambda_2 \cot \alpha'$$

$$+ 3 \cot \beta \cot \lambda_2 - 3 \cot \beta \cot \alpha').$$

It is obvious that the minimum factor of safety is obtained by minimizing the stability number, N_2, with respect to α' and λ_2, so that

$$\frac{\partial N_2}{\partial \alpha'} = 0$$

$$\frac{\partial N_2}{\partial \lambda_2} = 0.$$

The foregoing operations may be carried out with the aid of a computer, and N_2 minimum would be a function of K', m, n, p, and β. For given values of K', m, n, and p, the stability number is a function of β alone.

Computations have been carried out for values of K' ranging from 0.5 to 1.0, n ranging from 1.0 to 3.0, and p ranging from -0.5 to +0.5. Since the unit weights of soils in the two layers would not ordinarily differ greatly, $m = \gamma_2/\gamma_1$ is assumed to be unity.

<u>Case (b).</u> The approach followed in arriving at the expressions for disturbing moment, resisting moment, and the factor of safety is the same as that for Case (a).

The total disturbing moment, $M_{D_{II}}$, is the sum of the disturbing moments M_{D_1} and M_{D_2}.

$$M_{D_1} = \gamma_1 \left\{ \frac{H_2^2}{2} (\cot \lambda_2 - \cot \beta)(\cot \lambda_2 \tan \beta - 1) \right\} \times \left\{ R_2 \cos \alpha_2 - \frac{1}{3} (\cot \lambda_2 - \cot \beta) \right\} ;$$

$$M_{D_2} = \gamma_2 \left[\frac{H_2^2}{2} (\cot \lambda_2 - \cot \beta) \left\{ R_2 \cos \alpha_2 - \frac{1}{3} (2H_2 \cot \lambda_2 - H_2 \cot \beta) \right\} + \frac{2}{3} R_2^3 \sin^3 \alpha' \cos (\alpha^2 + \alpha') \right].$$

The total disturbing moment ($M_{D_{II}}$) is found by

$$M_{D_{II}} = M_{D_1} + M_{D_2}.$$

$$M_{D_{II}} = \gamma_1 \left\{ \frac{H_2^{\,2}}{2} (\cot \lambda_2 - \cot \beta)(\cot \lambda_2 \tan \beta - 1) \right\}$$

$$\times \left\{ R_2 \cos \alpha_2 - \frac{1}{3}(\cot \lambda_2 - \cot \beta) + \gamma_2 \left[\frac{H_2^{\,2}}{2} (\cot \lambda_2 \right.\right.$$

$$- \cot \beta) \left\{ R_2 \cos \alpha_2 - \frac{1}{3}(2H_2 \cot \lambda_2 - H_2 \cot \beta) \right\}$$

$$\left. + \frac{2}{3} R_2^{\,3} \sin^3 \alpha' \cos(\alpha_2 + \alpha') \right] .$$

From the geometry of the problem,

$$R_2 = \frac{H_2}{2 \sin \alpha' \sin \lambda_2}$$

$$\alpha_2 = 90 - (\lambda_2 + \alpha')$$

and as per the notation

$$n = \frac{H_2}{H_1}$$

$$m = \frac{\gamma_2}{\gamma_1} .$$

Substituting these values in the above equation for the disturbing moment, and on simplifying,

$$M_{D_{II}} = \frac{\gamma_2 H_2^{\,3}}{12} \left[\frac{1}{m} (\cot \lambda_2 - \cot \beta)(\cot \lambda \tan \beta - 1)(3 \cot \alpha' \right.$$

$$+ \cot \lambda_2 + 2 \cot \beta) + (\cot \lambda_2 - \cot \beta)(3 \cot \alpha' - \cot \lambda_2$$

$$\left. + 2 \cot \beta) + \operatorname{cosec}^2 \lambda_2 \right] .$$

The resisting moment consists of two parts, M_{R_1} and M_{R_2}, as in Case (a). The total resisting moment ($M_{R_{II}}$) is the sum of M_{R_1} and M_{R_2}.

$$M_{R_1} = \frac{C_1 H_2^{\ 2}}{2} (\cot \lambda_2 \tan \beta - 1)(\cot \lambda_2 + \cot \alpha') ;$$

$$M_{R_2} = \frac{C_1' H_2^{\ 2}}{8 \sin^2 \alpha' \sin^2 \lambda_2} \Big[2\alpha' (1 + K') - (1 - K')$$

$$\times \sin 2\alpha' \cos (2\alpha' - 2\lambda_2) \Big] .$$

$$M_{R_{II}} = M_{R_1} + M_{R_2}$$

$$M_{R_{II}} = \frac{C_1 H_2^{\ 2}}{2} (\cot \lambda_2 \tan \beta - 1)(\cot \lambda_2 + \cot \alpha')$$

$$+ \frac{C_1' H_2^{\ 2}}{8 \sin^2 \alpha' \sin^2 \lambda_2} \Big[2\alpha' (1 + K') - (1 - K')$$

$$\times \sin^2 \alpha' \cos (2\alpha' - 2\lambda_2) \Big] .$$

Putting $C_1' = (p + 1) C_1$,

$$M_{R_{II}} = C_1 H_2^{\ 2} \Big[\frac{1}{2} (\cot \lambda_2 \tan \beta - 1)(\cot \lambda_2 + \cot \alpha')$$

$$+ \frac{(p + 1)}{8 \sin^2 \alpha' \sin^2 \lambda_2} \Big\{ 2\alpha' (1 + K') - (1 - K')$$

$$\times \sin 2\alpha' \cos (2\alpha' - 2\lambda_2) \Big\} \Big] .$$

The factor of safety, F, is given by

$$F = \frac{\text{Resisting Moment } (M_{R_{II}})}{\text{Disturbing Moment } (M_{D_{II}})}$$

$$F = C_1 H_2^2 \left[\frac{1}{2} (\cot \lambda_2 \tan \beta - 1)(\cot \lambda_2 + \cot \alpha') + \frac{(p+1)}{8 \sin^2 \alpha' \sin^2 \lambda_2} \left\{ 2\alpha' (1 + K') - (1 - K') \sin 2\alpha' \times \cos (2\alpha' - 2\lambda_2) \right\} \right] \Bigg/ \frac{\gamma_2 H_2^3}{12} \left[\frac{1}{m} (\cot \lambda_2 - \cot \beta) \times (\cot \lambda_2 \tan \beta - 1)(3 \cot \alpha' + \cot \lambda_2 + 2 \cot \beta) + (\cot \lambda_2 - \cot \beta)(3 \cot \alpha' - \cot \lambda_2 + 2 \cot \beta) + \operatorname{cosec}^2 \lambda_2 \right].$$

The above expression for the factor of safety may be conveniently expressed as

$$F = \frac{C_1}{\gamma_2 H_2} N_2$$

where N_2 is termed a stability number.

$$N_2 = \frac{1}{2} (\cot \lambda_2 \tan \beta - 1)(\cot \lambda_2 + \cot \alpha') + \frac{(p+1)}{8 \sin^2 \alpha' \sin^2 \lambda_2} \times \left\{ 2\alpha' (1 + K') - (1 - K') \sin 2\alpha' \cos (2\alpha' - 2\lambda_2) \right\} \div \frac{1}{12} \left[\frac{1}{m} (\cot \lambda_2 - \cot \beta)(\cot \lambda_2 \tan \beta - 1)(3 \cot \alpha' + \cot \lambda_2 + 2 \cot \beta) + (\cot \lambda_2 - \cot \beta)(3 \cot \alpha' - \cot \lambda_2 + 2 \cot \beta) + \operatorname{cosec}^2 \lambda_2 \right].$$

It is obvious that the minimum factor of safety is obtained by minimizing the stability number, N_2, with respect to α' and λ_2, so that

$$\frac{\partial N_2}{\partial \alpha'} = 0$$

$$\frac{\partial N_2}{\partial \lambda_2} = 0 .$$

The foregoing operations may be carried out with the aid of a computer; and N_2 minimum would be a function of K', m, p, and β. For given values of K', m, and p, the stability number is a function of β alone.

Computations have been carried out for a given set of K', m, and p, as listed in Case (a).

Layer 1

The failure surface is circular, as shown in Figure 5. The solution for this case is available (Lo, 1965), but to keep the clarity and continuity of the present analysis, this is listed in detail.

For limiting equilibrium of the mass above the potential surface of rupture EF, the total disturbing moment about O_1 must be equal to the total resisting moment about the same point.

The disturbing moment of the mass of soil above EF is equal to M_{D_I}.

$$M_{D_I} = \frac{\gamma_1 H_1^3}{12} (1 - 2 \cot^2 \beta + 3 \cot \lambda_1 \cot \beta + 3 \cot \alpha \times \cot \lambda_1 - 3 \cot \alpha \cot \beta).$$

The resisting moment (M_{R_I}) is given by

$$M_{R_I} = R_1 \int_{\alpha_1}^{\alpha_2} C(\theta, Z)\, R_1\, d\theta$$

$$= R_1^2 \int_{\alpha_1}^{\alpha_2} \left\{ C_2 + (C_1 - C_2) \right\} \cos^2 i \, d\theta .$$

On integration and simplification, the expression for M_{R_I} would be

$$M_{R_I} = \frac{C_1 H_1^2}{4 \sin^2\alpha \sin^2 \lambda_1} \Big[(1 + K)\,\alpha + \tfrac{1}{2}(1 - K) \sin 2\alpha$$

$$\times \cos(2f - 2\lambda) \Big] .$$

The factor of safety, F, is given by

$$F = \frac{\text{Resisting Moment } (M_{R_I})}{\text{Disturbing Moment } (M_{D_I})}$$

$$F = \frac{C_1 H_1^2}{4 \sin^2\alpha \sin^2 \lambda_1} \Big[(1 + K)\,\alpha + \tfrac{1}{2}(1 - K) \sin^2 \alpha$$

$$\times \cos(2f - 2\lambda) \Big] \Big/ \frac{\gamma_1 H_1^3}{12} (1 - 2 \cot^2 \beta + 3 \cot \lambda_1$$

$$\times \cot \beta + 3 \cot \alpha \cot \lambda_1 - 3 \cot \alpha \cot \beta).$$

For isotropic material, $C_1 = C_i = C_2$, and the above expression for the factor of safety reduces to

$$F = 6\alpha\, C_1 \Big/ \gamma_1 H_1 \sin^2 \alpha \sin \lambda_1 (1 - 2 \cot^2 \beta + 3 \cot \lambda_1$$

$$\times \cot \beta + 3 \cot \alpha \cot \lambda_1 - 3 \cot \alpha \cot \beta).$$

This equation is identical to Taylor's solution for the case $\varphi = 0$ (Taylor, 1937).

The above equation for F may be conveniently written as

$$F = \frac{C_1}{\gamma_1 H_1} N_1$$

where N_1 is termed a stability number.

$$N_1 = 3\left[(1 + K)\alpha + \frac{1}{2}(1 - K)\sin 2\alpha \cos(2f - 2\lambda_1)\right]$$
$$\div \sin^2\alpha \sin^2\lambda_1 (1 - 2\cot^2\beta + 3\cot\lambda_1 \cot\beta$$
$$+ 3\cot\alpha \cot\lambda_1 - 3\cot\alpha \cot\beta).$$

It is obvious that the minimum factor of safety is obtained by minimizing the stability number N_1 with respect to α and λ_1, so that

$$\frac{\partial N_1}{\partial \alpha} = 0$$

$$\frac{\partial N}{\partial \lambda_1} = 0.$$

The foregoing operations may be carried out with the aid of a computer; and N_1 is solely a function of K, f, and β. For given values of K and f, the stability number is a function of β alone.

Computations have been carried out for $f = 55^{\circ}$ and K ranging from 0.5 to 1.0. The value of 55° for f is based on experimental data (Lo, 1965).

Charts

Numerical results, which are graphically presented in the following pages (Figures 7 through 29), are obtained with the aid of an IBM 360/65 computer available at Oklahoma State University.

Figure 30 is a reproduction of the chart presented by Lo.(1965). These charts can be used to solve slope stability problems in a two-layered system of anisotropic soils.

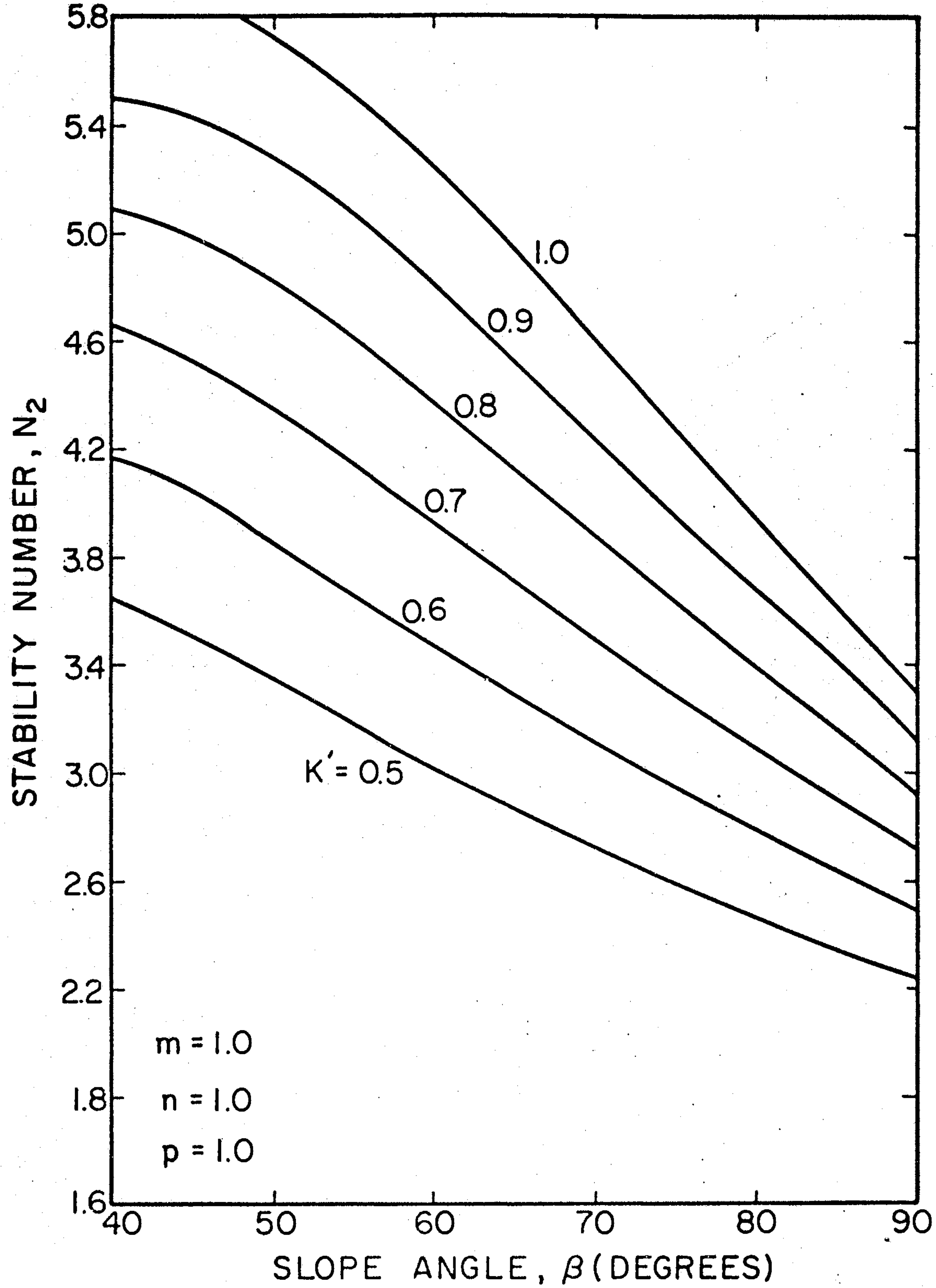


Figure 7. Slope Angle (β) versus Stability Number (N_2), Layer 2 (m = 1.0, n = 1.0, and p = 1.0)

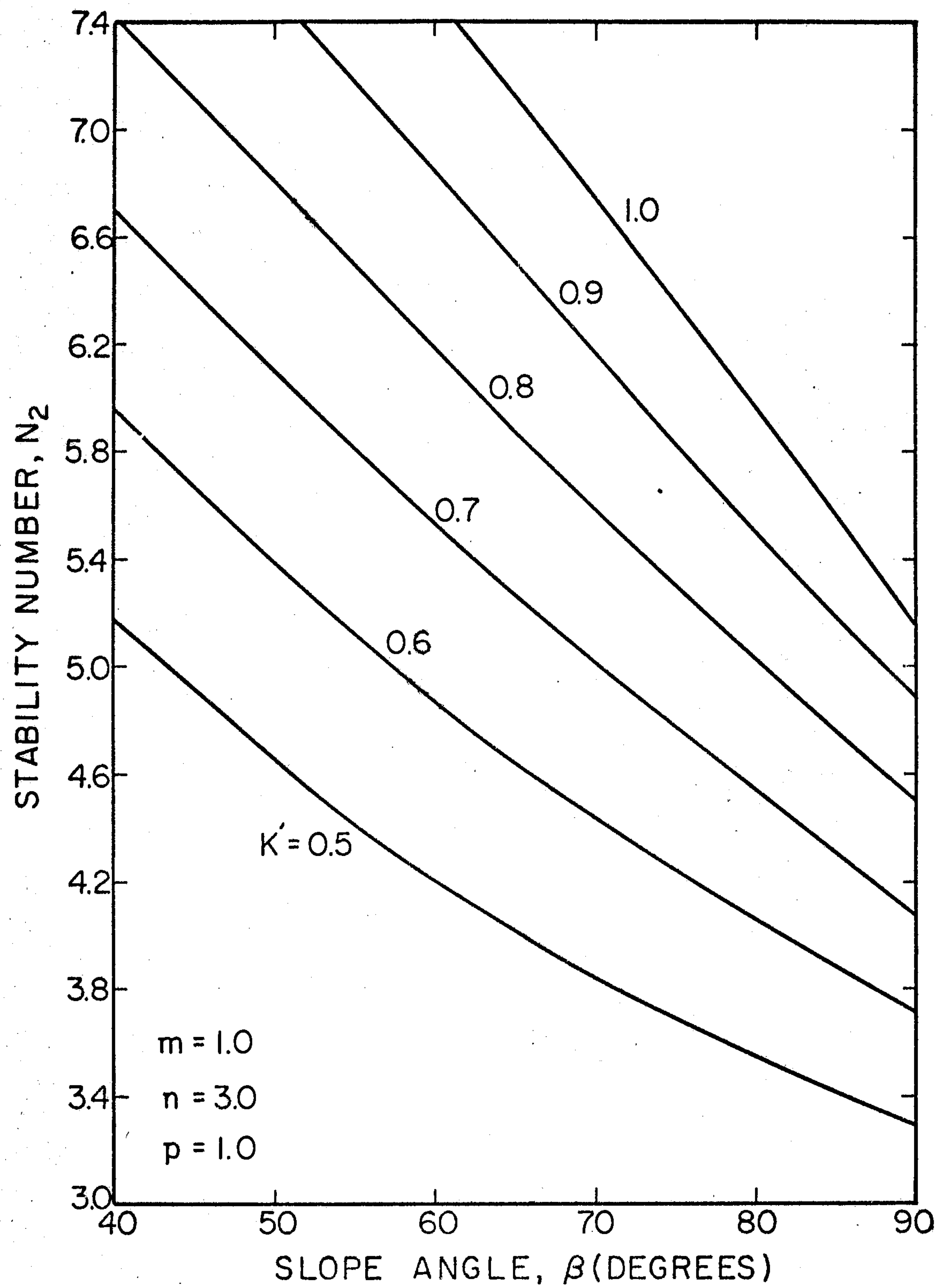


Figure 8. Slope Angle (β) versus Stability Number (N_2), Layer 2 (m = 1.0, n = 3.0, and p = 1.0)

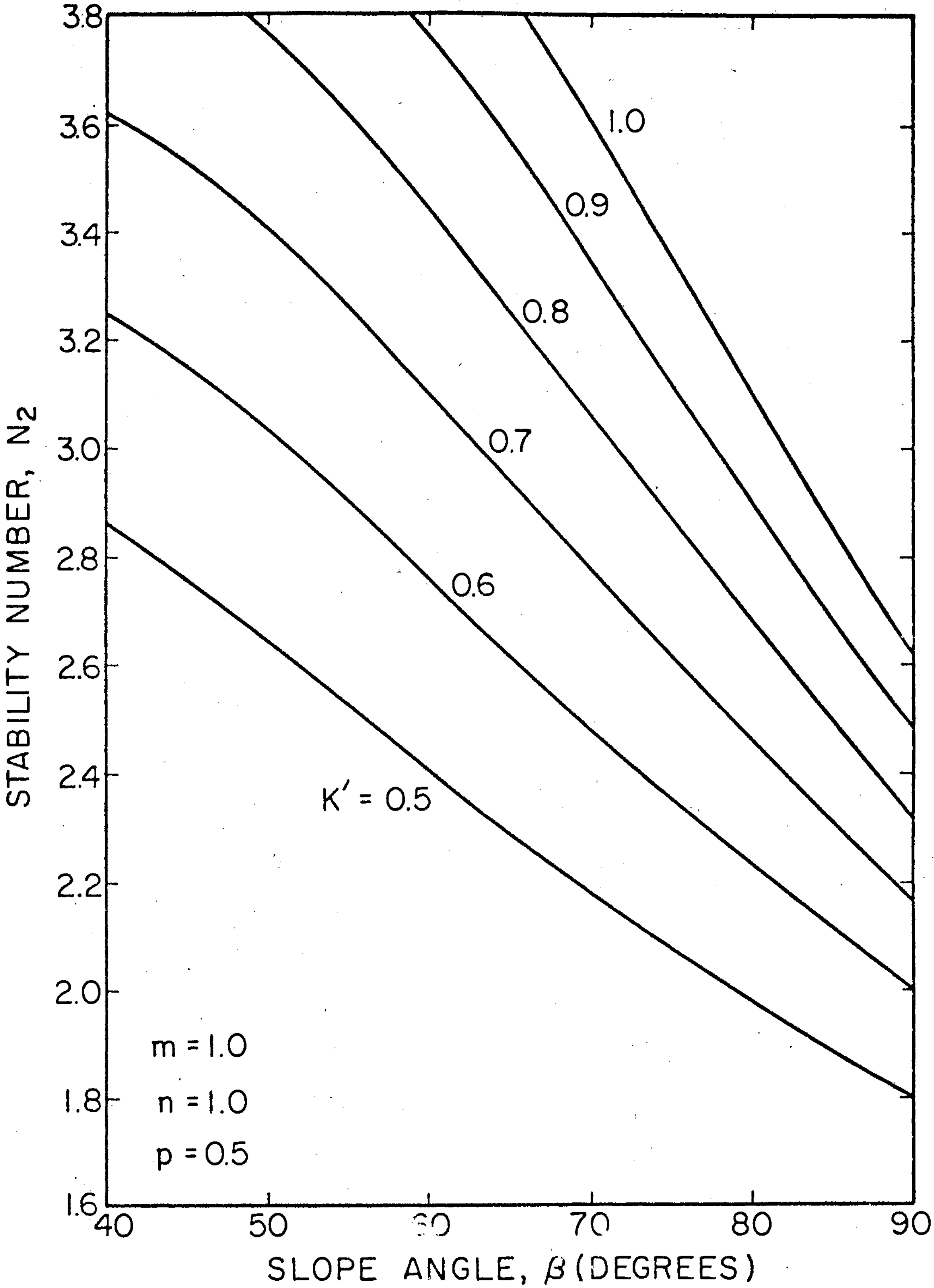


Figure 9. Slope Angle (β) versus Stability Number (N_2), Layer 2 (m = 1.0, n = 1.0, and p = 0.5)

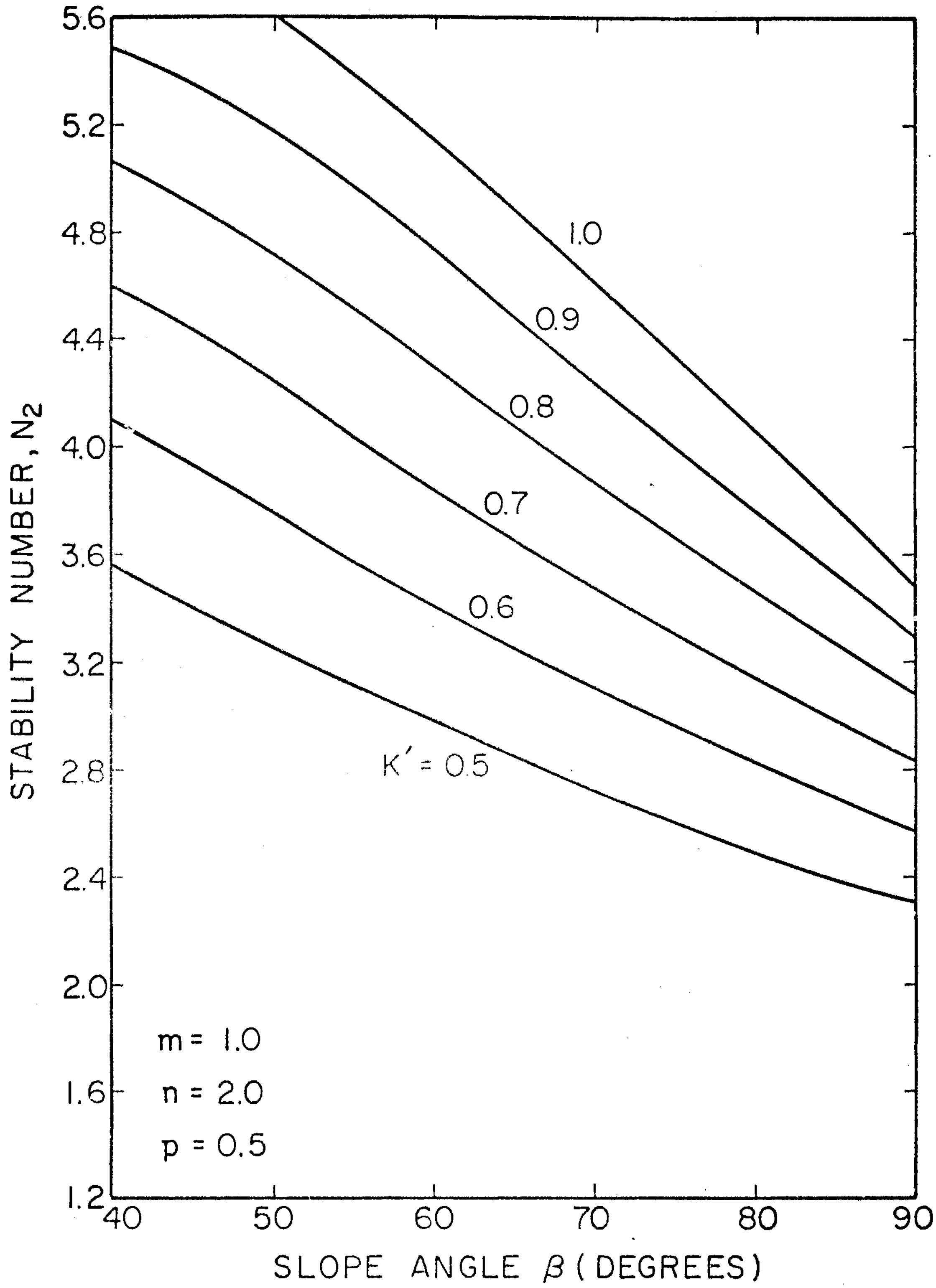


Figure 10. Slope Angle (β) versus Stability Number (N_2), Layer 2 (m = 1.0, n = 2.0, and p = 0.5)

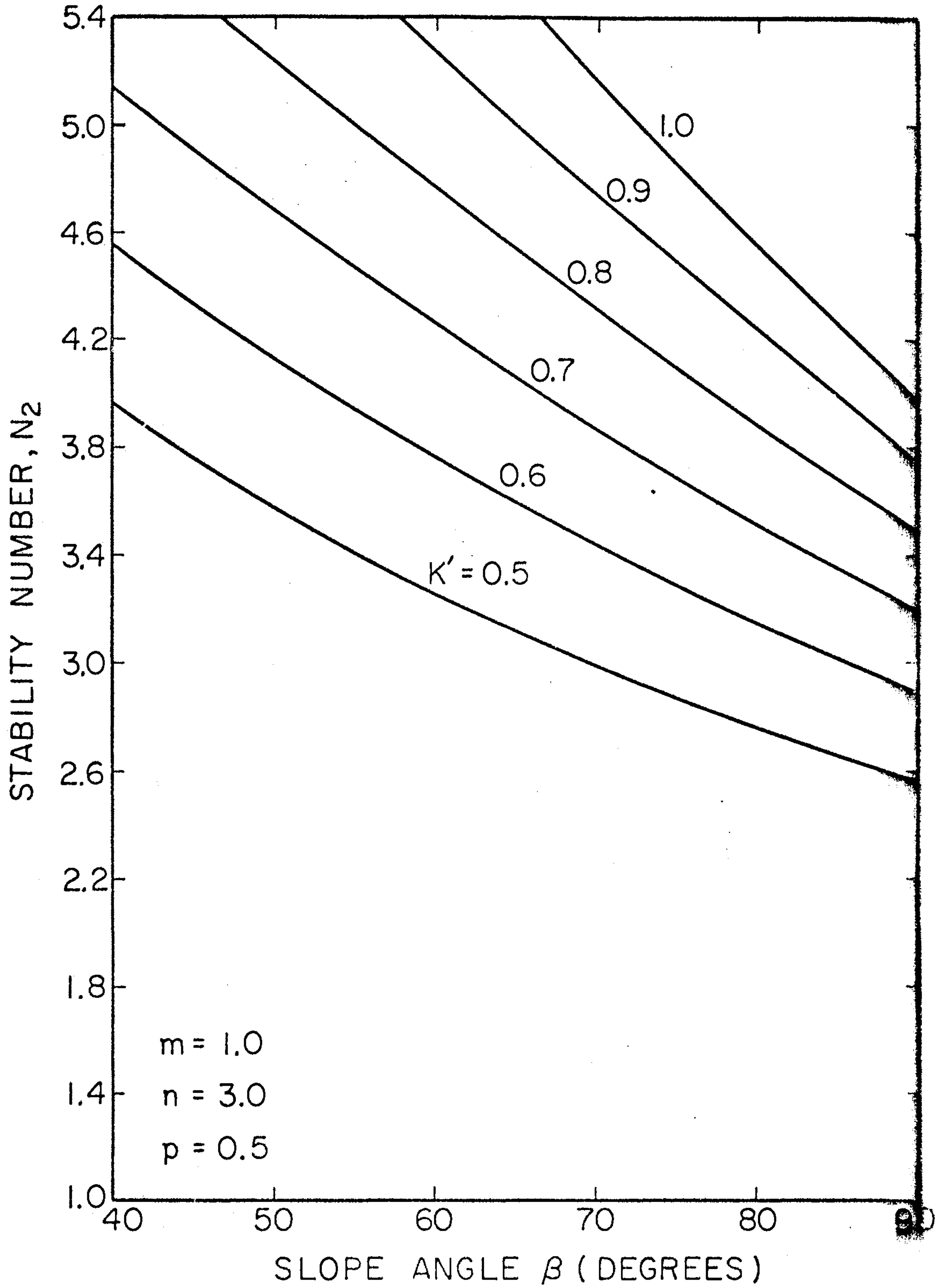


Figure 11. Slope Angle (β) versus Stability Number (N_2), Layer 2 (m = 1.0, n = 3.0, and p = 0.5)

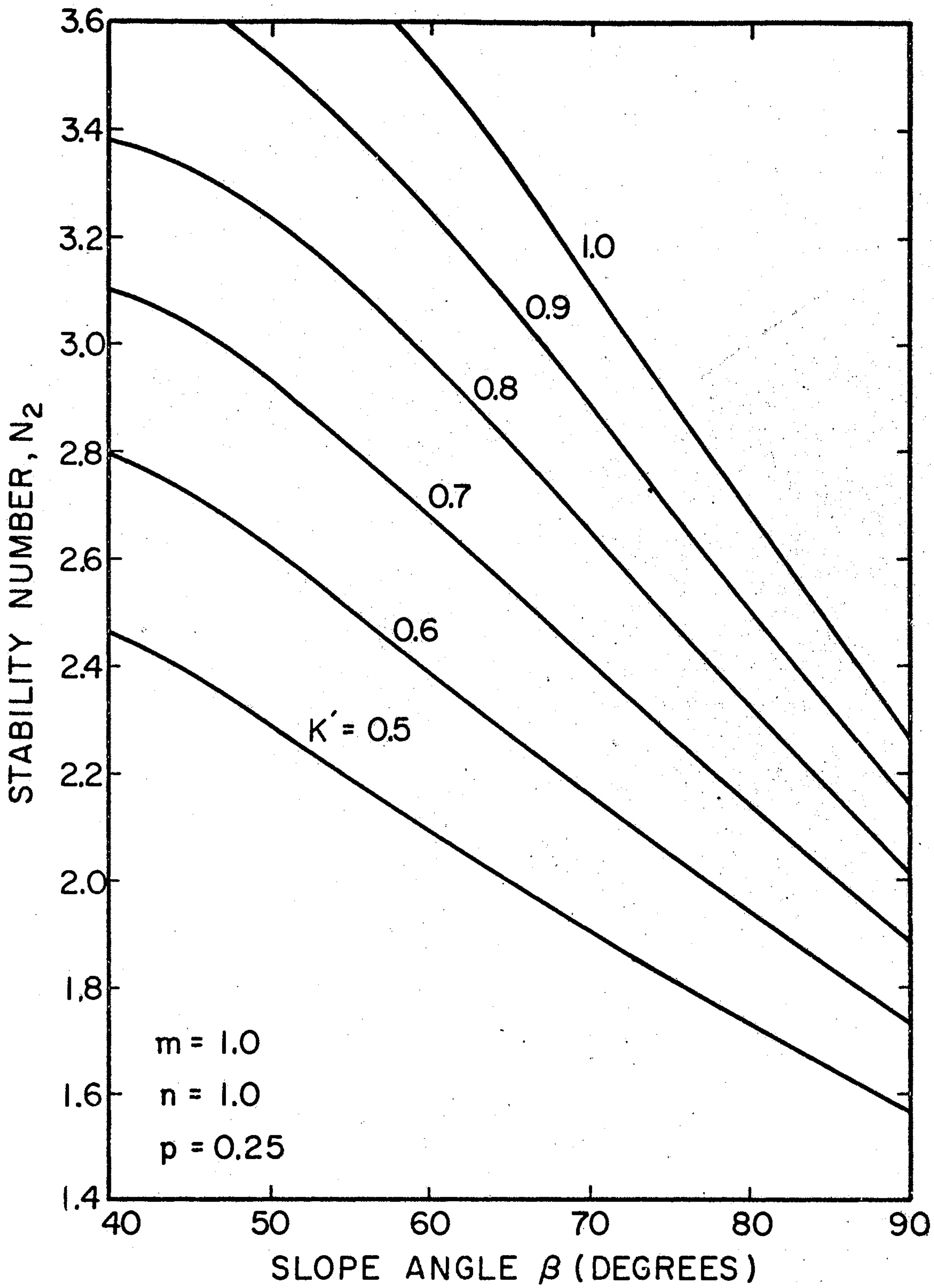


Figure 12. Slope Angle (β) versus Stability Number (N_2), Layer 2 ($m = 1.0$, $n = 1.0$, and $p = 0.25$)

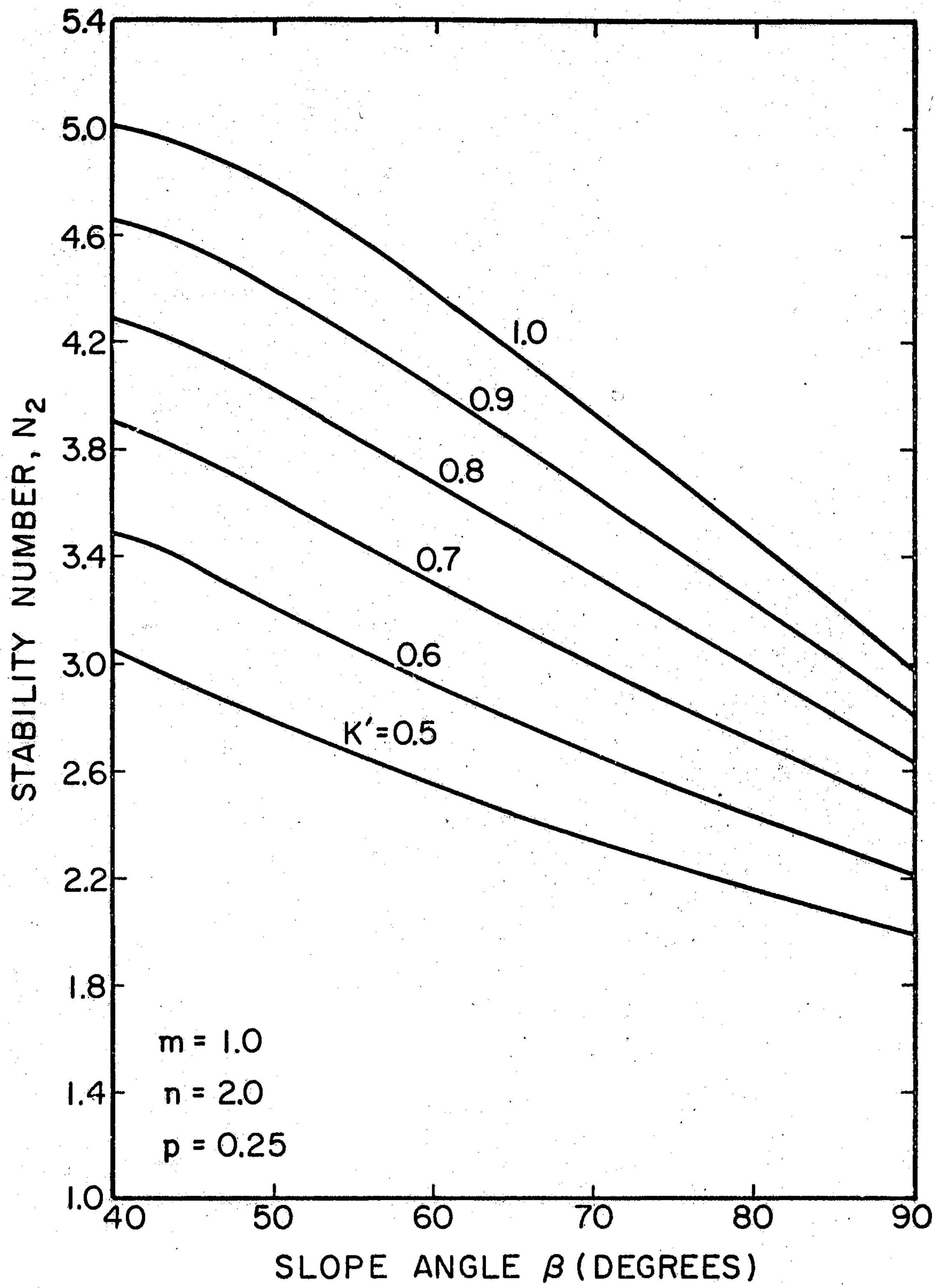


Figure 13. Slope Angle (β) versus Stability Number (N_2), Layer 2 (m = 1.0, n = 2.0, and p = 0.25)

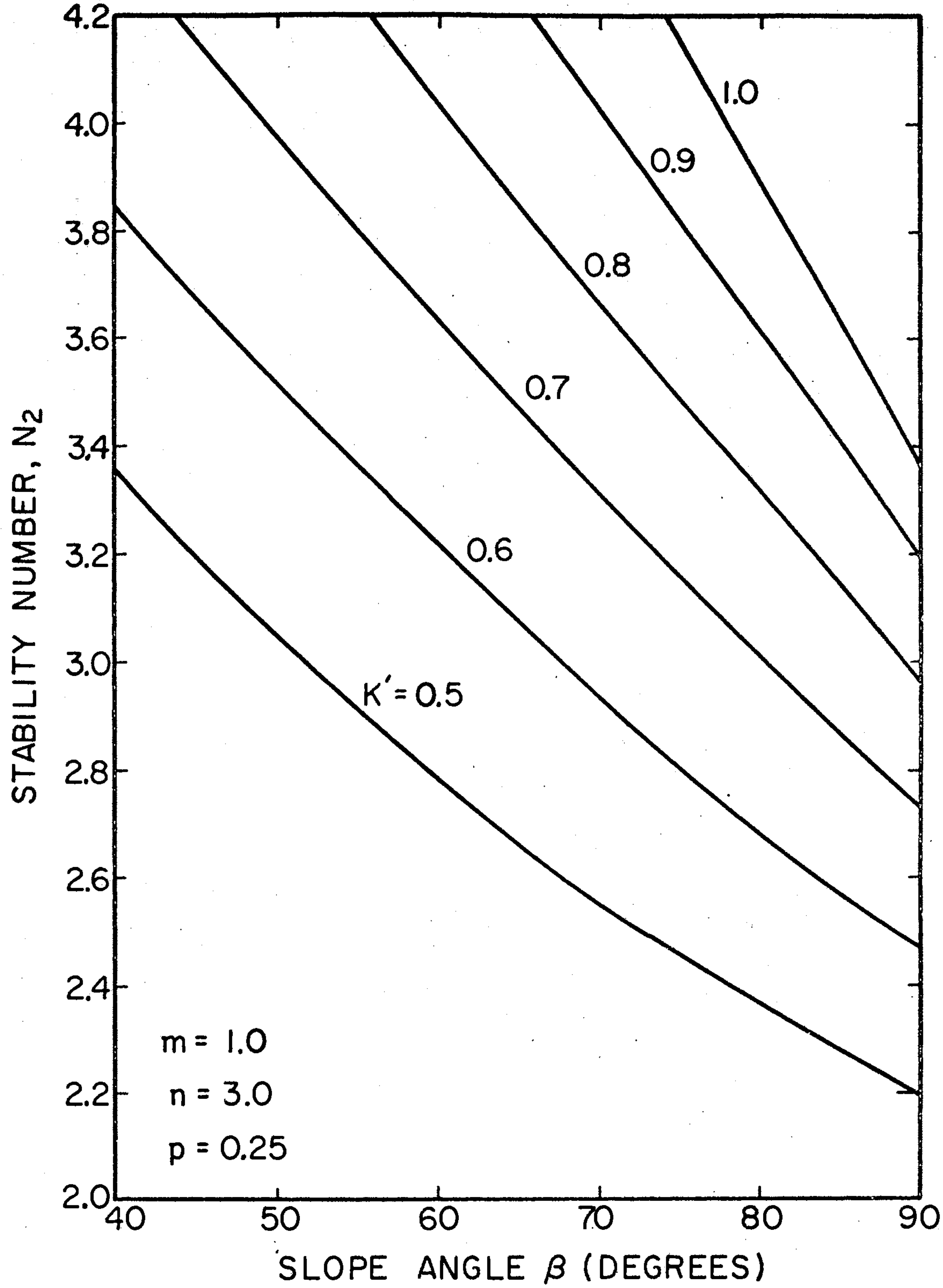


Figure 14. Slope Angle (β) versus Stability Number (N_2), Layer 2 (m = 1.0, n = 3.0, and p = 0.25)

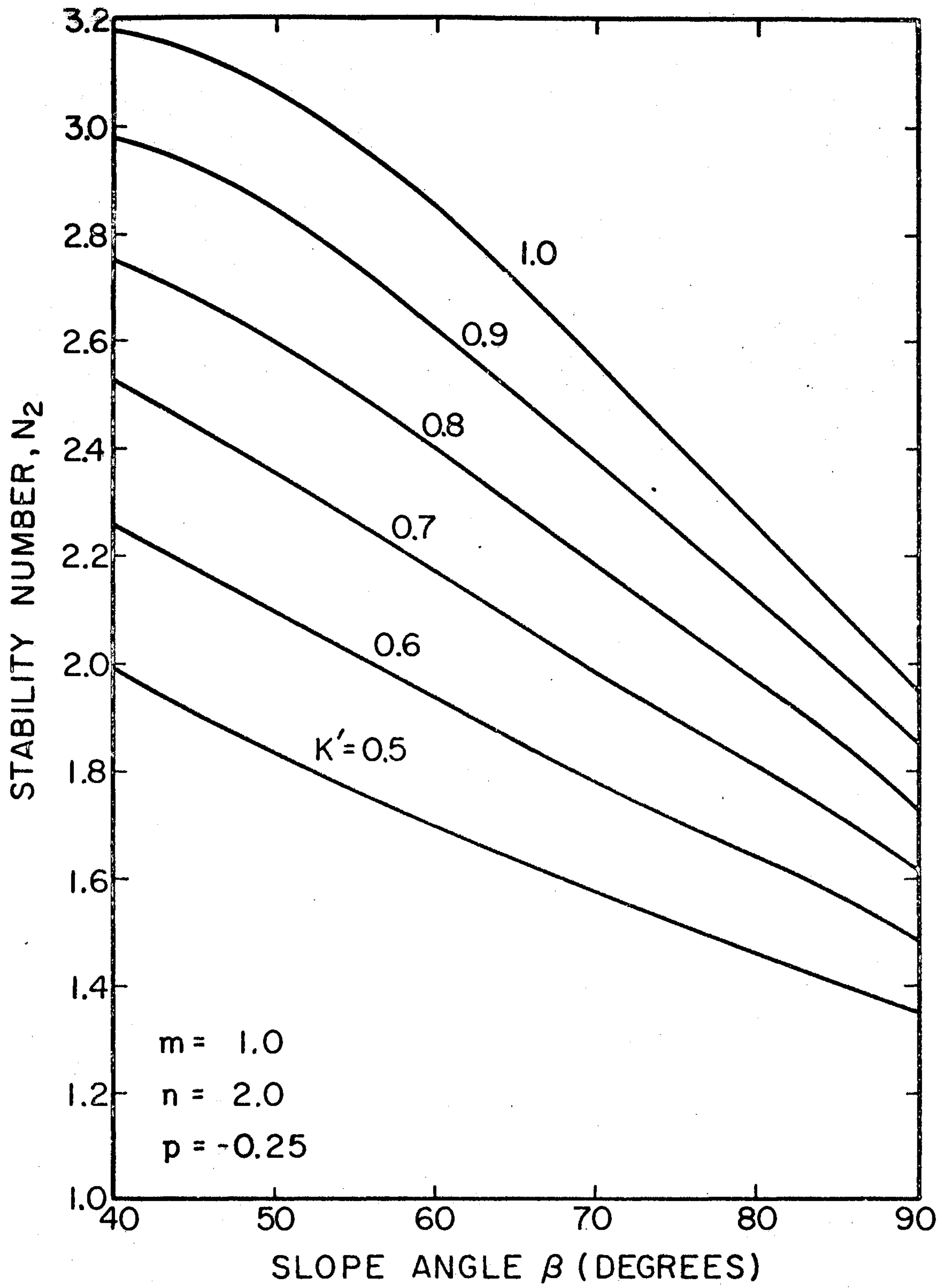


Figure 15. Slope Angle (β) versus Stability Number (N_2), Layer 2, ($m = 1.0$, $n = 2.0$, and $p = -0.25$)

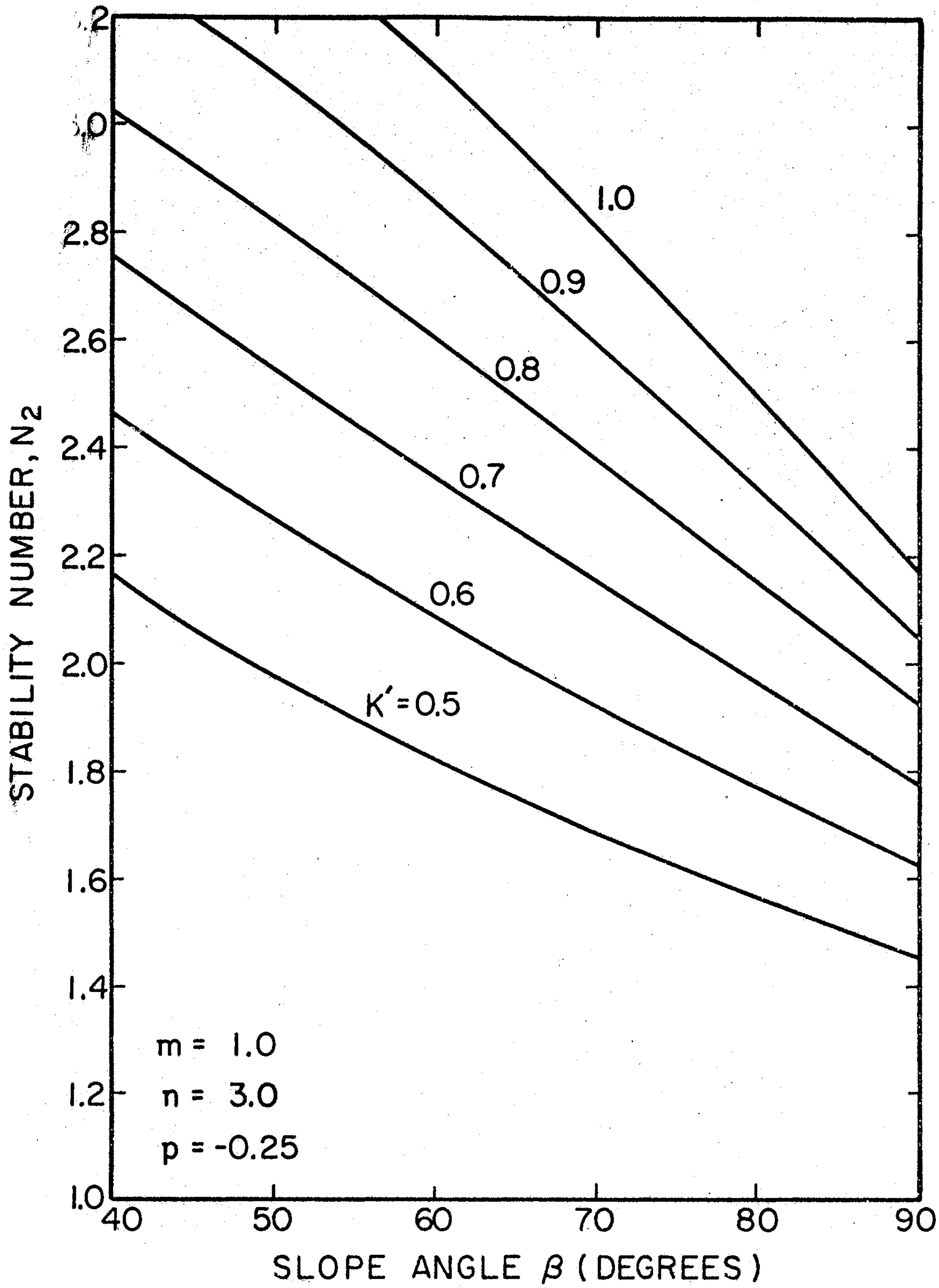


Figure 16. Slope Angle (β) versus Stability Number (N_2), Layer 2 (m = 1.0, n = 3.0, and p = -0.25)

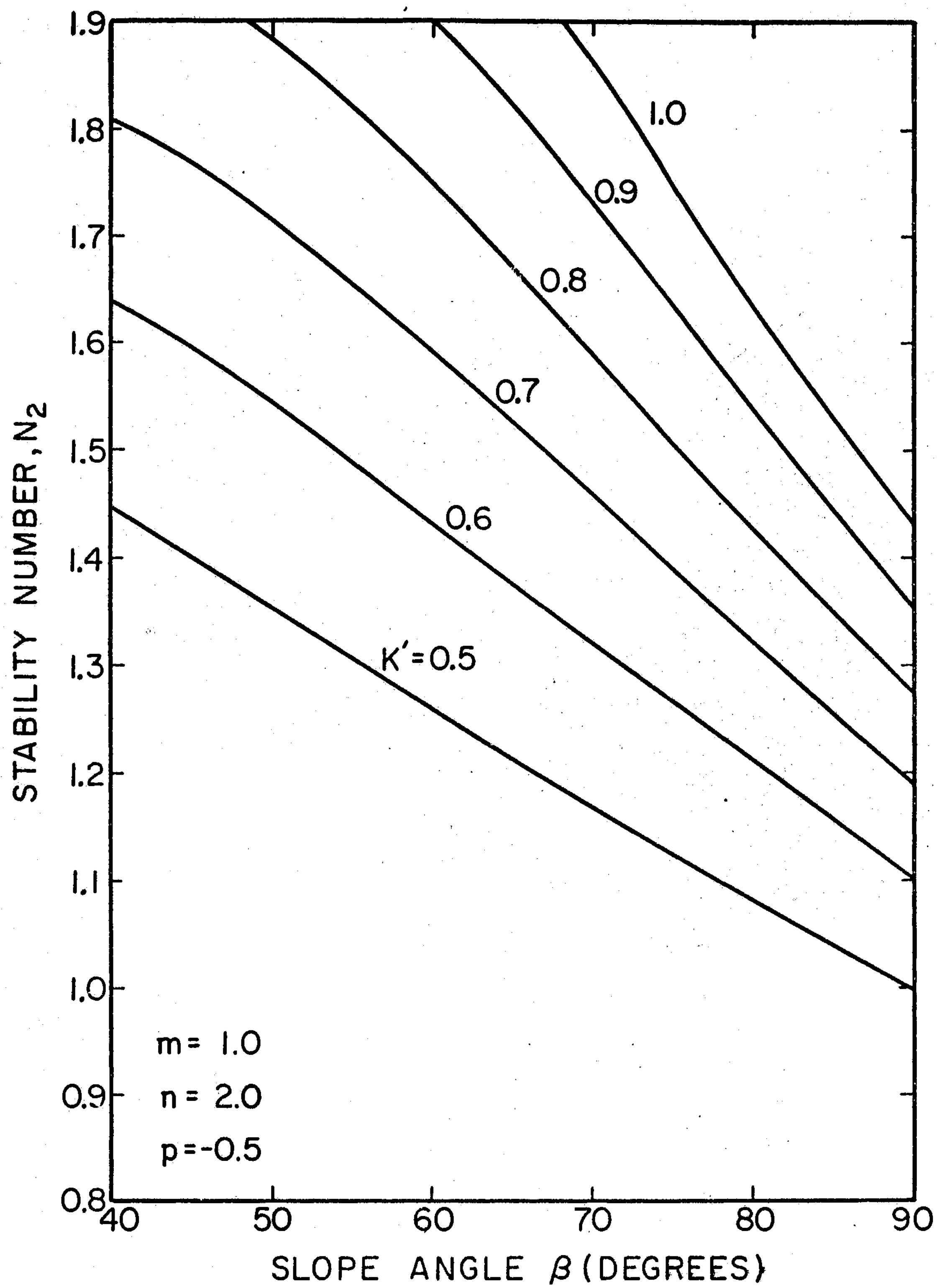


Figure 17. Slope Angle (β) versus Stability Number (N_2), Layer 2 ($m = 1.0$, $n = 2.0$, and $p = -0.5$)

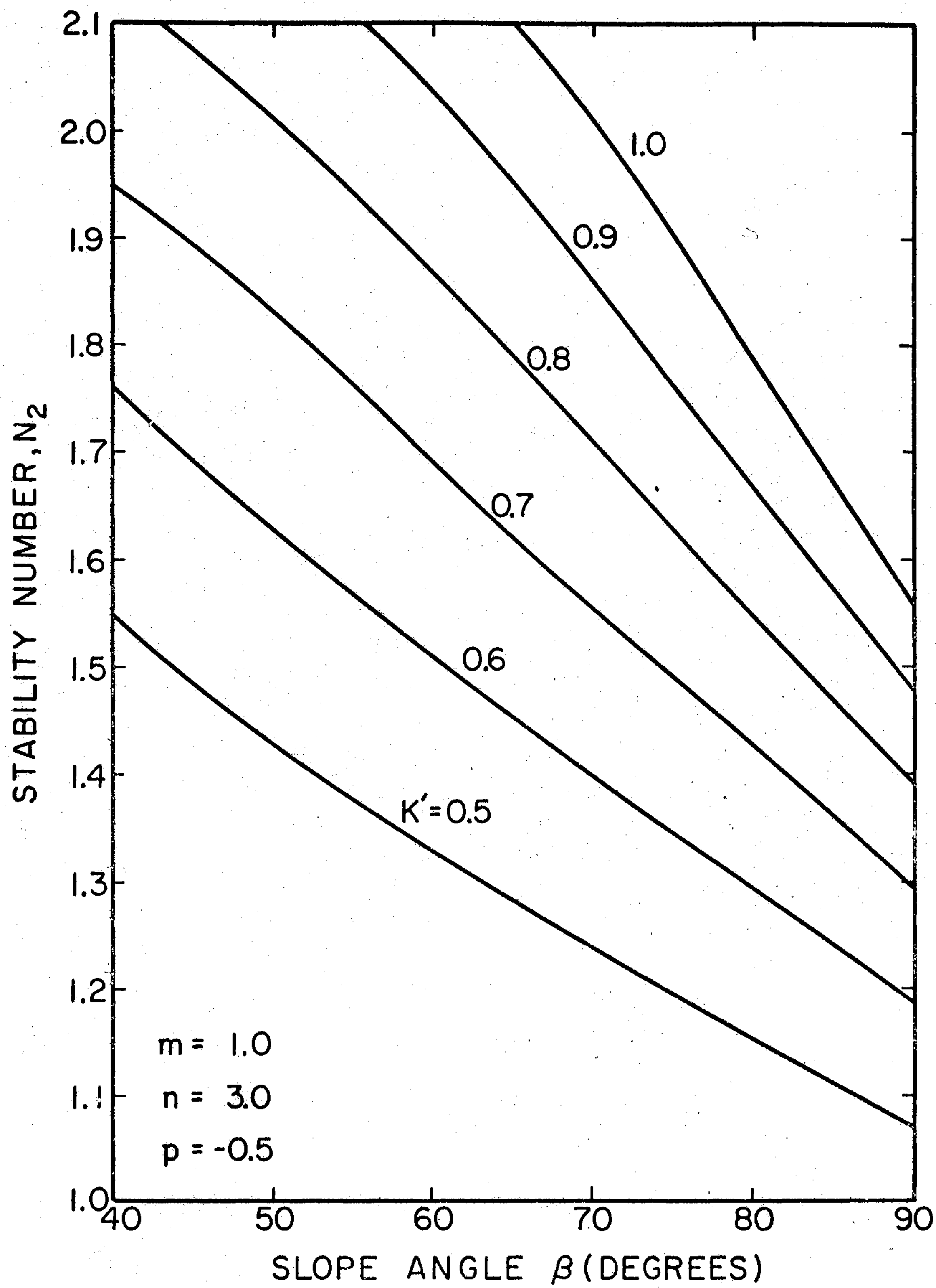


Figure 18. Slope Angle (β) versus Stability Number (N_2), Layer 2 ($m = 1.0$, $n = 3.0$, and $p = -0.5$)

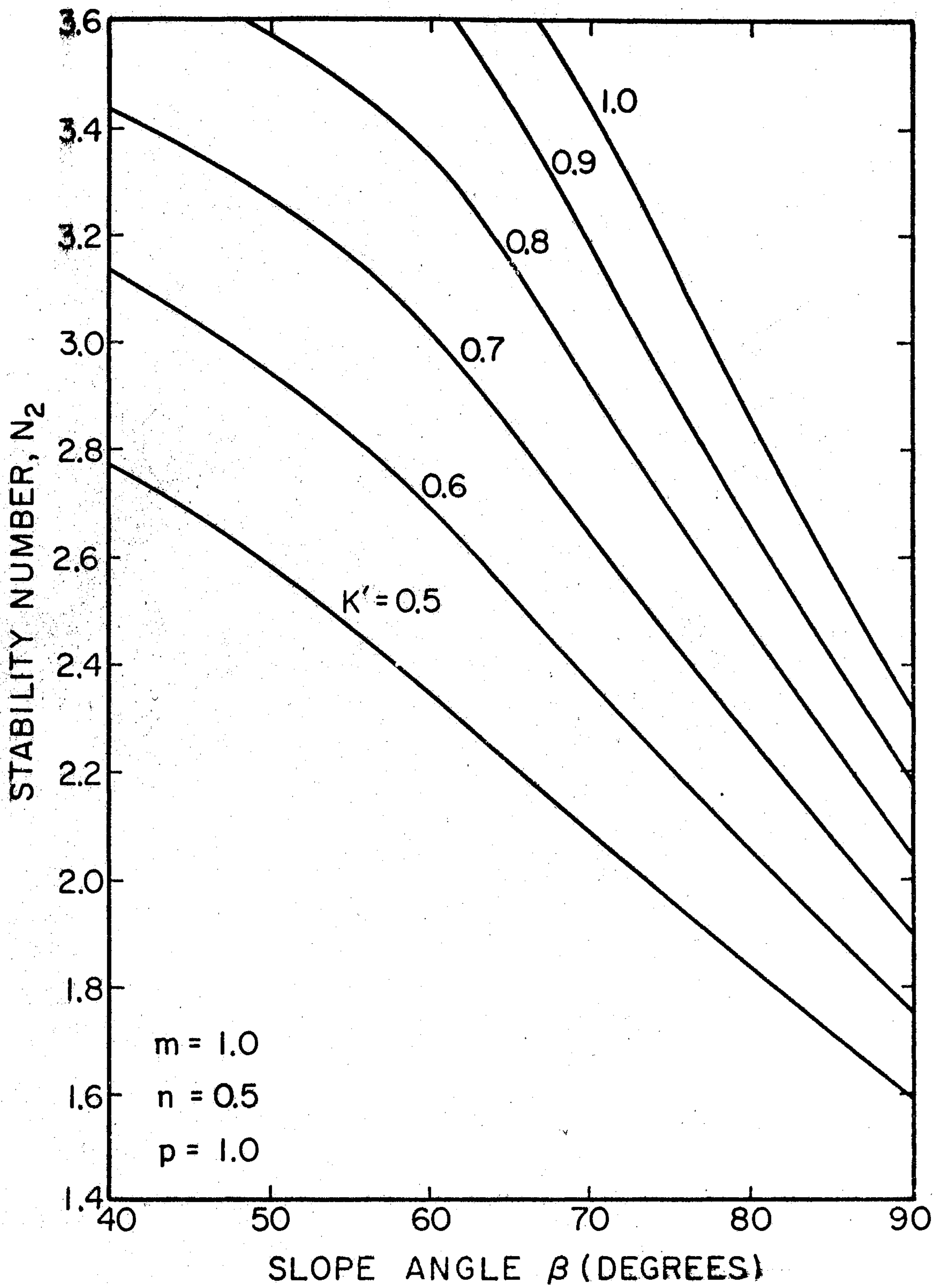


Figure 19. Slope Angle (β) versus Stability Number (N_2), Layer 2 (m = 1.0, n = 0.5, and p = 1.0)

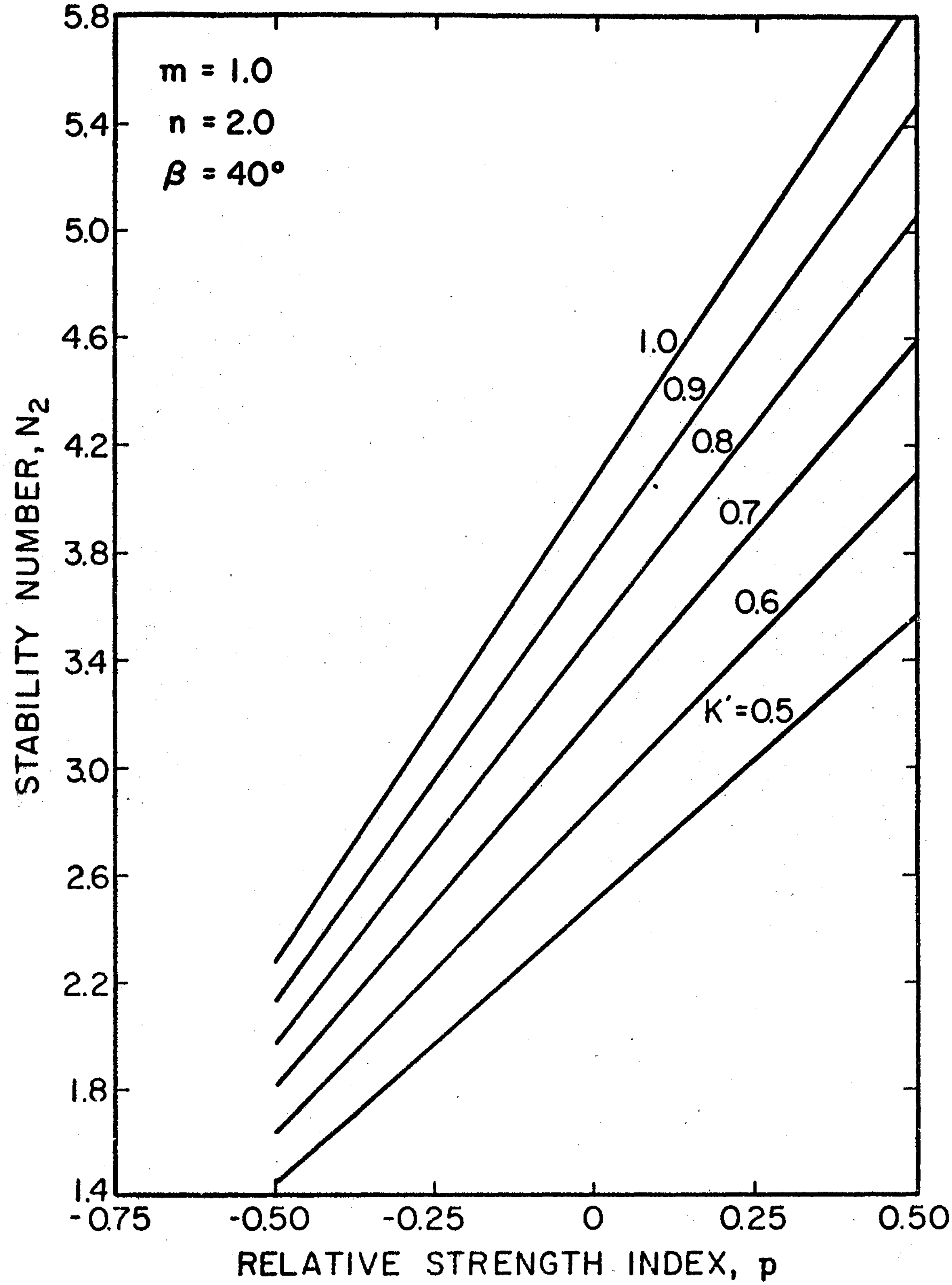


Figure 20. Relative Strength Index (p) versus Stability Number (N_2) ($m = 1.0$, $n = 2.0$, and $\beta = 40^{o}$)

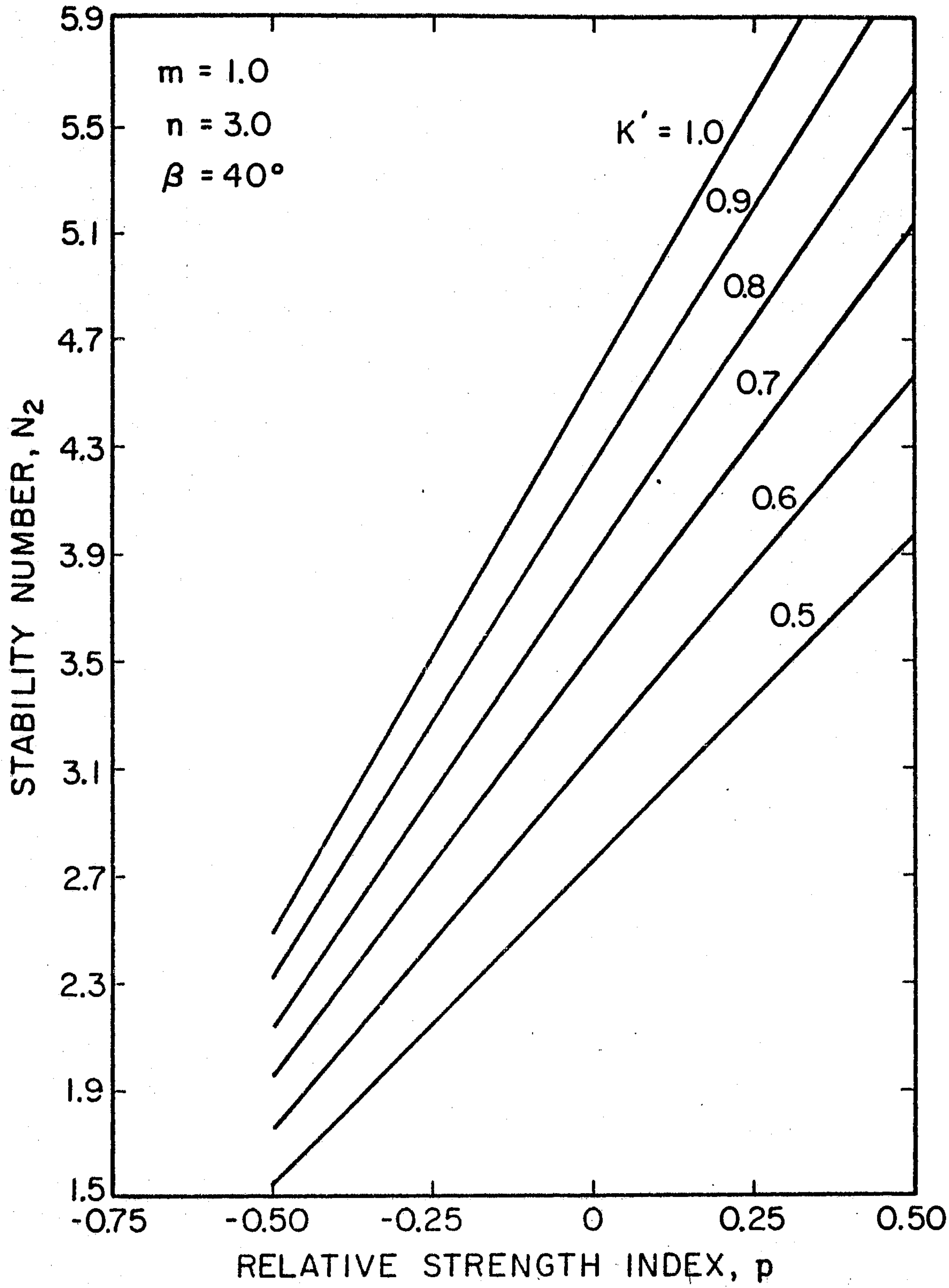


Figure 21. Relative Strength Index (p) versus Stability Number (N_2) (m = 1.0, n = 3.0, and β = 40°)

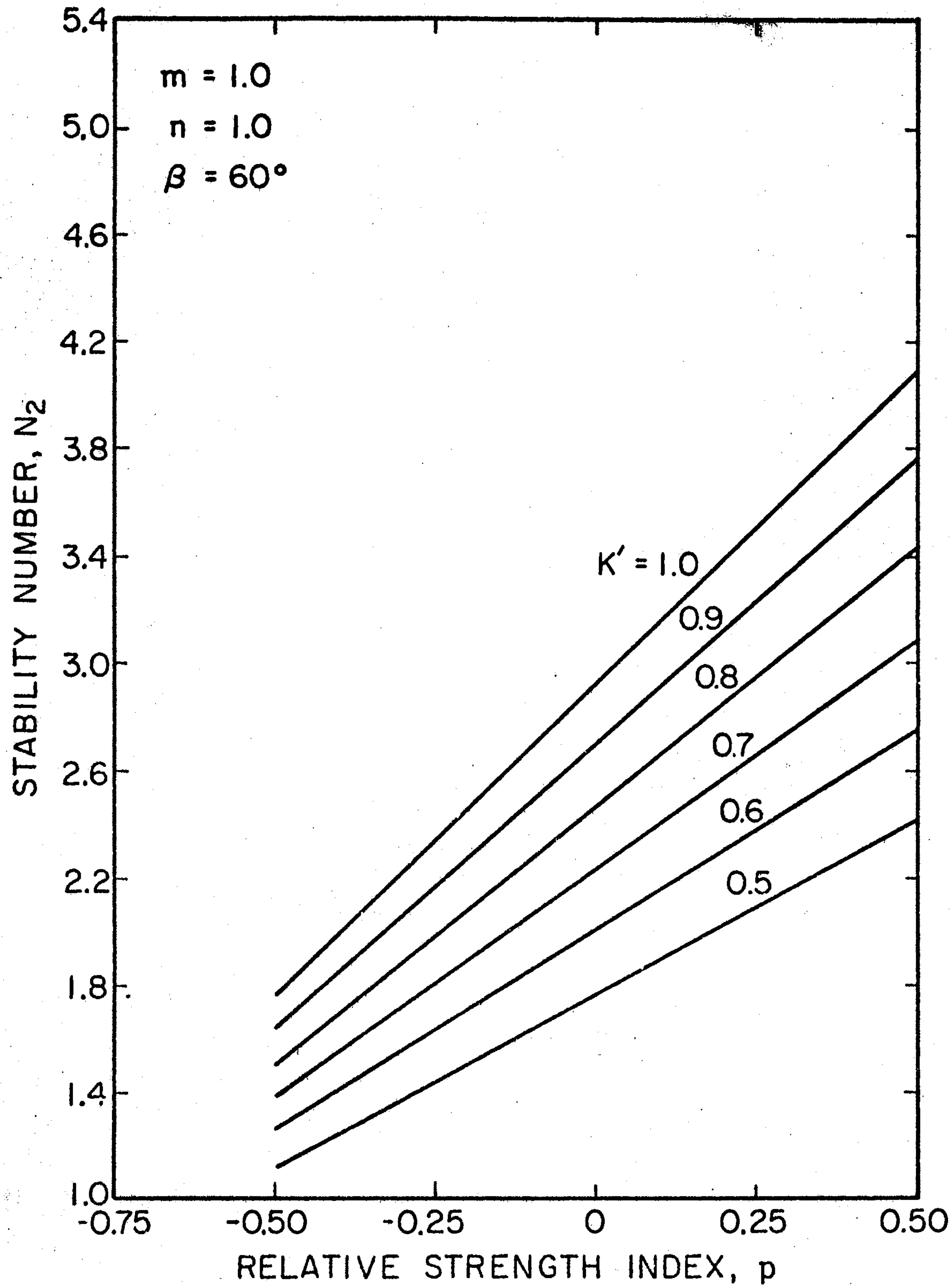


Figure 22. Relative Strength Index (p) versus Stability Number (N_2) (m = 1.0, n = 1.0, and β = 60°)

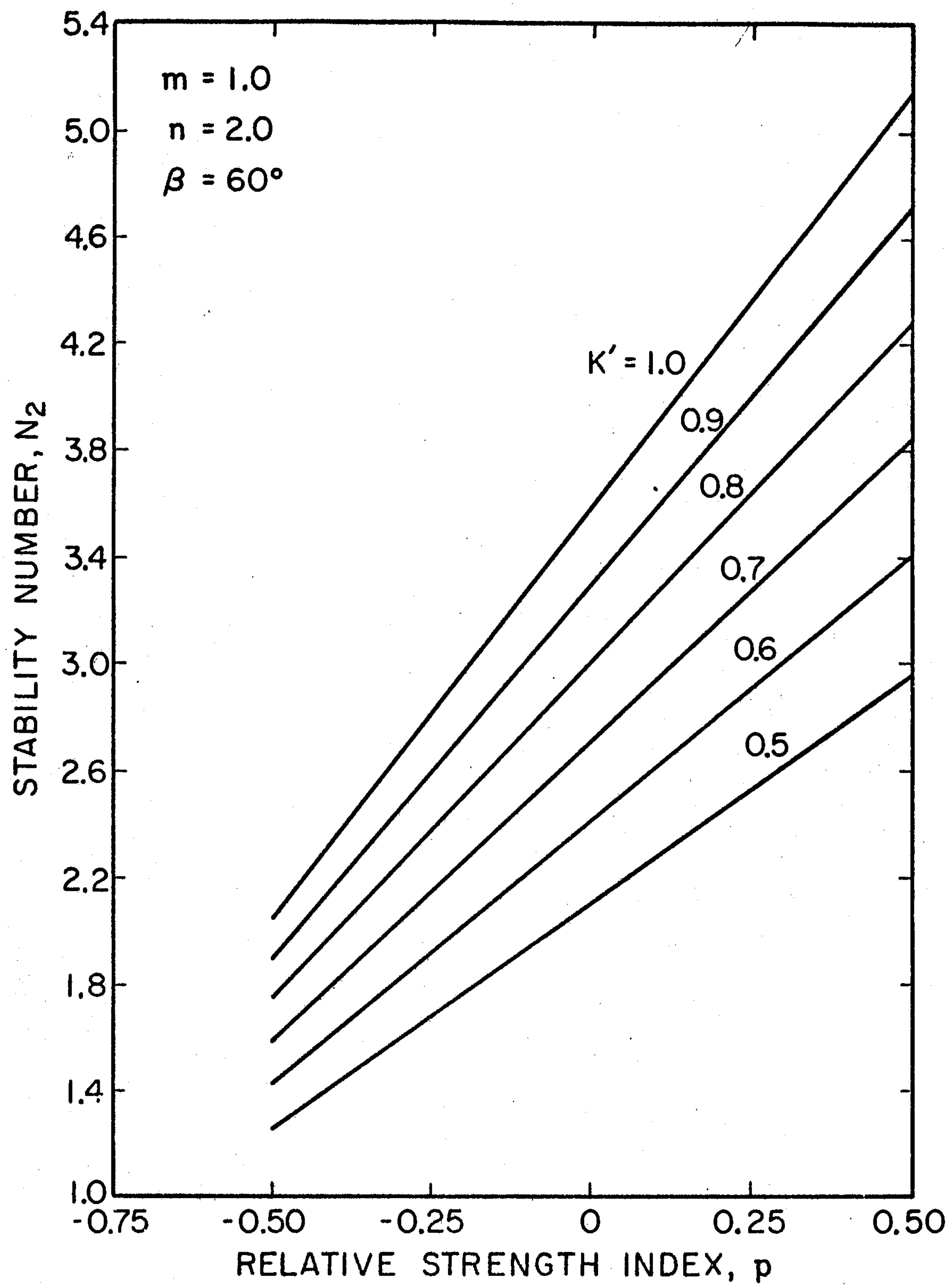


Figure 23. Relative Strength Index (p) versus Stability Number (N_2) (m = 1.0, n = 2.0, and $\beta = 60^{o}$)

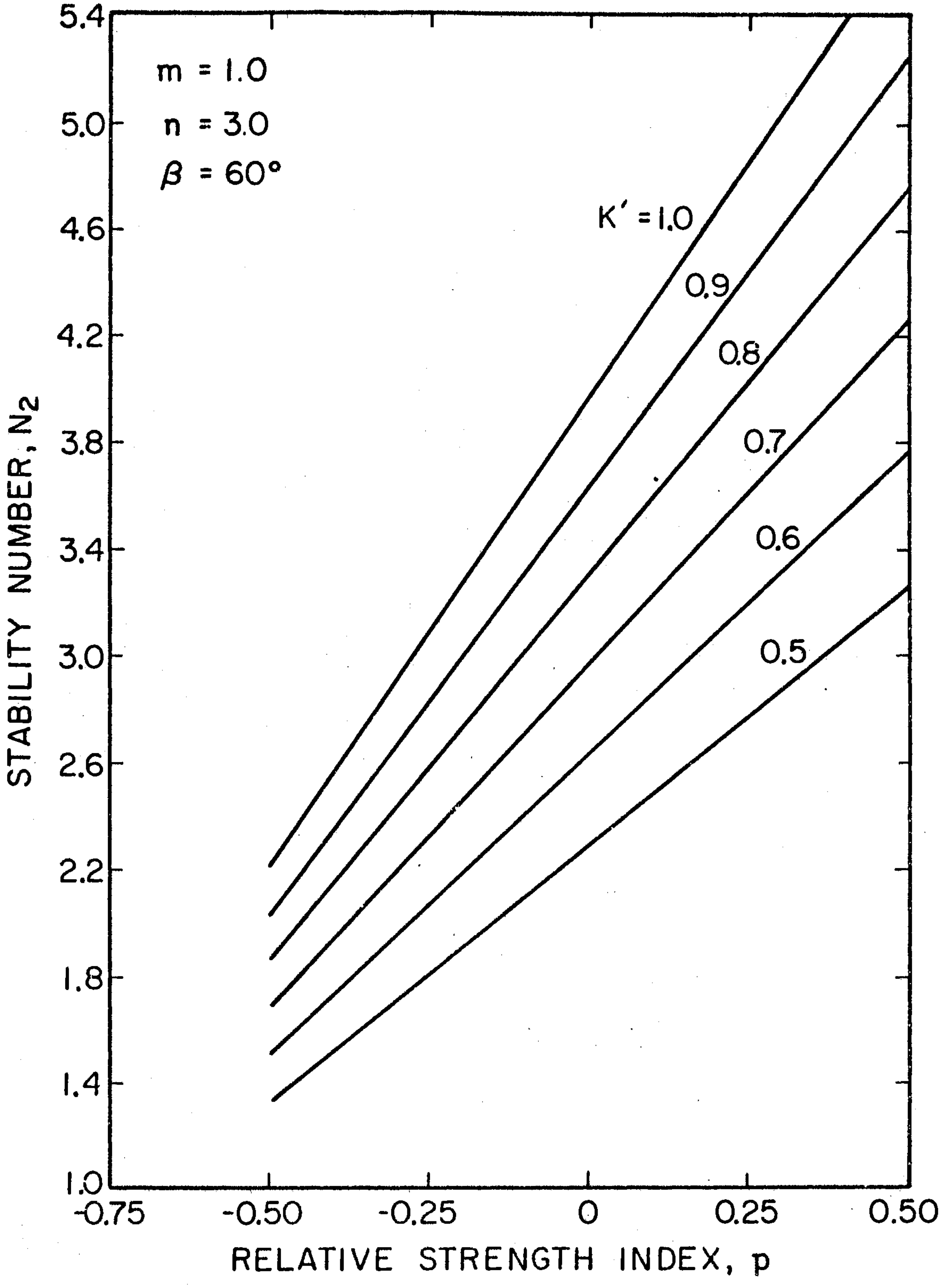


Figure 24. Relative Strength Index (p) versus Stability Number (N_2) (m = 1.0, n = 3.0, and β = 60°)

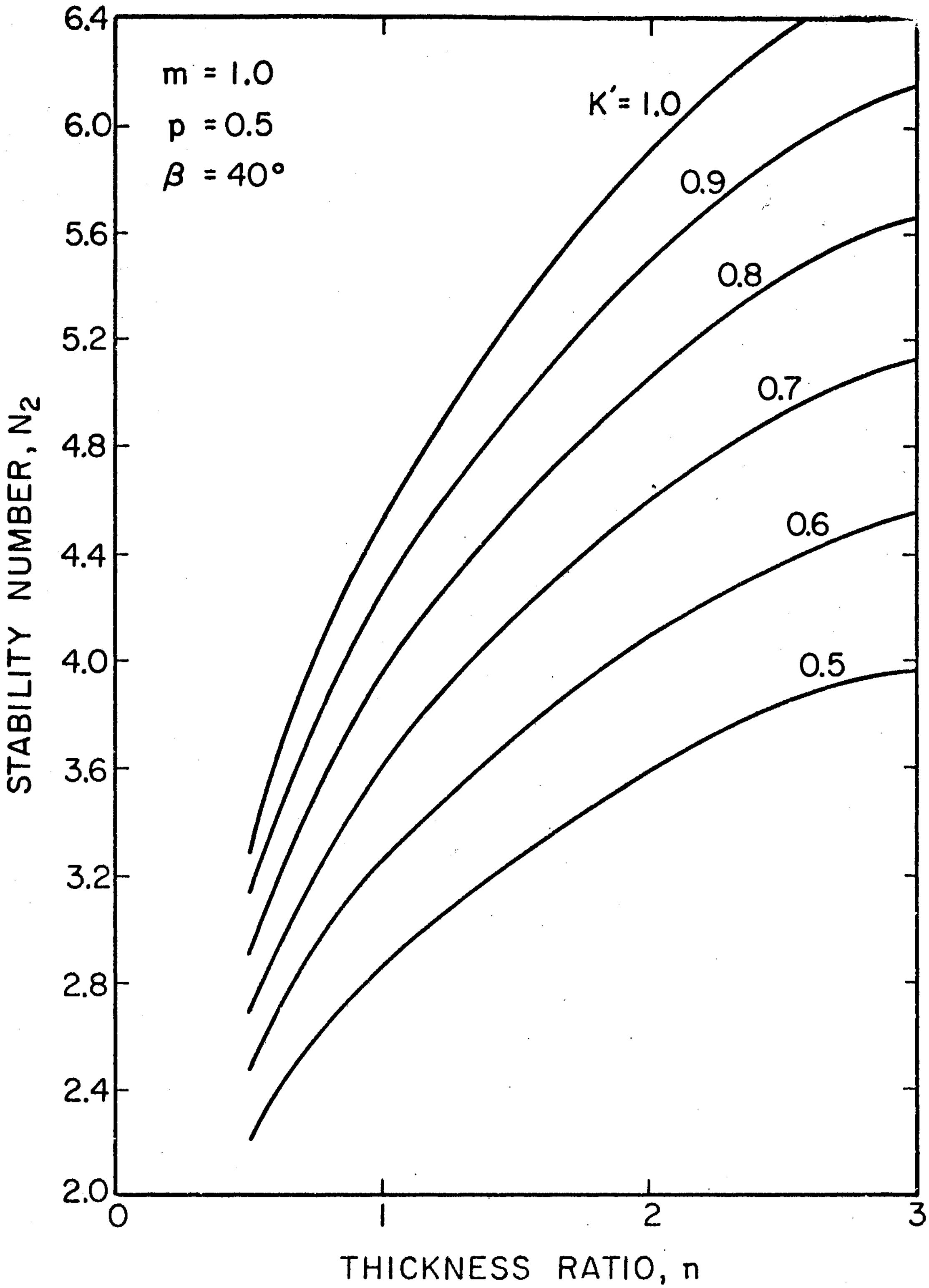


Figure 25. Thickness Ratio (n) versus Stability Number (N_2), (m = 1.0, p = 0.5, and $\beta = 40^o$)

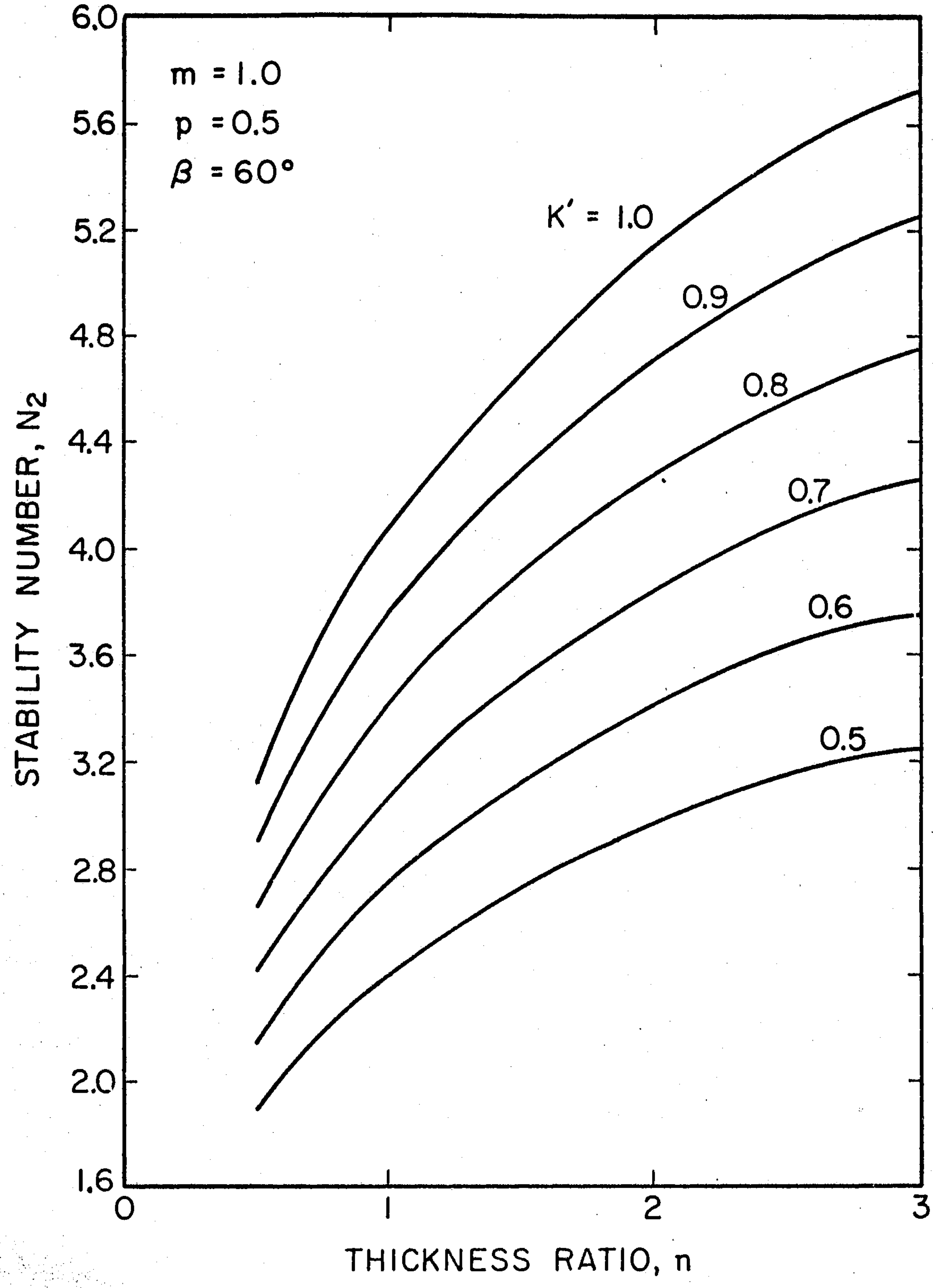


Figure 26. Thickness Ratio (n) versus Stability Number (N_2), (m = 1.0, p = 0.5, and β = 60°)

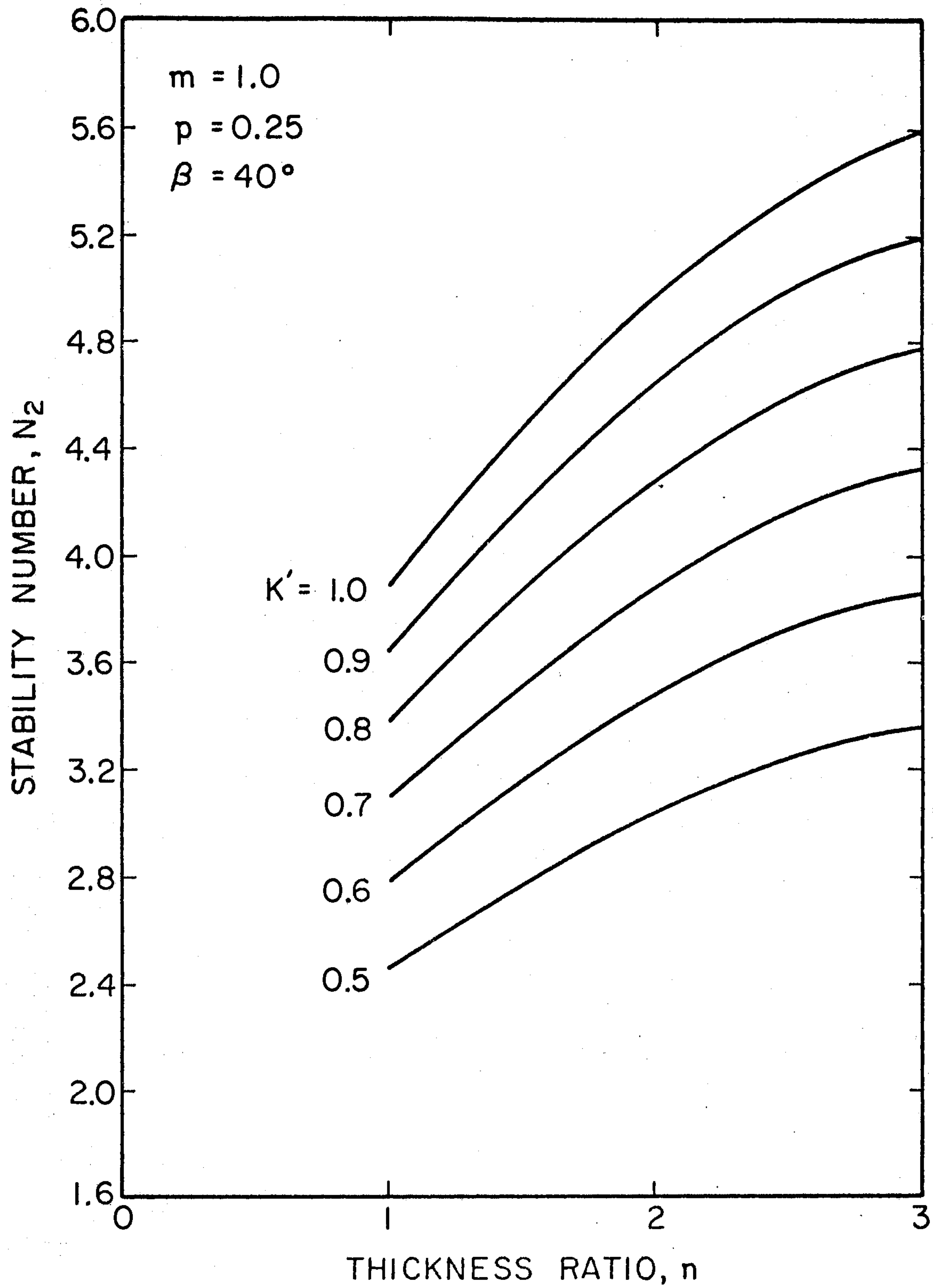


Figure 27. Thickness Ratio (n) versus Stability Number (N_2), (m = 1.0, p = 0.25, and β = 40°)

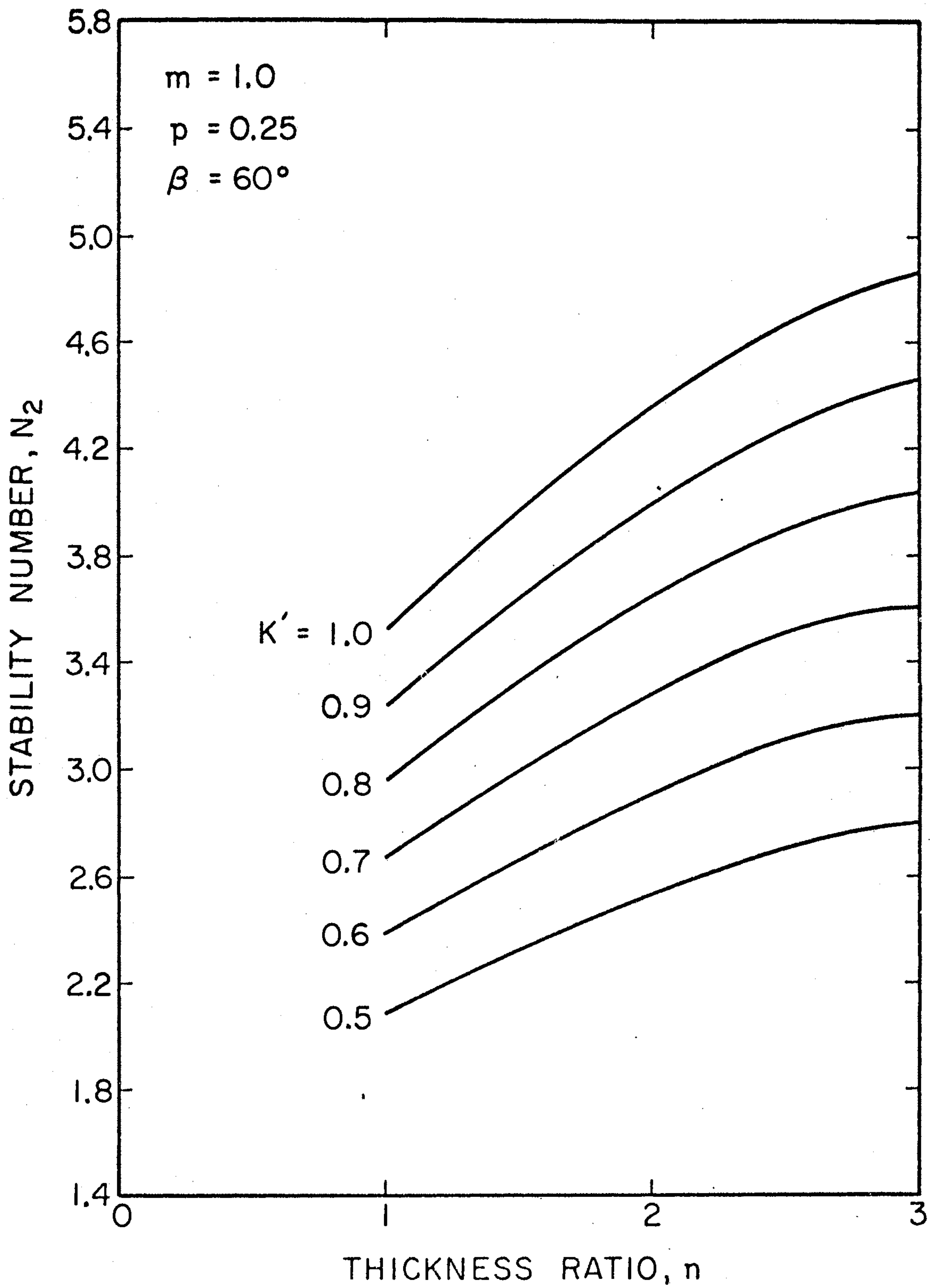


Figure 28. Thickness Ratio (n) versus Stability Number (N_2), (m = 1.0, p = 0.25, and β = 60°)

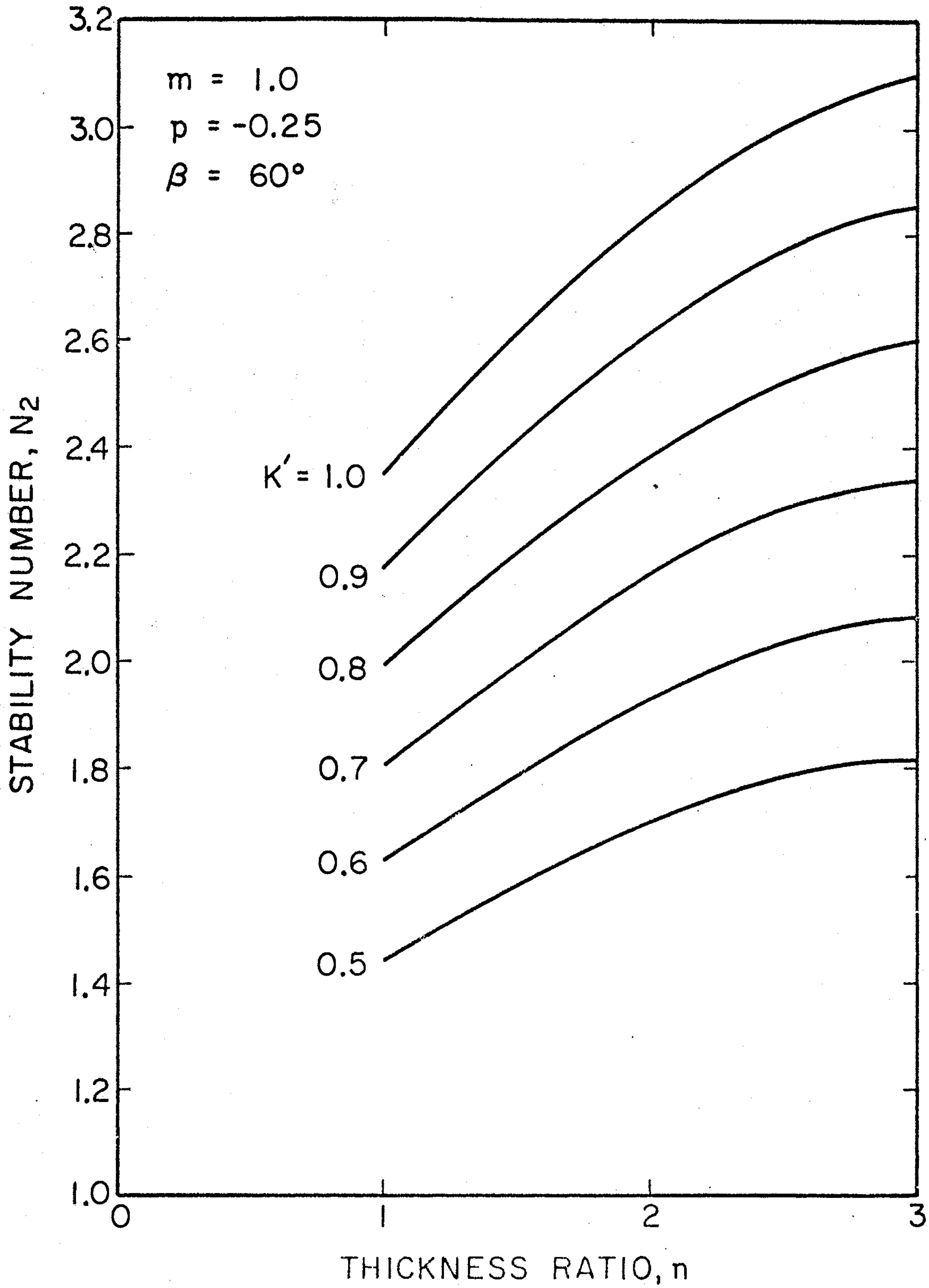


Figure 29. Thickness Ratio (n) versus Stability Number (N_2), ($m = 1.0$, $p = -0.25$, and $\beta = 60^{\circ}$)

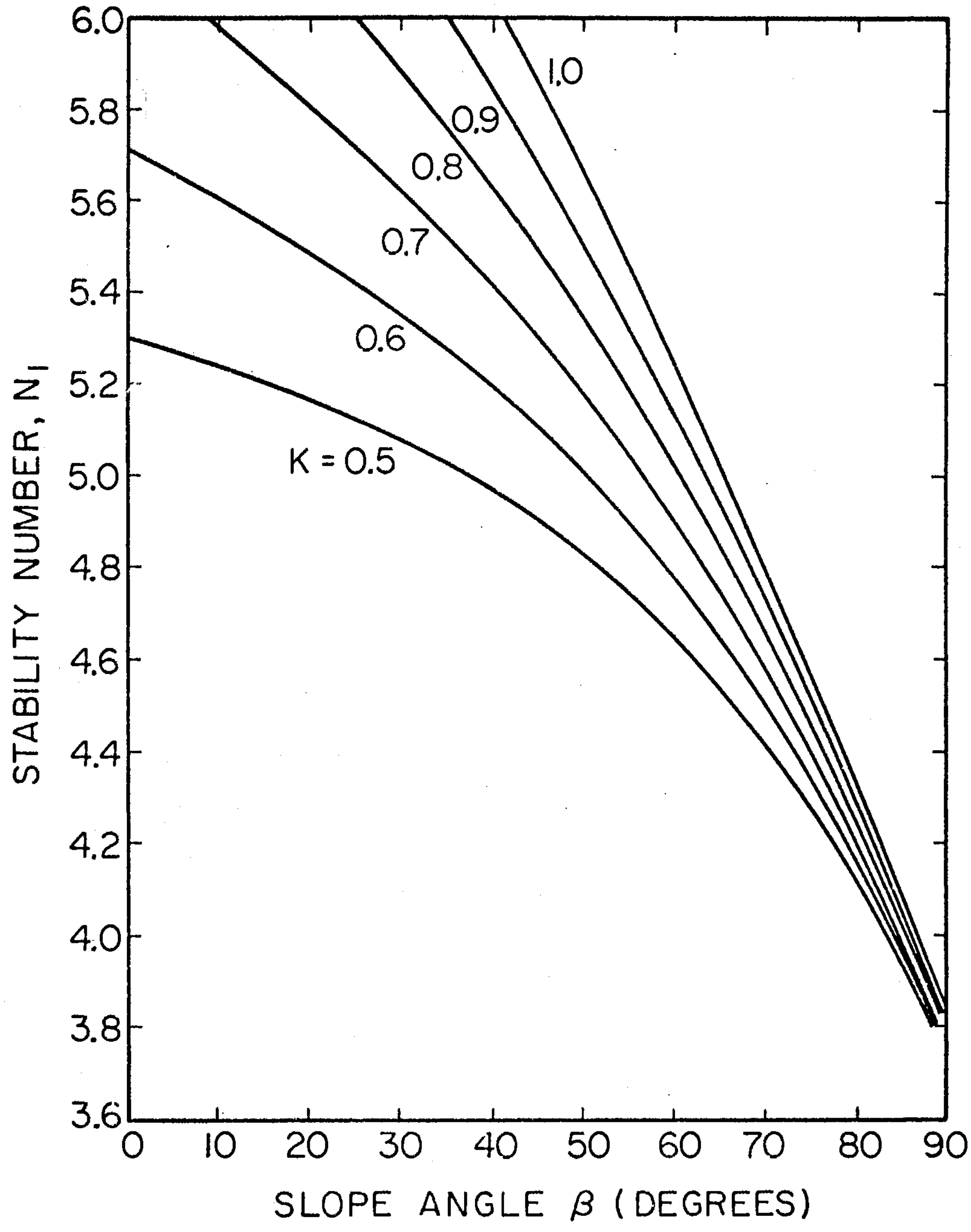


Figure 30. Slope Angle (β) versus Stability Number (N_2), Layer 1, (After Lo, 1965)

CHAPTER IV

DISCUSSION OF RESULTS AND CONCLUSIONS

While estimating the stability of slopes, it is often assumed that soil is homogeneous and isotropic. However, it is evident from the review of the published literature (Chapter II) that soil is rarely homogeneous and isotropic. Nonhomogeneity and anisotropy in natural soil deposits affect the stability of slopes in such deposits. There are two major kinds of deviation from the ideal homogeneous material. The first is the case in which the soil consists of layers of distinctly different soils (for example, layered clays), the second being a soil deposit which lacks any distinct stratification but whose properties vary from one point to another over a wide range.

In the present study, it is the former kind of nonhomogeneity that is studied. An analytical method for the evaluation of stability of earth slopes in a two-layered system of anisotropic soils is presented. The approach to this problem is based on the intuition that the overall stability of the slope is governed by the individual stability of the layers. Hence, the two layers are analyzed separately for their stability numbers (from which the factor of safety is obtained, $F = \frac{C}{\gamma H} N$). The stability of a given slope is then dependent on the layer having the lower factor of safety.

It is logical to expect that the ratio of thickness of layers ($n = H_2/H_1$), the anisotropy index ($K = C_2/C_1$, $K' = C_2'/C_1'$), and the relative strength (p) of the layers would influence the stability of the slope. To study these effects, numerical results are obtained for the following values of the above parameters:

$$n = 0.5,\ 1.0,\ 2.0,\ \text{and}\ 3.0$$

$$K \text{ and } K' = 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ \text{and}\ 1.0$$

$$p = -0.5,\ -0.25,\ 0.25,\ 0.5,\ \text{and}\ 1.0.$$

Since there would not be much difference in the unit weights of soil in the two layers, m (γ_2/γ_1) is taken to be unity. All the numerical results which are graphically presented (Figures 7 through 29) were obtained with the aid of IBM 360/65 computer available at Oklahoma State University. The computer program used in obtaining the minimum stability number is listed in Appendix I. The analytical method suggested in this report is valid for slopes steeper than 40°.

The stability number N_2 for the bottom layer is dependent on slope angle (β), coefficient of anisotropy (K'), coefficient of nonhomogeneity (m), thickness ratio (n), and relative strength index (p). Charts (Figures 7 through 19) are presented to show slope angle (β) versus stability number (N_2) for various values of K', n, and p. For all these charts, m is taken to be unity.

The stability number N_1 for the top layer is a function of slope angle (β) and coefficient of anisotropy (K). A chart (Figure 30) showing slope angle (β) versus stability number (N_1) is presented for various values of K (Lo, 1965).

To assess the influence of p (relative strength index) on the stability of the second layer, the stability number (N_2) is plotted against p varying from -0.50 to 0.50 in Figures 20 through 24. It is evident from these charts that the stability number increases linearly with p.

This is in accordance with the expectations that the stronger the stratum the more stable it will be.

Charts (Figures 25 through 29) demonstrate the influence of thickness ratio (n) on stability number (N_2). For a given slope (β) and strength (p), the stability number (N_2) increases with n. Nevertheless, there is a trend indicating that N_2 is less and less influenced with an increase in n from 0.5 to 3.0. For clarity in understanding this statement, four cases are tabulated below. Perhaps, for values of n greater than 3, its influence on N_2 is negligible.

TABLE III

INFLUENCE OF THICKNESS RATIO (n) ON STABILITY NUMBER (N_2)

				percent increase in N_2 as n increases from		
m	p	β	K'	0.5 to 1.0	1.0 to 2.0	2.0 to 3.0
1.0	0.5	60°	0.5	27.50	22.82	10.14
1.0	0.5	60°	1.0	30.24	25.92	11.40
1.0	0.5	40°	0.5	30.60	24.12	11.83
1.0	0.5	40°	1.0	38.85	29.95	12.37

Conclusions

From the above study, the following conclusions may be drawn with regard to stability slopes in a layered system:

1. Charts (Figures 7 through 30) enable the analysis of earth slopes (slopes steeper than 40°) in a two-layered system of anisotropic soils.

2. The overall stability of a slope is dependent on the individual stability of the layers.

3. The stability of the bottom layer is dependent on the thickness ratio (n), the coefficient of anisotropy (K'), the relative strength index (p), $m(\gamma_2/\gamma_1)$, and slope angle (β), whereas the stability of the top layer is a function of the coefficient of anisotropy (K) and slope angle (β).

4. The stability number (N_2) of the bottom layer for a given slope increases linearly with the relative strength index (p).

5. The influence of the thickness ratio (n) on the stability number (N_2) for a given slope reduces gradually as n increases from 0.5 to 3.0. Perhaps, for higher values ($n > 3.0$), its influence on N_2 is negligible. So, for thickness ratios greater than 3.0, the charts for n = 3.0 could be used to analyze the stability of a given slope. These would give a conservative estimate of the factor of safety.

6. The method presented in this thesis assumes the following variation for shear strength:

$$C_i = C_h + (C_v - C_h)\cos^2 i,$$

where

C_i = shear strength along a plane inclined at an angle, i, to the vertical, and

C_h and C_v = shear strengths along the horizontal and vertical planes, respectively.

However, this method could be extended for any other assumed variation for shear strength.

Recommendations for Further Research

During this study, some interesting topics were noted which merit further investigation. Some suggestions in this direction are listed in the following paragraphs.

1. It is not uncommon for a soils engineer to encounter C - φ soils in nature. Therefore, it may be worthwhile to develop an analytical method to solve stability problems in such soils.

2. Pore pressure effects and earthquake effects have not been considered in the present work. It is suggested that a theoretical method could be developed taking these factors into account to assess their influence on stability of earth slopes in layered soils.

3. The application of the finite element method of analysis, which has been found to be versatile in solving problems in some areas of soil mechanics, could be studied to analyze slope stability problems in a layered system of nonhomogeneous and anisotropic soils.

A SELECTED BIBLIOGRAPHY

Aas, G. "A Study of the Effect of Vane Shape and Rate of Strain on the Measured Values of In-Situ Shear Strength of Soils." Proceedings, Sixth International Conference on Soil Mechanics and Foundation Engineering, Vol. 1 (1965), 141-45.

Bishop, A. W. "The Use of the Slip Circle in the Stability Analysis of Slopes." Geotechnique, Vol. V, No. 1 (1955), 7-17.

Brooker and Ireland. "Earth Pressure at Rest Related to Stress History." Canadian Geotechnical Journal, Vol. 2, No. 1 (1965), 1-15.

Casagrande, A., and N. Carrillo. "Shear Failure of Anisotropic Soils." Journal of the Boston Society of Civil Engineers, Vol. 31, No. 4 (1944), 122-35.

Chen, W. F. "Soil Mechanics and the Theorems of Limit Analysis." Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 95, No. SM2 (1969), 493-518.

Chen, W. F., M. W. Giger, and H. V. Fang. "On the Limit Analysis of Stability of Slopes." Soils and Foundations, The Japanese Society of Soil Mechanics and Foundation Engineering, Vol. IX, No. 4 (1969).

Chen, W. F., and M. W. Giger. "Limit Analysis of Stability of Slopes." Journal of Soil Mechanics and Foundations Division, ASCE, Vol. 97, No. SM1 (1971), 19-26.

Coates, D. F. and G. C. McRostie. "Some Deficiencies in Testing Leda Clay." Symposium on Laboratory Shear Testing of Soils, Ottawa, ASTM, STP 361 (1963), 459-470.

Collin, A. (1846). Landslides in Clays. Translated by W. R. Schriever University of Toronto Press, 1956.

Coulomb, C. A. "Eassai Sur une Application des Regles de Maximis et Minimis a grelques Problemes de Statique Relatifs a L'archtecture." Memoires Academie Royale Sciences, Presentes Divers Savants, Paris, Vol. 7 (1776), 343-81.

Culmann, C. Die Graphische Statik. Zurich (1886).

Drucker, D. C., and W. Prager. "Soil Mechanics and Plastic or Limit Design." Quarterly of Applied Mathematics, Vol. 10 (1952), 157-165.

Fellenius, W. Erdstatische Berechnungen mit Reibring und Kohasion und unter Annahme Dreiszylindrischer Cleitflachen. W. Ernst and Sohn, Berlin. (See also Transactions, Second Congress on Large Dams, Washington, D. C. Vol. IV (1936), 445-62.)

Frohlich, O. K. "General Theory of Stability of Slopes." Geotechnique Vol. V, No. 1 (1955), 37-47.

Frohlich, O. K. "Basic Principles of the Computation of the Resistance of Earth Slopes to Sliding." Proceedings, Fifth International Conference on Soil Mechanics and Foundation Engineering Vol. II (1961), 595-602.

Frontard, M. Cycloides de Glissement des Terres. Comptes Rendues, Paris, (1922).

Gilboy, G. "Soil Mechanics Research." Transactions of American Society of Civil Engineers, Vol. 98 (1933), 218-308.

Gibson, R. E. and N. Morgenstern. "A Note on the Stability of Cuttings in Normally Consolidated Clays." Geotechnique, Vol. XII, No. 3 (1962), 212-16.

Hvorslev, M. J. "Physical Components of Shear Strength of Saturated Clays." Proceedings, American Society of Civil Engineers Research Conference on Shear Strength of Cohesive Soils, Boulder, Colorado (1960), 169-273.

Hultin, S. "Grusfyllninger for Kajbyggnader." Teknesk Tidskrift, Vol. 46, No. 292 (1916).

Janbu, N. "Stability Analysis of Slopes with Dimensionless Parameters." Harvard Soil Mechanics Series, No. 46 (1954).

Livneh, M. "Analysis of Slope Stability in an Anisotropic Cohesional Medium under $\varphi_u = 0$ Condition." Israel Journal of Technology, Vol. 5, No. 4 (1967), 274-82.

Lo, K. Y. "Stability of Slopes in Anisotropic Soils." Journal of Soil Mechanics and Foundations Division, ASCE, Vol. 91, No. SM4 (1965), 85-106.

Martin, T. R. "Research on the Physical Properties of Marine Soils, August, 1961 - July, 1962." Research Report R62-42, Soil Engineering Division, Publication No. 127, Massachusetts Institute of Technology, Cambridge, Massachusetts (1962).

Mitchell, J. K. "The Fabric of Natural Clays and Its Relation to Engineering Properties." Proceedings, Highway Research Board, Washington, D. C., Vol. 35 (1956), 693-713.

Morgenstern, N. and V. E. Price. "The Analysis of Stability of General Slip Surfaces." Geotechnique, Vol. XV, No. 1 (1965), 79-93.

Pacey, J. G., Jr., "The Structure of Compacted Soils." M.S. Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts, (1956).

Parcher, J. V. "A Critical Evaluation of Procedures Used to Analyze the Stability of Slopes." Ph.D. Thesis, University of Arkansas (1968).

Petterson, K. E. "The Early History of Circular Sliding Surfaces." Geotechnique, Vol. 5, No. 4 (1955), 275-96.

Poncelet, J. V. Memoire sur la Stabilite des Revetements et des leurs Fondations. Memoires de l'Officier de Genie, Paris (1840).

Quigley, R. M. and C. D. Thompson. "The Fabric of Anisotropically Consolidated Sensitive Marine Clay." Canadian Geotechnical Journal, Vol. 3, No. 2 (1966), 61-73.

Rankine, W. J. M. "On the Stability of Loose Earth." Philosophical Transactions of the Royal Society of London, Vol. 147 (1857), 9-27.

Ranganatham, B. V. and A. C. Mathai. "Effect of Anisotropy on the Stability of Earth Masses Under $\varphi_u = 0$ Condition." Third Asian Regional Conference on Soil Mechanics and Foundation Engineering, Haifa, Israel, Vol 1 (1967), 348-52.

Ranganatham, B. V., A. C. Sani, and V. Sreenivasulu. "Strength Anisotropy on Slope Stability and Bearing Capacity of Clays." Proceedings, Seventh International Conference on Soil Mechanics and Foundations Engineering, Mexico, Vol. 2 (1969), 659-67.

Rao, K. L. "Stability of Slopes in Earth Dams and Foundations Excavations." Proceedings, Fifth International Conference on Soil Mechanics and Foundation Engineering, Paris, Vol. II (1961), 691-95.

Skempton, A. S. "Horizontal Stresses in an Overconsolidated Eocene Clay." Proceedings, Fifth International Conference on Soil Mechanics and Foundation Engineering, Paris, Vol. 1 (1961).

Skempton, A. W. "Fourth Rankine Lecture--Long-Term Stability of Clay Slopes." Geotechnique, Vol, XIV, No. 2 (1964), 75-101.

Spencer, E. "Circular and Logarithmic Spiral Slip Surfaces." Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 95, No. SM1 (1969), 227-34.

Taylor, D. W. "Stability of Earth Slopes." Journal of the Boston Society of Civil Engineers, Vol. 24 (1937), 197-246.

Terzaghi, Karl. "Stability of Slopes in Natural Clay." Proceedings, First International Conference on Soil Mechanics and Foundation Engineering, Harvard, Vol. 1 (1936), 161-65.

Ward, W. H., S. G. Samuels, and M. E. Butler. "Further Studies of the Properties of London Clay." Geotechnique, Vol. 9, No. 2 (1959), 33-58.

APPENDIX A

HYPOTHETICAL PROBLEMS

1. Analyze the slope cut shown in Figure 31 for its stability. The cut is of 30' in height and is on a 40° slope in a layered system of non-homogeneous and anisotropic soils. The properties of the soil in the layers are as follows:

Top Layer: $C_1 = 800$ psf $C_2 = 480$ psf $\gamma_1 = 120$ pcf

Bottom Layer: $C'_1 = 600$ psf $C'_2 = 360$ psf $\gamma_2 = 120$ pcf.

Assume that the critical surface corresponds to a toe failure.

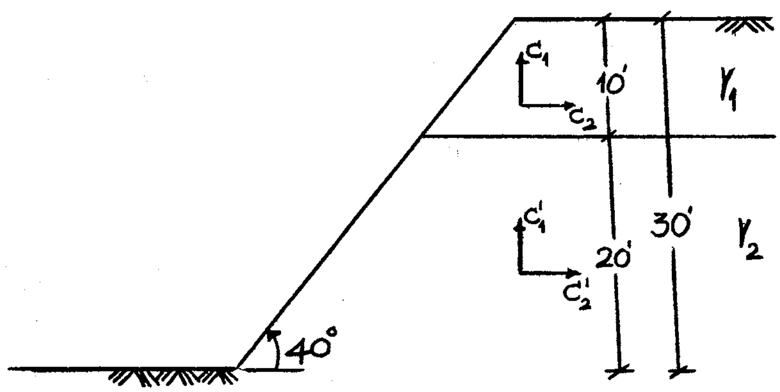


Figure 31. Slope Cut in a Layered System of Anisotropic Soils

Data: $C_1 = 800$ psf, $C_2 = 480$ psf, $K = \frac{C_2}{C_1} = \frac{480}{800} = 0.6$

$C'_1 = 600$ psf, $C'_2 = 360$ psf, $K' = \frac{C'_2}{C'_1} = \frac{360}{600} = 0.6.$

As per the approach followed in this thesis,

$$C'_1 = (p+1)\,C_1$$

$$600 = (p+1)\,800$$

$$(p+1) = \frac{600}{800}$$

$$p = 0.75 - 1.0$$

$$= -0.25.$$

$$m = \frac{\gamma_2}{\gamma_1} = \frac{120}{120} = 1.0$$

$$n = \frac{H_2}{H_1} = \frac{20}{10} = 2.0$$

$\beta = 40^{\circ}$ $M = 1.0$ $n = 2.0$

$p = -0.25$ $K = K' = 0.6.$

Layer 1:

Stability Number (N_1) = 5.198 (From Figure 30)

$$\text{Factor of Safety} = \frac{C_1}{\gamma_1 H_1} N_1$$

$$= \frac{800}{120 \times 10} \times 5.198$$

$$= 3.446.$$

Layer 2:

Stability Number (N_2) = 2.26 (From Figure 15)

$$\text{Factor of Safety} = \frac{C_1}{\gamma_2 H_2} N_2$$

$$= \frac{800}{120 \times 20} \times 2.26$$

$$= 0.753.$$

The stability of the cut is governed by the bottom layer (weaker), since the factor of safety for this layer is less than that for the top layer. In the present case, the cut is unstable, since the factor of safety is less than one.

2. An embankment 30 feet high (Figure 32) is made up of two soils, S_1 and S_2, whose properties are given below. The soil S_1 is used for constructing the lower 10 feet of the embankment, and soil S_2 is used for the rest.

Soil S_1: $C_1 = 500$ psf $\quad C_2 = 400$ psf $\quad \gamma_1 = 120$ pcf

Soil S_2: $C_1' = 1000$ psf $\quad C_2' = 800$ psf $\quad \gamma_2 = 120$ pcf.

Analyze this embankment for its stability. Assume that the critical failure surface corresponds to toe failure.

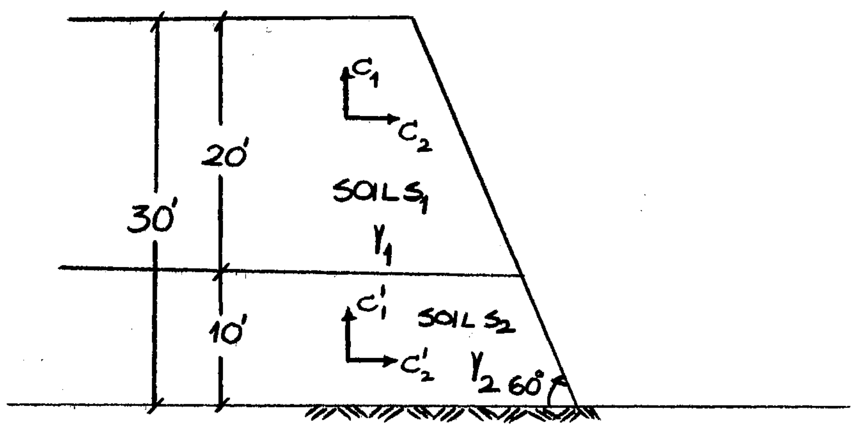


Figure 32. An Embankment with a Layered System of Anisotropic Soils

Data:

Soil S_1: $C_1 = 500$ psf, $C_2 = 400$ psf, $K = \frac{C_2}{C_1} = \frac{400}{500} = 0.8$

Soil S_2: $C'_1 = 1000$ psf, $C'_2 = 800$ psf, $K' = \frac{C'_2}{C'_1} = \frac{800}{1000} = 0.8$

As per the notation followed in this thesis,

$$C'_1 = (p + 1)\, C_1$$

$$1000 = (p + 1)\, 500$$

$$(p + 1) = 2.0$$

$$p = 1.0$$

$$m = \frac{\gamma_2}{\gamma_1} = \frac{120}{120} = 1.0$$

$$n = \frac{H_2}{H_1} = \frac{10}{20} = 0.5.$$

$\beta = 60^{o}$ $m = 1.0$ $n = 0.5$

$p = 1.0$ $K = K' = 0.8.$

Top Layer:

Stability Number = 5.023 (From Figure 30)

$$\text{Factor of Safety} = \frac{C_1}{\gamma_1 H_1} N_1$$

$$= \frac{500}{120 \times 20} \times 5.023$$

$$= 1.0465.$$

Bottom Layer:

Stability Number = 3.3415 (From Figure 19)

$$\text{Factor of Safety} = \frac{C_1}{\gamma_2 H_2} N_2$$

$$= \frac{500}{120 \times 10} \times 3.3415$$

$$= 1.3920.$$

The stability of the embankment is controlled by the top layer (weaker), as the factor of safety for this is less than that for the bottom layer.

APPENDIX B

COMPUTER PROGRAM

80/80 LIST

```
0000000001111111111222222222233333333334444444444555555555566666666667777777778
1234567890123456789012345678901234567890123456789012345678901234567890123456789 0
C      OKLAHOMA STATE UNIVERSITY               D.DHAVALA
C
C
C      STABILITY ANALYSIS OF SLOPES IN A TWO-LAYER SYSTEM OF
C      ANISOTROPIC SOILS
C
C
C      DESCRIPTION OF PARAMETERS
C
C
C      B         =SLOPE ANGLE
C      M         =COEFFICIENT OF NON-HOMOGENEITY
C      T         =THICKNESS RATIO
C      P         =RELATIVE STRENGTH INDEX
C      RP        =COEFFICIENT OF ANISOTROPY
C      SN2       =STABILITY NUMBER-LAYER 2
C      AP        =GEOMETRICAL PARAMETER DEFINING CRITICAL SLIP SURFACE.
C                 ALPHA PRIME
C      BP        =GEOMETRICAL PARAMETER DEFINING CRITICAL SLIP SURFACE.
C                 LAMDA TWO
C      XL(1)     =LOWER LIMIT ON AP
C      XL(2)     =LOWER LIMIT ON BP
C      XR(1)     =UPPER LIMIT ON AP
C      XR(2)     =UPPER LIMIT ON BP
C
C
C      SUBROUTINES REQUIRED
C
C      PATRN, EXPLOR, AND MERIT
C
C
       IMPLICIT REAL*8 (A-H,O-Z)
       COMMON /MNMERT/ AA,BB,B,R,RP,SN2
       DIMENSION X(9),XL(9),XR(9)
       N=2
       NP=2
       DELTA=0.01
       ROW=0.5
       F=0.001
       Q=3.1415926D0/18.0D0
   200 CONTINUE
       READ 201,RP
   201 FORMAT(1E10.3)
       IVAL=10.0*RP
       IF (IVAL .EQ. 0) GO TO 999
       DO 25 I=5,7
       B=I*Q
       XL(1)=0.1745
       XL(2)=0.1745
       XR(1)=0.8855
       XR(2)=0.5655
       X(1)=0.5236
       X(2)=0.4363
       CALL PATRN(N,NP,DELTA,F,XL,XR,Y,X,ROW,NN)
```

80/80 LIST

```
00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
CARD
      CALL MERIT(X,Y)
      WRITE(6,3) AA,BB,B,R,RP,SN2
    3 FORMAT(/,1X,'AA=',F15.8,/,1X,
     1'BB=',F15.8,/,1X,
     1'B=',F15.8,/,1X,
     1'R=',F11.4,/,1X,
     1'RP=',F11.4,/,1X,
     1'SN2=',F11.4,//)
   25 CONTINUE
      GO TO 200
  999 STOP
      END






      SUBROUTINE PATRN(N,NP,DELTA,F,XL,XR,Y,X,ROW,NN)
      IMPLICIT REAL*8 (A-H,O-Z)
C     PATTERN SEARCH FOR MULTIVARIABLES
C     THIS SUBROUTINE CONDUCTS A PATTERN SEARCH
C     WITHIN REGIONAL CONSTRAINTS IN A HIPERSURFACE
C     OF UPTO NINE INDEPENDENT VARIABLES.
C
C     CALLING PROGRAM REQIREMENTS.
C
C     PROVIDE A SUBROUTINE MERIT FROMWHICH AN ORDINATE Y IS RETURNED
C     WHEN COLUMN VECTOR ABSCISSA X IS RETURNED.
C
C     VARIABLES.
C     N=NUMBER OF INDEPENDENT VARIABLES.
C     NP=CONVERGENCE MONITOR.
C     NP=C WILL NOT PRINT.
C     NP 1 WILL PRINT EVERY ITERATION.
C     NP 2 WILL PRINT EVERY 2ND ITERATION.
C     DELTA =CURRENT STEP SIZE.
C     F=MINIMUM STEP SIZE
C     XL=LOWER BOUND OF SEARCH DOMAIN
C     XR=HIGHER BOUND OF SEARCH DCMAIN
C     Y=FUCTIONAL VALUE RESULTING FROM CURRENT MOVE
C     YY=FUNCTIONAL VALUE AT BASE POINT
C     YYY=FUNCTIONAL VALUE AT CURRENT BASE POINT
C     XXX=PREVIOUS BASE POINT
C     XX=BASE POINT RESULTING FROM CURRENT MOVE
       DIMENSION X(9),XX(9),XXX(9),XL(9),XR(9),NG(9)
       NF=0
      N1=0
      N2=0
      NN=0
       DELTA1=DELTA
        DO 45 I=1,N
   45  NG(I)=I
      IF (NP)5,5,6
```

80/80 LIST

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00000000011111111112222222222333333333344444444445555555555666666666677777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
    6  WRITE(6,7)(NG(I),I=1,N)
    7  FORMAT(2X,'NN',6X,'DELTA',9X,'Y',2X,9(7X,'X(',I2,')'))
    5 CALL MERIT(X,Y)
       NF=NF+1
      NN=NN+1
      IF(NP)31,31,32
   32 WRITE(6,33)NN,DELTA,Y,(X(I),I=1,N)
   33 FORMAT(1X,I4,9(2X,E11.4))
   31 CONTINUE
C     STRAT AT BASE POINT
    1 YY=Y
      DO 10 K=1,N
      XX(K)=X(K)
   10 CONTINUE
C     MAKE EXPLORATORY MOVES
        CALL EXPLOR(N,XX,YY,XL,XR,DELTA,ROW,NF)
C     IS PRESENT FUNCTIONAL VALUE BELOW THAT AT BASE POINT?
C
      IF(YY - Y)3,3,2
C     SET NEW BASE POINT
    2 DO 12 K=1,N
      XXX(K)=X(K)
      X(K)=XX(K)
   12 CONTINUE
      Y=YY
C     MAKE PATTERN MOVE
      DO 14 K=1,N
      XX(K)=2.0*XX(K)-XXX(K)
   14 CONTINUE
C     CHECK IF CONSTRAINT IS VIOLATED
      DO 20 I=1,N
      IF(XX(I)-XL(I))41,42,42
   41 XX(I)=XL(I)
   42 IF(XX(I)-XR(I))20,20,44
   44 XX(I)=XR(I)
   20 CONTINUE
      CALL MERIT(XX,YYY)
       NF=NF+1
      NN=NN+1
      YY=YYY
      IF (N2)21,22,21
   21 N2=N2+1
      IF(N2-NP)22,23,22
   22  WRITE(6,33)NN,DELTA,Y,(X(I),I=1,N)
      N2=0
   23  CALL EXPLOR(N,XX,YY,XL,XR,DELTA,ROW,NF)
      IF(YY-Y)1,1,2
    3 DDELTA=DELTA-F
      IF (DDELTA)13,15,15
C     DECREASE STEP SIZE
   15 DELTA=ROW*DELTA
      GO TO 1
   13  WRITE(6,100)NF,Y,NN,DELTA1,DELTA,ROW,F
  100 FORMAT(/,1X,'TOTAL NUMBER OF FUNCTION EVALUATIONS.......',I5,/,1X,
```

80/80 LIST

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000000000111111111122222222223333333333444444444455555555556666666666777777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
     1'LARGEST MERIT ORDINATE.....................',E15.8,/,1X,
     I'NUMBER OF BASE      EVALUATIONS............',I15,/,1X,
     3'ORIGINAL STEP SIZE.........................',F15.8,/,1X,
     4'FINAL STEP SIZE............................',F15.8,/,1X,
     5'REDUCTION FACTOR FOR STEP SIZE.............',F15.8,/,1X,
     6'FRACTIONAL REDUCTION OF UNCERTAINITY.......',F15.8,//)
      RETURN
      END
C
C
       SUBROUTINE EXPLOR(N,XX,YY,XL,XR,DELTA,ROW,NF)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION XX(9),XL(9),XR(9)
      DO 10 K=1,N
C     INCREASE ORDINATE, CALCULATE ORDINATE
      XX(K)=XX(K)+DELTA
C     CHECK IF CONSTRAINT IS VIOLATED
      IF(XX(K)-XL(K))21,22,22
   21 XX(K)=XL(K)
   22 IF(XX(K)-XR(K))24,24,23
   23 XX(K)=XR(K)
   24 CONTINUE
      CALL MERIT(XX,YYY)
       NF=NF+1
C     IS MOVE A SUCCESS ?
      IF(YYY-YY)1,1,2
C     RETAIN NEW CO ORDINATE AND NEW FUNCTIONAL VALUE
    2 YY=YYY
      GO TO 10
C     DECREASE ORDINATE,CALCULATE NEW ORDINATE
    1 XX(K)=XX(K)-2.0*DELTA
      IF(XX(K)-XL(K))25,26,26
   25 XX(K)=XL(K)
   26 IF(XX(K)-XR(K))28,28,27
   27 XX(K)=XR(K)
   28 CONTINUE
      CALL MERIT(XX,YYY)
       NF=NF+1
C     IS MOVE A SUCCESS ?
      IF(YYY-YY)3,3,4
C     RETURN CO-ORDINATE & NEW FUNCTIONAL VALUE
    4 YY=YYY
      GO TO 10
C     RESET CO-ORDINATE
    3 XX(K)=XX(K)+DELTA
   10 CONTINUE
      RETURN
      END
```

80/80 LIST

```
0000000001111111111222222222233333333334444444444555555555566666666667777777778
1234567890123456789012345678901234567890123456789012345678901234567890
      SUBROUTINE MERIT(X,Y)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /MNMERT/ AA,BB,B,R,RP,SN2
      REAL*8 DSIN,DCOS,M,N,AA,BB
      DIMENSION X(9),F(4)
C     AP=ALPHA PRIME
C     BP=LAMDA TWO
      AP=X(1)
      BP=X(2)
      M=1.0D0
      T=3.0D0
      P=-0.25D0
      SAP=DSIN(AP)
      CAP=DCOS(AP)
      C2AP=DCOS(AP+AP)
      COTAP=CAP/SAP
      SBP=DSIN(BP)
      CBP=DCOS(BP)
      COTBP=CBP/SBP
      COTBP2=COTBP*COTBP
      COTBP3=COTBP2*COTBP
      SB=DSIN(B)
      CB=DCOS(B)
      TANB=SB/CB
      COTB=CB/SB
      COTB2=COTB*COTB
      COTB3=COTB2*COTB
      S2AP=DSIN(AP+AP)
      SAP2=SAP*SAP
      CSAP2=1.0D0/SAP2
      SBP2=SBP*SBP
      CSBP2=1.0D0/SBP2
      Y1=2.0D0*(AP-BP)
      CY1=DCOS(Y1)
      SY1=DSIN(Y1)
      R=COTBP*TANB
      IF(R .GT. (4.0/3.0)) GO TO 50
      AA=0.25D0*(1.0D0+RP)*(1.0D0+P)*AP*CSAP2*CSBP2-0.125D0*(1.0D0-RP)*(
     11.0D0+P)*CSAP2*CSBP2*S2AP*CY1+0.5D0*(COTBP2*TANB+COTAP*COTBP*TANB-
     1COTBP-COTAP)
      BB=(COTAP*COTBP2*TANB/(4.0D0*M))+(COTBP3*TANB/(12.0D0*M))-(COTBP2/
     112.0D0)+((M-2.0D0)/(4.0D0*M))*COTAP*COTBP+((M-1.0D0)/(4.0D0*M))*CO
     1TBP*COTB+((1.0D0-M)/(4.0D0*M))*COTB*COTAP+((1.0D0-M)/(6.0D0*M))*CO
     1TB2+(1.0D0/12.0D0)*CSBP2
      F(1)=(0.25D0*(1.0D0+RP)*(1.0D0+P)*CSBP2-0.5D0*(1.0D0+RP)*(1.0D0+P)
     1*AP*COTAP*CSBP2+0.25D0*(1.0D0-RP)*(1.0D0+P)*COTAP*CSBP2*S2AP*CY1-0
     1.25D0*(1.0D0-RP)*(1.0D0+P)*CSBP2*C2AP*CY1+0.25D0*(1.0D0-RP)*(1.0D0
     1+P)*CSBP2*S2AP*SY1+0.5D0*(1.0D0-COTBP*TANB))*BB+(0.25D0*COTBP2*TAN
     1B/T+(M-2.0D0)*COTBP/(4.0D0*M)+(1.0D0-M)*COTB/(4.0D0*M))*AA
      F(2)=(-0.5D0*(1.0D0+RP)*(1.0D0+P)*AP*CSAP2*COTBP+0.25D0*(1.0D0-RP)
     1*(1.0D0+P)*CSAP2*COTBP*S2AP*CY1-0.25D0*(1.0D0-RP)*(1.0D0+P)*CSAP2*
     1S2AP*SY1+0.5D0*(-2.0D0*COTBP*TANB-COTAP*TANB+1.0D0))*BB+(0.5D0*COT
     1AP*COTBP+TANB/M+0.25D0*COTBP2*TANB/M+(M-2.0D0)*COTAP/(4.0D0*M)+(M-
     11.0D0)*COTB/(4.0D0*M))*AA
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80/80 LIST

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0000000001111111111222222222233333333334444444444555555555566666666667777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
      SN2=AA/BB
      Y=F(1)*F(1)+F(2)*F(2)
      Y=-Y
      RETURN
   50 AA=0.25D0*(1.0D0+RP)*(P+1.0D0)*AP*CSAP2*CSBP2-0.125D0*(1.0D0-RP)*(
     11.0D0+P)*CSAP2*CSBP2*S2AP*CY1+(COTAP+COTBP)/(2.0D0*T)
      BB=(0.5D0/(M*T)+0.25D0/(M*T*T)+0.25D0)*(COTBP-COTAP)*COTB-(0.5D0/(
     1M*T)+1.0D0/(M*T*T)+1.0D0/(6.0D0*M*T*T*T)+1.0D0/6.0D0)*COTB2+(0.5D0
     1/(M*T*T))*COTB3+1.0D0/12.0D0+(0.5D0/(M*T)+0.25D0)*COTBP*COTAP
      F(1)=(0.25D0*(1.0D0+RP)*(1.0D0+P)*CSBP2-0.5D0*(1.0D0+RP)*(1.0D0+P)
     1*AP*COTAP*CSBP2-0.25D0*(1.0D0-RP)*(1.0D0+P)*C2AP*CY1*CSBP2+0.25D0*
     1(1.0D0-RP)*(1.0D0+P)*S2AP*SY1*CSBP2+0.25D0*(1.0D0-RP)*(1.0D0+P)*S2
     1AP*CY1*COTAP*CSBP2-0.5D0/T)*BB+AA*((0.25D0+0.5D0/(M*T))*COTBP-(0.5
     1D0/(M*T)+0.25D0/(M*T*T)+0.25D0)*COTB)
      F(2)=(-0.5D0*(1.0D0+RP)*(1.0D0+P)*AP*CSAP2*COTBP-0.25D0*(1.0D0-RP)
     1*(1.0D0+P)*S2AP*SY1*CSAP2+0.25D0*(1.0D0-RP)*(1.0D0+P)*S2AP*CY1*CSA
     1P2*COTBP-0.5D0/T)*BB+((0.5D0/(M*T)+0.25D0)*COTAP+(0.5D0/(M*T)+0.25
     1D0/(M*T*T)+0.25D0)*COTB)*AA
      SN2=AA/BB
      Y=F(1)*F(1)+F(2)*F(2)
      Y=-Y
      RETURN
      END
```

VITA

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