A STUDY OF THE DESIGN OF AN INDUCTOR ALTERNATOR OF INTERMEDIATE FREQUENCY WITH GIVEN SPECIFICATIONS

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By

Astor Lewis Thurman

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APPROVED:

Research Professor Nicholas Oboukhoff

In Charge of Thesis

A. Naeter

Head, Department of Electrical Engineering

D. C. McIntosh

Dean of Graduate School

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PREFACE

The purpose of this paper is primarily to show the application of a method of designing medium and high frequency alternators of the inductor type as proposed by Dr. N. M. Oboukhoff in two of his bulletins published by the Engineering Experiment Station at the Oklahoma Agricultural and Mechanical College.[1] A sample design is presented, in which is shown a method of attack on the problems encountered in both the electrical and mechanical design of the inductor alternator.

In the past, the principal reason for the limited use of the inductor alternator has been the difficulty and uncertainty in its design. The Oboukhoff method has removed the uncertainty in the design of this type of machine, and now enables the designer to expect accuracy comparable to that in the design of the ordinary alternating current machine. His method has been checked in the actual design of inductor alternators, as well as with experimental data taken on machines already built.[2]

This method of design applies to all sizes of medium and high frequency alternators ranging from fractional kilowatt machines up to the larger alternators concerned with inductive furnaces in metallurgy. Among the many possible uses of medium and high frequency alternators, the most important are:

1. For induction furnaces in metallurgy and other heating purposes.
2. For supplying current to high speed induction motors.
3. For use in power control on transmission lines.
4. For energizing submarine signalling systems.
5. For use in modern mining prospecting methods.
6. For use in industrial and laboratory measurements and tests.

A more complete list of applications is listed by Oboukhoff in one of his bulletins.[3]

This thesis is written in the hope that it may be an aid to the designer of medium, intermediate, or high frequency inductor alternators.

GENERAL CONSIDERATIONS

The problems encountered in the design of the inductor alternator are more difficult and complicated than those in the alternator of ordinary commercial frequencies. In the first place, a method of design altogether different from that used for the ordinary alternator is necessary because of the unilateral flux existing in the machine. Extended research and theoretical study have been necessary for the development of a basic formula for γ, which gives a relationship between the ratio ϕ_{max}/ϕ_{min} and the physical dimensions of the machine.[4] On account of the existence of the unilateral flux and its fluctuation between $+\phi_{max}$ and $+\phi_{min}$, it is also necessary to use a different method for the computation of iron losses.[5] Next, the cooling problem is more difficult to solve because of the excessive heating of the iron due to high eddy current and hysteresis losses at the frequencies used. The nature of the cooling problem is such that it necessitates the use of circulating water or oil for larger size machines. Herein a design of the necessary cooling system is presented.

Because of the high eddy current losses, very thin laminations are necessary, this being one of the expensive items in the first cost of the machine. The lack of information concerning the losses with such thin laminations and at such high frequencies introduces an uncertainty in the determination of losses. Several thicknesses of laminations were considered with the aim of securing a most economical design.

The small air gap necessary in the inductor alternator introduces several vital considerations. The radial elongation due to centrifugal force and temperature rise, as well as the sag of the rotor, cannot be neglected. The problem of stresses due to centrifugal force also arises, and the solution is presented here along the lines of the design of high speed rotating discs.

The critical speed of the rotor due to its sag and any possible eccentricity must be calculated and must be safely above the speed at which the machine operates in order to avoid the effect of mechanical resonances.

It is possible to use a much higher current density in the armature conductors of the inductor alternator than in ordinary alternating current machines because of better cooling on account of a finer distribution of the winding around the periphery. Then, too, there is relatively more cooling surface per unit volume of stator slot. The finer distribution of the armature winding around the periphery of the machine allows an even dissipation of the heat.

Since the inductor alternator operates at resonance, it becomes necessary to determine the inductance of the machine in order to know the amount of capacitance required. A machine of this type must be designed with a view of holding the inductance to a reasonable value in order that tuning of the circuit may be possible. Dr. Oboukhoff presents a method for the computation of inductive reactance of the inductor alternator in his second bulletin on that subject,[6] the application of which was used for the determination of the inductance in this design.

Details of construction have been studied by the method of starting with several variants, working out separate results for each of them, and then choosing the most desirable one. This method necessitates numerous computations, but offers the important advantage of a careful study and choice of the best results.

An effort has been made to take into account all practical considerations, and to use a reasonable factor of safety in the selection of each final value. Adjustments may be made in most cases, involving small changes in the actual construction of the machine. When more accurate information is available regarding losses at high frequency, the designer of this type of machine will be able to proceed with less caution.

Both the English and c.g.s. system of units have been used in this paper as a matter of convenience in computation, but when necessary a conversion from one to the other has been shown.

NOTATION

B_{max} denotes the maximum flux density in the air gap corresponding to the median plane of a pole (rotor tooth) when it faces a stator tooth.

B_{min} denotes the minimum flux density in the median plane of an interpolar space at the stator periphery.

u denotes the linear peripheral velocity in cm. per second of a rotor.

m denotes the number of parallel connected circuits of the armature a.c. winding.

z denotes the number of conductors connected in series in one slot of the stator or the number of turns in series in one coil, two slots being used per coil.

S denotes the number of coils of the armature winding connected in series (i.e. in one circuit).

$E_{1\,max}$ denotes the maximum or crest value of the fundamental sine wave of e.m.f.

E_1 denotes the r.m.s. of same. $E_1 = \frac{E_{1\,max.}}{\sqrt{2}}$

ζ denotes the value of the flux linked with one coil of the armature winding per cm. length of the iron core in a stator and per ampere turn in a stator slot.

p_r denotes the pole (rotor tooth) pitch.

p_s denotes the stator slot pitch (or stator tooth pitch).

a_r denotes the peripheral width of a pole.

a'_r denotes the width of a pole at its root.

L_a denotes the coefficient of self induction of the armature winding.

f denotes the frequency of e.m.f.

T denotes its period.

$\omega = 2\pi f$.

L denotes the coefficient of self induction of one of the parallel circuits of the armature winding so that $L = mL_a$.

l denotes the gross iron length in cm. of the armature core.

b_r denotes the peripheral width of a rotor slot.

a_s denotes the peripheral width of a stator tooth.

b_s denotes the peripheral width of a stator slot (the opening at the air gap).

a'_s denotes the least width of a stator tooth.

b denotes the width of a stator slot which corresponds to a'_s.

$t = \frac{2a'r}{p_r}$.

$\alpha = \frac{B_{min}}{B_{max}}$.

ϕ denotes the flux linked with one coil of the armature winding.

ϕ_{max} denotes its maximum value.

ϕ_{min} denotes its minimum value.

γ denotes the coefficient of flux oscillation $= \frac{\phi_{max}}{\phi_{min}}$.

δ denotes the single air gap length.

B_r denotes the maximum flux density in poles.

B_s denotes the maximum flux density in stator teeth.

$B_{s\ min}$ denotes the minimum flux density in stator teeth.

B'_s denotes the maximum flux density in the stator tooth due to armature current.

B''_s denotes the resultant flux density in the stator tooth due to excitation flux combined with armature current flux.

ψ denotes the armature core iron space factor, i.e. the ratio of the net iron length to the gross iron core length l.

DESIGN

GIVEN SPECIFICATIONS:

Frequency............................5,500 cycles per second

Peripheral Velocity...................100-150 meters per second

Power Output..........................150 kilowatts

Output Voltage........................250-600 volts

Type of Load..........................Steady

PRELIMINARY STEPS

The first step in the design is the determination of the pole pitch, p_r, which is a function of the peripheral velocity, u, and the frequency, f. A value of 100 meters per second was first chosen for u with the idea of obtaining the maximum safety from stresses due to centrifugal force. Using this value, $p_r = u/f = 100/5500 = 0.01818$ meters $= 1.818$ cm.

Choosing a speed of 4,500 revolutions per minute, or seventy-five revolutions per second, the rotor circumference becomes $100/75 = 1.333$ meters $= 133.3$ cm. The number of poles then is $133.3/1.818 = 73.4$. Use seventy-four poles. Then the circumference must be $74 \times 1.818 =$ 134.4 cm., which changes the r.p.m. to $60 \times 100/1.344 = 4460$. Diameter of rotor $= 134.4/\pi = 42.8$ cm.

It is now necessary to choose a value for the air gap. If high standard workmanship is used, the air gap may be 0.7 to 0.8 mm. for machines this size. For more common workmanship a value for δ of 1.25 mm. may reasonably be selected.

The inside stator diameter, then, is rotor diameter plus 2δ, or $42.8 + 0.25 = 43.05$ cm.

Inside stator circumference $= 43.05\pi = 135.2$ cm.

A reference to Plate III will show the relation between the stator slot pitch, p_s, and the rotor pole pitch, p_r. It is seen from this that for each pole there are two stator slots and two stator teeth. Hence, p_s = circumference/(no. of stator slots) = 135.2/(2)(74) = 0.913 cm.

The ratio of δ/p_r determines the class of alternator as classified according to Oboukhoff on pages 34 and 35 in his first bulletin on the inductor alternator.[7] This classification is necessary in order to determine which formula to use for the determination of γ.

$$\delta/p_r = 0.125/1.818 = 0.0687,$$

which puts this design in class C, in which

$$0.049 < \delta/p_r < 0.0794.$$

For class C the optimum a_r (rotor pole width) is 0.361 p_r, as given by Oboukhoff in his first bulletin,[8] or

$$\text{optimum } a_r/p_r = 0.361. \quad 2a_r/p_r = 0.722.$$

A reasonable value for the peripheral width of a stator slot, b_s (the opening at the air gap), is one millimeter. Then $a_s = p_s - b_s$ = 0.913 - 0.1 = 0.813 cm. a_s/p_s = 0.813/0.913 = 0.89. $2a_r/p_r$ = 0.722.

The value for B_{max}, the maximum flux density in the air gap, is dependent upon the physical dimensions of the machine and is generally limited by the maximum flux density in the stator tooth, B_s, although the rotor tooth flux density, B_r, may sometimes be the limiting factor. One trial value for B_{max} will be sufficient to give a good indication as to the proper value to assume. Since B_r and B_s should lie somewhere in the region of 10,000-11,350,[9] a reasonable value for B_{max} would be from 4,000-7,000. Try B_{max} = 5,000.

All values necessary for the determination of γ are now known and may be substituted in formula (51) of the first bulletin.[10]

$$\gamma = 1 + \frac{p_r\left(3.25-2.25\frac{2a_r}{p_r}\right)\frac{2a_r}{p_r}\left(3.25-2.25\frac{a_s}{p_s}\right)\frac{a_s}{p_s}}{10\delta + B_{max}^2 10^{-7}}$$

$$= 1 + \frac{18.18[3.25-(2.25)(0.722)](0.722)[3.25-2.25(0.89)](0.89)}{10(1.25) + (5000)^2 10^{-7}}$$

$$\gamma = 2.577$$

It is now possible to make a fairly close determination of B_r by use of formula (35) of the first bulletin.

$$\left.\begin{aligned} B_r &= \frac{B_{max}}{t}\left[2 - k\left(1-\frac{1}{\gamma}\right)\right] \\ k &= \frac{2a_p}{a_p+a'_p}\;\frac{1}{1+0.5\left(1-\frac{1}{\gamma}\right)\left(\frac{2a_p}{a_p+a'_p}-1\right)} \\ a_p &= p_r/2 = 1.818/2 = 0.909 \end{aligned}\right\} \quad 11$$

Values for a'_p, the width of the wave form at its top, may be approximated by estimating the flux wave form of the alternator to be designed. As an average for this trial, the wave form II as shown on page 13 of the first bulletin[12] may be assumed. Referring to Figure (9) on that page, it is seen that $a'_p/a_p = 0.667$, which is little different from $2a_r/p_r = 0.722$, as obtained on page 6. $a'_p = 0.667\ (0.909) = 0.606$.

$$k = \frac{2(0.909)}{0.909+0.606}\left[\frac{1}{1+0.5\left(1-\frac{1}{2.577}\right)\left(\frac{2(0.909)}{0.909+0.606}-1\right)}\right] = 1.192$$

$$t = \frac{2a'_r}{p_r}$$

Use $a'_r = p_r/2$ to give a trapezoidal form to the tooth. Then $t = 1$.

$$B_r = \frac{5000}{1}\left[2 - 1.192(0.612)\right] = 6350,$$

which is reasonable.

To determine the value of B_s, it is first necessary to design the slots to hold the armature conductors. For economical construction, the stator winding should be divided into two halves, making it possible to obtain two voltages. Since 225 and 450 volts are common values, they will be selected, although a higher voltage would make the design less difficult, since it would lower the current in the armature winding.

The number of conductors connected in series in one slot should of a necessity be limited to one because of the small available space for the placing of the conductor, p_s being only 0.913 cm. The current density should range from six to twelve amperes per square mm. as indicated by Oboukhoff.[13]

Use 10 amperes per square mm.

$$I = \frac{150,000}{450} = 333.3 \text{ amperes.}$$

Cross-sectional area of conductor = 333.3/10 = 33.33 sq. mm.

Because of the small value of p_s, use a conductor of height = 2.5 width.

$$\text{Width} = \sqrt{\frac{33.33}{2.5}} = 3.65 \text{ mm.}$$

One millimeter is a sufficient allowance for the insulation on each side of the conductor for the small value of voltage generated in one slot. Then the total width of the slot would be 5.65 mm.

This gives a'_s, the least width of the stator tooth, a value of 0.913 - 0.565 = 0.348 cm.

The armature iron space factor, ψ, should be assumed as 0.8 because of the thinness of laminations in an alternator of this frequency.

The formula for B_s is[14]

$$B_s = \frac{k'}{\psi}\frac{p_s}{a'_s} B_{max},$$

$$\text{where } k' = \frac{1}{1+\left(1-\frac{1}{\gamma}\right)\left(\frac{a_p}{a_p+a'_p}-0.5\right)} = \frac{1}{1+(0.612)\left[\frac{0.909}{0.909+0.606}-0.5\right]}$$

$$k' = 0.994$$

$$B_s = \frac{(0.994)(0.913)(5000)}{(0.8)(0.348)} = 16,300,$$

which is too great.[15]

A lower value of B_{max} might now be assumed, but would necessarily be so low that the design would not be economical, since the length of the stator would be greater as B_{max} is made smaller if the same e.m.f. is to be obtained.

Furthermore, the dimension of 0.348 cm. as obtained above for p_s makes the stator teeth too narrow from the mechanical standpoint. Thus the use of 100 m.p.s. for peripheral velocity is not practical; likewise trials for higher velocities led to the same conclusion until 140 meters per second was assumed.

FUNDAMENTAL DIMENSIONS

Then $p_r = 14,000/5,500 = 2.545$ cm. Make $p_r = 2.55$ cm., with peripheral velocity of 140.2 meters per second.

Use a speed of 4,500 r.p.m. = 75 r.p.s.

Circumference = 14,020/75 = 187 cm.

Poles = 187/2.55 = 73.3. Use 74 poles.

Circumference = 74 x 2.55 = 188.7 cm.

r.p.m. = 60 x 14,020/188.7 = 4,460.

Rotor diameter = $188.7/\pi$ = 60.07 cm.

Inside stator diameter = 60.07 + 2(.125) = 60.32 cm.

Inside stator circumference = 60.32π = 189.5 cm.

$P_s = 189.5/2(74) = 1.28$ cm.

$\delta/p_r = 0.125/2.55 = 0.049016$, which puts this alternator on the boundary between class B and class C alternators.[7] <u>This being the case,</u>

a more accurate design may be obtained by taking an average between the results obtained from the formulas for each class.

For class B alternator:[8]

optimum $a_r/p_r = 0.4165$

For class C alternator:[8]

optimum $a_r/p_r = 0.361$

Average $= (0.4165 + 0.361)/2 = 0.38875$

Then optimum $a_r = (0.38875)(2.55) = 0.991$ cm.

$2a_r/p_r = 0.7775$, which shows that the approximate value of a flux wave form might reasonably be selected as the intermediate one between the trapezoids I and II, page 13 of Oboukhoff's publication, Part I; that is, it may be assumed that actual $2a_r/p_r = 0.75$.

It is practical to increase a_r in order to have a low value of B_r. This does not affect noticeably the value of γ , because optimum a_r/p_r has been obtained on the basis of γ becoming maximum; it is well known that a continuous function changes slowly in the neighborhood of its maximum or minimum.

Make $a_r = 1.03$ cm.

$a_r/p_r = 1.03/2.55 = 0.4035$

$a_s/p_s = (p_s - b_s)/p_s = (1.28-0.1)/1.28 = 0.921$

DETERMINATION OF γ

Several trials have shown the proper value for B_{max} to be 4,750.

Class C formula:

$$\gamma = 1 + \frac{p_r\left(3.25 - 2.25\frac{2a_r}{p_r}\right)\frac{2a_r}{p_r}\left(3.25 - 2.25\frac{a_s}{p_s}\right)\frac{a_s}{p_s}}{10\,8 + B_{max}^2\,10^{-7}}$$

$$= 1 + \frac{25.5\left(3.25 - 2.25(0.807)\right]0.807\left[3.25 - 2.25(0.921)\right](0.921)}{10(1.25) + 2.26}$$

$$\gamma = 3.115$$

Class B formula:

$$\gamma = 1 + \frac{p_r\left(2.5 - 1.5\frac{2a_r}{p_r}\right)\frac{2a_r}{p_r}\left(2.5 - 1.5\frac{a_s}{p_s}\right)\frac{a_s}{p_s}}{A\delta + B_{max}^2 10^{-7}}$$

$$\gamma = 1 + \frac{25.5[2.5 - 1.5(0.807)](0.807)[2.5 - 1.5(0.921)](0.921)}{10(1.25) + 2.26}$$

because $A = 10$ for $\delta/p_r > 0.025$

$$\gamma = 2.853$$

Use γ as an average between the two values.

$$\gamma = \frac{3.175 + 2.853}{2} = 3.014.$$

LENGTH OF ROTOR AND STATOR TEETH

In this type of alternator the flux wave form obviously may be approximated by the form of the rotor tooth. In other words, $a'_p/a_p = a'_r/p_r/2 = (2)(1.03)/(2.55) = 0.809.$[16] Since actually the trapezoid flux wave form will have rounded corners on its top, it is better to estimate a'_p/a_p as being equal to 0.75, the same value as used previously.

Interpolating from Table I on page 19 of Part I of Oboukhoff's publications for a wave form intermediate between trapezoidal curves I and II for $\gamma = 3.014$, σ is found to be 0.9595.

In the medium, intermediate, or high frequency alternators, the fundamental component of the e.m.f. wave is the only one which is present at resonance conditions. The working e.m.f., which is indicated in all cases by the subscript 1, or E_1, is used as the root mean square value of the fundamental component.

Combining formula (22) on page 25 with formula (27) on page 27 in the first Oboukhoff bulletin,[17] the following result is obtained:

$$E_1 = \sigma(1 - \frac{1}{\gamma})B_{max} z\, l\, Su 10^{-8}$$

in which E_1 is the desired voltage, l is given in centimeters, and u is given in centimeters per second. S is the number of coils of the armature winding connected in series. z is the number of conductors per slot.

Then $E_1 = 0.9595(1 - \frac{1}{3.014})(4750)(1)\, l\, (74)(14020)10^{-8} = 31.65 l =$ $31.65\, l$

The common type of the inductor alternator is of the form shown in Figures 1 and 1A of the first bulletin,[18] in which the stator is divided into two parts. The type in Figure 1A has the advantage of better cooling conditions for the exciting coil and will be used here. With both halves of the alternator connected in series, the e.m.f. for one-half will be 225 volts, which would require a length of 225/31.5 = 7.1 cm. The design will be made for a greater e.m.f., say by fifteen percent, to allow for some regulation. Then one-half gross armature iron length = (7.1)(1.15) = 8.18 cm., which gives the armature length for one-half of the machine.

Make the rotor teeth for one-half 8.2 cm. long. Let one-half the gross armature iron length of the stator be 8.8 cm. to take care of the end fringes.

TEETH AND SLOTS

Take the rotor tooth to be of trapezoidal form with the base, a'_r = $p_r/2$ = 1.275 cm., the top = 1.03 cm., and the height = 1.275 cm. = $p_r/2$. The flux density at the root of a rotor tooth is

$$B_r = \frac{B_{max}}{t}\left[2 - k(1 - \frac{1}{\gamma})\right]$$ [19]

where $t = 2a'_r/p_r = 1$.

k from Table I of the first bulletin is found to be 1.091. Then

$B_r = (4750/1)\left[2 - 1.091(0.6685)\right] = 6030$, which is perfectly acceptable.

A complete investigation has been made regarding the size and shape of the armature conductor. The round conductor has the advantage that a higher B_{max} may be allowed, as will be understood by reference to Appendix III of the first bulletin.[20] It has the disadvantage, however, of less surface per unit volume for the transmission of heat from the conductor to the iron; a'_s has the lowest value for this type of conductor, thus cutting down the mechanical strength. The size and shape of the stator teeth enter into the calculation of the iron losses. In this particular case, round slots were found to give larger values for iron losses than rectangular slots, due to the higher values of B_s. The calculation of losses is a more lengthy procedure in this case also, due to the variable cross section of the teeth. The final result of the investigation led to the use of same width for stator slot and tooth, or $p_s/2 = 1.28/2 = 0.64$ cm.

This would make conductor width = 0.44 cm. because of thickness of insulation.

The conductor height was made 0.94 cm.

Area = 4.4 x 9.4 = 41.35 sq. mm. = 0.064 sq. in.

= 81,600 cir. mils.

Current density = 333.3/41.35 = 8.06 amperes/sq. mm.

Several trials have shown this value of current density to be desirable in this particular case.

Slot dimensions = 0.64 x 1.14 cm. = 0.73 sq. cm.

ARMATURE COPPER LOSSES

To determine the length of the end connections, a drawing of the conductor was made to scale and measured; it was found to be 2.03 cm.

Length per slot = $8.8 + 2.03 = 10.83$ cm.; total number of slots = $2 \times 2 \times 74 = 296$.

Total winding length = $296 \times 10.83 = 3210$ cm. = 1261 in.

To be on the safe side, a factor of 1.6 is used to take care of the skin effect on the increase of the resistance at this high frequency, although the conductor is supposed to be stranded.

Total resistance of winding = length in inches/area in circular mils[21]

$$= \frac{(1.6)(1261)}{81,600} = 0.0248 \text{ ohms.}$$

$I^2R = (333.3)^2(.0248) = 2750$ watts.

Watts loss per cm. length = $2750/3210 = 0.853$ watts/cm.

HEAT TRANSFER AND COOLING IN CONDUCTORS AND SLOTS

To determine the cooling surface of the end connections, they are considered to be made up of two lengths of 0.1 cm. on each side and a semicircle of 1.28 cm. diameter, with inside diameter = 0.64 cm. and outside diameter = $0.64 + 2(0.64) = 1.92$. Then length of inside surface = $0.2 + (\pi/2)(0.64) = 1.205$ cm. Inside area = $(1.205)(1.14) = 1.373$ sq. cm. Length of outside surface = $0.2 + (\pi/2)(1.92) = 3.215$ cm. Outside area = $3.215(1.14) = 3.66$ sq. cm. Area of upper and lower portions = $2(2.03)(0.64) = 2.6$ sq. cm. Total area exposed to air = $1.373 + 3.66 + 2.6 = 7.633$ sq. cm. = 1.182 sq. in.

The cooling surface of the end connections is found to be 7.63 sq. cm. = 1.182 sq. in. A reasonable figure for the cooling of a conductor in air as used in actual practice[22] is one watt per second per ten square centimeters when the temperature difference, t_d, of the air and conductor is 40° C. and the air is flowing by at the rate of four meters per second. This velocity for the air is a safe assumption since the peripheral speed of the rotor is 140 meters per second. Hence the watts evacuated from the end connections if $t_d = 40^\circ$ C. is 0.7633 watts per second. Actual watts to be evacuated = (watts loss per cm.)(length in cm.) = 0.853 x 2.03 = 1.734 watts. $t_d = 40 \times 1.734/0.7633 = 90.9^\circ$ C.

For the insulation on the conductor, use treated cotton, the thermal conductivity of which is 0.006[23] watts per inch cube per degree C.

With an insulation thickness of 1 mm., heat conductivity = 0.006 x 25.4 = 0.1525 per sq. in. t_d across insulation = (watts to be dissipated)/(thermal conductivity x cooling surface).

t_d across insulation = $1.734/(0.1525)(1.182) = 9.62^\circ$ C.

Total temperature difference between conductor and air = $90.9 + 9.62 = 100.52^\circ$ C.

It is evident that the temperature difference cannot be 100° C., but actually will be much smaller; hence little of the heat will flow into the air. Since copper is a very good conductor of heat, most of the heat produced by the end connections will flow into the conductor embedded in the iron, where the conditions for the evacuation of heat are more favorable.

As a worst possible condition, consider that all the heat from the end connections must also flow into the iron. Then total heat to be evacuated = $(0.853)(8.8 + 2.03) = 9.25$ watts, where 8.8 is the length of conductor imbedded in iron and 2.03 is the length of end connection.

As will be seen by reference to Plate I and considering the length of 8.8 cm., the cooling surface of the embedded conductor = $8.8\left[(2) \times (1.14) + 2(0.64)\right] = 31.3$ sq. cm. = 4.86 sq. in.

Using the thermal conductivity as found above, the temperature drop across the insulation of the conductor, t_d = (watts to be dissipated)/(thermal conductivity x cooling surface) = $9.25/(0.1525)(4.86) = 12.5^\circ$ C.

Consider also that the heat must flow across two very thin air films, one which exists between the conductor and insulation, the other between the insulation and iron of the armature core. Use each as a length of 0.0025 cm.*, which is customary for contact films, making a total distance through the air of 0.005 cm. This arrangement is shown in a sketch on Plate II.

The conductivity of air is 0.000259 watts per sq. cm. per degree C. per cm. of length.[25] Conductivity per 0.005 cm. = 0.000259/0.005 = 0.0518. Necessary temperature drop across air film for the evacuation of the heat = watts to be dissipated/(thermal conductivity x cooling surface) = 9.25/(0.0518)(31.3) = 5.7° C.

The heat must flow from the copper conductor across one air film, across the insulation, across a second air film into the iron, from which it will be removed by cooling water. The necessary total temperature difference between the copper conductor and the iron = 12.5 + 5.7 = 18.2° C.

The treated cotton insulation can withstand temperature up to 130° C.,[26] while the temperature of the iron will be only about 85° C., as will be shown later.

DETERMINATION OF ζ

The actual form of the tooth and slot should now be determined. To prevent too great a flux concentration at any section, the dimension "a" in Plate III might be calculated in this particular case, when the tooth width is the same as the slot width, on the basis that one-fourth

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* This value is in agreement with the average data obtained for a joint in a transformer core as calculated from a curve in Still's "Elements of Electrical Design."[24]

of the flux of the stator tooth must pass through section "a" with an allowable flux density of B_s. The form of the tooth between "a" and the center of the slot may be determined in the same manner.

The computation of an additional flux density within a stator tooth is based on the coefficient ζ. Its determination may be accomplished by the method suggested and developed by Dr. Oboukhoff in the second bulletin.[27] ζ represents the value of the flux linked with one coil of the stator winding per cm. length of the coil core and per ampere turn of the stator slot. Its value enables one to determine both the effect of the armature winding flux on B_r and the inductance of the entire machine.

Formula (1) on page 2 in that bulletin gives $\zeta = \zeta_1 + \zeta_2 + \zeta_3 + \zeta_4$.
Formula (2) on page 4 gives $\zeta_1 = 0.838 \frac{t}{b}$ for a rectangular slot, where t is the height of the stator slot and b the width.

$\zeta_1 = 0.838(11.5/6.4) = 1.507.$

Formula (4) on the same page gives

$\zeta_3 = 1.257(a_r - b_s + 2\delta)/4\delta = 1.257\left[10.3 - 1.0 + 2(1.25)\right]/4(1.25) =$ 2.965.

Formula (5) on that page gives

$\zeta_4 = \zeta_3/\gamma_1$, where γ_1 is obtained by Formula (6) of that page, or

$$\gamma_1 = 1 + \frac{p_r\left[1+\gamma-\gamma\frac{2a_r}{p_r}\right]\frac{2a_r}{p_r}\left[1+\gamma-\gamma\frac{a_s}{p_s}\right]\frac{a_s}{p_s}}{2A\delta}$$

$$\frac{2\delta}{p_r} = \frac{2(1.25)}{25.5} = \underline{0.098}$$

$$A = 10 \quad \text{for} \quad 2\delta/p_r > 0.025$$

$$\gamma = 3 \quad \text{for} \quad 2\delta/p_r > 0.08$$

$$\therefore \gamma_1 = 1 + \frac{25.5\left[1+3-3(0.807)\right](0.807)\left[1+3-3(0.921)\right](0.921)}{2(10)(1.25)}$$

$$\gamma_1 = 2.478$$

$$\zeta_4 = \zeta_3/\gamma_1 = 2.965/2.478 = 1.197$$

The definition of ζ and ζ_2 enables the computation of ζ_2 . This value, ζ_2 , is connected with the flux across narrow slot openings on both sides of the tooth[28] (see lines f-g-h of Figures 1 and 2 on page 3 of the second bulletin).

From the equation, $B\ell = 0.4\pi NI$, the value of B may be determined. The definition of ζ states that NI is one; ℓ is the length of the air path across the opening, or 0.1 cm. Then $B = 0.4\pi/0.1 = 4\pi$.

In the equation, $\phi = BA = B\ell h$, ℓ is, from the definition of ζ, 1 cm., and h is 0.1 cm. as seen in the sketch on Plate III. Then $\phi = 4\pi(1)(0.1) = 0.4\pi$. Since ζ_2 represents the flux across the slot openings on both sides of the stator tooth, its value would be $0.8\pi = 2.514$. Then

$$\zeta = 1.507 + 2.514 + 2.965 + 1.197 = 8.183.$$

The maximum flux density in the stator tooth due to armature current, B'_s, may now be determined. Referring to the definition of ζ,

$$B'_s = \frac{\zeta NI_{max}}{A\psi} = (8.183)(1)(333.3\sqrt{2})/(0.64)(1)(0.8) = 7510$$

where A denotes the cross-sectional area of the stator tooth at the narrowest place, and ψ is the stacking factor.

On Plate II is seen a sketch of the rotor tooth in three different positions; at "a" it is opposite the stator tooth on the left side of the stator slot considered. Under this condition the excitation flux density in the stator tooth is maximum. Up to this point the flux density has been increasing, at which time it starts decreasing. Hence the e.m.f. produced in the armature winding changes sign at this point and is zero. Since the machine operates at resonance, the armature current and flux due to it are also zero at this point. At position "b" the armature e.m.f., current, and armature flux density in the stator tooth are maximum since the excitation flux is going through the maximum rate of change at this point. Here, the excitation flux density in the stator tooth would be an average between the maximum and minimum flux density, i.e., between B_s and $B_{s\ min}$ respectively, or $(B_s + B_{s\ min})/2$. At position "c" the armature e.m.f., current, and armature flux are again zero. As a fairly close approximation, the highest total value for B''_s, the resultant flux density in stator tooth I at position "b" may be determined as $(B_s + B_{s\ min})/2 + B'_s$. $B_{s\ min} = B_s/\gamma$, since $\phi_{min} = \phi_{max}/\gamma$.

$B''_s = (B_s/2)(1 + 1/\gamma) + B'_s = (11{,}350/2)(1 + 1/3.014) + 7510 = 7560 + 7510 = 15{,}070$, which is slightly high. By increasing the length of the rotor and armature core, this may be reduced to the desired value for a normal value of e.m.f. An allowance of fifteen percent was originally made, but this may be increased still more. Let the rotor teeth for one-half be 9.5 cm. long and one-half the gross armature iron length be 10 cm. The required length for the output voltage is = 7.1 cm. (see page 15). Hence the flux density in the stator tooth due to the excitation flux would be reduced to 7550(7.1)/9.5 = 5640 for the required output voltage. The resultant flux density in the stator tooth would then be $B''_s = 5640 + 7510 = 13{,}150$, which is permissible since it does not exceed the "knee" value and because of considerations introduced in the next paragraph.

This value of flux concentration would be present at only one section since it is obtained by using the maximum value of flux. The flux due to armature current would be maximum at section s_1, as shown on Plate II in tooth I, and would decrease going down the tooth to section s_2. This is because total flux must pass through section s_1, while at s_2 the flux would be less than the total flux by the amount which has gone across the armature conductor. The coefficient ς_1 corresponds to the magnetic flux across the stator slot[27] (see lines x-y-z of Figures 1 and 2 of the second bulletin), so the flux through section s_2 would be obtained by using $\zeta_0 = \zeta_2 + \zeta_3 + \zeta_4 = \zeta - \zeta_1$. $\zeta_0 = 8.183 - 1.507 = 6.676$. Then the flux density due to armature current at section s_2 = $B'_{s2} = \zeta_0 NI_{max}/A\psi = (6.676)(1)(333\sqrt{2})/(0.64)(1)(0.8) = 6130$. Then, the flux at the mid-point of the stator tooth height would be the average. The flux density due to armature current at the mid-point of the stator tooth height would be the average between the flux densities at sections s_1 and s_2, or $(7510 + 6130)/2 = 6820$.*

The average resultant flux density, $B''_{s\ ave}$, would be

$B''_{s\ ave} = 5640 + 6820 = 12,460$.

The armature losses would be increased slightly less than the ratio of 10/8.8,** but the necessary temperature difference would be slightly decreased from the values as previously calculated since the cooling surface is increased exactly in the ratio 10/8.8. New armature copper losses are slightly less than (2750)(10/8.8) = 3,125 watts. Assume 3,125 watts.

With ζ known, the value of the inductive reactance may also be computed.

$$X_L = \omega L = (8.183)(2\pi)(5500)(1)^2(74) \times 2(10)10^{-8}$$
$$= 4.18 \text{ ohms.}$$

It is necessary to know this inductance because of the resonance conditions under which the machine operates. This inductance value will

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* This is assuming that the stator tooth is of uniform width, while actually it is of greater width at section s_1 because of a greater diameter of stator at that height.

** The actual increase would be $(10 + 2.03)/(8.8 + 2.03)$, because the length of the end connections remains constant at 2.03 cm.

enter into the computation of the outside circuit.

IRON LOSSES AND HEAT EVACUATION

Keeping in mind the fact that in this type of alternator the stator teeth are the only portion of the iron in the magnetic circuit in which the flux density varies, it is only necessary to consider iron losses due to the fluctuation of the flux density in the stator tooth. The pulsating excitation flux density of the stator teeth produces eddy current and hysteresis losses in the same manner as the alternating current.

It pulsates between B_s and $B_{s\ min}$, but, of course, is unidirectional. Referring to Figure 8 of the first bulletin,[29] the value $(B_{max} + B_{min})/2$ is the line corresponding to the zero flux density line of an ordinary alternating current flux density curve and the amplitude of pulsations, B_a, is $\frac{B_{max} - B_{min}}{2}$. But $B_{s\ min} = B_s/\gamma$. Making this substitution, the value is found to be $B_a = (B_s/2)(1 - \frac{1}{\gamma}) = (11,350/2)(0.6685) = 3795$.

Cross-sectional area per stator tooth = 0.86 sq. cm.

Total volume of teeth = 296(0.86)(10) = 2,545 cu. cm.

= 2.545 cu. dm.

Although eddy current losses vary as the square of lamination thickness, it is desirable from the economical standpoint to use laminations as thick as is compatible with the required efficiency, prescribed losses, and temperature conditions of the machine.

The choice will depend partially upon the physical dimensions of the machine as compared to its desired output since the available cooling surface determines the amount of heat which may be dissipated. For a given machine, when the thickness of lamination is increased above a certain value, the height of the core above the slots must be increased to make available more area for the necessary larger cross section of the cooling pipes; thus the increase in the cost of the cooling system and larger volume of laminations offsets some of the advantage gained by the lower cost of thicker laminations.

The lamination thickness used was adapted to be 0.005". The use of this value is justified in computations which follow.

For calculating eddy-current losses in the ordinary a.c. machine, Kapp and Fleming give the formula:[30]

$W_E = 1.6 \, \Delta^2 f^2 B^2 10^{-8}$ watts/dm^3,

where Δ = lamination thickness in centimeters,

f = frequency in cycles per second, and

B = maximum flux density in gausses.

Because of uncertainty of iron loss computation in very thin laminations at high frequencies, increase the losses by forty percent.

Then $W_E = 2.24(0.005 \times 2.54)^2(5500)^2(3795)^2 10^{-8} = 1{,}565$ watts/dm^3.

Total eddy-current losses = (1.565)(2.545) = 3.99 kw. = 4 kw.

Hysteresis losses may be calculated by the Steinmetz formula,[31]

$W_H = \eta f B^{1.6} 10^{-4}$ watts/dm^3,

where η varies between 0.0025 and 0.003. To be safe, use η = 0.003 and increase losses by fifty percent instead of forty percent because there is more uncertainty about hysteresis than eddy-current losses.

$W_H = 0.0045(5500)(3795)^{1.6} 10^{-4} = 1{,}315$ watts/dm^3

W_H (total) = (1.315)(2.545) = 3.35 kw.

Now there must be some allowance made for the variation of the flux density in the stator due to armature current. This may be reasonably estimated without detailed computation because of the high temperature allowed for the entering cooling water, and because of the possibility of an easy adjustment of this temperature to the conditions of cooling, the heat evacuation may easily be accomplished. (It is obvious that one can use the water for cooling at a temperature of 20° C. even in hottest weather.) A reasonable assumption to make allowance for this additional would be to increase the total iron losses by additional fifty percent. New iron losses $=$ 1.5 (hysteresis losses $+$ eddy current losses) $=$ $1.5(3{,}350 + 4{,}000) = 11{,}200$ watts.

With the iron and copper losses already determined, it is possible to decide upon the method of cooling which should be used. In some special instances it is possible economically to cool the inductor alternator by means of natural or forced air ventilation, but this detail must be carefully considered in each machine designed. In the case of the present machine, it was found that water cooling would be more economical and more efficient than the use of forced air ventilation.

The cooling pipes should always be placed above and near to the stator tooth where most of the heat is produced. With 148 teeth and a pole pitch of 1.28 centimeters it is found advisable to use a cooling hole at every fourth tooth, or thirty-nine holes in all, because of the size of copper tubing available. Plumber's seamless drawn copper tubing was chosen,[32] the least available size of which is 5/8", with an inside diameter of 0.521" and an outside diameter of 0.654". For a more even distribution of heat evacuation the cooling system should be made up of two branches on each half of the machine with water entering at opposite sides of the rotor. Thirty-two holes on each side should have cooling pipes, leaving seven holes for the entrance of air for cooling the exciting coil. Thus, each branch would have sixteen cooling pipes with a total cooling area of $(16) \times (\pi)\dfrac{(0.654)(10)}{144\ (2.54)} = 0.9$ sq. ft. (The length of the embedded pipe in the armature core is 10 cm.)

In the type of cooling system used all the heat produced by the total iron losses and armature copper losses is to be removed from the iron by cooling water. Then total heat to be evacuated = copper losses + iron losses = 3,125 + 11,200 = 14,325 watts = 49,000 B. T. U./hr.

B. T. U. to be removed per half of stator = 49,000/2.

B. T. U. to be removed per branch = 49,000/(2 x 2) = 12,250 B. T. U./hr.

It is now necessary to determine the resistance offered to the flow of heat from the iron to the water. The heat must flow across an air film between the iron and copper tubing, through the copper tubing, and then through the liquid film created by the flow of water through the pipe. This is shown on Plate II.

The liquid film coefficient may be determined by means of a formula given in Marks' Handbook:[33]

$$h = 207\ (JV)^{0.8}/D^{0.2},$$

where h is the film coefficient, D is the diameter of the pipe in inches, V is the liquid velocity in feet per second, and J is the reciprocal of viscosity in centipoises at the arithmetic mean temperature of the pipe wall and water. Values for J at various temperatures are given by Marks.[33]

To get the necessary value for h, a value of two and one-half feet per second for V had to be used. Using 85° C. as the temperature of the pipe wall and 30° C. as the temperature of the water, the mean temperature would be 57.5° C. To be conservative, use 47° C., for which temperature J = 1.7.[33]

$$h = 207(1.7 \times 2.5)^{0.8}/(0.521)^{0.2} = 750.$$

As shown on Plate II, the total resistance, R, to the flow of heat from the iron to cooling water is the sum of the separate resistances, $R = 1/h + L_1/k_1 + L_2/k_2$, when L_1 = thickness of copper tubing, k_1 = thermal conductivity of copper, L_2 = length of air path, and k_2 = thermal conductivity of air. Use the same length of air path as in case of air between iron and insulation of armature conductor, or 0.001" = 0.0000833'. Thickness of copper tubing = 0.0665" = 0.00554'. Thermal conductivity of copper = 221 B. T. U./hr./sq.ft./°F./ft. of thickness.[34] Assuming 70° C. for the temperature of the air path, the thermal conductivity is 0.0154.[35]

$R = 1/750 + 0.00554/221 + 0.0000833/0.0154 = 0.006745.$

$U = 1/R = 1/0.006745 = 148.3.$

The necessary temperature difference between the iron and water would be

t_d = B. T. U./UA = 12,250/(148.3)(0.9) = 91.7° F. drop = 51° C. drop.

Water flow = Q = AV = (0.001481)(2.5) = 0.00371 cu. ft./sec.
= 13.35 cu. ft./hr.
= 859#/hr. per branch.

Temperature rise of the water between entering and leaving point is 12,250/859 = 14.27° F. = 7.92° C. = 8° C.

If entering water temperature is 30° C., leaving water temperature is 38° C. Mean temperature difference between iron and water = (85 - 30 + 85 - 38)/2 = 51° C., which will make 81° C. for stator iron temperature;[36] hence the heat may be evacuated.

Total water necessary for system would be

8 x 13.35 x 1728/231 = 799 gallons per hour.

With forty-two gallons per barrel and allowing fifteen minutes for the water to cool, the necessary water would be 799/42(60/15) = 4.76, or five barrels of water.

The cooling system may be adjusted to the demands of the alternator by varying the temperature, and, if necessary, the two cooling branches on each half may be changed to four; according to the computations shown the water velocity is slightly higher than is strictly necessary. Even if the iron temperature reaches 90° C., the temperature of the copper conductor will reach only 108° C. (see page 19), while the insulation is capable of withstanding 130° C.

DESIGN OF EXCITING COIL

The magnetic circuit consists of the double air gap, the stator tooth-slot zone, rotor tooth-slot zone, stator yoke, and rotor yoke. The number of ampere turns required for the double air gap is determined in a straightforward manner, while the ampere turns required for the iron path will depend upon the variable reluctance as it changes with the flux density in different cross sections.

With a stacking factor of 0.8, the equivalent air gap would be

$$\frac{1\,\delta}{\frac{(1 + 0.8)}{2}} = 1.25/0.9 = 0.1389 = 0.14 \text{ cm.}$$

On account of a possible uncertainty of the length of air gap, allow twenty-five percent additional; this will also take care of the joints where the laminations are in contact with the frame. For the double air gap, the total would be $1.25 \times 2 \times 0.14 = 0.35$ cm.

$$0.4\,\pi NI = B\ell = (4750)(0.35)$$

$$NI = (4750)(0.35)/1.257 = 1320 \text{ ampere turns.}$$

The total flux for the yokes is the flux through the teeth plus the flux from the slots. The flux per rotor pole would be $\phi_{max} + \phi_{min} =$

$B_r a'_r l$[37] = (11,350)(1.275)(2 x 10) = 290,000 lines. Then total number of lines of flux would be 74 x 290,000 = 2,144,000 lines.

Use a flux density of 5,000 for the iron in the yoke of the rotor. Minimum cross section of rotor yoke would be 2,144,000/5,000 = 429.0 sq. cm. Mean circumference is approximately 55 π = 172.8 cm.

Minimum height = 429/172.8 = 2.48 cm.

For the stator yoke the mean circumference is approximately 80 π = 252 cm. Minimum height = 429/252 = 1.7 cm. These values have been increased in order to obtain sufficient mechanical strength.

The actual dimensions are shown on Plate IV.

The required ampere turns for overcoming the reluctance of the iron path, determined by reference to suitable B-H curves, are 230, making a total requirement for the exciting coil of 1320 + 230 = 1550 ampere turns.

For the exciting coil, use a current density of three amperes per sq. mm.

Area of copper = 1550/3 = 517 sq. mm.

= 0.802 sq. in.

Using No. 18 B and S copper wire, with a cross-sectional area of 0.001276 sq. in.,[38] the number of turns required in 0.802/0.001276 = 628. Use 630 turns.

The mean diameter of the coil would be approximately seventy cm. or 27.6". Total length = 630(27.6 π/12) = 4550 ft.

From the table, R at 60° C. is 7.42 ohms/1000 ft.

At 90°, $R = \left[\frac{1 + 0.00427(90)}{1 + 0.00427(60)}\right] 7.42 = 8.16$ ohms/1000 ft.[39]

Total R = 4.55 x 8.16 = 37.1 ohms.

Required current in 1550/630 = 2.46 amperes. Required voltage = 37.1 x 2.46 = 91.5 volts. Use 110 volts d.c. for excitation voltage with a rheostat for varying the voltage.

From Still's "Elements of Electrical Design," page 36, the space factor is $s_f = \frac{\pi}{4} d^2/(d + 2t)^2$ when d is the diameter of the copper and t is the thickness of insulation.

$s_f = \frac{\pi}{4}(40.3)^2/(51)^2$, where 40.3 and 51 are the diameters in mils of the copper and copper plus insulation respectively.

Total area = (0.001276)(630)/0.491 = 1.637 sq. in.

= 10.58 sq. cm.

It is better to make the cross section of the coil in rectangular form for lower internal temperature, so make b = 2.5 cm., and h = 10.58/2.5 = 4.23 cm.

Watts to be dissipated = EI = 91.5 x 2.46 = 225 watts.

Cooling surface $= 2(70\pi)(4.23) + (70 - 4.23/2)\pi(2.5) + (70 + 4.23/2)\pi(2.5)$

= 2960 sq. cm.

Using 10 sq. cm. to dissipate one watt when velocity is four meters per second with temperature drop, t_d, of 40° C.,

$t_d = (225/296) \times 40 = 30.4°$ C.

Using formula from Still for the temperature difference between center of coil to outside,[40]

$$t_d = \frac{W}{8m\left[k_a\left(\frac{w}{l}\right) + k_b\left(\frac{l}{w}\right)\right]},$$

where W = watts to be dissipated, w = length of the cross section of the coil, l = width of the cross section, $k_a = k_b = 1/400(1 - \sqrt{sf})$, m = mean circumference of the coil in inches.

$$k_a = k_b = \frac{1}{400(1 - \sqrt{0.491})} = 0.00835$$

$w/l = 4.23/2.5 = 1.69$; $l/w = 2.5/4.3 = 0.59$

$$t_d = \frac{225}{8(27.6\pi)\left[0.00835(1.69 + 0.59)\right]} = 17.10° \text{ C.}$$

Total $t_d = 17.02 + 30.4 = 47.42^\circ$ C., which means that the exciting coil is satisfactory.[36]

DESIGN OF ROTOR DISC

The design of the rotor disc is very similar to the design of a turbine disc and may be accomplished in much the same manner. A method of determining the radial and tangential stresses in a disc of variable cross section has been developed by Stodola in his book, "The Steam Turbine."[41] From the material developed by Stodola, S. H. Weaver has obtained some suitable approximate formulas, which he presented in "The General Electric Review."[42] The formulas given by Weaver have greatly shortened the length of time required for solution of such a problem and give results of sufficient accuracy.[42]

Some dimensions of the rotor are already known, such as those of the teeth, the outside diameter and the length. In comparing the alternator rotor with the turbine wheel, the teeth are analogous to the buckets, covers, etc. Thus, the outer radial stress of the rotor equals the centrifugal force of the teeth divided by the outer cylindrical area of the rotor. If the rotor is put on the shaft with a shrink fit, the radial stress at the bore may be taken as zero, for the fit with which the rotor is placed on the shaft may be supposed to be almost neutralized at normal speed by the centrifugal expansion at the bore, and at some overspeed the stress is zero.[42] Thus, two radial stresses are known and the Weaver method of solution may be applied.

The rotor has been designed with the idea of obtaining a minimum of material and weight. A form of the rotor similar to the one shown in Plate IV was first assumed with dimensions to fit those already known.

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The stresses were then determined by use of the Weaver method, which gave an indication of the value to use for the disc thickness at the bore. With this thickness known, the disc was designed as one of uniform strength by use of the formula.[43]

$$y = y_a e^{-\frac{\mu\omega^2}{2\sigma}x^2}$$

where y_a is the thickness of the disc carried to the shaft center, ω is the angular velocity, μ is the specific mass, σ is the unit stress, x is the distance from the center of the shaft to the point considered, and y is the thickness of the disc at a radius x.

Weight of forged steel = 486#/cu. ft. = 7800 kg./cu. meter.

Specific mass = W/g = 7800/9.8 = 795.

Using a factor of safety of 2, which is permissible since the rotation is at constant speed, the allowable stress = 37,500/2 = 18,750#/sq. in. 1#/sq. in. = 703 kg./sq. m., so 18,750#/sq. in. = 13,190,000 kg./sq. m. $\omega = 2\pi \times 4460/60 = 466$ radians/sec. $\omega^2 = 217{,}500$.

$\frac{\mu\omega^2}{2\sigma} = (795)(217{,}500)/(2)(13{,}190{,}000) = 6.57$.

A value of 14 cm. was found for y at the bore by means of the Weaver method as explained above. Then $y = 14e^{-6.57x^2}$. Using this relation, a form shown by the dotted lines on the disc cross section in Plate IV is obtained. It is obvious that in this case the theoretical curve would not be practical because of the cost of machining, which would most probably be greater than the economy of saving in material. The actual form used has been chosen on the basis of practical considerations, and is shown by the full lines on the cross section of the disc on Plate IV.

In the disc of uniform strength, the elongation is given by the relation,[44]

$$\xi = \frac{1-\nu}{E}\sigma x,$$

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where v = reciprocal of the ratio of the elastic elongation to cross section contraction, which has a value of 0.3 for steel, E = modulus of elasticity, σ = unit stress, and x is the radius.

Since the disc used is not of uniform strength, but has a greater cross section at all points, the actual elongation will be less than this; the difference, however, may be disregarded. Therefore, actual radial elongation of rotor $< \frac{(1 - 0.3)}{29(10)^6}(18{,}750)\frac{(30.035)}{2.54} = 0.0136$ cm.

DECREASE OF AIR GAP DUE TO TEMPERATURE RISE

The coefficient of linear expansion for steel is 10×10^{-6} for 1° C.[45] Considering the room temperature as 30° C., the stator would have a temperature rise of 51° (see page 28). Since the inside stator diameter is 60.32 cm., the increase in the radius would be $\frac{(60.32)}{2}(51)(11.5)(10^{-6}) =$ 0.0177 cm. to make the radius 30.16 + 0.0177 = 30.1777 cm. The rotor may have about 10° or so greater rise in temperature than the stator because of the air friction and the lack of forced water cooling. Elongation of radius $= (60.07/2)(61)(11.5)(10^{-6}) = 0.0211$ cm. Net effect on air gap then $= 0.0211 - 0.0177 = 0.0034$ cm.

SAG OF ROTOR

To obtain the sag the weight of the rotor must be known. This weight is computed as follows:

Cross-sectional area of one tooth $= (1.275/2)(1.03 + 1.275) = 1.47$ sq. cm.

Total volume of teeth $= (74)(2)(9.5)(1.47) = 2065$ cu. cm.

Consider now a layer 1 cm. thick just under the teeth: Its mean diameter $= 60.07 - 2(1.275) - 2(\frac{1}{2}) = 56.52$ cm.

Volume $= (56.52)(\pi)(1)(28.9) = 5120$ cu. cm.

The next layer is made up of a cylinder 2.8 cm. thick and 13.85 cm. long bounded by two triangular areas 2.8 cm. high and 7.425 long. Mean diameter of 2.8 cm. thick cylinder $= 56.52 - 1 - 2(2.8/2) = 52.72$ cm. Volume $= (52.72)(\pi)(2.8)(13.85) = 6430$ cu. cm.

Mean diameter of triangular cross sections $= 56.52 - 1 - 2(2.8/3) = 53.66$ cm.

Volume of two triangular areas $= 2(53.66)(\pi)(2.8)(7.425)/2 = 3500$ cu. cm.

Next is a uniform cylinder 13 cm. wide, 17 cm. high.

Mean diameter $= 52.72 - 2.8 - 2(17/2) = 32.92$ cm.

Volume $= (32.92)(\pi)(14)(17) = 24{,}600$ cu. cm.

Mean diameter of hub $= 32.92 - 17 - 2(3.5)/2 = 12.42$ cm.

Volume of hub $= (12.42)(\pi)(28.9)(3.5) = 3.950$ cu. cm.

Volume of shaft $= (59.0)(9^2\,\pi/4) = 3{,}750$ cu. cm.

Total volume $= 2{,}065 + 5{,}120 + 6{,}430 + 3{,}500 + 24{,}600 + 3{,}950 + 3{,}750 = 49{,}410$ cu. cm. $= 3{,}010$ cu. in.

Weight $= 0.282 \times 3{,}010 = 850$#.

To take care of a possible unsymmetrical magnetic pull, increase the weight to 1,200#. The length from center to center of bearings is 59 cm., or 23.2". As a limiting condition, consider the weight concentrated at the center and calculate the sag by means of the simple beam formula,

$y = Wl^3/46EI$.

Consider $r = 4$ cm. $= 1.575$".

$I = \frac{\pi}{2} r^4 = \frac{\pi}{2}(1.575)^4 = 9.71$ in.^{4}

$y = (1{,}200)(23.2)^3/(48)(29)(10^6)(9.71) = 0.00111$".

Since there would be practically no bending in the length of shaft embedded in the rotor, a more logical assumption would be to consider the effective length of shaft as twice the distance from the center of the bearing to the hub on the rotor, or $2(9) + 2(6) = 30$ cm., or 11.8".

Then, $y = (1,200)(11.8)^3/(48)(29)(10^6)(9.71) = 0.000146"$.

The maximum decrease of air gap would be the sum of the radial elongation of the rotor due to centrifugal force, plus the decrease of air gap due to temperature rise, plus the sag $= 0.0136 + 0.0034 + (0.00111)(2.54) = 0.01982$ cm. The minimum air gap, then, would be $0.125 - 0.01982 = 0.10689$ cm., which is permissible.

DESIGN OF SHAFT

The shaft should be designed for combined twisting and bending. Considering the bending amount, one-half the length of the shaft = 59/2 cm. = 11.61". Bending amount = Wl = 1200 x 11.61 = 13,950 in. lb. T = 63,030 x h.p./N = (63,030)(150/0.746)/4,460 = 2,850 in. lb. Using the maximum normal stress theory, Leutwiler gives the formula,[46]

$$T''_e = M + \sqrt{M^2 + T^2} = 13,950 + \sqrt{(13,950)^2 + (2,850)^2} = 28,200''\#.$$

$$\frac{T''_e}{S_s} = \frac{\pi d^3}{16}$$

Using a diameter of 8 cm., or 3.15", $S_s = (28,200)(16)/(\pi)(3.15)^3 = 4,410\#/\text{sq.in.}$ Factor of Safety = 37,500/4,410 = 8.5, which is safe enough.

PRELIMINARY DESIGN OF BEARING

Leutwiler gives 40 to 60#/sq. in. for the allowable bearing pressure,[47] and the formula for pressure in pounds per square inch[48] of

$$p = P/cd^2$$

where P is the total pressure on the bearing, c is the ratio of length to diameter of bearing, d is bearing diameter, and p is unit pressure. He suggests a value for c from 2 to 3 for generators and motors.[49] Let c = 2.5. The diameter of the shaft has been previously determined in the shaft design as 8 cm. Use the journal diameter as 8 cm. and the remainder of the shaft as 9 cm. Then

$p = 600/(2.5)(8/2.54)^2 = 24.2\#/\text{sq. in.}$

Since the shaft diameter has been determined with a good factor of safety, it will be permissible to decrease c to 1.5, which gives a reasonable value for unit pressure on the bearing, or

$p = 600/(1.5)(8/2.54)^2 = 40.3\#/\text{sq. in.}$

Length of journal = 1.5d = (1.5)(8) = 12 cm.

In the original design, the journal length and dimensions of shaft were determined simultaneously since the actual length of the shaft, including journals, was necessary in the shaft design.

CHECK FOR CRITICAL VELOCITY

Timoshenko gives the formula for critical velocity,[50]

$$\omega_{cr} = \sqrt{\frac{k}{W}g}$$

where $k = 48EI/l^3$, g = acceleration due to gravity, W = weight on the shaft, and ω_{cr} = critical velocity in radians per second. From the value for sag for simple beams, $y = Wl^3/48EI$, it is seen that $k/W = 1/y$. Then

ω_{cr} in radians per second = $\sqrt{32.17 \times 12/y} = 19.63/\sqrt{y}$

ω_{cr} in r.p.m. = $(19.63/2\pi) \times 60 \times 1/\sqrt{y} = 187.7/\sqrt{y}$

Using the two limiting values for sag as previously determined, i.e., 0.00111" and 0.000146", the two resulting critical velocities would be

(1) ω_{cr} in r.p.m. = $187.7/\sqrt{0.000146}$ = 15,550 r.p.m.

(2) ω_{cr} in r.p.m. = $187.7/\sqrt{0.00111}$ = 5630 r.p.m.

Hence the critical velocity is safely above the operating speed, even if the sag were as great as 0.00111".

CORRECTION FOR γ

Since the rotor and stator were increased in length above the original values, the maximum flux density has been decreased. New B_{max} = $\frac{\text{(old rotor length)}}{\text{new rotor length}}(\text{old } B_{max}) = \frac{(8.2)}{9.5}(4750) = 4{,}100.$

The average air gap is decreased at operating speed because of the expansion of the rotor due to temperature rise and centrifugal force. The calculated decrease in air gap by these two factors is slightly less than $0.0136 + 0.0034 = 0.017$. Air gap at operating speed $= 0.125 - 0.017 = 0.108$. Use 0.11 for δ in the calculations for the correction of γ.

Old $\gamma = 3.014 = 1 + 2.014$.

The correction ratio for changing the value 2.014 is

$$\frac{10(1.25) + (4750)^2 10^{-7}}{10(1.1) + (4100)^2 10^{-7}} = 1.162,$$

as will be seen by reference to the formula for γ as used on page 13.

New $\gamma = 1 + (1.162)(2.014) = 3.46$.

The voltage depends primarily upon the factors $(1 - \frac{1}{\gamma})$, length of teeth, and B_{max} (see page 15).

The change in B_{max} has been offset by changing the length of teeth, so the voltage at operating speed would be $\frac{(1 - \frac{1}{3.46})}{(1 - \frac{1}{3.014})} \times 450 = (1.062)(450) = 479$ volts, which gives a sufficient margin of voltage regulation compared to the specified voltage.

DESIGN WITH LAMINATION THICKNESS OF 0.022 CENTIMETERS

A greater lamination thickness may be used for the stator laminations, but the cooling system must be redesigned because of the higher eddy current losses. From a descriptive catalogue published by Societe Francaise Radio-Electrique entitled "Alternateurs a Haute Friquence" data are obtained regarding the variation of lamination thickness with the frequency of the machine for some alternators manufactured and tested by that company. With these data a curve has been drawn on Plate V, and by extropolation the thickness of 0.22 mm. has been found for a frequency of 5,500 c.p.s.

With this thickness of laminations, the eddy current losses are increased in the ratio of $\left[\frac{0.022}{(0.005)(2.54)}\right]^2 = 3$. With the former thickness of laminations the eddy current losses were 4 kw. Hence the new eddy current losses $= 4 \times 3 = 12$ kw. Total iron losses $= 12{,}000 + 3{,}350$ $= 15{,}350$ watts. Increasing this fifty percent to take care of the variation of flux density in the stator due to the armature current, the iron losses become $(1.5)(15{,}350) = 23{,}050$ watts. Total watts to be dissipated = copper losses + iron losses $= 3{,}125 + 23{,}050 = 26{,}175$ watts = 89,400 B. T. U./hour.

The added heat produced necessitates an increase in the size of cooling pipes and a larger volume of stator laminations to take care of the larger cross-sectional area of the cooling pipes. Use 1" seamless copper tubing with an outside diameter of 1" and an inside diameter of 0.836".[32] Increase the outside diameter of the stator laminations to 75 cm., and the circumference of the cooling pipe center line to 68 cm., making the circumference of this center line $68\pi = 210$ cm. = 82.8". With thirty-nine holes placed above the stator teeth, the space in between would be = 1.12", or about one inch, which is satisfactory.

Use four branches of cooling pipes on each half of the machine with eight holes containing cooling pipes. B. T. U. to be removed per hour per branch = 89,400/(2)(4) = 11,180 B. T. U./hr.

Using a velocity of three feet per second for cooling water, the liquid film coefficient would be

$h = 207(1.7 \times 3)^{0.2}/(0.836)^{0.2} = 852.$

The length of copper path is now (1 - 0.836)/2 = 0.082" = 0.00683'.

$R = 1/852 + 0.00683/221 + 0.0000833/0.0154 = 0.00717.$

$U = 1/R = 1/0.00717 = 139.5.$

Cooling area of the cooling pipes per branch = $(8)(\pi)(1)(10)/(144)(2.54)$ = 0.686 sq. ft.

The necessary temperature difference, t_d, between the iron and water is

t_d = B. T. U./UA = 11,180/(139.5)(0.686) = 116.7° F. = 64.7° C.

Water flow = Q = AV = (0.00381)(3) = 0.01142 cu. ft./sec.

= 41.1 cu. ft./hr./branch.

= 2550#/hr./branch.

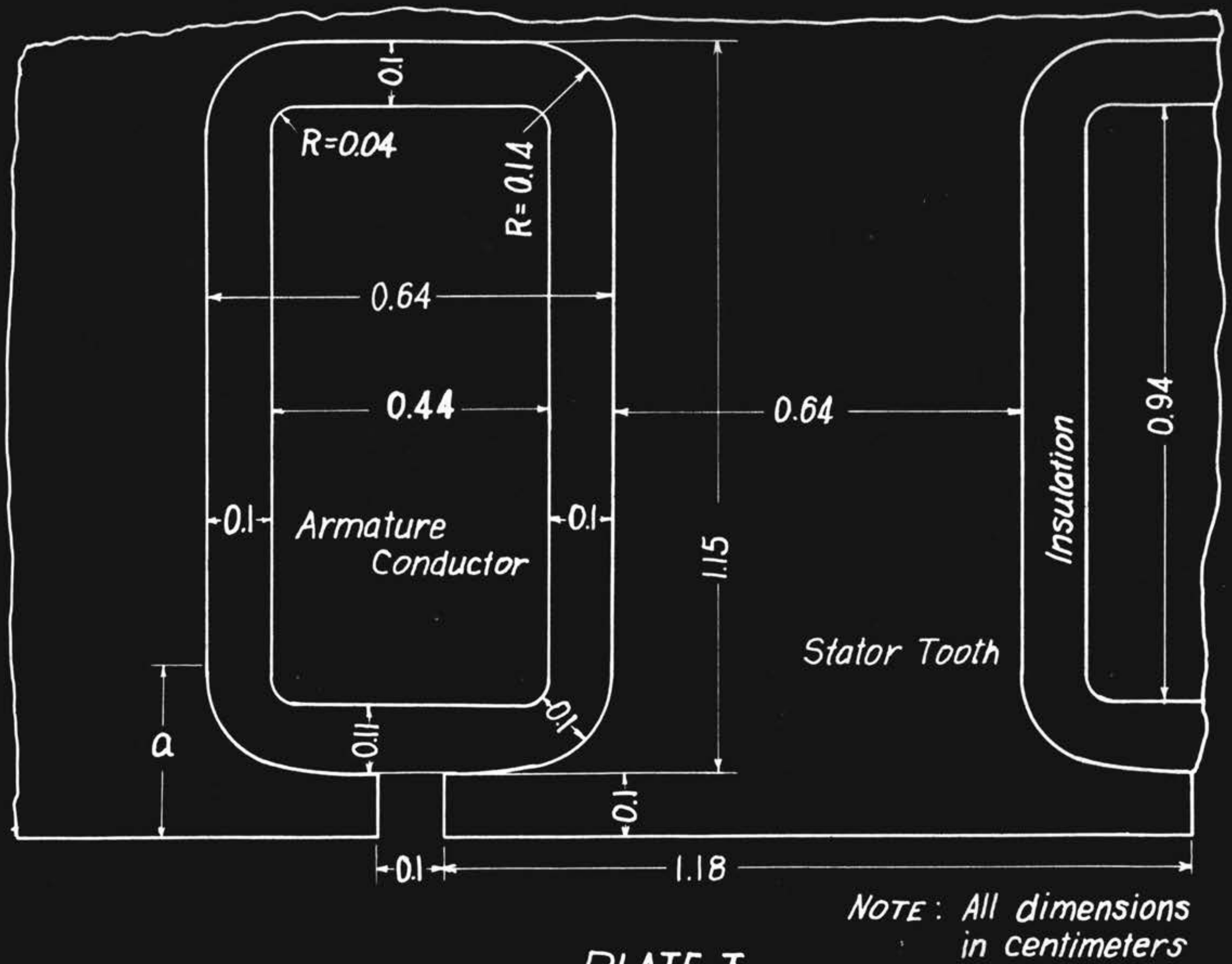
Temperature rise of water between entering and leaving = 11,180/2,550 = 4.38° F. = 2.43° C.

If the entering cooling water were maintained at a temperature of 20° C., the leaving water temperature would be 22.43° C. Mean temperature difference between iron and water = (85 - 20 + 85 - 22.5)/2 = 68.75° C. Hence the heat may be evacuated.

The total water required for all eight branches would be 8 x 2,550 = 20,400#/hr. = 2,450 gallons per hour. Allowing fifteen minutes for recooling of the water, this amount would be decreased to 2,450/4 = 612 gallons = 14.55 barrels or 15 barrels.

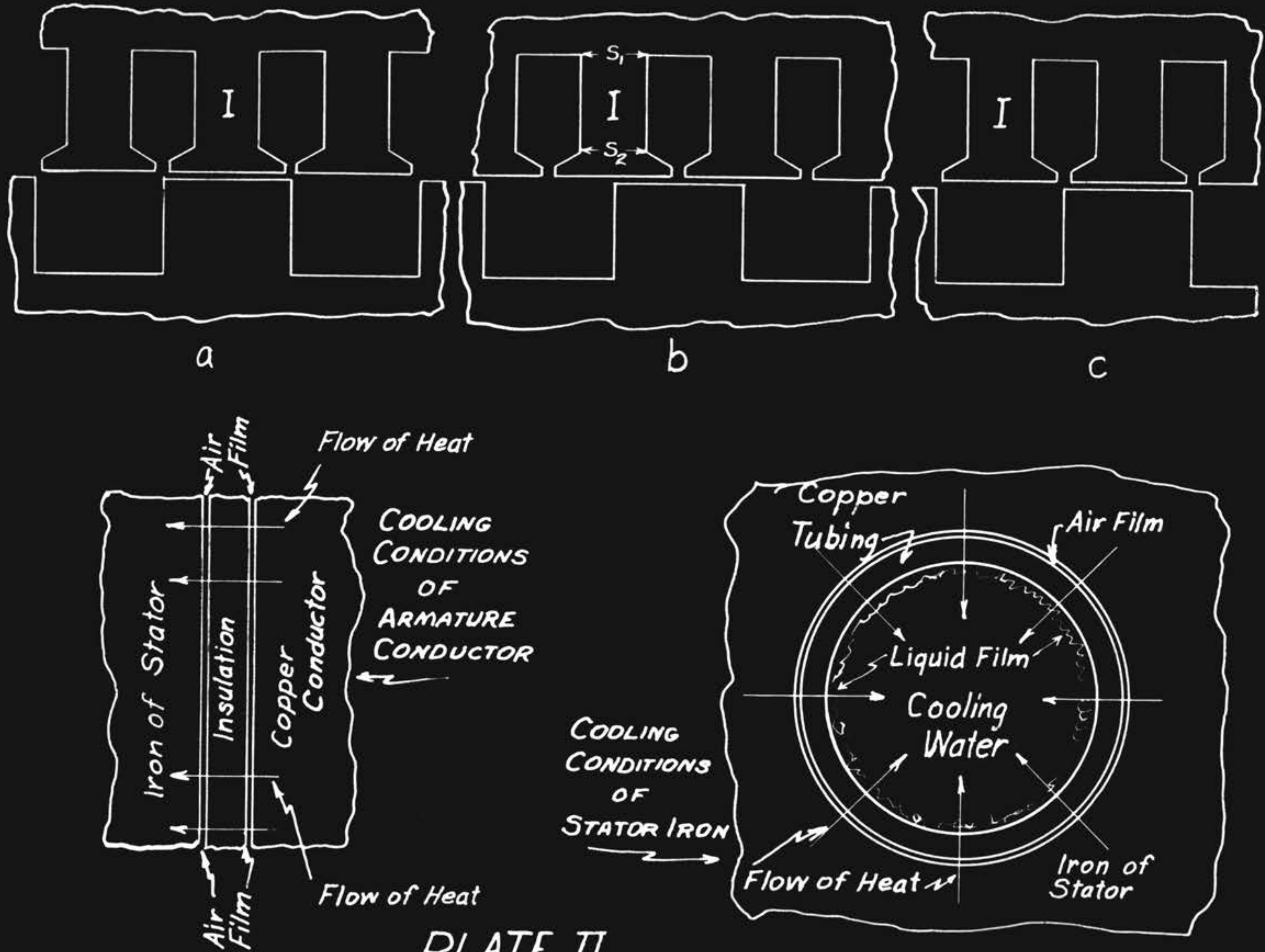
CONSIDERATIONS INTRODUCED BY VARIABLE LOAD

In certain applications, as for instance in some types of laboratory tests, it is important that the circuit of the high frequency inductor alternator remain at stable resonance under conditions of changing load. In such a case provisions must be made for sufficient adjustment to take care of the change in inductance as the operation point on the magnetization curve changes. At different loads the machine will have different values of inductive reactance due to the iron in the circuit, sometimes making it difficult to control an exact resonance condition as the load changes. A remedy for this is to reduce relatively the armature flux so that the resultant flux density would be at the lower points of the magnetization curve. This calls for a smaller current per slot and therefore for an increase of the number of parallel circuits. In this particular design it would result in an increase in the number of parallel circuits to four. To obtain the required voltage after this change, it will be necessary to increase accordingly the length of teeth. Then the machine will be able to maintain automatically the desired stability of resonance as the load changes.



0.1
R=0.04
R= 0.14
0.64
0.44
0.64
0.1
Armature
Conductor
0.1
1.15
Insulation
0.94
Stator Tooth
0.11
0.1
a
0.1
0.1
1.18
NOTE: All dimensions
in centimeters

PLATE I



I
S_1
I
S_2
I
a
b
c
Air Film
Flow of Heat
Cooling Conditions of Armature Conductor
Iron of Stator
Insulation
Copper Conductor
Air Film
Flow of Heat
Copper Tubing
Air Film
Liquid Film
Cooling Water
Cooling Conditions of Stator Iron
Flow of Heat
Iron of Stator

PLATE II

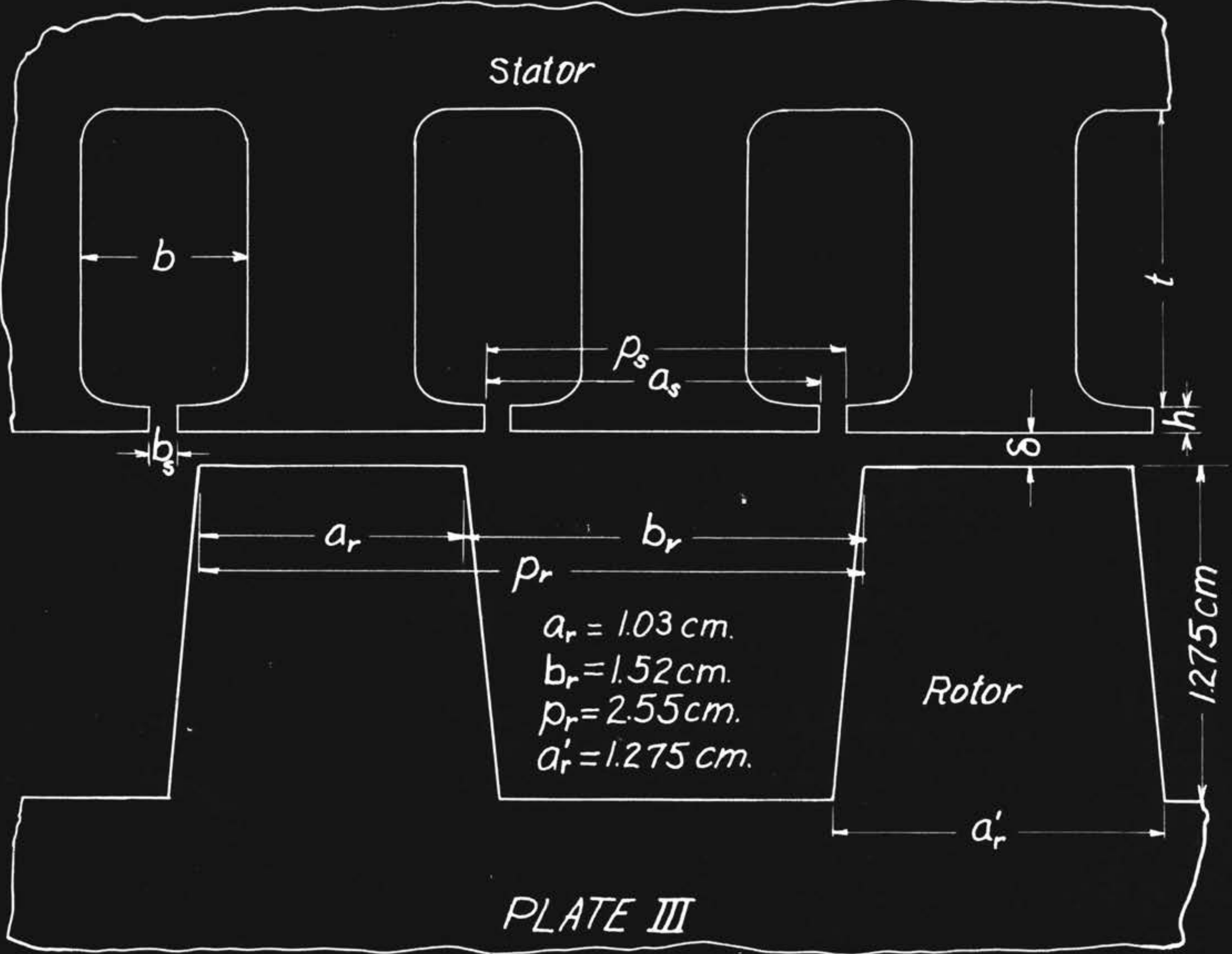
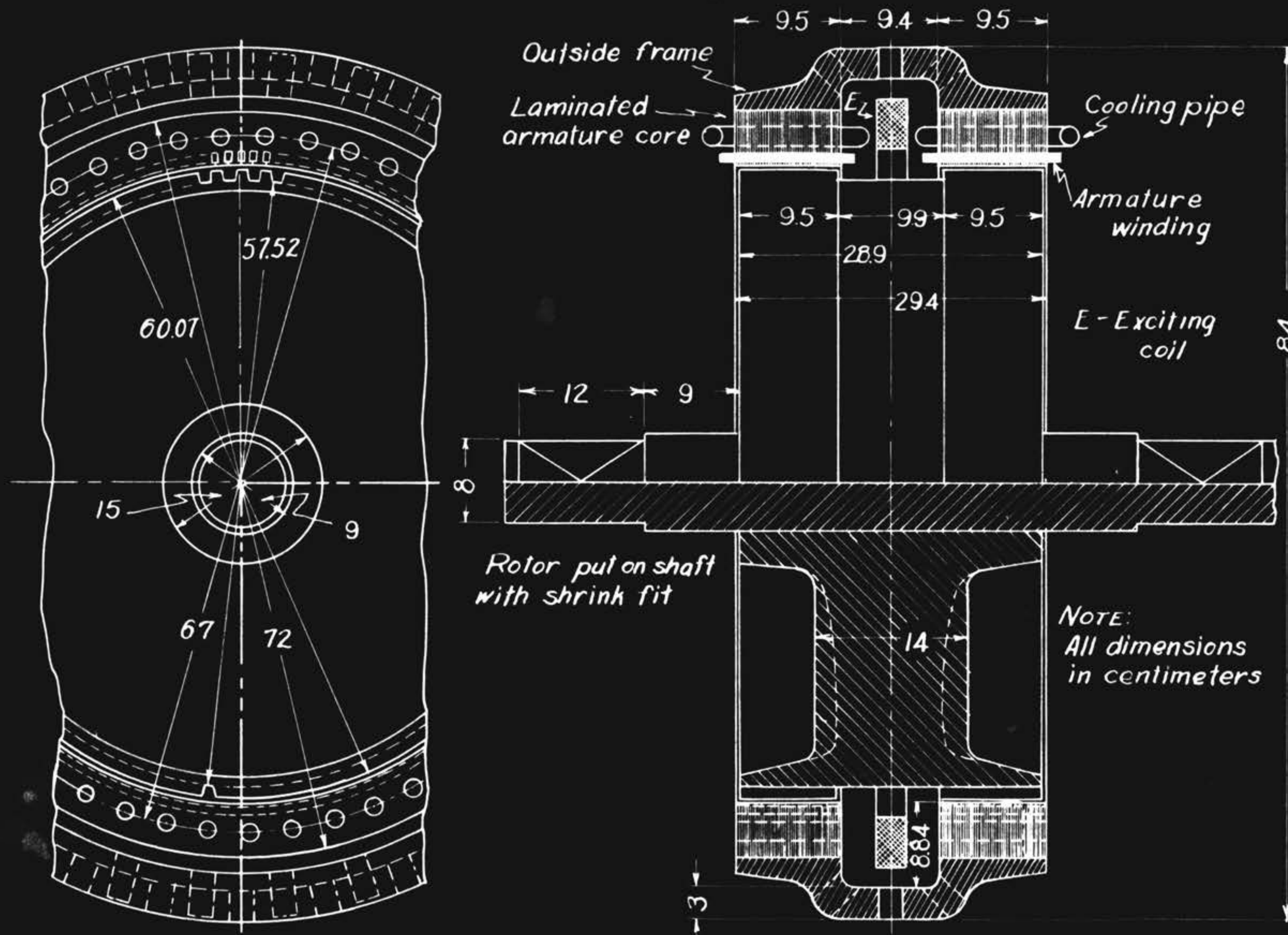
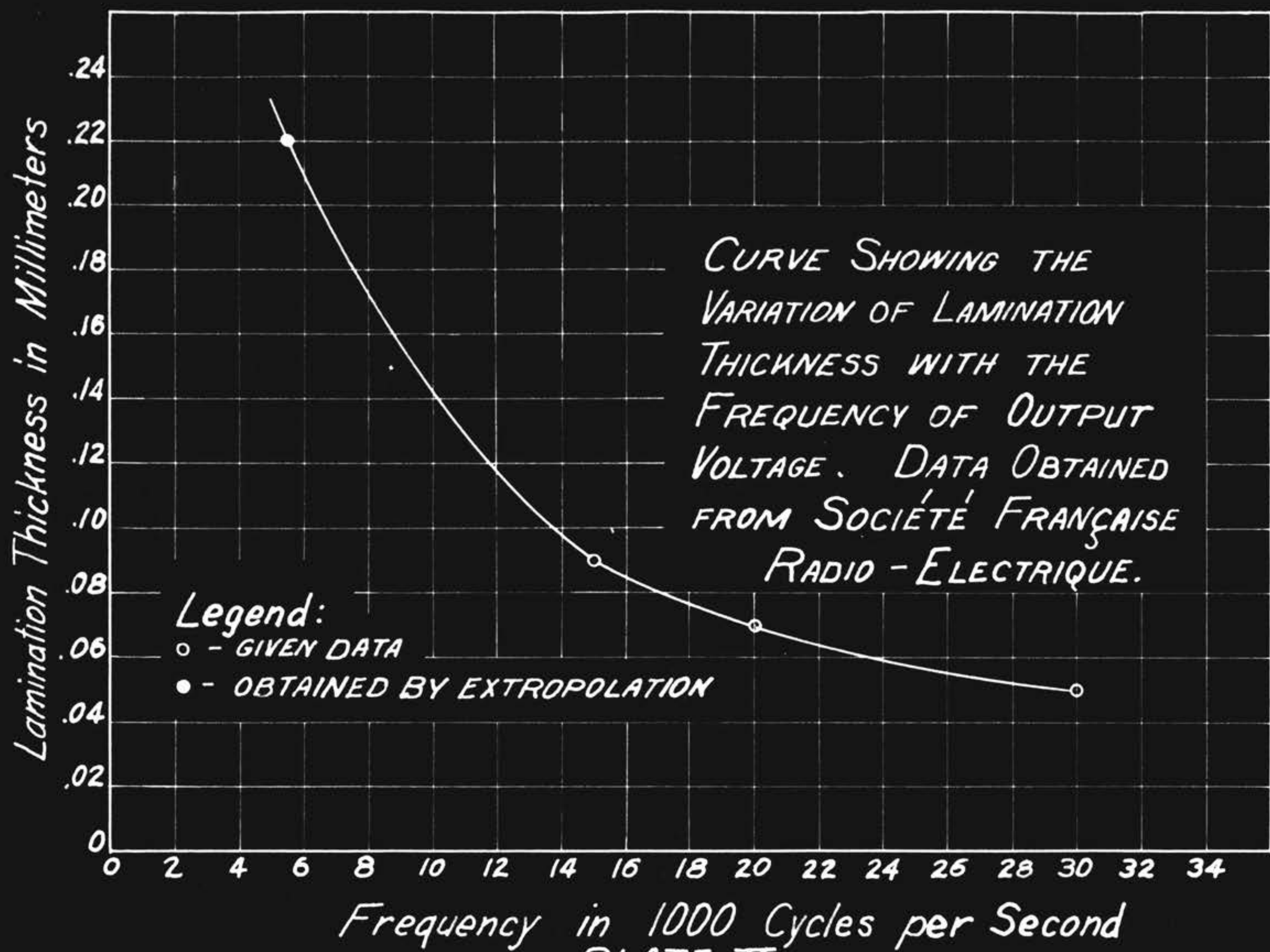

Stator
b
p_s
a_s
t
h
δ
b_s
a_r
b_r
p_r
$a_r = 1.03$ cm.
$b_r = 1.52$ cm.
$p_r = 2.55$ cm.
$a'_r = 1.275$ cm.
Rotor
1.275 cm
a'_r

PLATE III



Outside frame
Laminated armature core
Cooling pipe
Armature winding
E - Exciting coil
E
9.5
9.4
9.5
99
28.9
29.4
84
12
9
8
Rotor put on shaft with shrink fit
14
NOTE: All dimensions in centimeters
8.84
3
57.52
60.07
15
9
67
72

PLATE IV



CURVE SHOWING THE VARIATION OF LAMINATION THICKNESS WITH THE FREQUENCY OF OUTPUT VOLTAGE. DATA OBTAINED FROM SOCIÉTÉ FRANÇAISE RADIO - ELECTRIQUE.
Legend:
○ - GIVEN DATA
● - OBTAINED BY EXTROPOLATION
Lamination Thickness in Millimeters
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
Frequency in 1000 Cycles per Second

PLATE V

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- Freda Thurman -