APPLICATION OF MACHINE LEARNING: AN ANALYSIS OF ASIAN OPTIONS PRICING USING NEURAL NETWOPRK

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Title of Study: APPLICATION OF MACHINE LEARNING: AN ANALYSIS OF ASIAN OPTIONS PRICING BY NEURAL NETWORK

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Abstract: Pricing Asian Option is imperative to researchers, analysts, traders and any other related experts involved in the option trading markets and the academic field. Not only trading highly affected by the accuracy of the price of Asian options but also portfolios that involve hedging of commodity. Several attempts have been made to model the Asian option prices with closed-form over the past twenty years such as the Kemna-Vorst Model and Levy Approximation. Although today the two closed-form models are still widely used, their accuracy and reliability are called into question. The reason is simple; the Kemna-Vorst model is derived with an assumption of geometric mean of the stocks. In practice, Average Priced Options are mostly arithmetic and thus always have a volatility high than the volatility of a geometric mean making the Asian options always underpriced. On the other hand, the Levy Approximation using Monte Carlo Simulation as a benchmark, do not perform well when the product of the sigma (volatility) and square root maturity of the underlying is larger than 0.2. When the maturity of the option enlarges, the performance of the Levy Approximation largely deteriorates. If the closed-form models could be improved, higher frequency trading of Asian option will become possible. Moreover, building neural networks for different contracts of Asian Options allows reuse of computed prices and large-scale portfolio management that involves many contracts.

In this thesis, we use Neural Network to fill the gap between the price of a closed-form model and that of an Asian option. The significance of this method answers two interesting questions. First, could an Asian option trader with a systematic behavior in pricing learned from previous quotes improve his pricing or trading performance in the future? Second, will a training set of previous data help to improve the performance of a financial model? We perform two simulation experiments and show that the performance of the closed-form model is significantly improved. Moreover, we extend the learning process to real data quote. The use of Neural Network highly improves the accuracy of the traditional closed-form model. The model's original price is not so much accurate as what we estimate using Neural network and could not capture the high volatility effectively; still, it provides a relative reasonable fit to the problem (Especially the Levy Model).

Keywords: Asian Options; Option Pricing; Neural Networks (NNs); Machine Learning; Behavioral Finance

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CHAPTER I

INTRODUCTION

1.1. The Study Background and Purpose

What is Asian Option? An Asian option (or average priced option) is a special type of option contract. For Asian options, the payoff is determined by the average underlying price over some pre-set period of time. An Asian option is also path-dependent, meaning that either the settlement price or the strike of the option is formed by some aggregation of underlying asset prices during the option lifetime. This thesis will focus on European style Arithmetic Asian options where the settlement price at maturity is formed by the arithmetic average price of the underlying asset. (Levy, 1992)

For Asian options pricing, we try to use the neural network to set up a parameter for the input of the non-linear model with the change of the volatility. In the thesis, we propose and test a valuation methodology for improving the accuracy of options by using Neural Network integrated with Levy Approximation. Therefore, in our research, we will review the history of the financial derivative and the pricing research, and then study the classical Black-Scholes option model in order to improve the pricing accuracy of the Levy Approximation option pricing formula. Our study also reviews the various numerical methods of Asian pricing options. Based on the literature reviews, a method of Asian option pricing is given based on the filtering of real volatility with the neural network. The parameter of the pricing model is estimated by the neural network B-P algorithm, integrating with Levy Approximation, to construct a new Asian option pricing model. The real market quote of WTI Financial Futures Average Price Options is used to validate the new model, and its performance is compared. Finally, we clarifiy the new model will improve the efficiency of valuation and financial decision-making dramatically in the market of Asian Options.

1.2. Research Questions and Designs

During the last two decades, Asian Options serve as an important financial derivative for investors to control their investment risks in the future markets. Determining the theoretical price for options, especially Asian Options is regarded as one of the most important issues in the financial research. However, many classical and successful pricing models that have been presented, due to these traditional methods that are only simplification to the actual market, the results are less than optimal and the effects are not as good as what people expected (Levy, 1992).

For the Levy Approximation Model, when the value of $\sigma\sqrt{\tau}$ (volatility multiply by square root maturity) is larger than 0.2 its performance deteriorates (Levy, 1992). In extreme cases in my experiment, when the option have a high volatility and is deep in the money, the mean square error of the value using Levy Approximation could be larger than 10 making the model completely unreliable. The question addressed in this thesis is with the aid of neural network, could the performance of the Levy Approximation be improved and the deterioration of the performance be fixed. Moreover, we perceive neural network as a tool for simulating human mind to answer the question if an experienced Asian Option trader realized that there is a gap between the model and the real quotes. Could the trader fill the pricing gap according to his/her previous trading experience? At last, we want to see that if a model proposed decades ago could better applied with new tools and techniques.

In our research, we introduced neural network theory and model to analyze Asian Option and price it. Learning capability is a feature of neural networks, which can gain the rule from the sample. The nonlinear neural network is able to approximate any integrated function arbitrarily well (Cybenko, 1989). In the application of neural network to Asian Options, a new option-pricing model based on the adjusted Levy Approximation Model is established to improve the pricing performance. First, our new model integrates traditional pricing model–Antithetic Monte Carlo model to obtain a better forecasting result as benchmark of learning in order to cut down the forecasting errors. Second, we modify the implied volatility of the Levy Approximation model using Monte Carlo prices as a benchmark. Third, we build the neural network by mapping the real volatilities to the implied volatilities. After the network is built, we can always use the network to get the right implied volatility and improve the accuracy of Levy Approximation Model. The experiments are divided into simulation experiments and real data experiments. The only difference between the two is that in simulations, the Monte Carlo Prices are used as benchmark prices while in real data, the WTI Average Priced Option quotes are used as benchmark prices. The new pricing model improves the performance of Asian option pricing significantly. Compared to the traditional option pricing models, the error of the traditional method will be reduced and our pricing performance will be improved significantly.

1.3. Significance and Contributions of the Study

Asian options trading are a significant component of committing money or capital in derivatives markets. They perform a role of controlling risks and creating income. Trading in options unlike other derivatives provides unparalleled set of merits as follows; they offer inexpensive and productive ways of hedging one's portfolio against unfavorable and unforeseen price variations, and they provide a speculative method to trading among others. Trading on either call or put options are all the time inexpensive than the underlying stock. Majority of traders would rather trade on options than stocks in order to; conserve transaction costs, circumvent tax exposures, and circumvent stock market restrictions among others (Kolb, 1995).

Several attempts have been made to model the Asian option prices with closed-form over the past twenty years such as the Kemna-Vorst Model and Levy Approximation Model. Although today the two closed-form models are still widely used, their accuracy and reliability are called into question (Levy, 1992). The reason is simple, the Kemna-Vorst is derived with an assumption of geometric mean of the stocks. In practice Average Priced Options are mostly arithmetic and thus always have a volatility higher than the volatility of a geometric mean making the Asian options always underpriced. On the other hand, the Levy Approximation using Monte Carlo Simulation as a benchmark, do not perform well when the value of $\sigma\sqrt{\tau}$ (volatility multiply by square root maturity) is larger than 0.2 (Levy, 1992). Also, when the maturity of the option extends the performance of the Levy Approximation largely deteriorates. Although in practice a long maturity does not often exist, there is still requirements for Asian Style FLEX options with long Maturity. Monte Carlo Simulation or numerical methods are the existing best method for pricing Asian Options. However, numerical methods required long running time to achieve a reasonable price (Glasserman, 2003). In such a case, a fast and reliably closed-form pricing model is still preferable for adapting to the actively trading of underlying contract and the dynamic change of the volatility. If the closed-form model could be improved, not only the higher frequency trading of Asian option will become possible but also the large-scale Asian option portfolio management becomes achievable.

In this thesis, we use Neural Network to fill the gap between the price of a closed-form model and that of an Asian option. The significance of this method answers two interesting questions. First, could an Asian option trader with a systematic behavior in pricing learned from previous quotes improve his/her pricing or trading performance in the future? Second, will a training set of previous data help to improve the performance of a financial model? We perform a simulation experiment and a real data experiment to show that the performance of the closed-form model is significantly improved. The accuracy of the Levy Approximation Model with Neural Networks is higher than that of the other traditional models. The analysis shows that after learning and testing, the pricing of Asian Options on simulation or on the real dataset , all have a significant effect on the results. Moreover, the neural network algorithms used in study impact the results significantly.

Zhang, Patuwo, & Hu, (Zhang,1998) pointed out, artificial neural networks (ANNs) are one of the most accurate and widely used forecasting models that have enjoyed fruitful applications in forecasting social, economic, engineering, foreign exchange, stock problems, etc. Several distinguishing features of artificial neural networks make them valuable and attractive for a forecasting task. First, as opposed to the traditional model-based methods, artificial neural networks are data-driven self-adaptive methods in that there are few a priori assumptions about the models for problems under study. Second, artificial neural networks can generalize. After learning the data presented to them (a sample), ANNs can often correctly infer the unseen part of a population even if the sample data contain noisy information. Third, ANNs are universal functional approximators. It has been known that a network can approximate any continuous function to any desired accuracy (Hornik, 1989). Finally, artificial neural networks are nonlinear. The traditional approaches to time series prediction, such as the Box–Jenkins or ARIMA, assume that the time series under study are generated from linear processes. However, they may be inappropriate if the underlying mechanism is nonlinear. In fact, real world systems are often nonlinear.

In this study, we will validate our research design on neural network for Asian option pricing. Some of the advantages of the neural network in Asian option pricing over the parametric option pricing approaches are presented as follows:

(1) Since the neural network model does not rely on restrictive parametric assumptions such as log-normality or sample path continuity, it is robust to the specification of errors that frequently limited the scope when handling nonlinear or nonstandard problems in Asian Options Pricing.

(2) Neural network have the ability to provide an arbitrary function approximation mechanism which learns from observed data i.e. their ability to learn how to perform tasks based on data given for training or initial experience.

(3) A trained neural network can be conceived as an expert in the category of information it has been given to analyze Asian Options.

(4) Neural networks' generalization feature i.e. the neural networks are initially established through a training phase, whereby exemplar inputs are presented and the neural network is trained to extract relevant information patterns, the neural network has the capability to generalize so that unseen input (i.e. out-of-sample) patterns may also be processed.

(5) Neural networks exhibit flexibility feature i.e. range of tasks that neural networks can be applied to vastly exceed any one traditional technique. The markets are changing rapidly and unless a model has capability to constantly update its parameters based on changing market scenarios, the validity of the model in the long run is uncertain, neural network models have some capability to learn continuously from the data and revise the knowledge in its network weights.

(6) Neural network is nonlinear modeling on Asian Options pricing, i.e., the neural networks mapping process involves nonlinear functions that can consequently cover a greater range of problem complexity.

1.4. Chapters and Contexts

This thesis contains five main chapters. A software package developed by MATLAB® is attached to this booklet. The chapters are as follows:

Chapter 1: Introduction; Chapter 2: Related works or literature reviews; Chapter 3: Methodology and research model design; Chapter 4: Data presentation and results analysis; and Chapter 5: Conclusion.

CHAPTER II

RELATED WORKS

2.1. An Overview of Neural Networks and Asian option pricing Research

Robert R. Trippi and EfraimTurban (Trippi & Turban, 1992) said, neural networks are revolutionizing virtually every aspect of financial and investment decision making. Financial firms worldwide are employing neural networks to tackle difficult tasks involving intuitive judgement or requiring the detection of data patterns, which elude conventional analytic techniques. Many observers believe neural networks will eventually outperform even the best traders and investors. Neural networks have already being used to trade the securities markets, to forecast the economy and to analyze credit risk.

In order to avoid the shortcomings of traditional parametric models, neural network model has been widely applied and studied in option pricing. Huchison (Huchison, 1994) first used RBF and BP neural network models for pricing European options, His research demonstrated that the model is better than the Black-Scholes model. Huchison proposed a nonparametric method for estimating the pricing formula of a derivative asset using neural networks. Huchison pointed out that although not a substitute for the more traditional arbitrage-based pricing formulas, network-pricing formulas may be more accurate and computationally more efficient alternatives when the underlying asset's price dynamics are unknown, or when the pricing equation associated with the no-arbitrage condition cannot be solved analytically.

Jingtao Yao at el. (Yao, Li&Tan, 2000) Conducted forecasting of the option prices of Nikkei 225 index futures by using back propagation neural networks. Different results in terms of accuracy are achieved by grouping the data differently. The results suggest that for volatile markets a neural network integrated option-pricing model outperforms the traditional Black–Scholes model. In the neural network model, data partition according to moneyness is applied, and those who prefer high risk and high return may choose to use the neural network model results.

In 2004, Marco J. Morelli at el. (Marco J. Morelli, 2004) applied neural network algorithms to the problem of option pricing and adopted it to simulate the nonlinear behavior of such financial derivatives. Two different kinds of neural networks, i.e. multi-layer perceptions and radial basis functions, are used and their performances compared in detail. The analysis was carried out both for standard European options and American ones, including evaluation of the Greek letters, necessary for hedging purposes. Marco J. Morelli at el. ‘s detailed numerical investigation show that, after a careful phase of training, neural networks are able to predict the value of options and Greek letters with high accuracy and competitive computational time.

Gradojevic et al. (Gradojevic, 2009) proposed a nonparametric modular neural network (MNN) model to price the S&P-500 European call options. The modules are based on time to maturity and moneyness of the options. The option price function of interest is homogeneous of degree one with respect to the underlying index price and the strike price. When compared to an array of parametric and nonparametric models, the MNN method consistently exhibits superior out-of-sample pricing performance. In their study they found out that, modularity improves the generalization properties of standard feedforward neural network option pricing models.

Yi-Hsien Wang (Wang, 2009) integrated new hybrid asymmetric volatility approach into artificial neural networks option-pricing model to improve the forecasting ability of derivative securities price. Owing to the integration, the new hybrid asymmetric volatility method can reduce the stochastic and nonlinearity of the error term sequence and capturethe asymmetric volatility simultaneously. In Wang's ANNS option-pricing model, the results demonstrated that Grey-GJR–GARCH volatility provides higher predictability than other volatility approaches.

Zhang Hongyan, at el. (Zhang & Lin & Jiang, 2010) proposed a model of hybrid wavelet neural network based on the Black-Scholes model, and built some hybrid forecasting models combining the hybrid Wavelet neural network and genetic algorithm for test. In such an approach, options are classified according to their moneyness, and the weighted implied volatility rates are regarded as the input of the neural network. Zhang used a genetic algorithm to determine the optimal weights of the implied volatility rates of different kinds of options. Case study on Hong Kong derivative market shows that these hybrid models are better than the conventional Black-Scholes model and the other neural network models.

Lin and Yeh (Lin and Yeh, 2009) presents a novel neural network model for forecasting option prices using past volatilities and other options market factors. Out of different approaches to estimating volatility in the option-pricing model, his study uses backpropagation neural network to forecast prices for Taiwanese stock index options. The ability to develop accurate forecasts of grey prediction volatility enables practitioners to establish an appropriate hedging strategy at in-the-money option.

Michael Kohler, et al. (Kohler, 2010) used the Monte Carlo approach to generate artificial sample paths of these price processes, and then used the least squares neural networks regression estimates to estimate from this data the so-called continuation values, which are defined as mean values of the American options for given values of the underlying assets at time *t* subject to the constraint that the options are not exercised at time *t*. Results concerning consistency and rate of convergence of the estimates are presented, and the pricing of American options is illustrated by simulated data.

Muhammad Yahuza Bello and Haruna Chiroma (Bello & Chiroma, 2011) built a Feed Forward Artificial Neural Network Predictor (FFANNP) and tried to capture a pattern in the historical data of Ashaka Cement (Ashaka Cem) security of the Nigeria Stock Market (NSM). They predicted the closing prices of the security with a trend accuracy of 80%.The analysis showed that it is possible to train ANNs with controlled parameters using technical data to capture pattern in the historical data and generalize well on unseen data. The empirical studies proved that prediction of share prices of a stock in a stock market using ANNs model is better than the current analytical techniques presently in use by financial expert.

S. Shakya, at el. (Shakya, 2012), described how neural networks and evolutionary algorithms can be combined together to optimize pricing policies in their study. Particularly, they built a neural network based demand model and use evolutionary algorithms to optimize policy over build model. There are two key benefits in their approach. Use of neural network made it flexible enough to model a range of different demand scenarios occurring within different products and services, and the use of evolutionary algorithm made it versatile enough to solve very complex models. Shakya also evaluated the pricing policies found by neural network based model to that found by other widely used demand models. The results showed that proposed model is more consistent, adapts well in a range of different scenarios, and in general, finds more accurate pricing policy than other three compared models.

Mitra (Mitra, 2012) pointed out, the Black and Scholes formula for theoretical pricing of options exhibits certain systematic biases, as observed prices in the market differs from the formula. A number of studies attempted to reduce these biases by incorporating a correction mechanism in the input data. Amongst non-parametric approaches used to improve accuracy of the model, Artificial Neural Networks are found as a promising alternative. Mitra's study made an attempt to improve accuracy of option price estimation using Artificial Neural Networks where all input parameters are adjusted by suitable multipliers. The values of these multipliers were determined using known data that minimizes errors in valuation.

Fei Chen and Charles Sutcliffe (Chen & Sutcliffe, 2012) compared the performance of artificial neural networks (ANNs) with that of the modified Black model in both pricing and hedging short sterling options. Using high-frequency data, standard and hybrid ANNs are trained to generate option prices. Chen's study testified that the hybrid ANN is significantly superior to both the modified Black model and the standard ANN in pricing call and put options. Hedge ratios for hedging short sterling options positions using short sterling futures are produced using the standard and hybrid ANN pricing models, the modified Black model, and also standard and hybrid ANNs trained directly on the hedge ratios. The performance of hedge ratios from ANNs directly trained on actual hedge ratios is significantly superior to those based on a pricing model, and to the modified Black model.

Adam Fadlalla and Farzaneh Amani (Fadlalla & Amani, 2014) pointed out, accurate prediction of stock market price is of great importance to many stakeholders. Artificial neural networks (ANNs) have shown robust capability in predicting stock price return, future stock price and the direction of stock market movement. Adam Fadlalla and Farzaneh Amani tried to predict the next trading day closing price of the Qatar Exchange (QE) Index using historical data. A multilayer perception ANN architecture was used as a prediction model with 10 market technical indicators as input variables. The experimental results indicate that ANNs are an effective modeling technique for predicting the QE Index with high accuracy, outperforming the well-established autoregressive integrated moving average models. The ANN model also revealed that the weighted and simple moving averages are the most important technical indicators in predicting the QE Index, and the accumulation/distribution oscillator is the least important such indicator. The analysis results also indicated that the ANNs are resilient to stock market volatility.

In their study, Georgios Sermpinis, et al. (Sermpinis, 2016) introduced two NN hybrid techniques: an adaptive evolutionary multilayer perceptron (aDEMLP) and an adaptive evolutionary wavelet neural network (aDEWNN). The study demonstrated the forecasting and trading superiority of the aforementioned NNs over a set of benchmarks that dominate the relevant literature. It showed that the proposed models are free from any data-snooping bias. Overall, the proposed models are able to exploit the superior forecasting power associated with the nonlinear nature of NNs, while avoiding a complicated and objective NN training procedure.

2.2. Review of the Research

At present, although some scholars have conducted a lot of research on the theory and method of Asian option pricing, no one has found a reasonable closed form pricing model for Asian options. Even the Levy Approximation, and its extension, are difficult to adapt to the rapid change of the financial market environment. In the actual market the value of Asian options are complicated and time-consuming to price. On the other hand artificial neural network algorithm through simulation on neurons can establish a market data-driven nonlinear model for parameter estimation so as to obtain better pricing effects of the parameter model to make the option pricing more objective and more accurate, and provide a scientific basis for trading and hedging decisions. A lot of researches used neural network itself as a model and this is definitely applicable and achievable, but it also has some shortcomings as the followsing:

(1) the influence factors of the option pricing and the number of samples need to be improved to meet the requirement of the practice. Better experimental design or statistical method must be used to find the other convincing influential factors, which would improve the accuracy of BP neural network model.

(2) The hidden layer neurons number in the neural network model is difficult to be determined reasonably in the actual model, which will result in unacceptable errors of neural network prediction and self-learning, and at last the deviation between the algorithm and real quotes will be enlarged. Therefore, in the thesis we integrated both the benefits of neural network and closed-form option pricing model by applying a new mode. We only learn the parameters of the closed-form and leave the rest of error adjustment to the closed-form model. Since the mapping implied volatility is much easier than mapping all of the parameters to the exact price, we could successfully reduce the problem into a lower dimensional problem. Moreover, since the relationship between the implied and real volatility is much simpler, a simple neural network will be sufficient.

Mehdi Khasheiand and Mehdi Bijari (Khashei & Bijari, 2010) concluded that one of the major developments in neural networks over the last decade is the model combining or ensemble modeling. The basic idea of this multi-model approach is the use of each component model's unique capability to better capture different patterns in the data. Both theoretical and empirical findings have suggested that combining different models can be an effective way to improve the predictive performance of each individual model, especially when the models in the ensemble are quite different. In addition, since it is difficult to completely know the characteristics of the data in a real problem, hybrid methodology that has both linear and nonlinear modeling capabilities can be a good strategy for practical use. In the literature, different combination techniques have been proposed in order to overcome the deficiencies of single models and yield more accurate results. The difference between these combination techniques can be described using terminology developed for the classification and neural network literature.

Therefore, this is one reason we would like to use Levy Approximation Model to extract the implied volatility, and then use NNs to map the volatility, so we can get more accurate results by the closed-form Levy after adjusting the volatility. Due to the characteristics of Asian Option, it is difficult to achieve the expected results either by only using the traditional closed-form option pricing model or only neural networks to price Asian Option. This has been proved in the previous studies, in our reviews of Asian option pricing, we have not yet found anyone or any article use neural networks method to study it systematically and show significant results. None of them has showed convincing results that neural network could fulfil the requirement of practical Asian option pricing. They fail because such networks are difficult to build or may not serve as a systematic pricing method for Asian options. Another reason is that the Black-Scholes model is much more accurate than their models.

We have to say that the area is not well covered and studied. In particular, our efforts have been made to integrate the neural networks and the other traditional pricing models, such as Black-Scholes model. It has been regarded as a new innovative attempt in the field of financial analysis. However, none of the researches has conducted in the field of Asian option pricing. In the thesis, we successfully improve the pricing accuracy of Asian option by the integrated model and we provide a more reasonable and systematic way of using neural network for Asian option pricing.

2.3. Asian Options

Asian options are path dependent derivatives whose payoffs depend on the average of the underlying asset prices during the option life. They were originally issued in 1987 by

Bankers Trust Tokyo on crude oil contracts and hence with the name “Asian” option. The features or advantages of Asian options are as follows. (Kemna & Vorst,1990)

(1) Asian options are appropriate to meet the hedging needs of users of commodities, energies, or foreign currencies who will be exposed to the risk of average prices during a future period.

(2) Since the volatility for the average of the underlying asset prices is lower than the volatility for the underlying asset prices, Asian options are less expensive than corresponding vanilla options and are therefore more attractive for some investors.

(3) Asian options are also useful in thinly-traded markets to prevent the manipulation of the underlying asset price.

2.4. Option Pricing Models

2.4.1. The Black-Scholes (BS) Model

The Black–Scholes or Black–Scholes–Merton model is a mathematical model of a financial market containing derivative investment instruments. From the model, one can deduce the Black–Scholes formula, which gives a theoretical estimate of the price of European-style options. The formula led to a boom in options trading and legitimized scientifically the activities of the Chicago Board Options Exchange and other options markets around the world. It is widely used, although often with adjustments and corrections, by options market participants. Many empirical tests have shown that the Black–Scholes price is "fairly close" to the observed prices, although there are well-known discrepancies such as the "option smile". (Bodie, 2008)

As stated by the BS assumption that the underlying asset follows a geometric Brownian motion, hence Black-Scholes's formulas (Black-Schole, 1973) are as follows:

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t)$$

where ***W*** is Brownian, ***dW*** is the uncertainty in the price of stock, α is a constant expected rate of return, and σ is a constant volatility of the stock price

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma}{2}\right)(T-t)\right]$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Put call parity: $P(S,t) = Ke^{-r(T-t)} - S + C(S,t) = N(-d_2)Ke^{-r(T-t)} - N(-d_1)S$

Where

N (•) is the cumulative distribution function of the standard normal distribution

T-t is the time to maturity

S is the spot price of the underlying asset

K is the strike price

r is the risk free rate (annual rate, expressed in terms of continuous compounding)

σ is the volatility of returns of the underlying asset

2.4.2. The Kemna-Vorst Model

In 1990 and 1992, Kemna and Vorst proposed a Monte Carlo methodology which employs the corresponding geometric option as a control variable. The Kemna-Vorst Model presents a new strategy for pricing average value options, i.e. options whose payoff depends on the average price of the underlying asset over a fixed period leading up to the maturity date. Such options are of particular interest and importance for thinly-traded assets (e.g. crude oil), since price manipulation is inhibited, and both the investor and issuer enjoy a welcome degree of protection from the vagaries of the market. These options are often implicit in a bond contract, although they also appear in a straightforward form. The Kemna-Vorst Model suggests that the price of an average-value option always be lower than that of a standard European option. The pricing strategy involves Monte Carlo simulation with variance reduction elements and offers an enhanced pricing method to both arbitragers and hedgers, as well as to the issuers of such bonds. (Nielsen&Klaus Sandmann, 1995)

Kemna and Vorst (Kemna&Vorst, 1990) showed that the Asian option price, subject to the boundary condition characteristic for the option considered, is the solution to a second order partial differential equation in three variables, time, spot price of the underlying asset and the known information about the average value. Rather than solving the partial differential equation, Kemna and Vorst applied Kolmogorov's backward equation and show that the price of the Asian option can be written as the discounted expected value of the maturity payment of the option.

2.4.3. Monte Carlo Model

Paul Glasserman's book (Glasserman, 2003), *Stochastic Modeling and Applied Probability: Monte Carlo Methods in Financial Engineering* has given a good explanation of Monte Carlo models. Monte Carlo models are a wide range of computational algorithms that rely on the repeated random sampling or output of a random-base function to obtain the best numerical results. In a stochastic process or time series simulation, the functions generate different results in different iterations. They are often used in physical, mathematical and statistical problems and are most useful when it is difficult or impossible to use the other mathematical methods. These models contain a white noise term in their definition. Different random variables lead us to the different values and possible paths. For these types of modeling, it is possible to run the model for several times, which are generated from various and independent white noises. Every run results its specific solution.

2.4.4 Antithetic Monte Carlo Method

Due to the complexity of the Error when pricing averaged price options, variance reduction techniques is required. (Owen, 2013) Unwanted correlation between random numbers will impair the independence of different paths. This is the intuition of the antithetic method.

Variance reductions are used to improve the efficiency of Monte Carlo methods. So, the problem of estimating $\theta = \int g(x)f(x)\,dx$ is considered. In the control variate and importance sampling variance reduction methods the averaging of the function g(x) done in crude Monte Carlo was replaced by the averaging of g(x) combined with another function. The knowledge about this second function brought about the variance reduction of the resulting Monte Carlo method. In the antithetic variate method, two estimators are found by averaging two different functions, say, g_1 and g_2. These two estimators are such that they are negatively correlated, so that the combined estimator has smaller variance. (Lange, 1998)

2.4.5. Levenberg–Marquardt Algorithm

The Levenberg-Marquardt method is a standard technique used to solve nonlinear least squares problems. Least squares problems arise when fitting a parameterized function to a set of measured data points by minimizing the sum of the squares of the errors between the data points and the function. Nonlinear least squares problems arise when the function is not linear in the parameters. Nonlinear least squares methods involve an iterative improvement to parameter values in order to reduce the sum of the squares of the errors between the function and the measured data points. The Levenberg-Marquardt curve-fitting method is actually a combination of two minimization methods: the gradient descent method and the Gauss-Newton method. (Gavin, 2016)

CHAPTER III

PROBLEM STATEMENTS

3.1. Conceptual and Theoretical Framework

3.1.1. The Levy Model

The Levy Model (Levy, 1992) can be used to calculate prices and sensitivities of European style geometric Asian options. The spot price process is the familiar geometric diffusion given by the stochastic differential equation:

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW(t) \qquad (1)$$

Where *S(t)* denotes the price of the underlying asset at time t. Then, the arithmetic average $A(t_m)$ of prices at time t in the interval$[t_{0,} t_N]$ is given by the formula:

$$A(t_m) = \frac{1}{m+1}\sum_{i=0}^{m} S(t_i) \text{ , where } 0 \leq m \leq N, \qquad (2)$$

$A(t_N)$ is called the final average. Undetermined component of the final average at time t_m is given by

$$M(t_m) = [A(t_N) - \frac{A(t_m)(m+1)}{N+1}] \qquad (3)$$

M(t) is a sum of correlated log normal random variables and follows a lognormal distribution (Levy, 1992).

The moment generating function for this lognormal distribution is given by:

$$\Psi_x(k) = E^*[M(t)^k] = e^{k\alpha(t)+\frac{1}{2}k^2v(t)^2} \qquad (4)$$

where

$$\alpha(t) = 2lnE^*[M(t)] - \frac{1}{2}E^*[M(t)^2] \qquad (5)$$

$$v(t) = \sqrt{lnE^*[M(t)^2] - 2lnE^*[M(t)]} \qquad (6)$$

α is the expected value and ν is the standard deviation.

With the above definitions, the options price is given in closed form as:

$$C[S(t), A(t), t] = e^{-r(T-t)}\{E^*[M(t)]N(d_1) - [K - A(t)(m+1)/(N+1)]N(d_2)\} \quad (7)$$

where

$$d_1 = \frac{\frac{1}{2}lnE^*[M(t)^2] - \ln[K - A(t)(m+1)/(N+1)]}{v(t)} \qquad (8)$$

$$d_2 = d_1 - v(t) \qquad (9)$$

The parameter that we need to estimate for accurate option pricing is σ; the volatility of the underlying asset described by formula (1).

3.1.2. The Extraction of Levy Model

Based on the formula given in the previous section, we can define

$$Option\ Price = LevyPrice(Volatitliy, S_0, R_f, Matruity, Strike)$$

Where S_0 is the initial price of the underlying asset R_f is the risk free rate, Maturity is the time for the contract to expiration of the option and Strike price is the fixed price at which the option can be exercised.

Given an option Price, we fix the parameters $S_0, R_f, Matruity, and\ Strike$. Using the Levenberg-Marquardt method based on the non-linear least squares will give us an implied volatility. This implied volatility is used as the response variable for the neural network.

3.1. 3. Methodologies of Neural Network Analysis and Asian Option Pricing

3.1. 3.1. Two Methodologies.

In our study, the two methodologies to improve option pricing accuracy we focus attention on here are using simulation data from Antithetic Monte Carlo and real data for Asian options price. The difference between the two methodologies is that in real data experiments, the real data are used for benchmark to derive the implied volatility while in the simulation experiments, the simulated option price from Antithetic Monte Carlo is used.

In our study, the first step is to calculate the benchmark price by Antithetic Monte Carlo Model. The results from the closed form model with integrated neural network will be compared with the closed form without neural network. The built neural network could be used as a filter for the real volatility and provide a reliable way of mapping the real volatility to the implied volatility.

3.1.3.2. Simulation Approach

The Steps of the simulation experiment are as the following:

(1) Calculate the benchmark option prices by Antithetic Monte Carlo Model. Firstly, create one set of option prices with volatility ranging from 0.1 to 0.9 with interval of 0.05. This set of data is used to build the neural network. Then two sets of option prices are generated to form the test set. The two sets are generated using the same method with the volatilities increased by 20% and decreased by 20%. (We have an original set of volatilities ranging from 0.1 to 0.9 with interval of 0.05.We increased the volatility of the original set by 20% to get a second set of volatilities and decrease the original set by 20% to get the third set. Thus, can get three sets of option prices from these three sets of volatilities. Option price with volatility (0.1 ~ 0.9 with interval of 0.05), Option price with volatility increased by 20%, Option price with volatility decreased by 20%).

(2) Extract the implied Volatility Using the Levy Model. Note that the implied volatility is not the real volatility of the underlying asset but the volatility that serve as the response variable within the neural networks. The algorithm used is also the Levenberg–Marquardt algorithm.

(3) We take the Volatility ranging from 0.1 to 0.9 as input and the corresponding implied volatility as response variable to build the neural network. The data set is divided into training, validation and test set while building the neural network.

(4) We then take the set of option prices with volatility that increased by 20% and decreased by 20% as test sets. The option prices of the Levy model and the integrated

model are calculated and compared. This will give straightforward view how the price would change after the neural network is applied.

3.1.3.3. Simulation Methodology

The experiment is designed to primarily test the error of the closed-form model and to see that if there is any improvement made when implied volatility is computed using neural network. Firstly, 9 panels of synthetic data are generated. These simulated prices have a stock price to strike price ratio of 1.2, 1.0, and 0.8. Within each set of the Stock price to strike price ratio, we have different maturities of 1, 2 and 4-year contract prices. So, there will be nine panels in total. Within each panel, we first generate a series of prices with volatility ranging from 0.1 to 0.9 with an interval of 0.05. This set of data is used to build the network. We then generate two other sets of option prices with the same parameter except that the volatility is increased by 20% for one set and decreased by 20% for the other. These two sets are used as test sets for the trained neural network. We want to see that if the volatility changes, how well neural network is able to deal with the change of the error. The 0.1 to 0.9 set will be considered as the minimum number of data required to build a convincing neural network for adjusting the error. We also try to use part of the increased 20% and decreased 20% set as data sets to build the neural network. The result of the neural network is expected to be better since more data points are incorporated. The range of the set has to cover the whole range of the volatility to be priced. If this is achievable, more reasonable fitting for large portfolio management will become available.

3.1.3.4. Real Data Approach

The Steps of the Real Data Experiment is as the follows:

(1) Retrieve the Real data from Chicago Mercantile exchange. We will use one-month real data to build the neural network and take the whole week after the real data as the test set.

(2) Extract the implied Volatility Using the Levy Model. Note that the implied volatility is not the real volatility of the underlying asset but the volatility that serve as the response variable within the neural networks. The algorithm used is also the Levenberg–Marquardt algorithm. Also, note that the maturity, rate, and the underlying price will be the real data retrieved each day. (These parameters follow the same assumption across time).

(3) We calculate the real Volatility from the underlying asset as input for the neural network and the implied volatility as the response variable. We build neural networks and record the mean square error and R-value.

(4) We then use the neural network to fit the real volatility in the test set to the target volatility. Then the error of pricings, with real volatility and the target volatility, are compared using the real data as benchmark.

3.1.3.5. The Explanation of Real Data Experiment Design

The real data experiment will even be simpler than the simulation experiment. Since the Asian option contracts of WTI all are one month maturity. We build the neural network by mapping the real volatility to the implied volatility using data of one month period. Then we use the neural network to map the real volatility to the estimated implied volatility for the data of one week long. The error of the estimated option price is then measured by the real price of the option. The errors of Asian option prices with and without the learning process are compared using the real data quote as a benchmark.

To summarize, the objective of this thesis is to augment the accuracy of the Levy Model for option pricing (Levy, 1992) by integrating the model with a neural network to fit the real volatilities to the implied volatilities. The objective is to obtain an improved Levy Model to better price Asian options. We propose to evaluate the augmented model by two approaches namely, Antithetic Monte Carlo Model and real options price data.

CHAPTER IV

RESULTS

4.1. Data Collection and Analysis

Data collection involves many steps in order to provide suitable data for the next step in the modeling. We select the data from Bloomberg Database to ensure the authenticity and accuracy of the data. After acquiring the data from reliable sources, the data are screened, calculated or converted into a suitable format for modeling in Matlab.

For the real data experiment, this study use the WTI financial futures Asian option prices as the major financial product for study. The dataset covers the period of 3 months from August 1, 2016 to Oct 1, 2016. Different proportions of the data set are chosen as the training and testing data set. The training and testing of the data set is done in a rolling basis since Asian options are not actively traded. Different contracts with different strike prices and dates of maturity are priced using the proposed model in order to test the validity of the new augmented model.

4.2. Findings

The Levy approximation will not be a valid pricing model when the product of volatility and maturity reach a threshold and as the original author claimed since the log normal distribution assumption will be violated. Without the existence of a very accurate closed-form model, fitting techniques and learning algorithm will still play an important role in pricing such derivatives. In addition, the missing of a convincing closed-form model for pricing Asian option is also the reason that the products are not actively traded. Finaly, the levy approximation should never be used to calculate Asian option with long maturities because the theoretical price would be very far from the real price, still, a lot of people use levy approximation for pricing long maturity Asian options. With the implementation of the neural network, the problem will be less severe and the price could easily approximate a simulated Monte Carlo price or even a real option price.

4.2.1. Benchmark Calculation.

We choose the antithetic Monte Carlo Approach as the benchmark of our simulation experiments. From a prudent perspective, we use 300 periods and 100000 of trials to acquire the benchmark price. We repeat the simulation using Antithetic Monte Carlo for each set of the data five times and take the average. As shown in Figure 1, the error decreases as the number of trials increase. This justifies the use of a simulation as a benchmark. The parameters used for the simulations are S=100, K=100, τ=1 and σ=0.2.

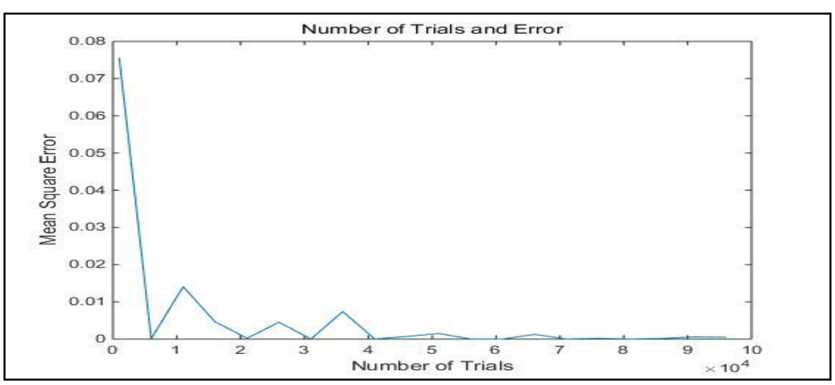

Figure 1. The relationship Between Number of Trials and Pricing Error

4.2.2. The Characteristic of Errors of the Closed Form.

We can see that for both the models as the volatility of the option increases, the error increases from the following Figure 2**.**

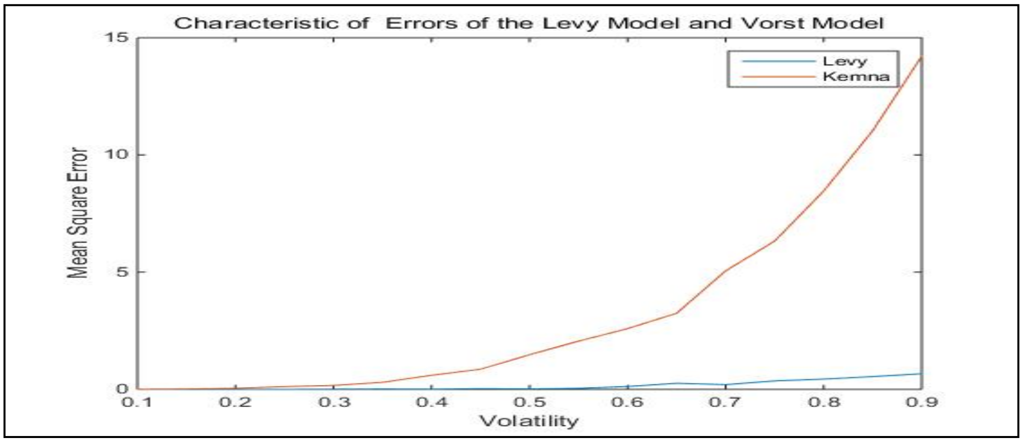

Figure 2. Characteristics of the Errors of the Closed Form Pricing Models

We choose S=100, K=100, and τ=1 as the input for the options. We can see that as the Volatility of a single option increases, their performance deteriorates, so there is room for neural networks to improve the performance of the two closed forms.

4.2.3. The Test of the Simulation Experiment

The Simulation Experiment test is as follows:

Three sets of data with an S/K ratio of 1.2, 1, and 0.8 are generated for testing. We also extended the experiment to option with three different maturities of one year, two years and three years. First, we run the Monte Carlo simulation to retrieve an option price with volatility ranging from 0.1 to 0.9 with an interval of 0.05 within each set. Using these prices as benchmark, we extract corresponding volatility using the closed forms of Levy Approximation. These implied volatilities are used as outputs for building the network.

(We divide the data points randomly) and we matched the training set using neural network. Within the same set, we change the volatility within the set by ±20 percentage. The changed volatilities are the test sets and we used neural network to examine the test sets and achieved a learned volatility. We also try to use different proportion of data for training set, validation set and test set. The more data available for building the network, the more accurate the neural network will be in terms of predicting. In this experiment, we will use the least possible data for building the neural network. We rework 5 different networks in and see how its average performance will be. We compared the error of Levy Model with and without the learning phase against the Antithetic Monte Carlo benchmark.

S/K: Set 1: 120/100 Set 2: 100/100 Set 3: 80/100

Volatility：0.1，0.15，0.2，0.25，0.3，0.35，…，0.85，0.9

This set of numbers is used to build the neural network. We will create another set as test set with the same parameters except that the volatility is increased by 20% or decreased by 20%.

4.2.4. The Results of the Simulation Experiment

The objective of the experiment is to find out when there is a change within the volatility of the option, if the neural network could still fit the error and improve the accuracy of Asian option pricing. For the entire group within the simulation, we can find that using neural networks, we can fit the error and improve the accuracy of pricing by a reasonable mean square error and a R value.

Here some results of simulation experiments presented as follows:

(1) S/K=0.8, τ=1

The mean estimated standard error for the Monte Carlo benchmark prices for this set of data is:

Original: 0.030132 for the set with volatility ranging from 0.1 to 0.9 with interval of 0.05.

+20%: 0.042798 for the set with the volatility increased by 20%

-20%:0.024396349 for the set with the volatility decreased by 20%

We calculated the estimated simulation error to make sure that the error will not be significant enough to disturb the mean square error when the simulated prices are used as benchmarks. The result showed that the errors for the simulated prices are trivial compared to the deficiency of the levy approximation model. The statistics are represented in table 1 while the experiment results are represented in figure 3 and 4.

We can see from the table that the neural network have a very high value of R-value meaning that the real volatilities and implied volatilities follows an positive correlation. The mean square error is also low as represented in table 1.

Table 1. Neural Network Statistics for options of S/K=0.8, τ=1

Neural Network Number	Training	Test
No. 1	MSE: 1.05531e-6	MSE: 5.17143e-6
	R: 9.99990e-1	R: 9.99996e-1
No. 2	MSE: 9.71353e-7	MSE: 6.52455e-6

	R: 9.99992e-1	R: 9.99999e-1
No. 3	MSE: 8.72818e-7	MSE: 6.07107e-6
	R: 9.99992e-1	R: 9.99998e-1
No. 4	MSE: 1.05610e-6	MSE:5.80716e-6
	R: 9.99991e-1	R:9.99920e-1
No. 5	MSE:1.36646e-6	MSE: 6.98383e-6
	R:9.99986e-1	R: 9.99988e-1

Figure 3. Average Performance of the Neural Network When the Volatilities Decreased by 20% for options of S/K=0.8, τ=1

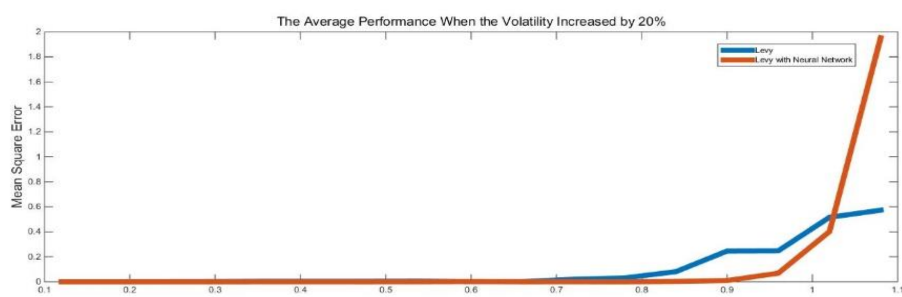

Figure 4. Average Performance of the Neural Network When the Volatilities Increased by 20% for options of S/K=0.8, τ=1

We can see that in figure 3 if the volatility lies in the range 0.1 to 0.9 then we can get a very good result using the neural network. But in figure 4 if the volatility from the test set exceed the range for a certain threshold then the neural network fails to give a reasonable price. For example in the graph, with a volatility of 0.96 and 1.02. The estimate error is still better than the original Levy Model. But when the volatility exceeds 20% and the volatility reaches 1.08 the neural network fit result in a higher error.

(2) S/K=0.8, τ=2

The mean estimated standard error for the Monte Carlo benchmark prices of data is

Original: 0.071923 for the set with volatility ranging from 0.1 to 0.9 with interval of 0.05.

+20%: 0.104372 for the set with the volatility increased by 20%

-20%: 0.0459 for the set with the volatility decreased by 20%

The results in table 2 is similar to that in table 1. The network also shows a positive correlation. The results in figure 5 and figure 6 show improvement compared to figure 3 and figure 4. This is due to the characteristic of proportional approximation error of the model.

Table 2. Neural Network Statistics for options of S/K=0.8, τ=2

Neural Network Number	Training	Test
No. 1	MSE: 1.08408-6	MSE: 5.89453e-6
	R: 9.99989e-1	R: 9.99999e-1
No. 2	MSE: 2.88701e-7	MSE: 7.87035e-6
	R: 9.99997e-1	R: 9.99915e-1
No. 3	MSE: 1.09936e-6	MSE: 3.08415e-6
	R: 9.99988e-1	R: 9.99996e-1
No. 4	MSE: 1.09128e-6	MSE:7.08578e-7
	R: 9.99988e-1	R:9.99999e-1

No. 5	MSE:1.09248e-6	MSE: 9.96125e-7
	R:9.99986e-1	R: 9.99999e-1

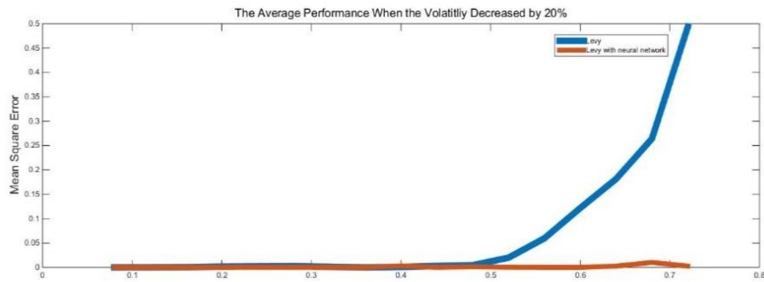

Figure 5. Average Performance of the Neural Network When the Volatilities Decreased by 20% for options of S/K=0.8, τ=2

Figure 6. Average Performance of the Neural Network When the Volatilities Increased by 20% for options of S/K=0.8, τ=2

As you can see when the maturity of the contract is increased as in figure 6 and figure 5, even when the volatility exceeds the original volatility by 20%, the increase of the error from the neural network is not as fast as the closed-form model. Note that the mean square error of the levy model reach a very high level.

(3) S/K=0.8, τ=4

The mean estimated standard error for the Monte Carlo benchmark prices for this set of data is

Original: 0.115915 for the set with volatility ranging from 0.1 to 0.9 with interval of 0.05.

+20%: 0.218933 for the set with the volatility increased by 20%

-20%: 0.080612 for the set with the volatility decreased by 20%

For table 3, the mean square error is relatively small and the R-value of networks shows a positive correlation as the previous tables.

Table 3. Neural Network Statistics for options of S/K=0.8, τ=4

Neural Network Number	Training	Test
No. 1	MSE: 1.35594e-6	MSE: 4.78321e-6
	R: 9.99987e-1	R: 9.99990e-1
No. 2	MSE: 1.43754e-6	MSE: 3.86518e-6
	R: 9.99985e-1	R: 9.99997e-1
No. 3	MSE: 1.81855e-6	MSE: 1.53480e-6
	R: 9.99982e-1	R: 9.99995e-1
No. 4	MSE: 1.85174e-6	MSE:1.21404e-6
	R: 9.99981e-1	R:9.99993e-1
No. 5	MSE:9.19666e-7	MSE: 9.04710e-6
	R:9.99989e-1	R: 9.99989e-1

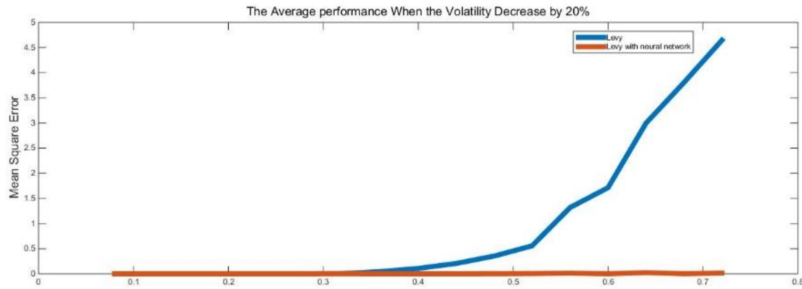

Figure 7. Average Performance of the Neural Network When the Volatilities Decreased by 20% for options of S/K=0.8, τ=4

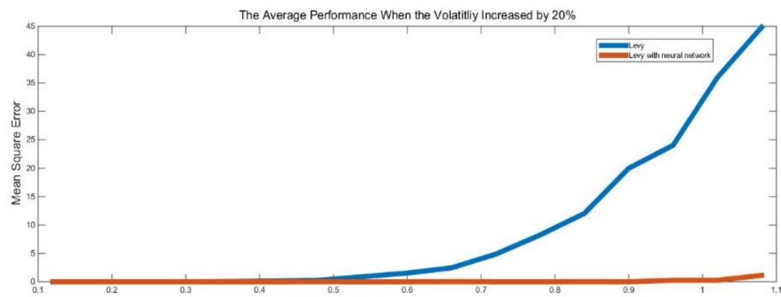

Figure 8. Average Performance of the Neural Network When the Volatilities Increased by 20% for options of S/K=0.8, τ=4

When the maturity of the contract reaches 4 as in figures 7 and 8, the levy approximation fails to price the option in a reasonable way. The rate of increase for the error of the neural network is much slower than the original model. In pricing long maturity out of the money options, neural network would result in more accurate pricing.

(4) S/K=1, τ=1

The mean estimated standard error for the Monte Carlo benchmark prices for this set of data is

Original: 0.047042 for the set with volatility ranging from 0.1 to 0.9 with interval of 0.05

+20%: 0.077002 for the set with the volatility increased by 20%

-20%: 0.041258 for the set with the volatility decreased by 20%

When the option is at the money, the mean square error could be better reduced compared to the situation that the option is out of the money as shown in table 4. Moreover, due to the larger error, the improvement of the options are also significant as shown in figure 9 and figure 10.

Table 4. Neural Network Statistics for options of S/K=1, τ=1

Neural Network Number	Training	Test
No. 1	MSE: 6.13313e-7	MSE: 4.28333e-6
	R: 9.99995e-1	R: 9.99994e-1
No. 2	MSE: 1.03061e-6	MSE: 6.50973e-6
	R: 9.99991e-1	R: 9.99732e-1
No. 3	MSE: 1.67824e-6	MSE: 1.04196e-6
	R: 9.99984e-1	R: 9.99994e-1
No. 4	MSE: 1.85259e-6	MSE:4.34712e-6
	R: 9.99983e-1	R:9.99994e-1
No. 5	MSE:1.73776e-7	MSE: 7.83487e-7
	R:9.99986e-1	R: 9.99990e-1

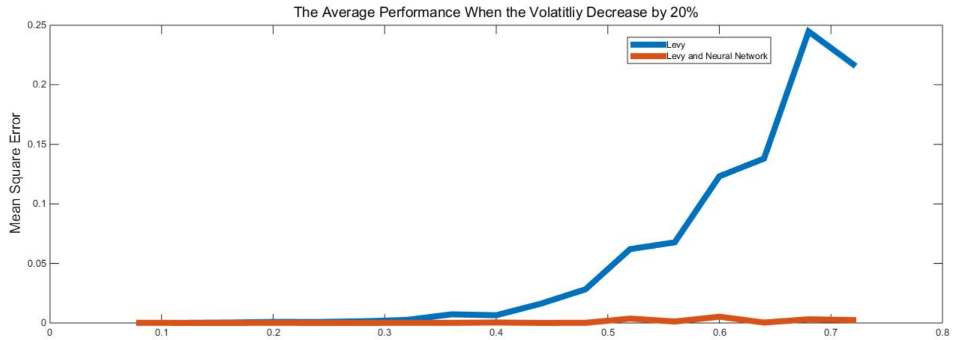

Figure 9. Average Performance of the Neural Network When the Volatilities Decreased by 20% for options of S/K=1, τ=1

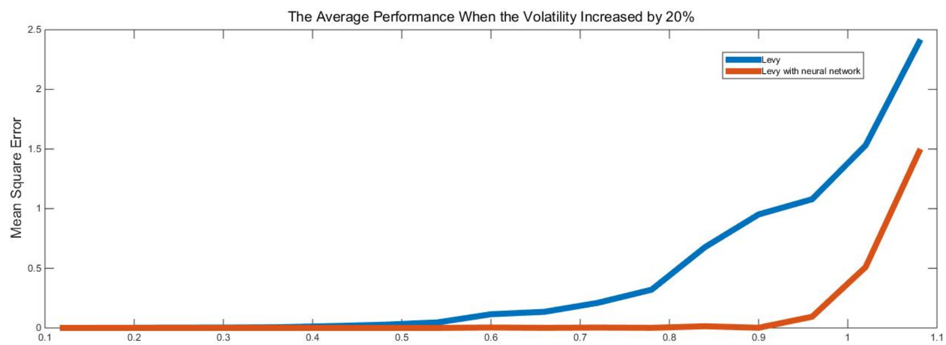

Figure 10. Average Performance of the Neural Network When the Volatilities Increased by 20% for options of S/K=1, τ=1

We can see that both in figure 9 and figure 10 even with a maturity of one year, for in-the-money options, the increase for the error is faster than the out-of-the money option. For one-year maturity, the proposed model outperforms the original model from beginning to the end.

(5) S/K=1, τ=2

The mean estimated standard error for the Monte Carlo benchmark prices for this set of data is

Original: 0.098507 for the set with volatility ranging from 0.1 to 0.9 with interval of 0.05

+20%: 0.148377 for the set with the volatility increased by 20%

-20%: 0.068199 for the set with the volatility decreased by 20%

The mean square error as shown in table 5 is similar to that in table 4. Due to the characteristic of at the money options, the mean square error is smaller and the R-value is higher than that of table 4 on average.

Table 5. Neural Network Statistics for options of S/K=1, τ=2

Neural Network Number	Training	Test
No. 1	MSE: 1.04853e-6	MSE: 9.63230e-6
	R: 9.99987e-1	R: 9.99997e-1
No. 2	MSE: 1.24888e-6	MSE: 7.31441e-7
	R: 9.99988e-1	R: 9.99993e-1
No. 3	MSE: 6.29583e-7	MSE: 4.88512e-6
	R: 9.99994e-1	R: 9.99996e-1
No. 4	MSE: 1.13355e-6	MSE:7.53868e-7
	R: 9.99988e-1	R:9.99999e-1
No. 5	MSE:.1.19055e-6	MSE: 1.34715e-7
	R:9.99988e-1	R: 9.99997e-1

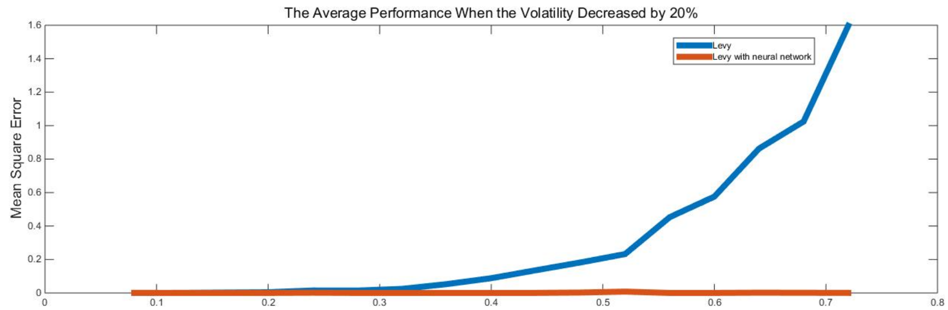

Figure 11. Average Performance of the Neural Network When the Volatilities Decreased by 20% for options of S/K=1, τ=2

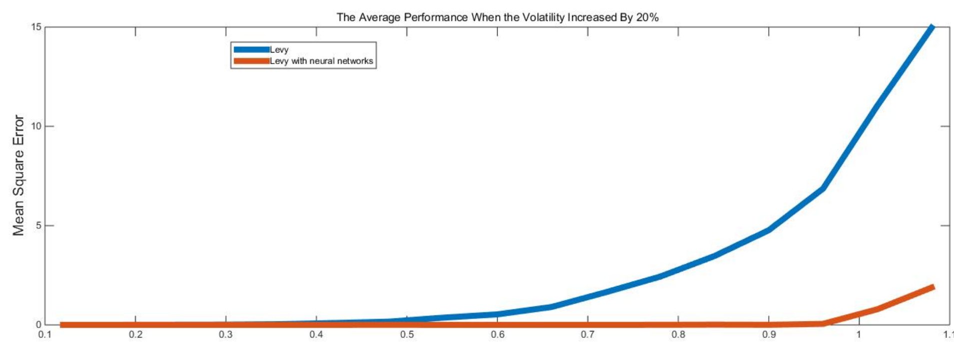

Figure 12. Average Performance of the Neural Network When the Volatilities Increased by 20% for options of S/K=1, τ=2

The error of the neural network model start to raise after the volatility exceeds 0.9 while the original model is earlier starting from around 0.5 and 0.6. Similar to the previous graph, in figure 11 and 12, the proposed model outperform the original one.

(6) S/K=1, τ=4

The mean estimated standard error for the Monte Carlo benchmark prices for this set of data is

Original: 0.173388 for the set with volatility ranging from 0.1 to 0.9 with interval of 0.05

+20%: 0.273918 for the set with the volatility increased by 20%

-20%: 0.109637 for the set with the volatility decreased by 20%

Within table 6, the mean square error is lower than that of the out of the money option. We can also see that in the figure 13 and figure 14, when the volatility reaches 0.9, the result of the original model is distorted while the adjusted one is significantly improved.

Table 6. Neural Network Statistics for options of S/K=1, τ=4

Neural Network Number	Training	Test
No. 1	MSE: 2.79467e-6	MSE: 2.07831e-6
	R: 9.99968e-1	R: 9.99979e-1
No. 2	MSE: 1.93887e-6	MSE: 7.18128e-6
	R: 9.99977e-1	R: 9.99559e-1
No. 3	MSE: 2.47110e-6	MSE: 4.16690e-6
	R: 9.99971e-1	R: 9.99969e-1
No. 4	MSE: 2.38209e-6	MSE:5.51298e-6
	R: 9.99971e-1	R:9.99941e-1
No. 5	MSE:.1.97543e-6	MSE: 6.12985e-7
	R:9.99975e-1	R: 9.99955e-1

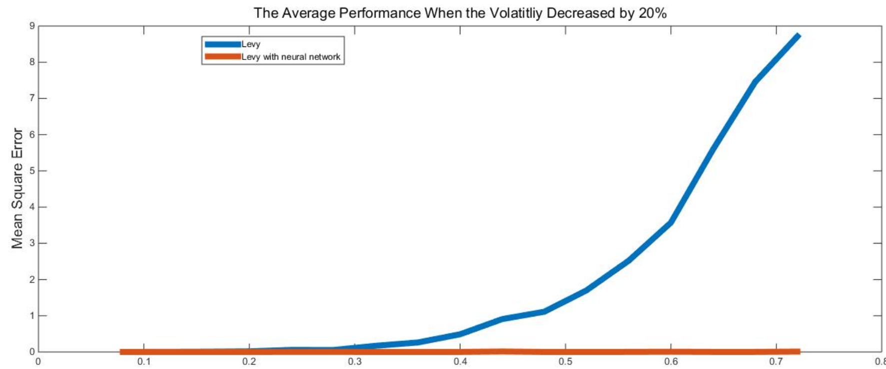

Figure 13. Average Performance of the Neural Network When the Volatilities Decreased by 20% for options of S/K=1, τ=4

Figure 14. Average Performance of the Neural Network When the Volatilities Increased by 20% for options of S/K=1, τ=4

For the in-the-money option in figure 13 and 14, the problem is also severe for the levy approximation. For the neural network, even though the result is improved, but the error becomes too large when the predicted volatility is too far from the training volatility.

(7) S/K=1.2, τ=1

The mean estimated standard error for the Monte Carlo benchmark prices for this set of data is

Original: 0.065445 for the set with volatility ranging from 0.1 to 0.9 with interval of 0.05

+20%: 0.09096 for the set with the volatility increased by 20%

-20%: 0.042568 for the set with the volatility decreased by 20%

For the in the money option, as shown in table 7 the average of mean square error for the training set and the test is less than the previous experiments meaning that the relationship between the real and implied volatilities become more obvious.

Table 7. Neural Network Statistics for options of S/K=1.2 τ=1

Neural Network Number	Training	Test
No. 1	MSE: 2.99848e-7	MSE: 1.88907e-6
	R: 9.99997e-1	R: 9.99983e-1
No. 2	MSE: 4.45387e-7	MSE: 3.20084e-6
	R: 9.99994e-1	R: 9.99938e-1
No. 3	MSE: 5.67161e-7	MSE: 2.97277e-6
	R: 9.99994e-1	R: 9.99979e-1
No. 4	MSE: 4.47649e-6	MSE:1.69264e-6
	R: 9.99996e-1	R:9.99976e-1
No. 5	MSE:.4.83649e-7	MSE: 8.12137e-7
	R:9.99995e-1	R: 9.99989e-1

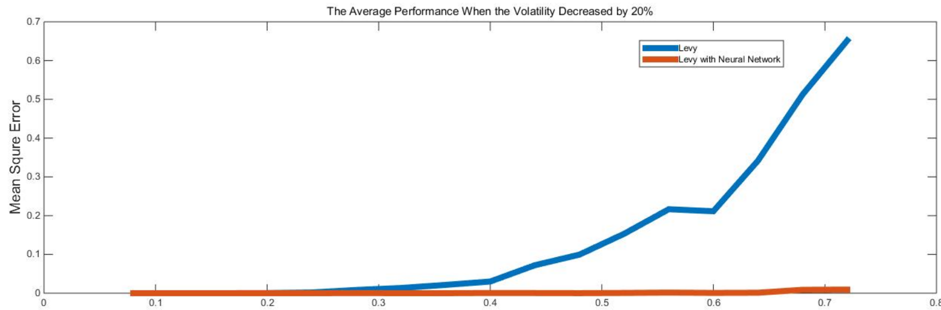

Figure 15. Average Performance of the Neural Network When the Volatilities Decreased by 20% for options of S/K=1.2, τ=1

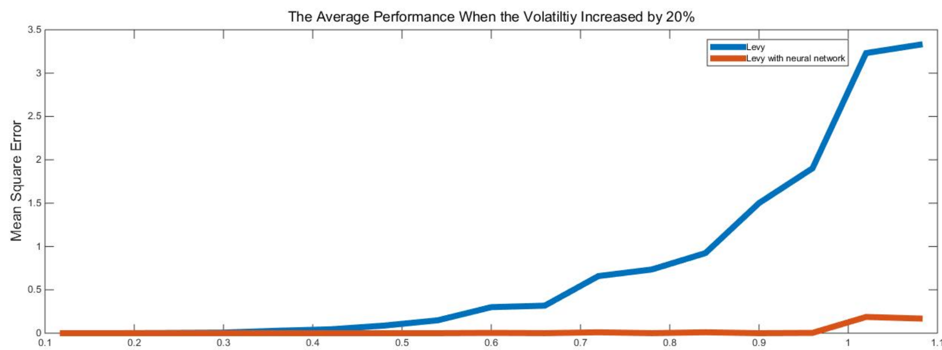

Figure 16. Average Performance of the Neural Network When the Volatilities Increased by 20% for options of S/K=1.2, τ=1

The neural network performs much better than the in-the-money and out-of-the-money option as shown in figure 15 and 16. Compared to the one year out-of-the money option, the error of in-the-money option follows the same pattern but have a higher error.

(8) S/K=1.2, τ=2

The mean estimated standard error for the Monte Carlo benchmark prices for this set of data is

Original: 0.107694 for the set with volatility ranging from 0.1 to 0.9 with interval of 0.05

+20%: 0.15261 for the set with the volatility increased by 20%

-20%: 0.08399 for the set with the volatility decreased by 20%

In table 8, the average of the mean square error is similar to that of table 7. We can see compared to out of the money and at the money options the mean square error of the in the money options are less in value meaning that the mapping of the real volatility to implied volatility is more successful.

Table 8. . Neural Network Statistics for options of S/K=1.2 τ=2

Neural Network Number	Training	Test
No. 1	MSE: 7.41007e-7	MSE: 8.08313e-7
	R: 9.99992e-1	R: 9.99995e-1
No. 2	MSE: 5.31102e-7	MSE: 3.77837e-6
	R: 9.99995e-1	R: 9.99998e-1
No. 3	MSE: 7.29289e-7	MSE: 2.75924e-6
	R: 9.99992e-1	R: 9.99994e-1
No. 4	MSE: 5.67640e-7	MSE:3.62116e-6
	R: 9.99993e-1	R:9.99999e-1
No. 5	MSE:.8.11229e-7	MSE: 5.03703e-6
	R:9.99989e-1	R: 9.99923e-1

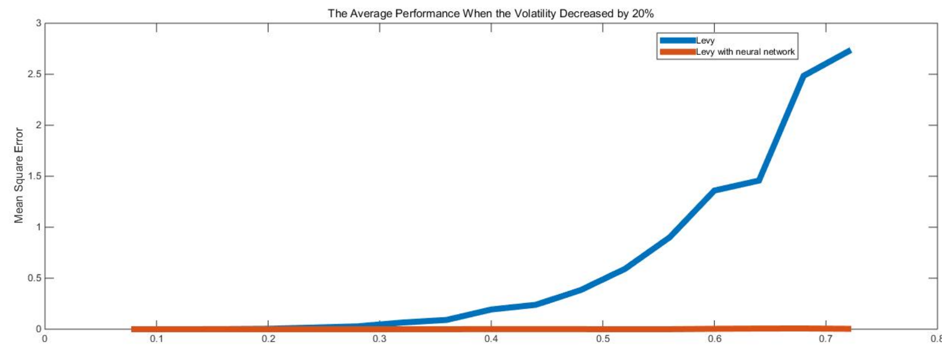

Figure 17. Average Performance of the Neural Network When the Volatilities Decreased by 20% for options of S/K=1.2, τ=2

Figure 18. Average Performance of the Neural Network When the Volatilities Increased by 20% for options of S/K=1.2, τ=2

Compared to the previous graphs, in figure 17 and 18 the result for mapping in the money will be better. Neural network seems to have a more stable prediction compared to the previous ones. The neural network successfully fixed the error even when the volatility changed 20%. We can also see that the original model fails to offer a reasonable price.

(9) S/K=1.2, τ=4

The mean estimated standard error for the Monte Carlo benchmark prices for this set of data is:

Original: 0.214279 for the set with volatility ranging from 0.1 to 0.9 with interval of 0.05

+20%: 0.285916 for the set with the volatility increased by 20%

-20%: 0.124688 for the set with the volatility decreased by 20%

Compared to the previous experiments, the result from table 9 is not the best but still lies in a reasonable range. However, as the option value increases, the implied volatility will also increases. In return, the mean square error of the in the money options will be more trivial than that of at the money and out of the money options proportional to the corresponding implied volatilities.

Table 9. Neural Network Statistics for options of S/K=1.2 τ=4

Neural Network Number	Training	Test
No. 1	MSE: 1.32955e-7	MSE: 4.90016e-6
	R: 9.99986e-1	R: 9.99830e-1
No. 2	MSE:1.35725e-6	MSE: 2.26924e-6
	R: 9.99985e-1	R: 9.99998e-1
No. 3	MSE: 1.36251e-6	MSE: 3.89050e-6
	R: 9.99986e-1	R: 9.99966e-1
No. 4	MSE: 9.64506e-7	MSE:5.34932e-6
	R: 9.99989e-1	R:9.99967e-1
No. 5	MSE:.1.51718e-6	MSE: 7.48646e-6
	R:9.99981e-1	R: 9.99995e-1

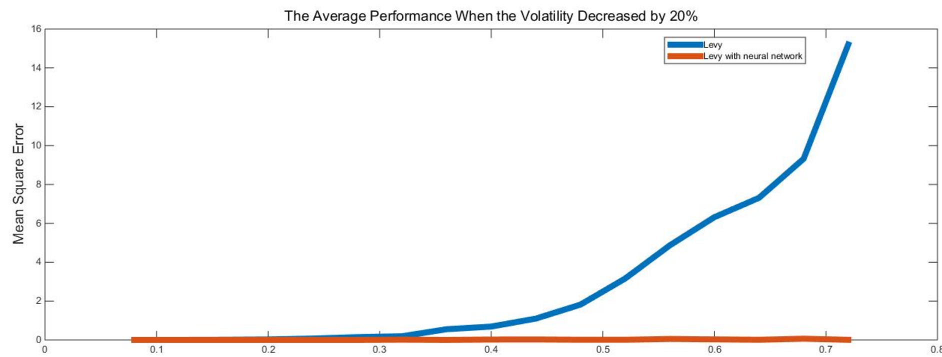

Figure 19. Average Performance of the Neural Network When the Volatilities Decreased by 20% for options of S/K=1.2, τ=4

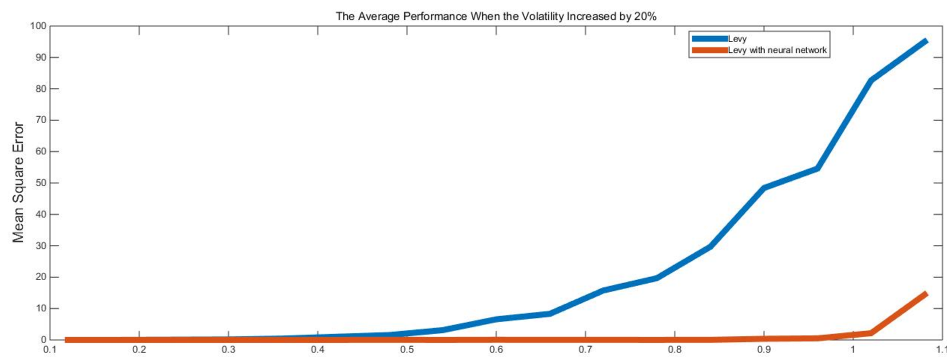

Figure 20. Average Performance of the Neural Network When the Volatilities Increased by 20% for options of S/K=1.2, τ=4

For figure 19 and 20, we can conclude that as the deepness of in the money increases, the method could achieve a better result by adjusting the volatility using the neural network and could improve the degree of accuracy of Asian option pricing.

In conclusion, on all of the panels above, neural network could successfully adjust the volatility and reduce the error of the Asian options. Also, note that the neural networks has a very high value and a very low mean square error for fitting, meaning that the volatility can be fitted using algorithms and neural network. More experiments will be performed with the real data in the next section.

4.2.5 Model Results for Real Data Test

For real data, we chose several different contracts of different maturity and underlying asset price to validate our method on real data. We test five contracts in total. Some of the contracts are already expired while some are still trading. We attempt to cover different types of contracts as well. For convenience proposes, the prices we display are prices of option that have a strike price of 30. The reasons are as follows. Firstly, with a strike price of 30, the premium will not be too low or too high. If the premium is too low or the option is out of the money, then the price not be obvious enough to show the pattern of the errors across a wide range of volatilities between the levy model and the adjusted model with neural network. (When the option premium is less than 0.01 theoretically, it will be sold at a price of 0.01 in the exchange.) The author also applied the same method for different strike prices and they show the same result. As the premium is higher, the improvement is more obvious while the premium of the option is lower, the improvement will be less. The experiment also shows that the error of the Levy Model is proportional to the premium. (This also reflects in the maturity and the strike price of the option since they affect the premium of an option). For testing purpose, data of one month period is used as the dataset to build the neural network. Then we use the neural network with the levy approximation model to price the option for the next one-week period. Other parameters use corresponding data on that specific date. The risk free rate uses treasury bills as a reference.

(1) Contract CSM 2018 (June 2018)

The contract CSM 2018 (Jun 2018) is tested with and without the neural network. The data ranging from 2016-Aug-02 to 2016-Sept-15 (One month period) is used as the data set to build the neural network. We then use the built neural network with levy approximation to price the option of future one-week time. We found that the there is still room for improvement for the levy approximation according to the real data. After the learning process is applied, the accuracy of the pricing model has improved. The results are shown in figure 21.

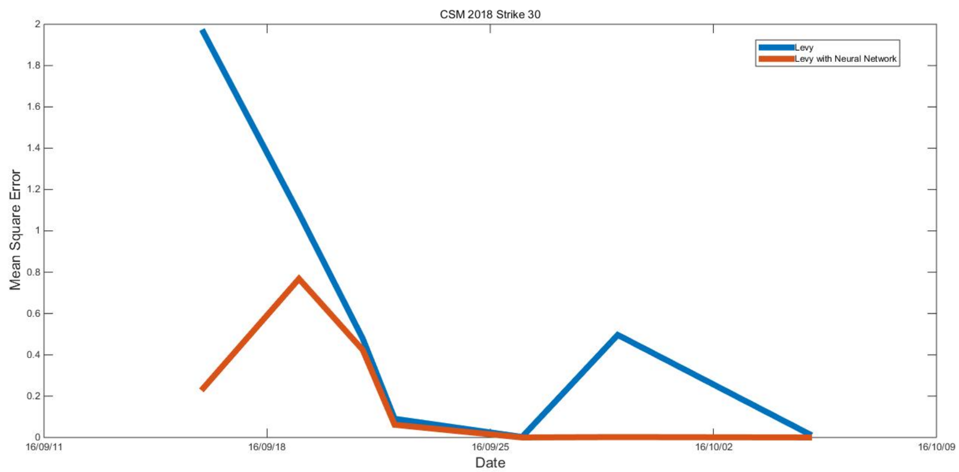

Figure 21. Contract CSM 2018 Average Pricing Accuracy Comparison

As shown in table 10, the mean square error is not as ideal as the previous simulation experiments. The reason is that the difference between the real volatility and implied volatility within real option prices is even larger. In such a case, the mean square error will seem less desirable. However, the result shows that the mapping could improve the option pricing accuracy. Also, note that these networks are not the optimized networks, and if carefully filtered the selection of the networks, we can even get a better result.

On the other hand, the R-value is not as good as that of the simulation experiment. The most possible explanation is that the real data might have more noises. For research purposes, the author only tests a few groups of assumed parameters but traders in the market bear different expectation of the parameters.

Table 10. Neural Network Statistics for CSM 2018

Neural Network Number	Training	Test
No. 1	MSE: 1.21348e-2	MSE: 9.73656e-3
	R: 7.06070e-1	R:6.82633e-1
No. 2	MSE:2.17956e-2	MSE:3.22945e-2
	R: 7.97997e-1	R:7.57166e-1

As a result, we can also see that the average performance of the model is better than the original. We can see on average the accuracy of option pricing is improved.

(2) Contract CSQ2018 (Aug 2018)

The data ranging from 2017-Mar-22 to 2017-Apr-26 (One month period) is used as the data set to build the neural network. We can see that at the last data point, the error is as high as the original model. The reason might be that the implied volatility of the Asian option is very close to the volatility of the Asian option. In that case, the neural network might give a very close solution. Still, on average, as shown in in figure 22 the neural network gives a more accurate result and we can argue that it is at least as good as the original pricing model. On the other hand, if the implied volatility (not unique) is chosen more carefully, we can even get a more precise result.

From table 11 compared to the previous tables, we can see that as the maturity of the contract is large enough, the levy model price will deviate from the real prices. In such as case, the neural network could better map the real and implied volatilities since the gap of the two is large compared to short maturity options.

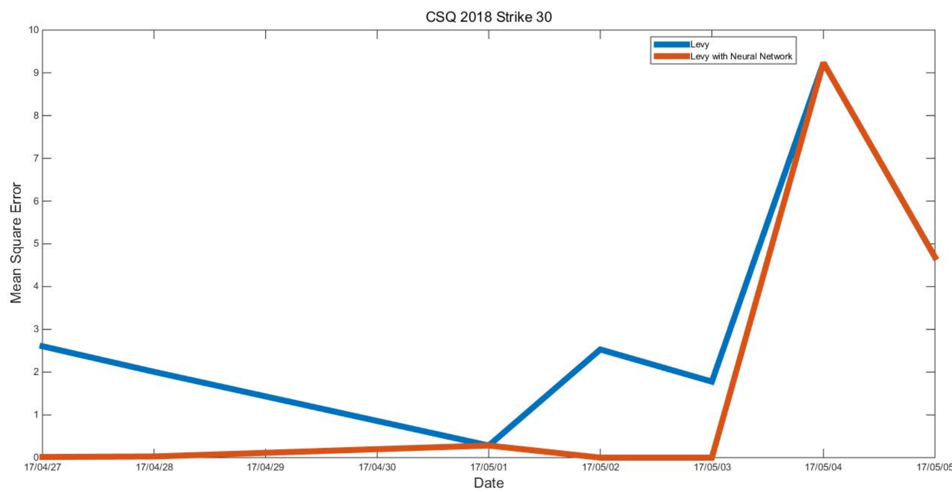

Figure 22. Contract CSQ 2018 Average Pricing Accuracy Comparison

Table 11. Neural Network Statistics for CSQ 2018

Neural Network Number	Training	Test
No. 1	MSE: 9.48765e-3	MSE: 2.49870e-2
	R: 9.33968e-1	R:9.94202e-1
No. 2	MSE: 1.54002e-2	MSE: 8.16726e-3
	R: 8.75441e-1	R: 9.89976e-1

(3) Contract CSN2017 (July 2017)

The data ranging from 2017-Mar-22 to 2017-Apr-26 (One month period) is used as the data set to build the neural network.

The R-value of the neural networks on average in table 12 is not as good as that of table 11 since the maturity of the option is shorter. Due to the noise from the option price, the volatility will not show a strong correlation.

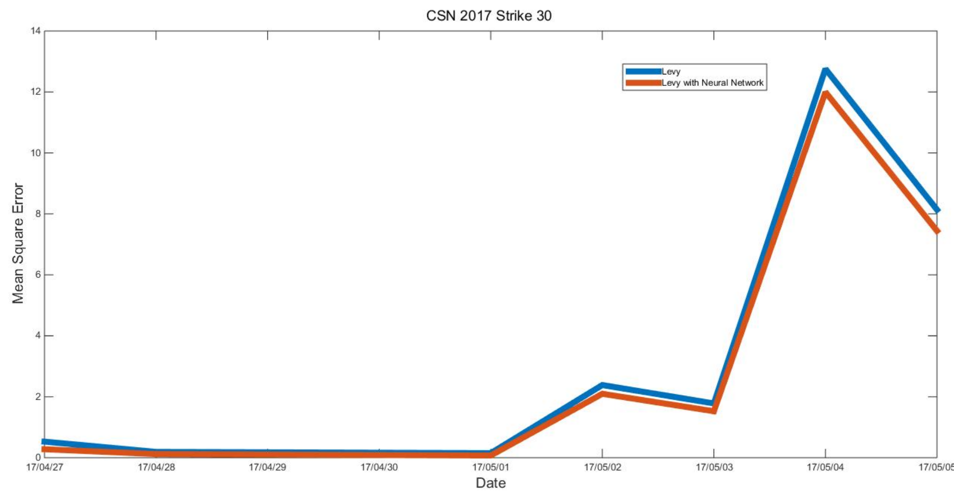

Figure 23. Contract CSN 2017 Average Pricing Accuracy Comparison

Table 12. Neural Network Statistics for CSN 2017

Neural Network Number	Training	Test
No. 1	MSE: 9.19668e-2	MSE: 2.72365e-2
	R: 7.20562e-1	R:9.34908e-1
No. 2	MSE: 1.06288e-2	MSE: 4.27134e-3
	R: 8.05127e-1	R: 9.96222e-1

For this contract, as shown in figure 23, all of the prices improved. However as the date comes further from the learning data set, the lower the accuracy of pricing will be since more information might involve in the market.

(4) Contract CSH2017 (March 2017)

The data ranging from 2017-Feb-14 to 2017-Mar-21 (One month period) is used as the data set to build the neural network.

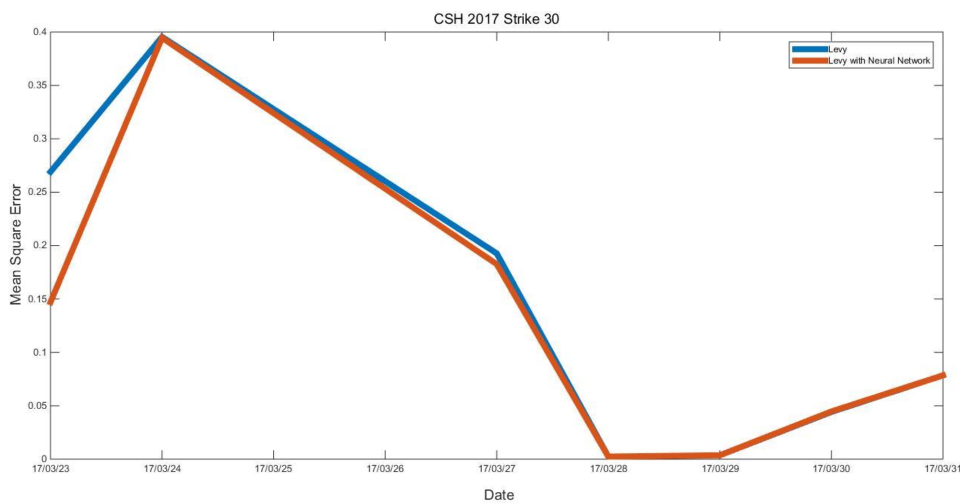

Figure 24. Contract CSH 2017 Average Pricing Accuracy Comparison

Table 13. Neural Network Statistics for CSH 2017

Neural Network Number	Training	Test
No. 1	MSE: 5.53375e-1	MSE: 8.88603e-1
	R: 8.80796e-1	R:8.36442e-1
No. 2	MSE: 6.82120e-1	MSE: 7.64490e-1
	R: 7.85836e-1	R: 9.39447e-1

For contracts that have a short maturity, the mean square error that measures the deviation from the real data quote is smaller since the premium is lower. We also achieve a better improvement after the neural network is applied. The mean square as shown in table 13 is similar to that of table 12. Since the mean square error is not low enough, the pricing accuracy of the option in figure 24 show that when the maturity is short, the volatilities are more difficult to map and the pricing accuracy are more difficult to improve.

(5) Contract CSK2019 (May 2019)

The data ranging from 2017-Mar-22 to 2017-Apr-26 (One month period) is used as the data set to build the neural network.

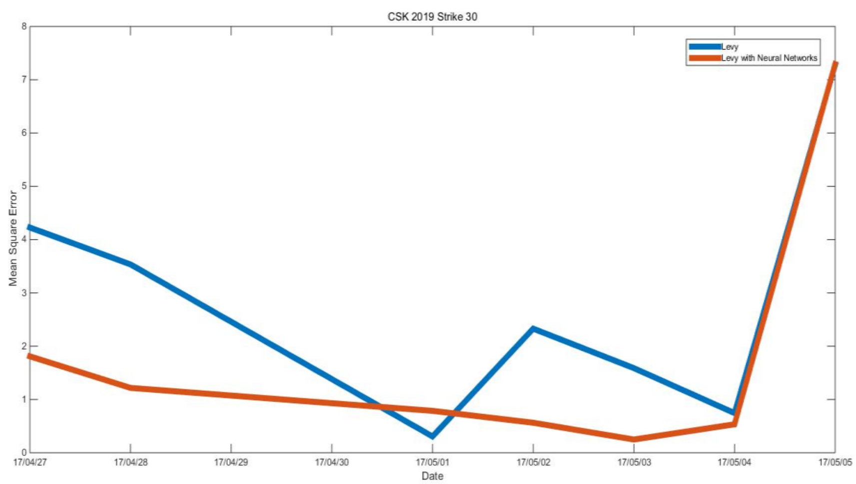

Figure 25. Contract CSK 2019 Average Pricing Accuracy Comparison

Table 14. Neural Network Statistics for CSK 2019

Neural Network Number	Training	Test
No. 1	MSE: 6.77547e-3	MSE: 4.39259e-2

	R: 9.29789e-1	R:9.33806e-1
No. 2	MSE: 2.15513e-3	MSE: 1.20900e-2
	R: 9.54820e-1	R: 9.57398e-1

Except for date 2017 May 11, all of the results improves in figure 25. Sometimes, on the next date, the quote will still be the quote for previous day since there is no trades taken place. In that case, situation of date May 1st might occur. Still on the other hand, the improvement for long maturity is significant as expected. Still in table 14, the R-value shows that the volatilities have a stronger positive correlation due to the long maturity of the option meaning that levy model option prices with longer maturity result in larger deviation from the market option prices.

4.3. Discussion and Conclusion

By using Neural Networks, the levy model should exhibit improvements after the learning process. These improvements should exist across options of all spot price, all volatility and all maturity. For the Levy model, in the original paper (Levy, 1992), the author showed that when the product of volatility and square root of maturity is lower than 0.2, the closed-form price approximates the Monte Carlo price accurately. (Note that an underlying asset with a volatility lower than 0.2 is a rare case in practice.) In such a circumstance, the prices of options with volatility higher larger than 0.2 or higher required correction because premiums are very far from the real premium. We anticipated that after applying neural network, the error of the options with high underlying asset volatility would be lower than that of the original closed-form. Another finding for the closed-form is that, it has part of error incurred by assumption of the model. The same thing happens to the Levy model. When the maturity and the volatility are large, the errors that incurred by the assumption will magnify, making the model meaningless. That is also why theses model performs better for out of the money options since the premium for an out-of-the-money option is smaller than that of an in-the-money option. The biases incurred by the assumption of the closed form model are somehow alleviated when the premium is low. One of our major goals is to fix such biases and improve the pricing accuracy. As shown in the previous section, after the neural network is applied and the volatility is adjusted, the pricing accuracy is significantly improved and the biases are fixed.

CHAPTER V

CONCLUSIONS AND FUTURE WORKS

As the conclusion of the thesis, this part summarizes the research results of our study. Compared with the original model, the prediction accuracy after integrating neural network model is significantly improved. Because there is no closed form formula available, neural network could always be applied. The method proposed in this research can significantly improve the efficiency of pricing and financial decision. The use of Neural Network highly improves the accuracy of the traditional closed-form model. The analysis shows that after learning and testing, the pricing of Asian Options on simulation or on the real data , all have a significant positive effect on the results.

It is well known that the study of the volatility surface is one of the most difficult problem in option pricing. This paper also provides an intuition that rather than fitting the volatility surface with some unreliable mathematical model, as an alternative we can remember the whole volatility surface with Neural Networks or other machine-learning algorithm. The methodology could also be applied to deal with the term structure of fixed income securities. This behavior resembles the human behavior that when we encounter a difficult problem we do not understand; we first start to remember it. Although the Method required certain amount of computation, once the neural network finished training, the neural network can be used for a long-time and output accurate result instantaneously.

REFERENCES

[1] Levy, E. ,*Pricing European Average Rate Currency Options*, Journal of International Money and Finance (1992), 11, 474-491.

[2]Robert Kolb, *Understanding Options 2*, Wiley & Sons,1995

[3]Zhang, G., Patuwo, B. E., & Hu, M. Y. ,*Forecasting with Artificial Neural Networks: The State of the Art*. International Journal of Forecasting, 1998. 14, p.35–62.

[4] Robert R. Trippi and Efraim Turban, *Neural Networks in Finance and Investing: Using Artificial Intelligence to Improve Real World Performance*, McGraw-Hill, Inc. New York, NY, USA ©1992

[5] Hutchison J M, LoA, Poggio *T.*, *A Nonparametric Approach to Pricing and Hedging Derivative and Securities Via Learning Networks*, Journal of Finance, 1994, 49(3):851-889.

[6] Jingtao Yao, Yili Li, Chew Lim Tan, *Option Price Forecasting Using Neural Networks*, Omega Volume 28, Issue 4, 1 August 2000, Pages 455–466.

[7] Marco J. Morelli, Guido Montagna, Oreste Nicrosini; C, Michele Treccani, Marco Farina, Paolo Amato, *Pricing Financial Derivatives with Neural Networks*, Physica A, 338 , 2004,pp. 160 – 165.

[8] Gradojevic, N., Gen ay, R., and Kukolj, D., *Option Pricing with Modular Neural Networks*, IEEE Transaction on Neural Networks, Vol. 20, No. 4, 2009, pp. 626-637.

[9] Yi-Hsien Wang, *Nonlinear Neural Network Forecasting Model for Stock Index Option Price: Hybrid GJR–GARCH Approach*, Expert Systems with Applications, Volume 36, Issue 1, January 2009, Pages 564–570

[10] Zhang Hong-yan, Lin Hui, and Jiang Cai-lou, *Study on Option Pricing by Applying Hybrid Wavelet Networks and Genetic Algorithm*, Journal of Systems Engineering, Vol.25 No.1, 2010: 43-49.

[11] Lin, C.-T., and Yeh, H.-Y., *Empirical of the Taiwan Stock Index Option Price Forecasting Model - Applied Artificial Neural Network*, Applied Economics, Vol. 41, No. 15, 2009, pp. 1965-1972.

[12] Michael Kohler, Adam Krzyzak and Nebojsa Todorovic, *Pricing Of High-Dimensional American Options by Neural Networks*, Mathematical Finance, Vol. 20, No. 3, July 2010, pp.383–410

[13] Muhammad Yahuza Bello, Haruna Chiroma, *Utilizing Artificial Neural Network for Prediction in the Nigerian Stock Market Price Index*, GESJ: Computer Science and Telecommunications 2011, No.1 (30)

[14] S. Shakya, M. Kern, G. Owusu, C.M. Chin, *Neural Network Demand Models and Evolutionary Optimisers for Dynamic Pricing*, Knowledge-Based Systems, 2012, 29,pp.44–53

[15] Mitra, S. K., *An Option Pricing Model that Combines Neural Network Approach and Black Scholes Formula*, Global Journal of Computer Science and Technology, Vol. 12, No. 4, 2012, pp. 6-16.

[16] Fei Chen and Charles Sutcliffe, *Pricing And Hedging Short Sterling Options Using Neural Networks,* Intelligent Systems in Accounting, Finance And Management, , 2012, 19, pp.128–149.

[17] Adam Fadlalla and Farzaneh Amani ,*Predicting Next Trading Day Closing Price of Qatar exchange Index Using Technical Indicators and Artificial Neural Networks* , Intelligent Systems in Accounting, Finance And Management, 2014,21, pp.209–223.

[18] Georgios Sermpinis, Thanos Verousis and Konstantinos Theofilatos, *Adaptive Evolutionary Neural Networks for Forecasting and Trading without a Data-Snooping Bias,* Journal of Forecasting, 2016, 35, pp.1–12.

[19] Mehdi Khasheiand Mehdi Bijari, *An Artificial Neural Network (p, d, q) Model for Timeseries Forecasting,* Expert Systems with Applications,2010,37, pp. 479–489.

[20] Kemna and Vorst, *A Pricing Method for Option Based on Average Asset Values*,Journal of Banking & Finance 1990, 14, pp. 113–129.

[21] Bodie, Zvi; Alex Kane; Alan J. Marcus. Investments (7th Ed.). New York: McGraw-Hill/Irwin.2008

[22] Black, Fischer; Myron Scholes (1973). "The Pricing of Options and Corporate Liabilities". Journal of Political Economy. 81 (3): 637–654.

[23] J.AaseNielsen andKlaus Sandmann,The Pricing of Asian Optionsunder Stochastic Interest Rates,Applied Mathematical Finance, Vol. 3, No. 3, pp. 209-236, 1995.

[24] Paul Glasserman, Monte Carlo Methods in Financial Engineering, Springer, 2003.

[25] Art B. Owen,*Monte Carlo theory, methods and examples*, 2013.

[26] K.Lange, *Numerical Analysis for Statisticians*, Springer-Verlag: NY.1998.

[27] Henri P. Gavin, *The Levenberg-Marquardt method fornonlinear least squarescurve-fitting problems*, Department of Civil and Environmental Engineering, Duke University, May 4, 2016.

[28] Cybenko, G. *Approximation by superpositions of a sigmoidal function. Mathematics of Control, Signals, and Systems2*, 303–314, 1989.

[29] Hornik, K., Stinchcombe, M., and White, H. *Multilayer feedforward networks are universal approximators. Neural Networks2*, 359–366, 1989.

APPENDICES

$b = r_f - Div$, where r_f is the risk free rate and Div is the dividend rate
S_0 is the initial asset price. $T - t$ is the time to maturity. σ is the volatility of the underlying asset.

$$SE = \frac{S_0(e^{(b-r_f)(T-t)} - e^{(-r_f)(T-t)})}{b(T-t)}$$

$$P_1 = \frac{e^{((2b+\sigma^2)(T-t))} - 1}{2b + \sigma^2}$$

$$P_2 = \frac{e^{(b(T-t)-1)}}{b}$$

$$M = \frac{2{S_0}^2}{b + \sigma^2}(P_1 - P_2)$$

$$D = \frac{M}{(T-t)^2}$$

$$V = \log(D) - 2(r_f(T-t) + \log SE)$$

$$d_1 = \frac{1}{\sqrt{V}}(\log\frac{D}{2} - \log K)$$

$$d_2 = d_1 - \sqrt{V}$$

$$Call\ Option\ Price = SE * N(d_1) - Ke^{-r_f(T-t)}N(d_2)$$

To be more specific the σ is what we estimate here.

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