GREEN WEIGHT, TAPER & VOLUME EQUATIONS FOR LOBLOLLY PINE IN OKLAHOMA, USA

By

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Bachelor of Science in Natural Resource Ecology and Management, Forestry Option

Stillwater, Oklahoma

2014

Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
MASTER OF SCIENCE
July, 2017

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FOR LOBLOLLY PINE IN OKLAHOMA, USA

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ACKNOWLEDGEMENTS

First of all: all thank you Ellyn for keeping my life moving, for tutoring me through Calculus I and II, for proofing my thesis, but more than that for being the best partner and wife a man could have. I would be much in the wrong if I did not mentioning my parents for giving me the start and support to get where I am. Not just in terms of education, that is piddly stuff in comparison to the life skills and critical thinking that the taught me. Thank you Janie, Daniel and Jonathan for always being there.

I am very grateful to my major advisor Tom who gave me an opportunity, on faith, that I cannot repay. I do not think I could have sought out a better graduate experience. I have been stretched farther and done more than I ever thought possible, but through it all I have had a great teacher and mentor. It was a privilege to work under you and I am blessed to have you as a friend.

Most of the technicalities of the project: the funds, the sampled trees, the logistics of selecting stands, are due to Rayonier. Specifically, John Paul McTague and Becki Rollins; without these two this work would never have happened. Thank you Becki for your valuable contributions as one of my committee members.

Thank you Rod, not just for the part you played on my committee, but also for all of the practical, ecological wisdom that you tried to convey to us knuckleheads. You had a huge impact on the way I think about forestry and ecology. The time spent in your classes was indispensable to my professional development.

The major source for boots on the ground was the folks from the Kiamichi Forestry Research Station. Thank you Bob for loaning us your resources, Randy for your knowledge of the area and good attitude, Keith for keeping it real, Casey for all of your hard work, Dennis for helping us keep up with the details, and Glenda for looking after us all. Additionally, without the several other support staff we would have been very much at a loss. This group gives their all to our department, up to and including a place to sleep for anyone who needs a place to stay in their part of OK. It has always been my experience that they treat you like family.

We did not know it going in but we needed the expertise of David Walker, Phil Radtke and their crew from Virginia Tech. They have turned taper and biomass sampling into a fine art.

Thanks as well to the crews who came up from the University of Georgia and from the SRS Wood Products Lab (that was one heck of a saw…).

People always include something or another (usually cheesy) about God. I also have something to say, but hopefully it is not cheesy. My relationship with Jesus, The Messiah, and the purpose I have in teaching others how to follow him makes my life worth living. Nothing else does.

Forestry is a way to put food on the table, to help people and to take care of the earth but it pales in comparison to the magnitude of the universe, much less the One who made it. Something, no Someone, of that magnitude is the only thing worth giving one's life to.

Acknowledgements reflect the views of the author and are not endorsed by committee members or Oklahoma State University.

Name: WILLIAM THOMAS HARGES

Date of Degree: JULY, 2017

Title of Study: GREEN WEIGHT, TABER & VOLUME EQUATIONS FOR LOBLOLLY PINE IN OKLAHOMA, USA

Major Field: NATURAL RESOURCE ECOLOGY AND MANAGEMENT

Abstract: Data from 158 loblolly pine (*Pinus taeda L.*) trees located in industrial plantations in the Ouachita Mountains of southeastern Oklahoma, USA were collected and used to develop taper, merchantable volume and merchantable green weight equations. These plantations are located near the north-west extreme but outside of the natural range of loblolly pine. Oklahoma loblolly pine was compared to predictions from previously published equations for loblolly pine's native range in order to evaluate whether differences for taper, volume and green weight characteristics existed between these populations. This study indicates that Oklahoma loblolly pine has less merchantable green weight and volume for a given dbh, total height and merchantability limit than predicted by some well-known models. Diameter for a given height was also different. Since differences were observed the exponential merchantable diameter ratio equation, a new merchantable height based model and a new taper equation, which was derived from the merchantable height based model, were selected from among several well-known models for prediction of merchantable green weights and volumes. This work provides equations that have good predictive ability for trees across a wide range of conditions present in this region of Oklahoma, and which should work well in neighboring areas in the Ouachita Mountain region.

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CHAPTER I

INTRODUCTION

Stem volume, taper and green weight are important to any forest management planning that incorporates timber production for any purpose—e.g., lumber, pulp or carbon sequestration. However, the relationship between predictor variables (dbh, height and a merchantability limit) and predicted variables (volume, merchantable diameter and green weight) vary due to a number of factors. Research on this variation investigates the effects of site characteristics (i.e., location, site index, soils, weather and climate), stand characteristics (i.e., tree density, age, fertilization, competition control and site preparation), genetic characteristics (seed source, family and individual), as well as interactions between all of these characteristics.

One of the primary studies on this variation is Amateis and Burkhart (1987b) which examined the effect of stand origin (i.e., the previous uses of the land and method of stand regeneration) on the relationship between tree diameter at breast height (diameter at 4.5 ft, dbh, *D*), total tree height (Ht, *H)*, and inside bark volume. This study used the combined variable equation (Spurr 1952, p. 111-119), the total height to dbh relationship, tree form (in the context of solids of rotation) and the taper relationship to examine the variables and relationships that they wanted to investigate. They found that stand origin (i.e., unthinned old-field plantation, cutover plantation and natural stands) affected all of these relationships and tree characteristics. They found no impact from geographic region, stand density, stand age or site index. Schmidtling and Clark (1989) studied the effects of seed source on variation in individual tree stem content (referring to both volume and green weight) via the relationships and tree characteristics examined by Amateis and Burkhart (1987b), finding significant impacts in all.

Jordan et al. (2008) modeled loblolly pine specific gravity for a large portion of the range of loblolly pine, finding that it decreased from south to north and from east to west with the lowest specific gravity found in the north-west corner of the range. Specific gravity decreased with site index and increased with age. Antony et al. (2010) corroborated the findings of Jordan et al. (2008). Zobel and Van Buijtenen (1989) show that loblolly pine specific gravity has little impact on green weight. Though, specific gravity may decrease (i.e., across the species range) water content increases so that, in spite of the change, the green weight of wood stays almost constant, but slightly heavier.

The results of wide-ranging provenance tests can help elucidate variability observed in loblolly pine characteristics. Provenance studies often only examine the relationships of variables to seed source, site and the interaction of these factors, while most of the previously mentioned studies have explored how the relationships between variables change. A large provenance study, Farjat et al. (2017), reported that dbh, height, volume, bole straightness, tree survival and incidence of fusiform rust are all affected by seed source, site and interactions of the two. Their results indicate an especially strong interaction between site and seed source for volume, height and bole straightness. Additionally, this study showed that minimum winter temperature was a good summary variable for site effects. Genetic expression was maximized at higher temperatures whereas the different seed sources performed similarly at lower temperatures. Another large provenance study, Wells and Wakeley (1966), examined the main and interaction effects of site and seed source on height, dbh, volume (calculated using a conoid), survival, insect damage and disease. These effects were deemed to be significant for most of the variables across many of the sites; however, the significance of the results differed between sites. Wells and Wakeley (1966) did not calculate overall significance for effects across all sites, nor did their follow-up reports. The factors and relationships examined by Wells and Wakeley (1966) were studied again at 25 years from the beginning of the experiment in Wells (1983); finding significant differences in all variables except for estimated volume, in many sites. At 25 years, distinct differences between coastal and inland areas were reported, as well as phenotypic differences between populations divided by the Mississippi river. Trees the gulf coast and eastern populations grow faster than those from the west or from inland areas, respectively; western populations show more drought resistance. A small provenance study, reported in Sherrill et al. (2008), found that dbh, height, form and volume were highly related to seed source while taper was moderately related. They also found that bark thickness could play an important role in the variability of inside bark stem content prediction, as it seems to be controlled by seed source to a large extent. They found that outside bark dbh was fraught with measurement error when compared to the inside bark dbh. In summary: seed source, location and interactions affect variables of interest to forest inventory.

These findings are primarily restricted to the main portion of the loblolly pine range; however they also hold for the Oklahoma and Arkansas region. Lynch et al. (2010) reported that trends for the relationships between dbh and height, as well their combined relationship with inside-bark volume are different for North Carolina and Oklahoma-Arkansas seed sources planted in south-east Oklahoma. Lynch et al. (2010) also reported significantly different relationships as tree density varies. The variation in loblolly characteristics has had a large impact on seed source selection in Oklahoma. Extensive progeny testing in Oklahoma revealed that eastern seed sources often grow faster than Oklahoma-Arkansas seed sources (Wells and Lambeth 1983, Lambeth et al. 1984). This information led Weyerhaeuser Company, owner of substantial timberlands in south-east Oklahoma, to plant North Carolina sourced seed in suitable locations, which included most of their timberland in south-east Oklahoma (Lambeth et al. 1984). North Carolina seed stock has even been shown to outperform Oklahoma-Arkansas seed stock under severe drought conditions (Will et al. 2010). Because of the success of the eastern seed sources, it has become common practice to plant non-local seed sources on private and industrial timberlands in Oklahoma.

Since loblolly pine tree attributes (i.e., dbh, height, form, bark thickness, specific gravity and taper) and the relationships between them change due to site, stand and genetic characteristics it is necessary to account for this change when predictions involving these attributes are made. Given this variation development of accurate, unbiased prediction equations for loblolly pine stem weight, volume and taper are critically important because loblolly pine is the most important timber tree in the South (Baker and Langdon 1990), for areas both inside and outside its natural range (Schultz 1999). Because of the importance of these equations, considerable effort has been devoted to developing accurate stem content prediction models for loblolly pine. Frequently, modelers account for the variability in loblolly stem characteristics by parameterizing prediction equations using data limited to the population of interest (i.e., Burkhart 1977, Baldwin 1987, Lenhart et al. 1987, Newbold et al. 2001, Sherrill et al. 2011); although, seed source or genetics are not usually explicitly accounted for in the model or the modeling process. Several individual tree stem content prediction equations and systems of equations have also been published for large portions of the South and covering a wide range of site qualities and stand conditions (i.e., Van Deusen et al. 1981, Amateis and Burkhart 1987a, Tasissa et al. 1997, Bullock and Burkhart 2003). However, to our knowledge, only one prediction system has been published that includes data from Oklahoma loblolly pine: Clark et al. (1991), which was updated by Souter (1999), Souter (2001a) and Souter (2001b). The Clark et al. (1991) system is rather complicated when compared to other prediction systems; additionally, the green density prediction portion of this system (Souter 2001b) was not developed using any data from Oklahoma. Only taper and volume data from natural stands was incorporated into the system (Clark et al. 1991, Souter 1999) for the region that applies to south-east Oklahoma.

Although only a small portion of the loblolly pine range is found in Oklahoma, industrial forestry organizations and private landowners have planted loblolly pine extensively in the state, both within and outside the natural range. These loblolly pine plantations represent a significant contribution to the forestry sector of the Oklahoma economy. Oklahoma's forest industry has an annual contribution of $2.95 billion and directly employs over 6,770 people—whose annual wages and salaries amounted to $351.67 million in 2012 (Joshi 2017). Loblolly pine plantations are an important part of the Oklahoma economy and forestry sector and their characteristics could differ importantly from South-wide loblolly populations. Due to lack of published loblolly pine stem content prediction systems based on data from Oklahoma plantations, such systems should be developed. This need, while motivated by the literature and the economic importance of the Oklahoma forestry sector, has been reported by practicing foresters in south-east Oklahoma and felt by the land managers and owners who they work for (John Paul McTague, Manager of Growth and Yield Research, Rayonier Inc., pers. comm., May 2015).

The purpose of this research is to provide a system of individual tree, bole attribute prediction equations for Oklahoma loblolly pine that satisfies the needs of plantation owners and managers. The prediction system we developed includes accurate equations to predict merchantable bole green weight, merchantable volume and taper of planted loblolly pine trees, primarily within the Kiamichi region of the Ouachita Mountains in Oklahoma. I utilized very recent and versatile methods to increase model utility and reflect advances in forest biometrics. I evaluated several additional variables for inclusion into the model. Comparisons of Oklahoma data to existing South-wide loblolly pine stem content and taper prediction systems are presented to evaluate differences between characteristics of populations in the far north-west corner and main portion of its natural range. My hypothesis was that Oklahoma loblolly pine would have some characteristics that were different than those present in the main portion of the range, particularly with respect to the relationship between stem content and typical predictor variables including dbh, height and merchantability limits.

CHAPTER II

METHODS

Data

The data for this project were collected from 22 stands belonging to Rayonier, Inc. across a broad range of ages and site indices (Table 1), located in the Kiamichi region of the Ouachita Mountains in Oklahoma (Figures 1-3). This region is particularly poor for loblolly pine growth, and so minimal management is done. However, typically stands in this region are ripped before planting; weed control, fertilization and release are all done minimally, but as needed; and stands are thinned one or two times over their life. We collected data from trees and stands whose characteristics (tree size, tree crown position, stand site index, stand age) spanned the ranges typical for managed industrial loblolly pine plantations in the Ouachita mountain region of Oklahoma (variables shown in Table 2, the region is shown in Figures 2 and 3). I obtained an approximately equal number of stands within reasonable categories of age (3 year increments) and site index (10 ft increments), given the range of available stands. Within each stand a sample sample of several trees that was representative of the stand (based on dbh and crown position) was selected for felling and measurement.

A horizontal point sample was performed by selecting trees with an angle gauge, using each sample tree as the sample point. I recorded the dbh of all trees that were within critical distance. The number of points per stand was variable and depended on the number of trees sampled in each stand. The basal area factor was also varied so that between 10 and 20 trees were selected at each point (this was done to reduce workload while avoiding bias). A summary of stand (management unit) and plot level variables is presented in Table 1.

Table 1. Summary of stand level variables for the 22 sampled stands that were calculated from Rayonier Inc. records together with quantities computed from data collected in the field. Age is stand age (years), SI is site index (dominant height in feet at base age 25 years), TPA is trees per acre and BA is basal area in ft^2 per acre.

Variable	Min	Max	Mean
Rayonier's Stand Variables			
Age	12	30	19.77
SI	54	77	65.59
TPA	126	678	334.1
Calculated Stand Variables			
TPA	94.1	662.4	293.6
BA	65	183.33	110.52
Calculated Plot Variables			
TPA	37.08	1044.03	310.92
BA	40	220	113.6

After selecting sample trees, diameters were measured in inches to the nearest 0.1 in. on standing trees at 0.5, 1, 2.5 and 4.5 ft from the ground line, the measure at 4.5 ft being dbh (*D*). We measured the diameters of felled sample trees at 0.5, 4.5, and 8 ft from the ground, and every 4 ft up the stem thereafter, to the tip of the the tree. In addition Pressler's Diameter (Belyea 1931 p. 96), Hossfeld's Height (Belyea 1931 p. 96-97, Ducey and Williams 2011) and diameter at the base of the live crown (stem diameter at the base of the lowest live branch) were measured. Section volumes were calculated using all paired diameter and height measures (including Pressler's Diameter and Hossfeld's Height) assuming a neiloid for the shape of the first segment, and a parabolic shape of subsequent segments, using Smalian's formula (Avery and Burkhart 2015, p. 101), until the last segment was reached. A conic shape was assumed for the portion of the tree from its tip to the top of the last full 4-foot segment.

Trees were sectioned at 0.5, 4.5, and 8 ft from the ground and at every 4 ft thereafter, until the tip of the tree was reached. Green weight of these bolts was directly measured in the field. Table 2 presents the ranges, means and standard deviations of individual tree variables for the sampled trees. The number of bolts for green weight, inside bark volume and outside bark volume differ.

Table 2. Descriptive statistics for the sampled trees where N is the number of bolts, GW is green weight (lb), D is dbh (in.), H is total height (ft), d is stem diameter (in.) outside bark and inside bark, respectively (as denoted by the context).

	Variable	min	max	mean	std. dev
Green Weight N = 1995	Bolt GW (lb)	<1	604	65	72
	Bole GW (lb)	20	3663	984	799
	D (in.)	3	21	11	4
	H (ft)	21	80	53	13
	d (in.)	0	20	6	4
Outside Bark N = 2577	Bolt Vol. (ft^3)	<1	7	1	1
	Bole Vol. (ft^3)	<1	63	18	14
	D (in.)	3	21	10	4
	H (ft)	21	80	52	13
	d (in.)	0	23	7	4
Inside Bark N = 2554	Bolt Vol (ft^3)	<1	6	1	1
	Bole Vol. (ft^3)	<1	53	15	12
	D (in.)	0	21	6	4
	H (ft)	3	21	10	4
	d	21	80	52	13

The geographic location of stands is on the north-west edge, but outside the range of, loblolly pine, illustrated in Figures 1-3.

Figure 1. Stands where trees were destructively sampled in south-east Oklahoma and the entire loblolly pine range.

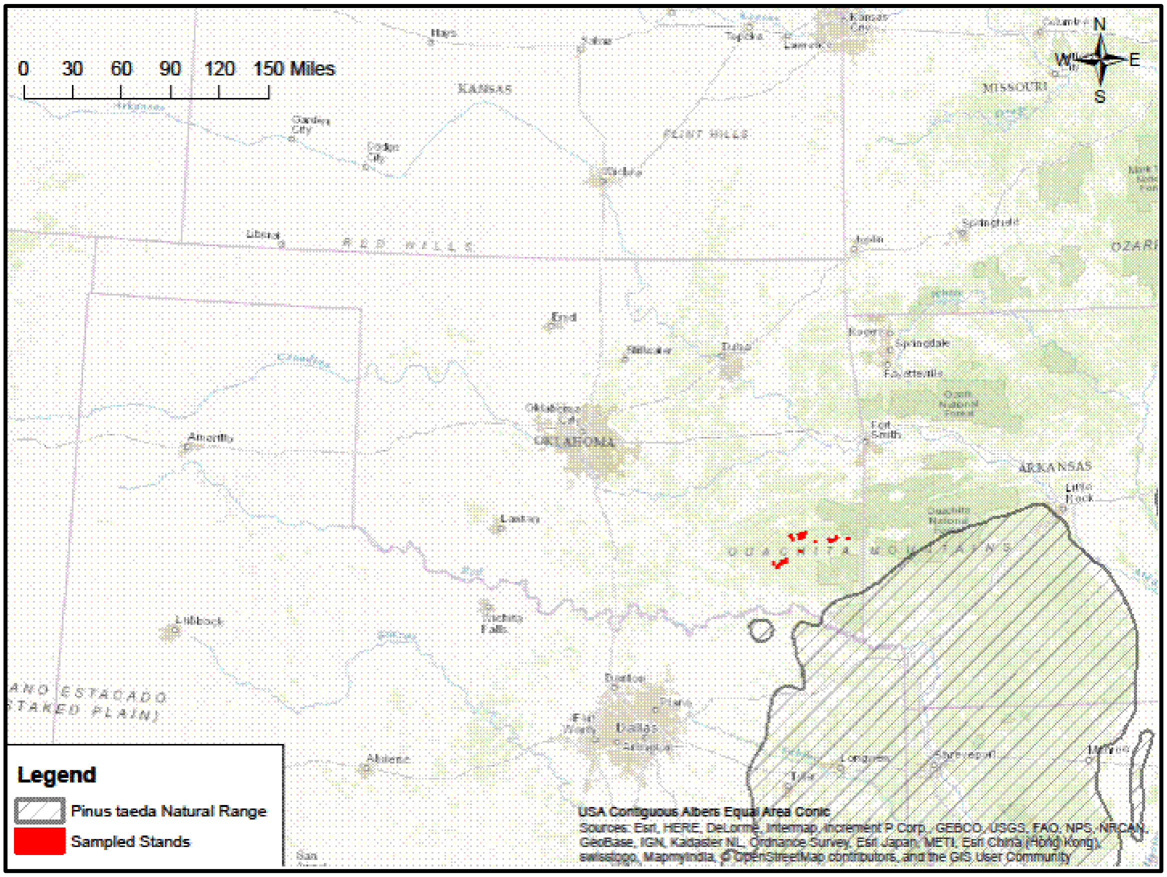

Figure 2. Sample sites in south-east Oklahoma with all of Oklahoma and a portion of the loblolly pine range.

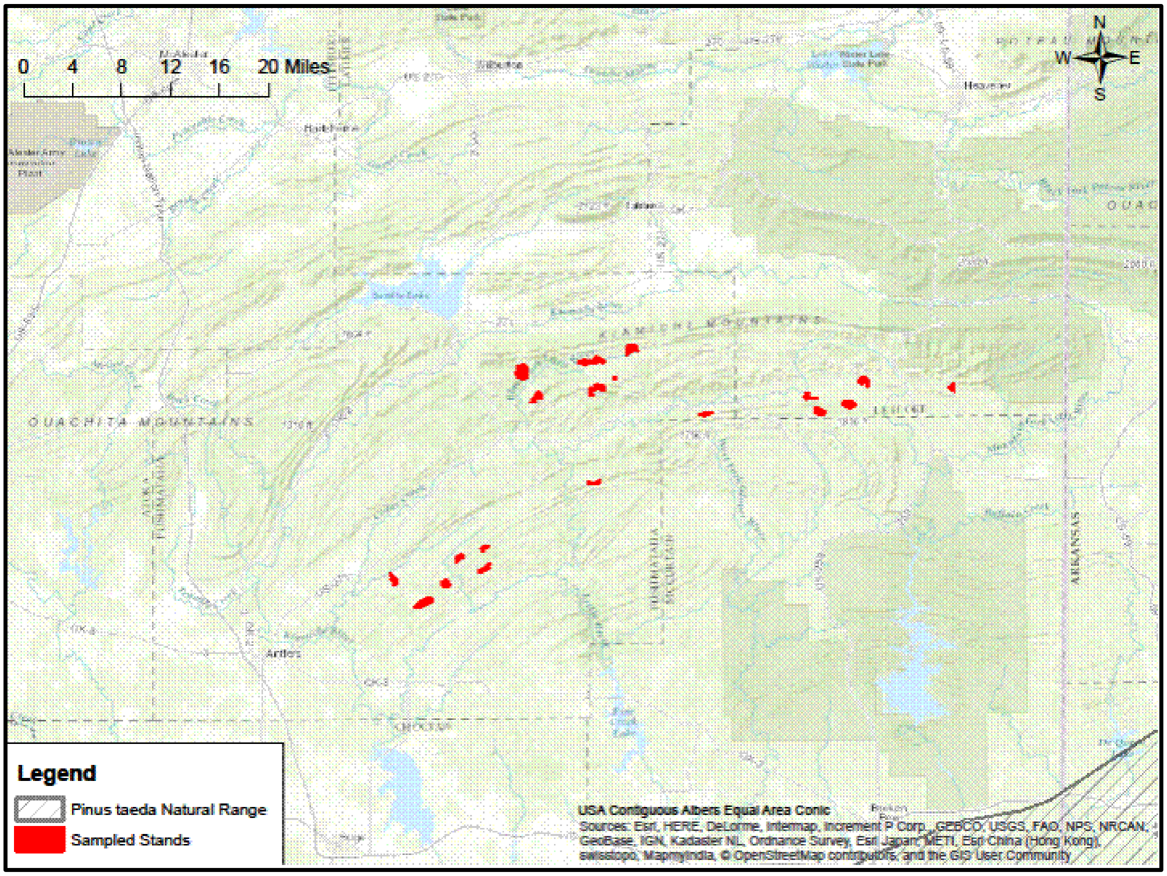

Figure 3. Close up view of sample sites showing Oklahoma counties, nearby towns and a portion of the range of loblolly pine (partially hidden under map citations).

Regional Comparisons

The data collected in Oklahoma were compared to predictions from several loblolly stem content models (Van Deusen et al. 1981, Pienaar et al. 1987, Clark et al. 1991, Souter 1999) commonly used for the main portion of the loblolly pine range. The results of the comparisons were used to evaluate the possible need of new models for the Oklahoma region of the Kiamichi Mountains. Predictions of total green weight, total outside bark volume, total inside bark volume as well as outside and inside bark diameters at 17.3 ft were generated from several models using Oklahoma data. The predicted values were then compared to the true values using paired t-tests. Additionally, I plotted mean bias of predicted weights, volumes and diameters at points of relative diameter or height to show how the models performed in a way that would be consistent with how they would be used. The application of normal fit statistics (mean bias) to portions of the data is sometimes referred to as “lack of fit statistics”, and is recommended for a more thorough model evaluation than fit statistics of the entire data set alone (Kozak and Kozak 2003). The combination of these two methods allows for both statistical evaluation of possible differences as well as the exploration of why those differences may be occurring, while avoiding complex testing procedures that are necessary for data that have a complex error structure. Stem measurements in the data used here are correlated within sample trees and are heteroskedastic.

Model Fitting, Selection and Evaluation

Previous approaches of estimating merchantable bole content (both green weight and volume) have included integration of tree form functions, often coupled with density functions (Parresol and Thomas 1989). Another method to estimate merchantable bole content is to multiply predicted total stem content by the predicted ratio of merchantable-to-total stem content up to a merchantability limit based on upper-stem diameter or height, (e.g., Honer 1964, Burkhart 1977). Either method can result in dimensionally compatible (henceforth "compatible" when referred to in the forestry context) systems for estimating volume and taper—given compatible equation forms. While the idea of dimensional compatibility has been in existence for some time in the fields of physics and engineering the definition and derivation of compatible stem taper and content prediction equations are more recent and attributable to Demaerschalk (1972 and 1973) and Clutter (1980). I selected the approach of compatible volume and taper equations with taper functions derived from merchantable volume equations because I was primarily interested in merchantable tree content and desired to utilize relatively simple equation forms that could be used for both green weight and volume.

There are several different methods of for parameterizing total content and ratio systems. Tree content ratios to merchantable heights or diameters can be estimated independently of total tree content equations (e.g., Bullock and Burkhart 2003); simultaneously with total tree content and taper models using nonlinear seemingly unrelated regression methods (Jorden et al. 2005); or simultaneously by fitting the product of total volume and ratio equations to cumulative tree content data. This final method has been used by several mensurationists, most recently in Zhao and Kane (2017).

The third method is advantageous as it accounts for correlation between the merchantable and total stem content equations without complex statistical methods. Total stem content is simply viewed as merchantable stem content with upper stem diameter equal to zero or merchantable height equal to total height. One possible disadvantage of this approach is that if several equations are fitted to the same data, as happens when both height and diameter ratio based content equations are fitted, multiple total tree content equations can result for the same individual tree dimensions (i.e., two total green weight equations and two total volume equations). I chose to use the method of simultaneously fitting the total and merchantable ratio equations to cumulative tree content because of its simplicity and compliance with least squares assumptions.

After selecting a total tree content equation and several merchantable ratio equations for comparison, I used non-linear least squares methods to estimate parameter values of the models. Methods were implemented using the R packages minpack.lm (Elzhov et. al 2016) and nlme (Pinheiro et. al 2017). When estimating the parameters of models with non-linear least squares, weighting should be implemented if heteroskedasticity is present. Fortunately this is simple to accomplish through options in R packages. All models were initially fitted with no weighting, as a baseline, and subsequently with one or two different weighting schemes (this differed from model to model due to convergence issues). Two variance models were employed in this exploration, both of which utilize a scale parameter multiplied by a function describing the structure of the variance. One of these functions depends only on the fitted value raised to a power while the other also incorporates an additive constant. The latter should perform better if, at a point, the variability reaches a lower asymptote when estimated from the mean predicted value alone.

Akaike's Information Criterion (Akaike 1974) and the Bayesian Information Criterion (Schwarz 1978) were the criteria used to select the best combination of prediction model and weighting scheme. Both were implemented in the base R program (R Core Team 2017). The best model estimates were compared using the AIC, BIC, the maximum likelihood estimator of bias (MB), the maximum likelihood estimator of the square root of the mean square error (RMSE) and the pseudo R^2. Because nonlinear models are used the true R^2 is not defined; however, calculation of the proportion of total variation explained by the nonlinear model can still be informative. These procedures were followed for tree content equations based on both merchantable stem diameter ratios (ratios of merchantable top diameter limits to dbh) and merchantable height ratios (ratios of merchantable height to total tree height). Models fitted were evaluated for conformity to the assumptions of constant error variance, normality and correct model specification using several different types of plots of the model residuals. Standardized residuals were employed in plotting as these give a more accurate picture of model performance. Standardized residuals are residuals that have been divided by the model standard error at the particular point where they occur. Fitted values, commonly called predicted values referring to the values of the dependent variable predicted by the model for the data used to fit the model, were also used in model evaluation. Plots used in model evaluation included: residuals versus the fitted values and residuals versus the independent variables.

No model validation using data splitting techniques or independent datasets was performed. Kozak and Kozak (2003) note that any form of cross validation results in a loss of information available to the modeling process. The use of new data for model validation is a better (although expensive option); however even these comparisons do not add information to the process (Kozak and Kozak 2003). Conventional validation techniques are only capable of statistically disproving that the data used to fit the model and those used to validate the model come from the same population (rather than proving that they do come from the same population), which is not a useful conclusion within the context of model selection.

After parameterizing the total content equations I was able use the compatible taper equations (derived from the height ratio based merchantable volume equations) to complete our system of merchantable stem content and taper equations. I utilized the parameters available from the content equations in these taper equations (as opposed to fitting the taper equations independently or the system simultaneously). The performances of the compatible taper equations were also compared using MB, RMSE and the pseudo R^2.

Merchantable content as well as taper model performance was evaluated at points of relative diameters (*d/D*) or heights (*h/H*) along the stem using lack of fit statistics corresponding to MB and RMSE. The incorporation of lack of fit statistics into graphical form enables a more intuitive evaluation of model performance. Finally, I created plots of residuals over plot and stand level variables to visually assess whether stand and/or plot variables, not included in the fitted models, could be used to improve the models.

While the order of the methods follows the logical order of investigation, the order of the results is slightly changed to facilitate the presentation of information. In the results the modeling section is presented first, followed by the regional comparisons, so that the fit statistics, etc. of the selected models could be compared to models for other regions. Model evaluation is presented last. The discussion follows the order of the methods section.

All modeling was carried out in R (R Core Team 2017) using the RStudio integrated development environment (RStudio Team 2016). Several previously unmentioned packages were also used: cowplot (Wilke 2016), dplr (Wickham and Francois 2016), MASS (Venables and Ripley 2002), ggplot2 (Wickham 2009), xlsx (Dragulescu 2014) and zoo (Grothendieck and Zeileis 2005).

Models Evaluated

I chose to use the Schumacher and Hall total content prediction equation (Schumacher and Hall 1933) due to its conformance with allometric theory, satisfactory performance in many previous studies (e.g., Zhao and Kane 2017) and its excellent performance on our data.

The Schumacher and Hall equation:

$$\mathbf{ln}(\boldsymbol{C_t}) = \mathbf{ln}(\boldsymbol{a_0}) + \boldsymbol{a_1}\mathbf{ln}(\boldsymbol{D}) + \boldsymbol{a_2}\mathbf{ln}(\boldsymbol{H})$$

Where: C_t is total tree content (originally volume),

D is dbh (in.),

H is total height (ft)

a_0, a_1 and a_2 are parameters to be estimated

Rather than using a logarithmic transformation to fit the equation using linear regression, as originally proposed by Schumacher and Hall (1933), I chose to fit the equation using non-linear regression methods (Moser and Beers 1969). This method of fitting Schumacher and Hall's equation eliminates transformation bias and reduces the standard error of the parameter estimates, compared to the logarithmic fit. Additionally I chose to compare several different merchantable tree content and ratio equations (Table 3).

- The Burkhart diameter ratio (R_{Bd}) (Burkhart 1977),
- The modified Burkhart height ratio (R_{Bh}) (Cao et al. 1980, Cao and Burkhart 1980),
- The exponential diameter ratio equation (R_{ed}) (Van Deusen et al. 1981, Parresol et al. 1987, Baldwin 1987, Tasissa 1997),

- Zhao & Kane ratio equation 1 ($R_{Z\&K1}$) (Zhao and Kane 2017);
- Zhao & Kane ratio equation 2 ($R_{Z\&K2}$) (Zhao and Kane 2017)
- The PMRC merchantable content equation which is presented in different forms for green weight and volume (C_{gd} or C_{vd}, respectively) (Pienaar et al. 1987, Harrison and Borders 1996).

Table 3. Merchantable content equations selected for evaluation, along with the equation number and a name that will be used to refer to the equation throughout the text (often as an abbreviation). The left hand side of the equation contains the previously mentioned and cited designator for the ratio and/or content equation employed.

Equation Name	Equ. #	Equation
Burkhart Merchantable Diameter	**1**	$C_t R_{Bd} = a_0 D^{a_1} H^{a_2} \left(1 + b_0 \frac{d^{b_1}}{D^{b_2}}\right) + \varepsilon$
Burkhart Merchantable Height	**2**	$C_t R_{Bh} = a_0 D^{a_1} H^{a_2} \left(1 + b_0 \frac{(H-h)^{b_1}}{H^{b_2}}\right) + \varepsilon$
Exponential Merchantable Diameter	**3**	$C_t R_{ed} = a_0 D^{a_1} H^{a_2} e^{\left(b_0 \frac{d^{b_1}}{D^{b_2}}\right)} + \varepsilon$
Zhao & Kane 1	**4**	$C_t R_{Z\&K1} = a_0 D^{a_1} H^{a_2} \left[1 - \left(1 - \frac{h}{H}\right)^{\alpha}\right]^{\left(1 - \theta e^{\left(-e^{(\phi D^{a_1} H^{a_2})}\right)}\right)} + \varepsilon$
Zhao & Kane 2	**5**	$C_t R_{Z\&K2} = a_0 D^{a_1} H^{a_2} - c_0 D^{c_1} H^{c_2} \left(1 - \frac{h}{H}\right)^{\alpha} + \varepsilon$
PMRC Merchantable Diameter	**6 & 7**	$\begin{cases} C_{gd} = a_0 D^{a_1} H^{a_2} - a_3 (H - 4.5) \left(\frac{d^{b_4}}{D^{b_5}}\right) + \varepsilon \\ \quad or \\ C_{vd} = a_0 D^{a_1} H^{a_2} - a_3 (H - 4.5) \left(\frac{d^{b_4}}{D^{b_4 - 2}}\right) + \varepsilon \end{cases}$

Where: d is merchantable diameter,

h is merchantable height,

$a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2,$ and α are parameters to be estimated,

e is the base of the natural logarithm

ε is an error term with mean zero, and all other variables are as previously defined

The weighting functions investigated are shown in Table 4.

Table 4. Variance estimation functions selected for comparison in this study along with the equation number and the name; note that the name has an abbreviated form in parentheses that will be used throughout the text.

Equation Name	Equation #	Equation
Power of the Mean (POM)	**8**	$\mathbf{Var(\varepsilon_i) = \sigma^2 \hat{y}_i^{2\delta}}$
Power of the Mean Plus a Constant (POM + C)	**9**	$\mathbf{Var(\varepsilon_i) = \sigma^2 (\hat{y}_i^{2\delta} + C)}$

(Pinheiro and Bates 2007, p. 210-213)

Where: ε_i is the model error for observation *i*

σ^2 is the estimated model variance scale parameter

$\hat{\mathbf{y}}_\mathbf{i}$ is the model's estimate for observation *i*

$\boldsymbol{\delta}$ is a parameter to be estimated

C is a constant to be estimated and

i is a vector from 1 to *n* (the total sample size)

I opted to use the taper equations derived by Lynch et al. (2017) from merchantable-to-total volume ratio equations based on upper-stem merchantable height limits. This method is similar to the method presented by Clutter (1980), which derived compatible taper equations from merchantable-to-total volume ratio equations based on upper-stem diameter limits by using a separable differential equation. The taper equations of Lynch et al. (2017) are derived from Zhao and Kane's models 1 and 2 (equations 4 and 5), presented in Table 5.

Table 5. Compatible taper equations selected for comparison in this study, derived in Lynch et al. (2017) from equations 2, 4 and 5 respectively, for volume inside and outside bark.

Equation Name	Equ. #	Equation
Lynch et al. Taper 1	10	$d(h) = \sqrt{\frac{\alpha}{kH} a_0 D^{a_1} H^{a_2} \left(1 - \frac{h}{H}\right)^{\alpha-1} \left(1 - \theta e^{\left(-e^{\left(\phi D^{a_1} H^{a_2}\right)}\right)}\right) \left[1 - \left(1 - \frac{h}{H}\right)^{\alpha}\right]^{\left(-\theta e^{\left(-e^{\left(\phi D^{a_1} H^{a_2}\right)}\right)}\right)}} + \varepsilon$
Lynch et al. Taper 2	11	$d(h) = \sqrt{\frac{\alpha}{kH} c_0 D^{c_1} H^{c_2} \left(1 - \frac{h}{H}\right)^{\alpha-1}} + \varepsilon$
Burk. Merch. Ht Comp. Taper	12	$d(h) = \sqrt{-\frac{a_0}{k} D^{a_1} H^{a_2} b_0 b_1 \left(\frac{(H-h)^{b_1-1}}{H^{b_2}}\right)} + \varepsilon$

Where: *d(h)* is diameter as a function of height and all other variables are as previously defined

CHAPTER III

RESULTS

Model Selection

The selection of the merchantable content equations and variance equations preceded as follows. Each total content equation was fit using two or three different weighting schemes; I selected the best combination for each content equation to use in comparisons between equations going forward. Table 6 contains comparisons of the AIC and BIC values from all equations that were fitted. All of the merchantable content and model variance prediction systems are defined in Table 6. The best combination for a given merchantable content equation is highlighted in yellow.

Table 6. AIC and BIC of paired merchantable content (table 3) and variance estimation equations (table 4). Paired equations are relabeled and renumbered for easier reference later in the text. Degrees of freedom for the models are shown, where degrees of freedom differed for volume and green weight models green weight degrees of freedom are in parentheses. Model combination selected for each merchantable content equation is highlighted in yellow. Model selection was based on lowest AIC or BIC value.

Model	Equ. #	df	Green Weight		Outside Bark Vol.		Inside Bark Vol.	
			AIC	BIC	AIC	BIC	AIC	BIC
Burk. Merch. Dia. (No Var)	13	7	24997.14	25036.33	11708.44	11749.42	11277.03	11317.95
Burk. Merch. Dia. (Var = POM)	14	8	23903.61	23948.40	No Convergence		No Convergence	
Burk. Merch. Dia. (Var = C + POM)	15	9	23600.51	23650.90	10184.35	10237.03	9358.36	9410.97
PMRC Merch. Dia. (No Var)	16	7 (6)	24996.96	25036.15	11741.45	11776.58	11331.95	11367.02
PMRC Merch. Dia. (Var = C + POM)	17	9 (8)	23592.93	23643.31	10190.59	10237.43	9394.40	9441.16
Exp. Merch. Dia. (No Var)	18	7	24966.27	25005.46	10456.92	10497.90	10254.37	10295.28
Exp. Merch. Dia. (Var =POM)	19	8	23569.70	23614.48	8638.22	8685.05	8018.34	8065.10
Exp. Merch. Dia. (Var = C + POM)	20	9	23484.96	23535.35	8323.06	8375.75	7623.96	7676.57
Burk. Merch Ht. (No Var)	21	7	23586.59	23625.78	7569.70	7610.68	8075.87	8116.79
Burk. Merch Ht. (Var =POM)	22	8	21267.06	21311.85	3509.28	3556.11	3404.44	3451.21
Burk. Merch Ht. (Var = C + POM)	23	9	21268.94	21319.33	3511.28	3563.97	3406.44	3459.05
Zhao & Kane 1 (No Var)	24	7	23588.78	23627.96	7585.70	7626.68	8094.35	8135.27
Zhao & Kane 1 (Var = POM)	25	8	21269.79	21314.58	3938.32	3985.15	3849.04	3895.80
Zhao & Kane 2 (No Var)	26	8	23580.75	23625.54	7553.11	7599.95	8061.10	8107.86
Zhao & Kane 2 (Var = POM)	27	9	21265.18	21315.57	3499.01	3551.70	3391.73	3444.33

The best combinations of merchantable content and variance equations for each merchantable content model were ranked for comparison (Table 7). Ranks are based on the models AIC and BIC values, compared to the AIC and BIC values of the other models in the comparison; i.e., the best model – the model with the lowest AIC and BIC - was assigned a 1, the next best a 2, etc. The equations were ordered from best to worst. All of the merchantable height based equations were better than the diameter based equations.

Table 7. Comparison of best performing combined merchantable content and variance models. A simple ranking system based on AIC and BIC (which largely agreed with one another) is used to make the comparison. Smaller ranks correspond to smaller AIC or BIC values and are better.

Merch. Limit	Model Name	Equ. #	Green Weight Ranks		Outside Bark Vol. Ranks		Inside Bark Vol. Ranks	
			AIC	BIC	AIC	BIC	AIC	BIC
Ht.	Zhao & Kane 2 (Var = POM)	27	1	3	1	1	1	1
	Burk. Merch Ht. (Var = POM)	22	2	1	2	2	2	2
	Zhao & Kane 1 (Var = POM)	25	3	2	3	3	3	3
Dia.	Exp. Merch. Dia. (Var = C + POM)	20	4	4	4	4	4	4
	PMRC Merch. Dia. (Var = C + POM)	17	5	5	6	6	6	6
	Burk. Merch. Dia. (Var = C + POM)	15	6	6	5	5	5	5

The AIC and BIC values indicate the overall best model was Zhao and Kane 2, however the differences were slight in many cases between all height ratio based merchantable content equations.

Fit statistics for the models ranked in Table 7 are compared in Table 8.

Table 8. Comparison of fit statistics for the best performing combined merchantable content and variance models, the best values are highlighted in yellow.

Merch. Limit	Model Name	Equ #	Green Weight			Outside Bark Volume			Inside Bark Volume		
			MB	RMSE	R	MB	RMSE	R	MB	RMSE	R
Ht.	Zhou & Kane 2 (Var =POM	27	1.7949	89.752	0.98250	0.0084	1.116	0.99156	0.0088	1.226	0.98601
	Burk. Merch Ht. (Var =POM)	22	1.6246	89.821	0.98247	0.0052	1.121	0.99150	0.0052	1.232	0.98587
	Zhou & Kane 1 (Var =POM)	25	1.0484	89.997	0.98241	0.0129	1.080	0.99211	0.0202	1.225	0.98603
Dia.	Exp. Merch. Dia. (Var=C+POM)	20	0.4732	127.624	0.96462	0.0149	1.911	0.97528	0.0101	1.897	0.96652
	PMRC Merch. Dia. (var=C+POM)	17	1.5837	131.016	0.96271	0.1369	2.802	0.94683	0.1609	2.663	0.93399
	Burk. Merch. Dia. (Var=C+POM)	15	1.4714	130.888	0.96278	0.1521	2.750	0.94879	0.1503	2.591	0.93752

The fit statistics show that Zhao and Kane 2 (equation 27) could be best for green weight (based on RMSE and R^2) and that Zhao and Kane 1 (equation 25) could be best for volume (based on RMSE and R^2). Zhao and Kane 1 (equation 25) was best according to more criteria than any other equation; however, the statistics do not show a clear pattern concerning which equation is best overall. The best merchantable diameter based content equation appears to be the Exponential Merchantable Diameter equation (equation 20). Merchantable height based content equations are better than merchantable diameter based content equations in almost every case (excepting green weight bias).

Fit statistics for the compatible taper equations derived in Lynch et al. (2017) were calculated using the parameter values obtained from fits of merchantable volume equations (equations 22, 25, and 27) and from taper measurements from stump height to the tip of the tree (Table 9). These clearly indicate Lynch et al. Taper 2 (equation 11) as the best.

Table 9. Comparison of compatible taper equation using fit statistics, best values are highlighted in yellow.

Compatible Vol. Equ.	Equ. #	Outside Bark Taper			Inside Bark Taper		
		MB	RMSE	R^2	MB	RMSE	R^2
Lynch et al. Taper 2	11	0.06159	0.66875	0.97518	-0.02843	0.64372	0.97208
Lynch et al. Taper 1	10	-0.01150	0.76083	0.96788	-0.09341	0.77581	0.95945
Burk. Merch. Ht Comp. Taper	12	0.06081	0.66904	0.97516	-0.02916	0.64437	0.97203

Based on all of these results I selected the Exponential Merchantable Diameter equation (equation 20), Zhao and Kane 2 (equation 27) and Lynch et al. Taper 2 (equation 11) as the final models. The parameters estimated for these equations are presented in Tables 10 and 11 and the variance-covariance matrices for the fitted models are presented in Appendix 1 (Tables 14-19). For convenience, the fitted content and diameter prediction equations that we selected are presented (Table 12).

Table 10. Parameters and standard errors from the Exponential Merchantable Diameter equation, the selected merchantable diameter ratio based model, fitted to Oklahoma data.

Exponential Merchantable Diameter (Equ. 20) Parameters						
	Green Weight		Vol. Outside Bark		Vol. Inside Bark	
Coef.	Values	SE	Values	SE	Values	SE
a_0	0.1180070	0.0090008	0.001685	0.0001022	0.0010510	7.155E-05
a_1	2.0333030	0.0209004	1.946389	0.0165584	1.9700670	0.0186889
a_2	1.0056780	0.0245445	1.12641	0.0193942	1.1860110	0.0219383
b_0	-1.0245930	0.0920309	-0.983365	0.062645	-1.0683730	0.0707728
b_1	5.2418560	0.0984784	4.890275	0.0508136	4.8238820	0.0539147
b_2	5.0128580	0.1052186	4.682557	0.0580126	4.6345840	0.0622172
C	53.2836865	NA	1.6915623	NA	1.5103016	NA
δ	0.9053368	NA	0.8845528	NA	0.9493773	NA
σ^2	0.2598936	NA	0.1558108	NA	0.1476603	NA

Table 11. Parameters and standard errors from Zhao and Kane 2, the selected merchantable height ratio based model fitted to Oklahoma data.

Zhou and Kane 2 (Equ. 27) Parameters						
	Green Weight		Vol. Outside Bark		Vol. Inside Bark	
Coef.	Values	SE	Values	SE	Values	SE
a_0	0.1524174	0.0075012	0.0024734	6.558E-05	0.0014977	4.912E-05
a_1	1.9901138	0.0127603	1.9275643	0.006893	1.9444217	0.0084378
a_2	0.9754948	0.0172392	1.0429602	0.0092413	1.1157035	0.0113971
c_0	0.1206957	0.0097052	0.0025981	7.508E-05	0.0015688	5.558E-05
c_1	1.9708939	0.0200089	1.9225321	0.0074892	1.9382129	0.0090917
c_2	1.0424848	0.0270678	1.0372152	0.0100259	1.1113800	0.0122632
α	2.1772090	0.0287737	2.5097158	0.0136271	2.3531334	0.0155343
δ	0.8742187	NA	0.8373359	NA	0.8365720	NA
σ^2	0.2342671	NA	0.1006951	NA	0.1193303	NA

Table 12. Final content prediction equations fitted to data from south-east Oklahoma. The simultaneously fitted variance functions are not shown.

Equation Name, Predicted Variable	Prediction Equation
Exponential Merchantable Diameter, Merchantable Green Weight	$0.118007 * D^{2.033303} * H^{1.005678} * e^{\left(-1.024593 * \frac{d^{5.241856}}{D^{5.012858}}\right)}$
Exponential Merchantable Diameter, Outside Bark Volume	$0.001685 * D^{1.946389} * H^{1.12641} * e^{\left(-0.983365 * \frac{d^{4.890275}}{D^{4.682557}}\right)}$
Exponential Merchantable Diameter, Inside Bark Volume	$0.001051 * D^{1.970067} * H^{1.186011} * e^{\left(-1.068373 * \frac{d^{4.823882}}{D^{4.634584}}\right)}$
Zhao & Kane 2, Green Weight	$0.1524174 * D^{1.9901138} * H^{0.9754948} - 0.1206957 * D^{1.9708939} * H^{1.0424848} * \left(1 - \frac{h}{H}\right)^{2.177209}$
Zhao & Kane 2, Outside Bark Volume	$0.0024734 * D^{1.9275643} * H^{1.0429602} - 0.0025981 * D^{1.9225321} * H^{1.0372152} * \left(1 - \frac{h}{H}\right)^{2.5097158}$
Zhao & Kane 2, Inside Bark Volume	$0.0014977 * D^{1.9444217} * H^{1.1157035} - 0.0015688 * D^{1.9382129} * H^{1.11138} * \left(1 - \frac{h}{H}\right)^{2.3531334}$
Lynch et al. Taper 2, Outside Bark Diameter	$\sqrt{\frac{2.5097158}{\frac{\pi}{576} * H} * 0.0025981 * D^{1.9225321} * H^{1.0372152} * \left(1 - \frac{h}{H}\right)^{2.5097158-1}}$
Lynch et al. Taper 2, Inside Bark Diameter	$\sqrt{\frac{2.3531334}{\frac{\pi}{576} * H} * 0.0015688 * D^{1.9382129} * H^{1.11138} * \left(1 - \frac{h}{H}\right)^{2.3531334-1}}$

Next the comparisons of Oklahoma loblolly pine to populations from across the range are presented. This comparison is presented now because I desired to include plots showing the lack of fit statistics for our fitted models along with those models fitted to the more southeastern data.

Regional Comparisons

In order to assess the possible need for new prediction equations in Oklahoma tests of the difference between Oklahoma data and models from across the loblolly pine range were performed. The results of these tests are presented in Table 13 along with the RMSE of the model predictions. In order to more fully assess the differences between populations, mean bias at relative locations along the stem for the green weight and volume equations tested in Table 13 are presented in Figures 4-7; these figures show the best equations derived from the Oklahoma data for comparison. Similar graphs comparing the performances of other southern taper equations to the best taper equation fitted to the Oklahoma data are given in Figures 8 and 9. In these comparisons the four recommended equations from Bullock and Burkhart (2003) are used. They are the parameterized equations 1 (to estimate total green weight), 6 (to estimate green weight ratio to a merchantable height), 7 (to estimate green weight ratio to a merchantable diameter) and mixed-67 (the term applied to the implicit taper function resulting from equating the merchantable diameter and height ratio equations and solving for merchantable diameter), as named within that document. Additionally, volume and taper equations for the Piedmont of Georgia and Alabama, volume equations for the southeastern USA, a taper equation for the Forest Service Arkansas growing region (a region that covers the Ouachita Mountains) and a taper equation for most of the range of loblolly pine are also used to evaluate the differences in Oklahoma data.

Table 13. Comparison of south-east Oklahoma data with external models fitted to data collected across the southern USA.

Variable	Citation	Stand Type	Region	Mean Difference (Obs. - Pred.)	2-sided p-value of Difference	RMSE
Tot. GW (lb)	Bullock and Burkhart (2003)	Plantation	The South	-33.70534	0.0002	115.6339
Tot. GW (lb)	Pienaar et al. (1987)	Plantation	Upper Coastal Plain	-4.606913	0.5795	104.0562
Tot. OB Vol. (ft^3)	Van Deusen et al. (1981)	Plantation	The South-East	0.02678351	0.8347	1.605644
Tot. OB Vol. (ft^3)	Pienaar et al. (1987)	Plantation	Upper Coastal Plain	-0.3711246	0.0061	1.714943
Tot. IB Vol. (ft^3)	Van Deusen et al. (1981)	Plantation	The South-East	0.8296415	0.0000	2.068261
Tot. IB Vol. (ft^3)	Pienaar et al. (1987)	Plantation	Upper Coastal Plain	-2.80596	0.0000	6.50117
Dia. OB 17.3 ft (in.)	Updated SE-282	Plantation	The South	0.1010184	0.0030	0.4288319
Dia. OB 17.3 ft (in.)	Updated SE-282	Natural	Arkansas Area	0.4611592	0.0000	1.038294
Dia. OB 17.3 ft (in.)	Pienaar et al. (1987)	Plantation	Upper Coastal Plain	0.9102257	0.0000	1.220754
Dia. IB 17.3 ft (in.)	Updated SE-282	Plantation	The South	0.2728319	0.0000	0.5211773
Dia. IB 17.3 ft (in.)	Updated SE-282	Natural	Arkansas Area	0.9102257	0.0000	1.220754
Dia. IB 17.3 ft (in.)	Pienaar et al. (1987)	Plantation	Upper Coastal Plain	-0.3420954	0.0000	0.6204798

Where: GW is Green Weight,
OB is Outside Bark and
IB is Inside Bark

RMSE is the sum of the squared errors divided by sample size. It does not refer to the standard error of the mean bias; this number is recommended for evaluating models for inventory by Avery and Burkhart (2015) p. 176.

The tests indicate statistically significant differences in every case except when the Oklahoma data was compared with predicted total green weight from the Pienaar et al. (1987) and outside bark volume predicted from the Van Deusen et al. (1981) system.

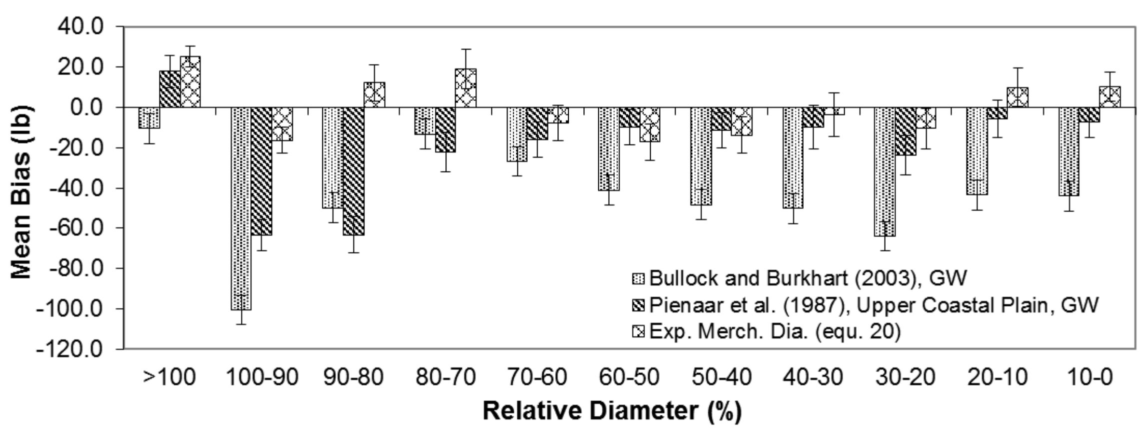

Figure 4. Mean biases for the Bullock and Burkhart (2003) equation 6, Pienaar et al. (1987) and the Oklahoma fit of equation 20 green weight predictions to a merchantable outside bark

diameter, where relative diameter is merchantable diameter as a percent of dbh (error bars represent ± one standard error from the means).

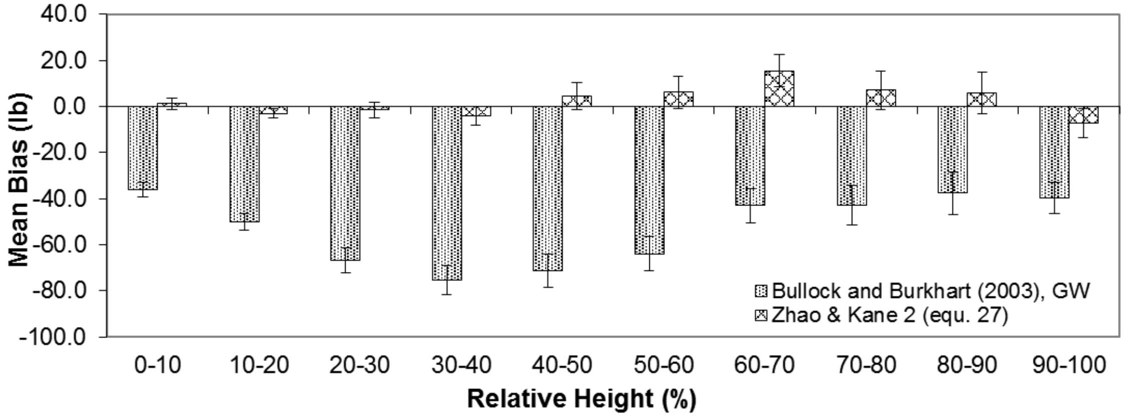

Figure 5. Mean biases for the Bullock and Burkhart (2003) equation 7 and the Oklahoma fit of equation 27 green weight predictions to a merchantable height, where relative height is merchantable height as a percent of total height (error bars represent ± one standard error from the means).

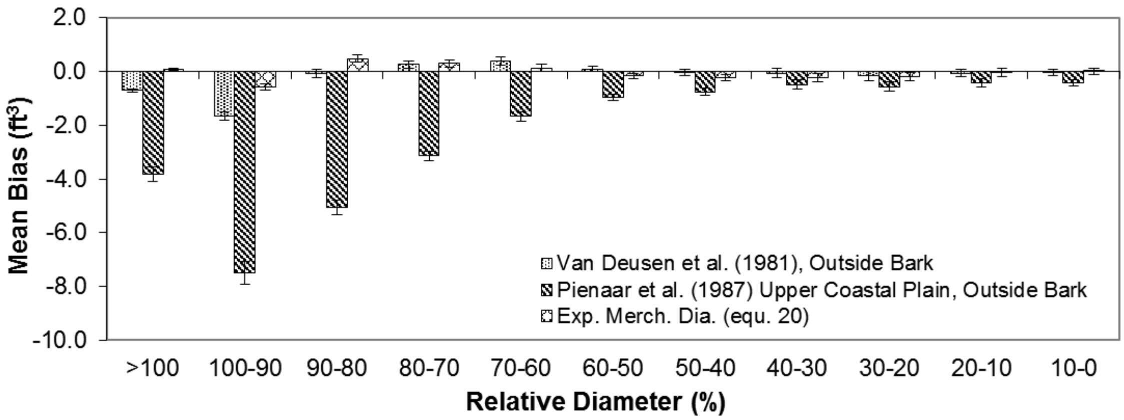

Figure 6. Mean biases for the Van Deusen (1981), Pienaar et al. (1987) and the Oklahoma fit of equation 20 outside bark volume predictions to a merchantable outside bark diameter, where relative diameter is merchantable diameter as a percent of dbh (error bars represent ± one standard error from the means).

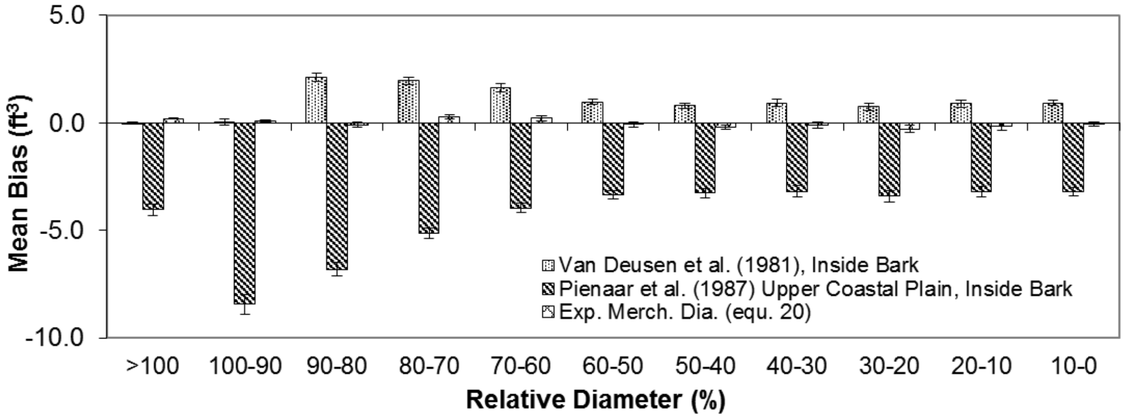

Figure 7. Mean biases for the Van Deusen (1981), Pienaar et al. (1987) and the Oklahoma fit of equation 20 inside bark volume predictions to a merchantable inside bark diameter, where relative diameter is merchantable diameter as a percent of dbh (error bars represent ± one standard error from the means).

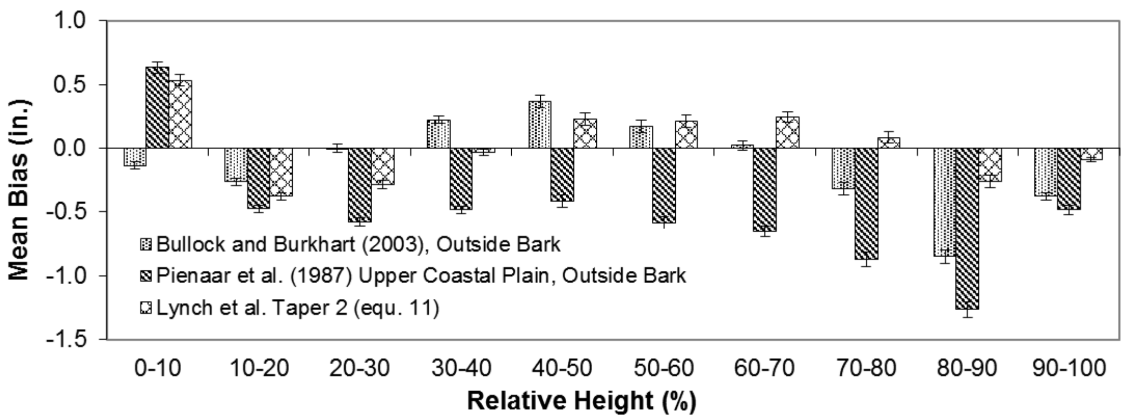

Figure 8. Mean biases for the Bullock and Burkhart (2003) equation mixed 67, Pienaar et al. (1987) taper equation and the Oklahoma fit of Lynch et al. Taper 2 outside bark diameter predictions to a merchantable height, where relative height is merchantable height as a percent of total height (error bars represent ± one standard error from the means).

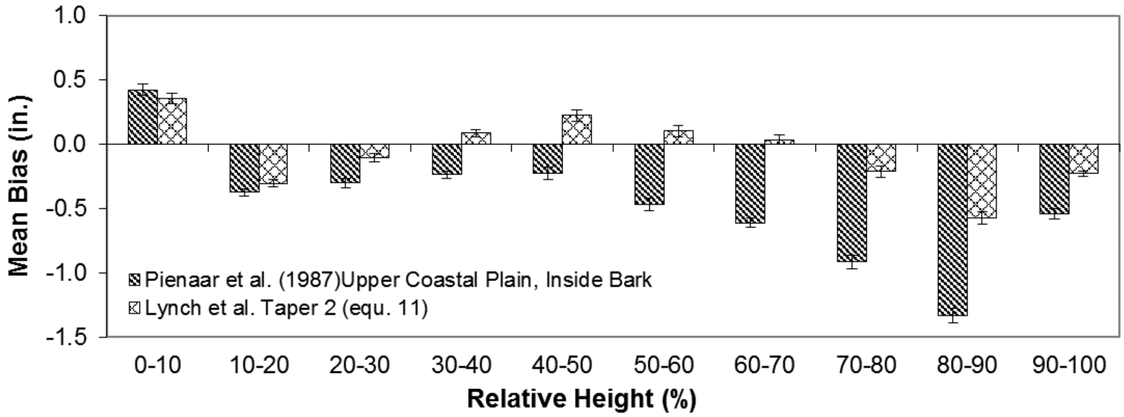

Figure 9. Mean biases for the Pienaar et al. (1987) and the Oklahoma fit of Lynch et al. Taper 2 inside bark diameter predictions to a merchantable height, where relative height is merchantable height as a percent of total height (error bars represent ± one standard error from the means).

The bias plots show that for the most part equations with parameters generated from loblolly pines within the range over-predict green weight and volume for southeastern Oklahoma. Although, the Pienaar et al. (1987) green weight predictions compares fairly well with the equation fitted to the Oklahoma data on the upper portion of the stem and for total stem weight. Bullock and Burkhart (2003) over-predicted green weight for both merchantable diameter and height based equations. Pienaar et al. (1987) consistently over-predicted green weight; though for the entire stem it was not statistically significant. Van Deusen et al. (1981) predicts outside bark volume well, while Pienaar et al. (1987) overpredicts outside bark volume (however, predictions become better towards the tip of the stem). Van Deusen et al. (1981) underpredicts inside bark volume, while Pienaar et al. (1987) shows the same pattern for this variable as it did for outside bark volume. The Bullock and Burkhart (2003) taper system overpredicts the taper of southeastern Oklahoma trees near the tip of the tree while the Pienaar et al. (1987) system underpredicts for the lowest portion of the bole and over-predicts for the rest of the stem. The Pienaar et al. (1987) system consistently over-predicts inside bark diameter for the southeastern Oklahoma trees.

Model Evaluation

Model evaluation and validation was performed after model selection and parameterization was complete. The residuals of the equations in Table 6 were visually evaluated for patterns using plots of residuals versus fitted values and plots of residuals versus the independent variables. The models that did not include equations that model variance will obviously show patterns indicating heteroskedasticity and so were not included. Plots of residual versus predicted values for the final models (equations 20 for content to a merchantable diameter, 27 for content to a merchantable height) are presented as Figures 10-15 in the text, while plots for all fitted models are contained in Appendix 2 (Figures 34-58). Plots such as these are an important part of the model building process because they are used to assess the conformity of the models to the assumptions of nonlinear least squares regression.

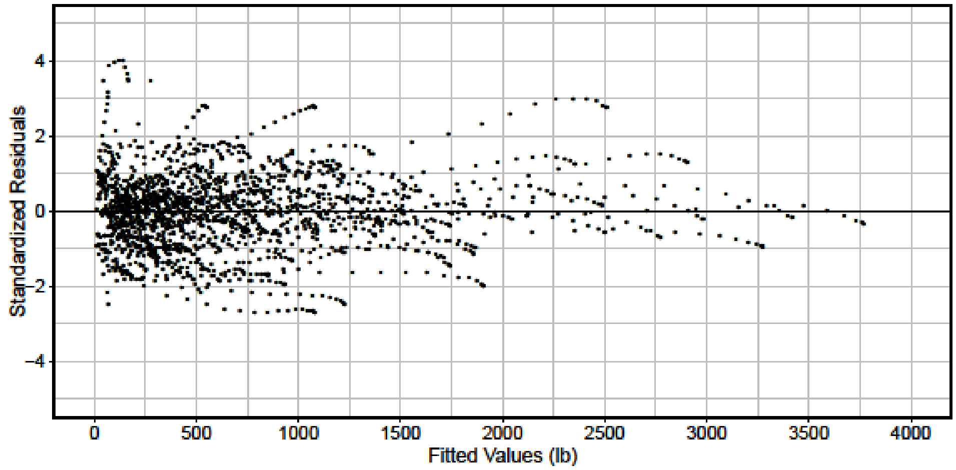

Figure 10. Residuals for Zhao and Kane 2 merchantable height equation fitted to cumulative green weight versus the fitted values associated with the data used to fit the model.

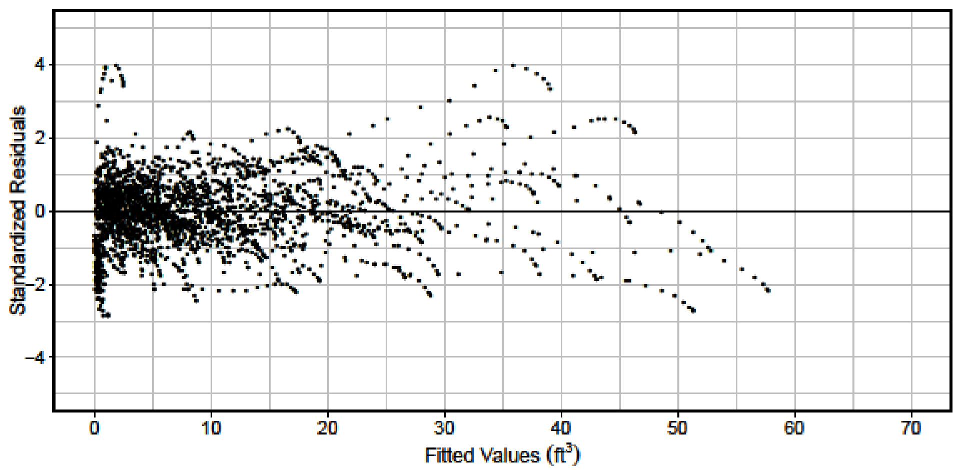

Figure 11. Residuals for Zhao and Kane 2 merchantable height equation fitted to cumulative outside bark volume versus the fitted values associated with the data used to fit the model.

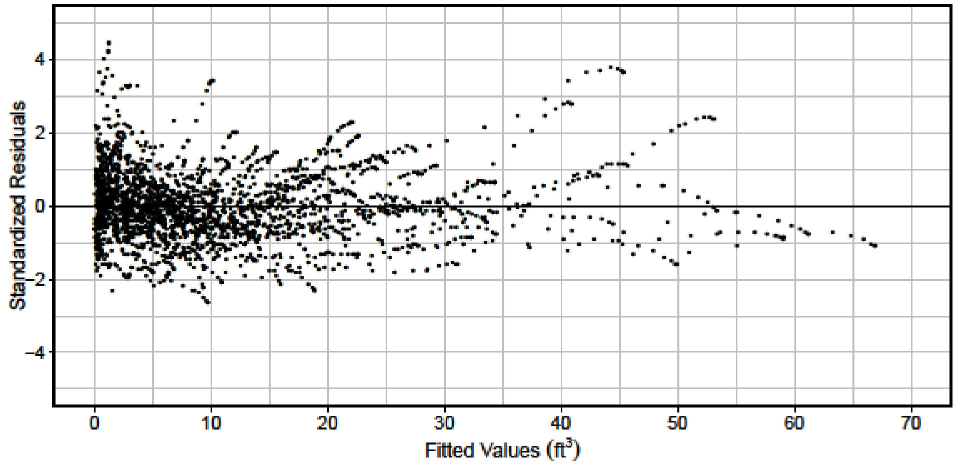

Figure 12. Residuals for Zhao and Kane 2 merchantable height equation fitted to cumulative inside bark volume versus the fitted values associated with the data used to fit the model.

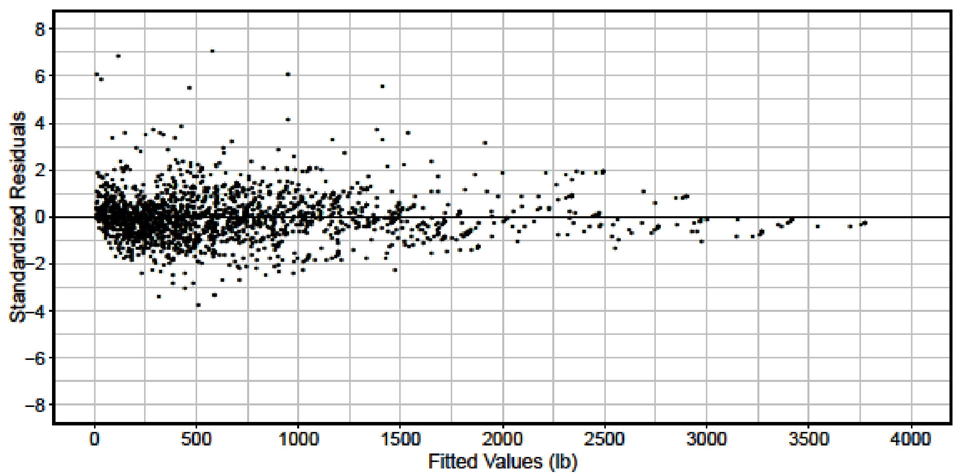

Figure 13. Residuals from the Exponential Merchantable Diameter equation fitted to cumulative green weight versus the fitted values associated with the data used to fit the model.

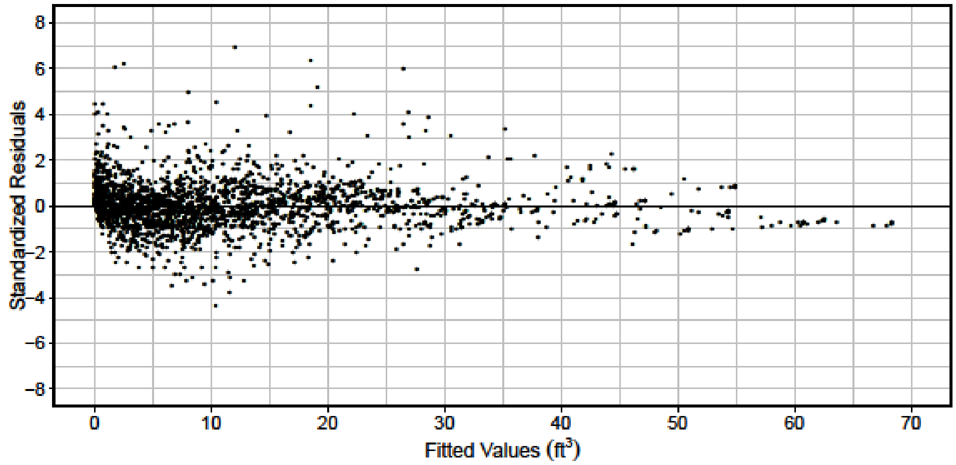

Figure 14. Residuals from the Exponential Merchantable Diameter equation fitted to cumulative outside bark volume versus the fitted values associated with the data used to fit the model.

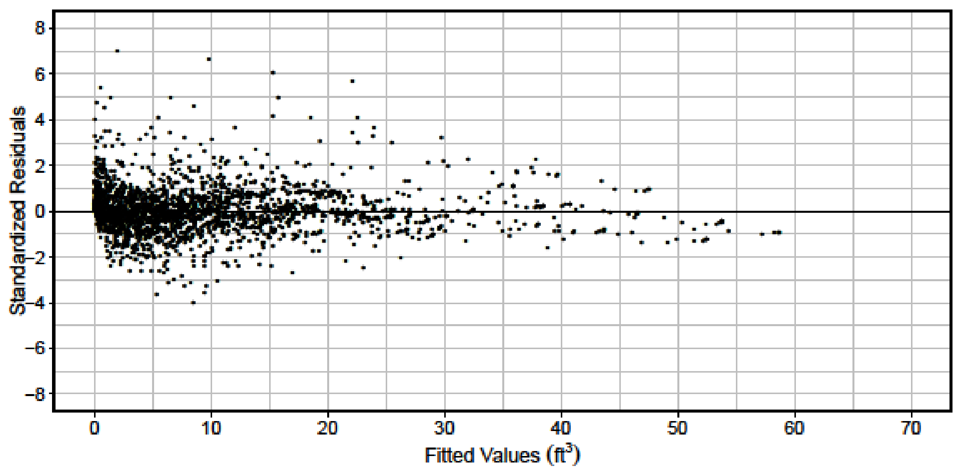

Figure 15. Residuals from the Exponential Merchantable Diameter equation fitted to cumulative inside bark volume versus the fitted values associated with the data used to fit the model.

I examined the plots for autocorrelation, systematic bias, and heteroskedasticity. In general the plots of the residuals versus the fitted values indicated some autocorrelation and no apparent heteroskedasticity or distinct bias.

The plots of the residuals for the merchantable diameter based model versus the fitted values (Figures 13-15) reveal no apparent autocorrelation, though residual variability may be larger at lower cumulative contents and smaller at larger cumulative contents. Some bias may be present in the residuals as fitted values increase. The pattern indicates bias shifting first positive, then negative, then slightly positive, then moving towards no bias.

The plots of the residuals for the merchantable height based models (Figures 10-12) indicate distinct autocorrelation in the residuals of individual trees. The autocorrelation was verified by examining the residuals for individual trees (this was also done for the Exponential Merchantable Diameter equation, neither set of resulting figures are shown). The pattern in the residuals for the height ratio based models follows individual tree stems, showing the correlated error variances for individual trees. Frequently the same tree has content residuals that tend to be similarly greater or less than zero. For each tree the absolute value of errors are smaller where the cumulative content is small, grow larger in magnitude as cumulative content does and then begin growing smaller in magnitude at some point. Unfortunately, the results for individual trees were not always consistent for trees growing in the same stand or management unit. These plots do not show any patterns of systematic bias or heteroskedasticity.

Plots of model residuals versus the independent variables are shown in Appendix 3 (Figures 59-76). Plots of the merchantable height based models (Figures 68-76) show the same trends mentioned in the last paragraph. The plots with the independent variables dbh (Figures 68, 71 and 74) and total height (Figures 69, 72 and 75) on the x-axis show these patterns in one dimension. The plots with merchantable height on the x-axis (Figures 70, 73 and 76) show that as h increases so do the positive residuals, to a point, then the magnitude of the residuals tend to decrease; the negative residuals seem to decrease, increase and then decrease along the stem. Plots of the residuals associated with the Exponential Merchantable Diameter equation (equation 20) plotted over independent variables (Figures 59-67) reveal that residuals tend to be more variable where merchantable diameter is large (Figures 61, 64 and 67), but no additional trends were detected.

In order to compare the height based models further and to show the ability of both selected equations to make accurate predictions to various merchantability limits on the stem, lack of fit statistics (measures of fit for portions of the data) were calculated. All the models in the subsequent figures were fitted to the Oklahoma data, thus they differ from figures in the model comparison section which compares the best Oklahoma model to models fitted to non-Oklahoma data. These results for models 22, 25, 27 and 20 are presented in Figures 16-27, similar figures are also presented for the compatible taper equations in Figures 28-31.

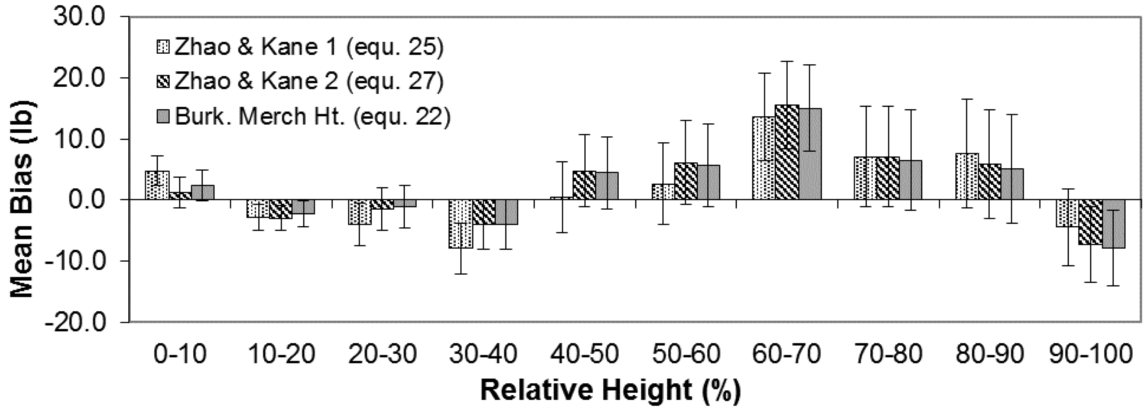

Figure 16. Mean biases for the Zhao and Kane 1, Zhao and Kane 2 and Burkhart Merchantable Height equations for merchantable green weight to a merchantable height, where relative height is merchantable height as a percent of total height (error bars represent ± one standard error from the means).

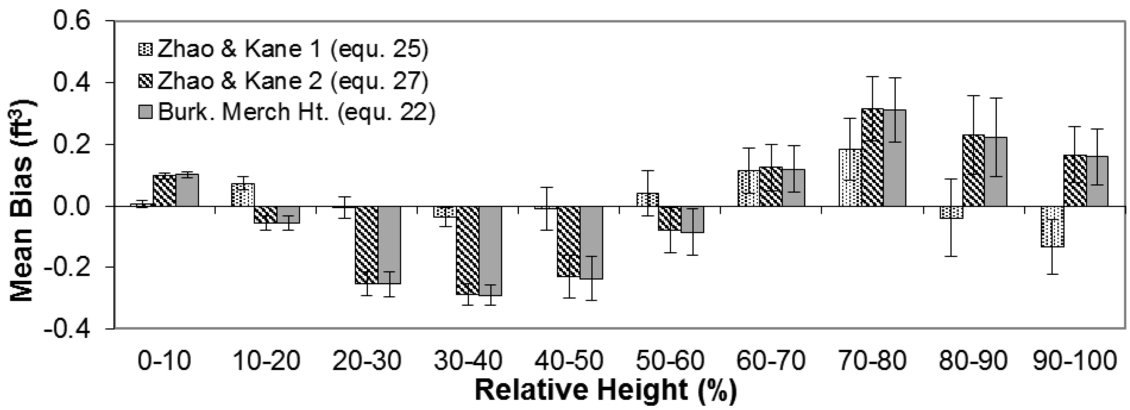

Figure 17. Mean biases for the Zhao and Kane 1, Zhao and Kane 2 and Burkhart Merchantable Height equations for merchantable outside bark volume to a merchantable height, where relative height is merchantable height as a percent of total height (error bars represent ± one standard error from the means).

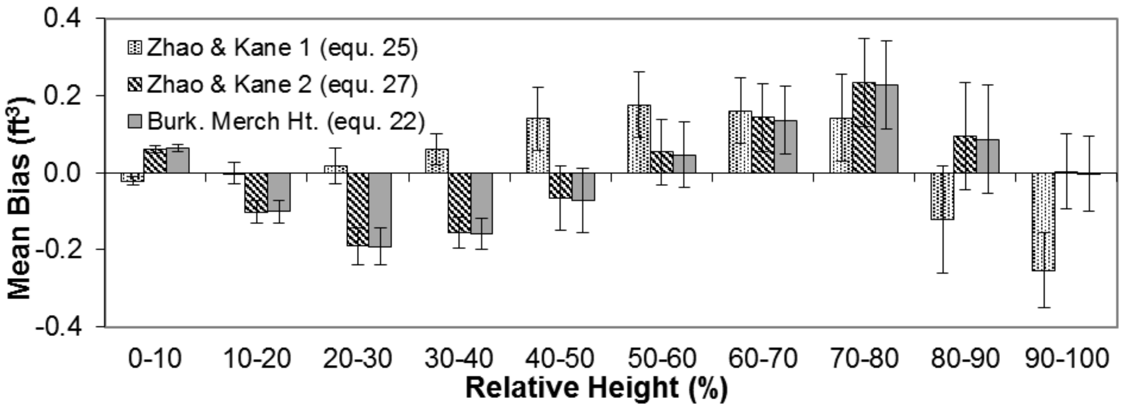

Figure 18. Mean biases for the Zhao and Kane 1, Zhao and Kane 2 and Burkhart Merchantable Height equations for merchantable inside bark volume to a merchantable height, where relative height is merchantable height as a percent of total height (error bars represent ± one standard error from the means).

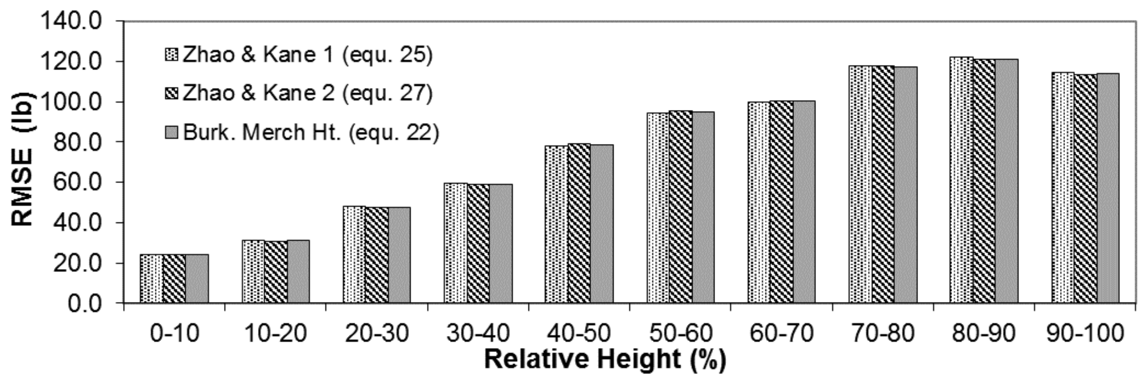

Figure 19. RMSE for the Zhao and Kane 1, Zhao and Kane 2 and Burkhart Merchantable Height equations to a merchantable green weight, where relative height is merchantable height as a percent of total height.

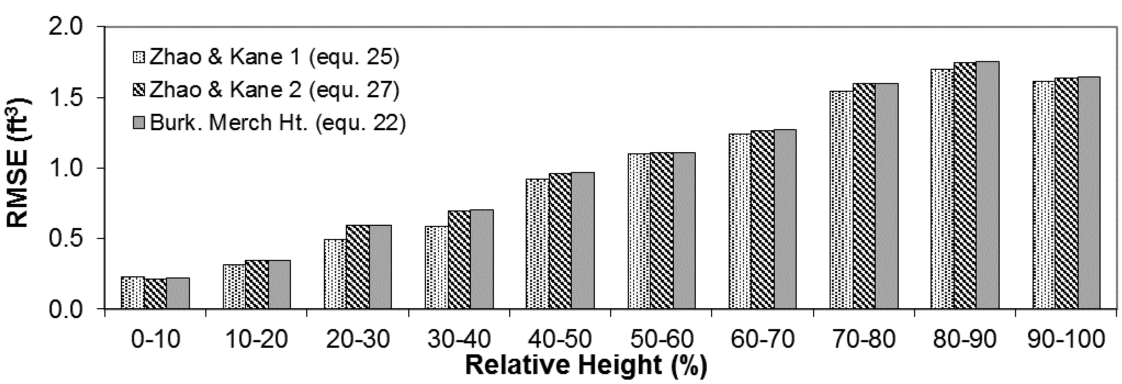

Figure 20. RMSE for the Zhao and Kane 1, Zhao and Kane 2 and Burkhart Merchantable Height equations for merchantable outside bark volume to a merchantable height, where relative height is merchantable height as a percent of total height.

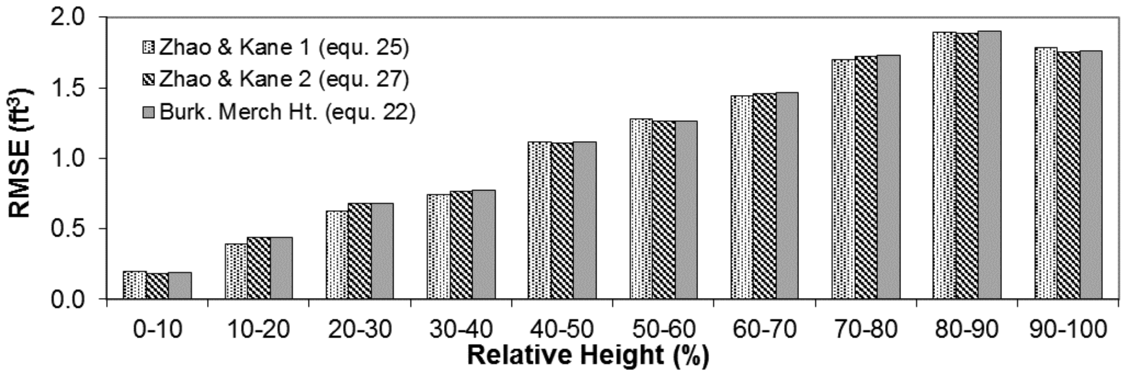

Figure 21. RMSE for the Zhao and Kane 1, Zhao and Kane 2 and Burkhart Merchantable Height equations for merchantable inside bark volume to a merchantable height, where relative height is merchantable height as a percent of total height.

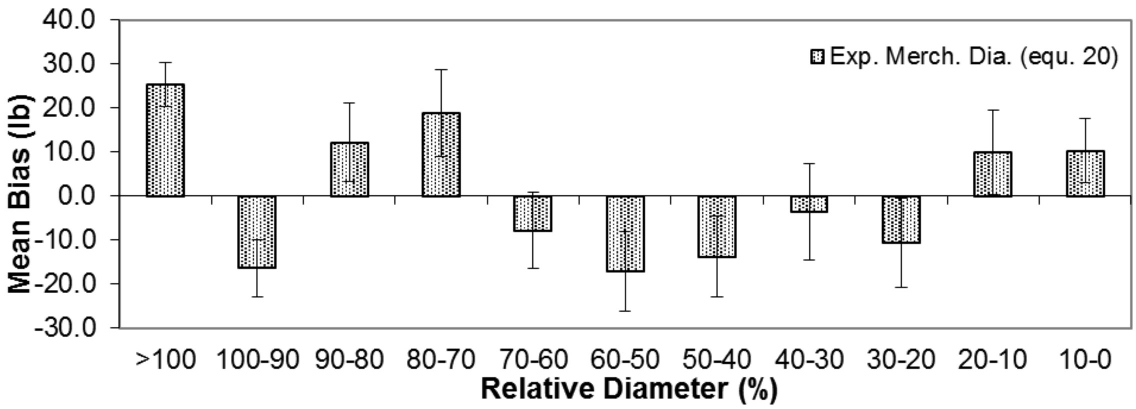

Figure 22. Mean biases for the Exponential Merchantable Diameter equation for merchantable green weight to a merchantable diameter, where relative diameter is merchantable diameter as a percent of dbh (error bars represent ± one standard error from the means).

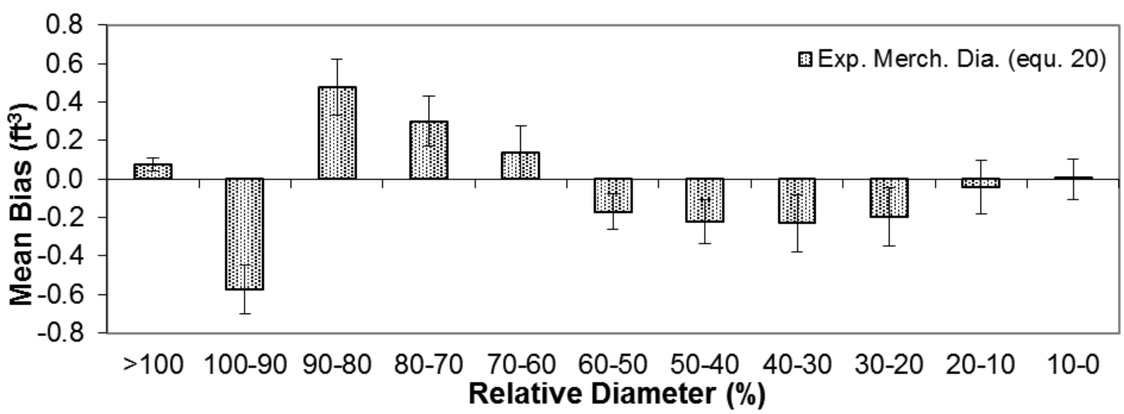

Figure 23. Mean biases for the Exponential Merchantable Diameter equation for merchantable outside bark volume to a merchantable diameter, where relative diameter is merchantable diameter as a percent of dbh (error bars represent ± one standard error from the means).

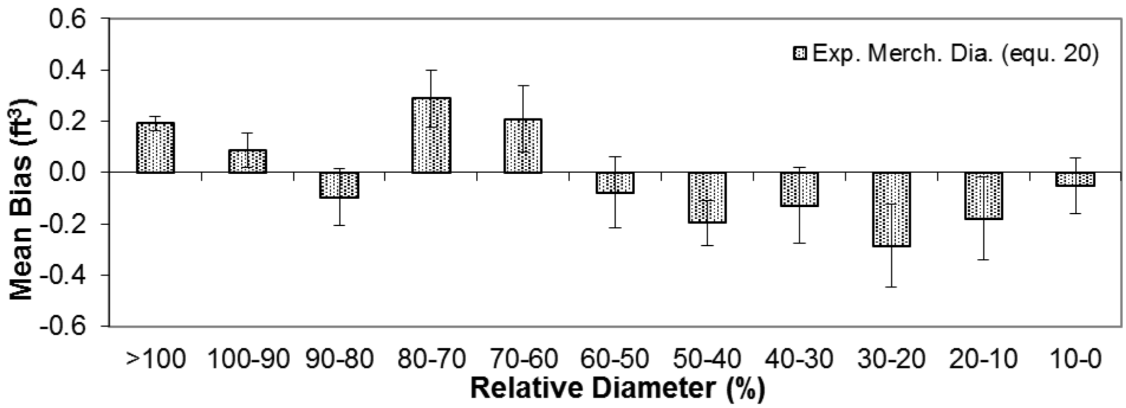

Figure 24. Mean biases for the Exponential Merchantable Diameter equation for merchantable inside bark volume to a merchantable diameter, where relative diameter is merchantable diameter as a percent of dbh (error bars represent ± one standard error from the means).

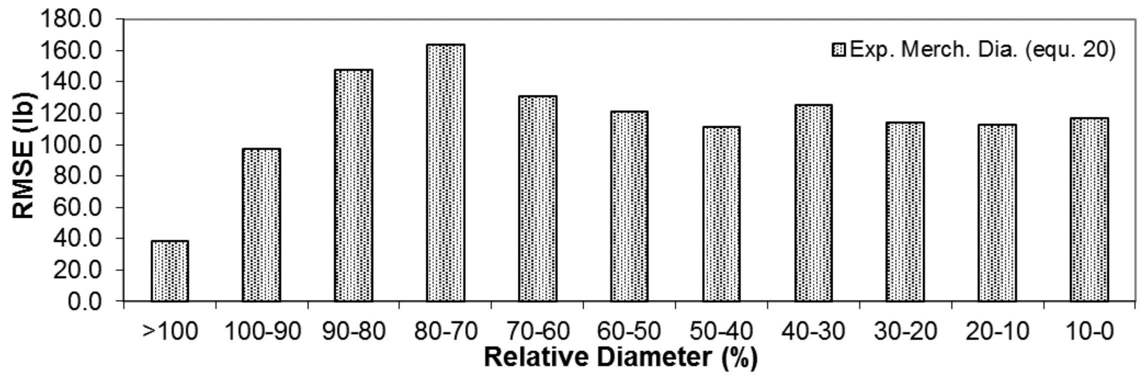

Figure 25. RMSE for the Exponential Merchantable Diameter equation for merchantable green weight to a merchantable diameter, where relative diameter is merchantable diameter as a percent of dbh.

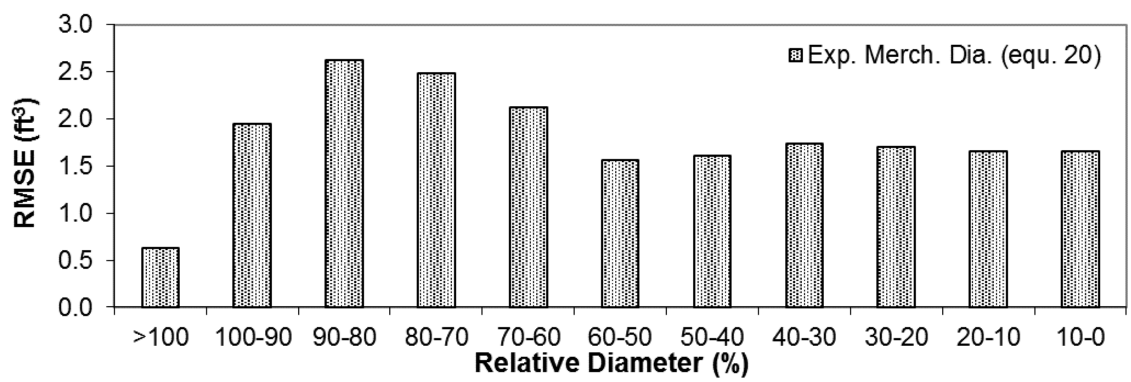

Figure 26. RMSE for the Exponential Merchantable Diameter equation for merchantable outside bark volume to a merchantable diameter, where relative diameter is merchantable diameter as a percent of dbh.

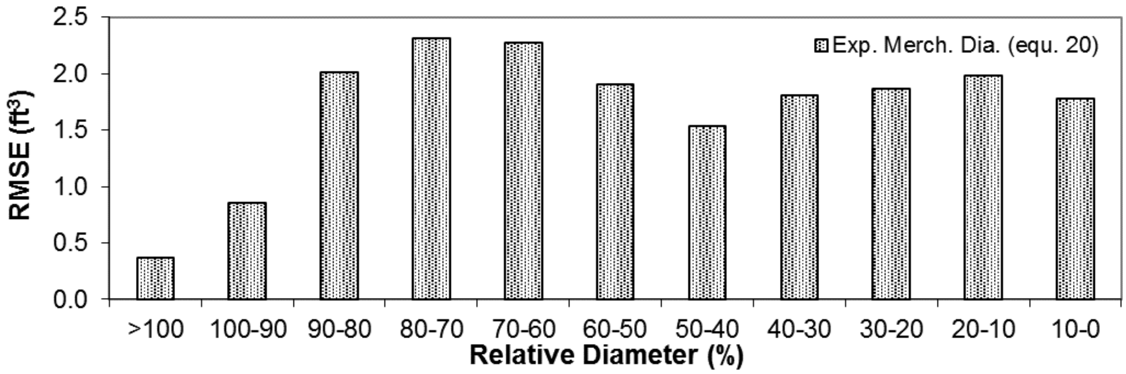

Figure 27. RMSE for the Exponential Merchantable Diameter equation for merchantable inside bark volume to a merchantable diameter, where relative diameter is merchantable diameter as a percent of dbh.

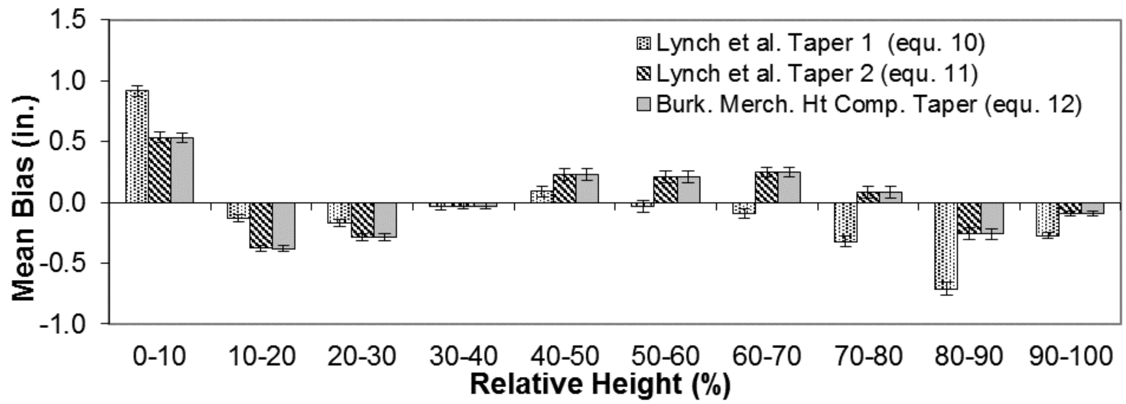

Figure 28. Mean biases for Lynch et al. Taper 1, Lynch et al. Taper 2 and Burkhart Merchantable Height compatible taper equations for merchantable outside bark diameter to a merchantable height, where relative height is merchantable height as a percent of total height (error bars represent ± one standard error from the means).

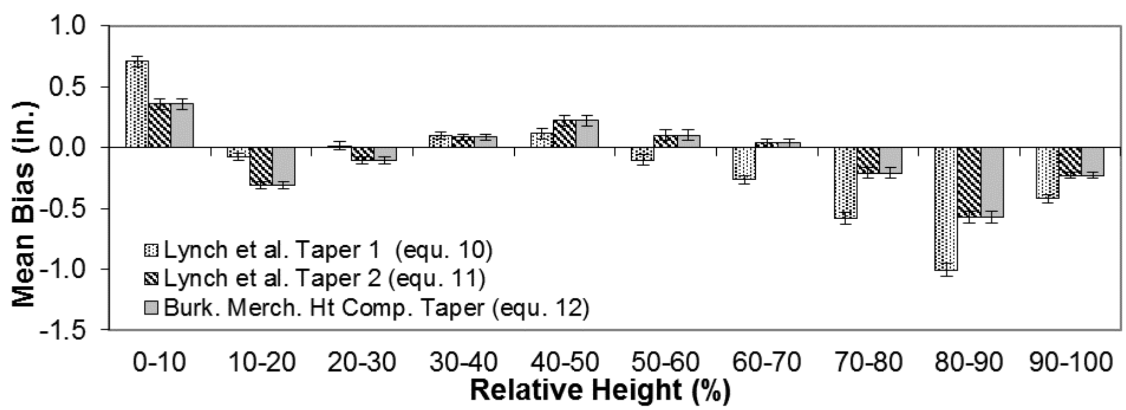

Figure 29. Mean biases for Lynch et al. Taper 1, Lynch et al. Taper 2 and Burkhart Merchantable Height compatible taper equations for merchantable inside bark diameter to a merchantable height, where relative height is merchantable height as a percent of total height (error bars represent ± one standard error from the means).

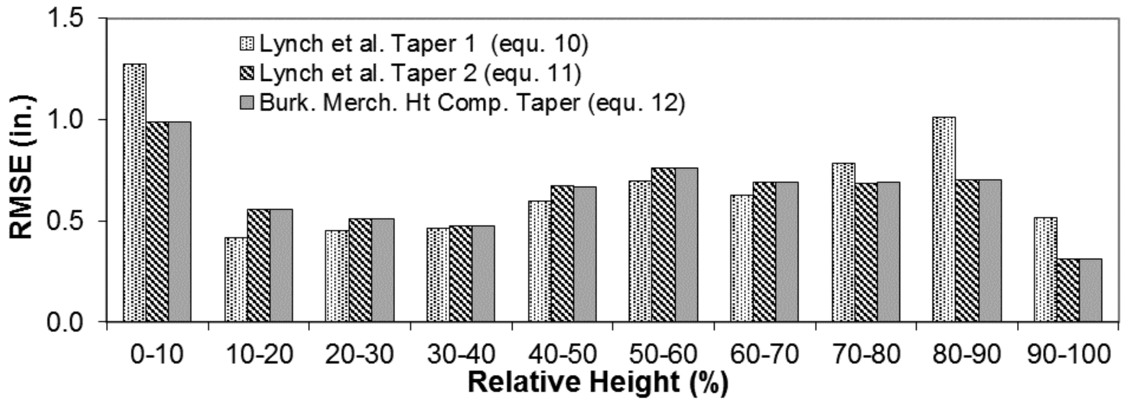

Figure 30. RMSE for Lynch et al. Taper 1, Lynch et al. Taper 2 and Burkhart Merchantable Height compatible taper equations for merchantable outside bark diameter to a merchantable height, where relative height is merchantable height as a percent of total height.

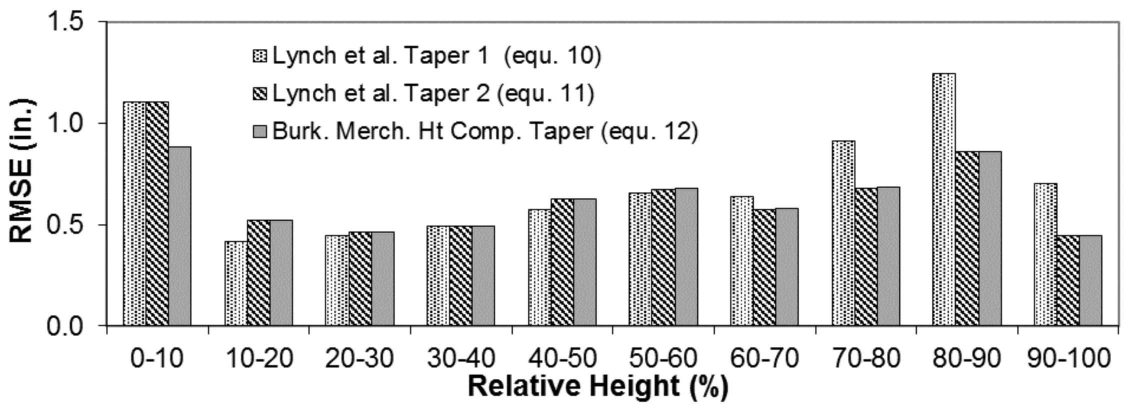

Figure 31. RMSE for Lynch et al. Taper 1, Lynch et al. Taper 2 and Burkhart Merchantable Height compatible taper equations for merchantable inside bark diameter to a merchantable height, where relative height is merchantable height as a percent of total height.

These results indicate that for the last portion of the tree, model 27 has some somewhat lower RMSE and model 25 has lower bias. Most of the comparisons made in these charts do not appear to show statistically significant differences between models, which means that judgments on model performance are not always clear-cut. Additionally, many of the MB values along the stem do not appear to be different from zero. That being said, the model that shows the least MB and RMSE for the upper stem should be selected.

The plots for equation 20 show slight upward bias for green weight predictions in the upper stem (Figures 22) and slight lower bias for volume predictions in the upper stem (Figures 23-24). MB for this model may not be different from zero for the upper stem. The MB for the upper stem volume predictions indicate good prediction. The RMSE is very constant along the stem (Figures 25-27).

The compatible taper equations have clear results: model 11 excels in both outside and inside bark prediction upper stem (for both RMSE and Bias); though it is hard to see from the plots because equations 11 and 12 perform similarly (Figures 28-31).

Finally, plots of the model residuals that incorporate several stand and plot variables were created to assess the impact of these variables on model performance. The residuals were plotted over stand age (in years), basal area (ft^2 per acre) of the stand (i.e., the management unit) and plot, stand trees per acre (both values from our field measurements and those available from Rayonier Inc. were used), plot trees per acre, site index (height in feet of dominant and codominant trees at the base age of 25 years), relative basal area at the plot and stand level (tree basal area divided by plot or stand basal area) and a calculated proxy for probable thinning. Stands having fewer trees per acre were more likely to be thinned and, conversely, stands with higher basal areas were less likely to be thinned. The Oklahoma data used in this study had a distribution of stand level trees per acre that was distinctly bimodal making this estimation of thinning status simple to perform. The clearest patterns were observed when more than one of these new variables were plotted in conjunction with one another as interaction plots. Interaction plots for Zhao and Kane 2 (equation 27) fitted to green weight and outside bark volume are presented as Figures 32 and 33. All other plots are presented in the Appendix 4 (Figures 77-86).

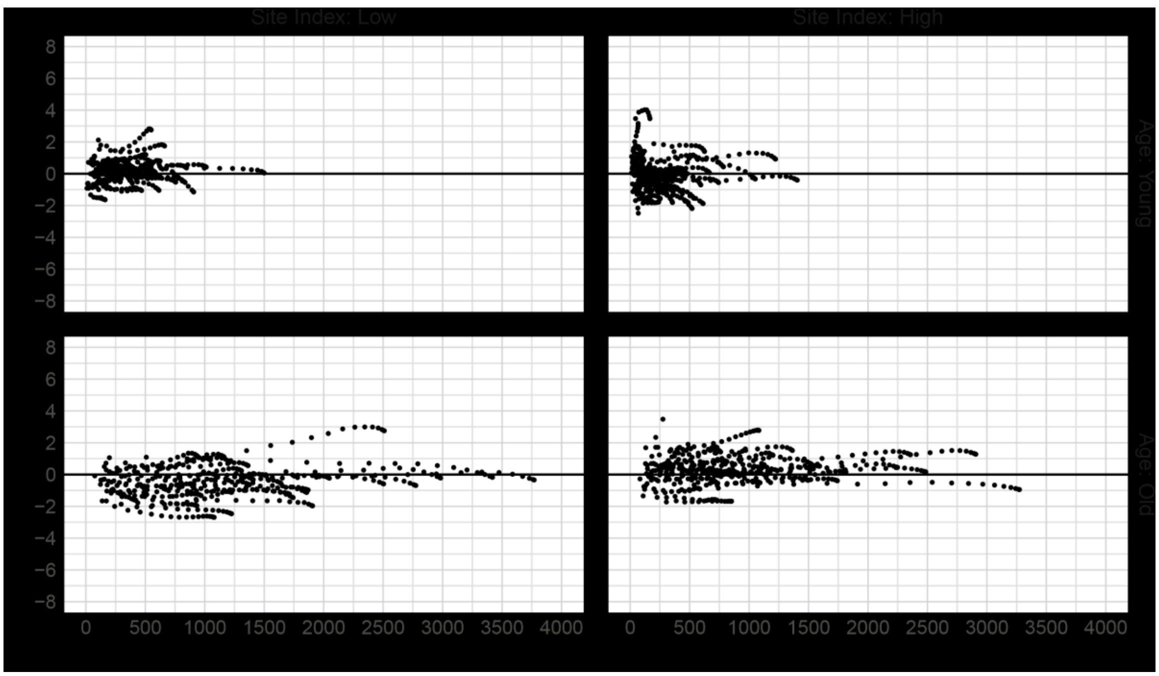

Figure 32. Interaction plot showing residuals from Zhao and Kane 2 (equation 27) fitted to green weight data at low and high levels of site index and young and old ages.

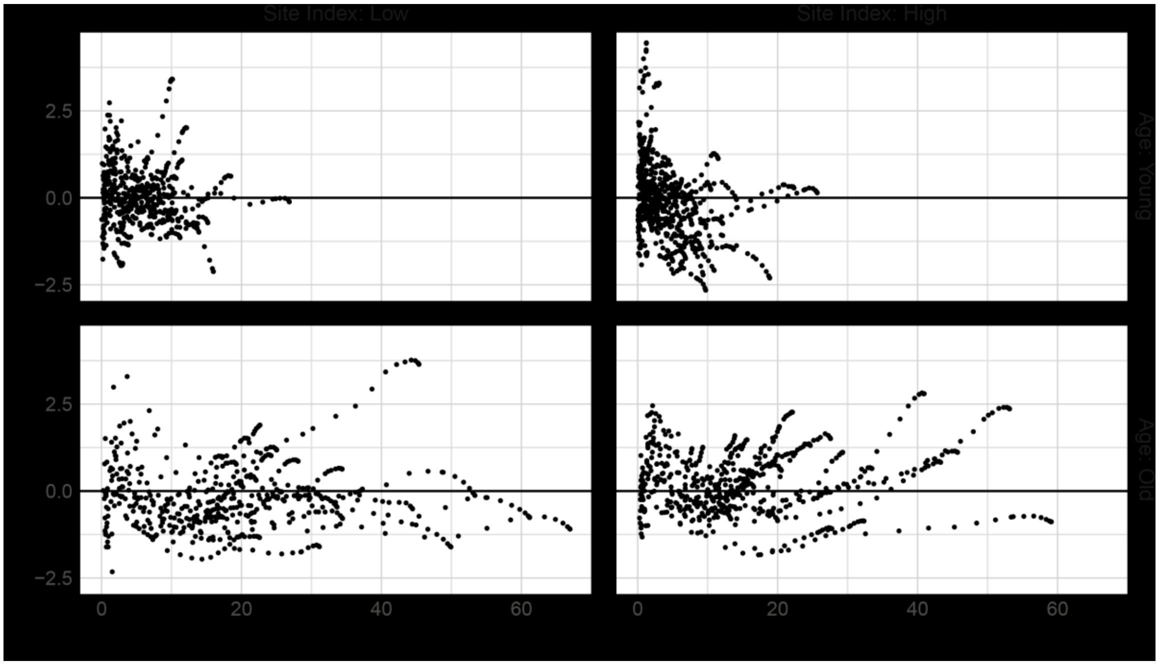

Figure 33. Interaction plot showing residuals from Zhao and Kane 2 (equation 27) fitted to outside bark volume data at low and high levels of site index and young and old ages.

Figure 32 shows only slight patterns of biasedness in the residuals; at older ages merchantable green weight at lower site indexes is slightly over-predicted while it is underpredicted at higher site indexes. Merchantable outside bark volume (Figure 33) does not appear to have any pattern and accurately predicts across all site indexes and ages.

CHAPTER IV

DISCUSSION

Regional Comparisons

The tests of similarity of Oklahoma loblolly pine characteristics to those of southern populations revealed that—for the most part—trees in Oklahoma have less total green weight and volume and less taper at 17.3 ft for a given DBH and total height (Table 13). This affirms my hypothesis that Oklahoma loblolly pine is different from that of the main portion of the loblolly pine range. Though, upper-stem diameter is shown to be underpredicted along much of the stem in Figure 7 so all results do not point in a single direction. These results are both statistically and practically important. Interestingly, some of the results are not statistically different and some of the fit statistics for models not fitted to the Oklahoma data indicate good model performance (e.g., Van Deusen et al. (1981), outside bark merchantable volume model).

When the south-east Oklahoma trees are compared to the population represented in Bullock and Burkhart (2003) green weight is consistently over-predicted for the entire stem and along the stem (i.e., the detected differences in green weight are negative, Figures 4-5). Upper-stem diameter, however, is accurately predicted over much of the stem, but is over-predicted near the tip of the tree (Figure 8). The consistent over-prediction of green weight along the stem indicates that other factors are influencing loblolly pine green weight in Oklahoma.

The tests of mean differences and plots of lack of fit statistics do not all corroborate between green weight and volume and between volume and taper when loblolly pines growing in the Upper Coastal Plain of Georgia and Alabama (Pienaar et al. 1987) are compared to those growing in south-east Oklahoma. Within the variables we do see corroboration. The plots of outside and inside bark volume biases along the stem show patterns that are similar to each other and consistent with the test results (Figures 6-7). Although predicted total green weight is not statistically different for the two populations the plot of the lack of fit statistics for this variable (Figure 4) indicates that Oklahoma trees are consistently smaller along the stem and consistent with the volume predictions along the lower portion of stems. Based on the slight inverse relationship of green weight to specific gravity (Zobel and Van Buijtenen 1989) and the extremely low specific gravity of loblolly pine in the north-west (Jordan et al. 2008) corner of its range it could be possible that the green weight could be larger than expected from volume prediction alone. However, this conclusion is a bit of a stretch given the results of Zobel and Van Buijtenen (1989).

Examination of tested differences between inside and outside bark volume leads to the conclusion that bark volume is greater in Oklahoma trees than it is in Upper Coastal Plain trees (Table 13); examination of outside and inside bark upper-stem diameter leads to the conclusion that Oklahoma trees have thicker bark (Figures 8-9). The underprediction of diameter by the PMRC outside bark diameter equation at 17.3 ft is inexplicable to the authors given the plotted lack of fit statistics. Given the over-prediction of outside bark stem volume it would be expected that outside bark volume would be over-predicted as well. These differences between results could be due to the stand origin and silviculture of the trees utilized by the PMRC.

When the south-east Oklahoma data are compared to the population represented in Van Deusen et al. (1981) the outside bark volume model performed surprisingly well (Figure 6). Inside bark volume is underpredicted (Figure 7) which could mean that Oklahoma trees have less bark volume than those in the south-east, USA.

When compared to the South (the entire loblolly pine range) via the updated Forest Service publication SE-282 for loblolly plantations both outside and inside bark diameters of Oklahoma trees are underpredicted (Table 13). A difference in magnitude between results could indicate that Oklahoma trees have thinner bark than those sampled for SE-282. One reason these results may not corroborate with those previously mentioned is that data used in this report were probably not collected exclusively from intensively managed stands.

It was surprising that the taper of Oklahoma trees is statistically different from that reported by SE-282 in the Arkansas region (this region includes the part of Oklahoma where I sampled). This difference could be due to differences between natural stands and plantations and also the density differences that come from these unique stand origins. These results are consistent with those reported in Amateis and Burkhart (1987b) which reported steeper taper in natural stands.

Model Selection and Evaluation

The models that I developed and evaluated using the data collected from stands in south-east Oklahoma performed well, in particular the merchantable content models based on the height ratio (Tables 7-8). I selected a new height ratio model (labeled equation 27) presented by Zhao and Kane (2017) based on its performance as evaluated by statistics of fit and lack of fit. In part, the decision to select equation 27 was influenced by the superior performance of the Lynch et al. equation 2 (labeled equation 11) (Table 9). The diameter based exponential merchantable content equation (equation 20) outperformed all other diameter ratio based equations and was selected; however, it did not perform as well as any of the height based equations (excepting green weight bias). Given the better performance of the merchantable height based equation it could be better to use this equation exclusively (instead of in addition to the merchantable diameter based equation). However, if one is to predict stem content to a merchantable top diameter limit, a taper equation would have to be used in conjunction with an upper-stem height based merchantable content equation, which would add to the total combined prediction error.

The Zhao & Kane Ratio Equation 1 and The Modified Burkhart Height Ratio are equivalent models and, as would be expected, their results were very close (Tables 6 and 8); however, both were evaluated because Burkhart's model has been well proven and the slight differences between them become apparent during analysis.

Merchantable content equations derived from merchantable heights typically outperform those derived from merchantable diameters because merchantable height ratios are more highly correlated to merchantable content ratios than are merchantable diameter ratios (Van Deusen et al. 1982, Reed and Green 1984, McTague and Bailey 1987). Another possible reason for the discrepancy between merchantable height and diameter based models is that the conditions that must be satisfied by height based models are well understood and documented (Zhao and Kane 2017) while conditions that must be satisfied by the diameter based models are not as well documented. Though, a similar process could be carried out for merchantable diameter based equations (Zhao and Kane 2017).

The exponential height ratio equation was originally included (Bullock and Burkhart 2003) in model selection and evaluation. Unlike other height based models it performed poorly, which was consistent with other studies (Bullock and Burkhart 2003). Additionally, during model evaluation it was found that this particular model does not satisfy condition IV of Zhao and Kane (2017) for cumulative relative content profile models. As a result the compatible taper function for the exponential height ratio equation illogically predicts reverse taper on a portion of the stem (these results are available from the committee chair).

In general the variance models fitted as a part of these systems (combined content and variance models) worked exceptionally well to weight the models, as evidenced by the small range of the standardized residuals (Figures 10-15). Since the standardized residuals of the Exponential Merchantable Diameter equation appeared to be larger at lower predicted merchantable contents as well as merchantable diameters this weighting scheme may not have completely accounted for all heteroskedasticity in this model (Figure 13-15). However, another explanation of this pattern could be the larger broad range of tree sizes and stand conditions sampled. Lower bolt contents were highly variable, probably resulting from the range of stand densities sampled; some stands were growing in almost open conditions while many had typical densities.

Autocorrelation was evident (and unaccounted for) in the height based models (Figures 10-12). These patterns could probably be resolved within a mixed effects modeling approach and/or by utilizing a continuous autoregressive structure (Burkhart and Tomé 2012, p. 32). However, either of these methods make prediction more complicated. I did attempt to fit a continuous autoregressive structure, but had convergence issues. The correlation of error terms does not seem to be a problem in our merchantable diameter ratio based models and due to the small overall spread of the residuals in the merchantable height ratio based models, I do not believe model variability estimates are unduly affected by within tree autocorrelation in these data. The correlation of errors should affect only the variances and error estimates associated with these models, they should not hinder their predictive ability (Reams 1994, Kozak 1997). Predictions from ordinary least squares are still unbiased (and nonlinear ordinary least squares asymptotically unbiased) in the presence of correlated errors. Additionally, the method I use to deal with heteroskedasticity does not interfere with prediction.

Additional Variables

Investigation of autocorrelation patterns along tree stems in the height ratio content prediction models revealed that whole trees were often over or underpredicted. Patterns were sometimes apparent for whole stands of trees; however, the results were inconsistent from one stand to another. These patterns led us to consider that incorporating stand, site, and plot variables into the model might help to account for the between stand variability.

After investigating the inclusion of these additional variables I conclude that incorporating them into the model would not provide a great improvement. Some authors split sample trees into groups based on age for prediction equations, our results suggest that this practice could be useful for merchantable green weight prediction equations. It could also be worthwhile to split sample trees into groups based on site index for green weight (Figure 32). These results corroborate with Jordan et al. (2008) who, as stated previously, found age and site index to be important predictors of specific gravity. That being said, our models perform very well and the spread of the Oklahoma fitted residuals is so small that any improvements would most likely be minor. Since differences between stands with high and low site indexes were only perceptible at older ages it seems that younger stands likely have similar populations of trees, with regard to stem form and green density, regardless of site quality.

CHAPTER V

CONCLUSIONS

In conclusion, I compared loblolly pine trees from the Ouachita mountain region of Oklahoma to models developed from loblolly populations in more southern and southeastern areas within the loblolly pine natural range and found that they have different green weight, volume and taper for a given diameter, height and merchantability limit. In general, Oklahoma grown loblolly pine from this region have smaller stem content (by weight and volume) for given values of dbh, total height and a merchantability limit than populations across the South. Total green weight in the Upper Coastal Plain is not different; although it is biased, especially in the lower parts of the stem. Merchantable or total outside bark volume in the southeast is not different.

I have created a system of equations that is capable of predicting merchantable green weight, merchantable outside bark volume, merchantable inside bark volume, outside bark diameter and inside bark diameter for plantation-grown loblolly pine trees in the Ouachita mountain region of Oklahoma. The parameter estimation methods that I used accounted for heteroskedasticity, but not for within tree autocorrelation, which should not be a large problem for our predictive models.

These models give accurate estimates of loblolly pine stem content and taper. The method that I used to model the heteroskedasticity of tree content, coupled with the variance covariance matrices provided, allows those who use these equations to more precisely, and practically understand the variability associated with the predictions that they are making. These equations should prove useful to plantation owners and managers in the Kiamichi region of the Ouachita Mountains in Oklahoma (and possibly into Arkansas) by providing more accurate predictions of loblolly pine merchantable green weights, volumes and diameters—making better financial assessments of loblolly pine plantations in the state possible.

REFERENCES

Akaike, H. 1974. A new look at the statistical model identification. *IEEE Transactions on Automatic Control* 19(6):716-723.

Amateis, R.L., and H.E. Burkhart. 1987. Cubic-Foot Volume Equations for Loblolly Pine Trees in Cutover, Site-Prepared Plantations. *Southern Journal of Applied Forestry* 11(4):190-192.

Amateis, R.L., and H.E. Burkhart. 1987. Tree volume and taper of loblolly pine varies by stand origin. *Southern Journal of Applied Forestry* 11(4):185-189.

Antony, F., L.R. Schimleck, R.F. Daniels, A. Clark, and D.B. Hall. 2010. Modeling the longitudinal variation in wood specific gravity of planted loblolly pine (Pinus taeda) in the United States. *Canadian Journal of Forest Research* 40(12):2439-2451.

Avery, T.E., and H.E. Burkhart. 2015. *Forest Measurements*. Waveland Press, Long Grove Illinois.

Baker, J.B., and O.G. Langdon. 1990. *Pinus taeda* L. Loblolly pine. P. 497-512 in Silvics of North America Volume 1, Agriculture Handbook 654, Burns, R.M., and B.H. Honkala (eds.). USDA, Forest Service, Washington, DC.

Baldwin, V. 1987. Green and dry-weight equations for above-ground components of planted loblolly pine trees in the West Gulf Region. *Southern Journal of Applied Forestry* 11(4):212-218.

Belyea, H.C. 1931. *Forest Measurement*. Wiley, New York, USA.

Bullock, B.P., and H.E. Burkhart. 2003. Equations for Predicting Green Weight of Loblolly Pine Trees in the South. *Southern Journal of Applied Forestry* 27(3):153-159.

Burkhart, H.E. 1977. Cubic-Foot Volume of Loblolly Pine to Any Merchantable Top Limit. *Southern Journal of Applied Forestry* 1(2):7-9.

Burkhart, H.E., and M. Tomé. 2012. *Modeling Forest Trees and Stands*. Springer Science & Business Media.

Cao, Q.V., H.E. Burkhart, and T.A. Max. 1980. Evaluation of Two Methods for Cubic-Volume Prediction of Loblolly Pine to Any Merchantable Limit. *For. Sci.* 26(1):71-80.

Cao, Q.V., and H.E. Burkhart. 1980. Cubic-foot volume of loblolly pine to any height limit. *Southern Journal of Applied Forestry* 4(4):166-168.

Clark, A.I., R.A. Souter, and B.E. Schlaegel. 1991. *Stem Profile Equations for Southern Tree Species*. USDA Forest Service Southeastern Forest Experiment Station, Asheville, North Carolina. Research Paper SE-282. 113 p.

Clutter, J.L. 1980. Development of Taper Functions from Variable-Top Merchantable Volume Equations. *For. Sci.* 26(1):117-120.

Demaerschalk, J.P. 1972. Converting volume equations to compatible taper equations. *For. Sci.* 18(3):241-245.

Demaerschalk, J.P. 1973. Integrated systems for the estimation of tree taper and volume. *Canadian Journal of Forest Research* 3(1):90-94.

Dragulescu, A.A. 2014. *xlsx: Read, write, format Excel 2007 and Excel 97/2000/XP/2003 files*. R package version 0.5.7. https://CRAN.R-project.org/package=xlsx

Ducey, M.J., and M.S. Williams. 2011. Comparison of Hossfeld's Method and Two Modern Methods for Volume Estimation of Standing Trees. *Western Journal of Applied Forestry* 26(1):19-23.

Elzhov, T.V., K.M. Mullen, A.-N. Spiess, and B. Bolker. 2016. *minpack.lm: R Interface to the Levenberg-Marquardt Nonlinear Least-Squares Algorithm Found in MINPACK, Plus Support for Bounds*. R package version 1.2-1. https://CRAN.R-project.org/package=minpack.lm

Farjat, A.E., A.K. Chamblee, F. Isik, R.W. Whetten, and S.E. McKeand. 2017. Variation among Loblolly Pine Seed Sources across Diverse Environments in the Southeastern United States. *For. Sci.* 63(1):39-48.

Grothendieck, G., and A. Zeileis. 2005. zoo: S3 Infrastructure for Regular and Irregular Time Series. *Journal of Statistical Software* 14(6):1-27.

Harrison, W., and B. Borders. 1996. *Yield prediction and growth projection for site-prepared loblolly pine plantations in the Carolinas, Georgia, Alabama and Florida*. University of Georgia Plantation Management Research Cooperative. Technical Report (Compilation) 1996-1.

Honer, T.G. 1964. The use of height and squared diameter ratios for the estimation of merchantable cubic foot volume. *The Forestry Chronicle* 40(3):324-331.

Jordan, L., K. Berenhaut, R. Souter, and R.F. Daniels. 2005. Parsimonious and Completely Compatible Taper, Total, and Merchantable Volume Models. *For. Sci.* 51(6):578-584.

Jordan, L., A. Clark, L.R. Schimleck, D.B. Hall, and R.F. Daniels. 2008. Regional variation in wood specific gravity of planted loblolly pine in the United States. *Canadian journal of forest research* 38(4):698-710.

Joshi, O.J. 2017. *Economic importance of forestry in Oklahoma in 2012*. Oklahoma Cooperative Extension Service. Extension fact sheet L-462. 2 p.

Kozak, A. 1997. Effects of multicollinearity and autocorrelation on the variable-exponent taper functions. *Canadian Journal of Forest Research* 27(5):619-629.

Kozak, A., and R. Kozak. 2003. Does cross validation provide additional information in the evaluation of regression models? *Canadian Journal of Forest Research* 33(6):976-987.

Lambeth, C.C., P.M. Dougherty, W.T. Gladstone, R.B. McCullough, and O.O. Wells. 1984. Large-Scale Planting of North Carolina Loblolly Pine in Arkansas and Oklahoma: A Case of Gain Versus Risk. *Journal of Forestry* 82(12):736-741.

Lenhart, J.D., T.L. Hackett, C.J. Laman, T.J. Wiswell, and J.A. Blackard. 1987. Tree Content and Taper Functions for Loblolly and Slash Pine Trees Planted on Non-Old-Fields in East Texas. *Southern Journal of Applied Forestry* 11(3):147-151.

Little Jr, E.L. 1979. *Checklist of United States trees (native and naturalized)*. USDA Forest Service, Washington, D.C. Agriculture Handbook No. 541. 375 p.

Lynch Thomas, B., E. Will Rodney, C. Hennessey Thomas, A. Heinemann Robert, and T. Holeman Randal. 2010. Relationships among Diameter at Breast Height and Loblolly Pine Attributes from Local and Nonlocal Seed Sources near the Western Edge of the Natural Range of Loblolly Pine. *Southern Journal of Applied Forestry* 34(4):149-153.

Lynch Thomas, B., D. Zhao, W. Harges, and J.P. McTague. 2017. Deriving compatible taper functions from volume ratio equations based on upper-stem height. *Canadian Journal of Forest Research* Accepted.

McTague, J.P., and R.L. Bailey. 1987. Simultaneous total and merchantable volume equations and a compatible taper function for loblolly pine. *Canadian Journal of Forest Research* 17(1):87-92.

Moser, J.W., and T.W. Beers. 1969. Parameter estimation in nonlinear volume equations. *Journal of Forestry* 67(12):878-879.

Newbold, R.A., V.C. Baldwin, and G. Hill. 2001. *Weight and Volume Determination for Planted Loblolly Pine in North Louisiana*. USDA Forest Service Southern Research Station, Asheville, North Carolina. Research Paper SRS-26. 19 p.

Parresol, B.R., J.E. Hotvedt, and Q.V. Cao. 1987. A volume and taper prediction system for bald cypress. *Canadian Journal of Forest Research* 17(3):250-259.

Parresol, B.R., and C.E. Thomas. 1989. A density-integral approach to estimating stem biomass. *Forest Ecology and Management* 26(4):285-297.

Pienaar, L.V., T. Burgan, and J.W. Rheney. 1987. *Stem volume, taper and weight equations for site-prepared loblolly pine plantations*. University of Georgia Plantation Management Research Cooperative, Athens, Georgia. Technical Report 1987-1.

Pinheiro, J., D.M. Bates, S. DebRoy, D. Sarkar, and R Core Team. 2017. *nlme: Linear and Nonlinear Mixed Effects Models*. R package version 3.1-131. https://CRAN.R-project.org/package=nlme

Pinheiro, J.C., and D.M. Bates. 2000. *Mixed-Effects Models in S and S-Plus*. Springer-Verlag, New York. 528 p.

R Core Team. 2017. *R: A Language and Environment for Statistical Computing*. R Version 3.4.0 Patched. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/

Reams, G.A. 1994. Correction for serial correlation in volume ratio models. *Research paper SO (USA). no. 281.*

Reed, D.D., and E.J. Green. 1984. Compatible stem taper and volume ratio equations. *For. Sci.* 30(4):977-990.

RStudio Team. 2016. *RStudio: Integrated Development for R*. 1.0.143. RStudio, Inc., Boston, MA. http://www.rstudio.com/

Schmidtling, R.C., and A. Clark III. 1989. Loblolly pine seed sources differ in stem form. P. 421-425 in Fifth Biennial Southern Silvicultural Research Conference, Miller, J.H. (ed.). U.S. Department of Agriculture, Forest Service, Southern-Forest Experiment Station, Memphis, TN.

Schultz, R.P. 1999. Loblolly -- the pine for the twenty-first century. *New Forests* 17(1):71-88.

Schumacher, F.X., and F.D.S. Hall. 1933. Logarithmic expression of timber-tree volume. *J. Agric. Res.* 47:0719-0734.

Schwarz, G. 1978. Estimating the Dimension of a Model. *The Annals of Statistics* 6(2):461-464.

Sherrill, J.R., B.P. Bullock, T.J. Mullin, S.E. McKeand, and R.C. Purnell. 2011. Total and Merchantable Stem Volume Equations for Midrotation Loblolly Pine (Pinus taeda L.). *Southern Journal of Applied Forestry* 35(3):105-108.

Sherrill, J.R., T.J. Mullin, B.P. Bullock, S.E. McKeand, R.C. Purnell, M.L. Gumpertz, and F. Isik. 2008. An evaluation of selection for volume growth in loblolly pine. *Silvae Genetica* 57(1):22-28.

Souter, R.A. 1999. *Taper and Volume Prediction in Southern Tree Species*. USDA Forest Service Forest Inventory and Analysis Research Work Unit, Asheville, NC. Technical Report.

Souter, R.A. 2001. *Sapling Taper and Volume Prediction in Southern Tree Species*. USDA Forest Service Forest Inventory and Analysis Research Work Unit, Asheville, NC. Technical Report.

Souter, R.A. 2001. *Taper, Volume and Weight Prediction in Southern Tree Species*. USDA Forest Service Forest Inventory and Analysis Research Work Unit, Asheville, NC. Technical Report.

Spurr, S.H. 1952. *Forest inventory*. Ronald Press Company, New York. xii + 476 pp. p.

Tasissa, G., H.E. Burkhart, and R.L. Amateis. 1997. Volume and Taper Equations for Thinned and Unthinned Loblolly Pine Trees in Cutover, Site-Prepared Plantations. *Southern Journal of Applied Forestry* 21(3):146-152.

Van Deusen, P.C., T.G. Matney, and A.D. Sullivan. 1982. A Compatible System for Predicting the Volume and Diameter of Sweetgum Trees to any Height. *Southern Journal of Applied Forestry* 6(3):159-163.

Van Deusen, P.C., A.D. Sullivan, and T.G. Matney. 1981. A prediction system for cubic foot volume of loblolly pine applicable through much of its range. *Southern Journal of Applied Forestry* 5(4):186-189.

Venables, W.N., and B.D. Ripley. 2002. *Modern Applied Statistics with S 4th edition*. Springer, New York.

Wells, O.O. 1983. Southwide Pine Seed Source Study--Loblolly Pine at 25 Years. *Southern Journal of Applied Forestry* 7(2):63-71.

Wells, O.O., and C.C. Lambeth. 1983. Loblolly Pine Provenance Test in Southern Arkansas 25th Year Results. *Southern Journal of Applied Forestry* 7(2):71-75.

Wells, O.O., and P.C. Wakeley. 1966. Geographic Variation in Survival, Growth, and Fusiform-Rust Infection of Planted Loblolly Pine. *For. Sci.* 12(3):a0001-z0001.

Wickham, H. 2009. *ggplot2: Elegant Graphics for Data Analysis*. Springer-Verlag, New York.

Wickham, H., and R. Francois. 2016. *dplyr: A Grammar of Data Manipulation*. R package version 0.5.0. https://CRAN.R-project.org/package=dplyr

Wilke, C.O. 2016. *cowplot: Streamlined Plot Theme and Plot Annotations for 'ggplot2'*. R package version 0.7.0. https://CRAN.R-project.org/package=cowplot

Will, R., T. Hennessey, T. Lynch, R. Holeman, and R. Heinemann. 2010. Effects of Planting Density and Seed Source on Loblolly Pine Stands in Southeastern Oklahoma. *For. Sci.* 56(5):437-443.

Zhao, D., and M. Kane. 2017. New Variable-Top Merchantable Volume and Weight Equations Derived Directly from Cumulative Relative Profiles for Loblolly Pine. *For. Sci.* 63(3):261-269.

Zobel, B.J., and J.P. Van Buijtenen. 1989. *Wood variation: its causes and control*. Springer-Verlag.

APPENDICES

Appendix 1: Variance-Covariance Matrices

Table 14. Variance-covariance matrix for the parameters of Zhao and Kane 2 (equation 27) fitted to Oklahoma merchantable green weight data.

Zhou and Kane 2 (Equ. 20) Var-Cov Matrix for Green Weight Parameters							
	a_0	a_1	a_2	c_0	c_1	c_2	α
a_0	5.6269E-05	4.3584E-05	-1.1858E-04	6.3697E-05	5.8834E-05	-1.6656E-04	1.1362E-05
a_1	4.3584E-05	1.6283E-04	-1.6911E-04	4.6700E-05	2.2767E-04	-2.3308E-04	2.9343E-06
a_2	-1.1858E-04	-1.6911E-04	2.9719E-04	-1.3327E-04	-2.3287E-04	4.1523E-04	-4.2207E-05
c_0	6.3697E-05	4.6700E-05	-1.3327E-04	9.4191E-05	7.7334E-05	-2.3950E-04	5.1823E-05
c_1	5.8834E-05	2.2767E-04	-2.3287E-04	7.7334E-05	4.0036E-04	-3.9981E-04	1.4245E-06
c_2	-1.6656E-04	-2.3308E-04	4.1523E-04	-2.3950E-04	-3.9981E-04	7.3267E-04	-1.1057E-04
α	1.1362E-05	2.9343E-06	-4.2207E-05	5.1823E-05	1.4245E-06	-1.1057E-04	8.2792E-04

Table 15. Variance-Covariance matrix for the parameters of Zhao and Kane 2 (equation 27) fitted to Oklahoma merchantable outside bark volume data.

Zhou and Kane 2 (Equ. 20) Var-Cov Matrix for Outside Bark Volume Parameters							
	a_0	a_1	a_2	c_0	c_1	c_2	α
a_0	4.3007E-09	2.0095E-07	-5.5352E-07	4.8138E-09	2.1241E-07	-5.8876E-07	-4.5536E-08
a_1	2.0095E-07	4.7514E-05	-4.8837E-05	2.2295E-07	5.0634E-05	-5.1848E-05	-1.2831E-07
a_2	-5.5352E-07	-4.8837E-05	8.5403E-05	-6.1957E-07	-5.1863E-05	9.0851E-05	-1.1384E-06
c_0	4.8138E-09	2.2295E-07	-6.1957E-07	5.6371E-09	2.4373E-07	-6.8707E-07	4.4809E-10
c_1	2.1241E-07	5.0634E-05	-5.1863E-05	2.4373E-07	5.6088E-05	-5.7111E-05	-9.6028E-08
c_2	-5.8876E-07	-5.1848E-05	9.0851E-05	-6.8707E-07	-5.7111E-05	1.0052E-04	-4.8624E-06
α	-4.5536E-08	-1.2831E-07	-1.1384E-06	4.4809E-10	-9.6028E-08	-4.8624E-06	1.8570E-04

Table 16. Variance-Covariance matrix for the parameters of the Zhao and Kane 2 equation (equation 27) fitted to Oklahoma merchantable inside bark volume data.

Zhou and Kane 2 (Equ. 20) Var-Cov Matrix for Inside Bark Volume Parameters							
	a_0	a_1	a_2	c_0	c_1	c_2	α
a_0	2.4124E-09	1.8447E-07	-5.1192E-07	2.6776E-09	1.9402E-07	-5.4153E-07	-4.0135E-08
a_1	1.8447E-07	7.1196E-05	-7.3573E-05	2.0311E-07	7.5449E-05	-7.7693E-05	-5.3592E-08
a_2	-5.1192E-07	-7.3573E-05	1.2989E-04	-5.6825E-07	-7.7713E-05	1.3743E-04	-1.7337E-06
c_0	2.6776E-09	2.0311E-07	-5.6825E-07	3.0896E-09	2.1989E-07	-6.2307E-07	-4.4263E-09
c_1	1.9402E-07	7.5449E-05	-7.7713E-05	2.1989E-07	8.2658E-05	-8.4678E-05	1.2033E-08
c_2	-5.4153E-07	-7.7693E-05	1.3743E-04	-6.2307E-07	-8.4678E-05	1.5039E-04	-6.5605E-06
α	-4.0135E-08	-5.3592E-08	-1.7337E-06	-4.4263E-09	1.2033E-08	-6.5605E-06	2.4132E-04

Table 17. Variance-Covariance matrix for the parameters of the Exponential Merchantable Diameter equation (equation 20) fitted to Oklahoma merchantable green weight data.

Exp. Merch. Dia. (Equ. 27) Var-Cov Matrix for Green Weight Parameters						
	a_0	a_1	a_2	c_0	c_1	c_2
a_0	8.1014E-05	3.1157E-05	-1.8972E-04	-2.0100E-04	4.4052E-05	1.2355E-04
a_1	3.1157E-05	4.3683E-04	-3.3051E-04	8.1033E-04	-2.5195E-05	-3.4255E-04
a_2	-1.8972E-04	-3.3051E-04	6.0243E-04	-7.1952E-05	-1.5543E-04	-1.2928E-04
c_0	-2.0100E-04	8.1033E-04	-7.1952E-05	8.4697E-03	-8.5604E-04	-4.0917E-03
c_1	4.4052E-05	-2.5195E-05	-1.5543E-04	-8.5604E-04	9.6980E-03	9.7553E-03
c_2	1.2355E-04	-3.4255E-04	-1.2928E-04	-4.0917E-03	9.7553E-03	1.1071E-02

Table 18. Variance-Covariance matrix for the parameters of the Exponential Merchantable Diameter equation (equation 20) fitted to Oklahoma merchantable outside bark volume data.

Exp. Merch. Dia. (Equ. 27) Var-Cov Matrix for Outside Bark Volume Parameters						
	a_0	a_1	a_2	c_0	c_1	c_2
a_0	1.0453E-08	2.6047E-07	-1.6988E-06	-1.5313E-06	2.5845E-07	8.9235E-07
a_1	2.6047E-07	2.7418E-04	-2.0494E-04	4.3241E-04	-2.6709E-05	-2.0257E-04
a_2	-1.6988E-06	-2.0494E-04	3.7614E-04	-4.1451E-05	-5.4446E-05	-4.0595E-05
c_0	-1.5313E-06	4.3241E-04	-4.1451E-05	3.9244E-03	-1.8940E-04	-1.7546E-03
c_1	2.5845E-07	-2.6709E-05	-5.4446E-05	-1.8940E-04	2.5820E-03	2.6581E-03
c_2	8.9235E-07	-2.0257E-04	-4.0595E-05	-1.7546E-03	2.6581E-03	3.3655E-03

Table 19. Variance-Covariance matrix for the parameters of the Exponential Merchantable Diameter equation (equation 20) fitted to Oklahoma merchantable inside bark volume data.

Exp. Merch. Dia. (Equ. 27) Var-Cov Matrix for Inside Bark Volume Parameters						
	a_0	a_1	a_2	c_0	c_1	c_2
a_0	5.1191E-09	2.2257E-07	-1.3499E-06	-1.2034E-06	2.2672E-07	6.9031E-07
a_1	2.2257E-07	3.4927E-04	-2.6362E-04	5.5485E-04	-4.6039E-05	-2.5526E-04
a_2	-1.3499E-06	-2.6362E-04	4.8129E-04	-5.4526E-05	-6.6252E-05	-4.9794E-05
c_0	-1.2034E-06	5.5485E-04	-5.4526E-05	5.0088E-03	-3.5333E-04	-2.2005E-03
c_1	2.2672E-07	-4.6039E-05	-6.6252E-05	-3.5333E-04	2.9068E-03	3.0443E-03
c_2	6.9031E-07	-2.5526E-04	-4.9794E-05	-2.2005E-03	3.0443E-03	3.8710E-03

Appendix 2: Residual plots of all fitted equations

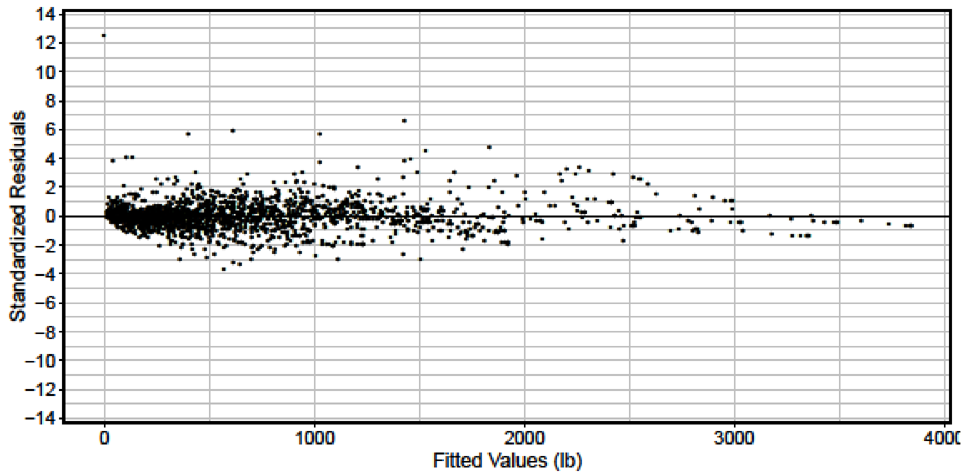

Figure 34. Residuals for Burkhart's merchantable diameter equation fitted to merchantable green weight with Var = POM versus the fitted values associated with the data used to fit the model.

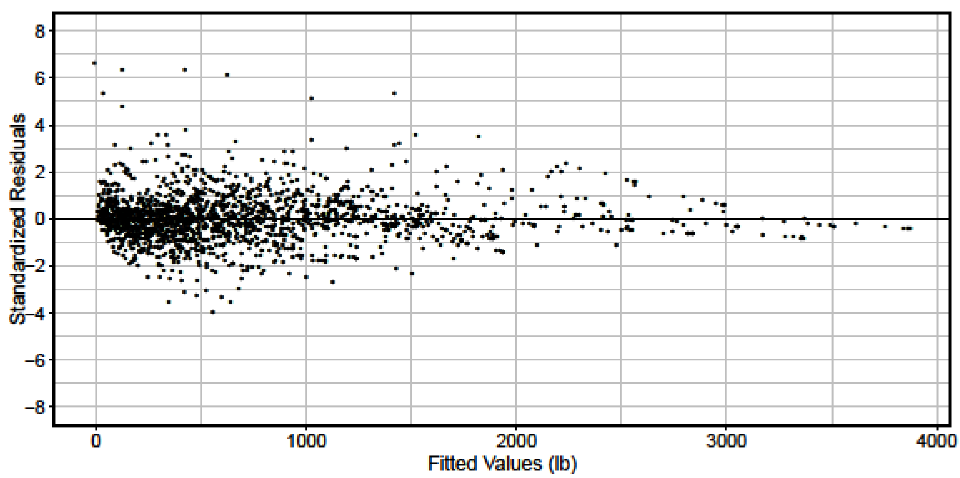

Figure 35. Residuals for Burkhart's merchantable diameter equation fitted to merchantable green weight with Var = C + POM versus the fitted values associated with the data used to fit the model.

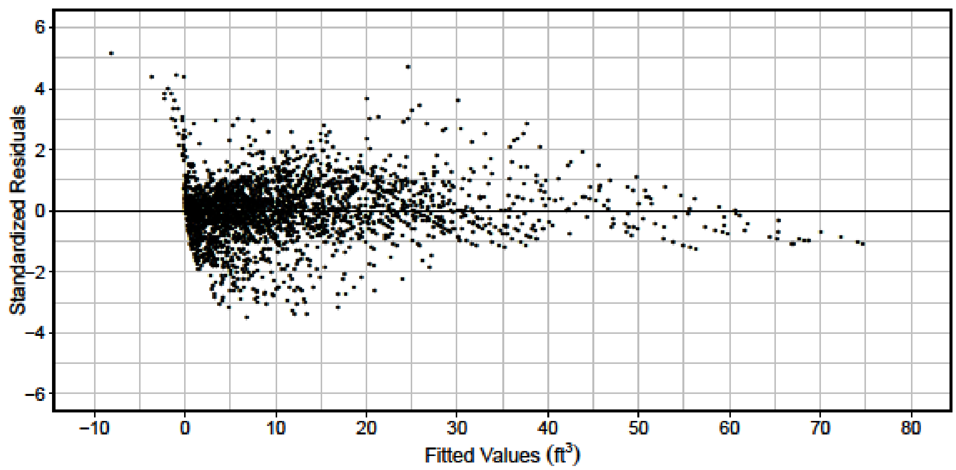

Figure 36. Residuals for Burkhart's merchantable diameter equation fitted to merchantable outside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

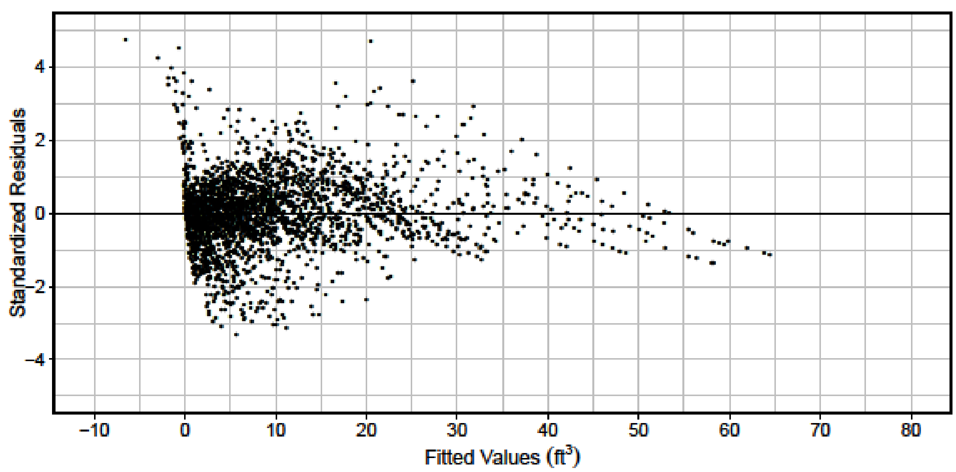

Figure 37. Residuals for Burkhart's merchantable diameter equation fitted to merchantable inside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

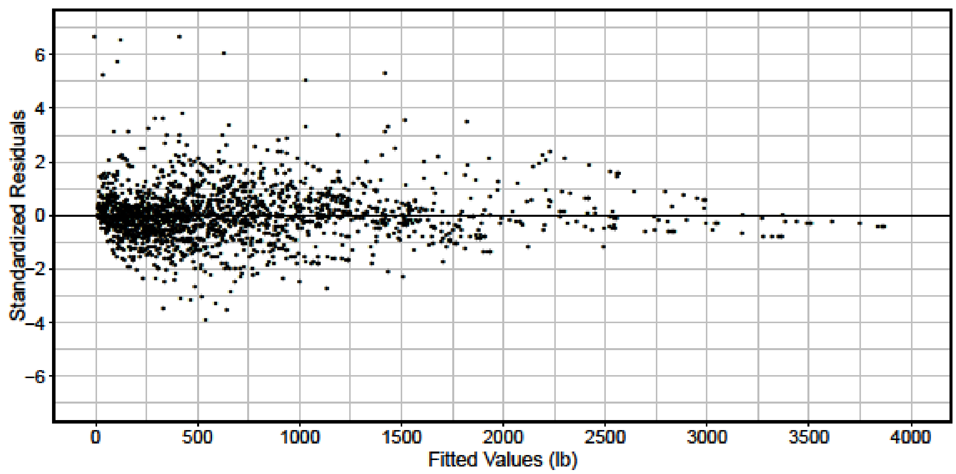

Figure 38. Residuals for the PMRC merchantable diameter equation fitted to merchantable green weight with Var = C + POM versus the fitted values associated with the data used to fit the model.

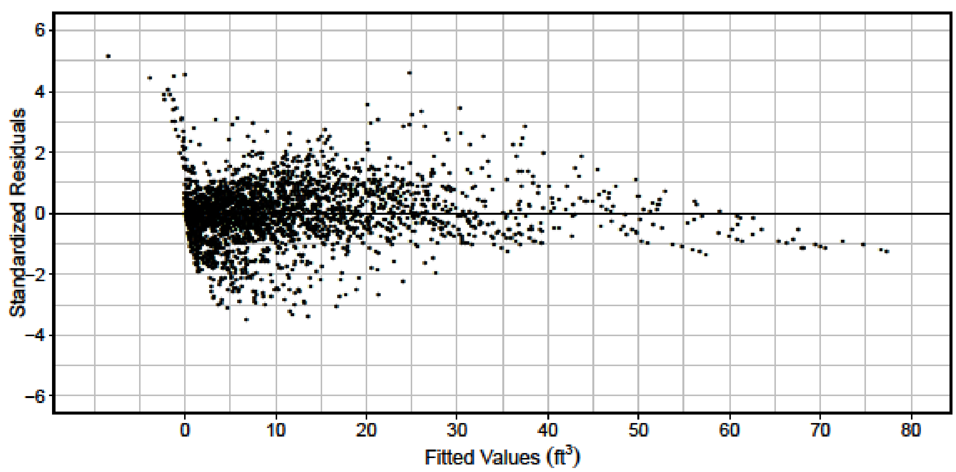

Figure 39. Residuals for the PMRC merchantable diameter equation fitted to merchantable outside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

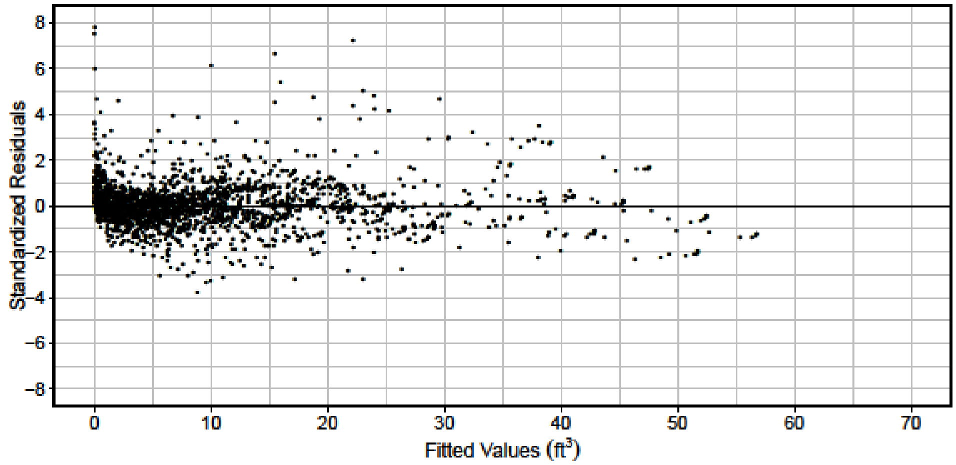

Figure 40. Residuals for the PMRC merchantable diameter equation fitted to merchantable inside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

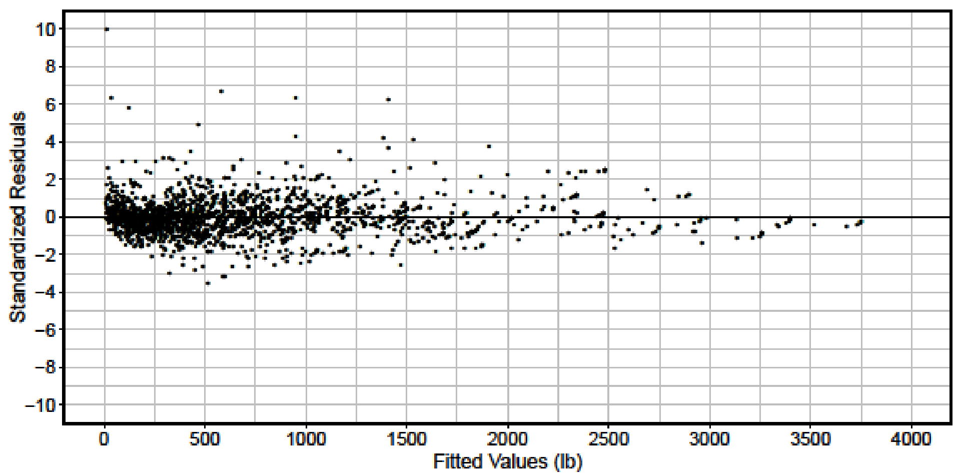

Figure 41. Residuals for the exponential merchantable diameter equation fitted to merchantable green weight with Var = POM versus the fitted values associated with the data used to fit the model.

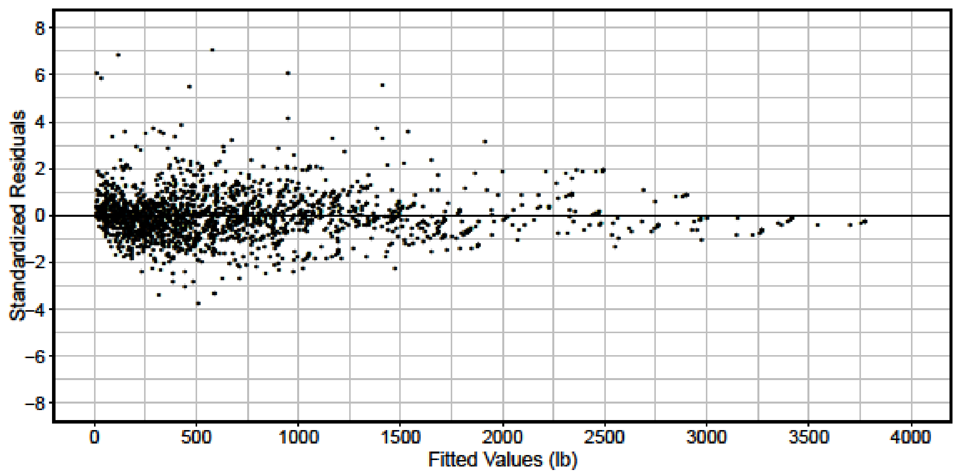

Figure 42. Residuals for the exponential merchantable diameter equation fitted to merchantable green weight with Var = C + POM versus the fitted values associated with the data used to fit the model.

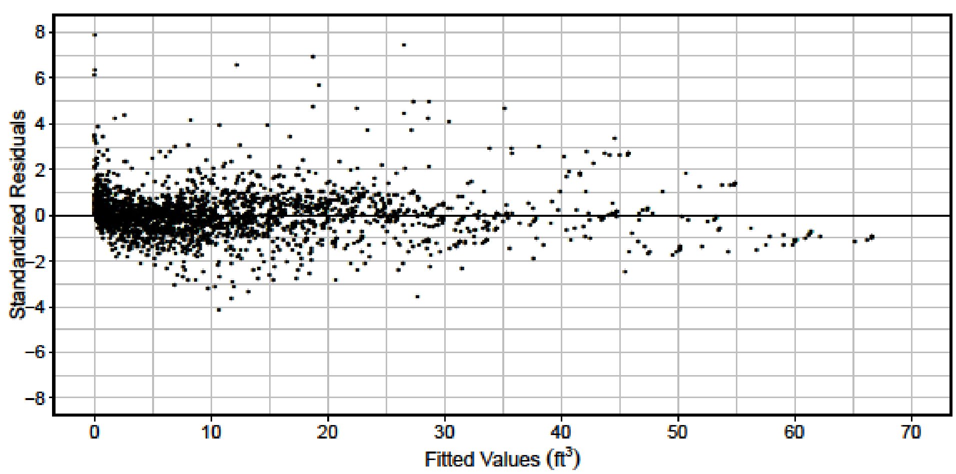

Figure 43. Residuals for the exponential merchantable diameter equation fitted to merchantable outside bark volume with Var = POM versus the fitted values associated with the data used to fit the model.

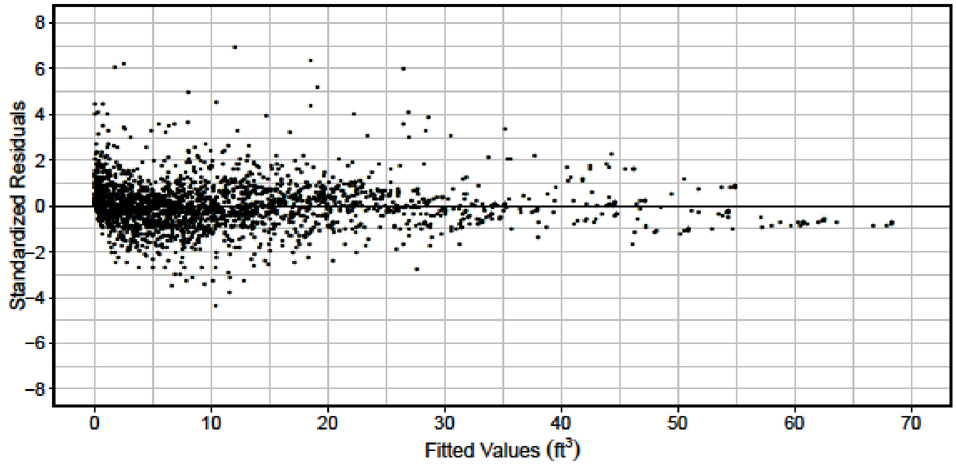

Figure 44. Residuals for the exponential merchantable diameter equation fitted to merchantable outside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

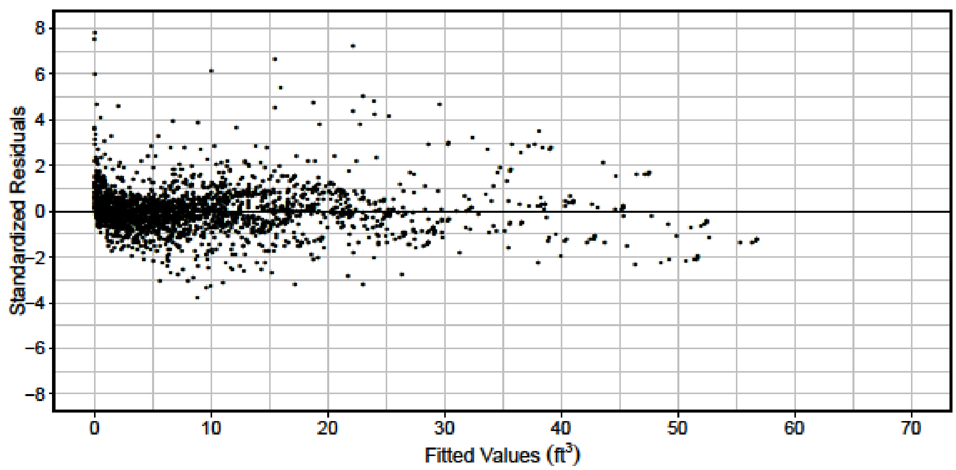

Figure 45. Residuals for the exponential merchantable diameter equation fitted to merchantable inside bark volume with Var = POM versus the fitted values associated with the data used to fit the model.

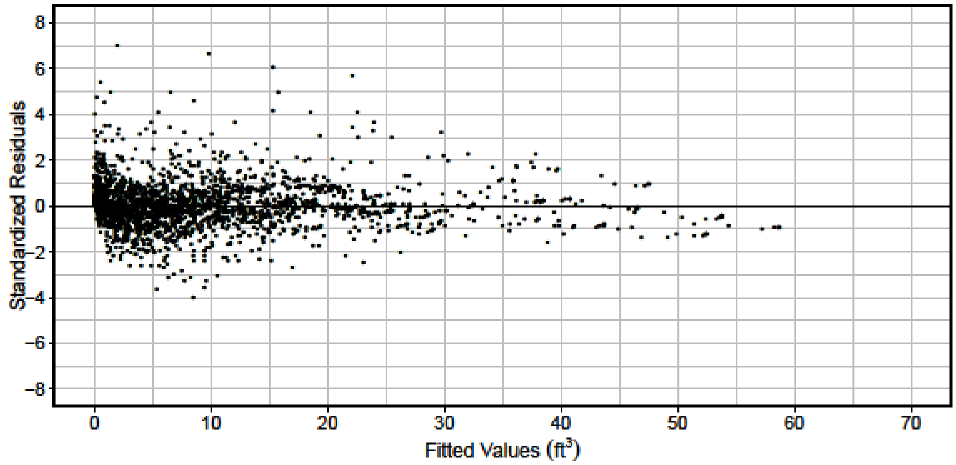

Figure 46. Residuals for the exponential merchantable diameter equation fitted to merchantable inside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

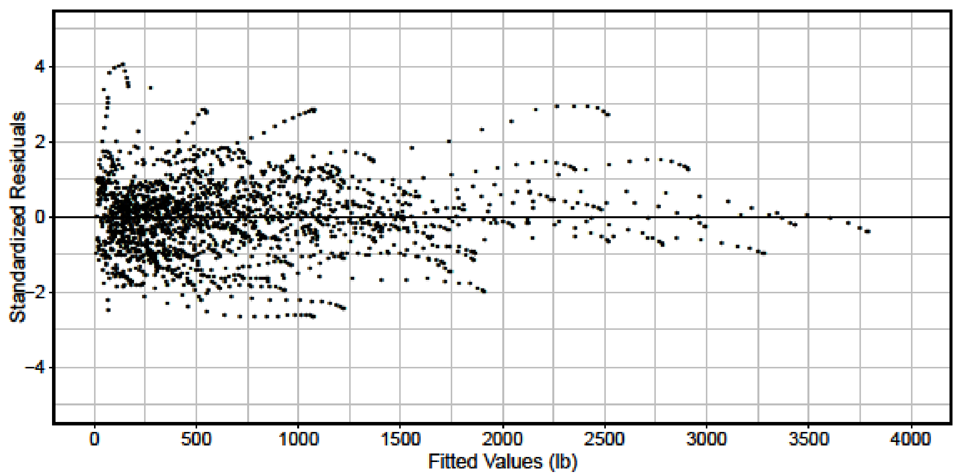

Figure 47. Residuals for the Burkhart merchantable height equation fitted to merchantable green weight with Var = POM versus the fitted values associated with the data used to fit the model.

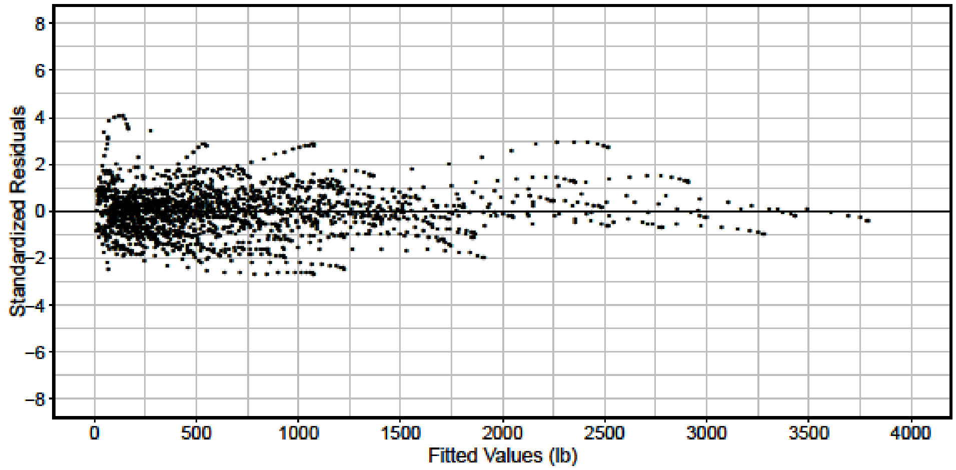

Figure 48. Residuals for the Burkhart merchantable height equation fitted to merchantable green weight with Var = C + POM versus the fitted values associated with the data used to fit the model.

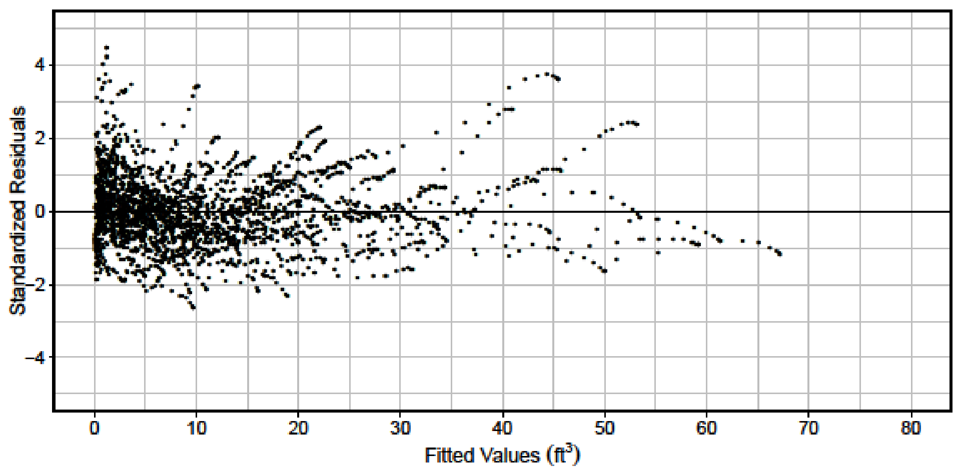

Figure 49. Residuals for the Burkhart merchantable height equation fitted to merchantable outside bark volume with Var = POM versus the fitted values associated with the data used to fit the model.

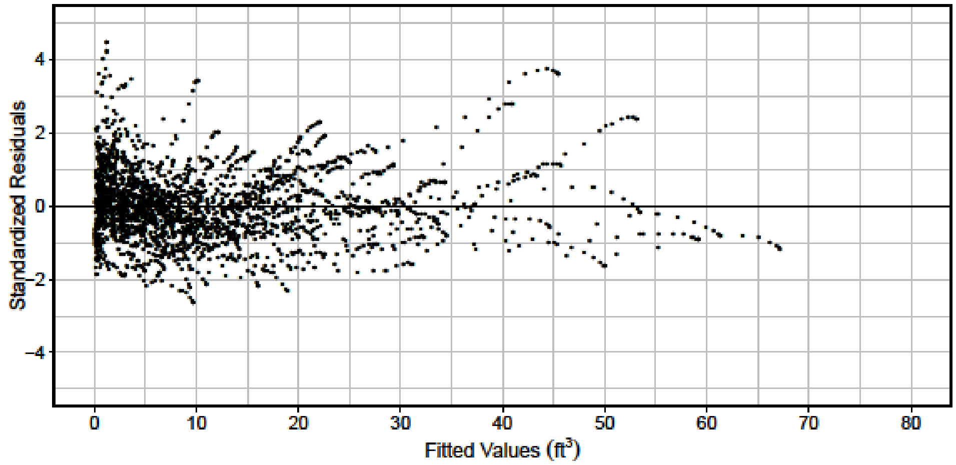

Figure 50. Residuals for the Burkhart merchantable height equation fitted to merchantable outside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

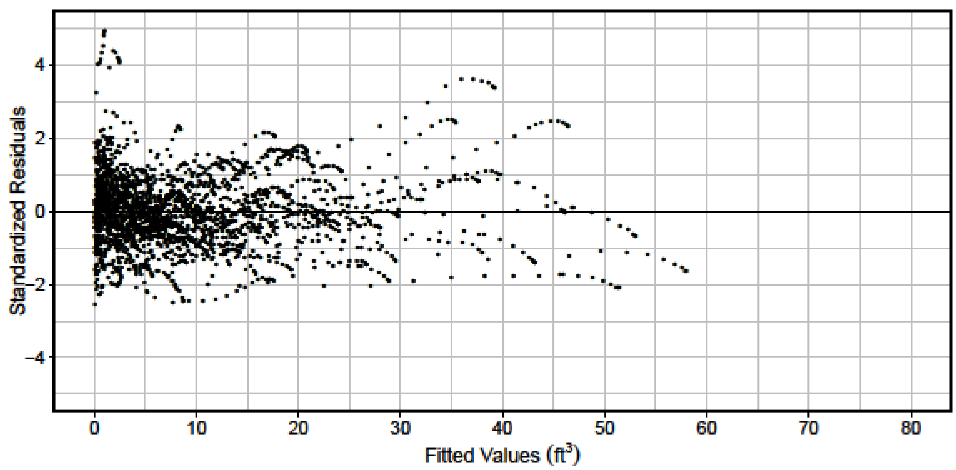

Figure 51. Residuals for the Burkhart merchantable height equation fitted to merchantable inside bark volume with Var = POM versus the fitted values associated with the data used to fit the model.

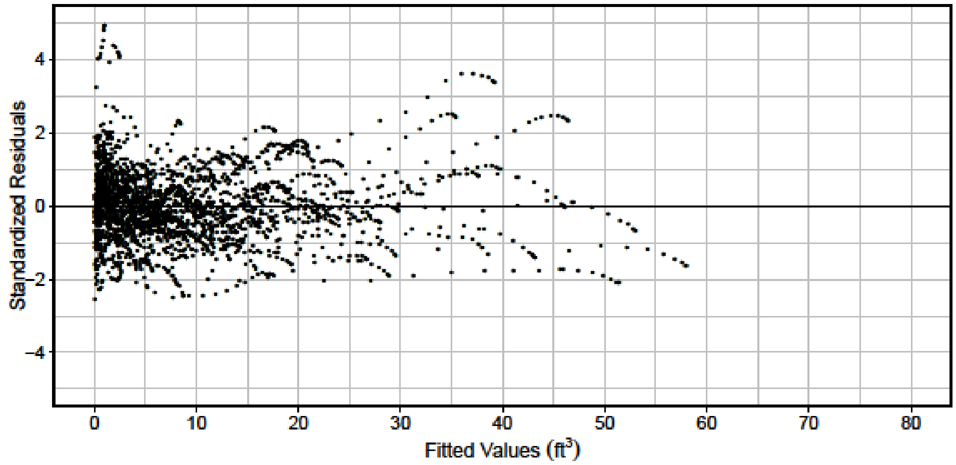

Figure 52. Residuals for the Burkhart merchantable height equation fitted to merchantable inside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

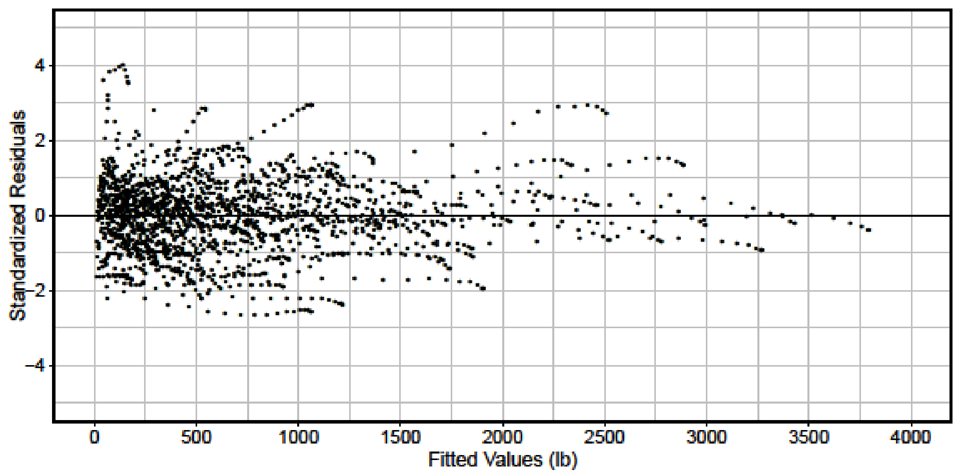

Figure 53. Residuals for the Zhao and Kane 1 merchantable height equation fitted to merchantable green weight with Var = C + POM versus the fitted values associated with the data used to fit the model.

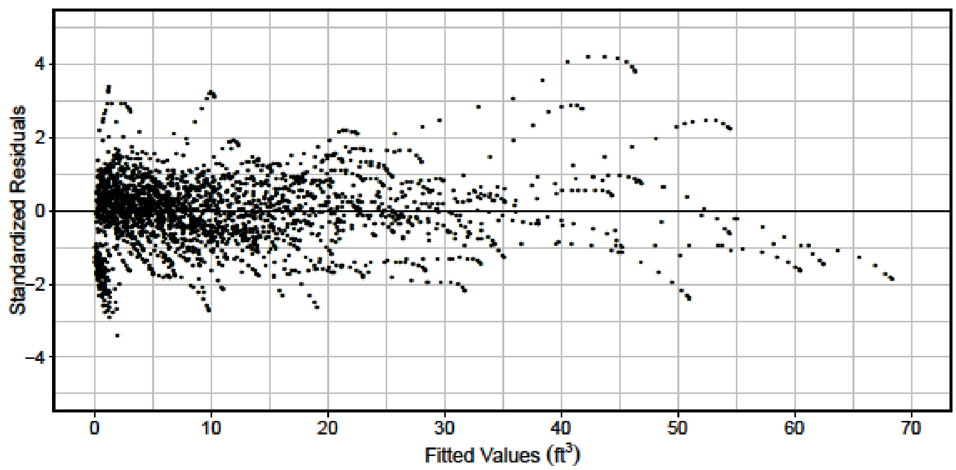

Figure 54. Residuals for the Zhao and Kane 1 merchantable height equation fitted to merchantable outside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

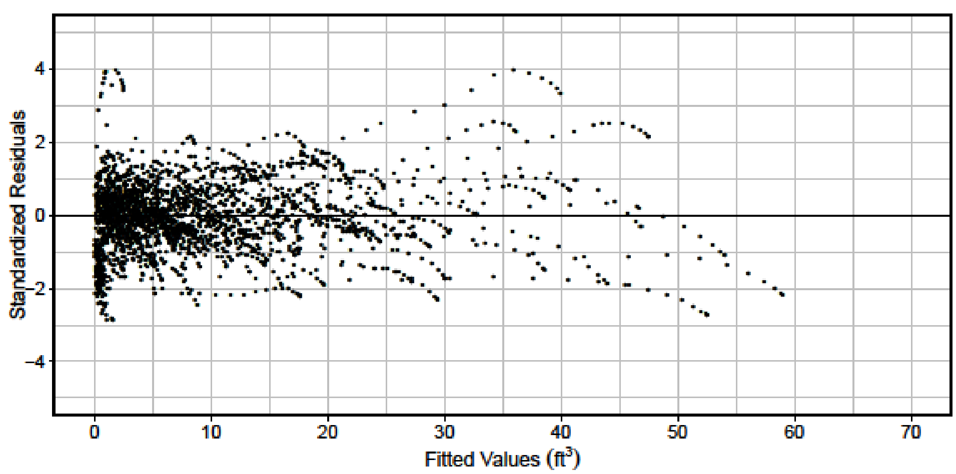

Figure 55. Residuals for the Zhao and Kane 1 merchantable height equation fitted to merchantable inside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

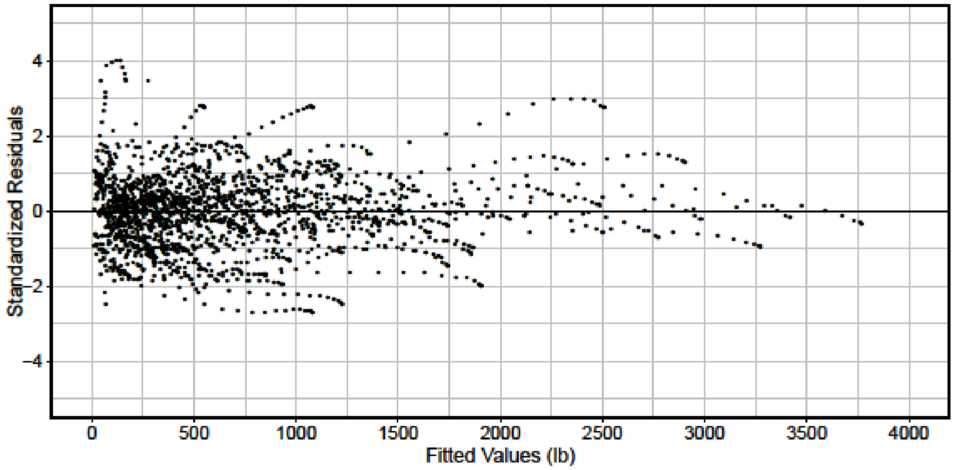

Figure 56. Residuals for the Zhao and Kane 2 merchantable height equation fitted to merchantable green weight with Var = C + POM versus the fitted values associated with the data used to fit the model.

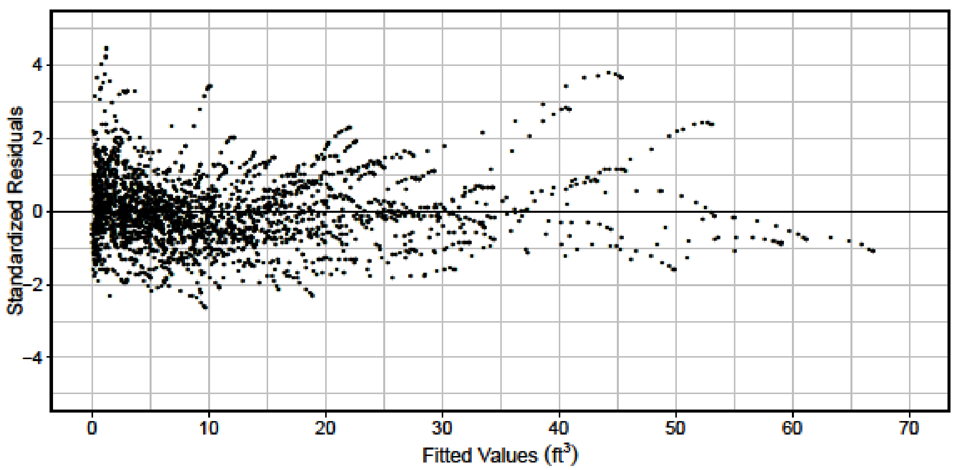

Figure 57. Residuals for the Zhao and Kane 2 merchantable height equation fitted to merchantable outside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

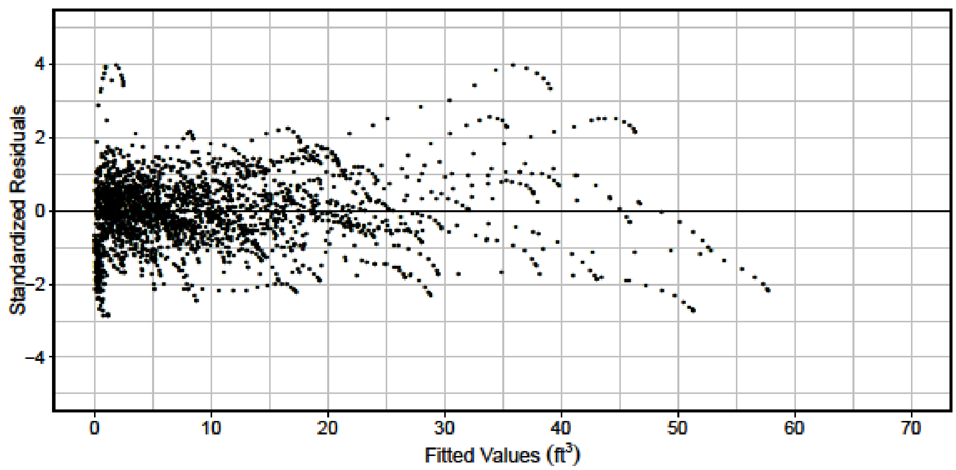

Figure 58. Residuals for the Zhao and Kane 2 merchantable height equation fitted to merchantable inside bark volume with Var = C + POM versus the fitted values associated with the data used to fit the model.

Appendix 3: Residual plots of final equations versus independent variables

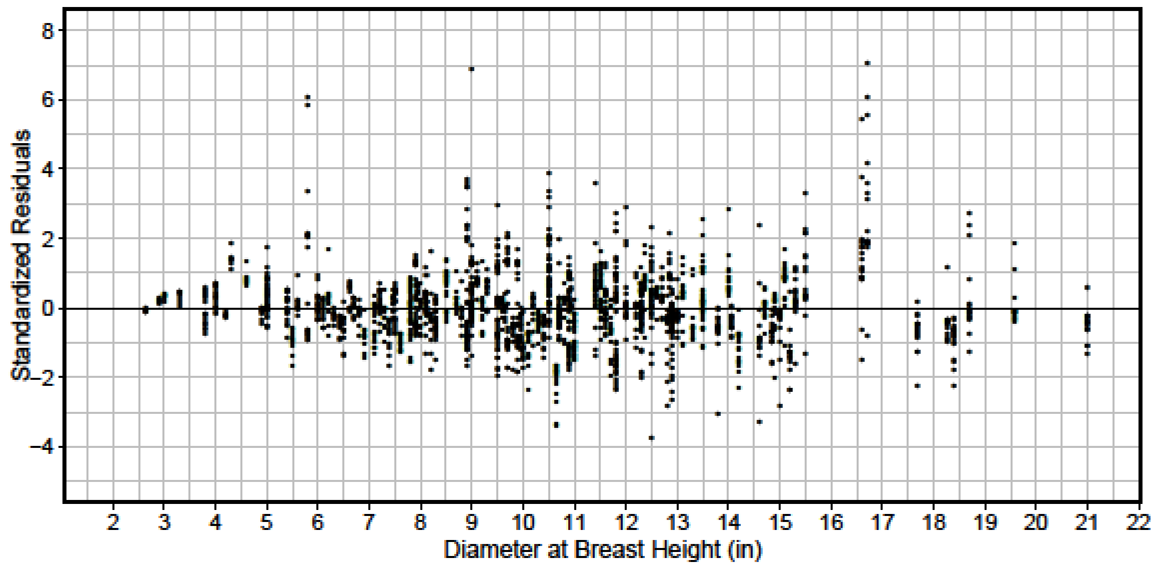

Figure 59. Residuals of the exponential merchantable diameter equation fitted to green weight data versus dbh.

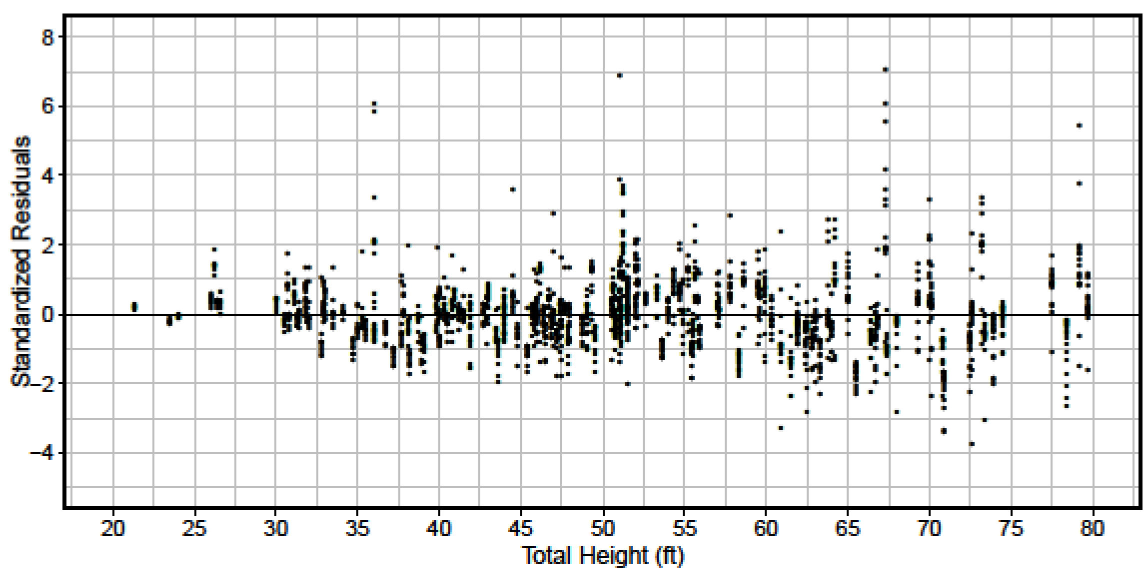

Figure 60. Residuals of the exponential merchantable diameter equation fitted to green weight data versus total height.

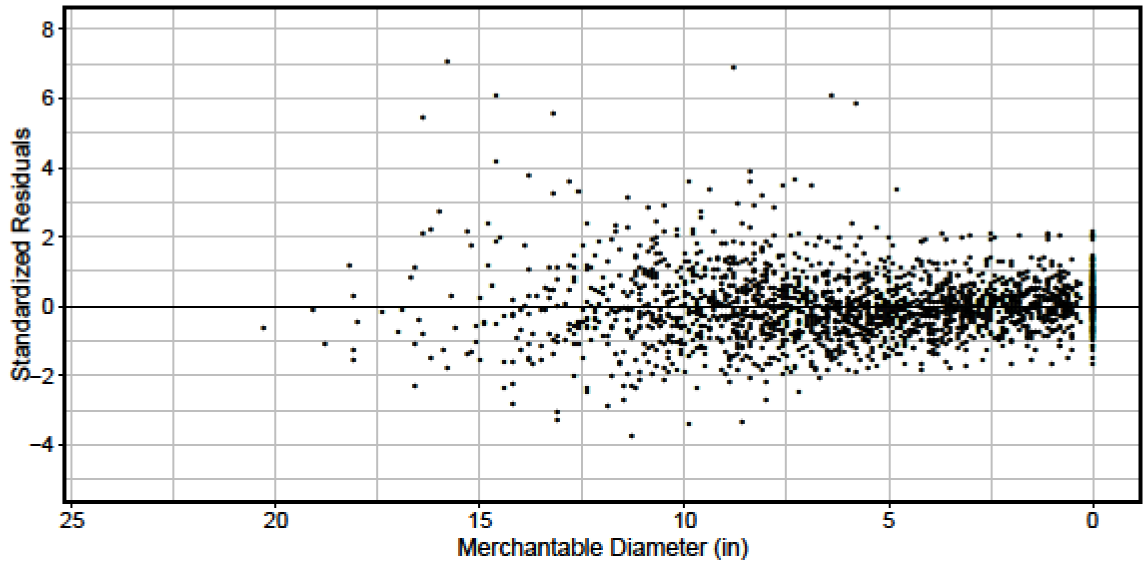

Figure 61. Residuals of the exponential merchantable diameter equation fitted to green weight data versus merchantable diameter.

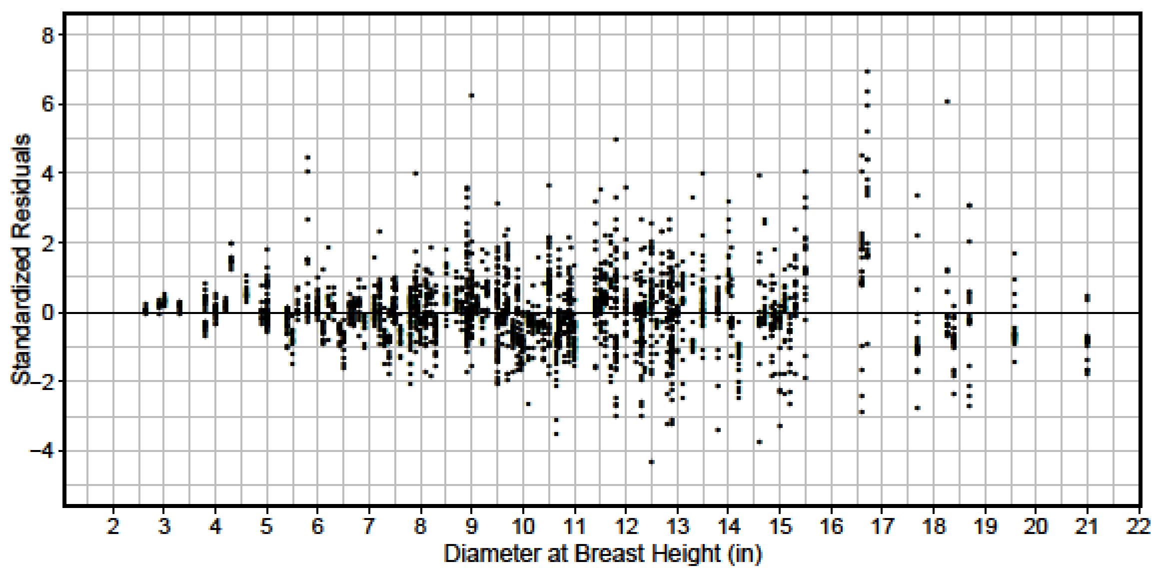

Figure 62. Residuals of the exponential merchantable diameter equation fitted to outside bark volume data versus dbh.

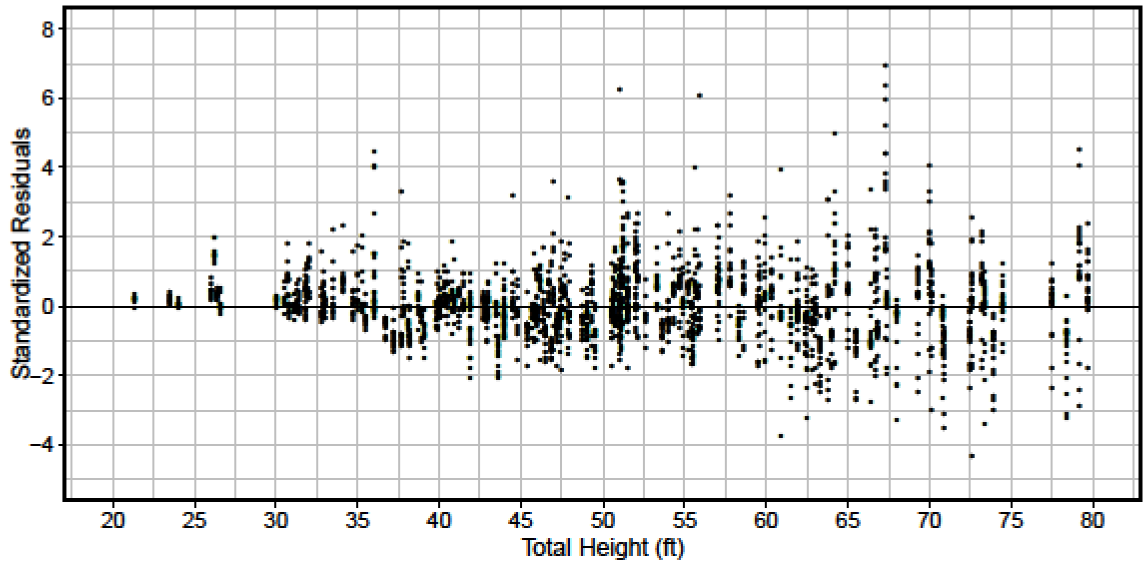

Figure 63. Residuals of the exponential merchantable diameter equation fitted to outside bark volume data versus total height.

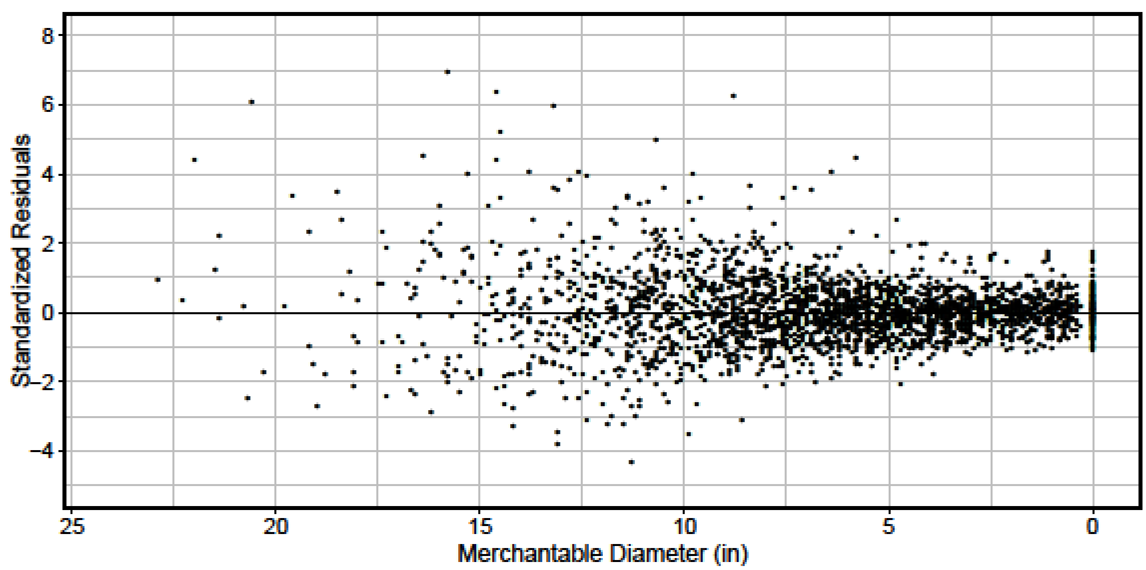

Figure 64. Residuals of the exponential merchantable diameter equation fitted to outside bark volume data versus merchantable diameter.

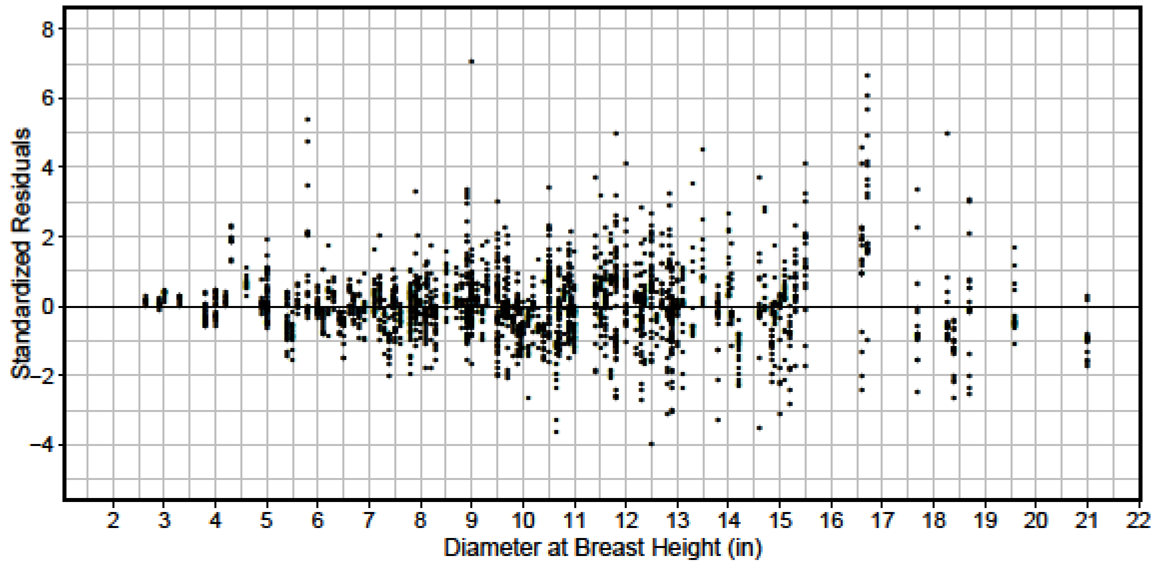

Figure 65. Residuals of the exponential merchantable diameter equation fitted to inside bark volume data versus dbh.

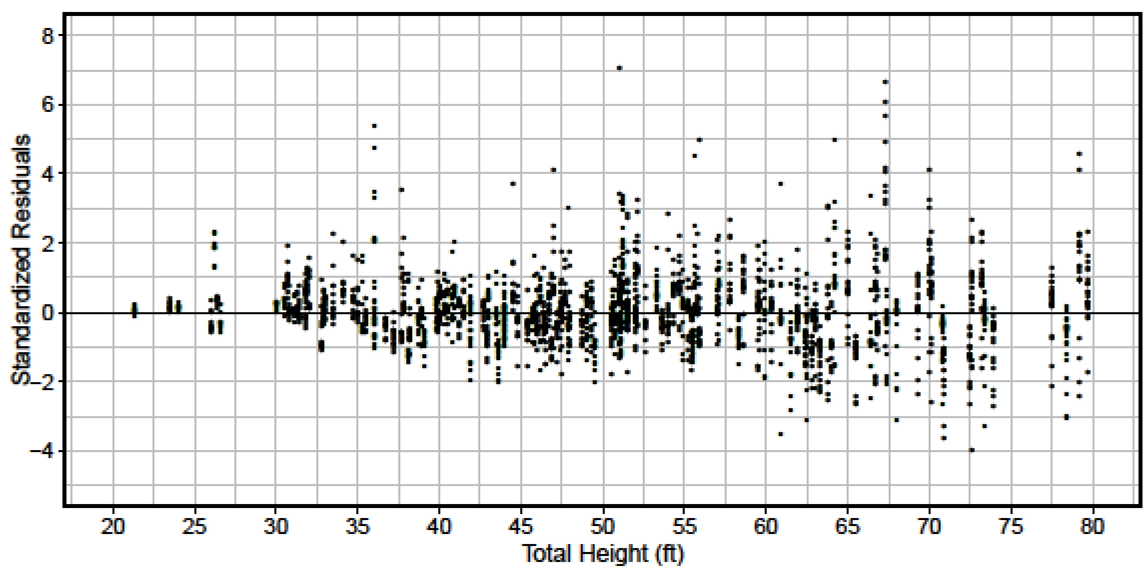

Figure 66. Residuals of the exponential merchantable diameter equation fitted to inside bark volume data versus total height.

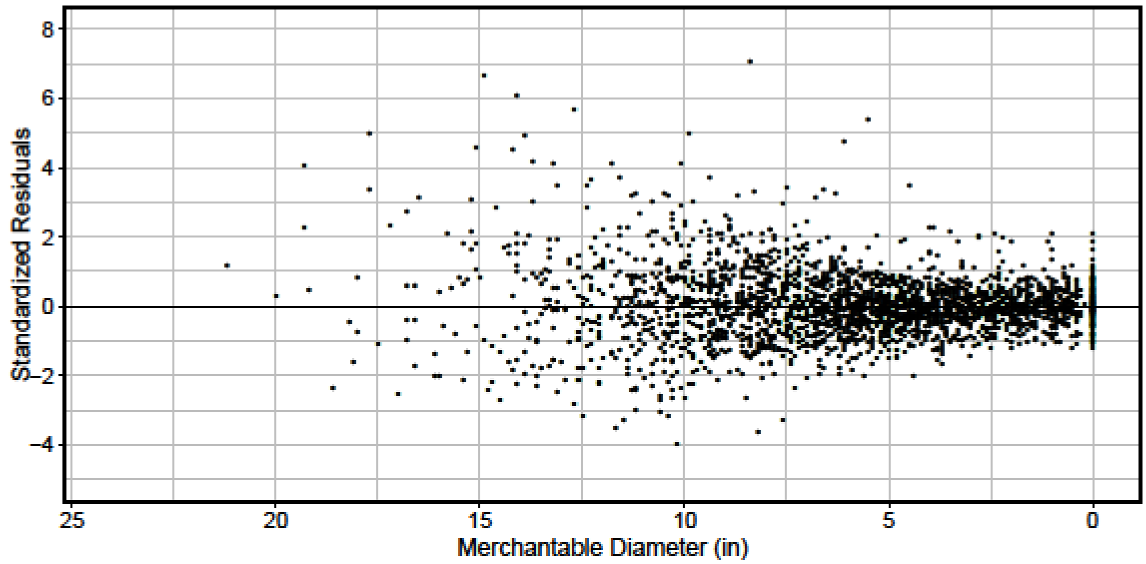

Figure 67. Residuals of the exponential merchantable diameter equation fitted to inside bark volume data versus merchantable diameter.

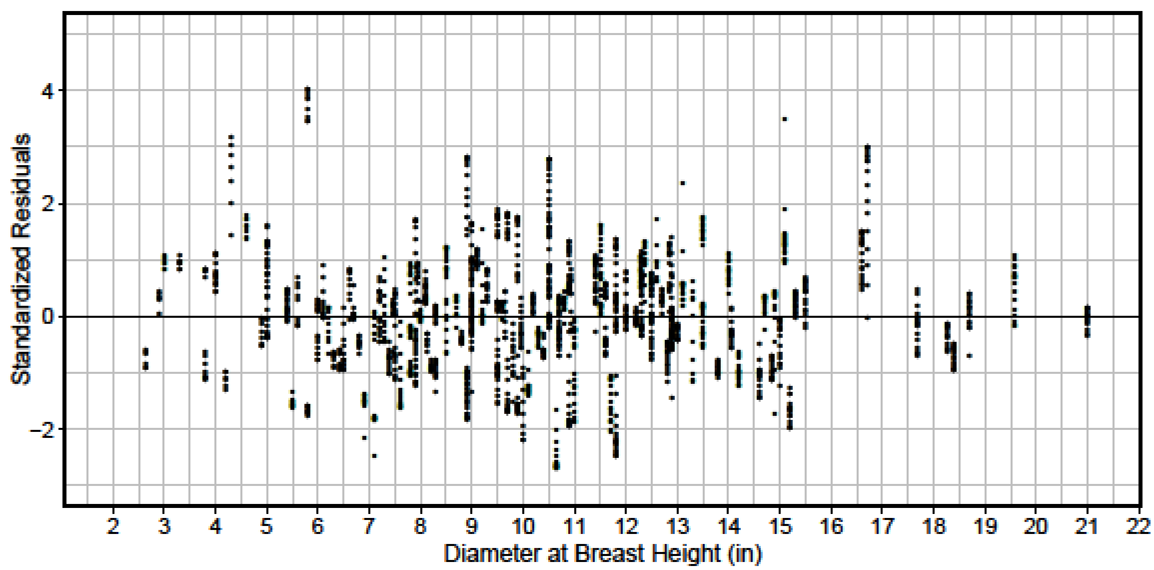

Figure 68. Residuals of the Zhao and Kane 2 merchantable height equation fitted to green weight data versus dbh.

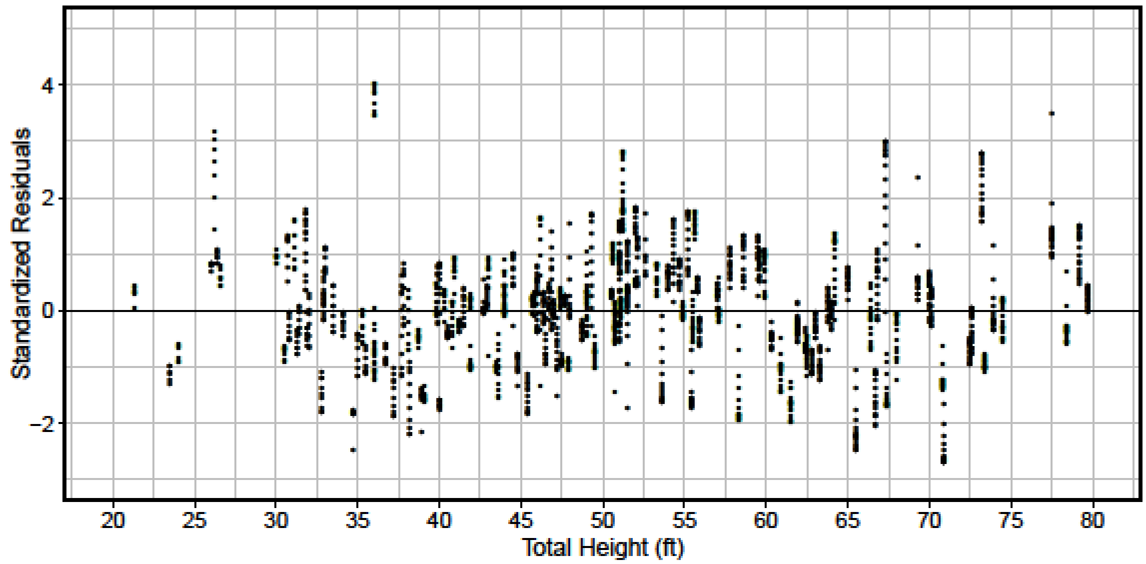

Figure 69. Residuals of the Zhao and Kane 2 merchantable height equation fitted to green weight data versus total height.

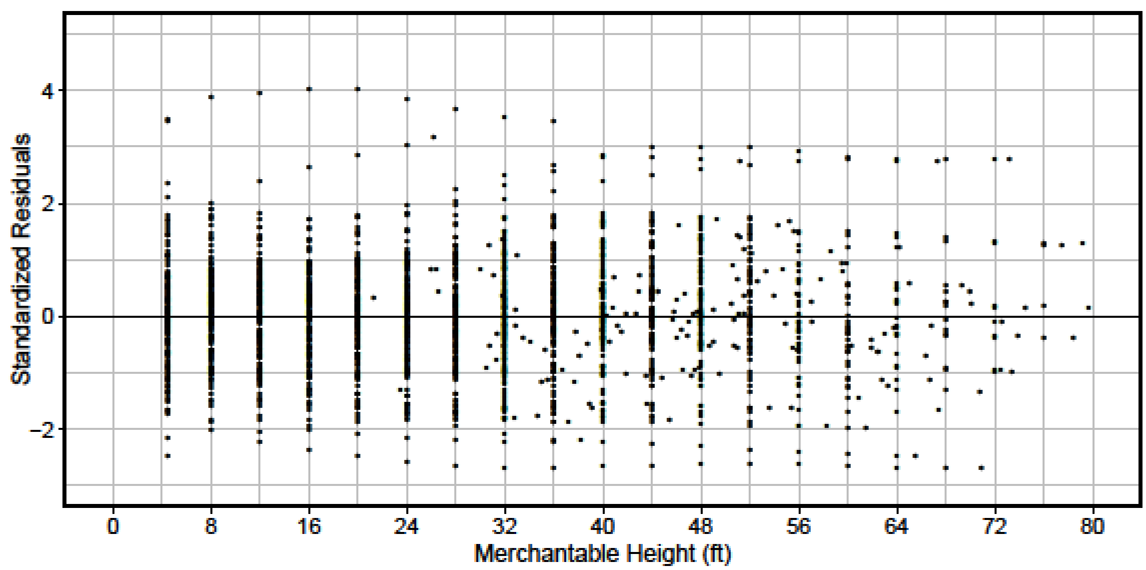

Figure 70. Residuals of the Zhao and Kane 2 merchantable height equation fitted to green weight data versus merchantable height.

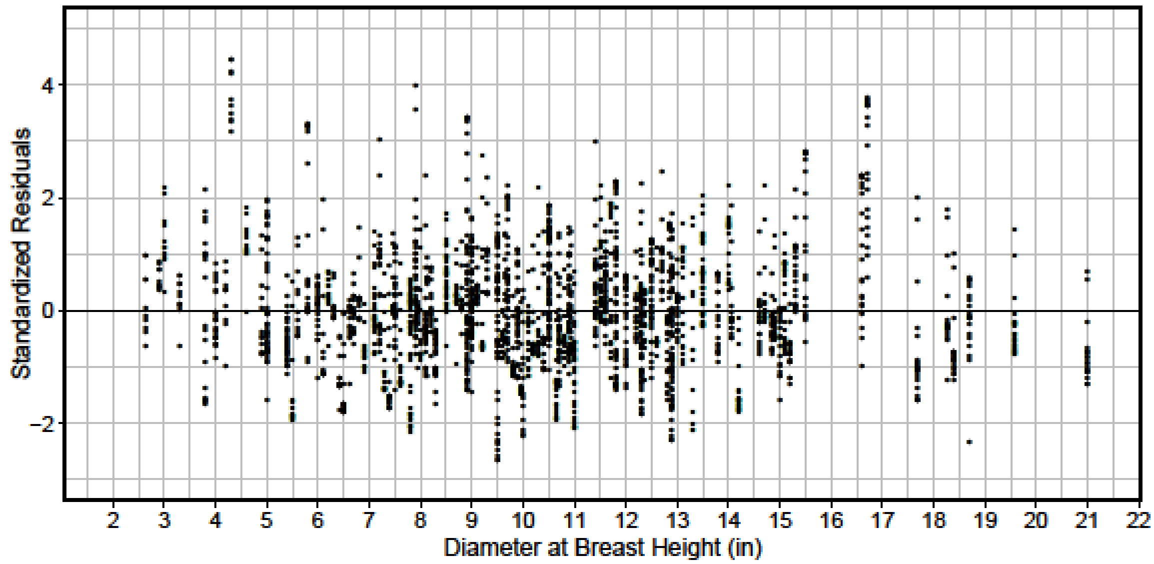

Figure 71. Residuals of the Zhao and Kane 2 merchantable height equation fitted to outside bark volume data versus dbh.

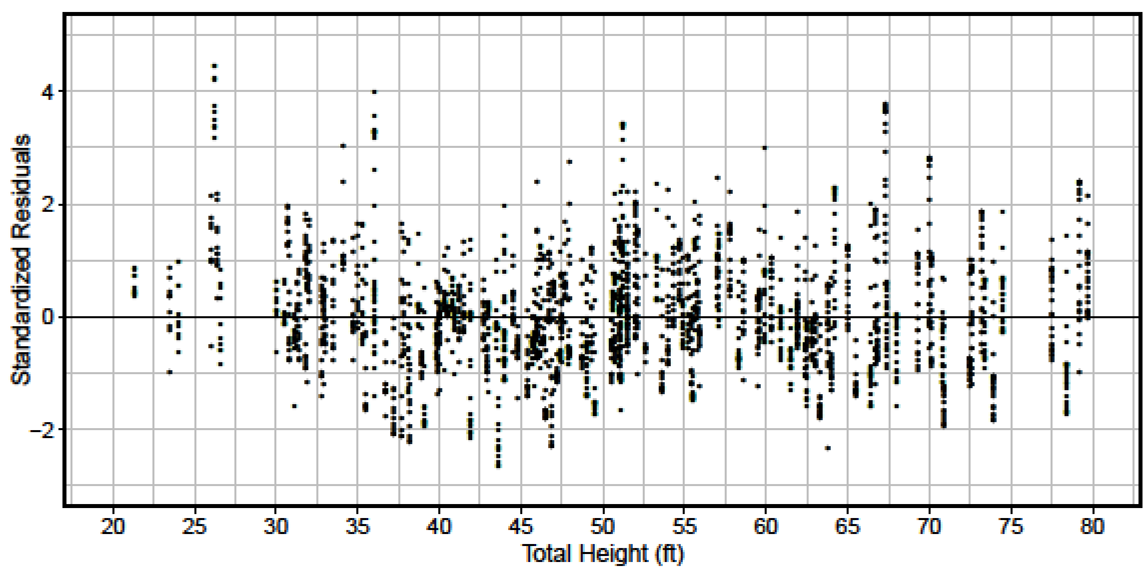

Figure 72. Residuals of the Zhao and Kane 2 merchantable height equation fitted to outside bark volume data versus total height.

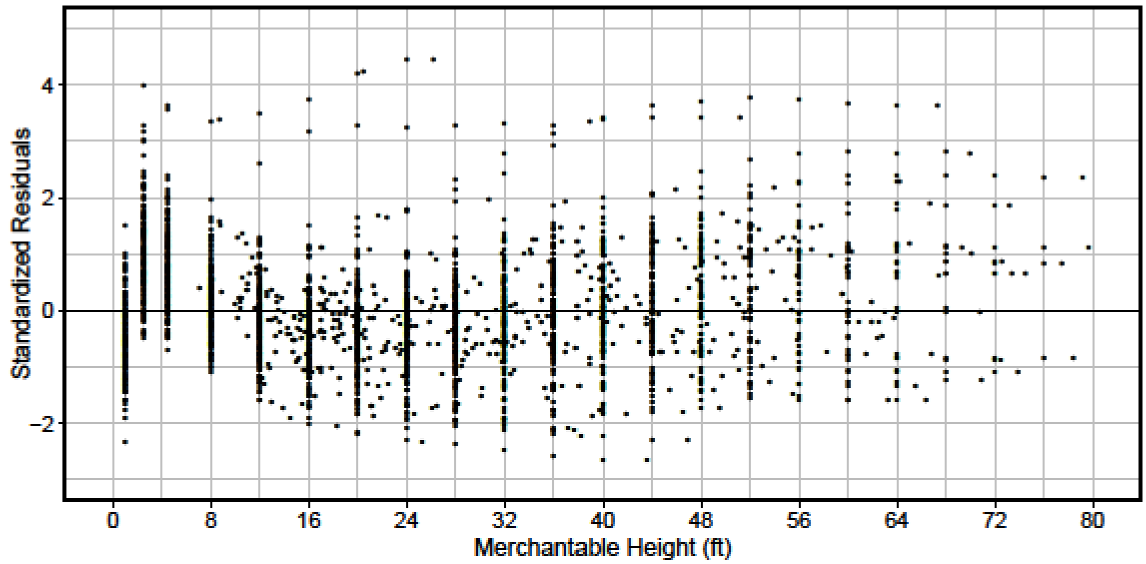

Figure 73. Residuals of the Zhao and Kane 2 merchantable height equation fitted to outside bark volume data versus merchantable height.

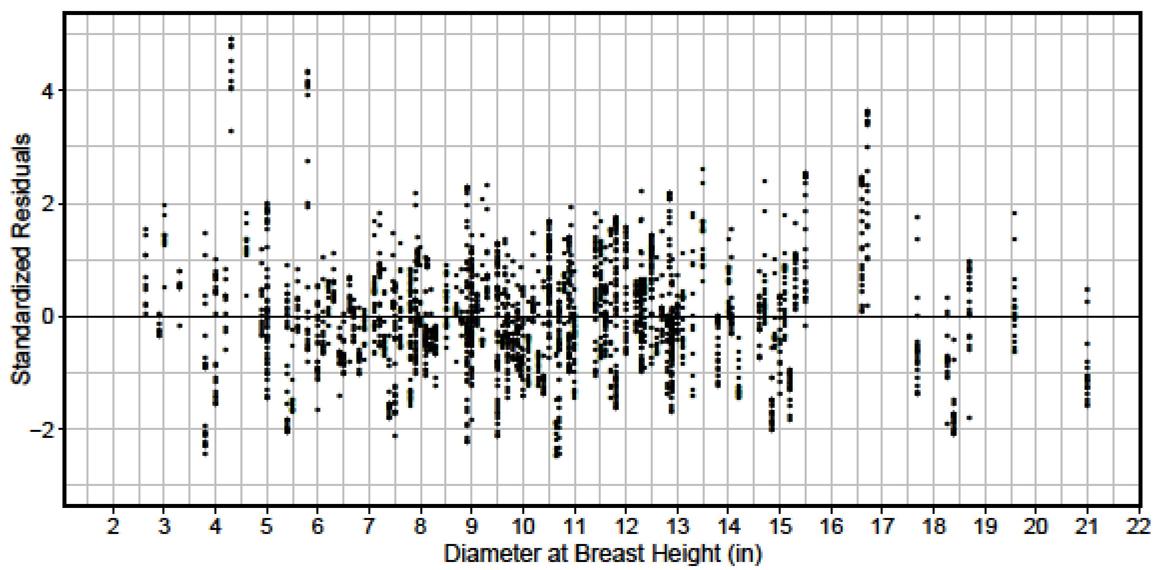

Figure 74. Residuals of the Zhao and Kane 2 merchantable height equation fitted to inside bark volume data versus dbh.

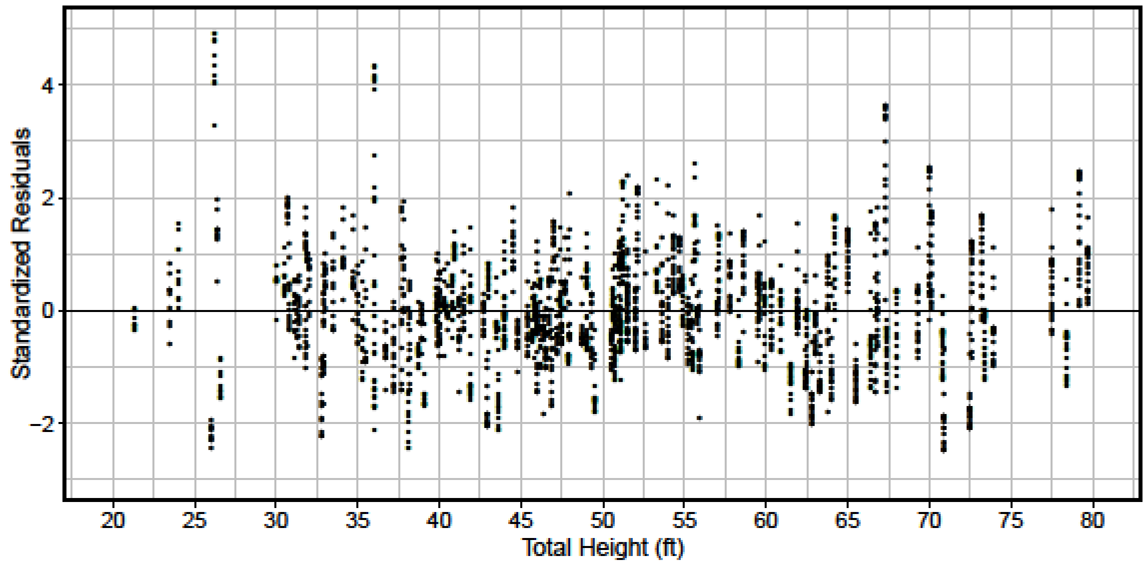

Figure 75. Residuals of the Zhao and Kane 2 merchantable height equation fitted to inside bark volume data versus total height.

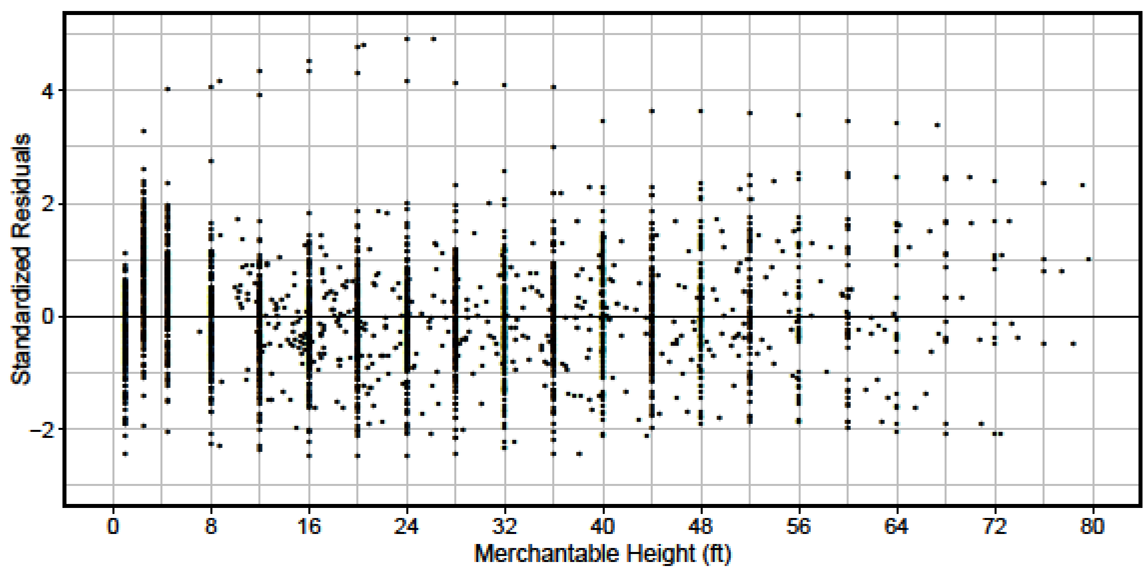

Figure 76. Residuals of the Zhao and Kane 2 merchantable height equation fitted to inside bark volume data versus merchantable height.

Appendix 4: Plots showing residuals of models versus site and stand variables

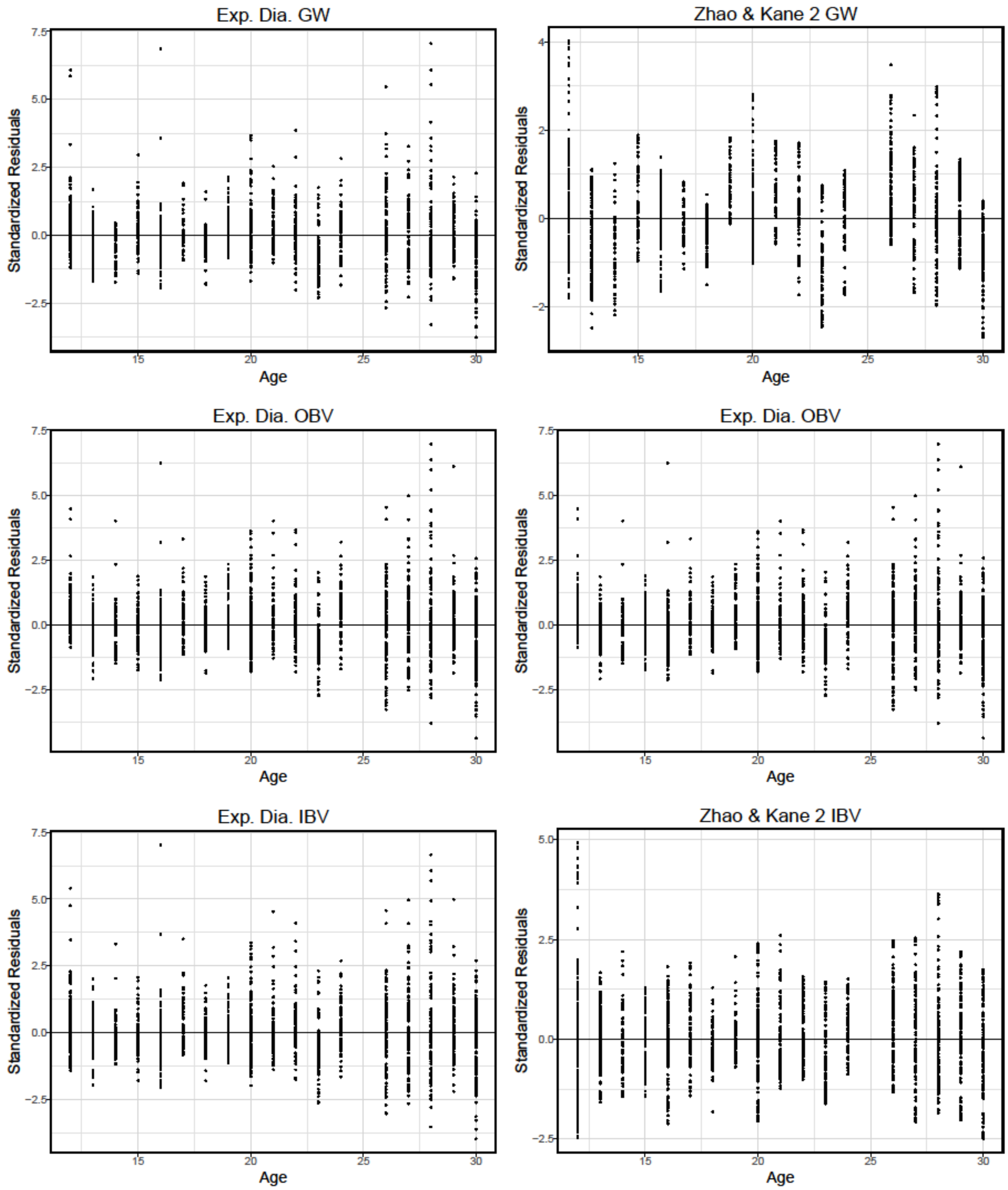

Figure 77. Residuals of selected models versus tree age.

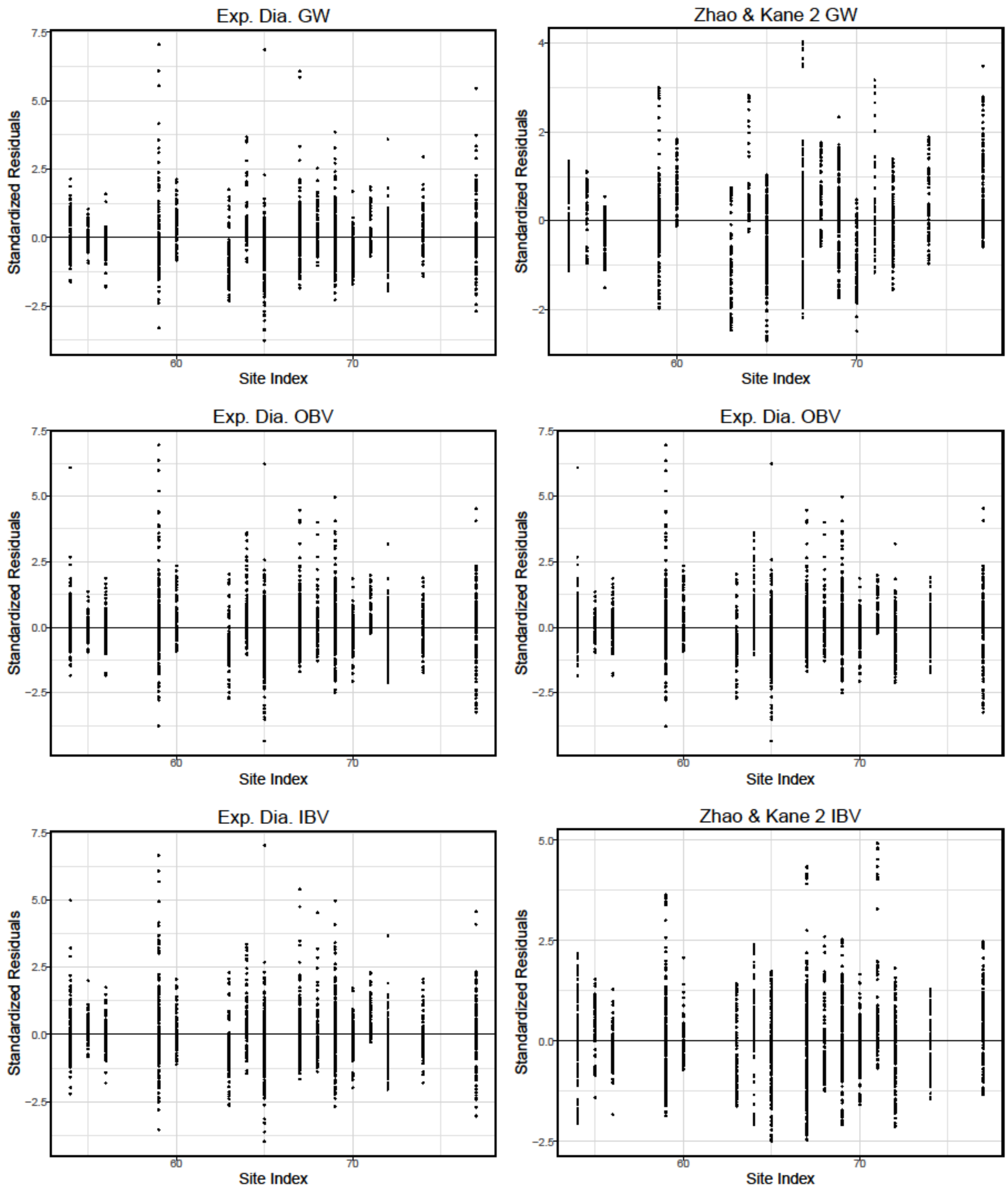

Figure 78. Residuals of selected models versus stand site index.

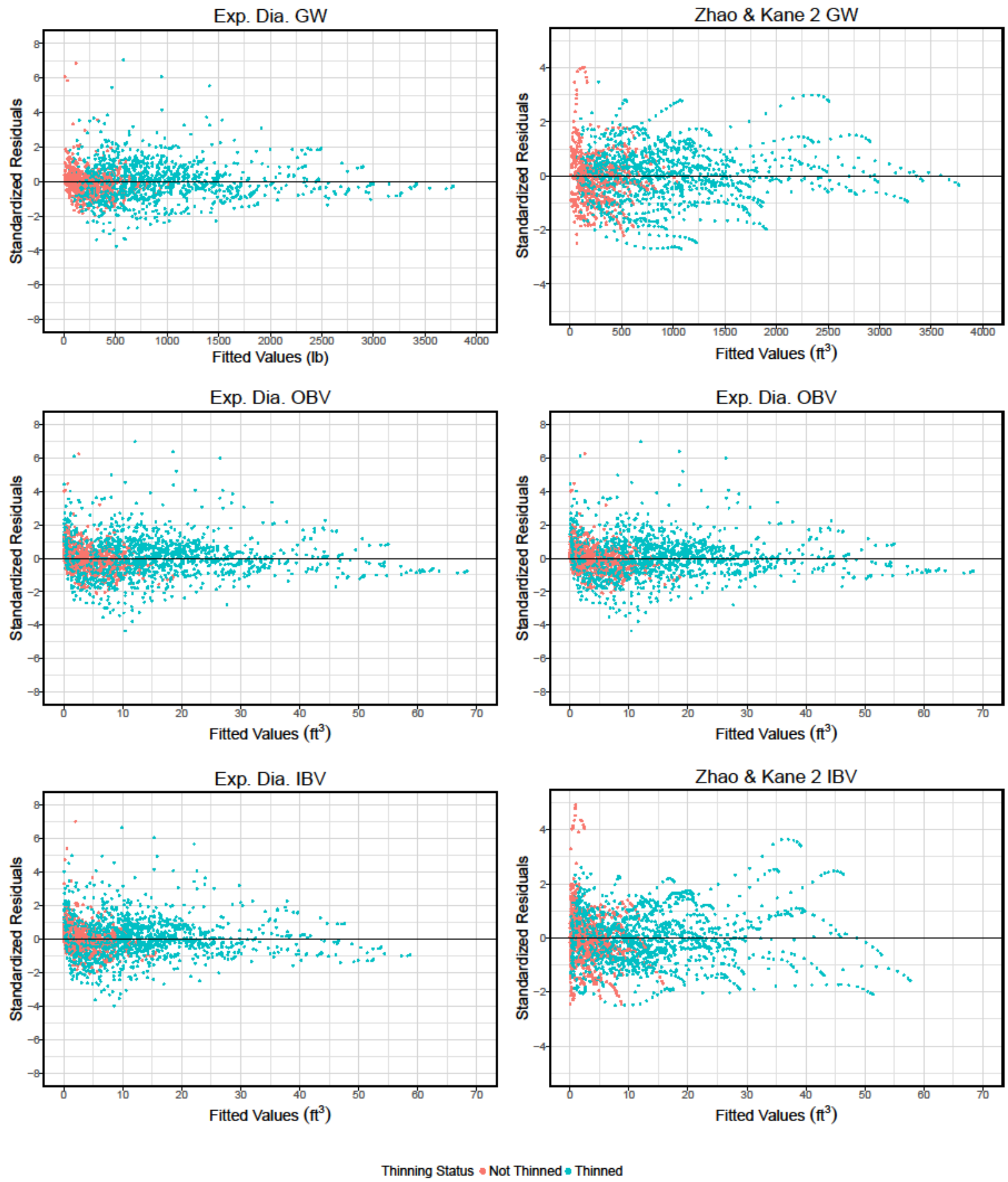

Figure 79. Residuals of selected models versus fitted values, points colored based on an estimate of thinning status—red shows not thinned, blue shows thinned.

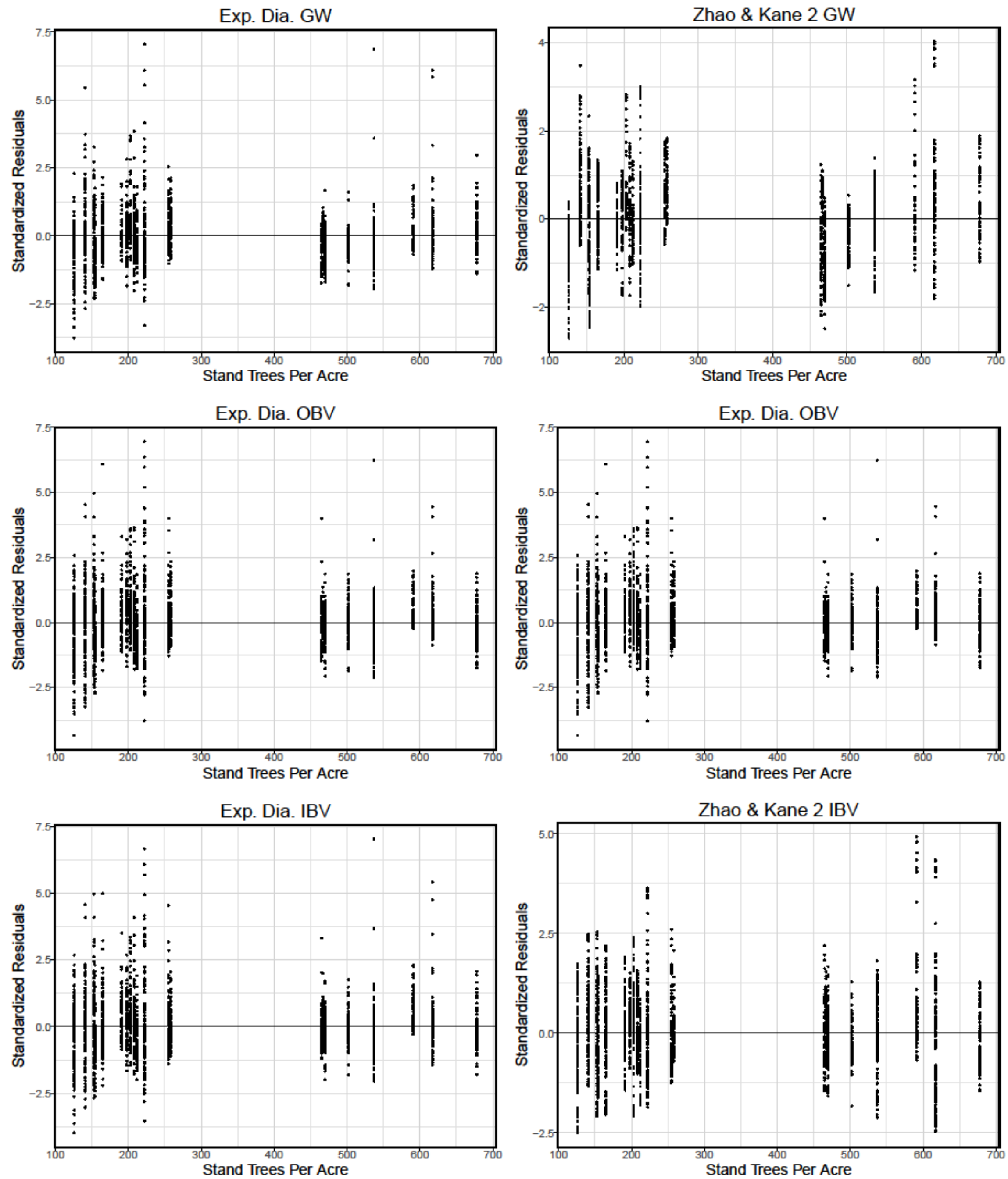

Figure 80. Residuals of selected models versus trees per acre from Rayonier Inc. records.

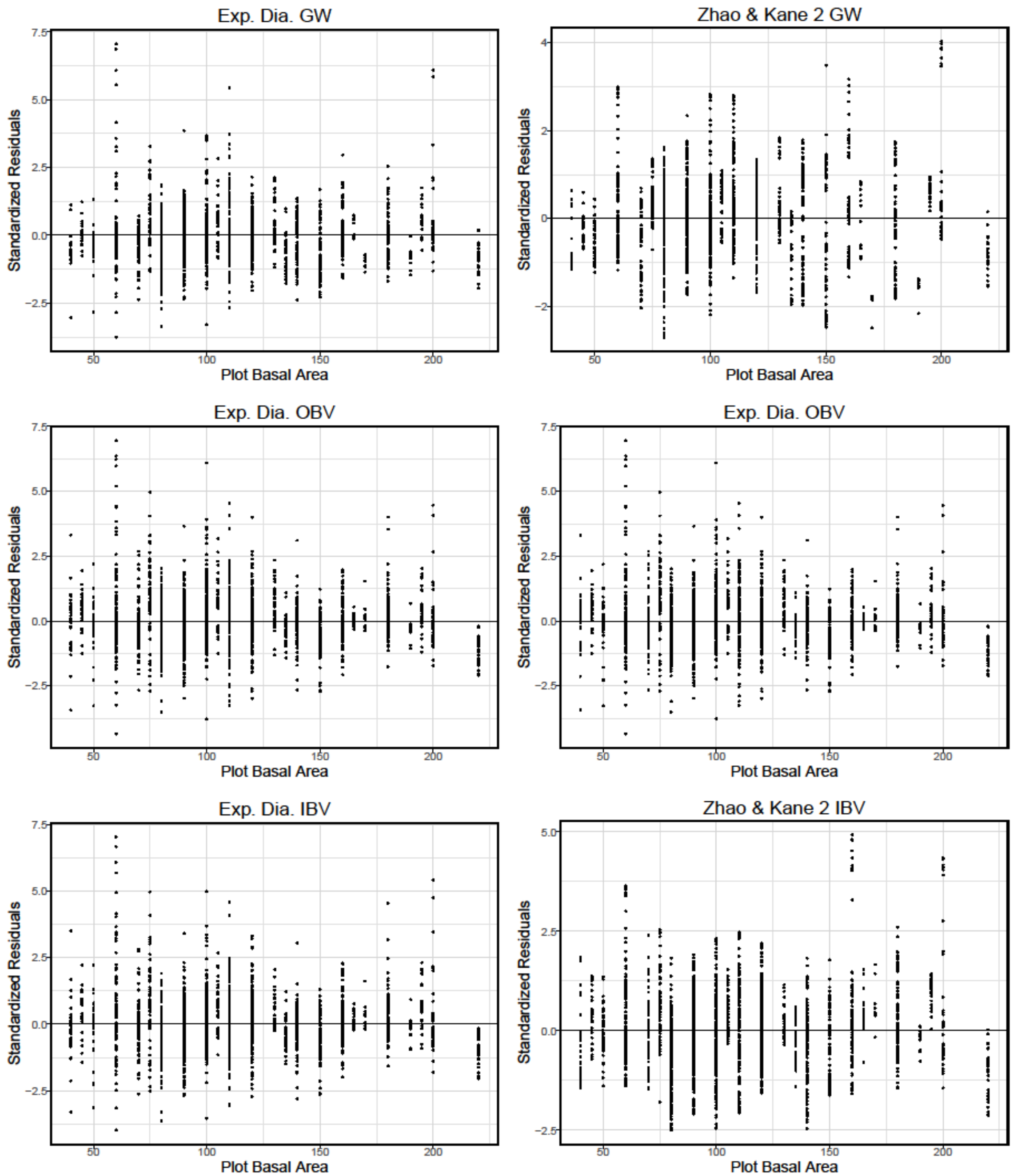

Figure 81. Residuals of selected models versus the basal area of the plot surrounding each tree.

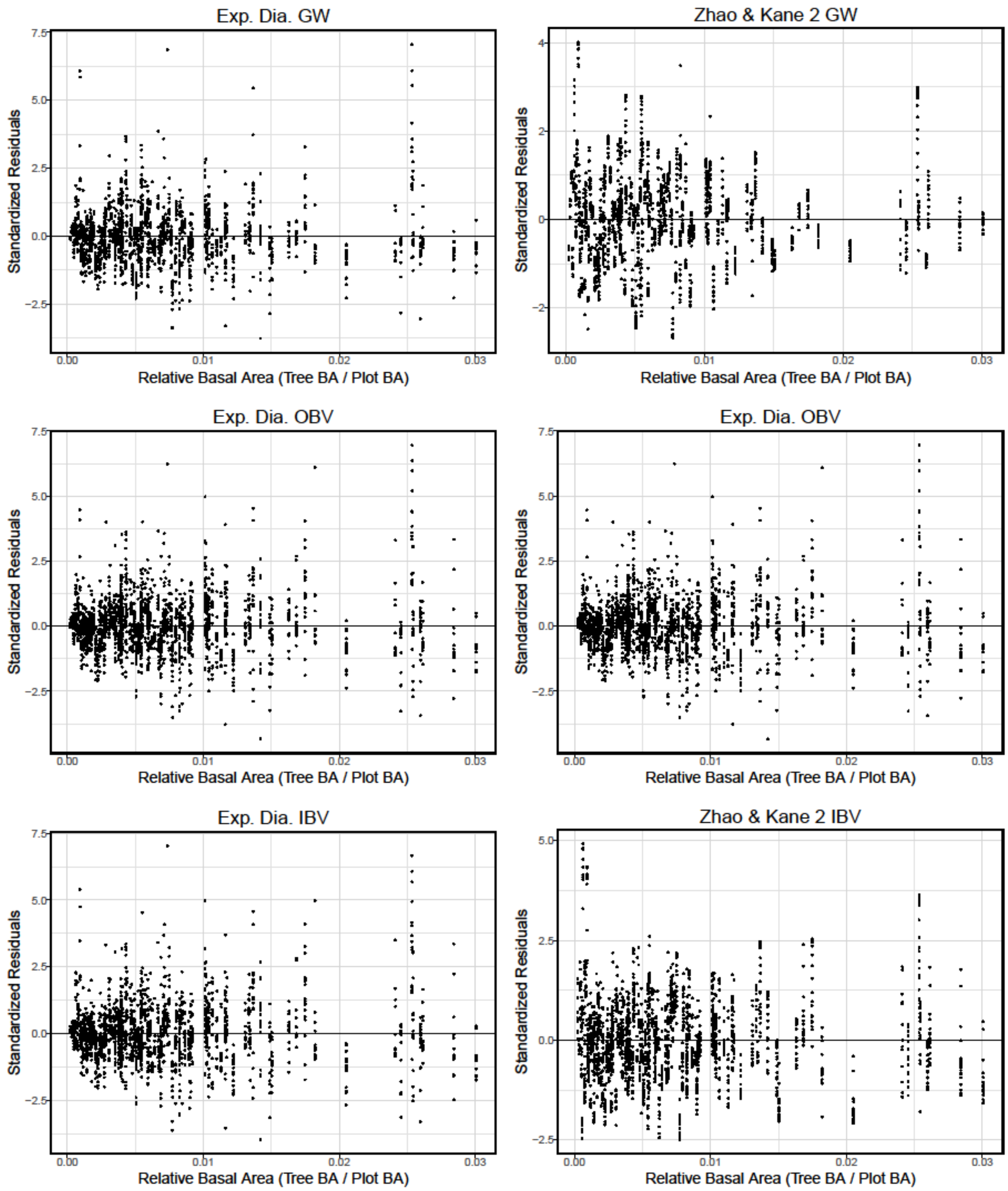

Figure 82. Residuals of selected models versus tree basal area divided by basal area of plot surrounding each tree.

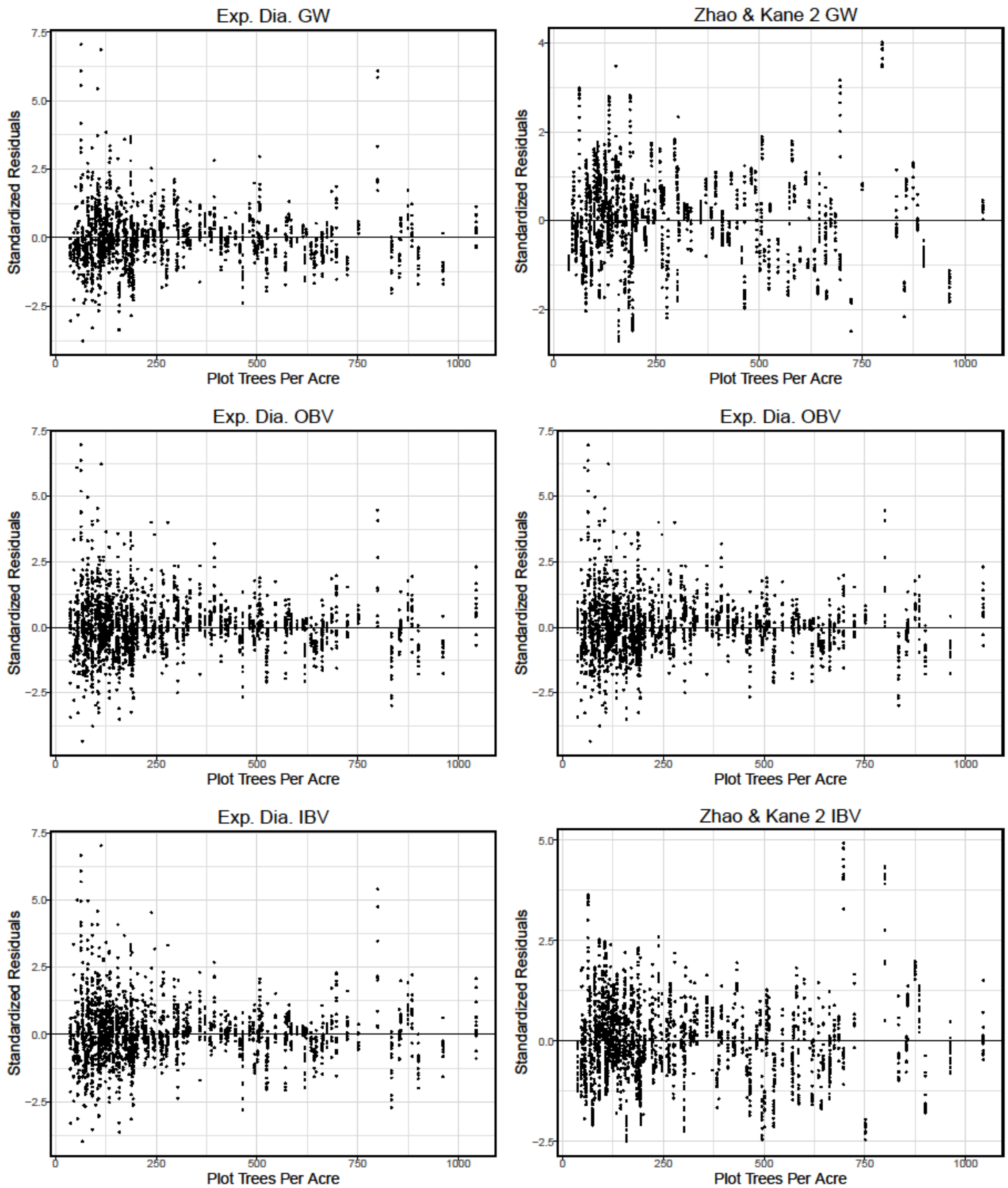

Figure 83. Residuals of selected models versus trees per acre of the plot surrounding each tree.

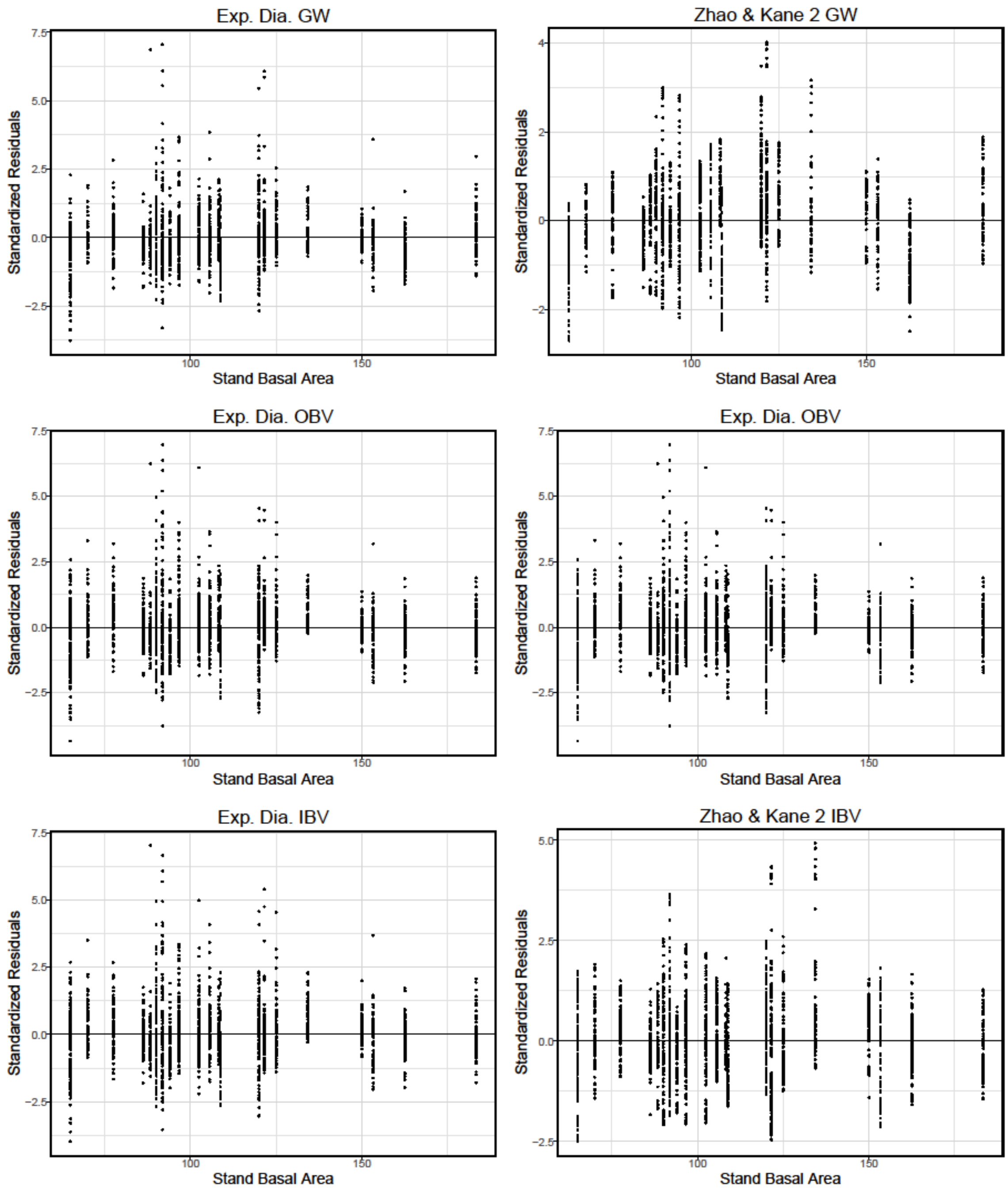

Figure 84. Residuals of selected models versus the stand basal area calculated from the plots that were measured around sample trees in a given stand.

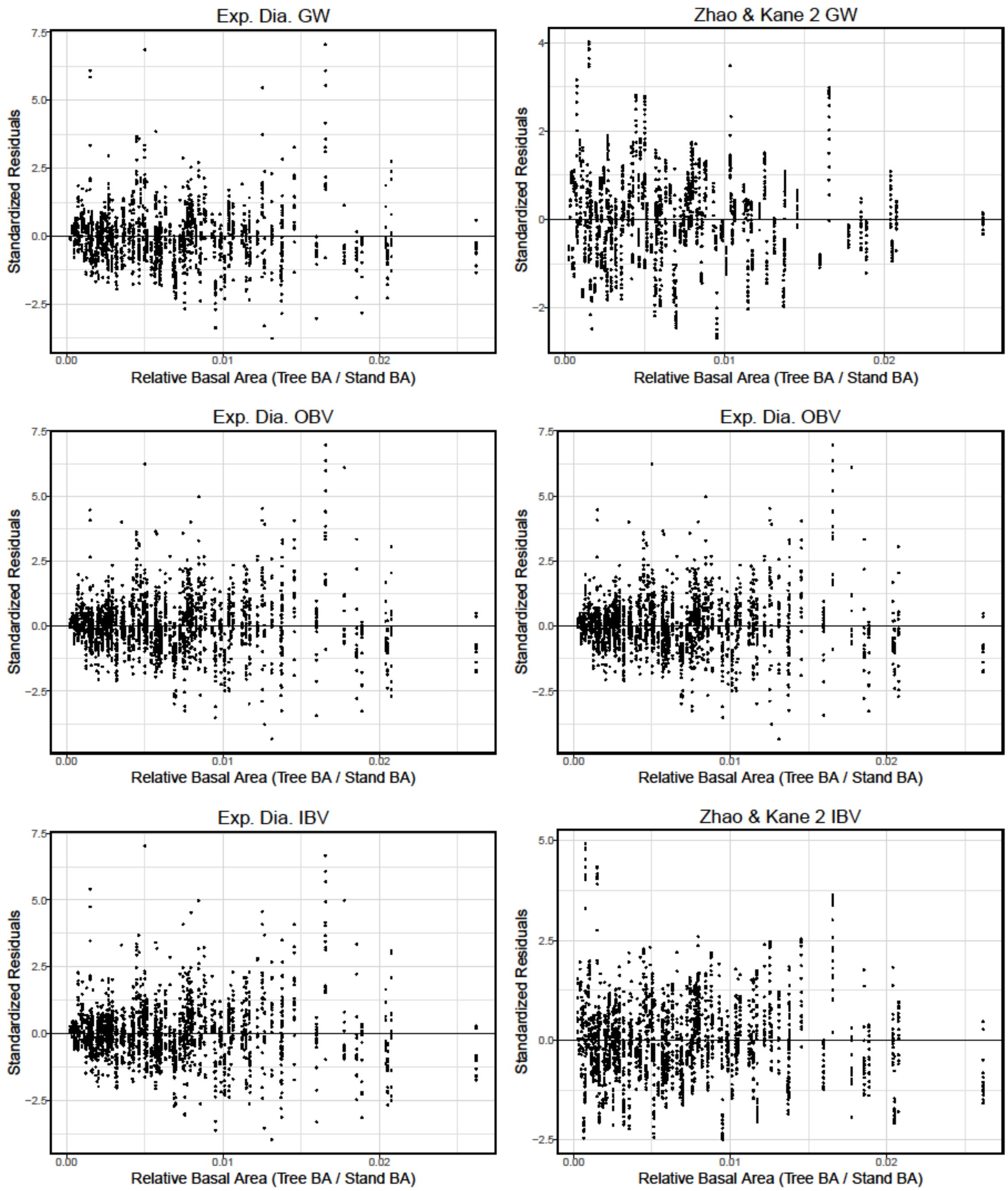

Figure 85. Residuals of selected models versus tree basal area divided by stand basal area.

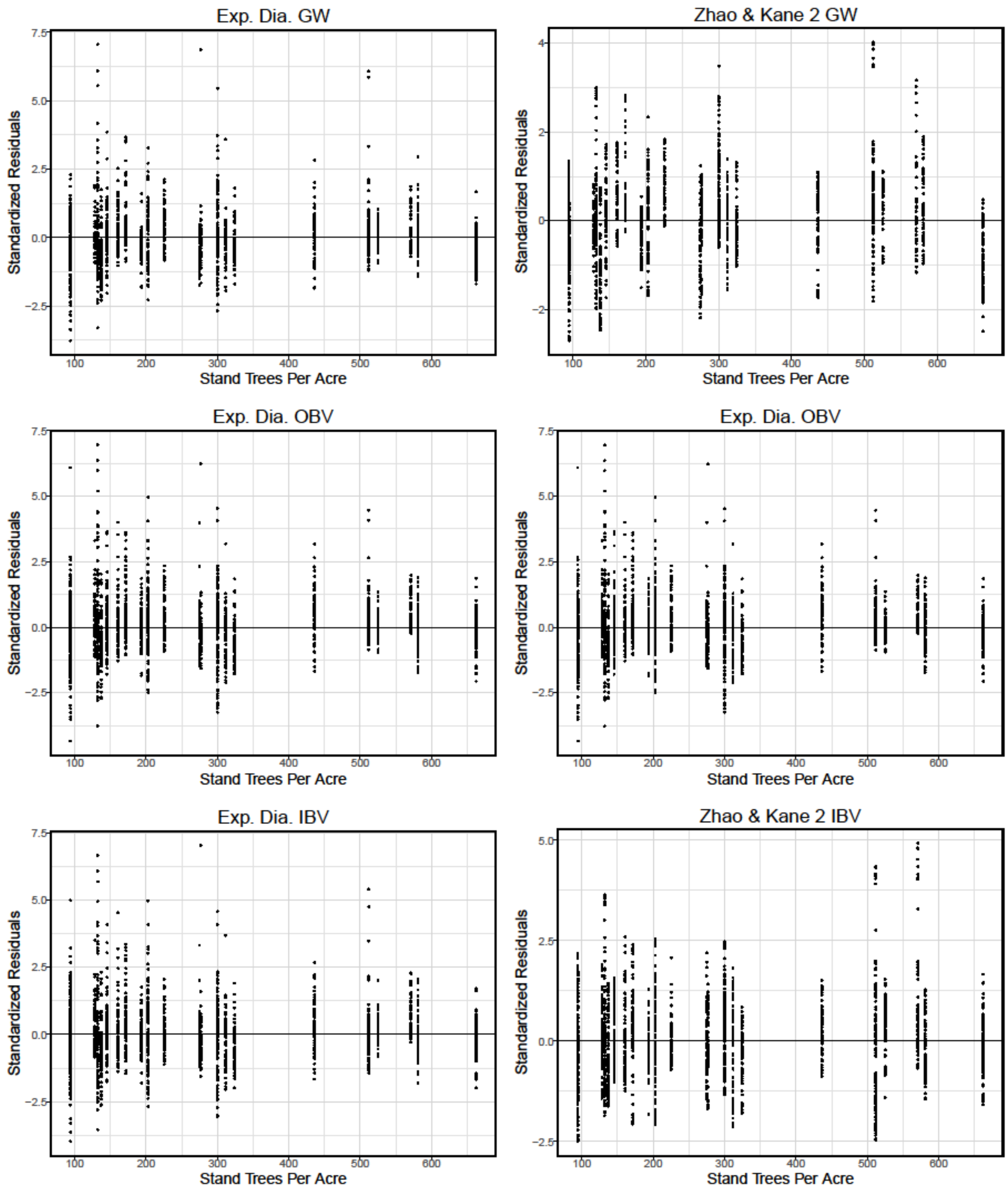

Figure 86. Residuals of selected models versus trees per acre calculated for stands from the plots surrounding sample trees within the stand.

Appendix 5: Observed versus predicted plot

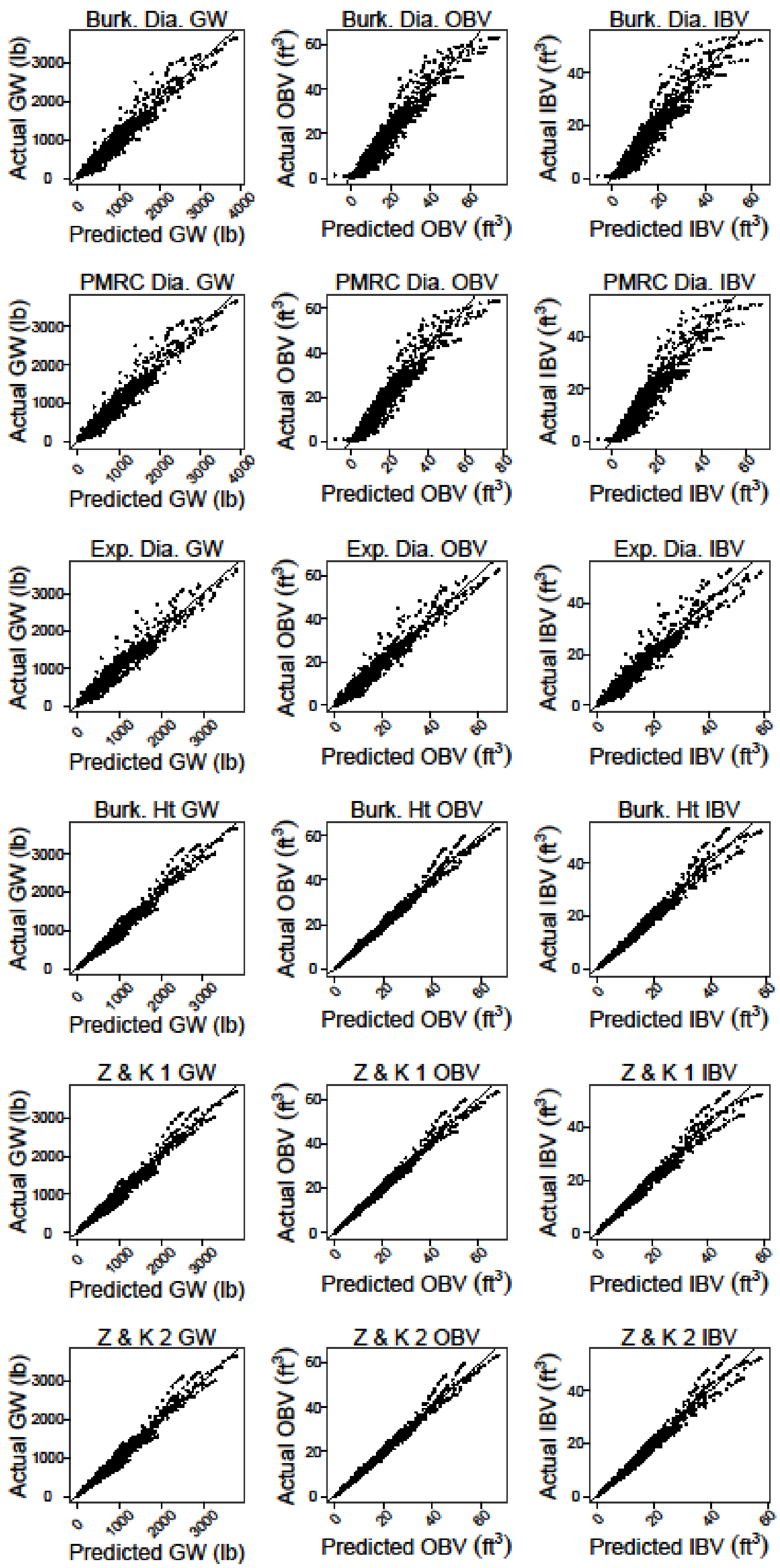

Figure 87. Actual values versus the predicted values for the top six merchantable content equations.

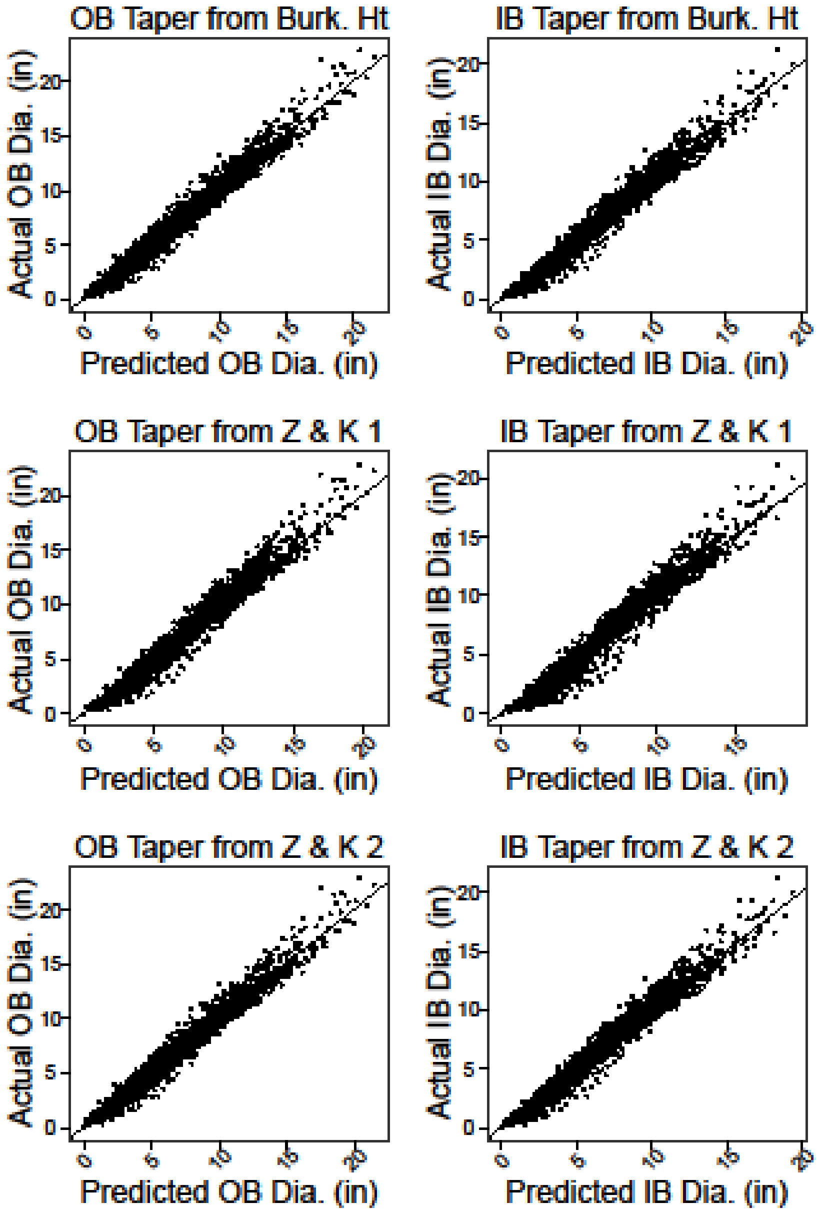

Figure 88. Actual values versus the predicted values for the compatible taper equations.

Appendix 6: Normality assessment plots

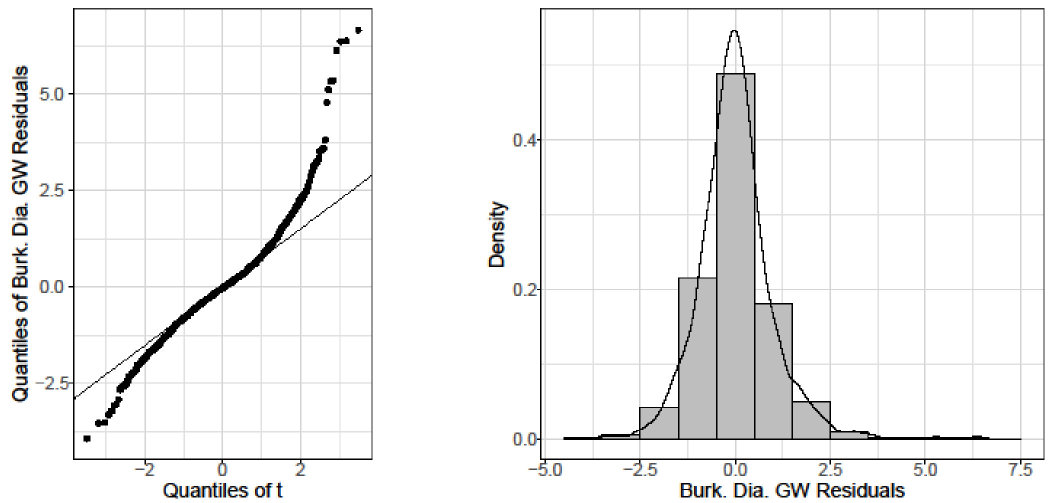

Figure 89. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Burkhart Merchantable Diameter equation (equation 15) fitted to merchantable green weight.

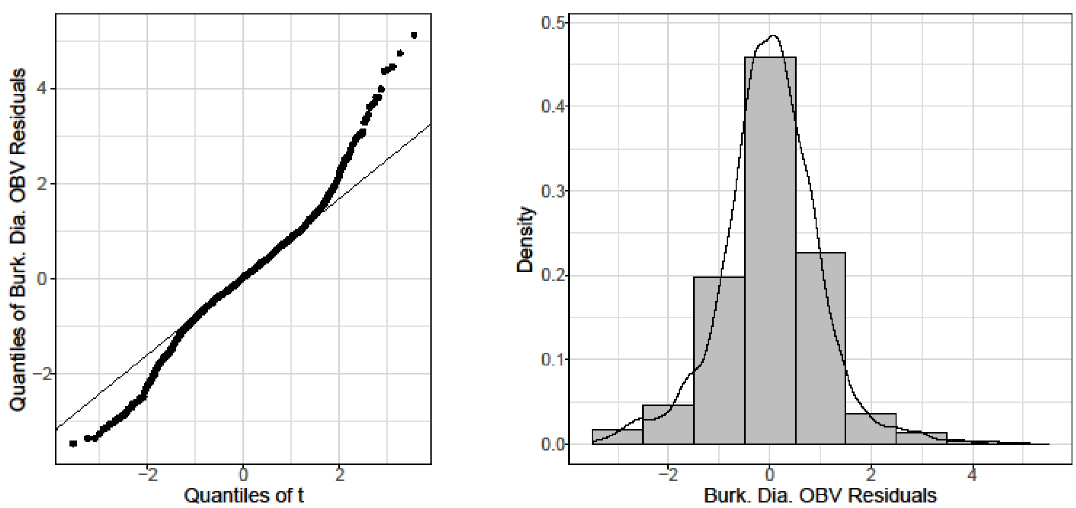

Figure 90. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Burkhart Merchantable Diameter equation (equation 15) fitted to outside bark diameter.

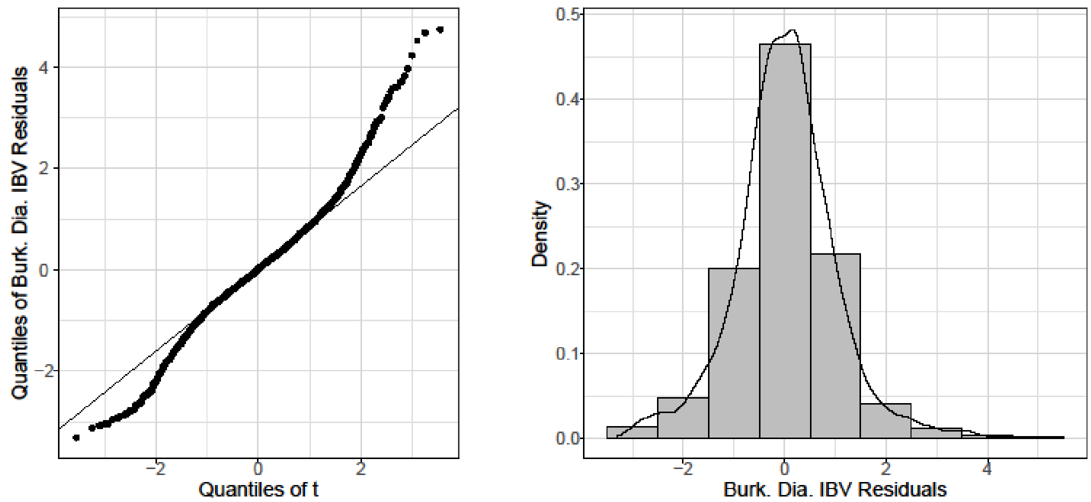

Figure 91. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Burkhart Merchantable Diameter equation (equation 15) fitted to inside bark diameter.

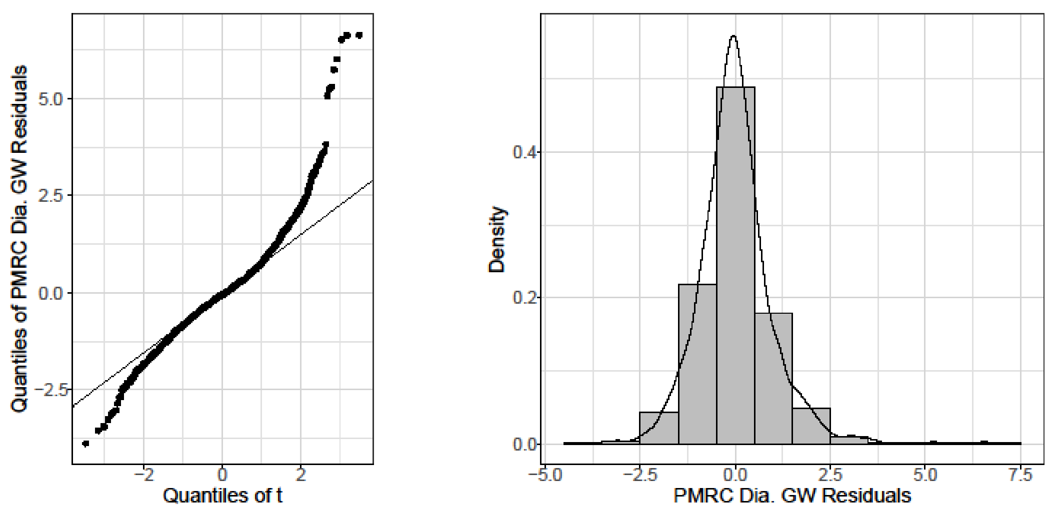

Figure 92. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the PMRC Merchantable Diameter equation (equation 17) fitted to merchantable green weight.

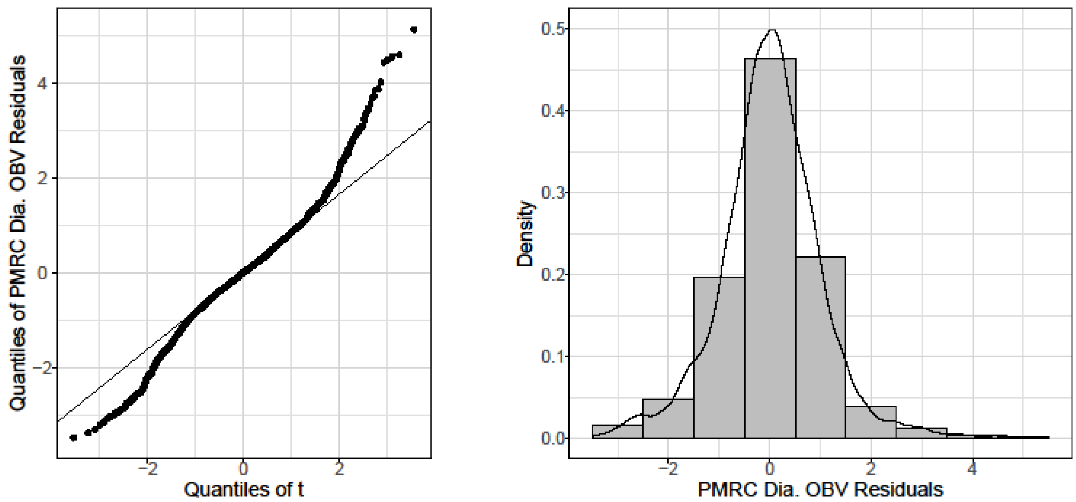

Figure 93. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the PMRC Merchantable Diameter equation (equation 17) fitted to merchantable outside bark volume.

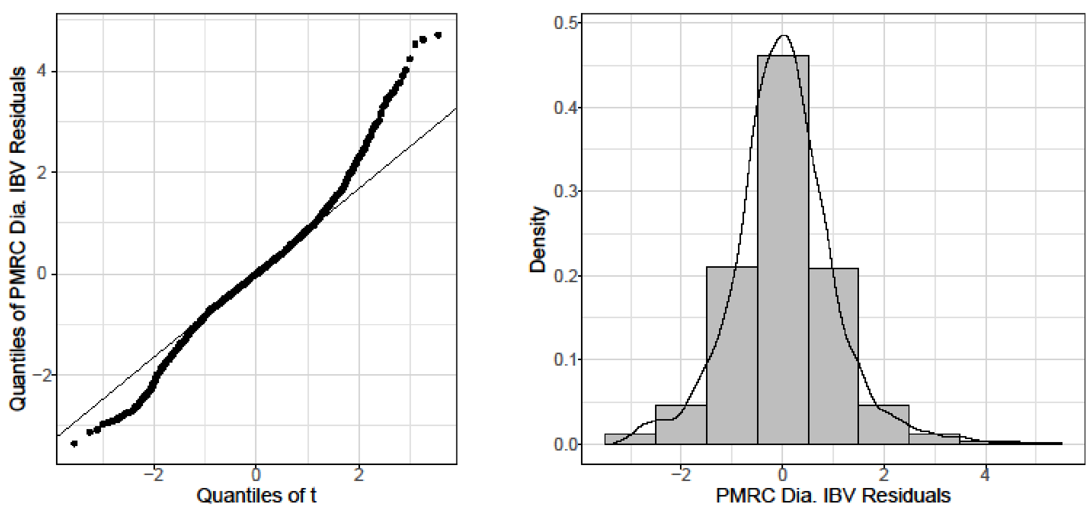

Figure 94. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the PMRC Merchantable Diameter equation (equation 17) fitted to merchantable inside bark volume.

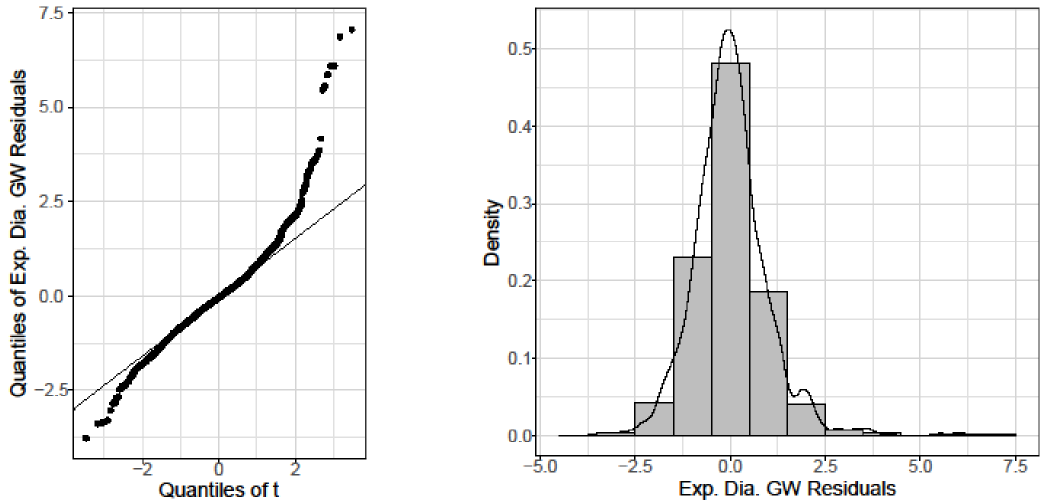

Figure 95. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Exponential Merchantable Diameter equation (equation 20) fitted to merchantable green weight.

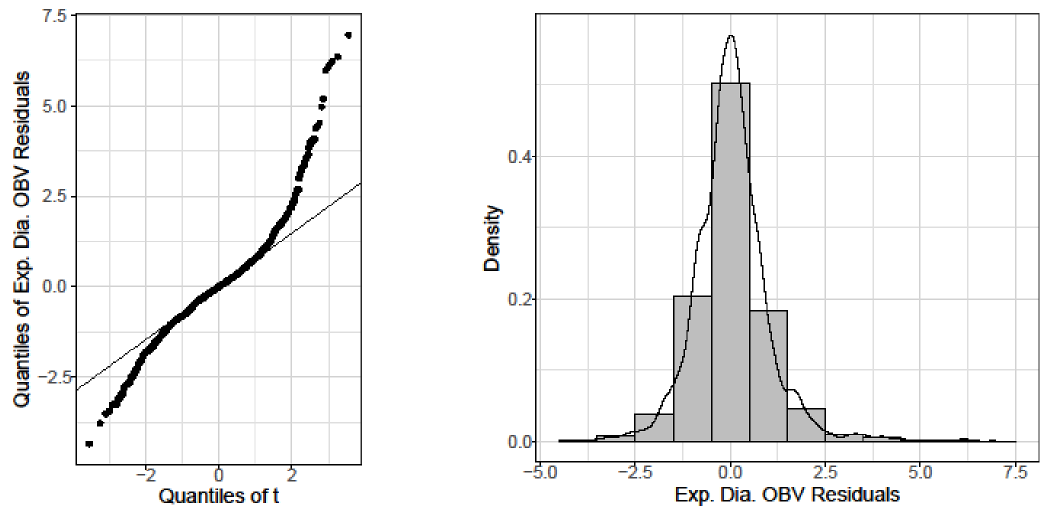

Figure 96. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Exponential Merchantable Diameter equation (equation 20) fitted to merchantable outside bark volume.

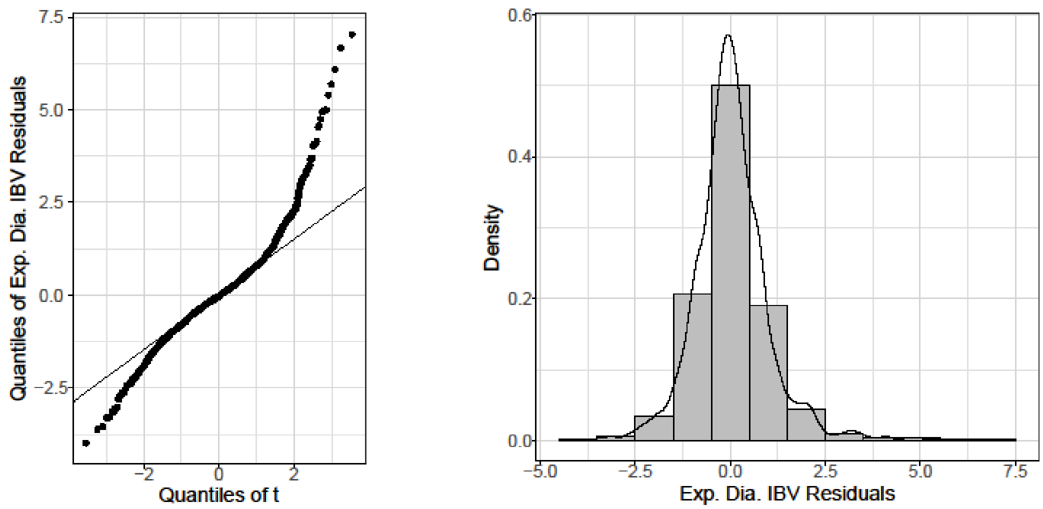

Figure 97. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Exponential Merchantable Diameter equation (equation 20) fitted to merchantable inside bark volume.

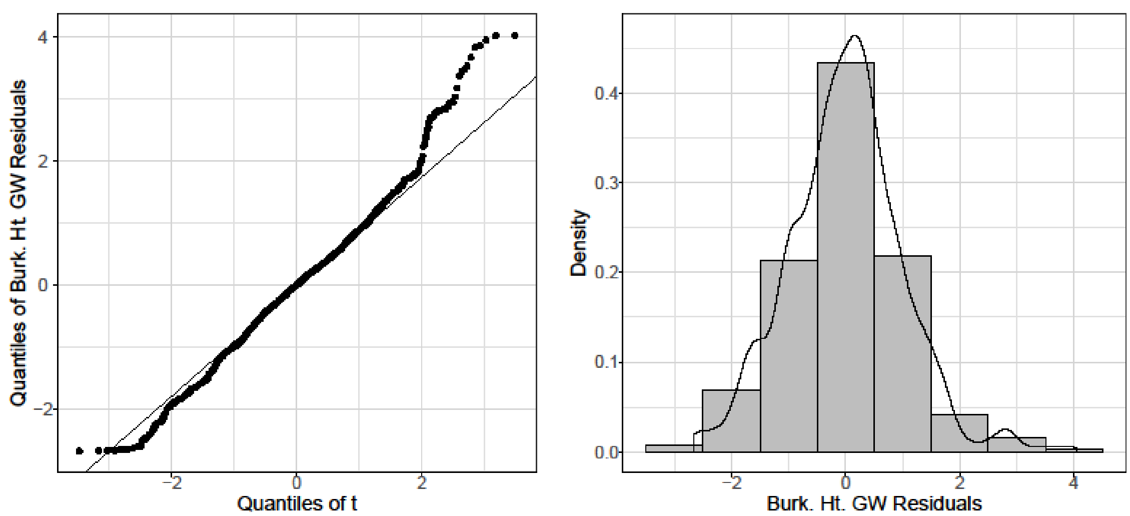

Figure 98. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Burkhart Merchantable Height equation (equation 22) fitted to merchantable green weight.

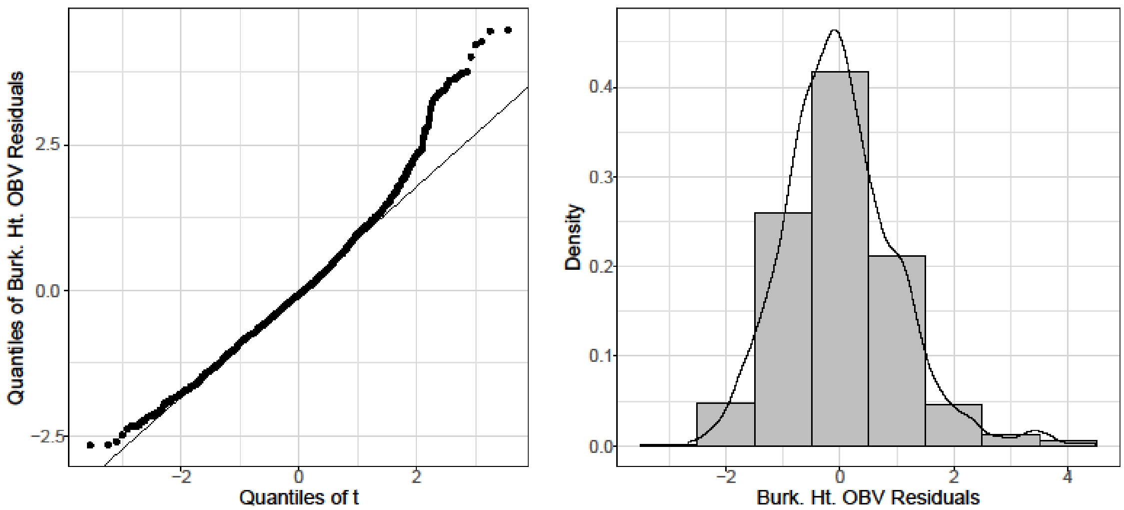

Figure 99. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Burkhart Merchantable Height equation (equation 22) fitted to merchantable outside bark volume.

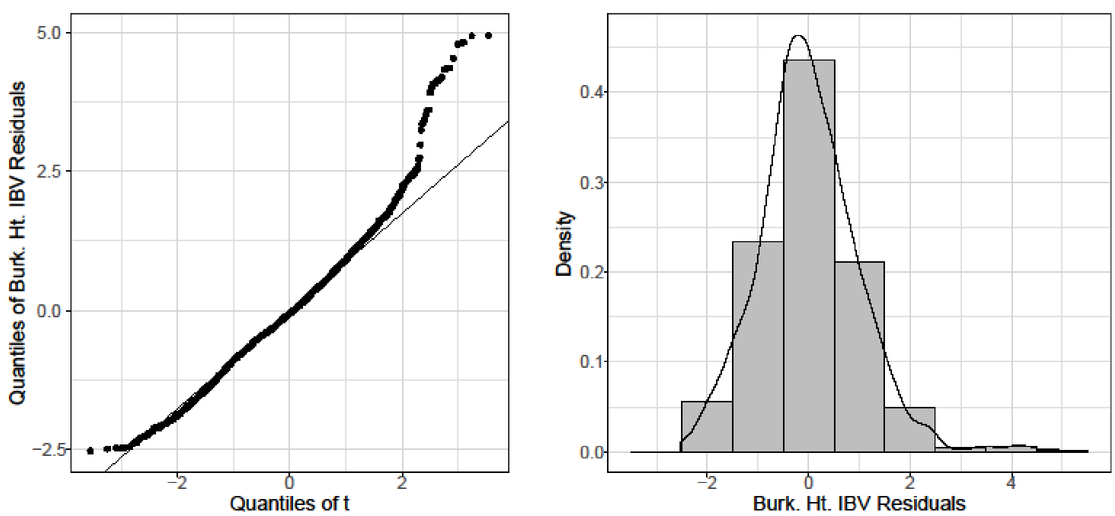

Figure 100. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Burkhart Merchantable Height equation (equation 22) fitted to merchantable inside bark volume.

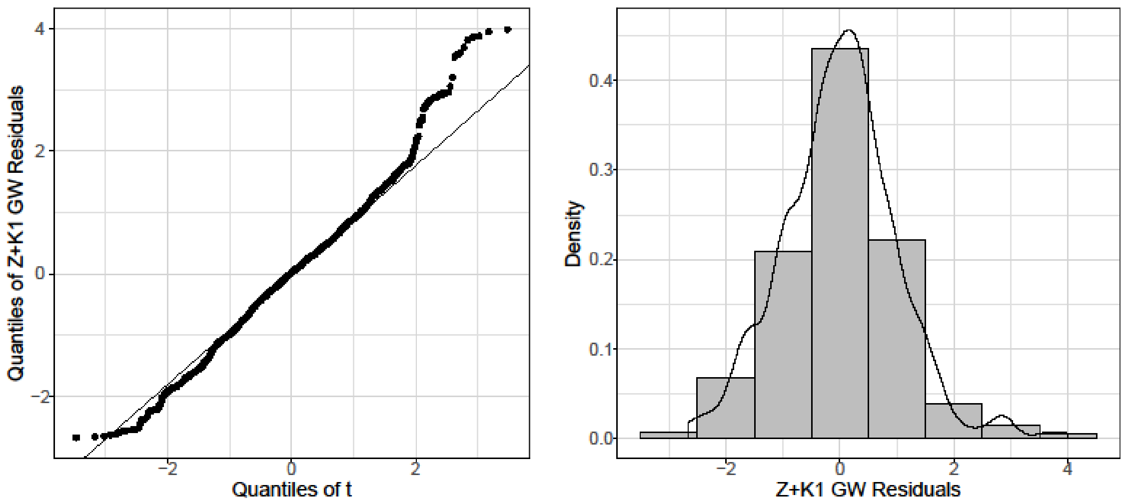

Figure 101. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Zhao and Kane 1 Merchantable Height equation (equation 25) fitted to merchantable green weight.

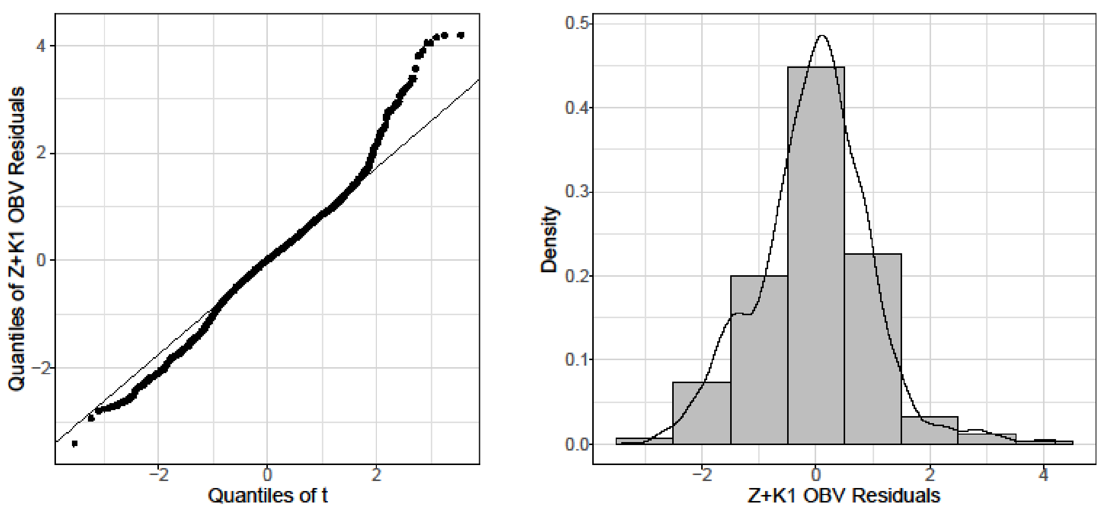

Figure 102. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Zhao and Kane 1 Merchantable Height equation (equation 25) fitted to merchantable outside bark volume.

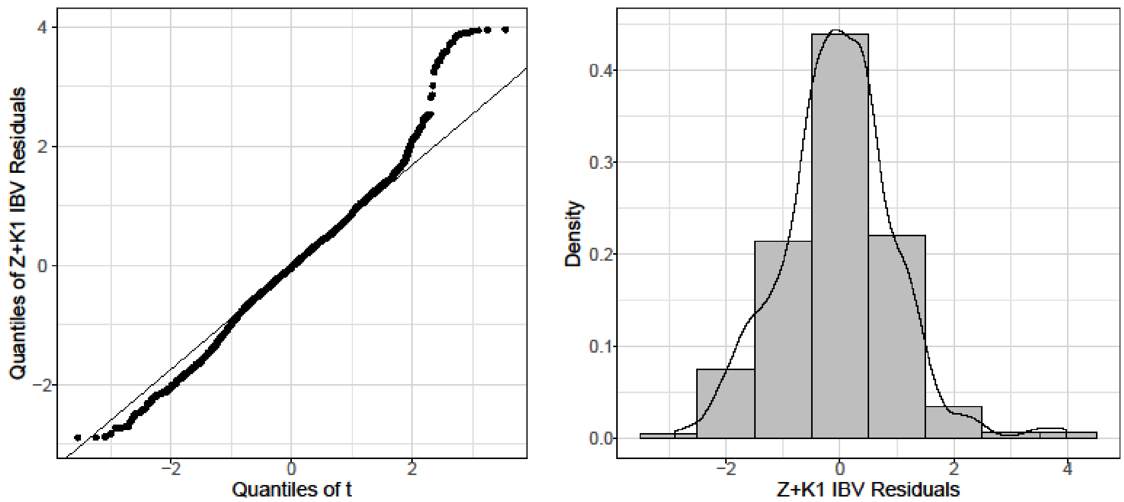

Figure 103. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Zhao and Kane 1 Merchantable Height equation (equation 25) fitted to merchantable inside bark volume.

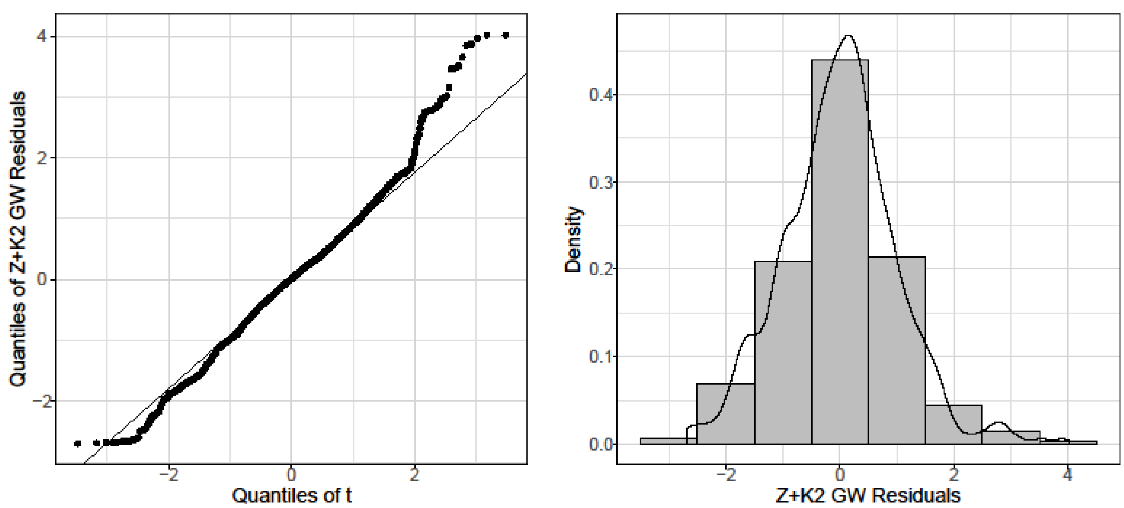

Figure 104. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Zhao and Kane 2 Merchantable Height equation (equation 27) fitted to merchantable green weight.

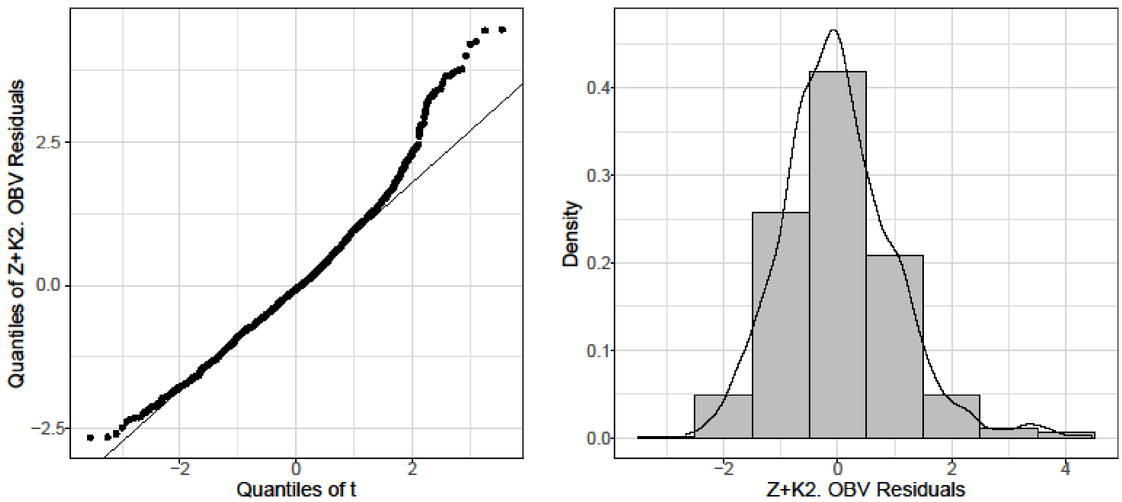

Figure 105. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Zhao and Kane 2 Merchantable Height equation (equation 27) fitted to merchantable outside bark volume.

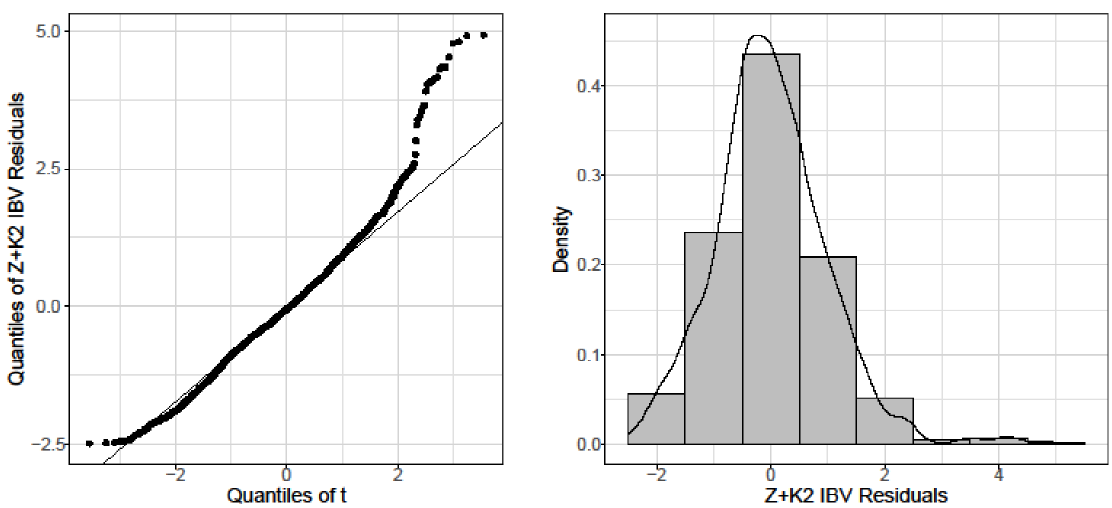

Figure 106. Q-Q plot and histogram of the standardized residuals (with an estimated density plot superimposed) of the Zhao and Kane 2 Merchantable Height equation (equation 27) fitted to merchantable inside bark volume.

VITA

William Thomas Harges

Candidate for the Degree of

Master of Science

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Publication(s):

Lynch Thomas, B., D. Zhao, W. Harges, and J.P. McTague. July 11, 2017. Deriving compatible taper functions from volume ratio equations based on upper-stem height. *Canadian Journal of Forest Research* Published on the web.