# INFLUENCE OF TEACHER CERTIFICATION <br> PATHWAY AND KNOWLEDGE OF ALGEBRA FOR <br> TEACHING ON MATHEMATICAL BELIEFS OF 

ALGEBRA I TEACHERS

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# TEACHER CERTIFICATION PATHWAY AND KNOWLEDGE OF ALGEBRA FOR TEACHING ON MATHEMATICAL BELIEFS OF ALGEBRA I TEACHERS 

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# of Study: TEACHER CERTIFICATION PATHWAY AND KNOWLEDGE OF ALGEBRA FOR TEACHING ON MATHEMATICAL BELIEFS OF ALGEBRA I TEACHERS 

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#### Abstract

This quantitative survey-based study explores Oklahoma Algebra I teachers’ beliefs about algebra and their Knowledge of Algebra for Teaching (KAT). The study included 144 teachers currently teaching Algebra I during the 2016-2017 academic year. Emails were sent to Oklahoma mathematics teachers with a link to an online questionnaire, then only analyzing data from those teaching Algebra I. The data from the online questionnaire was analyzed using one-way ANOVAs and Chi-Square tests for Independence. The results are organized into two distinct articles, each answering different research questions. As a whole, results indicate that regardless of certification pathway, algebra beliefs of teachers are not significantly different, but their KAT is significantly different ( $p<.01$ ) regarding their certification pathway. Additionally, results indicate those teachers who have a high KAT are dependent upon having a more problem-solving view of the nature of algebra ( $p<.05$ ) and also dependent on holding a mathematics degree ( $p<.001$ ).


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## CHAPTER I

## INTRODUCTION

Due to increased graduation requirements, the number of students who are required to take an algebra course at the high school level is increasing with each passing academic year (Teitelbaum, 2003). Along with this increase, teachers are having difficulty straying away from the traditional algorithmic teaching techniques of algebra (Van Roekel, 2008). This mode of teaching algebra can drive students away from mathematics before they are able to make sense of the mathematical knowledge presented to them (Kaput, 2000). The majority of experiences students have in an algebra classroom do not allow students to make connections or construct ideas for themselves. Students are not given opportunities to reflect on their experiences in algebra or explain their understanding to other students "but more importantly, to understand its importance - and usefulness - to their own lives" (Kaput, 2000, p. 2). There is more to algebra than solving for variables and manipulating equations. Dennis Van Roekel (2008), the president of the National Education Association (NEA), said, "We need to shift our own thinking and beliefs about algebra to see the broader value of this subject to today's students who will be tomorrow's workers" (p. 3).

Algebra I is a gatekeeper of higher mathematics, and it correlates to college graduation rates (Kim, Kim, DesJardins, \& McCall, 2015). With the number of students enrolling and trying their best to succeed in algebra, it is more imperative than ever that mathematics teachers have teacher characteristics that lead to the highest level of student achievement. This study will examine those teacher characteristics known to be the most impactful on the students' success levels in the mathematics classroom, and more specifically, an Algebra I classroom.

## Statement of the Problem

Algebra I is considered by many as the gatekeeper of high school mathematics achievement, a predictor of post-secondary success, and a key step to college and career readiness (Stoelinga \& Lynn, 2013). Atanda (1999) determined that enrollment in an algebra course in the eighth grade is highly related to the decision to apply for and attend college. Additionally, the trend of requiring more years of mathematics to fulfill high school graduation requirements is on the rise. In 2013, the successful completion of at least three years of mathematics was required by 42 states while the successful completion of four years of mathematics was required by 16 of those 42 states (Stoelinga \& Lynn, 2013). The state of Oklahoma, specifically, requires three successful years of mathematics courses in order to graduate from high school ("Core Curriculum," 2017). Stoelinga and Lynn (2013) went on to explain that Algebra I is not explicitly stated as a course requirement, but the completion of the Algebra I course and passing of some states end-of-instruction exams tends to be the critical step in meeting these increasingly rigorous graduation requirements.

With increased graduation requirements that are much different from two decades ago, algebra is not just a course for students who plan on attending college (Eddy et al., 2015). Previously, there was not a push for all students to enroll in Algebra I. The number of students who are enrolled in Algebra I has risen in recent years, particularly in eighth grade where enrollment has more than doubled from $16 \%$ to $34 \%$ since 1986 to the present (Pappano, 2012). However, Pappano (2012) declared that failure rates in the United States are as high as $40 \%$ or $50 \%$ in the Algebra I course.

Since Algebra I is a course surrounded by high-stakes and higher than desired failure rates, one would assume that the most qualified teachers are assigned to teach such a foundational course. However, there is a shortage of secondary mathematics teachers entering the profession from a traditional teacher preparation program and an increase in teachers from alternative certification pathways and emergency certifications (Ludlow, 2011). Henry et al. (2013) found that individuals who enter the teaching profession through one of these alternative certification pathways could have significantly negative effects on student achievement in secondary mathematics. Teachers with alternative and emergency certifications are widespread throughout the country and their numbers are growing rapidly. Particular teacher certification pathways weigh heavily on content knowledge; others weigh heavily on pedagogy; and some have a mixture of both. Emergency certification into the classroom may contain neither content knowledge nor pedagogy.

Alarmingly, the state of Oklahoma issues an ever-increasing number of emergency teacher certifications. In a news report by Cameron (2015), the Oklahoma State Board of Education approved only 32 emergency certifications during the 2011-

2012 school year, while during the first five months of the 2015-2016 school year, the Board of Education approved 948 emergency certifications. This means the number of emergency certified teachers in the state of Oklahoma has increased by almost $3,000 \%$ in the last five years. The placement and content among these teachers in school districts and classrooms were unknown, but the likelihood of mathematics classrooms being impacted is highly probable because of the shortage of mathematics teachers in Oklahoma.

An effective teacher needs to have a strong background in the content area and a deep understanding of how students learn (Darling-Hammond \& Baratz-Snowden, 2007). Teachers who do not know a subject well are likely to have insufficient subject matter knowledge to help students learn; however, knowing a subject well is not always sufficient for student understanding either (Ball, Thames, \& Phelps, 2008). Research findings indicate that students of teachers who have a strong mathematical knowledge for teaching (MKT) have higher achievement gains in all areas of mathematics (Hill, Rowan, \& Ball, 2005). Hill et al. (2005) found that MKT requires special knowledge of the content being taught in order to truly enhance student achievement in mathematics. Producing teachers with high MKT along the right certification pathway can positively impact a student's success in mathematics, especially, in the gatekeeper course of Algebra I. Although, there has been research done on MKT, there is a void of research on the specific area of algebra and how the Knowledge of Algebra for Teaching (KAT) impacts student learning.

## Purpose of the Study

While the push for students to enroll in Algebra I during the eighth grade continues, a majority of these students do not leave the course with a deep understanding of Algebra I concepts to succeed in higher-level mathematics courses. Some researchers found that student success in mathematics can depend on the teachers' beliefs about mathematics and their beliefs on how mathematics should be taught and learned (Ampadu, 2012). Others found that the teachers' mathematical knowledge of teaching (MKT) and certification pathway can help make a strong impact on the level of mathematical success for students (Ball et al., 2008; Darling-Hammond, Chung, \& Frelow, 2002; Goldhaber \& Brewer, 2000; Henry et al., 2014; Hill et al., 2005; Mandeville \& Liu, 1997). Many of the studies performed focus solely on elementary level mathematics. Lack of attention has been paid to middle-level and secondary-level algebra. The purpose of this study is to investigate the influence of teachers' Knowledge of Algebra Teaching (KAT) and pathway to certification has on their beliefs in the Algebra I classroom. The participants are teachers of Algebra I across the state of Oklahoma.

## Research Questions

This dissertation is made up of two independent studies. The first study, titled "Algebra I Teachers' Beliefs and Knowledge of Algebra for Teaching," answered the following questions:

1. Who is the Algebra I teaching force in Oklahoma?
a. What are the characteristics of the Algebra I teachers?
> b. What beliefs do Algebra I teachers in Oklahoma hold about mathematics, teaching mathematics, and learning mathematics?
> c. What is the Algebra I teachers' Knowledge of Algebra for Teaching (KAT)?
2. Is there a significant difference between an algebra teacher's certification pathway and the beliefs he or she holds?
3. Is there a significant difference between an algebra teacher's certification pathway and his or her Knowledge of Algebra for Teaching (KAT)?
4. Is there an association between an Algebra I teachers' Knowledge of Algebra Teaching (KAT) and their beliefs about algebra, about teaching algebra, and about learning algebra across certification pathways?

The second study, titled "Relationship of Algebra I Teachers' Beliefs and their Knowledge of Algebra for Teaching (KAT)," answered the following questions:

1. Is there an association between algebra teachers' Knowledge of Algebra Teaching (KAT) and their beliefs about algebra, about teaching algebra, and about learning algebra?
2. Is there an association between an Algebra I teachers' content knowledge of mathematics and their beliefs about algebra, about teaching algebra, and about learning algebra?
3. Is there an association between an Algebra I teachers' content knowledge of mathematics and their Knowledge of Algebra for Teaching (KAT)?

## Significance of the Study

This research study is important because the teaching, understanding, and learning of algebra for students is a critical component for future mathematics success, college admissions, and career readiness. Improving teacher effectiveness in the algebra classroom may be the initial step in producing more efficient learners of algebra and, therefore, producing more college-and-career-ready adults. Encouraging more teachers to understand the content of algebra is not enough; rather, encouraging them to understand how to teach it more successfully may help students see the practical uses of algebra and provide the motivation to work on higher-level mathematics. Results of this research may assist school administrators when deciding who will teach their students in Algebra I courses. Also, the results may create more awareness for the impact teachers' beliefs about mathematics has on their teaching and students' learning.

## Conceptual Framework

A plethora of research has analyzed the relationship between teacher characteristics and student achievement. Kosgei, Mise, Odera, and Ayugi (2013) defined teacher characteristics to be "qualities that can be measured with tests or derived from their academic or professional records" (p. 77). Darling-Hammond (2000) used data from a 50 -state survey of policies and case studies to examine ways in which teacher qualities are related to student achievement. Teacher characteristics including demographic (i.e., ethnicity and gender) and non-cognitive (i.e., beliefs and attitudes) are other contributing factors (Goe, 2007). The results of most studies on teacher characteristics have varied; however, trends have taken place in the categories of (a) general academic ability, (b) subject matter knowledge, (c) knowledge of teaching and
learning, (d) teaching experience, (e) certification status, and (f) teacher beliefs (DarlingHammond, 2000; Ernest, 1989).

Taking past literature on teacher characteristics impacting teacher quality, which ultimately can impact student achievement was considered in this study. Research has indicated teacher preparation and certification pathway to be the strongest correlation to student achievement (Darling-Hammond, 2000). This study uses a conceptual framework (see Figure 1.1) incorporating teacher preparation and certification pathway, along with teacher beliefs and teaching experience, that influence the quality of Algebra I teachers.


Figure 1.1. This figure illustrates the factors influencing teacher quality of Algebra I teachers.

## Assumptions, Limitations, and Delimitations

It is assumed that all participants in this study held a valid teaching certificate or emergency certification for teaching mathematics in the state of Oklahoma during the time the questionnaire was completed. It also was assumed that they were employed fulltime in an Oklahoma middle or high school during the time they completed the online survey, and they gave honest responses to the questionnaires to the best of their abilities. Another assumption for this study was that no outside assistance was received on the Knowledge of Algebra Teaching (KAT) portion.

A strict effort was made to ensure that the sample population of teachers was representative of the Algebra I teacher population in the state of Oklahoma. The researcher, however, had no power over who chose to participate in the study. Since data were collected from a sample who chose to participate, the population was limited. With the sample being only Algebra I teachers from the state of Oklahoma, the generalizability across other states or the country as a whole may be limited. A final limitation is that the gender of the teachers were not identified in the questionnaire.

The study was delimited to teachers for grades 6 through 12 in Oklahoma who were responsible for a full year of Algebra I instruction to students during the 2016 2017 academic year. Finally, the data were self-reported enabling a chance for response bias.

## Summary

This introductory chapter provides background information related to the current study along with the purpose and significance of the study. In Chapter II, the researcher presents a review of the literature related to teacher characteristics on student
achievement, MKT, algebra as a gatekeeper, teacher certification pathways, teacher beliefs, and the theoretical framework that guides the study. Chapter III and IV are independent studies that investigate Algebra I teachers' educational backgrounds, certification pathways, beliefs, and Knowledge of Algebra Teaching (KAT). Chapter V provides a summary of the complete dissertation.

## CHAPTER II

## REVIEW OF LITERATURE

This paper reviews the professional literature related to the subject of algebra, teacher characteristics, mathematics teachers' certification pathways, mathematical knowledge for teaching, and their mathematical beliefs. First, research related to the practical reasons students learn algebra and how success in the subject can impact their futures in college and careers is presented leading into how teacher characteristics can affect student achievement. Next, an overview of the particular certification pathways that are available for prospective teachers in the United States, and more specifically, Oklahoma, is presented. Then, a body of work describing the main ideas of a teacher's Mathematical Knowledge for Teaching (MKT) and Knowledge of Algebra Teaching (KAT) is discussed. The last section of literature will review research pertaining to mathematical teacher beliefs that occur in the classroom. The paper concludes with a final summary of the literature and identification of the gaps that will be addressed by this research.

## Algebra as a Gatekeeper

The word gatekeeper is not new by any means. Discourse regarding the word was published in The Republic, a book by Plato written over 2,300 years ago (Stinson, 2004, p. 9). In The Republic, Plato made several arguments that mathematics was "virtually the first thing everyone has to learn...common to all arts, science, and forms of thought."

Stinson continued to go into detail about the history of the gatekeeper definition. Mathematics education was beginning to become attacked in the 1950s by society for the business sector graduating students who had minimal computational skills, college students entering the workforce with inadequate knowledge, and mathematics curriculum being "watered down."

During this time of mathematical education being scrutinized by so many, the issue of equity came about as well. The advanced mathematics learning world was dominated by white males at that time. The National Council of Teachers of Mathematics (NCTM, 2000) published the Curriculum and Evaluation Standards for School Mathematics with bold statements such as the following:

The social injustices of past schooling practices can no longer be tolerated. Current statistics indicate that those who study advanced mathematics are most often white males. Women and most minorities study less mathematics and are seriously underrepresented in careers using science and technology. Creating a just society in which women and various ethnic groups enjoy equal opportunities and equitable treatment is no longer an issue. Mathematics has become a critical filter for employment and full participation in our society. We cannot afford to have the majority of our population mathematically illiterate: Equity has become an economic necessity. (p. 3)

This quotation leads impeccably into the understanding of the word "gatekeeper" in mathematics.

Gatekeeper in mathematics means that all students need to be provided the opportunity to take part in mathematics, basic and advanced, in order to gain economic
access, full citizenship, and higher education (Stinson, 2004). There have been two reports from the U.S. government to state quantitatively how mathematics, specifically algebra, is considered to be a gatekeeper. Atanda (1999) summed up one of the studies completed by the National Education Longitudinal Study of 1988 to provide the following two impactful statements:

- Students who enrolled in algebra as eighth-graders were more likely to reach high-level math courses (e.g., algebra 3, trigonometry, or calculus) in high school than those students who did not enroll in algebra as eighth-graders.
- Students who enrolled in algebra as eighth-graders and completed a high-level math course during high school were more likely to apply to a 4-year college than those students who did not enroll in algebra as eighth-graders but who also completed a high-level math course during high school. (p. 33)

Hence, the influence that an algebra course has on a student entering postsecondary education and having a career is strong.

Practical reasons to learn algebra. The practical use of algebra in the minds of students has seemed to disappear with each passing generation. Since algebra is known to be the gatekeeper of advanced mathematics, students must learn the subject with a strong sense of understanding. Fennema and Romberg (1999) stated, "Students have learned some mathematics if and only if they are able to apply this knowledge to learning new topics and to solving new and unfamiliar topics" (p.45). Therefore, the first main practical reason for students to learn algebra is the gatekeeper purpose.

Several practical reasons for students to learn algebra were given by Usiskin (2004). Those practical reasons were (a) algebra is the language of generalization; (b)
algebra enables a person to answer all the questions of a particular type at one time; (c) algebra is the language of relationships among quantities; (d) algebra is a language for solving certain types of problems; (e) algebra is the study of structures with certain properties; ( f ) algebra shows that our universe possesses order; and (g) algebra is a prerequisite for virtually all other mathematics.

Algebra being a prerequisite for all other mathematics leads back to the discussion above. The language of generalization, as conceptualized by Kaput (2000), is extending reasoning to more than just one explicit case by exposing commonalities for all cases. Usiskin (2004) used the example of multiplying fractions to show this practical use. After careful examinations of several cases, one might determine the rule for multiplication of fractions. Instead of using long English sentences to explain the multiplication of the numerators and the multiplication of the denominators results in the product of the two fractions, the language of algebra allows us to write $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)=$ $\frac{(a)(c)}{(b)(d)}=\frac{a c}{b d}$ to show the rule (p. 148). Mathematics educators should try not to focus on the formulas before the understanding of the concept is developed. It is possible to get by without knowing the language of algebraic formulas, but quick calculations of them will decrease the chance of being misled from a misinterpretation by another.

Usiskin (2004) speaks of algebra being the language of relationships among quantities. Phrases, such as growing exponentially, varying directly, rate at which a rate is changing, and extrapolation versus interpolation, contain content from algebra. These phrases show up regularly in news articles and on television. The example he gave was using the algebraic formula of area equaling length times width. From knowing properties and operations on numbers, other equations can be created from the
manipulation of the original. "What this means is that from one formula we can deduce other" (Usiskin, 2004, p. 149).

Deduction is a powerful tool that leads into Usiskin's final practical reasoning of algebra use. Algebra led to the development of calculus, which allowed calculus to explain the laws of Kepler. From the explanation of Kepler's laws, the elliptical orbit of the planets was discovered. Algebra does not just perform manipulation on numbers, but the language of algebra explains aspects of our universe.

The practical reasons for students to learn algebra revolve around the language and the real-world applications that can be tied to the language. When Fennema and Romberg (2000) were describing the importance of teaching mathematics for understanding, they said "mathematics should become a language for thought rather than merely a collection of ways to get answers" (p. 46). The practicality of learning algebra is to produce thinking.

Impact of Success in Algebra. Much research has been done on the gatekeeper aspect of algebra. The research that has been done contains information on the implications of success in algebra to be limited to the idea of performance in college, career readiness, impact on career salary, student perception on higher mathematics, and the effect on other students in the algebra classroom (Eddy et al., 2015; Gaetner, Kim, DesJardins, \& McClarty, 2014; Kim, Kim, DesJardins, \& McCall, 2015; Loveless, Williams, Ball, Hoffer, Venkataraman, \& Hedber, 2008; Murnane, Willett, \& Levy, 1995; Sieglern et al., 2012).

While the importance of success in algebra is what most research predicts or attempts to prove, Eddy et al. (2015) proposed the unification of the leading algebra
standards and assessment framework. The research in this article looked at the inconsistencies of what it means when a student studies algebra in different classrooms across the country. The Algebra for All movement is strong, which calls for identification of what concepts should be learned in algebra. Several frameworks were looked at in this study including NCTM principles and standards, Research and Development (RAND) corporation, Common Core State Standards (CCSS), and Texas Essential Knowledge and Skills (TEKS). The unification of these frameworks provided better research opportunities and evaluation for the learning of algebra. Six key ideas of algebra were determined throughout this unification process: (a) Variables, (b) Functions, (c) Patterns, (d) Modeling, (e) Technology, and (f) Multiple Representations (Eddy et al., 2015, p. 84). Eddy et al. (2015) reiterated the purpose of the present article is not to create another framework but to unify existing classroom instruction and assessment frameworks that characterize algebra so that all students have the opportunity to be on the correct pathway to college and career. (p. 84)

Loveless et al. (2008) discussed the thought of enrolling students in algebra too early. With all of the research suggesting that students who take algebra earlier have higher math skills, Loveless et al. (2008) stated that "these findings, however, are clouded by selection effects - by the presence of unmeasured factors influencing who takes algebra early and who takes it late" (p. 2). The lack of basic arithmetic skills while being placed in an algebra course in eighth grade determined that thousands of students in the United States were misplaced in the year 2007 according to Loveless et al. Teachers in the study reported that classes of the diverse mathematics preparation are the cause for
ineffective teaching and created more unprepared students for higher mathematics courses to come. Before the discussion of what current and past research says about the impact of student's success in algebra, the researcher found it imperative to involve the findings of these studies in order to describe what should be expected in terms of content from an algebra course.

Success in algebra can have an impact on one's probability of attending college and one's future career salary (Gaertner et al., 2014; Kim et al., 2015; Murnane et al., 1995). Success is measured by Kim et al. (2015) with the completion of Algebra II in high school. By using Florida's longitudinal student unit record data, Kim et al. tracked students who were enrolled in $7^{\text {th }}$ grade in the 1995-96 school year through the 2005-06 school year. The relationship between completing Algebra II in high school and attainment of a degree from a two-year college or a four-year college was then examined. Kim et al. made it clear that previous studies have supported that the idea of all types of math courses have some kind of statistically significant positive effect on college attendance. The completion of Algebra II posed the conclusion of it significantly affecting two-year college attendance, but not significantly affecting four-year college degree attainment.

The body of research done on high school courses taken relating to college outcome is very large. A similar study by Gaertner et al. (2013) was done just two years before Kim et al. (2015) looking at success in Algebra II related to college and career outcomes. Gaertner et al. (2005) took the research further in trying to determine if the completion of a higher mathematics course like Algebra II had an effect on not only college outcomes, but career outcomes as well. The results were similar to those of Kim
et al. (2015) that the completion of Algebra II helped students gain entry into two-year and four-year colleges. The results did not show that the completion of Algebra II had any effect on career outcomes, such as salary earnings. The main statement made by Gaertner et al. (2013) was "Algebra II completion may boost wages by way of college degree attainment" (p. 161).

Murnane et al. (1995) performed a study almost 20 years before Gaertner et al. (2013) and Kim et al. (2015) for which the results were explained to reflect wage determination for those who mastered the basic algebra skills, such as working with fractions and interpreting line graphs. Resulting wages for males and females differed while female wages were directly affected. For males, success in the basic cognitive skills of algebra was not positively related to their wages two years after graduation. For females, success in those algebra skills predicted much stronger relationships to wage as far as six years after graduation.

Studying factors that influence the success of algebra can also contribute to finding connections related to the impact of students' algebra success. Siegler et al. (2012) hypothesized that a student's knowledge of fractions at a young age would predict algebra knowledge and overall mathematics achievement in high school, and also general intellectual ability. Using the British Cohort Study (BCS) and the Panel of Income Dynamics-Child Development Supplement (PSID-CDS) longitudinal data, Siegler et al. found that the results from the United Kingdom group and the United States group yielded extremely similar results. Results showed that fraction knowledge at the age of 10 was the strongest mathematical predictor of algebra knowledge. Siegler et al. (2012) also concluded,
although algebra is a major part of high school mathematics and fractions constitute a smaller part, the correlation between high school students' knowledge of fractions and their overall mathematics achievement was stronger than the correlation between their algebra knowledge and their overall mathematics achievement. (p. 693)

Considering that fractions predict success in algebra, and algebra predicts attendance to a two-year or four-year colleges, this study provided key information to the importance of elementary school teaching to produce successful students in algebra.

## Teacher Characteristics

Teacher characteristics are defined to be "qualities that can be measured with tests or derived from their academic or professional records" (Kosgei, Mise, Odera, \& Ayugi, 2013, p. 77). Additionally, teacher characteristics can include demographics and noncognitive factors (Goe, 2007). Literature on two categories of teacher characteristics, general academic ability and teaching experience, are discussed in this section.

General academic ability. Darling-Hammond and Youngs (2002) stated that "rigorous research indicates that verbal ability and content knowledge are the most important attributes of highly qualified teachers" (p.18). This relationship especially was noticed in a study done by Ferguson and Ladd (1996) in which a teacher's ACT score was significantly related to third and fourth grade classes gains in reading and mathematics achievement. Although evidence has been found in some cases that general academic ability of the teachers relates to student achievement, it is not always true. There have been studies conducted that resulted in negative findings related to general
academic ability and student achievement when certain variables were controlled, such as college ratings and the sample size (Hanushek, 1992; Murnane \& Phillips, 1981).

Teaching experience. Measuring the effectiveness for years of teaching experience on student achievement is difficult. Experienced teachers produce student achievement at a higher level due to the fact that the experienced teachers have mastered the content and acquired the classroom management skills to handle all types of classroom issues (Gibbons, Kimmel, \& O’Shea, 1997). Clotfelter, Ladd, and Vigdor (2007) found clear evidence in the state of North Carolina that teachers with more experience raise student achievement in the areas of mathematics and reading more efficiently than those who have less experience. Other researchers have been cautious of studying years of teaching experience due to attrition rates. Five-year teacher education programs tend to produce teachers more confident than those in 4-year programs (Andrew \& Schwab, 1995). This leads to some beginning teachers being just as effective as veteran teachers in the profession (Darling-Hammond, 2000). Also, Loeb, Kalogrides \& Beteille (2012) found a trend of teachers being more likely to raise student achievement scores if that school raised student achievement scores the previous year. Ferguson (1991) disclosed that students at the high school level perform significantly better on tests when the teacher has had more than nine years of teaching experience. Thus, researchers have found that a teacher's effectiveness increases exponentially every year during the first decade of teaching (Loeb et al., 2012). However; length of experience is a topic that Goldhaber (2002) tended to rule out in his study. Goldhaber stated that teachers with high test scores are not likely to stay in the profession without moving from an experience pay scale to a performance-based
incentive pay scale. Goldhaber's findings indicate that the profession will become more attractive to motivated individuals and will, therefore, increase student achievement.

## Mathematics Teacher Certification Pathways in the United States

The United States allows the individual states to control the profession of teaching by coordinating which teacher certification programs signify as relevant and appropriate toward the teaching profession. While each state has its own procedure for certification, most of the certification pathways involve exams in content knowledge and pedagogy combined with coursework and internships in teaching (Boyd, Goldhaber, Lankford, \& Wyckoff, 2007). The idea of teacher certification was created for a particular reason - to keep poor teachers out of the classroom; however, certification comes with consequences. Boyd et al. (2007) state, "Because the path to certification can be arduous, it may reduce the appeal of teaching for some people who could potentially become good teachers" (p. 46). The pathways to teacher certification has become a great topic of discussion and an area of research highly developed over the last decade.

Again, each state decides the steps to certification for teachers to enter the classroom. There are two main distinctions between certification pathways in the United States labeled as traditional and alternative. Boyd et al. (2007) emphasized that most U.S. school districts look for teacher candidates from the colleges and universities across the nation who are graduates of a teacher preparation program. This is the most common pathway to a teacher certification. After completing the state-approved teacher preparation program, the passing of certification exams is required. "States assume that by completing the state-approved preparation programs, teachers have met the
preparation component of certification, including required course content and field experiences" (Boyd et al., 2007, p. 48).

Walsh and Jacobs (2007) wrote that although the traditional certification pathway had been popular for several decades, the evidence in the 1980s showed significantly fewer talented individuals were choosing the teaching profession. Thus, began the alternative certification movement in several states across the United States. Walsh and Jacobs (2007) gave the following reason as to why alternative certification began:

The idea behind alternative certification was straightforward: expedite entry into the public school classroom for well-educated individuals who were eager to teach but unwilling (or could not afford) to spend a great deal of time and money in education coursework, and strengthen the classroom support given to new teachers via mentoring and other induction activities. (p. 17).

Alternative certification programs are now ever-present and allow opportunities to those individuals who wish to teach during or after earning a bachelor's degree.

As long as a bachelor's degree has been achieved or is currently being completed, more than 40 states offer these alternative programs that vary from short summer programs to 1- or 2-year post baccalaureate programs that are heavily involved with mentoring and supervision (Darling-Hammond et al., 2002). Some alternative programs require extra coursework to be taken after the bachelor's degree in which explicit pedagogical topics are covered. The differences in requirements from state to state are unnerving to many. According to the National Council on Teacher Quality (2007), Mississippi and Georgia require only nine credit hours to be completed whereas Utah requires 30 additional hours. The National Council on Teacher Quality also claims that

27 states require a Master's degree to complete the program and one state, Florida, actually forbids education coursework during the alternative programs.

Regardless of which alternative certification program an individual may choose, approximately one out of every five teachers enters into the teaching profession through an alternative program (Walsh \& Jacobs, 2007). The number of alternative certification programs that are accepted by each state varies as well. Programs, such as The New Teacher Project (TNTP), American Board for Certification of Teacher Excellence (ABCTE), New York City Teaching Fellows (NYCTF), Teach Kentucky, and Mississippi Teacher Corps (MTC), are some other alternative certification programs that are offered throughout the county.

One of the most commonly known alternative certification programs is Teach for America (TFA). Beginning in 1990, TFA had about 500 teachers in six widespread communities across the country and has now expanded to more than 7,000 teachers in 35 communities that are mostly rural and urban (Heilig \& Jez, 2010). The program requires a two-year commitment from the teachers, and, like many other alternative certification programs, training to teach in the classroom takes place between graduation and the first day of their teaching assignments. Heilig and Jez (2010) explained the process into the classroom to contain a brief student teaching experience, minor lessons in pedagogy and classroom management, and little to no information on which type of classroom or grade level they will enter until arriving at the teaching site. Although it seems like a much simpler task, "TFA teachers must continue coursework in local colleges to pursue full teaching credentials" (Heilig \& Jez, 2010, p. 1).

Emergency certification is yet another way that individuals may enter the classroom. Emergency certification is known to occur when a qualified teacher cannot be found for a classroom. The certificates are issued to individuals in order to teach but have not fulfilled all of the requirements to become certified regularly. The emergency certification is issued to an individual for a short period of time; however, it is expected that the person receiving the emergency certification will be attempting to acquire the necessary credentials (Williamson, Backman, Guy, Kat, \& Turley, 1984). The school will usually continue to search for qualified individuals to take the place of the emergency certified teacher.

One explanation that explains why so many different certification pathways exist is because "teacher demand has increased and funding inequalities have grown over the past 15 years" (Darling-Hammond, Holtzman, Gatlin, \& Heilig, 2005, p. 2). This has created teacher shortages, especially in the urban and poor rural school districts. In order to have a teacher in every classroom, these "urban and poor rural districts have hired a growing number of individuals on emergency permits or waivers who lack formal preparation for teaching" (Darling-Hammond et al., 2005, p. 2).

Pathways in Oklahoma. According to the Oklahoma State Department of Education website, Oklahoma has seven pathways to become certified: Traditional Path, Alternative Placement Program, Troops for Teachers, Career Development Program for Paraprofessionals, American Board for Certification of Teacher Excellence (ABCTE), Teach for America (TFA), and Emergency Certification ("Teacher Certification Paths," 2016). All of the pathways mentioned allow the certified individual to teach in the area of mathematics.

The traditional pathway for Oklahoma teacher certification listed on the Oklahoma State Department of Education (2006) website has the following requirements:

- Graduated from an accredited institution of higher education that has a state approved teacher education program (SATEP) for the certification area sought.
- Successfully completed a higher education teacher education program approved by the Office of Educational Quality and Accountability (OEQA).
- Meets all other requirements as established by the Oklahoma State Board of Education.
- Successfully passed the three required competency examinations
- Oklahoma General Education Test (OGET)
- Oklahoma Professional Teaching Examination (OPTE)
- Oklahoma Subject Area Tests (OSAT) (para. 1)

The Alternative Placement Program, Troops for Teachers, and Career Development Program for Paraprofessionals do not require that a SATEP be completed, but they all have the following requirements according to the Oklahoma Department of Education website:

- Have a minimum of a baccalaureate degree from an accredited college/university.
- Have a major in a field of study that corresponds to an area of Oklahoma certification for a Secondary Certificate, Elementary/Secondary Certificate, or a Career and Technology Education Certificate.
- Have at least a 2.5 retention Grade Point Average.
- Document of one to two years of work experience in your degree field area or completion of post-baccalaureate coursework related to your degree field area.
- Successfully passed the two required competency examinations
- Oklahoma General Education Test (OGET)
- Oklahoma Subject Area Test (OSAT) (para. 1)

The ABCTE website gives the following information and requirements in order to become a certified teacher in the state of Oklahoma ("Become a Teacher in Oklahoma, Get Teacher Certification Preparation," 2016):

- Complete the ABCTE teacher preparation program within the $8-10$ month timeframe.
- Obtain a full-time teaching position.
- Apply to the Oklahoma State Board of Education for a one-year, nonrenewable secondary or middle level teaching license.
- Apply concurrently for the ABCTE Mentoring Program and complete 1 year of ABCTE's mentoring program.
- After completing the first year of teaching and mentorship program, apply for the Oklahoma Standard License.

Teach for America, described earlier, has three prerequisites for application ("Getting Certified | Teach For America," 2015):

- A bachelor's degree from an accredited college or university by the first day of summer training with TFA.
- A cumulative undergraduate GPA of at least 2.50 on a 4.00 scale and pass any courses in progress on your transcript at the time of the TFA interview.
- Candidates must be U.S. citizens, nationals, legal permanent residents, or deferred action for childhood arrivals (DACA) recipients.

Once the TFA coursework and mentoring are completed for the required two years, an Oklahoma standard certificate will be issued.

Lastly, the final certification pathway in the state of Oklahoma is an emergency certification and "should only be requested when the district has exhausted every option to find an appropriately certified person for the open position" ("Teacher Certification Paths," 2016, para. 1).

Other states' insight on pathways. Many research studies have been conducted across the country examining the effects of teacher certification pathways, specifically in California, Florida, New York, North Carolina, South Carolina, and Texas (Boyd, Grossman, Lankford, Loeb, Michelli, \& Wycoff, 2006; Goe, 2002; Henry et al., 2014; Lincove, Osborne, Mills, \& Bellow, 2015; Mandeville \& Liu, 1997; Sass, 2011). In each of the states where the research was done, varying results were found.

In the California study (Goe, 2002), North Carolina study (Henry et al., 2014), and South Carolina study (Mandeville \& Liu, 1997), teacher certification was found to have significantly strong effects on the achievement of the students. Henry et al. (2014) found that in the state of North Carolina, teachers who chose alternative certification into the profession were less effective than teachers who chose a traditional pathway, especially in the areas of high school math, science, and social studies. California contains one of the highest percent of emergency certified teachers. Goe (2012) determined a significant negative relationship between student achievement in California schools and the percent of emergency permit (EP) teachers while controlling for
contributors to student achievement. The South Carolina study done by Mandeville and Liu (1997) is the most relatable for this paper since it looked directly at the impact of teacher certification on mathematics achievement in the middle school grades. Statistically significant differences were found between the performances of seventh grade students on low-level mathematics problems and high-level mathematics problems when taught by teachers with more specialized mathematics training.

Potential issues. The debate over teacher certification and its effectiveness in the classroom has been occurring for decades. With the recent literature on education and policy implementing need for highly qualified teachers for all students, teacher preparation reforms have resulted in upgrading credential and course requirements for traditional and alternative pathways (Touranki, 2009). Since the creation of the first alternative certification programs, research has been conducted to determine the effects that the specific teacher certification has had on student achievement, teacher efficacy, and teacher attrition in various states.

Several studies have not found significant differences between student achievement or teacher efficacy in regards to teacher certification pathway (Goldhaber \& Brewer, 2000; Tournaki et al., 2009; Unruh \& Holt, 2010). Goldhaber and Brewer (2000) compared students of teachers of traditional certification in their subject area to students of teachers with emergency certification or no certification in their subject area. In the subject areas of mathematics and science, the students of traditionally certified teachers compared to students of emergency certified teachers do no worse than one another in achievement levels. Interestingly enough, a year later, Darling-Hammond et al. (2001) claimed the Goldhaber and Brewer study to be untrue by analyzing and
criticizing the study's methods and findings. Darling-Hammond et al. (2001) argued, "students of certified teachers in mathematics do substantially better than those of uncertified teachers in every analysis" (p. 59).

Evans (2012) conducted an impactful study. It centered around 34 teachers in an alternative certification program in New York. The study focused on the mathematical problem-solving abilities and perceptions of the new teachers. All 34 were enrolled in a proof-based algebra course for a semester in which they were required to keep a journal of their own problem-solving abilities along with their students' problem-solving abilities. Evans found a significant improvement in problem solving for the teachers' own abilities, while the teachers described their students to still be struggling in problem solving due to lack of persistence and poor literacy skills. The teachers felt the proofbased algebra course had helped aid them in their own problem-solving experiences and, in return, facilitated student learning.

Since there was an increase in teachers' problem-solving scores over the course of the semester it can be argued that a strong mathematics requirement for alternative certification mathematics teachers, combined with their own teaching experiences, can lead to stronger problem-solving abilities, which is important given the emphasis of teaching mathematics from problem solving perspectives. (Evans, 2011, p. 40)

This leads into the question, "Can mathematics be taught through a problem-solving approach without having the appropriate education or training?"

Alternative certification pathways research was summarized very generally by Carlyn Ludlow (2011). After giving explanations on the supply of traditional teachers, a
paradigm shift in teacher certification, and descriptions of alternative routes to teacher certification, Ludlow (211) reached four conclusions. Those conclusions of alternative certification were the following:

1. Alternative pathways to certification are organizationally different in each state and represent each state's educational policy directives.
2. No statistically significant difference in student achievement exists between traditionally and alternatively certified teachers.
3. Research is inconclusive in alternative pathway's enrollment of higher quality teachers.
4. Alternative pathways to certification program participants are more diverse and alternative pathway teachers have higher probability to teach in high-minority schools. (Ludlow, 2011, p. 15)

Potential issues, such as affecting student achievement or teacher efficacy, were main points of discussion throughout the last section. More issues unrelated to student achievement and teacher efficacy that may turn up depending upon teacher certification pathway selection are the next focus of emphasis.

The first potential issue is retention rate. According to Roth and Swail (2000), the years of retention of teachers between short-term alternative programs, five-year master in education programs, and four-year traditional education programs are very dissimilar. About 33 percent of alternatively certified teachers remain in the profession after three years, while about 80 percent of those who complete a five-year master in education and about 50 percent who complete a four-year traditional education program stay employed as teachers after three years.

A second potential issue is the alternative routes to recruit a diverse group of individuals into the classroom. Research has shown that genuine alternatively certified teachers are high in minorities and are placed to teach in hard-to-staff schools, and minority students benefit from the minority teachers (Stoddart \& Floden, 1995). Alternative routes are driven by personal perspectives as a main source for teaching and for knowledge, which may be why minority students learn better from minority teachers.

The teaching profession seems to be moving in the direction contrary to that of others, such as law or medicine, for which completion of professional training from an accredited college or university is required before licensing (Stoddart \& Floden, 1995). This brings the attention to the next potential issue. Teaching is turning more and more into an "on-the-job" training profession rather than learning the tools of the trade before entering the classroom. A strength of alternatively certified teachers is their strong content knowledge background while they enter the classroom, but this does not always warrant successful teaching (Roth \& Swail, 2000). Another explanation of this particular potential issue is the lasting effect of student teaching through a traditional program compared to no student teaching experience in an alternative certification program.

The final potential issue was stated by Constantine, Player, Silva, Hallgren, and Deke (2009) who found that students of alternatively certified teachers scored lower in math when compared to students of traditionally certified teachers. This sounds familiar to studies that have been conducted before, but Constantine et al. suggested the lower performance was a result of the alternatively certified teacher taking additional courses while teaching.

Potential issues will result from any type of certification pathway into teaching. Educators should determine which pathway will cause less issues for the teachers and students alike. The number of different pathways to certification is a downfall in the United States, and measures to improve teacher education programs will not have much effect if states continue to hire teachers without the proper preparation (DarlingHammond et al. 2002).

## Mathematical Knowledge for Teaching (MKT)

Pedagogical content knowledge (PCK) became an area of strong interest in the late 1980s in a way to blend content and pedagogy for instruction to adapt to abilities of learners and display topics or problems in an organized manner (Shulman, 1987). Shulman's groundwork in PCK started a revolution in education research. Researchers in mathematics education have now spent the last 15 years assessing the types of mathematical knowledge that is most necessary for effective teaching (Ball, 2003; Ball et al., 2008; Hill \& Ball, 2009; Hill, Ball, \& Schilling, 2008; Hill et al., 2005; Hurrell, 2013; Li, 2011).
"How teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing" (Ball, 2003, p. 1). The PCK conception from Shulman (1987) combined with that of Ball (2003) to produce Mathematical Knowledge for Teaching (MKT). According to Ball et al. (2008), MKT is defined as the mathematical knowledge necessary to teach mathematical ideas, reason with the mathematics, be highly fluent in terminology, and have thoughtfulness in the nature of mathematics. Ball et al. (2008) were not satisfied with the PCK as given by Shulman
(1987) and felt the need to sensibly map it out and find ways to measure it to include how this knowledge is used for effective teaching.

Hill et al. (2008) not only focused their attention on teachers' subject matter knowledge, but they also turned their attention toward Knowledge of Content and Students (KCS), which they defined as knowledge of how students think and learn mixed with content knowledge of their own. In order to sum up MKT for other mathematics education researchers to interpret quickly, a domain map was created to display how KCS relates to subject matter knowledge and PCK (see Figure 2.1).

A few years before Hill et al. (2008) created the domain map of MKT, research was done on the effects of a teacher's MKT on student achievement. Hill et al. (2005) used a sample of schools that were engaged in instructional improvement and were highpoverty elementary schools in urban areas. After explicitly stating clear limitations in the study, results still showed that teachers' mathematical knowledge for teaching positively predicts gains in student mathematical achievement in grades 1 and 3. Hill et al. (2005) admitted that they were surprised about MKT having a positive effect on students' mathematical achievement in first grade when they expected effects of MKT to only show through in more complex mathematical


Figure 2.1. Mathematical Knowledge for Teaching (MKT) domain map (Hill et al., 2008, p. 377).

Hill et al. reported that it was interesting to find out that a "teachers' MKT plays a role even in the teaching of very elementary mathematics content" (p. 399). Continuing the discussion from earlier about the impact of a students' understanding of fractions on success in algebra brings about more thoughts as to how a third-grade teachers' MKT is connected to success in algebra.

The amount of research that has been done on MKT affecting success in algebra is one that needs to be dug deeper into by mathematics education researchers. Xuhui Li (2009) is one of those researchers that is continuing that dig into understanding a teachers' MKT in algebra courses. He considers the strong foundational role that algebra has in high school mathematics to be of much importance. He extends his thoughts to research on algebra-related MKT to go beyond algebra itself. Li believes in order to truly understand algebra-related MKT, the researcher must "address the unique characteristics and dynamics of individual algebraic topics, strands, or types of activities" (p. 2). The
most recent study done by Li (2011) was on quadratic equation solving. The case study took place of the most diverse school districts in California on a $9^{\text {th }}$ grade teacher. Li observed that the teachers' knowledge of subject matter tended to weigh in more than the pedagogical content knowledge. The success of the students' understanding of solving quadratic equations was not measured in this study. Simply analyzing which domain of MKT the teacher fell into most frequently was the main focus of the study. Again, much more research is required to fully understand the impact of a teachers' MKT on algebra success since it is an area that is so lightly touched.

## Teacher Beliefs

In educational research, the term beliefs is becoming a very common theme to attempt to understand. It is so common, in fact, that A.G. Thompson (1992) has stated that most researchers assume that the readers know what beliefs are. For a clearer understanding though, Richardson (1996) defines teacher beliefs to be "psychologically held understandings, premises, or propositions about the world that are thought to be true" (p. 259). It is widely known that beliefs can be classified according to the specific content area in which teachers are teaching. Pajeras (1992) stated, "Attention to the beliefs of teachers should be a focus of educational research and can inform educational practice in ways that prevailing research agendas have not and cannot" (p.307). Because of this, the study of teacher beliefs and how it influences the instructional practice in the classroom has gained strong motion in the past two decades.

Research shows that holding a particular belief system about learners, teaching, resources, knowledge, and curriculum are individualized for each teacher (Gudmundsdottir \& Shulman, 1987; Lovat \& Smith, 1995). As a result of each teacher
having their own beliefs about how a subject should be taught and learned, it has been shown by Clark and Peterson (1986) that those beliefs are the filter through which teachers make decisions for their teaching and learning rather than trusting their pedagogical knowledge or curriculum standards. NCTM (2000) makes a bold statement about teacher beliefs affecting students' evaluations of their own abilities and on their definitive outlooks on mathematics as a whole. The teacher beliefs could even go further into affecting the students' motivation to engage in mathematical tasks.

Teachers' mathematical beliefs can originate from a multitude of origins. Carroll (1995) believes that teachers' beliefs form strongly after spending years of their lives sitting and observing their former mathematics teachers. Along with learning not just the content in a mathematics classroom, future teachers are taking in instructional strategies as well. For those who enroll in a teacher education program, the beliefs of teaching and learning mathematics are implanted harshly in that individual. Researchers have discussed that the traditionalist view of some teacher education programs do not aid any positive effects on the teachers' already heavily weighted traditional view of mathematics and their mathematical beliefs (Brown \& Rose, 1995; Foss \& Kleinsasser, 1996).

For teachers who may not view teaching mathematics in a traditional way as effective, their beliefs may be affected by several other factors. It is expected from parents and, sometimes other colleagues, to teach mathematics in a traditional way. Perry, Howard, and Tracey (1999) found that with expectations such as focusing on examinations, following the textbook closely, and limiting movement in the classroom, those teachers with progressive educational beliefs are forced to conform to the
traditional instruction. Other studies have shown ethnic background, social class, and gender issues to affect the teachers' beliefs about mathematics (Butt \& Raymond, 1989).

As stated before by Brown and Rose (1995), the beliefs of mathematics teachers are individualized and personal. Studies have been performed to try to identify what could be considered right-and-wrong beliefs by mathematics teachers, but the conclusions have always been that there are too many broad differences between all teachers. Before trying to determine the right-and-wrong beliefs systems for mathematic teachers, Ernest (1989) researched the main areas of teachers' mathematical beliefs. His models for teachers' beliefs about mathematics included researching their beliefs on the nature of mathematics as well as their beliefs on the teaching and learning of mathematics. Overall, several researchers tend to focus on the study of these three categorizations of teachers' mathematical beliefs.

To sum up the idea of teachers' mathematical beliefs, "its teaching and learning reflect a teacher's priorities for the practices of mathematics classrooms and play a significant role in shaping teachers' characteristic patterns of instructional behavior" (Lepik \& Pipere, 2011, p. 116). For a visual reference, Raymond (1997) developed a model of relationships between mathematics beliefs and practice (see Figure 2.2).

In the next two sections, literature will be provided on the three areas of teachers' mathematical beliefs previously mentioned - the nature of mathematics, the teaching of mathematics, and the learning of mathematics.


Figure 2.2. A model of the relationship between beliefs and teaching practices (Raymond, 1997).

Beliefs about the nature of mathematics. A teacher's belief system about the nature of mathematics do not have to be held directly the same for one's entire teaching career, nor do they have to be fully developed into a philosophy. Research has shown that teachers of mathematics tend to hold different views on what they believe mathematics to represent. Ernest (1989) presents the three main views that teachers commonly hold about the nature of mathematics as a whole:

1. Mathematics is a continually expanding field of human inquiry where it is an unfinished product and its results remain open to revision.
2. Mathematics is a static, but unified body of knowledge, consisting of interconnecting structures and truths that is discovered, not created.
3. Mathematics is a useful, but unrelated collection of facts, rules, and skills. (p. 21)

Each of the views listed above are known respectively as the problem-solving view, the Platonist view, and the instrumentalist view.

Each of the different beliefs of mathematics can also be known as a philosophy by the mathematics teacher. Ernest (1989) went on to explain how each of the three philosophies can create practical outcomes in the mathematics classroom:

An active, problem solving view of mathematical knowledge can lead to the acceptance of children's methods and approaches to tasks. In contrast, a static

Platonist or instrumentalist view of mathematics can lead to the teacher's
insistence on there being a single 'correct' method for solving each problem. (p. 21)

The way teachers view the nature of mathematics show abruptly through the teaching instructions in their classrooms. Francis (2014) goes on to state that beliefs supporting problem-solving and critical thinking are deemed the most advantageous for instruction by mathematics teachers.

When teachers believe mathematics to be a set of facts and procedures to be learned and memorized, the idea of an inquiry-based classroom is lost. Teachers have attempted many efforts to stress inquiry-based mathematics lessons in their classrooms, but many struggle to bring any changes in their practices or student success (Cohen, 1990; Wilson \& Lloyd, 2000). Since most teachers' beliefs and practices are rooted with school mathematics traditions, education initiatives must identify ways that encourage teachers to change their beliefs (Lloyd, 2002).

Beliefs about teaching and learning mathematics. Most researchers have focused on elementary school teachers' beliefs on students' thinking. Darling-Hammond and

Sclan (1996) determined that most prospective elementary school teachers care primarily about the children themselves, but not about the mathematics they are teaching to the children. There have been pieces of literature done on middle grade teacher beliefs and secondary education teacher beliefs that impact their instructional judgements.

Sowder et. al (1998) contended that teachers need to explore more realistic problems in order to make sense of mathematical ideas for themselves before introducing the ideas to the students. This allows for a wide understanding of sometimes simple ideas so the inclusion of a variety of student reactions can occur. The most important point made by Sowder et. al (1998) is it provides a framework for teachers to analyze the work of the students beyond just correct and incorrect answers. The beliefs of how the teachers thought their students learned topics could be uncovered by how much the teachers engage in that type of analysis.

Borasi, Fronzi, Smith, and Rose (1999) researched middle school mathematics teachers' beliefs when the teachers were required to take part in inquiry-based lessons from the perspective of the students. It was confirmed from this study that it is important for teachers to experience these types of learning situations before implementing them in the classroom as the role of the teacher. Although perfect implementation in the teacher's own classroom was not always certain, the study gave way to understanding more about teachers' beliefs concerning how their students learn mathematics. There are often many factors that contribute to teachers' beliefs about learning mathematics, but Borasi et al. found that this technique has a substantial effect on possibly changing the teachers' beliefs.

Turning the focus toward the secondary level, Even (1999) highlighted how changing teachers' beliefs can lead to changes in teaching practices. Even stated that teachers look for "reliable and relevant rules that can be put to immediate use" (p. 236). The traditional teachers who believe mathematics is a system of rules to be memorized is the point Even is trying to make. The study also came to the conclusion that providing flexibility and teaching multiple ways to find a solution is a critical factor in mathematics teaching.

Another framework, referred to as the Phase-Dimension Framework (PDF) was described in an article by Artzt and Armour-Thomas (1999) between teachers' beliefs and their instructional practice. The framework allowed the researchers to assess teacher behaviors on teaching conceptually versus teaching theoretically. The outcomes of the study suggested that reflecting about one's own teaching practice can be helpful to promote more positive situations in the classroom, especially when focusing the reflection on student understanding.

Some of the previous studies did not address the view of beliefs very often, but the studies all suggest that teachers' beliefs incorporate several different kinds of behavior. In general, it is often noted that changing teacher beliefs about teaching and learning is a very difficult and sometimes impossible task. Depending on how important or functionally connected the teachers' beliefs are to the mathematics itself is a large influence on changing the beliefs of the teaching and learning of mathematics (Chapman, 2002).

## Summary

Mathematics education, and more specifically Algebra I education, is an area of research that could always use more understanding in order to strengthen teacher quality. The need for strong teachers in an Algebra I classroom is evident from the literature recently presented in order to produce everyday problem solvers once students leave the classroom. This leads to concerns about how prepared teachers in an Algebra I classroom truly are to produce high levels of student achievement. This also leads to concerns about what types of beliefs teachers hold about the teaching and learning of

## Algebra I.

Enrollment in gatekeeper courses, such as Algebra I, initiates that growth into the higher-level mathematics courses that are necessary for college readiness (Atanda, 1999). For those students to gain the knowledge required to succeed in those higher-level mathematics courses, the teacher plays a key factor. With that said, "the use of mathematical knowledge in teaching is often taken for granted" (Ball \& Bass, 2000, p. 86). The certification pathway into an Algebra I class can vary between teachers, which results in little to no understanding of mathematical knowledge for some.

The mathematical beliefs of the teachers are so imperative that they shape the teachers' decisions and actions in the classroom daily (Nathan \& Koedinger, 2000). It is just as important to understand the beliefs teachers hold as it is to understand their educational backgrounds, certification pathways, and their mathematical knowledge for teaching. Nathan and Koedinger (2000) also said that being able to incorporate changes in the teachers' classroom practices requires a crucial understanding of the beliefs those teachers hold about the teaching and learning of their content.

The current study helps to address the gaps in literature related to Algebra I teachers' certification pathway and their Knowledge of Algebra Teaching (KAT) on their beliefs in the classroom. This study also paints the picture of the Algebra I teacher workforce in the state of Oklahoma to provide more insight on whom is teaching this gatekeeper course to students.

## CHAPTER III

# ALGEBRA I TEACHERS' BELIEFS AND KNOWLEDGE OF ALGEBRA FOR TEACHING 

Target Journal: Investigations in Mathematics Learning

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#### Abstract

:

Research indicates that teachers' mathematical beliefs and mathematical knowledge for teaching impacts practices in the classroom. Research also suggests that success in Algebra I is the gatekeeper to higher-level mathematics. With the increased number of certification pathways in some states, it is important to identify those Algebra I teachers' beliefs and knowledge of algebra for teaching. A study of current Algebra I teachers revealed that regardless of certification pathway, mathematical beliefs are not significantly different. Additionally, significant differences did exist in regards to certification pathway and Knowledge of Algebra for Teaching (KAT) levels.


## Introduction

The teacher shortage crisis in public schools over the last two decades has created concern for many school systems and policymakers (Flynt \& Morton, 2009; Ingersoll \& Smith, 2003). Enrollment in teacher preparation programs in the United States decreased over 35 percent from 2009 - 2014 and enrollment only continues to decline with each
passing year (Aragon, 2016). With the shortage of teachers to place in the classroom, schools are forced to lower their standards for teacher quality, especially in the areas of mathematics and science (Ingersoll \& Smith, 2003; Murnane et al., 1991; Liu et al., 2008). This shortage has opened up a multitude of pathways for teachers to enter the mathematics classroom. Particularly in the state of Oklahoma where there are now seven different pathways to certification (Oklahoma State Department of Education, 2016).

This leads to questions about who the teachers are currently in classrooms and what practices they are using to instruct students. There are several factors that contribute to a teacher's practices in the classroom, however, teacher beliefs and pedagogical content knowledge are considered to be two of those main factors (Pajares, 1992; Wilkins, 2008; Ball, Thames, \& Phelps, 2008). With a push for mathematics to be taught with a more conceptual and problem-solving approach by researchers and educators alike, the overarching view of mathematics by teachers is still seen as procedural and full of algorithms to be memorized (Cai, 1994). Teachers with these beliefs are less likely to understand how to guide students' construction of mathematical ideas (Battista, 1994). Along with teacher beliefs, pedagogical content knowledge connects content knowledge to teaching practices (Shulman, 1986; Ball \& Bass, 2000).

Teacher practices are especially important in gatekeeper courses such as Algebra I (Stinson, 2004). The success of students in Algebra I has been linked to performance in college, career readiness, impact on career salary, and perceptions of higher mathematics (Eddy et al., 2015; Gaertner, Kim, DesJardins, \& McClarty, 2014; Kim, Kim, DesJardins, \& McCall, 2015; Murnane, Willett, \& Levy, 1995; Siegler et al., 2012).

This study aims to portray a picture of who is teaching Algebra I in Oklahoma and highlight the pathways to certification taken by those teachers. Additionally, this study explored the algebra beliefs of these Algebra I teachers along with their own understanding of algebra and the teaching of algebra concepts.

## Related Literature

The research on teacher beliefs in this study are broken down into three main categories: beliefs about nature of mathematics, beliefs about learning mathematics, and beliefs about teaching mathematics. Mathematical Knowledge for Teaching (MKT) and teacher certification pathway are the two other main focuses of this study. Thus, a discussion of how the combination of teacher beliefs and MKT influence teacher practices follows.

## Teacher Beliefs

The term beliefs refers to "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Richardson, 1996, p. 259). Research indicates that teachers' belief systems are highly individualized (Gudmundsdottir \& Shulman, 1987; Lovat \& Smith, 1995). Additionally, researchers found that these individual beliefs held by teachers greatly determine their teaching practices in the classroom regardless of their pedagogical knowledge or curriculum standards (Clark \& Peterson, 1986; Lepik \& Pipere, 2011). Furthermore, Love \& Kruger (2005) found teacher beliefs to impact student achievement when teachers believed in a sense of community in the classroom and flexible teaching strategies.

Teacher beliefs about nature of mathematics. Mathematicians and mathematics educators have debated for years over how the nature of mathematics can be defined (Skemp, 1976; Hersh, 1979; Ernest, 1989). Describing the nature of mathematics the definition given by Ernest (1989) provided the following three views commonly held by mathematics teachers:

1. Mathematics is a continually expanding field of human inquiry where it is an unfinished product and its results remain open to revision.
2. Mathematics is a static, but unified body of knowledge, consisting of truths which are discovered, not created.
3. Mathematics is a useful, but unrelated collection of facts, rules, and skills (p. 21).

Each of these views are known respectively as the dynamic problem-solving view, the Platonist view, and the static instrumentalist view. Ernest (1989) explained further that, An active, problem solving view of mathematical knowledge can lead to the acceptance of children's methods and approaches to tasks. In contrast, a static Platonist or instrumentalist view of mathematics can lead to the teacher's insistence on there being a single 'correct' method for solving each problem. (p. 21)

Teachers who hold a dynamic problem-solving view about the nature of mathematics show strong impact through the teaching instructions in the classroom and bring about more understanding and desire to learn mathematics (Francis, 2014). Similarly, Lerman (1990) recognized that children will be able to apply prior knowledge
of mathematics in a creative manner when problem-solving views of mathematics are held.

Teacher beliefs about teaching of mathematics. Researchers have found that typical, daily mathematics instruction is taught by the introduction of a new procedure with step-by-step instructions, then followed up by homework problems that are meant to mimic the procedure (Stipek et al., 2001). The opposing style of constructivist teaching focuses more on posing relevant problems to solve, learning the essence of primary concepts, and valuing the student's viewpoints on solving the problems (Kim, 2005). Even when a professional development program was implemented to encourage the use of a more constructivist teaching style, researchers found that teachers assimilated new practices back to traditional practices due to their own experiences of how they were taught mathematics (Cohen \& Ball, 1990; Raymond, 1997).

Similarly, Van Zoest, Jones, and Thornton (1994) studied two groups of preservice mathematics teachers. One group was in a mentorship program heavily based on a constructivist approach to teaching mathematics. The second group of pre-service mathematics teachers were not in the mentorship program. After comparing beliefs of each group after the mentorship was completed, it initially seemed like the pre-service teachers in the program were persuaded to teach mathematics in a constructivist way. At the conclusion of the study, it was determined that both groups had resorted to a more traditional set of beliefs about teaching mathematics.

Stipek et al. (2001) found a direct correlation between teachers' beliefs and their classroom practices. The study showed those teachers who held traditional beliefs about mathematics teaching tended to stress getting correct answers, achieving good grades,
and speed of finding solutions, rather than teaching for conceptual understanding. These teachers were also found to assert mistakes as a negative in the classroom instead of using those mistakes as a learning opportunity.

Teacher beliefs about learning of mathematics. A teachers' belief of how students learn mathematics is a major factor in how teachers carry out their instruction in the classroom. Teachers need to be able to perceive types of mathematics activities that will best develop the learning of the students (Ball, 2003).

In order to describe the relationship between beliefs and practice, Peterson, Fennema, Carpenter, and Loef (1989) found that teachers who believe their students learn mathematics through a problem-solving approach, used more word problems in instruction. Similarly, they found those same teachers spent more time developing number sense rather than teaching number facts. Conversely, another study conducted by Even and Tirosh (2002) found that teachers rarely base their practices on how they believe their students learn mathematics, but rather on how their students can immediately put the rules and information to use.

Teachers' beliefs about mathematics, the teaching of mathematics, and the learning of mathematics can have implications on classroom practice. Although beliefs are not the only factor impacting practice, they can be considered highly influencing.

## Mathematical Knowledge for Teaching

Researchers in mathematics education have now spent the last couple of decades assessing the types of mathematical knowledge that is necessary for effective teaching (Ball, 2003; Ball et al., 2008; Hill, Ball, \& Schilling, 2008; Hill et al., 2005; Hurrell, 2013; Li, 2011). According to Ball et al. (2008), Mathematical Knowledge for Teaching
(MKT) places an emphasis on both subject matter and pedagogical content knowledge. MKT moves away from just knowing mathematical content to being able to teach the mathematical content. Focusing specifically on secondary mathematics teachers, MKT research showed that simply taking a certain number of higher level mathematics courses does not always guarantee a suitable level of mathematical knowledge to teach mathematics (Even, 1999). Wilson \& Heid (2010) developed a framework that defines what mathematical understanding for secondary teaching to be:

To facilitate the learning of secondary school mathematics, teachers need a particular kind of understanding. Mathematical understanding for teaching at the secondary level is the mathematical expertise and skill a teacher has and uses for the purpose of promoting students' understanding of, proficiency with, and appreciation for mathematics. It requires that teachers not only know more mathematics than they teach but also know it more deeply. (p. 2)

A recent framework was developed to investigate mathematical knowledge for teaching algebra (McCrory, Floden, Ferrini-Mundy, Reckase, \& Senk, 2012). The researchers suggested three categories of knowledge were needed to effectively teach algebra. These categories include (a) school algebra - knowledge of concepts and ideas taught in high school algebra, (b) advanced mathematics - knowledge of college level mathematics, and (c) algebra for teaching - pedagogical content knowledge of teaching algebra. McCrory et al. (2012) found that secondary teachers are strictly measured for knowledge of teaching through content tests or by number of mathematics courses taken, where neither of these measures give a full understanding of a teachers' knowledge of algebra for teaching.

## Certification Pathways

Certification pathways has been a topic of study since the 1980s when alternative certification programs became abundant with research finding mixed results on the impact of these different pathways on student achievement. Several research studies have found no significant difference exists between student achievement based on their teacher's certification pathway (Goldhaber \& Brewer, 2000; Tournaki, Lyublinskaya, \& Carolan, 2009; Unruh \& Holt, 2010). For example, Goldhaber and Brewer (2000) found no significant difference in achievement levels of students with teachers certified through a traditional teacher preparation in their subject area compared to students of teachers with no certification in their subject areas. In contrast, researchers in a California study (Goe, 2002), North Carolina study (Henry et al., 2014), and South Carolina study (Mandeville \& Liu, 1997), found teacher certification to have significantly strong effects on student achievement when teachers were certified through a traditional teacher program, especially in the areas of mathematics and science.

With the multitude of certification pathways in Oklahoma, one may question the teachers in mathematics classrooms. According to the Oklahoma State Department of Education website, Oklahoma has seven pathways to become certified: Traditional Path, Alternative Placement Program, Troops for Teachers, Career Development Program for Paraprofessionals, American Board for Certification of Teacher Excellence, Teach for America, and Emergency Certification (Oklahoma State Department of Education, 2016). Figure 3.1 provides the requirements for each of these pathways.

The research on mathematical knowledge for teaching is especially important for algebra teachers because of the recent push for all students to take algebra (Pappano,
2012). Additional research is needed to further examine the beliefs of teachers who are
in an Algebra I classroom. The purpose of this study is to investigate the influence an

| Pathways to Certification in Oklahoma | Requirements |
| :---: | :---: |
| 1. Traditional Path | - Graduated from an accredited institution of higher education that has a state approved teacher education program (SATEP) for the certification area sought. <br> - Successfully completed a higher education teacher education program approved by the Office of Educational Quality and Accountability (OEQA). <br> - Meets all other requirements as established by the Oklahoma State Board of Education. <br> - Successfully passed the three required competency examinations <br> - Oklahoma General Education Test (OGET) <br> - Oklahoma Professional Teaching Examination (OPTE) <br> - Oklahoma Subject Area Tests (OSAT) |
| 2. Alternative Placement Program <br> 3. Troops for Teachers <br> 4. Career Development <br> Program <br> for Paraprofessionals | - Have a minimum of a baccalaureate degree from an accredited college/university. <br> - Have a major in a field of study that corresponds to an area of Oklahoma certification for a Secondary Certificate, Elementary/Secondary Certificate, or a Career and Technology Education Certificate. <br> - Have at least a 2.5 retention Grade Point Average. <br> - Document of one to two years of work experience in your degree field area or completion of post-baccalaureate coursework related to your degree field area. <br> - Successfully passed the two required competency examinations - Oklahoma General Education Test (OGET) <br> - Oklahoma Subject Area Test (OSAT) |
| 5. American Board for Certification of Teacher Excellence | - Complete the ABCTE teacher preparation program within the 8-10 month timeframe. <br> - Obtain a full-time teaching position. <br> - Apply to the Oklahoma State Board of Education for a one-year, non-renewable secondary or middle level teaching license. <br> - Apply concurrently for the ABCTE Mentoring Program and complete 1 year of ABCTE's mentoring program. <br> - After completing the first year of teaching and mentorship program, apply for the Oklahoma Standard License. |
| 6. Teach for America | - A bachelor's degree from an accredited college or university by the first day of summer training with TFA. <br> - A cumulative undergraduate GPA of at least 2.50 on a 4.00 scale and pass any courses in progress on your transcript at the time of the TFA interview. <br> - Candidates must be U.S. citizens, nationals, legal permanent residents, or deferred action for childhood arrivals (DACA) recipients. |
| 7. Emergency Certification | - Decision left to the school district when every option to find an appropriately certified person for the open position is exhausted. |

Figure 3.1. Certification pathways and requirements in the state of Oklahoma. This information can found and was retrieved from http://sde.ok.gov/sde/teacher-certification-paths.

Algebra I teachers' certification pathway has on their beliefs and Knowledge of Algebra for Teaching (KAT). The following research questions were utilized to guide the study:

1. Who is the Algebra I teaching force in Oklahoma?
a. What are the characteristics of the Algebra I teachers?
b. What beliefs do Algebra I teachers in Oklahoma hold about mathematics, teaching mathematics, and learning mathematics?
c. What is the Algebra I teachers' Knowledge of Algebra for Teaching (KAT)?
2. Is there a significant difference between an algebra teacher's certification pathway and the beliefs he or she holds?
3. Is there a significant difference between an algebra teacher's certification pathway and his or her Knowledge of Algebra for Teaching (KAT)?
4. Is there an association between an Algebra I teachers' Knowledge of Algebra Teaching (KAT) and their beliefs about algebra, about teaching algebra, and about learning algebra across certification pathways?

## Methodology

## Research Design

This study used a survey research design to quantitatively describe the beliefs and Knowledge of Algebra for Teaching (KAT) of Algebra I teachers in the state of Oklahoma (Creswell, 2013). The sample of teachers in the study can be used to generalize to all Algebra I teachers in Oklahoma.

## Participants

After an open records request was made to the Oklahoma State Department of Education, all Oklahoma public school mathematics teachers $(\mathrm{N}=2,488)$ were emailed a link to an online questionnaire. The email addresses of specifically Algebra I teachers were not given, although the number of Algebra I teachers $(\mathrm{N}=1,455)$ was given. The questionnaire was completed by 144 Algebra I teachers from across the state of Oklahoma, which resulted in a $9.9 \%$ response rate from Algebra I teachers.

The geographic regions in the state of Oklahoma were divided into eight different regions by the Oklahoma State Department of Education called the REAC ${ }^{3} \mathrm{H}$ regions (http://sde.ok.gov/sde/sites/ok.gov.sde/files/documents/files/REAC3H\ Network\ R egions\%20map.pdf ). The data in Table 3.1 shows that the sample was representative of the state population of mathematics teachers according to geographic distribution, education level, teaching experience, and ethnicity.

For the purpose of this study, the sample was broken down into four different grouping variables based on certification pathway. The four different pathways used were the following:

1. Mathematics Education $(\mathrm{n}=67)$ - any teacher who completed a degree in mathematics or secondary education mathematics while completing a teacher education program leading to certification.
2. Mathematics $(\mathrm{n}=16)$ - any teacher who holds a Bachelor's degree in mathematics while becoming alternatively certified.

Table 3.1


Highest Education Level

| Bachelor's | 1853 | 69.40 | 71 | 49.31 |
| ---: | ---: | ---: | ---: | ---: |
| Master's | 801 | 30.00 | 72 | 50.00 |
| Doctorate | 15 | 0.56 | 1 | 0.69 |

## Teaching Experience

(Years)

| to 5 | 701 | 26.25 | 27 | 18.75 |
| ---: | ---: | ---: | ---: | ---: |
| 6 to 10 | 508 | 19.03 | 30 | 20.83 |
| 11 to 15 | 457 | 17.12 | 35 | 24.31 |
| 16 to 20 | 363 | 13.60 | 16 | 11.11 |
| 21 to 25 | 252 | 9.44 | 14 | 9.72 |
| 26 to 30 | 203 | 7.60 | 15 | 10.42 |
| 31 to 35 | 92 | 3.45 | 3 | 2.08 |
| 36 to 40 | 61 | 2.29 | 2 | 1.39 |
| over 40 | 33 | 1.24 | 2 | 1.39 |

Ethnicity

| African American | 68 | 2.55 | 2 | 1.39 |
| ---: | ---: | ---: | ---: | ---: |
| American Indian or Alaskan |  |  |  |  |
| Native | 148 | 5.54 | 13 | 9.03 |
| Hispanic | 49 | 1.84 | 3 | 2.08 |
| Asian or Pacific Islander | 25 | 0.94 | 0 | 0 |
| White | 2,316 | 86.74 | 122 | 84.72 |
| More than One | 64 | 2.40 | 4 | 2.78 |

[^0]3. Elementary Education $(\mathrm{n}=23)$ - any teacher who completed a degree in elementary education while completing a teacher education program leading to certification.
4. Other $(\mathrm{n}=38)$ - any teacher who did not follow one of three previous paths mentioned above.

Referring back to the pathways listed in Figure 3.1, the mathematics education and elementary education grouping variables in this study follow the traditional pathway in Oklahoma. The mathematics grouping variable follow the alternative placement program in Oklahoma. Finally, the other grouping variable may follow one of the other five pathways to certification.

## Measures

Three different instruments constituted the data sources in the online questionnaire. Participants were asked to provide demographic information, respond openly about their beliefs about algebra, and participate in a 20 -question assessment that measures their Knowledge of Algebra for Teaching (KAT). Since the last three questions on the KAT were open ended and the process of uploading solutions was timeconsuming, several teachers did not complete that portion. Scoring on the KAT was adjusted to not include the last three open-ended questions, therefore, those participants who completed all parts of the questionnaire excluding the three open-ended questions were still considered in this study.

Demographics. The online questionnaire (see Appendix A) collected information on the Algebra I teachers' current grade being taught, school name, and district. This allowed the teachers to be filtered in the correct $\mathrm{REAC}^{3} \mathrm{H}$ region. Additionally, the
teachers were asked to state their age, ethnicity, number of years they have taught mathematics, educational background, and certification pathway.

Algebra Beliefs Questionnaire. The algebra beliefs questionnaire (see Appendix B) used in this study is a modification of Raymond's (1997) beliefs questionnaire by changing all mentions of "mathematics" to "algebra". The questionnaire has three subscales - beliefs about the nature of algebra, beliefs about learning algebra, and beliefs about teaching algebra. While Raymond did not validate the instrument, two mathematics educators examined the revised instrument to ensure that the questions measured the individual beliefs specified. Cronbach's alphas were calculated for each of the three subscales using the data from this study. Those Cronbach's alphas for beliefs about the nature of algebra, learning algebra, and teaching algebra were $.81, .75$, and .54 , respectively. Each subscale has a series of semantic differential ranging from 1-13 and a group of 5-point Likert-type questions. The 5-piont Likert questions were scaled to match the 13-point range of the semantic differential questions. The beliefs about the nature of algebra subscale has 8 questions of each type with a range of potential scores being from 16 to 176 with higher scores more indicative of a problem-solving view of algebra. For the beliefs about learning algebra, there were 7 semantic differential questions and 10 Likert-type questions with a range of scores being from 17 to 187 with higher scores more indicative of a discovery view of algebra. The beliefs about the teaching of algebra subscale has 8 semantic differential questions and 7 Likert-type questions with a range of potential scores being from 15 to 165 with higher scores more indicative of a discovery view of algebra.

Survey of Knowledge of Algebra for Teaching (KAT). The Survey of Knowledge of Algebra for Teaching (Floden, Ferrini-Mundy, Senk, Reckase, \& McCrory, 2012) measures the knowledge most efficient in the teaching of algebra. During the validation study, this instrument had a Cronbach alpha reliability of .84 . The KAT measures three dimensions of a teachers' mathematical knowledge for teaching with the range of scores included - teaching knowledge (25.72 - 53.23), knowledge of school algebra (29.11 57.63), and advanced knowledge of mathematics (37.02-60.11). The teaching knowledge dimension includes five questions about mathematical knowledge specific to teaching algebra concepts. The knowledge of school algebra dimension involves questions typically taught in a middle or high school algebra course. The advanced knowledge of mathematics dimension includes questions typically taught in college level mathematics courses. The instrument also provides a final score, incorporating all three dimensions together with a score range of $26.72-57.62$.

## Data Analysis

Results were analyzed using both descriptive and inferential statistics using SAS ${ }^{\circledR}$ software, version 9 of the SAS system (SAS Institute, 2013). Descriptive statistics were used to show information across certification pathways. Inferential statistics included the use of a one-way ANOVA to find any significant differences between the four certification pathways in terms of beliefs scores and KAT scores. All assumptions for one-way ANOVA's were checked including the use of the Levene's test to check the homogeneity of variances between groups. Where significant differences were found between groups, Tukey's HSD test was then run to determine the differences between
exact groups. Box and whisker plots were used to visualize data and make comparisons across certification pathways.

## Results

In order to describe who Algebra I teachers in Oklahoma are, a variety of characteristics were used such as age, ethnicity, years of teaching experience, and highest education level (see Table 3.1). Of the 144 teachers sampled, the average age was nearly 43 years old. The ethnicity of the teachers are predominantly White with the second largest ethnicity being American Indian or Alaska Native. The years of teaching experience of those teachers were largely clumped between $1-15$ years with just under 64 percent of Algebra I teachers falling in that category. Also, nearly 20 percent of those teachers are novice with only $1-5$ years of teaching experience. The number of teachers who held a Bachelor's degree and those who held a Master's degree were 49 percent and 50 percent, respectively. Furthermore, 32 percent of teachers with a Master's degree hold one in the area of mathematics education.

The certification pathways of Algebra I teachers in this study were broken down into four groups - traditional mathematics education certification, Bachelor's degree in mathematics with alternative certification, traditional elementary education certification, or any other pathway different from the previous three. The percentage of teachers who followed a traditional mathematics education certification was 47 percent and teachers who followed a traditional elementary education pathway was 16 percent. This means that 63 percent of teachers in this study were certified through a traditional certification pathway. Teachers who hold a Bachelor's degree in mathematics and were alternatively
certified to teach make up 11 percent of this sample. The remaining 26 percent of teachers hold non-traditional teacher certifications in non-mathematics areas.

The three different beliefs being measured in this study are beliefs about the nature of algebra, learning of algebra, and teaching of algebra, where descriptive statistics of each of the four certification pathways on beliefs are given in Table 3.3. Overall, Algebra I teachers in Oklahoma did not have mean belief scores that were considered to be problem-solving or constructivist views. In general, the means and standard deviations in each certification pathway were very similar and the teachers fell noticeably in the middle of each spectrum of the beliefs categories. Although, a consistency was found in that the mean belief scores of the nature of algebra were consistently the highest of the three types of beliefs regardless of certification pathway. After using one-way ANOVA, no significant differences were found at the $\alpha=.05$ level between any of the four certification pathways in any of the three areas of beliefs (see Table 3.4).

Table 3.2
Descriptive Statistics of Beliefs about Nature, Learning, and Teaching of Algebra.

|  | Nature of Algebra | Learning of Algebra | Teaching of Algebra |
| :---: | :---: | :---: | :---: |
| Pathway | $M(S D)$ | $M(S D)$ | $M(S D)$ |
| Mathematics Education | 6.60 (.60) | 6.20 (.62) | 5.76 (.75) |
| Mathematics | 6.61 (.53) | 5.94 (.65) | 5.65 (.92) |
| Elementary Education | 6.40 (.48) | 6.32 (.45) | 5.48 (.75) |
| Other | 6.35 (.69) | 6.26 (.59) | 5.78 (.64) |

[^1]Table 3.3

| $l$ |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| One-Way ANOVA Results of Certification Pathway on Beliefs about Nature, Learning, and <br> Teaching of Algebra. |  |  |  |  |  |
| Variable and Source | $d f$ | SS | $M S$ | $F$ | $p$ |
| Nature of Algebra |  |  |  |  |  |
| Between groups | 3 | 1.97 | .66 | 1.80 | .1497 |
| Within groups | 140 | 51.04 | .36 |  |  |
| Learning of Algebra |  |  |  |  |  |
| Between groups | 3 | 1.45 | .48 | 1.32 | .2691 |
| Within groups | 140 | 51.11 | .37 |  |  |
| Teaching of Algebra |  |  |  |  |  |
| Between groups | 3 | 1.58 | .53 | .96 | .4121 |
| Within groups | 140 | 76.43 | .55 |  |  |

When analyzing the KAT scores for Algebra I teachers in this study, the knowledge of teaching (Tscore), knowledge of school algebra (Sscore), advanced knowledge of mathematics (Ascore), and Final Score mean scores were 38.58, 43.73, 48.29 , and 42.35 respectively. Those Algebra I teachers who were certified through a traditional mathematics education pathway and those with a Bachelor's in mathematics plus alternative certification scored at or above the overall means in every dimension of the KAT. The mathematics education group obtained the highest mean score on all of the KAT scores, excluding the Ascore $(M=51.19)$. The mathematics group had the highest mean score for the Ascore dimension of the KAT $(M=52.21)$. The elementary education group obtained the lowest mean score on all four of the KAT scores, including the final score $(M=36.93)$. It is important to notice that the elementary education group had the lowest standard deviations in all four of the scoring sections, emphasizing that their scores are not spread out much about the mean scores. It is also notable that those teachers who were certified through a mathematics education $(M=45.13, S D=5.97)$ pathway and those were alternatively certified with a Bachelor's in mathematics ( $M=$
44.74, $S D=5.08$ ) had final mean scores that only differed by .39 . The full descriptive statistics are given for the four different KAT scores depending on certification pathway in Table 3.5. Also, a visual representation of all four dimensions of the KAT scores between certification pathways using box and whisker plots is provided in Figure 3.2. The plots show the differences in the range of scores between certification pathways along with showing which pathways scored higher and lower on each dimension of the KAT. From an initial look, it appears that elementary education and other pathways tend to have the lowest scores in every aspect of the KAT. Additionally, mathematics education and mathematics pathways appear to have the highest mean score in every aspect of the KAT. It was noticed that mathematics education and mathematics tend to score near the same in each dimension of the KAT.

Table 3.4
Descriptive Statistics of Knowledge of Algebra for Teaching (KAT) Scores.

|  | $\frac{\text { Tscore }}{M(S D)}$ | $\underline{\text { Sscore }}$ | $\frac{\text { Ascore }}{M(S D)}$ | $\frac{\text { Final Score }}{M(S D)}$ |
| :--- | :---: | :---: | :---: | :---: |
| Pathway | $40.38(5.85)$ | $46.16(6.51)$ | $51.19(6.85)$ | $45.13(5.97)$ |
| Mathematics Education | $48.58(6.18)$ | $45.15(5.38)$ | $52.21(7.67)$ | $44.74(5.08)$ |
| Mathematics | $38.99(5.40)$ | $39.46(4.71)$ | $42.20(4.78)$ | $36.93(3.76)$ |
| Elementary Education | $34.99 .64(6.36)$ | $41.54(4.86)$ | $45.38(6.91)$ | $39.86(5.30)$ |
| Other | 37.64 |  |  |  |

Note. Tscore = Teaching Knowledge; Sscore = Knowledge of School Algebra; Ascore = Advanced Knowledge of Mathematics.
Range of Tscore $=25.72-53.23$, Sscore $=29.11-57.63$, and Ascore $=37.02-60.11$


Figure 3.2. Certification Pathway scores on the Tscore, Sscore, Ascore, and Final Score portions of the Knowledge of Algebra for Teaching (KAT).

With the use of a one-way ANOVA, significant differences were found between multiple different certification pathways when comparing all four dimensions of KAT levels (see Table 3.6). All dimensions were significant at the $\alpha=.001$ level, except Tscore, which was significant at $\alpha=.01$. Thus, post hoc comparisons using the Tukey HSD test were made on all four dimensions of the KAT scores with those test results for Tscore, Sscore, Ascore, and Final Score displayed in Table 3.7. The significant comparison results for each of the four dimensions are discussed below.

Table 3.5
One-Way ANOVA Results of Certification Pathway on KAT Tscore, Sscore, Ascore, and Final Score.

| Variable and Source | $d f$ | $S S$ | $M S$ | $F$ |
| :--- | :---: | :---: | :---: | :---: |
| Tscore |  |  |  |  |
| $\quad$ Between groups | 3 | 549.09 | 183.03 | $5.15^{* *}$ |
| $\quad$ Within groups | 140 | 4974.13 | 35.53 |  |
| Sscore |  |  |  |  |
| $\quad$ Between groups | 3 | 1031.01 | 343.67 | $10.49^{* * *}$ |
| $\quad$ Within groups | 140 | 4584.68 | 32.75 |  |
| Ascore |  |  |  |  |
| $\quad$ Between groups | 3 | 1976.04 | 658.68 | $14.78^{* * *}$ |
| $\quad$ Within groups | 140 | 6237.29 | 44.55 |  |
| Final Score |  |  |  |  |
| $\quad$ Between groups | 3 | 1520.99 | 506.99 | $17.35^{* * *}$ |
| $\quad$ Within groups | 140 | 4091.61 | 29.23 |  |

Note. Tscore $=$ Teaching Knowledge; Sscore $=$ Knowledge of School Algebra; Ascore $=$ Advanced Knowledge of Mathematics
**significant at $p<.01$
$* * *$ significant at $p<.001$

Tscore. The teaching knowledge scores of the KAT had two significant differences between pathways. Post hoc comparisons indicated that the mean score for teachers who completed a traditional mathematics education certification pathway ( $M=$ $40.38, S D=5.85$ ) was significantly different than those teachers who completed an elementary education certification pathway ( $M=34.99, S D=5.40$ ). These results suggest that those teachers certified through a mathematics education pathway have a much higher teaching knowledge than those certified through elementary education.

Sscore. The knowledge of school algebra dimension of the KAT posed significant differences between in three different comparisons. Those Algebra I teachers who were certified through a mathematics education pathway $(M=46.16, S D=6.51)$ scored significantly higher than those who were certified through an elementary education pathway ( $M=39.46, S D=4.71$ ) or other pathway $(M=41.54, S D=4.86)$.

Results also suggested a significant difference between those alternatively certified through a mathematics only pathway $(M=45.15, S D=5.38)$ scoring higher than those certified through an elementary education pathway. The only comparison that was not significantly different from elementary education was the pathway considered to be other.

Ascore. The advanced knowledge of mathematics dimension of the KAT, along with the final score, showed the most number of significant differences between certification pathways. The only pathways that were not significantly different in Ascore were mathematics education versus mathematics and elementary education versus other. This implies that the mathematics education pathway scores $(M=51.19, S D=6.85)$ and mathematics pathway scores $(M=52.21, S D=7.76)$ were significantly higher than those of elementary education $(M=42.20, S D=4.78)$ and other $(M=45.38, S D=6.91)$ pathways. These results indicate that teachers who have an educational background with more depth of mathematical content score higher than those who do not have mathematics as an educational background.

Final score. Similarly to the Ascore results, significant differences occurred between certification pathways. Those Algebra I teachers with certification through a mathematics education $(M=45.13, S D=5.97)$ pathway and mathematics $(M=44.74, S D$ $=5.08)$ pathway scored significantly higher than those Algebra I teachers who are certified to teach through an elementary education $(M=36.93, S D=3.76)$ pathway or other $(M=39.86, S D=5.30)$ pathway. Results suggest that those teachers who were certified through a mathematics education pathway or those alternatively certified with a mathematics degree have an overall higher KAT than those who are certified in elementary education or another pathway not in mathematics.

Table 3.6
Tukey HSD Comparison for KAT Scores across Certification Pathways

|  | ME | M | EE | O |
| :---: | :---: | :---: | :---: | :---: |
| Tscore |  |  |  |  |
| ME ( $M=40.38$ ) | - | 1.81 | 5.39* | 2.75 |
| M ( $M=38.58$ ) |  | - | 3.59 | . 94 |
| EE ( $M=34.99$ ) |  |  | - | -2.65 |
| O ( $M=37.64$ ) |  |  |  | - |
| Sscore |  |  |  |  |
| ME ( $M=46.16$ ) | - | 1.01 | 6.70* | 4.62* |
| M ( $M=45.15$ ) |  | - | 5.68* | 3.60 |
| EE ( $M=39.46$ ) |  |  | - | -2.08 |
| O ( $M=41.54$ ) |  |  |  | - |
| Ascore |  |  |  |  |
| ME ( $M=51.19$ ) | - | -1.02 | 8.99* | 5.81* |
| M ( $M=52.21$ ) |  | - | 10.00* | 6.83* |
| EE ( $M=42.20$ ) |  |  | - | -3.81 |
| O ( $M=45.38$ ) |  |  |  | - |
| Final Score |  |  |  |  |
| ME ( $M=45.13$ ) | - | . 40 | 8.20* | 5.27* |
| M ( $M=44.74$ ) |  | - | 7.81* | 4.87* |
| EE ( $M=36.93$ ) |  |  | - | -2.94 |
| O ( $M=39.86$ ) |  |  |  | - |

Note: $\mathrm{ME}=$ Mathematics Education; $\mathrm{M}=$ Mathematics; $\mathrm{EE}=$ Elementary Education; $\mathrm{O}=$ Other * $\alpha<.05$

In order to explore any association between beliefs and KAT of Algebra I teachers across certification pathways, scatterplots were created to show teachers' belief scores and KAT scores classified into high and low categories. Being classified into the high category was dependent upon scoring higher than the overall means for all 144 Algebra I teachers in the sample, while being classified as low was dependent upon scoring lower than the overall means. The scatterplots are separated into three different figures based on the belief about the nature of algebra (Figure 3.3), learning of algebra (Figure 3.4), and teaching of algebra (Figure 3.5). All figures, regardless of type of belief, have the same KAT final score mean $(M=42.35)$ for the x -axis separating the
high and low teachers. Listed in the caption of each figure, depending on the type of belief, is the mean score that represents that $y$-axis separating the high and low teachers. Additionally, the upper left quadrant shows the percentage of teachers falling into the high beliefs and high KAT category depending upon certification pathway.

First, visual examination across all three sets of scatterplots show that elementary education and other certification pathways tend to have a low KAT in comparison to those with a strong mathematics content background. Second, examining the relationship between teachers' KAT and nature of mathematics beliefs based on pathway (see Figure 4.3) revealed that nearly half ( $45 \%$ ) with mathematics education certification, nearly onefourth (20\%) with mathematics alternative certification, only $4 \%$ with elementary education certification, and almost one-fifth (18\%) with other certification, fall into the high-high category of KAT and nature of algebra beliefs. The scatterplots suggest that those teachers possessing a high KAT and a high problem-solving view of mathematics typically have strong mathematics content knowledge.


Figure 3.3. Certification pathways regarding to KAT and nature of algebra beliefs $(M=6.50)$.

Similarly, the highest percentage of Algebra I teachers who fall into the high KAT final score and high learning of algebra beliefs (see Figure 4.4) were those certified through a mathematics education pathway (28\%) or mathematics with alternative certification (20\%) pathways. Only a few Algebra I teachers that had an elementary education certification (4\%) or other (10\%) certification pathway held high KAT and high learning beliefs. The percentages suggest that those teachers possessing a high KAT and a high discovery view of learning mathematics normally enter the classroom via either a traditional mathematics education certification pathway or have the equivalent of a degree in mathematics and hold an alternative certification in mathematics.


Figure 3.4. Certification pathways regarding to KAT and learning of algebra beliefs ( $M=6.21$ ).

The percentages of Algebra I teachers in the high KAT and high teaching of algebra beliefs category (see Figure 4.5) were mathematics education pathway (37\%), mathematics with alternative certification (13\%), elementary education (0\%), and other (15\%). The percentages from the scatterplots indicate that those teachers who are a high KAT along with a high discovery view of algebra are certified through a mathematics education pathway. Overall, the scatterplots suggest that those Algebra I teachers certified through an elementary education pathway had the highest percentages of low

KAT and low beliefs of nature of algebra (48\%), learning of algebra (43\%), and teaching of algebra (28\%).


Figure 3.5. Certification pathways regarding to KAT and teaching of algebra beliefs ( $M=5.39$ ).

## Discussion and Implications

The purpose of this study was to (a) paint a picture of who is teaching Algebra I in Oklahoma and (b) explore the algebra beliefs of these Algebra I teachers along with their own understanding of algebra and the teaching of algebra concepts. This study is filling a gap in the research literature by looking at associations between teachers' beliefs and KAT determined by their certification pathway. Particularly focusing on Algebra I
teachers allows the research to indicate the type of teachers that should be teaching this gatekeeper course.

Regarding to certification pathway of Algebra I teachers in the state of Oklahoma, this study contained a sample of 144 where 77 teachers did not receive their certification through a traditional mathematics education teacher preparation program. Of those 77 teachers, many of them did not complete a certification pathway with a strong mathematics content background, but were still placed in an Algebra I classroom to teach algebraic content.

The teaching experience of Algebra I teachers is another important characteristic to discuss. In Oklahoma, one out of every five Algebra I teachers are novice teachers with only $1-5$ years of teaching experience. This brings up the ideas of whether novice teachers should be teaching Algebra I, which is a gatekeeper course. If novice teachers do teach this course, strict attention should be paid to the teacher to ensure support whenever needed and ensure the teachers have a strong knowledge of algebra and pedagogical strategies to teach algebraic concepts.

Findings in the current study indicate that Algebra I teachers in the state of Oklahoma hold similar beliefs about the nature, teaching, and learning of algebra. Previous research has shown that teachers' prior school experience in a mathematics classroom tend to be the main influence in beliefs, which leads teachers to teach more traditionally and procedurally (Raymond, 1997; Prawat, 1992). Furthermore, even when teachers are taught and encouraged to teach in a more constructivist manner, they must have the opportunity to be surrounded by other educators and teachers who share similar constructivist beliefs in the teaching and learning of mathematics (Prawat, 1992).

On the other hand, the findings in this study indicate that certification pathway are linked to the teaching and content knowledge of Algebra I teachers. Findings from past research have shown that effective classroom instruction is strongest when the teacher holds high subject content knowledge, curricular knowledge, and pedagogical content knowledge (Ball et. al, 2008; Shulman, 1986). Those Algebra I teachers who were certified through a traditional mathematics education teacher preparation program and those were alternatively certified after receiving a degree in mathematics consistently had a higher level of algebraic content and teaching knowledge than those who were certified through an elementary education teacher preparation programs or any other type of certification pathway. Previous research suggests that in order to effectively teach mathematics at the middle and secondary level, teachers need a deep knowledge of advanced mathematics including calculus, linear algebra, and other courses (McCrory et al., 2012). The current study suggests similar findings. Elementary education certified teachers and other non-mathematics based majors may not have the depth of mathematical content background to effectively teach algebra courses. With so many teachers in the Algebra I classroom with an educational background in non-mathematics content, the rigor and depth of the content on the regional certification exam should be carefully established before distributing mathematics certifications.

Implications of this study include, notably, that there should be a more strict and rigorous process to enter the mathematics classroom as a teacher. For those teachers who do not follow a traditional mathematics certification pathway, simply passing a content knowledge exam may not be enough to be designated a teacher of that subject area, especially in mathematics, unless this exam is rigorous and focuses on content well
beyond Algebra I. Usiskin et al. (2001) argued that middle and secondary mathematics teachers should understand three major categories of mathematical understanding: "concept analysis - the phenomenology of mathematical concepts, problem analysis - the extended analysis of related problems, and connections and generalizations within and among the diverse branches of mathematics" (p.3). These categories are a mixture of content and pedagogical content knowledge that would ensure middle level and secondary level mathematics teachers are prepared to teacher content effectively to students.

Since Algebra I is considered the gatekeeper to higher level mathematics, schools need to ensure that the highest quality teachers are instructing this course. Algebra I teachers should be able to bridge mathematics across different topics and concepts that will link those ideas of standard school algebra to more advanced mathematics (McCrory et al., 2012). There is a need for the state of Oklahoma to examine their Algebra I teacher workforce and the pathways to certify those teachers. Otherwise, how can we improve student achievement in Algebra I or prepare the students for other high school, or college mathematics courses?

## CHAPTER IV

## RELATIONSHIP OF ALGEBRA I TEACHERS' BELIEFS AND THEIR KNOWLEDGE OF ALGEBRA FOR TEACHING (KAT)

Target Journals: A. Journal of Mathematics Teacher Education
B. Journal of Teacher Education

Authors: Travis Mukina, Juliana Utley


#### Abstract

: This article presents a study of teachers' mathematical beliefs and Knowledge of Algebra for Teaching (KAT) of Algebra I teachers in Oklahoma. A Chi-Square Test for Independence was run on the sample of 144 teachers to determine if a high level of Knowledge of Algebra for Teaching score can predict a high problem-solving or constructivist view of algebra. A comparison of Algebra I teachers with a deeper background in mathematics content was compared to their beliefs as well. The study finds that those Algebra I teachers who have high KAT scores are dependent upon having a more problem-solving view of the nature of algebra. Additionally, the study indicates those Algebra I teachers with a high KAT are dependent upon having a deep mathematics content background. The results of this study suggest (1) those teachers in an Algebra I classroom are better prepared to teach algebra concepts with a problem-solving approach if having a greater KAT and (2) teachers with a deeper mathematics content background are more likely to have a high KAT.


## Introduction

During the 2015-2016 academic year in Oklahoma, 43 percent of incoming teachers were alternatively certified or emergency certified, where 1 in 3 of emergency certified teachers are employed in Oklahoma City or Tulsa Public Schools. (Baines, Hanna, \& Wickham, 2016). The teacher shortage crisis in the country is only predicted to get worse with the enrollment in teacher preparation programs decreasing over 35 percent from 2009 - 2014 (Aragon, 2016). Researchers (e.g. Ingersoll \& Smith, 2003; Murnane et al., 1991) have suggested that due to this shortage, overall teacher quality is decreasing as schools continue to fill their classrooms with underqualified teachers, especially in the areas of mathematics and science.

Since teacher quality has been found to be a significant factor in student achievement, researchers examined the phenomenon of out-of-field teaching (DarlingHammond, 2000; Ingersoll, 1999). Ingersoll (1999) focused on the definition of out-offield teachers to be those "teachers assigned to teach subjects for which they have little training or education" (p.26). Ingersoll (1999) found that nearly one-third of secondary mathematics teachers do not hold a degree in mathematics or mathematics education. The idea that out-of-field teachers' subject matter knowledge can affect teacher quality has been discredited by some researchers; however, research indicates that student achievement is significantly higher in Algebra I compared to other general mathematics courses when the teacher is fully certified in middle and secondary mathematics (Hawk, Coble, \& Swanson, 1985).

These out-of-field teachers, from traditional and alternative certification pathways, may be underprepared to teach the mathematics course they are assigned.

Student success and strong conceptual understanding of Algebra I content has shown implications related to performance in college, career readiness, impact on career salary, and student perception on higher mathematics (e.g., Eddy et al., 2015; Gaetner, Kim, DesJardins, \& McClarty, 2014; Kim, Kim, DesJardins, \& McCall, 2015; Loveless et al., 2008; Siegler et al., 2012).

Since not all mathematics teachers are trained through the same certification pathway and may not share similar educational backgrounds, teachers' mathematical beliefs can affect how teachers conceptualize mathematics and its learning and teaching (e.g., Andrews, 2007; Cooney, Shealy, \& Arvold, 1998). Teachers' beliefs about mathematics and its teaching are influenced by their own experiences in school (Borko et al., 1992) and those beliefs "play a significant role in shaping teachers' characteristic patterns of instructional behavior" (Thompson, 1992, p. 130).

This study aims to examine any relationship between teachers' algebra beliefs and their Knowledge of Algebra for Teaching (KAT). Additionally, it examines whether teachers' subject matter knowledge in mathematics has a relationship to their algebra beliefs and their KAT. Thus, the research questions guiding this study are:

1. Is there an association between algebra teachers' Knowledge of Algebra Teaching (KAT) and their beliefs about algebra, about teaching algebra, and about learning algebra?
2. Is there an association between an Algebra I teachers' content knowledge of mathematics and their beliefs about algebra, about teaching algebra, and about learning algebra?
3. Is there an association between an Algebra I teachers' content knowledge of

# mathematics and their Knowledge of Algebra for Teaching (KAT)? 

## Conceptual Framework

Teacher quality and teacher characteristics in the classroom are found to be especially important in mathematics more so than in any other subject (Nye, Konstantopoulos, \& Hedges, 2004). Teacher quality can be described in terms of certification and degrees held, but can also be described in terms of teacher practices in the classroom to further student achievement (Goe, 2007). Research on teacher quality based on qualifications such as certification pathway, level of education and subject matter knowledge differ across subjects (Betts, Zau, \& Rice, 2003; Boyd, Grossman, Lankford, Loeb, \& Wyckoff, 2006). When a teachers' deep subject matter knowledge is combined with teacher education coursework, significant increases in student achievement in mathematics have been shown (e.g., Betts, Zau, \& Rice, 2003; Clotfelter, Ladd, \& Vigdor, 2006; Goldhaber \& Brewer, 1999). Teacher characteristics including demographic (ethinicity and gender) and non-cognitive (beliefs and attitudes) factors affect teacher quality (Goe, 2007). Researchers have found mixed results identifying these types of teacher characteristics to have an impact on student achievement (Ehrenberg, Goldhaber, \& Brewer, 1995; Goddard, Sweeltand, \& Hoy, 2000).

This study uses a conceptual framework based on the work from DarlingHammond (2000) and Ernest (1989) with those specific teacher characteristics that were found to be directly or indirectly related to student achievement through teacher practice. Trends have taken place in the categories of (a) subject matter knowledge, (b) knowledge of teaching and learning, (c) teacher beliefs, and (d) certification pathway (Darling-

Hammond, 2000; Ernest, 1989). The interrelated constructs in Figure 4.1 form the conceptual framework of factors influencing teacher quality of Algebra I teachers for this study.


Figure 4.1. Factors influencing teacher quality of Algebra I teachers.

## Related Literature

## Subject Matter and Pedagogical Knowledge for Teaching

Knowing subject matter and being able to use it is at the heart of teaching all students (Ball, 2000). During the 1980s, there was a plethora of research that showed a teachers' knowledge of a subject could affect their students' learning opportunities (Lampert, 1986; Leinhart \& Smith, 1985; Shulman, 1986; Wineburg \& Wilson, 1988). Banks and Necco (1987) found that student achievement gains occurred specifically in algebra classes more often than general mathematics classes when the teacher held a
degree in mathematics. Shulman (1986) believed teachers should not just have high subject matter knowledge, but be able to "explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions" (p. 9). Shulman (1986) defined this to be pedagogical content knowledge (PCK).

For over the last decade, mathematics education researchers studied teachers' combined subject matter knowledge and PCK to determine the high impact they both have for high quality teaching (Ball, 2003; Hill, Ball, \& Schilling, 2008; Hill et al., 2008). The exploration of both subject matter knowledge and PCK brought about the term of content knowledge for teaching (MKT), explained to be the following:

- Knowing the content that the students are supposed to learn.
- Knowing ways to unpack, represent, and make that content learnable.
- Knowing how students think about the specific content.
- Knowing ways to teach the specific content. (Ball, Thames, \& Phelps, 2008, p. 3).

Recently, MKT of secondary mathematics teachers is a focus for researchers, specifically in the area of algebra (Li, 2011; McCrory, Floden, Ferrini-Mundy, Reckase, \& Senk, 2012). McCrory et al. (2012) found that secondary teachers' knowledge of teaching is strictly measured through content tests or by the number of mathematics courses on their transcripts, where neither of these measures given are a certain to produce effective teaching of algebra. Research has shown that a strong MKT is not always guaranteed by taking a certain number of higher level mathematics courses (Even, 1999). Studies have linked high levels of student achievement with teachers who attend
professional development where content-based pedagogy is being taught (Brown, Smith, \& Stein, 1996; Cohen \& Hill, 2000; Wiley \& Yoon, 1995).

## Teacher Beliefs

Beliefs are referred to as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Richardson, 1996, p. 259). Research on teacher beliefs found beliefs to be highly individualized for every teacher, greatly influential in teaching practices, and impactful on student achievement (Brown \& Rose, 1995; Clark \& Peterson, 1986; Gudmundsdottir \& Shulman, 1987; Lepik \& Pipere, 2011; Lovat \& Smith, 1995; Love \& Kruger, 2005). The three main areas of teacher beliefs are discussed in here including beliefs about nature of mathematics, learning of mathematics, and teaching of mathematics.

Teacher beliefs about nature of mathematics. Historically, philosophical views about the nature of mathematics have fallen on two extremes. On one extreme, mathematics is viewed as "static, fixed, and either discovered or waiting to be discovered" (Amirali \& Halai, 2010, p. 47) while the other extreme view of mathematics is "seen and interpreted as socially constructed phenomena" (Amirali \& Halai, 2010, p. 47). Furthermore, Ernest (1989) suggests that views on the nature of mathematics typically fall into one of the following three views:

1. Dynamic problem-solving view - Mathematics is a continually expanding field of human inquiry where it is an unfinished product and its results remain open to revision.
2. Platonist view - Mathematics is a static, but unified body of knowledge, consisting of truths which are discovered, not created.
3. Static instrumentalist view - Mathematics is a useful, but unrelated collection of facts, rules, and skills. (p. 21)

Furthermore, Ernest (1991) states that teachers' nature of mathematics views can have implications on their teaching practices. Teachers with a dynamic problem-solving view of mathematics use a non-directive and open-teaching style (Lerman, 1990). Teachers with a Platonist view of mathematics use related rules and facts attempting to find links between concepts (Ernest, 1991). Teachers with a static instrumentalist view consider themselves an authority in the classroom and their role is to impart mathematical knowledge to their students (Ernest, 1991). Francis (2014) found that instruction of teachers holding a dynamic problem-solving view of the nature of mathematics brought more understanding to students and more desire to learn mathematics.

Teacher beliefs about learning mathematics. Studies on teacher beliefs about learning mathematics revealed that a majority of teachers believe that learning mathematics is most effective in a traditional lecture-style method (Peterson, Carpenter, Fennema, \& Loef, 1989). This is in contrast to mathematics educators who believe the role of a mathematics teacher "is to guide and support students' invention of viable mathematical ideas rather than transmit 'correct' adult ways" (Clements \& Battista, 1990, p. 35). Civil (1990) revealed that mathematics teachers emphasized neatness and speed as the best way to solve a problem in the classroom. Other researchers have found that many mathematics teachers perceive mathematics learning best occurs when it is based on memorization of rules and algorithms (Lappan \& Even, 1989; Southwell \& Khamis 1992).

Teacher beliefs about teaching mathematics. Research has shown that teacher beliefs about the teaching of mathematics is similar to the traditional beliefs of learning mathematics where teaching is about providing step-by step procedures and then students mimicking those procedures (Stipek et al., 2001). In a study of 249 secondary mathematics teachers, Howard, Perry, and Lindsay (1997) found teacher beliefs tended to fall into two types of beliefs about teaching that they called transmission and constructivist. They found that the majority of the teachers fell into the transmission view of teaching mathematics where they believed that mathematics is based on memorization of rules and procedures, while the remaining few teachers held a constructivist view of teaching mathematics where students were encouraged to explore, propose, and explain solutions.

Similarly, Van Zoest, Jones, and Thornton (1994) studied two groups of preservice mathematics teachers where one group was enrolled in a mentorship program based on a philosophy of constructivist teaching and the other group served as the control group. Results of this study revealed that regardless of completing the mentorship program, both groups had resorted to a more traditional set of beliefs about teaching mathematics. These traditional sets of beliefs about teaching mathematics are so engraved in teachers' beliefs that reforming those beliefs has proven to be difficult. (Perry, Tracey, \& Howard, 1999)

## Methodology

## Participants

An online questionnaire was sent to all Oklahoma public school mathematics teachers, where only teachers currently teaching Algebra I during the 2016-2017 academic year completed the questionnaire. The Oklahoma State Department of Education divided the state into eight geographic regions called the $\mathrm{REAC}^{3} \mathrm{H}$ regions (http://sde.ok.gov/sde/sites/ok.gov.sde/files/documents/files/REAC3H\ Network\ R egions\%20map.pdf ) where all of the 516 school districts are assigned. The questionnaire was completed by 144 Algebra I teachers from across the eight regions, which resulted in a 5.4 percent response rate and was representative of the state (see Table 4.1). There was almost an even distribution of teachers with a Bachelor's degree $(49.3 \%, n=71)$ and teachers with a Master's degree $(50 \%, n=72)$. Teachers with 10 or more years of teaching experience comprised about 60 percent of the sample while those with $0-5$ years or 6-10 years of teaching experience each comprised 20 percent.

Table 4.1
Representativeness of Teachers in Oklahoma Reac ${ }^{3} h$ Regions ${ }^{1}$

|  | Population <br> Reac $^{3} h$ Region |  | Number | Percentage |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 113 | 4.23 | Number | Sample |  |
| 2 | 191 | 7.15 | 6 | 4.17 |  |
| 3 | 550 | 20.60 | 20 | 13.89 |  |
| 4 | 341 | 12.73 | 26 | 18.06 |  |
| 5 | 185 | 6.93 | 11 | 7.63 |  |
| 6 | 229 | 8.58 | 9 | 6.25 |  |
| 7 | 205 | 7.68 | 17 | 11.81 |  |
| 8 | 856 | 32.06 | 8 | 5.56 |  |

Note. ${ }^{1}$ The Oklahoma Reac ${ }^{3} \mathrm{~h}$ regions were used to determine the geographical representation of the state. A map of the Reac ${ }^{3} \mathrm{~h}$ regions can be found at http://ok.gov/sde/reac3h-network.

## Instrumentation

The participants completed three different instruments through an online questionnaire. Participants were asked to provide demographic information, respond openly about their beliefs about algebra, and participate in a 20 -question assessment that measure their Knowledge of Algebra for Teaching (KAT). The last three questions on the KAT were open ended and the process of uploading solutions was time-consuming, which resulted in teachers not completing these. Thus, the range of scores on the KAT were adjusted to not include those last three open-ended questions so that teachers who did not complete that portion were still included in the study.

Demographics. The first questions on the questionnaire (see Appendix A) required the Algebra I teacher to declare the current grade(s) they were teaching, school name, and the district. This allowed the researcher to classify each teacher into the appropriate REAC ${ }^{3} \mathrm{H}$ region to check representativeness for the state of Oklahoma. The next demographic information collected included the teacher's years of experience teaching mathematics, Bachelor's and/or Master's degrees achieved, and pathway to certification.

Algebra Beliefs Questionnaire. The algebra beliefs questionnaire (see Appendix B) used in this study is a modification of Raymond's (1997) beliefs questionnaire where all mentions of "mathematics" were changed to "algebra". Although the current instrument has not yet been validated, it was thoroughly examined by two mathematics educators at Oklahoma State University to certify the questions were appropriately measuring teacher beliefs. The beliefs questionnaire contains three subscales - beliefs about the nature of algebra, beliefs about learning algebra, and beliefs about teaching
algebra. The Cronbach's alphas for each of the three subscales in this study were calculated as $.81, .75$, and .54 , respectively, using the data from this study. Each of the three subscales have a series of semantic differential questions and a group of 5-point Likert-type questions. The beliefs about the nature of algebra subscale has a possible score range of 16 to 176 containing 8 of each type of question. For the beliefs about learning algebra and teaching algebra, the possible range of scores are 17 to 187 (7 semantic differential questions and 10 Likert-type questions) and 15 to 165 (8 semantic differential and 7 Likert-type questions), respectively. The higher score on the beliefs about learning and teaching of algebra portion, the more constructivist view of algebra is suggested (Clements \& Battista, 1990).

Survey of Knowledge of Algebra for Teaching (KAT). The Survey of Knowledge of Algebra for Teaching (Floden, Ferrini-Mundy, Senk, Reckase, \& McCrory, 2012) measures the knowledge that is most effective in teaching algebra - teaching knowledge specific to algebra concepts (Tscore), knowledge of middle and high school algebra (Sscore), and advanced knowledge of mathematics typically taught in college level mathematics (Ascore). A final score is given that incorporates all three dimensions of the KAT simultaneously. The instrument had a Cronbach alpha reliability of .84 during the validation study. The Cronbach alpha for this study was .73 . Out of the 17 questions on the KAT, five of the questions cover Tscore, eight cover Sscore, and four include topics in Ascore. The range of scores for the final score and each of the dimensions when using all 20 questions and the adjusted scores when the three open-ended questions were removed.

## Data Analysis

Results were analyzed using inferential statistics with SPSS (Version 18.0). Inferential statistics included the use of a Chi-Square Test of Independence with cross tabulation. The test calculated whether there was an association between Algebra I teachers' KAT score level and their beliefs about algebra by comparing the observed outcomes to the expected outcomes. The test also calculated whether there was an association between an Algebra I teachers' mathematical content background and their beliefs about algebra. All assumptions for the Chi-Square Test of Independence were ensured including checking that all the expected cell counts were greater than five. Scatterplots were used to visualize data and make comparisons.

## Findings

In order to explore any association between beliefs and KAT of Algebra I teachers, scatterplots and a Chi-square test of independence was run on contingency tables of high and low beliefs scores and KAT scores of those teachers. The Algebra I teachers' beliefs scores and KAT scores were classified into high and low categories depending on the scores falling above or below the overall means of the 144 Algebra I teachers in the sample. The overall means were - KAT final score $(M=42.35)$, beliefs about the nature of algebra $(M=6.50)$, beliefs about the learning of algebra $(M=6.21)$, and beliefs about the teaching of algebra $(M=5.39)$.

First, a visual representation of scatterplots was used to show the number of teachers from the sample that fell into each high-low category of KAT final score and each type of belief. The scatterplots in Figure 4.2 are based on the belief about the nature
of algebra, learning of algebra, and teaching of algebra. Each scatterplot, regardless of type of belief, have the same KAT final score mean $(M=42.35)$ for the x -axis separating the high and low teachers. The overall mean scores for each belief, listed above, represent the $y$-axis separating the high and low teachers.

The scatterplot of teachers' KAT and nature of algebra beliefs indicate about 28 percent of teachers falling into the high KAT and high nature of algebra beliefs category, suggesting that these teachers have a problem-solving view of mathematics. A fairly equal percentage (30\%) were found in the low KAT and low nature of algebra beliefs category. This suggests that these teachers have a more instrumentalist view of mathematics. With more than half of the sample falling in one of these categories, the trend of this scatterplot may suggest an association between KAT and nature of algebra beliefs. The percentage of teachers falling into high KAT and high learning of algebra category compared to those teachers falling into the high KAT and high teaching of algebra were $19 \%$ and $23 \%$, respectively. This suggests these teachers have a more constructivist view of learning and teaching mathematics. No trends appeared to take place in the learning and teaching of algebra scatterplots due to the data being fairly evenly distributed in each high-low category.


Figure 4.2. KAT and beliefs about algebra of 144 Algebra I teachers.

To confirm the visual inspection of the data, a Chi-square test of independence was performed. The Chi-square test of independence indicated significant association between an Algebra I teachers' KAT final score and nature of algebra beliefs, $\chi^{2}(1, n=$ $144)=5.76, p=.016, p h i=.2($ see Table 4.3). These results suggest that those Algebra I teachers with a high KAT final score are more likely to have a high nature of algebra beliefs score, indicating a more dynamic problem-solving view of algebra. The results also suggest the same for those Algebra I teachers who have a low KAT final score more likely to have a static instrumentalist view about the nature of algebra. The effect size of this association is considered to be a small to medium effect (Cohen, 1988). A Chi-
square test of independence showed no association between an Algebra I teachers' KAT final score and belief about the learning of algebra, $\chi^{2}(1, n=144)=.36, p=.547$, $p h i=$ .05 and belief about the teaching of algebra, $\chi^{2}(1, n=144)=2.07, p=.150, p h i=.12$. Results indicate that those teachers who have a high KAT do not necessarily have a relationship to having a constructivist view of learning and teaching of algebra.

Additional analyses were run on the three sub-dimensions of the KAT and three beliefs about algebra. Table 4.3 shows there was no significant dependency between any of the three KAT dimensions to any of the three types of beliefs about algebra.

Table 4.2
Cross Tabulation of KAT and Beliefs

|  | Nature of Algebra |  |  |  | Learning of Algebra |  | Teaching of Algebra |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Low | High | $\chi^{2}$ | $\phi$ | Low | High | $\chi^{2}$ | $\phi$ | Low | High | $\chi^{2}$ | $\phi$ |
| KAT Final Score |  |  |  |  |  |  |  |  |  |  |  |  |
| High | 24 | 42 | 5.76* | . 2 | 38 | 28 | . 36 | . 05 | 35 | 31 | 2.07 | . 12 |
| Low | 44 | 34 |  |  | 41 | 37 |  |  | 32 | 46 |  |  |
| KAT Tscore |  |  |  |  |  |  |  |  |  |  |  |  |
| High | 25 | 32 | . 43 | . 05 | 35 | 23 | 1.18 | . 09 | 27 | 32 | . 02 | . 01 |
| Low | 43 | 44 |  |  | 44 | 42 |  |  | 40 | 45 |  |  |
| KAT Sscore |  |  |  |  |  |  |  |  |  |  |  |  |
| High | 21 | 34 | 2.92 | . 14 | 34 | 21 | 2.49 | . 13 | 28 | 26 | 1.26 | . 09 |
| Low | 47 | 42 |  |  | 43 | 46 |  |  | 38 | 52 |  |  |
| KAT Ascore |  |  |  |  |  |  |  |  |  |  |  |  |
| High | 26 | 39 | 2.48 | . 13 | 35 | 30 | 0 | . 01 | 29 | 35 | . 01 | . 01 |
| Low | 42 | 37 |  |  | 43 | 36 |  |  | 37 | 43 |  |  |

Note. ${ }^{*} \mathrm{p}<05$.

In order to explore any associations between an Algebra I teachers' level of subject matter knowledge and their beliefs of algebra, the sample of Algebra I teachers was broken down into two levels of mathematical content knowledge based on educational background as follows:

1. Mathematics Degree $(\mathrm{n}=83)$ - any teacher who obtained a Bachelor's degree in mathematics or the equivalent.
2. No Mathematics Degree $(\mathrm{n}=61)$ - any teacher who did not obtain a Bachelor's degree in mathematics or the equivalent.

A Chi-square test of independence indicated significant association between an Algebra I teachers' KAT final score and their mathematical content knowledge, $\chi^{2}(1, n=$ $144)=27.72, p<.0001, p h i=.44$ (see Table 4.4). These results suggest that those Algebra I teachers who hold a mathematics degree are more likely to have a high KAT. The results also indicate the same for those Algebra I teachers who do not hold a mathematics degree are more likely to have a low KAT. The effect size of this association is considered to a medium to large effect (Cohen, 1988).

The Chi-square test of independence showed a significant association between Algebra I teachers' mathematical content knowledge and every dimension of KAT, including Tscore $\left(\chi^{2}(1, n=144)=14.42, p<.001, p h i=.31\right)$, Sscore $\left(\chi^{2}(1, n=144)=\right.$ 24.63, $p<.001$, phi $=.41$ ), and Ascore $\left(\chi^{2}(1, n=144)=24.26, p<.001, p h i=.41\right)$. The effect size of every test is considered to be a medium to large effect (Cohen, 1988). The results indicate that a higher teaching knowledge of algebra (Tscore), knowledge of middle and high school algebra concepts (Sscore), and advanced knowledge of mathematics (Ascore) is strongly associated with having a degree in mathematics.

A Chi-square test of independence indicated no association between an Algebra I teachers' mathematics content background and belief about the nature of algebra, $\chi^{2}(1, n$ $=144)=1.16, p=.28, p h i=.09$, belief about the learning of algebra, $\chi^{2}(1, n=144)=$ 1.38, $p=.24, p h i=.10$, and beliefs about the teaching of algebra, $\chi^{2}(1, n=144)=.22, p$ $=.64, p h i=.04$. Results indicate that those teachers with who hold a mathematics degree do not show a relationship to having a dynamic problem-solving or constructivist view of algebra.

## Table 4.3

Cross Tabulation of Mathematics Background with Beliefs and KAT

|  | Mathematics Degree |  | $\chi^{2}$ | $\Phi$ |
| :---: | :---: | :---: | :---: | :---: |
|  | No | Yes |  |  |
| Nature of Algebra |  |  |  |  |
| Low | 32 | 36 | 1.16 | . 09 |
| High | 29 | 47 |  |  |
| Learning of Algebra |  |  |  |  |
| Low | 30 | 49 | 1.38 | . 10 |
| High | 31 | 30 |  |  |
| Teaching of Algebra |  |  |  |  |
| Low | 27 | 40 | . 22 | . 04 |
| High | 34 | 43 |  |  |
| KAT Final Score |  |  |  |  |
| Low | 49 | 30 | 27.72** | . 44 |
| High | 12 | 53 |  |  |
| KAT Tscore |  |  |  |  |
| Low | 47 | 38 | 14.42** | . 31 |
| High | 14 | 45 |  |  |
| KAT Sscore |  |  |  |  |
| Low | 52 | 37 | 24.63** | . 41 |
| High | 9 | 46 |  |  |
| KAT Ascore |  |  |  |  |
| Low | 48 | 31 | 24.26** | . 41 |
| High | 13 | 52 |  |  |

## Discussion

The purpose of this study was to (a) explore the association between an Algebra I teachers' KAT and their beliefs about the nature of algebra, learning of algebra, and teaching of algebra and (b) explore the association between an Algebra I teachers' KAT and their mathematics content background. This research is significant because it is important to know Algebra I teachers' beliefs and MKT since they both can have an impact on instruction.

Findings in this study suggest that Algebra I teachers in the state of Oklahoma hold a problem-solving view of mathematics when they have a higher KAT. More than three-fourths ( $81 \%$ ) of the teachers with a deep subject matter knowledge (mathematics education and mathematics pathways) tended to hold this dynamic problem-solving view and held high KAT. This finding coincides with previous research that found when teachers have taken more mathematics content courses along with teaching methods courses there was an increase in their MKT (Hill et al., 2008; Hill, Rowan, Ball, 2005; Darling-Hammond, 2000). Additionally, Francis (2014) suggested that teachers holding beliefs that support problem-solving views of mathematics is considered one of the most valuable factors affecting teacher quality. Research (Kim, 2005) has shown that when teachers have a dynamic problem-solving view of mathematics (a) they tend to create a classroom environment based on constructivist teaching practices and (b) their students' achievement in mathematics has been associated with higher gains, particularly in algebra. Teachers who hold a dynamic problem-solving view tend to approach problems in multiple ways and have expectations for their students to solve problems in a variety of ways (Amirali \& Halai, 2010).

In terms of the mathematics background, having a mathematics degree seems to have a large effect on the KAT of Algebra I teachers, but not on their beliefs. Teacher certification pathway has shown a significantly positive effect on mathematics achievement of students when the teacher is certified in mathematics education (Rice, 2003). Additionally, previous studies (e.g. Corkin, Ekmecki, \& Fan, 2016; Hill et al., 2005; Rice, 2003) have found that teacher coursework in the specific content area and pedagogy yields a higher quality of teachers in the classroom. These results suggest that many of the Algebra I teachers in the state possess a low knowledge of algebra and its teaching. This calls into question the depth and rigor of the subject area test that allows these teachers to teach Algebra I and above. Additionally, with the high number of elementary certified teachers teaching Algebra I, how exposed are they to the tools and methods to effectively teaching Algebra I concepts before entering the classroom?

An implication of this study includes ensuring those teachers in an Algebra I classroom have a high Knowledge of Algebra for Teaching (KAT) for the most effective teaching to take place in the classroom. Although the beliefs about the nature of algebra, learning of algebra, and teaching of algebra are known to be difficult to change (Gill, Ashton, \& Algina, 2004), attention still needs to be paid to them. Along with increasing KAT, professional development for mathematics teachers should focus on forming those beliefs about problem-solving and constructivist views. Due to the effect that a teachers' mathematics content background has on their KAT, it is advised that more rigorous standards for certification pathways be explored to enhance the student achievement in Algebra I. Additionally, for those teachers already holding a mathematics certification in the state, but who do not hold a sufficient mathematics educational background should be
required to continue to deepen their content knowledge. This can take place through targeted professional development and/or Master's coursework rich in content and pedagogy related to algebra concepts. The enactment of these implications would only increase the teacher quality of Algebra I teachers in the state, and mathematics teachers in general, which ultimately would impact student achievement.

## CHAPTER V

## SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

With Algebra I being the gatekeeper course for higher mathematics achievement along with college and career readiness, questions about the quality of teachers in those classrooms arise (Stoelinga \& Lynn, 2013). Research has shown that the factors such as subject matter knowledge, knowledge of teaching and learning, and certification pathway can influence teacher practices in the classroom and affect student achievement (DarlingHammond, 2000; Goldhaber \& Brewer, 2000; Goldhaber \& Brewer, 1997; Hill, Rowan, \& Ball, 2005; Henry et al., 2014). Teachers' mathematical beliefs are highly individualized with many teachers falling back into the traditionalist view of mathematics regardless of their certification pathway or educational background (Gudmundsdottir \& Shulman, 1987; Perry, Howard, \& Tracey, 1999). Since Algebra I can play such a key role in the future mathematical success of students, is close enough attention given to those teachers' qualities and beliefs before entering those classrooms?

The research in this study explored those specific Algebra I characteristics and teacher qualities in the state of Oklahoma by measuring their Knowledge of Algebra for Teaching (KAT), beliefs about the nature of algebra, learning of algebra, and teaching of algebra. Additionally, comparisons of those measurement were made across different
certification pathways and mathematical backgrounds. The questions that guided this research study were:

1. Who is the Algebra I teaching force in Oklahoma?
a. What are the characteristics of the Algebra I teachers?
b. What beliefs do Algebra I teachers in Oklahoma hold about mathematics, teaching mathematics, and learning mathematics?
c. What is the measure of the Algebra I teachers' Knowledge of Algebra for teaching (KAT)?
2. Is there a significant difference between an algebra teacher's certification pathway and the beliefs he or she holds?
3. Is there a significant difference between an algebra teacher's certification pathway and his or her Knowledge of Algebra for Teaching (KAT)?
4. Is there an association between an Algebra I teachers' Knowledge of Algebra Teaching (KAT) and their beliefs about algebra, about teaching algebra, and about learning algebra across certification pathways?
5. Is there an association between algebra teachers' Knowledge of Algebra Teaching (KAT) and their beliefs about algebra, about teaching algebra, and about learning algebra?
6. Is there an association between an Algebra I teachers' content knowledge of mathematics and their beliefs about algebra, about teaching algebra, and about learning algebra?
7. Is there an association between an Algebra I teachers' content knowledge of mathematics and their Knowledge of Algebra for Teaching (KAT)?

In this study, quantitative data was collected from 144 teachers currently teaching Algebra I during the 2016-2017 academic year in the state of Oklahoma. Data about the teachers were collected through an online questionnaire using demographics questions, an algebra beliefs questionnaire, and the Survey of Knowledge of Algebra for Teaching (KAT) instrument (Floden, Ferrini-Mundy, Senk, Reckase, \& McCrory, 2012). Results were used to portray a picture of Algebra I teachers in Oklahoma using demographics, certification pathways taken, and mathematics background. The quantitative results from the algebra beliefs questionnaire and KAT were analyzed to find any differences or associations between those teachers' beliefs, KAT, certification pathways, and mathematics backgrounds. The results of the study were organized into two articles, which are summarized in the following pages.

## Summary of Findings

Overall, there were three main findings from this study. The first finding indicates that teachers' algebra beliefs in Oklahoma do not differ regardless of certification pathway, but teachers' KAT level are dependent upon certification pathway and mathematics content background. The second finding indicates that Oklahoma Algebra I teachers with a higher KAT are associated with having a more problem-solving of the nature of algebra, but not with having a constructivist view of the teaching and learning of algebra. The third finding indicates those Algebra I teachers certified through a traditional mathematics education program consistently have a higher KAT, higher problem-solving view of mathematics, and higher constructivist view of teaching and learning algebra than those certified through any other pathway. The focus and significant findings from each chapter article are discussed in the rest of this section.

The main focus of chapter three titled "Algebra I Teachers' Beliefs and Knowledge of Algebra for Teaching" was to portray a picture of Algebra I teachers in Oklahoma by identifying teacher characteristics, beliefs about algebra, and KAT and how certification pathways can affect those beliefs and KAT. The research questions answered in this chapter were:

1. Who is the Algebra I teaching force in Oklahoma?
a. What are the characteristics of the Algebra I teachers?
b. What beliefs do Algebra I teachers in Oklahoma hold about mathematics, teaching mathematics, and learning mathematics?
c. What is the measure of the Algebra I teachers' Knowledge of Algebra for teaching (KAT)?
2. Is there a significant difference between an algebra teacher's certification pathway and the beliefs he or she holds?
3. Is there a significant difference between an algebra teacher's certification pathway and his or her Knowledge of Algebra for Teaching (KAT)?
4. Is there an association between an Algebra I teachers' Knowledge of Algebra Teaching (KAT) and their beliefs about algebra, about teaching algebra, and about learning algebra across certification pathways?

The main teacher characteristics of Algebra I teachers in Oklahoma were examined by age, teaching experience, educational background, and certification pathway. While nearly 20 percent of teachers have only been teaching mathematics for 1 - 5 years, the average age of Algebra I teachers in Oklahoma is 43 years old. With exactly 50 percent of teachers holding a Master's degree, it is important to point out that

32 percent of them hold a Master's degree in mathematics education. The majority of certification pathways taken in the state are through a traditional teacher preparation pathway with 47 percent certified through a mathematics education pathway and 16 percent certified through an elementary education pathway. An additional 11 percent of teachers hold a Bachelor's degree in mathematics, but were certified alternatively.

Overall, the teachers' beliefs about the nature of algebra were consistently higher, meaning a higher problem-solving view of algebra, than those beliefs about learning and teaching algebra. Similarly, data revealed, that regardless of certification pathway, teachers held similar beliefs about the nature of algebra, learning of algebra, and teaching of algebra. These results suggest that even those teachers certified through a traditional teacher education program may fall back more into an instrumentalist view of mathematics and a non-constructivist view of teaching and learning mathematics. The findings of Raymond (2007) and Prawat (1992) suggested the same with teachers' prior school experience being the main influence on teacher beliefs pushing more traditional and procedural teaching techniques.

Findings in this study indicated certification pathway to have a strong effect on the content knowledge and teaching knowledge of Algebra I teachers. Teachers certified through a mathematics education pathway and those certified through a mathematics alternative pathway consistently had higher algebraic content knowledge and teaching algebra knowledge. Previous research (McCrory et al., 2012) points out the importance of these findings suggesting teachers at the middle and secondary level need to have a deep knowledge of advanced mathematics in order to effectively teacher mathematics. Additional previous research showing effective classroom instruction being strongest
when a high subject content knowledge and pedagogical knowledge are held points out the importance of these findings (Ball et al., 2008; Shulman, 1986). This research may also suggest elementary education certified teachers and other non-mathematics based majors do not have a deep enough mathematical content background to effectively teach algebra.

The main purpose of chapter four titled "Examining Algebra I Teachers' Certification Pathway and Knowledge of Algebra for Teaching (KAT) on Beliefs" was to explore any possible associations between teachers' beliefs about algebra, KAT, and their educational background in mathematics. The research questions answered in this chapter were:

1. Is there an association between algebra teachers' Knowledge of Algebra Teaching (KAT) and their beliefs about algebra, about teaching algebra, and about learning algebra?
2. Is there an association between an Algebra I teachers' content knowledge of mathematics and their beliefs about algebra, about teaching algebra, and about learning algebra?
3. Is there an association between an Algebra I teachers' content knowledge of mathematics and their Knowledge of Algebra for Teaching (KAT)?

This findings in this study suggested that Algebra I teachers who have a higher overall KAT tend to have higher nature of mathematics views, meaning a more problemsolving view. This association was mainly comprised (81\%) of teachers certified through a mathematics education pathway and those certified through a mathematics alternative certification pathway. This suggests those teachers holding a mathematics degree are
most likely to have a dynamic problem-solving view of mathematics. Previous research found similar results of teachers with more mathematics content courses combined with mathematics methods courses, there was an increase in their MKT (Hill et al., 2008; Hill, Rowan, Ball, 2005; Darling-Hammond, 2000). Furthermore, this aligns with research suggesting a dynamic problem-solving view of mathematics to be one of the most valuable teacher qualities in mathematics by approaching problems in multiple ways and pushing for students to solve problems in a variety of ways (Amirali \& Halai, 2010; Francis, 2014).

Additionally, results indicate that having a deep mathematics content background has a strong association with all dimensions of KAT, including teaching knowledge, school algebra knowledge, and advanced mathematical knowledge, but not on their beliefs. Similar research findings suggest the same (Corkin, Ekmecki, \& Fan, 2016; Hill et al., 2005; Rice, 2003). These results indicate there are many teachers in the state who hold a low content knowledge of algebra and its teaching since there are a high number of elementary certified teachers in an Algebra I classroom.

The results of this study showed that a teachers' mathematics content background did not have an association with the teachers' beliefs about the learning and teaching of algebra. These results suggest that regardless of mathematics content background, teacher beliefs are most likely influenced by their own previous experiences in a mathematics classroom, when most teachers were taught using a traditional teaching style. These findings align with previous research by Raymond (1997).

## Implications

The results from these two studies have implications for Algebra I teachers. First,
the requirements to become certified to teach in an Algebra I classroom should be more strict and rigorous. Middle and secondary mathematics teachers need to understand the connection within different branches of mathematics and convey importance of the learning of mathematics (Usiskin, 2001). Teachers who do not follow a traditional mathematics education certification pathway are asked to just pass a mathematics content knowledge exam, but is this enough in order to ensure this effective teaching necessary in an Algebra I classroom? Since Algebra I is the gatekeeper to higher level mathematic, schools need to make sure high teacher quality in these classrooms is put first. There is a need for the state of Oklahoma to truly examine who is in Algebra I classrooms and the pathways those teachers can gain certification in order to teach this course.

Second, the most effective teaching in an Algebra I classroom takes place when the teacher has a high KAT. Ensuring that all teachers of Algebra I in Oklahoma have a high KAT is a necessity. Although the beliefs about the nature of algebra, learning of algebra, and teaching of algebra are known to be difficult to change (Gill, Ashton, \& Algina, 2004), attention still needs to be paid to them. Professional development for teachers should always focus on increasing KAT, but a higher emphasis on forming beliefs to fit more problem-solving and constructivist views should be put in place. With the study showing such strong associations with mathematics content background to high levels of KAT, it is advisable that the state of Oklahoma implement more rigorous standards throughout the certification process to enhance student achievement in Algebra I. For those teachers who do not have a deep content mathematics background, but already hold a mathematics certification in the state of Oklahoma, steps to deepen their content knowledge should occur. Teachers can achieve this deeper level of
understanding mathematics content by attending professional development or complete Master's coursework focused on content pedagogy specific to algebra concepts. These implications would increase the Algebra I teacher quality in Oklahoma and would help students achieve a higher level of understanding algebra concepts to carry with them into higher level mathematics courses.

## Recommendations for Future Research

While the data from this study on Algebra I teachers added to the body of literature on beliefs, KAT, certification pathway, and mathematics background, further research is still needed on how to improve the teacher quality of Algebra I teachers. The following recommendations for future research from this study lead to these next studies:

- It is known that classroom practice can be affected by mathematical beliefs of teachers. Future research should examine key factors that cause resistance to the changing of teachers' beliefs.
- Although a challenge, researchers should explore the option of validating an instrument that measures teachers' beliefs about the nature of algebra more specifically according to the instrumentalist, Platonist, and problem-solving continuum.
- Some participants in this study held a high overall KAT, a high KAT in every dimension, problem-solving view of algebra, and constructivists views of teaching and learning algebra. Interviews with those teachers would be interesting to determine if their practices in the classroom match their beliefs.


## Concluding Remarks

Algebra I teachers should be of the highest teacher quality based on their beliefs, MKT, and mathematical content background. With the shortage of mathematics teachers, teacher quality is becoming more difficult than ever to maintain in mathematics classrooms (Aragon, 2016). This research study provides evidence that Algebra I teachers mathematical background and certification pathway can highly develop a teachers' KAT. However, the teachers' beliefs about algebra will most likely be similar regardless of mathematical background or certification pathway. More rigorous processes need to be setup in order to ensure that Algebra I teachers come into the classroom ready to teach algebra concepts multiple ways and bridge the concepts to other mathematics courses. Unless the KAT of teachers continues to increase and the beliefs of Algebra I teachers move more toward a problem-solving and constructivist views, teachers may not have the strong understanding of algebra in to teach for the understanding of all students (NCTM, 2000). Without the high level of KAT, problemsolving views, and constructivist views, student achievement may be hindered.

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## APPENDICES

## Appendix A

## Demographics Survey

1. Are you currently teaching Algebra I during the 2016 - 2017 academic year?
2. At what grade levels are you currently teaching Algebra I? Please check all that apply.

- $7^{\text {th }}$ Grade
- $8^{\text {th }}$ Grade
- High School

3. State the name of the school in which you are teaching during the 2016-2017 academic year.
4. State the name of the school district in which you are teaching during the 2016 2017 academic year.
5. Please enter your current age.
6. Overall, how many total years have you been a school teacher (including the current year)?
7. How many years have you been teaching mathematics (including the current year)?
8. How many years have you been teaching Algebra I?
9. Please indicate your ethnicity by selecting all that apply.

- African American
- American Indian, Alaska Native
- Asian
- Hispanic, Latino
- Native Hawaiian, Pacific Islander
- White, non-Hispanic
- Other, please specify

10. What was your major area of emphasis in your Bachelor's Degree?

- Early Childhood Education
- Elementary Education
- Mathematics
- Mathematics with Mathematics Teacher Certification
- Secondary Mathematics Education
- Other, please specify

11. Do you have a Master's degree? If yes, please state the major area of emphasis of your Master's degree.
12. Do you have a Doctorate degree? If yes, please state the major area of emphasis of your Doctorate degree.
13. Please choose the option below that best describes your pathway to certification.

- Traditional Undergraduate Teacher Education Program with Certification in Secondary Mathematics (major could have been Secondary Education or Mathematics - included student teaching)
- Traditional Undergraduate Teacher Education Program with Certification in a field other than Secondary Mathematics (included student teaching)
- Master's Degree that led to Certification in Secondary Mathematics
- Bachelor's Degree in Mathematics and pursued alternative certification (no student teaching)
- Bachelor's Degree in a field other than mathematics and pursued alternative certification (no student teaching)
- Troops for Teachers
- Teacher for America
- Emergency Certification
- Other, please describe

14. Complete the following statement so that it reflects your belief about algebra as a discipline: To me algebra is ...

## Appendix B

## Algebra Beliefs Questionnaire

Directions: On questions 1, 3, and 5, place an X at the point on each line which best represents your beliefs about the topic at hand. Note that the choices at the ends of each line are not necessarily direct opposites, but they are certainly different options. Thus, for example, if you feel both options are equally important or accurate in describing your opinion, then you would place the X directly in the middle of the line. The other questions should be self-explanatory.

## 1. Mark an $X$ at any point on the line segment between the two words/phrases which describe algebra to you.

To me, algebra is:

| Surprising |  | Predictable |
| :--- | :--- | :--- |
| Doubtful |  | Certain |
| Absolute |  | Relative |
| Dull |  | Intriguing |
| Aesthetic | Applicable |  |
| Changing | Fixed |  |
| Exploring |  |  |
| Relationships | Following |  |
| Studying | Procedures |  |
| Patterns |  | Memorizing |

2. Respond to the following statements by choosing the response that best represents your answer to the statement. (SA=Strongly Agree, A=Agree, $\mathbf{U}=$ Uncertain, $\mathbf{D}=$ Disagree, $\mathbf{S D}=$ Strongly Disagree)
a. Algebra is mostly facts and procedures that need to $\quad$ SA A $\quad$ U $\quad$ D $\quad$ SD be memorized.
b. Getting right answers in algebra is more important than $\operatorname{SA} \quad$ A $\quad$ U $\quad$ D $\quad$ SD understanding why the answer works.
c. Algebra is useful in day to day life. SA A $\quad$ U D
d. Algebra is creative. $\quad$ SA A $\quad$ U $\quad$ D $\quad$ SD
e. Problem solving is an important aspect of algebra. $\quad$ SA $A$
f. Algebra is a predictable subject. SA A $\quad$ U D SD

| g. Algebra is easy for me to do. | SA | A | U | D | SD |
| :--- | :--- | :--- | :--- | :--- | :--- |
| h. Algebra is mostly exploring patterns and relationships. | SA | A | U | D | SD |

## 3. Mark an $X$ at any point on the line segment between the two words/phrases which describe learning algebra to you.

Learning algebra mostly requires:
Independent work $\qquad$ Group work

Practice $\qquad$ Insight

A lot of work $\qquad$ Luck

Being good at math $\qquad$ Trying hard
$\qquad$ Strong students

Confidence $\qquad$ Discipline
Memorizing
4. Respond to the following statements by choosing the response that best represents your answer to the statement. (SA=Strongly Agree, A=Agree, U=Uncertain, D=Disagree, SD=Strongly Disagree)
a. Students can discover algebra on their own, without it $\quad$ SA $A$ being show to them.
b. Memorizing is one of the many important tools in $\quad$ SA $A$ learning algebra.
c. Students' algebraic achievement is directly related to $\begin{array}{lllllll}\text { SA } & \text { A } & \text { U } & \text { D } & \text { SD }\end{array}$ the appropriateness of the algebra teaching.
d. Drill and practice exercises help students understand $\quad$ SA $\quad$ A $\quad$ U $\quad$ D $\quad$ SD algebra.
e. Students learn algebra better when they work together $\begin{array}{llllll}\text { SA } & \text { A } & \text { U } & \text { D } & \text { SD }\end{array}$ on the assignments.
f. Hands-on materials are appropriate for all algebra $\quad$ SA $\quad$ A $\quad$ U $\quad$ D $\quad$ SD students.
g. For algebra students, knowing how to perform an $\quad$ SA A $\quad$ U $\quad$ D $\quad$ SD algebraic procedure is more important than understanding why the procedure works.
h. Students learn algebra more from doing the homework $\begin{array}{lllllll}\text { SA } & \text { A } & \text { U } & \text { D } & \text { SD }\end{array}$ than from the classroom algebra lessons.
i. Students should be able to figure out for themselves $\quad$ SA $A$ whether an answer is algebraically reasonable.
j. The best way to learn algebra is through exploring and $\quad$ SA $\quad$ A $\quad \mathrm{U} \quad \mathrm{D} \quad$ SD discussing.

## 5. Mark an $X$ at any point on the line segment between the two words which

 describe teaching mathematics to you.Being a good algebra teacher mostly entails or depends on:

| A Good Textbook |  | Using Manipulatives |
| :--- | :--- | :--- |
|  |  | Helping Students See |
| Helping Students |  | the Usefulness of Math |
| Like Math |  | Understanding Students |
| Understanding Math |  | Lecture-Based |
| Engaging |  |  |
| Presentations | Presentations |  |
| Teacher Direction | Student Participation |  |
| Consistency | Variety |  |
| Teacher Effort | Student Effort |  |
| Explicit Planning | Flexible Lessons |  |

6. Respond to the following statements by choosing the response that best represents your answer to the statement. (SA=Strongly Agree, A=Agree, U=Uncertain, $D=$ Disagree, $\mathbf{S D}=$ Strongly Disagree)
a. Good algebra teachers help students see multiple ways $\begin{array}{llllll}\text { SA } & \text { A } & \mathrm{U} & \mathrm{D} & \text { SD }\end{array}$
b. I feel confident in my ability to help students learn $\quad \begin{array}{llllll}\text { SA } & \text { A } & \text { U } & \text { D } & \text { SD }\end{array}$ algebra.
c. To teach algebra effectively, I must follow the textbook SA $\quad$ A $\quad$ U $\quad$ D $\quad$ SD closely.
d. The role of the teacher in algebra is to explain concepts. SA $\quad$ A $\quad$ U $\quad$ D $\quad$ SD
e. Increased teacher effort in teaching algebraic produces $\begin{array}{llllll}\text { SA } & \text { A } & \text { U } & \text { D } & \text { SD }\end{array}$ no change in students' algebraic understanding.
f. It is important to assign daily homework in algebra. $\quad$ SA $\quad$ A $\quad$ U $\quad$ D $\quad$ SD
g. The teacher should do most of the talking when teaching SA $\quad$ A $\quad \mathrm{U} \quad \mathrm{D} \quad$ SD algebra.

## Appendix C

## Terms of Use for KAT Forms

1. The KAT project grants permission for use of its two assessment forms for purposes of research and evaluation of professional development or preservice teacher education. The forms are not to be used for the evaluation of individual teachers.
2. The KAT project will provide scoring instructions. For the multiple choice items, the project will provide answer keys. For the constructed responses items, the project will provide scoring rubrics and responses illustrative of scoring levels.
3. The KAT project will also provide IRT item weighting for the computation of estimated IRT scale scores on the overall form and on each of the component subscales. Comparisons can be made to the mean scale scores for the KAT validation sample.
4. Any report produced using these assessments should include the following credit statement:

The Knowledge of Algebra Teaching (KAT) assessment was developed by R.E. Floden, J. Ferrini-Mundy, S. Senk, M. Reckase, R. McCrory, with support from a grant from the National Science Foundation (REC 0337595). Further information about the KAT assessment is available at www.educ.msu.edu/kat .
5. We request that those using the assessment forms submit an Excel file with deidentified item scores. We would also appreciate receiving a copy (hard copy or scanned pdf) of the constructed responses, with the score given on each response clearly marked. These responses will be used in our continued work to refine the rubric and scoring instructions.

Terms of use accepted:

Date $\qquad$

## Please type in name, title, affiliation here, sign and return.

## Appendix D

# Oklahoma State University Institutional Review Board 

| Date: | Friday, October 21, 2016 |  |
| :---: | :---: | :---: |
| IRB Adolication No | ED16166 |  |
| Pronosal Title: | Influence of Teacher Certification Pathway and Knowledge of Algebra for Teaching on Mathematical Beliefs of Algebra I Teachers |  |
| Reviewed and Processed as: | Expedited |  |
| Status Recommended by Reviewer(s): Approved |  | Protocol Expires: 10/20/2017 |
| Principal Investigator(s): |  |  |
| Travis Mukina | Juliana Utiey 233 Willard |  |
| Stillwater, OK 74078 | 8 Stillwater, OK 74078 |  |

The IRB application referenced above has been approved. It is the judgment of the reviewers that the rights and welfare of individuals who may be asked to participate in this study will be respected, and that the research will be conducted in a manner consistent with the IRB requirements as cutlined in section 45 CFR 46.
$\square$ The final versions of any printed recruitment, consent and assent documents bearing the IRB approval stamp are attached to this letter. These are the versions that must be used during the study.

As Principal Irvestigator, it is your responsibility to do the following:

1. Conduct this study exactly as it has been approved. Any modifications to the research protocol must be submitted with the appropriate signatures for IRB approval. Protocol modifications requiring approval may include changes to the tile, Pl adviscr, funding status or sponsor, subject population composition or size, recruitment, inclusion/exclusion criteria, research sibe, research procedures and consent/assent process or forms 2. Submit a request for continuation if the study extends beyond the approval period. This continuation must receive IRB review and approval before the research can continue.
3.Report any adverse events to the IRB Chair promptly. Adverse events are those which are unanticipated and impact the subjects during the course of the research; and
4.Notify the IRB office in writing when your research project is complete.

Please note that approved protocols are subject to monitoring by the IRE and that the IRB office has the authority to inspect research reconds associated with this protocol at any time. If you have questions about the IRB procedures or need any assistance from the Board, please contact Dawnet Watkins 219 Scott Hall (phone: 405-744-5700, dawnett. watkins (3. okstate.edu).


## VITA

Travis Daniel Mukina
Candidate for the Degree of
Doctor of Philosophy

## Dissertation: INFLUENCE OF TEACHER CERTIFICATION PATHWAY AND KNOWLEDGE OF ALGEBRA FOR TEACHING ON MATHEMATICAL BELIEFS OF ALGEBRA I TEACHERS

Major Field: Professional Education Studies of Mathematics Education
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Completed the requirements for the Doctor of Philosophy in Professional Education Studies of Mathematics Education at Oklahoma State University, Stillwater, Oklahoma in May 2017.

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[^0]:    ${ }^{1}$ The Oklahoma Reac ${ }^{3} h$ regions were used to determine the geographical representation of the state. A map of the Reac ${ }^{3} \mathrm{~h}$ regions can be found at http://ok.gov/sde/reac3h-network.

[^1]:    Note. The range of scores were from $1-13$ in each belief category.

