

70-22,998

MITCHELL, Billy Joe, 1935-
VISCOSITY OF FOAM.

The University of Oklahoma, Ph.D., 1970
Engineering, hydraulic

University Microfilms, A XEROX Company, Ann Arbor, Michigan

THE UNIVERSITY OF OKLAHOMA
GRADUATE COLLEGE

VISCOSITY OF FOAM

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the
degree of

DOCTOR OF PHILOSOPHY

By

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Norman, Oklahoma

1969

VISCOSITY OF FOAM

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ACKNOWLEDGMENT

The author wishes to express his appreciation to the following people and organizations for their valuable contributions to this dissertation.

Special thanks to Dr. John M. Campbell for his suggestions and inspiration.

The financial assistance by the Tenneco Oil Company was invaluable. To Dr. Ralph E. Gilchrist of that company is extended sincere gratitude.

Finally, my wife, Caroline, for her understanding.

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FOAM FLOW IN CAPILLARY TUBES

CHAPTER I

INTRODUCTION

The application of foam as a drilling fluid in the process of drilling boreholes in the earth's crust with the rotary system is relatively new. This process has had limited success due primarily to the lack of a theory pertinent to flow characteristics of foam and secondarily to the lack of sufficient experience with foam drilling on which empirical correlations could be founded. Thus the status of the art of applying foam as a rotary drilling fluid is relatively underdeveloped.

Because of the many roles that a drilling fluid must fulfill in the rotary drilling system, perhaps an all encompassing theory may never be developed; however, at this juncture a fundamental understanding of the flow characteristics of foam in tubes could improve current operations and broaden its application.

CHAPTER II

DISCUSSION AND REVIEW OF PREVIOUS INVESTIGATIONS

Introduction to Foam and Foam Flow

Foam is defined for use in this paper as a fluid consisting of distilled water, a surface active agent and water-saturated atmospheric air. The aqueous solution of water and surface active agent shall be the continuous phase with the air appearing as bubbles as the discontinuous phase. Mist is defined as a fluid consisting of identical components but is distinguished in that the air is the continuous phase with the aqueous solution appearing as discontinuous droplets. The usual definitions of gas and liquid apply.

A foam in a rotary drilling operation could be quite different from the foam defined above. It could consist of large inert solids (drill solids), small chemically-active solids (bentonite), active chemicals (sodium chloride, lignosulfonates, hydrogen sulfide, etc.), and

gases (air, nitrogen, natural gas, and carbon dioxide).

The popular method of manufacturing foam in the field is to inject the gas phase through a nozzle to control the rate and pressure into a line that is transporting the aqueous phase. A foam may or may not be created at the point of injection; however, further turbulence in the piping system does so.¹

In the laboratory, foams are successfully manufactured by simultaneously injecting and flowing both the aqueous phase and the gas phase through a cylinder containing a porous material.²

In the application of foam as a drilling fluid in the rotary drilling system it must, without collapsing to a simple water and air two-phase system (slug flow), lift all solids and liquids from the borehole. The foam must accomplish this task without eroding or corroding the equipment in the borehole or eroding the wall of the borehole because of excessive velocity of flow. Lifting capacity of fluids is primarily a function of its flow velocity, density, and viscosity. Because these properties are closely related to the absolute pressure to which the foam is subjected, the general problem is the prediction of pressure drop during flow.

The flow regimes shown in Figure 1 are the only two-phase types identified by most authors.³ These types are:

(1) pure liquid flow, (2) gas slug flow with or without interspaced bubbles, (3) foam flow with gas slugs, (4) foam flow, (5) mist flow, and (6) pure gas flow. If a foam generator precedes the flow tube, it may be expected that types of flow, 2 and 3, perhaps would be suppressed to a large extent. Then the combination of the various fluids may have viscosities similar to the appearance of the curve shown in Figure 2. In this figure, μ_l is the viscosity of liquid base and is assumed to be Newtonian in behavior, μ_g is the viscosity of the gas and is also assumed to be Newtonian, μ_f is the viscosity of foam, μ_m is the viscosity of mist, μ_c is the viscosity of the fluid system when the system is at the critical foam quality.

By definition a Newtonian fluid has a constant viscosity at any constant temperature and pressure, thus the portion of curve represented by the intersection of the curve and the dependent axis is a point and is represented by μ_l . The portion of the curve between the values of the independent variable of zero and approximately .50 ($0 < \Gamma < .50$) is justified with Einstein's⁴ theoretically developed equation. Hatschek's⁵ theoretically derived equation justifies the portion of the curve between $0 < \Gamma < .74$. Hatschek's⁶ second theoretically derived equation justifies the equation between $0 < \Gamma < \Gamma_c$. The

portion of the curve between $\Gamma_c \leq \Gamma \leq 1.0$ represents mist flow. It could be thought that one only needs to think of reversing the previous equations in regard to the fluids in the internal and external phases to describe this portion of the curve; however, in steel tubes water wets the wall of the tube and this fact could invalidate such reasoning. The point on the curve at the intersection of the principal curve and the line at $\Gamma=1.0$ is the viscosity of the gas.

Terms generally applied to foam are foam quality which is defined as the ratio of gas volume to the total volume ($\Gamma = V_g / V_t$) and foam texture which is related to the size of the gas bubbles. Coarse foams are those having large bubbles while fine foams have small bubbles. Wet foams are of low quality and dry foams are of high quality.

Previous Investigations

One of the most enlightening of the earlier papers written by Grove, et. al.⁷ contains an investigation of the viscosity of fire-fighting foam. They measured the effects of pressure (up to 90 psig), rate of shear, and foam quality on the apparent viscosity in a flow-type viscometer. The viscometer was essentially a rotometer with the inner floating rod attached to a spring scale.

Correlations within the paper are based on the Reynold's Number and a self-derived friction factor. Reynold's Numbers ranged from 42 to 3200. Velocity of flow ranged between 1.83 and 17.8 feet per second. The density of foam ranged between 16.5 and 46.6 pounds per cubic foot. The friction factors were calculated with the Fanning turbulent flow equation. The data were gathered at room temperature.

Grove, et. al. found that foam viscosities were nearly independent of the rate of flow of foam and that foams of the same densities have equal viscosities although confining pressures and gas-water ratios may vary over a wide range. Thus they found that foam flow in turbulent flow was independent of shear rate. They, as others after them, found that viscosities depend greatly and vary inversely with the qualities of the foam at equal, absolute pressures. They show that viscosities increased 13.7 times as the foam decreased in density from 46.6 to 16.5 pounds per cubic foot.

Sibree⁸ was one of the first investigators to show that the viscosity of foam became constant for all shear rates above a critical value. He also investigated foam bubble size distribution; however, this phase of his experiment was conducted at room temperatures and atmospheric pressures. Shear forces and shear rates were

measured with a rotating-cylinder viscometer. Values of viscosities to an arbitrary scale are correlated with cylinder angular velocity. It is of interest to note that Sibree found that the Hatschek formula that is used to predict the viscosities of coarse emulsions could not be applied directly to foam. However, the same general trend did develop in that foam asymptotically approaches a limiting value as shear rate increases. Also, he showed that foam shows Newtonian behavior below a critical shear stress and plug-like flow above the critical shear stress.

Hatschek's formula, as modified by Sibree, for describing the viscosity of foam at atmospheric pressure is:

$$\mu_f = \mu_0 \left\{ \frac{1}{1 - (2.3 \Gamma)^{1/3}} \right\}$$

Where Γ is the quality of the foam

μ_0 is the viscosity of the base liquid

μ_f is the viscosity of the foam

Foam qualities in this phase of his investigation ranged from 51.5 to 73.0 percent.

In his bubble-size-distribution investigation he showed that the bubbles in foam are of the same size as those in coarse emulsions and range in size from 20 to 500 microns in diameter. The most frequently occurring size was approximately 125 microns in diameter. A foam quality

for this distribution was not reported. The bubble sizes were measured on a glass slide with a reflected light microscope.

Khan and Marsden⁹, working at room temperatures and atmospheric pressure and with foam qualities ranging from 70 to 96%, showed that apparent viscosity of their foam decreased with increasing shear rate and usually fell in the range of 50 to 500 centipoises when measured with a modified rotational viscometer. They also report that at a given shear rate apparent viscosity increases linearly with foam quality. Measurements taken with a vibrating reed viscometer of kinematic apparent viscosity is independent of foam quality but apparent viscosity increases with foam quality from about three to eight centipoises. Results of measurements taken with their capillary viscometer were not reported.

Their pertinent apparatus consisted of a foam generator (a cylinder containing grain packing), a capillary tube viscometer, a modified rotational viscometer, and a vibrating reed viscometer. The rotational viscometer was modified by attaching ribs parallel to the longitudinal axis of the rotor and also to the axis of the stator.

Khan and Marsden report that texture (bubble size) of the foam can be varied by changing the size of the

grains within the foam generator near the affluent end. No data were reported in regard to bubble size distributions or grain size used. Maximum pressure available for the investigation was 100 pounds per square inch gauge.

Fried¹⁰ measured viscosity of foam in both a modified rotational viscometer and a capillary flow tube. His findings were consistent with other investigators. He found that the viscosity of the foam increased with increasing tube diameters and decreased with decreasing foam quality.

Raza and Marsden¹¹ measured viscosities and electrical potentials that developed while flowing foams through capillary flow tubes. Electrical potentials as high as 30 volts were reported. In their viscosity investigation they reported that foam flow could be described as pseudo-plastic, however, very little data are presented at the lower shear rates to fully support this conclusion. Raza and Marsden state that below a critical flow rate which is dependent on both foam quality and tube diameter that flow is Newtonian and that above the critical flow rate the foam passes through the tube as a viscous plug with most of the shear occurring at the walls. Pressure was limited to 100 pounds per square inch gauge and room temperatures.

Within these conditions of pressure and temperature,

Raza and Marsden report that foam is compressible and that Boyle's Law applies directly to foam neglecting gas solubility in the solution and liquid expansion. They present the following equation for predicting foam quality at some other pressure than which it was measured.

$$\Gamma = \frac{1}{\left[1 + \frac{P}{P_a} \left(\frac{1}{\Gamma_a} - 1 \right) \right]}$$

Where Γ is the calculated foam quality at pressure P

Γ_a is the measured foam quality at pressure P_a

Comments on Previous Investigations of Foam

A review of previous investigations shows that the following qualitative and quantitative information has been reported:

1. The viscosity of foam increases with increasing quality up to an undefined critical flow rate.
2. The viscosity of foam becomes constant for all values of flow rate above a critical flow rate.
3. Foam shows Newtonian behavior below a critical flow rate and plug flow behavior above it.
4. A modified Hatschek's formula describes the viscosity of foam at atmospheric pressures and room temperature.

5. Texture of foam is affected by grain size in a foam generator.
6. Foam flowing in capillary tubes generates large electrical potentials.
7. The viscosity of foam increases with increasing capillary tube diameters.
8. Foam is compressible and Boyle's Law may be applied to the calculation of foam quality at various pressures.

The investigators to date have conducted their studies at or below pressures of one hundred pounds per square inch and at room temperatures. This greatly limits the scope of their research when one considers that pressures in excess of 700 pounds per square inch and temperatures in excess of 250 degrees Fahrenheit may be encountered in foam applications in oil well drilling.

Perhaps the most important limitation of reported investigations to date is that no one has reported shear stress data over a wide range of shear rates that spans the various types of flow that could exist. Also, a study in which several parameters are correlated, has not been presented. The foaming agents are not described in detail.

To date a general-flow theory for foam has not been presented. A definition of foam could not be found. All definitions of emulsions as reported by Becher¹² exclude foam as an emulsion. Possibly emulsion theory may not be directly applicable to foam due primarily to the fact that emulsions have immiscible droplets of liquid dispersed in a second liquid. Whereas, foam has gas bubbles dispersed in a liquid. The above statement is emphasized when one considers the difference in gas properties as compared with those of liquids.

Investigations of Emulsions

Because of the lack of a theoretical basis for describing foam, a few of the pertinent theories pertaining to emulsions are presented.

Becher presents a series of nine definitions of the term, "emulsions", and points out the weaknesses of these. His formulation of the definition is more inclusive and is presented.

"An emulsion is a heterogeneous system, consisting of at least one immiscible liquid intimately dispersed in another in the form of droplets, whose diameters, in general, exceed 0.1 micron. Such systems possess a minimal stability,

which may be accentuated by such additives as surface-active agents, finely-divided solids, etc."¹²

It may be noted that foam is excluded as an emulsion because the dispersed phase is a gas and not a liquid. Also, none of the other definitions set forth by other authorities include foam as an emulsion.

McBain, Ford, and Wilson¹³ distinguished three types of surface-tension curves and are reproduced in Figure 3. Most soaps and detergents produce type three curve. Since McBain, et. al.'s data were a measure of surface tensions of aqueous solution in contact with air, these data apply directly to foam.

Fischer and Gans¹⁴ investigated the range of surface tensions of aqueous solutions containing many surface-active compounds. Their results are presented in Figure 4.

These data were taken with the solutions in contact with air and thus apply directly to foam.

Particle size and particle-size distribution are pertinent to the description of many bulk properties of emulsions. Schwarz and Bezemer¹⁶ derived the equation

$$\frac{dn}{dx} \frac{100}{N} = \frac{100}{6} \left\{ \frac{e^{a/x}}{1 + \frac{a}{X} + \frac{a^2}{X} + \frac{a^3}{X}} \right\} \frac{a^4}{x^5} e^{-a/x}$$

Refer to Figure 5.

and if $a/x \ll 1$,

then

$$\frac{dn}{dx} \frac{100}{N} = \frac{100}{6} \frac{a^4}{X^5} e^{-a/x}$$

where n is frequency of the droplets of diameter x

N is the number of drops in the count

X is the diameter of the largest occurring drop

a is a constant calculated from a histogram of experimental data

Einstein developed an equation that relates the viscosity of emulsions with the viscosity of the continuous liquid phase and its quality, using hydrodynamic theory⁴. Since his derivation is basic to the research in this study for the determination of the viscosity of foam in flow through circular flow tube, its derivation is presented in part in the chapter entitled "Theory."

Taylor derived a formula based on hydrodynamic theory which relates the viscosity of the liquid in the dispersed phase, and the viscosity of liquid constituting the continuous phase.¹⁷

$$\mu_e = \mu_o \left\{ 1 + 2.5 \Gamma \left[\frac{\mu_i + 2/5 \mu_o}{\mu_i + \mu_o} \right] \right\}$$

Where Γ is the quality of the foam

μ_i is the viscosity of the internal phase

μ_o is the viscosity of the external phase

μ_e is the viscosity of the emulsion

Nawab and Mason¹⁸ have experimental data which support the equations applicability to emulsion solutions.

The primary expectations of the formula is to account for the circulation of liquid within each droplet by considering the shearing flow force to which it is subjected. The liquid droplets are assumed incompressible, which of course is a major deviation in the case of gases.

Broughton and Squires¹⁹ show that in some emulsions particle size distribution can affect their viscosity. The following curves in Figure 6 show the data from which this conclusion was drawn. It may be noted also that the mean particle size decreases with narrower distributions. Observations of the data show that an eight-fold increase in apparent viscosities at equal rates of shear between the most sharply and broadly present distributions occurred.

Sherman²⁰ presents experimental data that show that the viscosities of emulsions can increase one and one-half times with a decrease in the size range of the droplets of three times. Thus a decrease in droplet size increases the apparent viscosity.

Smoluchowski²¹ theoretically investigated the effects of the electrical specific conductivity, the dielectric constant, and the electrokinetic potential (zeta potential) on emulsions in which the surface-active agent was a lyophilic type possessing an electric charge. His con-

clusion was that electro-viscous effects would always tend to increase the apparent viscosity of the emulsion.

Van der Waarden²² experimentally investigated the variable discussed above and presented data which show that some emulsions exhibit a several fold change in viscosity if high percent by weight (5 to 35%) of surface-active agent is used. He also points out that the electrical effects can be reduced to near zero levels by the addition of an electrolyte or by the selection of some types of ionic surface-active agent.

Raza¹¹ essentially agrees with Van der Waarden but in addition he states that dry foam yields higher streaming potential than a wet foam. He defines wet foam as a foam with a quality of 70 percent or less and dry foam as a foam with a quality of 80 percent or more. He defines quality as volume percent of the gaseous phase to the total volume of the foam.

Sherman²³, by way of a literature search, lists six physical phenomena which may affect the rheological properties of emulsions:

1. Viscosity of the liquid of the internal phase; μ_i
2. Viscosity of the liquid of the external phase; μ_o
3. Relative volume concentrations of the two phases

4. Type of surface-active agent and the nature of the interfacial film formed.
5. Electrical forces
6. Particle size distribution

To this list perhaps particle size should be added if one feels that particle size is not intended in the particle size distribution.

Helmholtz Double Layer - An Interfacial Theory

Helmholtz³⁴ proposed a physical model which is commonly referred to as the double layer theory that accounts for phenomena observed in the investigations of stability of emulsions and the electrical forces.

Emulsifying agents are partially categorized as cationic, anionic, and non-ionic and also as hydrophilic and lipophilic in regard to the polar portion structure. Other classifications exist, of course. Polarity here simply means that the molecule dissolves more readily in polar solvents and it does not mean that molecule necessarily has a dipole moment. Hydrophilic means that the polar portion of the molecule is more soluble in the water phase and lipophilic means that it is more soluble in the oil phase. Thus at the interface of a droplet of an oil-in-water emulsion, the hydrophilic surfactant molecule is found with its non-polar portion in the oil phase and its polar portion in the water phase. The classification of

cationic, anionic, and non-ionic refers to the charge of the polar portion of the molecule. Cationic is positive, anionic is negative, and non-ionic is neutral electrically. Figure 7 shows these ideas.

The double layer recognizes the foregoing comments and adds that since the charged portion of the hydrophilic molecule was closely bound to the interface and completely fills the surface area of the droplet that an oppositely charged molecule from the continuous phase should be directly adjacent. However, these adjacent molecules are more mobile and since the surface area of sphere increases with its radius, these molecules are more diffused than the surfactant molecules. Considering the above ideas, it was concluded that a potential should exist between the interface of the droplet and the continuous phase.

Stern²⁵ modified the double-layer theory by proposing that the total potential could be divided into a fixed layer and a mobile layer. The fixed layer is called either the Nernst or the Stern potential while the portion of the potential that is mobile is called the zeta potential. The boundary between these two layers is closely associated with surface of shear between the droplet and the continuous phase.

Raza¹¹ further points out in an illustration that the potential values between the interface and the solution

should occur as in Figure 8.

Raza¹¹ concludes that the total streaming potential which is related to the zeta potential in foam flow could be nearly reduced to a zero value in a glass capillary tube by the selection of a proper ionic surfactant. It appears that a proper electrical grounding of the flow tube would also reduce the effect of the streaming potential.

Comments on Emulsion Theory Pertinent to Foam

Foam is not an emulsion. The basic distinctions are these:

1. The dispersed gas phase is highly compressible as compared to a liquid in the dispersed phase. This means that each droplet is subject to changes in size and in interfacial surface where the surfactant molecules are located.
2. The variance in density and viscosity of a dispersed gas bubble and a liquid bubble would effect the creaming rate of these droplets. The flow characteristic should be altered.
3. Absorption of the surfactant molecule must certainly be altered. For instance, a hydro-

philic surfactant molecule in foam would orient its polar portion in the aqueous phase but the non-polar portion would be protruding into the gas phase instead of into a soluble liquid phase.

4. Due to the difference in viscosity of the dispersed phase (gas versus liquid) the gas bubbles could be considerably more easily deformed than a liquid droplet. This could have a marked effect on the coalescence and inversion rates.
5. Solubility variation between a gas and an immiscible liquid in an aqueous phase could affect their respective interfaces. For instance, the difference in concentration of the air versus oil molecules adjacent to the interface should alter the strength of the surface of the droplets.
6. As a further comparison between dispersions of gas and liquids, the velocity of the gas molecules would have the higher velocities at

the same temperature and pressure with which to break up the surfactant molecules at the interface. Perhaps these higher velocities would affect foam stability.

Summary of Introduction

Emulsions and foams may have similar properties. Investigations have shown that in regard to the six intensive properties listed as important in the determination of the viscosity of emulsion only four have been investigated with respect to foam to varying degrees. Of these six items, foam quality, bubble size, distribution, electrical effects have been reported, while internal phase and external phase viscosities, and interfacial forces have not.

No theoretical equations have been presented to relate these factors to the viscosity of foam.

CHAPTER III

THEORY

Einstein Flow Theory - An Energy Balance

In 1906 Albert Einstein invented a theory using hydrodynamic principles by which the dimensions of a molecule suspended in a solvent could be ascertained⁴. Within and as an integral part of the theory he developed equations which could be used to determine the effect of particles suspended in a solvent upon the viscosity of the suspension.

His theoretically-derived relationship which is pertinent to foam flow is:

$$\mu^* = \mu_o (1.0 + 2.5\Gamma)$$

where μ^* is the viscosity of the suspension

μ_o is the viscosity of the pure solvent

Γ is the quality of foam

Because of the relative obscurity of this paper and the theory and due to its publication date, the major points of the theory are reiterated. The original paper was published in the German language.

Hatschek's Flow Theory from Stokes' Law

Hatschek's flow theory accounts for the viscosity characteristics of foam in the foam quality range between the values of 0% and 74%.⁵ The theory is not repeated because it is easily read in Hatschek's original publication. The basic assumptions are these:

- (1) Stokes' Law applies.
- (2) Only spherical solid particles (non-deformable) are present.
- (3) These particles are equal in size and uniform in their distribution within the solvent.
- (4) The viscosities of the pure solvent, i.e., without particles and the solvent containing the particles, are proportional to the power required to sustain equal velocities of the two fluids in a linear horizontal displacement motion.
- (5) The fluid is considered not to have dilation motion.

His final equation is

$$\mu_f = \mu(1.0 + 4.5\Gamma)$$

In 1929 Rabinowitsch invented a theory that relates shear stress with shear rate for the flow of fluids

in tubes. As related to flow in a viscometer tube, his theoretically derived relationship is:

$$\left(-\frac{dv}{dr}\right)_w = \frac{3}{4} \left(\frac{\delta v}{D}\right) + \frac{1}{4} \left(\frac{D(\Delta P_f)}{4L}\right) \left[\frac{d(\delta v/D)}{d(D\Delta P_f/4L)}\right]$$

Definition of these terms is given in the nomenclature.

A literature search indicates that a majority of authors define viscosity of a fluid as

$$\mu = \frac{1}{g_c} \frac{\tau}{\phi}$$

where μ is the viscosity of the fluid

τ is the shear-stress in the fluid

ϕ is the shear-rate in the fluid

Apparent viscosity is defined as follows:

$$\mu_a = \frac{1}{g_c} \frac{\Delta \tau}{\Delta \phi}$$

These definitions will be adhered to in this work.

The three theories and the definition given above will be combined to formulate a composite theory for the flow of foam in circular tubes.

Einstein's Theory and Model

Einstein's model from which his theory is derived envisions the following:

1. Solid particles are suspended in a homogeneous fluid which will hereafter be referred to as the solvent.

2. The particles are always spherical, homogeneous, of equal size (volume), and cannot possess momentum (weightless).
3. The volume of the particles is large as compared with the volume of the molecules composing the solvent.
4. The volume of the particles is small as compared with the volume of any region of the fluid regardless of the size of the fluid region selected.
5. The spacing of the particles which is constant is sufficiently large such that they do not mutually interfere in their motion.
6. The motion of a particle does not affect any region of the fluid which is located at an infinite distance from its surface.
7. There is no relative motion between the surface of the sphere and fluid contacting its surface.

8. The fluid has viscous properties.
9. The fluid cannot possess momentum.
10. Hydrodynamic equations are used to determine the motion of the particles and the solvent.
11. All functions, regardless of the variable under consideration, within any region of the solvent are continuous and linear functions except at the surface of a particle or within the boundary of a particle.
12. The motion of the fluid is determined by the superposition of three principal directions of dilation for a fluid. The problem is solved for a three-dimensional expansion of the suspension.

His mathematical model is presented in the appendix.

Hatschek Flow Theory - An Interference Hypothesis

This particular flow theory of Hatscheck⁶ justifies the portion of the viscosity curve between $.7404 \leq \eta \leq \eta_c$ as shown in Figure 2. The following is a quotation from the Hatscheck⁶ paper:

"If we imagine a system of spheres in a given space, and let their radius or their number increase until they are in closest contact, i. e., until each sphere touches twelve others, the aggregate volume of the spheres is 74.04 percent of the total space occupied. The twelve points of contact are the centers of the faces of a dodecahedron (two different dodecahedra are possible, into which question it is not necessary to go), and, if the volume of the spheres is increased further, flattening must take place and the faces of the dodecahedron be developed more and more."

"While the author has found a general treatment of the viscosity of a system of two liquid phases impossible, it is comparatively easy to proceed synthetically from the stereometric considerations just developed. Figure 10A shows diagrammatically a film of such a two-phase system, the dispersed phase being shown white and the films of the continuous phase, black. As explained, the former assumes necessarily the shape of dodecahedra, the hexagons being sections through these. If the system is now sheared, the

polyhedra must slide over one another, and in a certain position the system must assume the altered shape shown in Figure 10B. A simple calculation, for which the reader is referred to the original paper, will show that such rectangular prisms fill space again continuously and are therefore possible.

In that event it is obvious that shearing only takes place in the horizontal films of continuous phase, and it is a simple matter to calculate the work done on this assumption. It is also obvious that neither the interfacial tension (which has not time to act) nor the viscosity of the dispersed phase enter into the calculation, which is given in extenso in the original paper. The formula finally obtained, if the viscosity of the continuous phase is taken as unity, is the following

$$\eta = \frac{\sqrt[3]{A}}{\sqrt[3]{A} - 1}$$

in which the symbols mean:

η = coefficient of viscosity of system

A = ratio: $\frac{\text{Volume of system}}{\text{Volume of dispersed phase}}$ "

Applying symbols adopted in this work his equation becomes,

$$\mu_1 = \mu_2 \left(\frac{1}{1 - f^{1/3}} \right)$$

The above quotation defines the essential assumptions required to derive his equation. His mathematical model

Rabinowitsch Flow Theory - A Momenta Balance

In 1929 Rabinowitsch²⁶ invented a theory that described the laminar flow of fluids in a long circular tube. His model consists of a homogeneous, isotropic, continuous fluid flowing through a straight, circular tube. Further, the flow is isothermal, steady state and without slippage at the wall of the tube. The fluid need not be Newtonian. His theory provides a means for determining the absolute flow properties of a fluid if measurements are made with a tube viscometer. His Mathematical Model is presented in the appendix.

CHAPTER IV

EXPERIMENTS AND APPARATUS

The apparatus used in the experiments for the study of foam is shown in Figure 13. The major systems comprising the apparatus are: (1) aqueous injection system, (2) air injection system, (3) high pressure air metering system, (4) foam generation system, (5) dye injection system, (6) photomicrographic system, (7) dye detection system (photoelectric cells), (8) flow tube system, (9) pressure detection systems, (10) differential pressure detection systems, (11) pressure control system, (12) foam observation system, (13) low pressure air metering system, and low pressure water metering system, (14) temperature monitoring system. Each of the systems are described in turn.

(1) Aqueous Injection System

The aqueous injection system consisted of (a) atmospheric pressure, a stainless steel reservoir with a capacity of 26,000 cc, (b) two variable flow rate, positive displacement plunger pumps capable of pressures to 1500 psi and 9800 psi, and (c) a high pressure stainless steel reservoir with a capacity of 3000 cc and with the associated

high pressure air lines for maintaining pressures in the reservoir. The plunger pumps were used for the higher rates of flow through the flow tube while the high pressure stainless steel reservoir was used for the lower flow rate tests. Maximum and minimum liquid flow rates used in the tests were 755.7 cc/second and 0.0421 cc/second respectively.

(2) Gas Injection System

A ten horsepower gas compressor capable of pressures of 3500 psig supplied gas to the air storage cylinders which had a capacity of 2.7 cubic feet. The storage cylinders also served as a fluid trap and a pulsation suppression system. The sand pack filter served to remove foreign particles. The pressure gauge shown just after the sand pack filter was a precision laboratory gauge and was used to monitor the pressure available for the flow tube systems. It was checked for calibration with a dead weight tester three times during the experiments and once afterward and was not found to be within a measurable error. All other gauges were in turn calibrated with this gauge.

(3) High Pressure Air Metering System

The high pressure air metering system consisted of an orifice plate gas meter, recorder, and driving equipment. This system proved inadequate for the precision

required and thereafter served only as an assist to pre-determine air flow rates before a test and in this capacity it was very useful.

(4) Foam Generation System

The foam generation system consisted of the foam generator, the two electrical grounds, and the by-pass line and valves. The foam generator consisted of a stainless steel thick wall vessel with an internal diameter of 7/9 inch and an internal length of 10½ inches filled with 20 and 40 mesh glass beads and extremely fine sand which passed through 200 mesh screen (200 meshes to the inch). Glass wool and wire mesh served to retain the sand particle within the chamber.

(5) Dye Injection System

This system injected ordinary concentrated, water base, drafting ink into the foam to serve as a marker for the photoelectric cells. The ink in droplet quantities was metered through a micrometer valve from its stainless steel storage cylinder.

(6) Photomicrographic System

The photomicrographic system consisted of a visual cell which permitted photographs to be taken with either reflected or refracted light. A polaroid camera 110B

model with various types of black and white and color films of various speeds mounted on a stereo-microscope served for taking the photomicrographs shown in Figures 14 through 17. The film exposure light source was an electric flash with a light flash duration of $1/3,000$ seconds. The stereo-microscope was of the continuous variable magnification type with a maximum magnification of 210x. Refer to Figures 18 and 19.

(7) Dye Detection System

The principle components of the dye detection system are: (a) the photoelectric cells, the dye detection cell, the light source, the recorder, and the image storable oscilloscope. The light source was tungsten. The image storable oscilloscope was employed for the higher flow rate tests while the recorder was employed for the lower flow rate tests. The purpose of this system was to measure the transit time of the dye front to pass through that portion of the flow tube between the detection chambers. This system could detect dye in such minute concentrations as to be unobservable to the eye. A sample record is shown in Figure 20. The equipment is shown in Figure 21.

The second purpose of the cells was to qualitatively delineate the consistency of the foam. Minor air or water slugs and most types of flows were recorded as Part B of Figure 20, while consistent foam recorded as Part A. None of the other equipment could sense these minor, transient

disturbances.

(8) Flow Tube System

The flow tubes were of stainless steel with dimensions as shown in Table 1 . The internal diameter dimensions are shown as: (a) those reported by the manufacturer and (b) those determined by calibration tests. The calibration tests disclosed that the internal diameter dimensions should be reduced by the percentages listed for use in flow calculations. Note the consistency in the percentage of reduction.

The unusually long lengths of the flow tubes were chosen to negate entrance and exit pressure disturbances normally associated with smaller diameter flow tubes. The tubes are primarily manufactured for use as thin-wall, low-pressure syringe needles. An attempt was made to determine the internal diameters by filling the tubes with mercury and then comparing filled and empty weights. This procedure could not be used due to the length or flexibility of the tubes and scales. The discussion of the calibration of the tube's internal diameters is deferred to the section entitled, "Calibration."

(9) Pressure Detection System

It was essential that the pressures be detected and monitored accurately at each end of the flow tube in order to determine the absolute pressure range within the flow

tube. This was accomplished by observing the two gauges and the records of the pressure transducer connected to the outlet end of the flow tube. The two gauges as well as the pressure transducer were regularly checked for accuracy with the laboratory gauge. None was ever found to be within a detectable error.

(10) Differential Pressure Detection System

The differential pressure detection system served to quantitatively measure the pressure loss across the flow tube. The system's pressure detection devices were:

(a) a differential pressure transducer with a capacity of 10 psi differential, (b) a high pressure manometer, and (c) the two pressure gauges. Calibration of the transducer showed that it was accurate and linear from zero to seven psi thus it was employed to monitor the lowest flow rates. The mercury manometer was capable of measuring differential pressures to 19.7 psi, and, it was employed in the intermediate flow rate tests. The two pressure gauges were used in the highest flow rate tests. The signal from the pressure transducer was displayed on an oscilloscope screen. Some of the signals were recorded. The pressure transducer was calibrated with the high pressure manometer filled with both mercury and water.

(11) Pressure Control System

The absolute pressures within the flow tube were controlled through a high pressure air operated, adjustable choke valve and its associated air regulator.

(12) Foam Observation System

Foam was observed for consistency and texture through the visual cells located at each end of the tube. Foam that was fine-textured and consistent always appear to be snow white while flowing through the visual cells. The cells were manufactured of plexiglass.

(13) Low Pressure Air and Water Metering System

This system metered the volume of water and air at atmospheric pressure temperature that passed through the flow tube. The foam, after passing through the choke valve, entered a flask which served as an air-liquid separator, where the liquid was retained and weighed and the air was withdrawn and passed through a precision wet test meter. The flow rate was determined by recording the weight of liquid change in the flask and the volume of air through the wet test meter during a time interval as recorded by a stop watch. These readings were recorded while the system was in dynamic equilibrium.

(14) Temperature Monitoring System

The temperature of the system was indicated by (a) a resistance wire, (b) a thermocouple, and (c) a thermometer. All of these were in contact with the flow tube and sealed with asbestos. Temperature variations were minor. Temperatures ranged between 71.0° F and 80.8° F through the experiments. These temperature variations were accounted for in the calculations.

Calibration

Calibration of the various systems has been explained with the exception of the flow tube's internal diameter dimensions. It was deemed necessary that the composite of the systems demonstrate that several Newtonian liquids of various viscosities could be measured as Newtonian liquids and at the correct viscosity levels and throughout a wide range of shear rates. For these calibration tests, the liquids chosen were: (a) distilled water, (b) nujol (a mineral oil), (c) glycerine, (d) glycerine and water mixtures, and (e) ethelene glycol. Before and after each run through the flow tube system, the viscosity of the liquids was checked with a rotary type viscometer and an Ostwald capillary tube type viscometer. Theory and experiments show that when shear stress is plotted versus shear rate on logarithm coordinates that Newtonian fluids data appear

linear with a slope equal to unity. If this criterion is met then the viscosity is determined by dividing the shear rate value into the shear stress value at any data point. Figure 22 shows that the system functioned properly within experimental error throughout a wide range of shear stress. It is further necessary to show that the calculated flow tube viscosities are equivalent to those measured by the Ostwald and rotary viscometers. Figure 23 shows the degree of agreement and linearity of the data. After this data was collected and analyzed, the flow tube internal diameters were determined. The procedure was now reduced to measuring the necessary flow data and calculating the viscosity for both single phase liquid and two phase foam and determining the intercept of a line as determined by the method of least squares constructed through the data points and ordinate of a plot of viscosity versus foam quality. The internal diameters of the tubes were adjusted such that the intercept value was equivalent to the previously determined liquid viscosities. Refer to Tables 2, 3, and 4.

Surface Active Agent and Solution

The surface active agent was a commercial foamer manufactured by Proctor and Gamble and was known by the code of R-7. This surfactant is non-ionic and resistant to contamination by salts. It is also bio-degradable and

as a result small (0.038% by volume) of ethyl alcohol was used as a preservative. Aqueous solutions having concentrations of this surfactant of 1.0% by volume were used for all tests.

Procedure

Prior to a group of tests the system was checked for calibration by passing only the liquid surfactant solution through it at various flow rates. If these data fell within tolerance limits, then the foam tests were commenced by introducing air. After sufficient time for equilibrium, the flow rates of the water and air were determined by weighing the water and measuring the volume of air passing through a precision wet test meter during a time period as measured by a stop watch. During this period the absolute pressure was recorded and the differential pressure was noted by one of the three systems for that purpose. Also the foam was checked for consistency through the visual cells and by the recording from the photoelectric cells. The next step was to introduce the dye into the flow line and record the transit time for it to pass between the photoelectric cells. If a photo-micrographic film was to be exposed, it was done so at this point. This would complete a test. The second and following tests only required that the length of stroke

of the plunger pump(s) be increased or decreased as required. Also the upstream air pressure was increased or decreased to permit nearly equal average flowing pressures within the flow tube for all tests.

The procedure for making the exposures shown in Figure 14 through 17 was to have the foam flowing at high pressure at stable conditions and simply shut in the foam. Thus the exposure shown as "123 minutes after shut-in" refers to the time elapsed between shut-in and exposure of the film. The atmospheric pressure was measured with a mercury barometer prior to a group of tests.

The system was grounded at the points as shown in the figure to reduce stray currents and streaming potentials. The ground was the flange of a surface casing which was set to a depth of sixty feet in a water well.

Procedure for Calculation

The following presentation describes the procedure and methods used for the calculations of foam properties. Since a good part of the data represents the properties of the foam at atmospheric pressure, certain formulas and principles were used to estimate these properties at actual flowing conditions. The discussion is commenced with a mass balance from the entrance of the tube through the wet test meter.

Mass Balance

Please refer to Figure 24.

If dynamic equilibrium is to be achieved within the flow tube, mass either in the form of liquid or gas cannot collect therein. Thus, all of the mass introduced at the left must collect in the container or pass through the wet test meter. Since the pressure and temperature of the mass entering the system is not identical to that exiting, mass will be transported between the airstream (bubbles) and the liquid stream. The equation that may be concluded is the following:

$$\text{Mass Entering} = \text{Mass Exiting}$$

$$\begin{aligned} \text{Mass Entering} = & \text{Mass of the liquid in the liquid stream} \\ & + \text{mass of the liquid in the gas stream} \\ & + \text{mass of the gas in the liquid stream} \\ & + \text{mass of the gas in the gas stream.} \end{aligned}$$

Mass Exiting = Mass of the liquid collected in the container - mass of the gas displaced from the container by the liquid + mass of the liquid carried through the wet test meter + mass of the gas absorbed in the liquid in the container + mass of the gas passed through the wet test meter.

Due to the low solubility of air in water at atmospheric temperatures and pressures, these solubility terms were neglected. Accordingly the mass of the liquid and air at the efflux end of the tube is the mass collected in the container and the mass of air and liquid which was passed through the wet test meter less the mass of the air which is displaced by the liquids collected in the container.

The mass of the liquid in the container may be determined directly:

$$M_{l,c} = W_{l,c} \text{ g/g}_c \quad . . . (1)$$

The mass of the liquid and gas carried through the wet test meter may be estimated by use of the perfect gas law and Dalton's law of partial pressures,

Charles' Law

$$\begin{aligned} P_g + P_l &= P \quad , \quad P_g = P - P_l \\ P_l &= P_v \end{aligned} \quad . . . \quad (2)$$

Gas Law:

$$M_g = \frac{P_g V_g \bar{M}}{z_g R T_g} \quad . . . \quad (3)$$

Substituting yields,

$$M_{g,m} = \frac{(P - P_v) V_m \bar{M}_g}{z_g R T_g} \quad . . . \quad (4)$$

$$M_{l,m} = \frac{P_v V_m \bar{M}_l}{z_l R T_l} \quad . . . \quad (5)$$

Now the masses of gas carried through the wet test meter that were displaced from the container are by analogy;

$$M_{g,d} = \frac{(P - P_v)(V_{l,c}) \bar{M}_g}{z_g R T_g} \quad . . . \quad (6)$$

$$M_{l,d} = \frac{P_v (V_{l,c}) \bar{M}_l}{z_l R T_l} \quad . . . \quad (7)$$

Thus the total mass of liquid flowing through the tube is,

$$M_{l,r} = M_{l,c} + M_{l,g} - M_{l,d} \quad \cdot \cdot \cdot (8)$$

or

$$M_{l,r} = W_{l,c} * \frac{g}{g_c} + \frac{P_v V_m \bar{M}_l}{z_l R T_l} - \frac{P_v V_{l,c} \bar{M}_g}{z_l R T_l} \quad \cdot \cdot \cdot (9)$$

Thus the total mass of gas flowing through the tube is

$$M_{g,r} = M_{g,m} - M_{g,d} \quad \cdot \cdot \cdot (10)$$

$$M_{g,r} = \frac{(P - P_v) V_m \bar{M}_g}{z_g R T_g} - \frac{(P - P_v) V_{l,c} \bar{M}_g}{z_g R T_g} \quad \cdot \cdot \cdot (11)$$

The temperatures of the gas and liquid were taken from the wet test meter. The pressure at which the gas and liquid vapor were measured was atmospheric. The deviation factor (z_g) for the gas was taken from Reference 27. The molecular weights were taken from Reference 29. The vapor pressures of the liquid were assumed to be the vapor pressures of water. The volume of gas that the liquid displaced in the container was calculated with the equation,

$$V_{l,c} = \frac{M_{l,c}}{\rho_l} \quad \cdot \cdot \cdot (12)$$

The density of the liquid was measured with a hydrometer and was found to be .99 gm/cc at atmospheric pressure and 28° C.

Volume of Gas and Liquid Flowing within the Tube

The volume of gas in the tube at the average flowing conditions of pressure and temperature consists of the volume of the gas and vapor in the gas stream (bubbles). Application of the gas law yields,

$$V_{g,T} = \frac{M_{g,T} R T_T}{\bar{M}_g P_g Z_g} \quad . . . \quad (13)$$

and Dalton's law:

$$P_g = P - P_l = P - P_v \quad . . . \quad (14)$$

Thus

$$V_{g,T} = \frac{M_{g,T} R T_T}{\bar{M}_g (P_T - P_v) Z_g} \quad . . . \quad (15)$$

By analogy the volume of liquid in the vapor phase is

$$V_{l,g} = V_{g,T} * \frac{P_v}{(P_T - P_v)} \quad . : . \quad (16)$$

The volume of the liquid flowing in the tube is,

$$V_{l,T} = \frac{M_{l,T}}{\rho_{l,T}} - V_{l,g} \quad . . . \quad (17)$$

and the volume of the vapor and gas phase flowing in the tube is

$$V_{g,\tau} = V_{g,\tau} + V_{l,g} \quad . . . \quad (18)$$

The average pressures were calculated by the equation

$$P_A = (P_{g1} - P_{g2})/2 + P_{ATM} \quad . . . \quad (19)$$

Calculation of Foam Quality

The quality of the foam was calculated with the equation,

$$\Gamma = V_{g,\tau} / (V_{g,\tau} + V_{l,\tau}) \quad . . . \quad (20)$$

Calculation of Shear Rate

The shear rates within the tube were calculated by the equation,

$$\phi = \frac{8(V_{g,\tau} + V_{l,\tau})}{D t (\pi D^2/4)} \quad . . . \quad (21)$$

Calculation of Shear Stress

The shear stress within the tube was calculated by the equation,

$$\tau = \frac{D \Delta P_d}{4 L} \quad . . . \quad (22)$$

Calculation of the Viscosity of the Foam

The viscosity of the foam was calculated with

$$\text{Poiseuille's Law: } \mu_f = 9 \cdot \frac{\tau}{\phi} \quad . . . \quad (23)$$

Calculation of the Reynold's Number for Foam

The density of the foam (ρ_f) was calculated with the equation,

$$\rho_f = \frac{M_{l,\tau} + M_{v,\tau}}{V_{l,\tau} + V_{v,\tau}} \quad . . . \quad (24)$$

The velocity of flow of the foam was calculated with the equation,

$$v_f = \frac{(V_{l,\tau} + V_{v,\tau}) / t}{\pi D^2 / 4} \quad . . . \quad (25)$$

The Reynold's Number of the foam was calculated with the equation,

$$R_{n,f} = \frac{\rho_f D v_f}{\mu_f} \quad . . . \quad (26)$$

Calculation of the Reynold's Number of the Gas and Liquid

The Reynold's number for the gas flow was calculated with the equations,

$$\rho_{g,\tau} = \frac{V_{v,\tau}}{M_{v,\tau}} \quad . . . \quad (27)$$

$$v_{g,\tau} = \frac{(V_{v,\tau}) / t}{\pi D^2 / 4} \quad . . . \quad (28)$$

The values of $\mu_{g,\tau}$ were taken from Reference 28.

$$R_{n,g} = \frac{\rho_{g,\tau} D V_{g,\tau}}{\mu_{g,\tau}} \quad . . . \quad (29)$$

The Reynold's number for the liquid flow was calculated with the equation,

$$R_{n,l} = \frac{\rho_l D V_{l,\tau}}{\mu_{l,\tau}} \quad . . . \quad (30)$$

DISCUSSION OF RESULTS

Flow Tube Results

The composite of the data shows that the viscosity of foam is dependent on foam quality and shear rate. Please refer to Figure 25. The abscissa of this figure is foam quality and the ordinate is foam viscosity while the interior lines represent shear rate of the foam. This figure which is a composite of the data shows that the viscosity of the foam is a linear function of the foam quality and is independent of the shear rates of the foam in the range of foam quality beginning at the ordinate and ending at 0.54 foam quality fraction. A least squares fit showed that the equation for these data was,

$$\mu_f = \mu(1.0 + 3.6 \Gamma)$$

This compares favorably with Einstein's and Hatschek's theoretically derived equations of

$$\mu_f = \mu(1.0 + 2.5 \Gamma) \quad \text{and} \quad \mu_f = \mu(1.0 + 4.5 \Gamma)$$

The range of this linear section does not agree closely with Hatschek's prediction of the range of $0 \leq \Gamma \leq .74$. However, these data suggest the following explanation. Imagine a set of evenly-sized bubbles in a liquid placed

in a cubic packing arrangement in a tube. As the bubble size increases, laminar flow will be permitted without bubble distortion until the quality becomes 52.3%. At this quality the bubbles will be in contact while in a cubic packing arrangement. As the bubbles increase in size (quality increases), laminar flow is only possible with increasing bubble deformation. Thus the foregoing discussion suggests that the foam viscosity will be functionally related to foam quality differently when these additional forces are affecting the flow. The value of foam quality fraction of 0.54 agrees favorably with the value of .523 that the argument suggested.

The lower curves in Figure 25 in the quality range of 0.54 fraction to .96 fraction is comparable with Hatschek's equation in Figure 2. This curve represents those qualities and shear rates where the viscosity of the foam are becoming independent of shear rate. Another statement of this phenomenon is that on this portion of the curve and at any one foam quality the foam is approaching Newtonian behavior. This result is consistent with Sherman²³ and Raza¹¹.

Because of the interior curves, it is concluded that foam viscosity is dependent on shear rate and foam quality. It may be deduced that foam at any one quality must be considered as a separate fluid from a foam at another

quality in that the viscosity, shear rate relationships and the points where the viscosity becomes independent of shear rates are in extreme cases decidedly different.

The radial lines of various shear rates extend only from a foam quality of 0.54 fraction to the lower, right curve. They do not merge with this curve. The lower right curve separates the foam flow region from the mist and slug flow region. The interpretation of the data is that an increase in foam quality greater than that point formed by the intersection of a constant shear rate line and the lower, right curve degrades the foam to slug or mist flow. A second interpretation is that any attempt to reduce the shear rate and maintain a constant foam quality while stationed at one of these intersections permits rapid coalescence of the bubble or rapid coalescence of the water and thus changing the system to slug or mist flow. Both slug and mist flow viscosities at any air-liquid through-put rate were apparently lower than the corresponding foam viscosity as deduced during the tests with the equipment; however, the equipment did not produce a steady state slug flow or mist flow and as a result precise quantitative results were not possible.

Figures 26, 27, and 28 are primarily displayed to substantiate Figure 25, however, these figures show that foam viscosity within the range of tube internal diameters

does not depend on tube diameter. This is in variance with the study made by Raza¹¹. The largest of the three tubes used in this experiment was 3.12 times larger in diameter than the smallest while Raza¹¹ shows a corresponding value of 2.01. The range of the shear rates were from 640 sec^{-1} to $640,000 \text{ sec}^{-1}$. The total number of foam flow data points is 288 and they appear on the composite of the three figures. Figures 29, 30, and 31 were used in the construction of Figure 25.

Figures 32 through 44 show the data plotted in a convenient form for analysis with Rabinowitsch's theoretically derived equation. These figures show that foam at any one foam quality approaches Newtonian in behavior at the higher shear rates (slope ≈ 1.0) while at the lower shear rates the foam appears pseudoplastic (a flattening of the curve, slope < 1.0). Again, no effect of the tube diameters can be noted within experimental error.

Bubble Size Results

The results of the bubble size distribution study are shown in Figures 14 through 17. A magnification of 210 times was not sufficient to permit distinguishing of a bubble size distribution under flowing conditions. However, the figures show that coalescence was a relatively slow process within the cell where the environment was

similar to that in which the foam was created.

Flow Velocity Within the Tube

The data from photoelectric system showed that the photoelectric cell measured flow rates were comparable to those calculated with the mass balance technique. This comparison is shown in Figure 45. This comparison shows that in general the mass balance derived flow rate which is an average flow rate is less than the photoelectric cell derived flow rate which could be and probably is the maximum flow velocity within the tubes. Of course, in true laminar flow the average velocity should be one-half the value of the maximum velocity. Figure 20 indicates the preciseness with which the data could be taken with this particular equipment and arrangement. The sharp breaks in the curves represent the arrival of the dye marker at the photoelectric cells. The paper recorder speed was 1 inch/second in this figure.

Statistical Analysis of the Foam Data

The foam data are statistically analyzed in the range of quality $0 < \Gamma \leq .54$ as predicated by the theories of Einstein and Hatschek. The procedure was to use a least squares method to determine the coefficient of Γ in the Einstein and Hatschek linear equations. The total variance

and standard deviation of the data were calculated and then the statistic F test was used to determine the fit of the data. A linear regression coefficient was calculated and presented. Confidence intervals as to the fit of the least squares determined equations were established, using the student's "t" distribution. Because the confidence intervals are a function of Γ , the intervals were placed in tabular form.

Statistical Results

(1) The Einstein and linear Hatschek equations are

$$\mu_f = \mu (1.0 + 2.5 \Gamma) \quad \text{AND} \quad \mu_f = \mu (1.0 + 4.5 \Gamma)$$

The empirically derived foam data equation based on 45 data points are

$$\mu_f = \mu (1.0 + 3.6 \Gamma)$$

Total Variance = 0.435

Standard Deviation = 0.660

Linear Regression Coefficient = 0.676

(A value of 1.0 is perfect correlation while a value of zero is no correlation.)

The F statistic = 1.76

(This number is better than the 1% confidence level which is 2.03.)

Confidence Intervals based on 95% using the student "t" distribution

Γ	VALUE OF μ_f BY BEST FIT	95% CONFIDENCE INTERVAL
.1	1.11	.872 to 1.34
.2	1.41	1.23 to 1.58
.3	1.71	1.52 to 1.89
.4	2.00	1.74 to 2.25
.5	2.30	1.95 to 2.65

(2) The non-linear Hartschek equation is

$$\mu_f = \mu \left(\frac{1}{1 + \Gamma^{.33}} \right)$$

and was linearized by the following substitution

$$x = \log_e \Gamma$$

$$y = \log_e (1 - \mu / \mu_f)$$

The resulting empirically derived foam data equation based on 87 points is

$$\mu_f = \mu \left(\frac{1}{1 + \Gamma^{.49}} \right)$$

Total Variance of the linearity x and y = .0250

Standard Deviation of the linearity x and y = .0161

Linear Regression Coefficient of x and y = .966

Confidence Intervals using student "t" distribution

Γ	Value of μ_f By Best Fit	95% Confidence Interval
.6	3.69	3.60 to 3.80
.7	5.11	4.85 to 5.37
.8	7.96	7.11 to 8.81
.9	16.4	15.6 to 17.2

Future Work

Two studies are needed in the immediate future to assure the successful use of foam in drilling and completion operations. The first is the development of basic information that describes the rock chip lifting capacity of high pressure foam. This study would require complicated and expensive equipment and a significant quantity of man hours. The second is a study to develop a technique of using foam. It most likely would require a large digital computer in order to analyze and correlate the many equations and variables that would be disclosed as important.

CONCLUSIONS

The work completed in this study reasonably supports the following conclusions:

1. The viscosity of foam increases as foam quality increases at any single value of shear rate.
2. The viscosity of foam is independent of shear rate in the foam quality range between zero and 54%; i. e., the foam is Newtonian in behavior at any single quality in this range.
3. The viscosity of foam is dependent on both foam quality and shear rate for those foam qualities greater than 54%.
4. The viscosity of foam is dependent only on foam quality at all qualities as shear rates approach infinitely high rates.
5. An air-water-surfactant fluid system can only exist as foam between the foam qualities of

zero and 97%. For foam qualities greater than 97%, the system converts to either slug flow or mist flow.

6. Foam behaviour is Newtonian at higher shear rates and plastic at lower shear rates.
7. Coalescence of foam is a slow process if the foam is quiescent in the environment in which it was created. However, if it is exposed to air and atmospheric conditions, then coalescence is rapid.
8. Foam bubble sizes are submicroscopic (210x) under flowing conditions at high pressures and are much smaller than those sizes reported by others where measurements were made at atmospheric and non-flowing conditions.
9. The viscous rheological properties of foam may be measured within steady state conditions at pressures to 1,000 pounds force per square inch absolute with circular flow tubes.

10. The photoelectric cell technique of measuring foam flow velocity within a circular tube gave quantitative results.
11. The consistency of the texture of foam was quantitatively measured with the photoelectric cell, transmitted light equipment system.
12. Surfactants are capable of producing even texture foams up to pressures of 1,000 pounds force per square inch absolute.
13. The combination of the pressure measuring device and/or differential pressure measuring devices that consisted of differential pressure transducer and oscilloscope system, the high pressure mercury manometer, and the dial pressure gauges were capable of measuring all pressures reported.
14. The foam generator, as described, produced foams of similar texture at various air and water through-put rates and pressures to 1,000 pounds force per square inch absolute.

15. Of the various systems used, the sphinx valve, pressure regulator and air bypass bleed was the only successful system that could maintain a constant back pressure that was required to produce steady flow of the foam.
16. The data collected conforms with Einstein's flow theory in the foam quality range of zero percent to fifty-four percent. (Restrictions are discussed within the chapter entitled results.)
17. The data collected conforms with Hatschek's flow theory in the range of fifty-four percent to 97% foam quality. (Restrictions and supplements are cited within the chapter entitled results).
18. The data show that "slippage" of foam at the wall of the tube does not occur. ("Slippage" refers to the theory proposed by M. Mooney.)

NOMENCLATURE

ENGLISH SYMBOLS

- a - average or apparent
- A - area, flow tube area, ft^2
- C - constants, container
- d - differential calculus operator
- D - flow tube I.D., inches
- f - foam
- g_c - gas, gravitational constant $\frac{32.2 \text{ Lbm ft}}{\text{Lb}_f \text{ Sec}^2}$
- L - flow tube length, ft.
- m - mass, lbm; meter
- $\dot{m}$ - mass, rate, lbm/sec
- M - mass, Lbm; molecular weight
- ΔP_f - pressure, differential pressure across tube, psi
- q_t - volumetric flow rate, cu. ft./sec.
- r - radius, radial coordinate
- R - gas constant, 10.71 (psi $\text{ft}^3/\text{miles } ^\circ\text{R}$)
- t - time or total
- T - absolute temperature - $^\circ\text{R}$, total
- V - volume, cc or ft^3
- v - velocity ft/sec., vapor
- w - tube wall
- x, y, z - cartesian coordinate
- Z - gas deviation factor

NOMENCLATURE (continued)GREEK SYMBOLS

μ	- base fluid viscosity
ρ	- density, lbm/ft^3
Γ	- foam quality; fraction
Δ	- incremental difference
ϕ	- shear rate, $1/\text{sec}$
τ	- shear stress, lb_f/ft^2
μ	- viscosity, cps or $\frac{\text{lbm-ft}}{\text{sec}}$

ATM - atmospheric

Rn - Reynold's number

Table 1

TABLE OF FLOW TUBE DIMENSIONS

<u>TUBE NO.</u>	<u>I. D. INCHES*</u>	<u>I. D. INCHES #</u>	<u>PERCENT REDUCTION</u>	<u>TOTAL, LENGTH, FT.</u>	<u>LENGTH BETWEEN CELLS FEET</u>
1	.033	.0296	10.4	7.41	6.90
2	.054	.0483	10.5	7.92	7.44
3	.106	.0924	12.8	7.71	7.11

* As reported by Manufacturer

As calibrated

Table 2

FLOW TUBE CALIBRATION

<u>FLUID TYPE</u>	<u>VISCOSITIES AS MEASURED BY FLOW TUBE, cps.</u>	<u>VISCOSITIES AS MEASURED BY ROTARY VISCOMETER, cps.</u>
Glycerine	326.	349.
Nujol	130.	161.
Glycerine & Water No. 1	81.4	97.2
Ethylene Glycol	47.9	51.1
Glycerine & Water No. 2	11.9	13.7
Water	0.71	.82

Table 3

Calibration of Flow Tube Internal Diameters

Tube Diameter: 0.0296 inches. Fluid Type: Foam Solution

Test No.	Shear Rate Sec ⁻¹	Shear Stress ₂ lbs/ft ²	Flow Tube Viscosity CPS	Ostwald Viscosity CPS
1	5400	0.86	0.76	0.82
2	3800	.059	0.75	0.82
3	3100	0.048	0.75	0.82
4	1400	0.023	0.80	0.82
5	610	0.0099	0.78	0.82
6	960	0.012	0.62	0.82
7	2300	0.035	0.72	0.82
8	9600	0.17	0.86	0.82
9	7200	0.13	0.89	0.82
10	6000	0.10	0.82	0.82
11	1200	0.025	1.00	0.82
12	3000	0.048	0.77	0.82
13	4700	0.089	0.90	0.82
14	6600	0.12	0.85	0.82
15	6500	0.13	0.97	0.82
16	8900	0.16	0.85	0.82

Tube Diameter: 0.0483 inches Fluid Type: Foam Solution

1	550	0.0077	0.68	0.82
2	5200	0.081	0.74	0.82
3	1700	0.027	0.75	0.82
4	4200	0.067	0.76	0.82
5	4000	0.063	0.75	0.82
6	5500	0.089	0.77	0.82
7	7500	0.13	0.80	0.82
8	9600	0.17	0.84	0.82
9	1100	0.21	0.92	0.82
10	5900	0.13	1.10	0.82
11	3500	0.054	0.75	0.82
12	1500	0.025	0.78	0.82
13	760	0.013	0.79	0.82
14	8200	0.14	0.80	0.82
15	5000	0.079	0.76	0.82
16	2700	0.043	0.74	0.82

Table 3 (continued)

Calibration of Flow Tube Internal Diameters

Tube Diameter: 0.0924 in. Fluid Type: Foam Solution

Test No.	Shear Rate <u>Sec¹</u>	Shear Stress <u>lbs/ft²</u>	Flow Tube Viscosity <u>CPS</u>	Ostwald Viscosity <u>CPS</u>
1	1200	0.018	0.72	0.82
2	2800	0.041	0.70	0.82
3	3200	0.047	0.70	0.82
4	980	0.013	0.62	0.82
5	460	0.084	0.88	0.82
6	830	0.012	0.67	0.82
7	1600	0.020	0.61	0.82
8	2600	0.030	0.55	0.82
9	3100	0.038	0.58	0.82
10	3800	0.069	0.86	0.82
11	4400	0.089	0.98	0.82

Table 4

Experimental and Calculation Results For Equipment Calibration

Fluid Type: Mixture No. 1 of Glycerine and water.

<u>TEST No.</u>	<u>FLOW PERIOD</u> sec.	<u>VOL.</u> cc.	<u>PRESSURE DIFF'TIAL</u> psi.	<u>SHEAR RATE</u> sec ⁻¹	<u>SHEAR STRESS</u> lbs/ft ²
1	30.9	20.	173.	2550.	3.55
2	49.9	20.	135.	1580.	15.
3	57.0	15.	93.	1038.	1.91
4	51.2	10.	70.	769.	1.44
5	58.3	5.	29.5	339.	0.605
6	140.7	5.	14.0	140.	0.287
7	97.1	2.	5.5	81.3	0.113
8	220.7	2.	2.7	35.8	0.0554
9	161.	3.	6.9	73.4	0.141

Fluid Type: Nujol (Mineral Oil)

1	56.7	20.	190.	1393.	3.90
2	73.2	20.	142.	1078.	2.91
3	58.3	10.	91.	687.	1.87
4	99.3	10.	56.3	398.	1.15
5	163.6	8.	25.0	193.	.513
6	106.5	10.	51.5	370.	1.06
7	241.9	2.	3.0	32.6	0.0615
8	67.2	10.	81.	588.	1.66
9	67.0	6.	57.	353.	1.17

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Table 4 (continued)

Fluid Type: Glycerine

<u>TEST No.</u>	<u>FLOW PERIOD</u>	<u>VOL.</u>	<u>PRESSURE DIFF'TIAL</u>	<u>SHEAR RATE</u>	<u>SHEAR STRESS</u>
	sec.	cc.	psi.	sec ⁻¹	lbs/ft ²
1	29.0	20.	660.	2720.	13.5
2	45.7	20.	533.	1730.	10.9
3	56.0	20.	453.	1410.	9.29
4	81.0	20.	321.	974.	6.58
5	151.0	10.	79.	261.	1.62

Fluid Type: Water

1	125.	40.	1.13	1263.	.0232
2	99.	100.	2.71	3986.	.0555
3	94.	60.	1.73	2518.	.0355
4	200.	30.	0.377	592.	.00773
5	250.	15.	0.151	236.	.00310
6	103.	50.	1.21	1914.	.0248

Fluid Type: Mixture No. 2 Glycerine and Water

1	52.5	40.	118.	3007.	2.42
2	34.5	20.	96.	2289.	1.97
3	58.0	30.	62.	2040.	1.27
4	74.7	10.	25.	524.	0.513
5	141.	10.	12.5	279.	0.256
6	116.7	4.	7.5	135.	0.154
7	250.5	2.	2.	31.	0.041
8	43.5	5.	21.	453.	.431
9	68.0	4.	11.	232.	.226
10	93.7	3.	6.	126.	.123

Fluid Type Ethylene Glycol

1	50.5	40.	51.	3126.	1.046
2	55.5	30.	36.	2135.	0.738
3	67.3	30.	28.	1760.	0.574
4	137.5	40.	18.	1149.	0.369
5	85.8	10.	7.	462.	0.144
6	459.0	2.	1.	17.2	0.0205

Table 5

FOAM RHEOLOGY DATA

tube i.d = 0.0296 in.

Test No.	P_a	ΔP_t	r	ϕ	τ	μ	g_t (*10 ⁻⁷)
1	717.	10.7	0.433	1750.	0.128	3.5	25.7
2	731.	8.20	0.424	1660.	0.0928	2.8	24.3
3	715.	6.09	0.364	1540.	0.0729	2.3	22.6
4	734.	4.15	0.319	1410.	0.0497	1.7	20.8
5	773.	2.49	0.163	1110.	0.0296	1.3	16.3
6	761.	12.5	0.428	1620.	0.149	4.4	23.8
7	710.	16.6	0.472	1790.	0.199	5.3	26.2
8	645.	39.0	0.554	7280.	0.467	3.1	107.
9	596.	95.0	0.702	11000.	1.14	4.9	162.
10	567.	145.	0.744	12800.	1.74	6.5	189.
11	515.	213.	0.789	15700.	2.55	7.8	230.
12	445.	315.	0.839	20500.	3.77	8.8	301.
13	409.	374.	0.858	23200.	4.48	9.2	341.
14	393.	422.	0.872	25700.	5.05	9.4	377.
15	690.	9.76	0.196	4070.	0.117	1.4	59.7
16	691.	7.68	0.0722	3570.	0.092	1.2	52.5
17	718.	12.9	0.189	4040.	0.154	1.8	59.3
18	720.	17.0	0.335	4960.	0.204	2.0	72.8
19	714.	8.72	0.134	3810.	0.104	1.3	56.0
20	709.	53.0	0.550	10200.	0.635	3.0	149.
21	743.	17.0	0.338	6920.	0.204	1.4	102.
22	672.	101.	0.649	13100.	1.21	4.4	192.
23	632.	136.	0.699	15300.	1.63	5.1	224.
24	736.	169.	0.723	16600.	2.02	5.8	243.
25	742.	145.	0.703	15500.	1.74	5.4	227.
26	655.	402.	0.836	28000.	4.81	8.2	412.
27	714.	309.	0.802	23200.	3.70	7.6	341.
28	754.	245.	0.771	20100.	2.93	7.0	295.
29	743.	40.0	0.514	9440.	0.479	2.4	139.
30	808.	12.9	0.247	6080.	0.154	1.2	89.3
31	795.	8.72	0.0808	4980.	0.104	1.0	73.2
32	770.	17.0	0.303	6840.	0.204	1.4	100.
33	596.	167.	0.805	10900.	2.00	8.8	160.
34	568.	205.	0.831	12400.	2.45	9.5	182.
35	535.	245.	0.842	14300.	2.93	9.8	210.
36	563.	381.	0.873	18800.	4.65	12.	276.
37	523.	422.	0.890	19300.	5.05	13.	283.
38	496.	459.	0.902	21000.	5.05	13.	309.
39	595.	374.	0.880	16900.	4.48	13.	284.
40	574	427.	0.888	17900.	5.51	14.	263.

Table 5 (continued)

FOAM RHEOLOGY DATA

tube i.d. = 0.0296^{in.}

Test No.	P _a	ΔP _t	r	φ	τ	μ	g _t (*10 ⁻⁷)
41	532.	448.	0.900	20100.	5.36	13.	294.
42	506.	487.	0.903	21000.	5.83	13.	308.
43	501.	103.	0.430	27300.	1.23	2.2	401.
44	551.	85.0	0.415	23900.	1.02	2.0	351.
45	551.	85.0	0.413	23700.	1.02	2.1	348.
46	555.	62.0	0.572	24500.	0.742	1.5	360.
47	531.	78.0	0.561	13500.	0.934	3.3	198.
48	534.	70.0	0.545	13000.	0.838	3.1	191.
49	516.	92.0	0.551	17500.	1.10	3.0	256.
50	496.	92.0	0.694	9240.	1.10	5.7	136.
51	478.	84.0	0.774	4500.	1.01	11.	66.0
52	495.	74.0	0.796	4170.	0.886	10.	61.2
53	500.	65.0	0.747	4070.	0.778	9.2	59.8
54	508.	63.0	0.696	4110.	0.754	8.8	60.4
55	514.	53.0	0.807	3230.	0.635	9.4	47.5
56	514.	55.0	0.779	3360	0.659	9.4	49.3
57	518.	52.0	0.726	3530.	0.623	8.4	51.8
58	586.	18.0	0.301	6440	0.216	1.6	94.6
59	547.	53.0	0.572	7380.	0.635	4.1	108.
60	541.	65.0	0.595	5780.	0.778	6.4	84.9
61	514.	72.0	0.681	7720.	0.862	5.3	113.
62	594.	126.	0.647	16200.	1.51	4.5	238.
63	581.	139.	0.695	18100.	1.66	4.4	265.
64	573.	152.	0.668	19600.	1.82	4.5	287.
65	570.	155.	0.712	18500.	1.86	4.8	271.
66	563.	164.	0.713	19400.	1.99	4.9	285.
67	548.	194.	0.732	20600.	2.32	5.4	303.
68	527.	229.	0.752	22600.	2.74	5.8	332.
69	499.	274.	0.774	25000.	3.28	6.3	368.
70	471.	323.	0.788	26900.	3.87	6.9	396.
71	436.	386.	0.809	30000.	4.62	7.4	440.
72	392.	468.	0.831	33800.	5.60	7.9	497.
73	358.	533.	0.847	37700.	6.38	8.1	554.
74	675.	15.0	0.306	11700.	0.180	0.73	172.
75	637.	50.0	0.592	19600.	0.599	1.5	288.
76	611.	94.0	0.626	22000.	1.13	2.4	323.
77	527.	239.	0.721	29300.	2.86	4.7	430.
78	503.	281.	0.736	31200.	3.36	5.2	458.
79	475.	333.	0.774	32700	3.99	5.8	480.
80	448.	382.	0.777	35900.	4.57	6.1	527.

Table 5 (continued)

FOAM RHEOLOGY DATA

tube i.d. = 0.296^{in.}

Test No.	P _a	ΔP _t	r	φ	τ	μ	g _t (*10 ⁻⁷)
81	422.	433.	0.791	38700.	5.18	6.4	568.
82	396.	482.	0.802	41200.	5.77	6.7	605.
83	369.	537.	0.816	44500.	6.43	6.9	654.
84	346.	583.	0.831	47300.	6.98	7.1	795.
85	643.	70.0	0.422	31500.	0.838	1.3	462.
86	617.	109.	0.341	49100.	1.31	1.3	721.
87	587.	151.	0.542	36600.	1.81	2.4	537.
88	564.	196.	0.581	38800.	2.35	2.9	570.
89	533.	242.	0.605	41400.	2.90	3.3	608.
90	506.	290.	0.631	42300.	3.47	3.9	621.
91	477.	337.	0.646	45400.	4.03	4.3	666.
92	449.	388.	0.664	48400.	4.65	4.6	710.
93	423.	438.	0.687	50300.	5.24	5.0	739.
94	399.	481.	0.700	54600.	5.76	5.0	802.
95	366.	544.	0.721	56800.	6.51	5.3	861.
96	662.	59.0	0.214	33100.	0.706	1.0	486.
97	637.	56.0	0.384	32900.	0.670	0.98	483.
98	610.	100.	0.435	36100.	1.20	1.6	530.
99	582.	146.	0.486	39500.	1.75	2.1	580.
100	555.	187.	0.508	41800.	2.24	2.6	614.
101	525.	230.	0.559	43500.	2.75	3.0	639.
102	499.	279.	0.577	46800.	3.34	3.4	687.
103	467.	341.	0.598	52000.	4.08	3.8	746.
104	433.	402.	0.581	50500.	4.81	4.6	741.
105	393.	477.	0.713	55700.	5.71	4.9	818.
106	355.	548.	0.679	64000.	6.56	4.9	940.

tube i.a. = 0.0483

107	775.	0.876	0.0181	1420.	0.0160	0.54	91.0
108	694.	4.38	0.319	2140.	0.0801	1.8	137.
109	634.	24.0	0.613	3770.	0.439	5.6	241.
110	576.	48.0	0.715	5100.	0.878	8.2	327.
111	667.	16.0	0.521	3120.	0.293	4.5	200.
112	717.	2.57	0.138	1690.	0.0470	1.3	108.
113	636.	24.0	0.599	3660.	0.439	5.7	234.
114	622.	27.0	0.626	3920.	0.494	6.0	251.
115	585.	41.0	0.687	4650.	0.750	7.7	298.

Table 5 (continued)

FOAM RHEOLOGY DATA

tube i.d. = 0.0483 in.

Test No.	P_a	ΔP_t	r	ϕ	τ	μ	g_t ($\times 10^{-7}$)
116	547.	64.0	0.743	5710.	1.17	1.3	366.
117	643.	11.0	0.561	1780.	0.201	5.4	114.
118	565.	51.0	0.761	2910.	0.933	1.5	187.
119	619.	23.0	0.673	2070.	0.421	9.8	132.
120	668.	19.0	0.450	6270.	0.384	2.7	402.
121	630.	67.0	0.578.	8340.	1.23	7.0	534.
122	723.	10.0	0.184	4170.	0.183	2.1	267.
123	570.	68.0	0.660	10000.	1.24	5.9	642.
124	545.	81.0	0.691	10800.	1.48	6.5	695.
125	485.	114.0	0.739	13300.	2.09	7.5	854.
126	714.	12.0	0.183	5110.	0.220	2.0	327.
127	672.	21.0	0.443	7850.	0.384	2.4	486.
128	635.	33.0	0.523	8870.	0.604	3.3	568.
129	588.	59.0	0.616	11000.	1.08	4.7	703.
130	558.	75.0	0.661	12100.	1.37	5.4	775.
131	737.	50.0	0.817	1560.	0.915	28.	99.9
132	774.	25.0	0.745	1120.	0.457	19.	72.0
133	659.	7.01	0.476	2360.	0.128	2.6	151.
134	616.	28.0	0.648	3410.	0.512	7.2	219.
135	598.	35.0	0.661	3790.	0.640	8.1	243.
136	564.	52.0	0.730	4370.	0.951	10.	280
137	494.	101.	0.724	4350.	1.85	20.	279.
138	413.	152.	0.858	8790.	2.78	15.	563.
139	356.	202.	0.887	10600.	3.70	17.	680.
140	735.	1.02	0.0168	1090.	0.0187	0.82	70.1
141	703.	1.68	0.118	1220.	0.0307	1.2	78.1
142	677.	10.0	0.305	5400.	0.183	1.6	346.
143	729.	8.0	0.162	3990.	0.146	1.8	255.
144	713.	31.0	0.552	7690.	0.567	3.5	493.
145	723.	63.0	0.651	10100.	1.15	5.5	646.
146	674.	108.	0.702	13400.	1.98	7.1	856.
147	714.	108.	0.738	13400.	1.98	7.1	857.
148	698.	6.72	0.155	4350.	0.123	1.4	279.
149	761.	2.70	0.0582	3080.	0.0494	0.77	197.
150	769.	2.19	0.0385	2810.	0.0401	0.68	180.
151	630.	40.0	0.538	8020.	0.732	4.4	514..
152	734.	74.0	0.616	12600.	1.35	5.1	809.
153	701.	78.0	0.654	13300.	1.43	5.1	851.
154	734.	101.	0.694	14600.	1.85	6.1	933.
155	745.	33.0	0.533	9530.	0.604	3.0	611.

Table 5 (continued)

FOAM RHEOLOGY DATA

tube i.d = 0.0483 in.

Test No.	P_a	ΔP_t	Γ	ϕ	τ	μ	g_t (*10 ⁻⁷)
156	742.	12.	0.341	6700.	0.220	1.6	429.
157	626.	20.1	0.429	7600.	0.384	2.4	487.
158	633.	21.0	0.386	7420.	0.384	2.5	476.
159	766.	6.13	0.141	7130.	0.112	0.75	457.
160	689.	8.76	0.276	8650.	0.160	0.89	554.
161	715.	45.0	0.442	10600.	0.823	3.7	681.
162	717.	60.0	0.641	17300.	1.10	3.0	1110.
163	752.	130.	0.692	19500.	2.38	5.8	1250.
164	745.	179.	0.765	21400.	3.27	7.3	1370.
165	762.	3.21	0.0312	4240.	0.0588	0.66	272.
166	729.	10.0	0.0241	9730.	0.183	0.90	624.
167	693.	9.00	0.181	7390.	0.165	1.1	474.
168	669.	37.0	0.515	12200.	0.677	2.6	784.
169	621.	7.3	0.621	11000.	0.134	5.8	70.6
170	634.	36.0	0.770	1880.	0.659	17.0	120.
171	652.	80.0	0.870	2290.	1.46	31.	147.
172	712.	99.0	0.891	2130.	1.81	41.	136.
173	722.	111.	0.909	2240.	2.03	43.	143.
174	351.	227.	0.868	16400.	4.15	12.	1050.
175	523.	88.0	0.750	8690.	1.61	8.9	557.
176	610.	36.0	0.646	6100.	0.659	5.2	391.
177	648.	12.0	0.507	4510.	0.220	2.3	289.
178	648.	12.0	0.466	4340.	0.220	2.4	278.
179	771.	1.61	0.0576	2410.	0.0294	0.58	154.
180	698.	52.0	0.700	7080.	0.951	6.4	454.
181	681.	69.0	0.722	7450.	1.26	8.1	478.
182	644.	11.0	0.771	860.	0.201	11.	55.1
183	632.	19.0	0.755	1100.	0.384	15.	70.2
184	614.	39.0	0.823	1340.	0.714	25.	85.9
185	583.	62.0	0.843	1770.	1.13	31.	113.
186	553.	82.0	0.879	2230.	1.50	32.	143.
187	520.	108.	0.912	2650.	1.98	36.	170.
188	480.	138.	0.923	3270.	2.52	37.	209.
189	504.	235.	0.969	4150.	4.30	50.	266.
190	459.	274.	0.966	5560.	5.01	43.	356.
191	671.	86.0	0.793	6090.	1.57	12.	390.
192	726.	43.0	0.704	4450.	0.787	8.5	285.
193	693.	67.0	0.773	5340.	1.23	11.	342.
194	663.	89.0	0.799	6420.	1.63	12.	411.

Table 5 (continued)

FOAM RHEOLOGY DATA

tube i.d. = 0.0483 in.

Test No.	P_t	ΔP_t	r	ϕ	τ	μ	g_t (*10 ⁻⁷)
195	623.	125.	0.843	7920.	2.29	14.	508.
196	592.	151.	0.846	8550.	2.76	15.	548.
197	559.	179.	0.864	9690.	3.27	16.	621.
198	528.	210.	0.879	10900.	3.84	17.	700.
199	488.	249.	0.896	12400.	4.56	18.	797.
200	456.	285.	0.903	13600.	5.21	18.	870.
201	421.	316.	0.912	15300.	5.78	18.	978.
202	392.	361.	0.921	16700.	6.60	19.	1070.
203	765.	20.0	0.556	5190.	0.366	3.4	332.
204	728.	42.0	0.674	7160.	0.768	5.1	459.
205	697.	70.0	0.724	8500.	1.28	7.2	544.
206	660.	100.	0.762	9800.	1.83	8.9	628.
207	624.	131.	0.806	12300.	2.40	9.3	787.
208	590.	159.	0.812	12500.	2.91	11.	804.
209	554.	193.	0.834	14300.	3.53	12.	918.
210	523.	229.	0.853	16000.	4.19	13.	1030.
211	486.	265.	0.868	17700.	4.85	13.	1140.
212	460.	302.	0.879	19500.	5.53	14.	1250.
213	425.	339.	0.891	21600.	6.20	14.	1380.
214	387.	402.	0.908	24300.	7.35	14.	1560.
215	733.	56.0	0.707	10100.	1.02	4.8	650.
216	709.	73.0	0.723	11700.	1.34	5.4	752.
217	682.	98.0	0.751	13100.	1.79	6.5	840.
218	625.	148.	0.797	15700.	2.71	8.2	1010.
219	594.	179.	0.813	17400.	3.27	9.0	1120.
220	562.	210.	0.829	19000.	3.48	9.7	1220.
221	532.	244.	0.855	22700.	4.46	9.4	1450.
222	461.	325.	0.868	24800.	5.95	11.	1590.
223	402.	408.	0.889	28900.	7.46	12.	1850.
224	763.	28.0	0.5991	9940.	0.512	2.5	637.
225	735.	45.0	0.654	11000.	0.823	3.6	706.
226	696.	79.0	0.698	13000.	1.45	5.3	830.
227	659.	108.	0.739	15100.	1.98	6.3	968.
228	627.	139.	0.767	17100.	2.54	7.1	1190.
229	591.	175.	0.791	18900.	3.20	8.1	1210.
230	555.	211.	0.814	21300.	3.86	8.7	1360.
231	524.	247.	0.828	22900.	4.52	9.4	1470.
232	479.	301.	0.849	26500.	5.51	9.9	1700.
233	430.	363.	0.863	22900.	6.64	11.	1920.
234	402.	408.	0.878	32300.	7.46	11.	2070.

Table 5 (continued)

FOAM RHEOLOGY DATA

tube i.d. = 0.0483

Test No.	P_a	ΔP_t	r	ϕ	τ	μ	g_t (*10 ⁻⁷)
235	779.	27.0	0.483	9810.	0.494	2.4	628.
236	737.	38.0	0.526	10900.	0.695	3.1	698.
237	700.	77.0	0.636	14200.	1.41	4.7	912.
238	670.	120.	0.697	17000.	2.20	6.2	1090.
239	593.	174.	0.760	20800.	3.18	7.3	1330.
240	546.	244.	0.818	25400.	4.46	8.4	1630.
241	504.	289.	0.876	27400.	5.29	9.3	1750.
242	468.	343.	0.840	31900.	6.28	9.4	2040.
243	427.	408.	0.859	35800.	7.46	10.	2290.
244	395.	457.	0.869	39100.	8.36	10.	2510.
245	706.	84.0	0.624	15900.	1.54	4.6	1020.
246	639.	150.	0.711	21400.	2.74	6.1	1370.
247	600.	189.	0.750	24100.	3.46	6.9	1540.
248	570.	218.	0.768	26000.	3.99	7.3	1670.
249	534.	258.	0.738	29300.	4.72	7.7	1880.
250	480.	328.	0.823	33900.	6.00	8.5	2170.
251	430.	407.	0.843	38500.	7.45	9.3	2470.

tube i.d. = 0.0924^{in.}

252	699.	5.99	0.570	1910.	0.215	5.4	857.
253	751.	6.42	0.589	2000.	0.231	5.5	897.
254	713.	9.78	0.642	2270.	0.352	7.4	1020.
255	745.	15.0	0.670	2490.	0.539	10.	1120.
256	709.	19.0	0.708	2750.	0.683	12.	1230.
257	639.	30.0	0.755	3330.	1.08	16.	1490.
258	650.	55.0	0.810	4240.	1.98	22.	1900.
259	690.	60.0	0.821	4320.	2.16	24.	1940.
260	614.	80.0	0.841	5040.	2.88	27.	2260.
261	655.	105.	0.879	5980.	3.78	30.	2680.
262	747.	0.657	0.0397	854.	0.0236	1.3	383.
263	658.	2.48	0.394	1470.	0.0893	2.9	660.
264	752.	4.89	0.558	1880.	0.176	4.5	841.
265	700.	1.90	0.534	631.	0.0683	5.2	283.
266	709.	14.0	0.733	1070.	0.503	22.	481.
267	732.	25.0	0.796	1310.	0.899	33.	568.
268	719.	40.0	0.833	1610.	1.44	43.	723.
269	762.	55.0	0.902	1010.	1.98	93.	455.

Table 5 (continued)

FOAM RHEOLOGY DATA

tube i.d. = 0.0924^{in.}

Test No.	P_t	ΔP_t	Γ	ϕ	τ	μ	g_t (* 10 ⁻⁷)
270	702.	61.0	0.911	1110.	2.19	95.	498.
271	652.	70.0	0.918	1240.	2.52	97.	556.
272	647.	99.0	0.940	1450.	3.56	120.	651.
273	620.	113.	0.949	1610.	4.06	120.	724.
274	571.	132.	0.952	1800.	4.75	130.	808.
275	483.	159.	0.960	2230.	5.72	120.	1000.
276	503.	151.	0.969	2250.	5.43	120.	1010.
277	442.	159.	0.973	2610.	5.72	100.	1170.
278	647.	76.0	0.887	2380.	2.73	55.	1070.
279	606.	91.0	0.899	2640.	3.27	59.	1180.
280	546.	115.	0.915	3110.	4.14	64.	1400.
281	498.	136.	0.923	3500.	4.89	67.	1570.
282	401.	152.	0.943	4670.	5.47	56.	2090.
283	325.	206.	0.959	6540.	7.41	54.	2940.
284	346.	229.	0.959	6500.	8.24	61.	2920.
285	510.	135.	0.944	2170.	4.85	110.	971.
286	625.	96.0	0.925	1590.	3.45	100.	714.
287	683.	102.	0.975	4050.	3.67	43.	1820.
288	645.	45.0	0.989	10600.	1.62	7.3	4730.

Table 6

FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVES

Foam Quality: $0 < \gamma < .25$		
<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
.02	1420	.16
.14	1690	.047
.18	4170	.183
.18	5110	.220
.02	1090	.0187
.12	1220	.0307
.16	3990	.146
.16	4350	.123
.14	7130	.112
.03	4240	.0588
.02	9730	.183
.18	7390	.165
.16	1110	.0298
.20	4070	.117
.07	3570	.093
.19	4040	.154
.14	3810	.104
.25	6080	.154
.21	3310	.706

Table 6 (continued)FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVESFoam Quality: $.30 \leq F \leq .35$

<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
0.32	2100.	0.080
0.31	5400.	0.18
0.34	6700.	0.22
0.32	1400.	0.050
0.34	5000.	0.20
0.34	6900.	0.20
0.33	6800.	0.20
0.34	49000.	1.30

Table 6 (continued)

FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVES

Foam Quality: $.25 \leq \gamma \leq .35$

<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
0.32	2140	.0801
0.31	5400	.183
0.34	6700	.220
0.28	8650	.160
0.32	1410	.0497
0.34	4960	.204
0.34	6920	.204
0.33	6840	.204
0.30	6440	.216
0.31	11700	.180
0.34	49100	1.31

Table 6 (continued)

FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVES

Foam Quality: $.35 \leq \Gamma < .45$

<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
0.44	7580	.384
0.43	7600	.384
0.39	7420	.384
0.44	10600	.823
0.39	1470	.0893
0.44	1750	.128
0.42	1660	.0982
0.36	1540	.0729
0.43	1620	.149
0.43	27300	1.23
0.42	23900	1.02
0.41	23700	1.02
0.42	31500	.838
0.38	32900	.670
0.44	36100	1.20

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Table 6 (continued)

FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVES

Foam Quality: $.45 \leq F < .55$		
<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
0.52	3120	.293
0.45	6270	.348
0.52	8870	.604
0.54	8020	.732
0.53	9530	.604
0.52	12200	.677
0.51	4510	.220
0.47	4340	.220
0.53	631	.0683
0.47	1790	.199
0.51	9440	.479
0.55	13000	.838
0.54	36600	1.81
0.47	39500	1.75
0.51	41800	2.24
0.53	10900	.695

Table 6 (continued)FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVESFoam Quality: $.55 \leq \gamma \leq .60$

<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
0.56	1800.	0.20
0.58	8300.	1.20
0.55	7700.	0.57
0.57	1900.	0.22
0.59	2000.	0.23
0.56	1900.	0.18
0.55	7300.	0.47
0.55	10000.	0.63
0.57	25000.	0.74
0.56	13000.	0.93
0.55	17000.	1.1
0.57	7400.	0.63
0.59	20000.	0.60
0.56	43000.	2.8
0.58	47000.	3.3
0.58	50000.	4.8
0.56	5200.	0.37
0.59	9900.	0.51

Table 6 (continued)

FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVES

Foam Quality: $.60 \leq f < .65$		
<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
0.60	3700.	0.44
0.63	3900.	0.49
0.62	11000.	1.1
0.61	3800.	0.44
0.62	13000.	1.4
0.64	17000.	1.1
0.62	1100.	0.13
0.64	2300.	0.35
0.60	5800.	0.78
0.63	22000.	1.1
0.61	41000.	2.9
0.63	42000.	3.5
0.60	52000.	4.1
0.64	14000.	1.4
0.62	16000.	1.5

Table 6 (continued)FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVESFoam Quality: $.65 \leq \alpha < .70$

<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
0.69	4700.	0.75
0.67	2100.	0.42
0.66	10000.	1.2
0.69	11000.	1.5
0.66	12000.	1.4
0.65	3400.	0.51
0.66	3800.	0.64
0.65	10000.	1.2
0.65	13000.	1.4
0.69	15000.	1.8
0.69	20000.	2.4
0.65	6100.	0.66
0.64	2300.	0.35
0.67	2500.	0.54
0.65	13000.	1.2
0.69	9200.	1.1
0.68	7700.	0.86
0.65	16000.	1.5
0.69	18000.	1.7
0.69	20000.	1.8
0.65	45000.	4.0
0.66	48000.	4.6
0.68	50000.	5.2
0.68	64000.	6.6
0.67	7200.	0.77
0.65	11000.	0.82

Table 6 (continued)FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVESFoam Quality: $.70 \leq F < .75$

<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
0.71	5100.	0.88
0.74	5700.	1.2
0.74	13000.	2.1
0.73	4400.	0.95
0.72	4300.	1.8
0.74	13000.	2.0
0.70	13000.	2.0
0.75	8700.	1.6
0.70	7100.	0.95
0.72	7500.	1.3
0.71	2700.	0.68
0.73	1100.	0.50
0.70	11000.	1.1
0.74	13000.	1.7
0.70	15000.	1.6
0.72	17000.	2.0
0.70	15000.	1.7
0.70	4100.	0.75
0.73	3500.	0.62
0.71	18000.	1.9
0.71	19000.	2.0
0.73	21000.	2.3
0.72	29000.	2.9
0.74	31000.	3.4
0.70	55000.	5.8
0.72	59000.	6.5
0.71	56000.	5.7
0.72	8500.	1.3
0.71	10000.	1.0
0.72	12000.	1.3
0.70	13000.	1.4
0.74	15000.	2.0
0.70	17000.	2.2
0.71	21000.	2.7

Table 6 (continued)FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVESFoam Quality: $.75 \leq \rho < .80$

Quality Fraction	Shear Rate sec^{-1}	Shear Stress Lb_f/ft^2
0.76	2900.	0.93
0.75	1100.	0.46
0.77	21000.	3.3
0.77	1900.	0.66
0.75	8700.	1.6
0.76	3300.	1.1
0.79	16000.	2.6
0.77	20000.	2.9
0.77	4500.	1.0
0.79	4200.	0.89
0.75	4100.	0.78
0.78	3400.	0.66
0.75	23000.	2.7
0.77	25000.	3.3
0.79	27000.	3.9
0.77	33000.	4.0
0.78	36000.	4.6
0.79	39000.	5.2
0.77	860.	0.2
0.76	1100.	0.35
0.79	6100.	1.6
0.77	5300.	1.2
0.75	13000.	1.8
0.77	17000.	2.5
0.79	19000.	3.2
0.76	21000.	3.2
0.75	24000.	3.5
0.77	26000.	4.0
0.78	29000.	4.7

Table 6 (continued)

FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVES

Foam Quality: $.80 \leq F < .86$

<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/ft²</u>
0.82	1600.	0.91
0.81	4200.	2.0
0.82	4300.	2.2
0.84	5000.	2.9
0.80	1300.	0.90
0.83	1600.	1.4
0.84	21000.	3.8
0.84	28000.	4.8
0.80	23000.	3.7
0.80	11000.	2.0
0.83	12000.	2.5
0.84	14000.	2.9
0.81	30000.	4.6
0.83	34000.	5.6
0.80	41000.	5.8
0.82	45000.	6.4
0.83	47000.	7.0
0.82	1300.	0.71
0.84	1800.	1.1
0.80	6400.	1.6
0.83	7900.	2.3
0.81	12000.	2.4
0.81	13000.	2.9
0.83	14000.	3.5
0.80	16000.	2.7
0.81	17000.	3.3
0.83	19000.	3.8
0.81	21000.	3.9
0.83	23000.	4.5
0.82	25000.	4.5
0.84	32000.	6.3
0.82	34000.	6.0
0.84	39000.	7.4
0.85	27000.	5.5
0.85	16000.	4.2
0.85	8600.	2.8
0.85	38000.	6.4

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Table 6 (continued)

FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVES

Foam Quality: $.86 \leq F < .90$

<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
0.86	8800.	2.8
0.89	11000.	3.7
0.87	2300.	1.5
0.89	2100.	1.8
0.87	16000.	4.2
0.88	6000.	3.8
0.86	23000.	4.5
0.87	26000.	5.1
0.87	19000.	4.6
0.89	19000.	5.1
0.88	17000.	4.5
0.89	18000.	5.1
0.88	2200.	1.5
0.86	9700.	3.3
0.88	11000.	3.8
0.87	18000.	4.8
0.88	19000.	5.5
0.89	22000.	6.2
0.86	23000.	4.5
0.87	25000.	5.9
0.89	29000.	7.5
0.86	30000.	6.6
0.88	32000.	7.5
0.88	27000.	5.3
0.86	36000.	7.5
0.87	39000.	8.4

Table 6 (continued)

FOAM FLOW DATA FOR THE CONSTRUCTION OF
ABSOLUTE FOAM FLOW CURVES

Foam Quality: $.90 \leq \Gamma < .96$

<u>Quality Fraction</u>	<u>Shear Rate Sec⁻¹</u>	<u>Shear Stress Lb_f/Ft²</u>
0.90	1000.	2.0
0.91	1100.	2.2
0.92	1200.	2.5
0.94	1500.	3.6
0.95	1600.	4.1
0.95	1800.	4.7
0.90	2600.	3.3
0.92	3100.	4.1
0.92	3500.	4.9
0.94	4700.	5.5
0.94	2200.	4.9
0.92	1600.	3.5
0.90	21000.	5.5
0.90	20000.	5.4
0.90	21000.	5.8
0.91	2700.	2.0
0.92	3300.	2.5
0.90	12000.	4.6
0.90	14000.	5.2
0.91	15000.	5.8
0.92	17000.	6.6
0.91	24000.	7.4

Table 7

FOAM VELOCITIES IN THE FLOW TUBES

Tube diameter: 0.0296 inches

<u>TEST No.</u>	<u>Calculated*</u> <u>Feet/Second</u>	<u>Measured**</u> <u>Feet/Second</u>
1	1.5	1.4
2	1.9	1.7
3	6.2	5.6
4	7.2	6.1
5	8.6	9.6
6	4.8	4.8
7	5.1	4.5
8	4.0	3.7
9	2.1	2.0
10	3.1	2.7
11	7.9	7.5
12	7.1	7.5
13	6.3	7.5
14	4.8	5.2
15	4.0	4.8
16	3.4	3.7
17	2.2	1.7
18	7.1	7.9
19	3.0	4.0

* Calculation method is explained on page 62.

** As measured with the photoelectric cells.

Table 7 (continued)

FOAM VELOCITIES IN THE FLOW TUBES

Tube Diameter: 0.0403

<u>TEST</u>	<u>CALCULATED*</u> <u>Feet/Second</u>	<u>MEASURED**</u> <u>Feet/Second</u>
1	3.8	4.4
2	3.6	3.8
3	2.2	2.0
4	2.3	1.4
5	3.1	2.6
6	4.4	4.9
7	8.3	9.9
8	1.1	1.5
9	1.1	1.3
10	1.1	1.0
11	1.2	1.1
12	0.94	1.7
13	2.1	2.3
14	5.3	4.4
15	3.7	6.6
16	3.8	6.1

* Calculation method is explained on page 62.

** As measured with the photoelectric cells.

Table 8

VISCOSITY OF SURFACTANT SOLUTION*
(1.0% surfactant)

Test No.	<u>SURFACTANT SOLUTION</u>		<u>DISTILLED AERATED WATER</u>	
	<u>Time Seconds</u>	<u>Viscosity CPS</u>	<u>Time Seconds</u>	<u>Viscosity CPS</u>
1	59.2	.888	54.3	.822
2	58.6	.879	55.2	.828
3	58.6	.879	55.0	.825
4	58.2	.873	55.0	.825
5	58.0	.870	54.8	.822
6	56.4	.846	55.3	.829
7	56.8	.852	54.9	.824
8	56.6	.849	55.0	.825
9	56.6	.849	54.8	.822
10	56.5	.848	54.8	.822
11	57.8	.867	55.4	.831
12	57.7	.866	55.2	.828
13	57.8	.867	55.4	.831
14	57.6	.864	55.2	.828
15	57.6	.864	55.4	.831
16	56.2	.852	56.2	.843
17	56.3	.854	56.3	.845
18	56.2	.852	56.2	.843
19	56.0	.855	56.0	.840
20	55.9	.855	55.9	.838

* Temperature: $28^{\circ}\text{C} + 1/2^{\circ}\text{C}$
 Barometric Pressure: 12:02 psia
 Viscosities were measured with an Ostwald type viscometer.

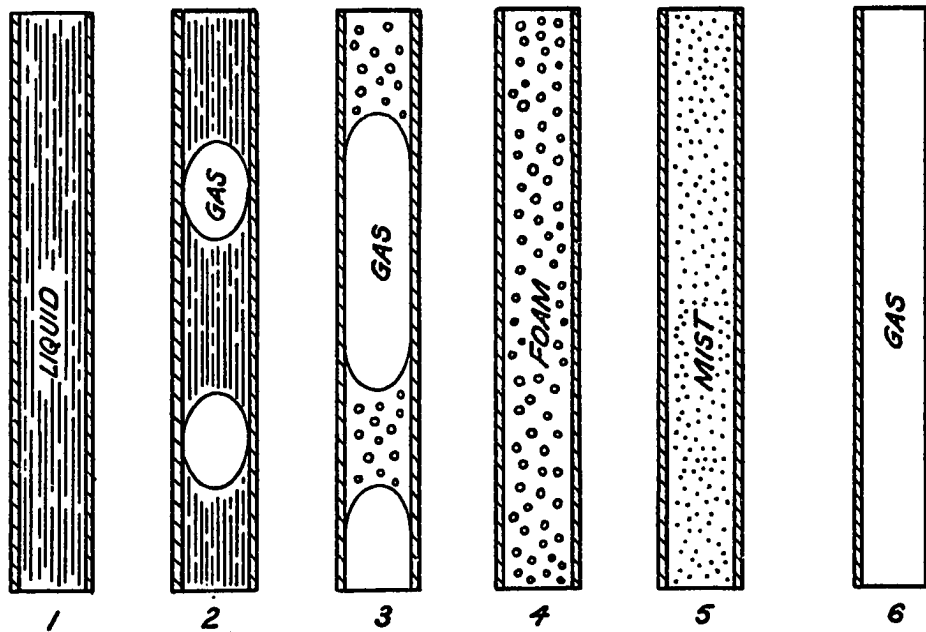


FIGURE 1 TYPES OF FLOW REGIMES IN TUBES
— SINGLE AND MULTI-PHASE FLOW

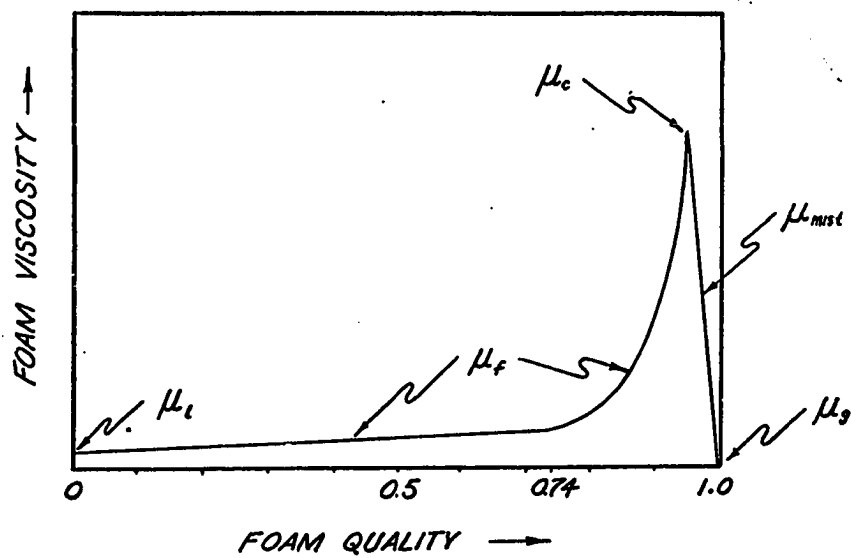


FIGURE 2 HYPOTHETICAL FOAM VISCOSITY-
QUALITY RELATIONSHIP

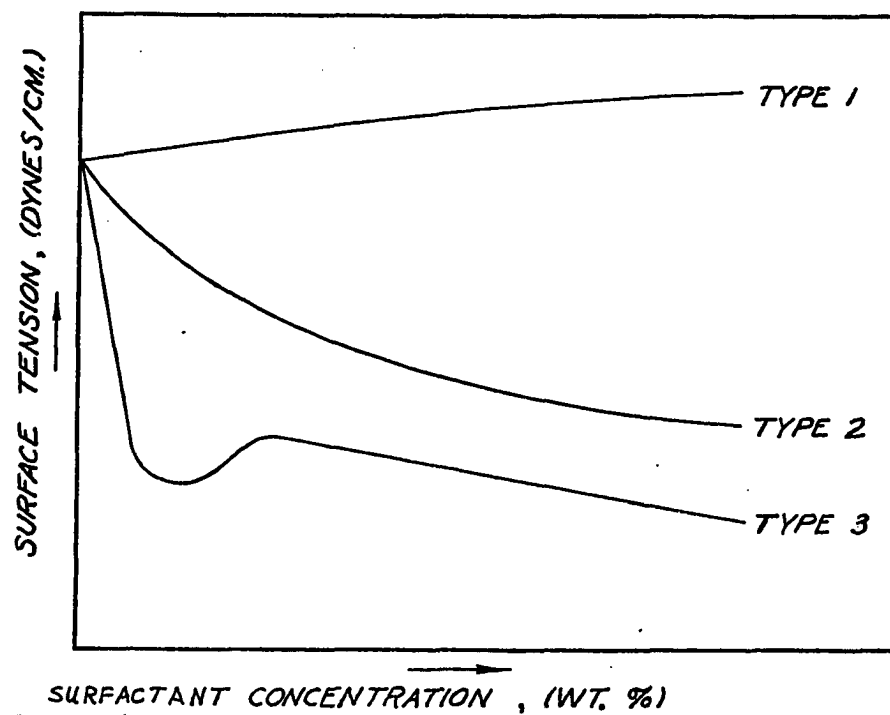


FIGURE 3 TYPES OF SURFACTANTS
IN AQUEOUS SOLUTIONS
(N^e BAIN)

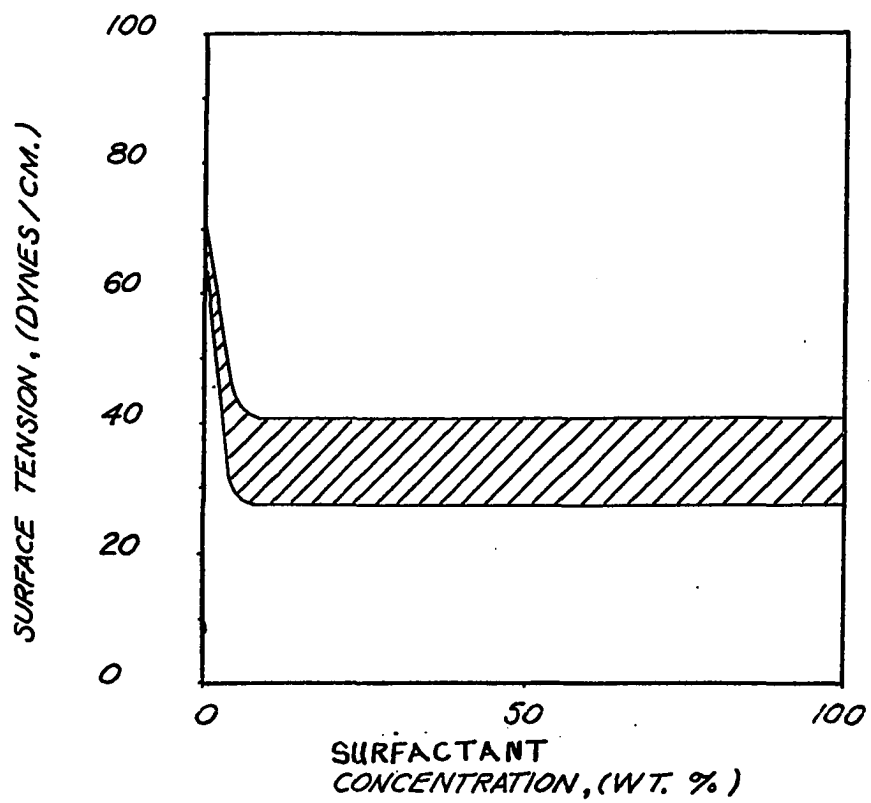


FIGURE 4 SURFACE TENSIONS FOUND IN AQUEOUS SOLUTIONS OF MOST SURFACTANTS

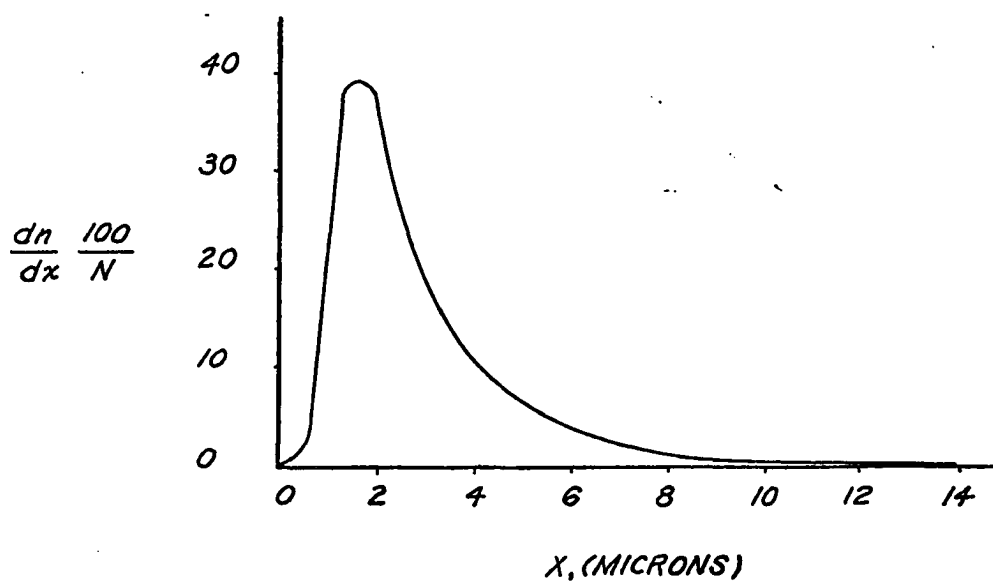


FIGURE 5 HISTOGRAM OF SITOREE'S DATA PRESENTED BY SCHWARZ AND BEZEMER¹⁶ WITH $\alpha = 8.7 \mu$ AND $X = 19 \mu$

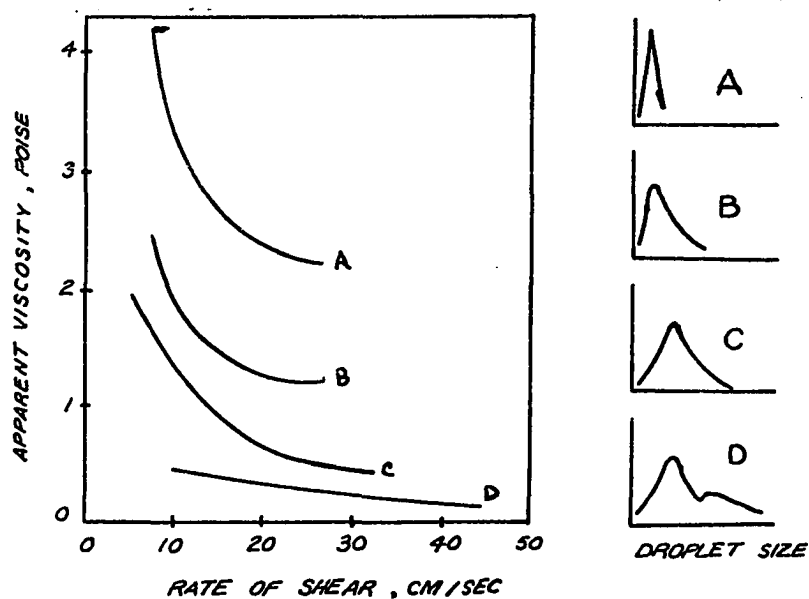


FIGURE 6 APPARENT VISCOSITY AS A FUNCTION OF SHEAR RATE AND DROPLET SIZE DISTRIBUTION

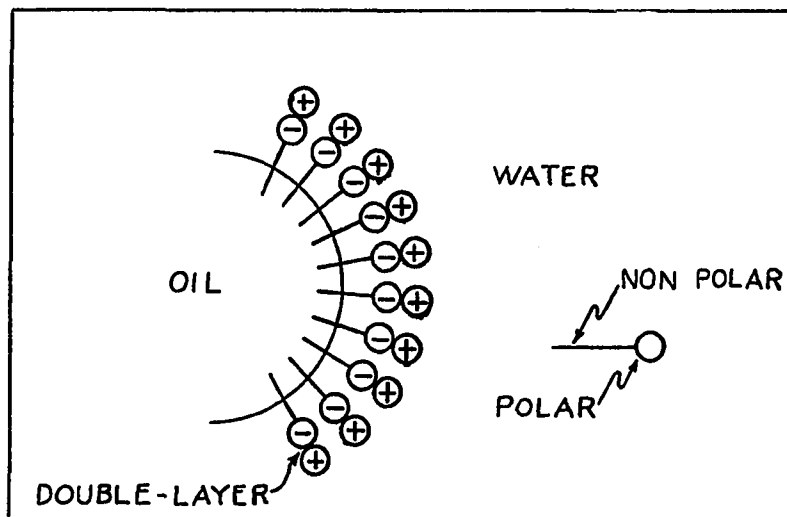


FIGURE 7 HELMHOLTZ DOUBLE LAYER MODEL

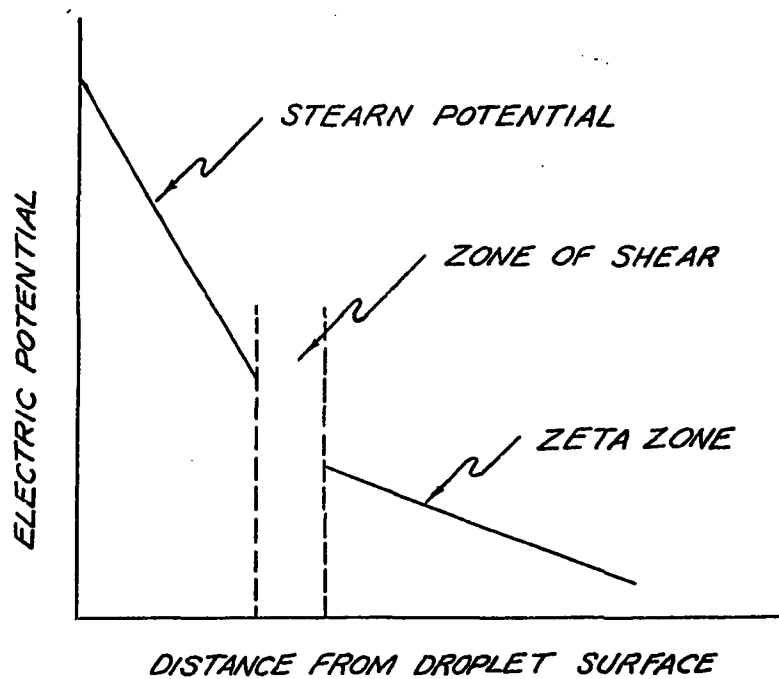


FIGURE 8 ELECTRIC POTENTIAL SURROUNDING DROPLETS IN AN EMULSION

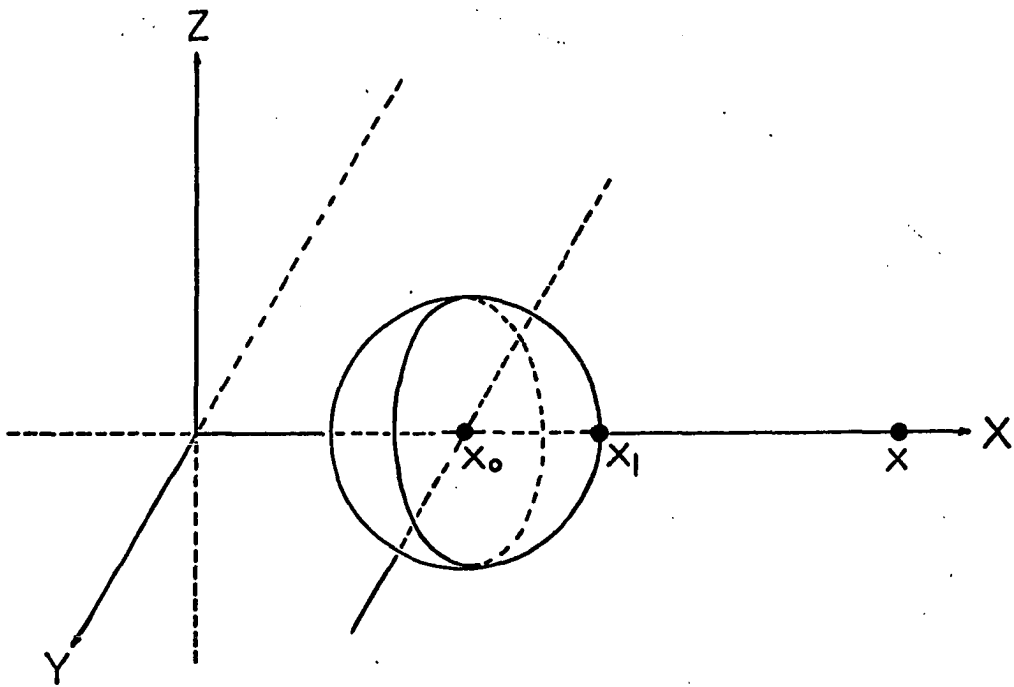
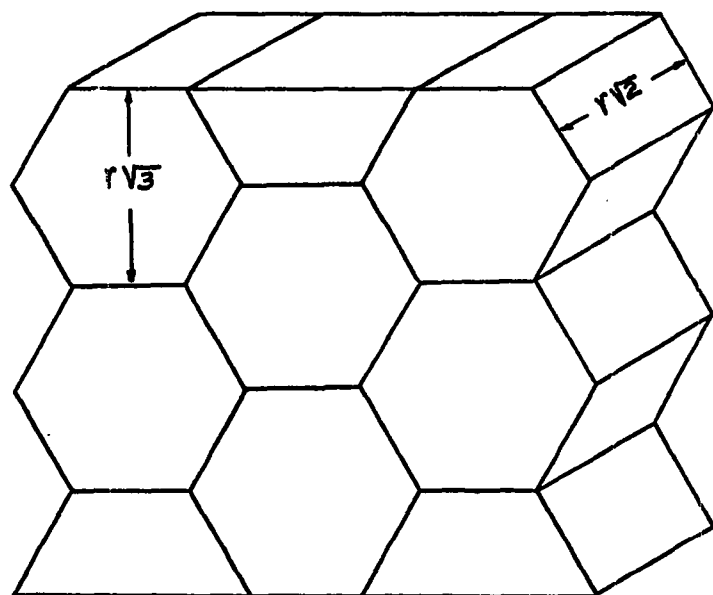
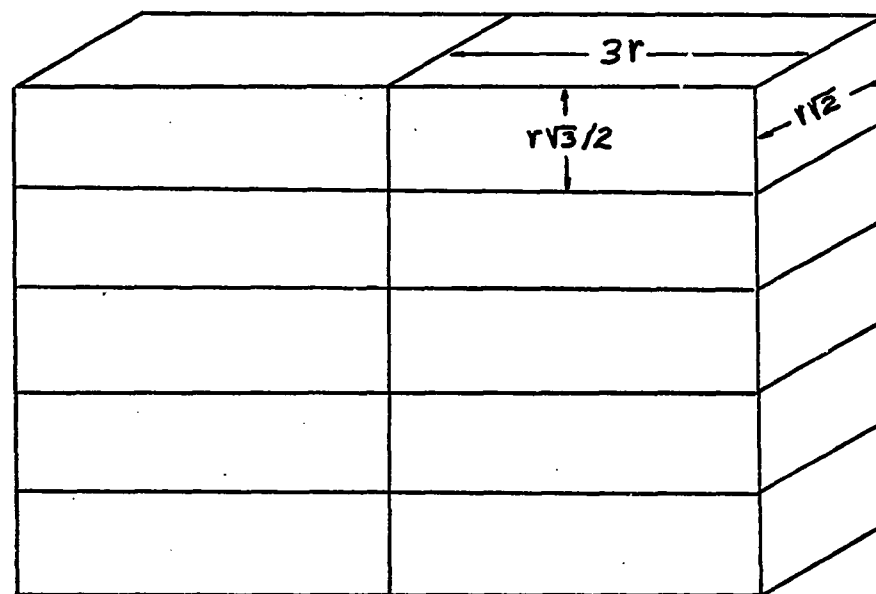


FIGURE 9 EINSTEIN'S FLOW MODEL



(A)



(B)

FIGURE 10 HATSCHEK'S (A) HEXAGONAL PRISMS AND (B) RIGHT ANGLE PRISMS.

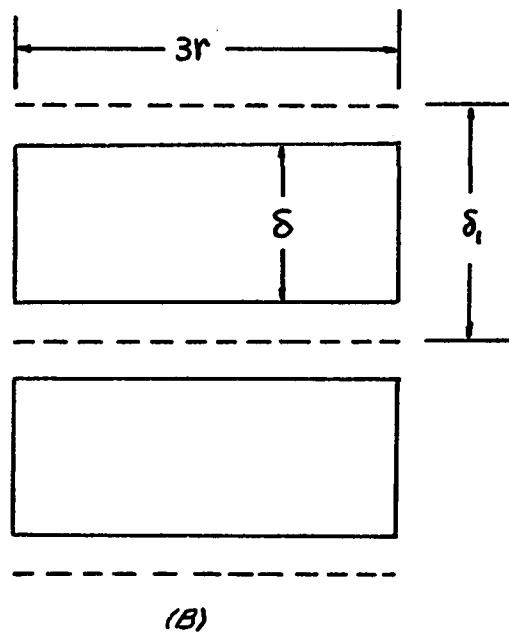
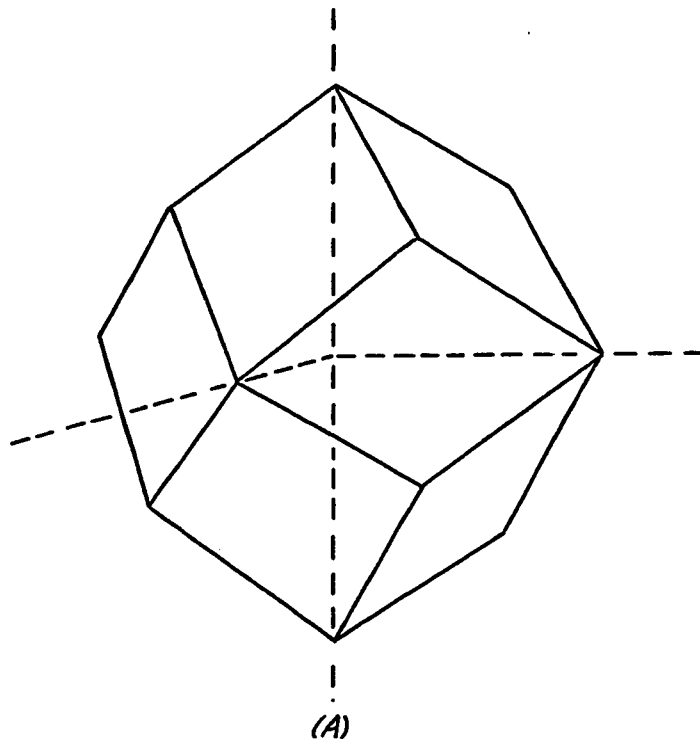


FIGURE 11 HATSCHEK'S (A) DODECAHEDRONS AND (B) RECTANGLES FOR DETERMINING FOAM QUALITY

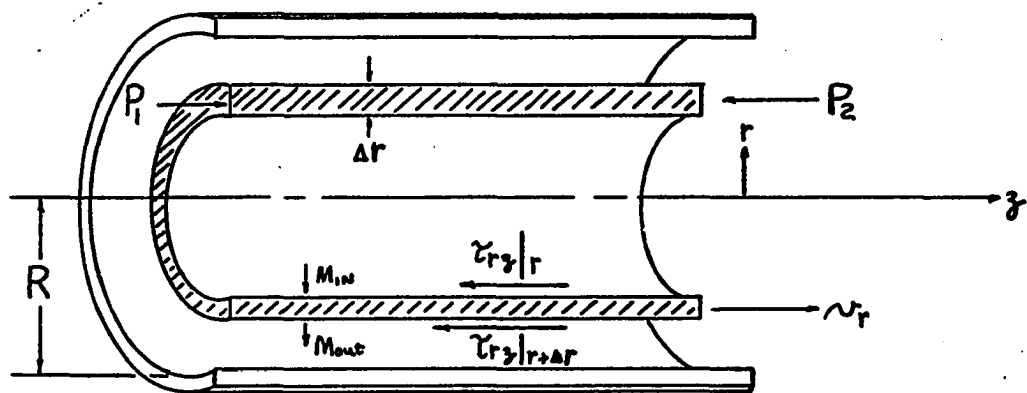


FIGURE 12 RABINOWITSCH FLOW MODEL

FOAM SYSTEM

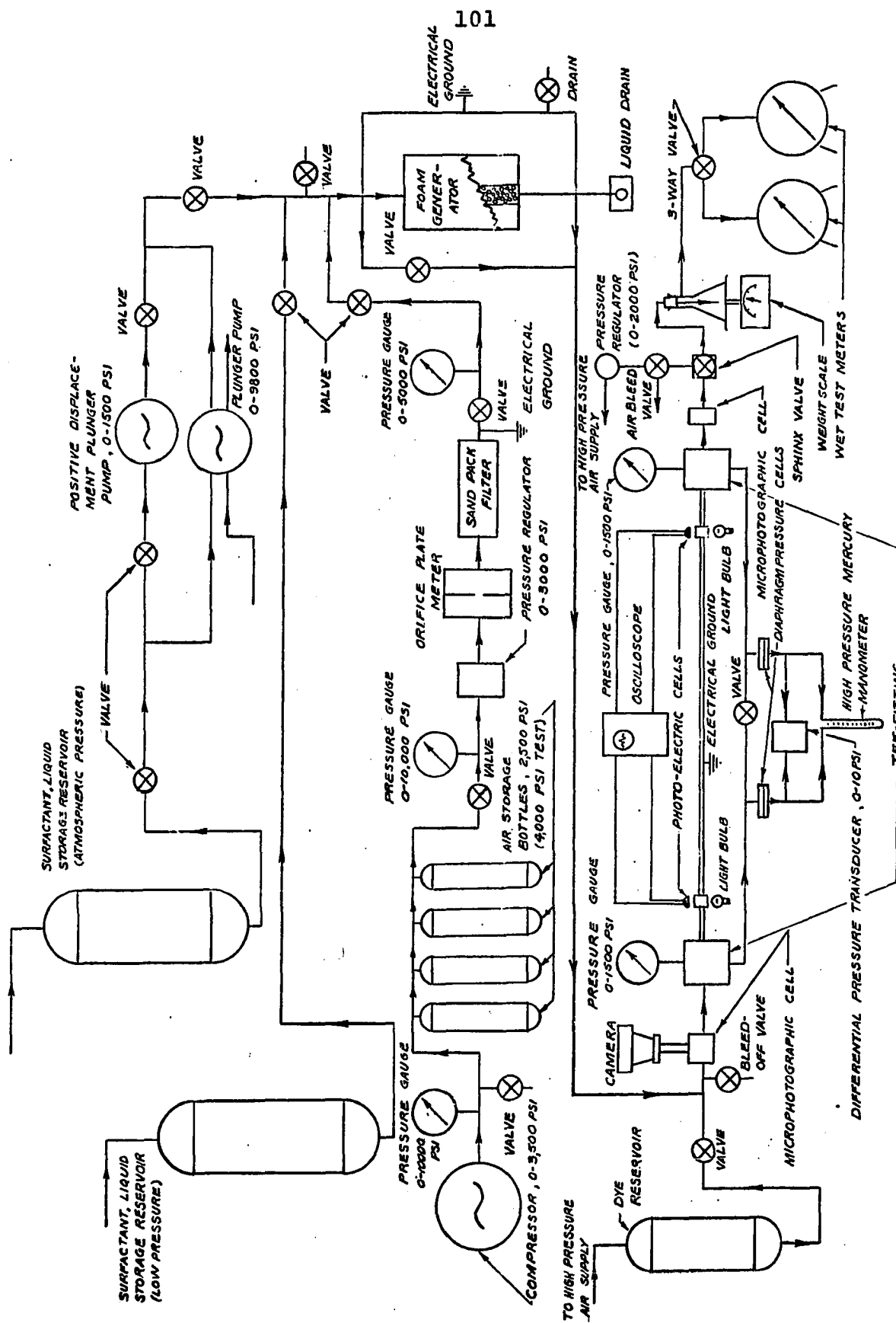
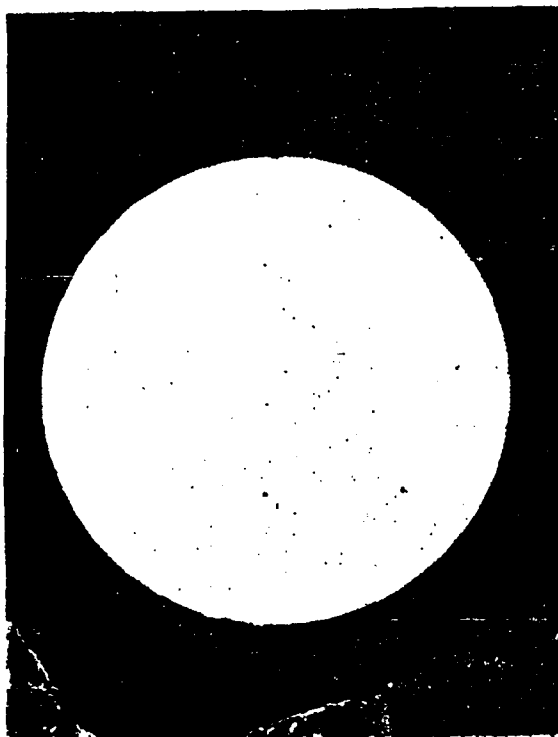


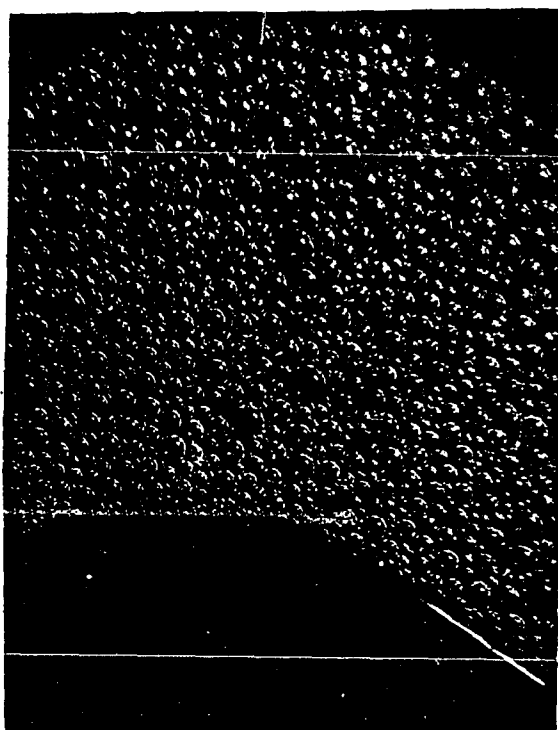
FIGURE 13



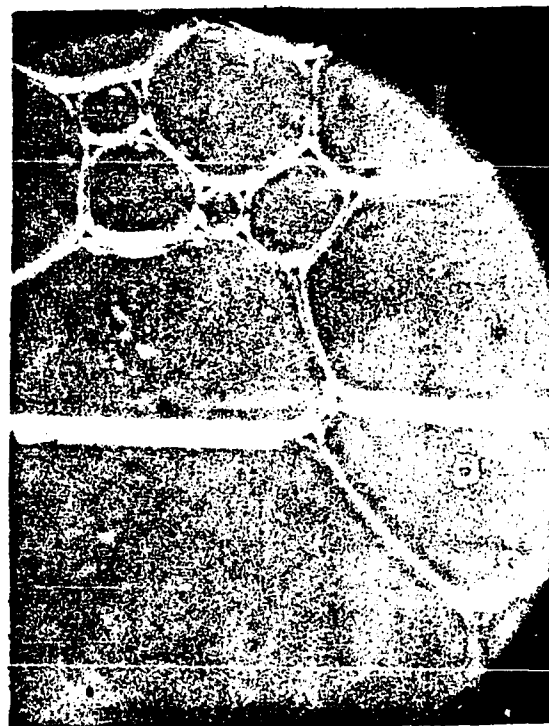
210X $t=0$ $\Gamma=.81$
FIGURE 14



210X $t=123$ MINUTES
FIGURE 15



210X $t=5$ HOURS
FIGURE 16



210X $t=12$ HOURS
FIGURE 17

MICROPHOTOGRAPHS OF FOAM

MICROPHOTOGRAPHIC — CELL
(FULL SCALE)

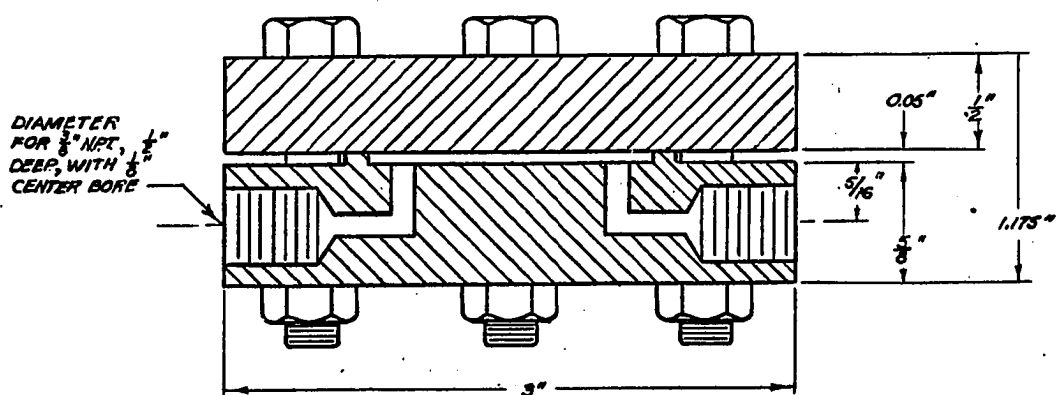
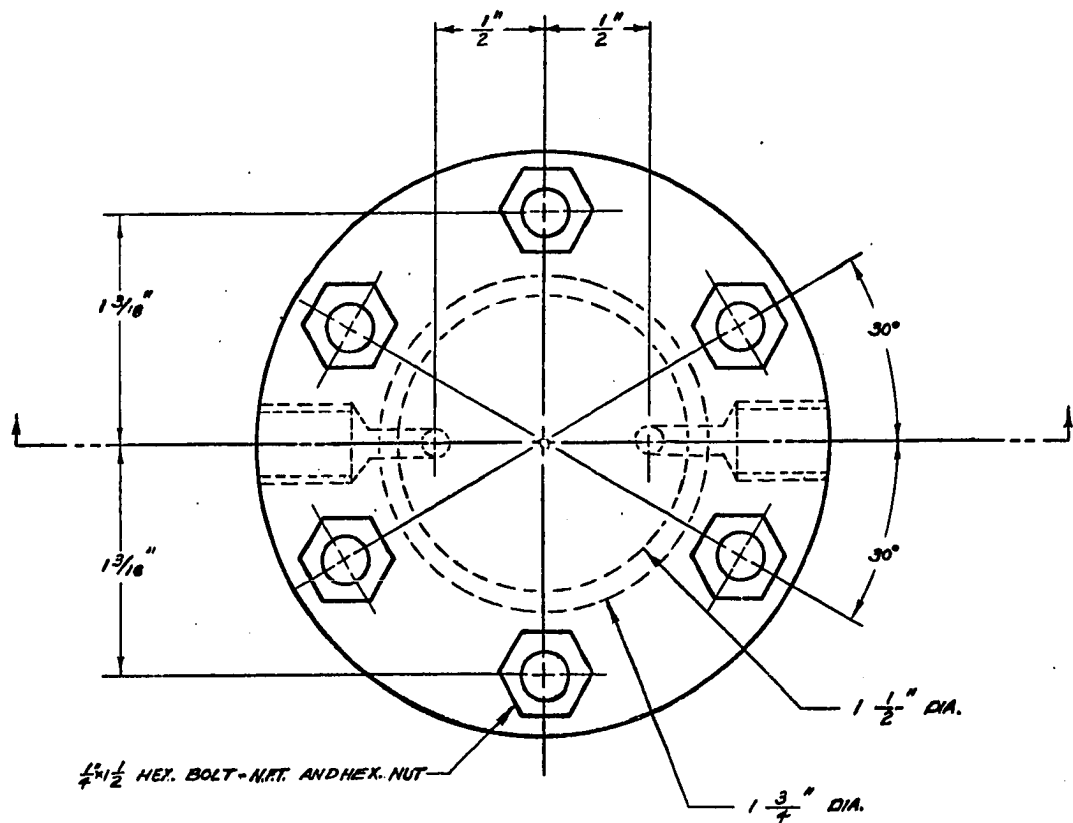


FIGURE 18

DETECTION SYSTEM

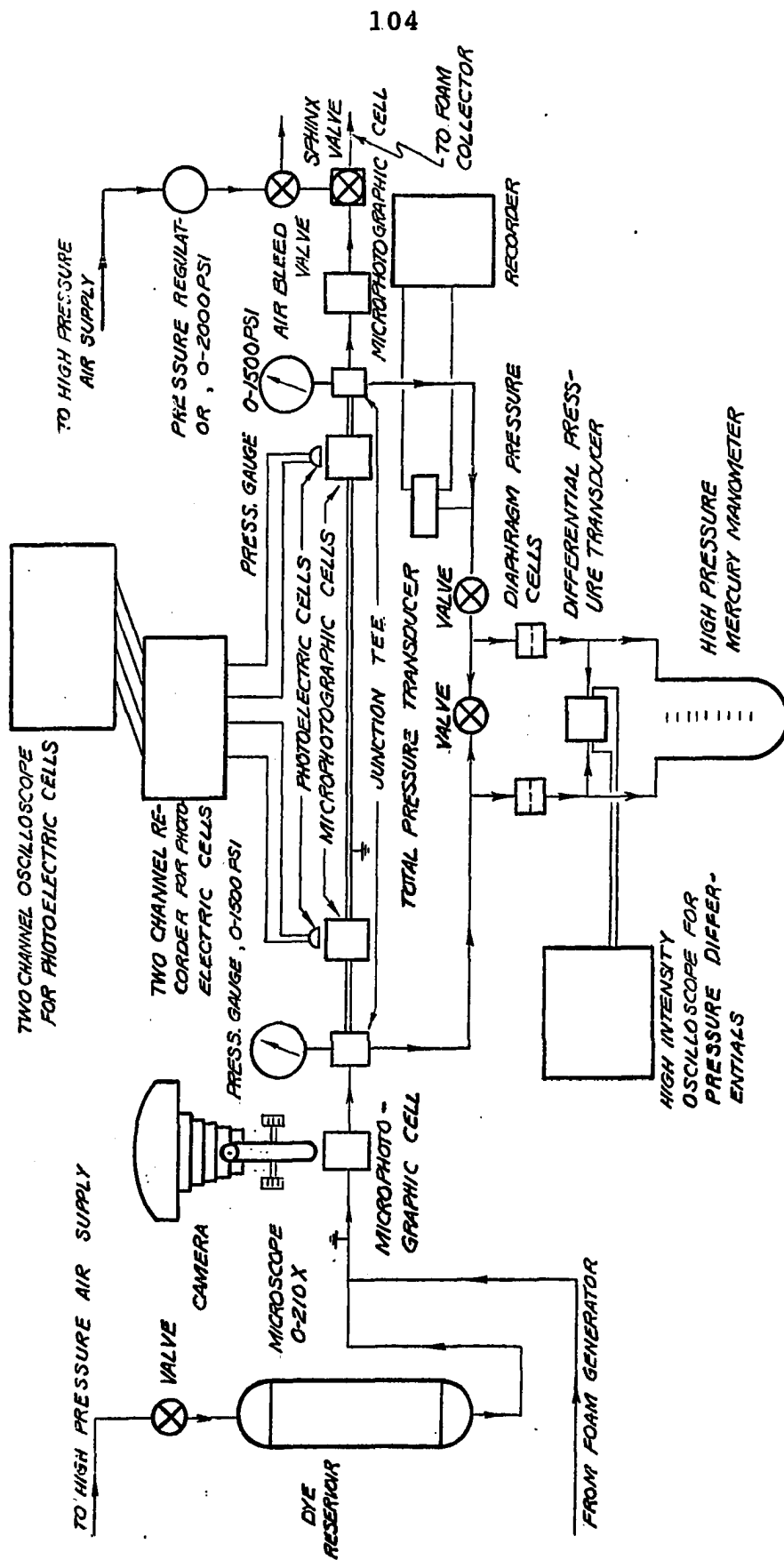
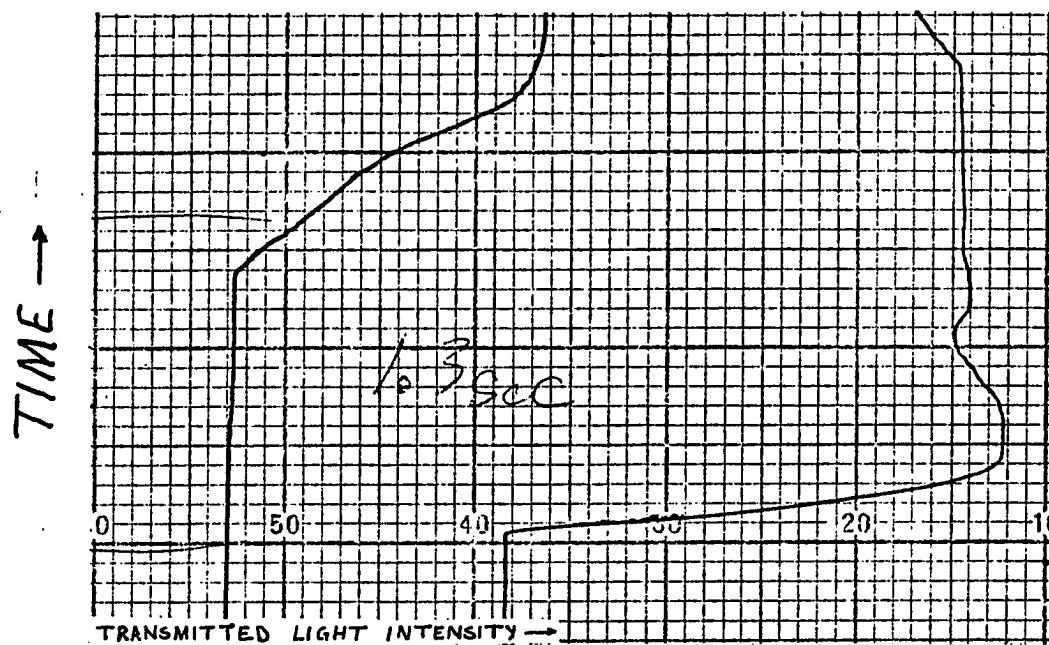
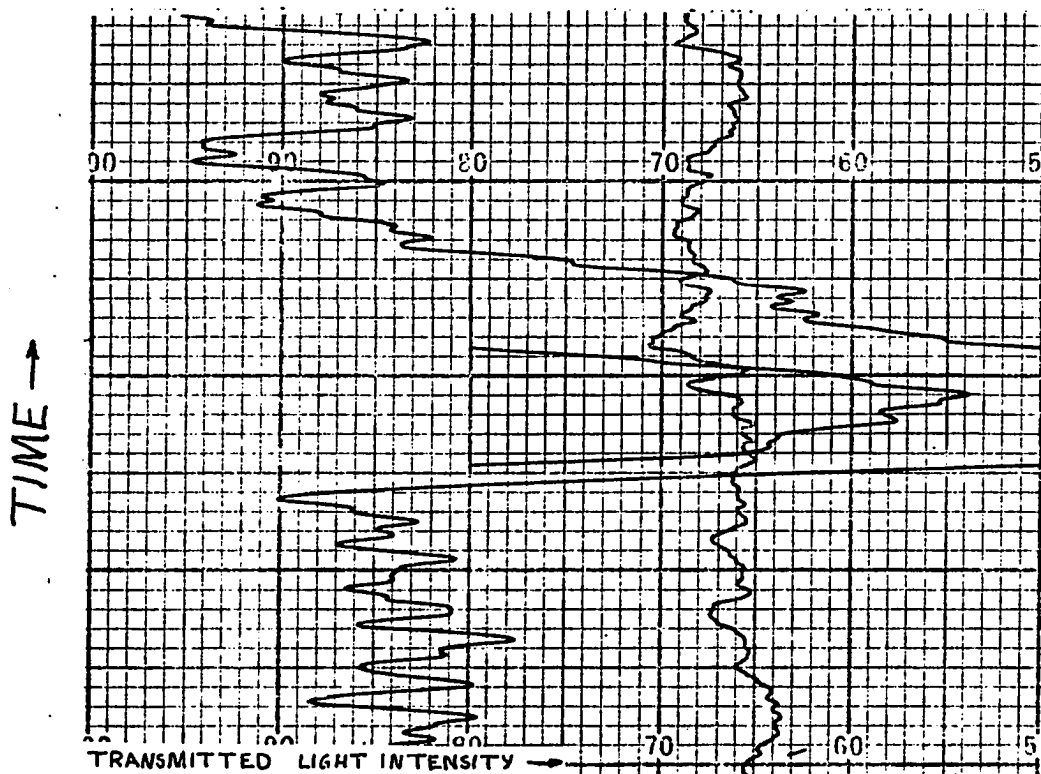


FIGURE 19



(A)

FOAM OF SMOOTH TEXTURE



(B)

FOAM OF UNEVEN TEXTURE

FIGURE 20 EXAMPLE RECORDS OF FOAM TEXTURE

PHOTOELECTRIC DETECTION EQUIPMENT

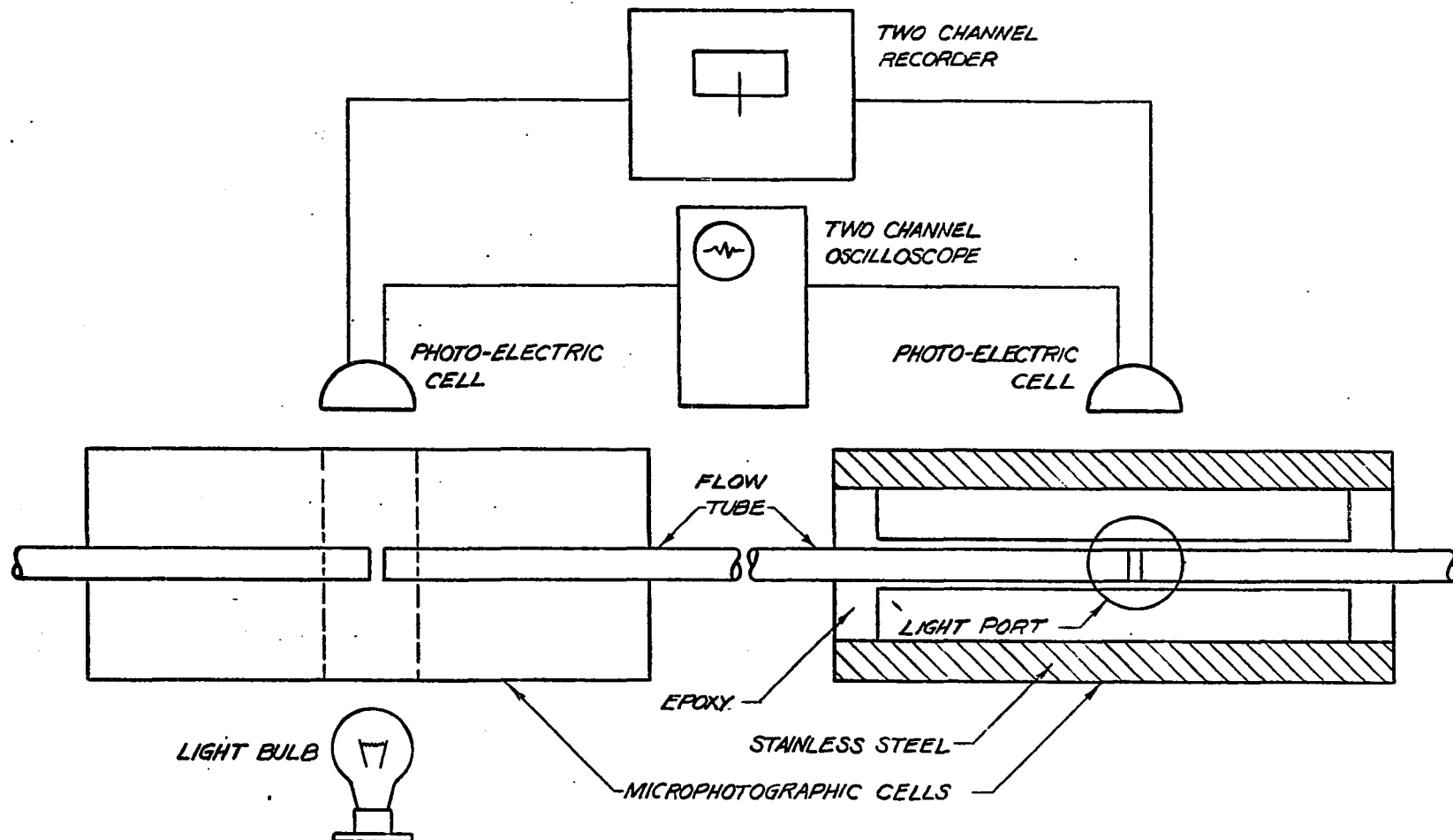


FIGURE 21

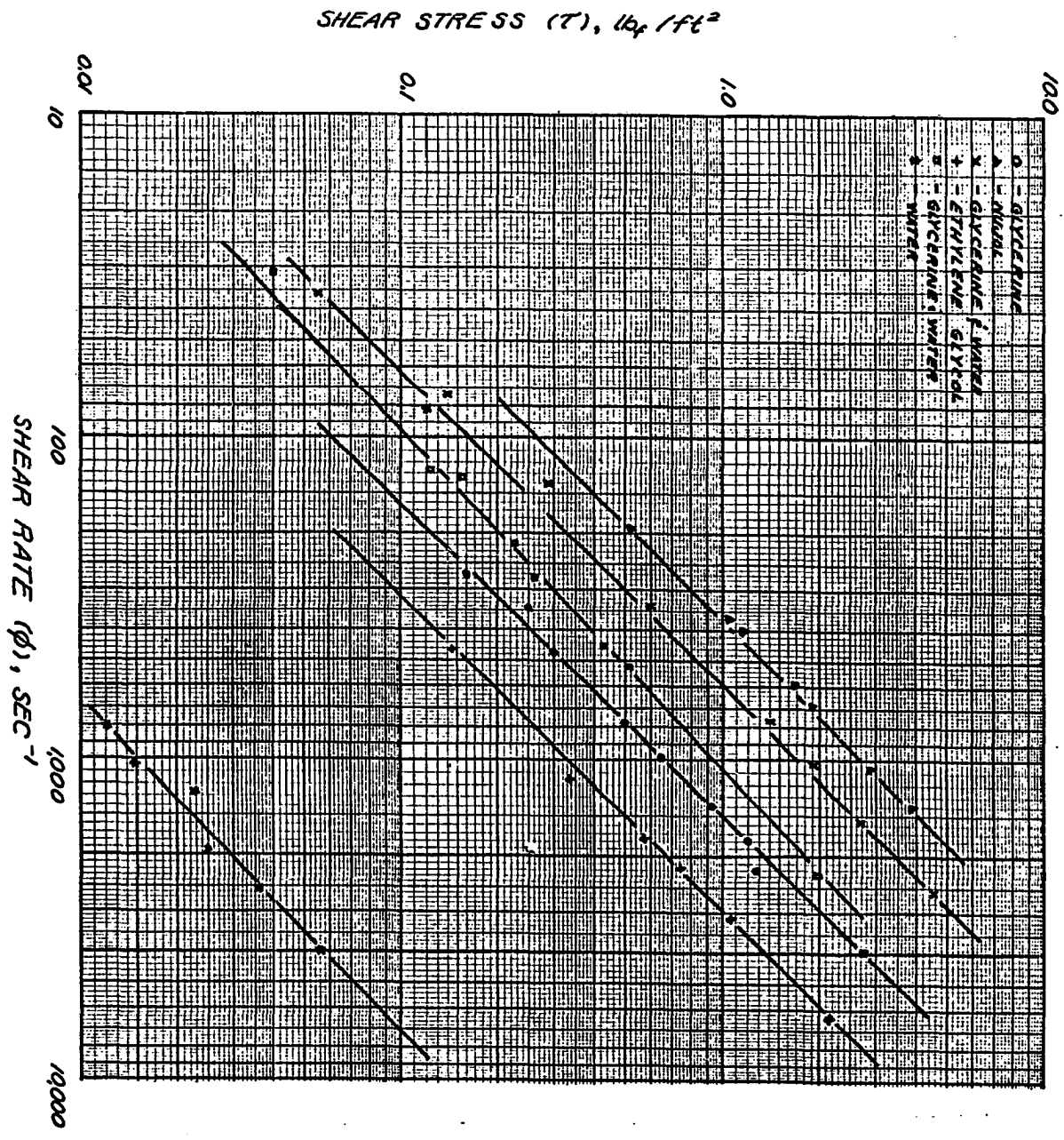


FIGURE 22 CALIBRATION OF SYSTEMS

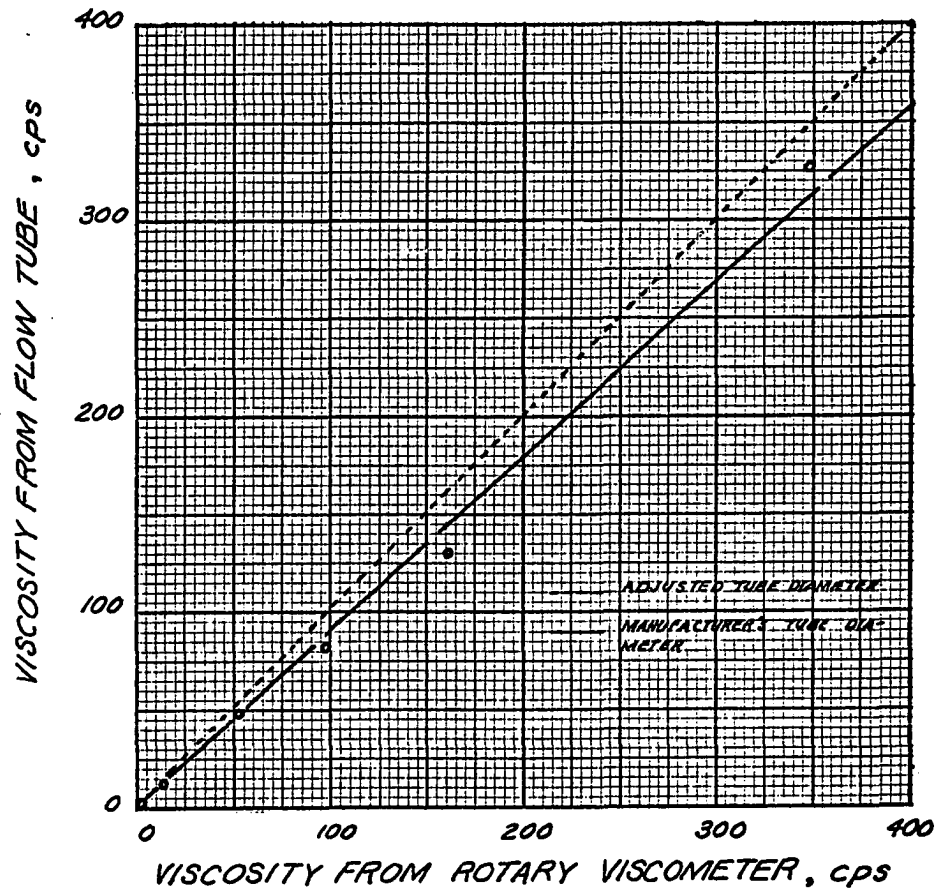


FIGURE 23 CORRECTION OF FLOW TUBE INTERNAL DIAMETER

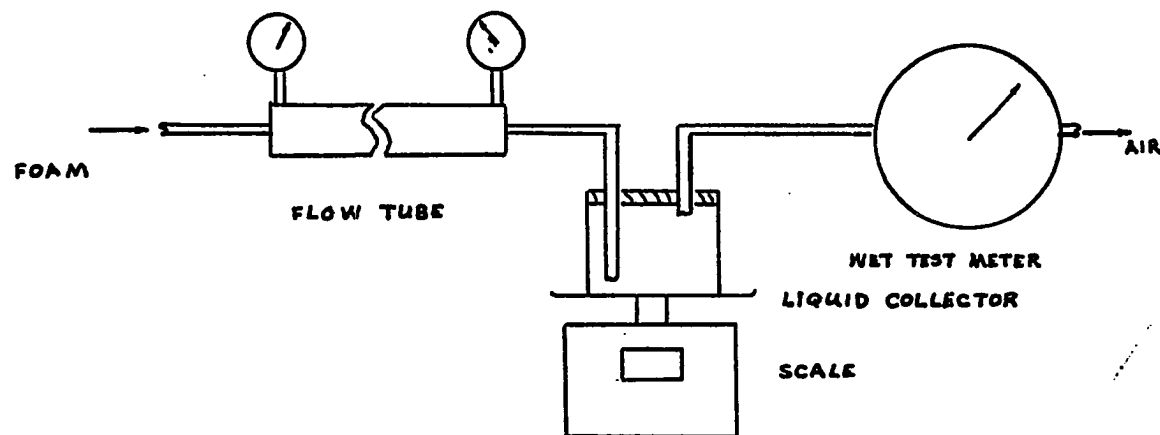


FIGURE 24 AIR AND LIQUID METERING SYSTEM

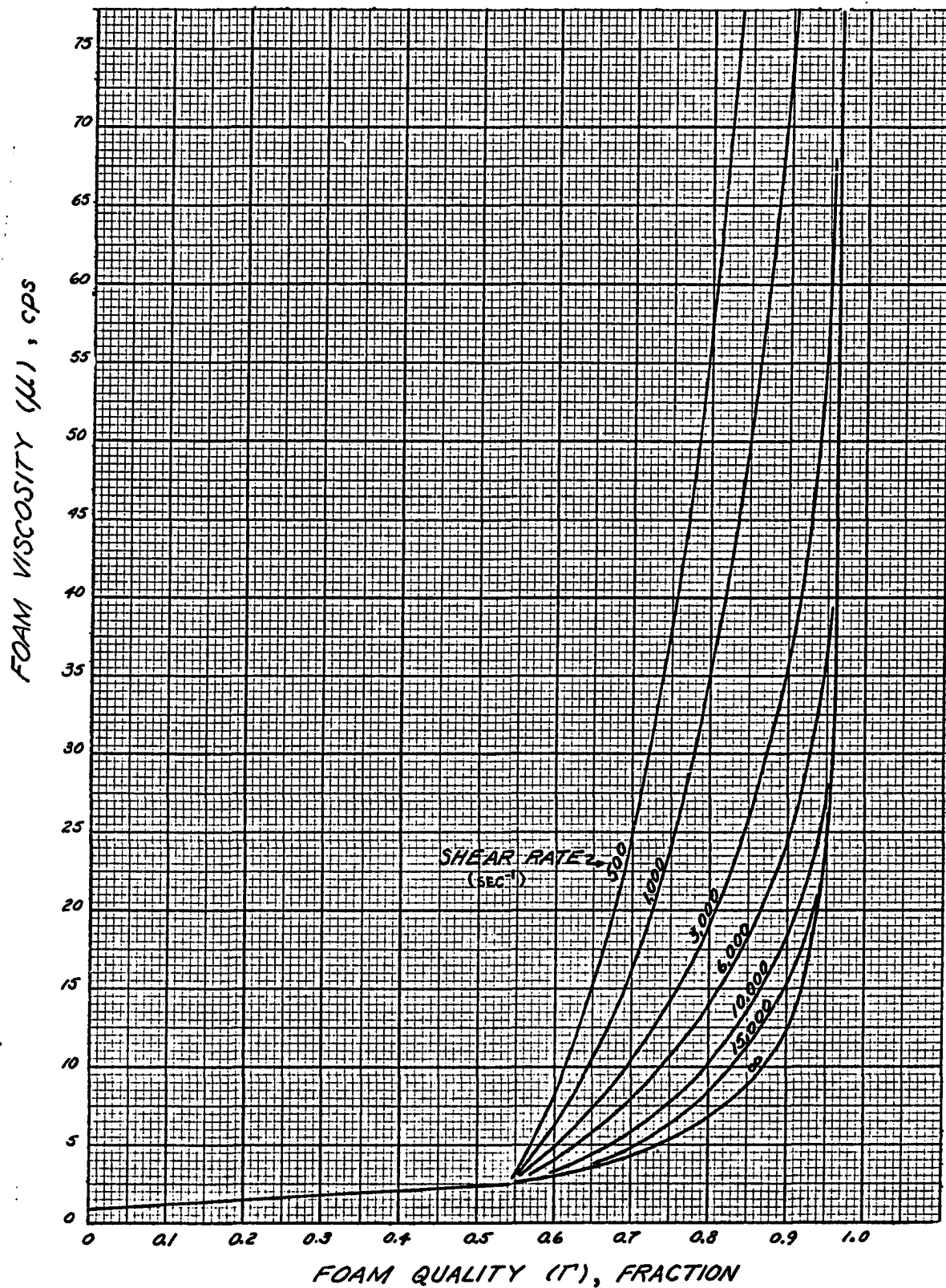


FIGURE 25 FOAM VISCOSITY

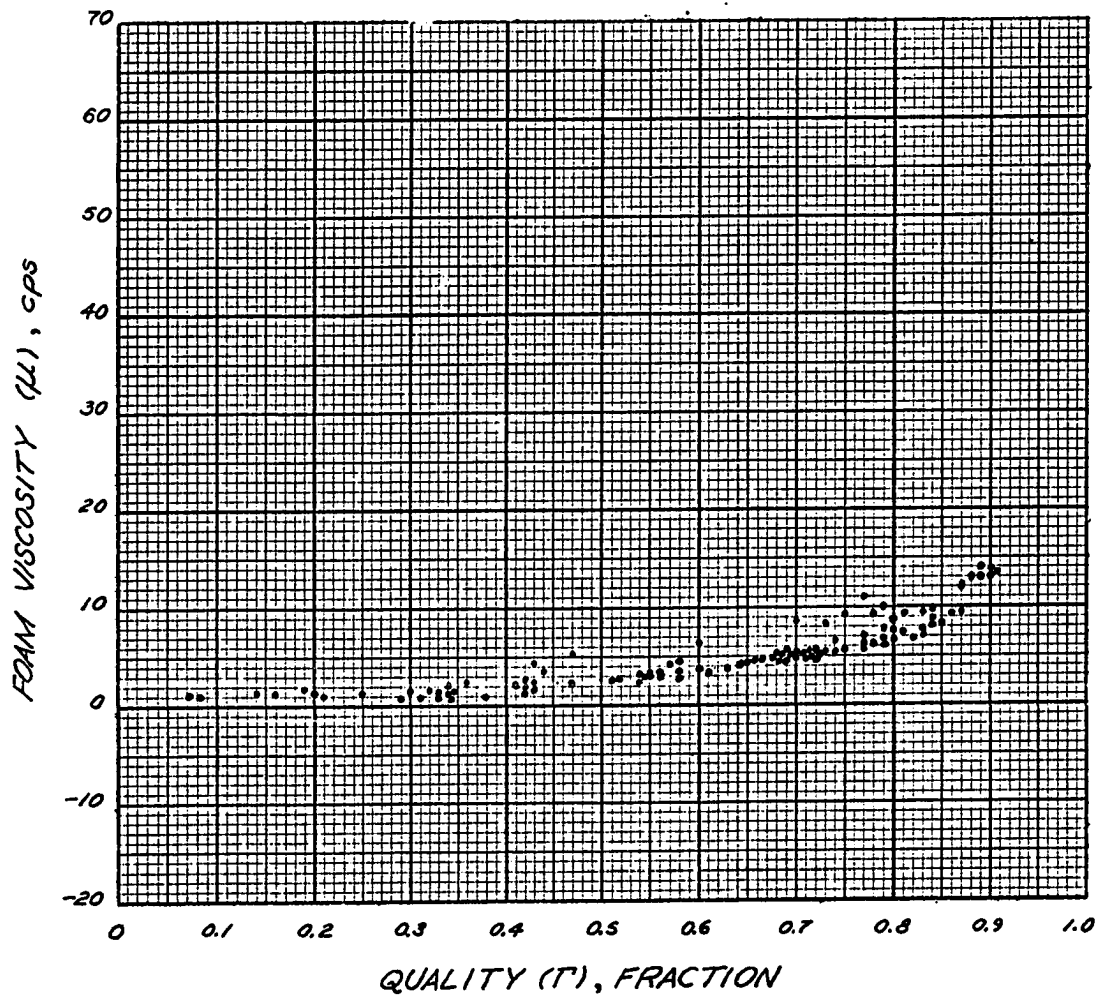


FIGURE 26 FOAM VISCOSITY (TUBE I.D. - 0.0296 in.)

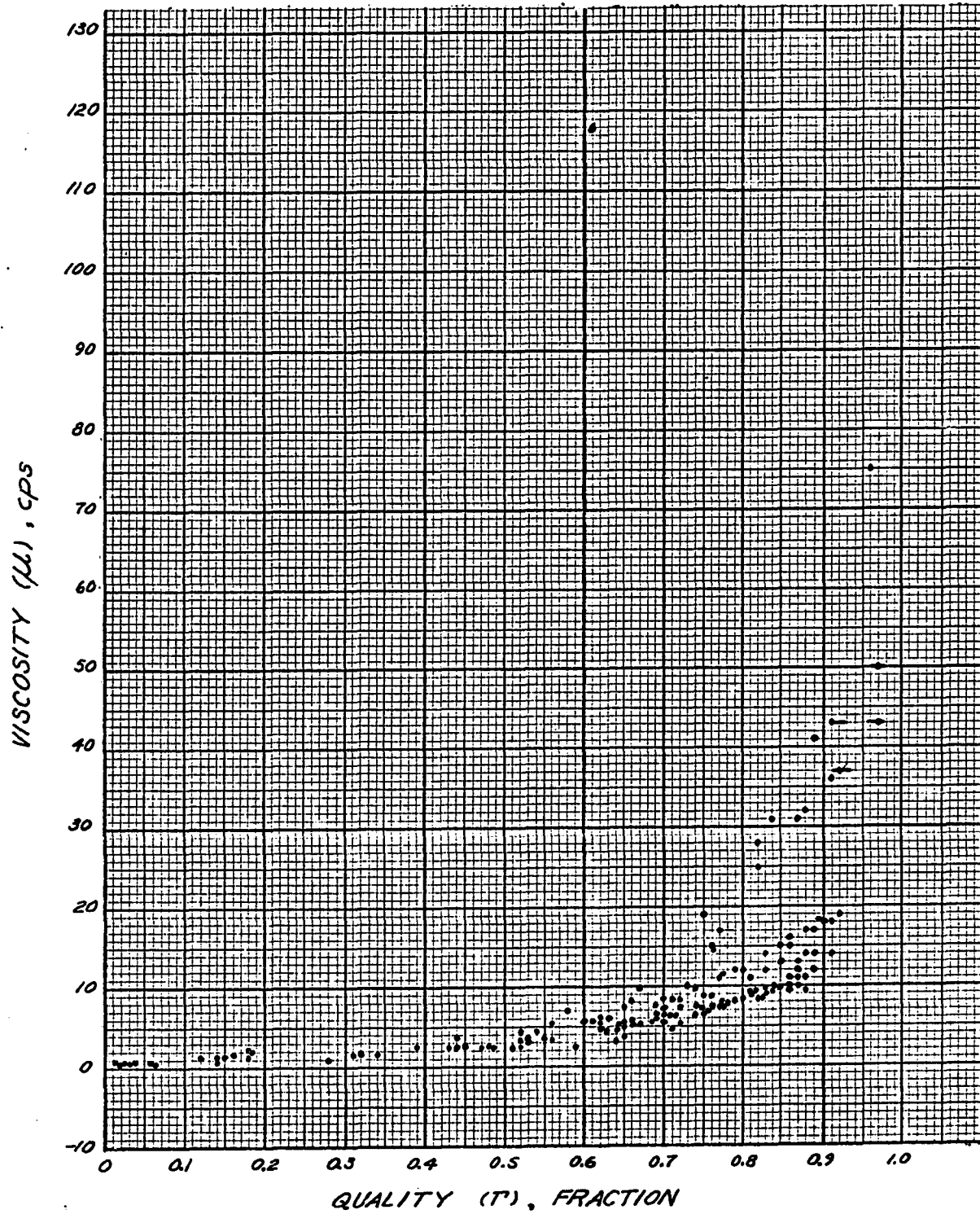


FIGURE 27 FOAM VISCOSITY, (TUBE I.D. - 0.0483 in.)

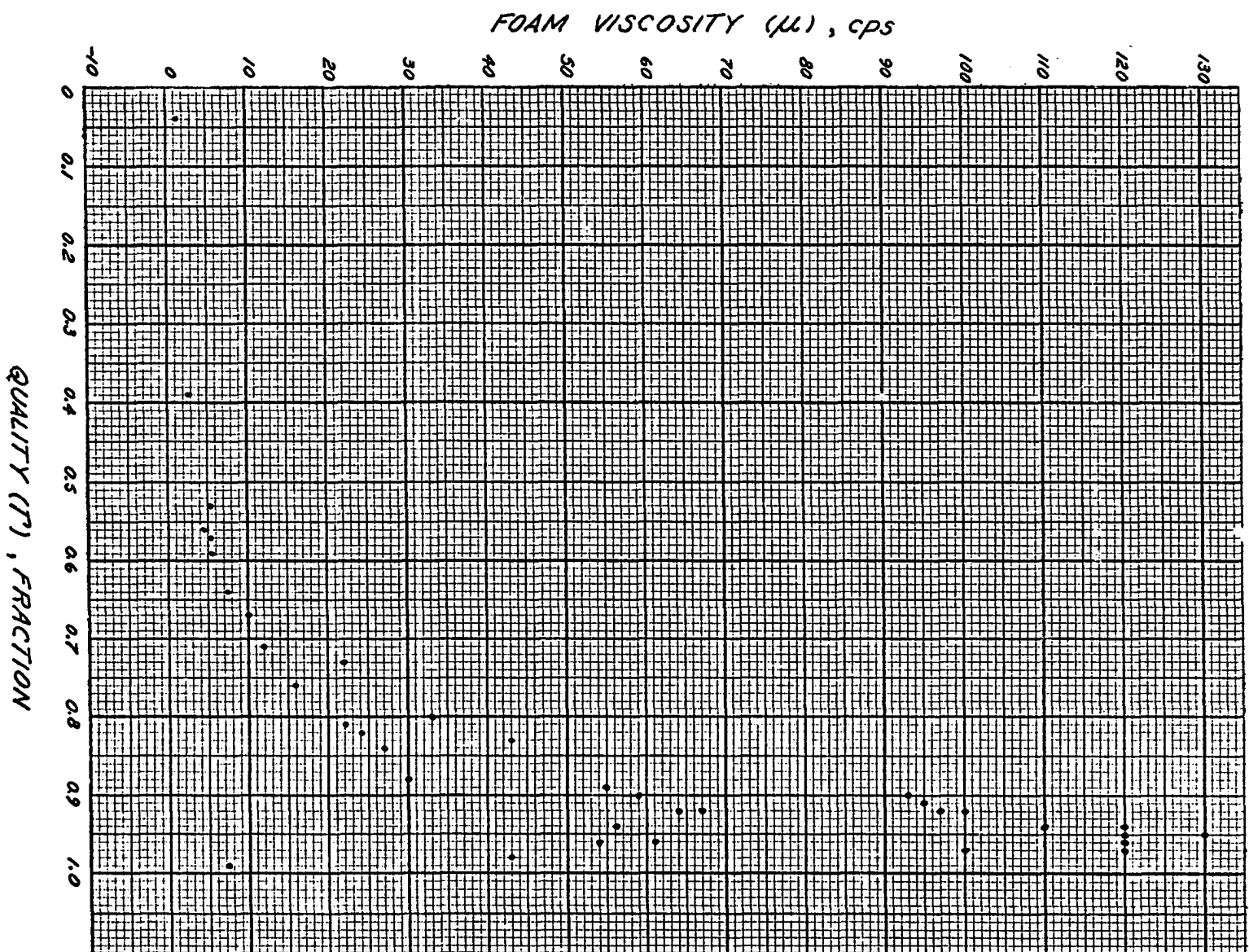
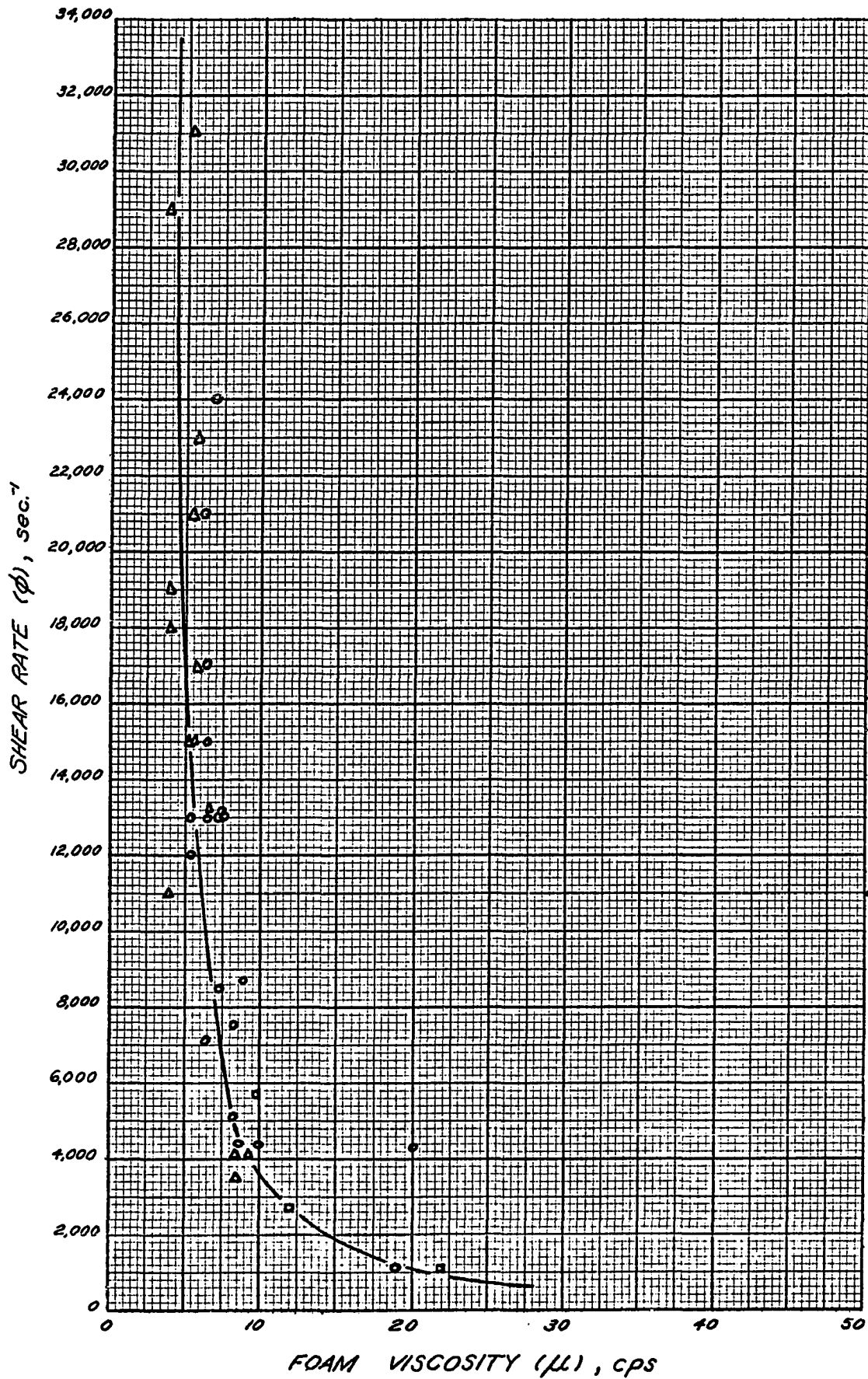
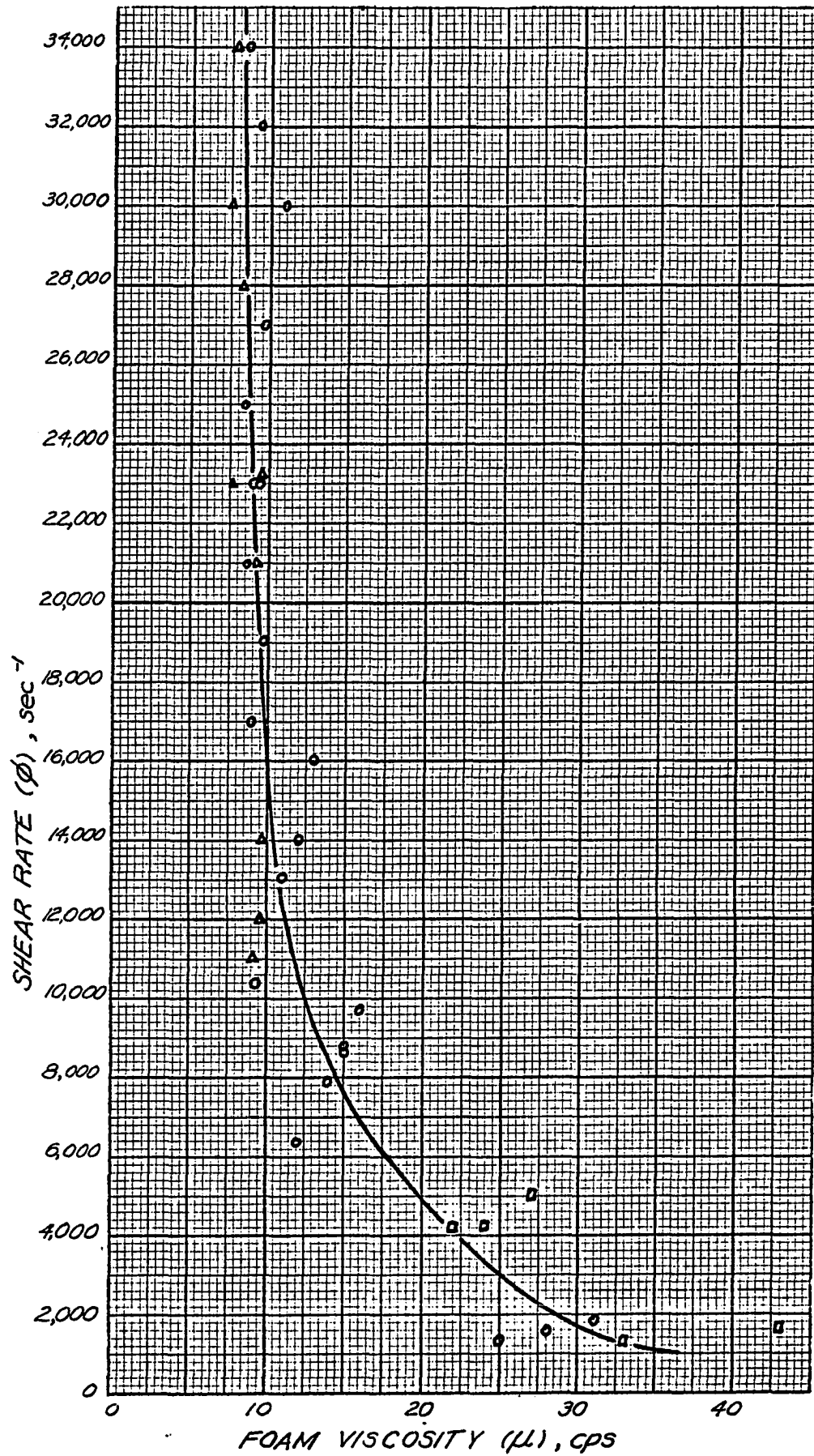
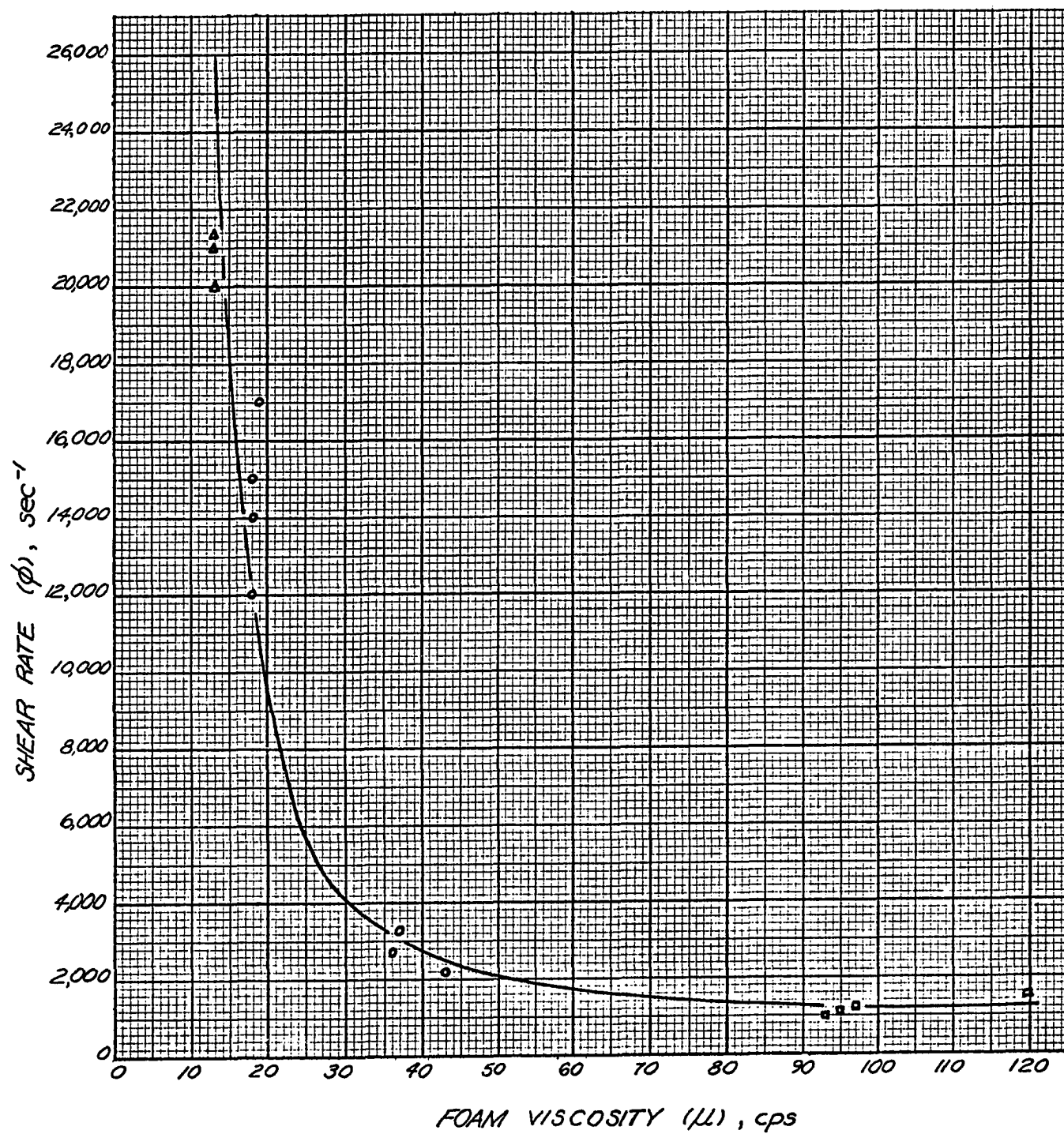


FIGURE 28 FOAM VISCOSITY (TUBE I.D. - 0.0928 in)

FIGURE 29 FOAM VISCOSITY ($0.70 \leq T \leq 0.75$)

FIGURE 30 FOAM VISCOSITY ($0.80 \leq \Gamma \leq 0.86$)

FIGURE 31 FOAM VISCOSITY ($.90 \leq \Gamma \leq .94$)

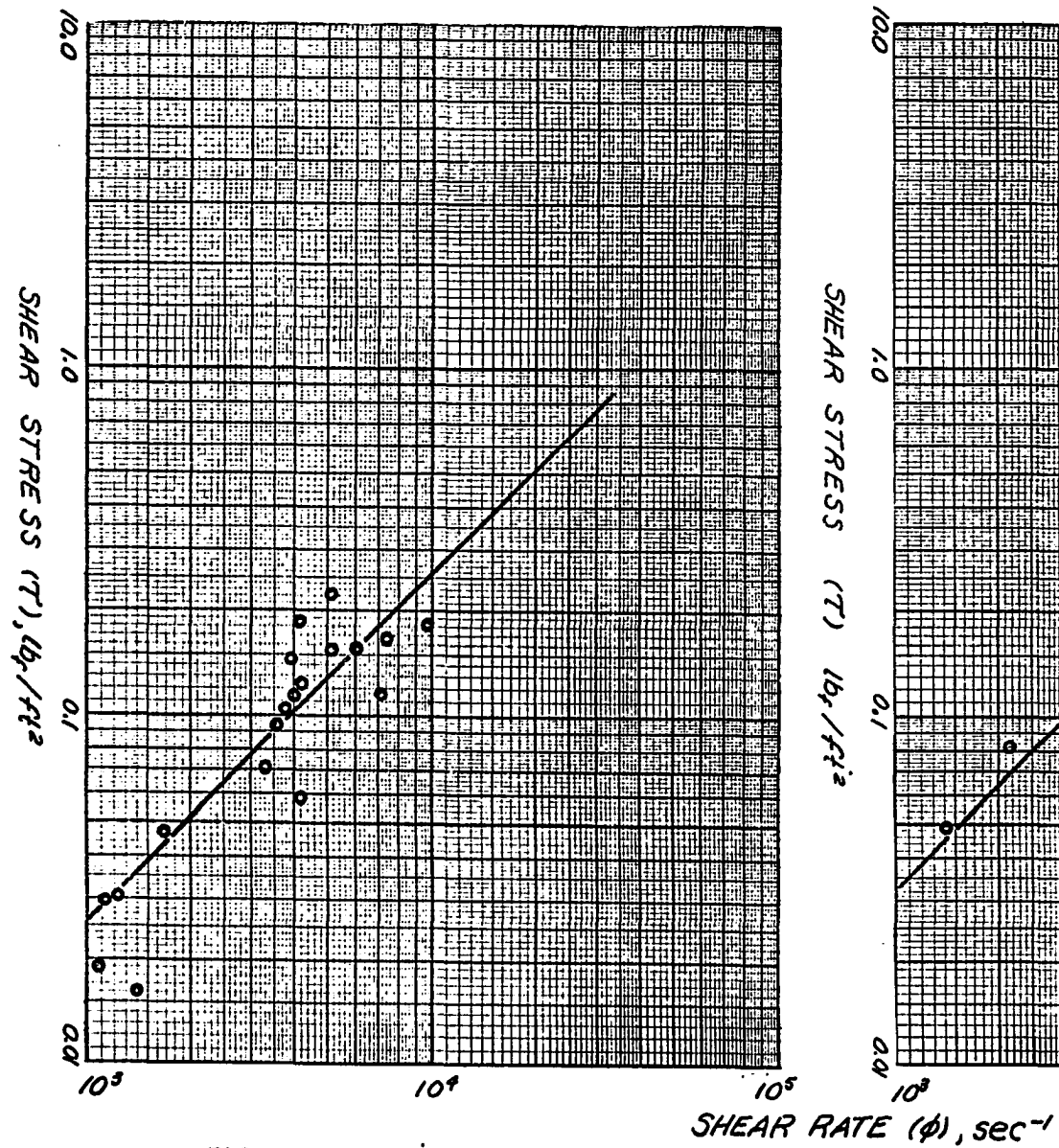


FIGURE 32 ($0 \leq P \leq .25$)

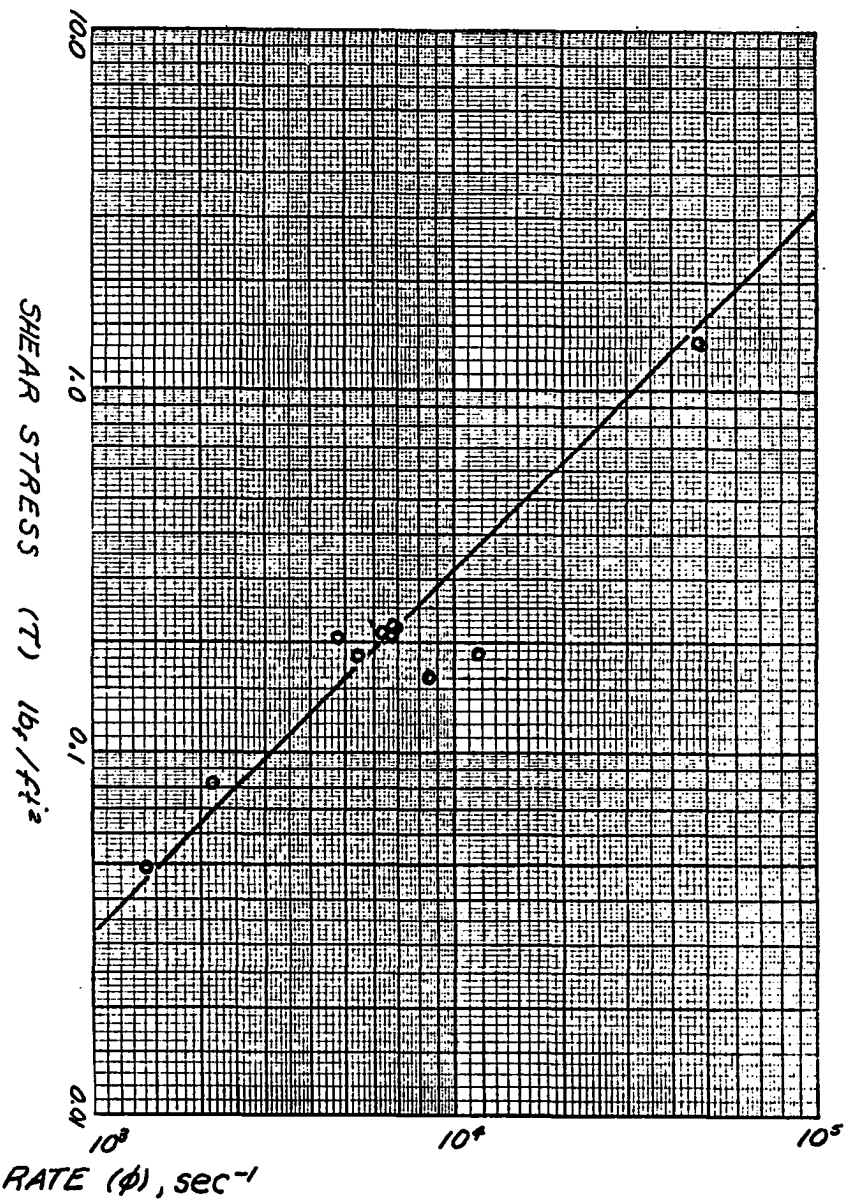


FIGURE 33 ($.25 \leq P \leq .35$)

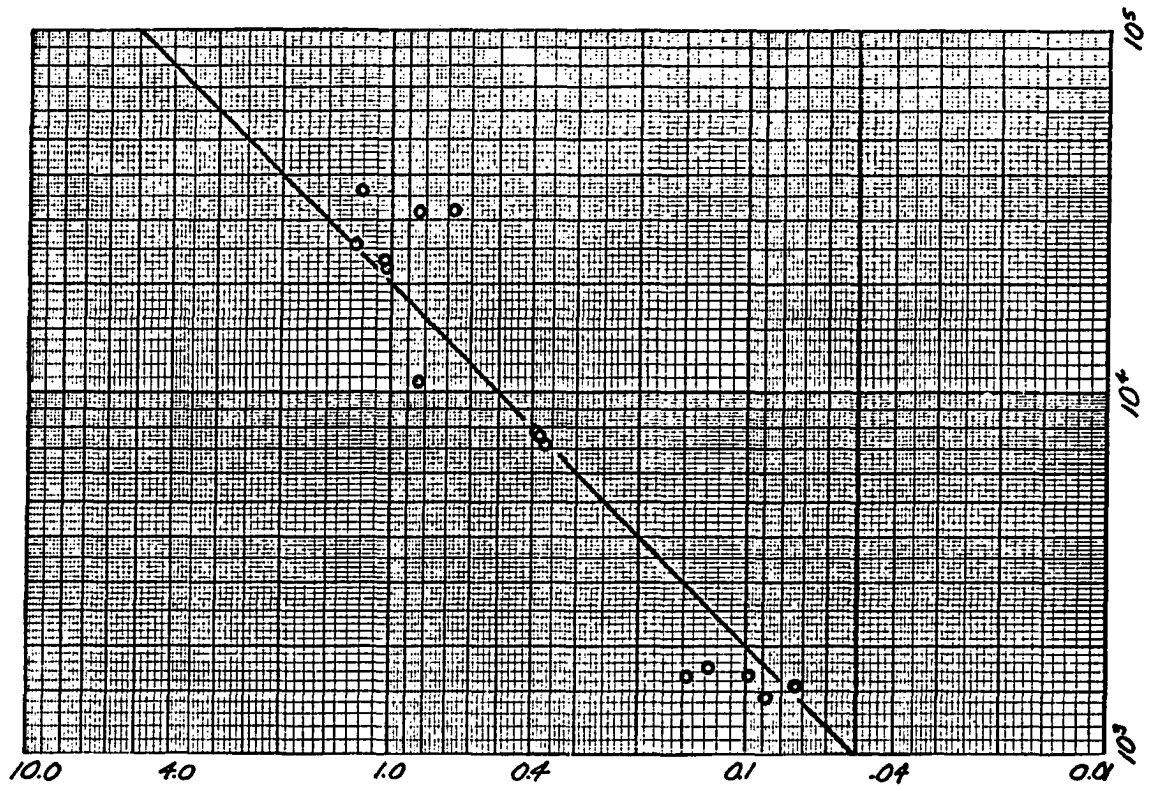


FIGURE 35 ($0.35 \leq \tau \leq 4.5$)

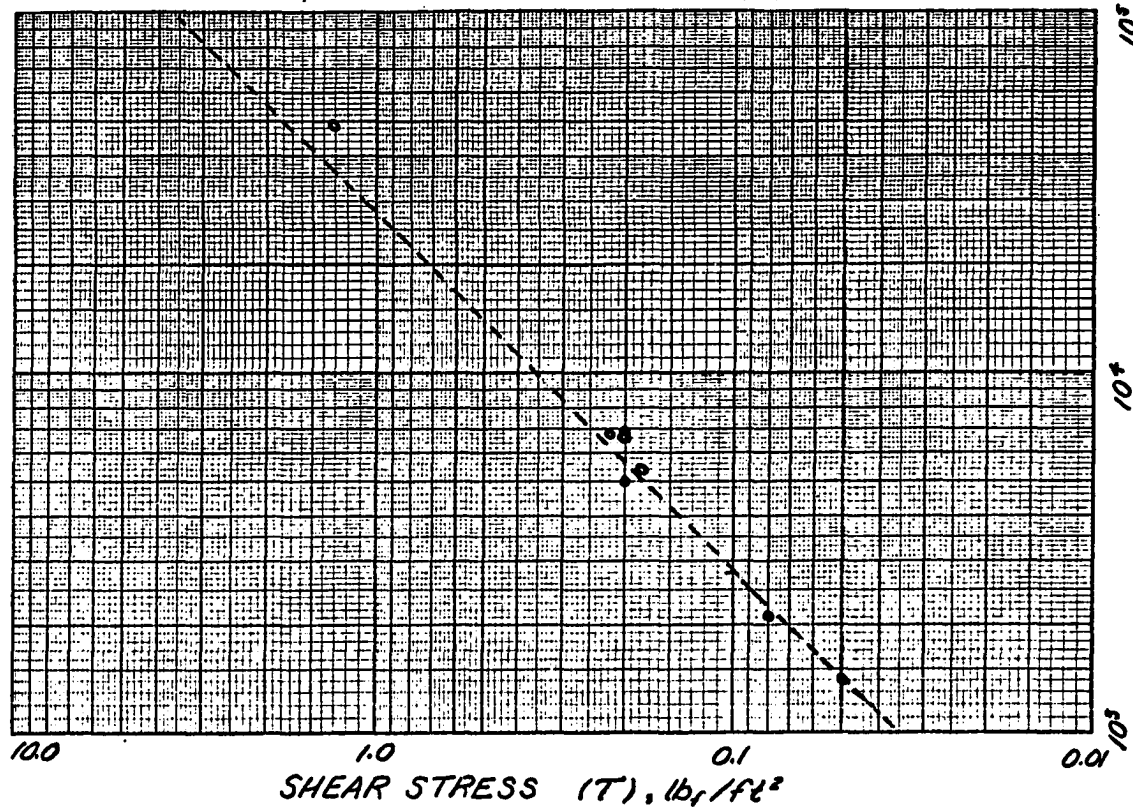


FIGURE 34 ($0.30 \leq \tau \leq 0.35$)

FIGURE 36 (1.45 ≤ $T \leq 1.55$)

SHEAR RATE ($\dot{\gamma}$), sec^{-1}

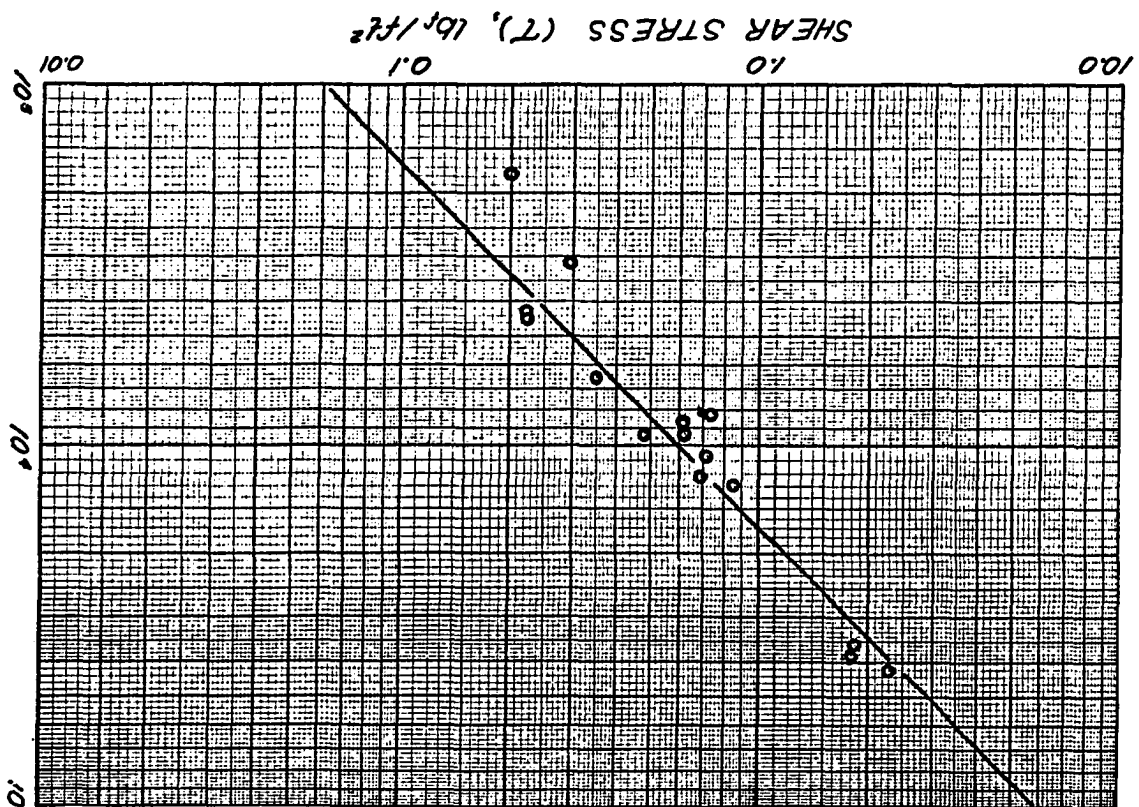
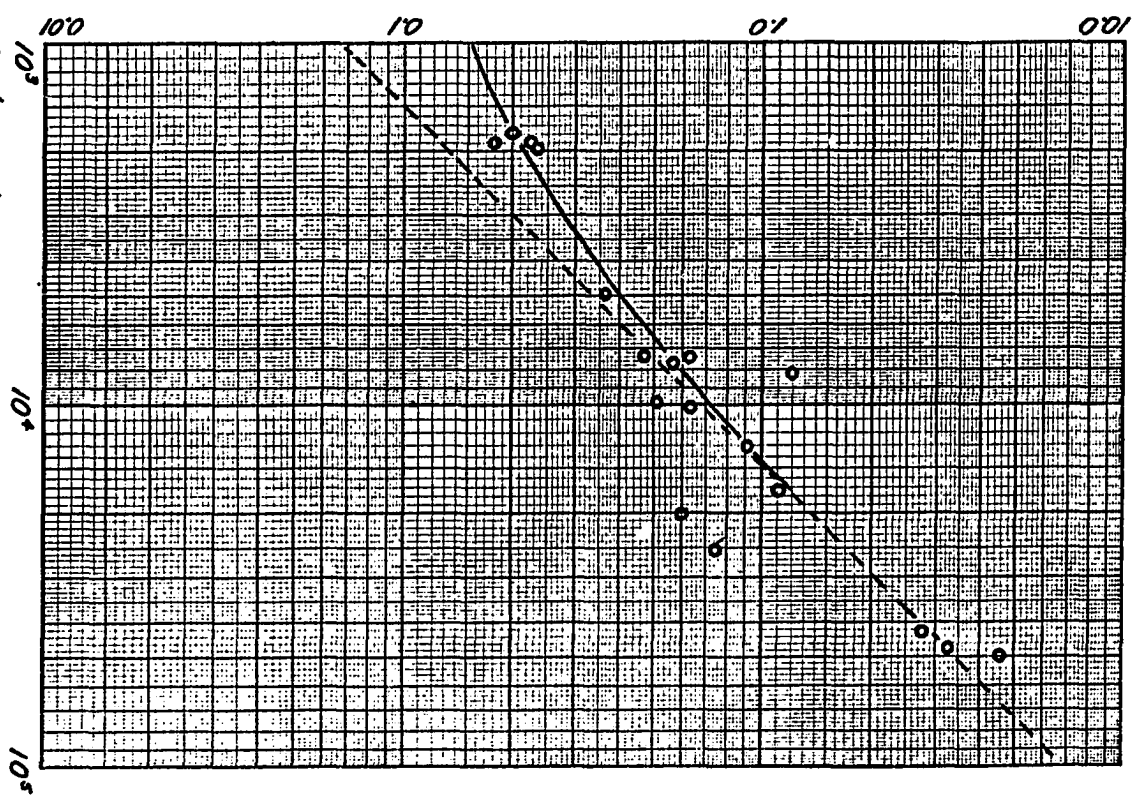
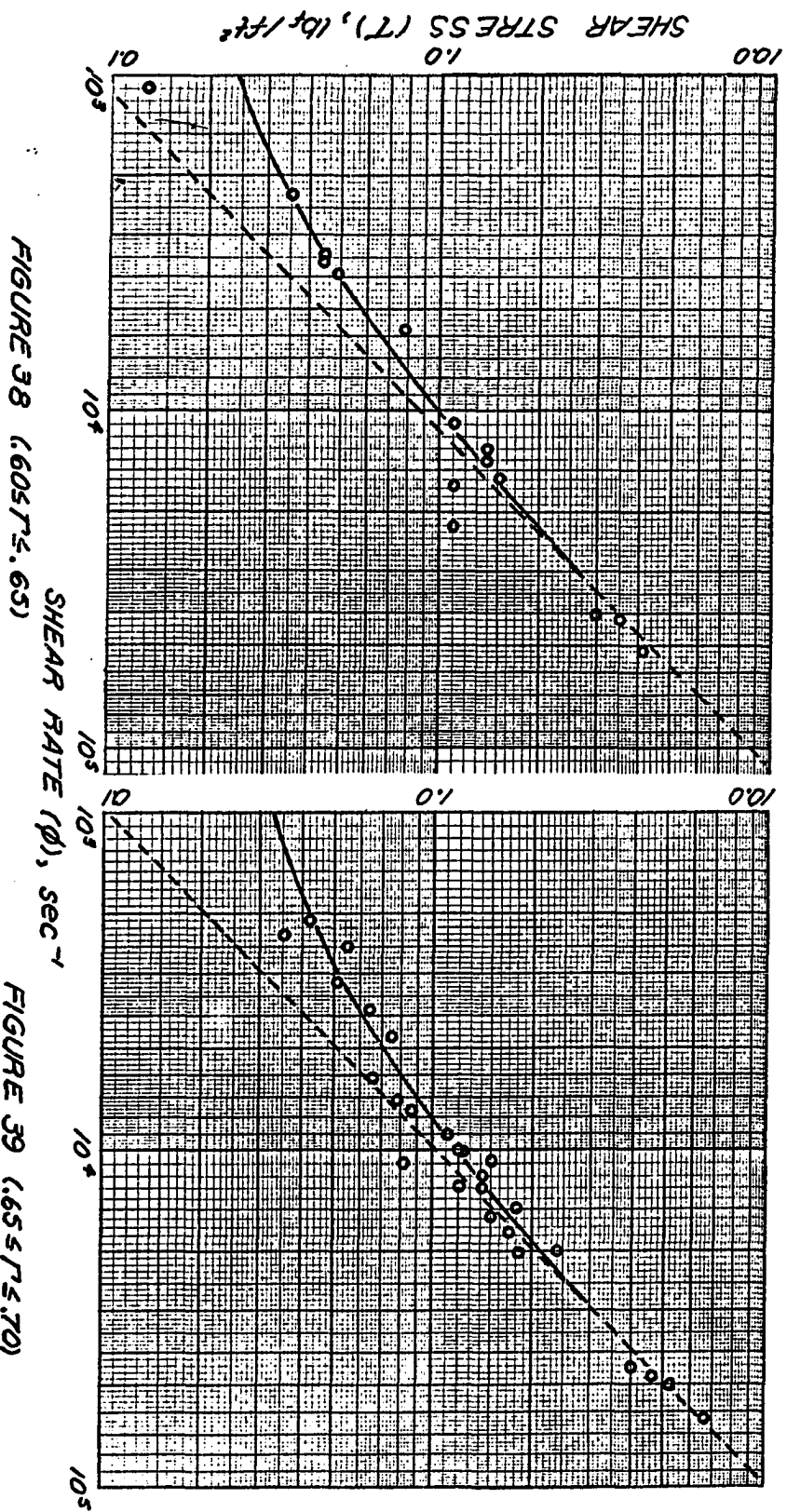
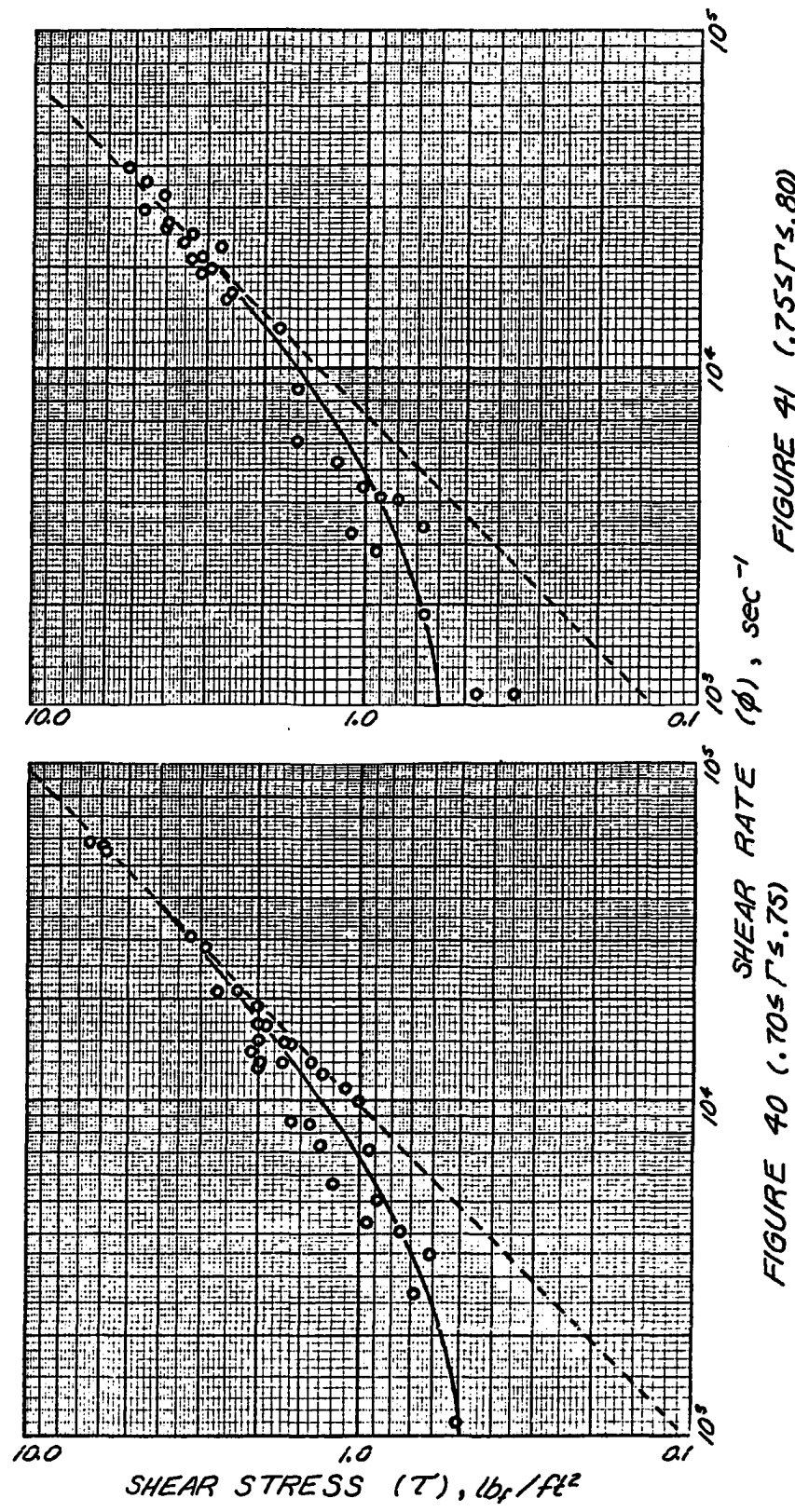


FIGURE 37 (1.55 ≤ $T \leq 1.60$)





FIGURE 41 ($0.75 \leq \Gamma \leq 0.80$)FIGURE 40 ($0.70 \leq \Gamma \leq 0.75$)

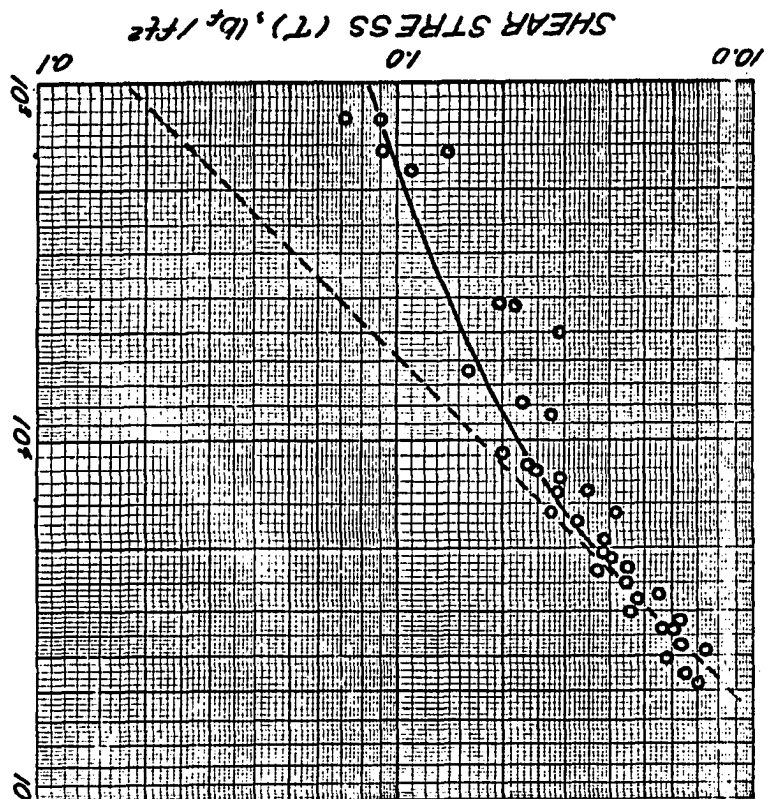


FIGURE 42 ($80 \leq T \leq 86$)

SHEAR RATE (ϕ), sec^{-1}

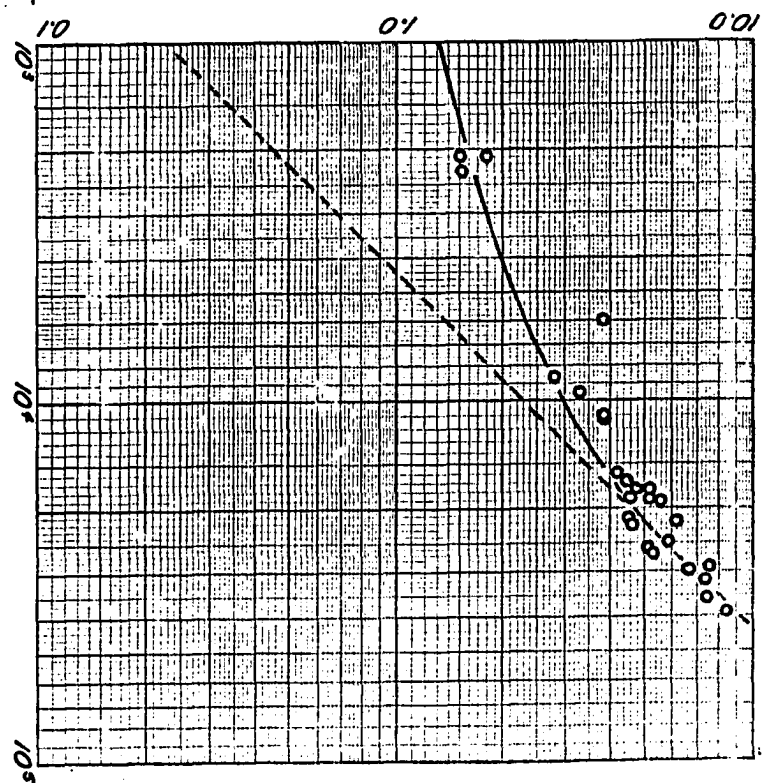
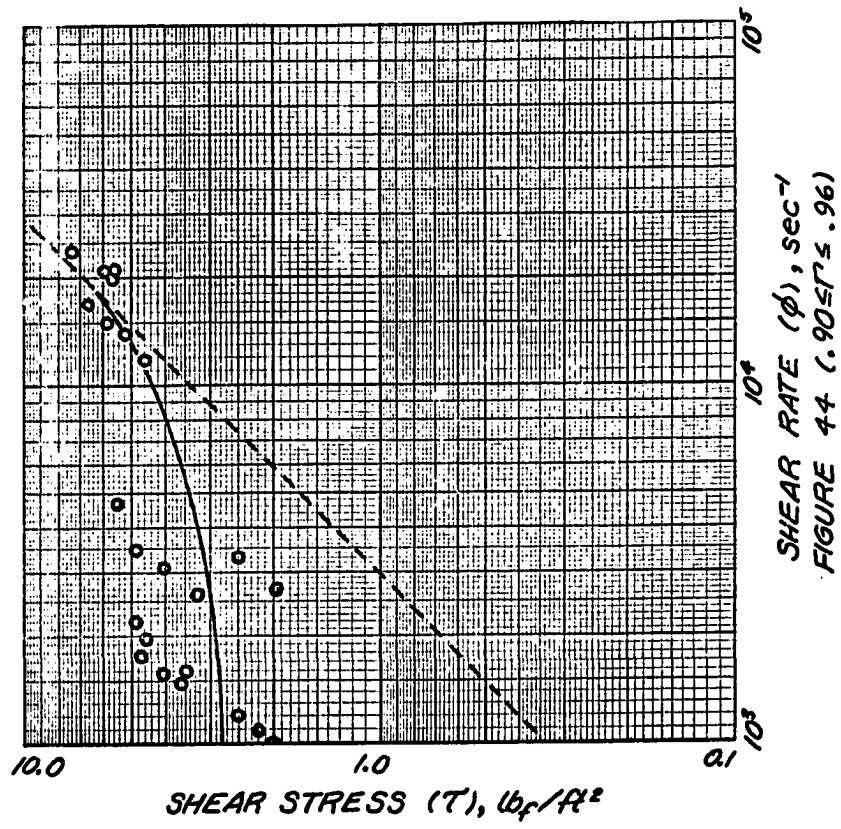


FIGURE 43 ($86 \leq T \leq 90$)



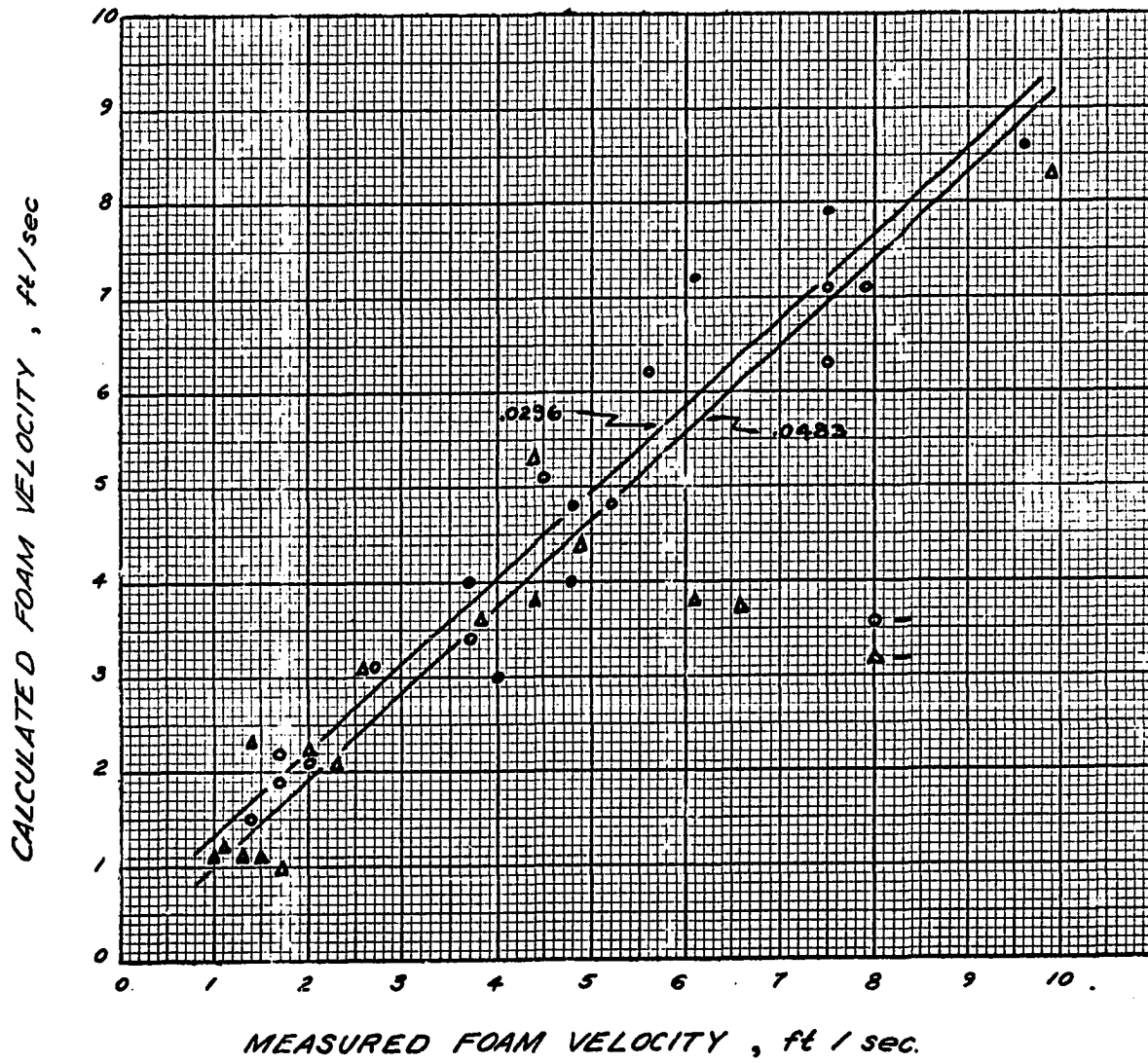


FIGURE 45 FOAM FLOW VELOCITY COMPARISON

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APPENDIX

EXAMPLE CALCULATION

This example calculation sets forth the procedure that may be used to determine the pressure gradient due to foam flow in a circular tube with special attention directed toward the selection of the correct foam viscosity. The selected problem is to find the pressure gradient in a pipe of two inches internal diameter containing a 75% quality foam with a foam velocity of 20 feet per second at an absolute pressure of 500 psia and temperature of 60° F.

The average shear rate of the foam is determined,

$$\phi = \frac{8v}{D}$$
$$\frac{1}{\text{sec}} = \frac{8 \text{ ft/sec}}{\text{in} * \text{ft/in}}$$
$$\phi = \frac{8 * 20}{2 * 1/12} = 960 \text{ sec}^{-1}$$

The viscosity of a 75% quality foam corresponding with this shear rate is from Figure 25,

$$\mu = 26. \text{ centipoises}$$

The density of the foam may be calculated with the equation,

$$\rho_f = \rho_w (1 - F) + \rho_a (F)$$

$$\frac{\text{lb}_m}{\text{ft}^3} = \frac{\text{lb}_m}{\text{ft}^3} (\text{dimensionless}) + \frac{\text{lb}_m}{\text{ft}^3} (\text{dimensionless})$$

In this equation, ρ is quality fraction, P_w is assumed to be $62.4 \text{ lb}_m/\text{ft}^3$, and P_a may be calculated with the gas law or simply found in the literature.

$$\rho_f = 62.4 (1 - .75) + 2.5 (.75)$$

$$\rho_f = 17.5 \text{ lb}_m / \text{ft}^3$$

The Reynold's number for this problem is,

$$R_n = \frac{\rho D v}{\mu}$$

$$\text{dimensionless} = \frac{\text{lb}_m/\text{ft}^3 * \text{in} * \text{ft}/\text{in} * \text{ft}/\text{sec}}{\text{cps} * \text{lb}_m/\text{ft} - \text{sec}/\text{cps}}$$

$$R_n = \frac{17.5 * 2. * 1/12 * 20.}{26. * 6.72 * 10^{-4}}$$

$$R_n = 476.$$

Since the Reynold's number is less than 2,000, the flow regime within the tube is laminar and the pressure gradient may be calculated with the Hagen-Poiseuille law,

$$\frac{\Delta P_f}{L} = \frac{32 \mu v}{g_c d^2}$$

$$\frac{\text{lb}_f}{\text{ft}} = \frac{\text{cps} * \text{lb}_m/\text{ft} - \text{sec}/\text{cps} * \text{ft}/\text{sec}}{\text{lb}_m - \text{ft} / \text{lb}_f - \text{sec}^2 * \text{in}^2}$$

$$\frac{\Delta P_f}{L} = \frac{32. * 26. * 6.72 * 10^{-4} * 20.}{32.2 * 2^2}$$

$$\frac{\Delta P_f}{L} = 0.0874 \text{ psi/ft}$$

Thus, the pressure gradient at the specified conditions is 0.0874 psi/ft.

EINSTEIN'S MATHEMATICAL MODEL

Previous assumptions in Einstein's theory are explained in the chapter entitled "Theory." These will not be reiterated at this point. Also, the theory is only presented in part. This theory constituted his doctoral thesis and as such was quite lengthy. The explanations presented here only include the theory that is proposed for a single particle in a continuous liquid solvent; whereas, general theory considered many particles suspended in a continuous liquid solvent. His model is shown in Figure 9.

Einstein commences his derivation by considering a single region of solvent, G , in a three-dimensional coordinate system (x, y, z) with its center located at the point x_0, y_0, z_0 . The velocity of the region upon dilation with respect to the coordinate axis is only a function of position and time.

The velocity components are:

$u = u(x, y, z, t)$ in the direction of the x axis

$v = v(x, y, z, t)$ in the direction of the y axis

$w = w(x, y, z, t)$ in the direction of the z axis

Using the Taylor expansion theorem, the functions, u, v, w , may be expanded around a point x_0, y_0 , and z_0 .

$$u_x - u_0 = \frac{\partial(u)}{\partial(x)} (x - x_0) + \frac{\partial^2 u}{\partial(x^2)} \frac{(x - x_0)^2}{2!} + \text{ect.} \quad \dots (1)$$

After discarding the second order and all higher order terms and recalling from our assumptions that all functions are linear within any region, G , with respect to location and time, thus,

$$\frac{\partial u}{\partial x} = \text{CONSTANT}$$

and letting

$$\begin{aligned} x - x_0 &= \xi \\ y - y_0 &= \eta \\ z - z_0 &= \zeta \end{aligned} \quad \dots (2)$$

and considering symmetry with respect to the axis equation Number 1 becomes,

$$\begin{aligned} u_x - u_0 &= A \xi & u &= A \xi \\ v_y - v_0 &= B \eta & v &= B \eta \\ w_z - w_0 &= C \zeta & w &= C \zeta \end{aligned} \quad \dots (3)$$

if it is understood that u, v, w , represent the relative velocity components between the two points. Also u_x, v_y, w_z

are the velocity components of the region G in the direction of the coordinate axis due to the location of the region projected onto that axis. The total change in the velocity in the x axis direction as an example would be given by

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz + \frac{\partial u}{\partial t} dt \quad . . . (4)$$

He basically uses the principle of superposition to circumvent Equation 4.

If a solid sphere is introduced within the region G with its center at the point x_0, y_0, z_0 and has a radius P which is small as compared with the Region G, then the equations of motion of the region that are affected by the sphere become,

$$\begin{aligned} u - u_1 &= A \xi \\ v - v_1 &= B \eta \\ w - w_1 &= C \zeta \end{aligned} \quad . . . (5)$$

where u, v, w are the velocity components of the portion of the fluid that are in contact with the surface of the sphere. This is pictorially seen in one dimension in Figure 9. Equation 5 may be clarified by the considerations shown in Figure 9. Taylor's theorem is used to expand the functions about the point x_0, y_0, z_0 to x, y, z.

$$\begin{aligned} u_x - u_0 &= A \xi = A(x - x_0) \\ u_x - u_1 &= A(x - x_1) \end{aligned} \quad . . . (6)$$

but $u_0 = u_1$ (the particle is solid, thus all portions must move at the same velocity).

Therefore,

$$u_x - u_1 = A (x - x_0)$$

then,

$$u = A\xi + u_1 \quad . . . (7)$$

It is assumed that vorticity and rotational motion of the particle or of any region of fluid does not occur when the suspension is set in motion due to the symmetry of the fluid surrounding the sphere. Thus the equation of motion of a region of the fluid containing a sphere is Equation 7.

Einstein follows the derivation with an argument of boundary conditions for the single sphere.

The boundary conditions at the surface of the sphere are,

$$\begin{aligned} u - u_1 &= 0 \\ v - v_1 &= 0 \end{aligned} \quad . . . (8)$$

$$w - w_1 = 0$$

$$\text{at } \rho = P$$

where $\rho = (\xi^2 + \eta^2 + \zeta^2)^{\frac{1}{2}} > 0$ and is a radius vector describing a sphere. Note that as ξ, η, ζ approach infinity in Equation 7 that u_1, v_1, w_1 must vanish if the fluid at an infinite distance from the sphere is not to be affected by the sphere. Equation 7 represents the solution to the problem; however, u_1 is not determined at this point.

Einstein now applies hydrodynamic theory to the problem to determine u_1, v_1, w_1 .

The hydrodynamic equations for a region of fluid, if body forces and inertial forces are neglected, are

$$\begin{aligned}\frac{\partial p}{\partial \xi} &= \mu \left(\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial \zeta^2} \right) \\ \frac{\partial p}{\partial \eta} &= \mu \left(\frac{\partial^2 v}{\partial \xi^2} + \frac{\partial^2 v}{\partial \eta^2} + \frac{\partial^2 v}{\partial \zeta^2} \right) \\ \frac{\partial p}{\partial \zeta} &= \mu \left(\frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \eta^2} + \frac{\partial^2 w}{\partial \zeta^2} \right) \\ 0 &= \frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} + \frac{\partial w}{\partial \zeta}\end{aligned} \quad \dots (9)$$

Using a method presented by G. Kirchhoff during a classroom lecture, he found the solution equations to Equations 5 and 9 to be

$$\begin{aligned}p &= -\frac{5}{3} \mu p^3 \left[A \frac{\partial^2 (1/\rho)}{\partial \xi^2} + B \frac{\partial^2 (1/\rho)}{\partial \eta^2} + C \frac{\partial^2 (1/\rho)}{\partial \zeta^2} \right] + \text{CONST.} \\ u &= A\xi - \frac{5}{3} \mu^3 A \frac{\xi}{\rho^3} - \frac{\partial \psi}{\partial \xi} \\ v &= B\eta - \frac{5}{3} \mu^3 B \frac{\eta}{\rho^3} - \frac{\partial \psi}{\partial \eta} \\ w &= C\zeta - \frac{5}{3} \mu^3 C \frac{\zeta}{\rho^3} - \frac{\partial \psi}{\partial \zeta} \\ \text{WHERE } \psi &= A \left[\frac{5}{6} \mu^3 \frac{\partial^2 \rho}{\partial \xi^2} + \frac{1}{6} \mu^3 \frac{\partial^2 (1/\rho)}{\partial \xi^2} \right] \\ &\quad + B \left[\frac{5}{6} \mu^3 \frac{\partial^2 \rho}{\partial \eta^2} + \frac{1}{6} \mu^3 \frac{\partial^2 (1/\rho)}{\partial \eta^2} \right] \\ &\quad + C \left[\frac{5}{6} \mu^3 \frac{\partial^2 \rho}{\partial \zeta^2} + \frac{1}{6} \mu^3 \frac{\partial^2 (1/\rho)}{\partial \zeta^2} \right]\end{aligned} \quad \dots (10)$$

It is shown that Equation 10 represents the only solution to Equation 4. This may be shown by direct substitution and because

$$\frac{\partial^2(1/\rho)}{\partial \xi^2} - \frac{\partial^2(1/\rho)}{\partial \eta^2} - \frac{\partial^2(1/\rho)}{\partial \zeta^2} = 0$$

$$\frac{\partial^2 \rho}{\partial \xi^2} + \frac{\partial^2 \rho}{\partial \eta^2} + \frac{\partial^2 \rho}{\partial \zeta^2} = \frac{2}{\rho}$$

$$\frac{\partial^2(\xi/\rho^3)}{\partial \xi^2} + \frac{\partial^2(\xi/\rho^3)}{\partial \eta^2} + \frac{\partial^2(\xi/\rho^3)}{\partial \zeta^2} = \dots (11)$$

$$\frac{\partial}{\partial \xi} \left[\frac{\partial^2(1/\rho)}{\partial \xi^2} + \frac{\partial^2(1/\rho)}{\partial \eta^2} + \frac{\partial^2(1/\rho)}{\partial \zeta^2} \right] = 0$$

He also shows that this solution satisfies the boundary conditions; i. e., that u, v, w , must vanish for $\rho = P$ and upon extending to infinity u, v, w reduce to Equation 1.

Equation 10 simplifies if higher order terms are neglected, if the continuity equation and assumptions are applied to show that many of the terms vanish or simply subtract out of the equations, and it becomes

$$\begin{aligned}
p &= -5\mu P^3 \frac{A\xi^2 - B\eta^2 - C\zeta^2}{\rho^6} \\
u &= A\xi - \frac{5}{2} P^3 \frac{\xi}{\rho^3} \left[A\xi^2 + B\eta^2 + C\zeta^2 \right] \\
v &= B\eta - \frac{5}{2} P^3 \frac{\eta}{\rho^3} \left[A\xi^2 + B\eta^2 + C\zeta^2 \right] \\
w &= C\zeta - \frac{5}{2} P^3 \frac{\zeta}{\rho^3} \left[A\xi^2 + B\eta^2 + C\zeta^2 \right]
\end{aligned} \quad \dots (12)$$

Now he combines Equation 12 to gain a relationship between the viscosities of the solvent and the suspension containing the sphere. This is accomplished by comparing the work of expanding the solvent with the work of expanding the suspension. To do this he creates a model that consists of a spherical region, R , about the single solid sphere within the solvent. Then the work rate of the expansion of this region is given by

$$\dot{W} = \int_R (X_n u + Y_n v + Z_n w) ds \quad \dots (13)$$

where x_n , y_n , z_n are pressures normal to the surface and s is the surface area of the solid sphere.

The transformation function for x_n , y_n , z_n are

$$\begin{aligned}
x_n &= - \left(x_\xi \frac{\xi}{\rho} + x_\eta \frac{\eta}{\rho} + x_\zeta \frac{\zeta}{\rho} \right) \\
y_n &= - \left(y_\xi \frac{\xi}{\rho} + y_\eta \frac{\eta}{\rho} + y_\zeta \frac{\zeta}{\rho} \right) \\
z_n &= - \left(z_\xi \frac{\xi}{\rho} + z_\eta \frac{\eta}{\rho} + z_\zeta \frac{\zeta}{\rho} \right)
\end{aligned}
\quad \dots (14)$$

It should also be noted that

$$\begin{aligned}
x_\xi &= p - 2\mu \frac{\partial u}{\partial \xi} \\
y_\eta &= p - 2\mu \frac{\partial v}{\partial \eta} \\
z_\zeta &= p - 2\mu \frac{\partial w}{\partial \zeta}
\end{aligned}
\quad \dots (15)$$

and

$$\begin{aligned}
y_\xi &= z_\eta = -\mu \left(\frac{\partial v}{\partial \xi} + \frac{\partial w}{\partial \eta} \right) \\
z_\xi &= x_\zeta = -\mu \left(\frac{\partial w}{\partial \xi} + \frac{\partial u}{\partial \zeta} \right) \\
x_\eta &= y_\zeta = -\mu \left(\frac{\partial u}{\partial \eta} + \frac{\partial v}{\partial \zeta} \right)
\end{aligned}
\quad \dots (16)$$

Substitution of these functions and the equations of motion (u , v , w) into Equation 12 yields the equation,

$$W \propto u \left(v + \frac{1}{2} \phi \right) \quad . . . (17)$$

where v is the volume of the region R and ϕ is the volume of the solid sphere.

In a procedure similar to the foregoing one Einstein develops the equation that relates the viscosity of a fluid containing numerous evenly distributed, identical size spheres within the solvent. His final equation is

$$\mu^* = \mu (1 + 2.5 \Gamma) \quad . . . (18)$$

One view of Einstein's Equation 17 is that additional work due to internal friction occurs within the solvent when it contains solid spheres. This additional internal work of friction should manifest itself as an additional rise in temperature of the suspension.

HATSCHEK'S MATHEMATICAL MODEL

Hatschek's mathematical model partially consists of determining the size and packing arrangement of right angle prisms that have been distorted due to flow from an hexagonal prism that in turn is assumed to approximate an orderly packed set of dodecahedrons of the cubic structure system. The remainder of the model consists of applying an energy balance to the packing arrangement of the right angle prisms in order to relate the viscosity of foam to the quality of foam. The applicable figures are shown in Figures 10 and 11.

He establishes the height of the hexagonal prism $r\sqrt{3}$ by assuming that the dimension of the dodecahedron in the direction of flow will be conserved in the hexagonal prism. If r is the side length of the hexagonal prism and δ is the side length of the dodecahedron, then

$$\delta = 3r/\sqrt{8}$$

Since the volumes of all three solids are to be equivalent through the distortions, the width of the hexagonal

prism must be $r\sqrt{2}$. This width is also conserved in the right angle prism.

If laminar flow of the hexagonal prisms is to occur, then each staggered row must be distorted to fit between adjacent rows. This requires that each right angle prism be one-half the height of the hexagonal prism from which it was generated. Thus, the height of the right angle prism is $r\sqrt{3}/2$. The length of the right angle prism is specified if the volume of the dodecahedron is conserved in the right angle prism through these distortions. The volume of the dodecahedron is $6\delta^3/\sqrt{27}$ or $r^3\sqrt{27}/2$. The frontal area of the right angle prism is $r^2\sqrt{3}/2$ and thus the length of the prism is $3r$.

If the particles retain their right angle prismatic shape, then distortion due to flow may only take place in the horizontal laminas of the continuous phase.

Imagine two solids of cubic structures of unit dimensions. Further suppose that one of the two cubes is filled with thin right angle prisms while the other is continuous. The work necessary to maintain a differential velocity between each prism within the cube is

$$W_i \propto \mu n v^2$$

where n is the number of prisms, v is the velocity differential, and μ is the viscosity of the continuous phase.

If d is the distance between laminas, then,

$$W_l \propto \mu \frac{1}{d} v^2$$

The similar work for the continuous cube is

$$W \propto \mu v^2$$

If the same form for the two equations is to be maintained, then, by comparison

$$\mu_t = \mu \frac{1}{d}$$

where μ_t is the viscosity of the lamina system which may be thought of as foam.

If the distance between laminas is δ_1 and the height of the prisms is δ , then the following ratio holds,

$$\frac{\delta}{\delta_1 + \delta} = \frac{1}{d}$$

If f is the total volume of the system divided by the volume of the solid prisms, then

$$f = \frac{\delta_1^3}{\delta^3}$$

Elimination of δ_1 and δ between the two preceding equations yields

$$\frac{1}{d} = \frac{\sqrt[3]{f}}{\sqrt[3]{f} - 1}$$

Elimination of $\frac{1}{d}$ between this equation and $\mu_s = \mu \frac{1}{d}$ yields

$$\mu_s = \mu \frac{\sqrt[3]{f}}{\sqrt[3]{f} - 1}$$

This is Hatschek's equation which expresses the viscosity of foam as a function of the reciprocal of foam quality and the viscosity of the fluid which forms the continuous phase.

RABINOWITSCH'S MATHEMATICAL MODEL

The change in momenta across the cylindrical surface of the tube as shown in Figure 12 is

$$\Delta(2\pi L r \tau_{rz}) \quad . . . (19)$$

The change in momenta across the ends of the shell surfaces is the difference in the pressure forces

$$(2\pi r \Delta r) \Delta P \quad . . . (20)$$

The momenta must balance and if both terms are divided by Δr and the limit is taken as $\Delta r \rightarrow 0$; then

$$\frac{d(r\tau_{rz})}{dr} = \frac{r\Delta P}{L} \quad . . . (21)$$

Integration of this equation yields,

$$\tau_{rz} = \frac{\Delta P}{2L} r + \frac{C}{r} \quad . . . (22)$$

The constant of integration must be zero if the shear stress is to be finite at the center of the tube; thus,

$$\tau_{rz} = \frac{\Delta P}{2L} r \quad . . . (23)$$

The shear stress at the wall of the pipe is

$$\tau_{wz} = \frac{\Delta P}{2L} r_w = \frac{D\Delta P}{4L} \quad . . . (24)$$

then

$$\tau_{rz} = \frac{2\tau_{wz}}{D} r \quad . . . (25)$$

and then differentiation gives

$$d\tau_{rz} = \frac{2\tau_{wz}}{D} dr \quad . . . (26)$$

Now it was assumed that the shear rate within the fluid at a point can only be a function of the shear stress at that point.

$$\phi_{rz} = C\phi(\tau)_{rz} \quad . . . (27)$$

From the usual definition of viscosity and in consideration of a circular tube the shear rate takes the form

$$\left(-\frac{dv}{dr}\right)_{rz} = \phi_{rz} = C\phi(\tau)_{rz} \quad . . . (28)$$

Then

$$dv_{rz} = C\phi(\tau)_{rz} dr_{rz} \quad . . . (29)$$

Substitution of Equation 26 into 29 to eliminate the factor dr_{rz} yields:

$$dv_{rz} = C\phi(\tau)_{rz} \frac{D}{2\tau_{wz}} d\tau \quad . . . (30)$$

The velocity distribution within the flow tube may now be determined by solving the preceding differential equation.

$$v_{rz} = \frac{CD}{2\tau_{wz}} \int_{\tau}^{\tau_w} \phi(\tau)_{rz} d\tau + \text{CONSTANT} \quad . . . (31)$$

If, as it was assumed, there was no fluid slippage at the wall of tube then the constant of integration is zero.

The mean value theorem of calculus is applied to find the average velocity, v , of the fluid, then,

$$v = \frac{\int v_{rz} dA}{\int dA} \quad . . . (32)$$

also, $A = 2\pi r dr$

$$A = 2\pi \frac{D}{2\tau_{wz}} \tau * \frac{D}{2\tau_{wz}} d\tau_{rz} \quad . . . (33)$$

$$\int_{r=0}^{r=r_w} dA = \pi r_w^2$$

The boundary conditions are,

$$\begin{aligned} r=r_w, \tau=\tau_w, v=0 \\ r=0, \tau=0, v=\text{max.} \end{aligned} \quad . . . (34)$$

Substituting Equation 31 and 33 into 32 and factoring yields,

$$\frac{v}{D} = \frac{C}{\tau_{wz}^3} \int_0^{\tau_w} \left[\int_{\tau}^{\tau_w} \phi(\tau) d\tau \right] \tau d\tau \quad . . . (35)$$

Integration of this equation by parts results in an equation for laminar pipe flow. The integration scheme is

$$\begin{aligned} \text{LET } u &= \int_{\tau}^{\tau_w} \phi(\tau) d\tau & \text{THEN } du &= -\phi(\tau) d\tau \\ & & & \dots (36) \\ dv &= \tau d\tau & \text{THEN } v &= \tau^2/2 \end{aligned}$$

The fundamental mathematical equation for integration by parts is

$$\int_0^x u dv = uv \Big|_0^x - \int_0^x v du \quad \dots (37)$$

Substituting into the mathematical equation yields,

$$\int_0^{\tau_w} u dv = \left[\frac{\tau^2}{2} \int_{\tau}^{\tau_w} \phi(\tau) d\tau \right]_0^{\tau_w} - \int_0^{\tau_w} \frac{\tau^2}{2} (-) \phi(\tau) d\tau \quad \dots (38)$$

The first term on the right side of the equal sign is equal to zero for all values of τ ; thus,

$$\left(\frac{2v}{CD} \right) \tau_w^3 = \int_0^{\tau_w} \frac{\tau^2}{2} \phi \tau d\tau \quad \dots (39)$$

Equation 38 is differentiated with respect to τ_w and is simplified to the following equation:

$$3 \left(\frac{2v}{D} \right) \tau_w^2 + \frac{d(2v/D)}{d\tau_w} \tau_w^3 = C \phi(\tau) \tau^2 \quad \dots (40)$$

It is assumed that τ is to be evaluated at the tube wall and recalling,

$$C\phi(\tau_{wz}) = \left(-\frac{dv_r}{dr} \right)_{wz} \quad . . . (28)$$

$$\tau_{wz} = \frac{D\Delta P}{4L} \quad . . . (24)$$

thus,

$$\tau = \tau_{wz}$$

$$\left(-\frac{dv_r}{dr} \right)_w = \frac{3}{4} \left(\frac{8v}{D} \right) + \frac{1}{4} \left(\frac{D\Delta P}{4L} \right) \left(\frac{d(8v/D)}{d(D\Delta P/4L)} \right) \quad . . . (41)$$

This equation relates the shear rate of a fluid with the shear stress as measured at the wall of the tube and permits the determination of an absolute flow curve for that fluid.