THE UTILIZATION OF VERTICAL ACCELERATION DATA IN DETERMINING ACCUMULATED FATIGUE DAMAGE ON AIRCRAFT STRUCTURES

By

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Submitted to the Faculty of the Graduate School of
the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
June, 1962

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PREFACE

It has been theorized that if the vertical acceleration history of an aircraft were recorded, the data could be utilized to determine the accumulated fatigue damage on the aircraft structure, to predict fatigue failures, and to schedule structural modifications.

This thesis discusses the processing of vertical acceleration data and the additional information which would be necessary to conduct a fatigue analysis. An analysis procedure is presented and discussed.

Indebtedness is acknowledged to Professor Roger L. Flanders and Professor Raymond E. Chapel for their valuable guidance, to Mr. N. A. Piercy, Mr. R. W. Blakeburn, and Mr. John Huff, of Service Engineering Division, Oklahoma City Air Materiel Area, and Mr. Woods, I. B. M. Representative, for their assistance in preliminary research and in reviewing preliminary copies of this manuscript, and to Mr. William E. Jones, Deputy Chief, Service Engineering, Oklahoma City Air Materiel Area, for his assistance while I was temporarily employed at Oklahoma City Air Materiel Area.

J. D. T.

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NOMENCLATURE

A/C . . . . Aircraft.

A . . . . Wing area.

C . . . . Configuration type.

C G . . . . Center of gravity.

c . . . . Average wing chord.

cps . . . . Cycles per second.

D . . . . Fatigue damage due to n cycles.

D' . . . . Fatigue damage due to one particular cycle or spectrum.

D_T . . . . Total fatigue damage.

d . . . . Fatigue damage rate.

d' . . . . Fatigue damage rate due to a particular gust spectrum.

EDP . . . . Electronic Data Processing.

EDPE . . . . Electronic Data Processing Equipment.

FPS . . . . Feet per second (velocity).

FPS^2 . . . . Feet per second per second (acceleration).

G . . . . Vertical acceleration; vertical acceleration range.

GAG . . . . Ground-air-ground cycle.

g . . . . Unit of acceleration (load factor) due to gravity, 32.2 feet per second per second.

H . . . . Height (pressure altitude); height range.

IAS . . . . Indicated airspeed.

K_M . . . . .Ratio of absolute value of maximum acceleration peak, ω to minimum acceleration peak β for a maneuver.

K_t . . . . .Stress concentration factor.

K_W . . . . .Gust factor.

M . . . . .Mission type.

m . . . . .Slope of the lift curve.

n_z . . . . .Normal acceleration.

n . . . . .Number of stress cycles endured.

N . . . . .Number of stress cycles required to cause failure.

$\bar{n}_{z_p}$ . . . . .Mean normal acceleration for vertical acceleration range p.

R . . . . .Rate of change of gross weight.

S . . . . .Stress.

S_{1g} . . . . .Stress level at n_z = 1.0g.

S_{al} . . . . .Stress due to air loads.

S_{gl} . . . . .Stress due to ground loads.

S_a . . . . .Alternating stress.

S_m . . . . .Mean stress.

S_A . . . . .Alternating stress at $S_m = 0$.

S_u . . . . .Ultimate stress.

S_e . . . . .Endurance limit.

S_y . . . . .Yield stress.

T . . . . .Time since the recorder was turned on.

T(with subscript) . . .Time from recorder-on to time at point indicated by subscript.

T_i . . . . .Time spent at flight condition "i".

U_{d_e} . . . . .Derived gust velocity

V . . . . Indicated airspeed, airspeed range.

V_e . . . . Equivalent airspeed.

W . . . . Gross weight.

W_{max} . . . . Maximum take-off gross weight.

$\overline{W}_c$ . . . . Mean weight for weight range c.

$\frac{1}{X}$ . . . . Slope of the Log S- Log N curve.

γ . . . . Data handling factor.

ν . . . . Dynamic factor.

ρ . . . . Air density.

μ . . . . $\frac{2\frac{W}{S}}{gcm}$.

Δ . . . . A change or a small increment.

$\Delta S/\Delta N$ . . . . . . Change of stress ÷ change in vertical acceleration.

Σ . . . . Summation.

ω . . . . Peak during maneuver cycle. (Maximum).

β . . . . Peak during maneuver cycle. (Minimum).

SUBSCRIPTS

a . . . . Alternating.

a . . . . Velocity range.

b . . . . Height range.

c . . . . Weight range.

d . . . . Mission type.

e . . . . Configuration.

i . . . . Instantaneous value.

m . . . . Average.

r . . . . Line number, or range, on log S-log N curve.

CHAPTER I

INTRODUCTION

The failure of aircraft structures due to metal fatigue is a major problem. Although this type of failure was almost unheard of in early types of aircraft, a number of recent developments have combined to magnify the problem. These include higher design stresses, higher speeds and higher operating temperatures.

Fatigue failures result from large numbers of stress cycles, and failure occurs sooner if the cyclic stress level is high. Stress cycles result from maneuvers caused by the pilot as well as from the gusts or turbulence in the atmosphere. While the frequency of maneuver cycles has remained about the same for the same type of mission, a decrease in the margin of safety has the effect of increasing the stress levels resulting from maneuver loads. The probability of encountering a vertical gust is directly proportional to the distance traveled. Therefore, an increase in speed will increase the number of vertical gusts encountered per unit of time. The increase in speed also has a secondary effect on fatigue life. Extremely high speeds will cause aerodynamic heating of the aircraft structure. The number of stress cycles required to cause a fatigue failure decreases as the temperature of a metal is increased.

If no preventive action is taken, fatigue damage continues to accumulate until a catastrophic failure occurs. By timely inspections, modifications and repairs, the service life of an aircraft can be increased appreciably. However, in order to determine the proper time for these preventive measures, the accumulated fatigue damage must be known for each critical point on each individual aircraft.

The three types of information required for fatigue damage calculations are:

1. Constants for each type of aircraft.
2. A history of the high-rate-of-change variables which affect stress levels.
3. A history of the low-rate-of-change variables which affect stress levels.

The first type of information consists of constants for each critical point on each type of aircraft and must be obtained by testing. These constants include information such as the fatigue characteristics of each critical point, the change in stress as a function of the change in vertical acceleration, and the dynamic characteristics of the aircraft structure.

The second type of information consists of data which must be recorded continuously during each flight. These rapidly varying parameters are airspeed, altitude and vertical acceleration at the aircraft center of gravity (C. G.).

The third type of information consists of the parameters that vary less rapidly, such as gross weight, aircraft configuration, mission being performed and aircraft identification number. The gross weight does vary continuously, but the rate of change over a short period of time may be considered to be constant. Therefore, it is only necessary to record the gross weight periodically. A linear change between recorded data values is then assumed. The allowable interval between data points depends upon the type of aircraft and the type of mission being performed. In the case of a large bomber, it is suggested that gross weight be recorded at takeoff, at the end of climbout, before and after refuelings, after bomb drops, and every two hours during high level cruise. Errors due to this linear assumption will not exceed one per cent, and will generally be much less than that.

The second type of data must be continuously recorded in analog form, but there are two feasible methods for recording the third type. One method is to make entries on a form, giving the gross weight, the type of mission being performed, the configuration and the time since the recorder was turned on. (This last item is necessary in order to correlate the tabulated data with the analog data.) An alternate method is to have an audio channel on the analog recorder and require periodic vocal recordings of configuration, mission type and gross weight. This audio method has several disadvantages. In some instances the voice would not be audible. Also, this procedure makes it necessary to replay the recorded data at the same speed at which it was recorded, rather than at some accelerated speed. Offsetting these are several advantages, such as always having the audio data associated with the proper analog data, and having it correctly associated as to time since the beginning of the flight. The time correlation of two related groups of data is frequently a major problem in data analysis. The audio method is preferred because of the perfect time correlation. However, the necessity for data reduction at accelerated speeds, such as 100 to 1, and the expense of the added audio capability of the recorder will dictate the use of a form. This form, designated here as Form A, will be used by some crew member to record configuration, mission type, gross weight, and time since takeoff.

The equipment that is required for this project - the determination of accumulated fatigue damage on inservice aircraft - is divided into four categories, as listed below:

1. Test equipment.
2. Airborne equipment.
3. Data reduction equipment.
4. Data analysis equipment.

The first category of equipment is used to obtain the aircraft constants which were previously mentioned. This category would include large capacity fatigue testing equipment such as the equipment that is available in the Oklahoma State University Materials Testing Laboratory. It also includes specially designed test apparatus for full scale fatigue tests of complete aircraft structures as well as the various airborne recorders and transducers used to determine dynamic factors and stress levels during flight. These items will not be discussed in detail here.

The second category consists of 1) an analog recorder, 2) transducers to measure airspeed, vertical acceleration and the time since the recorder was turned on, and 3) the "hardware" necessary for the installation of these items. The recorder must be a very accurate analog recorder with at least four recording channels. It must have a capacity for recording data for the duration of the longest expected flights. Typical of aircraft instrumentation requirements, the recorder must be of light weight construction, with low power requirements. It must be designed so that large changes in temperature will not affect the accuracy of the recorded data. Similar requirements apply to the transducers.

The third category of equipment is the data reduction equipment. (It should be noted here that the magnitude of data that would be recorded in this program makes it necessary to use a statistical approach to data handling. This is in accordance with the usual approaches to fatigue analysis. Instead of using the exact values for each one of the data points, data are divided into increments or ranges. The number of data points falling in each range must be determined and handled as a unit during computations.) The reduction equipment consists of a record player to play back the data off of the analog tape, an editor which will pick out certain points of data which are of interest, and an analog to digital converter which will translate the chosen data points into digital values. The principal types of desired information are the amount of time spent at each flight condition* and the number and magnitude of vertical acceleration peaks that occur during each flight condition. To determine the amount of time spent at each flight condition it is necessary for the editor to monitor the airspeed and altitude data. Each time the airspeed or altitude range changes the new range must be recorded along with the time since the beginning of the flight. For the peak acceleration data, it is necessary to edit the acceleration data and record peak accelerations as well as the corresponding airspeed, altitude and time since the recorder was turned on. This information makes up what may be termed a peak data word.

The fourth type of equipment is the data analysis equipment, consisting of the digital computer and associated equipment that is necessary to perform the fatigue analysis. The proposed fatigue analysis

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* A flight condition is defined as a particular combination of ranges of airspeed, altitude, gross weight, configuration and mission type.

procedure is summarized below and is presented graphically in figure 1-1.

Summary of Analysis Procedure

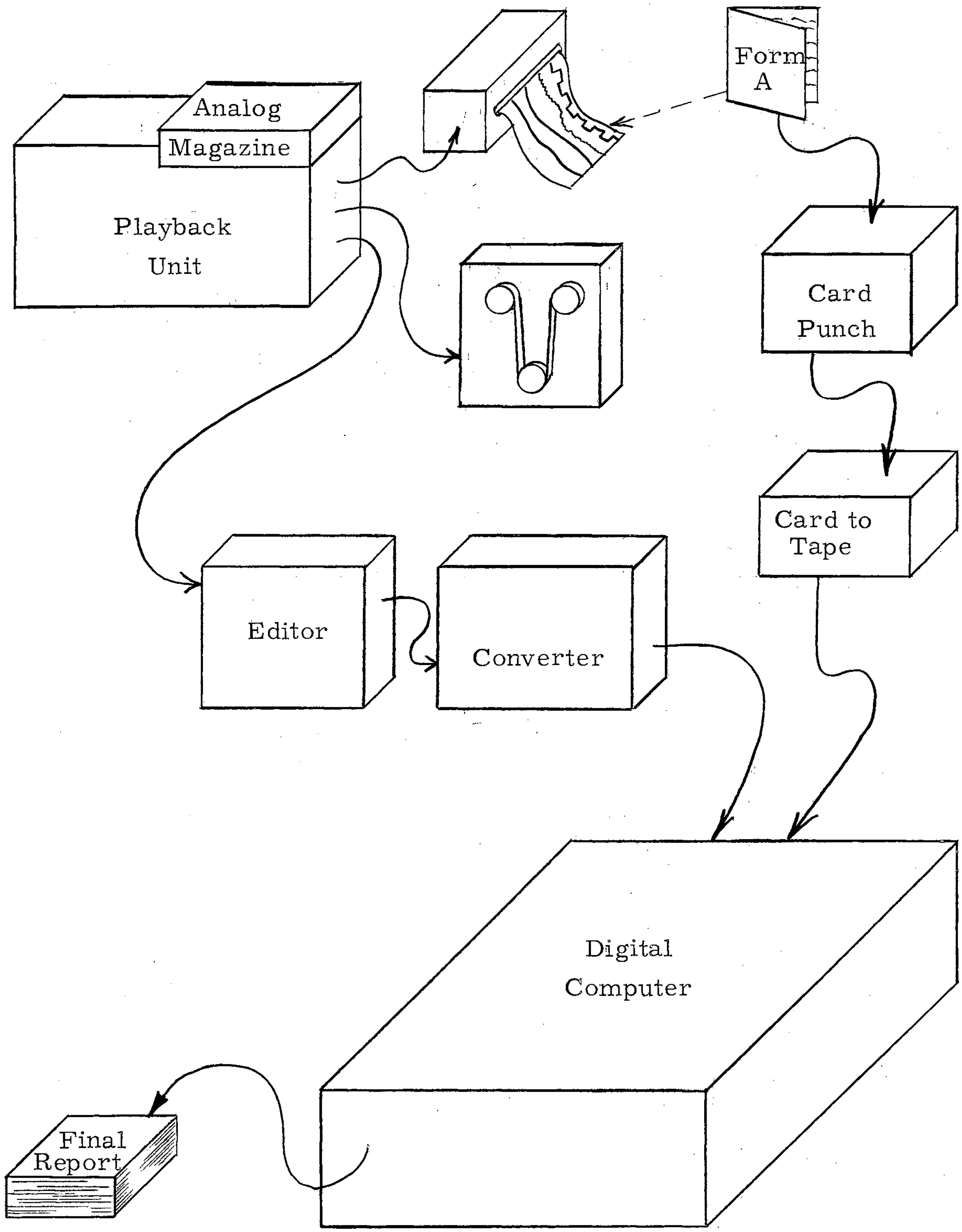
The airspeed, altitude, vertical acceleration and time since takeoff are converted from analog to digital data. The gross weight, mission type, configuration and time since takeoff are converted from tabulated data to digital form and recorded on a magnetic tape. The first calculation is the determination of the time at which gross weight changes occurred. Then the data are combined into the total number of vertical accelerations of each magnitude for each flight condition. This total is made up of data from many flights. The total amount of time spent at each of these flight conditions is also tabulated. The vertical acceleration data are divided into gust and maneuver cycles and then converted into stress cycles for each critical point. The stress data are converted into values of mean and alternating stresses. The damage rates for each critical point are then calculated for each flight condition. These damage rates may be used to calculate accumulated damage on each individual aircraft, to evaluate the original fatigue test spectra, to evaluate new missions or to predict the service life expectancy.

Chapter II presents a review of information on metal fatigue, Chapter III discusses aircraft fatigue problems in particular and Chapter IV outlines the fatigue analysis procedure in detail.

Appendix A describes a procedure for determining the exact weight ranges to be used.

Figure 1-1

ELECTRONIC DATA PROCESSING (EDP) FLOW CHART



CHAPTER II

A REVIEW OF METAL FATIGUE

There are several theories concerning the basic principles of metal fatigue. These theories maybe divided into three groups (1). * One group of theories postulates some sequence of movement and interaction which produces such experimentally observed actions as slip bands. The second group are statistical theories which predict the probability of failure of metallic bonds under certain cyclic loadings, with failure being defined as the breaking of a certain number of bonds. The third group of theories are those which are based on continuum of either elasticity or plasticity theories.

Certain facts are known about fatigue of metals, and any theories about metal fatigue will have to be compatible with these facts. Some of these are listed below (2).

1. Fatigue failures occur in most metals, metallic alloys, wood, plywood, and in some plastics.
2. Fatigue failure is dependent upon the number of stress cycles, rather than the time under load. Under normal conditions the cyclic rate (from 100 cps to 10, 000 cps) does not affect the fatigue life.
3. Some metals have an endurance limit, a stress level below which cyclic loading will not cause failure.
4. For most materials a stress concentration will greatly reduce the

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* See Reference No. 1 in Bibliography

fatigue life.

5. The alternating stress required to cause a failure in a particular number of cycles decreases as the mean stress becomes more tensile.

Presentation of Fatigue Test Data

The most frequently used method of presenting Fatigue Test Data is the S-N curve. It is a graph showing the number, N, of stress cycles, S, required to cause failure for each magnitude of alternating stress. An S-N curve is made up entirely of test data from identical specimens tested at the same mean stress, the same temperature and usually the same test speed. The same type of testing device is used for all tests, and at least 5 specimen tests should be conducted for each data point on the curve.

The S-N curve is usually plotted on semi-log paper, and forms a curve as shown in figure 2-1. However, if the S-N data is plotted on log-log graph paper, it forms a series of straight lines as is shown in figure 2-2. The equation

$$N_i = N' \left(\frac{S_{A_i}}{S_A'} \right)^M \tag{2-1}$$

can be developed from a logS-logN curve. In this equation

N_i = Number of cycles to failure for a particular value of S_{A_i}.

S_{A_i} = The alternating stress for a stress cycle which has $S_m = 0$.

S_A' = The first alternating stress greater than S_{A_i} at which a change in direction occurs on the LogS-LogN curve.

$\frac{1}{M}$ = Slope of LogS-LogN curve at S_{A_i}.

Figure 2-1

SEMILOG S-N CURVE

Stress (KSI) Versus Number of Cycles to Failure

Data for 7075S-T6 Plate, $K_M = 0$, $K_y = 2.0$ (From pg. 222, Ref. 3)

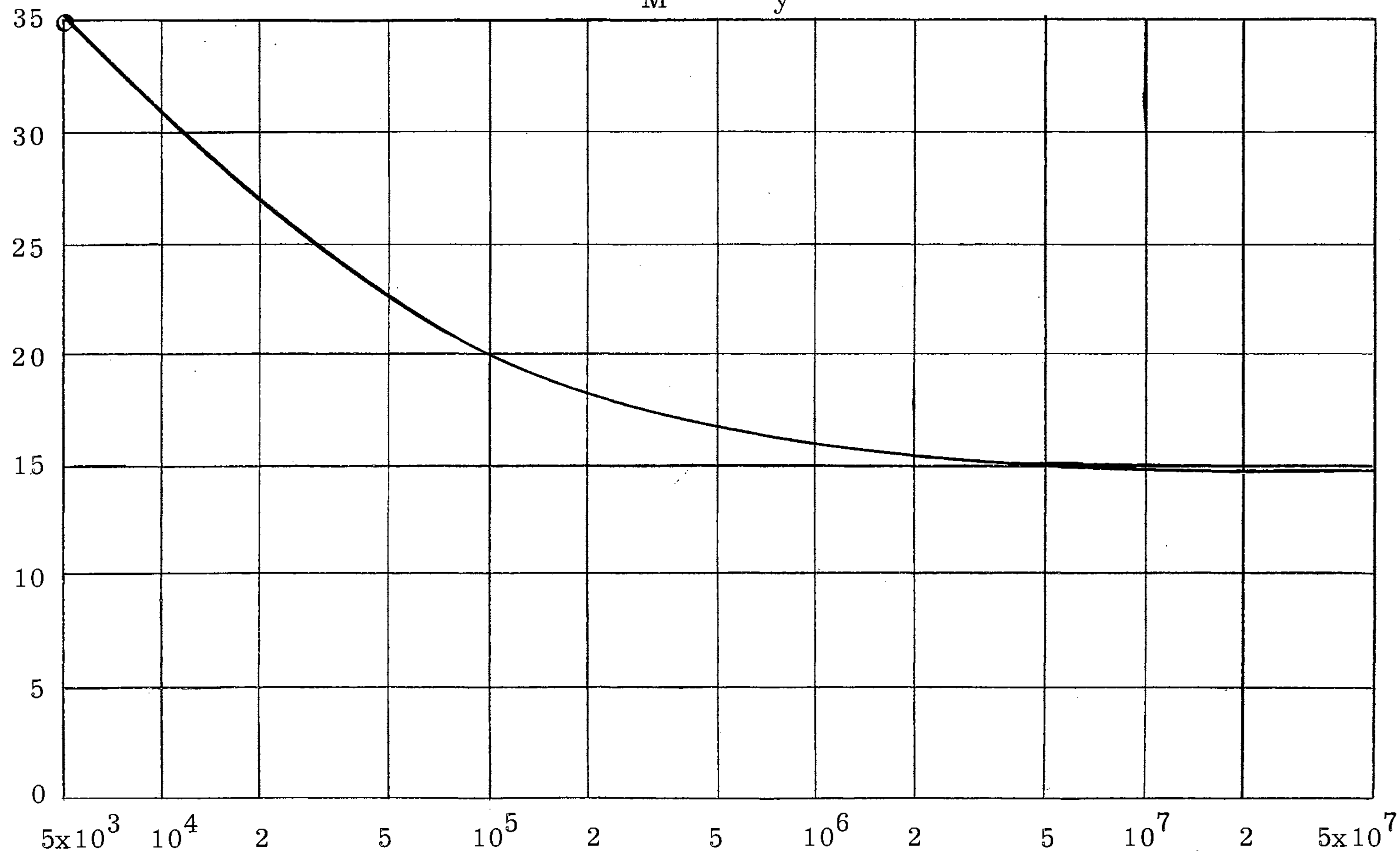
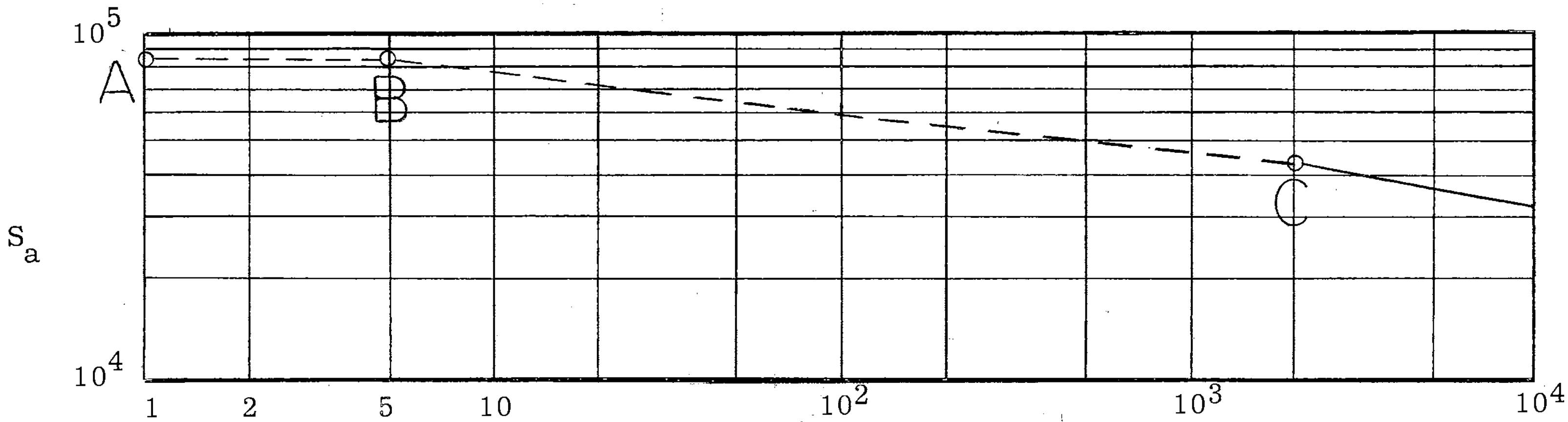


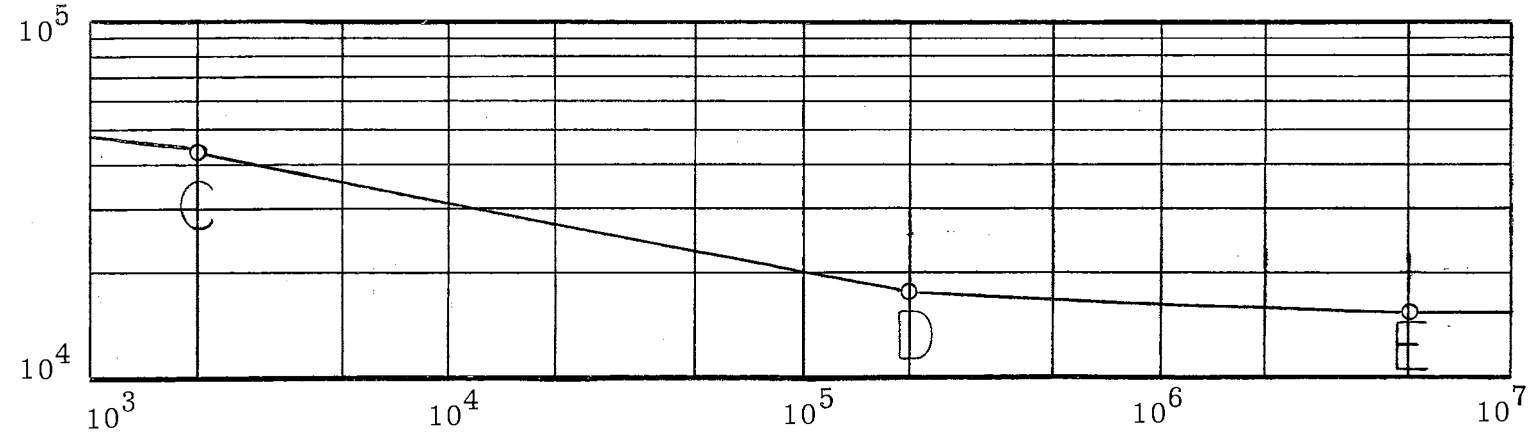
Figure 2-2

LOG-LOG S-N CURVE

(This is the Same Data as in Fig. 2-1)



(a) N ⟶



(b) N ⟶

There are a number of other methods of presenting fatigue test data. In some cases it is presented in tabular form, giving the alternating stress ($S_m = 0$) that will cause failure at 10^7 cycles, 10^6 cycles, etc. In some cases only the endurance limit* is given. Other methods include: families of S-N curves, with curves for various values of the stress concentration factor, K_t; or for various values of S_m. When enough data points are available, they may be plotted as alternating stress verses mean stress. When this is done, all of the data may have the same value of K_t, and each curve indicates a particular life, or all data may be from tests with the same life, and each curve represents a particular value of K_t. Examples of these curves are given in figure 2-3.

All of the above methods of presenting data, with the exception of LogS-LogN curves, were used in reference 3.

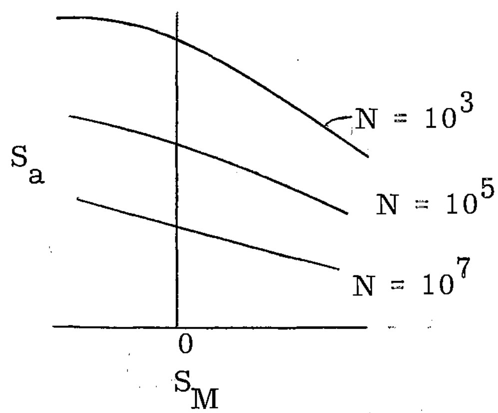
Due to the large scatter inherent in fatigue test data, any presentation of data indicating the number of cycles required to cause failure should also state what portion of the failures will occur prior to that life. Data is usually given for a life that at least 90% of the specimens will exceed, but the average life is also used in some instances.

Figure 2-4 is a graph of stress level verses time. It is presented in order to better define some of the more frequently used symbols.

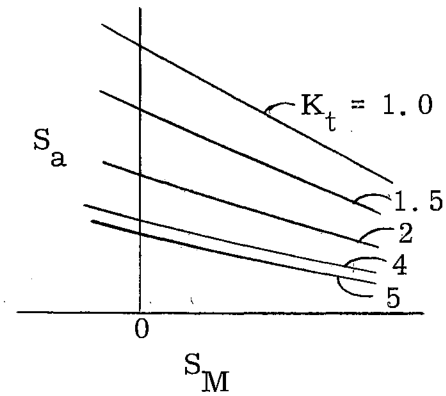
* The endurance limit is the highest alternating stress ($S_m = 0$) that will not cause failure, regardless of the number of repetitions.

Figure 2-3

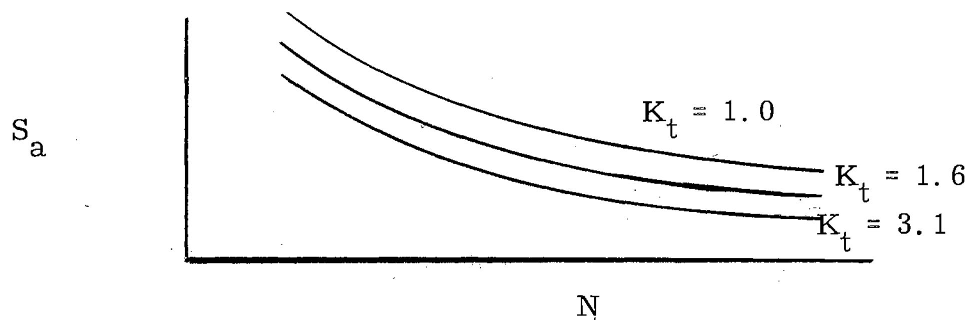
EXAMPLES OF DATA PRESENTATION



(a) $K_t = 2.0$



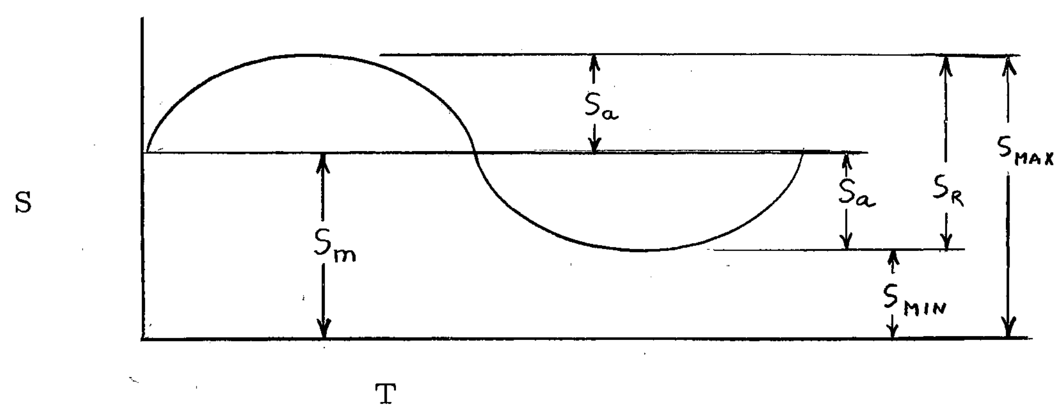
(b) $N = 10^5$



(a) $S_M = 0$

Figure 2-4

STRESS VERSUS TIME (ONE CYCLE)



Extrapolation of Test Data

A reliable S-N curve requires at least 20 fatigue tests, and then only provides data for one particular value of K_t and one value of S_m. It becomes obvious that data can not be obtained for every combination of S_a, S_m and K_t, and some method of extrapolating test data is required. Several methods are available, such as the Gerber Law, the Soderberg Straight Line Law and the Modified Goodman Law. * In the case of a particular critical point on an aircraft structure, the value of K_t will be a constant, but the stress cycles will have large variations of both S_a and S_m. Each of the three laws above provides for determining an "equivalent" stress cycle for any combination of S_a and S_m. The equivalent stress cycle has a mean stress of zero. Thus, since K_t is constant at a particular point and since any stress cycle, regardless of the values of S_a and S_m, can be converted to an equivalent stress cycle with $S_m=0$, then only one S-N curve is required for each critical point. As the test specimens required for this S-N curve are expensive duplications of structural joints and splices, the cost of test data prohibits use of more than one S-N curve per critical point.

In comparing the relative merits of the Gerber, Modified Goodman and Soderberg Laws, approximately 100 graphs of cyclic test data were considered (3). The data were plotted as S_a verses S_m, with curves representing various values of N for a particular value of K_t, or with curves representing various values of K_t for a particular value of N. Examples are shown in figure 2-3.

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* It should be noted that most fatigue critical points on aircraft structures have K_t values of approximately 3.5 (4, 5, 6).

The Modified Goodman Law (18) implies that the alternating stress that will cause a fatigue failure in N_i cycles decreases linearly as the mean stress increases from 0 to S_u. The Soderberg Law is similar to the Modified Goodman Law, except that it uses the values of S_y instead of S_u. The Gerber Law uses a parabola instead of a straight line, with the end points of the parabola at $\pm S_u$. These theories are presented graphically in figure 2-5.

While the Gerber Law best fits graphs of test data where $K_t = 1$, the Modified Goodman Law is much more realistic when the value of K_t is above 2. As previously noted, fatigue-critical aircraft structures usually have a value of K_t of approximately 3.5. It is therefore proposed that the Modified Goodman Law be used to extrapolate test data for this type of an analysis.

Using the Modified Goodman Law, the equation for S_{A_i} is:

$$S_{A_i} = \frac{S_{a_i} \; S_u}{S_u - S_{m_i}} = \frac{S_{a_i}}{1 - \dfrac{S_{m_i}}{S_u}} \tag{2-2}$$

where the terms are defined as:

S_{A_i} = The alternating stress, for $S_m = 0$, that will cause failure in the same number of cycles, N_i, that would cause failure of stress cycles of S_{a_i}, S_{m_i}.

S_{a_i} = Instantaneous alternating stress.

S_{m_i} = Instantaneous mean stress.

S_u = Ultimate stress.

Figure 2-5 (a)

MODIFIED GOODMAN CURVE

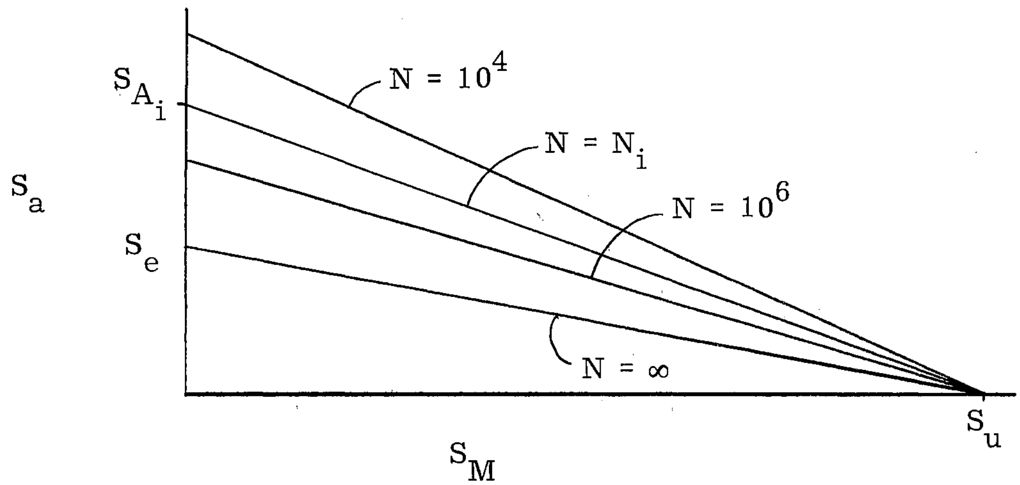
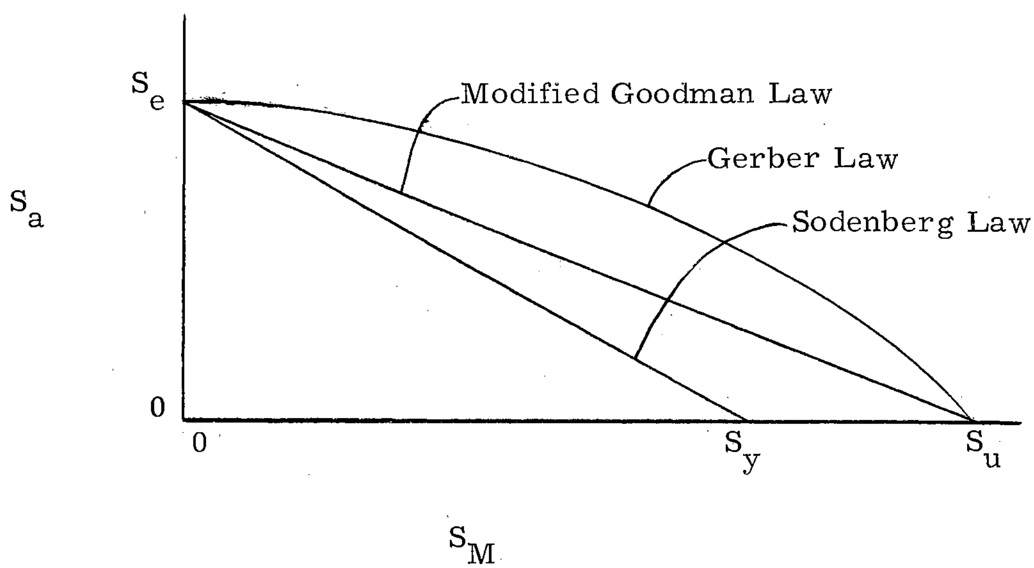


Figure 2-5 (b)

COMPARISON OF EXTRAPOLATION CURVES

FOR N = ∞



Cumulative Damage Theories

Most of the cumulative fatigue damage relations so far advanced have one or more of the following limitations:

1. No physical mechanism is clearly defined so the relation contains factors identifiable with concepts useful in design.
2. Too many experimental data are required for engineering application.
3. The mathematical calculations are too cumbersome.

A satisfactory cumulative damage theory has been sought for the last 35 years. Several have been proposed. The most frequently used is the one by Palmgren (7) later suggested by Miner (8). This theory states that the damage incurred is

$$D_i = \frac{n_i}{N_i} \tag{2-3}$$

where $\frac{n_i}{N_i}$ is the cycle ratio at stress S_i. If cycling at stress S_i is continued until $n_i = N_i$, $D_i = 1$ and the failure occurs. Their theory is that when several stress levels occur, $D_T = \Sigma_i D_i = \Sigma \frac{n_i}{N_i}$ and failure will occur when $D_T = 1$.

With this theory, the only data required is the S-N curve, the expected load spectra, and a detailed stress analysis. Test data from some multi-stress level tests do not substantiate this theory (9, 10).

Rechart and Newmark (9, 10) outlined empirical formulas based upon the theory that the damage rate, in terms of the cycle ratio, is considered stress dependent. This procedure is considered by the author of Ref. 11 to be a more accurate one than Miner's, but requires an extremely large amount of data.

Marco and Starkey (12) suggested a modification of Miner's

Theory, where $D_i = \left(\frac{n_i}{N_i}\right)^{\alpha_i}$. They suggested that there may be "a correlation between stress level and the number of fatigue nuclei which result in independent cracks" which would explain the variation of α_i with S_i, where α_i is a constant to be determined experimentally. They also emphasized the necessity of taking the statistical nature of fatigue data into consideration, since a particular specimen will have an S-N curve which will probably differ from the average S-N curve for similar specimens.

Freudenthal (13) suggests the formula: $N_T = \dfrac{1}{\Sigma_i \dfrac{\rho_i}{N_i}}$ (2-4)

but in the case of a greater damaging effect due to the action of a few high level stresses, he recommends: $N_R = \dfrac{1}{\Sigma_i \dfrac{\rho_i W_i{}^{\alpha}}{N_i}}$ (2-5)

where "W_i (>1) reduces the cycle ratio remaining at any stress S_i on account of the relatively greater damage by cycle ratios at all stress levels where $S > S_i$". This procedure requires the evaluation of W_i by test.

The more recently developed theories for cumulative damage generally include damage rate, in terms of cycle ratio, being stress dependent, and generally being higher at higher stress levels. Strengthening by understressing is considered as an additional effect to be added, if applicable.

Corten and Dolan (14) for instance have suggested for damage at a fixed stress amplitude, $D_i = (M_i\ r_i\ n_i)^{\alpha}$ where:

M_i = number of damage nuclei at the stress level S_i

r_i = coefficient of damage propagation at stress level S_i

n_i = number of cycles at S_i.

Assuming that "(1) damage nuclei, once introduced at any stress level, remain and propagate at any other stress level, and that (2) the rate of propagation is determined solely by the stress level", the expression for the total number of cycles at varied stress amplitudes required for failure is

$$N_T = \frac{N_1}{\Sigma_i \rho_i \frac{S_i}{S_1}} \tag{2-6}$$

Over 800 cumulative-fatigue-damage tests were run, each using only two cyclic stress levels, (15) and the resulting values of $\Sigma \frac{n_i}{N_i}$ were between 0.568 and 1.440, with 72% falling between 0.80 and 1.20. This gives good agreement with Miner's Cumulative Damage Theory.

Fatigue tests were conducted on a bar with $K_t = 3.5$, see ref. (4). The bar represented a wing spar, and the test spectrum represented actual stresses on a transport type aircraft wing. The value for $\Sigma \frac{n_i}{N_i}$ for these tests was 4. *

Although the exact value of D_T at failure is unknown, the use of Miner's Linear Cumulative Damage Theory is proposed for this analysis procedure. As will be discussed later in this thesis, the value of D_T at failure of the cyclic test aircraft may be determined and used in life predictions.

In the analysis procedure proposed in the following chapters, it will be necessary to calculate D', the damage due to one cycle. Miner's equation becomes:

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* It was also noted during these tests that the GAG cycle caused 75% of the fatigue damage.

$$D'_i = \frac{1}{N_i} \tag{2-7}$$

Substituting N_i from Equation 2-1 into the above equation we get

$$D'_i = \frac{1}{N'\left(\frac{S_{A_i}}{S_{A'}}\right)^M} = \frac{1}{N'}\frac{\left(S_{A'}\right)^M}{\left(S_{A_i}\right)^M} \tag{2-8}$$

Substituting S_{A_i} from Equation 2-2, the equation becomes

$$D_i' = \frac{1}{N'}\frac{\left(S_{A'}\right)^M}{\left(\frac{S_{a_i}\ S_u}{S_u - S_{m_i}}\right)^M}$$

$$D_i' = \frac{\left(S_{A'}\right)^M}{N'}\frac{1}{\left(\frac{S_{a_i}\ S_u}{S_u - S_{m_i}}\right)^M} = \frac{\left(S_{A'}\right)^M}{N'}\left(\frac{S_u - S_{m_i}}{S_{a_i}\ S_u}\right)^M \tag{2-9}$$

or

$$D'_i = \frac{\left(S_{A'}\right)^M}{N'}\left[\left(\frac{1}{S_{a_i}}\right)\left(1 - \frac{S_{m_i}}{S_u}\right)\right]^M \tag{2-10}$$

Let

$$K = \frac{\left(S_{A'}\right)^M}{N'} \qquad J = \frac{1}{S_u}$$

$$X = \frac{1}{S_{a_i}} \qquad Y = S_{m_i}$$

where J and K are constants for certain conditions and X and Y are variables.

Then

$$D_i' = K_i\left[X(1 - J_iY)\right]^{M_i}, \tag{2-11}$$

the damage due to n_i cycles becomes

$$D_i = D_i' n_i , \tag{2-12}$$

and the damage rate for a particular condition becomes

$$d = \frac{\Sigma D_i}{T} . \tag{2-13}$$

Miscellaneous Effects

Any method of fatigue analysis or life calculations can only give an estimate as to the average life of an item. The life is greatly affected by small variations, such as in the surface finish, heat treatment, residual stresses, stress concentrations, temperature of cyclic stressing, faults in the material, corrosion, rest periods, order of loadings, grain direction, grain size, etc. In other words, the quality of workmanship will greatly effect the life of a structure. Keeping these in mind, it is probable that there will be large variations in the life of similar composite structures, with the first failing at approximately 20% of the average life (16).

CHAPTER III

FATIGUE OF AIRCRAFT STRUCTURES

Fatigue failures are due to a large number of stress cycles. In order to predict fatigue failures, the number of stress cycles and their magnitudes must be known, as well as the S-N data for each critical point. This information is presently being accumulated by the use of panel tests of critical areas, and by testing of complete aircraft structures.

The number and magnitude of vertical accelerations, as well as related data, will be obtained by installing airborne recorders, as described in Chapter I. These vertical acceleration data must be converted to stress data at each critical point.

The principal loads on an aircraft are thrust, lift, drag and the weight of the aircraft. Lift is affected by W, the weight of the aircraft, and by n_z, the vertical acceleration. Drag is affected by velocity and altitude as well as by external stores. Thrust is fairly constant for a given flight condition. The items affecting stress at a point may be listed as:

V - Airspeed

H - Altitude

n_z - Vertical Acceleration

W - Weight

C - Configuration of the aircraft

As discussed in Chapter I, the first three items can be automatically recorded in flight, and the last two must be noted on some type of form by a crew member during flight.

In the case of stress cycles caused by gusts, vibrations are induced in the wing structure. A dynamic magnification factor, ν, must be obtained for each critical point, relating the number of stress cycles at that point to the number of vertical acceleration cycles at the aircraft C. G. For many aircraft this information is already available.

From stress analysis data, the 1 g stress and the change in stress due to a change in acceleration ($\frac{\Delta_s}{\Delta_n}$) must be obtained for every flight condition for each critical point. As there must be approximately 10 ranges of altitude, weight, configuration and velocity, there will be 10^4 possible values of $S_{1\ g}$ and $\frac{\Delta_s}{\Delta_n}$ for each critical point. Some of these would be the same, but in the electronic data processing equipment, a place would have to be provided for each value.

Two other factors are required for an aircraft fatigue analysis. They are K_M and γ. K_M is the ratio of the maximum n_z peak to the minimum n_z peak during a maneuver, and is obtained by measurement of recorded n_z data. Its use is described in Chapter IV, Step 3. The data handling factor, γ, is a correction for errors inherent in the data processing system, and is assumed to be 1 until analysis of the system output indicates otherwise.

The data as presently planned for processing are collected on the basis of the "peak count" philosophy. This system does not lend itself for easy calculation of the actual stress history such that the true alternating and mean stresses may be derived. Therefore, statistical studies must be accomplished to actually compare the true mean and alternating stresses to the stresses calculated by the analysis procedure. This ratio plus any correction deemed necessary as a result of using Miner's theory, the Modified Goodman Law, or any factor required for the S-N curves should be included in this γ factor.

CHAPTER IV

ANALYSIS PROCEDURE

The proposed analysis procedure consists of the following steps:

1. Calculate the instantaneous gross weight.
2. Sort the vertical acceleration peak data according to flight conditions, compile the time spent at each flight condition, and list the maximum peaks (for GAG cycle) for each flight.
3. Divide the peaks into gust cycles and maneuver cycles.
4. Calculate the stress at each critical point due to each vertical acceleration.
5. Compute S_a and S_m for each cycle.
6. Compute the damage rate for each flight condition.
7. Use the data calculated above to:
   a. List A/C (by tail number) in order of accumulated damage.
   b. Compute average mission and damage per average mission.
   c. Evaluate fatigue test spectra.
   d. Compute damage expected during new missions.

These seven steps are discussed in detail in the following sections.

Step 1. Calculating W_i

Three types of data words enter the first computation (instantaneous gross weight computation) from the analog to digital converter. These words are:

Type 1 - Identification word - identifies the flight and aircraft.

Type 2 - Peak data word - a 15 character word giving elapsed time, in ten second intervals since "recorder on", the normal acceleration in three digits, the velocity and altitude ranges.

Type 3 - Profile data word - a word which gives the airspeed or altitude range just entered and the time since "recorder on".

Three other types of data words enter Program 1 from the data on form A. These words are:

Type 4 - Identification word - similar to type 1.

Type 5 - Mission configuration data word - a word which gives the mission or configuration blocks just entered and the time since "recorder on."

Type 6. - Weight data word - a word which gives the instantaneous gross weight and the time since "recorder on".

The instantaneous gross weight will be calculated from the weight data words. It is only necessary to determine the times at which changes occur in the weight ranges. The actual weight at any time is not required. The calculation procedure will be in three parts. First, for two consecutive weight data words, it must be determined whether or not a change in weight range has occurred. Second, if a change has occurred between the two consecutive weight data words, the rate of change of weight must be determined. Third, the time at which the weight range change occurred must be calculated and a new data word generated. The output from the weight calculation program will be a data word giving the weight range just entered and the time (since "recorder-on") that this range was entered.

In the first part of the calculation, a weight data word will be compared with the break-points for the weight ranges, and a digit added to the word, indicating the weight range in which the data word occurs. Then this word is held, say at address A, while the next weight data word is considered, and the weight range digit added. Then this data word is compared to the one held at address A. If they both occurred in the same weight range, the word held at A is dropped and the new data word is held at A, and the same procedure is repeated for the next weight data word. If the second data word were in a different weight range than the first, then the rate of change of weight must be calculated.

The rate of change of weight is calculated by subtracting the time and weight of data word number N from the time and weight of data word number N + 1. This will give a positive number for time, and either a positive or a negative value for the change in gross weight. The change in weight is then divided by the period of time, giving change in weight per unit time. A negative rate of change, R, indicates a decrease in weight and will be the usual case. A positive rate of change, R, indicates an increase in weight, as in the case of refueling.

The time at which the change in range occurred is $T = T_1 - (W_1 - W) \frac{1}{R}$ as shown in figure 4-1. A data word must be generated, giving the weight range just entered and the time (T). This will be referred to as data word type 7, weight range data word.

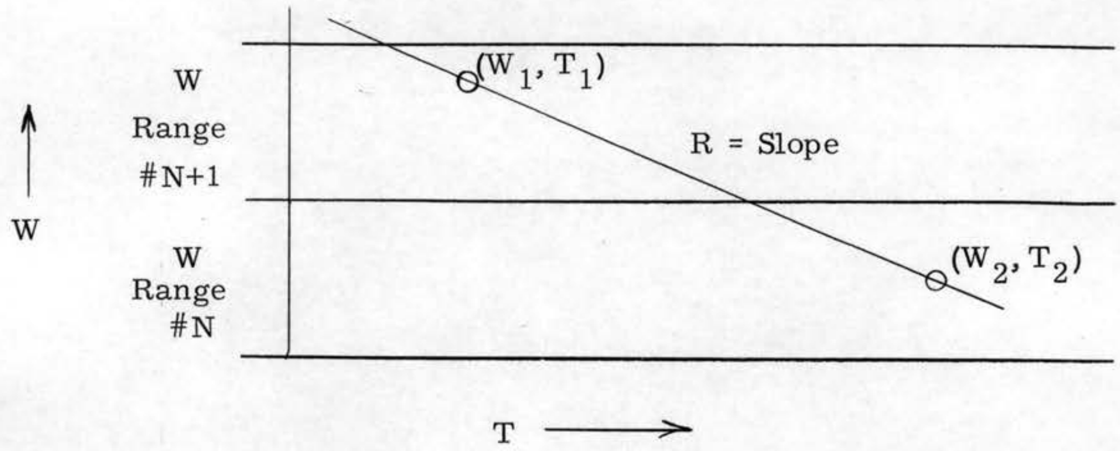
The output of program number 1 will be five types of data words. They are:

Type 1. Flight identification word.

Type 2. Peak data word.

Figure 4-1

GROSS WEIGHT VERSUS TIME



Type 3. Profile data word.

Type 5. Mission configuration data word.

Type 7. Weight range data word.

It is estimated that there would be a total of from 50 to 200 type 1, 3, 5, or 7 data words per flight, depending upon the length of flight. The number of peak data words would be much larger.

Step 2. Sorting Data.

Three types of outputs are desired from the raw data. They are:

Output 1 - Time spent at each flight condition.

Output 2 - Peak accelerations experienced at each flight condition.

Output 3 - A listing, by flight, of the maximum normal accelerations for each of six flight conditions, (for GAG cycle calculation).

The first type of output will be one table for each Mission-

Configuration-Weight combination, giving the total amount of time spent at each airspeed-altitude block. This will be approximately 400 tables.

The second type of output will be one table for each Height-Mission-Configuration-Weight combination, giving the number of peaks of each magnitude for each airspeed. This will be approximately 4000 tables.

The third type of output will be one table listing the maximum positive normal acceleration that occurred during each of the three highest gross weight ranges for flight loadings, giving both the actual value of n_z and the flight condition in which it occurred. These data are also required for the highest gross weight range during taxi. This will consist of six data words per flight, and will also serve as an indication of the number of flights represented by the report.

If an "average mission" is desired, it can be determined from the first type of output. Divide the time spent at each flight condition by the number of missions represented by the data.

Step 3. Separation of Acceleration Data into Gust and Maneuver Cycles

There are two primary types of cyclic loading encountered by an aircraft, those due to atmospheric turbulence, and those due to maneuvers induced by the pilot.* The first type, gust cycles, are generally symmetrical about the 1 g level, while the maneuver cycles are of a different type. In order to discuss maneuver cycles, it is necessary to first consider the basic concept of acceleration.

* These two types of cycles are actually cycles superimposed on the GAG cycle which is the cycle from the highest positive stress during the entire mission to the highest negative stress. One usually is a ground load while the other is a flight load.

An airplane in the special condition of zero vertical velocity will remain in this state as long as $n_z = 1$ g, neglecting antisymmetrical maneuvers. But as n_z becomes greater than 1 g for some period of time, T, and then returns to 1 g, the aircraft will have gained some vertical velocity. It will continue to have a constant vertical velocity until a vertical acceleration of less than 1 g reduces this vertical velocity to zero.

Consider as an example a simple pull-up maneuver (see figure 4-2), during which n_z is greater than 1 g from T_1 to T_2. The resulting vertical velocity will be $\Delta V_1 = \int_{T_1}^{T_2} (n_z - 1)\, g dT$. Then at time T_2, n_z becomes less than 1 g, until time T_3, such that the change in vertical velocity ΔV_2 will be equal to $-\Delta V_1$, where $\Delta V_2 = \int_{T_2}^{T_3} (n_z - 1)\, g dT$ and the vertical velocity will again return to 0.

Test data from previous airloads programs indicate that during a pilot-initiated pull-up maneuver ω is much greater than β and the time period T_1 to T_2 is much less than the time period T_2 to T_3.

Figure 4-2 represents the vertical acceleration during a simple pull-up. In figure 4-3 this pull-up is represented with straight lines. From test data it is observed that the second period of time is frequently 3 times as long as the first. It is found that the corresponding ratio of β to ω must be approximately 2:5. This ratio should be determined experimentally for each aircraft in the program. Since the g data is recorded in analog form on the data magazine, this trace may be played out on some paper recorder such as the Sanborn, and the ratio determined more exactly. This ratio is referred to as K_M.

The existance of this ratio is important in separating gust data from maneuver data. It was first proposed to separate gust data from maneuver data by subtracting the number of peaks in the range below

Figure 4-2

VERTICAL ACCELERATION TRACE

(For a Pull-up Maneuver)

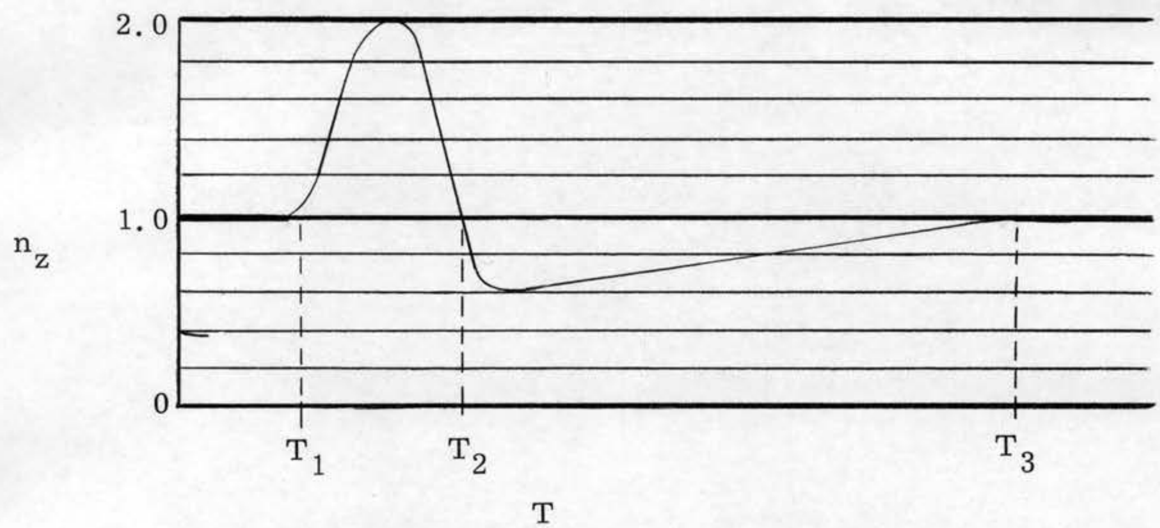
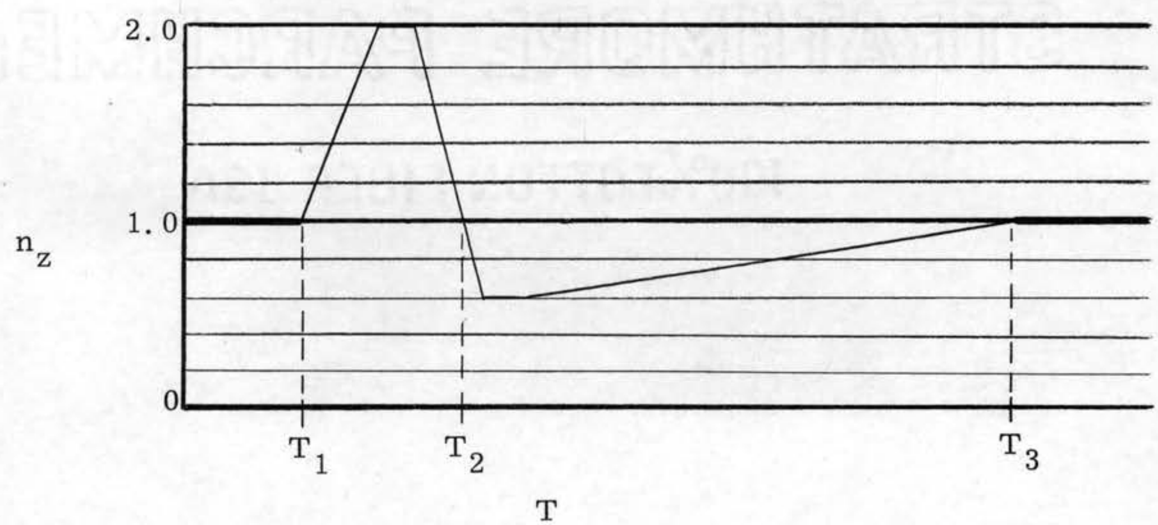


Figure 4-3

LINEAR APPROXIMATION OF VERTICAL ACCELERATION DATA

(Same Pull-up Maneuver as in Fig. 4-2)



1 g from the equivalent range above 1 g, and considering these as equivalent gust cycles with a mean acceleration of 1 g. The remaining positive peaks were considered to be equivalent maneuver cycles. This procedure does not take into account the existence of K_M, and therefore is in error in two areas. First, the equivalent maneuver cycles would be analyzed as being from ω to 1 g, when they were actually from ω to β. A cycle from ω to β would have a larger alternating stress and a smaller mean stress than would a cycle from ω to 1 g. Secondly, by considering all of the peaks below the 1 g level to be gust cycles, (when some are actually part of maneuvers) the result would be to consider some maneuvers of low positive magnitude to be gusts, thus applying erroneous values for the mean and alternating stresses in these cases.

A more refined method for the separation of gust and maneuver data is presented in the following four steps;

1. Subtract the number of n_z peaks in each acceleration range below 1 g (the equivalent gust cycles) from the number of peaks in the corresponding range above 1 g. The remainder will be the equivalent maneuver cycles.

2. Determine β for each equivalent maneuver cycle and determine the number of these in each acceleration range below 1 g.

3. Subtract these peaks from the number of equivalent gust cycles, leaving the corrected equivalent gust cycles.

4. Add the peaks from step 2 to the equivalent maneuver cycles found in step one, obtaining the corrected number of equivalent maneuver cycles.

The following is an example of this procedure.

EXAMPLE 4-1

Step 1

$\bar{n}_z$	.1	.3	.5	.7	1.3	1.5	1.7	1.9
Number of Occurrences	0	6	17	112	186	39	14	1
Equivalent Gust Cycles					112	17	6	0
Equivalent Maneuver Cycles					74	22	8	1

Step 2

K_M is assumed to be 0.4

$\bar{n}_z$	$1-K_M(\bar{n}_z-1)$	NUMBER OF OCCURRENCES	TOTALS
2.0	0.60		
1.9	0.64	1	
1.8	0.68		$\bar{n}_z = 0.7$
1.7	0.72	8	31
1.6	0.76		
1.5	0.80	22	
1.4	0.84		
1.3	0.88	74	$\bar{n}_z = 0.9$
1.2	0.92		74

All equivalent maneuver positive peaks between 1.50 g and 2.00 g have negative peaks between 0.60 g and 0.80 g. All below 1.50 g have negative peaks in the range where peaks are not counted.

Step 3

$\bar{n}_z$	1.3	1.5	1.7	1.7
Equivalent Gust Cycles	112	17	6	0
Correction	31			
Corrected Equivalent Maneuver Cycles	81	17	6	0

Step 4

$\bar{n}_z$	1.3	1.5	1.7	1.9
Equivalent Maneuver Cycles	74	22	8	1
Correction	31			
Corrected Equivalent Maneuver Cycles	105	22	8	1

Steps 4 and 5. <u>Calculating Stresses</u>

Stress calculations for gust cycles are slightly different than for maneuver cycles. In either case, a value for 1 g stress and for $\frac{\Delta_s}{\Delta_n}$, the rate of change of stress with respect to vertical acceleration, will be required for each critical point for each flight condition.

In the case of maneuver loads, the values of S_a and S_m are

$$S_a = \frac{1}{2} \frac{\Delta_s}{\Delta_n} (n_z - 1)(1 + K_M)\, \gamma \tag{4-1}$$

$$S_m = S_{1\,g} + \frac{1}{2} \left(\frac{\Delta_s}{\Delta_n}\right) (n_z - 1)(1 - K_M)\, \gamma \tag{4-2}$$

For gust loads, S_a and S_m are determined from:

$$S_a = \frac{\Delta_s}{\Delta_n} (n_z - 1)(\gamma)(\nu) \tag{4-3}$$

$$S_m = S_{1\,g} \tag{4-4}$$

Where ν is the dynamic magnification factor and γ is the data handling factor.

As previously noted in Step 2, the highest acceleration peak for each of the three highest W ranges during ground loads and for each of the three highest W ranges during air loads are recorded separately, and used to calculate the GAG stress cycle for each critical point. This must be calculated separately for each flight.

For each critical point the procedure is to calculate the six stress values and then use the two extreme values to make up the GAG stress cycle. The equations for S_a and S_m are:

$$S_a = \frac{S_{\text{air load}} - S_{\text{ground load}}}{2} \tag{4-5}$$

$$S_m = \frac{S_{\text{air load}} + S_{\text{ground load}}}{2} \tag{4-6}$$

The computation for GAG damage must be done manually.

Step 6. <u>Damage Rate Calculations</u>

At this point in the analysis procedure the stress data will already be categorized according to the number of occurrences of each S_a - S_m combination for each flight condition for each critical point, and separated into gust data and maneuver data. Using Miner's Linear

Damage Theory, as previously discussed, the equation for the damage rate at a particular critical point for a particular flight condition is:

$$d_i = \frac{\Sigma D_i}{T_i} \tag{4-7}$$

where d_i is the damage rate at a particular critical point for a particular flight condition.

D_i = total damage caused at a particular critical point during a particular flight condition.

T_i = total time spent at a particular flight condition.

Step 7A. Aircraft Damage

One of the primary objectives of this program is to schedule aircraft by tail number for inspections and modifications. To accomplish this objective, it is necessary to know for each aircraft the total amount of damage, D_T, at the particular critical point being considered.

The value of D_T can be obtained for a particular airplane by dividing the flight profiles for all of the flights for this particular aircraft into a listing of time spent at each flight condition, T_i. The equation for D_T for a particular joint for a particular aircraft becomes:

$$D_T = \Sigma d_i T_i \tag{4-8}$$

An automatic procedure can be established to divide previous flights into "time per flight condition" type of data when the exact form of the flight history is known.

Step 7B. Average Mission

As mentioned previously, the average mission can be determined by dividing time spent at each flight condition by the number of flights represented. Similarly, the average damage per mission can be determined by dividing the total of the D's by the number of flights represented, and then adding the average GAG damage.

Step 7C. <u>Evaluation of Fatigue Test Spectra</u>

The cyclic test induced certain known S_a - S_m cycles at each critical point. The number of cycles and stress levels can be introduced into the procedure for calculating damage, as previously described, and the damage at each critical point due to a test spectrum can be calculated. This total damage can be compared with the damage per average mission, as determined in step 7B, and the life of the test specimen re-evaluated in terms of average missions.

Step 7D. <u>New Mission Damage</u>

Any proposed new mission can be divided into "time at each flight condition" data, and the total damage for the mission computed as for step 7A.

CHAPTER VI

CONCLUSIONS

The analysis of aircraft, based upon vertical acceleration data, can be accomplished by utilizing the procedure described in this report. However, a number of types of information are required. In addition to flight data, these include S-N curves, stress analysis data for each critical point, the dynamic magnification factor, the data handling factor, and K_M. The analysis would be expensive and the accuracy of life predictions questionable.

However, data listing similar aircraft according to the magnitude of accumulated fatigue damage would be reasonably accurate, as would be comparisons of the amount of damage caused by different mission plans.

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* I have not personally examined the references indicated, but have read discussions of them in one or more other references.

APPENDIX A

DETERMINING THE UPPER AND LOWER LIMITS FOR EACH GROSS WEIGHT INTERVAL*

In stress calculations it is necessary to consider that all of the gust and maneuver data from a particular weight interval occurred at one particular weight, $\overline{W}_i$. In most cases there will be some ΔW_i, some difference between the W_i and $\overline{W}_i$. Ideally, the gross weight intervals should be established so that the ΔW_i will cause as little error as possible in the damage calculated for each cycle.

This can best be done by establishing the weight interval limits in the following manner:

Step 1. Plot W_i versus d_i, the damage due to a typical series of vertical gusts.

Step 2. Divide the curve into equal ranges of damage rates.

Step 3. Determine, from the graph, the gross weights corresponding to the upper and lower limits of each damage rate interval.

In order to plot the damage due to a particular gust spectrum, the gust spectrum must be defined and the values of $\frac{\Delta_s}{\Delta_n}$, $S_{1\,g}$, S_u, ν and γ must be either known or assumed. The gust spectrum is made up of several gusts at each of several gust velocities. Determine the values of n_z for each vertical gust velocity for each of several gross

* A similar procedure can be used to establish the limits for airspeed, altitude and vertical acceleration ranges.

weights. Then, for each of the values of n_z, calculate S_{a_i} and S_{m_i}. From these, calculate S_{A_i}. The damage rates are then calculated from S_{A_i}. (The necessary equations have been discussed in the body of this report, and are used in example 1.) The damage rates for each value of W is then plotted.

Divide the largest damage rate into equal intervals. If X intervals are used, each interval of damage will be $\frac{100}{X}$% of the largest damage rate. The upper and lower limits of each damage interval are noted.

The particular gust spectrum causes a different damage rate at each gross weight. The graph of d_i vs W_i may be consulted to determine the gross weights corresponding to each damage rate noted above. These values of W_i are the limits of the gross weight intervals.

The value of gross weight, $\overline{W}_i$ to be used in stress calculations for a particular weight interval is the average of the upper and the lower limits for the weight interval. One exception to this procedure is in the case of ground loads during the highest weight interval. Most of these data will be recorded at or very near the upper limit of the interval, and therefore in calculating ground loads it is desirable to use $\overline{W}_1$(ground loads) as the upper limit of the highest W range.

As seen from figure 4-1, the d_i - W_i plot for example 1, the value of $\overline{W}_i$, as described above, will be approximately the W corresponding to the average damage rate for the W interval, and the calculated damage for any gust, no matter at what W_i it occurs, will be within $\frac{50}{W}$% of the correct damage rate. As stress cycles will occur at all values of W, the errors will tend to cancel each other.

EXAMPLE A-1

An Example of Determining Limits for Gross Weight Intervals.

A number of conditions must be assumed for this example. The critical joint is assumed to be made of 7075S-T6, and figure 2-1, the S-N curve for 7075S-T6 at $S_m = 0$, and K_T - 2.0, is assumed to be the S-N curve for this joint. S_u is assumed to be 84,000 PSI. It is also assumed that as W varies from 0 to 450,000#, $S_{1\,g}$ varies proportionately from 0 to 28,000 PSI, and $\frac{\Delta_s}{\Delta_n}$ varies proportionately from 0 to 28,000 PSI per g.

The gust spectrum is assumed to be that which would cause a 450,000# airplane to have the number and magnitude of vertical accelerations shown in Table A-1.

All calculations are for one V-H-C combination, assuming V-H-C remains constant.

The vertical acceleration due to a particular gust is calculated (17) using the equation:

$$n_z = 1 \pm \rho \frac{V_e U_{de} m K_w}{2 \frac{W}{A}} \tag{A-1}$$

The symbols are defined as:

ρ = air density

V_e = airspeed feet per second, equivalent air speed

U_{d_e} = gust velocity FPS, EAS

m = slope of lift curve

K_W = gust factor, $= \frac{0.88\mu}{5.3+\mu}$

$\mu = \frac{2\ W'S}{g\,c\,m}$

TABLE A-1 (For Example A-1)

Example of Peak n_z Data

n_z Range	$\bar{n}_z$	Number of Peaks Per 1000 Min	Peaks Due to
2.0 to 1.8	1.9	2.00	Gust A
1.8 to 1.6	1.7	4.67	Gust B
1.6 to 1.4	1.5	30.33	Gust C
1.4 to 1.2	1.3	293.0	Gust D

TABLE A-2 (For Example A-1)

n_z Due to Gust A, B, C, and D at Various W_i

W_i (Kips)	Gust A	Gust B	Gust C	Gust D
450	1.90	1.70	1.50	1.30
400	2.00	1.78	1.55	1.33
350	2.11	1.86	1.61	1.37
300	2.24	1.96	1.68	1.41
250	2.39	2.08	1.77	1.46
200	2.63	2.26	1.90	1.54

$g = 32.2\ FPS^2$

c = average wing chord $(\frac{\text{area}}{\text{span}})$

A = wing area, sq. ft.

For this example, this equation for n_z was reduced to the form:

$$n_z = 1 + X \frac{K_W}{W}$$

where

$$K_W = \frac{.88\mu}{5.3 + \mu} \quad \text{and} \quad \mu = YW$$

At $n_z = 1.9$ and $W = 450,000\#$ the value of μ was assumed to be 21. From these values, Y was determined to be .0466, and is a constant for a constant V-H-C condition.

There are four gust magnitudes considered in this example for the 450,000# example aircraft,

gust A caused $n_z = 1.9$

gust B caused $n_z = 1.7$

gust C caused $n_z = 1.5$

gust D caused $n_z = 1.3$

The values of n_z for other gross weights were calculated using the following procedure: (Calculation for 400,000# A/C, gust A)

$$n_z = 1 + X \frac{K_W}{W}$$

$$\mu = .0466W = .0466\ (400) = 18.64$$

$$K_W = \frac{.88\ (18.64)}{23.94} = .69$$

For gust A for a 400,000# aircraft,

$$X = \frac{1.9 - 1}{\frac{K_W}{W}} = \frac{(.9)\ 450}{.69} = 580$$

$$n_z = 1 + 580 \frac{.69}{400} = 1.00 + 1.00 = 2.00$$

other n_z values for gusts A, B, C, and D are listed in Table A-2.

For gust loads, $S_m = S_{1\,g}$ and $S_a = S_{1\,g}(n_z - 1)$

For Gust A, the stresses at this hypothetical critical joint on a 400,000# aircraft are: $S_{1\,g} = \frac{(28,000 \text{ PSI})(400,00 \text{ lbs})}{450,000 \text{ lbs}} = 24,800$ PSI

$S_a = 24,800(2.00 - 1.00) = 24,800$ PSI.

Values of S_m and S_a for other values of W for Gusts A, B, C and D are given in Table A-3.

The values of S_{A_i} are calculated from the following equation, developed from the Modified Goodman Law:

$$S_{A_i} = \frac{S_{a_i} S_u}{S_u - S_{m_i}}$$

For Gust A and a 400,000# aircraft,

$$S_{A_i} = \frac{(24,800)(84,000)}{84,000 - 24,800} = 35,300 \text{ PSI}$$

Values of S_{A_i} for other conditions are given in Table A-4.

From the values of S_{A_i} damage per cycle may be calculated from the equation:

$$D_i = \frac{1}{N_B} \left(\frac{S_{A_i}}{S_{A_B}} \right)^b$$

The constants for this equation are obtained from Figure 2-1, the S-N curve for 7075S-T6, with $K_t = 2.0$ and $S_m = 0$.

Point A on this curve is the stress for N = 1, and B, an assumed point, is for the same stress, S_u, but at N = 5. Points C and D are intersections of straight-line portions of the curve, and E is the endurance limit. Lines 1, 2 and 3 are from points B to C, C to D, and D to E, respectively. Values of N and S_a for points A-E are

TABLE A-3 (For Example A-1)

Values of S_m, S_s and n_z due to Gust A, B, C, and D for Various W_i

W_i (Kips)	S_m	Gust A		Gust B		Gust C		Gust D	
		n_z	s_a	n_z	s_a	n_z	s_a	n_z	s_a
450	28.0	1.90	25.2	1.70	19.6	1.50	14.0	1.30	8.40
100	24.8	2.00	24.8	1.78	19.3	1.55	13.7	1.33	8.20
350	21.7	2.11	24.2	1.86	18.7	1.61	13.2	1.37	8.00
300	18.6	2.24	23.1	1.96	17.8	1.68	12.6	1.41	7.60
250	15.5	2.39	21.5	2.08	16.7	1.77	11.9	1.46	7.20
200	12.4	2.63	20.2	2.26	15.6	1.90	11.2	1.54	6.70

TABLE A-4 (For Example A-1)

Values of S_{A_i} for the S_a - S_m Combinations Listed in Table A-3

W_i (Kips)	$\frac{S_U}{S_U - S_M}$	S_{A_i}			
		Gust A	Gust B	Gust C	Gust D
450	1.50	37.8	29.4	21.0	12.6
400	1.42	35.3	27.4	19.4	11.7
350	1.35	32.7	25.2	17.8	10.8
300	1.28	29.5	22.8	16.1	9.8
250	1.22	26.2	20.3	14.5	8.8
200	1.17	23.7	18.3	13.1	7.8

TABLE A-5 (For Example A-1)

S-N Data for Points on Figure 2-1

(K_t - 2, S_m = 0)

Data Point	N	S_A
A	1	84,000
B	5	84,000
C	2,000	42,500
D	200,000	18,100
E	5,000,000	15,200

TABLE A-6 (For Example A-1)

Values for $\frac{1}{M}$ and b from Figure 2-1

Line End Points	Line N_r	Slope ($\frac{1}{M}$)	Inverse of Slope (b)
B-C	1	-0.114	+ 8.75
C-D	2	-0.186	+ 5.38
D-E	3	-0.055	+18.10

listed in Table A-5. Values of $\frac{1}{M}$ and b are listed in Table A-6.

The slope, $\frac{1}{M}$, was calculated using the equation:

$$\frac{1}{M} = \frac{\log \frac{S_C}{S_B}}{\log \frac{N_C}{N_B}}$$

$$b_n = - \frac{1}{M_n}$$

For a sample calculation, the damage due to one Gust cycle (due to Gust A, 400,000# aircraft) is calculated below:

S_A = 35,000 PSI, which is in range r = 2, between S_{A_C} and S_{A_D}. Therefore, the constants for the damage equation are:

$b_2 = 5.38$

$N_C = 2,000$

$N_D = 200,000$

$S_{A_C} = 42,500$

$S_{A_D} = 18,100$

$$\frac{1}{M_2} = \frac{\log \frac{S_D}{S_C}}{\log \frac{N_D}{N_C}} = -0.186$$

$$b_2 = - \frac{1}{M_2} = 5.38$$

With these constants, the damage equation is

$$D = \frac{1}{2,000} \left(\frac{35,200}{42,500} \right)^{5.38} = .000184$$

Other damage rates are listed in Table A-7.

From these damage per cycle values and the number of peaks per 1000 minutes, damage rates are calculated. The damage rate

$$d_i = \frac{\Sigma D n}{T}$$

The damage rates for each gross weight are shown in Table A-8, and plotted in figure A-1. It is noted that the maximum damage rate is <u>.00120.</u>

Assuming that, regardless of the gross weight, damage calculation errors for a gust spectrum must not exceed approximately 6% of the damage rate that this spectrum would cause at W_{max}, it is necessary to have 8 W ranges. Dividing the max. damage rate by 8, each range will have a variation from the mean value of $\pm .000075$. The gross weights corresponding to these damage rates are given in Table A-9.

TABLE A-7 (For Example A-1)

Damage per Gust Cycle ($x10^7$)

W(Kips)	2.0g	1.9g	1.7g	1.5g	1.3g
450	4735	2760	705	117	0
400	1840	1040	270	31.6	0
350	670	385	100	0	0
300	230	132	11.6	0	0
250	65	14.7	0	0	0
225	13.1				
200	0	0	0	0	0
150	0	0	0	0	0

TABLE A-8 (For Example A-1)

Damage Rate for Various Weights

Weight in Kips	Damage
450	.0012030
400	.0008236
350	.0004993
300	.0002426
250	.0001184
200	.0000697

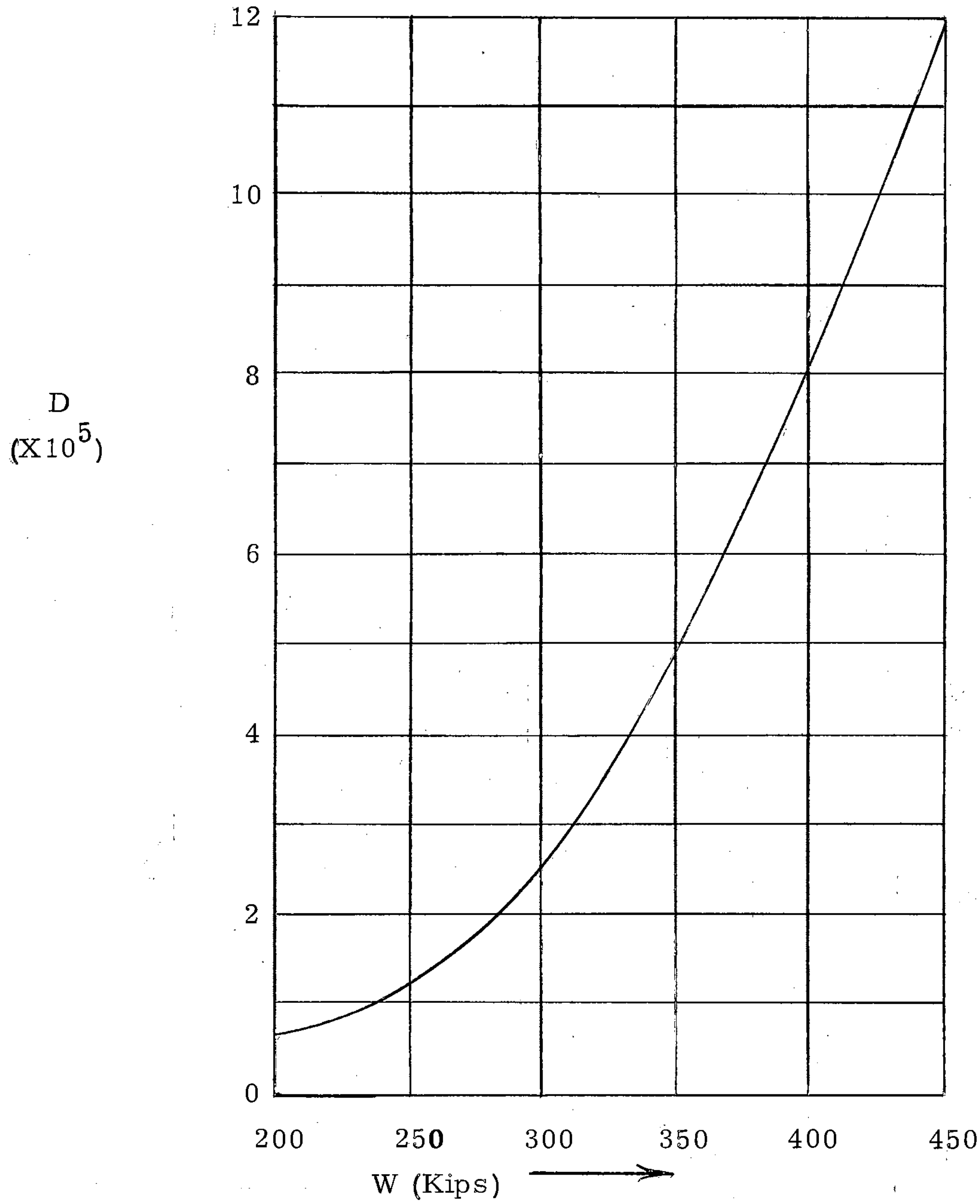
TABLE A-9 (For Example A-1)

Weight Ranges for Equal Change in Damage Rates

Interval	Max Damage Rate	Max Weight (in kips)
1	.0012030	450
2	.0010500	430
3	.0009000	410
4	.0007500	389
5	.0006000	368
6	.0004500	343
7	.0003000	313
8	.0001500	267

Figure A-1

DAMAGE RATE VERSUS WEIGHT



VITA

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