ANALYSIS OF RECTANGULAR GRIDS BY STIFFNESS METHODS

By

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BY STIFFNESS METHODS

Thesis Approved:

Thesis Adviser

Dean of the Graduate School

PREFACE

This thesis is an outgrowth of lectures on space structures delivered by Professor J. J. Tuma at the Oklahoma State University in spring, 1960. The relationship between the slope deflection method, the moment and shear distribution method and the joint moment and shear carry-over method stressed in these lectures are recorded in this thesis and fully defined. The nomenclature and the illustrations follow closely Professor Tuma's presentation.

The writer, in completing this work, wishes to express his gratitude to the following individuals:

To Professor Jan J. Tuma for his sincere guidance in preparation of this thesis and for his instruction and advice given to the writer throughout his graduate study. The writer is also thankful to Professor Tuma for providing an opportunity to study at Oklahoma State University.

To Professors R. L. Flanders and K. S. Havner for acting as his advisers.

To his parents for their patience and encouragement during his study.

Finally, to Mrs. Arlene Starwalt for careful typing of the manuscript and E. Citipitioglu for preparing the neat sketches and checking the manuscript.

J. H. T.

Stillwater, Oklahoma
July, 1962

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NOMENCLATURE

i , j , k , m , n	Joints
m_{jxx} , m_{jyy}	Starting moments
q_{ijz}	Carry-over joint shear factor from i to j
r_{ijxx} , r_{ijyy}	Joint moment carry-over factor from i to j
s_{ijzx} , s_{ijzy}	Carry-over joint shear-moment factor from i to j
t_{ijxz} , t_{ijyz}	Carry-over joint moment-shear factor from i to j
u , v	Segments
du	Elemental length
$C_{ijx}^{(0)} , C_{ijy}^{(0)}$	Carry-over moment factor from i to j
$C_{ijz}^{(\Delta)}$	Carry-over shear factor from i to j
$C_{ijxz}^{(0\Delta)} , C_{ijyz}^{(0\Delta)}$	Carry-over moment-shear factor from i to j
$C_{ijzx}^{(\Delta 0)} , C_{ijzy}^{(\Delta 0)}$	Carry-over shear-moment factor from i to j
$D_{ijx}^{(0)} , D_{ijy}^{(0)}$	Distribution moment factor at i of member ij
$D_{ijz}^{(\Delta)}$	Distribution shear factor at i of member ij
E	Modulus of elasticity
G	Modulus of rigidity
I_y , I_z	Moment of inertia of the cross section
J	Torsional constant of the cross section

K_{ijx} , K_{ijy}	Stiffness moment factor at i of member ij
L_{ij}	Length of member ij
L	Length of any member
M_{ijx} , M_{ijy}	End moment at i of member ij
M_{jx} , M_{jy}	Moment at j
$M^{(W)}$	Partial end moment due to loads
$M^{(\theta)}$	Partial end moment due to end rotation
$M^{(\Delta)}$	Partial end moment due to end displacements
M	Moment
O	Origin
P_{uz} , P_{jz}	Concentrated load
Q_{ux} , Q_{uy}	Applied couple
S_{ijx} , S_{ijy}	Fixed end moment due to unit end deflection or fixed end shear due to unit end rotation at i of member ij
T_{ijz}	Stiffness shear factor at i of member ij
X , Y , Z	Coordinate axes
BM_x , BM_y	Bending moment of basic structure
$CD^{(0)}_{ijx}$, $CD^{(0)}_{ijy}$	Carry-over distribution moment factor at i of member ij
$CD^{(\Delta)}_{ijx}$, $CD^{(\Delta)}_{ijy}$	Carry-over distribution shear factor at i of member ij
CK_{ijx} , CK_{ijy}	Carry-over stiffness moment factor at i of member ij
FM_{ijx} , FM_{ijy}	Fixed end moment at i of member ij
FV_{ijz}	Fixed end shear at i of member ij

JM_{jxx} , JM_{jyy}	Joint moment at j
JV_{jzz}	Joint shear at j
ΣK_{jx} , ΣK_{jy}	Joint stiffness rotation moment factor at j
ΣS_{jx} , ΣS_{jy}	Joint stiffness deflection moment factor at j or joint stiffness rotation shear factor at j
ΣT_{j}	Joint stiffness deflection shear factor at j
ΣFM_{jx} , ΣFM_{jy}	Sum of fixed end moments at j
ΣFV_{jz}	Sum of fixed end shears at j
θ_{jx} , θ_{jy}	Joint rotation
Δ_{jz}	Joint deflection
$\left\|\Delta M_{jx}\right\|^{(W)}$ $\left\|\Delta M_{jy}\right\|^{(W)}$	Moment unbalance due to loads at j
$\left\|\Delta V_{jz}\right\|^{(W)}$	Shear unbalance due to loads at j

CHAPTER I

INTRODUCTION

1-1. General.

A comparative study of the algebraic constants involved in the analysis of rectangular, planar grids loaded normal to their plane is presented. Three classes of constants are considered:

(a) slope-deflection constants

(b) moment distribution constants

(c) carry-over constants.

The relationships and the application are explained and a procedure of analysis is described.

1-2. Historical Background

The introduction of slope-deflection equations to the analysis of rectangular grids is being credited to Szego (1), Marcus (2) and Beyer (3). Recently a restatement of this application was recorded by Martin and Hernandez (4). The study of planar grids by the numerical moment distribution method was reported by Ferguson (5), Lothers (6), Reddy and Jaeger (7). The carry-over moment procedure was developed by Tuma (8) and presented in his lectures.

1-3. Statement of the Problem.

A rectangular, planar grid acted on by loads perpendicular to its plane is considered. The cross section of the straight members forming this grid are constant or variable. The joints of the grid are rigid joints, free to rotate and to displace. The exterior points of the grid are free, simply supported or fixed. Loads are stationary, or moving loads of constant or variable magnitude. The supports rest on rigid or elastic foundation.

1-4. Limitation of the Problem.

The analysis is based on the following assumptions:

(a) Material of the structure is homogeneous, isotropic, and continuous.

(b) All deformations are small and elastic.

(c) Material follows Hooke's Law.

(d) Modulus of elasticity and of rigidity are known numbers.

(e) Modulus of elasticity in tension and in compression is the same number.

1-5. Sign Convention.

All analytical values (geometric quantities, forces, moments, deflections and slopes) are related to a set of orthogonal, coordinate axes X , Y , Z and they are positive if acting in the positive direction on these axes.

1-6. Nomenclature.

All symbols are defined where they first appear and they are rearranged alphabetically in the list of symbols.

CHAPTER II

SLOPE DEFLECTION METHOD

2-1. Deformation Equations.

A rectangular, planar grid loaded as shown in Fig. 2-1 is considered.

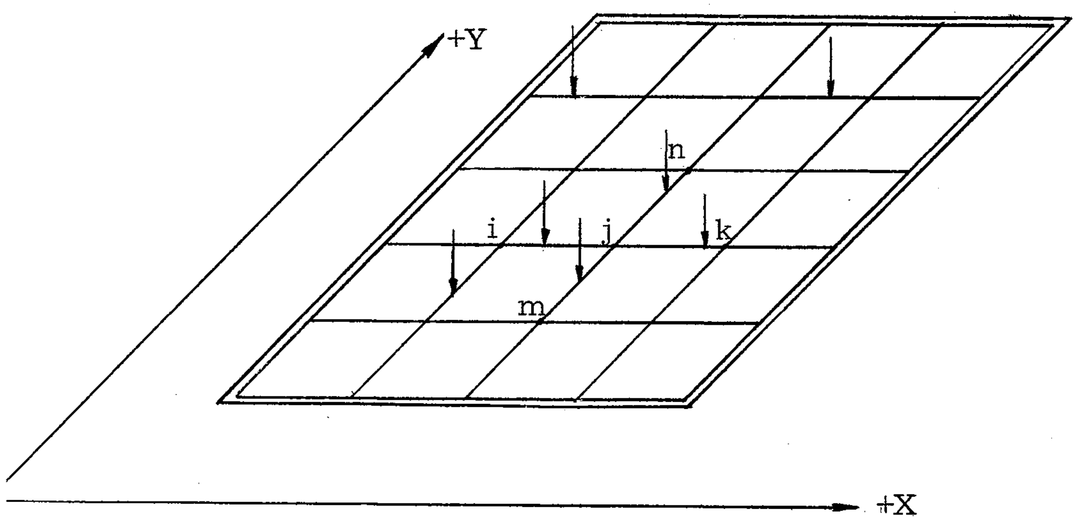


Fig. 2-1

Rectangular Grid

The initially planar grid deforms into an elastic surface defined by the elastic curve of each member. The end deformations of each member are the vertical displacements Δ's and the angular rotations θ's . The mathematical relationships between loads, end moments, end forces and end deformations are known as the slope deflection equations.

The moment slope-deflection equations for the members of joint j (Fig. 2-2) are recorded in Table 2-1. The force slope-deflection equations for the same members are recorded in Table 2-2. From these tables the following observations are being made:

(A) End Moments

(1) Each member has two types of end moments:

(a) flexural moments

(b) torsional moments.

(2) Each end moment is designated by the symbol M and three subscripts, identifying the near end, the far end and the axis or rotation, respectively.

(3) Each end flexural moment consists of three partial moments:

(a) angular displacement moment $M^{(\theta)}$

(b) linear displacement moment $M^{(\Delta)}$

(c) moment due to loads $M^{(W)}$.

(4) Each end torsional moment consists of two parts only:

(a) angular displacement moment $M^{(\theta)}$

(b) moment due to loads $M^{(W)}$.

(B) End Forces

(1) Each member has one type of end force called end shears.

(2) Each end shear is designated by the symbol V and three subscripts identifying the near end, the far end and the direction of the end force, respectively.

(3) Each end shear consists of three partial shears:

(a) Angular displacement shear $V^{(\theta)}$.

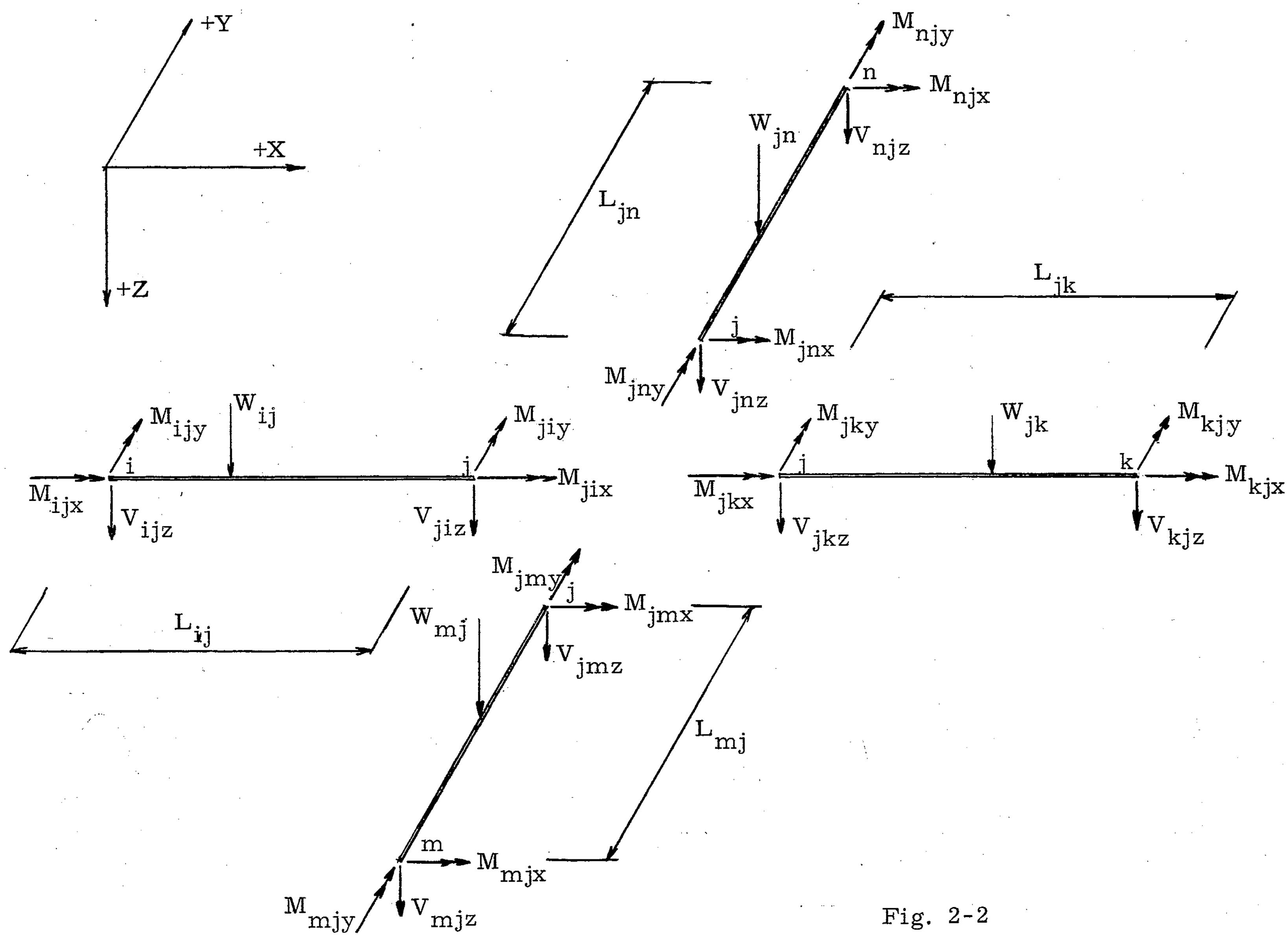


Fig. 2-2

Members of Joint j

(b) Linear displacement shear $V^{(\Delta)}$.

(c) Shear due to loads $V^{(W)}$.

(C) End Deformations

(1) Each joint undergoes three deformations

(a) two rotations (θ)

(b) one deflection (Δ)

measured in the direction of respective axes.

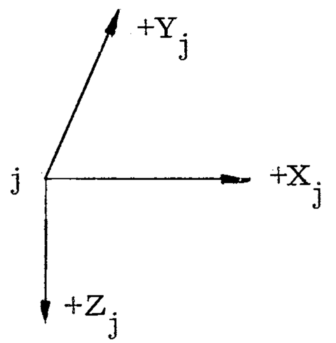
(2) Each deformation is identified by two subscripts. The first subscript designates the end and the second one the direction. The direction subscript of deflection is omitted to simplify the typing of the text.

2-2. Elastic Constants.

In addition to moment-, force-, and deformation-symbols, a group of elastic constants is being utilized in the moment- and force-equations recorded in Tables 2-1 and -2. These constants derived by means of special conditions are recorded in Tables 2-3, -4, -5, -6, -7, -8, -9, -10.

2-3. Joint Equilibrium.

Each joint of the grid offers three conditions of compatibility and three conditions of static equilibrium. The compatibility conditions already utilized in Tables 2-1, -2 are: "Rotations and deflections of all member ends connected at a given rigid joint are the same with respect to a given axis."



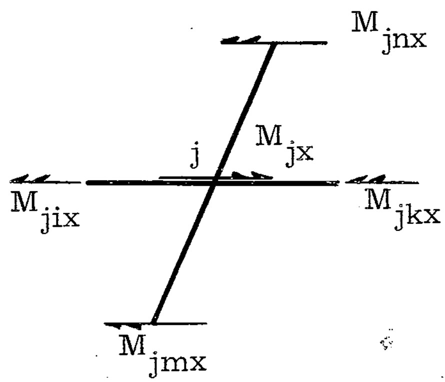


Fig. 2-3a

Equilibrium of
X-Moments at j

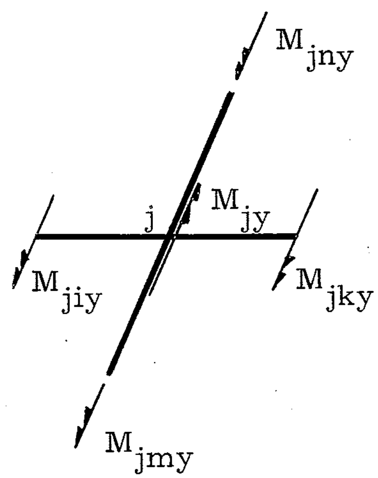


Fig. 2-3b

Equilibrium of
Y-Moments at j

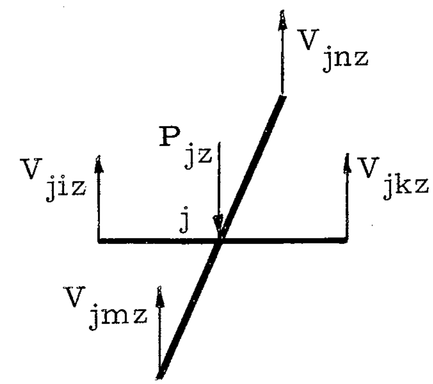


Fig. 2-3c

Equilibrium of
Z-Shears at j

TABLE 2-1 — END MOMENT SLOPE DEFLECTION EQUATIONS

Member ij

$$M_{ijx} = + K_{ijx}\theta_{ix} + CK_{jix}\theta_{jx} + FM_{ijx}$$

$$M_{jix} = + K_{jix}\theta_{jx} + CK_{ijx}\theta_{ix} + FM_{jix}$$

$$M_{ijy} = + K_{ijy}\theta_{iy} + CK_{jiy}\theta_{jy} + S_{ijy}(\Delta_i - \Delta_j) + FM_{ijy}$$

$$M_{jiy} = + K_{jiy}\theta_{jy} + CK_{ijy}\theta_{iy} + S_{jiy}(\Delta_i - \Delta_j) + FM_{jiy}$$

Member jk

$$M_{jkx} = + K_{jkx}\theta_{jx} + CK_{kjx}\theta_{kx} + FM_{jkx}$$

$$M_{kjx} = + K_{kjx}\theta_{kx} + CK_{jkx}\theta_{jx} + FM_{kjx}$$

$$M_{jky} = + K_{jky}\theta_{jy} + CK_{kjy}\theta_{ky} + S_{jky}(\Delta_j - \Delta_k) + FM_{jky}$$

$$M_{kjy} = + K_{kjy}\theta_{ky} + CK_{jky}\theta_{jy} + S_{kjy}(\Delta_j - \Delta_k) + FM_{kjy}$$

Member mj

$$M_{mjx} = + K_{mjx}\theta_{mx} + CK_{jmx}\theta_{jx} - S_{mjx}(\Delta_m - \Delta_j) + FM_{mjx}$$

$$M_{jmx} = + K_{jmx}\theta_{jx} + CK_{mjx}\theta_{mx} - S_{jmx}(\Delta_m - \Delta_j) + FM_{jmx}$$

$$M_{mjy} = + K_{mjy}\theta_{my} + CK_{jmy}\theta_{jy} + FM_{mjy}$$

$$M_{jmy} = + K_{jmy}\theta_{jy} + CK_{mjy}\theta_{my} + FM_{jmy}$$

Member jn

$$M_{jnx} = + K_{jnx}\theta_{jx} + CK_{njx}\theta_{nx} - S_{jnx}(\Delta_j - \Delta_n) + FM_{jnx}$$

$$M_{njx} = + K_{njx}\theta_{nx} + CK_{jnx}\theta_{jx} - S_{njx}(\Delta_j - \Delta_n) + FM_{njx}$$

$$M_{jny} = + K_{jny}\theta_{jy} + CK_{njy}\theta_{ny} + FM_{jny}$$

$$M_{njy} = + K_{njy}\theta_{ny} + CK_{jny}\theta_{jy} + FM_{njy}$$

TABLE 2-2 END FORCE SLOPE DEFLECTION EQUATIONS

Member ij

$$V_{ijz} = + S_{ijy}\theta_{iy} + S_{jiy}\theta_{jy} + T_{ijz}(\Delta_i - \Delta_j) + FV_{ijz}$$

$$V_{jiz} = - S_{jiy}\theta_{jy} - S_{ijy}\theta_{iy} - T_{jiz}(\Delta_i - \Delta_j) + FV_{jiz}$$

Member jk

$$V_{jkz} = + S_{jky}\theta_{jy} + S_{kjy}\theta_{ky} + T_{jkz}(\Delta_j - \Delta_k) + FV_{jkz}$$

$$V_{kjz} = - S_{kjy}\theta_{ky} - S_{jky}\theta_{jy} - T_{kjz}(\Delta_j - \Delta_k) + FV_{kjz}$$

Member mj

$$V_{mjz} = - S_{mjx}\theta_{mx} - S_{jmx}\theta_{jx} + T_{mjz}(\Delta_m - \Delta_j) + FV_{mjz}$$

$$V_{jmz} = + S_{jmx}\theta_{jx} + S_{mjx}\theta_{mx} - T_{jmz}(\Delta_m - \Delta_j) + FV_{jmz}$$

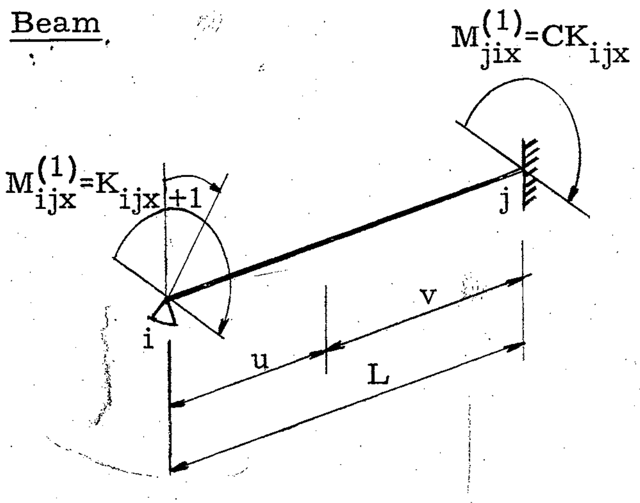
Member jn

$$V_{jnz} = - S_{jnx}\theta_{jx} - S_{njx}\theta_{nx} + T_{jnz}(\Delta_j - \Delta_n) + FV_{jnz}$$

$$V_{njz} = + S_{njx}\theta_{nx} + S_{jnx}\theta_{jx} - T_{njz}(\Delta_j - \Delta_n) + FV_{njz}$$

TABLE 2-3 CONDITION 1

Beam



Conditions

$\theta_{ix} = +1$ $\quad \theta_{iy} = 0$ $\quad \Delta_i = 0$

$\theta_{jx} = 0$ $\quad \theta_{jy} = 0$ $\quad \Delta_j = 0$

Torsional Load = 0

Flexural Load = 0

Angular Stiffness Factors

a) General Form

$$M_{ijx}^{(1)} = K_{ijx} \qquad M_{ijy}^{(1)} = 0$$

$$M_{jix}^{(1)} = CK_{ijx} \qquad M_{jiy}^{(1)} = 0$$

$$V_{ijz}^{(1)} = 0 \qquad V_{jiz}^{(1)} = 0$$

b) Algebraic Form - I Variable

$$K_{ijx} = \frac{1}{\int \frac{du}{GJ_x}}$$

$$CK_{ijx} = -\frac{1}{\int \frac{du}{GJ_x}}$$

c) Algebraic Form - I Constant

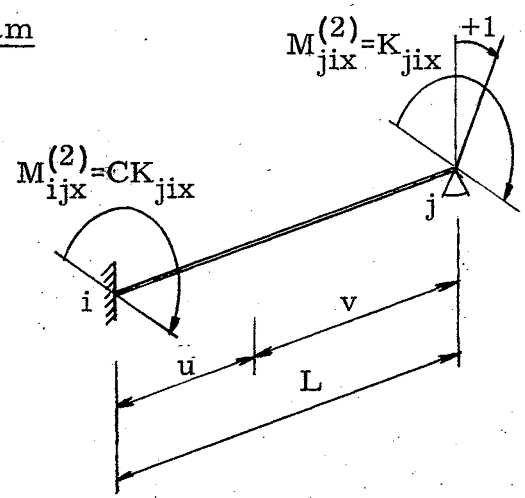
$$K_{ijx} = \frac{GJ_x}{L}$$

$$CK_{ijx} = -\frac{GJ_x}{L}$$

The limits of all integrals in this table are from i to j unless noted otherwise.

TABLE 2-4 CONDITION 2

Beam



Conditions

$\theta_{ix} = 0$ $\theta_{iy} = 0$ $\Delta_i = 0$

$\theta_{jx} = +1$ $\theta_{jy} = 0$ $\Delta_j = 0$

Torsional Load = 0

Flexural Load = 0

Angular Stiffness Factors

a) General Form

$M_{ijx}^{(2)} = CK_{jix}$ $M_{ijy}^{(2)} = 0$

$M_{jix}^{(2)} = K_{jix}$ $M_{jiy}^{(2)} = 0$

$V_{ijz}^{(2)} = 0$ $V_{jiz}^{(2)} = 0$

b) Algebraic Form - I Variable

$$K_{jix} = \frac{1}{\int \frac{dv}{GJ_x}}$$

$$CK_{jix} = - \frac{1}{\int \frac{dv}{GJ_x}}$$

c) Algebraic Form - I Constant

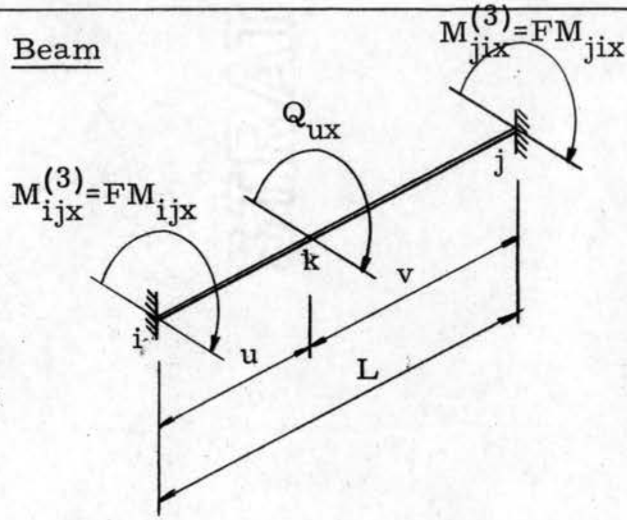
$$K_{jix} = \frac{GJ_x}{L}$$

$$CK_{jix} = - \frac{GJ_x}{L}$$

The limits of all integrals in this table are from i to j unless noted otherwise.

TABLE 2-5 CONDITION 3

Beam



Conditions

$\theta_{ix} = 0$ $\theta_{iy} = 0$ $\Delta_i = 0$

$\theta_{jx} = 0$ $\theta_{jy} = 0$ $\Delta_j = 0$

Torsional Load = Q_{ux}

Flexural Load = 0

Fixed End Moments

a) General Form

$M^{(3)}_{ijx} = FM_{ijx}$ $M^{(3)}_{ijy} = 0$

$M^{(3)}_{jix} = FM_{jix}$ $M^{(3)}_{jiy} = 0$

$V_{ijz} = 0$ $V_{jiz} = 0$

b) Algebraic Form - I Variable

$$FM_{ijx} = - \frac{Q_{ux} \int_j^k \frac{dv}{GJ_x}}{\int \frac{du}{GJ_x}}$$

$$FM_{jix} = - \frac{Q_{ux} \int_i^k \frac{du}{GJ_x}}{\int \frac{du}{GJ_x}}$$

c) Algebraic Form - I Constant

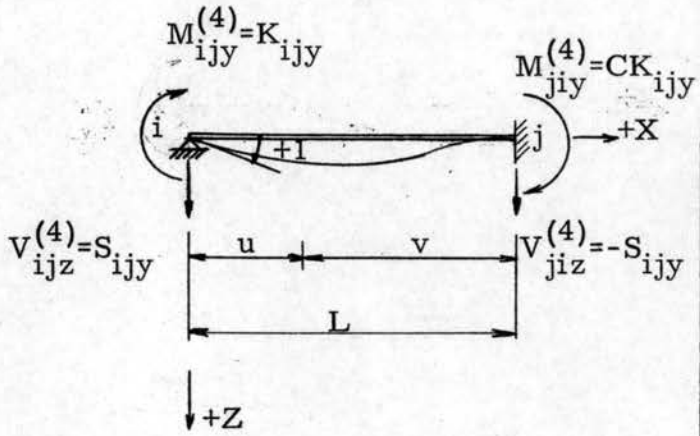
$$FM_{ijx} = - Q_{ux} \frac{v}{L}$$

$$FM_{jix} = - Q_{ux} \frac{u}{L}$$

The limits of all integrals in this table are from i to j unless noted otherwise.

TABLE 2-6 CONDITION 4

Beam



Conditions

$\theta_{ix} = 0 \quad \theta_{iy} = +1 \quad \Delta_i = 0$

$\theta_{jx} = 0 \quad \theta_{jy} = 0 \quad \Delta_j = 0$

Torsional Load = 0

Flexural Load = 0

c) Algebraic Form - I Constant

$$K_{ijy} = \frac{4EI_y}{L}$$

$$CK_{ijy} = \frac{2EI_y}{L} \qquad S_{ijy} = \frac{6EI_y}{L^2}$$

Angular Stiffness Factors

a) General Form

$M^{(4)}_{ijx} = 0 \qquad M^{(4)}_{ijy} = K_{ijy}$

$M^{(4)}_{jiy} = 0 \qquad M^{(4)}_{jiy} = CK_{ijy}$

$V^{(4)}_{ijz} = S_{ijy} \qquad V^{(4)}_{jiz} = - S_{ijy}$

b) Algebraic Form - I Variable

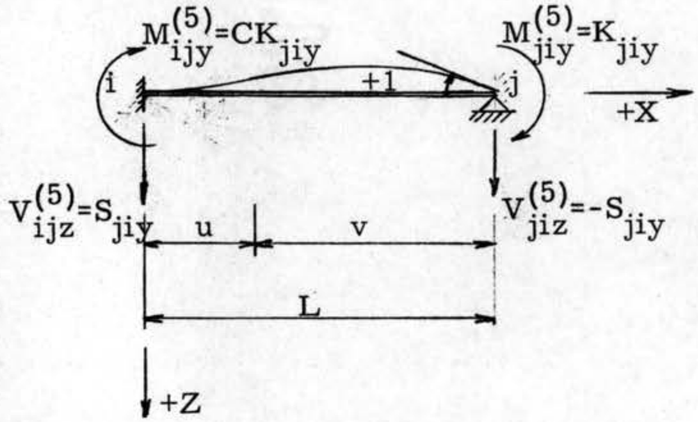
$$K_{ijy} = \frac{L^2 \int \frac{u^2\,du}{EI_y}}{\int \frac{u^2\,du}{EI_y} \cdot \int \frac{v^2\,du}{EI_y} - \left(\int \frac{uv\,du}{EI_y} \right)^2}$$

$$CK_{ijy} = \frac{L^2 \int \frac{uv\,du}{EI_y}}{\int \frac{u^2\,du}{EI_y} \cdot \int \frac{v^2\,du}{EI_y} - \left(\int \frac{uv\,du}{EI_y} \right)^2}$$

$$S_{ijy} = \frac{K_{ijy} + CK_{ijy}}{L}$$

The limits of all integrals in this table are from i to j unless noted otherwise.

TABLE 2-7 CONDITION 5



Conditions

$\theta_{ix} = 0$ $\theta_{iy} = 0$ $\Delta_i = 0$

$\theta_{jx} = 0$ $\theta_{jy} = +1$ $\Delta_j = 0$

Torsional Load = 0

Flexural Load = 0

Angular Stiffness Factors

a) General Form

$$M_{ijx}^{(5)} = 0 \qquad M_{ijy}^{(5)} = CK_{jiy}$$

$$M_{jix}^{(5)} = 0 \qquad M_{jiy}^{(5)} = K_{jiy}$$

$$V_{ijz}^{(5)} = S_{jiy} \qquad V_{jiz}^{(5)} = -S_{jiy}$$

b) Algebraic Form - I Variable

$$K_{jiy} = \frac{L^2 \int \frac{v^2 du}{EI_y}}{\int \frac{u^2 du}{EI_y} \cdot \int \frac{v^2 du}{EI_y} - \left(\int \frac{uv\, du}{EI_y} \right)^2}$$

$$CK_{jiy} = \frac{L^2 \int \frac{uv\, du}{EI_y}}{\int \frac{u^2 du}{EI_y} \cdot \int \frac{v^2 du}{EI_y} - \left(\int \frac{uv\, du}{EI_y} \right)^2}$$

$$S_{jiy} = \frac{K_{jiy} + CK_{jiy}}{L}$$

The limits of all integrals in this table are from i to j unless noted otherwise.

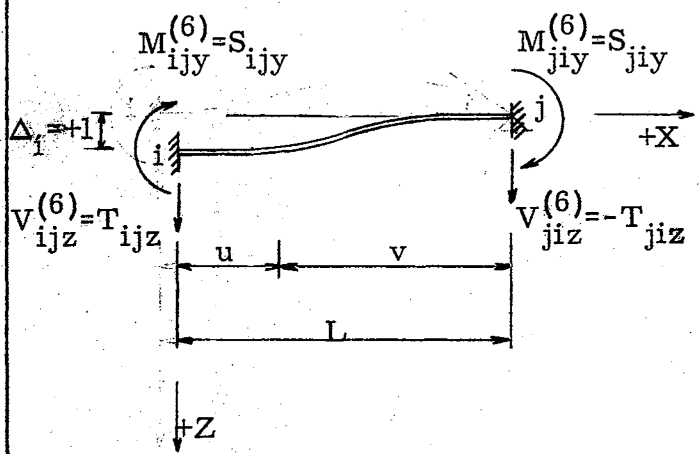
c) Algebraic Form - I Constant

$$K_{jiy} = \frac{4EI_y}{L}$$

$$CK_{jiy} = \frac{2EI_y}{L} \qquad S_{jiy} = \frac{6EI_y}{L^2}$$

TABLE 2-8 CONDITION 6

Beam



Conditions

$\theta_{ix} = 0 \qquad \theta_{iy} = 0 \qquad \Delta_i = +1$

$\theta_{jx} = 0 \qquad \theta_{jy} = 0 \qquad \Delta_j = 0$

Torsional Load = 0

Flexural Load = 0

Linear Stiffness Factors

a) General Form

$M_{ijx}^{(6)} = 0 \qquad M_{jiy}^{(6)} = S_{ijy}$

$M_{jix}^{(6)} = 0 \qquad M_{jiy}^{(6)} = S_{jiy}$

$V_{ijz}^{(6)} = T_{ijz} \qquad V_{jiz}^{(6)} = -T_{jiz}$

b) Algebraic Form - I Variable

$$S_{ijy} = \frac{K_{ijy} + CK_{ijy}}{L}$$

$$S_{jiy} = \frac{K_{jiy} + CK_{jiy}}{L}$$

$$T_{ijz} = T_{jiz} = \frac{S_{ijy} + S_{jiy}}{L}$$

(For K_{ijy}, K_{jiy} and CK_{ijy}, CK_{jiy} use Tables 2-6, -7)

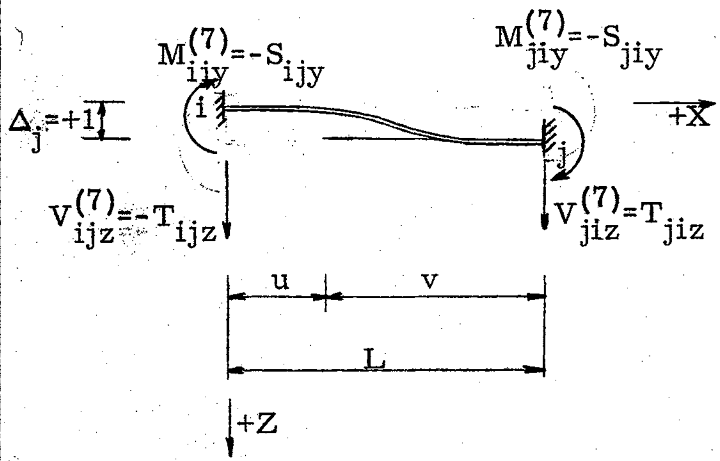
c) Algebraic Form - I Constant

$$S_{ijy} = S_{jiy} = \frac{6EI_y}{L^2}$$

$$T_{ijz} = T_{jiz} = \frac{12EI_y}{L^3}$$

TABLE 2-9 CONDITION 7

Beam



Conditions

$\theta_{ix} = 0 \quad \theta_{iy} = 0 \quad \Delta_i = 0$

$\theta_{jx} = 0 \quad \theta_{jy} = 0 \quad \Delta_j = +1$

Torsional Load = 0

Flexural Load = 0

Linear Stiffness Factors

a) General Form

$M_{ijx}^{(7)} = 0 \qquad M_{ijy}^{(7)} = -S_{ijy}$

$M_{jix}^{(7)} = 0 \qquad M_{jiy}^{(7)} = -S_{jiy}$

$V_{ijz}^{(7)} = -T_{ijz} \qquad V_{jiz}^{(7)} = T_{jiz}$

b) Algebraic Form - I Variable

$$S_{ijy} = \frac{K_{ijy} + CK_{ijy}}{L}$$

$$S_{jiy} = \frac{K_{jiy} + CK_{jiy}}{L}$$

$$T_{ijz} = T_{jiz} = \frac{S_{ij} + S_{ji}}{L}$$

(For K_{ijy}, K_{jiy} and CK_{ijy}, CK_{jiy} use Tables 2-6, -7)

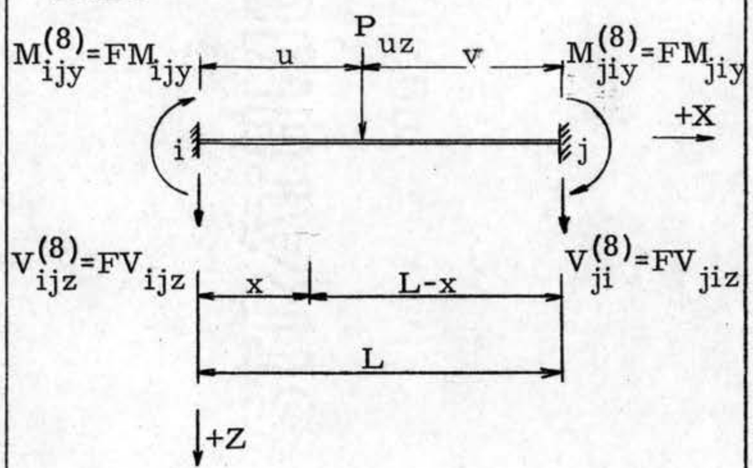
c) Algebraic Form - I Constant

$$S_{ijy} = S_{jiy} = \frac{6EI_y}{L^2}$$

$$T_{ijz} = T_{jiz} = \frac{12EI_y}{L^3}$$

TABLE 2-10a CONDITION 8a

Beam



Conditions

$\theta_{ix} = 0$ $\theta_{iy} = 0$ $\Delta_i = 0$

$\theta_{jx} = 0$ $\theta_{jy} = 0$ $\Delta_j = 0$

Torsional Load = 0

Flexural Load = P_{uz}

The limits of all integrals in this table are from i to j unless noted otherwise.

Fixed End Moments

a) General Form

$M_{ijx}^{(8)} = 0$ $M_{ijy}^{(8)} = FM_{ijy}$

$M_{ijx}^{(8)} = 0$ $M_{jiy}^{(8)} = FM_{jiy}$

$V_{ijz}^{(8)} = FV_{ijz}$ $V_{jiz}^{(8)} = FV_{jiz}$

b) Algebraic Form - I Variable

$$FM_{ijy} = -K_{ijy}\tau_{ijy} + CK_{jiy}\tau_{jiy}$$

$$FM_{jiy} = K_{jiy}\tau_{jiy} - CK_{ijy}\tau_{ijy}$$

$$\tau_{ijy} = \int \frac{BM_y (L-x)dx}{L\,EI_y}$$

$$\tau_{jiy} = \int \frac{BM_y\, x\, dx}{L\,EI_y}$$

BM_y = Bending moment of simple beam ij due to loads

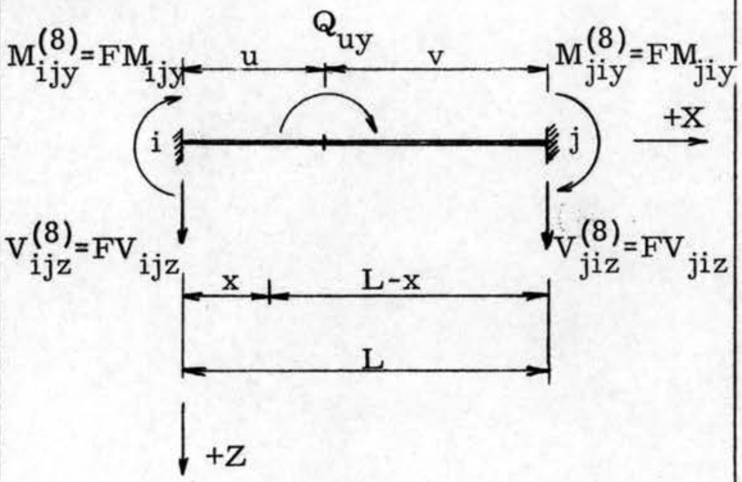
c) Algebraic Form - I Constant

$$FM_{ijy} = -\frac{P_{uz}\, uv^2}{L^2} \qquad FM_{jiy} = \frac{P_{uz}\, u^2 v}{L^2}$$

$$FV_{ijz} = -\frac{P_{uz}\, v^2(L+2u)}{L^3} \qquad FV_{jiz} = -\frac{P_{uz}\, u^2(L+2v)}{L^3}$$

TABLE 2-10b CONDITION 8b

Beam



Conditions

$\theta_{ix} = 0 \quad \theta_{iy} = 0 \quad \Delta_i = 0$

$\theta_{jx} = 0 \quad \theta_{jy} = 0 \quad \Delta_j = 0$

Torsional Load = 0

Flexural Load = Q_{uy}

The limits of all integrals in this table are from i to j unless noted otherwise.

Fixed End Moments

a) General Form

$$M_{ijx}^{(8)} = 0 \qquad M_{ijy}^{(8)} = FM_{ijy}$$

$$M_{jix}^{(8)} = 0 \qquad M_{jiy}^{(8)} = FM_{jiy}$$

$$V_{ijz}^{(8)} = FV_{ijz} \qquad V_{jiz}^{(8)} = FV_{jiz}$$

b) Algebraic Form - I Variable

$$FM_{ijy} = -K_{ijy}\tau_{ijy} + CK_{jiy}\tau_{jiy}$$

$$FM_{jiy} = K_{jiy}\tau_{jiy} - CK_{ijy}\tau_{ijy}$$

$$\tau_{ijy} = \int \frac{BM_y (L-x) dx}{L\, EI_y}$$

$$\tau_{jiy} = \int \frac{BM_y\, x\, dx}{L\, EI_y}$$

BM_y = bending moment of simple beam ij due to loads

c) Algebraic Form - I Constant

$$FM_{ijy} = \frac{Q_{uy}\, v}{L^2}(3u - L) \qquad FM_{jiy} = -\frac{Q_{uy}\, u}{L^2}(3v - L)$$

$$FV_{ijz} = -FV_{jiz} = 6\frac{uv}{L^3} Q_{uy}$$

TABLE 2-11 JOINT X - MOMENT EQUILIBRIUM EQUATION

Algebraic Form:

$$+\ \theta_{nx}CK_{njx} \qquad\qquad + \ \Delta_n S_{jnx}$$

$$\theta_{ix}CK_{ijx} + \theta_{jx}\Sigma K_{jx} + \theta_{kx}CK_{kjx} + \Delta_j \Sigma S_{jx} + \Sigma FM_{jx} - M_{jx} = 0$$

$$+\ \theta_{mx}CK_{mjx} \qquad\qquad - \ \Delta_m S_{jmx}$$

Equivalents:

$$\Sigma K_{jx} = K_{jix} + K_{jkx} + K_{jmx} + K_{jnx} \qquad \Sigma S_{jx} = S_{jmx} - S_{jnx}$$

$$\Sigma FM_{jx} = FM_{jix} + FM_{jkx} + FM_{jmx} + FM_{jnx}$$

TABLE 2-12 JOINT Y - MOMENT EQUILIBRIUM EQUATION

Algebraic Form:

$$
\begin{array}{llllll}
 & + \theta_{ny}CK_{njy} & & + \Delta_i S_{jiy} & & \\
\theta_{iy}CK_{ijy} & + \theta_{jy}\Sigma K_{jy} & + \theta_{ky}CK_{kjy} & + \Delta_j \Sigma S_{jy} & + \Sigma FM_{jy} & - M_{jy} = 0 \\
 & + \theta_{my}CK_{mjy} & & - \Delta_k S_{jky} & &
\end{array}
$$

Equivalents:

$$\Sigma K_{jy} = K_{jiy} + K_{jky} + K_{jmy} + K_{jny}$$

$$\Sigma S_{jy} = S_{jky} - S_{jiy}$$

$$\Sigma FM_{jy} = FM_{jiy} + FM_{jky} + FM_{jmy} + FM_{jny}$$

TABLE 2-13 JOINT Z - FORCE EQUILIBRIUM EQUATIONS

Algebraic Form:

$$\qquad\qquad - \theta_{nx} S_{njx} \qquad\qquad\qquad\qquad\qquad\qquad - \Delta_n T_{jnz}$$

$$- \theta_{iy} S_{ijy} + \theta_{jy} \Sigma S_{jy} + \theta_{jx} \Sigma S_{jx} + \theta_{ky} S_{kjy} - \Delta_i T_{jiz} + \Delta_j \Sigma T_j - \Delta_k T_{jkz} + \Sigma FV_{jz} - P_{jz} = 0$$

$$\qquad\qquad + \theta_{mx} S_{mjx} \qquad\qquad\qquad\qquad\qquad\qquad - \Delta_m T_{jmz}$$

Equivalents:

$$\Sigma S_{jy} = S_{jky} - S_{jiy}$$

$$\Sigma S_{jx} = S_{jmx} - S_{jnx}$$

$$\Sigma T_j = T_{jiz} + T_{jkz} + T_{mjz} + T_{jnz}$$

$$\Sigma FV_{jz} = FV_{jiz} + FV_{jkz} + FV_{jmz} + FV_{jnz}$$

$$\theta_{jix} = \theta_{jkx} = \theta_{jmx} = \theta_{jnx} = \theta_{jx}$$

$$\theta_{jiy} = \theta_{jky} = \theta_{jmy} = \theta_{jny} = \theta_{jy} \qquad (2\text{-}1)$$

$$\Delta_{jiz} = \Delta_{jkz} = \Delta_{jmz} = \Delta_{jnz} = \Delta_{jz}$$

This relationship explains why only two angular deformations and one linear deformation occur at a given joint. The joint equilibrium equations state (Fig. 2-3) that: "Sum of all end moments, end forces and joint loads at a given joint with respect to a given axis is equal to zero."

$$- M_{jix} - M_{jkx} - M_{jmx} - M_{jnx} + M_{jx} = 0$$

$$- M_{jiy} - M_{jky} - M_{jmy} - M_{jny} + M_{jy} = 0 \qquad (2\text{-}2)$$

$$- V_{jiz} - V_{jkz} - V_{jmz} - V_{jnz} + P_{jz} = 0$$

Thus there are as many equations of static equilibrium as joint deformations.

2-4. Deformation Matrix.

The joint equilibrium equations (Eq's. 2-2) may be expressed in terms of elastic constants, and deformations (Tables 2-11, -12, -13) and written for each joint of the grid. The matrix thus obtained is called the stiffness matrix. The solution of the joint equilibrium matrix gives the numerical values of θ's and Δ's . The final moments are obtained by substituting these numerical values in the slope-deflection equations (Tables 2-1, -2).

2-5. Procedure of Analysis.

The procedure of analysis may be summarized in the following steps:

(a) Designate all joints by symbols and introduce redundant slopes and deflections.

(b) Record geometric values such as lengths of span, functions of cross sections, position coordinates of loads and material constants E and G .

(c) Compute elastic constants: fixed end moments FM's , fixed end shears FV's , stiffness factors K's , CK's , S's , T's .

(d) Write joint equilibrium equations for all joints.

(e) Consider end conditions and eliminate unnecessary unknown θ's and Δ's .

(f) Invert the stiffness matrix for θ's and Δ's .

(g) Compute the end moments and the end shears.

(h) Check for the equilibrium at all joints.

CHAPTER III

MOMENT AND SHEAR DISTRIBUTION METHOD

3.1. General.

The moment and shear distribution method as a general method of numerical, successive approximation for analyzing planar frames with straight members has been developed by Cross (9). The extension of this method to the analysis of orthogonal, planar grids is discussed herein. In applying the moment and shear distribution method to this structure, six classes of constants must be considered:

(a) load constants

(b) X-angular constants

(c) Y-angular constants

(d) Z-linear constants

(e) XZ-cross constants

(f) YZ-cross constants .

These constants are derived from Tables 2-11, -12, -13 by means of special conditions.

3-2. Load Constants.

If all deformations in joint equations (Table 2-11, -12, -13) are assumed to be equal to zero, three load constants are obtained.

(a) Unbalance of X-moments at j

$$\left|\Delta M_{jx}\right|^{(W)} = + \Sigma FM_{jx} - M_{jx} . \tag{3-1x}$$

(b) Unbalance of Y-moments at j

$$\left|\Delta M_{jy}\right|^{(W)} = + \Sigma FM_{jy} - M_{jy} . \tag{3-1y}$$

(c) Unbalance of Z-shears at j

$$\left|\Delta V_{jz}\right|^{(W)} = + \Sigma FV_{jz} - P_{jz} . \tag{3-1z}$$

Because these constants represent the unbalance at j , a corrective procedure, which will eliminate this unbalance, must be developed.

3-3. X-Angular Constants.

If the joint j is allowed to rotate about the X-axis (all other deformations are assumed equal to zero), the slope at j (Table 2-11)

$$\theta_{jx} = - \frac{\left|\Delta M_{jx}\right|^{(W)}}{\Sigma K_{jx}} \tag{3-2x}$$

and the end moments due to loads and this rotation become (Table 2-1),

$$\begin{aligned}
M_{jix} &= - \frac{K_{jix}}{\Sigma K_{jx}} \left|\Delta M_{jx}\right|^{(W)} + FM_{jix} \\
M_{jkx} &= - \frac{K_{jkx}}{\Sigma K_{jx}} \left|\Delta M_{jx}\right|^{(W)} + FM_{jkx} \\
M_{jmx} &= - \frac{K_{jmx}}{\Sigma K_{jx}} \left|\Delta M_{jx}\right|^{(W)} + FM_{jmx} \\
M_{jnx} &= - \frac{K_{jnx}}{\Sigma K_{jx}} \left|\Delta M_{jx}\right|^{(W)} + FM_{jnx}
\end{aligned} \tag{3-3x}$$

$$M_{ijx} = -\frac{CK_{jix}}{\Sigma K_{jx}} \left|\Delta M_{jx}\right|^{(W)} + FM_{ijx}$$

$$M_{kjx} = -\frac{CK_{jkx}}{\Sigma K_{jx}} \left|\Delta M_{jx}\right|^{(W)} + FM_{kjx}$$

$$M_{mjx} = -\frac{CK_{jmx}}{\Sigma K_{jx}} \left|\Delta M_{jx}\right|^{(W)} + FM_{mjx} \qquad (3\text{-}4x)$$

$$M_{njx} = -\frac{CK_{jnx}}{\Sigma K_{jx}} \left|\Delta M_{jx}\right|^{(W)} + FM_{njx} \quad .$$

New constants introduced in Eq's. (3-3x, -4x) are defined below:

(a) Moment Joint Stiffness ΣK_{jx} is the joint moment required at j , to produce a unit rotation of that joint about the X-axis

$$\Sigma K_{jx} = K_{jix} + K_{jkx} + K_{jmx} + K_{jnx} \quad . \qquad (3\text{-}5x)$$

(b) Moment Distribution Factors

$$D^{(0)}_{jix} = -\frac{K_{jix}}{\Sigma K_{jx}} \qquad D^{(0)}_{jmx} = -\frac{K_{jmx}}{\Sigma K_{jx}}$$

$$D^{(0)}_{jkx} = -\frac{K_{jkx}}{\Sigma K_{jx}} \qquad D^{(0)}_{jnx} = -\frac{K_{jnx}}{\Sigma K_{jx}} \qquad (3\text{-}6x)$$

are the moments developed at near ends of members forming joint j by a unit moment unbalance about the X-axis at that joint. The sum of distribution factors

$$\Sigma D^{(0)}_{jx} = D^{(0)}_{jix} + D^{(0)}_{jkx} + D^{(0)}_{jmx} + D^{(0)}_{jnx} = +1 \quad . \qquad (3\text{-}7x)$$

(c) Moment Carry-Over Distribution Factors

$$CD^{(0)}_{jix} = -\frac{CK_{jix}}{\Sigma K_{jx}} = C^{(0)}_{jix} D^{(0)}_{jix}$$

$$CD^{(0)}_{jkx} = -\frac{CK_{jkx}}{\Sigma K_{jx}} = C^{(0)}_{jkx} D^{(0)}_{jkx}$$

$$CD^{(0)}_{jmx} = -\frac{CK_{jmx}}{\Sigma K_{jx}} = C^{(0)}_{jmx} D^{(0)}_{jmx} \qquad (3\text{-}8x)$$

$$CD^{(0)}_{jnx} = -\frac{CK_{jnx}}{\Sigma K_{jx}} = C^{(0)}_{jnx} D^{(0)}_{jnx}$$

are the moments developed at the far ends of members forming the joint j by a unit moment unbalance about the X-axis at that joint.

(d) Moment Carry-Over Factors

$$C^{(0)}_{jix} = \frac{CD^{(0)}_{jix}}{D^{(0)}_{jix}} = \frac{CK_{jix}}{K_{jix}}$$

$$C^{(0)}_{jkx} = \frac{CD^{(0)}_{jkx}}{D^{(0)}_{jkx}} = \frac{CK_{jkx}}{K_{jkx}}$$

$$C^{(0)}_{jmx} = \frac{CD^{(0)}_{jmx}}{D^{(0)}_{jmx}} = \frac{CK_{jmx}}{K_{jmx}} \qquad (3\text{-}9x)$$

$$C^{(0)}_{jnx} = \frac{CD^{(0)}_{jnx}}{D^{(0)}_{jnx}} = \frac{CK_{jnx}}{K_{jnx}}$$

are the ratios of the moments developed at the far ends to their counterparts induced at the near ends. For members of constant cross section, the torsional carry-over factor

$$C^{(0)}_{jix} = C^{(0)}_{jkx} = -1 \qquad (3\text{-}10x)$$

and the flexural carry-over factor

$$C^{(0)}_{jmx} = C^{(0)}_{jnx} = +\frac{1}{2} \quad . \qquad (3\text{-}11x)$$

3-4. Y-Angular Constants.

If the joint j is now allowed to rotate about the Y-axis, the slope at j becomes (Table 2-12)

$$\theta_{jy} = -\frac{|\Delta M_{jy}|^{(W)}}{\Sigma K_{jy}} \qquad (3\text{-}2y)$$

and the end moments due to loads and this new rotation may be obtained from Eq's. (3-3x, -4x) by substituting y for x . New constants thus obtained are identical to those in Eq's. (3-5x, -6x, -7x, -8x, -9x) if y is interchanged for x . For members of constant cross section, the torsional carry-over factor

$$C^{(0)}_{jmy} = C^{(0)}_{jny} = -1 \qquad (3\text{-}10y)$$

and the flexural carry-over factor

$$C^{(0)}_{jiy} = C^{(0)}_{jky} = +\frac{1}{2} \; . \qquad (3\text{-}11y)$$

3-5. Z-Linear Constants.

If finally the joint j is free to displace vertically but locked against rotation (Table 2-13), the deflection at j

$$\Delta_{j} = -\frac{|\Delta V_{jz}|^{(W)}}{\Sigma T_{j}} \qquad (3\text{-}2z)$$

and the end shears due to loads and this deflection become

$$V_{jiz} = -\frac{T_{jiz}}{\Sigma T_j} \left|\Delta V_{jz}\right|^{(W)} + FV_{jiz}$$

$$V_{jkz} = -\frac{T_{jkz}}{\Sigma T_j} \left|\Delta V_{jz}\right|^{(W)} + FV_{jkz}$$

$$V_{jmz} = -\frac{T_{jmz}}{\Sigma T_j} \left|\Delta V_{jz}\right|^{(W)} + FV_{jmz} \qquad (3\text{-}3z)$$

$$V_{jnz} = -\frac{T_{jnz}}{\Sigma T_j} \left|\Delta V_{jz}\right|^{(W)} + FV_{jnz}$$

$$V_{ijz} = \frac{T_{jiz}}{\Sigma T_j} \left|\Delta V_{jz}\right|^{(W)} + FV_{ijz}$$

$$V_{kjz} = \frac{T_{jkz}}{\Sigma T_j} \left|\Delta V_{jz}\right|^{(W)} + FV_{kjz}$$

$$V_{mjz} = \frac{T_{jmz}}{\Sigma T_j} \left|\Delta V_{jz}\right|^{(W)} + FV_{mjz} \qquad (3\text{-}4z)$$

$$V_{njz} = \frac{T_{jnz}}{\Sigma T_j} \left|\Delta V_{jz}\right|^{(W)} + FV_{njz} \quad .$$

New constants introduced in Eq's. (3-3z, -4z) are similar in sense to those defined by Eq's. (3-5x, -6x, -7x, -8x, -9x).

(a) Shear Joint Stiffness ΣT_{jz} is the joint force required at j to produce a unit displacement of that joint in the Z-direction

$$\Sigma T_j = T_{jiz} + T_{jkz} + T_{jmz} + T_{jnz} \quad . \qquad (3\text{-}5z)$$

(b) Shear Distribution Factors

$$D^{(\Delta)}_{jiz} = -\frac{T_{jiz}}{\Sigma T_j} \qquad D^{(\Delta)}_{jmz} = -\frac{T_{jmz}}{\Sigma T_j}$$

$$D^{(\Delta)}_{jkz} = -\frac{T_{jkz}}{\Sigma T_j} \qquad D^{(\Delta)}_{jnz} = -\frac{T_{jnz}}{\Sigma T_j} \qquad (3\text{-}6z)$$

are the end shears developed at the near ends of members forming joint j by a unit shear unbalance at that joint. The sum of shear distribution factors

$$\Sigma D^{(\Delta)}_{jz} = D^{(\Delta)}_{jiz} + D^{(\Delta)}_{jkz} + D^{(\Delta)}_{jmz} + D^{(\Delta)}_{jnz} = +1 \ . \qquad (3\text{-}7z)$$

(c) Shear Carry-Over Distribution Factors

$$\begin{aligned} CD^{(\Delta)}_{jiz} &= \frac{T_{jiz}}{\Sigma T_j} = \frac{T_{ijz}}{\Sigma T_j} = C^{(\Delta)}_{jiz}\, D^{(\Delta)}_{jiz} \\ CD^{(\Delta)}_{jkz} &= \frac{T_{jkz}}{\Sigma T_j} = \frac{T_{kjz}}{\Sigma T_j} = C^{(\Delta)}_{jkz}\, D^{(\Delta)}_{jkz} \\ CD^{(\Delta)}_{jmz} &= \frac{T_{jmz}}{\Sigma T_j} = \frac{T_{mjz}}{\Sigma T_j} = C^{(\Delta)}_{jmz}\, D^{(\Delta)}_{jmz} \\ CD^{(\Delta)}_{jnz} &= \frac{T_{jnz}}{\Sigma T_j} = \frac{T_{njz}}{\Sigma T_j} = C^{(\Delta)}_{jnz}\, D^{(\Delta)}_{jnz} \end{aligned} \qquad (3\text{-}8z)$$

are the shears developed at the far ends of members forming the joint j , by a unit shear unbalance at that joint.

(d) Shear Carry-Over Factors

$$\begin{aligned} C^{(\Delta)}_{jiz} &= \frac{CD^{(\Delta)}_{jiz}}{D^{(\Delta)}_{jiz}} = -1 \\ C^{(\Delta)}_{jkz} &= \frac{CD^{(\Delta)}_{jkz}}{D^{(\Delta)}_{jkz}} = -1 \\ C^{(\Delta)}_{jmz} &= \frac{CD^{(\Delta)}_{jmz}}{D^{(\Delta)}_{jmz}} = -1 \\ C^{(\Delta)}_{jnz} &= \frac{CD^{(\Delta)}_{jnz}}{D^{(\Delta)}_{jnz}} = -1 \end{aligned} \qquad (3\text{-}9z)$$

are the ratios of the end shears developed at the far ends to their counterparts induced at the near ends. These constants are equal to -1, regardless of the variation of the cross section.

3-6. XZ-Cross Constants.

New moments developed about the X-axis by means of Eq's. (3-3x, -4x) consist of two parts:

(a) Initial fixed end moment FM

(b) Correction moment DM or CDM .

The correction moments introduce new unbalance of shears at j ,

$$\left|\Delta V_{jz}\right|^{(0x)} = - \frac{S_{mjx}}{\Sigma K_{mx}} \left|\Delta M_{mx}\right|^{(W)} - \frac{\Sigma S_{jx}}{\Sigma K_{jx}} \left|\Delta M_{jx}\right|^{(W)} + \frac{S_{njx}}{\Sigma K_{nx}} \left|\Delta M_{nx}\right|^{(W)}$$

which is the sum of the products of moment-shear carry-over factors and the joint moment unbalances. The moment-shear carry-over factors

$$C^{(0\Delta)}_{mjxz} = - \frac{S_{mjx}}{\Sigma K_{mx}} \quad \Bigg| \quad C^{(0\Delta)}_{jjxz} = - \frac{\Sigma S_{jx}}{\Sigma K_{jx}} \quad \Bigg| \quad C^{(0\Delta)}_{njxz} = + \frac{S_{njx}}{\Sigma K_{nx}} \qquad (3\text{-}11x)$$

are the shear unbalances at j due to a unit moment unbalance at m , j , and n , respectively.

Similarly, new shears developed by means of Eq. (3-3z, -4z) consist of two parts:

(c) Initial fixed end shear

(d) Correction shear DV or CDV .

The correction shears introduce new unbalance of moments about the X-axis at j ,

$$\left|\Delta M_{jx}\right|^{(\Delta z)} = + \frac{S_{jmx}}{\Sigma T_{m}} \left|\Delta V_{mz}\right|^{(W)} - \frac{\Sigma S_{jx}}{\Sigma T_{j}} \left|\Delta V_{jz}\right|^{(W)} - \frac{S_{jnx}}{\Sigma T_{n}} \left|\Delta V_{nz}\right|^{(W)}$$

(3-12x)

which is the sum of products of shear-moment carry-over factors and the joint shear unbalances. The shear moment carry-over factors

$$C^{(\Delta 0)}_{mjzx} = + \frac{S_{jmx}}{\Sigma T_m} \quad \Bigg| \quad C^{(\Delta 0)}_{jjzx} = - \frac{\Sigma S_{jx}}{\Sigma T_j} \quad \Bigg| \quad C^{(\Delta 0)}_{njzx} = - \frac{S_{jnx}}{\Sigma T_n} \qquad (3\text{-}13x)$$

are the moment unbalances at j due to a unit shear unbalance at m , j , and n , respectively.

3-7. YZ-Cross Constants.

A similar situation arises about the Y-axis, where the shear unbalance,

$$\left|\Delta V_{jz}\right|^{(0Y)} = + \frac{S_{ijy}}{\Sigma K_{iy}} \left|\Delta M_{iy}\right|^{(W)} - \frac{\Sigma S_{jy}}{\Sigma K_{jy}} \left|\Delta M_{jy}\right|^{(W)} - \frac{S_{kjy}}{\Sigma K_{ky}} \left|\Delta M_{ky}\right|^{(W)} \qquad (3\text{-}10y)$$

the moment-shear carry-over factors YZ ,

$$C^{(0\Delta)}_{ijyz} = + \frac{S_{ijy}}{\Sigma K_{iy}} \quad \Bigg| \quad C^{(0\Delta)}_{jjyz} = - \frac{\Sigma S_{jy}}{\Sigma K_{jy}} \quad \Bigg| \quad C^{(0\Delta)}_{kjyz} = - \frac{S_{kjy}}{\Sigma K_{ky}} \qquad (3\text{-}11y)$$

the moment unbalance

$$\left|\Delta M_{jy}\right|^{(\Delta Z)} = - \frac{S_{jiy}}{\Sigma T_i} \left|\Delta V_{iz}\right|^{(W)} - \frac{\Sigma S_{jy}}{\Sigma T_j} \left|\Delta V_{jz}\right|^{(W)} + \frac{S_{jk}}{\Sigma T_k} \left|\Delta V_{kz}\right|^{(W)} \qquad (3\text{-}12y)$$

and the shear-moment carry-over factor ZY ,

$$C^{(\Delta 0)}_{ijzy} = - \frac{S_{jiy}}{\Sigma T_i} \quad \Bigg| \quad C^{(\Delta 0)}_{jjzy} = - \frac{\Sigma S_{jy}}{\Sigma T_j} \quad \Bigg| \quad C^{(\Delta 0)}_{kjzy} = + \frac{S_{jk}}{\Sigma T_k} \qquad (3\text{-}13y)$$

3-8. Procedure of Analysis.

The application of these constants in the analysis may be summarized in the following steps:

(a) Designate all joints by symbols and record geometric values and material constants.

(b) Compute elastic constants FM's , FV's , K's , CK's , S's , T's , ΣK's , ΣS's , ΣT's , D's , and C's .

(c) Prepare three distribution tables (X-moments, Y-moments, and Z-shears) with number of columns equal to number of end values required.

(d) Record distribution factors, carry-over factors, fixed end moments and fixed end shears in the corresponding columns of each table.

(e) Using the moment and shear distribution procedure, calculate the resulting moments and shears at each end.

(f) Check for the equilibrium of joints.

It is important to note that:

(1) Each distribution cycle consists of three distributions (about X-, Y-, Z-axis) at each joint, each type performed in a separate table.

(2) Each in-table carry-over cycle consists of three independent transmissions from one end to the other end, each type performed in a separate table.

(3) Each between-tables carry-over cycle consists of transmissions from the moment table -x and -y to the shear table -z and vice versa.

(4) Each joint unbalance consists of two parts: carried-over values from the adjacent joints (in-table unbalance) and the sum of carried-over values from the same joint in other table or tables (between-tables unbalance).

CHAPTER IV

JOINT MOMENT AND SHEAR CARRY-OVER METHOD

4-1. General.

The application of the joint moment and shear carry-over method to the analysis of orthogonal, planar grids is discussed in this chapter. The historical background of this method was recorded by Tseng (10) and is not repeated here.

4-2. Elastic Constants.

Similarly as in the case of the moment and shear distribution method, the application of the joint moment and shear carry-over method requires calculation of elastic constants, by means of which the carry-over procedure is performed. In the derivation of the analytical expressions for these constants, the joint equations (Table 2-11, -12, -13) and new equivalents are utilized.

The product of the joint rotation and the joint moment stiffness

$$\theta_{jx} \; \Sigma K_{jx} = JM_{jxx} \qquad (4\text{-}1x)$$

$$\theta_{jy} \; \Sigma K_{jy} = JM_{jyy} \qquad (4\text{-}1y)$$

is defined as the joint moment required at j to produce a rotation θ_j, when all other deformations are equal to zero.

Similarly, the product of the joint deflection and the joint shear stiffness

$$\Delta_j \, \Sigma T_j = JV_{jzz} \tag{4-1z}$$

is defined as the joint force required at j to produce a deflection Δ_j, when all other deformations are equal to zero. These joint equivalents are introduced at all joints of the grid.

4-3. X - Joint Constants.

With new symbols

$$r_{ijxx} = -\frac{CK_{ijx}}{\Sigma K_{ix}} \qquad r_{mjxx} = -\frac{CK_{mjx}}{\Sigma K_{mx}}$$

$$r_{kjxx} = -\frac{CK_{kjx}}{\Sigma K_{xx}} \qquad r_{njxx} = -\frac{CK_{njx}}{\Sigma K_{nx}} \tag{4-2x}$$

$$s_{ijzx} = 0 \qquad s_{mjzx} = +\frac{S_{jmx}}{\Sigma T_m}$$

$$s_{kjzx} = 0 \qquad s_{njzx} = -\frac{S_{jnx}}{\Sigma T_n} \tag{4-3x}$$

$$s_{jjzx} = -\frac{\Sigma S_{jx}}{\Sigma T_j}$$

$$m_{jxx} = -\Sigma FM_{jx} + M_{jx} \tag{4-4x}$$

The X-joint equilibrium equation (Table 2-11) takes the carry-over form shown in Table 4-1.

The joint constants r's , s's , and m's have the following physical meaning:

(a) The Starting Moment m_{jxx} is the X-joint moment at j due to loads, when the far joints are locked against rotation and deflection, and the near joint j is restrained against deflection.

TABLE 4-1 JOINT CARRY-OVER EQUATIONS

JM_{jxx}	=	$r_{mjxx}JM_{mxx}$, $r_{ijxx}JM_{ixx}$, $s_{mjxz}JV_{mzz}$, $s_{jjzx}JV_{jzz}$, $s_{njzx}JV_{nzz}$, $r_{kjxx}JM_{kxx}$, $r_{njxx}JM_{nxx}$ → m_{jxx}
JV_{jzz}	=	$t_{mjxz}JM_{mxx}$, $t_{jjxz}JM_{jxx}$, $t_{njxz}JM_{nxx}$, $q_{mjzz}JV_{mzz}$, $q_{ijzz}JV_{izz}$, $t_{ijyz}JM_{iyy}$, $t_{jjyz}JM_{jyy}$, $t_{kjyz}JM_{kyy}$, $q_{kjzz}JV_{kzz}$, $q_{njzz}JV_{nzz}$ → v_{jzz}
JM_{jyy}	=	$s_{jjzy}JV_{jzz}$, $s_{ijzy}JV_{izz}$, $r_{ijyy}JM_{iyy}$, $r_{mjyy}JM_{myy}$, $r_{njyy}JM_{nyy}$, $r_{kjyy}JM_{kyy}$, $s_{kjzy}JV_{kzz}$ → m_{jyy}

TABLE 4-2x X - CARRY-OVER MOMENT FACTORS

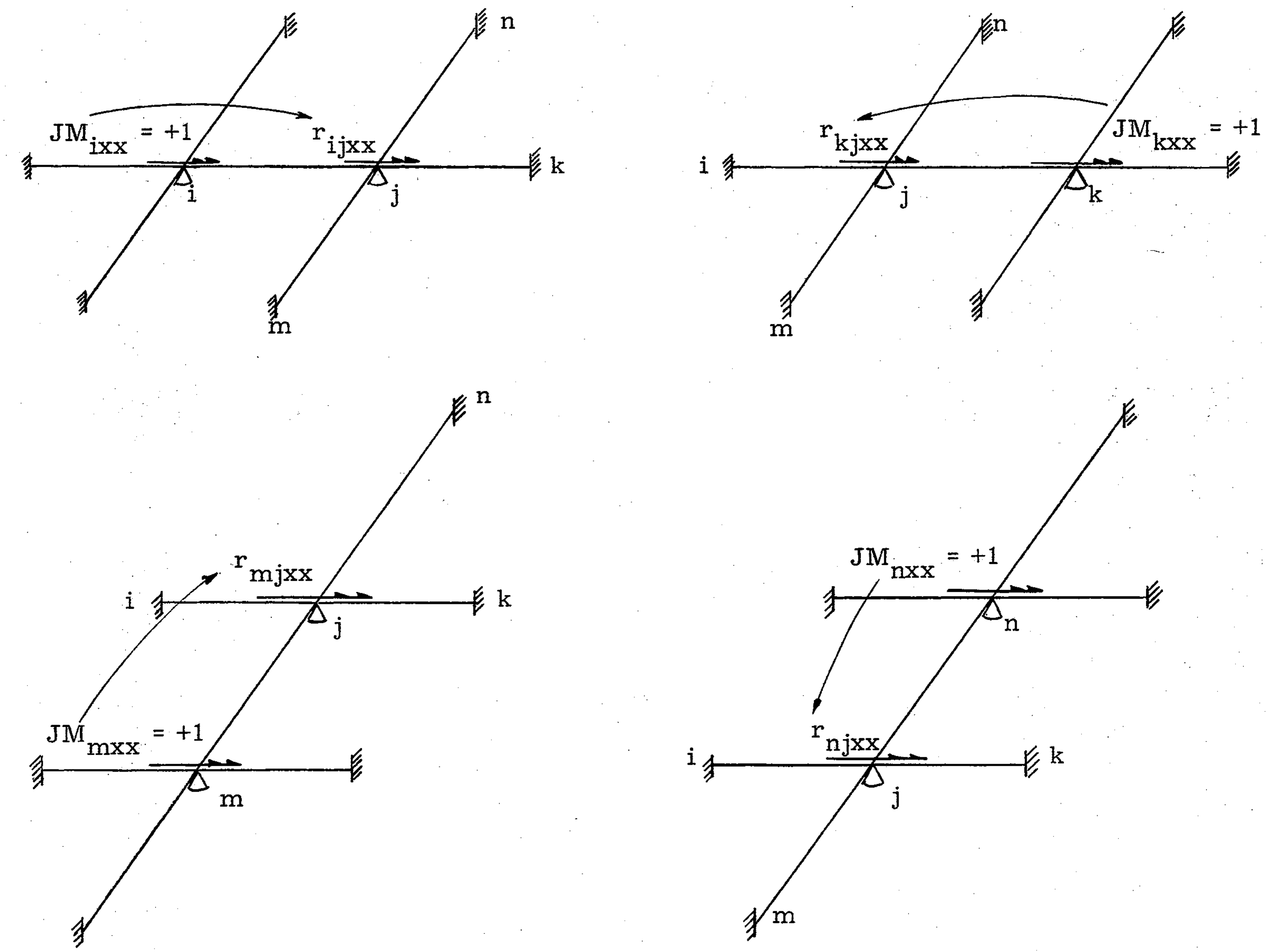
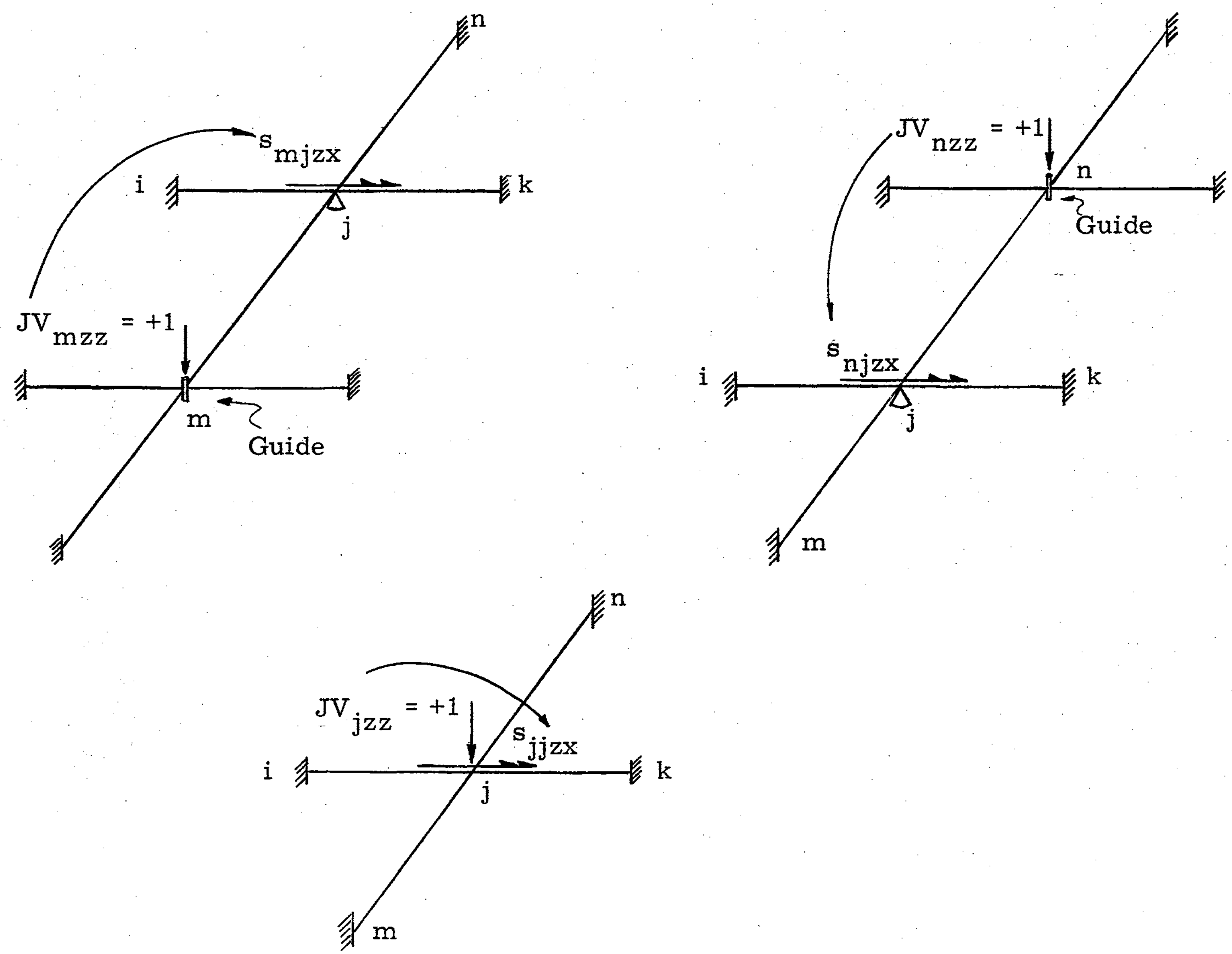


TABLE 4-3x ZX - CARRY-OVER SHEAR MOMENT FACTORS



(b) The Carry-Over Moment Factor r_{ijxx} (r_{kjxx}, r_{mjxx}, r_{njxx}) is the X-joint moment at j due to a unit X-joint moment at i (k, m, n) when all joints are restrained against deflection and the uninvolved joints are fixed.

(c) The Carry-Over Shear-Moment Factor s_{mjzx} (s_{jjzx}, s_{njxx}) is the X-joint moment at j due to a unit Z-joint shear at m(j, n) when all other joints are restrained against deflection and all far ends are fixed.

4-4. Y-Joint Constants.

With similar symbols

$$r_{ijyy} = -\frac{CK_{ijy}}{\Sigma K_{iy}} \qquad r_{mjyy} = -\frac{CK_{mjy}}{\Sigma K_{my}}$$

$$r_{kjyy} = -\frac{CK_{kjy}}{\Sigma K_{ky}} \qquad r_{njyy} = -\frac{CK_{njy}}{\Sigma K_{ny}} \qquad (4\text{-}2y)$$

$$s_{ijzy} = -\frac{S_{jiy}}{\Sigma T_i} \qquad s_{mjzy} = 0$$

$$s_{kjzy} = \frac{S_{kjy}}{\Sigma T_k} \qquad s_{njzy} = 0 \qquad (4\text{-}3y)$$

$$s_{jjzy} = -\frac{\Sigma S_{jy}}{\Sigma T_j}$$

$$m_{jyy} = -\Sigma FM_{jy} + M_{jy} \qquad (4\text{-}4y)$$

the joint Y-joint equilibrium equation (Table 2-12) takes the carry-over form shown in Table 4-1. The physical meaning of these constants is similar to those defined in the preceding article.

TABLE 4-2y

Y - CARRY-OVER MOMENT FACTORS

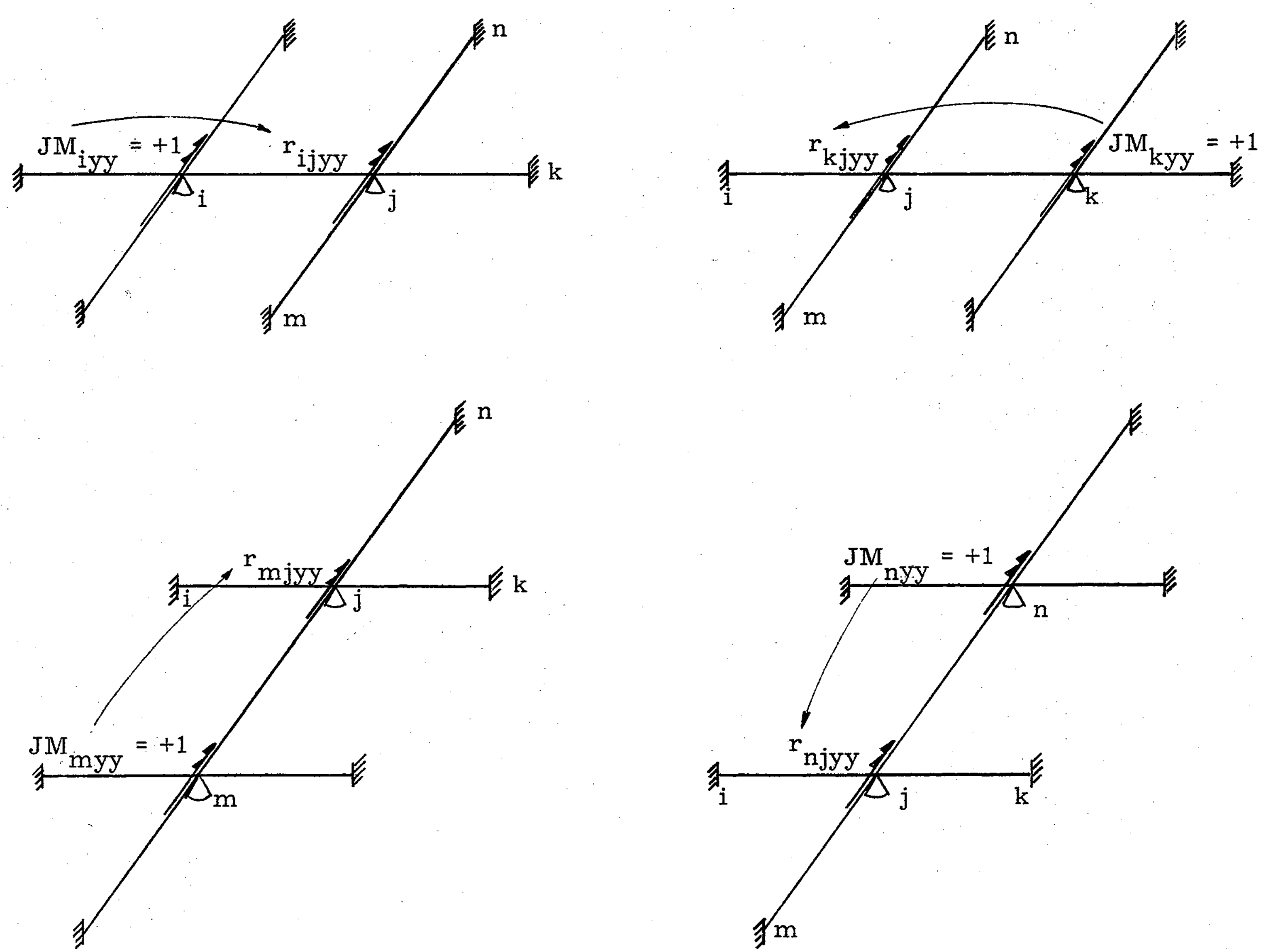
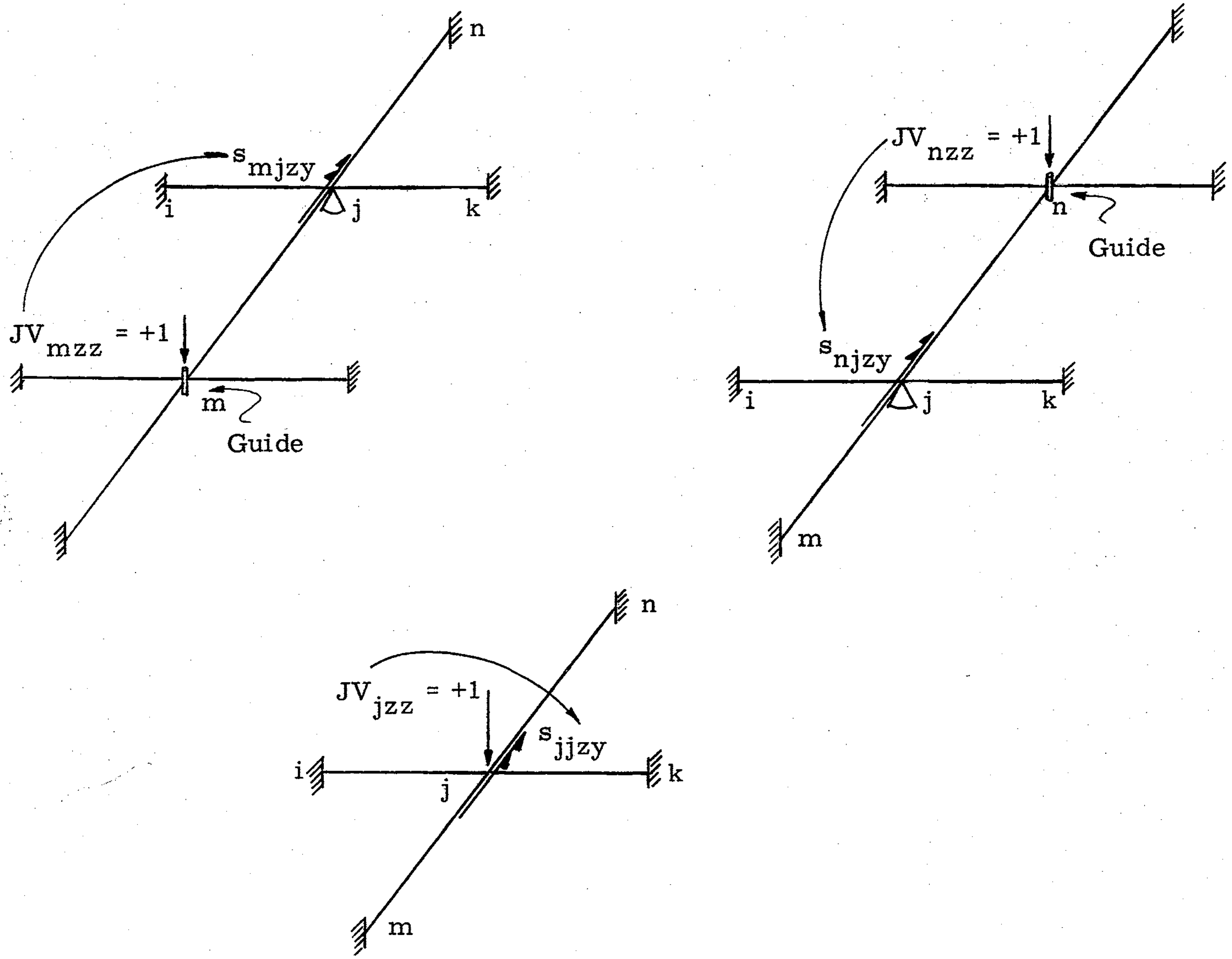


TABLE 4-3y

ZY - CARRY-OVER SHEAR MOMENT FACTORS



4-5. Z-Joint Constants.

With similar symbols

$$q_{mjzz} = \frac{T_{mjz}}{\Sigma T_m} \qquad q_{njzz} = \frac{T_{njz}}{\Sigma T_n}$$

$$q_{ijzz} = \frac{T_{ijz}}{\Sigma T_i} \qquad q_{kjzz} = \frac{T_{kjz}}{\Sigma T_k} \tag{4-2z}$$

$$t_{jjxz} = -\frac{\Sigma S_{jx}}{\Sigma K_{jx}}$$

$$t_{mjxz} = -\frac{S_{mjx}}{\Sigma K_{mx}} \qquad t_{njxz} = \frac{S_{njx}}{\Sigma K_{nx}}$$

$$t_{ijyz} = \frac{S_{ijy}}{\Sigma K_{iy}} \qquad t_{kjyz} = -\frac{S_{kjy}}{\Sigma K_{ky}} \tag{4-3z}$$

$$t_{jjyz} = -\frac{\Sigma S_{jy}}{\Sigma K_{jy}}$$

$$v_{jzz} = -\Sigma FV_{jz} + P_{jz} \tag{4-4z}$$

the Z-joint equilibrium equation (Table 2-13) takes the carry-over form shown in Table 4-1. The physical meaning of constants q's , t's , and v's is defined in the following paragraphs.

(a) The Starting Shear v_{jzz} is the Z-joint shear at j due to loads, when all joints are locked against rotation and the far joints are restrained against deflection.

(b) The Carry-Over Shear Factor q_{ijzz} (q_{kjzz} , q_{mjzz} , q_{njzz}) is the Z-joint shear at j , due to a unit Z-joint shear at i (k , m , n) , when all joints are locked against rotation and the uninvolved joints are restrained against deflection.

TABLE 4-2z Z - CARRY-OVER SHEAR FACTORS

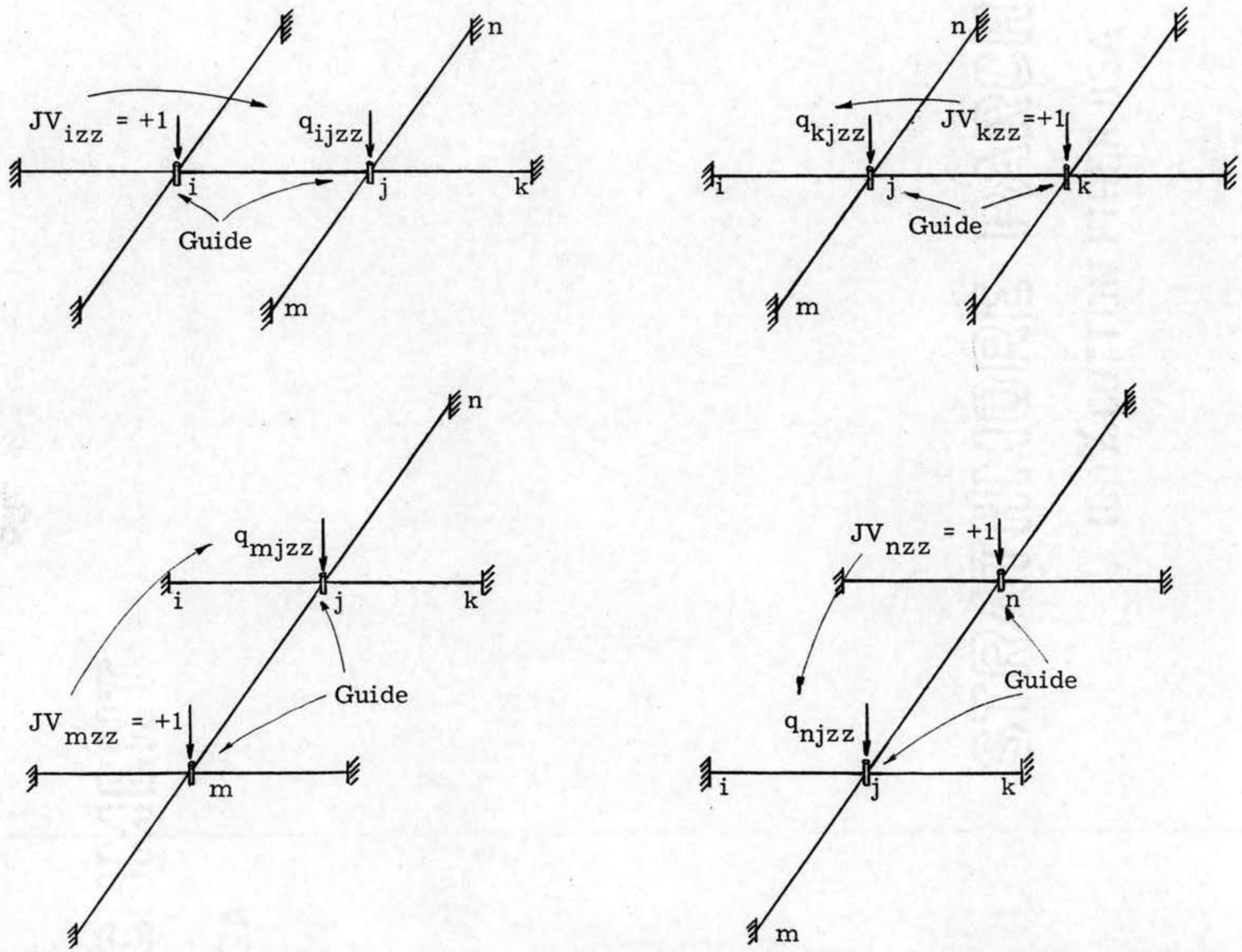


TABLE 4-3z XZ - CARRY-OVER MOMENT SHEAR FACTORS

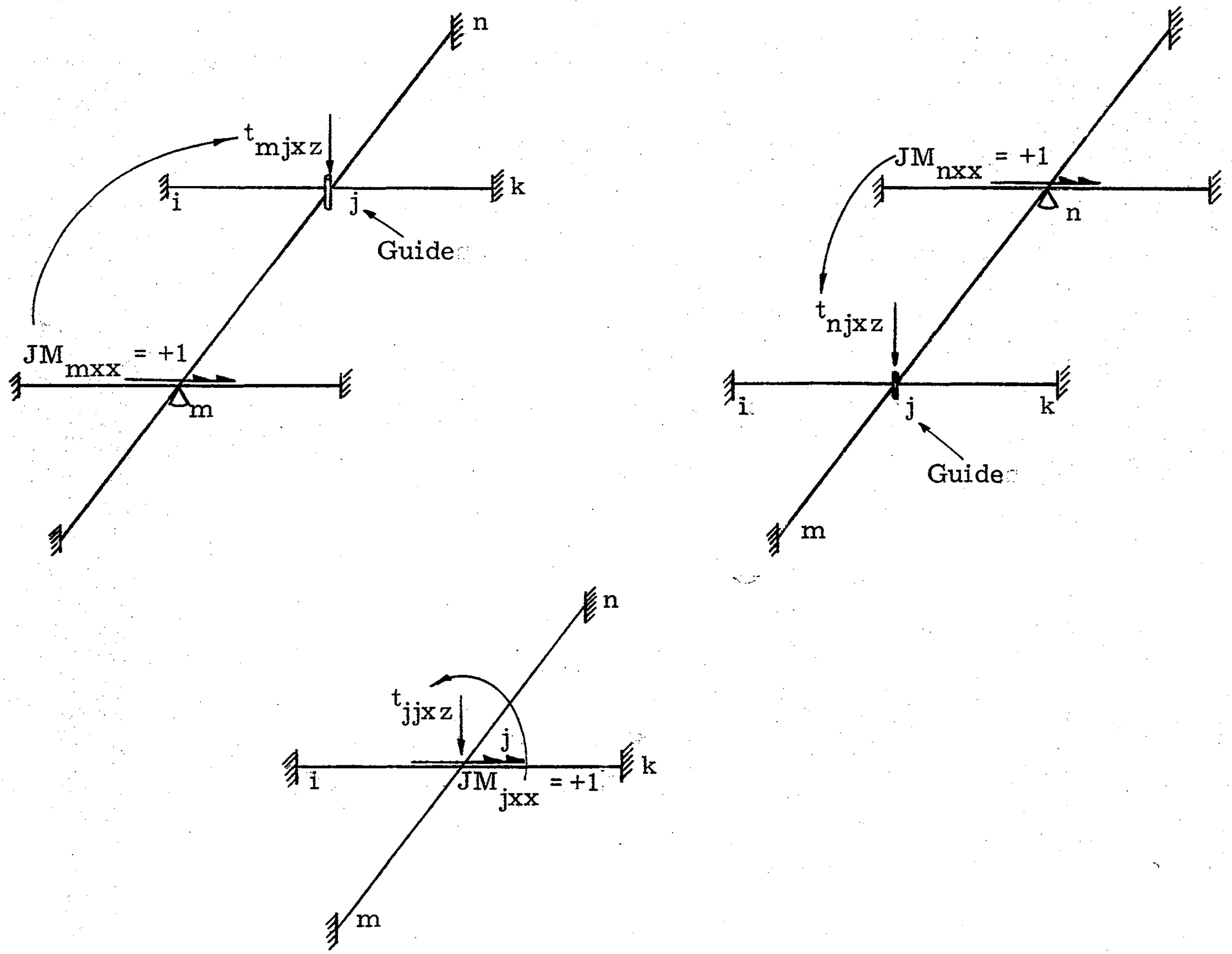
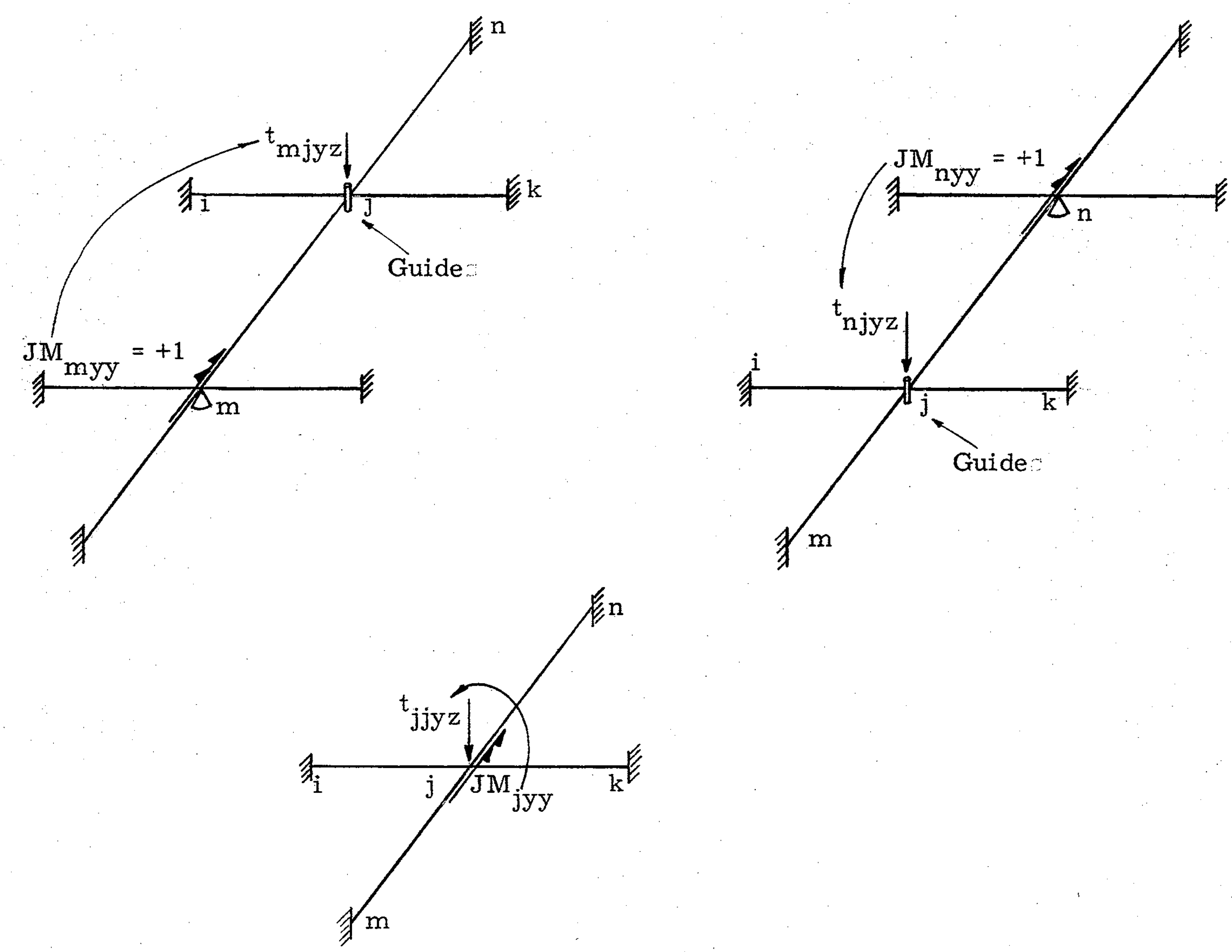


TABLE 4-4z YZ - CARRY-OVER MOMENT SHEAR FACTORS



(c) <u>The Carry-Over Moment-Shear Factor</u> t_{mjxz} (t_{jjxz} , t_{njxz}) is the Z-joint shear at j , due to a unit X-joint moment at m(j, n) , when all other joints are locked against rotation and all far ends are restrained against deflection.

(d) <u>The Carry-Over Moment-Shear Factor</u> t_{ijyz} (t_{jjyz} , t_{kjyz}) is the Z-joint shear at j , due to a unit Y-joint moment at i(j, k) , when all other joints are locked against rotation and all far ends are restrained against deflection.

4-6. Procedure of Analysis.

The application of these constants in the analysis may be summarized in the following steps:

(a) Designate all joints by symbols and record geometric values and material constants.

(b) Compute elastic constants FM's , FV's , K's , CK's , S's , T's , ΣK's , ΣS's , ΣT's , r's , q's , s's , and t's .

(c) Prepare three carry-over tables (JM_{xx} , JM_{yy} , JV_{zz}) with number of columns corresponding to the number of values required (equal or less than number of joints).

(d) Record all carry-over factors, starting moments, and starting shears in the corresponding columns of each table.

(e) Using the carry-over procedure, calculate the final joint moments and joint shears.

(f) Substitute these final values in the slope deflection equations to obtain the final end moments and end shears.

(g) Check for equilibrium of joints.

It is important to observe that:

(1) Three tables are used and two types of carry-over procedure are applied

(a) carry-over in-table (r's and q's)

(b) carry-over between-tables (s's and t's).

(2) Each new starting value consists of two parts:

(a) the sum of carried-over values from the adjacent joints

(b) the sum of carried-over values from the same joints in other table or tables.

CHAPTER V

SUMMARY AND CONCLUSIONS

5-1. Summary.

The elastic constants of the major stiffness methods used in analyzing rectangular grids are derived, defined and physically interpreted. Their application is explained and a procedure of analysis is described.

5-2. Conclusions.

(1) Three major stiffness methods are available for the analysis of rectangular grids:

(a) slope deflection method

(b) distribution method

(c) carry-over method.

(2) All three methods are based on the slope deflection equation, which relates the end statical vectors to the end deformation vectors through linear elastic constants called stiffness factors.

(3) The stiffness factors are of three types:

(a) angular (K's)

(b) linear (T's)

(c) mixed (S's)

(4) These factors are functions of each member, independent of loads and formation of structure.

(5) Governing equations of these methods are:

(a) joint deformation vector equations

(b) joint balance vector equations

(c) joint starting value vector equations.

(6) Each method requires a solution of a set of linear equations by a direct or a successive inversion.

(7) If an electronic computer (with a subroutine capable of inverting the given matrix) is available, the slope deflection method is preferred (Chapter II).

(8) If long hand computation must be carried on, the carry-over method is preferred (Chapter III).

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