TRANSFER FUNCTIONS OF SMALL TWO-PHASE A-C SERVOMOTORS FROM TIME AND FREQUENCY RESPONSE MEASUREMENTS

By

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Thesis

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PREFACE

The wide acceptance of the two-phase induction motor as a control element in the areas of instrumentation and small power has resulted in more precise designs of these motors. In order to realize the full benefit of these more precise designs, it is desirable that transfer functions which accurately represent these motors be available.

Several methods of obtaining a transfer function from either the motor parameters or the response curve are discussed. The limitation imposed upon these methods by the non-linearities of the motor is also discussed.

The author wishes to gratefully acknowledge the instruction, encouragement, and advice of his advisor, Professor Paul A. McCollum. It was through his interest in the subject that I initially became engaged in the study of servomotors.

I would like to express my thanks to Dr. H. T. Fristoe and Texas Instruments for obtaining and supplying, respectively, the semiconductor devices used in the test setup.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Applications of servosystems in the areas of instrumentation and small power frequently make use of two-phase a-c motors. The importance of these applications has caused many improvements to be made in two-phase motors, resulting in more precise designs that are more nearly ideal motors than their early predecessors.

The two-phase servomotor is basically a poly-phase induction motor which has two input windings spaced 90 electrical degrees apart. Under normal operating conditions, one of these windings is excited with a current of the correct magnitude and frequency. This winding is usually referred to as the reference winding. The other winding, usually referred to as the control winding, is then excited with a current 90 degrees out of time phase with the current of the reference winding. These two currents are of the same magnitude for balance operation, but may be of different magnitude for unbalance operation.

The fractional horsepower two-phase induction servomotor, having a squirrel-cage rotor, has the advantage over the fractional horsepower d-c servomotor, in being higher in reliability because it requires no slip rings or brushes, and usually being lower in cost due to the simplicity of the rotor. Also, the servosystem using a-c servomotors has the advantage of being able to utilize a-c amplifiers in the control circuit thereby avoiding the inherent drift problems associated with d-c amplifiers.

The two-phase induction servomotor has the disadvantage in that the control field power required for large torque applications becomes prohibitive. Therefore, two-phase servomotors are usually used in applications where the output torque requirements is small.

The two-phase servomotor is inherently a non-linear device to some degree. Because of the non-linearity, mathematical analysis becomes somewhat difficult. During the past several years, a host of articles have appeared throughout the literature proposing various schemes and methods of analyzing two-phase motors. Most of these methods of analysis have the common fault in that linear operating conditions and servomotor characteristics are assumed. Due to these assumptions, transfer functions derived by these methods are only approximate and often insufficient for highly accurate design and synthesis work. However, these methods of analysis find numerous applications in preliminary design work. Due to their applicability, three of these methods will be discussed.

Since transient tests are rapidly becoming standard engineering tools, several techniques for determining the transfer function of a two-phase servomotor from experimentally determined transient response curves are derived and demonstrated in this thesis. The accuracy obtainable by these methods is directly proportional to the accuracy of the transient response curve and the number of sampling points used.

These methods have a possible disadvantage in that they often require relatively many calculations for an accurate transfer function. However, the calculations are of a type that can readily be performed on a digital computer.

The accuracy of the transfer function is dependent upon the accuracy of the experimentally determined transient response curve. A technique for determining the transient response curve is also developed in this thesis.

CHAPTER II

CHARACTERISTICS OF TWO-PHASE SERVOMOTORS

Two-phase servomotors can be considered as a special class of poly-phase induction motors. As such, many of the analysis techniques for poly-phase induction motors is applicable to two-phase servomotors. A brief summary of the characteristics of two-phase induction motors is given in this chapter. Differences in construction and operation between conventional two-phase motors and two-phase servomotors is also discussed.

The induction motor derives its name from the fact that the currents flowing in the secondary member (usually the rotor) are induced by the action of the magnetic fields set up in the machine by currents flowing in the primary member (usually the stator). Most induction motors that are used as servomotors have squirrel-cage rotors for the secondary member, because this type is lower in cost and higher in reliability; it requires no slip rings or brushes.[1]

The two-phase induction motor has two input windings spaced 90 electrical degrees apart (Figure 1). These windings are excited with currents, 90 degrees apart in time phase, which in turn generate magnetic fields in the air gap. The two components of the magnetic

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[1]R. J. W. Koopman, "Operating Characteristics of Two-Phase Servomotors," American Institute of Electrical Engineers Transactions, Volume 68, 1949, pp. 319-28.

field are also in time and space quadrature. They can be combined vectorally, yielding the resultant magnetic field.[2] In well designed poly-phase induction motors, the magnitude of the rotating field does not vary appreciably as it rotates.[3] This is usually not the case for a single-phase induction motor, however.

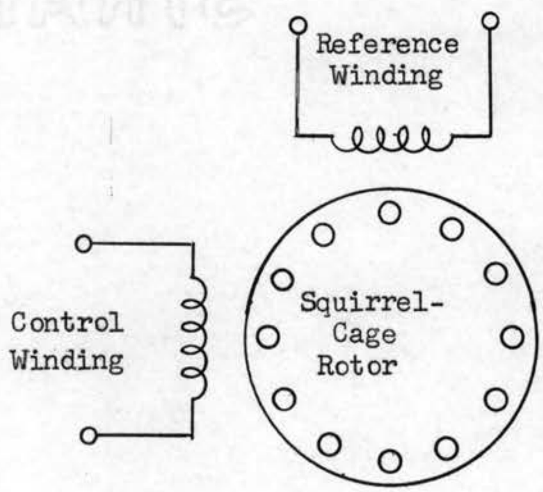

Figure 1. The Winding Arrangement for a Two-Phase Motor

The resultant magnetic field is rotating at a speed, N_s, such that

$$N_s = 120\ f/p,$$

where f is the frequency of the applied voltage and p is the number of poles.[4]

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[2]G. S. Brown and D. P. Campbell, Principles of Servomechanisms (New York, 1948).

[3]R. J. W. Koopman, "Operating Characteristics of Two-Phase Servomotors," American Institute of Electrical Engineers Transactions,

[4]John G. Truxal, Control Engineers' Handbook (New York, 1958).

Since the motor must develop sufficient torque to overcome windage, friction, and certain parasitic torques as well as the load torque, it cannot reach synchronous speeds. This difference between actual and synchronous speed is designated by a parameter known as slip, S, which is defined by the equation,

$$S = (N_s - N_a)/N_s,$$

where N_s = synchronous speed, and N_a = actual speed. A value of slip of 1/6 is usually considered typical for small servomotors operating under no-load conditions.

A idealized two-phase induction motor would have performance characteristics such that the stall torque is proportional to control voltage, and the torque for any control voltage decreases at a definite uniform rate with speed (Figure 2 and 3).

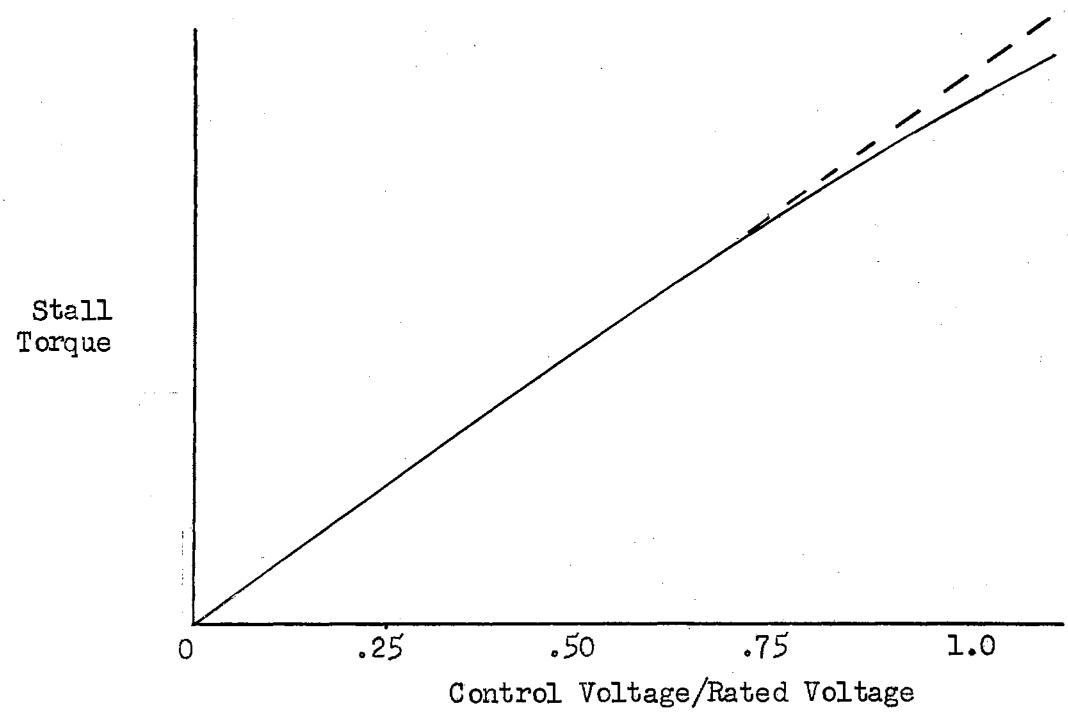

Figure 2. Dimensionless Control Voltage--Stall Torque Characteristic

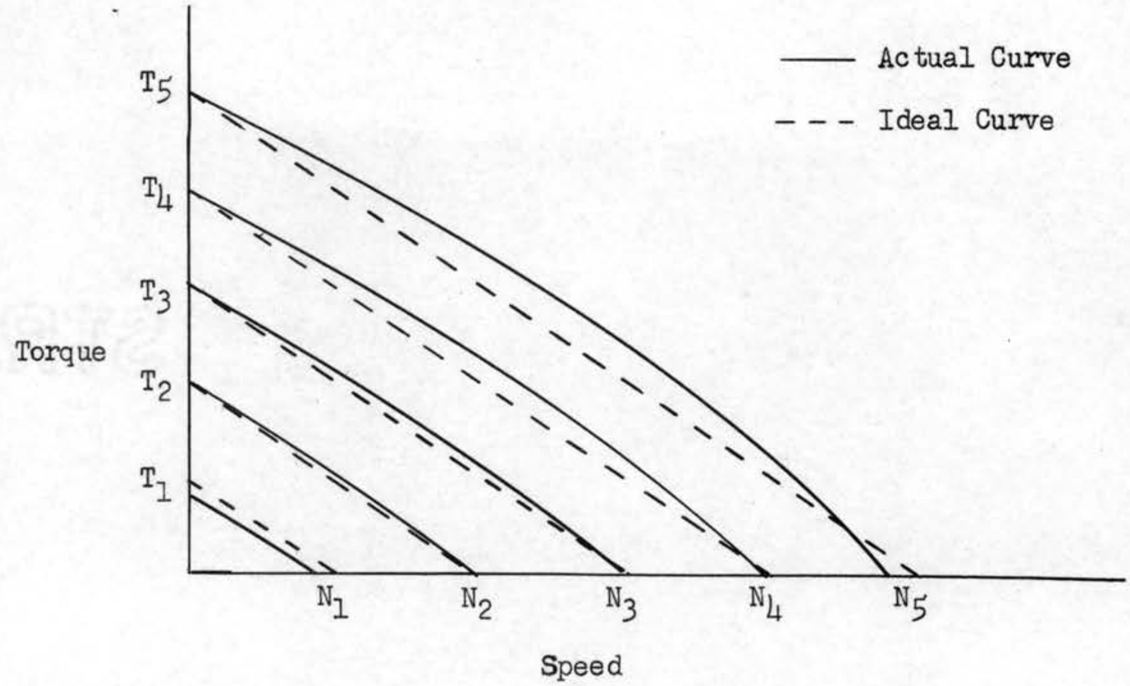

Figure 3. Dimensionless Speed--Torque Characteristic

The transfer function of a linear motor having these characteristics will be analyzed using differential equations and operational methods in Chapter III. The idealized induction motor characteristics cannot be achieved due to the inherent non-linearities of the windage and friction losses and the non-linearities of the magnetic path of the field.

The parameters for the performance characteristics of the two-phase servomotors can be adjusted to approximate linear performance by increasing the rotor resistance. Unfortunately, a compromise is necessary in the degree of linearity that can be achieved because the straighter the speed-torque curve (higher rotor resistance), the less stall torque is available, which means that less power is delivered to the load, smaller initial acceleration, and sluggish servo response.

The negative slope of the speed-torque curve (Figure 3) indicates internal damping which can be considered as viscous friction in the motor and can be calculated by

$$D = - \Delta T / \Delta N = - (T_1 - T_2)/N_1 - N_2)$$

where D = damping coefficient

= slope of speed-torque curve (dyne-cn./rad./sec.)

T = torque

N = speed .

Another performance characteristic parameter is the motor stiffness, K_e, which is defined as the ratio of the stall torque to the applied voltage, $K_e = T/V$. In well designed servomotors operating under balance conditions, the assumption that the motor stiffness, K_e, is constant will usually be valid over a relative wide range of applied voltages (Figure 2).

Since a servomotor normally operates at a higher speed than is required for a given load, it is necessary to employ gearing. The high motor speed means high rotational energy plus the added loss in the gears. Since the speed of rotation of the motor is inversely proportional to the number of poles, the design technique for minimizing rotor inertial energy is to wind the motor with as many poles as is practical.

The increased number of poles reduces the speed and provides a torque increase. However, the percentage torque gain is never as large as the speed reduction due to reduced efficiency.

Because of the high speed, the kinetic energy of the rotor is an important parameter of the motor. To achieve fast response, servomotors are designed for a maximum ratio of stall torque to rotor inertia.

Since inertia varies as the fourth power of the rotor diameter while developed output torque varies as the square of the rotor diameter, servomotors are designed with a relative small diameter rotor for their frame size.

The performance characteristics may also be improved by reducing the air gap and by reducing the windage and frictional losses. The reduction in the air gap and windage and frictional losses are associated with closer manufacturers tolerances which results in increasing the price of the servomotor.

It may be desirable to operate a servomotor under distinctly unbalance conditions. That is, the main winding may be intentionally overenergized in order to obtain more output torque per control field watt. The above discussion of performance characteristics, as well as the majority of manufacturer's data, is limited primarily to balance operation. Sidney Davis in his article, "Converting Ideal to Working Data for Application of Two-Phase Servomotors," describes an analytical method for converting the conventional manufacturer's data (based on balance operation) into useful data for unbalance operation.[5] Since this paper does not analyze the operation of servomotors from the performance characteristics, the details of this method will not be discussed. Unfortunately, unbalance conditions tend to add to the non-linearities of the two-phase servomotors.

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[5]Sidney A. Davis, "Converting Ideal to Working Data for Application of Two-Phase Servomotors," Electrical Manufacturing, Volume 58, Part I, September, 1956, pp. 110-15.

CHAPTER III

OBTAINING THE TRANSFER FUNCTION FROM THE MOTOR PARAMETERS

The most direct method for approximating the transfer function of two-phase servomotors is based upon the idealized motor. That is, a motor which has a stall torque proportional to the applied voltage, and a torque for any applied voltage that decreases at a definite uniform rate with speed (Figure 2 and 3). Any transfer function derived under these ideal conditions is restricted to the linear operating range of the servomotor.

Assuming that the load is characterized by both inertia and friction and that the steady-state speed-torque curve sufficiently describes the motor operation under transient conditions, the idealized motor-torque expression is

$$T_m = \partial T / \partial n \cdot n + \partial T / \partial e_i \cdot e_i$$

$$= \partial T / \partial n \cdot d\,\theta_o/dt + \partial T / \partial e_i \cdot e_i$$

where T_m = the motor's developed torque

n = the motor's speed

e_i = the applied voltage .

The above expression shows the dependence of torque upon both voltage and speed. Some simplification in the torque expression results if several constants derived from the speed-torque characteristics are defined. Let $\partial T/\partial n = -D$, which is the slope of the speed-torque curve (Figure 3) and $\partial T/\partial e_i = K_e$, which is defined as the motor stiffness (Figure 2). Substituting these constants into and transforming the resultant motor-torque expression yields

$$T_m(s) = -Ds\theta_m(s) + K_e E_i(s) .$$

The dynamic load-torque equilibrium equation is

$$T_L = Jd^2\theta_m/dt^2 + fd\theta_m/dt$$

where T_L = load torque

J = load inertia

f = load friction .

If inertia and friction are referred to a common shaft, the above expression transforms into

$$T_L(s) = Js^2\theta_m(s) + fs\theta_m(s) .$$

Equating the load and developed torques yields

$$-Ds\theta_m(s) + K_e E_i(s) = Js^2\theta_m(s) + fs\theta_m(s) .$$

Solving the above expression by relating the output motion to the voltage applied to the variable phase yields

$$\theta_m(s)/E_i(s) = K_e/(Js^2 + fs + Ds)$$

$$= K_m/s(t_b s + 1)$$

where $K_m = K_e/(f + D)$

$t_b = J/(f + D)$.

The above analysis is based upon the assumption that an applicable set of characteristic curves for the servomotor are available. Since most manufacturers supply the necessary curves for balance operation and since balance operation most nearly approaches the ideal motor, this method is applied most readily to balance conditions.

The frequency response of such a system will have a 90 degree phase lag at low frequencies which will approach 180 degrees phase lag as the frequency increases (Figure 4). The amplitude response of such a system will decrease with increasing frequency, being asymptotic to a slope of -20 decibels per decade at low frequencies and approaching a slope of -40 decibels per decade at high frequencies (Figure 4).

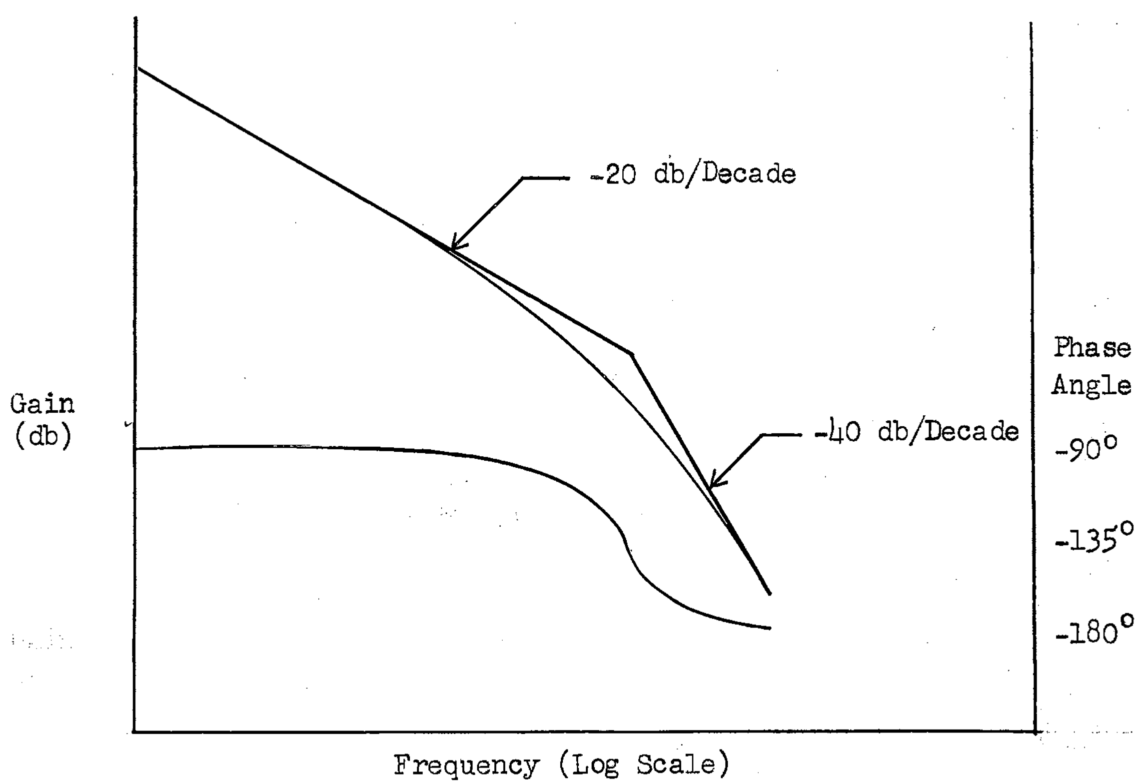

Figure 4. Frequency Response of a Two-Phase Servomotor

If the frequency response of a two-phase motor is measured experimentally, the response at low frequencies will very nearly match those given above. At high frequencies, however, the phase shift will increase beyond 180 degrees. Such a system must contain at least one more time constant. A more probable response to a step voltage input for the system would be

$$\theta_m (s)/E_i (s) = H/s (t_a s + 1) (t_b s + 1) .$$

In his paper, "Transfer Functions for a Two-Phase Induction Motor," Lloyd O. Brown, Jr. proposes an analytical method of analysis which results in a transfer function having two time constants.[1] The magnitude of these time constants is determined from a knowledge of the inertia and friction of the load and motor plus the steady-state speed-torque characteristics.

This analysis is based on the following assumptions: the magnetic material of the paths within the machine has a constant permeability so that the magnetizing currents are proportional to the applied voltages; the currents of both the rotor and the stator are distributed over the entire surface; the current density of a single phase of the stator varies sinusoidally with angular displacement around the stator; and the self impedances of the stator windings are negligible when compared to the magnetizing impedances of the machine.

First, a voltage of rated magnitude and frequency is applied to the reference phase of the machine. After the transients subside, a

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[1]Lloyd O. Brown, Jr., "Transfer Functions for a Two-Phase Induction Servomotor," American Institute of Electrical Engineers Transactions, Volume 70, Part II, 1951, pp. 1890-93.

voltage of rated magnitude and frequency with the correct phase shift is applied to the control field. The transient velocity of the rotor and load is then obtained.

For simplicity, the voltages applied to each phase of the machine will be assumed to be of a phase such that the simplest expressions will result throughout the analysis. These voltages are expressed as

$$V_c = 0 \qquad t < 0$$

$$V_c = V \sin (wt + \tan^{-1} wL_m/R_m) \qquad t \geq 0$$

$$V_{re} = V \cos (wt + \tan^{-1} wL_m/R_m) \quad .$$

Based upon the above assumptions, the magnetic flux density of the machine as a function of position and time referred to the rotor element will be

$$B (t, \theta_r) = -1/2 \, P_c I_s \sin (wt - a - \theta_r)$$

where P_c = proportionality constant between stator current and flux density

a = angle between the stator and rotor reference line as a function of time

θ_r = angle between an element of the rotor and the reference line of the rotor.

The instantaneous voltage in the rotor elements will be proportional to the magnetic flux cutting that element and the relative velocity between the element and the field. The corresponding instantaneous current will be of the same frequency as the applied voltage but shifted in phase due to the rotor inductance. It may be expressed as

$$i_r = C \text{ EXP. } (-R_r t/L_r) + A_1 \sin (wt - a - \theta_r)$$

$$+ A_2 \cos (wt - a - \theta_r)$$

where A_1 and A_2 are defined by the equations

$$L_r \, dA_1/dt - A_2 (w - w_r) L_r + A_1 R = -1/2 \, P_c I_s (w - w_r)$$

or

$$L_r \, dA_1/dt + A_1 R = (A_2 L_r - 1/2 \, P_c I_s) (w - w_r)$$

and

$$L_r \, dA_2/dt + A_1 (w - w_r) + A_2 R = 0 \quad .$$

In order to simplify the equations which define A_1 and A_2, the linearizing assumption that

$$(A_2 L_r - 1/2 \, P_c I_s) (w - w_r) = K_3$$

is made. The constant C is evaluated by the initial conditions of the problem.

The element torques are proportional to the product of the element currents and the flux density. The total torque of the rotor is the integral from 0 to 2π of the element torques with respect to θ_r. This total torque is written as

$$T_r = K_1 \text{ EXP. } (-R_r t/L_r) \cos (wt - a) + K_2 A_1 \quad .$$

The total torque is then equated to the mechanical requirements of the motor and load such that

$$J\ dw_r/dt + fw_r = K_1\ \text{EXP.}\ (-R_r\ t/L_r)\ \cos\ (wt - a) + K_2A_1\ .$$

The above expression when linearized reduces to

$$J\ dw_r/dt + fw_r = K_1\ \text{EXP.}\ (-R_r\ t/L_r) + K_2A_1\ .$$

An approximation to the transient velocity is then found by solving these equations simultaneously. This approximation yields

$$w_T = M_1 + M_2\ \text{EXP.}\ (-ft/J) + M_3\ \text{EXP.}\ (-R_r\ t/L_r)\ .$$

From the initial conditions of zero acceleration and velocity, the coefficients of the exponentials are evaluated in terms of the constant M_1. This constant must be the final value of the velocity and can be expressed as the steady state torque divided by the coefficient of friction for the system.

By applying the LaPlace transformation termwise to the transient velocity and collecting terms over a common denominator, the response of the system to a step input voltage become

$$KG\ (s) = \theta_o/V_2 = M_1\ w_1\ w_2/V_2 \times 1/s\ (s + w_1)(s + w_2)$$

where w_1 = reciprocal of the mechanical time constant of the system

w_2 = reciprocal of the electrical rotor time constant.

Another effective way of approximating the transfer function of two-phase motors is based upon the equivalent circuit of a general two-phase induction motor. This circuit is suitable for the analysis of either balance or unbalance operating conditions. The generalized form of this circuit is

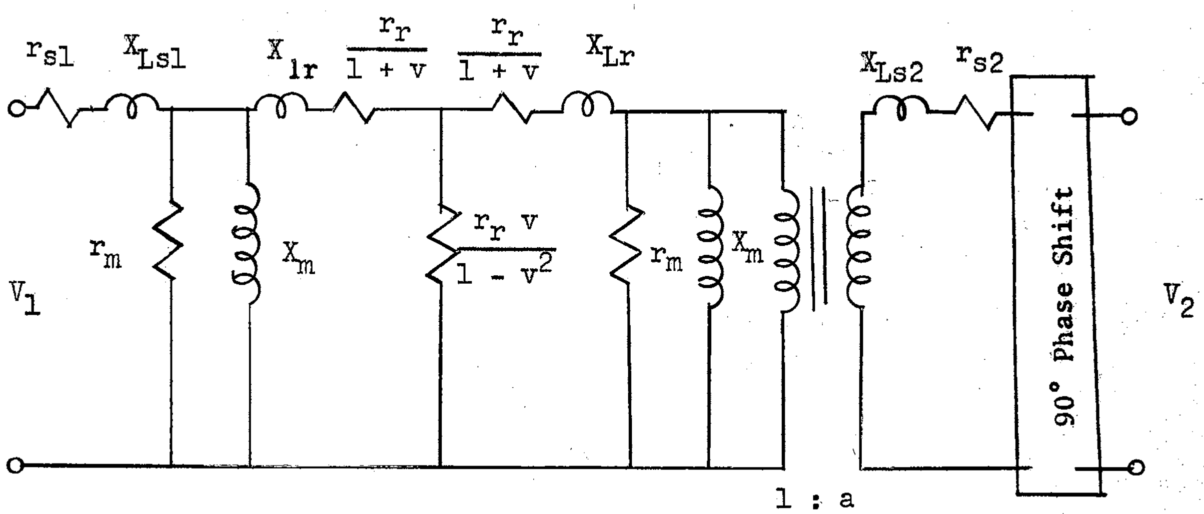

Figure 5. Equivalent Circuit for Analysis of Two-Phase Motor[2]

where V_1 = applied voltage on phase 1

V_2 = applied voltage on phase 2

a = ratio of phase 2 effective turns to phase 1 effective turns

r_{s1} = stator resistance phase 1

r_{s2} = stator resistance phase 2

X_{ls1} = stator leakage reactance phase 1

X_{ls2} = stator leakage reactance phase 2

r_m = core loss resistance

X_m = air gap reactance

Z_m = air gap impedance

X_{lr} = rotor leakage

r_r = effective rotor resistance

v = ratio of actual speed to synchronous speed

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[2]John G. Truxal, Control Engineers' Handbook (New York, 1958).

For operation under balance conditions, this generalized equivalent circuit reduces to

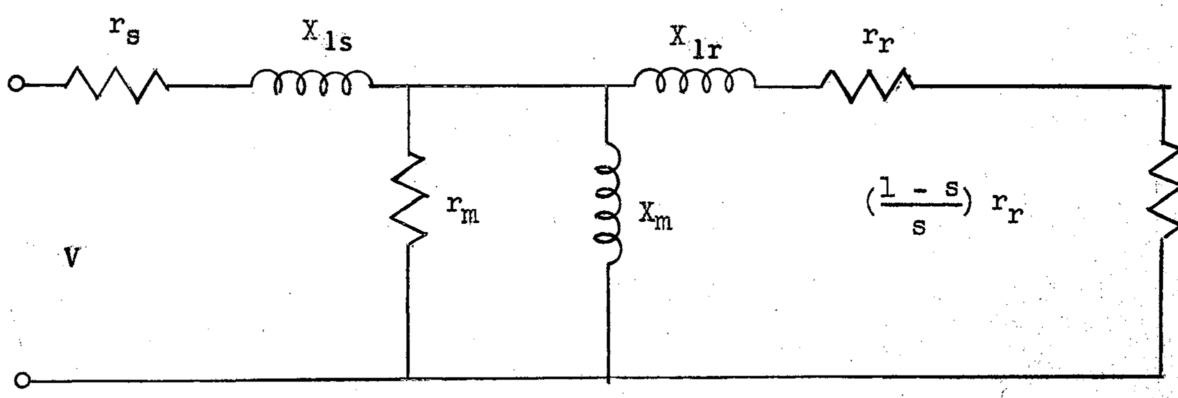

Figure 6. Equivalent Circuit for Two-Phase Motor Operating Under Balance Conditions[3]

The total power to the rotor equals

$$P_{rf} = m\ I_r^2\ r_r + m\ I_r^2\ r_r\ (1 - s)/s$$

where m = number of stator phases

I_r = rotor current .

The power delivered to the load equals

$$P_{dev} = m\ I_r^2\ (r_r/s - r_r)$$

and the total torque equals

$$T = (1352/N_s)\ P_{rf}$$

where N_s is the synchronous speed. From these expressions, a transfer

[3]Ibid.

function for the two-phase motor can be derived.

Analysis based upon the equivalent circuit are applicable only over the range where the parameters are linear. This method of analysis if often very tedious unless simplifying assumptions are made. It also depends upon the accuracy of determining the necessary parameters.

CHAPTER IV

DETERMINING THE TRANSFER FUNCTION BY EXPERIMENTAL METHODS

In order to be able to produce a more realistic design for a system, it is desirable that the actual measured transfer functions of the various components be available. Several techniques of measuring the transient response are well known, but most of these methods are hampered by the necessity of placing a load on the motor. Due to the relative low output torque obtainable from most two-phase servomotors, a small load applied by the measuring instrument may have an appreciable effect. The ideal response measuring instrument would be one that adds neither mass, compliance, or friction to the motor under test. A device which approximates this ideal response measuring device was developed and analyzed. The only additional load placed on the motor by this transient response measuring device is a small amount of mass and windage friction of a thin fiber disk which is mounted on the output shaft of the motor. This response measuring system reproduces the transient response of the motor tested, which has an output power in the neighborhood of 0.01 horsepower, within an estimated accuracy of $\pm$ 5 percent. It can be calibrated with a pulse generator and it is applicable to a wide range of motor speeds (200 to 10,000 r.p.m.).

After the transient velocity response to a step control voltage has been obtained, the second step is to represent the experimentally determined curve by a suitable functional expression in order that analytical operations may be performed. A representation of the experimentally determined transient response curve by an exponential function of time is desired for the transfer function analysis of two-phase servomotors. This representation can be achieved by Legendre's Principle of Least Squares Curve Fitting.[1, 2] This principle is used to determine the constant, a, of a selected exponential time function of the type, $x = K (1 - e^{-at})$, by the criteria that the sum of the mean squared errors between the transient response curve and the selected function over the range is a minimum. If a simple exponential function of this type does not yield a suitable approximation, then an exponential function of the form, $x = K (1 - Ae^{-at} + Be^{-bt})$, should be selected because it should yield a more accurate representation. A representation of the experimentally determined transient time response curve as a function of frequency is also useful, since the frequency response technique of analyzing servosystems is often used to facilitate both the general analysis and synthesis operations. This frequency representation can be produced from the time representation by using the Fourier transform, or it can be obtained directly from the experimentally determined transient time response curve by Samulon's method.[3] Samulon's method yields a theoretically exact representation

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[1]F. B. Hildebrand, Introduction to Numerical Analysis (New York, 1956).

[2]I. S. Sokolnikoff and R. M. Redheffer, Mathematics of Physics and Modern Engineering (New York, 1958).

[3]H. A. Samulon, "Spectrum Analysis of Transient Response Curves," Proceedings of the Institute of Radio Engineers, Volume 39, 1951, pp. 175-186.

of the experimentally determined transient response curve as a function of frequency.

After an exponential time function representation or a frequency function representation of the transient response curve has been obtained, the transfer function can be obtained by performing analytical operations on the time representation of the transient response or by performing graphical operations on the frequency representation of the transient response.

For the exponential time response, these analytical operations are demonstrated by the following example:

Assume that the time representation obtained was of the form

$$x = K\,(1 - e^{-at}) \qquad \text{r.p.m.}$$

where K = steady state speed (r.p.m.). This expression is the representation of the response to a voltage step of rated magnitude, E. By normalizing this expression, the response to a unit step of voltage is obtained which is

$$x_s = \frac{K}{E}\,(1 - e^{-at}) \qquad \text{r.p.m./volt}$$

The response to a unit impulse of voltage is obtained by taking the time derivative of the unit step response. This operation yields

$$x_i = dx_s/dt = \frac{aK}{E}\,e^{-at} \qquad \text{r.p.m./volt-sec.}$$

The transfer function of the system is the Laplace transform of the unit impulse response. This transformation yields

$$KG\,(s) = (K/E)/\,(1 + s/a)\,. \qquad \text{r.p.m./volt-sec.}$$

This same procedure is applicable to more complex exponential time functions.

Samulon's method yields a transfer function as a frequency representation which has not been normalized. The normalized transfer function can be obtained by dividing the frequency representation obtained from the experimental response by the magnitude of the step voltage applied. Since the results obtained by Samulon's method are not in a form which is readily transformable, it is necessary to plot the frequency response. By using Bode diagram analysis techniques, the transfer function is obtained from the graph of the frequency response.[4, 5]

Frequently, it is desirable to operate servomotors with voltage magnitudes other than the rated magnitude applied to the control winding. It would be convenient if the assumption that the transfer function derived from the step voltage of rated magnitude is applicable. For well designed servomotors, this assumption is usually valid for voltages from 20 percent to 100 percent of the rated magnitude.

The transfer function obtained during the investigation at hand is restricted to small two-phase motors operating under no-load conditions. This transfer function will be modified by the addition of a load to the motor. As an example, consider the response function to a 20 volt step voltage input obtained during the investigation at hand. This function is

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[4] George J. Thaler and Robert G. Brown, Servomechanism Analysis (New York, 1953).

[5] H. Chestnut and R. W. Mayer, Servomechanisms and Regulating System Design, Volume I (New York, 1951).

$$G(s) = 1440/s\,(1 + 0.103\,s), \qquad \text{r.p.m.}$$

If a load is applied to the system, this function can be considered as being modified by the load such that the modified function is

$$G(s) = 1440/\left[s\,(1 + 0.103\,s) + Y(s)\right] \qquad \text{r.p.m.}$$

where Y (s) is the load function.[6] For a linear load containing both friction and inertia

$$Y(s) = K\,(J_L s^2 + f_L s)$$

where K = proportionality factor

J_L = load inertia

f_L = load friction .

By assuming that the speed-torque curve for the motor is also linear, K can be approximated by

$$K = -(1/\ \partial T/\ \partial n) = (+1/D)$$

where D = the negative slope of the speed-torque curve.

For non-linear loads, a appropriate representation of Y (s) might not be readily obtainable. An alternate method of analysis might be to obtain an experimental transient response of the motor and load combined. An approximate transfer function could then be obtained by the same methods used to analyze the transient response of the motor.

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[6] George J. Thaler and Robert G. Brown, Analysis and Design of Feedback Control Systems (New York, 1960).

CHAPTER V

EXPERIMENTAL TRANSIENT RESPONSE MEASUREMENTS

The transient response of two-phase a-c servomotors can be measured in any one of several ways. For accurate results, it is imperative that the method used reproduces the transient response without loading the servomotor appreciably.

The method used for this analysis meets the above criterion reasonably well. This method consists of the following functional elements arranged as shown in Figure 7.

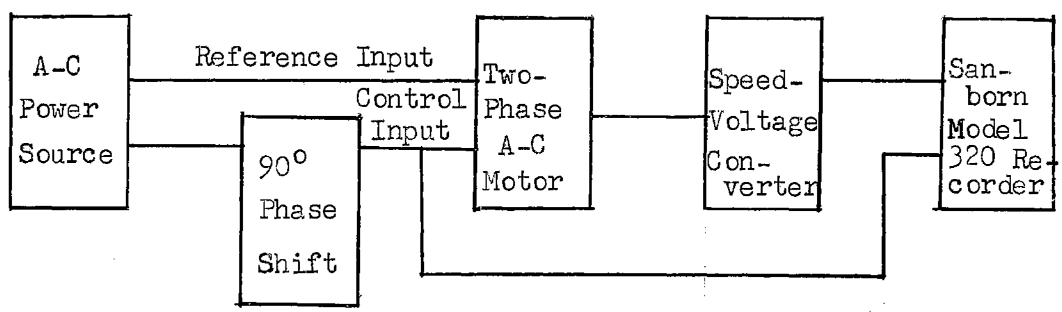

Figure 7. Block Diagram of the Transient Response Measuring System

The speed-voltage converter is designed so that a minimum amount of torque is required from the servomotor. The driving mechanism for the speed-voltage converter consists of a thin fiber disk, 4 inches in diameter. This disk has 60, 1/16 inch holes drilled in it, and it is mounted to the shaft of the servomotor. A light source is mounted on the motor side of the disk, and a sensing circuit is placed on the opposite side of the disk. The sensing circuit and light source are located such that they align with the holes in the disk (Figure 8). The only additional load applied to the motor by the driving mechanism of the speed-voltage converter is a small amount of inertia due to the disk plus the windage and friction losses of the disk.

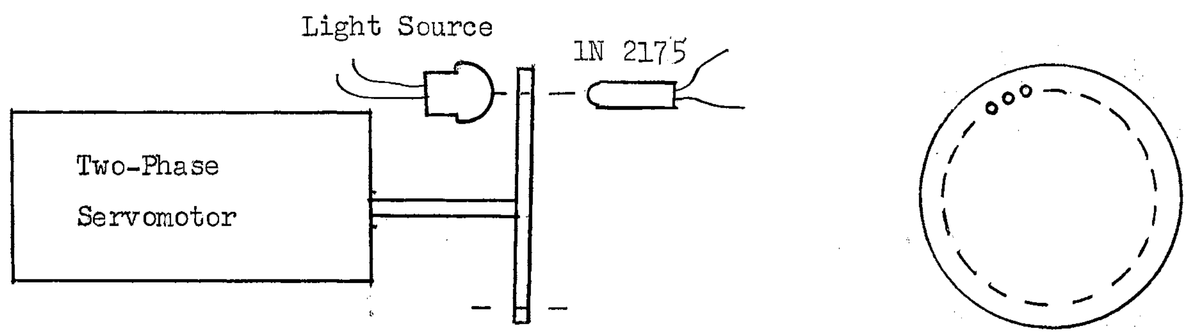

Figure 8. Speed-Voltage Converter Driving Mechanism

The circuit diagram of the speed-voltage converter is shown in Figure 9. This circuit contains six functional parts which are: a sensing element, a d-c amplifier, a pulse shaping circuit, a differentiator, a mono-stable multivibrator, and an integrator and filter.

The sensing element is a Texas Instrument "N-P-N Diffused Silicon Photo-Duo-Diode", type 1N 2175.[1] The bias voltage applied to the 1N 2175 is 5 volts d-c. With this bias voltage applied, the dark current is approximately 0.001 microamp when measured at 25° C. The light current

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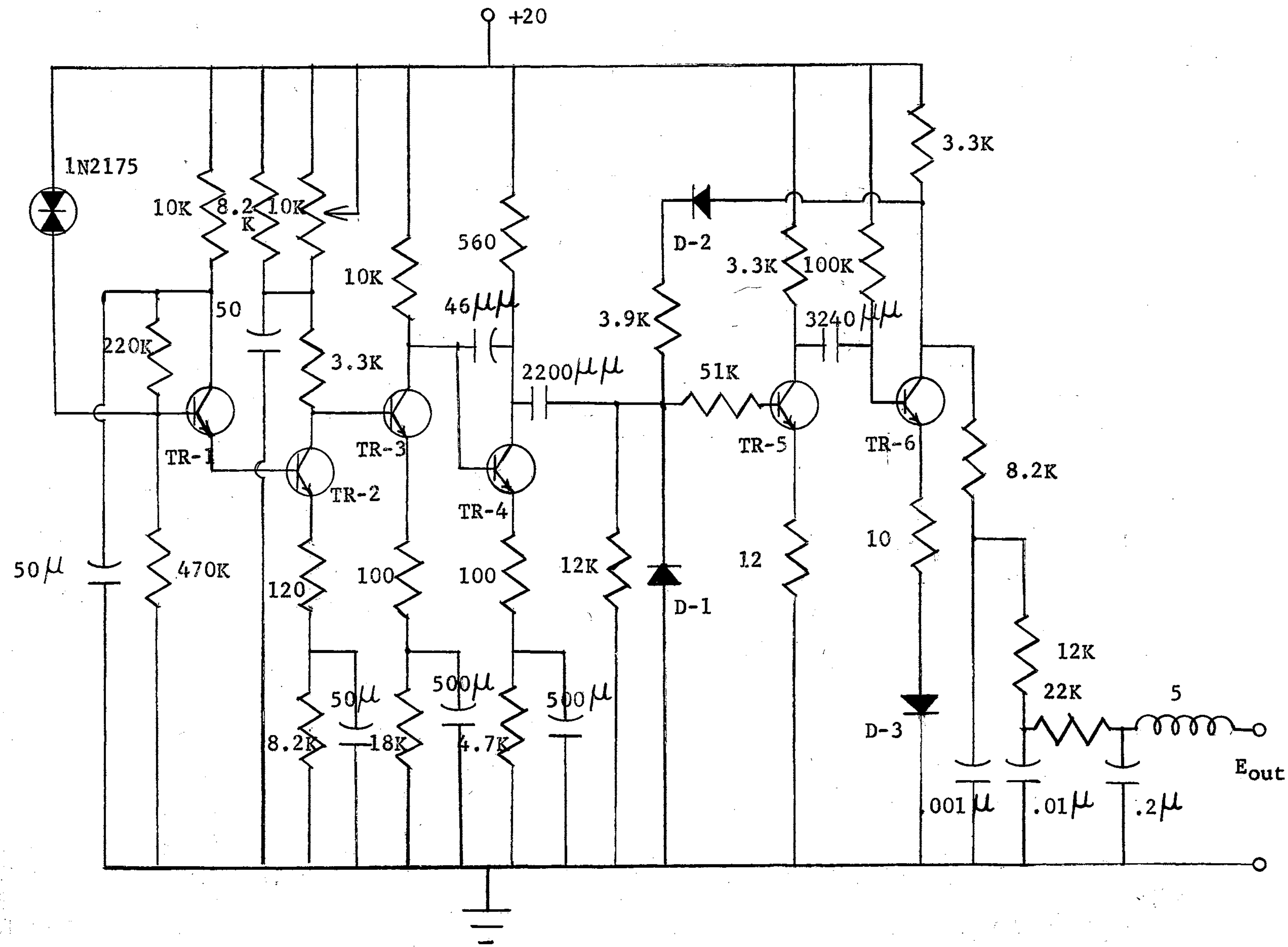
[1]"The Photo-Duo-Diode: Theory, Measurements of Parameters, and Operation," Application Notes, Texas Instruments, Inc., Dallas, Texas, April, 1961.


+20
1N2175
10K
8.2K
10K
10K
560
3.3K
100K
3.3K
D-2
50
220K
3.3K
46μμ
3.9K
3240μμ
2200μμ
51K
TR-1
TR-2
TR-3
TR-4
TR-5
TR-6
8.2K
50μ
470K
120
100
100
12K
12
10
D-1
8.2K
50μ
18K
500μ
4.7K
500μ
12K
22K
5
D-3
.001μ
.01μ
.2μ
Eout

is directly proportional to the bias voltage, temperature, and light intensity. The pulse shape transmitted by the sensing element to the d-c amplifier stage is shown in Figure 10.

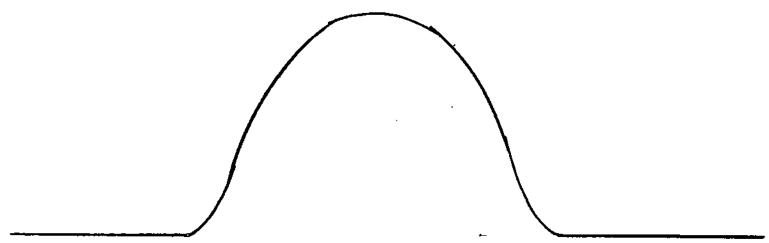


Figure 10. Sensing Elements Pulse Waveform

The d-c amplifier consists of a common-collector stage followed by a common-emitter stage. Bias stability is assured by the use of a large emitter resistor in the common emitter stage which is bypassed for signal frequencies. The transistors used throughout this circuit are small signal communication types made of silicon with betas of approximately 80. A 10,000 ohm potentiometer is placed in parallel with the 8,200 voltage divider resistor so that the d-c voltage level can be adjusted to compensate for the different types of drive signals used in the converter. This stage has a voltage gain of approximately 50.

The pulse shaping circuit is basically a d-c amplifier with an excessive amount of voltage feedback present. When the amplifier is overdriven, the effect of this excessive voltage feedback is to square the signal waveform. Since this amplifier is normally overdriven, the output signal waveform of this stage is a square wave independent of the input signal waveform. A limited amount of feedback between transistors in this stage increases the stability and reduces the sensitivity to output load changes which might occur.

The next part of the circuit is a passive differentiator and clipper. Both a positive and negative pulse is generated when a square wave is differentiated, but due to the action of diode #1, the negative pulse is clipped. The time constant for the positive pulse is approximately equal to 26 microseconds.

The next part is a mono-stable multivibrator. During the stable state of this multivibrator, tr-5 is cutoff and tr-6 is operating under saturated conditions. The positive pulse reverses the states of the transistors until the R-C coupling network discharges sufficiently to allow the transistors to return to their stable states. This R-C discharge time is approximately equal to 172 microseconds. The diode D-2 prevents the reverse base to emitter voltage from destroying the transistor tr-6.

The final part is a passive integrator and filter network. The time constants are choosen so that the output voltage is a d-c voltage directly proportional to the input pulse rate.

The pulse width of the mono-stable multivibrator can be adjusted so that speeds up to 10,000 r.p.m. can be measured. It should be noted, however, that an increase in the maximum speed measurable will be associated with a proportional decrease in sensitivity.

The rise time for the speed-voltage converter was measured at the full speed of the motor so that the maximum output d-c level and corresponding maximum rise time would be observed. The circuit rise time, as measured on the Sanborn Model 320 recorder, was found to be less than 50 milliseconds.

The circuit is designed so that the d-c level of the output voltage is independent of the input signal waveform. This design feature facilitated the use of an audio generator as a driver for calibration purposes. The frequency of the audio generator is converted into equivalent r.p.m., and the associated calibration curve plotted as shown in Figure 11.

The experimentally determined transient response curve for a Diehl Manufacturing Company, Type Fb-25, SS code 2, two-phase, 60 cycles, 20 volts, a-c motor is shown in Figure 12.

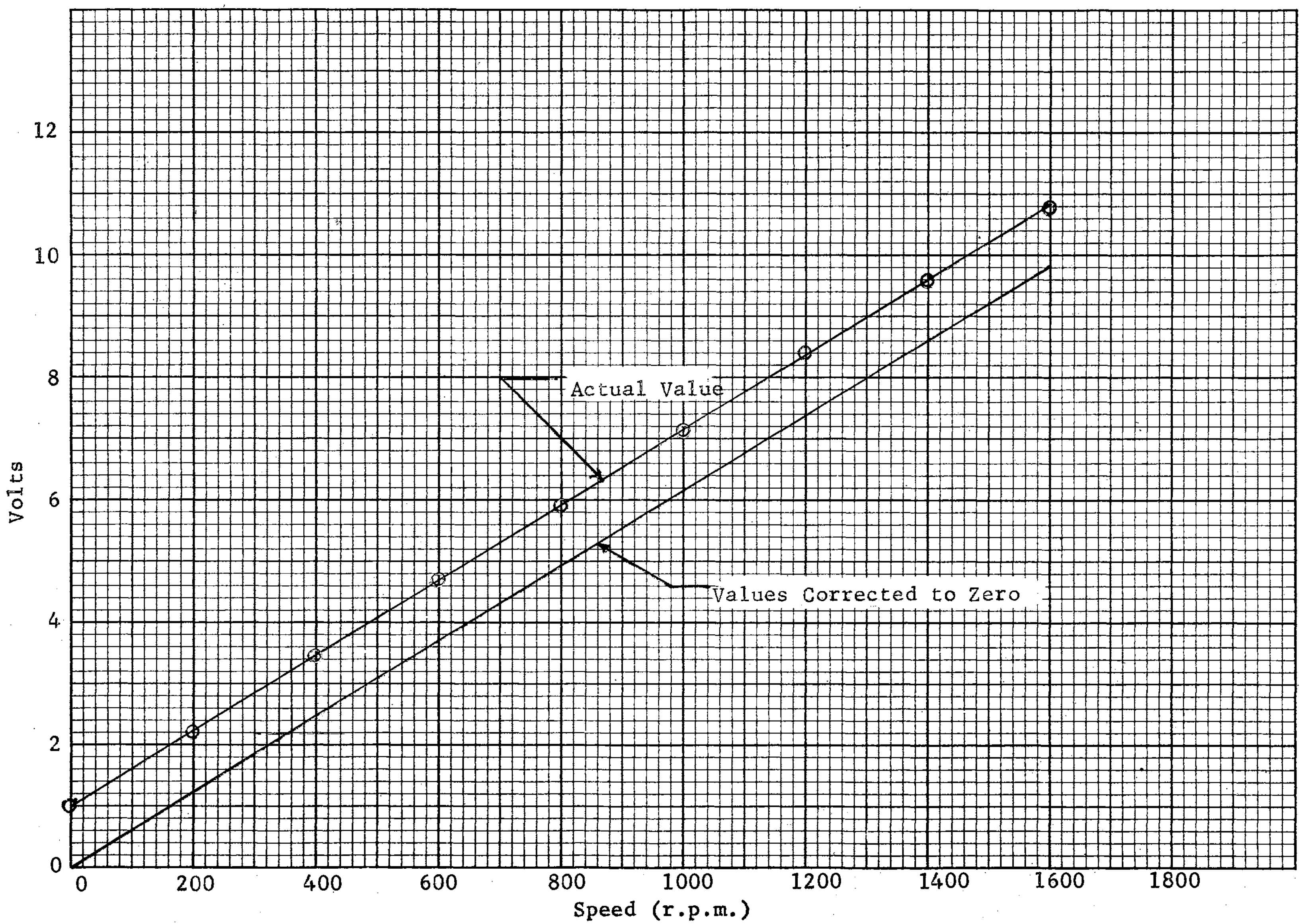


Figure 11. Calibration Curve for Speed-Voltage Converter

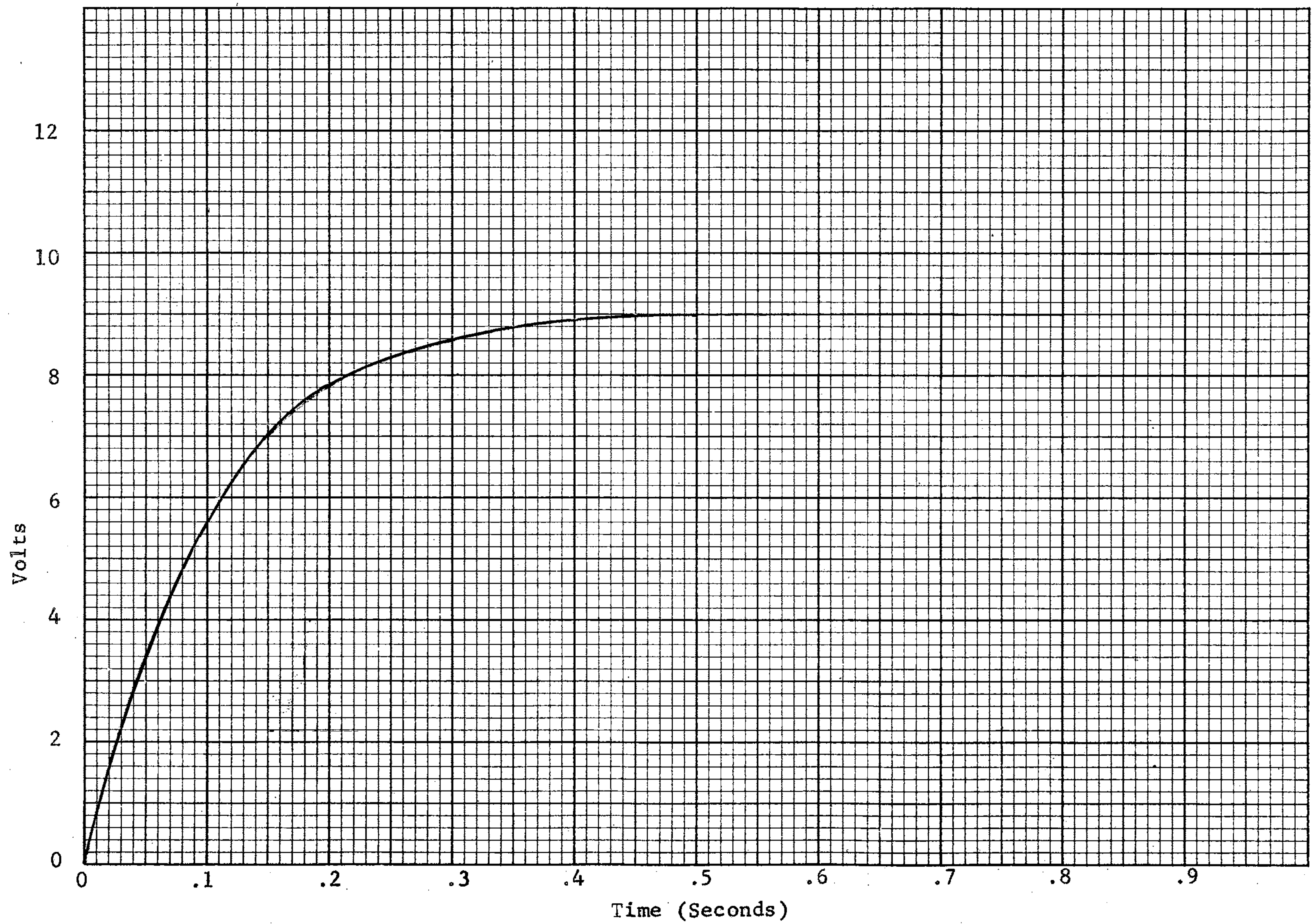


Figure 12. Transient Response of Diehl Manufacuturing Company, Type Fb-25, Two-Phase Servomotor

CHAPTER VI

CURVE FITTING BY THE METHOD OF LEAST SQUARES

By approximating the velocity transient response with an exponential function, the transfer function can be produced as previously described (Chapter IV). The least squares method of approximation is generally suitable for this application. This method is demonstrated below:

The experimentally determined transient response curve f (t) is to be fitted by a selected function, g (t). Since step control voltage response functions of the type

$$G_1 (s) = K/s (1 + Ts) \qquad \text{and}$$

$$G_2 (s) = K/s (1 + T_1 s) (1 + T_2 s)$$

are desirable for the analysis of two-phase servomotors, the selected function G (s), which is the transform of g (t), is chosen so that

$$G (s) = K/s (1 + Ts) \qquad \text{and}$$

$$G (s) = K/s (1 + T_1 s) (1 + T_2 s)$$

for the first and second approximation respectively.

For the first approximation, where

$$G (s) = K/s (1 + Ts) \quad ,$$

the corresponding time function is

$$g\ (t) = K\ [1 - \text{EXP.}\ (-at)]$$

where $a = 1/T$.

When the value of time becomes large, g (t) must approach K. Therefore, K must equal the steady-state gain of the servomotor. The unknown quantity now is the constant a.

Next, a value of a is chosen which will minimize the sum of the squares of the deviations of the observed values from the corresponding values of g (t) at t if computed from the chosen value of a.

Let t_1, t_2, t_3, ... t_n be the observed values of t corresponding respectively to the observed values of f (t), which are represented by y_1, y_2, y_3, ... y_n. Under these conditions, n equations can be written which are:

$$y_1 = K\ (1 - \text{EXP.} - a_1 t_1)$$

$$y_2 = K\ (1 - \text{EXP.} - a_1 t_2)$$

$$y_n = K\ (1 - \text{EXP.} - a_1 t_n) \quad .$$

The problem is to make all of the above equations as accurate as possible by the proper choice of a. This is done by minimizing the sum of the squares of their alleged inaccuracies. The sum of the squares of the inaccuracies can be represented by

$$S = [y_1 - K\ (1 - \text{EXP.} - a_1 t_1)]^2 + [y_2 - K\ (1 - \text{EXP.} - a_1 t_2)]^2 + \text{------} \ [y_n - K\ (1 - \text{EXP.} - a_1 t_n)]^2 \ .$$

Taking the derivative of S with respect to a_1 yields

$$dS/da_1 = -2K \left[(t_1 \text{ EXP. } (-a_1 t_1)\right] \left[y_1 - K + K \text{ EXP. } (-a_1 t_1)\right]$$

$$+ \left[t_2 \text{ EXP. } (-a_1 t_1)\right] \left[y_2 - K + K \text{ EXP. } (-a_1 t_2)\right]$$

$$+ \text{---}$$

$$+ \left[t_n \text{ EXP. } (-a_1 t_n)\right] \left[y_n - K + K \text{ EXP. } (-a_1 t_n)\right] \quad .$$

If equal increments of time are chosen, the equation will reduce to

$$dS/da_1 = -2Kt \left[\text{EXP. } (-a_1 t)\right] \left[y_1 - K + K \text{ EXP. } (-a_1 t)\right]$$

$$+ 2 \text{ EXP. } (-a_1\ 2t) \left[y_2 - K + K \text{ EXP. } (-a_1\ 2t)\right]$$

$$+ \text{---}$$

$$+ n \text{ EXP. } (-a_1\ nt) \left[y_n - K + K \text{ EXP. } (-a_1\ nt)\right] \quad .$$

Next, the observed values of y_n, t, and K are substituted in the above expression and this expression is set equal to zero. The constant, a_1, is then solved for by using algebraic methods.

As an example, this method will be applied to the experimentally determined transient response curve for a Diehl Manufacturing Company, Type Fb - 25, SS code 2, two-phase, 60 cycles, 20 volt a-c motor.

From steady-state conditions, K = 1440 r.p.m. (Figure 13). If time increments of 0.1 seconds are chosen, the corresponding value of y_i (i = 1, 2, 3, ... n) in r.p.m. are:

$$y_1 = 865$$

$$y_2 = 1245$$

$$y_3 = 1375$$

$$y_4 = 1424$$

Substituting these values into the equation for dS/da_1 yields

$$f(x) = 57.6 X^7 + 43.2 X^5 + 28.15 X^3 - 1.95 X^2 + 10.5 X - 5.75 = 0$$

where $X = \text{EXP.} (-0.1a_1)$.

A root of this equation exists when X = 0.38. Therefore,

$$a_1 = -10 \ln 0.38 = 9.67 \quad .$$

The response function to a step control voltage obtained is

$$G(s) = 1440/s (1 + 0.103 s), \quad \text{r.p.m.}$$

The transform of this function and the experimentally determined transient response are plotted in Figure 13.

When the frequency of some high gain, fast response two-phase motors is measured experimentally, the response at low frequencies will be very nearly represented by this type of a response function. At high frequencies, however, the phase shift will increase beyond 180 degrees. Such a system must contain another time constant.

For the second approximation, the g (t) is represented by

$$g(t) = K\left[1 - [a_2/(a_2 - a_1)] \text{EXP.} (-a_1 t) + [a_1/(a_2 - a_1)] \text{EXP.} (-a_2 t)\right]$$

where $a_1 = 1/T_1$, which was determined by the first approximation to equal 9.67

$a_2 = 1/T_2$.

The only unknown remaining in this expression is a_2.

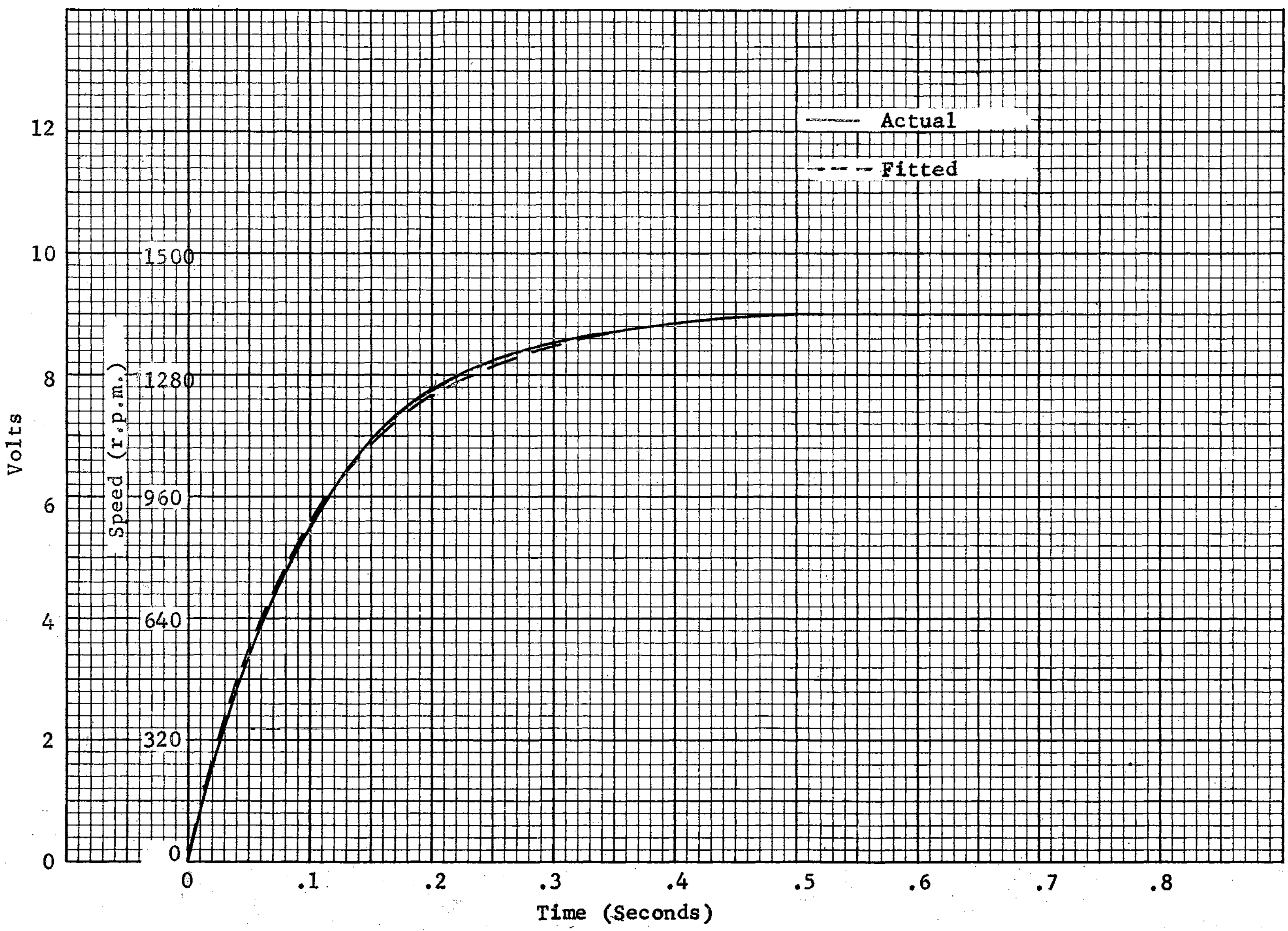


Figure 13. Experimentally Determined Tranisent Curve and the Least Squares Fitted Curve

The procedure for minimizing the sum of the squares of the inaccuracies is repeated once again. This time dS/da_2 must be obtained and set equal to zero to minimize the sum of the squares of the inaccuracies of the adjusted g (t). The value of a_2 obtained is approximately equal to 250. Therefore, the adjusted response function is

$$G(s) = 1440/s\,(1 + 0.103\, s)\,(1 + 0.004\, s), \quad \text{r.p.m.}$$

The extremely large value of a_2 obtained in this example indicates that the particular servomotor investigated is closely represented by the first approximation.

CHAPTER VII

SAMULON'S METHOD OF TRANSIENT ANALYSIS

A theoretical exact technique for determining the transfer function of a two-phase servomotor from an experimentally determined transient response curve involves the use of Samulon's method of spectrum analysis of transient response curves in conjunction with Shannon's sampling theorem and Bode diagrams.[1, 2, 3, 4, 5] Samulon's method yields a representation of the transient response as a function of frequency.

Shannon's sampling theorem states that if a function f (t) contains no frequencies higher than f_{co} cycles per second, it can be completely described by sampling its ordinates at a series of points spaced $1/(2f_{co})$ seconds apart.

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[1]George J. Thaler and Robert G. Brown, Servomechanism Analysis (New York, 1953).

[2]H. Chestnut and R. W. Mayer, Servomechanisms and Regulating System Design, Volume I (New York, 1951).

[3]Claude E. Shannon, "Communication on the Presence of Noise," Proceedings of the Institute of Radio Engineers, Volume 37, Part I, 1949, pp. 10-22.

[4]H. A. Samulon, "Spectrum Analysis of Transient Response Curves," Proceedings of the Institute of Radio Engineers, Volume 39, 1959, pp. 175-186.

[5]Simon Ramo, M. Grabble, Dean Wooldridge, Handbook of Automation, Computation, and Control, Volume I (New York, 1958).

A mathematical proof showing that this is not only approximately but exactly true will be presented next. This proof starts with the Fourier integral theorem which is proven in most texts on operational mathematics.[6] This theorem states that

$$f\ (t) = 1/2\pi \int_{-\infty}^{+\infty} F\ (\omega)\ \text{EXP.}\ (j\,\omega t)\ d\omega$$

where F (ω) is the frequency spectrum of f (t). Since it is assumed that no frequencies higher than f_{co} cycles per second exist, the above equation reduces to

$$f\ (t) = 1/2\pi \int_{-2\pi f_{co}}^{+2\pi f_{co}} F\ (\omega)\ \text{EXP.}\ (j\,\omega t)\ d\omega \quad .$$

By letting $t = n/2f_{co}$ where n is any integer, the preceding equation reduces to

$$f\ (n/2f_{co}) = 1/2\pi \int_{-2\pi f_{co}}^{+2\pi f_{co}} F\ (\omega)\ \text{EXP.}\ (j\,\omega\, n/2f_{co})\ d\omega \quad .$$

Since the sampling points are spaced ($1/2f_{co}$) seconds apart, f ($n/2f_{co}$) are the values of f (t) at the sampling points.

The integral on the right will be recognized as the n^{th} coefficient of the Fourier series expansion of the function F (ω), when the fundamental period is taken from $+f_{co}$ to $-f_{co}$. This means that the values of the samples f ($n/2f_{co}$) determine the Fourier coefficients in the series expansion of F (ω). Thus they determine F (ω), since

[6]Murray F. Gardner and John L. Barnes, Transients in Linear Systems, Volume I (New York, 1958).

$F(\omega)$ is zero for frequencies greater than f_{co}, and for lower frequencies $F(\omega)$ is determined if its Fourier coefficients are determined. Also $F(\omega)$ determines the original function $f(t)$ completely, since a function is determined if its spectrum is known.

There is one and only one function whose spectrum is limited to a band f_{co}, and which passes through the given values at sampling points separated $1/2f_{co}$ seconds apart. This function can be reconstructed from the samples by using a pulse of the type $\sin(2\pi f_{co}t)/2\pi f_{co}t$. This pulse is unity at $t = 0$ and zero elsewhere.

If at each sampling point a pulse of this type is placed whose amplitude is adjusted to equal that of the sample, the sum of these pulses is the required function. This process can be described as follows:

Let A_n be the n^{th} sample. The function $f(t)$ is represented by

$$f(t) = \sum_{n=-\infty}^{+\infty} A_n \sin(2f_{co}t - n)\pi / \pi(2f_{co}t - n)$$

or

$$f(t) = \sum_{n=0}^{+\infty} A_n \sin 2\pi f_{co}(t - T)/2\pi f_{co}(t - T)$$

where $T = 1/2f_{co}$.

Therefore, the Fourier spectrum of the n^{th} term of the sum is

$$\Phi(\omega) = \int_{-\infty}^{+\infty} F(t)\, \text{EXP.}(-j\omega t)\, dt$$

$$= \int_{-\infty}^{+\infty} A_n \left[\sin 2\pi f_{co}(t - T)/2\pi f_{co}(t - T)\right] \text{EXP.}(-j\omega t)dt$$

$$= A_n\, \text{EXP.}(-jn\omega T/2f_{co}) \quad .$$

Hence, the Fourier spectrum of the curve F (t) is

$$\Phi(\omega) = \sum_{n=0}^{\infty} A_n \text{ EXP. } (-jn\,\omega\, T/2f_{co}) \quad .$$

In his article, "Spectrum Analysis of Transient Curves," H. A. Samulon choose to write the Fourier spectrum of the transient response in terms of B_n where $B_n = A_n - A_{n-1}$.[7] By neglecting the constant time delay, the spectrum analysis transforms to

$$\Phi(\omega) = 1/j4f \sin \pi f/2f_{co} \sum_{n=0}^{\infty} B_n \text{ EXP. } (-jn\,\pi f/f_{co}) \quad .$$

Since the measured curve is the response to an ideal step, the complex transfer function of the system is

$$H(\omega) = (\pi f/2f_{co})/\sin(\pi f/2f_{co}) \sum_{n=0}^{\infty} B_n \text{ EXP. } (-jn\,\pi f/f_{co})$$

which can be represented as

$$G(j\omega) = (\pi f)/\sin(\pi f/2f_{co}) \text{ EXP. } (j\omega T/2) \sum_{n=0}^{\infty} B_n \text{ EXP. } (-jn\,\omega T) \quad .$$

This equation would be exact if the system response contained no frequency components greater than f_{co}.

As an example, this method of analysis is applied to the experimentally determined transient response curve used in Chapter VI.

A cutoff frequency of 20 cycles per second or 125.6637 radians per second is assumed. Therefore, the value of T, the sampling time, is

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[7] H. A. Samulon, "Spectrum Analysis of Transient Response Curves," _Proceedings of the Institute of Radio Engineers_, Volume 39, 1951, pp. 175-186.

$1/2f_{co} = 0.025$ seconds. The values of the transient response curve, A_n, which corresponds to the Sampling points, nT, are tabulated in Tables I to VI.

The vectorial summation for judiciously chosen frequencies was performed with the aid of a desk calculator by resolving the vectors into their real and imaginary components. These resolved vectors are tabulated and summed in Tables I to VI.

The corresponding $(\pi \omega/2 \omega_{co})/\sin(\pi \omega/2 \omega_{co})$ term can be considered as a magnitude correction term. For small values of ω, this magnitude correction term will be negligible. As ω approaches ω_{co}, however, this term will have an appreciable effect upon the magnitude.

The term EXP. $(j \omega T/2)$ can be considered as a phase correction term. The effect of this term is usually appreciable and should be taken into account.

The magnitude (in db) and phase of $G(j\omega)$ is plotted in Figure 14. The 3 db attenuation point occurred at approximately 9.67 radians per second. The corresponding $G(s)$ term is

$$G(s) = 1440/(1 + 0.1034\, s)$$

and the corresponding normalized open loop transfer function is

$$HG(s) = 72/(1 + 0.1034\, s)\,. \qquad \text{r.p.m./volt-sec.}$$

The derived formula for $G(j\omega)$ is based upon the assumption that the spectrum of the response curve does not contain frequencies above a certain limit, f_{co}. This assumption will never be fulfilled with full mathematical rigor for a practical network, however. It can be shown that the amount of the error will be largest near to the assumed cutoff frequency. This condition is demonstrated in Figure 14 where as ω approaches ω_{co}, the error in the magnitude and the phase shift of the frequency response curve becomes appreciable. As an example, at $\omega = 100$ radians per second, the error in the magnitude of the frequency response curve is approximately 65 percent of the frequency response while the error at $\omega = 50$ radians per second is less than 10 percent of the magnitude of the frequency response. For accurate results, the magnitude of the frequency response curve at f_{co} should be attenuated at least 40 db. However, by assuming a cutoff frequency lower than the 40 db attenuation frequency, a close approximation for the lower frequencies will be obtained.

TABLE I

VECTOR SUMMATION FOR THE COMPLEX TRANSFER FUNCTION (ω = 1)

nT	A_n	B_n	$\sum_{n=0}^{\infty} B_n$ EXP. $(-j\,\omega\,nT)$ Imaginary +	Imaginary −	Real +	Real −
.025	304	304		7.60	303.90	
.050	536	232		11.60	231.72	
.075	720	184		13.78	183.48	
.100	865	145		14.47	144.27	
.125	1,000	135		16.82	133.94	
.150	1,100	110		16.43	108.76	
.175	1,185	75		13.05	73.85	
.200	1,245	60		11.92	58.80	
.225	1,288	43		9.59	41.91	
.250	1,320	32		7.91	31.00	
.275	1,350	30		8.13	28.87	
.300	1,375	25		7.39	23.86	
.325	1,390	15		4.79	14.21	
.350	1,400	10		3.43	9.39	
.375	1,410	10		3.66	9.30	
.400	1.424	14		5.45	12.89	
.425	1,431	7		2.88	6.38	
.450	1,436	5		2.17	4.50	
.475	1,438	2		0.91	1.78	
.500	1,440	2		0.96	1.75	
				162.93	1,424.56	

$$\sum_{n=0}^{\infty} B_n \text{ EXP. } (-j\omega nT) = 1433.99 \text{ EXP. } (-j\ .114)$$

$$\text{EXP. } (+j\ \omega\ T/2) = \text{EXP. } (+j\ 0.0125)$$

$$G\ (j\,\omega) = 1433.99 \text{ EXP. } (-j\ 0.1015)$$

$$= 1433.99 \text{ EXP. } (-j\ 5.81^{\circ})$$

$$20 \log_{10} |\, G\ (j\ \omega)\,| = 63.1310$$

TABLE II

VECTOR SUMMATION FOR THE COMPLEX TRANSFER FUNCTION ($\omega = 5$)

nT	A_n	B_n	$\sum_{n=0}^{\infty} B_n$ EXP. ($-j\ \omega\, nT$) Imaginary +	Imaginary −	Real +	Real −
.025	304	304		38.00	301.62	
.050	536	232		57.86	224.78	
.075	720	184		67.38	171.21	
.100	865	145		69.51	127.25	
.125	1,000	135		78.97	109.47	
.150	1,100	110		74.97	80.48	
.175	1,185	75		57.56	48.00	
.200	1,245	60		50.49	32.40	
.225	1,288	43		38.79	18.54	
.250	1,320	32		30.37	10.08	
.275	1,350	30		29.43	5.83	
.300	1,375	25		24.94	1.75	
.325	1,390	15		14.98		0.85
.350	1,400	10		9.84		1.79
.375	1,410	10		9.54		3.00
.400	1,424	14		12.72		5.83
.425	1,431	7		5.95		3.70
.450	1,436	5		3.89		3.14
.475	1,438	2		1.38		1.44
.500	1,440	2		1.19		1.60
				677.76	1,176.41	21.35

$$\sum_{n=0}^{\infty} B_n \text{ EXP. } (-j\ \omega\, nT) = 1363.16 \text{ EXP. } (-j\ 0.531)$$

$$\text{EXP. } (+j\ \omega\ T/2) = \text{EXP. } (+j\ 0.0625)$$

$$G\ (j\,\omega) = 1363.16 \text{ EXP. } (-j\ 0.4685)$$

$$= 1363.16 \text{ EXP. } (-j\ 26.83^{\circ})$$

$$20 \log_{10} \left| G\ (j\,\omega) \right| = 62.69$$

TABLE III

VECTOR SUMMATION FOR THE COMPLEX TRANSFER FUNCTION ($\omega = 10$)

nT	A_n	B_n	$\sum_{n=0}^{\infty} B_n$ EXP. $(-j\ \omega\ nT)$ Imaginary +	Imaginary −	Real +	Real −
.025	304	304		75.20	294.54	
.050	536	232		111.22	203.60	
.075	720	184		125.41	134.63	
.100	865	145		122.01	78.30	
.125	1,000	135		128.11	42.56	
.150	1,100	110		109.72	7.70	
.175	1,185	75		73.79		13.41
.200	1,245	60		54.54		25.00
.225	1,288	43		33.43		27.03
.250	1,320	32		19.13		27.90
.275	1,350	30		11.40		27.75
.300	1,375	25		3.49		24.75
.325	1,390	15	1.65			14.91
.350	1,400	10	3.52			9.36
.375	1,410	10	5.73			8.19
.400	1,424	14	10.61			9.13
.425	1,431	7	6.72			3.11
.450	1,436	5	4.89			1.05
.475	1,438	2	2.00		0.08	
.500	1,440	2	1.91		0.57	
			36.58	867.45	761.98	191.58

$$\sum_{n=0}^{\infty} B_n \text{ EXP } (-j\ n\ \omega\ T) = 1007.23 \text{ EXP. } (-j\ 0.97)$$

$$\text{EXP. } (+j\ \omega\ T/2) = \text{EXP } (+j\ 0.125)$$

$$G\ (j\ \omega) = 1007.23 \text{ EXP. } (-j\ 0.845)$$

$$= 1007.23 \text{ EXP. } (-j\ 48.42^{\circ})$$

$$20 \log_{10} \left| G\ (j\ \omega) \right| = 60.06$$

TABLE IV

VECTOR SUMMATION FOR THE COMPLEX TRANSFER FUNCTION ($\omega = 20$)

nT	A_n	B_n	$\sum_{n=1}^{\infty} B_n$ EXP. $(-j\ \omega nT)$			
			Imaginary		Real	
			+	-	+	-
.025	304	304		145.73	266.79	
.050	536	232		195.23	125.35	
.075	720	184		183.54	13.00	
.100	865	145		131.80		60.42
.125	1,000	135		80.62		108.28
.150	1,100	110		15.34		108.90
.175	1,185	75	26.42			70.19
.200	1,245	60	45.46			39.14
.225	1,288	43	42.05			8.99
.250	1,320	32	30.55		9.18	
.275	1,350	30	21.10		21.33	
.300	1,375	25	6.91		24.03	
.325	1,390	15		3.27	14.64	
.350	1,400	10		6.59	7.52	
.375	1,410	10		9.39	3.43	
.400	1,424	14		13.84		2.10
.425	1,431	7		5.57		4.24
.450	1,436	5		2.04		4.56
.475	1,438	2	0.16			1.99
.500	1,440	2	1.09			1.67
			173.74	792.96	485.27	409.48

$$\sum_{n=0}^{\infty} B_n \ \text{EXP.}\ (-j\ n\ \omega\ T) = 653.02\ \text{EXP.}\ (-j\ 1.449)$$

$$\text{EXP.}\ (+j\ \omega\ T/2) = \text{EXP.}\ (+j\ .25)$$

$$G\ (j\ \omega) = 653.02\ \text{EXP.}\ (+j\ 1.199)$$

$$= 653.02\ \text{EXP.}\ (-j\ 68.7^{\circ})$$

$$20 \log_{10} |G\ (j\ \omega)| = 56.2982$$

TABLE V

VECTOR SUMMATION FOR THE COMPLEX TRANSFER FUNCTION ($\omega = 50$)

nT	A_n	B_n	$\sum_{n=1}^{\infty} B_n$ EXP. ($-j\ \omega$ nT) Imaginary +	Imaginary −	Real +	Real −
.025	304	304		288.49	95.85	
.050	536	232		138.55		186.09
.075	720	184	105.41			150.80
.100	865	145	138.91		41.57	
.125	1,000	135	4.05		134.94	
.150	1,100	110		103.29	37.79	
.175	1,185	75		46.57		58.78
.200	1,245	60	32.88			50.19
.225	1,288	43	41.54		11.09	
.250	1,320	32	1.92		31.94	
.275	1,350	30		27.84	11.15	
.300	1,375	25		16.10		19.12
.325	1,390	15	7.84			12.78
.350	1,400	10	9.73		2.29	
.375	1,410	10	0.89		9.96	
.400	1,424	14		12.83	5.59	
.425	1,431	7		4.67		5.22
.450	1,436	5	2.48			4.34
.475	1,438	2	1.96		0.40	
.500	1,440	2	0.24		2.00	
			347.85	638.34	384.57	487.32

$$\sum_{n=0}^{\infty} B_n \text{ EXP. } (=j\ \omega\ nT) = 309.27 \text{ EXP. } (-j\ 1.91)$$

$$\text{EXP. } (+j\ \omega\ T/2) = \text{EXP. } (+j\ 0.625)$$

$$G\ (j\omega) = 309.27 \text{ EXP. } (-j\ 1.285)$$

$$= 309.27 \text{ EXP. } (-j\ 78.63^{\circ})$$

$$20 \log_{10} |G\ (j\omega)| = 49.8076$$

TABLE VI

VECTOR SUMMATION FOR THE COMPLEX TRANSFER FUNCTION ($\omega = 100$)

nT	A_n	B_n	$\sum_{n=0}^{\infty} B_n$ EXP. $(-j\ \omega\, nT)$ Imaginary +	Imaginary −	Real +	Real −
.025	304	304		181.54		243.84
.050	536	232	222.26		66.51	
.075	720	184		172.79	63.22	
.100	865	145	79.46			121.29
.125	1,000	135	8.10		134.73	
.150	1,100	110		70.86		84.52
.175	1,185	75	73.01		17.16	
.200	1,245	60		55.01	23.95	
.225	1,288	43	21.37			37.32
.250	1,320	32	3.83		31.77	
.275	1,350	30		20.67		21.75
.300	1,375	25	24.62		4.25	
.325	1,390	15		13.36	6.80	
.350	1,400	10	4.44			8.96
.375	1,410	10	1.79		9.84	
.400	1,424	14		10.23		9.55
.425	1,431	7	6.95		.77	
.450	1,436	5		4.31	2.53	
.475	1,438	2	0.78			1.84
.500	1,440	2	0.47		1.94	
			447.08	528.77	363.47	529.07

$$\sum_{n=0}^{\infty} B_n \quad \text{EXP. } (=j\ \omega\, nT) = 184.41 \text{ EXP } (=j\ 2.6836)$$

$$\text{EXP. } (+j\ \omega\, T/2) = \text{EXP } (+j\ 1.25)$$

$$G\ (j\,\omega) = 184.41 \text{ EXP. } (-j\ 1.4336)$$

$$= 184.41 \text{ EXP. } (-j\ 82^{\circ})$$

$$\sin\ (\omega T/2)/(\pi\omega/2\omega_{co}) = .7592$$

$$|G\ (j\,\omega)| = 243.40$$

$$20 \log_{10}\ |G\ (j\,\omega)| = 47.7264$$

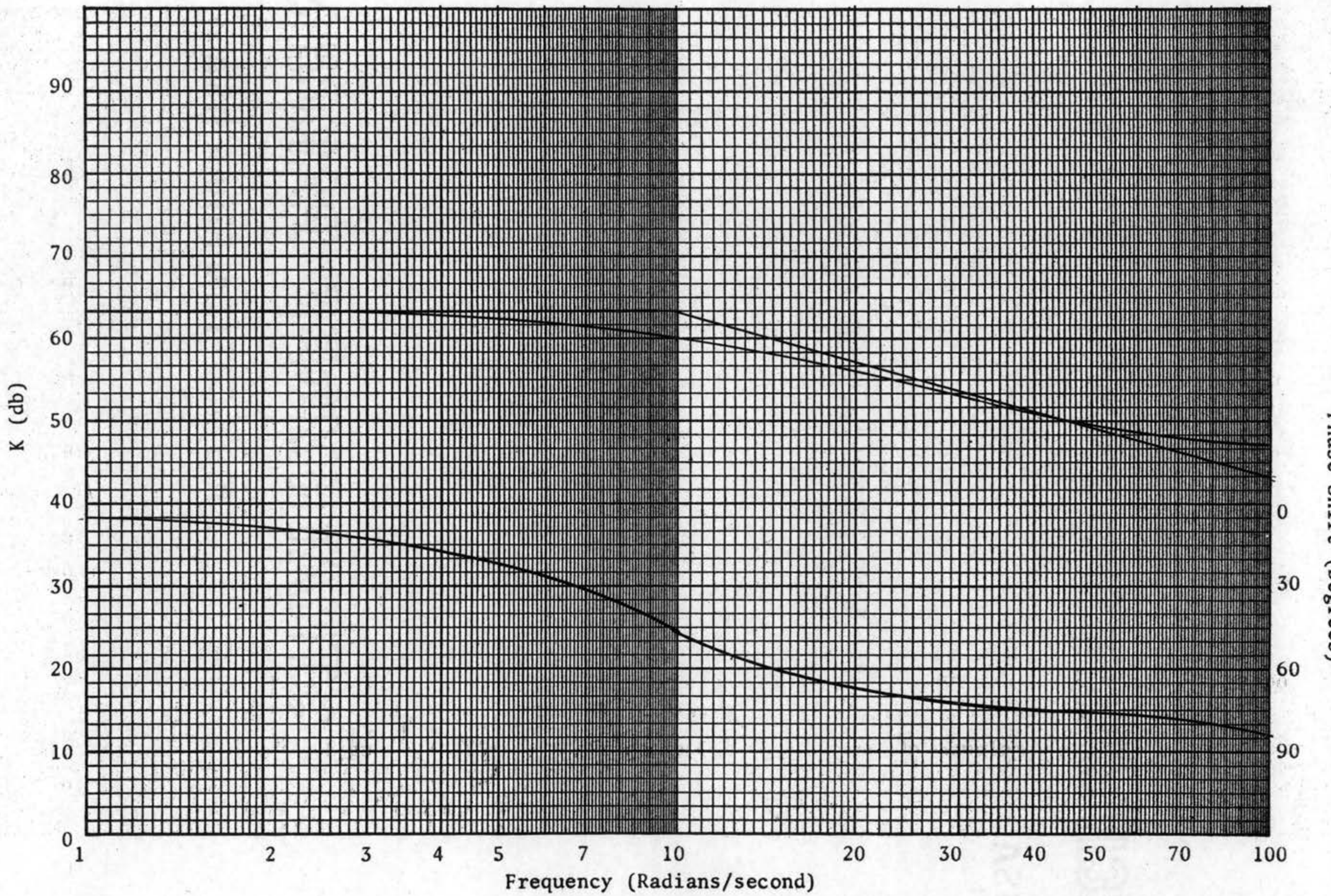

Figure 14. Frequency Response Curve Determined by Samulon's Method

CHAPTER VIII

SUMMARY

Methods of obtaining transfer functions of small two-phase servomotors were investigated, and one particular method was developed experimentally.

In order to be able to better understand the factors effecting the transfer function, the operating characteristics of both the conventional two-phase motor and the two-phase servomotors were investigated. Differences between the conventional two-phase motors and two-phase servomotors have been discussed as well as factors affecting the non-linearities in the operating characteristics of the two-phase motors. During the investigation, the frequently used assumption that the torque for any control voltage decreases at a definite uniform rate with speed was found to be applicable only for a motor having a high rotor resistance and operating under balance conditions. This assumption, however, gives a valid approximation for most well designed servomotors operating under balance conditions.

Three methods of obtaining the transfer function from the motor parameters was discussed. The first of these methods is based upon the idealized motor which is defined as a linear motor which has a stall torque proportional to the control voltage and which has a torque for any control voltage that decreases at a definite uniform rate with speed. A transfer function determined by this method is applicable only to a servomotor operating under conditions such that it can be approximated by an ideal motor. The second method discussed is based upon the electrical transients of the circuit and a knowledge of the applied load. A transfer function analysis by this method proved to be quite tedious as well as involving a knowledge of the circuit parameters which is seldom known. This transfer function would only be applicable over the range of operation where these circuit parameters are linear. The third method discussed is based upon the equivalent circuit of the servomotor. This method is applicable to both balance and unbalance operation. It is complicated and limited to the range of operation where the motor parameters are linear.

Legendre's principle of least squares curve fitting and Samulon's method of transient analysis were discussed and demonstrated. This investigation indicated that the transfer function could be obtained from the transient response curve with little knowledge of the motor being tested. The transfer function obtained by these methods for the particular data at hand varied by less than 1 percent from each other, and the experimentally determined transient response curve for the motor tested was reproduced within 1 percent by Legendre's principle of least squares curve fitting (Figure 13).

Since the calculations are based upon the transient response curve, an accurate response curve is desirable. Due to the relative small size of most two-phase servomotors, instrumentation is often a problem. Considerable time and effort was devoted to obtaining an accurate and yet relative straight forward method of determining the transient response curve for the system. The method developed for this investigation reproduces the transient response of a Diehl Manufacturing Company, Type Fb-25, SS code 2, 60 cycle, 20 volt servomotor within an estimated accuracy of ± 5 percent. The external load applied to the motor was held to a minimum.

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