ECONOMICS OF FERTILIZATION FOR SELECTED OKLAHOMA CROPS

By

THOMAS F. POGUE

Bachelor of Science

New Mexico State University

University Park, New Mexico

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Thesis Approved:

Odell L. Walker
Thesis Adviser

Clark Edwards

Robert MacVicar
Dean of the Graduate School

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CHAPTER I

INTRODUCTION

Problem Setting

Commercial fertilizer is an increasingly important factor of production in Oklahoma agriculture. In recent years, farmers have used over ten times as much fertilizer annually as they used twenty years ago (10,000 tons in 1942 vs. 144,000 tons in 1959).[1] This increased use of fertilizer is partially explained by the increased acreage under irrigation, the depletion of soil nutrients caused by a half century of cropping, the higher costs of other productive resources, the increased availability of fertilizers, and greater farmer awareness of the effects of fertilization. These influences will very probably continue to be important. The substitution of fertilizer (and other inputs) for land precipitated by acreage controls is another important reason for greater fertilizer use. Thus, the question of how much fertilizer to use is and will continue to be an important question for many Oklahoma farmers and agricultural scientists. This is essentially a question of (1) how to produce, i.e., whether to fertilize or not, and (2) how much to produce, i.e., how much fertilizer to use.

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[1] "Fertilizer Consumption in Oklahoma for the Past 10 Years," Oklahoma State Department of Agriculture, 1952, 1960. These figures do not reflect the trend to higher analysis fertilizer. Thus, they probably understate the increase in fertilizer use over the past two decades.

Further discussion of the problem introduced above centers about (1) the determinants of the optimum rate of fertilization, (2) the adequacy and relevance of past and present research designed to supply information about these factors, and (3) methods of using this information for decision making in the farm firm (decision making techniques).

The Determinants of the Optimum Rate of Fertilization

To determine the optimum level of fertilization, the farm manager needs to know or have estimates of fertilizer productivities, fertilizer costs, and crop prices.

Fertilizer Productivities. How do crop yields change as the rate of fertilization changes? Factors other than fertilizer influence crop yields and some or all of these factors may interact with fertilizer to influence its productivity. That is,

$$Y = f(X_1, X_2, \ldots, X_n) \tag{1}$$

where Y is the crop yield and $X_1, X_2, \ldots, X_n$ are factors influencing this yield and it is possible that

$$\frac{\delta Y}{\delta X_i} = g(X_i, X_j, \ldots, X_n) \text{ , } X_i = \text{fertilizer} \tag{2}$$

Furthermore, some of these factors are subject to managerial control, others are not. For example, corn yields are a function of land, labor, management, weather, and capital in the form of seed, machinery, fertilizer, etc. Seeding rates and weather are controllable and uncontrollable factors, respectively, usually affecting the productivity of fertilizer. Labor and machinery are examples of factors which probably do not affect the productivity of fertilizer (for the range of inputs common on Oklahoma farms).

For this dissertation, it is assumed that the manager seeks only the answer to the question of how much fertilizer to use (except the single case for which seeding rate is also variable). That is, it is assumed that the decision to produce the crop has been made; and, consequently, amounts of some factors have been allocated for its production. The actual amounts of these fixed factors do not affect the optimum level of variable factor (fertilizer) use unless the fixed factor interacts with the variable factor, i.e., unless the productivity of the variable factor is affected by the level of the fixed factor.[2] If, for instance, X_3, X_4, . . ., X_n are these allocated factors, and X_1 and X_2 are the variable factors (fertilizer), equation (1) becomes

$$Y = f(X_1, X_2 \mid X_3, \ldots, X_n) \qquad (3)$$

Equation (3) tells how Y varies as X_1 and X_2 vary. This is the information that the farmer needs to answer the question posed at the beginning of this section.

Fertilizer Costs. Total fertilizer costs are necessary for assessing the cost of yields obtainable from fertilizer, given some allotment of other productive resources--land, labor and machinery for planting, cultivating, and harvesting the crop, etc. Two categories of costs determine total fertilizer costs. First, there are costs that do not vary as the fertilization rate varies (over a range). For example, costs for labor and machinery used in applying fertilizers are essentially constant (fixed) over a wide range of fertilization rates. Secondly, there are costs which

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[2] See Chapter II, pp. 18-19 for a discussion of the role of interaction in decision making and equation (2) above for a statement of interaction conditions.

vary as fertilization rates vary. These variable costs of fertilizing are essentially (1) the in-field cost of fertilizer and (2) the cost of capital invested in fertilizer.[3] Since the farmer buys competitively, the price (in-field cost) of a unit of fertilizer is not expected to vary as the fertilization rate varies.

Fixed costs of fertilizing affect only the question of whether to fertilize. For the fertilizer-crop combinations considered in this study, fixed costs are small relative to variable costs. Thus, the definition of fertilizer costs used here refers to the variable costs of fertilizer. Fixed costs (the question of whether to fertilize) are considered separately and briefly in Chapter V. Fixed costs are estimated from charges for custom fertilizing to avoid allocative problems associated with fixed equipment and labor requirements.

Crop Prices. Crop prices determine the value of changes in yields attributable to fertilizing. For this dissertation, crop prices are defined net of harvesting, hauling, and other harvesting and marketing charges. Again, because the farmer sells his products competitively, crop prices are not expected to vary as output varies. This last statement is true only if all yields, with and without fertilization, are of the same quality. For the crops considered in this study, product quality either does not vary as output varies or can be held constant by harvesting procedures.

Given the answers to the above questions, i.e., given estimates of the factor productivity and product and factor prices, the farm manager is able to predict, more or less certainly, the effect on net returns of

[3]See pp. 16-17 of Chapter II for a definition of capital cost.

fertilizing at any particular rate. If the farm manager's goal is to achieve some desired level of net returns to the fixed factors ($X_3, \ldots, X_n$), then he can now choose a rate of fertilization (production strategy) that most nearly achieves his goal(s).[4] The manager's choice of a production strategy is also influenced by the confidence that he has in his predicted outcomes. In summary, the selection of an optimum production strategy (levels of X_1 and/or X_2) may be viewed as a process of maximizing the attainment of managerial goals (NR), subject to the restraints of fertilizer (P_{x_i}) and crop (P_y) prices, the production function [$\bar{X} = f(X_1, X_2)$], and the degree of certainty with which each of the preceding are known.[5]

Adequacy and Relevance of Present and Past Research

Much agronomic and economic research purports to answer the above questions, i.e., to supply information about production functions (input-output relationships), fertilizer costs, and crop prices that will help farmers decide how to produce and how much to produce. Is available information (the result of past research) and information that will be forthcoming from present research of the quantity and quality needed by Oklahoma farm managers? This section discusses present and past research

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[4]See Chapter II, pp. 12-13 for the definition of goal used in this thesis.

[5]For the remainder of this dissertation, these symbols will be used:

P_y = the expected price of a unit of Y net of per unit harvesting and handling costs.

P_{x_i} = the total cost of a unit of composite factor X_i (in-field cost).

X_i = a unit of the decision factor (fertilizer) and its auxiliary (variable) services.

NR = the difference between total variable costs and total returns resulting from a particular production strategy; i.e., NR = TR - TVC.

Y = $f(X_1, X_2, X_3, \ldots, X_n)$ a continuous production function with finite continuous derivatives up to the order two.

designed to supply information about (1) input-output relationships and (2) fertilizer costs and crop prices.

Input-Output Relationships. In Oklahoma, a large number of agronomic experiments have been conducted to explain the effects of fertilization, seeding rates, and other cultural practices on crop yields. Publications giving crop yield responses to discrete levels of fertilization or partially factorial treatments of fertilizer and seed, irrigation, or other similar variables have been and continue to be widely disseminated by our extension service. This information is undoubtedly useful to farm managers. However, much present and forthcoming input-output information is inadequate because

(1) input-output relationships are often specified for particular levels of a number of other factors of production: soil, weather, management, and cultural practices not included as variables. Some of these fixed factors profoundly affect the productivity of fertilizer (e.g., soil, weather); yet few agronomic experiments give more than perfunctory attention to the effects of soil and weather on productivity. Thus, there is little basis for generalizing these results for soil and weather conditions other than those existing in the experiment. One way to account for effects of other factors would be to design experiments with weather, soil, cultural practices, and managerial techniques as variables. This means experiments on a much greater scale, spatially and temporarily, and consequently, more expensive experiments. Alternatively, it might be possible to measure the effects of weather on past experiments (since weather records are generally available) and thereby increase the meaningfulness of available information. The latter alternative is explored in this study.

(2) it is determined for discrete input levels. Fertilizer and seed are highly divisible factors; that is, the relationship between the variable input and crop yield is essentially continuous rather than discrete. Interpolation between discrete experimental input levels is always troublesome and frequently gives misleading answers.

In summary, present and forthcoming information about input-output relationships is useful; but, it may often be made more useful. A continuous input-output relationship that accounts for the effects of factors interacting with fertilizer (primarily weather) would more nearly meet the needs of farm decision makers.

Fertilizer Costs and Crop Prices. Fertilizer costs are generally known when production plans are made. Crop prices are known with degrees of knowledge varying from almost complete certainty (pre-season contracting) to almost complete uncertainty (certain highly perishable fruit and vegetable crops). Government support prices, pre-season contracting, and the price forecasting activities of the Agricultural Marketing Service lessen price uncertainty for some products. Unfortunately, price uncertainty is a problem only slightly amenable to present research techniques and only slightly resolved by present research efforts. The physical laws that determine input-output relationships are more easily defined than the elusive hodge-podge of sociological, psychological, cultural, and physical propensities that determine market prices. And, once discovered, input-output relationships are likely to undergo only gradual and directional changes. However, a mechanism for predicting or explaining market prices may be rendered obsolete at any moment. Thus, with some exceptions, price uncertainty remains essentially the problem of the individual manager.

Decision Making Techniques

Decision making techniques are needed to guide the systematic consideration of pertinent information and the subsequent formulation of an optimal production strategy. Such techniques are required even if the farm manager has perfect knowledge of all the determinants of production strategy. Knowledge of appropriate decision making techniques becomes increasingly critical as the manager's information about determinants of production strategy becomes less certain. The previous section is concluded with the statement that, in many cases, only the individual manager can deal with price uncertainty and, then, only because he has no alternative. Despite the importance of appropriate decision making techniques, farm managers are probably less than fully aware of them for two reasons. First, decision making techniques are rather steadily being defined for a growing range of managerial problems. Second, our farmer eduction programs do not emphasize these techniques.

Objectives

The problem attacked by this study is defined above as one of resource (factor) allocation. Specifically, how much fertilizer (i.e., what production strategy) should be used in producing certain Oklahoma crops. The information that farm managers need to formulate an optimum production strategy has been listed and declared quantitatively or qualitatively inadequate. The first major objective of this study is to increase the usefulness of some existing information; i.e., to specify the input-output relationship between fertilizer (and in two cases, seed) and yield for a number of Oklahoma crops, quantifying when possible the effects of weather on this relationship. Corollary objectives are to

(1) demonstrate the use of statistical procedures for increasing the usefulness of existing agronomic, climatological, and price data, and

(2) make explicit some considerations about the use of these statistical techniques as tools in economic research.

In addition to partially meeting farm managers' real need for better decision making information, accomplishing this objective may provide insights into some of the problems peculiar to research of this nature. Particularly, what characteristics of the original data would justify the use of the rather refined and expensive technique that may be needed? Also, results may emphasize the need for experimental designs that yield useful data for decision makers.

Given higher quality input-output data, there is still the need for choice guides or decision making techniques to indicate the best means of achieving a desired end. Relevant questions are: How are these decision making techniques modified to accommodate uncertainty (imperfect knowledge)? How is information about price and weather conditions and the input-output relationship actually used in decision making? How do managerial goals modify production decisions? The second major objective of this study is to show how information about input-output relationships, prices, and weather is used to determine optimum production strategies for varying states of knowledge and differing managerial goals. Accomplishing this objective will give one measure of the criticalness of the various determinants of production strategy for each of several important Oklahoma crops. Future research and extension could then be directed to increasing farmers' knowledge of the more critical determinant, either input-output relationships or prices.

Format of Remainder of Thesis

The problem attacked by and the objectives of this study are stated above. The following outline gives the organization of this thesis and furnishes a preview of the methods by which the objectives of this study are attained.

Chapter II

Knowledge situations and farmer goals considered in this thesis are defined and discussed. Choice criteria are developed for several knowledge-goal combinations. Explanation of each decision making technique stresses the applicability of the technique to actual farm decision problems.

Chapter III

Methods of and considerations in obtaining information about product and factor prices and factor productivities are stated and explained.

Chapter IV

The regression equations obtained are presented and comments are made about the usefulness of the regression equations as estimates of production functions.

Chapter V

The choice criteria of Chapter II and the empirical information of Chapter IV are combined in an economic analysis of fertilizer use (for the particular fertilizer-crop combinations of this study).

Chapter VI

A summary of the study is given. Specific conclusions are made about fertilizer use and research aimed at making fertilization more profitable.

CHAPTER II

DECISION MAKING TECHNIQUES

The problem of using fertilizer optimally was posed in Chapter I. The determinants of the optimum factor use level (production strategy) were defined as

(1) product (P_y) and factor (P_{x_i}) prices,

(2) the production function $Y = f(X_1, X_2, X_3, \ldots, X_n)$,

(3) the farm manager's knowledge of (1) and (2), and

(4) the farm manager's goal (NR).

Techniques for formulating production strategy, given the four determinants listed above, are presented in this chapter.

Scope of Chapter

Decision making techniques vary as knowledge and managerial goals vary. Thus, these techniques are classified according to the knowledge situations and goals for which they may be appropriate. The knowledge situations and managerial goals to be used in this dissertation are necessarily defined before the above classification is carried out.

The classification of knowledge used in this dissertation is perfect knowledge and imperfect knowledge.[1]

[1] See Walker (1), Knight (2), and Johnson and Haver (3) for a fuller discussion of the effects of knowledge on decision making principles. The following classification of knowledge is essentially that of Luce and Raiffa (4, p. 13).

Perfect knowledge describes the situation in which the outcome of a particular production strategy is known with certainty. That is, the expected outcome is realized.

Imperfect knowledge gives rise to

(1) the risk situation in which there are several possible outcomes and the probability of each outcome is known. These probabilities may come from either a priori, statistical, or subjectively determined probability distributions.

(2) uncertainty situations in which probabilities of outcomes are unknown. However, the possible outcomes are known.

Each model used in this dissertation assumes that all possible outcomes are known.[2] Farm managers have some knowledge of outcomes in both of the above knowledge situations. The case of no knowledge of outcomes (uncertainty by many definitions) is not considered. This is a reasonable omission since the manager would necessarily have knowledge of any outcome that would seem possible to him. Finally, these classifications should be regarded as subjectively defined domains on a continuum of knowledge. That is, different individuals with the same amount of information may view possible outcomes with differing degrees of certainty.

Maximum utility for himself and his family is certainly the goal of any farm manager. Utility is generally considered to be a function of a number of variables, one of which is net returns. Thus, a farm manager's goal is to maximize the function

$$\text{Utility} = f(Z_1, Z_2, \ldots, Z_n) \qquad (1)$$

[2]Implications of this assumption are examined in footnote 13, p. 26, following the discussion of particular decision models.

where Z_1, say, is net returns. However, for this dissertation, the term managerial goal is more restrictively defined. Specifically, a managerial goal is a level of net returns bringing about or associated with a preferred (maximum) level of utility.[3] The managerial goals used in this dissertation are

(1) maximum net returns,

(2) maximum security level where the security level for any production strategy is the minimum possible level of NR for that strategy, and

(3) minimum regret where regret is the *ex post* cost of making a wrong decision.

These goals are more fully defined in later sections of this chapter.

The utility derived from a given outcome varies widely among individuals. Therefore, any managerial goal (level of NR) would be completely appropriate for only a few farm managers. However, it is possible that a particular level of NR (managerial goal), say maximum NR, brings about or is coincident with approximately maximum utility for a fairly large number of individuals. Subsequent discussions of managerial goals aim at establishing the goals considered in this study as appropriate goals for a fairly large group of farmers.

In the remainder of this chapter, choice guides are developed for several combinations of the above knowledge situations and managerial goals. Specifically:

(1) Each knowledge situation (perfect knowledge, then imperfect knowledge) is discussed and defined more fully.

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[3]Generally, the level of net returns from any outcome approximates the utility derived from that outcome to the extent that the determinants of utility have a price.

(2) Managerial goals considered in each knowledge situation are discussed.

(3) Choice criteria that are appropriate for each goal are stated and explained.

Decision Making with Perfect Knowledge

In a perfect knowledge situation, there is a one to one correspondence between strategies and outcomes. That is, a farm manager knows that a particular production strategy will always yield a certain addition to the firm's net returns. Since there is no reason for discounting the returns from a production strategy because of uncertainty, these returns may be viewed as true measures of the desirability of each alternative (strategy).[4] Thus, the only likely objective of management in the perfect knowledge situation[5] is maximization of NR. Maximization of NR is described here as a mechanical process in which the production function is always a restraint, factor and capital supply may be restraints, and product and factor prices are known and constant.

In Chapter I (p. 5), net returns (NR) were defined as the returns to other (fixed) factors of production resulting from the use of variable factors (X_i) to produce Y. That is,

Net Returns = Total Returns = Total Variable Costs (2)

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[4]Alternatively, NR probably does not interact with the other Z_i of the farmer's utility function (equation 1). With no change in the other Z_i from changes in NR, an outcome with a higher NR is always preferred. Therefore, the goal of maximum NR is most likely in the perfect knowledge situation.

[5]A view of perfect knowledge situations, consistent with later sections, is that perfect knowledge situations are those in which management can exercise control over all variables (determinants of NR).

If there are two variable factors X_1 and X_2,[6] equation (2) becomes

$$NR = P_y Y - P_{x_1} X_1 - P_{x_2} X_2 \qquad (3)$$

where P_y and P_{x_i}, the market value of a unit of product and the cost of a unit of the variable factor, respectively, are known and constant (because of the assumptions of perfect knowledge and pure competition). To maximize NR (the objective stated above), the manager should use more of a variable factor, i.e., increase the output of Y, as long as the resulting additions to NR are greater than zero. That is, a manager would use more of X_1 and X_2 as long as their addition to total returns (marginal returns) is greater than their addition to total costs (marginal costs).

If the amount of Y resulting from inputs of X_1 and X_2 is given by a continuous production function (as it is for the examples in this dissertation), i.e., if

$$Y = f(X_1, X_2) \qquad (4)$$

equation (2) becomes

$$NR = P_y \left[f(X_1, X_2) \right] - P_{x_1} X_1 - P_{x_2} X_2 \qquad (5)$$

Net returns, as given in equation (5) are maximum if X_1 and X_2 are used at the levels for which[7]

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[6]Only two factors are variable in later examples; thus, the general case (n variable factors) is not developed here. However, the procedure for maximizing net returns is the same, in principle, for any number of variable factors. The reader may refer to Leftwich (5) Chapter 9 for a lucid explanation of the general process.

[7]Equations (6) and (7) are necessary conditions for a maximum. Sufficient conditions are that equations (6) and (7) hold and

$$\frac{\delta^2 Y}{\delta X_1^2} < 0 \ , \quad \frac{\delta^2 Y}{\delta X_2^2} < 0$$

$$\left[\frac{\delta^2 Y}{\delta X_1 \, \delta X_2} \right]^2 < \frac{\delta^2 Y}{\delta X_1^2} \cdot \frac{\delta^2 Y}{\delta X_2^2}$$

Since all of the production functions (surfaces) in this study are concave, second order conditions are met for any level of variable factor. The procedure for determining values of variables that maximize a continuous function (in this case, the net returns function) is given in any elementary calculus text.

$$\frac{\delta NR}{\delta X_1} = \frac{\delta f(X_1, X_2)}{\delta X_1} P_y - P_{x_1} = 0 \quad (6)$$

and

$$\frac{\delta NR}{\delta X_2} = \frac{\delta f(X_1, X_2)}{\delta X_2} P_y - P_{x_2} = 0 \quad (7)$$

Alternatively, net returns are maximum if X_1 and X_2 are used so that

$$\frac{\delta f(X_1, X_2)}{\delta X_1} P_y = P_{x_1} \text{ or } \frac{\delta f(X_1, X_2)}{\delta X_1} = \frac{P_{x_1}}{P_y} \quad (8)$$

and

$$\frac{\delta f(X_1, X_2)}{\delta X_2} P_y = P_{x_2} \text{ or } \frac{\delta f(X_1, X_2)}{\delta X_2} = \frac{P_{x_2}}{P_y} \quad (9)$$

Implied in the conditions stated in equations (8) and (9) is the assumption that the supply of X_i is subject to no effective restriction. However, this is seldom the case. The supply of X_i may be absolutely limited or limited only in the sense that X_i must be purchased with capital having a positive price (i.e., interest rate). In this dissertation, it is assumed that X_i is limited in the sense that it is a form of capital which has a price. Whether the farm manager supplies his own capital or borrows operating capital, the cost of capital is either its opportunity cost, i.e., the level of returns to capital in its best alternative use (if capital supply is limited) or one plus the interest charge on capital (if capital supply is unlimited). The abbreviation $K \geq 1$ will be used for cost of capital in this dissertation. For example, suppose a farm manager pays 10 per cent interest on operating capital but can borrow no more than \$30,000. If, in investing the full \$30,000, the farm manager gets a \$1.20 yield on the last dollar invested, then the opportunity cost of capital is \$1.20, and $K = 1.2$. Capital would not be reallocated unless it would return more than \$1.20 in the new use. If, however, he can borrow all he wishes at 10 per cent interest, he will use capital to the point where the last dollar invested yields \$1.10. For this farmer the cost of capital is simply what he has to pay for a unit of capital, i.e., \$1.10, and $K = 1.1$. Thus, in any case, the manager should invest in variable resources so that the returns from the last dollar invested equal K--the opportunity cost of capital or one plus the interest charge on capital. If P_y and P_{x_i} are given in dollars, this means that equations (5), (8), and (9) must be modified to read:

$$NR = P_y \; f(X_1, X_2) = P_{x_1} K \, X_1 - P_{x_2} \, K \, X_2 \qquad (10)$$

$$\frac{\delta f(X_1, X_2)}{\delta X_1} \; P_y = P_{x_1} \; K \text{ or } \frac{\delta f(X_1, X_2)}{\delta X_1} = \frac{P_{x_1} \; K}{P_y} \qquad (11)$$

$$\frac{\delta f(X_1, X_2)}{\delta X_2} \; P_y = P_{x_2} \; K \text{ or } \frac{\delta f(X_1, X_2)}{\delta X_2} = \frac{P_{x_2} \; K}{P_y} \qquad (12)$$

where $\frac{\delta f(X_1, X_2)}{\delta X_i} \; P_y$ is the marginal revenue from a unit of X_i and $P_{x_i} K$ is the marginal cost in dollars (including capital cost) of a unit of X_i.

Decision Making with Imperfect Knowledge

The farm manager seldom has perfect knowledge of the determinants (except factor prices) of the optimum fertilizer use level (production strategy). Variability of outcome stems from imperfect knowledge of (1) the production function, (2) the levels of some factors of production, and (3) crop prices. These three causes of uncertainty are considered successively in the next paragraph.

Knowledge of fertilizer-crop production functions is limited and difficulty gained because many of the factors of production affecting fertilizer productivities are not easily measured and/or controlled. For this reason, future efforts to more accurately define production functions will probably meet with limited success and empirical crop production functions will continue to be rather rough summaries of complex productive processes. Similarly, because the levels of many factors of production, e.g., temperature, rainfall, etc., are not known (for a particular season), the outcome (crop yield) of any strategy (fertilization rate) is unknown when decisions are made. This is true even if the farm manager knows the true production function for the particular fertilizer-crop combination. A farm manager using this true production function must guess what weather will prevail during the coming season and fertilize accordingly. For example, he may guess that weather will be "average." If other than "average" weather prevails during the season, he will have either overfertilized or underfertilized. Finally, because of fluctuations in weather and general economic conditions, crop prices vary considerably and are generally unknown when production strategies are formulated. Thus, price uncertainty is similar to imperfect knowledge of some factor levels in that farm managers base plans on one crop price and typically receive another. In summary, uncertainty--fostered by imperfect knowledge of production functions, factor levels, and crop prices, and characterized by an often large difference between expected and realized outcomes--is the environment in which production decisions are usually made.

The harshness of uncertainty is lessened by the fact that some of the determinants of total yield (Y) and its value (total revenue) do not affect a variable factor's addition to the firm's net returns, and therefore, do not influence the choice of a production strategy. The price level does not affect the optimum production strategy. Rather, /see equations (11) and (12)/ the optimum level of a factor is determined by the ratios between factor prices and between product and factor prices. Similarly, the production function does not need to account for the effects of factors that only shift the level of the function, i.e., the level of output. Account must be taken of factors other than the decision factors only if they affect the productivity of the decision factor.[8] That is, the level of X_j affects the optimum amount of X_i only if

$$\frac{\delta Y}{\delta X_i} = f(X_j) \tag{13}$$

Goals in Imperfect Knowledge Situations

Decision making in imperfect knowledge situations is further complicated because different managerial goals /levels of NR/ may seem both feasible and desirable to different managers. Under perfect knowledge, production strategies affect only the level of NR. However, in imperfect knowledge situations, production strategies may also affect the level of other Z_i in equation (1). For example, income stability may be an important determinant of utility for some farm managers. Since production strategies maximizing NR frequently give less than minimum income variability, some farm managers might maximize utility with a strategy giving greater income stability and less than maximum NR. Since greater NR does not always mean greater utility, maximum NR is not always associated with maximum or approximately maximum utility. Thus, in the imperfect knowledge situation the farm manager appraises a production strategy's effect on other Z_i as well as its effect on NR and chooses a strategy giving net returns, not necessarily maximum, that approximately maximize utility or are associated with maximum utility.

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[8] See Chapter IV, pp. 47-48 for a discussion of factors that may and may not affect the productivity of a variable factor.

Walker (1, p. 44-49) points out that a farmer's definition of utility (and maximum utility) is determined by a number of factors, e.g., his value system, psychological traits, and resource situation. A farm manager's attitude toward indebtedness and his view of the importance of conservation are two examples of value judgments that may affect the utility he derives from a particular outcome. His attitudes toward risk and his work preferences (including his desire for leisure time) are psychological traits affecting his definition of utility. Land tenure and equity are resource situations that help define utility. Furthermore, these factors are often affected by the state of other factors; that is, they interact. For example, all farm managers desire a degree of financial security. The desired degree is a function of the farmer's age, attitude toward risk, family position, equity position, and many other variables. Hence, a given course of action and resulting net returns might promise a satisfactory degree of security to a young farmer with a small equity, and yet be too risky for an older farmer with a larger equity and considerable family responsibilities. The possible losses relative to possible gains would be smaller for the younger farmer. Also, the older farmer, with more family responsibilities and less time to build a new business would naturally see more risk in the situation, and, therefore, seek a more conservative strategy. Because these and many other factors bear more or less critically on the case of each farm manager, many farmer goals are possible. For this dissertation, however, only three of these possible goals will be considered.

Maximize NR (Money Income) Over Time. A farm manager might have this goal if he feels he can withstand the worst possible series of unfavorable outcomes. That is, the farm manager must feel that his tenure, equity position, family responsibilities, etc. are such that he can wait until the actual distribution of outcomes conforms to the distribution of out-combes used as the basis for his decisions. He must be willing to bear the risk that the probability distribution of outcomes he envisions is inaccurate or that an unforseen event will make it invalid. The goal of maximum money income over time is more appropriate for business entities having longer planning spans than many farm firms. However, maximum NR over time will become an increasingly useful goal if the trend to farms with greater resources, particularly corporation farms, continues. A limitation of the goal of maximum NR is that the probability of each outcome must be known; i.e., it assumes a risk situation.

Maximize the Security Level for Each Time Period. The security level for any strategy is defined as the least desirable of the possible outcomes of that strategy. The security level for all strategies is the maximum of these minimums. Thus, the manager seeking the maximum security level would follow the strategy with the greatest minimum level of net returns.[9] This manager views any outcome as critical and is, therefore, willing to accept less than maximum net returns (if maximum net returns can be defined) to achieve greater certainty and stability of income. This could be a desirable objective in several instances. A goal of the maximum security level might be appropriate for a renter if the landlord views income stability as a sign of good management. Also, the renter, less certain of his tenure than an owner, would naturally be more concerned about short

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[9]Only the goal of maximum security level will be considered in this dissertation. A related and quite reasonable goal would be one of maximizing NR (net returns over time) subject to the restriction that every outcome (NR) be greater than or equal to a prescribed amount. This prescribed amount would necessarily have to be possible.

run net returns than the long run distribution of income. A farm manager with sizable and unavoidable periodic outlays might also seek a maximum sure level of income. He may have children in college or regular payments on machinery or additional land. Since he considers only the possible outcomes and not the probability of each outcome, the farm manager does not have to know the entire probability distribution of outcomes to choose a strategy when his goal is the maximum security level. Thus, this is a feasible goal for the farmer who feels that conditions are too dynamic or uncertain and data too limited to predict the distribution of outcomes over any lengthy future period. However, even if the probability distribution of outcomes is known, the manager may still choose to maximize his security level. That is, he may not use some of his information about the probability distribution of outcomes because maximum money income over time is not as important as net returns in each time period. The price of his security-seeking is some loss in net returns over time. Maximizing the security level when it is also possible to maximize net returns over time should be viewed as a conservative policy of management.

Minimize Regret (the cost of making a wrong decision) in Each Time Period. The farm manager who is actually concerned about "what could have been" might act to minimize regret. There is little evidence that farmers are concerned with the magnitude of the cost of making the wrong decision. However, many farmers do make decisions *as if* they are motivated by the desire to minimize regret (1, p. 35). For example, farmers who insure property usually know that they will probably pay more than the value of the property in insurance premiums before the property is lost. That is, the cost of insuring would be greater than the value of the property times the probability that it will be lost. However, they *do* insure, perhaps thinking that if they don't insure and the property is lost, their regret will be the difference between the value of the lost property and the insurance premium. If they do insure and the property isn't lost, their regret will be the value of the insurance premium, much less than the value of the property. Thus, insuring is behavior suggesting a goal of minimum regret. Regret minimizing strategies may be determined with incomplete knowledge of the probability distribution of outcomes. Thus, minimum regret is an attainable goal in uncertainty situations. If the probability distribution of outcomes is known, minimum regret becomes a rather conservative goal, i.e., some possible net returns over time are foregone. These lost returns are the cost of minimizing regret when it is possible to maximize net returns.

Choice Criteria for Imperfect Knowledge Situations

How does a farm manager choose strategies that best attain the above goals? Decision making criteria for the two imperfect knowledge situations, risk and uncertainty, will now be considered.

Risk. In the risk situation the farm manager knows or believes that he knows all possible values of all variables determining outcome (NR) and their respective probabilities. If the possible values of each determinant are independent of the values of the other determinants, the manager sees as many possible outcomes as there are different combinations of the values of the determinants. Furthermore, because the farm manager knows the probability distribution of values of the variables, hence outcomes, he is able to specify a unique value for NR over time.

When the probability distribution of outcomes is known, i.e., in a risk situation, the farm manager may still have a goal other than maximum

NR over time (see pages 20-23 above). However, in seeking a goal other than maximum NR over time, he doesn't use all available information about the probability distribution of outcomes. That is, he behaves as if an uncertainty situation exists. Only the choice criterion for maximum NR over time will be given in this section. The choice criteria for maximizing the security level or minimizing regret are the same for both the risk and uncertainty situations. They will be given in the section on uncertainty.

In the perfect knowledge situation, the levels of all factors of production could either be controlled or predicted. In the risk situation the level of one (or more) factor(s) of production cannot be controlled. It occurs randomly and according to a known probability distribution. Thus, the level of such a factor cannot be predicted for any instance; but, its expected value may be determined. For this section, X_2 in equation (5) will be considered the uncontrollable factor (P_{x_2} is zero). Equation (5) still gives NR for any particular values of X_1, X_2, P_{x_1}, and P_y. However, expected net returns over time are

$$E(NR) = E\left[P_y f(X_1, X_2) - P_{x_1} K X_1\right] \tag{14}$$

Since price uncertainty is not now being considered, P_y, P_{x_1}, and X_1 are known. Thus, equation (14) becomes

$$E(NR) = P_y\, E\left[f(X_1, X_2)\right] - P_{x_1} K X_1 \tag{15}$$

where

$$E\left[f(X_1, X_2)\right] = \sum_{j=1}^{m} P_j\, f(X_1, X_2)_j \tag{16}$$

and j = 1, 2, . . ., m are the different levels of the production function corresponding to the j possible levels of the uncontrollable factor, X_2.

P_j is the probability of the j^{th} value of X_2 and

$$\sum_{j=1}^{m} P_j = 1 \tag{17}$$

Since the level of X_1 is subject to managerial control, the probability of any value of X_1 is one. Thus, the probability of $f(X_1, X_2)_j$ is also P_j.[10] Substituting equation (16) into equation (15) gives

$$E(NR) = P_y \left[\sum_{j=1}^{m} P_j \; f(X_1, X_2)_j \right] - P_{x_1} K X_1 \tag{18}$$

E(NR) is maximum if X_1 is used so that[11]

$$\frac{\delta E(NR)}{\delta X_1} = P_y \sum_{j=1}^{m} P_j \frac{\delta f(X_1, X_2)_j}{\delta X_1} - P_{x_1} K = 0 \tag{19}$$

or

$$\sum_{j=1}^{m} P_j \frac{\delta f(X_1, X_2)_j}{\delta X_1} = E(MPP_{x_1}) = \frac{P_{x_1} K}{P_y} \tag{20}$$

$E(MPP_{x_1})$ is read expected marginal physical product of X_1.[12]

Equation (20) is the choice criterion (indicator of optimum production strategy) for maximizing expected net returns over time when one factor of production is a random variable with a discrete probability distribution.

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[10] Equation (16) gives $E[f(X_1, X_2)]$ for the case where the probability distribution of X_2 is discrete. If the probability distribution of X_2 is continuous, i.e., if $P_j = g(X_2)$, then

$$E(Y) = E[f(X_1, X_2)] = \int_{-\infty}^{\infty} f(X_1, X_2)\, g(X_2)\, dX_2.$$

[11] These are first order or necessary conditions only. Sufficient conditions, similar to those given in footnote 7, could be given. However, they need not be considered because all empirical production surfaces obtained in this study are concave.

[12] The first matrix on page 27 furnishes an example of the process of determining E(NR) for an uncontrollable factor with a discrete probability distribution. In that matrix, j = 2, $P_1 = .4$, and $P_2 = .6$.

Uncertainty. In the uncertainty situation, the farm manager knows or believes he knows all possible values of all factors determining net returns; but, he does not know the distribution of the values of one or more of the factors. Thus, it is impossible to specify a strategy for maximizing net returns over time. Choice criteria for two farmer goals, maximum security level and minimum regret, are given in this section.

The criterion for choosing the strategy that maximizes the security level is called the maximin criterion. A farm manager would follow the strategy indicated by the maximin criterion if he wishes to maximize the minimum possible level of NR. Thus, each outcome (level of NR) of a particular strategy must be considered.

Given some values of P_{x_1}, P_y, and K, suppose that X_{11} and X_{12} are the farm manager's alternative strategies (levels of X_1) and X_{21} and X_{22} are the possible levels of the uncontrollable factor, X_2. Each interaction between an X_{1_i} and an X_{2_j} results in an outcome, NR_{ij}. This information may be arrayed in a matrix[13] such as the following:

	X_{21}	X_{22}
X_{11}	3	4
X_{12}	2	6

[13]This matrix (as well as the production functions used in the perfect knowledge and risk situations considered previously) implies a unique outcome (NR_{ij}) for each production strategy-weather condition combination; i.e., only one possible entry in each cell of the above matrix. However, there would typically be several (a distribution of) outcomes for each production strategy-weather condition combination. The statistical techniques used to estimate outcomes allow interval estimates of outcomes (at a probability level) as well as point estimates. In this dissertation, only the point estimate is used. Additionally, outcomes (NR_{ij}'s) are assumed to be statistically different at an acceptable probability level, i.e., the confidence limits on the outcomes do not overlap. Luce and Raiffa (4, pp. 309-324) suggest similar techniques for resolving the complex of experimental and decision problems.

By the maximin criterion, X_{11} is the optimum pure strategy for this example. That is, X_{11} gives the farmer the greatest security level, 3. A mixed strategy (a combination of X_{11} and X_{12}) will not yield a greater security level because 3, the minimum in row X_{11} is also the maximum in column X_{21}.

If the probability distribution of outcomes is known, i.e., if X_{21} and X_{22} will each occur with a known probability, the farm manager may still seek the maximum security level (see p. 22). For example, if the probabilities of X_{21} and X_{22} are .4 and .6, respectively, the previous matrix becomes

	X_{21} (.4)	X_{22}(.6)	E(NR)[14]
X_{11}	3	4	3.6
X_{12}	2	6	4.4

E(NR) is the expected net returns in any time period, i.e., it is the average value of NR when strategy X_{1_i} is followed long enough for the realized distribution of outcomes to equal the probability distribution of outcomes. Thus, in this example, a farm manager loses .8 units of NR, in each time period, <u>on the average</u>, by maximizing security (using strategy X_{11}) rather than maximizing NR (using strategy X_{12}).

A second situation is presented in the outcome matrix

	X_{21}	X_{22}
X_{11}	5	3
X_{12}	2	6

In this example, X_{11} is the optimum pure strategy by the maximin criterion. However, because the minimum in row X_{11}, 3, is not the maximum in column X_{22}, a mixed strategy of $P_1 X_{11} + P_2 X_{12}$, if possible, will give a higher

[14]Entries in this column are obtained by the operation described by equation (18).

security level. The values of P_1 and P_2 may be determined as follows:

If X_{21} occurs,

$$NR = 5P_1 + 2P_2 \quad (21)$$

If X_{22} occurs,

$$NR = 3P_1 + 6P_2 \quad (22)$$

Simultaneous solution of (22), (23), and

$$P_1 + P_2 = 1 \quad (23)$$

gives $P_1 = 2/3$ and $P_2 = 1/3$. Using X_{11} and X_{12} in these proportions gives NR = 4 whether X_{21} or X_{22} prevails. Thus, the manager is able to attain a higher security level, 4 vs. 3, if he can follow a mixed strategy ($2/3 X_{11} + 1/3 X_{12}$) rather than a pure strategy (X_{11}).

Again, a manager may elect to maximize security even though the probabilities of X_{21} and X_{22} occuring are known. Thus, if X_{21} and X_{22} have probabilities .4 and .6, the above matrix becomes

	X_{21} (.4)	X_{22} (.6)	E(NR)
X_{11}	5	3	3.8
X_{12}	2	6	4.4
$2/3 X_{11} + 1/3 X_{12}$	4	4	4

The cost (in foregone net returns in each time period) of maximizing the security level rather than NR is .6 for the pure strategy of X_{11} and .4 for the mixed strategy of $2/3 X_{11} + 1/3 X_{12}$.

The preceding examples define and explain the use of the maximin criterion.[15] In Chapter V, the maximin criterion is used to evaluate alternatives predicted from the findings of this study.

[15] The reader may refer to Walker (1, especially pp. 24-34) for a more complete discussion of the use of the maximin criterion as a choice guide in farm decision making.

The criterion that indicates the optimum strategy for minimizing regret will be called the regret criterion. If the maximum NR_{ij} for each set of conditions, X_{2_j}, is subtracted from each of the other NR_{ij}'s for conditions X_{2_j}, a regret matrix is formed. For example, given the outcome matrix

	X_{21}	X_{22}
X_{11}	3	4
X_{12}	2	6

a regret matrix

	X_{21}	X_{22}
X_{11}	0	-2
X_{12}	-1	0

may be formed. The negative elements (R_{ij}) represent the cost of having followed the wrong strategy for the realized X_{2_j}. The larger the negative value (or absolute value) of an R_{ij} the more the farmer regrets having followed strategy X_{1_i} when conditions X_{2_j} are realized. If $R_{ij} = 0$, the farmer has no regret; he has followed the strategy giving the highest NR_{ij} (lowest regret or R_{ij}) for the realized conditions X_{2_j}. For this example, X_{12} is the optimum pure strategy.[16] But, when the regret criterion is used, a mixed strategy (if possible) will always be preferred to a pure strategy because the maximum regret in the row will never be the minimum

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[16] A serious criticism of the regret criterion (more easily seen in Chapter V, p. 72) is that adding or deleting an undesirable alternative may change the optimum strategy. For example, if A_2 is preferred (by the regret criterion) among alternatives A_1, A_2, A_3, and A_4, the deletion of an undesirable alternative, say A_4, will often make another strategy, A_1 or A_3, optimum. Luce and Raiffa (4, pp. 280-282) briefly and effectively discuss the regret criterion and some of its shortcomings.

regret in the column. Thus, in this example, a mixed strategy of 1/3 X_{11} + 2/3 X_{12} gives a regret level of -2/3, less in absolute value than -1, the regret level for the pure strategy X_{12}. Notice that the regret minimizing strategies (pure and mixed) are different from the strategy for maximizing the security level (see p.27). Again, if the probabilities of each outcome are known, the manager may still rationally minimize regret. For this example, the pure strategy for minimizing regret, X_{12}, also maximizes expected net returns (see p. 27). However, expected net returns for the mixed strategy, 4.1, are less than maximum possible expected net returns, 4.4.

This completes the development of choice guides for the knowledge-goal situations considered in this study. In Chapter V, these guides are applied to empirical information to determine economic optima.

CHAPTER III

TECHNIQUES FOR AND CONSIDERATIONS IN OBTAINING ESTIMATES OF PRODUCTION FUNCTIONS AND PRICES

An objective of this study, stated in Chapter I, is to increase the quality of available (for use by Oklahoma farmers) information about technical production relationships and prices. Chapters I and II show why this information is critical and how it is used in the decision making process. This chapter states the procedures used in this study to make available price and input-output data more useful to farm managers. The procedure used to estimate the several fertilizer-crop production functions will be discussed first. Price and weather data sources will then be briefly discussed.

Estimating the Production Function

In Chapter I, the general form of the production function was given as $Y = f(X_1, X_2, \ldots, X_n)$. This section states some considerations in and the procedure for statistically quantifying this general relationship. Stated differently, this section is a discussion of the means of obtaining an empirical statement of a production function, given satisfactory empirical data. A sequential discussion of (1) the data, (2) considerations in choosing the weather variables, (3) considerations in choosing a functional form for the production function, (4) the statistical procedure, and (5) considerations in evaluating empirical production functions follows.

The Data

The raw material for the estimating procedure is experiment station results giving observed crop yields with seed and/or fertilizer at various levels. In some cases, weather is considered a constant. In others, an effort is made to quantify the effects of weather on yields. Therefore, the most complicated case is one in which Y (yields) is a function of three variable factors (weather, fertilizer, and seed) and an undefined number of fixed factors. That is,

$$Y_{ijk} = f(X_{1_i}, X_{2_j}, X_{3_k} \mid X_4, X_5, \ldots, X_n) \qquad (1)$$

where $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, s$, and $k = 1, 2, \ldots, t$ are the different levels of each factor. Using this general notation, each observation in the experiment station data is a Y_{ijk} corresponding to a particular level of each factor, namely, X_{1_i}, X_{2_j}, X_{3_k}.

The data described above must meet certain criteria to be useful in estimating a crop production function. Specifically, data must make it possible to

(1) estimate the effect on yield of varying the level of each variable factor. This requires, at least, a partially factorial design, i.e., observations of the effects of several different levels of one factor when the other factors are also variable over some, preferably large, range. For example, a partially factorial design with respect to X_1 might have observations designated by Y_{111}, Y_{134}, Y_{156}, Y_{312}, Y_{334}, Y_{366}, etc. A completly factorial design would have all possible combinations of the levels of each variable.

(2) estimate the effect on yield of varying the level of each factor over the full range in which there is interest. The range of interest is

basically that range of factor levels in which the marginal productivity of each variable factor is decreasing.

(3) name the "fixed" factors, $X_4 . . ., X_n$, and to specify, to an extent, the effect on yield and variable factor productivities of different levels of these fixed factors.

(4) measure factors accurately. This is particularly important for weather variables.

These criteria are used in Chapter IV to gauge the adequacy of data used in this study.

Considerations in Choosing the Weather Variables

There is little doubt that weather is a critical determinant of yields and factor productivities for most crops. However, it is often difficult to include a weather variable in empirical production functions for several reasons. First, response data seldom cover a sufficient number of years; i.e., observations of the effects of weather are not available for the full range in which there is interest (see criterion 2 above). If observations do cover an adequate range, the usefulness of observations and subsequent estimates is often limited by inadequate measurement of the weather factor. For example, weather records do not distinguish between rainfall that runs off and rainfall that becomes available for plant growth; yet run off is essentially useless for plant growth. Similarly, small amounts of rainfall that evaporate before penetrating to root depth bias available rainfall records. Finally, weather records (rainfall, temperature, etc.) are seldom available for the exact location of the experiment.

Despite these shortcomings, an effort may logically be made to quantity the effects of weather on crop yields and the productivity of a decision factor if there is

(1) substantial correlation between the levels of the weather variable and yields, and

(2) a physiological basis for linking the weather variable and yields as cause and effect.

The second condition is critical only if there is interest in explaining the variable factor's effect on yield. If the primary objective of the study is to predict the effect on yields of different levels of the variable factor, the first condition is sufficient. Prediction is the primary objective of this study; therefore, weather variables that are highly correlated with yields are used. However, an effort is made to establish physiological support for using the chosen weather variables.

Considerations in Choosing a Functional Form for the Production Function

A quadratic function of the general form (for the three variable case)

$$\hat{Y}_{ijk} = b_o + b_1 X_{1_i} + b_2 X_{1_i}^2 + b_3 X_{2_j} + b_4 X_{2_j}^2 + b_5 X_{3_k} + b_6 X_{3_k}^2 + \text{Interaction Terms} \quad (2)$$

where $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, s$, and $k = 1, 2, \ldots, t$ are the levels of factors X_1, X_2, and X_3, respectively, was used in this study. It was chosen for the following reasons; or, alternatively, it met the following criteria:

(1) It allows the diminishing marginal productivity prescribed as a condition for a unique solution to the problem of maximizing NR. In addition to this mathematical necessity for diminishing marginal productivity, physiological considerations demand eventually decreasing returns from the addition of fertilizer or seed to a given amount of land (and/or other fixed factors).

(2) It allows some production with no inputs of the variable factors. This is reasonable in the case of fertilizer; but, it must be ignored in the case of seed.

(3) It allows interaction among variables without requiring it (as Cobb-Douglas equations do, for example).

(4) The effects of the variables are additive (the equation is linear in its parameters), a necessary condition if linear regression techniques are to be used.

(5) Plots of the observations conform fairly well to the functional form.

Figure 1 illustrates well the importance of this last point. The function $\hat{Y} = f(X_1)$ gives estimates of Y differing considerably from observed levels of Y <u>for values of X_1 well within the range of the data</u> primarily because the functional form does not fit the data. That is, a linear relationship does not explain the data well. Thus, the functional form[1] has a substantial effect on the "goodness of fit" of the function to the data and the accuracy of subsequent predictions.

The Statistical Procedure

The technique of least squares regression has been used to fit the chosen (quadratic) functional form to the data described in the first section of this chapter. This technique[2] estimates the parameters or constants (b_i) of equation (2) so that $\sum_{ijk=1}^{rst} (Y_{ijk} - \hat{Y}_{ijk})^2$, sum of

[1]Plaxico et. al. (6) and Baum et. al. (7, pp. 76-96) briefly but effectively discuss the problem of choosing a functional form.

[2]An IBM 650 computer was used to run the regression analyses of this study. The program used: Correlation Analysis with Annotated Output, Parts I, II, III; IBM 650 Library Program File Numbers 6.0.014, 6.0.032, and 6.0.037, respectively.

squares of the deviations, is minimized. Y_{ijk} is the sample (observed) yield and $\hat{Y}_{ijk}$ is the predicted yield for factor inputs of X_{1_i}, X_{2_j}, X_{3_k}.[3]

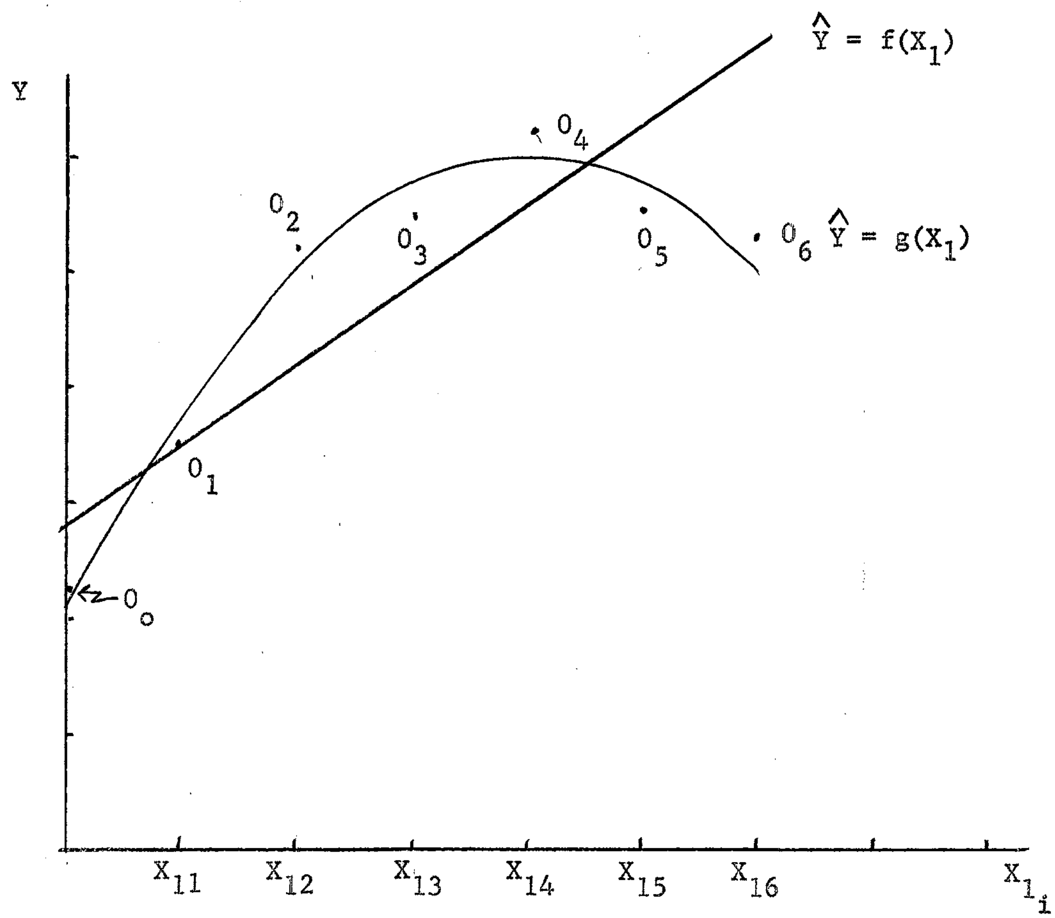

Figure 1. Linear and Parabolic Functional Forms "Fitted" to the Same Set of Data (O_i's).

Considerations in Evaluating the Empirical Production Function

An empirical production function is less valid when any of the conditions prescribed in the previous sections are not met by the data, variables, functional form, etc. This error is wholly apart from the error arising because the entire process, at best, gives only an estimate of the relationship between yield and its determinants. Another, more subtle, source of error (misinterpretation of results) arises because the empirical

[3]See Ostle (8, p. 117 ff) for a more complete explanation of the least squares regression technique.

production function directly accounts for interaction among the variable factors (via the interaction terms), and implicitly accounts for interaction between fixed factors and the variable factors (via the b_i's of equation 2). Thus, the production function quantifies the effects of the fixed factors on the marginal productivity of the variable factors for one particular level of the fixed factors. If one or more of these effects (interactions) change as the level of the fixed factors change; that is, if

$$\frac{\delta Y}{\delta X_v} = f(X_f) \quad (3)$$

for some variable factor(s) X_v and fixed factor(s) X_f, then the production function has no application to situations where the level of the fixed factor (X_f) is different from its level in the experiment. If, however,

$$\frac{\delta Y}{\delta X_v} \neq f(X_f) \quad (4)$$

the production function is applicable to situations where X_f is fixed at other than the experimental levels.[4] In this case, the prescribed level of X_v will be the same, and correct. However, the predicted total product (and total revenue) from the use of X_v at its optimum level may be incorrect. Recall (Chapter I) that the absolute level of production affects the decision to produce, but not the optimum factor use level.

The coefficient of determination (R^2) and a t-test of the significance of its parameters are the usual statistical tests of the validity of an empirical production function. The coefficient of determination gives a measure of the amount of the total variation in the input-output data explained by the empirical production function.[5] The t-test of the

[4]See Chapter IV, pp. 47-48 for a discussion of fixed factors which may and may not affect the productivity of fertilizer.

[5]See Ostle (8, p. 174 ff) for explanation of derivation and use of R^2.

significance of each parameter of the fitted equation allows us to decide, at a certain confidence level, whether the parameter is essential to the explanation of variation given by the production function.[6] For example, fitting a quadratic function to a set of input-output data might give the production function

$$\hat{Y} = 3 + \underset{P<.05}{.4X} \; \underset{P<.05}{-.1X^2} \qquad R^2 = .96 \tag{5}$$

The value of R^2 (.96) tells us that 96 per cent of the variation in the sample (original data) has been explained by the equation. $P < .05$ applies to the regression coefficients of X and X^2, .4 and -.1, respectively. It tells us that the probability is less than .05 that the regression coefficients are zero. Note that these are tests of the empirical production functions' ability to explain relationships implied by a given set of input-output data. They are not tests of the function's ability to explain the <u>actual</u> input-output relationship. Thus, the original input-output data is once again seen to be the critical determinant of the validity of the empirical production function.

It is possible to have the parameters of the production function significantly different from zero (at a probability level) and yet have the marginal physical product (MPP) of a variable in the equation not significantly different from zero (at the same probability level). A method of estimating the variance of the marginal physical product[7] is presented

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[6]See Ostle (8, p. 122 ff) for explanation of t-test of the significance of the regression coefficient.

[7]The term variance of the marginal physical product, as used in Doll's article and this dissertation, refers only to the variance of the estimate. However, in some of the empirical production functions obtained in this study, the marginal physical product also has variance attributable to weather fluctuations. Confidence limits on the marginal physical product function are determined in Chapter V without considering variation due to weather. That is, confidence limits are determined for a particular (the expected) value of weather.

in an article by Doll et. al. (7, p. 596 ff). An estimate of the variance of the marginal physical product makes it possible to

(1) test (at a probability level) the hypothesis that the marginal physical product of X is positive for some range of inputs, and

(2) set confidence limits on the estimate of the expected marginal physical product derived from the production function.

A graph of the marginal physical product function and its confidence limits (Figure 2) clearly shows the range of price ratios $\frac{P_{x_i} K}{P_y}$ for which the MPP of X_1 is different from zero and use of X_1 is profitable. Thus, given an expected price ratio (e.g., PR_1), the farm manager may specify a range of economically optimum levels of factor use (X_{11} to X_{12} in Figure 2).

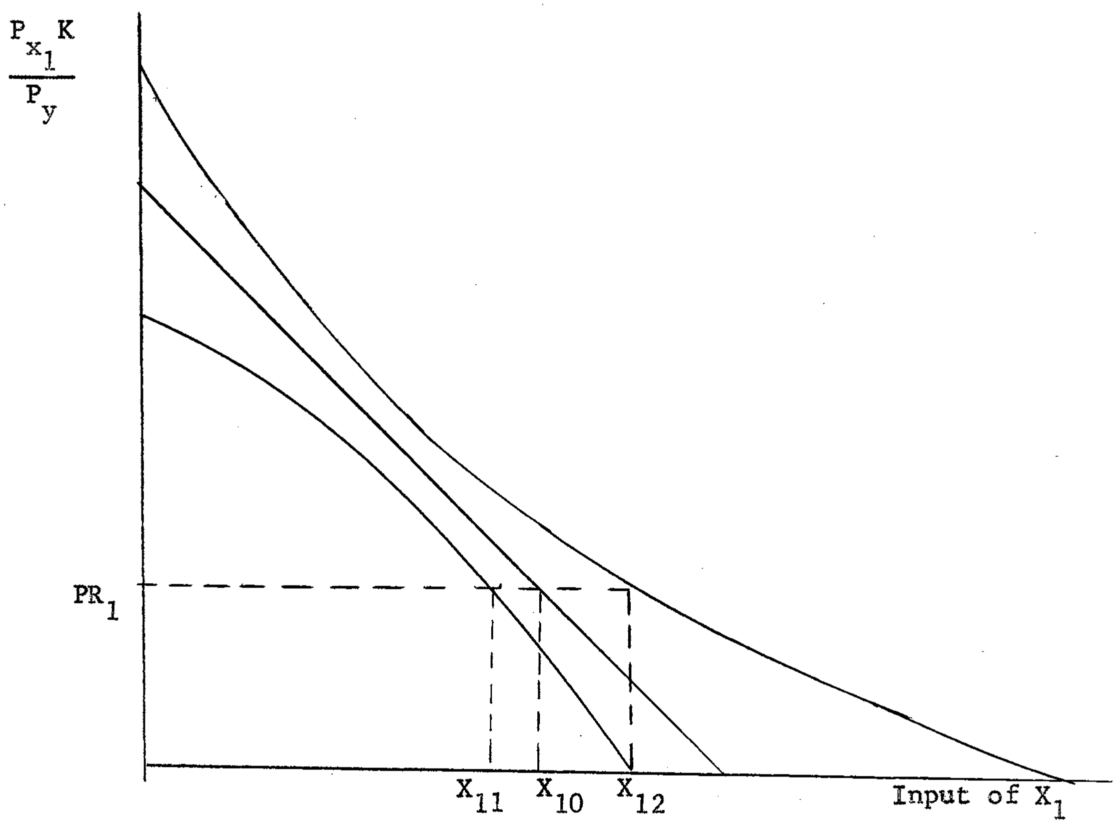

Figure 2. Use of Confidence Limits on the MPP Function to Determine a Range of Economically Optimum Factor Levels

Price and Weather Data Sources

Weather Data

The weather data are U. S. Weather Bureau observations for the several fertilizer experiment sites. It is assumed that (1) the range and (2) the frequency distribution of future weather will be the same as they have been during the period for which records have been kept. With this assumption it is possible to classify weather into several discrete levels (L_1, L_2, . . ., L_n) and attach probabilities to each level. If P_{L_i} is the probability of weather condition L_i, then from assumption (2)

$$\sum_{i=1}^{n} P_{L_i} = 1 \tag{6}$$

Weather data are available for periods of 45-65 years depending on the location of the experiment (see Appendix Table II). It is likely that the observed range of weather conditions does not cover all possibilities. However, the chance is small that weather conditions different from those observed will occur in any particular year.

Price Data

The price data used in this study are the 1955-60 Oklahoma net farm prices for factors and products. Price data are processed only to the extent that modal (or most likely) and limiting factor-factor and factor-product price ratios are established for each crop. This is because the primary purpose of this study is to estimate production functions. The distinction between net and gross farm prices is important. For example, if the cost of harvesting the additional yield is ignored, fertilization of cotton is extremely profitable. When harvesting costs are considered, the profitability of fertilizing cotton is questionable.

This concludes the general discussion of the sources of and methods of processing the several classes of data used in this study. Empirical production functions obtained from input-output data are presented in the next chapter. These production functions and price and weather data are used in the economic analysis of fertilizer use given in Chapter V.

CHAPTER IV

EMPIRICAL RESULTS

The estimating procedure outlined in Chapter III has been used to derive empirical production functions for several Oklahoma crops: spinach, snapbeans, cotton, corn, oats, and wheat. Table I gives a summary of the decision factors and weather variables considered for each crop. Asterisks mark the variables having a statistically significant effect on crop yields (based on the particular data used). Table I should give the reader a helpful, though perfunctory, introduction to the empirical scope of this study. Empirical results (estimates of crop production functions) and a discussion of their validity form the body of this chapter. Only those equations with a decision factor significantly affecting yield are discussed. However, it is useful to note the significance of each factor. For example, the equation for the Perkins cotton data will not be included in the following commentary. But, it is worth noting that temperature affects yields while fertilizer does not. Also, a temperature variable is seen to be valid for two geographical areas in one case (Lone Grove and Perkins cotton) and invalid for different geographical areas in another case (Stillwater and Miami wheat).

The validity of a least squares estimate of an input-output equation (production function) is critically affected by (see Chapter III)

(1) the raw data on which the estimate is based (the input-output data used in this study are given in Appendix Tables III to IX).

TABLE I

CROPS, DECISION FACTORS, AND WEATHER VARIABLES CONSIDERED IN THIS STUDY

Crop (1)	Location (2)	Decision Factors (3)	(4)	(5)	(6)	Weather Variables (7)	(8)
Fall Spinach	Bixby Oklahoma	Nitrogen*	Seed*				
Fall Snap Beans	"	Nitrogen	Seed*				
Spring Spinach	"	Nitrogen	Seed				
Spring Snap Beans	"	Nitrogen*	Seed*				
Cotton	Lone Grove	4-12-4*	Nitrogen	Phos- phorous*	Potassium	Av. max. Aug. Temp.*	
Cotton	Perkins	4-12-4				Av. max. Aug. Temp.*	
Irrigated Corn	Muskogee	Nitrogen*				Av. June Temp.*	
Oats	Stillwater	Nitrogen*				Aug.-Nov. Rain- fall*	
Wheat	Miami	Nitrogen				Av. max. Temp. Feb. 1-Apr. 1 Apr. 25-May 14	
Wheat	Stillwater	Nitrogen*	Phos phorous*			Oct. rainfall*[1]	Av. max. Temp. Feb. 1-Apr. 1 Apr. 25-May 14*

*These variables have a statistically significant effect on crop yields.

[1]Rainfall is not included in the predictive equation chosen because temperature is more highly correlated with yields. Therefore, it should be the better predictor of the two. Coefficients of all of the parameters of equations including both weather variables are not significant at a satisfactory probability level.

(2) the physiological relevance of each independent factor; i.e., is it logical that the variable (rainfall, temperature, fertilizer, seeding rate, etc.) affects yields?

(3) the statistical validity of the estimate as measured by

(a) the amount of the total variation of the sample that is explained by the estimate - R^2,

(b) the test of the significance of each parameter of the equation, and

(c) the test of the significance of the marginal physical product.[1]

These criteria point to each of the following equations as "best" among the several equations defined for each set of input-output data. They also determine the format of the remainder of this chapter. Thus, a discussion (or statement) of

(1) the characteristics of the experiment supplying the raw data for the estimate,

(2) the reasons for choosing the weather[2] variables (or choosing to not include a weather variable),

(3) the regression equation relating input and output and the degree to which it meets the criteria for statistical validity, and

(4) considerations in using the equation as a production function for decision making in farm firms will be given for each equation.

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[1]The functional form fitted to the data was also listed as a determinant of value of the estimate. It is not considered here because the same form was fitted to each equation.

[2]Seeding and fertilization rates clearly affect yield; therefore, only weather variables need be examined for physiological relevance.

In summary, the purpose of this chapter is to present the empirical findings of this study and to discuss their value as estimates of fertilizer-crop production functions. Economic considerations do not enter until Chapter V.

Equation 1: Fall Spinach

Characteristics of the Experiment

Location of Trials: Bixby, Oklahoma, Vegetable Research Station

Soil Type: Reinach Silt Loam

Years Covered by Data: Fall, 1958

Cultural Practices: Irrigation was sufficient to give optimum moisture conditions throughout the season. 375 pounds per acre of 0-16-80 were applied prior to planting. Nitrogen was applied at the rate of 100 pounds per week for zero to four weeks depending on the treatment level to be achieved.[3]

Independent Variables in the Regression Equation: Nitrogen and Seed.

Reasons for Not Choosing a Weather Variable

Nitrogen fertilizer and the seeding rate were the only independent variables considered for this data. This is because soil moisture, thought to be the only weather variable critically affecting the productivity of fertilizer, was presumably kept at the optimum level by irrigation. Apart from this physiological consideration, it would be operationally impossible to specify the effects of a weather variable from one observation.

[3] In strictest terms, this means that P_x (price of nitrogen) varies as the rate of use varies. However, the variation is small and will not be considered in the analysis (Chapter V).

The Regression Equation

$$\hat{Y} = 3.3 + \overset{.10}{.008N} - \overset{.06}{.0000204N^2} + \overset{.01}{1.046S} - \overset{.01}{.0721S^2} + \overset{.24}{.000463NS}$$

$$R^2 = .81$$

where

$\hat{Y}$ = tons of spinach per acre

N = pounds of available nitrogen added per acre, $0 \leq N \leq 400$[4]

S = pounds of seed per acre (Hybrid 7, 90 per cent germination), $1.21 \leq S \leq 9.68$[4]

Equation 1 scores well on all of the tests of statistical significance (except the relatively high probability, .24, that the t-value for the coefficient of the nitrogen-seed interaction term could occur for a zero coefficient). The marginal physical products of fertilizer and seed are positive at the 95 per cent confidence level. The range of the two variables in the experimental data includes the full range for which there is economic interest. Thus, in economic analysis, it is not necessary to extrapolate beyond the range of the raw data. This is a requirement for validity of the regression equation (see Chapter III).

Considerations in Using the Equation

Since the raw data cover only one season, the residual effects of the fertilizer and the effects of other climatic variables than soil mositure cannot be determined. Because of irrigation, soil moisture is presumably a controllable variable, held constant at the optimum level. Other climatic variables (than rainfall) are not thought to have a

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[4]The ranges stated for each variable are the ranges covered by the experimental data.

critical effect on spinach yields. Thus, the limited number of observations used in this equation are not a serious shortcoming.

This equation has been determined for a particular level of many factors: soil fertility, management, cultural practices, rainfall, temperature, etc. Some of these "fixed" factors affect the marginal productivity of fertilizer and seed and, thus, the usefulness of this function for decision making. To use this input-output equation in decision making the farm manager must either (1) assume that the levels of the "fixed" factors will be the same for his farm and the season at hand as they were in the experiment, or (2) assume that they do not affect the marginal productivity of fertilizer and seed. Assumption (1) would certainly be groundless. However, a farm manager would have grounds for assuming that many fixed factors, e.g., management, temperature, and possibly cultural practices, have little, if any, effect on the productivity of fertilizer. If he has sufficient irrigation water available, he may very logically assume that soil moisture can be controlled. However, differing soil characteristics may not be handled this easily. Certainly, the marginal productivity of fertilizer would vary as soil characteristics vary. Thus, this equation could be used with considerable confidence for predicting the effects of fertilizer on spinach yields <u>when</u> soil characteristics are substantially the same as they were in the experiment. If a farmer's land differs from that of the experimental plot, he could still use the equation as a guide, but he should probably make some compensating adjustments in his strategy.

Some "fixed" factors--management, cultural practices, and soil conditions--have essentially the same effect on any fertilizer-crop relationship. Time can be saved by stating these effects now and remembering that they are the same (or are assumed to be the same) for fertilizer-crop relationships considered in later portions of this Chapter. Cultural practices and soil conditions probably do have some effect on the marginal productivity of fertilizer. That is, soil texture and fertility, weed control, irrigation, etc., may be expected to influence the change in yield due to a unit change in the amount of fertilizer used. Management probably has little, if any, effect on the marginal productivity of fertilizer. However, management could concievably affect the marginal productivity of fertilizer through its interaction with cultural practices and soil conditions. For example, a farmer's managerial ability influences his selection or use of cultural practices and his attitudes toward his basic productive resource, the soil. In fact, then, the direction and magnitude of the effects of management, soil conditions, and cultural practices on fertilizer productivity may vary considerably. However, for this dissertation, it is assumed (1) that a farm manager can have considerable confidence in these production functions even if the quality of management on his farm is substantially different from that on the experimental farm and (2) that the usefulness of these production functions may be limited if soil conditions and cultural practices on his farm are essentially different from those on the experimental farm. The following discussions of input-output equations will consider only the effects of other "fixed" factors than management, cultural practices, and soil conditions.

Equation 2: Spring Snap Beans

Characteristics of the Experiment

Location of Trials: Stillwell, Oklahoma, Eastern Oklahoma Field Station.

<u>Soil Type</u>: Bodine Cherty Loam.

<u>Years Covered by Data</u>: Spring, 1959 and 1960, i.e., two seasons.

<u>Cultural Practices</u>: Irrigation was sufficient to give optimum soil moisture conditions throughout the season. 300 pounds per acre of 0-20-10 were applied prior to planting.

<u>Independent Variables in the Regression Equation</u>: Nitrogen and Seed.

Reasons for Not Choosing a Weather Variable

This equation is very similar to equation 1, fall spinach, in that all significant climatic variation is presumably eliminated with irrigation.

The Regression Equation

$$\hat{Y} = 3.39 + \overset{.05}{.0258N} - \overset{.05}{.000162N^2} + \overset{.05}{.00145S} - \overset{.10}{.000033S^3}$$

$$R^2 = .79$$

where

$\hat{Y}$ = tons of snap beans per acre

N = pounds of available nitrogen added to each acre, $0 \leq N \leq 133$

S = pounds of seed per acre (90 per cent germination), $13 \leq S \leq 207$

This equation scores quite well on all tests of statistical significance. Equations with a nitrogen-seed interaction term were fitted. However, the coefficient of the interaction term was not significant. Thus, physiological considerations would call for an interaction term that is not statistically evident. The comments about the significance of the marginal physical product of fertilizer and seed in equation 1 (see p. 46) are entirely appropriate for this equation.

Considerations in Using the Equation

The comments that were made about the use of equation 1 (see pp. 46-48) are entirely appropriate for this equation.

Equation 3: Cotton

Characteristics of the Experiment

Location of Trials: Lone Grove, Oklahoma

Soil Type: Durant Loam

Years Covered by Data: 1930-45, i.e., 16 weather observations.

Independent Variables in the Regression Equation: Pounds of 4-12-4 per acre and average of the daily maximum temperature in August.

Reasons for Choosing the Weather Variable

If a given variable (in this case, weather) is to be included in the input-output equation (see p. 33, Chapter III),

(1) there must be statistical evidence (correlation) of its effect on yield, and

(2) there should be a physiological basis for inferring that the variable effects yield.

Harper (9, p. 18) states "A study /regression/ of May, June, July, and August rainfall and the average maximum temperatures for these months on cotton production revealed that August temperature was the best single indicator for cotton yields." Thus, the weather variable, average maximum August temperature meets condition (1) above. The correlation between temperature and yields is negative, i.e., the higher the temperature, the lower the yields from any level of fertilization. This is reasonable from a physiological standpoint since plants require more moisture with higher temperatures and moisture conditions steadily deteriorate as temperature increases.

The Regression Equation

$$\hat{Y} = 6361.29 + \overset{.05}{2.29F} - \overset{.10}{.0004F^2} - \overset{.05}{59.64T} - \overset{.07}{.000185FT^2}$$

$$R^2 = .61$$

where

$\hat{Y}$ = pounds of seed cotton per acre

F = pounds of 4-12-4 per acre,[5] $0 \leq F \leq 1000$

T = average daily maximum August temperature, $90.5 \leq T \leq 102.6$

The t-test of the parameters and the R^2 for this equation are significant. Experimental observations cover the full range of the two variables in which there is economic interest. However, the marginal physical product is not significantly different from zero at the 95 per cent confidence level for any level of fertilization. This means that a manager cannot be confident (at the 95 per cent level) that the productivity of fertilizer is positive.

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[5] Experiments with a constant ratio of plant nutrients (such as this experiment) do not allow estimation of the substitutability of one plant nutrient for another. However, a number of combinations of plant nutrients may generally be used to obtain a particular yield; i.e., plant nutrients are usually substitutable (over a range). Farm managers need knowledge of substitutability to determine the least cost combination of factors for a chosen output. This criticism of "fixed factor ratio" experiments would be valid even if the ratio used is technologically efficient for all levels of the composite factors. However, it is unlikely that this is true. More likely, the interaction of the several factors (nutrients) will be such that some levels of yield could be obtained from smaller physical amounts of the several factors. Thus, "fixed factor ratio" experiments

(1) do not supply information enabling farm managers to substitute factors (when possible) to obtain a least cost combination for a chosen output, and

(2) usually do not furnish estimates of factor productivities based on technologically efficient proportions of the factors.

Considerations in Using the Equation

Equation 3 may be viewed as a "good" estimate of the fertilizer-cotton production for several reasons. First, some of the effects of weather are specified. Second, observations show the effects of fertilizer and temperature on cotton yields for the full range of fertilizer and temperature in which there is interest. Third, there is a relatively large number of observations. The farmer using this equation would have the same problems with regard to management, cultural practices, and soil conditions as were outlined for equation 1. The fact that the marginal physical product is not significantly different from zero at the 95 per cent probability level may or may not be important. If a manager requires a certain amount of statistical assurance that fertilizer increases cotton yields, he may decide from this equation that no level of 4-12-4 on cotton is profitable. On the other hand, because this estimate of the marginal physical product of 4-12-4 is a maximum likelihood estimate, another manager may decide from this equation that some levels of 4-12-4 on cotton are profitable.[6]

Equation 4: Cotton

Characteristics of the Experiment

<u>Location of Trials</u>: Lone Grove, Oklahoma

<u>Soil Type</u>: Durant Loam

<u>Years Covered by Data</u>: 1931-45; 15 weather observations

<u>Cultural Practices</u>: A fixed level of available nitrogen and potassium (24 pounds per acre) was applied to each plot in each year.

<u>Independent Variables in the Regression Equation</u>: Phosphorous and the average daily maximum temperature in August.

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[6] These points are discussed more fully on pp. 78-81 of Chapter V.

Reasons for Choosing the Weather Variable

The reasons given (see p. 50) for choosing the temperature variable for equation 3 are also entirely appropriate for this equation.

The Regression Equation

$$\hat{Y} = 3333 + \overset{.018}{29.00P} - \overset{.05}{.0747P^2} - \overset{.012}{29.037T} - \overset{.06}{.00218PT^2}$$

$$R^2 = .46$$

where

$\hat{Y}$ = pounds of seed cotton per acre

P = pounds of available P_2O_5 per acre, $0 \leq P \leq 80$

T = average daily maximum temperature for August, $90.5 \leq T \leq 102.6$

The t values and R^2 for this equation are significant. Also, the marginal physical product of phosphorous is positive at the 95 per cent confidence level for a range of phosphorous levels. Observations of the two variables cover the full range in which there is interest.

Considerations in Using the Equation

The comments on the use of equation 3 (p. 52) are appropriate for this equation (except the remarks about the failure of the marginal physical product of fertilizer to be significantly different from zero at the 95 per cent confidence level).

Equation 5: Corn

Characteristics of the Experiment

Location of Trials: Muskogee, Oklahoma

Soil Type: McLain Silt Loam

Years Covered by Data: 1949-1953; five weather observations.

<u>Cultural Practices</u>: Irrigation supplemented rainfall but was not sufficient to maintain optimum soil moisture conditions throughout the season. 400 pounds of 5-10-10 per acre were used as starter in each season. Treatments were applied about six weeks after planting.

<u>Independent Variables in the Regression Equation</u>: Available nitrogen and average June temperature.

Reasons for Choosing the Weather Variable

Yields at all levels of fertilization are highly correlated with average June temperature. Thus, this weather variable is entirely satisfactory for a predictive equation.

Nitrogen primarily influences the amount of foliage put out by the growing plant. Foliage growth is more rapid during the early part of the growing season--May and June for corn. Thus, higher temperatures in June and, therefore, poorer moisture conditions may be expected to depress the yield response to any level of fertilization.

The Regression Equation

$$\hat{Y} = 44.18 + \overset{.001}{1.986}N - \overset{.005}{.00217}N^2 - \overset{.001}{.0002088}NT^2$$

$$R^2 = .80$$

where

$\hat{Y}$ = bushels of corn per acre

N = pounds of available nitrogen per acre, $0 \leq N \leq 180$

T = average June temperature, $75.6 \leq T \leq 85.4$

The t values and R^2 for this equation are excellent. Furthermore, the marginal physical product of nitrogen is significantly different from zero for a wide range of nitrogen applications. By these tests, equation 5 is considerably better than either equations 3 or 4. However, the data used to estimate this equation are less impressive than the data for the previous two equations. Specifically, this equation is based on only five observations of the weather variable; and, these observations do not cover the full range temperature possibilities--$75.6 \leq T \leq 85.4$ vs. $73.3 \leq T \leq 85.4$.[7]

Considerations in Using the Equation

The most serious shortcoming of this estimate is its empirical basis, i.e., the original response data. The small number of weather observations is probably more critical than the lack of observations over the full range of possible temperatures.

Equation 6: Winter Oats

Characteristics of the Experiment

Location of Trials: Stillwater, Oklahoma

Soil Type: Kirkland Silt Loam

Years Covered by Data: 1953-55; 3 weather observations

Cultural Practices: 40 pounds of P_2O_5 per acre were applied at seeding in 1953.

Independent Variables in the Regression Equation: Available nitrogen and average monthly rainfall for August, September, October, and November.

Reasons for Choosing the Weather Variable

August through November rainfall is very highly correlated with yields, and, therefore, a suitable variable in a predictive equation.

[7]Temperature may be expected to fall outside these limits. However, it is assumed, for this study, that the weather (in this case, temperature) observed over the last 50 to 60 years constitutes the domain of possible weather.

Rainfall for this period is also a physiologically relevant variable since this is the period in which the stand and root systems largely determining later grain and foliage production are established.

The Regression Equation

$$\hat{Y} = -105.61 + \overset{.001}{.89N} - \overset{.001}{.00145N^2} + \overset{.001}{136.26R} - \overset{.001}{31.55R^2} - \overset{.005}{.725NR} + \overset{.001}{.1934NR^2}$$

$$R^2 = .98$$

where

$\hat{Y}$ = bushels of oats per acre

N = pounds of available nitrogen per acre, $0 \leq N \leq 160$

R = average monthly rainfall for August through November,

$1.19 \leq R \leq 2.64$

This equation is excellent by statistical criteria. However, the original data severely limit its usefulness since observed rainfall values do not cover the full range of possible rainfall values, i.e., $1.19 \leq R \leq 6.76$.

Considerations in Using the Equation

Since the value of R is known when nitrogen is applied (February or March), this equation could be used for years when $1.19 \leq R \leq 2.64$. However, use of this equation for years when $R > 2.64$ would involve dangerous extrapolation.

Equation 7: Wheat

Characteristics of the Experiment

<u>Location of Trials</u>: Stillwater, Oklahoma

<u>Soil Type</u>: Kirkland Loam

Years Covered by Data: 1931-1959; 28 weather observations

Independent Variables in the Regression Equation: Pounds of available nitrogen and phosphorous per acre and the average maximum temperature from February 1 to April 1 and April 25 to May 14.

Reasons for Choosing the Weather Variable

The average maximum temperature from February through May is highly correlated (negatively) with yields.

The Regression Equation

$$\hat{Y} = 682.2 + \overset{.001}{.42P} - \overset{.01}{.00829P^2} - \overset{.05}{19.27T} + \overset{.05}{.1385T^2} - \overset{.10}{.000000766NT^2P}$$

$$R^2 = .21$$

where

$\hat{Y}$ = bushels of wheat per acre

P = pounds of P_2O_5 per acre, $0 \leq P \leq 45$

N = pounds of available nitrogen per acre, $0 \leq N \leq 33$

T = average maximum temperature: February 1 to April 1 and April 25 to May 14, $54.3 \leq T \leq 68.4$

Contrary to equations 5 and 6, the response data used in this estimate are excellent. There are 28 weather observations covering the full range of possible weather. The parameters of the estimated equation are significant at a satisfactory probability level. However, R^2, though statistically significant, is low by usual standards. Insect and bird damage, different variables of wheat, and other weather variables contribute some of this unexplained variation.

Considerations in Using the Equation

The low R^2 for this equation means that realized yields are likely to differ considerably from predicted yields. However, the coefficients of the parameters of the equation are significantly different from zero at a satisfactory probability level. Thus, the estimates of economically optimum fertilization rates may be viewed as reliable. Alternatively stated, farm managers can have considerable confidence in production strategies based on this equation, if they can first, from other information, determine that growing wheat is profitable.

CHAPTER V

ECONOMIC OPTIMA AND THEIR IMPLICATIONS

A brief review of the organization of this thesis seems appropriate at this point. Chapter I defines the problem attacked by this study as one of resource allocation. Specifically, what rate of fertilization most nearly accomplishes managerial goals, given some level of knowledge about product and factor prices and the relationship between fertilizer inputs and crop yields? It lists major objectives of this study as those of (1) obtaining statistical estimates of production functions (including, where possible effects of relevant weather variables) for a number of Oklahoma crops, and (2) using these estimates and price and weather information to determine optimum production strategies for several combinations of knowledge states and managerial goals. Chapter II states choice criteria (indicators of the optimum production strategy) for different managerial goals and knowledge situations. These criteria show why estimates of production functions and prices are needed by decision makers. Chapter III discusses the procedure used to estimate the production functions and the sources of price and weather data. Chapter IV presents the empirical results of the study, i.e., the production functions. Thus, at this point, the first objective of this study--to obtain statistical estimates of production functions for some Oklahoma crops--has been accomplished. Chapter V accomplishes the second major objective. That is, the choice criteria of Chapter II are applied to the empirical results of Chapter IV to decide (1) whether to fertilize and (2) how much to fertilize.

The next section of this Chapter treats the first decision (whether to fertilize). In considering the second decision (how much to fertilize), otpimum production strategies and net returns are determined for different (1) managerial goals, (2) factor-product price ratios, (3) estimates of the production function, and (4) decision making techniques. Considerations in and methods of determining optimum production strategies and computing net returns are discussed in general for each case. Then, the results (the strategies and their net returns) are presented in tables. Net returns from each production strategy are computed using input-output equations developed in this study. Each table also includes entries for comparisons made in later parts of this Chapter and in Chapter VI. Every equation is not included in every table because some input-output equations are not appropriate for some knowledge-goal situations.

The procedure outlined above develops economic optima for several assumed economic environments. That is, optimum fertilization rates and resulting net returns are determined for specific prices, knowledge states, and managerial goals which, very probably, differ substantially from the prices, knowledge states, and goals appropriate for many farm managers. Thus, later sections of this chapter contain useful conclusions (derived from these results or optima) about fertilization of the crops considered in this study. Specifically, interpretations of results are made that

(1) show the effect of managerial goals, prices,[1] factor productivities,

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[1]Factor prices are usually known when fertilizer is applied; therefore, we are primarily interested in the effects of crop price uncertainty and variability. However, since the effects of different capital costs are shown, the influence of factor price variability is shown indirectly.

and knowledge of these determinants on production strategies and net returns.

(2) provide support for some generalizations about the profitability of fertilizing the crops considered in this study.

(3) provide a basis for estimating the economic importance of the problem. That is, do fertilizer productivities substantially affect optimum production strategies and do fertilization rates (production strategies) critically affect farm firm profits? Alternatively, are the improved estimates of fertilizer productivities obtained in this study worth their cost?

Economic Optima (Results)

Before deciding upon the correct rate of fertilization, the farm manager must determine whether fertilizing a crop can possibly yield a net addition to the per acre returns to the other factors used in growing the crop. If fertilizing can yield net additions to the total returns from the crop, then it becomes meaningful to define a production strategy (fertilization rate) that maximizes attainment of managerial goals, given some level of prices, knowledge, and fertilizer productivity. Thus, the next section establishes the profitability of fertilization for the crops considered in this study. Following sections define optimum production strategies for different and specific economic environments.

The Decision to Fertilize

In Chapter I (p. 4) some costs (fixed costs) of fertilizing are said to affect only the decision to fertilize. These costs are essentially the labor and machinery costs of applying fertilizer. Available information[2] indicates that the per acre charge for custom application of fertilizer is less than $3.00. Thus, a fertilization rate would be profitable if it yields net returns greater than $3.00 per acre. For crops and prices considered in this dissertation, net returns from fertilizer are always greater than $3.00 per acre (except nitrogen fertilization of spinach when low crop prices are received--see columns 4, 7, and 10 of Table II). Since fixed costs of fertilizing are small and highly variable (among farms and areas), they have not been deducted from the net returns figures given in the following sections. Rather, the reader may discount net returns entries by what he feels are the appropriate fixed costs of fertilizing. The optimum strategies given are correct for any case where fixed costs are less than net returns. Finally, all entries in Table II are computed for the expected value of the production function.

Optimum Strategies and Net Returns for Different Managerial Goals

To choose a strategy, a manager must have an objective. This section gives the optimum strategies and resulting net returns for three managerial goals: maximum net returns, maximum security level, and minimum regret. The relevance of these goals is examined in Chapter II. Modal prices, K = 1.1, and expected value of the input-output equation are used in each of these examples.

Maximum Net Returns. Two cases are considered under this heading: (1) outcomes do not vary (perfect knowledge) and (2) outcomes vary, but the probability distribution of the outcomes is known (risk). Equation 1, fall spinach, is used as an example of case (1). Given:

[2] "Oklahoma Custom Rates, 1960", Oklahoma Agricultural Extension Service Leaflet L-50.

TABLE II

NET ADDITIONS TO PER ACRE RETURNS TO FIXED FACTORS FROM FERTILIZING AT OPTIMUM RATES FOR DIFFERENT CROP PRICES[1]

	High Crop Prices			Modal Crop Prices			Low Crop Prices		
	Returns to Fixed Factors[2] if Variable Factor		Addition From Use of Variable Factor	Returns to Fixed Factors if Variable Factor		Addition From Use of Variable Factor	Returns to Fixed Factors if Variable Factor		Addition From Use of Variable Factor
Equation Number	Is Not Used	Is Used Optimally[3]		Is Not Used	Is Used Optimally		Is Not Used	Is Used Optimally	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1[4]	380.65	435.89	55.24	211.47	229.91	18.44	119.83	122.46	2.63
2[5]	611.75	729.10	117.35	222.60	267.56	44.96	142.70	169.39	26.69
3	49.21	54.47	5.26	43.74	47.58	3.84	40.56	43.62	3.06
4	43.63	58.59	14.96	38.79	56.62	17.83	35.97	47.39	11.42
5	64.06	128.18	64.12	55.23	107.79	52.56	46.39	87.52	41.13
6[6]	--	16.26	--	--	11.61	--	--	7.13	--
7	40.56	51.45	10.89	33.15	41.59	8.44	32.18	40.29	8.11

[1] Prices used in computing this table are given in Appendix Table I.

[2] Fixed factors here are all factors used in growing the crop except those associated with fertilization; not to be confused with "fixed factors" associated with fertilization.

[3] Optima based on a goal of maximum NR, K = 1.1, expected weather and modal factor prices.

[4] All entries computed for seed at 8 pounds per acre.

[5] Computed for seed at 169 pounds, 93 pounds, and 44 pounds for high, modal, and low crop prices, respectively.

[6] Blank cells occur because equation predicts negative yields when no fertilizer is used.

N = pounds of available nitrogen per acre

S = pounds of seed per acre, held constant at 8 pounds[3]

$\hat{Y}$ = tons of spinach per acre = f(N, S)

$= 3.3 + .008N - .0000204N^2 + 1.046S - .0721S^2 + .000463NS$

$= 3.3 + .008N - .0000204N^2 + 1.046(8) - .0721(64) + .000463(8)N$

P_n = price of a pound of nitrogen = .126

P_y = price of a ton of spinach, net of harvesting costs = $30

K = 1.1

Net returns are /see equation (10), Chapter II/

$$NR = 30[3.3 + .008N - .0000204N^2 + 1.046(8) - .0721(64) + .000463(8)N] - (.126)(1.1)N \quad (1)$$

Net returns are maximum if /see equation (11), Chapter II/

$$\frac{\delta f(N,S)}{\delta N} = \frac{P_n K}{P_y} \quad (2)$$

that is, if

$$.008 - .000408N + .000463(8) = \frac{(.126)(1.1)}{30} \quad (3)$$

Solving (3) for N gives 173 pounds per acre as the optimum strategy (level of N). Substituting 173 for N in equation (1) gives $229.91 as maximum net returns for the conditions given above. Columns 5 and 6 of Table III give these and the similar results for equation 2, snap beans.

Equation 3, cotton, may be used as an example of case (2). Given:

F = pounds of 4-12-4 fertilizer per acre

$\hat{Y}$ = pounds of seed cotton per acre = f(F,T)

$= 6361.29 + 2.29F - .0004F^2 - 59.64T - .000185FT^2$

[3]Seed is held constant in this equation because any recommended changes in seeding rate in response to price changes are too small to be effected. That is, all optimum seeding rates (based on prices observed over last 8 years) are very near 8 pounds per acre.

P_f = price of a pound of 4-12-4 = .018

P_y = price of a pound of seed cotton, net of harvesting and ginning costs = .0729

K = 1.1

$$E(T) = \sum_{j=1}^{m} T_j P_j = 96.6$$

$$E(T^2) = \sum_{j=1}^{m} T_j^2 P_j = 9344.9$$

Expected net returns are [see equation (15), Chapter II]

$$E(NR) = P_y E[f(F,T)] - P_f KF \qquad (4)$$

or

$$E(NR) = .0729\,[E(6361.29 + 2.29F - .0004F^2 - 59.64T - .000185FT^2)] - (.018)(1.1)F \qquad (5)$$

Since the coefficients of the input-output equation $[\bar{Y} = f(F,T)]$ were obtained by least squares regression, they are expected values. F is a controllable (decision) factor; therefore, its expected value is F. Thus, equation (5) may be written

$$E(NR) = .0729\left[6361.29 + 2.29F - .0004F^2 - 59.64\,[E(T)] - .000185\,F[E(T^2)]\right] - (.018)(1.1)F \qquad (6)$$

or, evaluating[4] E(T) and $E(T^2)$,

$$E(NR) = .0729\left[6361.29 + 2.29F - .0004F^2 - 59.64(96.6) - .000185F(9344.9)\right] - (.018)(1.1)F \qquad (7)$$

For this example, expected marginal physical product of F, $E(MPP_F)$, is

$$2.29 - .0008F - .000185F(9344.9) \qquad (8)$$

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[4] See Appendix Table II for weather distributions. Note that $E(T^2)$ is not $[E(T)]^2$ = 9331.6. This point is primarily of academic interest for the estimated production functions in this study. For example, using $[E(T)]^2$ rather than $E(T^2)$ in this example gives 369 pounds of F per acre as the optimum strategy rather than 364 pounds per acre (see next page). However, this point could conceivably be of consequence in other predictive equations. Ref: Unpublished paper by Clark Edwards.

TABLE III

PRODUCTION STRATEGIES AND RESULTING NET RETURNS AND SECURITY LEVELS FOR ALTERNATIVE MANAGERIAL GOALS

Equation (1)	Input (2)	Output (3)	Price Ratio[1], $\frac{P_x K}{P_y}$ (4)	Optimum Production Strategy (Level of X) and Resulting Net Returns and Security Level if Managerial Goal is: Maximum Net Returns — Strategy (5)	Maximum Net Returns — Resulting NR (6)	Maximum Net Returns — Resulting Security Level (7)	Maximum Security Level — Strategy (8)	Maximum Security Level — Resulting NR (9)	Maximum Security Level — Resulting Security Level (10)
1[2,3]	lbs. available N/acre	tons of spinach per acre	.00462	173	229.91	-	-	-	-
2[3]	lbs. available N/acre	tons of snap beans per acre	.00277	71	228.69	-	-	-	-
	lbs. of seed/acre 90% germination	tons of snap beans per acre	.0084	93		-	-	-	-
3	lbs. of 4-12-4 per acre	lbs. of seed cotton per acre	.272	364	47.58	15.72	86	43.35	17.89
4	lbs. of P_2O_5 per acre	lbs. of seed cotton per acre	1.449	48	56.62	29.63	41	55.55	30.63
5	lbs. available N/acre	bushels of corn per acre	.111	139	107.79	18.42	81	98.60	73.17
6[4]	lbs. available N/acre	bushels of oats per acre	.220	176[5]	11.61	-	-	-	-
7	lbs. P_2O_5 per acre	bushels of wheat per acre	.0624	24	41.59	29.38	25[6]	41.39	29.40

(See following page for footnotes).

Footnotes for Table III

[1]All entries are computed for a seeding rate of 8 pounds per acre.

[2]A goal of maximum security level is not appropriate for this equation because perfect knowledge of outcomes is assumed. Because of the variance of the estimate, these equations could just as appropriately be used in the risk model. Thus, the assumption of perfect knowledge for these equations requires the assumption that the variance of the estimate is zero.

[3]All entries are computed for nitrogen at 16 pounds per acre.

[4]The value of the weather variable (average monthly rainfall for August-November) is known when fertilizer is applied (February-March). Thus, a maximin strategy to deal with weather uncertainty is not appropriate for this equation.

[5]Computed for weather variable at its expected value.

[6]The requirement for increased use of fertilizer to attain mazimum security is explained by decreasing total yields and net returns as temperature increases and the increasing productivity of P_2O_5 as temperature increases.

and E(NR) is maximum if F is used so that /see equation (20), Chapter II/

$$E(MPP_F) = 2.29 - .0008F - .000185(9344.9)F = .272 \quad (9)$$

Solving equation (9) for F gives the optimum fertilization rate (level of F) as 364 pounds per acre. Substituting 364 for F in equation (7) gives $47.58 per acre as maximum E(NR). Similarly, optimum strategies and E(NR) have been computed for equations (4), (5), and (7). Columns 5 and 6 of Table III summarize these results. Column 7 gives the security level for the strategies maximizing net returns (column 5).

<u>Maximum Security Level</u>. The maximin criterion is introduced and explained in Chapter II. This criterion may be a useful choice guide for the manager who is concerned with each outcome rather than a series of outcomes.

For all of the input-output equations considered in this study, the lowest level of net returns for each strategy occurs at the same level of the uncontrollable variable--weather. The hypothetical outcome matrix given below is representative of all input-output equations used in this dissertation.

		Weather			
		X_{21}	X_{22}	X_{23}	X_{24}
Strategies	X_{11}	4	3	2	1
	X_{12}	5	4	3	2
	X_{13}	6	5	4	3
	X_{14}	3	2	1	0

Thus, the optimum maximin strategy (for equations in this study) is always the one which maximizes net returns for a particular level of weather (X_{24} in above example).

Equation 5, Corn, is used to illustrate the method of computing the optimum maximin strategies for the input-output equations of this study. Given:

N = pounds of available nitrogen per acre

T = average June temperature

$\hat{Y}$ = bushels of corn per acre = $f(N,T)$

$= 44.18 + 1.9864N - .00217N^2 - .0002088NT^2$

P_n = price of a pound of nitrogen = \$.126

P_y = price of a bushel of corn = \$1.25

$K = 1.1$

Corn yield from any level of nitrogen greater than zero decreases as temperature increases. 85.4 and 7293.16 are the highest values of T and T^2, respectively, observed over the past 52 years.[5] Thus, minimum net returns from any level of nitrogen (production strategy) occur when T^2 is greatest, i.e., when $T^2 = 7293.16$. Minimum net returns from any strategy are

$$NR = 1.25\,[44.18 + 1.9864N - .00217N^2 - .0002088(7293.16)N] - (.126)(1.1)N \qquad (10)$$

Equation (10) is maximum if N is used so that

$$1.25\,[1.9864 - .00434N - .0002088(7293.16)] - (.126)(1.1) = 0 \qquad (11)$$

Solving equation (11) for N gives 81 pounds of nitrogen per acre as the optimum strategy by the maximin criterion, given the above prices and K = 1.1. Column 8, Table III, gives the optimum maximin strategies for equations 3, 4, 5, and 7 (equations 1, 2, and 6 do not have an uncontrollable variable). Column 9 gives the expected value of net returns

[5] See Appendix Table II for weather distribution.

if the maximin strategies are used each year <u>and</u> the uncontrollable variables (T for this example) occur according to the probability distribution used to determine the optimum strategies and net returns given in columns 5 and 6 of Table III. Column 10 gives the security level resulting from the maximin strategies of column 8.

<u>Minimum Regret</u>. The regret criterion may be a useful choice guide for the farm manager who is actually concerned about the cost of a wrong decision (see p. 22, Chapter II).

Equation 5, irrigated corn, is used to illustrate the procedure for determining the optimum strategy by the regret criterion. Regret for any outcome (level of net returns) is defined as the difference between this outcome and the most desirable possible outcome, given the realized level of the uncontrollable variable.

Given the conditions stated on page 69, two alternative strategies of 50 and 180 pounds of nitrogen per acre,[6] and the possible levels of T^2 given in column 1 of Table IV, a regret matrix may be constructed. Net returns from each strategy for each level of T^2 are obtained by substituting the values of T^2 in column 1 into the equation

$$NR = 1.25\,[44.18 + 1.9864(50) - .00217(50)^2 - .0002088(50)T^2] - (.126)(1.1)(180) \qquad (13)$$

for N = 50, and the equation

$$NR = 1.25\,[44.18 + 1.9864(180) - .00217(180)^2 - .0002088(180)T^2] - (.126)(1.1)(180) \qquad (13)$$

for N = 180. These net returns are given in columns 2 and 3 of Table IV.

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[6] For a continuous variable such as fertilizer any number of strategies could be defined. Only two are used in this example to simplify calculations.

TABLE IV

NET RETURNS AND REGRET MATRICES: EQUATION 5, NITROGEN FERTILIZATION OF IRRIGATED CORN[1]

Levels of T^2 (1)	Net Returns From N=50 (2)	Net Returns From N=180 (3)	Regret N=50 (4)	Regret N=180 (5)	Net Returns From Mixed Strategy, i.e., .42 of N=50 .58 of N=180 (6)	Regret From Mixed Strategy (7)
5476	94.20	132.00	37.80	0	127.36	15.88
5776	90.28	117.91	27.63	0	117.53	11.60
6084	86.26	103.44	17.18	0	107.46	7.22
6400	82.13	88.60	6.47	0	97.12	2.72
6724	77.91	73.38	0	4.53	86.52	2.63
7056	73.58	57.79	0	15.79	75.65	9.16
7396	69.15	41.81	0	27.34	53.29	15.86
Expected Net Returns						
Optimum Pure Strategy (N=180)				103.29		
Mixed Strategy				82.58		
Security Level						
Optimum Pure Strategy (N=180)				41.81		
Mixed Strategy				53.29		

[1]All entries computed assuming $\frac{P_n K}{P_y} = .111$.

Regret (columns 4 and 5) for each strategy is obtained by subtracting maximum possible net returns from the net returns for each strategy at each level of T^2. The optimum pure strategy by the regret criterion is the one that minimizes possible regret. For this example, the maximum possible regret from using 50 pounds of N is \$37.80; the maximum possible regret from using 180 pounds of N is \$27.34. Therefore, 180 pounds of nitrogen per acre is the more desirable strategy by the regret criterion. However, a mixed strategy will always give lower regret (see p. 29). For this example, the mixed strategy is given by simultaneously solving the equations

$$P_1 + P_2 = 1 \qquad (14)$$

$$R = 37.80P_1 \qquad (15)$$

$$R = 27.34\ P_2 \qquad (16)$$

where R is the regret from each strategy, P_1 is the proportion of land fertilized with 50 pounds of nitrogen per acre and P_2 is the proportion fertilized with 180 pounds of nitrogen per acre. The mixed strategy for this example is to fertilize .58 of the land with 180 pounds of N per acre and .42 of the land with 50 pounds of N per acre. The net returns from this mixed strategy at each T^2 is given in column 7. The mixed strategy has a lower maximum possible regret, \$15.88, than the optimum pure strategy, \$27.34.

For fertilizer, seed, or any highly divisible factor, the optimum pure or mixed strategy indicated by the regret criterion is very arbitrary. Clearly, more and/or different alternative (or candidate) strategies can be chosen. The alternative strategies in the above example (50 and 180) are the approximate limits of useful strategies. Introducing a third candidate strategy or changing one or both of the original alternative strategies would definitely change the optimum mixed strategy for the above example and probably change the optimum pure strategy.

Optimum Strategies and Net Returns for Different Crop Prices

For most agricultural commodities, crop prices vary considerably over the years. Optimum rates of fertilization vary, sometimes substantially, sometimes only slightly, as crop prices vary. Optimum production strategies and resulting net returns are computed for high, modal, and low crop prices and modal factor prices, expected weather, K = 1.1, and a managerial goal of maximum net returns. These optimum strategies and the resulting net returns are presented in columns 5 through 12 of Table V. The procedure for determining the optimum strategy and computing net returns is the one that has been used in a previous section of this Chapter. Therefore, it will not be repeated.

Optimum Strategies and Resulting Net Returns for Different Opportunity Costs of Capital (Values of K)

K has been held at 1.1 in the previous sections. Table VI, columns 5 through 10 present optimum strategies and net returns for values of K from 1.0 to 1.5. Alternatively, these may be viewed as optimum strategies and net returns for different factor prices. Optimum strategies and net returns are computed using modal crop prices and a managerial goal of maximum net returns. Computational procedures for this section are similar to those for previous sections. The reader can easily make needed modifications.

Optimum Strategies and Resulting Net Returns When Production Decisions are Based on the Raw Data

All of the input-output data used in this study have either been published or are to be published in discrete form. A major objective of this study is to make this existing data more useful for decision making purposes. This objective implies that production strategies based on the raw data are not optimum and result in a significant loss in possible net returns. Therefore, logical questions are: (1) what production strategies are suggested as optimum in the raw data and (2) what level of net returns

TABLE V

OPTIMUM PRODUCTION STRATEGIES AND RESULTING NET RETURNS FOR HIGH, MODAL, AND LOW CROP PRICES

				High Crop Prices			Modal Crop Prices		Low Crop Prices		
Equa-tions (1)	In-put (2)	Out-put (3)	(P_xK) (4)	Optimum Strategy (5)	NR Optimum Strategy (6)	NR From Strategy in Column (8) (7)	Optimum Strategy (8)	NR From Optimum Strategy (9)	Optimum Strategy (10)	NR From Optimum Strategy (11)	NR From Strategy in Column (8) (12)
1	Columns 2		.1386	224	435.89	433.02	173	229.91	87	122.46	119.89
2	and 3 in this table		.1386 .418	76 169	658.46	634.42	71 93	228.69	68 44	151.00	148.03
3	are the same		.0198	403	54.47	54.42	364	47.58	338	43.62	43.60
4	as cols. 2		.1056	49.4	58.59	58.57	48	56.62	47.6	47.39	47.38
5	and 3 in		.1386	143	128.18	128.12	139	107.79	134	87.52	87.46
6	Table III		.1386	185	16.26	16.18	176	11.61	165	7.13	7.04

TABLE VI

OPTIMUM PRODUCTION STRATEGIES AND NET RETURNS FOR DIFFERENT CAPITAL COSTS (VALUES OF K)

Equation (1)	Input (2)	Output (3)	Price Ratio $\frac{P_x}{P_y}$ (4)	K = 1.0 Optimum Strategy (5)	K = 1.0 NR (6)	K = 1.1 Optimum Strategy (7)	K = 1.1 NR (8)	K = 1.5 Optimum Strategy (9)	K = 1.5 NR (10)
1	Pounds of N per acre	Tons of Spinach	$\frac{.126}{30}$	183		172		127	
	Pounds of Seed per Acre		$\frac{1.00}{30}$	7.61	224.63	7.55	219.54	7.32	211.17
2	Pounds of N per acre	Tons of Snap Beans	$\frac{.126}{50}$	72		71		68	
	Pounds of Seed per Acre		$\frac{.38}{50}$	105	229.22	93	224.26	40	211.70

would result from these strategies? The optimum strategies implied by or stated in the raw data are given in column 5 of Table VII.[7] Net returns from following each of these strategies are given in column 6. These net returns are computed using the input-output equations developed in this study, modal prices, expected weather, and K = 1.1. For equation 4 (as an example):

the modal price of a pound of phosphorous, P_p is \$.096,

the modal price of a pound of seed cotton, Y, is \$.0729,

$E(T) = 96.6$,

$E(T^2) = 9344.9$,

the optimum strategy suggested by the raw data is 32 pounds of phosphorous per acre, and

$$\hat{Y} = 3333 + 29.0P - .0747P^2 - 29.037T - .00218PT^2.$$

Expected net returns from 32 pounds of phosphorous per acre are

$$E(NR) = .0729[3333 + 29.0(32) - .0747(32)^2 - 29.037(96.6) - .00218(32)(9344.9)] - (.096)(1.1)(32) = 50.06 \quad (17)$$

The other entries (levels of net returns) in column 6 were computed similarly. Column 8 gives net returns from the optimum strategies based

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[7] The investigator's description of the fertilizer trials often includes a recommended level of fertilization which is used here as the optimum strategy based on the raw data. If some level of fertilization is not recommended, an "implied" optimum strategy is obtained by first computing the average marginal product of a unit of fertilizer in moving between the several fertilizer levels used in the experiment. The "implied" optimum strategy is the greatest factor level in the original fertilizer trials for which

$$\frac{\Delta Y}{\Delta X} P_y \geq P_y K$$

where $\frac{\Delta Y}{\Delta X}$ is the average marginal product of a unit of X (fertilizer) for the range between fertilizer levels.

TABLE VII

OPTIMUM PRODUCTION STRATEGIES SUGGESTED BY RAW DATA AND RESULTING NET RETURNS

Equation (1)	Input (2)	Output (3)	Price Ratio: $\frac{P_x K}{P_y}$ P_x, P_y are Modal K=1.1 (4)	Optimum Strategy for Price Ratio in Column (4) (based on raw data) (5)	Net Returns From Following Strategy Given in Column (5) (6)	Optimum Strategy for Price Ratio in Column (4) (based on regression equations) (7)	Maximum[1] Possible Net Returns for Given Price Ratio (8)	Loss in Net Returns From Basing Strategy on Raw Data (9)
1				100	226.59	173	229.91	3.32
2	Entries in columns			66 51	221.48	71 93	228.69	7.21
3	2, 3, 4 are same as			400	47.55	364	47.58	.03
4	those for Table III			90	101.23	139	107.79	6.56
5				32	50.06	48	56.62	6.56
6[2]				40	-5.26	176	11.61	16.87
7				20	39.68	24	41.69	1.91

[1]Taken from column 6, Table III.

[2]The entries in columns 6, 8, and 9 of this row are extrapolations (the expected value of the weather variable, used in these calculations, does not fall in the range of the data) and, as such, warrant little confidence.

on the regression equation estimates of fertilizer productivities, here defined as maximum possible net returns. Column 9, column 8 minus column 6, is the loss in net returns from basing production strategies on the raw data estimates of fertilizer productivities rather than the regression equations estimates of fertilizer productivities.

Optimum Strategies and Net Returns When Choice Criterion is Maximum Physical Product

A farm manager may conceivably possess adequate knowledge of all the determinants of production strategy, and yet, follow a strategy indicated by a choice criterion that is inconsistent with his goals. The view that maximizing total product (and total revenue) also maximizes net returns is often given as an example of imperfect knowledge of choice criteria. That is, the production strategy for which $\frac{\delta Y}{\delta X} = 0$ is chosen rather than the strategy for which $\frac{\delta Y}{\delta X} P_y = P_x K$. Product maximizing strategies, taken from the raw data, are presented in column 5 of Table VIII. The net returns from these strategies (column 6) are computed using the input-output equations developed in this study, modal prices, expected weather, and $K = 1.1$; i.e., they are computed in the same way as previous examples. Column 7 gives the losses in net returns from (1) using "poorer" estimates of fertilizer productivities than furnished by the regression equations and (2) using an inappropriate (for maximizing NR) choice criterion (maximum total product). Column 8 gives the loss in net returns from maximizing output.

Ranges of Fertilization Rates for Which the Marginal Physical Product of Fertilizer is Significantly Different from Zero

The statistical test of the significance of the marginal physical product is discussed on pp. 38-39. Figure 3 is a graph of a hypothetical

TABLE VIII

PRODUCTION STRATEGIES FOR MAXIMIZING TOTAL OUTPUT (BASED ON RAW DATA) AND RESULTING NET RETURNS

Equation (1)	(2)	(3)	(4)	Output Maximizing Strategies (based on raw data) (5)	Net Returns From Strategy Given in Column 5 (6)	Loss in Net Returns[1] From (1) Maximizing Output and (2) Using Raw Data (7)	Loss in Net Returns From Maximizing Output[2] (8)
1				300	220.14	9.77	6.45
2	Headings and entries			66 200	205.20	23.49	16.28
3	in columns 2, 3, 4			400	47.55	.03	0
4	same as columns 2, 3,			32	50.06	6.56	0
5	4, Table III			180	103.29	4.50	2.06[3] /gain/
6				40	-5.26	16.87	0
7				20	39.68	1.91	0

[1]Column 7 is obtained by subtracting column 6 this table from column 8, Table VII.

[2]Column 8 is obtained by subtracting column 9 in Table VII from column 7 in this table.

[3]This gain occurs because the strategy for maximizing total product happens to be nearer the optimum strategy for maximizing NR (as defined from regression equations) than the optimum strategy for maximizing NR determined from the raw data estimate of fertilizer productivity.

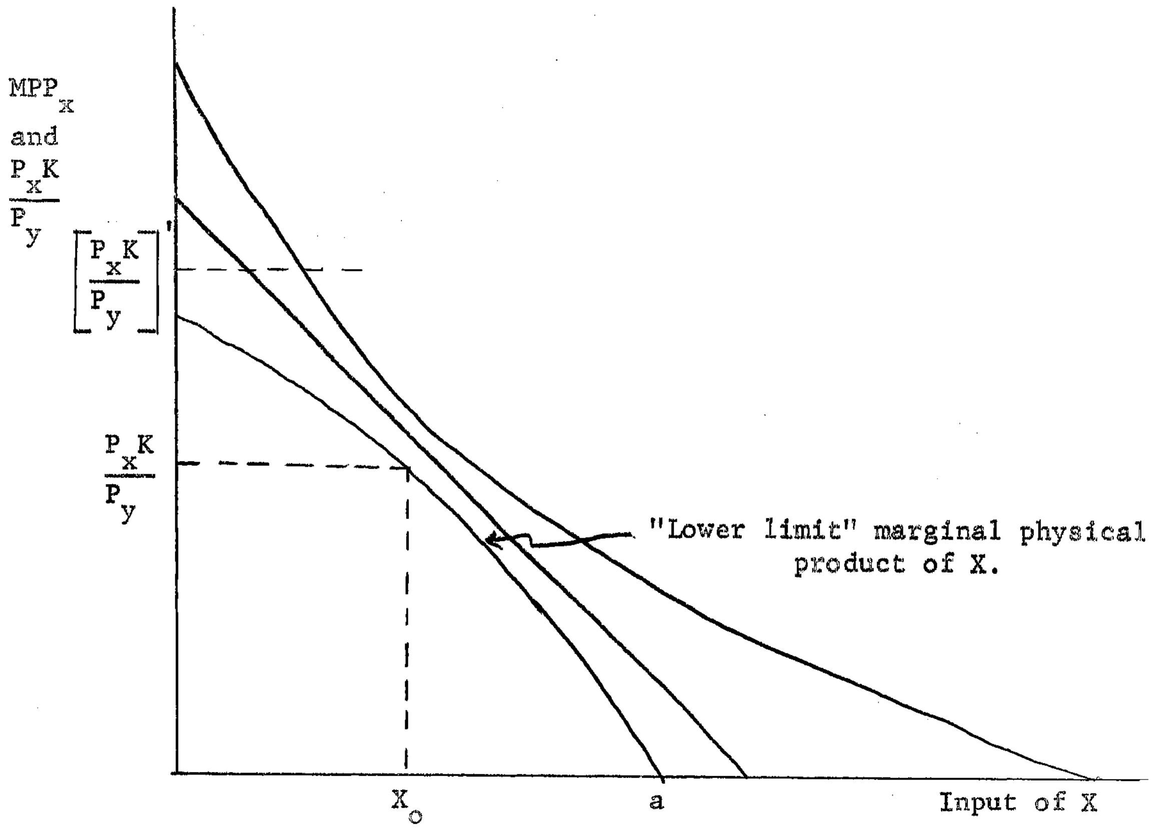

Figure 3. Hypothetical Marginal Physical Product Function and Confidence Limits

marginal physical product function and its confidence limits.[8] The lower curve in Figure 3 represents the minimum marginal physical product of X (at, say, the 95 per cent confidence level) for the range of inputs, $0 \leq X \leq a$. A farm manager would base production decisions on the "lower limit" (minimum) physical product if he requires statistical assurance (in this example, the lower limit is determined for 95 per cent confidence level) that the realized marginal physical product of X will be great enough to justify its cost.[9] The "lower limit" marginal physical product

[8]The 95 per cent confidence limits used in this study are quite arbitrary. A farm manager could rationally require more or less statistical assurance that the marginal physical product of fertilizer is positive.

[9]"Lower limit" marginal physical product is adopted as an expedient expression for the lower confidence limit (at some probability level) of the marginal physical product of X.

for each level of X would be higher or lower as the chosen confidence level is lower or higher than 95 per cent. Clearly, the "lower limit" marginal physical product is maximum when X = 0 and is zero when X = a. Column 4 of Table IX gives the maximum "lower limit" marginal physical product for each equation. Column 5 gives the amount of fertilizer (or seed) for which the "lower limit" marginal physical product is zero. For equation 1, for example, the information in columns 4 and 5 means the "lower limit" marginal physical product of nitrogen

(1) varies between 0 and .00237, and

(2) is positive for nitrogen levels between 0 and 200 pounds per acre.

These statements are generalizations or inferences from the regression equation. Since the regression equation is a statistical estimate, generalizations of this nature <u>are valid only</u> for a particular confidence (probability) level.

Column 6, Table IX answers the question: How much fertilizer (X) should be used if a manager requires that $\frac{\delta Y}{\delta X} = \frac{P_x K}{P_y}$ holds for $\frac{\delta Y}{\delta X}$ equal to the "lower limit" marginal physical product of X. Modal prices, expected weather, and K = 1.1 are used to compute these strategies. Figure 3 gives the graphic solution of the problem. X_o is the optimum input of X for the prescribed conditions and is the general case of the entries in column 6 of Table IX. If $\frac{P_x K}{P_y}$ is greater than the maximum "lower limit" marginal physical product, e.g., $\left[\frac{P_x K}{P_y}\right]'$, no X would be used and the entry in column 6 is 0.

TABLE IX

INTERVAL ESTIMATES OF THE MARGINAL PHYSICAL PRODUCT OF VARIABLE FACTORS (MPP) AT THE 95 PER CENT CONFIDENCE LEVEL AND PRODUCTION STRATEGIES BASED ON "LOWER LIMIT" MARGINAL PHYSICAL PRODUCTS

Equation Number (1)	Variable Input (X) (2)	Unit of Input (3)	Maximum "Lower Limit" MPP_x at 95% Confidence Level (4)[1]	Amount of X For Which "Lower Limit" $MPP_x = 0$ (5)	Minimum Input of X to Maximize Net Returns for Modal Prices, K=1.1 (6)	Range of Observed Price Ratios, K=1.1 (7)
1	Available N/Acre	1 lb.	.00237	200.0	0	.00244 - .00938
2	Available N/Acre	1 lb.	.016	70.0	62.5	.00104 - .00443
	Seed/Acre 90% Germination	13 lbs.	.075	11.5	(see footnote 2)	.040 - .178
3	4-12-4/Acre	1 lb.	Negative	--	0	--
4	P_2O_5/Acre	1 lb.	.881	16.5	0	1.118 -1.692
5	Available N/Acre	1 lb.	.421	112.0	90	.091 - .152
6	Available N/Acre	10 lbs.	4.594	15.5	8.0	1.86 -2.9
7	P_2O_5/Acre	10 lbs.	1.95	2.0	1.4	.466 - .691

[1]When the marginal physical product is affected by weather, MPP_x is computed for the expected value of the weather variable; i.e., $E(MPP_x)$ is used.

[2]For the range of inputs $13 \leq$ seed ≤ 208, the "lower limit" marginal physical product is less than $\frac{P_x K}{P_y}$ for modal prices and K = 1.1. An extrapolation estimating the "lower limit" marginal physical product for $0 \leq$ seed ≤ 13 would be needed to determine the minimum amount of seed that could possibly maximize net returns.

Optimum Strategies and Net Returns When
Weather is Known or Can be Predicted

Equation 6, winter oats, is the only equation for which the value of the weather variable is known at the time that production strategies are implemented. Thus, for this equation, weather is a known constant. Net returns are

$$NR = P_y \left[-105.61 + .89N - .00145N^2 + 136.26R - 31.55R^2 - .725NR + .1934NR^2 \right] - P_n KN \quad (18)$$

where

P_y = price of oats per bushel, net of harvesting costs,

N = pounds of nitrogen per acre,

R = average monthly rainfall for months of August through November,

P_n = price of a pound of nitrogen, and

K = opportunity cost of capital.

Since oats are fertilized in February or March, the optimum strategy for any season is obtained by substituting known values of P_n, R, R^2 and K and the expected value of P_y in the equation

$$.89 - .0029N - .725R + .1934R^2 = \frac{P_n K}{P_y} \quad (19)$$

and solving for N. Expected net returns may be obtained by solving equation (18) for this optimum level of N and the values of P_n, P_y, R, R^2, and K used in equation (19).

If the value of the weather variable is not known when fertilizer is applied, a farm manager may use a predicted weather value rather than the expected weather value used above. The optimum strategy would be determined by the above procedure, using a predicted rather than a known value of the weather variable. This might be a feasible alternative [to maximizing E(NR)] in equation (5), for example, because a fairly accurate prediction of the weather variable (average June temperature) may be available when nitrogen is applied (about May 10).

The Ranges of Weather for Which Fertilization is Profitable

Some of the effects of weather on fertilizer productivities have been specified for cotton, corn, wheat, and oats. It may be useful to point out the ranges of weather for which fertilization is profitable for each of these crops. Fertilization is profitable up to the level of X_1 at which

$$\frac{\delta Y}{\delta X_1} = \frac{P_{x_1} K}{P_y} \tag{20}$$

If

$$\frac{\delta Y}{\delta X_1} = g(X_2) \tag{21}$$

where X_2 is a weather variable, fertilization is profitable for a given $\frac{P_{x_1} K}{P_y}$ for all X^2 such that

$$g(X_2) \geq \frac{P_{x_1} K}{P_y} \tag{22}$$

In equation 4, for example, Y is pounds of seed cotton, X_1 is pounds of phosphorous, X_2 is [average maximum August temperature]^2, T^2, and

$$\frac{\delta Y}{\delta P} = 29 - .1494P - .00218T^2 = g(T^2) \tag{23}$$

For modal prices and K = 1.1, phosphorous fertilization of cotton is profitable for all T^2 such that [by equation (22)]

$$29 - .1494P - .00218T^2 \geq 1.449 \tag{24}$$

At least one pound of phosphorous would be profitable as long as

$$.000218T^2 \leq 29 - .1494 - 1.449 \tag{25}$$

or

$$T^2 \leq 12592.8 \quad (26)$$

Since the maximum T^2 observed over the past 47 years is 10526.76 (for T = 102.6), fertilization (with phosphorous) of cotton is profitable for all probable values of the weather variable and the assumed prices.

By a similar procedure, it can be shown that fertilization of cotton, corn, and wheat (equations 3, 4, 5, and 7) is profitable, given modal prices and K = 1.1, for the full range of probable weather. Nitrogen fertilization of oats is not profitable for $R \geq 4.37$. However, little weight can be given to this conclusion because it is based on an extrapolation beyond the range of the data. (See the discussion of equation 6 in Chapter IV.)

Analysis of Results

The several variables determining production strategies and net returns from these strategies are named and discussed in Chapters I and II. Some effects of these variables on strategies and net returns, implied by the results presented in the first part of this chapter, are now made explicit. Specifically, this section presents conclusions about

(1) the influence of managerial goals, factor and product prices, factor productivities, and managerial knowledge of these determinents on production strategies and net returns;[10] and

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[10]More accurately, these four variables are categories or classifications, each containing many variables, treated in this dissertation as singular variables (except the cursory attention given to the interaction between knowledge and goals). This convention will be continued; but, the reader is reminded that the following, seemingly concrete conclusions, "correct to two decimal places", are based on glimpses of complex physical and economic mechanisms.

(2) the profitability of factor use for the factor-product combinations of this study.

Summary: Effects of Managerial Goals on Production Strategies and Net Returns

The effects of managerial goals on production strategies and net returns are negligible (wheat, equation 7), small (cotton, equations 3 and 4) or considerable (corn, equation 5) for the fertilizer-crop combinations considered. Thus, for some crops in this study, farm managers may obtain greater utility by selecting the choice criterion that is most compatible with their goals. Table X, derived from Tables III and IV compares strategies and net returns for different managerial goals for equation 5, irrigated corn. The reader may easily make similar comparisons (except the effects of the regret criterion) for other equations.

Summary: The Effects of Prices on Production Strategies and Net Returns

Product prices generally exert considerable influence on optimum production strategies and net returns (see Table V). Wheat (equation 7) is the exception. The optimum rate of phosphorous fertilization of wheat does not change for the range of crop prices considered in this study. However, for the other equations, crop prices dictate differences in optimum strategies ranging from 1.8 pounds of P_2O_5 per acre of cotton (equation 4) to 137 pounds of available nitrogen per acre of spinach (equation 1). Similarly, variations in net returns due to crop price variation range from $9.13 per acre on oats (equation 6) to $507.46 per acre on snap beans (equation 2).[11] A notable characteristic of these

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[11] This statement assumes the optimum strategy for each price level is followed.

TABLE X

OPTIMUM STRATEGIES AND NET RETURNS FOR DIFFERENT MANAGERIAL GOALS: EQUATION 5

Goal	Optimum Strategy	E(NR)	Security Level	Regret
Maximize E(NR)	139	107.79	18.42	--
Maximize Security Level	81	98.60	73.17	--
Minimize Regret[1]				
Pure	180	103.29	41.81	27.34
Mixed	.42 @ 50 .58 @ 180	82.58	53.29	15.88

[1]Recall that these strategies are optimum only if choice is limited to N = 50 and N = 180. If the two strategies N = 139, N = 81 are included, the regret criterion may indicate a different optimum pure strategy and will surely indicate a different mixed strategy. Clearly, this characteristic of the regret criterion complicates comparisons of effects of managerial goals.

changes in net returns is that they are not usually attributable to changes in total cost and total revenue fostered by price induced changes in inputs (production strategies) and subsequent changes in output. Rather, variation in net returns stem from the revaluation, as crop prices change, of an essentially constant output. Comparing some of the entries in Table V makes the meaning and validity of this statement more evident. Columns 7 and 12 give net returns if the optimum strategies for modal prices are followed and high and low crop prices, respectively, are realized. A range of losses in net returns from basing production strategies on modal prices--\$.02 (equations 3 and 4) to \$24.04 (equation 2)--may be defined by considering the differences between entries in columns 6 and 7 and columns 11 and 12. If equations 1 and 2 are not considered, the upper limit of this range becomes \$.09. Operationally, for equations 3 through 7, this means that the farm manager may ignore price variability when determining optimum fertilization rates--base production strategies on a naive estimate (mode or mean) of product prices--without significantly depressing net returns.[12] However, for equations 1 and 2, losses in net returns from following the optimum strategy for modal prices (and realizing high or low prices) range from \$2.57 to \$24.04 per acre. Therefore, for spinach and snap beans, farm managers may significantly increase net returns by tailoring production strategies to crop prices.

Table VI shows the effects of changes in factor prices; i.e., a change in the K value is the same as a change in the factor price. For the equations considered, changes in factor prices cause substantial changes in production strategies (except rate of seeding for equation 1) and net returns. Thus, a 50 per cent increase in factor prices decreases net returns by \$13.46 and \$17.52 for spinach and snap beans, respectively, (compare columns 6 and 10, Table VI). However, factor price variations have small influence on net returns from the field crops of this study (equations 3 through 7).

In conclusion, prices substantially affect the level of net returns from crop production. However, Variations in net returns (from factor and product price changes) may be traced to crop prices as determinants of product value rather than fertilizer and crop prices as determinants or production strategies. Alternatively stated, variations in prices cause optimum production strategies to vary; but, the variation in strategies does not substantially affect net returns. Recall that this conclusion

[12]While this statement is true for deviations from optimum induced by the price variations considered, greater deviations may be expected to more significantly depress net returns.

is valid only for the field crops (equations 3 through 7). For the vegetable crops, prices significantly influence optimum strategies.

Summary: The Effects of Factor Productivity on Production Strategies and Net Returns

The strategies given in columns 5 and 7 of Table VII are optimum strategies for the same goals and prices, based on different estimates of factor productivities. With the exception of equation 3, the improved estimate of factor productivities furnished by the regression equations enables the farm manager to substantially increase net returns from fertilization, column 8 vs. column 6. For example, the improved estimate of the productivity of phosphorous on cotton (equation 4) would enable the farmer with 100 acres of cotton to increase his net income by $656 per year (given expected weather and modal factor and product prices).

Summary: Effects of Managerial Knowledge on Production Strategies and Net Returns

Production strategies may vary over a considerable range with small effect on net returns (for the crops considered in this study). Thus, farm managers do not need perfect knowledge of the determinants of production strategy to approximately achieve a desired level of net returns. Rather, a range of strategies may yield net returns that are sufficiently near the optimum. This range varies for different input-output combinations, generally decreasing as the relative value of the input increases and/or the value of the marginal product increases. For example (see Table V), a farm manager can substantially increase net returns from seed used in producing snap beans if he can obtain a more accurate estimate of crop prices than is furnished by an averaging of observed prices. However, for other equations, this naive estimate of crop prices is quite satisfactory. Similarly, the knowledge of factor productivities added by this study usually increases a farmer's ability to reach his goals (see Table VII). But, for equation 3, the raw data furnish enough information for the farm manager to satisfactorily define a production strategy.[13]

The farm manager may possibly be imperfectly aware of the decision making principles to follow in achieving his goals. For two goals, maximum net returns and maximum security, the effects of using an inappropriate choice criterion may be obtained by comparing security levels and net returns given in Table III. Column 7, Table VIII gives the loss in net returns from using the raw data and an inappropriate choice criterion (maximum total product). Column 8 gives only the losses stemming from use of the wrong choice criterion. These variable, but generally substantial effects of choice criteria on net returns emphasize the need for the development of a variety of decision making techniques to better fit the knowledge-goal situations of individual farm managers.

Summary: Profitability of Fertilizing Crops Considered in this Study

If a farm manager is willing to base production strategies on the expected value of the marginal physical product (the estimate of MPP furnished by the regression equations), some level of fertilization is profitable for all prices and goals considered in this study (see Tables III,

[13]The raw data are themselves substantial packets of information. The farmer using them is certainly not ignorant of factor productivities when choosing production strategies. Also, it is very useful to know that a range of productivities or prices does not significantly affect net returns. Such statements as these may be made only after the research worker has gathered enough information to define or pin down the range over which prices and productivities may be treated as constant (further knowledge becomes unnecessary).

IV, and V). However, fertilization may not always be profitable if a manager requires statistical assurance (at some probability level) that the marginal physical product is positive. For example, given expected weather, modal prices, and K = 1.1, net returns from fertilizing spinach and cotton are not positive at the 95 per cent confidence level (see column 6, Table IX). These results are rather arbitrary; that is, different prices, weather, or confidence limits would give different answers. With slightly lower factor prices or confidence limits, fertilizing spinach with nitrogen and cotton with phosphorous would be profitable. However, the variance of the estimate for equation 3 is large enough that 4-12-4 fertilizer on cotton becomes profitable only at very low confidence levels and/or very small fertilizer-crop price ratios. Finally, fertilizing cotton, corn, and wheat is profitable for any observed weather levels. Fertilizing oats (the remaining crop for which some weather effects are specified) is not profitable for all possible weather (see p. 85).

CHAPTER VI

SUMMARY AND CONCLUSIONS

The following summary describes, in general terms, the results of actions taken to meet the objectives of this study set forth in Chapter I. After this brief summary, concluding remarks summarize the implications of these results.

Summary

The primary purpose of this study was to supply information about fertilizer-crop production functions and decision making techniques that will enable farm mangers to define production strategies that more nearly achieve their goals. To this end:

(1) Empirical production functions for spinach, snap beans, cotton, corn, oats, and wheat were estimated by linear regression techniques. The field crop equations account for some of the effects of an uncontrollable weather variable. While some of the equations have a low R^2 (e.g., wheat and cotton equations), the regression coefficients for all equations are significant[1] at satisfactory probability levels. Thus, while predicted total product may be expected to differ considerably from realized total product, predicted optimum strategies possess considerable reliability.

(2) Choice criteria for three managerial goals (maximum net returns maximum security level, minimum regret) and three knowledge classifications

[1]Determined by the Student t-test.

(perfect knowledge, risk, uncertainty) were stated and explained. Knowledge classifications were treated as subjectively defined (by the decision maker) points on a continuum of knowledge. Maximum net returns was given as the most likely goal for a manager with perfect knowledge of outcomes. All three goals were considered possible (and probable) objectives for risk situations. However, only maximum security level and minimum regret were given as possible goals under uncertainty. Game theoretic techniques were applied to decision making under uncertainty. An effort was made to establish the goals implied by these choice criteria as approximations of maximum utility for substantial numbers of farmers.

(3) A number of examples have been constructed to (a) illustrate the use of these estimates and choice criteria, (b) provide a basis for some conclusions about fertilization of particular crops, and (c) give a measure of the significance of the problem attacked by this study.

In accomplishing this central objective:

(1) Several statistical tools have been applied to the problem of improving the decision making information available to Oklahoma farmers. For example, fertilizer-crop production functions were estimated by linear regression techniques, and confidence limits for the marginal productivities of fertilizer were determined.

(2) Some theoretical considerations in using these techniques (see Chapter III) have been empirically reinforced by the results given in Chapter IV. For example, the negative oat yields from low fertilization rates and average weather predicted by equation 6 dramatically emphasize the need for at least partially factorial observations of the effects of independent variables on the dependent variable.

(3) Some criteria for useful (in regression analysis) input-output data have been implied or specified.

Conclusions

Comparisons of optimum and non-optimum production strategies have shown that strategies significantly affect realized net returns and, hence, the extent to which farmers are able to attain their goals.[2] However, variations in optimum strategies induced by variation in goals, prices, and productivities do not always have a significant influence on net returns. The results of this study indicate that variation in production strategies due to variation in

(1) prices is significant for equations 1 and 2 only (spinach and snap beans).

(2) productivities is significant for all equations except 3 (4-12-4 on cotton).

(3) managerial goals is significant for all equations.[3] Therefore, for each equation, farmer knowledge of choice criteria influences the attainability of managerial goals.

The above remarks indicate that future research and extension activities may usefully aim at increasing farmers' knowledge of factor

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[2] A deviation from the optimum production strategy, per se, is not important. Rather, the question is: do possible deviations, limited or prescribed by managerial knowledge of the determinants of production strategies, substantially influence attainment of managerial goals? This criterion is used here to gauge the significance of the possible deviations from optimum production strategies discussed in Chapter V.

[3] Recall that these conclusions are based on the "possible" or "range of possible" prices, productivities, and goals defined for this study. Certainly other possible values of these variables exist. Therefore, reader and author must refrain from generalizing these conclusions.

productivities and decision making techniques. A number of derivative needs, e.g., explanation of more of the effects of interacting and uncontrollable variables (weather, soil), more accurate definition of farmer goals, and development of "better fitting" decision making techniques, quickly appear as this central objective is more definitively stated. Data inadequacies, a recurrent note in Chapter IV, further stress the need for experiments yielding more useful (in economic analysis) information.

For agricultural extension workers and Oklahoma farm managers, it is operationally significant that fertilization is profitable for all crops in this study even when weather and price variability are taken into account.[4] When the variance of the estimate of the production function is considered, some fertilization of corn, oats, and wheat is profitable (using a 95 per cent confidence limit, average prices, and expected weather). However, under similar conditions, it is not profitable to fertilize spinach or cotton. These rather arbitrary conclusions are made to point out that the results of this study may be used as statistical support for statements about the profitability of fertilizing.

[4] Profitable means simply that fertilization makes a positive net addition to total returns from growing the crop.

APPENDIX

APPENDIX

Price, weather and input-output data used in this study are given in Tables I through IX. Sources of price and weather data are indicated in the tables. Input-output data sources are published and unpublished descriptions of the agronomic experiments outlined in the discussion of each equation (see Chapter IV).

APPENDIX TABLE I

NET FARM PRICES USED IN THIS STUDY

Factor or Product	Unit of Measurement	Modal Price	Range of Prices	Years From Which Price Observations Are Drawn
available[1] Nitrogen	lb.	.126	.120-.145	1954-60
available[1] Phosphorous	lb.	.096	.088-.104	1954-60
Spinach Seed[2]	lb.	1.00	.85-1.05	--
Snap Bean Seed[2]	lb.	.38	.35-45	--
Spinach[3]	ton	30.00	17.00-54.00	1952-59
Snap Beans[3]	ton	50.00	36.00-125.00	1952-59
Seed Cotton[4,5]	lbs.	.0729	.0820-.0676	1954-59
Corn[4]	bu.	1.25	1.05-1.45	1954-59
Oats[4]	bu.	.63	.55-.71	1954-59
Wheat[4]	bu.	1.70	1.65-2.08	1954-59

[1]Source: Agricultural Prices, Crop Reporting Board, Agricultural Marketing Service, U. S. Department of Agriculture, 1954-60.

[2]Estimates obtained from Dr. Samuel Wiggins, Oklahoma State University.

[3]Source: Vegetables for Processing, Crop Reporting Board, Agricultural Marketing Service, U. S. Department of Agriculture, 1952-59.

[4]Source: Prices Received by Oklahoma Farmers 1910-1957, Oklahoma State University, Agricultural Experiment Station, Processed Series, P-297, June, 1958 and Supplement Published in May, 1960.

[5]Seed cotton prices are derived assuming seed cotton is 1/3 lint and 2/3 seed.

APPENDIX TABLE II

OBSERVATIONS OF WEATHER VARIABLES USED IN EQUATIONS 3 THROUGH 7

	Weather Variable					Weather Variable			
Year	Stillwater Av. Max. Temp. Feb. 1-Apr. 1; Apr. 25-May 14 (T)	Muskogee Av. June Temperature (T)	Ardmore (Lone Grove) Av. Max. Aug. Temp. (T)	Stillwater Av. Monthly Rainfall Aug.-Nov. (T)	Year	Stillwater Av. Max. Temp. Feb. 1-Apr. 1; Apr. 25-May 14 (T)	Muskogee Av. June Temperature (T)	Ardmore (Lone Grove) Av. Max. Aug. Temp. (T)	Stillwater Av. Monthly Rainfall Aug.-Nov. (T)
1894				2.16	1915	54.3	74.9	87.8	4.33
1895				3.18	1916	60.1	75.0	99.7	1.88
1896	64.2			2.16	1917	57.8	76.0	94.8	3.04
1897	61.3			1.75	1918	65.7	81.5	101.9	4.32
1898	63.2			2.61	1919	62.0	77.3	95.4	3.70
1899	59.0			2.63	1920	65.5	74.4	90.9	4.42
1900	62.8			3.53	1921	66.4	78.4	98.4	2.03
1901	64.1			1.53	1922	61.7	79.0	98.0	1.93
1902	63.4			3.88	1923	61.6	78.5	98.6	5.70
1903	57.6			2.39	1924	56.5	79.9	99.9	1.93
1904	65.1			2.36	1925	67.7	83.3	98.6	2.42
1905	58.5			3.74	1926	62.0	76.0	93.9	3.67
1906	56.9			4.96	1927	65.8	76.2	92.8	4.14
1907	63.4			1.94	1928	65.6	73.9	95.0	2.62
1908	62.8	75.0		6.20	1929	28.7	75.8	97.5	2.32
1909	61.8	76.8		2.56	1930	65.5	75.8	99.7	2.03
1910	64.8	75.8		1.75	1931	60.2	79.4	94.2	3.92
1911	64.1	84.4		3.25	1932	63.7	78.0	96.1	2.32
1912	56.8	74.8		1.65	1933	63.3	79.2	91.9	2.87
1913	59.0	78.2	102.1	3.11	1934	64.7	82.6	102.2	4.51
1914	60.8	83.0	91.5	2.02	1935	62.1	74.0	96.4	2.42

(Continued)

APPENDIX TABLE II (Continued)

Year	Weather Variable: Stillwater Av. Max. Temp. Feb. 1-Apr. 1; Apr. 25-May 14 (T)	Muskogee Av. June Temperature (T)	Ardmore (Lone Grove Av. Max. Aug. Temp. (T)	Stillwater Av. Monthly Rainfall Aug.-Nov. (T)	Year	Weather Variable: Stillwater Av. Max. Temp. Feb. 1-Apr. 1; Apr. 25-May 14 (T)	Muskogee Av. June Temperature (T)	Ardmore (Lone Grove Av. Max. Aug. Temp. (T)	Stillwater Av. Monthly Rainfall Aug.-Nov. (T)
1936	64.3	82.4	102.6	2.04	1948	58.1	77.0	94.6	1.76
1937	60.5	79.0	97.0	2.24	1949	60.6	78.6	93.3	2.04
1938	66.4	77.3	98.1	2.38	1950	62.3	77.7	90.6	1.48
1939	64.8	78.2	99.4	1.70	1951	63.3	75.6	100.4	3.68
1940	64.3	76.4	90.5	3.26	1952	63.8	82.7	101.2	1.96
1941	60.9	75.8	94.2	5.40	1953	65.6	85.4	94.1	2.34
1942	65.5	77.2	93.6	4.24	1954	64.6	80.6	99.8	1.19
1943	66.3	80.1	102.0	2.33	1955	67.0	75.6	95.8	2.16
1944	62.4	79.6	98.1	3.18	1956	67.2	78.8	102.4	1.30
1945	63.6	73.3	92.2	6.06	1957	61.7	77.6	99.5	2.86
1946	68.4	75.8	95.8	2.55	1958	55.4	78.6	95.5	2.49
1947	61.5	77.1	97.5	1.21	1959	65.6	77.4	94.7	6.76
						$\bar{T} = 62.6$	$\bar{T} = 78.0$	$\bar{T} = 96.6$	$\bar{R} = 2.92$
						$\bar{T}^2 = 3925.9$	$\bar{T}^2 = 6088.6$	$\bar{T}^2 = 9344.9$	$\bar{R}^2 = 10.1$

$\bar{T} = \frac{1}{n} \sum_{i=1}^{n} T_i$ where T_i is the observed value in the i^{th} year and n is the total number of years (observations).

$\bar{T}^2 = \frac{1}{n} \sum_{i=1}^{n} (T_i)^2$ where T_i and n are defined as above.

$\bar{R}$ and $\bar{R}^2$ are computed similarly.

Source: Monthly and Annual Summaries of Climatological Data published by the U. S. Weather Bureau.

APPENDIX TABLE III

INPUT-OUTPUT DATA FOR EQUATION 1, YIELDS (TONS OF SPINACH)[1] FROM SEVERAL RATES OF SEEDING AND FERTILIZING FALL SPINACH, 1958

Pounds of Seed Per Acre	Pounds of Available Nitrogen Per Acre				
	0	100	200	300	400
1.21	3.70	5.08	4.61	5.74	4.47
2.42	6.53	6.32	7.26	5.96	5.95
4.84	6.54	6.82	7.58	8.53	7.01
9.68	6.46	8.64	7.08	9.29	8.35

[1]Average of 6 repetitions.

Source of Data: (15)

APPENDIX TABLE IV

INPUT-OUTPUT DATA FOR EQUATION 2, YIELDS (TONS OF SNAP BEANS)[1] FROM SEVERAL RATES OF SEEDING AND FERTILIZING SPRING SNAP BEANS, 1959-60

Seeding Rate (lbs. per acre)	Pounds of Available Nitrogen Per Acre					
	0	33	50	67	100	133
13	3.08	3.95	--	4.24	--	4.09
26	4.07	4.78	4.70	4.74	4.25	4.35
52	4.13	4.58	5.23	5.22	5.08	5.33
104	4.33	5.33	5.55	5.28	5.28	4.71
208	4.80	--	6.25	--	5.85	--

[1]Averages of 6 to 12 repetitions. These are pooled yields covering two years. Statistical analysis indicated no significant difference in yields over years.

Source of Data: (15)

APPENDIX TABLE V

INPUT-OUTPUT DATA FOR EQUATION 3, YIELDS (POUNDS OF SEED COTTON)[1] FROM FERTILIZING COTTON WITH 4-12-4, 1930-1945

	Pounds of 4-12-4						
Year	0	200	300	400	600	800	1000
1930	260	333	348	342	369	370	356
1931	1008	1312	1392	1452	1455	1335	1490
1932	549	740	741	784	736	750	777
1933	522	578	554	608	560	735	822
1934	294	274	261	267	285	216	268
1935	286	338	472	445	428	435	429
1936	343	436	448	396	383	230	402
1937	658	866	860	826	808	776	820
1938	726	828	868	840	880	878	795
1939	343	378	328	345	352	341	362
1940	895	1460	1590	1758	1578	1698	1548
1941	713	862	922	922	855	862	877
1942	761	837	818	908	1030	922	994
1943	324	366	356	395	384	399	388
1944	603	771	730	810	794	804	786
1945	652	1070	1230	1172	1038	1054	1342

[1] Yields for zero rate are averages of 5 repetitions. Yields for other rates are averages of 2 repetitions.

Source of Data: (9)

APPENDIX TABLE VI

INPUT-OUTPUT DATA FOR EQUATION 4, YIELDS (POUNDS OF SEED COTTON)[1] FROM FERTILIZING COTTON WITH PHOSPHOROUS, 1931-1945

	Pounds of Available Phosphorous Per Acre					
Year	0	16	32	48	64	80
1931	1082	1231	1516	1416	1464	1413
1932	717	742	799	829	832	844
1933	581	518	564	583	603	612
1934	347	412	492	470	468	396
1935	498	390	435	444	499	537
1936	317	396	432	461	419	350
1937	544	710	779	765	859	745
1938	548	765	785	855	813	800
1939	386	422	500	510	453	424
1940	570	968	1358	1382	1379	1506
1941	525	510	648	716	735	735
1942	668	783	743	891	885	897
1943	351	448	543	553	506	524
1944	452	672	670	719	759	711
1945	405	666	999	860	897	777

[1]These yields also reflect effects of 24 pounds per acre of both available nitrogen and potassium.

Source of Data: (9)

APPENDIX TABLE VII

INPUT-OUTPUT DATA FOR EQUATION 5, YIELDS (BUSHELS OF CORN)[1] FROM FERTILIZING CORN WITH NITROGEN, 1949-1953

Year	Pounds of Available Nitrogen Per Acre					
	0	60	90	120	150	180
1949	55.8	79.7	105.6	96.5	103.2	100.8
1950	68.2	97.2	107.7	106.6	109.2	108.8
1951	44.0	89.0	95.2	97.9	104.8	109.6
1952	27.1	56.9	60.7	65.2	72.5	71.7
1953	20.2	64.4	68.0	65.6	66.1	70.9

[1]These yields include effects of 400 pounds of 5-10-10 per acre.

Source of Data: (11)

APPENDIX TABLE VIII

INPUT-OUTPUT DATA FOR EQUATION 6, YIELDS (BUSHELS, PER ACRE) FROM FERTILIZING WINTER OATS WITH NITROGEN, 1953-1955

Year	Pounds of Available Nitrogen Per Acre				
	0	20	40	80	160
1953	42.3	43.0	47.2	44.8	39.3
1954	33.8	36.8	47.8	53.6	48.0
1955	10.6	18.9	22.1	26.2	23.6

Source of Data: (12)

APPENDIX TABLE IX

INPUT-OUTPUT DATA FOR EQUATION 7, YIELDS (BUSHELS PER ACRE) FROM FERTILIZING WHEAT WITH NITROGEN AND PHOSPHOROUS, 1931-1959

	Levels of Available Nitrogen, Phosphorous								
Year	0,0	0,30	16,30	16,30	30,15	30,45	30,45	33,30	33,30
1931	25.6	25.2	28.4	32.3	25.0	28.5	27.4		
1932	19.3	23.9	28.6	22.7	30.2	35.1	27.6		
1933	12.3	22.1	22.9	25.1	20.8	30.1	31.2		
1934	12.7	18.7	18.0	21.9	12.7	14.1	13.9		
1935	14.0	24.1	26.1	27.0	27.7	23.4	23.6		
1936	19.3	19.4	20.2	20.6	21.8	21.7	18.1		
1937	22.0	28.8	30.3	32.2	28.3	28.7	26.8		
1938	3.4	11.7	11.7	12.4	10.2	12.4	12.7		
1939	15.3	25.8	24.4	26.7	25.2	28.7	28.0		
1940	15.2	28.6	30.6	33.6	28.2	35.3	31.3		
1941	.9	8.1	8.7	8.2	6.4	10.3	10.1		
1942	2.6	10.7	10.9	9.9	12.5	16.6	14.7		
1943	4.3	9.2	11.9	10.9	11.3	10.1	8.6		
1944	16.1	24.9	24.1	23.1	23.3	24.5	22.1		
1945	6.7	6.9	6.1	9.9	8.1	6.1	3.9		
1946	11.7	12.9	20.9	15.1	28.4	23.5	26.2		
1947	18.7	20.4	22.8	24.1	21.2	15.2	13.5		
1948	18.1	33.0			24.9			34.4	34.4
1949	9.8	15.9			20.9			17.4	19.7
1950	20.3	24.8			23.4			26.4	21.4
1951	8.4	18.5			25.9			21.4	24.2
1952	8.7	15.8			12.0			17.1	16.7
1953	14.7	24.5			21.6			32.0	32.1
1954	12.7	15.6			15.0			12.5	15.3
1955	7.8	8.0			3.3			5.4	2.5
1956	19.6	19.2			12.3			15.1	15.6
1957	13.3	15.3			20.8			15.8	17.0
1958	28.7	24.2			37.5			36.9	35.7
1959	28.1	27.0			44.5			39.5	39.4

Source of Data: (13)

SELECTED BIBLIOGRAPHY

(1) Walker, O. L. "Game Theory Applications in Agricultural Decisions." Ph.D. Thesis, Iowa State College Library, 1959.

(2) Knight, F. H. Risk, Uncertainty, and Profit. New York: Houghton Mifflin Co., 1921.

(3) Johnson, G. L., and C. B. Haver. "Decision Making Principles in Farm Management." Kentucky Agricultural Experiment Station Bulletin 593, January, 1953.

(4) Luce, R. Duncan, and Howard Raiffa. Games and Decisions: Introduction and Critical Survey. New York: John Wiley and Sons, Inc., 1957.

(5) Leftwich, Richard H. The Price System and Resource Allocation. New York: Holt. Rinehart, and Winston, 1960.

(6) Plaxico, J. S., Paul Andrilinas, and L. S. Pope. "Economic Analysis of a Concentrate Roughage Ratio Experiment." Oklahoma State University Agricultural Experiment Station Processed Series, P-310, January, 1959.

(7) Doll, John P., Emil H. Jebe, and Robert D. Munson. "Computation of Variance Estimates for Marginal Physical Product and Marginal Rate of Substitution." Journal of Farm Economics, XLII (August, 1960).

(8) Ostle, Bernard. Statistics in Research. Ames: Iowa State College Press, 1956.

(9) Harper, H. J. "Effect of Soil Fertility and Climatic Environment on Cotton Production in Central Oklahoma." Oklahoma State University Agricultural Experiment Station, Unpublished Manuscript, 1960.

(10) Heady, E. O., John T. Pesek and William G. Brown. "Crop Response Surfaces and Economic Optima in Fertilizer Use." Iowa State College Agricultural Experiment Station Research Bulletin 424, March, 1955.

(11) Brensing, O. H., and H. J. Harper. "Nitrogen Fertilization of Corn with Supplemental Irrigation." Oklahoma State University Agricultural Experiment Station Bulletin B-557, June, 1960.

(12) Eck, H. V., and Bobby A. Stewart. "Nitrogen Fertilization of Winter Oats." Oklahoma State University Agricultural Experiment Station Bulletin B-491, May, 1957.

(13) Harper, H. J. "65 Years of Continuous Wheat." Oklahoma State University Agricultural Experiment Station Bulletin B-531, May, 1959.

(14) Walker, O. L., Earl O. Heady, Luther G. Tweeten, and John T. Pesek. "Application of Game Theory Models to Decisions on Farm Practices and Resource Use." Iowa State University Agricultural and Home Economics Experiment Station Research Bulletin 488, December, 1960.

(15) Wiggins, S. C., and C. E. Marshall. "The Effect of Nitrogen Fertilizer and Plant Spacing on Yields of Spinach and Snap Beans." Oklahoma State University Agricultural Experiment Station Processed Series P-339, February, 1960.

VITA

Thomas F. Pogue

Candidate for the Degree of

Master of Science

Thesis: ECONOMICS OF FERTILIZATION FOR SELECTED OKLAHOMA CROPS

Major Field: Agricultural Economics

Biographical:

Personal Data: Born in Roswell, New Mexico, December 28, 1935, the son of Talmadge F. and Lela Pogue.

Education: Attended grade school in Kentucky, Kansas, Texas, and New Mexico; graduated from Clovis, New Mexico High School in 1953; received the Bachelor of Science degree from New Mexico State University, major in Agricultural Economics, in June, 1957; completed requirements for the Master of Science degree in August, 1960.

Professional Experience: Entered Air Force in 1957; enrolled at Oklahoma State University in February, 1960 upon completion of tour of duty; has been a research assistant with the Department of Agricultural Economics since that date.