ANALYSIS OF CONTINUOUS COLUMN BEAMS ON ELASTIC SUPPORTS BY CARRY OVER MOMENTS

By

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F.P.N.

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NOMENCLATURE

h, i, j, k, l . . . . Letters designating intermediate supports.

m_j . . . . Starting moments.

w(x). . . . . Equation of load variation.

BR . . . . Basic reaction due to loads.

BV . . . . Basic shear due to loads.

E . . . . Modulus of elasticity.

F . . . . Flexibility.

F' . . . . Modified Flexibility.

F* . . . . Final flexibility.

G . . . . Carry over value.

G' . . . . Modified carry over value.

G* . . . . Final carry over value.

I . . . . Moment of inertia.

L . . . . Length of span.

M_i, M_j . . . . . . Moments at i and j respectively.

Q . . . . Deflection coefficient.

γ . . . . The reciprocal of the equivalent spring constant.

λ . . . . The square root of the axial force divided by EI.

τ^L . . . . Load function.

$\tau^{L'}$. . . . . Modified load function.

τ^{L*} . . . . Final load function.

CHAPTER I

INTRODUCTION

1.1 Historical Background

The basic formulas for column beams were first published by Muller-Bresleau in about 1902.(1) They were complicated to such an extent as to be of little practical value. During the First World War, Muller-Bresleau worked out his formulas in more detail and supplied tables of complex functions that made possible their use in practical design.(2) At the same time the English, working independently, developed similar equations and tables. The English equations and tables were put into their most useful form by Arthur Berry.(3) The resulting tables usually are known as the Berry Functions.

During the First World War, the United States Navy adopted Berry's equations and tables. The Engineering Division of the Army Air Corps proved in 1922 that Berry's Equations and the Muller-Bresleau equations were fundamentally the same, differing only in choice of orgin and nomenclature.(4)

In 1935, J.E. Younger published the development of the three moment equation.(5) His equation was derived for any general loading, allowed for displacement of supports, but was limited to beams of constant cross section within any one given span. A similar equation was published by Niles and Newell in 1938, although it was limited to a loading variation of first degree only.(6) Younger utilizes the Berry Functions and a copy of them can be found in his book. Work along similar lines was done by Hawranek and Stienhardt in their publication in 1958, although they were concerned primarily with the problem of buckling.[7]

1.2 The Three Moment Equation

The standard three moment equation, modified only in the sense of nomenclature, is given here along with a definition and physical interpretation of the constants involved. Referring to the typical two span segment $\overline{ijk}$ in Figure 1, the three moment equation is as follows:

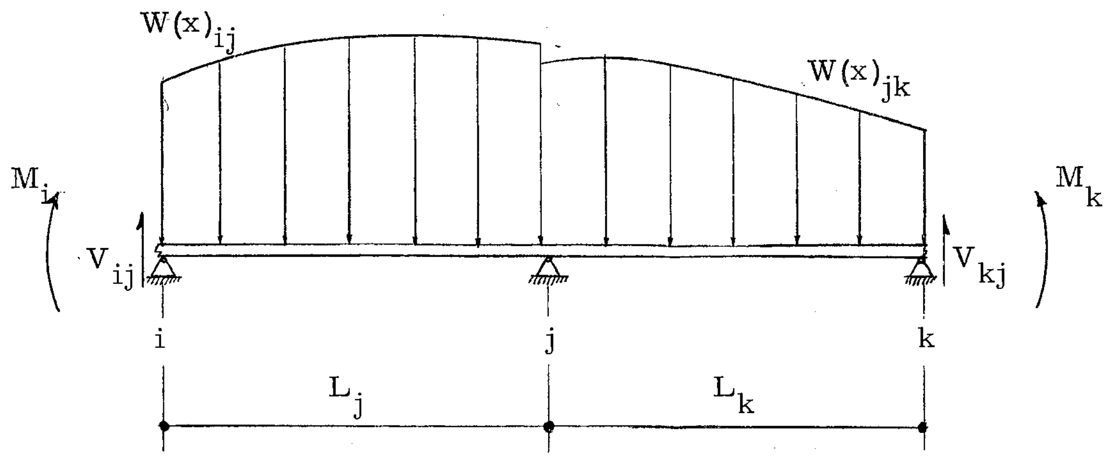


FIGURE 1

$$M_i G_{ij} + M_j(F_{ji} + F_{jk}) + M_k G_{kj} + \Sigma\tau_j^L + \Sigma\tau_j^\Delta = 0.$$

The algebraic expression for each of the constants that appear is:

$$G_{ij} = \frac{L_j}{E_j I_j}\left(\frac{\lambda_j L_j \operatorname{cosec} \lambda_j L_j - 1}{(\lambda_j L_j)^2}\right) \qquad \text{Eq. } 1$$

$$F_{ji} = \frac{L_j}{E_j I_j}\left(\frac{1 - \lambda_j L_j \cot \lambda_j L_j}{(\lambda_j L_j)^2}\right) \qquad \text{Eq. } 2$$

$$F_{jk} = \frac{L_k}{E_k I_k}\left(\frac{1 - \lambda_k L_k \cot \lambda_k L_k}{\left(\lambda_k L_k\right)^2} \right) \qquad \text{Eq. 3}$$

$$G_{kj} = \frac{L_k}{E_k I_k}\left(\frac{\lambda_k L_k \operatorname{cosec} \lambda_k L_k - 1}{\left(\lambda_k L_k\right)^2} \right) \qquad \text{Eq. 4}$$

$$\tau_j^L = \tau_{ji}^L + \tau_{jk}^L$$

$$\tau_{ji}^L = \int_o^{L_j} \left(\frac{\sin \lambda_j X}{\left(\lambda_j L_j\right)^2 \sin \lambda_j L_j} - \frac{X}{L_j \left(\lambda_j L_j\right)^2} \right) W(x)_{ij}\, dX \qquad \text{Eq. 5}$$

$$\tau_{jk}^L = \int_o^{L_k} \left(\frac{\sin \lambda_k X}{\left(\lambda_k L_k\right)^2 \sin \lambda_k L_k} - \frac{X}{L_k \left(\lambda_k L_k\right)^2} \right) W(x)_{jk}\, dX \qquad \text{Eq. 6}$$

$$\tau_j^\Delta = \tau_{ji}^\Delta + \tau_{jk}^\Delta$$

$$\tau_{ji}^\Delta = - \frac{\Delta_j - \Delta_i}{L_j} \qquad \tau_{jk}^\Delta = - \frac{\Delta_j - \Delta_k}{L_k} \qquad \text{Eq's 7 \& 8}$$

where

$W(x)_{ij}$ is the load variation in span $\overline{ij}$ from i to j.

$W(x)_{jk}$ is the load variation in span $\overline{jk}$ from k to j.

$$\lambda_j = \sqrt{\frac{N_j}{E_j I_j}} \qquad \lambda_k = \sqrt{\frac{N_k}{E_k I_k}}$$

1. 3 Physical Interpretation

1. F's (flexibilities) and G's (carry over values)

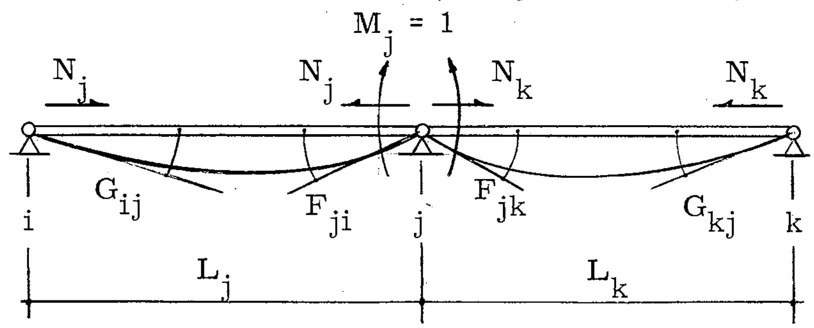


FIGURE 2

2. τ^L's (load functions)

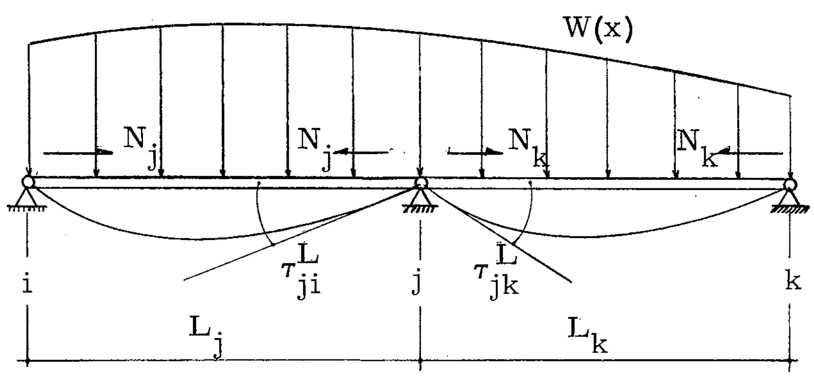


FIGURE 3

3. τ^Δ's (deflection functions)

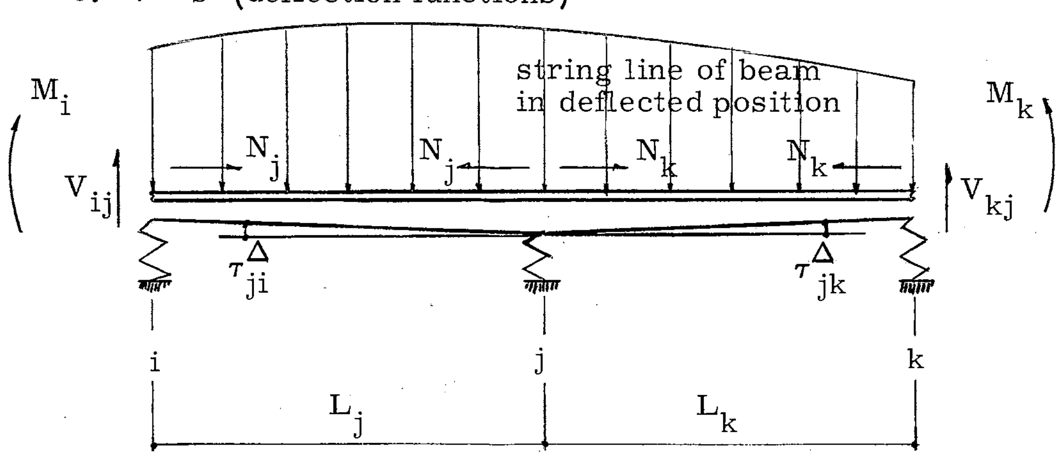


FIGURE 4

The elastic constants and load functions may be defined as follows:

G_{ij} is the end slope of a simple beam $\overline{ij}$ at i due to a unit moment applied at j, the existing axial loads being present.

F_{ji} is the end slope of a simple beam $\overline{ij}$ at j due to a unit moment applied at j, the existing axial forces being present.

τ_{ji} is the end slope of a simple beam $\overline{ij}$ at j due to loads, the existing axial loads being present.

The elastic constants and load functions corresponding to span $\overline{jk}$ are similarily defined.

It is readily seen that the primary drawback to the three moment equation is the necessity for an additional equation or equations to solve for the unknown support deflections. The purpose of this thesis is to eliminate this problem by developing a five moment equation involving redundant moments and load terms only. The procedure is similar to that used by Tuma and Havner,(8) who developed the five moment equation for continuous beams on elastic supports. The procedure is as follows:

1. Write expressions for the end shears of each span in terms of the redundant end moments, axial forces, loads and support deflections.
2. Write expressions for the support reactions using the equations obtained in step 1.
3. Write three separate three moment equations for supports i, j, and k.
4. Solve the equations resulting from steps 1, 2, and 3 for the unknown support deflections.

Using the equations for the support deflections, an expression can be written for the deflection function of the three moment equation. Substituting this expression into the three moment equation written at joint j, the five moment equation is obtained.

CHAPTER II

THE FIVE MOMENT EQUATION

2.1 Development of the Five Moment Equation

Using the typical four span segment of a continuous column beam shown in Figure 5, it is possible to arrive at a general expression for the deflection term of the three moment equation in terms of known

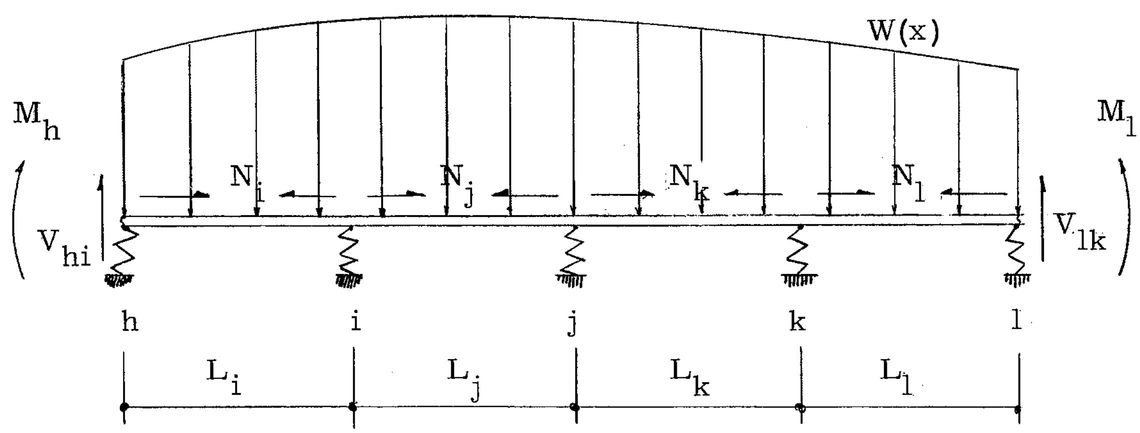


FIGURE 5

load functions and the redundant end moments. By isolating any typical span $\overline{ij}$, an expression can be written for the end shears of that span.

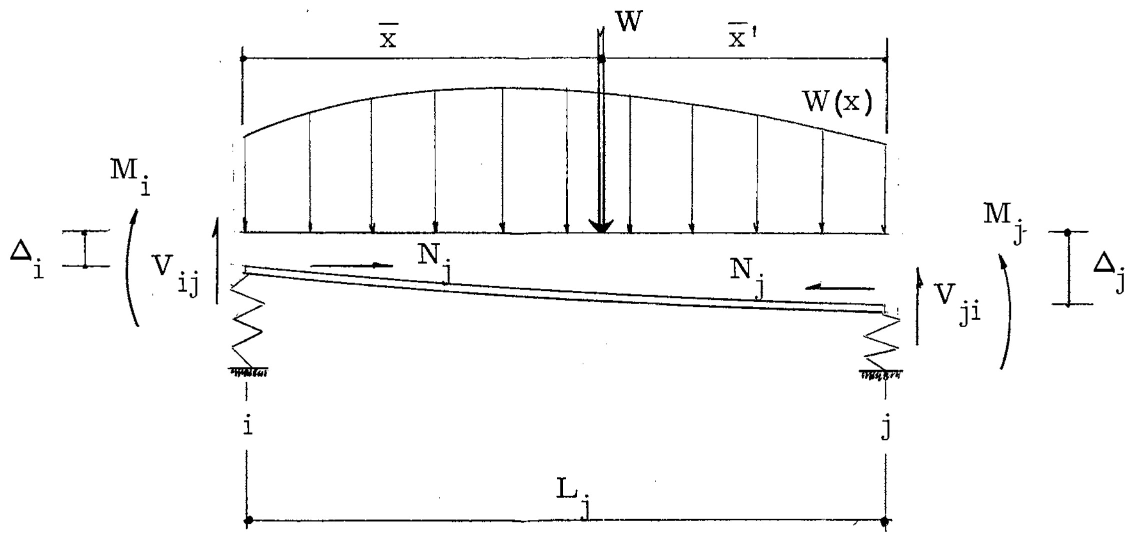


FIGURE 6

$$\Sigma M_j = 0$$

$$V_{ji}L_j - W\bar{X}' - M_i + M_j - N_j(\Delta_j - \Delta_i) = 0$$

Solving for V_{ji} :

$$V_{ji} = BV_{ji} + \frac{M_i - M_j}{L_j} + \frac{N_j}{L_j}(\Delta_j - \Delta_i)$$

Where: BV_{ji} is the end shear of the simple beam $\overline{ij}$ at j due to loads. Similarly, the end shears for each of the spans in Figure 5 can be expressed as follows:

$$V_{ih} = BV_{ih} - \frac{M_i - M_h}{L_i} + \frac{N_i}{L_i}(\Delta_i - \Delta_h) \qquad \text{Eq. 9}$$

$$V_{ij} = BV_{ij} + \frac{M_j - M_i}{L_j} - \frac{N_j}{L_j}(\Delta_j - \Delta_i) \qquad \text{Eq. 10}$$

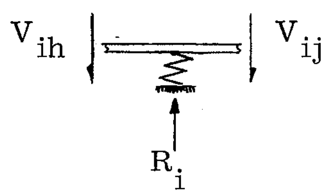
$$V_{ji} = BV_{ji} - \frac{M_j - M_i}{L_j} + \frac{N_j}{L_j}(\Delta_j - \Delta_i) \qquad \text{Eq. 11}$$

$$V_{jk} = BV_{jk} + \frac{M_k - M_j}{L_k} + \frac{N_k}{L_k}(\Delta_j - \Delta_k) \qquad \text{Eq. 12}$$

$$V_{kj} = BV_{kj} - \frac{M_k - M_j}{L_k} - \frac{N_k}{L_k}(\Delta_j - \Delta_k) \qquad \text{Eq. 13}$$

$$V_{kl} = BV_{kl} + \frac{M_l - M_k}{L_l} + \frac{N_l}{L_l}(\Delta_k - \Delta_l) \qquad \text{Eq. 14}$$

Having the end shears, one can isolate a single support and write an expression for the reaction of that support.



$$\Sigma F_v = 0$$

$$R_i = V_{ij} + V_{ih}$$

Substituting Equations 9 and 10 for the end shears, the equation yielding the reaction at joint i is:

$$R_i = BV_{ih} + BV_{ij} + \frac{M_h}{L_i} - M_i\left(\frac{1}{L_i} + \frac{1}{L_j}\right) + \frac{M_j}{L_j}$$

$$+ \frac{N_i}{L_i}(\Delta_i - \Delta_h) - \frac{N_j}{L_j}(\Delta_j - \Delta_i)$$

Denoting the sum of the basic shears at joint i equal to BR_i, the following expressions can be written for each of the reactions at supports i, j, and k:

$$R_i = BR_i + \frac{M_h}{L_i} - M_i\left(\frac{1}{L_i} + \frac{1}{L_j}\right) + \frac{M_j}{L_j} + \frac{N_j}{N_i}(\Delta_i - \Delta_h)$$

$$- \frac{N_j}{L_j}(\Delta_j - \Delta_i) \qquad \text{Eq. 15}$$

$$R_j = BR_j + \frac{M_i}{L_j} - M_j\left(\frac{1}{L_j} + \frac{1}{L_k}\right) + \frac{M_k}{L_k} + \frac{N_j}{L_j}(\Delta_j - \Delta_i)$$

$$- \frac{N_k}{L_k}(\Delta_k - \Delta_j) \qquad \text{Eq. 16}$$

$$R_k = BR_k + \frac{M_j}{L_k} - M_k\left(\frac{1}{L_k} + \frac{1}{L_l}\right) + \frac{M_l}{L_l} + \frac{N_k}{L_k}(\Delta_k - \Delta_j)$$

$$- \frac{N_l}{L_l}(\Delta_l - \Delta_k) \qquad \text{Eq. 17}$$

Isolating the two span segment $\overline{hij}$ and writing the corresponding three moment equation, an expression for τ_{ih}^{Δ} can be obtained.

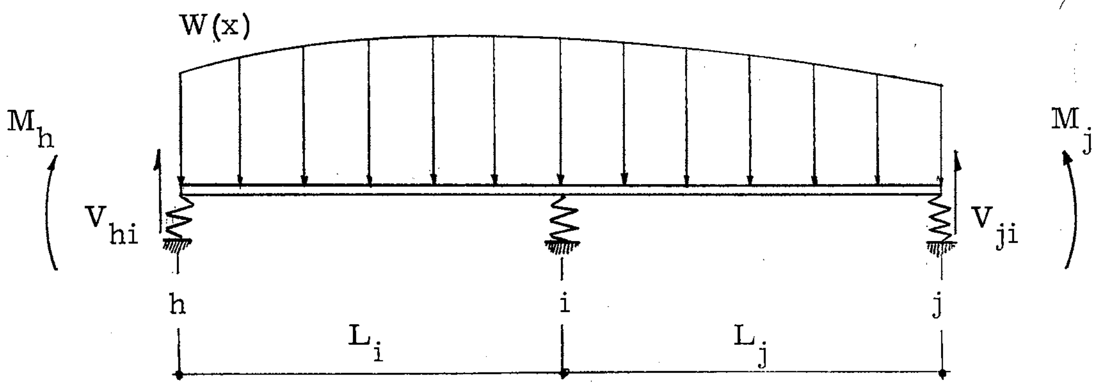


FIGURE 7

$$-\tau_{ih}^{\Delta} = M_h G_{hi} + (F_{ih} + F_{ij}) M_i + M_j G_{ji} + \Sigma\tau_i^{L} + \tau_{ij}^{\Delta}$$

Eq. 18

Similarly, from spans $\overline{jkl}$:

$$-\tau_{kl}^{\Delta} = M_j G_{jk} + (F_{kj} + F_{kl}) M_k + M_l G_{lk} + \Sigma\tau_k^{L} + \tau_{kj}^{\Delta}$$

Eq. 19

Utilizing Equations 7 and 8 and substituting Equations 18 and 19 into Equations 15 and 17 respectively:

$$R_i = BR_i + M_h\left(\frac{1}{L_i} + N_iG_{hi}\right) - M_i\left(\frac{1}{L_i} + \frac{1}{L_j} - N_i\,\Sigma F_i\right)$$

$$+ M_j\left(\frac{1}{L_j} + N_iG_{ji}\right) + \tau_{ij}^{\Delta}(N_i - N_j) + N_i\Sigma\tau_i^L$$

$$R_k = BR_k + M_j\left(\frac{1}{L_k} + N_lG_{jk}\right) - M_k\left(\frac{1}{L_k} + \frac{1}{L_l} - N_l\Sigma F_k\right)$$

$$+ M_l\left(\frac{1}{L_l} + N_lG_{lk}\right) + \tau_{kj}^{\Delta}(N_k - N_l) + N_l\,\Sigma\tau_k^L\ .$$

Since the reaction of support i is equal to the spring constant of support i multiplied by the displacement of that support:

$$R_i = C_i\,\Delta_i \qquad \text{Eq. 20}$$

utilizing Equations 7, 8, and 20:

$$C_i\,\Delta_i = BR_i + M_h\left(\frac{1}{L_i} + N_iG_{hi}\right) - M_i\left(\frac{1}{L_i} + \frac{1}{L_j} - N_i\,\Sigma F_i\right)$$

$$+ M_j\left(\frac{1}{L_j} + N_iG_{ji}\right) + \frac{\Delta_j}{L_j}(N_i - N_j) - \frac{\Delta_i}{L_j}(N_i - N_j)$$

$$+ N_i\ \Sigma\tau_i^L$$

rearranging:

$$\Delta_i = BR_i\gamma_{ij} + M_h\gamma_{ij}\left(\frac{1}{L_i} + N_iG_{hi}\right) - M_i\gamma_{ij}\left(\frac{1}{L_i} + \frac{1}{L_j} - N_i\Sigma F_i\right)$$

$$+ M_j\gamma_{ij}\left(\frac{1}{L_j} + N_iG_{ji}\right) - \frac{\Delta_j\gamma_{ij}}{L_j}(N_j - N_i) + \gamma_{ij}N_i\Sigma\tau_i^L$$

Eq. 21

similarly:

$$\Delta_k = BR_k\gamma_{kj} + M_j\gamma_{kj}\left(\frac{1}{L_k} + N_1G_{jk}\right) - M_k\gamma_{kj}\left(\frac{1}{L_k} + \frac{1}{L_1} - N_1\Sigma F_k\right) + M_1\gamma_{k_j}\left(\frac{1}{L_1} + N_1G_{1k}\right) - \frac{\Delta_j\gamma_{kj}}{L_k}(N_k - N_1) + N_1\gamma_{kj}\Sigma\tau_k^L \qquad \text{Eq. 22}$$

where:

γ_{ij} = the reciprocal of the equivalent spring constant.

$$\gamma_{ij} = \frac{1}{C_i + \frac{N_i - N_j}{L_j}}, \quad \gamma_{kj} = \frac{1}{C_k + \frac{N_1 - N_k}{L_k}}. \qquad \text{Eq. 22a \& 22b}$$

substituting Equations 21 and 22 into Equation 16, the equation for R_j becomes:

$$R_j = C_j\Delta_j = BR_j + \frac{M_i}{L_j} - M_j\left(\frac{1}{L_j} + \frac{1}{L_k}\right) + \frac{M_k}{L_k} + \Delta_j\left(\frac{N_j}{L_j} + \frac{N_k}{L_k}\right) - \frac{N_j}{L_j}\Bigg[\gamma_{ij}BR_i + M_h\gamma_{ij}\left(\frac{1}{L_i} + N_iG_{hi}\right) - M_i\gamma_{ij}\left(\frac{1}{L_i} + \frac{1}{L_j} - N_i\Sigma F_i\right) + M_j\gamma_{ij}\left(\frac{1}{L_j} + N_iG_{ji}\right) + \gamma_{ij}N_i\Sigma\tau_i^L - \frac{\Delta_j\gamma_{ij}(N_j - N_i)}{L_j}\Bigg] - \frac{N_k}{L_k}\Bigg[\gamma_{kj}BR_k + M_j\gamma_{kj}\left(\frac{1}{L_k} + N_1G_{jk}\right) - M_k\gamma_{kj}\left(\frac{1}{L_k} + \frac{1}{L_1} - N_1\Sigma F_k\right) + M_1\gamma_{kj}\left(\frac{1}{L_1} + N_1G_{1k}\right) + N_1\gamma_{kj}\Sigma\tau_k^L - \frac{\Delta_j\gamma_{kj}}{L_k}(N_k - N_1)\Bigg]$$

rearranging:

$$\Delta_j C_j = BR_j - M_h\left[\frac{\gamma_{ij}N_j}{L_j}\left(\frac{1}{L_i} + N_i G_{hi}\right)\right] + M_i\left[\frac{1}{L_j} + \frac{N_j\gamma_{ij}}{L_j}\left(\right.\right.$$

$$\left.\left.\frac{1}{L_i} + \frac{1}{L_j} - N_i\Sigma F_i\right)\right] - M_j\left[\frac{1}{L_j} + \frac{1}{L_k} + \frac{N_j\gamma_{ij}}{L_j}\left(\right.\right.$$

$$\left.\left.\frac{1}{L_j} + N_i G_{ji}\right)\right] + N_k\gamma_{kj}\left(\frac{1}{L_k} + N_l G_{jk}\right) + M_k\left[\right.$$

$$\left.\frac{1}{L_k} + \frac{N_k\gamma_{kj}}{L_k}\left(\frac{1}{L_k} + \frac{1}{L_l} - N_l\Sigma F_k\right)\right] - M_l\left[\frac{N_k\gamma_{kj}}{L_k}\left(\frac{1}{L_l}\right.\right.$$

$$\left.\left. + N_l G_{lk}\right)\right] - \frac{N_j\gamma_{ij}N_i\Sigma\tau_i^L}{L_j} - \frac{N_k\gamma_{kj}N_l\Sigma\tau_k^L}{L_k}$$

$$\Delta_j\left[\frac{N_j}{L_j} + \frac{N_k}{L_k} + \frac{\gamma_{ij}N_j}{L_j^2}(N_j - N_i) - \frac{\gamma_{kj}N_k}{L_k^2}(N_k - N_l)\right]$$

denoting:

$$\gamma_{jj} = \frac{1}{C_j - \dfrac{N_j C_i \gamma_{ij}}{L_j} - \dfrac{N_k C_k \gamma_{kj}}{L_k}} \qquad \text{Eq. 22-c}$$

the final equation for Δ_j becomes:

$$\Delta_j = \gamma_{jj}BR_j - M_h\gamma_{jj}\left[\frac{\gamma_{ij}N_j}{L_j}\left(\frac{1}{L_i} + N_i G_{ji}\right) + M_i\gamma_{jj}\left[\right.\right.$$

$$\left.\frac{1}{L_j} + \frac{N_j\gamma_{ij}}{L_j}\left(\frac{1}{L_i} + \frac{1}{L_j} - N_i\Sigma F_i\right)\right] - M_j\gamma_{jj}\left[\frac{1}{L_j}\right.$$

$$+ \frac{1}{L_k} + \frac{N_j\gamma_{ij}}{L_j}\left(\frac{1}{L_j} + N_i G_{ji}\right) + \frac{N_k\gamma_{kj}}{L_k}\left(\frac{1}{L_k}\right.$$

$$\left.\left. + N_l G_{jk}\right)\right] + M_k\gamma_{ji}\left[\frac{1}{L_k} + \frac{N_k\gamma_{kj}}{L_k}\left(\frac{1}{L_k} + \frac{1}{L_l} - N_l\Sigma F_k\right)\right] -$$

$$-M_l \gamma_{jj}\left[\frac{\gamma_{kj} N_k}{L_k}\left(\frac{1}{L_l} + N_l G_{lk}\right)\right] - \frac{N_i N_j \gamma_{ij} \gamma_{jj} \Sigma \tau_i^L}{L_j}$$

$$- \frac{N_k N_l \gamma_{kj} \gamma_{jj} \Sigma \tau_k^L}{L_k} . \qquad \text{Eq. 23}$$

Denoting:

$$Q_{hj} = - \frac{\gamma_{ij}\gamma_{jj} N_j}{L_j}\left(\frac{1}{L_i} + N_i G_{hi}\right) \qquad \text{Eq. 24}$$

$$Q_{ij} = \gamma_{jj}\left[\frac{1}{L_j} + \frac{N_j \gamma_{ij}}{L_j}\left(\frac{1}{L_i} + \frac{1}{L_j} - N_i \Sigma F_i\right)\right] \qquad \text{Eq. 25}$$

$$Q_{kj} = \gamma_{jj}\left[\frac{1}{L_k} + \frac{N_k \gamma_{kj}}{L_k}\left(\frac{1}{L_k} + \frac{1}{L_l} - N_l \Sigma F_k\right)\right] \qquad \text{Eq. 26}$$

$$\Sigma Q_j = Q_{ji} + Q_{jk} \qquad \text{Eq. 27}$$

$$Q_{ji} = - \gamma_{jj}\left[\frac{1}{L_j} + \frac{N_j \gamma_{ij}}{L_j}\left(\frac{1}{L_j} + N_i G_{ji}\right)\right]$$

$$Q_{jk} = - \gamma_{jj}\left[\frac{1}{L_k} + \frac{N_k \gamma_{kj}}{L_k}\left(\frac{1}{L_k} + N_l G_{jk}\right)\right]$$

$$\Sigma Q_j^* = \gamma_{jj} BR_j + Q_{ji}^* + Q_{jk}^* \qquad \text{Eq. 28}$$

$$Q_{ji}^* = - \frac{N_i N_j \gamma_{ij} \gamma_{jj} \Sigma \tau_i^L}{L_j}$$

$$Q_{jk}^* = - \frac{N_k N_l \gamma_{kj} \gamma_{jj} \Sigma \tau_k^L}{L_k}$$

Equation 23 can be written

$$\Delta_j = Q_{hj} M_h + Q_{ij} M_i + \Sigma Q_j M_j + Q_{kj} M_k + Q_{lj} M_l + \Sigma Q_j^* .$$

$$\text{Eq. 29}$$

Using Equations 21 and 29, the following expression is obtained for Δ_i.

$$
\begin{aligned}
\Delta_i = {} & \gamma_{ij} BR_i + M_h\left[\frac{\gamma_{ij} Q_{hj}}{L_j}(N_j - N_i) - \gamma_{ij}\left(\frac{1}{L_i} + N_i G_{hi}\right)\right] \\
& - M_i \gamma_{ij}\left[\frac{1}{L_i} + \frac{1}{L_j} - N_i \Sigma F_i + \frac{Q_{ij}}{L_j}(N_j - N_i)\right] \\
& + M_j \gamma_{ij}\left[\frac{1}{L_j} + N_i G_{ji} - \frac{\Sigma Q_j}{L_j}(N_j - N_i)\right] \\
& - \frac{M_k \gamma_{ij} Q_{kj}}{L_j}(N_j - N_i) - \frac{M_l \gamma_{ij} Q_{lj}}{L_j}(N_j - N_i) \\
& + \gamma_{ij} N_i \Sigma \tau_i^L - \frac{\gamma_{ij} \Sigma Q_j^*}{L_j}(N_j - N_i) \, . \qquad \text{Eq. 30}
\end{aligned}
$$

Similarly, from Equations 22 and 29

$$
\begin{aligned}
\Delta_k = {} & \gamma_{kj} BR_k - M_h\left[\frac{\gamma_{kj} Q_{hj}}{L_k}(N_k - N_l)\right] - \frac{M_i \gamma_{kj} Q_{ij}}{L_k}(N_k - N_l) \\
& + M_j \gamma_{kj}\left[\frac{1}{L_k} + N_l G_{jk} - \frac{\Sigma Q_j}{L_k}(N_k - N_l)\right] \\
& - M_k \gamma_{kj}\left[\frac{1}{L_k} + \frac{1}{L_l} - N_l \Sigma F_k - \frac{Q_{kj}}{L_k}(N_k - N_l)\right] \\
& + M_l \gamma_{kj}\left[\frac{1}{L_l} + N_l G_{lk} - \frac{Q_{lj}}{L_k}(N_k - N_l)\right] + N_l \gamma_{kj} \Sigma \tau_k^L \\
& + \frac{\Sigma Q_j^* \gamma_{kj}}{L_k}(N_k - N_l) \, . \qquad \text{Eq. 31}
\end{aligned}
$$

Using Equations 29, 30, and 31, the following solution is obtained for the deflection function of the three moment equation.

$$\tau_{ji}^{\Delta} = -\frac{M_h}{L_j}\left[Q_{hj}\left(1 + \frac{(N_j - N_i)\gamma_{ij}}{L_j}\right) - \gamma_{ij}\left(\frac{1}{L_i} + N_i G_{hi}\right)\right]$$

$$-\frac{M_i}{L_j}\left[Q_{ij}\left(1 + \frac{(N_j - N_i)\gamma_{ij}}{L_j}\right) + \gamma_{ij}\left(\frac{1}{L_i} + \frac{1}{L_j} - N_i \Sigma F_i\right)\right]$$

$$-\frac{M_j}{L_j}\left[\Sigma Q_j\left(1 + \frac{(N_j - N_i)\gamma_{ij}}{L_j}\right) - \gamma_{ij}\left(\frac{1}{L_j} + N_i G_{ji}\right)\right]$$

$$-\frac{M_k}{L_j}\left[Q_{kj}\left(1 + \frac{(N_j - N_i)\gamma_{ij}}{L_j}\right)\right] - M_l\left[Q_{lj}\left(1 + \frac{(N_j - N_i)\gamma_{ij}}{L_j}\right)\right]$$

$$-\frac{\Sigma Q_j^*}{L_j} + \frac{\gamma_{ij} BR_i}{L_j} + \frac{\gamma_{ij} N_i \Sigma \tau_i^L}{L_j} - \frac{\gamma_{ij}}{L_j^2}(N_j - N_i)\Sigma Q_j^* .$$

Rearranging:

$$\tau_{ji}^{\Delta} = -\frac{M_h}{L_j}\left[Q_{hj} C_i \gamma_{ij} - \gamma_{ij}\left(\frac{1}{L_i} + N_i G_{hi}\right)\right]$$

$$-\frac{M_i}{L_j}\left[Q_{ij} C_i \gamma_{ij} + \gamma_{ij}\left(\frac{1}{L_i} + \frac{1}{L_j} - N_i \Sigma F_i\right)\right]$$

$$-\frac{M_j}{L_j}\left[\Sigma Q_j C_i \gamma_{ij} - \gamma_{ij}\left(\frac{1}{L_j} + N_i G_{ji}\right)\right] - \frac{M_k Q_{kj} \gamma_{ij} C_i}{L_j}$$

$$-\frac{M_l Q_{lj} \gamma_{ij} C_i}{L_j} - \frac{\Sigma Q_j^* C_i \gamma_{ij}}{L_j} + \frac{\gamma_{ij}}{L_j}\left(BR_i + N_i \Sigma \tau_j^L\right) .$$

Eq. 32

Similarly, the expression for τ_{jk}^{Δ} becomes:

$$\tau_{jk}^{\Delta} = -\frac{M_h Q_{hj} C_k \gamma_{kj}}{L_k} - \frac{M_i Q_{ij} C_k \gamma_{kj}}{L_k}$$

$$-\frac{M_j}{L_k}\left[\Sigma Q_j C_k \gamma_{kj} - \gamma_{kj}\left(\frac{1}{L_k} + N_l G_{jk}\right)\right]$$

$$-\frac{M_k}{L_k}\left[Q_{kj} C_k \gamma_{kj} + \gamma_{kj}\left(\frac{1}{L_k} + \frac{1}{L_l} - N_l \Sigma F_k\right)\right] -$$

$$- \frac{M_1}{L_k}\left[Q_{1j}C_k\gamma_{kj} - \gamma_{kj}\left(\frac{1}{L_1} + N_1G_{1k}\right)\right] - \frac{\Sigma Q_j^*}{L_k} C_k\gamma_{kj}$$

$$+ \frac{\gamma_{kj}}{L_k}\ (BR_k + N_1\Sigma\tau_k^L)\ . \qquad \text{Eq. 33}$$

Thus the final equation of the deflection function in terms of the axial forces, loads, and redundant moments is obtained by combining Equations 32 and 33:

$$\Sigma\tau_j^{\Delta} = M_hG'_{hj} + M_iG_{ij} + M_j\Sigma F'_j + M_kG'_{kj}$$

$$+ M_1G'_{1j} + \Sigma\tau'_j \qquad \text{Eq. 34}$$

where:

$$G_{hj} = - Q_{hj}\left[\frac{C_k\gamma_{kj}}{L_k} + \frac{C_i\gamma_{ij}}{L_j}\right] + \frac{\gamma_{ij}}{L_j}\left[\frac{1}{L_i} + N_iG_{hi}\right]$$

$$G'_{ij} = - Q_{ij}\left[\frac{C_k\gamma_{kj}}{L_k} + \frac{C_i\gamma_{ij}}{L_i}\right] - \frac{\gamma_{ij}}{L_j}\left[\frac{1}{L_i} + \frac{1}{L_j} - N_i\Sigma F_i\right]$$

$$F'_{ji} = - \Sigma Q_j\frac{C_i\gamma_{ij}}{L_j} + \frac{\gamma_{ij}}{L_j}\left[\frac{1}{L_j} + N_iG_{ji}\right]$$

$$F'_{jk} = - \Sigma Q_j\frac{C_k\gamma_{kj}}{L_k} + \frac{\gamma_{kj}}{L_k}\left[\frac{1}{L_k} + N_1G_{jk}\right]$$

$$G'_{kj} = - Q_{kj}\left[\frac{C_k\gamma_{kj}}{L_k} + \frac{C_i\gamma_{ij}}{L_j}\right] - \frac{\gamma_{kj}}{L_k}\left[\frac{1}{L_k} + \frac{1}{L_1} - N_1\Sigma F_k\right]$$

$$G'_{1j} = - Q_{1j}\left[\frac{C_k\gamma_{kj}}{L_k} + \frac{C_i\gamma_{ij}}{L_j}\right] + \frac{\gamma_{kj}}{L_k}\left[\frac{1}{L_1} + N_1G_{1k}\right]$$

$$\tau'_{ji} = - \frac{\Sigma Q_j^*\gamma_{ij}C_i}{L_j} + \frac{\gamma_{ij}}{L_j}\ (BR_i + N_i\Sigma\tau_i^L)$$

$$\tau'_{jk} = - \frac{\Sigma Q_j^*\gamma_{jk}C_k}{L_k} + \frac{\gamma_{kj}}{L_k}\ (BR_k + N_1\Sigma\tau_k^L)\ .$$

Substituting Equation 34 into the three moment equation for spans $\overline{ijk}$, the five moment equation is obtained.

$$G'_{hj}M_h = (G'_{ij} + G_{ij})M_i + (\Sigma F'_i + \Sigma F_i)M_j + (G'_{kj} + G_{kj})M_k + G'_{lj}M_l + \Sigma\tau^L_j + \Sigma\tau'_j = 0. \qquad \text{Eq. 35}$$

Denoting:

$$G^*_{hj} = G'_{hj}$$

$$G^*_{ij} = G'_{ij} + G_{ij}$$

$$G^*_{kj} = G'_{kj} + G_{kj}$$

$$G^*_{lj} = G'_{lj}$$

$$F^*_j = F'_j + F_j$$

$$\tau^*_j = \tau^L_j + \tau'_j$$

Equation 35 becomes:

$$G^*_{hj}M_h + G^*_{ij}M_i + \Sigma F^*_j M_j + G^*_{kj}M_k + G^*_{lj}M_l + \Sigma\tau^*_j = 0 \qquad \text{Eq. 36}$$

Equation 36 is the final form of the five moment equation for a continuous column beam on elastic supports.

2.2 The Carry Over Moment Equation

The carry over moment equation is obtained by solving the five moment equation (Eq. 36) for the moment M_j. Thus:

$$M_j = -\frac{G^*_{hj}}{\Sigma F^*_j}M_h - \frac{G^*_{ij}}{\Sigma F^*_j}M_i - \frac{G^*_{kj}}{\Sigma F^*_j}M_k - \frac{G^*_{lj}}{\Sigma F^*_j}M_l - \frac{\Sigma\tau^*_j}{\Sigma F^*_j} \qquad \text{Eq. 37}$$

Denoting:

$$-\frac{G^*_{hj}}{\Sigma F^*_j} = r_{hj} \qquad -\frac{G^*_{kj}}{\Sigma F^*_j} = r_{kj}$$

$$-\frac{G^*_{ij}}{\Sigma F^*_j} = r_{ij} \qquad -\frac{G^*_{lj}}{\Sigma F^*_j} = r_{lj}$$

$$-\frac{\Sigma \tau^*_j}{\Sigma F^*_j} = m^*_j$$

the final form of the carry over moment equation is:

$$M_j = r_{hj}M_h + r_{ij}M_i + r_{kj}M_k + r_{lj}M_l + m^*_l \qquad \text{Eq. 38}$$

The carry over factors and starting moment which appear in Equation 38 may be defined as follows:

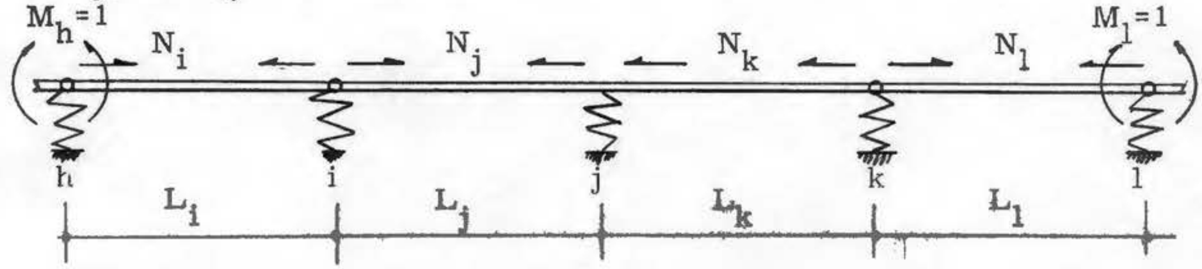
r_{hj} is the moment developed at joint j due to $M_h = 1$, if M_i, M_k, and M_l are zero.

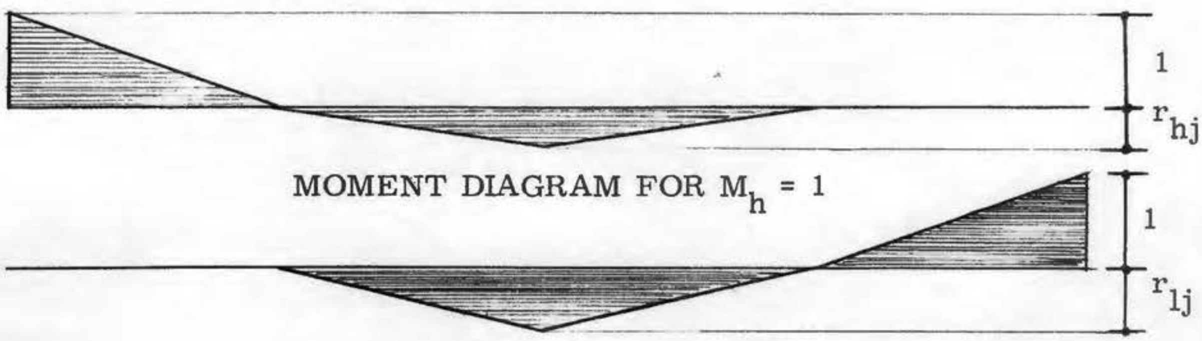
r_{ij} is the moment developed at joint j due to $M_i = 1$ if M_h, M_k, and M_l are zero.

m^*_j is the moment developed at j due to loads if the support moments at h, i, k, and l are zero.

A physical interpretation of these constants is given in Figures 8, 9, and 10.

1. r_{jh} and r_{lj} (far carry over factors)



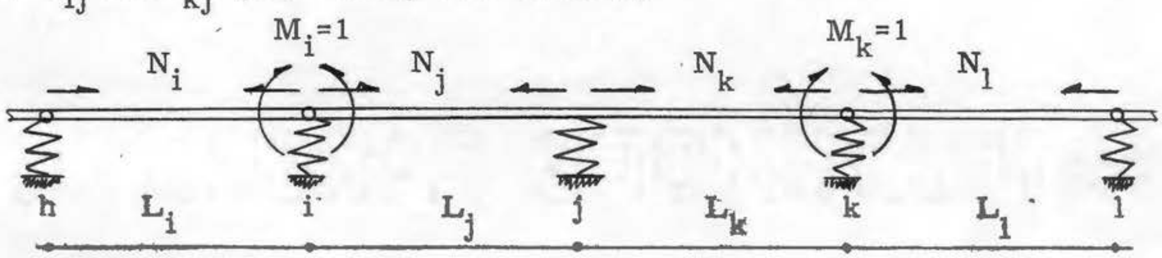


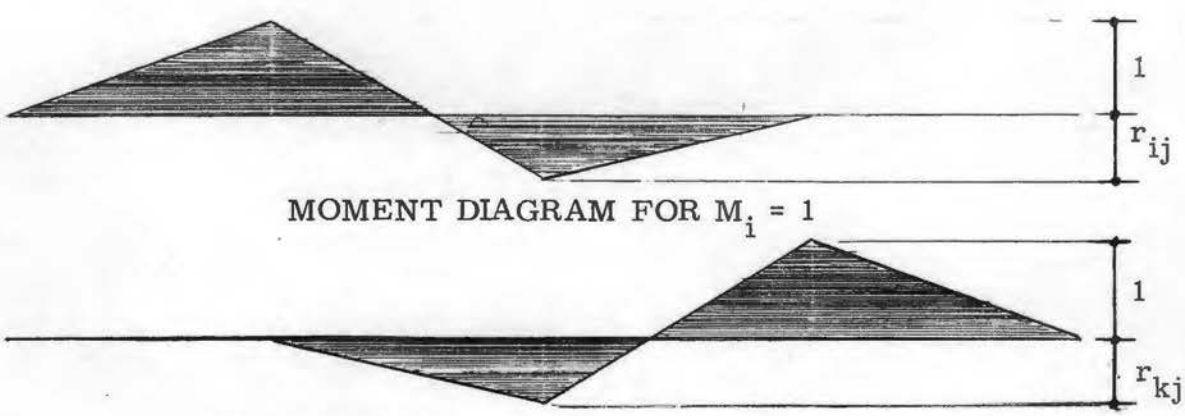
MOMENT DIAGRAM FOR M_h = 1

MOMENT DIAGRAM FOR M_l = 1

FIGURE 8

2. r_{ij} and r_{kj} (near carry over factors)

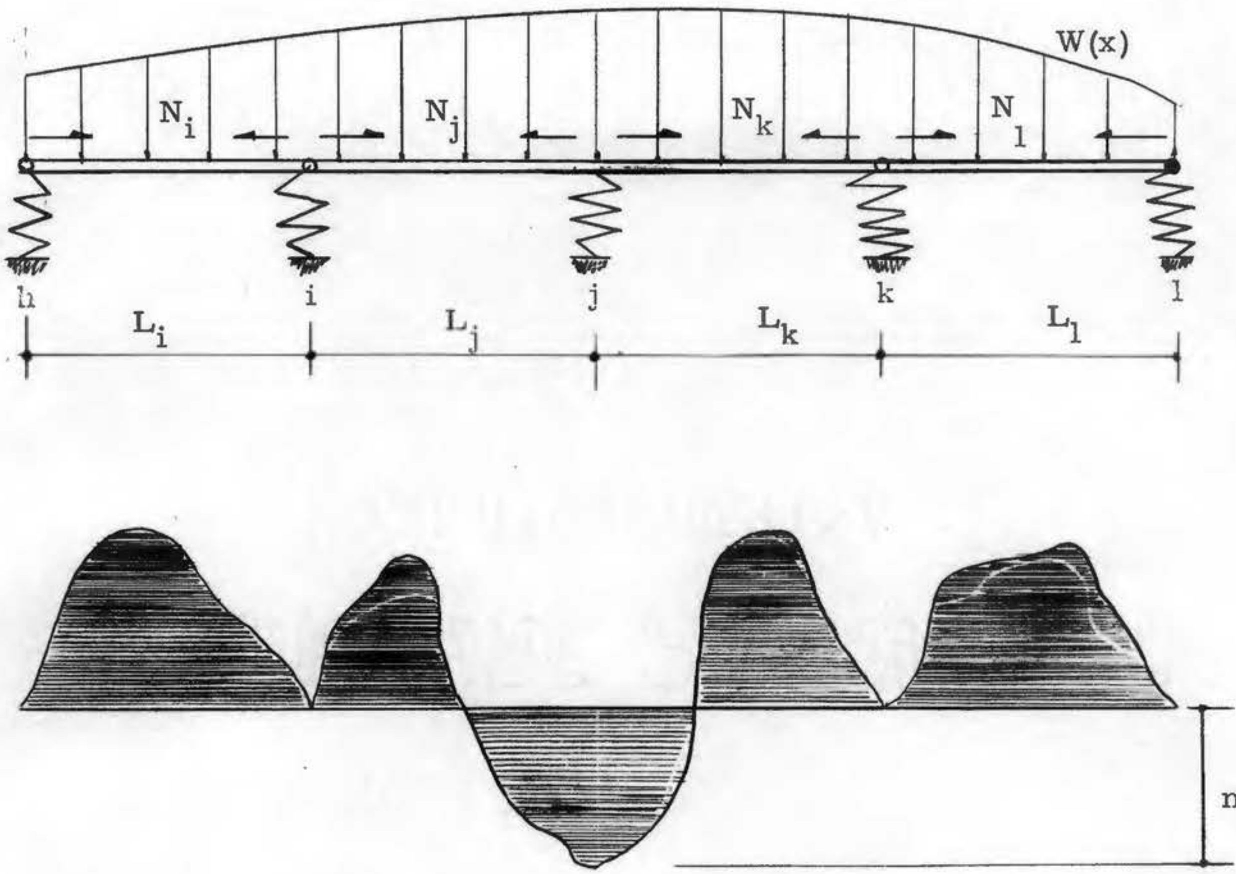




MOMENT DIAGRAM FOR M_i = 1

MOMENT DIAGRAM FOR M_k = 1

FIGURE 9



BENDING MOMENT DIAGRAM DUE TO LOADS

FIGURE 10

2-4. Conclusions

The problem of a continuous column-beam on elastic supports can be formulated as a five moment equation which relates the redundant moments over five consecutive supports to the axial and transverse loads. This is accomplished through the elimination of the deflection function of the well known three moment equation by expressing the deflections in terms of redundant moments and load functions.

To apply the five moment equation to the analysis of a continuous column-beam, it is necessary to compute three groups of beam and load constants, each group determined from the previous one. These are:

1. Angular functions F, G, τ (as defined for the three moment equation) and equivalent spring constants $1/\gamma$.
2. Q functions.
3. Modified angular functions F', G', τ'.

Thus, use of the five moment equation requires considerably more computation for the formulation of the matrix than does the three moment equation. However, for the three moment equation, the solution matrix requires, in addition to the moment equation at each support, a shear equation for each unknown support deflection. This implies that twice as many unknowns are involved as for the solution matrix for the five moment equation. It is reasonable to assume, therefore, that in problems involving a large number of spans, the five moment equation can be advantageously applied. To make a more definite relative evaluation of the two solutions would require an extensive comparative analysis beyond the scope of this thesis.

CHAPTER III

EXAMPLE PROBLEM

3.1 A Guyed Tower

The guyed tower shown in Figure 11a is analyzed as a continuous column-beam on elastic supports. The cross section of the tower mast is as shown in Figure 11b, and the guy arrangement, relative to the direction of the applied wind load, is shown in Figure 11c. The analogous column beam on elastic supports is shown in Figure 12.

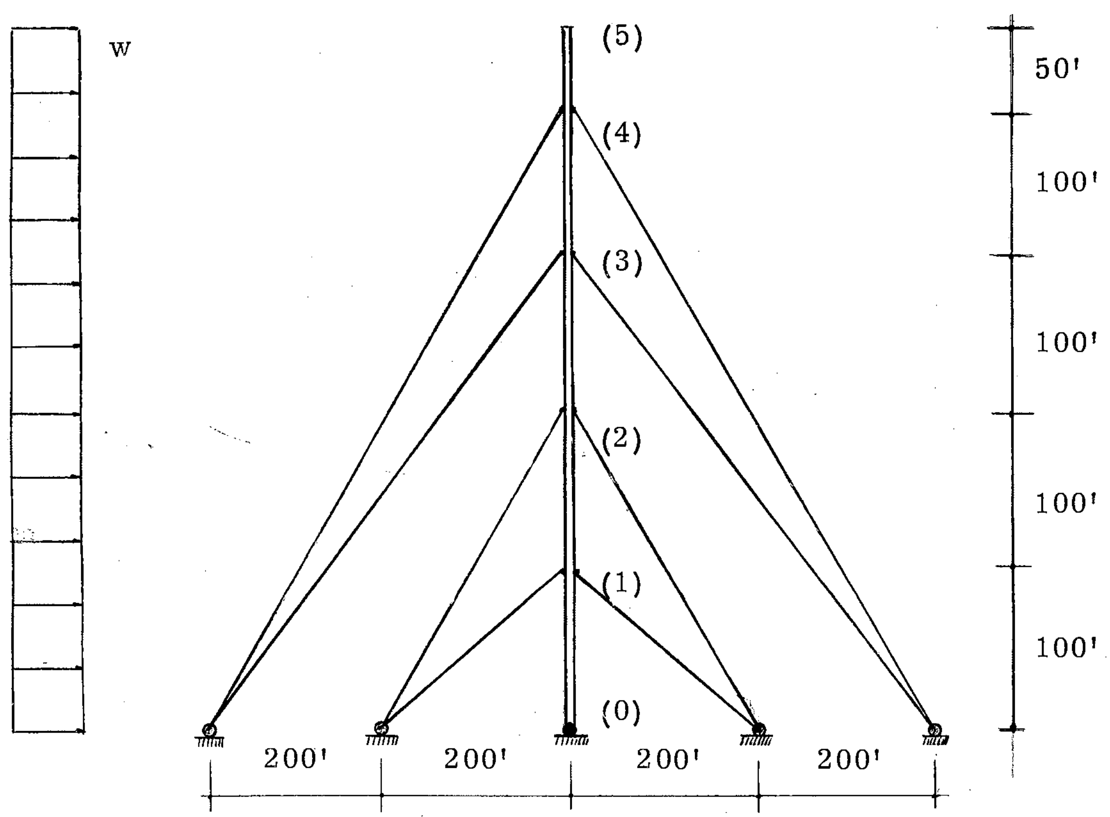


FIGURE 11a

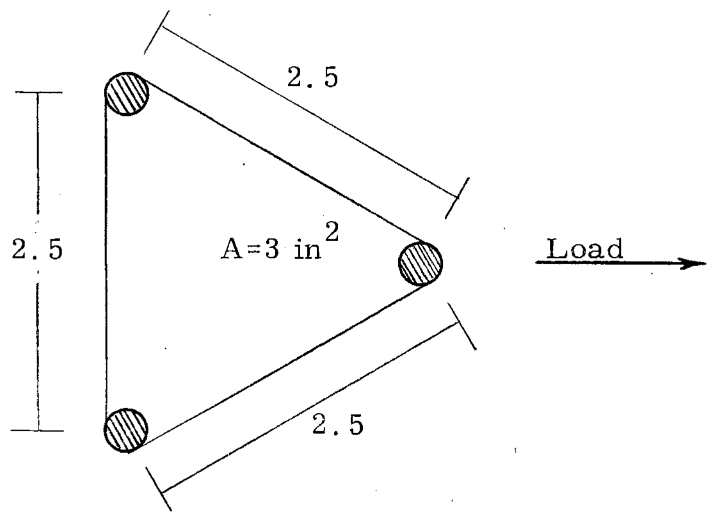


FIGURE 11b

MAST CROSS SECTION

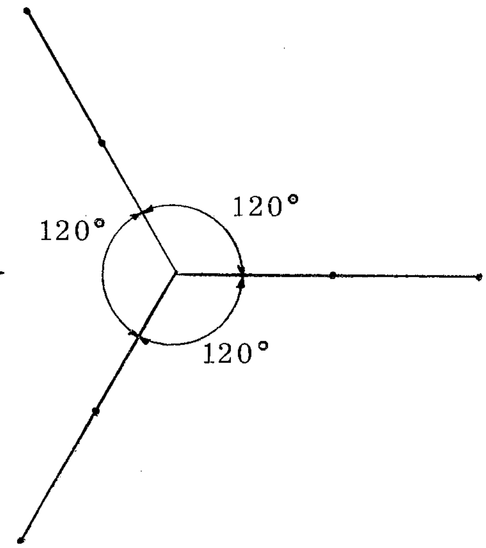


FIGURE 11c

GUY WIRE LAYOUT

Guy Wire Properties

Level 1 and 2

$A = .3603 \text{ in}^2$

$\phi = 7/8''$

$E = 20 \times 10^3 \text{ ksi}$

$P_{yield} = 70.0 \text{ kips}$

Level 3 and 4

$A = .4792 \text{ in}^2$

$\phi = 1''$

$E = 20 \times 10^3 \text{ ksi}$

$P_{yield} = 91.4 \text{ kips}$

Properties of Mast Cross Section

$A_T = 9 \text{ in}^2$

$I = A_T \frac{d^2}{2} = .0651 \text{ ft}^4$

where d is the length of one side.

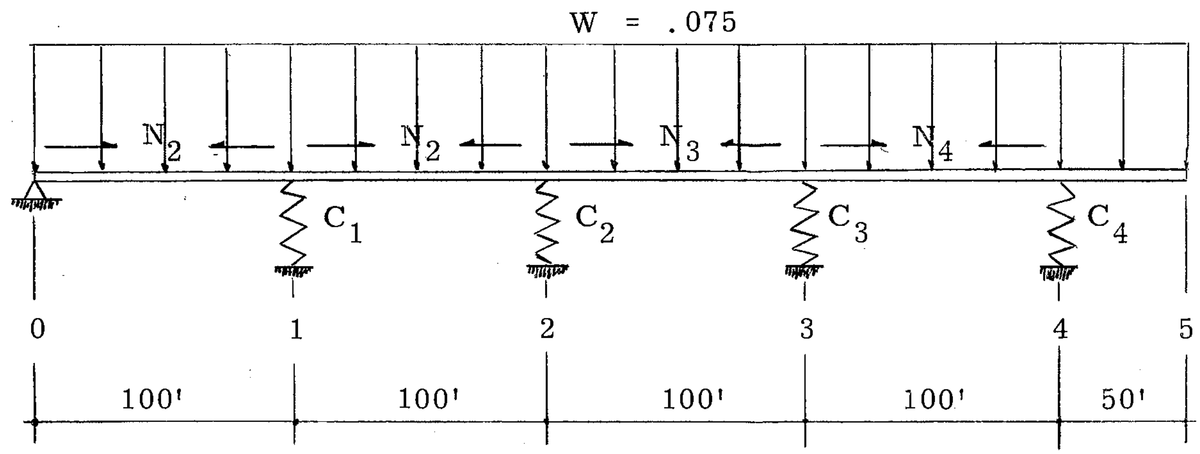


Figure 12

ANALOGOUS COLUMN-BEAM ON ELASTIC SUPPORTS

Support Spring Constants

$C_0 = \infty$

$C_1 = 38.88$

$C_2 = 19.23$

$C_3 = 18.39$

$C_4 = 12.70$

Axial Forces

$N_1 = 24.07$

$N_2 = 20.85$

$N_3 = 12.96$

$N_4 = 7.23$

Flexibilities and Carry Over Values (Eq's 1-4)

$F_{01} = F_{10} = 1.3056 \times 10^{-4}$

$F_{12} = F_{21} = 1.2941 \times 10^{-4}$

$F_{23} = F_{32} = 1.2672 \times 10^{-4}$

$F_{34} = F_{43} = 1.2488 \times 10^{-4}$

$F_{45} = F_{54} = 0.6131 \times 10^{-4}$

$G_{01} = G_{10} = 0.6833 \times 10^{-4}$

$G_{12} = G_{21} = 0.6731 \times 10^{-4}$

$G_{23} = G_{32} = 0.6491 \times 10^{-4}$

$G_{34} = G_{43} = 0.6328 \times 10^{-4}$

$G_{45} = G_{54} = 0.3065 \times 10^{-4}$

Load Functions (Eq's 5, 6)

$\Sigma \tau_1^L = 250.21 \times 10^{-4}$

$\Sigma \tau_2^L = 244.45 \times 10^{-4}$

$\Sigma \tau_3^L = 238.44 \times 10^{-4}$

$\Sigma \tau_4^L = 232.97 \times 10^{-4}$

<u>γ's</u> (Eq's 22a, 22b, 22c)

$\gamma_{01} = 0.0$

$\gamma_{21} = 0.0522$

$\gamma_{11} = 0.0260$

$\gamma_{12} = 0.0257$

$\gamma_{32} = 0.0548$

$\gamma_{22} = 0.0529$

$\gamma_{23} = 0.0518$

$\gamma_{43} = 0.0796$

$\gamma_{33} = 0.0550$

<u>Q's</u> (Eq's 24 - 28)

$\Sigma Q_1 = -5.2308 \times 10^{-4}$

$Q_{31} = -0.0307 \times 10^{-4}$

$Q_{21} = +2.6472 \times 10^{-4}$

$Q_{02} = -0.0330 \times 10^{-4}$

$Q_{12} = +5.3290 \times 10^{-4}$

$\Sigma Q_2 = -10.6522 \times 10^{-4}$

$Q_{32} = +5.3583 \times 10^{-4}$

$Q_{42} = -0.0393 \times 10^{-4}$

$Q_{13} = -0.0421 \times 10^{-4}$

$Q_{23} = +5.5541 \times 10^{-4}$

$\Sigma Q_3 = -11.0736 \times 10^{-4}$

$Q_{43} = +5.5475 \times 10^{-4}$

$\Sigma Q_1^* = +0.1913$

$\Sigma Q_2^* = +0.3965$

$\Sigma Q_3^* = +0.4123$

<u>G' 's and F' 's</u> (Eq 34)

$G'_{21} = -0.1136 \times 10^{-4}$

$G'_{31} = +0.0569 \times 10^{-4}$

$G'_{12} = -0.1424 \times 10^{-4}$

$G'_{32} = -0.2075 \times 10^{-4}$

$G'_{42} = +0.0582 \times 10^{-4}$

$G'_{13} = +0.0599 \times 10^{-4}$

$G'_{23} = -0.1874 \times 10^{-4}$

$G'_{43} = -0.2307 \times 10^{-4}$

$\Sigma F'_1 = +0.1617 \times 10^{-4}$

$\Sigma F'_2 = +0.3013 \times 10^{-4}$

$\Sigma F'_3 = +0.3607 \times 10^{-4}$

τ' 's (Eq 34)

$\Sigma \tau'_1 = 2.28 \times 10^{-4}$ $\Sigma \tau'_2 = -16.79 \times 10^{-4}$

$\Sigma \tau'_3 = +18.74 \times 10^{-4}$

Final Modified Flexibilities, Carry Over Values, and Load Functions.

$G^*_{21} = 0.5595 \times 10^{-4}$ $G^*_{43} = 0.4021 \times 10^{-4}$

$G^*_{31} = 0.0569 \times 10^{-4}$ $\Sigma F^*_1 = 2.7614 \times 10^{-4}$

$G^*_{12} = 0.5307 \times 10^{-4}$ $\Sigma F^*_2 = 2.8626 \times 10^{-4}$

$G^*_{32} = 0.4416 \times 10^{-4}$ $\Sigma F^*_3 = 2.8764 \times 10^{-4}$

$G^*_{42} = 0.0582 \times 10^{-4}$ $\Sigma \tau^*_1 = 252.4900 \times 10^{-4}$

$G^*_{13} = 0.0599 \times 10^{-4}$ $\Sigma \tau^*_2 = 227.6000 \times 10^{-4}$

$G^*_{23} = 0.4617 \times 10^{-4}$ $\Sigma \tau^*_3 = 257.3400 \times 10^{-4}$

Carry Over Factors and Starting Moments

$r_{21} = -0.2026$ $r_{42} = -0.0203$

$r_{31} = -0.0206$ $r_{13} = -0.0209$

$r_{12} = -0.1854$ $r_{23} = -0.1609$

$r_{32} = -0.1543$ $r_{43} = -0.1402$

$m^*_1 = -91.43$ $m^*_2 = -79.51$

$m^*_3 = -89.75$

Since M_4, computed by statics, is equal to 93.75, the modified starting moments are:

$m^{*\prime}_1 = -91.43$ $m^{*\prime}_2 = -77.61$

$m^{*\prime}_3 = -76.61$

Solution Matrix

$$\begin{bmatrix} 1.0000 & 0.2026 & 0.0206 \\ 0.1854 & 1.0000 & 0.1543 \\ 0.0209 & 0.1609 & 1.0000 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} -91.43 \\ -77.61 \\ -76.61 \end{bmatrix}$$

Carry Over Procedure

	1	2	3
r's	- .1854	-.2026 -.1609	- .1543
r's	- .0209		.0206
m*'s	- 91.43	- 77.61	- 76.61
	+ 1.58 9.89 + 11.47	16.95 11.82 - 48.84	+ 1.91 7.86 + 9.77
	- .20 + .74 + .54	- 2.13 - 1.51 - 3.64	- .24 + .59 + .35
	0.00	- .10 - .05 - .15	0.00
Σ	- 79.42	- 52.63	- 66.49

The final moments are:

$M_1 = -79.42$

$M_2 = -52.63$

$M_3 = -66.49$

$M_4 = -93.75$

The primary drawback to the solution of a guyed tower problem is that the axial forces are initially unknown. An analysis, neglecting the axial forces, was performed. The axial forces, induced due to deformation of the guy's, were computed and used as starting values in the analysis including the effect of the axial forces.

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APPENDIX A

CARRY OVER VALUES	FLEXIBILITIES	LOAD FUNCTIONS
$G_{ij} = \frac{L_j}{EI_j}\left\lvert\frac{\lambda_j L_j \operatorname{cosec} \lambda_j L_j - 1}{(\lambda_j L_j)^2}\right\rvert$	$F_{ji} = \frac{L_j}{EI_j}\left\lvert\frac{1 - \lambda_j L_j \cot \lambda_j L_j}{(\lambda_j L_j)^2}\right\rvert$	$\tau_{ji}^1 = \frac{1}{(\lambda_j L_j)^2}\int_i^j\left(\frac{\sin \lambda_j x}{\sin \lambda_j L_j} - \frac{x}{L_j}\right) W(x)dx$
$G_{kj} = \frac{L_k}{EI_k}\left\lvert\frac{\lambda_k L_k \operatorname{cosec} \lambda_k L_k - 1}{(\lambda_k L_k)^2}\right\rvert$	$F_{jk} = \frac{L_k}{EI_k}\left\lvert\frac{1 - \lambda_k L_k \cot \lambda_k L_k}{(\lambda_k L_k)^2}\right\rvert$	$\tau_{jk}^1 = \frac{1}{(\lambda_k L_k)^2}\int_k^j\left(\frac{\sin \lambda_k x}{\sin \lambda_k L_k} - \frac{x}{L_k}\right) W(x)dx$

EQUIVALENT SPRING CONSTANTS

$$\frac{1}{\gamma_{ij}} = C_i + \frac{N_i - N_j}{L_j} \qquad \frac{1}{\gamma_{jj}} = C_j - \frac{N_j \gamma_{ij} C_i}{L_j} - \frac{N_k \gamma_{kj} C_k}{L_k} \qquad \frac{1}{\gamma_{kj}} = C_k + \frac{N_l - N_k}{L_k}$$

Q FUNCTIONS

$$Q_{hj} = -\frac{\gamma_{ij}\gamma_{jj}N_j}{L_j}\left\lvert\frac{1}{L_i} + N_i G_{hi}\right\rvert$$

$$Q_{lj} = -\frac{\gamma_{kj}\gamma_{jj}N_k}{L_k}\left\lvert\frac{1}{L_l} + N_l G_{lk}\right\rvert$$

$$Q_{ij} = \gamma_{jj}\left[\frac{1}{L_j} + \frac{N_j\gamma_{ij}}{L_j}\left(\frac{1}{L_i} + \frac{1}{L_j} - N_i \Sigma F_i\right)\right]$$

$$Q_{kj} = \gamma_{jj}\left[\frac{1}{L_k} + \frac{N_k\gamma_{kj}}{L_k}\left(\frac{1}{L_k} + \frac{1}{L_l} - N_l \Sigma F_k\right)\right]$$

$$Q_{ji} = -\gamma_{jj}\left[\frac{1}{L_j} + \frac{N_j\gamma_{ij}}{L_j}\left(\frac{1}{L_j} + N_i G_{ji}\right)\right]$$

$$Q_{jk} = -\gamma_{jj}\left[\frac{1}{L_k} + \frac{N_k\gamma_{kj}}{L_k}\left(\frac{1}{L_k} + N_l G_{jk}\right)\right]$$

$$\Sigma Q_j^* = \gamma_{jj} BR_j + Q_{ji}^* + Q_{jk}^*$$

$$Q_{ji}^* = -\frac{N_i N_j \gamma_{ij}\gamma_{jj}\Sigma\tau_i^1}{L_j} \qquad Q_{kj}^* = -\frac{N_k N_l \gamma_{kj}\gamma_{jj}\Sigma\tau_k^1}{L_k}$$

MODIFIED CARRY OVER VALUES (G''s)

$$G'_{hj} = -Q_{hj}\left[\frac{C_i\gamma_{ij}}{L_j} + \frac{C_k\gamma_{kj}}{L_k}\right] + \frac{\gamma_{ij}}{L_j}\left|\frac{1}{L_i} + N_i G_{hi}\right|$$

$$G'_{lj} = -Q_{lj}\left|\frac{C_i\gamma_{ij}}{L_j} + \frac{C_k\gamma_{kj}}{L_k}\right| + \frac{\gamma_{kj}}{L_k}\left|\frac{1}{L_l} + N_l G_{lk}\right|$$

$$G'_{ij} = -Q_{ij}\left|\frac{C_i\gamma_{ij}}{L_j} + \frac{C_k\gamma_{kj}}{L_k}\right| - \frac{\gamma_{ij}}{L_j}\left|\frac{1}{L_i} + \frac{1}{L_j} - N_i\Sigma F_i\right|$$

$$G'_{kj} = -Q_{kj}\left|\frac{C_i\gamma_{ij}}{L_j} + \frac{C_k\gamma_{kj}}{L_k}\right| - \frac{\gamma_{kj}}{L_k}\left|\frac{1}{L_k} + \frac{1}{L_l} - N_l\Sigma F_k\right|$$

MODIFIED FLEXIBILITIES (F''s)

$$F'_{ji} = -\frac{\Sigma Q_j}{L_j}\; C_i\gamma_{ij} + \frac{\gamma_{ij}}{L_j}\left|\frac{1}{L_j} + N_i G_{ji}\right|$$

$$F'_{jk} = -\frac{\Sigma Q_j}{L_k}\; C_k\gamma_{kj} + \left|\frac{\gamma_{kj}}{L_k}\;\frac{1}{L_k} + N_l G_{jk}\right|$$

MODIFIED LOAD FUNCTIONS (τ''s)

$$\tau'_{ji} = -\frac{\Sigma Q_j^*\gamma_{ij}C_i}{L_j} + \frac{\gamma_{ij}}{L_j}\left|BR_i + N_i\Sigma\tau_i^1\right|$$

$$\tau'_{jk} = -\frac{\Sigma Q_j^* C_k\gamma_{kj}}{L_k} + \frac{\gamma_{kj}}{L_k}\left|BR_k + N_l\Sigma\tau_k^1\right|$$

FINAL FLEXIBILITIES AND CARRY OVER VALUES; CARRY OVER FACTORS AND STARTING MOMENT

$$G^*_{hj} = G_{hj}' \qquad G^*_{lj} = G_{lj}'$$

$$G^*_{ij} = G_{ij}' + G_{ij} \qquad G^*_{kj} = G_{kj}' + G_{kj}$$

$$F^*_{ji} = F_{ji}' + F_{ji} \qquad F^*_{jk} = F_{jk}' + F_{jk}$$

$$\tau^*_{ji} = \tau_{ji}' + \tau_{ji} \qquad \tau^*_{jk} = \tau_{jk}' + \tau_{jk}$$

$$r_{hj} = -\frac{G^*_{hj}}{\Sigma F^*_j} \qquad r_{lj} = -\frac{G^*_{lj}}{\Sigma F^*_j}$$

$$m^*_j = -\frac{\Sigma\tau^*_j}{\Sigma F^*_j}$$

$$r_{ij} = -\frac{G^*_{ij}}{\Sigma F^*_j} \qquad r_{kj} = -\frac{G^*_{kj}}{\Sigma F^*_k}$$

VITA

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