A METHOD OF ANALYSIS OF THE RELEASE CYCLE OF A RELAY USING AN ANALOG COMPUTER

By

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A METHOD OF ANALYSIS OF THE RELEASE CYCLE OF A RELAY USING AN ANALOG COMPUTER

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PREFACE

Since the coming of the missile age, the complexity of electronic equipment has increased many times over. Accordingly the electromagnetic relay has followed the same course. The design of the basic relay is of course the same, however the conditions governing its operation and the specifications guiding its manufacture have become extremely particular. Close tolerances in time over wide ranges of environment have brought about these changes. The reliability of the relay has become even more important, since the failure of a single relay could, for example, result in the destruction of large missiles.

The ever-increasing number of problems in the relay field have created an exhaustive program of research into methods of analysis and the prediction of performance. This paper presents a method of analysis of the dynamic behavior of a relay during the release cycle. It is felt that with this method an investigation of many important parameters could be derived which could be beneficial to the designer as well as the manufacturer. In all the analyses which have been investigated so far, none can represent all of the basic parameters during the transient conditions. It is felt that the analog computer can simulate them to a close degree, and the recordings shown in Chapter V are presented as verification.

The author wishes to express his sincere appreciation to Professor Charles F. Cameron for his guidance and assistance in carrying out this work.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure Page

CHAPTER I

INTRODUCTION

Background

The electromagnetic relay has an exacting job to perform in the electronic equipments of today's missile age. Tolerances closer than ever before are placed on them and predicted performance over a variety of environmental conditions must be known. In the past the contact requirements[1] were usually all that was necessary to obtain a relay to do a satisfactory job. Now the requirements are much more severe; not only are contact requirements stated but accurate information on the core and actuating mechanism must be known. The requirements also state a wide variety of shock, vibration and temperature conditions that will exist. No longer is the static behavior of a relay sufficient to predict the required performance; necessary and sufficient information also must be known during the transient behavior of the relay. It is during this time that the design becomes very critical.

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[1] A glossary of the relay terms used in this paper is included in Appendix A. Its purpose is to avoid repetitions definitions in the text.

The National Association of Relay Manufacturers has devoted its existence to the promotion of standardization of testing and specifications for relays as well as the advancement of the art of manufacturing. Each year the Association meets at Oklahoma State University and as part of their meeting papers are presented on new techniques in testing and developments in the field. It became obvious to the writer, after reviewing many of these papers that were presented, that there is a scarcity of measuring techniques for the transient behavior and a lack of equipment for this purpose. The people in the industry are aware of this and a more concerted effort is being put in this direction. Analysis along the mathematical and experimental lines is becoming more frequent. Among these analyses the digital computer has been employed[2], however the author found no evidence of the use of an analog computer to analyze the transient behavior of the relay.

The Problem

It is the history of this work that prompted the author to prepare this thesis. The purpose is to present a method of analysis of the transient behavior of the release cycle of a relay using an analog computer. The work carried out was divided into four parts.

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[2] Olin Smith and George Papaiconomou, "Computer-Calculated Curves Predict Solenoid Performance," Product Engineering, January 22, 1962, pp. 59-66.

1. The collection of data that will aid in the development of the computer program.
2. The derivation of a set of equations that will represent the release cycle of a relay.
3. Preparing the computer program and time and amplitude scaling it.
4. The verification of the computer program itself by comparing the results to earlier experimental data and to data obtained through research.

In Chapter V a compilation of curves is presented for different parameters involved in the relay operation, the coil current decay, flux density, magnetic pull on the armature, reluctance of the total magnetic circuit, and the displacement, velocity and acceleration of the armature.

It was found that the analysis of a relay during the transient behavior is a complex undertaking. The relations of single variables on the parameters, such as release time, can be determined with experimentation, but it is difficult to represent a combination of variables and predict performance. This is because the relay can be represented by three circuits, electrical, mechanical and magnetic. Each of these possesses a dependence on the others and, in turn, cannot be analyzed separately. One advantage of the analog computer is that the magnetic and mechanical circuits can be represented by similar networks as the electrical circuit and they can all be simulated in one diagram.

CHAPTER II

BACKGROUND INFORMATION

Assumptions Made

In order to perform a transient analysis on an electromagnetic relay, assumptions of one form or another invariably have to be made. This is true in part to the number of variables associated with each of the three basic circuits and in part to the dependence of these circuits on each other. Certain assumptions were made in the programming of the release cycle of the relay on the analog computer but it is believed that they had insignificant effect on most of the results obtained in Chapter V.

The first assumption that was made was with regard to the eddy currents created in the armature. These currents are created when a conductor moves relative to a magnetic field or when it is present in a field of varying intensity. In the case of the relay, both of these conditions exist and there are undoubtedly eddy currents created during transient conditions. The question that arose was relative to the effect these currents have on the over-all behavior of the relay. These currents oppose the field which induces them and also create heat losses. It was felt that both these factors were very small and that the eddy currents could be neglected. The basis for this decision was:

1. The energy loss due to eddy currents is proportional to the volume of the armature and the armature volume is small in the relay considered.
2. The ratio of the armature volume to the volume of the magnetic core volume is very small therefore the opposing forces can be considered negligible.
3. The eddy currents are directly related to frequency; the relay considered in this case is DC and the frequencies associated with the current decay and armature motion are small.

The residual magnetism of the core was also considered negligible. A certain amount of magnetism will remain in the core after the generated field is removed. This is due to the retentivity of the material. Hard iron and certain steel alloys have relatively high retentivities and make good permanent magnets. Soft iron and sheet steels have relatively low retentivities, make good electrical magnets and are used in relays. When the field is removed from the electric magnet the poles reorientate themselves randomly, however some remain fixed in their original north-south state due to the retentivity of the material. This magnetism can only be zero when the retentivity is zero, however such materials do not exist that make good electrical magnets. In order to overcome this problem in relays, a small residual air gap of some sort is designed into the structure. This gap is made large enough to overcome the residual magnetism. In this problem, the residual magnetism will not be considered a factor and will not be designed into the program.

Temperature was not considered to be a factor in the problem. It was felt that complications arising from severe temperature changes on a relay in themselves warrant a separate investigation. Temperature affects the geometrical quantities of a relay as well as the resistance of the coil. A wide temperature change could then in turn have significant effects on the friction of the armature and pull-in drop-out currents as well as the operate, release and transit times.

For the main part of the analysis, the friction of the armature was considered negligible and the spring tension constant for various air gaps. For simplicity the relay was considered to have one set of contacts, a two-position single-throw switch. The contacts at each end limited the armature travel and in turn determined the residual air gap and the total air gap. These stops were considered to be non-elastic in nature.

Laboratory Work Performed

Before preparing the program for the computer, considerable time was spent in the Oklahoma State University relay research laboratory learning firsthand the behavior of the relay with different conditions set for spring tension, air gaps, discharge resistance and the size of coils. Although most of the information was available from previous analysis[3] it was felt that seeing some of these basic characteristics in the laboratory would be an aid in setting up the computer.

The method used for most of the laboratory analysis was the transient coil current display, because of the many details which may be observed from the waveforms. The equipment[4] used was already set up in the laboratory and only minor adjustments had to be made so that the variables of interest could be changed.

Figures 1 and 2 represent typical traces of the coil current for the operate and release cycles of a relay. From them information can be obtained about the over-all relay behavior due to particular adjustments in the variables. The motion of the armature can be depicted from the characteristic "cusp" or "dip" in the trace. The variables, spring tension, discharge resistance, coil turns and air gaps, all have significant influences on the position and size of the "cusp" or "dip". The characteristics in the build up and decay coil current are due to a change in the total reluctance of the magnetic circuit due to the armature motion. When the armature is open the reluctance of the circuit is greater than when it is closed. The inductance, determined by the reluctance, will be less, and the instantaneous time constant is therefore less than that when the air gap is closed. From these facts it is

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[3] Final Reports on the Investigations of Dynamic Characteristics of Relays, Sandia Corporation, 1959, 1960, 1961.

[4] See Appendix B for the equipment set-up used to obtain and record the coil current waveform.

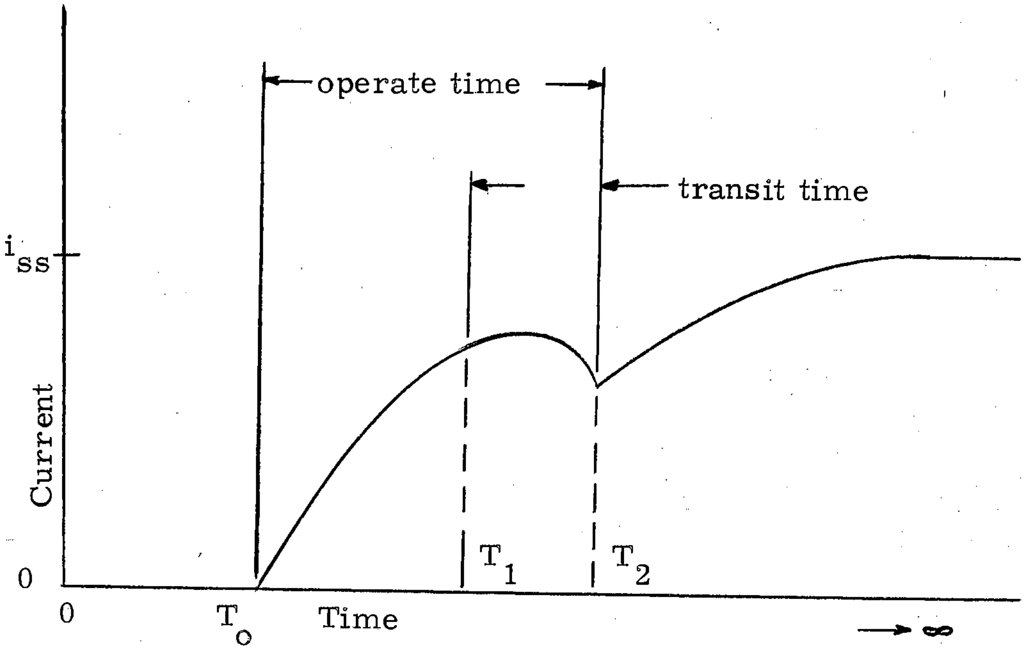

Fig. 1. Coil Current of an Electromagnetic Relay During Operate

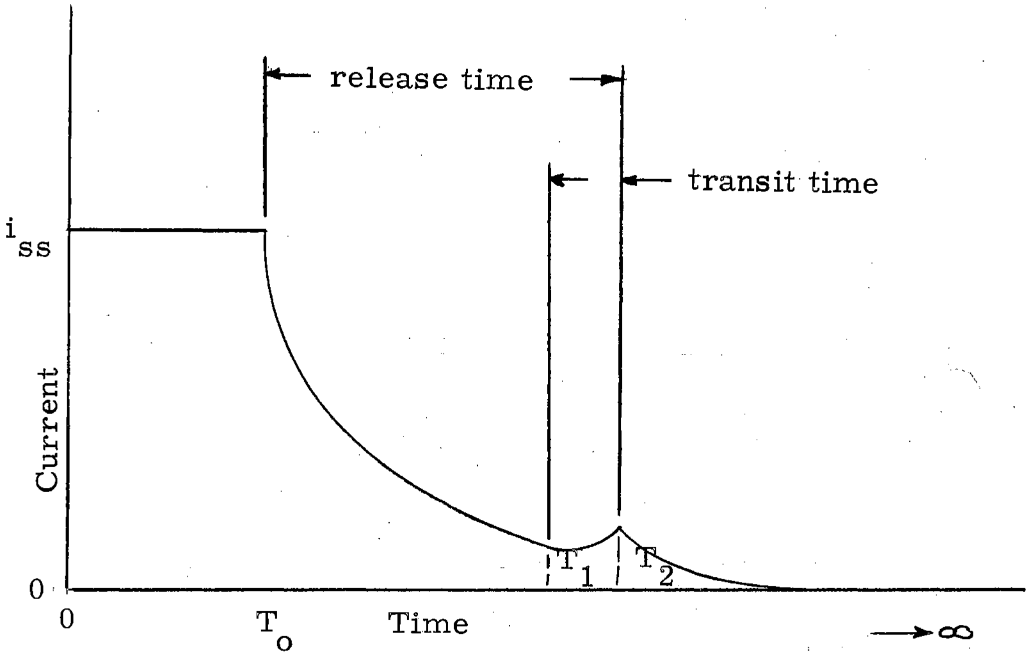

Fig. 2. Coil Current of an Electromagnetic Relay During Release

evident that the current traces will exhibit some characteristics during the armature motion.

The photographs shown in Figs. 3 and 4 represent some of the work which was performed in the laboratory. It was desired to collect some information experimentally which could be compared to the computer results. The spring tension and residual gap were chosen and the curves shown are for different values of each variable. The data for each circumstance is given under each set of curves. The release time in both sets of curves was measured with a ruler and dividers and the composite graph shown in Fig. 5 was plotted. This graph will be compared to results from the computer to show the validity of the computer results.

After completing the laboratory analysis of the variables mentioned, the following relations were determined and verified from existing literature.

Spring Tension - This variable has a marked effect on the operate, release and transit time of the relay. As the spring tension was decreased the transit time on operate became less and the transit time on release became larger. The operate time was increased with increasing values of spring tension and the released time was decreased. The spring tension affects the shape of the "cusp" or "dip" since it affects the transit times.

Air Gap - This has no noticeable effect on the release time but considerable on the operate time. For decreasing air gaps the operate

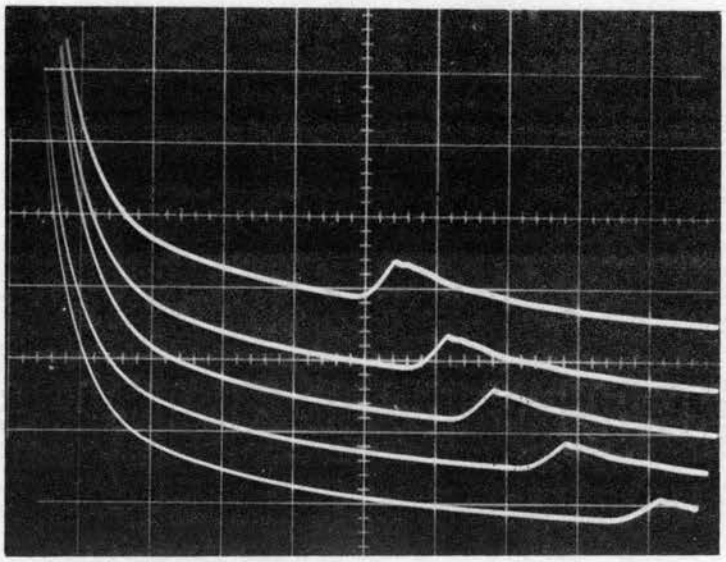

Fig. 3. Coil Current Decay for Various Values of Spring Tension

air gap = 0.01 inches
coil current = 120 ma.
total circuit resistance = 100 ohms
number of coil turns = 4,000

residual air gap approx. = 0
spring tension (left to right) = 300, 250, 200, 150, 100 grams
scope sweep time = 5 ms/cm

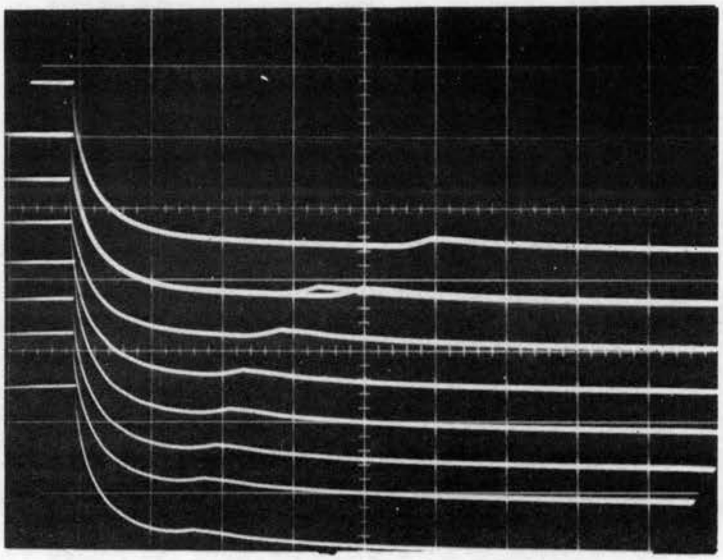

Fig. 4. Coil Current Decay for Various Values of Residual Air Gap

air gap = 0.01 inches
coil current = 120 ma.
total circuit resistance = 100 ohms
number of coil turns = 4,000
scope sweep time = 5 ms/cm

spring tension = 250 grams
residual air gap (top to bottom) = approx. 0, 0.0005, 0.001, 0.0015, 0.002, 0.0025, 0.003, 0.0035 inches

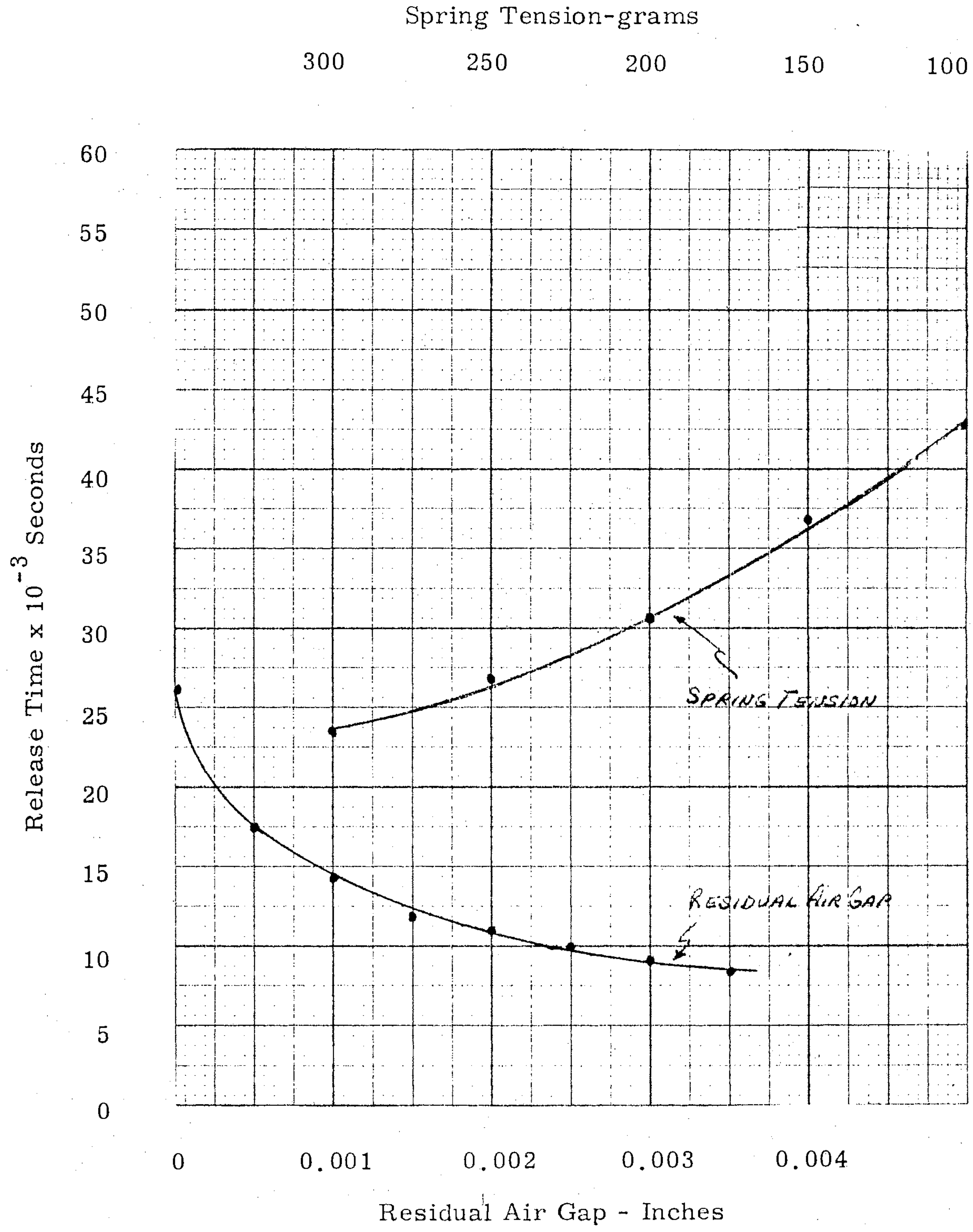

Fig. 5. Experimental Curves for the Release Time of a Relay for Various Values of Spring Tension and Residual Air Gap

time decreased and the transit time on operate decreased. The transit time on release also became less as the air gap was decreased. The air gap affected the shape of the "cusp" and "dip" noticeably.

Residual Air Gap - This had no noticeable effect on the operate time but considerable on the release time. It also noticeably affected both release times; for less residual air gap the times were increased. It had noticeable effect on the shape of the "cusp" and "dip".

Discharge Resistance - This resistance has no effect on the operate time but has significant effect on the release time. As the resistance went up the time went down, showing that the RL time constant of the circuit was affected only during release. The "cusp" and "dip" were not changed and no change was noticed in the transit times.

Number of Coil Turns - The change in the number of turns changed the inductance of the circuit, and had effects on both the operate and release times. It did not affect the transit times or the shape of the "cusp" or "dip".

An Analysis of a Relay Core

In order to generate the permeability of the core of the relay, a general idea of the behavior must be known. A relay was set up in the laboratory and the curves in Fig. 6 were made. These represent the coil current for various values of residual air gap up to the time of armature motion. From them the values of inductance at different points on the decay were calculated so that the permeability could be

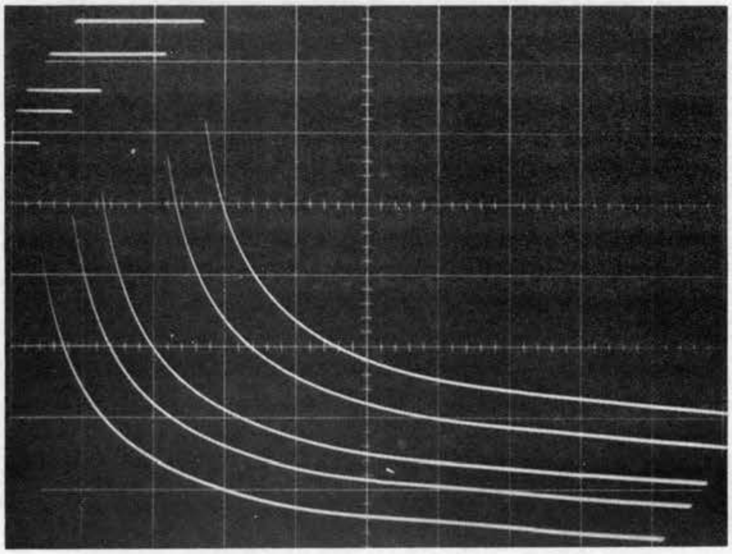

Fig. 6. Coil Current Decay of an Electromagnetic Relay for Various Values of Residual Air Gap, Shown up to the Armature Motion

discharge resistance = 1,000 ohms	sweep time = 2 ms/cm.
spring tension = 195 grams	vertical cal. = 30 ma./cm.
coil current = 135 ma.	residual air gap (left to right) =
air gap = 0.01 inches	approx. 0, 0.0005, 0.001,
	0.0015, 0.002 inches

determined.

The first trace in the photograph was used for the residual air gap at approximately zero. A log plot analysis was made by picking various points on the curve and employing the equation for the decay of a linear RL circuit

$$I/I_o = e^{-Rt/L} .$$

Eight points were picked off and the values of current and time were recorded in Table I. Each point was plotted on semilog graph paper, I/I_o versus time. Figure 7 shows all the points plotted. A straight line was drawn from the point $I/I_o = 1$ and $t = 0$ through each point. Each line represents a linear RL circuit, since a semilog plot of a linear RL circuit is a straight line.[5] The curve which would be represented by drawing a line through the points $I_1 - I_7$ would indicate the changing time constant for a nonlinear RL circuit.

By drawing a horizontal line across the graph in Fig. 7 at $I/I_o = 0.367$ the time constant for each current point $I_1 - I_7$ was determined, since the 63.2% value of current is represented by $I/I_o = 0.367$. The values of time where the horizontal line intersects each individual RL line were determined and recorded in Table I. From these values the inductance at each current point was determined from the relation $L/R = t_c$, where t_c is that value of time for the 63.2% current decay.

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[5] Gladwyn V. Lago and Donald L. Waidelich, Transients in Electrical Circuits, The Ronald Press Company, New York, 1958, p. 28.

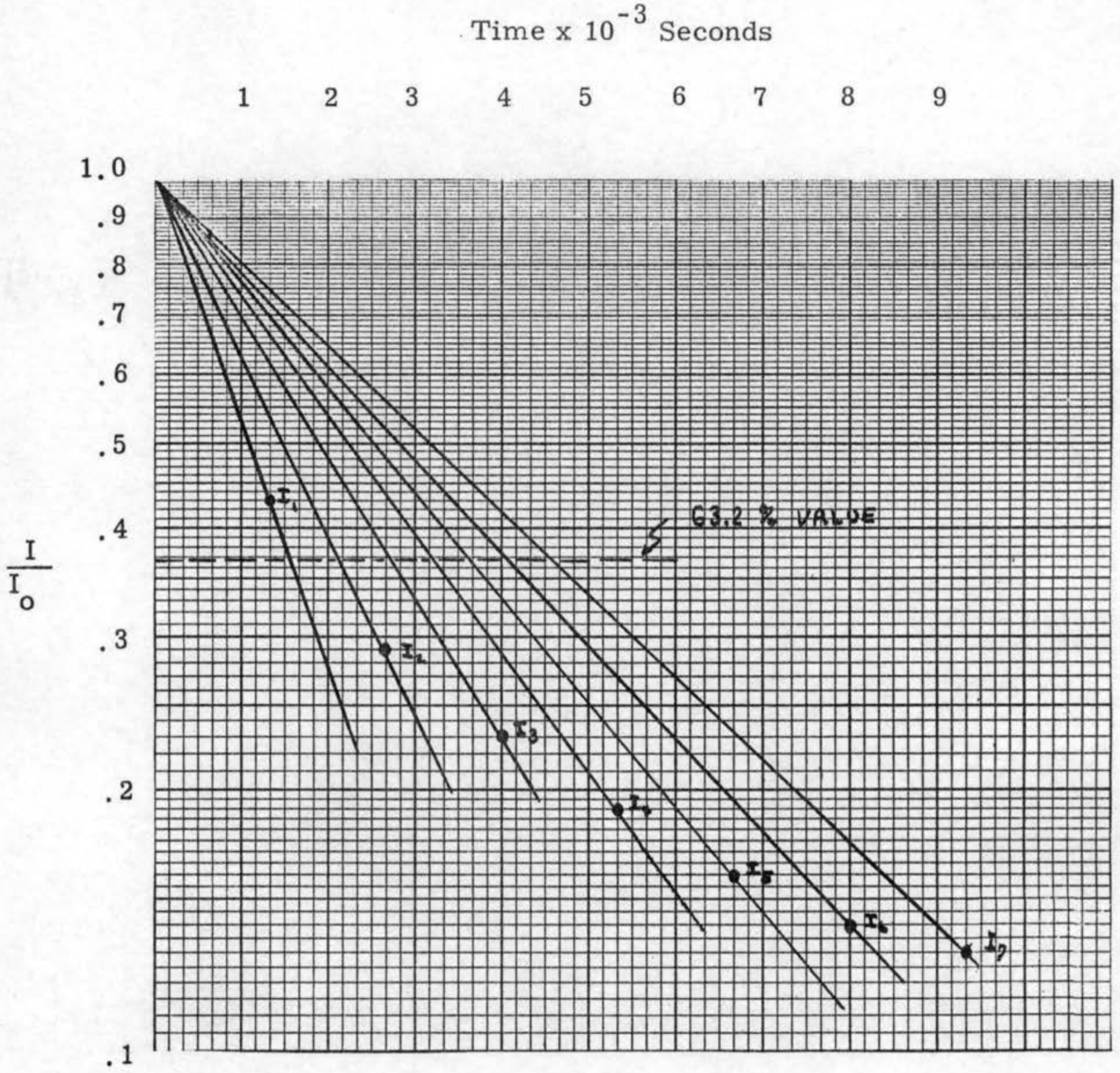

Fig. 7. Log Plot of the Nonlinear
RL Circuit of a Relay

TABLE I

DATA CALCULATED FROM THE COIL CURRENT DECAY OF FIG. 6

Data Taken from Fig. 6			Data Calculated		
Time 10^{-3}	Current	I/I_o	t_c	Inductance-h	Rel. Permeability
0	I_o = 135	1	0	----	----
1.33	I_1 = 57.3	0.43	1.58	1.58	46.3
2.67	I_2 = 39.4	0.29	2.167	2.17	63.6
4.00	I_3 = 31.2	0.23	2.75	2.75	80.6
5.33	I_4 = 25.8	0.19	3.19	3.19	93.8
6.67	I_5 = 22.2	0.16	3.61	3.61	105.5
8.00	I_6 = 18.0	0.14	4.05	4.05	118.7
9.33	I_7 = 17.1	0.13	4.58	4.58	134.8

The inductance was calculated and recorded in Table I and from the values the permeability of the core was determined from the equations

$$L = \frac{N^2}{Rel_{core}} \text{ henrys} \quad \text{and} \quad Rel_{core} = \ell / uA$$

where the total reluctance was considered in the core.

Figure 8 shows the calculated curve for the permeability of the core plotted against current decay. The INSERT shows the predicted behavior on operate and then release. From this curve the dotted line was made, so that a good idea of the "peak" value of permeability could be found. This value will be used in Chapter IV to generate the permeability.

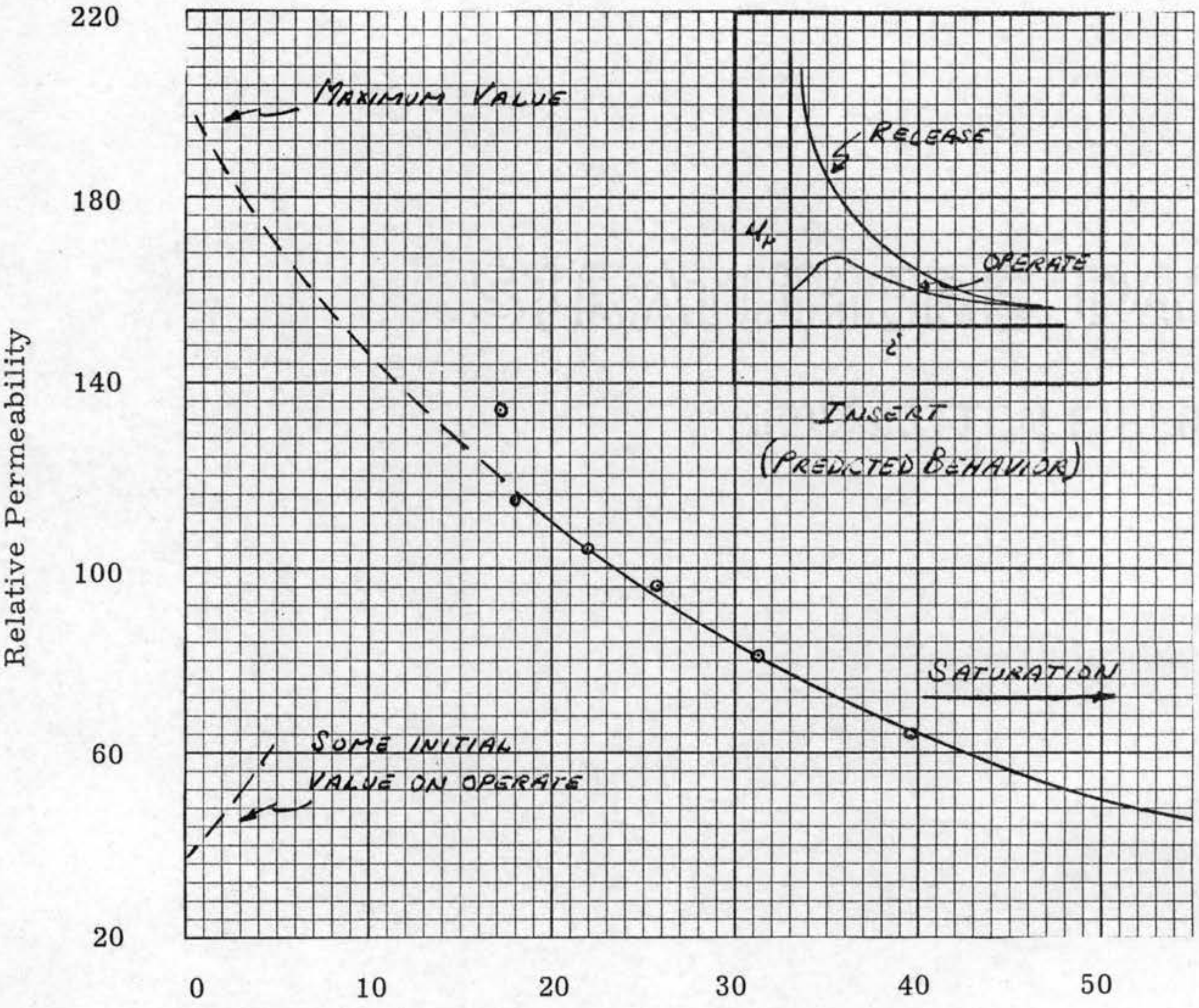

Fig. 8. The Relative Permeability of a Relay Core Plotted Against Coil Current

CHAPTER III

THE RELAY CIRCUIT

In order to describe the operation of a relay circuit mathematically, three basic circuits must be considered. Ideally, it should be possible to present one equation which would relate all three circuits simultaneously during the dynamic operation of the relay. An equation has been presented which satisfies the transient coil current build-up curve.[6] This equation relates the rate of change of four different variables associated with the three circuits; the rate of change of flux with respect to current, current with respect to time, flux with respect to armature motion, and armature motion with respect to time. This equation can be used to analyze the coil current waveforms of a relay but is a rather difficult equation to handle mathematically. It takes the form

$$i = \frac{E - \left(\frac{\partial \phi}{\partial i}\frac{di}{dt} + \frac{\partial \phi}{\partial x}\frac{dx}{dt}\right)}{R}$$

where i, E, and R represent current, voltage and resistance.

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[6] C. F. Cameron and D. D. Lingelbach, The Dynamics of Relays, Electronics Industries, 1959.

Since the above equation would be awkward to use, this chapter will be devoted to expressing the operation of the release cycle of a relay in terms of a series of equations, all interrelated. These equations will relate the electrical, mechanical, and magnetic circuits in terms of the variables of each.

Considering the circuit diagram of the relay shown in Fig. 9, the electrical equation during the time when the switch SW is closed is

$$E = R_d \times i_1 = R_s \times i = L \frac{di}{dt}$$

where $i_2 = i_1 + i$

R_d = discharge resistance

R_s = series resistance

L = inductance of ferromagnetic circuit.

The instant after the switch SW is opened, the equation becomes

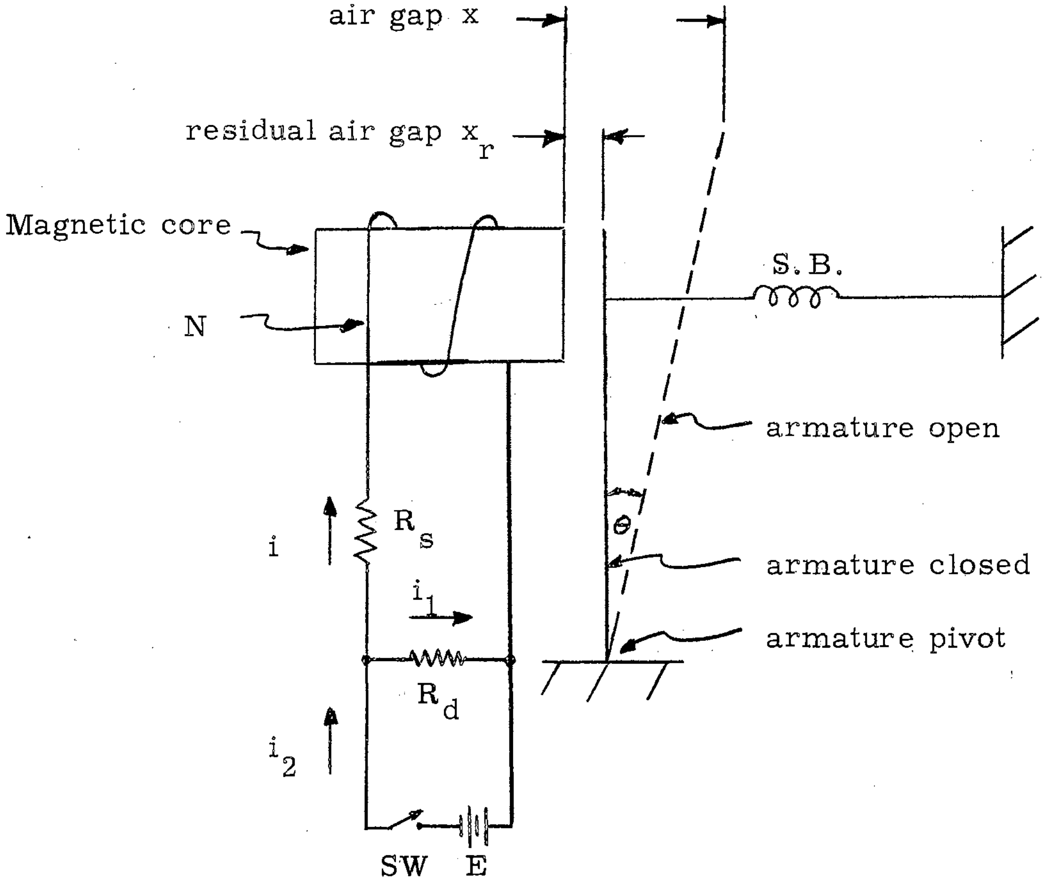
$$E = 0 = R_d \times i_1 = R_s \times i \; L \frac{di}{dt}$$

and rewriting and substituting for the conditions $i_2 = 0$, $-i_1 = i$, and $R_t = R_d + R_s$, the equation becomes

$$(1) \qquad L \frac{di}{dt} + R_t i = 0 \, .$$

The inductance of the circuit is not a constant, since the permeability of the iron core changes when the current intensity changes. The inductance can be related to the flux changing with respect to the current in the coil,

$$L = N \frac{d\psi}{di} \text{ henrys}$$



S.B. = release spring on armature, referred to as spring tension

N = number of turns of coil

R_d = resistance across coil, referred to as discharge resistance

R_s = series resistance of coil

Fig. 9. Pictorial Diagram of a Relay

where N = number of turns of coil

ψ = total flux in webers .

If fringing effects around the poles of the coil are considered negligible, then the total flux can be related to the flux density

$$d\psi = A\,dB$$

and di can be related to the current intensity by the expression

$$di = \frac{1}{N^2} \times dH$$

where B = flux density in webers/meter^2

H = current intensity in amperes/meter^2 .

Relating the above expressions, the inductance can then be stated as

$$L = \frac{N^2 A}{\ell} \frac{dB}{dH} \text{ henrys}$$

where

$$u_d = \frac{dB}{dH} .$$

The term u_d is referred to as the differential permeability and represents the changing permeability of the magnetic circuit during the current decay. The reluctance of a magnetic circuit can be stated in general as ℓ/uA, where ℓ and A are the length and cross sectional area of the particular circuit. From this the expression of the magnetic circuit, the inductance can be rewritten as

$$L = \frac{N^2 A}{\ell} \frac{dB}{dH} = \frac{N^2}{Rel_t} \text{ henrys}$$

where ℓ in the original equation was the length of the coil. The magnetic circuit of the relay can be considered a series circuit, where the reluctances of the core and air gap are additive. Rel_t can then be expressed as $Rel_{core} + Rel_{gap}$ and written out in an equation of the form

$$Rel_t = \frac{\ell}{uA} + \frac{x}{u_o A} \text{ reciprocal henrys}$$

where u = permeability of core material

u_o = permeability of air = $4\pi \times 10^{-7}$ henrys/meter

ℓ = length of coil in meters

x = length of air gap in meters .

The equation of the inductance can then be stated:

$$(2) \qquad L = \frac{N^2}{Rel_t} = \frac{N^2 A}{\ell/u + x/u_o} \text{ henrys} .$$

Equation (2) represents the inductance and relates the electrical circuit to the varying permeability of the magnetic circuit and to the changing air gap of the mechanical circuit. It is almost exact, in that only three approximations were made: the coil is uniform, the fringing of flux around the poles is negligible, and the remaining reluctances of the magnetic circuit are considered small compared to the air gap and core reluctances.

In describing the motion of the armature certain assumptions were made which simplified the equation somewhat. It was assumed that since θ, the angle of movement of the armature, is small (see

Fig. 9), the armature can be considered parallel to the pole at all times. It was also assumed that the mass of the armature is concentrated over the pole. The forces necessary to overcome friction were considered small compared to the spring and magnetic forces and therefore negligible.

The motion of the armature of the relay can be described by the equation

$$F_t = m x \frac{d^2x}{dt^2}$$

where F_t = magnetic and mechanical forces on the armature, m = mass of armature, and x a distance perpendicular from the pole.

The equation relating the quantities of the mechanical circuit to the attractive force of the magnet circuit can be stated:

$$\text{(3)} \qquad S.B. - F_m = m \frac{d^2x}{dt^2}$$

where S.B. = spring tension on armature (spring bias) and F_m is the magnetic pull on the armature. Equation (3) is a representation of the armature position, and the following relations can be stated.

$S.B. < F_m$ armature is closed

$S.B. = F_m$ armature is in equilibrium

$S.B. > F_m$ armature will or is open .

The gap force of the magnetic circuit can be stated as

$$F = \frac{B^2A}{2u_o} \quad \text{newtons} ,$$

and for a long solenoid the flux density at either end can be expressed as

$$B = \frac{uNi}{2\sqrt{D^2 + \ell^2}} \quad \text{webers/meter}^2$$

where ℓ is the length and D the diameter of the solenoid. From these two equations an equation specifying the magnetic pull at the pole can be written

$$F_p = \frac{u^2N^2i^2A}{4(D^2+\ell^2)u_o} \quad \text{newtons} \ .$$

Since the force on the armature from the magnet will obey the inverse square law, that is $F \alpha 1/x^2$, the equation for the magnetic pull on the armature can be written in the form

$$(4) \qquad F_m \alpha \frac{1}{x^2} \quad \frac{u^2N^2i^2A}{4(D^2 + \ell^2)u_o} \quad \text{newtons} \ .$$

Equation (4) gives the expression for the magnetic pull on the armature in terms of residual air gap (indirectly through the air gap x), air gap, current, and permeability, expressing mechanical, electrical, and magnetic parameters.

All of the basic relay parameters are present in Equations (1) through (4) and through the proper handling of them the release cycle of the relay can be simulated.

$$(1) \qquad L\frac{di}{dt} + R_t i = 0$$

$$(2) \qquad L = \frac{N^2A}{\ell/u + x/u_o}$$

(3) $$S.B. - F_m = m \frac{d^2x}{dt^2}$$

(4) $$F_m \alpha \frac{1}{x^2} \quad \frac{u^2N^2i^2A}{4(D^2 + \ell^2)u_o}$$

where all physical quantities are in the MKS system.

L = inductance = henrys

i = current = amperes

R = resistance = ohms

N = number of turns

A = cross-sectional area of coil = meters^2

ℓ = length of coil in meters

u = permeability of core = henrys/meter

u_o = permeability of air = $4\pi \times 10^{-7}$ henrys/meter

x = air gap = meters

x_r = residual air gap = meters

t = time in seconds

m = mass = kilograms

F_m = magnetic force = newtons

S.B. = spring force on armature = newtons

D = diameter of coil = meters .

CHAPTER IV

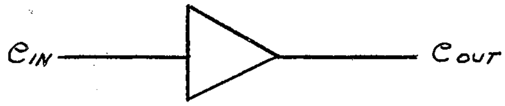
THE COMPUTER PROGRAM

Introduction to the Computer

The procedure for programming Equations (1) through (4) on an analog computer involves two basic steps: (1) rewriting the equations in the form of a computer diagram, and (2) scaling the equation parameters so that they are acceptable to the computer. Before proceeding with these steps, a brief run-down on the equipment used is in order.

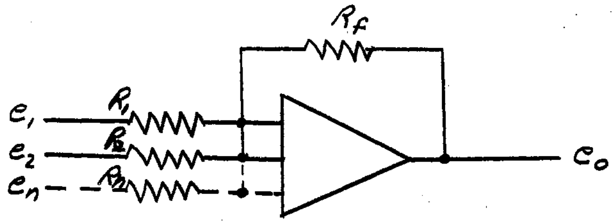
Two Donner Model 3400 desk type analog computers were available for this work. Each unit contained a problem board, coefficient potentiometers and all the necessary diodes, condensers and resistors to patch the problem. Each computer contains ten chopper-stabilized direct coupled amplifiers and has an accuracy of 0.1 percent. Four functional multipliers were available, and it was necessary to use all of them. These units were electronic and had an accuracy of around five percent and therefore degraded the results to a total accuracy of around five percent.

The computer language evolves around the operational DC amplifier which is represented by the symbol



These are used for inverting, amplifying, summing, and with the proper diodes they are used to limit functions. From the diagram shown, R_1, R_2, . . . R_n represent input resistors and R_f the feedback resistor in a typical amplifier set-up. The number of inputs can vary from one to about four or five, depending on the use. The amplifier has the following properties:

1. The output is inverted.
2. The dynamic range is ± 100 volts.
3. The gain is determined by R_f/R_{input} .
4. The output takes the form $e_o = -R_f(e_1/R_1 + e_2/R_2 + \ldots + e_n/R_n)$.



In order to perform integration, the feedback resistor is replaced by a condenser, and the output of the integrator will have the form $e_o = -\int_0^t \frac{1}{C_f}(e_1/R_1 + e_2/R_2 + \ldots + e_n/R_n)$. Initial conditions can be applied by establishing a voltage on the capacitor before the problem is started.

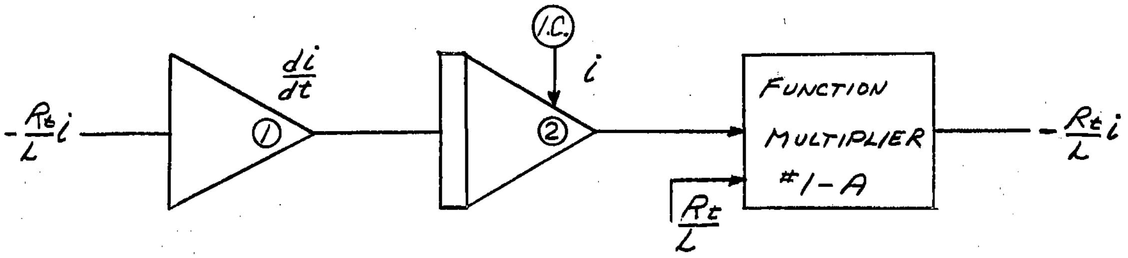
The condensers and resistors are available so that they can be plugged into the problem board as needed. The coefficient potentiometers are used to set odd values of resistance and to adjust the reference voltages supplied.

The Computer Program

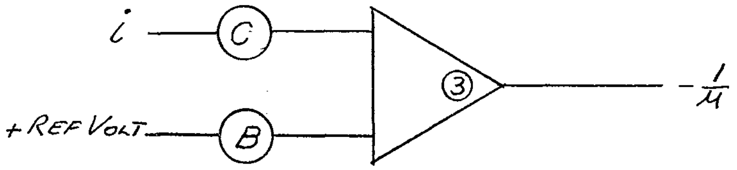
Through the proper use of Equations (1) through (4) from Chapter II, it is possible to simulate the release cycle of the relay on an analog computer. Rewriting Equation (1) in the form

$$\frac{di}{dt} = - \frac{R_t}{L} i ,$$

it can now be programmed using an inverter, an integrator, and a function multiplier. The program takes the form shown, where the initial condition set on the integrator represents the steady state coil current before decay. Since the inductance is a variable, it has to be multiplied with the current through a function multiplier.



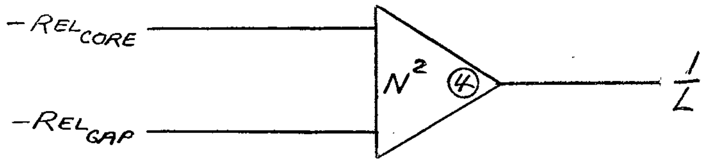
In Chapter III the permeability of the magnetic core was shown to be approximately proportional to the inverse of the coil current on decay. Using this relationship the permeability is generated through the following set-up where the values set on the potentiometers B and C will set the limits on the permeability curve. Potentiometer B will set the value of 1/u when the current approaches zero, and potentiometer C the value 1/u when the coil current is maximum.



Rewriting Equation (2) in the form

$$1/L = \frac{\text{Rel}_{\text{core}} + \text{Rel}_{\text{gap}}}{N^2} ,$$

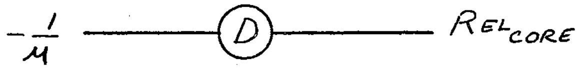
the inductance can be generated by the following circuit.



The reluctance of the core can be generated from the equation

$$\text{Rel}_{\text{core}} = \ell/Au$$

and is shown in the circuit



where potentiometer D represents ℓ/A . Similarly for the reluctance of the air gap ,

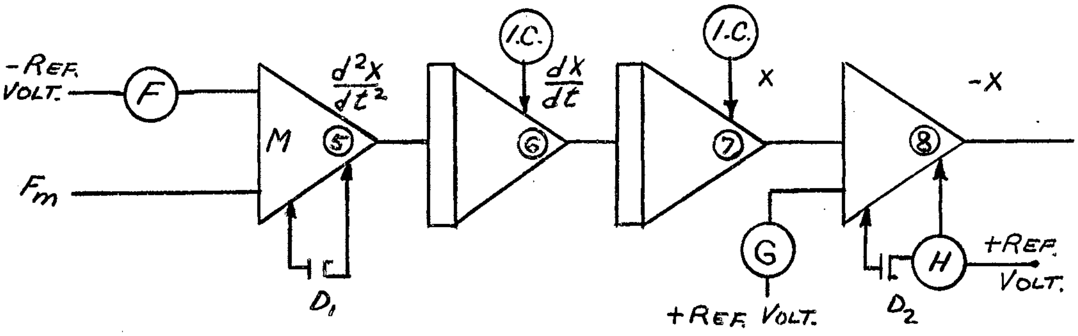
$$\text{Rel}_{\text{gap}} = x/uA .$$

The program is shown below where potentiometer E represents ℓ/uA.

The motion of the armature can be generated by integrating Equation (3) twice and setting the proper limits on the air gap. Rewriting Equation (3) in the form

$$\frac{d^2x}{dt^2} = \frac{1}{m}\,(\text{S.B.} - F_m) \quad ,$$

the program is shown below. The diode D_1 is used to limit conduction of amplifier 5 to the condition where the spring bias must be greater than the magnetic pull, since there should be no armature motion before this condition exists. Diode D_2 limits the output of amplifier 8 to a prescribed level by potentiometer H, which represents the armature stop on the relay. Potentiometer G represents the residual air gap of the relay, setting a small voltage at the output of amplifier 8, where this voltage is zero for zero residual air gap. The spring tension is represented by potentiometer. F.



Equation (4) was originally specified as the following:

$$F_m \propto \frac{1}{x^2} \quad \frac{u^2 N^2 i^2 A}{4(D^2 + \ell^2)u_o} \quad .$$

In order to make this equation realizable, the relationship that the force has on the armature must be known. Figure 10 shows this relationship and is a representation for the inverse square law. Since the force generated at the pole has finite limits, then a more close approximation of this curve is needed. Considering that the air gap approaches but is never equal to zero, a line representing some "minimum air gap" is shown in Fig. 10. If this "minimum air gap" is considered very small, then Fig. 11 can be assumed where the curve shown is a continuation of the inverse square curve. This maximum force, F_{max}, can then be assumed to be that maximum force generated by the magnetic circuit. This curve can now be represented by the equation $F_x = F_{max}/2^x$, where F_x is the force at some distance x. For very small values of air gap, in the order of 1/100 of a centimeter, the curve of Fig. 12 is drawn and shows that for very small values of air gap 2^x varies linearly. The equation can now be specifically stated

$$F_m = \frac{u^2 N^2 i^2 A}{4 \times 2^x (D^2 + \ell^2)\, u_o} .$$

Since the diameter of the solenoid is much less than the length, the $D^2 < \ell^2$ and the denominator can take the form $4xu_o \ell^2$. The program takes the form

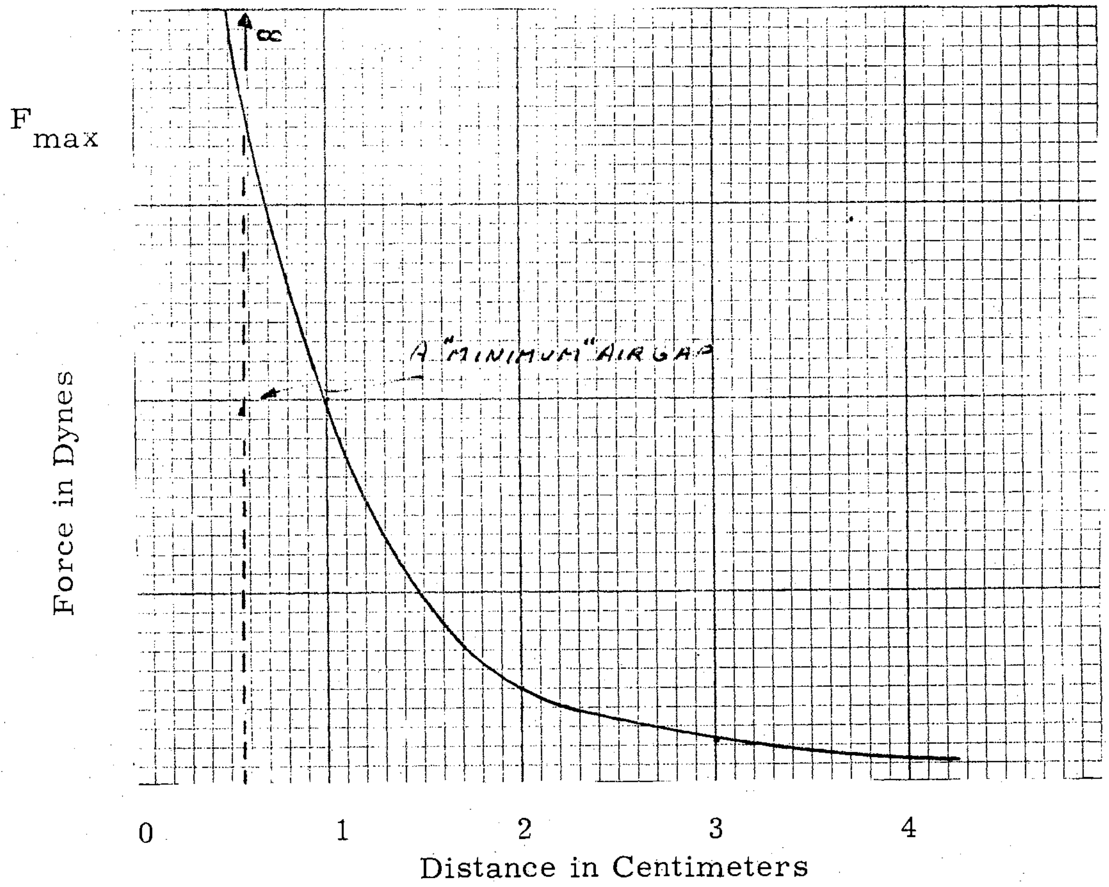


Fig. 10. Force Versus Distance Inverse-Square Law

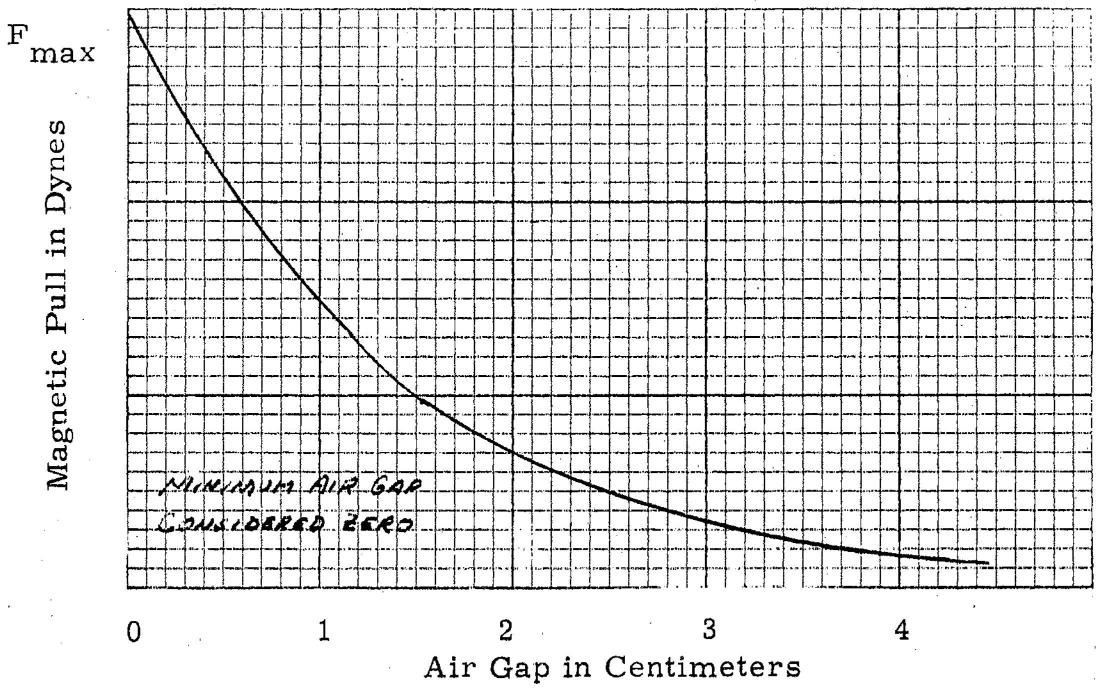


Fig. 11. Magnetic Pull Versus Air Gap

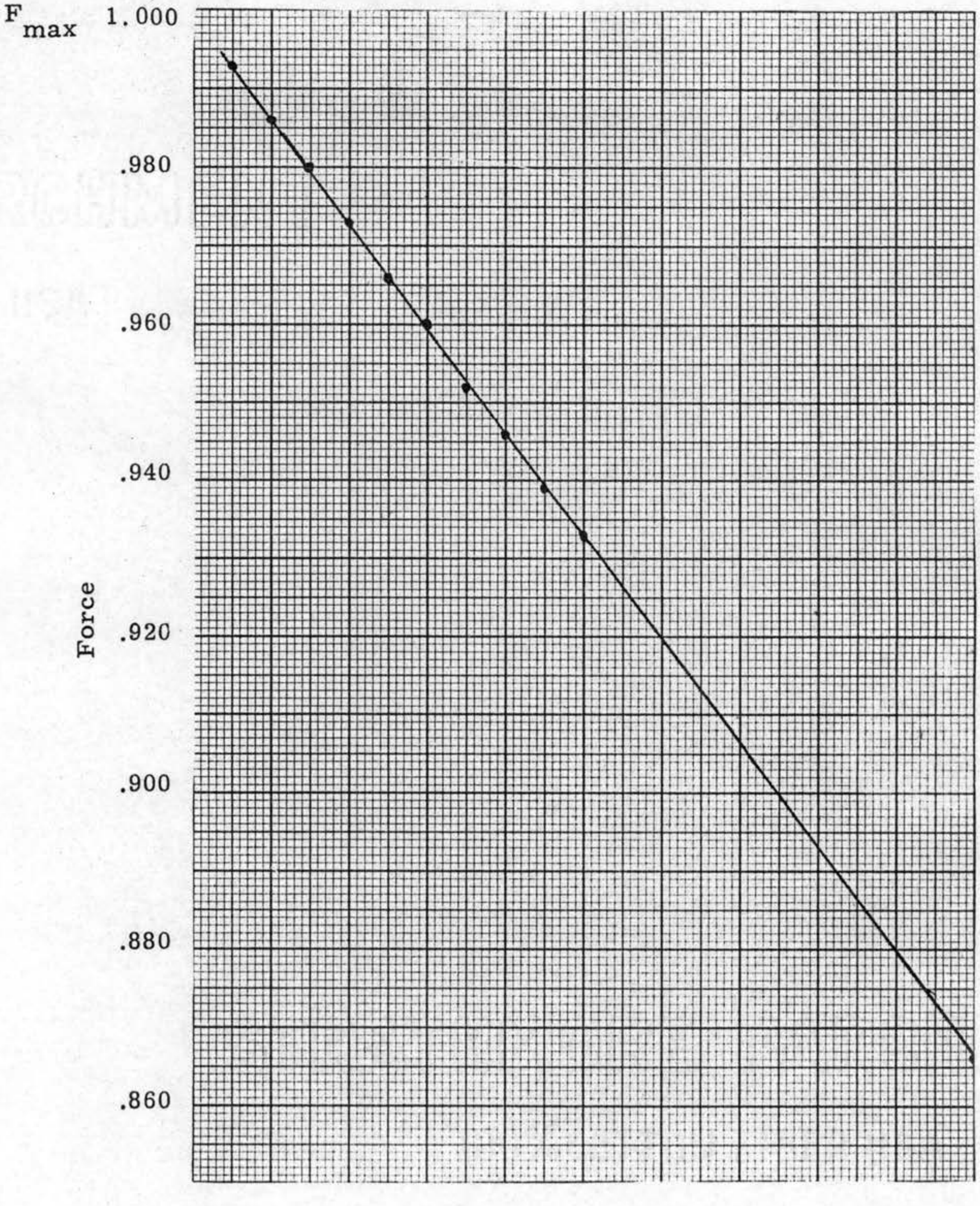
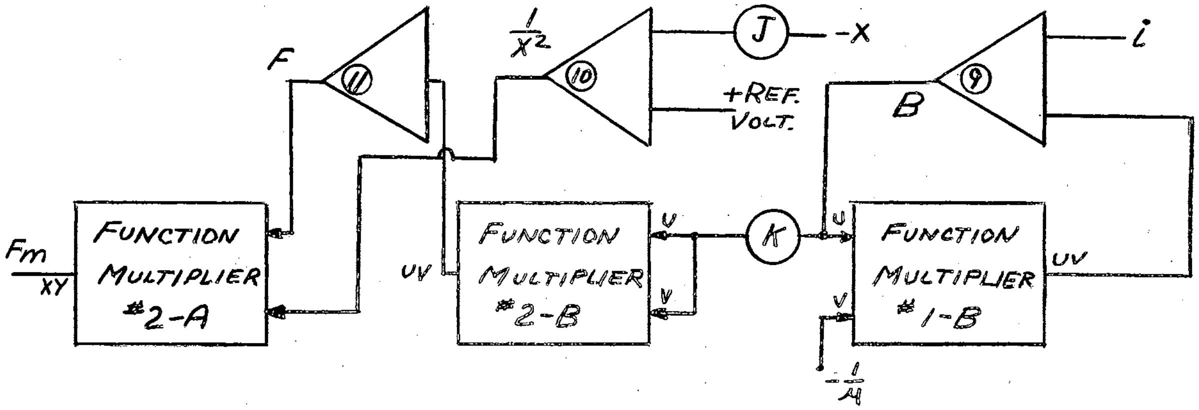


Fig. 12. Force Relative to Air Gap Plotted e^x



where function multiplier #1-B and amplifier 9 represent a divider circuit, function multiplier #2-A multiplies the force times the output of amplifier 10, where amplifier 10 presents the curve relationship shown in Fig. 8 inverted. Function multiplier #2-B represents a squaring device.

Scaling the Equations

In order to run the program, physical quantities must be known. Considering the relay parameters used in Chapter III, the following set of values was chosen for this problem:

i_{ss} = 100 milliamperes

R_s = 100 ohms

R_d = 900 ohms

R_t = 1000 ohms

ℓ = 6 cm

A = 1 cm^2

N = 4000 turns

X = 0.025 cm (approximately 0.01 inches)

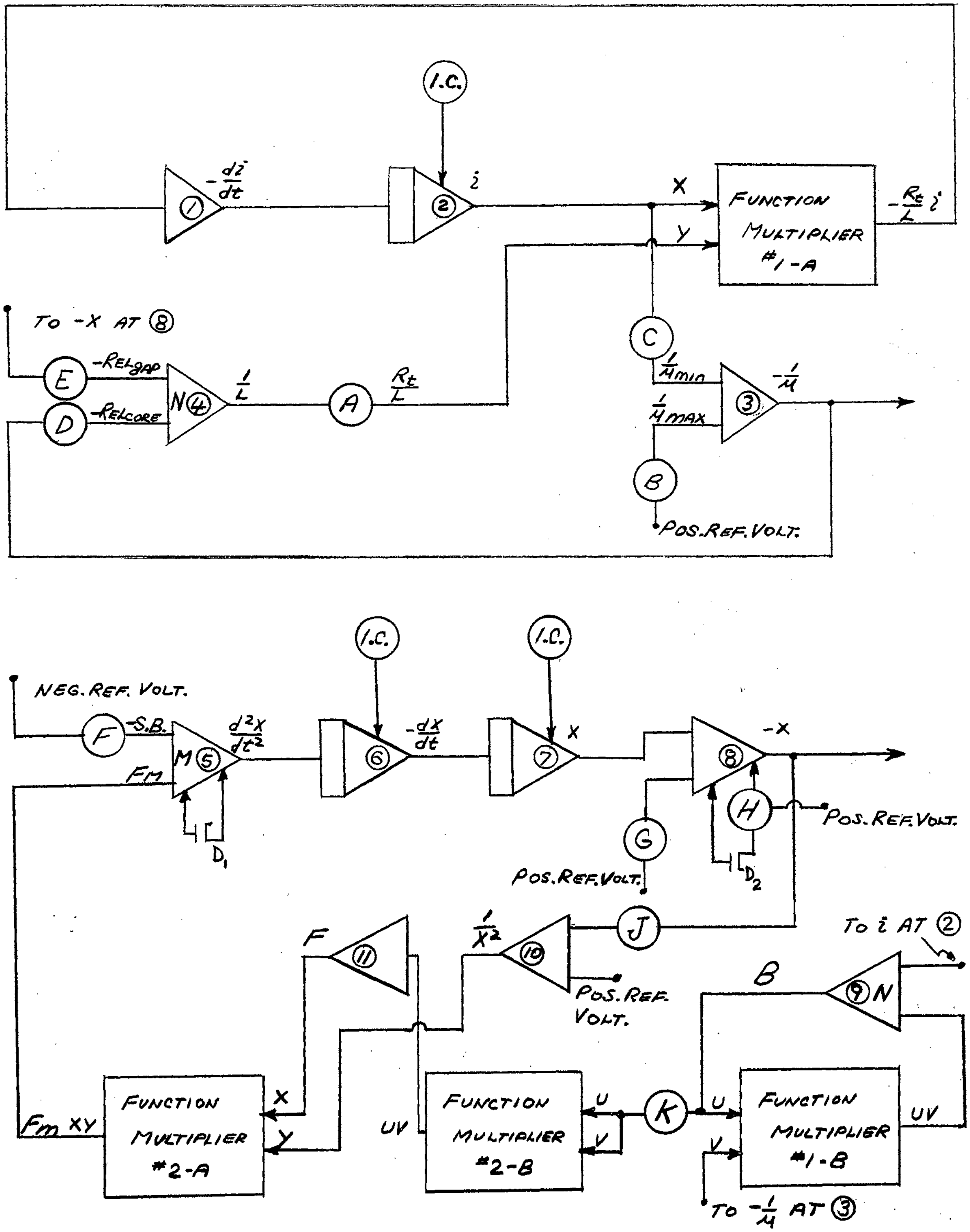


Fig. 13. Preliminary Computer Program for the Release Cycle of an Electromagnetic Relay

X_r = to be varied at will from zero to x

u_o = permeability of air = $4\pi \times 10^{-7}$ henrys/meter

m = 10 grams

u = permeability of core

As i approaches zero, $u = 70 \times 4\pi \times 10^{-7}$ henrys/meter

For i maximum, 100 ma. $u = 200 \times 4\pi \times 10^{-7}$ henrys/meter

S.B. = to be varied at will from zero to 408 grams (102 grams = 1 newton)

The purpose of the following calculations was to determine the values of reluctance under different conditions of current and air gap. With this data it was possible to determine the corresponding values of inductance using Equation (2). This information will be used to scale amplifier 4.

Calculation of Reluctance

$$\text{Rel}_{\text{core}}(\text{i = maximum value}) = \ell/uA = 6 \times 10^{-2}/70 \times 4\pi \times 10^{-7} \times 10^{-4}$$

$$= 6.66 \times 10^{6} \text{ reciprocal henrys}$$

$$\text{Rel}_{\text{core}}(\text{i at zero}) = 6 \times 10^{-2}/200 \times 4\pi \times 10^{-7} \times 10^{-4}$$

$$= 2.34 \times 10^{6} \text{ reciprocal henrys}$$

$$\text{Rel}_{\text{gap}}(\text{air gap maximum}) = 0.025 \times 10^{-2}/4\pi \times 10^{-7} \times 10^{-4}$$

$$= 2.0 \times 10^{6} \text{ reciprocal henrys}$$

$$\text{Rel}_{\text{gap}}(\text{air gap at zero}) = 0$$

Calculation of Inductance from Values of Reluctance

$$L = \frac{N^2}{Rel_{core} + Rel_{gap}} \quad \text{henrys}$$

$$L\ (i_{ss},\ 0.025\ \text{cm}) = 16 \times 10^6/(6.66 + 2.0) \times 10^6 = 1.85\ \text{henrys}$$

$$L\ (i_{ss},\ 0\ \text{cm}) = 16 \times 10^6/6.66 \times 10^6 = 2.40\ \text{henrys}$$

$$L\ (i = 0,\ 0.025\ \text{cm}) = 16 \times 10^6/(2.34 + 2.0) \times 10^6 = 3.66\ \text{henrys}$$

$$L\ (1 = 0,\ 0\ \text{cm}) = 16 \times 10^6/\ 2.34 \times 10^6 = 6.80\ \text{henrys}$$

maximum possible value of inductance = 6.60 henrys

minimum possible value of inductance = 1.85 henrys

Calculation of Maximum Magnetic Pull on the Armature

The maximum magnetic pull exists on the armature when the air gap is considered at zero and the current at its maximum value. Using these values and Equation (4), the pull was calculated so that amplifiers 9 and 10 could be scaled.

$$F_m = \frac{u^2N^2i^2A}{4u_o\ell^2 2^x} = \frac{(70 \times 4\pi \times 10^{-7})^2 \times (4000)^2 \times (0.1)^2 \times 10^{-4}}{4 \times (4\pi \times 10^{-7}) \times (0.06)^2 \times 1}$$

$$= 7.05\ \text{newtons}\ .$$

Once the dynamic limits of the inductance were known, the problem was time-scaled so that it would be compatible to the computer time. Using the Donner computer and a graphic Sanborn recorder, the problem should be scaled so that it runs around 1 to 10 radians per second (0.6 to 6 seconds), well within the response characteristics of both units. Solving Equation (1) where L is considered a constant, the expression $e^{-R_t t/L}$ was obtained. Using the exponent R_t/L and an average value of inductance, "ball park" figures were obtained for scaling the problem. Using the average value of inductance and a resistance of 1000 ohms,

$$\frac{L_{av}}{R_t} = \frac{(6.80 + 2.08)/2}{1000} = 4.44 \times 10^{-3} \text{ seconds} .$$

By scaling Equation (1) and its associated circuitry by a factor of 10^3, the speed of operation was decreased to around 4.4 seconds. The equation was scaled in the following manner:

let the equation be $\frac{di}{d\tau} = -\frac{R_t}{L} i$, and let $\tau = \frac{t}{10^3}$

and $10^3 \times \frac{di}{dt} = -\frac{R_t}{L} i$ or $\frac{di}{dt} = -\frac{R_t}{10^3 L} i$.

No time scaling of Equation (2) or (4) is necessary since neither has time dependence originating from its own variables.

In order to scale Equation (3), it is necessary to determine the volts-centimeter relationship for the output of amplifier 8 in terms of the input variables. The maximum return force possible on the armature is 408 grams (408/102 newtons). Using this information the distance that the armature would travel in a specified period of time was determined (considering no armature backstop). From this the volts-centimeter relationship was determined. Plugging the values into Equation (3) for $m = 10^{-2}$ kilograms and S.B. = 4 newtons

$$\frac{d^2x}{dt^2} = \frac{4}{10^{-2}} = 400 \ .$$

Integrating,

$$\frac{dx}{dt} = 400t + C_1 \ .$$

Where

$$\left.\frac{dx}{dt}\right|_{t=0} = 0 = C_1 \quad \text{and}$$

integrating again,

$$x = 200 \times t^2 + C_2 \ ,$$

and similarly for $t = 0 = x = C_2$,

$$x = 200\ t^2 \ .$$

From this equation the distance traveled by the armature in 10^{-3} seconds was calculated.

$$\begin{aligned} x = 200 \times (10^{-3})^2 &= 200 \times 10^{-6} \text{ meters} \\ &= 200 \times 10^{-4} \text{ centimeters} \\ &= 0.02 \text{ centimeters} \ . \end{aligned}$$

By arranging amplifier 8 so that it reads a specified voltage in 1 second (problem is scaled 10^3), that voltage represented 0.02 centimeters. This technique was carried out and will be explained more thoroughly later in the chapter.

The Scaled Program

Enough information has been determined and is known so that the final computer diagram could be assembled. The first step was to set Equation (1) up where the coefficient of i has its maximum value. This was done so that after the amplifier gains were set to satisfy the equation, no higher value would exist which would exceed the dynamic limits of the amplifier. Setting up the equation with the minimum value of inductance,

$$\frac{di}{dt} = - \frac{10^3}{10^3 \times 1.88} = 0.53\, i \quad .$$

The first variable scaled on the final program shown in Fig. 15 was the initial condition set on integrator 2. A scale factor of 1 milliampere = 0.8 volts was used and the I.C. was set at 80 volts, representing 100 milliamperes. Once this condition was established, it was possible to scale amplifiers 3 and 4.

Amplifier 3 was scaled in the following manner:

1. Amplifier 3 was given a gain of 2.

2. With potentiometer C set at 0%, potentiometer B was set for the maximum limit of permeability, where $1/u_{max} = 1/200 = 0.005 =$ -20 volts at e_3 .

3. Potentiometer C was adjusted for the lower limit of permeability, where $1/u_{min} = 1/70 = 0.0143 =$ -57.2 volts. With the conditions set on e_3 from step 2, the voltage at e_3 now reads -77.2 volts. (The scaling of the permeability was set so that 0.001 = 4 volts.)

Amplifier 4 was similarly scaled:

1. Amplifier 4 was given a gain of 2.

2. With potentiometer E set at 0%, potentiometer D was adjusted for the maximum core reluctance, where $\text{Rel}_{\text{core}} = 6.66 \times 10^6$ reciprocal henrys = 66.6 volts; e_3 was set to read 66.6 volts. The corresponding value of the Rel_{core} for $i = 0$ was automatically set when the current went to zero by the characteristics of the system.

3. Potentiometer E was set after amplifier 8 was scaled.

After amplifiers 3 and 4 had been scaled, amplifier 1 and integrator 2 were scaled in the following manner:

1. Amplifier 1 was given a gain of 2 (maximum voltage at e_1 would then be $2 \times 0.53 \times i = 2 \times 0.53 \times 80$ volts = 84.8 volts).

2. Integrator 2 was then set with a gain of 1/2 so that the coefficient would again be 0.53 i.

3. Potentiometer A was then adjusted to give 84.8 volts at e_1.

Amplifiers 5 and 8 and integrators 6 and 7 were scaled next using the following procedure:

1. Initial conditions on integrators 6 and 7 were set at zero.

2. All amplifiers and integrators were given a gain of one.

3. Function multiplier #2-A was disconnected from amplifier 5 so that maximum spring bias would occur on the armature.

4. Potentiometer F was set at - 40 volts, where 10 volts = 1 newton.

5. Potentiometers G and H were set at 0%.

6. The program was run off and the output of amplifier 8 was checked at 1 second to determine the volts-centimeter relationship (discussed earlier in the chapter). On the first run-off the value of voltage was very low, so a gain of 10 was given to amplifier 8 from the input of 7. Figure 14 shows the computer run-off of this and the corresponding volts-centimeter relationship. On the recording 1 cm vertical equaled 20 volts and the speed on the horizontal scale was 20 mm/sec. Correspondingly, a voltage of 40 volts was given for 0.02 cm. This set the scale relationship for amplifier 8 of 20 volts/ 0.01 cm with a scaled time of 1 second.

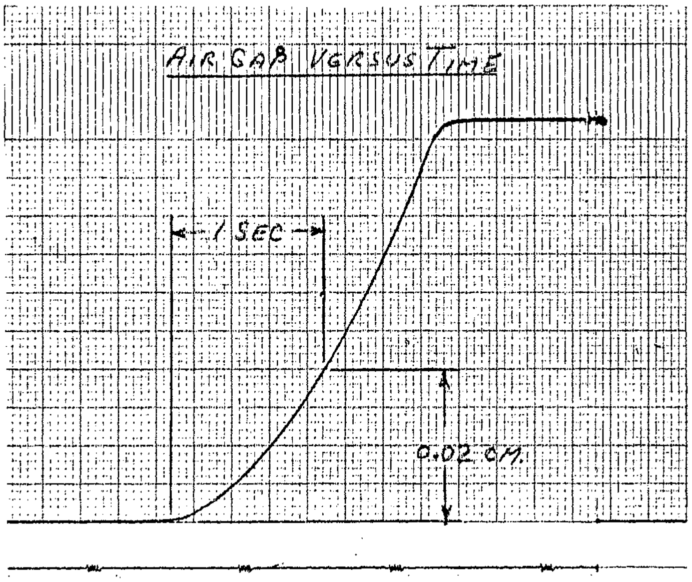


Fig. 14. The Volts-Centimeter Relationship for Amplifier 8

7. Using this scale factor potentiometer H was set to limit the output of amplifier 8 to 0.025 x 20 = 50 volts for the specified air gap of 0.025 cm. Later on potentiometer G will be set for the desired air gap; however, at this time it was set at 0% representing 0 residual air gap.

Potentiometer E was then adjusted with the output of 8 set at - 50 volts so that the output of amplifier 4 read 20 volts with potentiometer D disconnected. This set the Rel_{gap} for maximum air gap for 2×10^6 reciprocal henrys = 20 volts.

The final scaling which was performed to complete the computer set-up was the scaling of the function multipliers #2-A, #2-B, and #1-B, and also the amplifiers 9 and 10. The procedure was as follows:

1. Potentiometer J is set at 0%, representing the armature resting on the pole of the magnet.

2. Potentiometer K was then adjusted so that 7.05 newtons of magnetic pull resulted at the output of the function multiplier (7.05 newtons = 70.5 volts). This was read with an external voltmeter.

3. The output of amplifier 8 was set at - 50 volts, corresponding to the armature being against the backstop.

4. Potentiometer J was then adjusted so that the putput of function multiplier #2-A now corresponds to the value given in Fig. 12 for an air gap of 0.025 cm. This was taken as 0.985 x 70.5 volts = 69.4 volts.

This completed the time and amplitude scaling of the computer program. Table II was made up so that during the running of the program the relations of the parameters could be easily seen.

TABLE II

PARAMETER RELATIONSHIPS TO COMPUTER PROGRAM

e_1	$\frac{di}{dt}$ = 1 ma = 0.8 volts
e_2	i 1 ma = 0.8 volts
e_3	-1/u 0.001 (relative) = -4 volts
e_4	1/L 1 henry^{-1} = 25.5 volts
e_5	acceleration on armature (cm/sec^2)
e_6	velocity of armature cm/sec
e_7	relative position of armature
e_8	position of armature with backstop 0.01 cm. = 20 volts
e_9	relative flux density on surface of magnet pole webers/M^2
e_{10}	proportional variable for force versus distance relationship
A	N and R_t (relative relationships)
B	sets limit on maximum permeability
C	sets limit on minimum permeability
D	sets reluctance of core
E	sets reluctance of air gap
F	sets spring tension (S.B. = 10 volts = 102 grams)
G	residual air gap setting (0.01 cm = 2 volts at e_8)
H	air gap (2 volts = 0.001 cm)
J	used to set the relationship for amplifier 10
#1-A	$-\frac{R_t}{L} i$
#2-A	force on armature of relay
#2-B	force on surface of magnetic pole

Multiply computer time by 10^{-3} to obtain relay time.

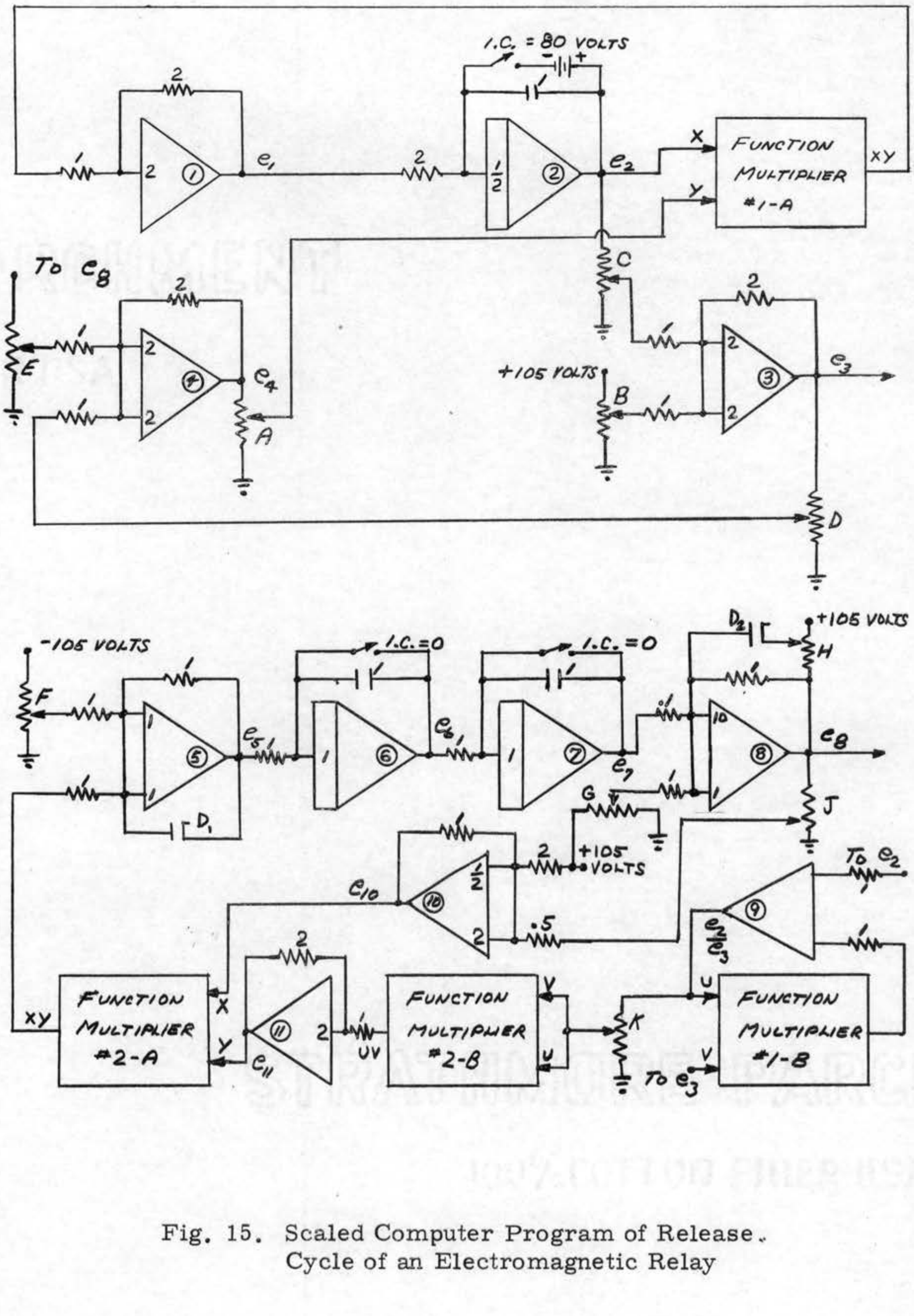


Fig. 15. Scaled Computer Program of Release Cycle of an Electromagnetic Relay

CHAPTER V

THE COMPUTER RESULTS

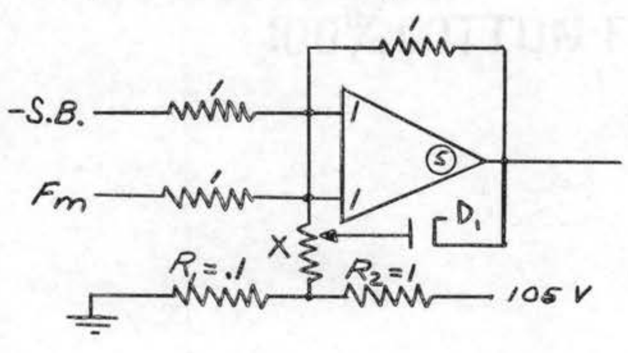
Setting Up the Program

The computer program shown in Fig. 15 was set up using both Donner computers that were available. A Tektronics model 651 oscilloscope was used to trouble shoot the circuit and a Sanborn model 320 dual channel "hot pen" recorder was used to display the results shown in Figs. 16 through 26. All three integrators in the program were placed on one computer so that the initial conditions could be controlled using a single control, alleviating the necessity of muting the two computers together.

In order to trouble shoot the program systematically it was assembled in sections, and each section checked individually for proper operation, indicative to the equation that it represents. Amplifier 1 and integrator 2 were set up first and a positive voltage was applied through a potentiometer to the input Y of function multiplier #1-A in place of the 1/L connection. This voltage was adjusted so that e_1 read 84.8 volts. The computer was then run off and both the amplifier and integrator were observed to be functioning properly, neither circuit saturated and the voltage decayed exponentially. Amplifiers 3 and 4 were assembled next and no difficulty was encountered. The armature motion circuit consisting of amplifiers 5 and 8 and integrators 6 and 7 were set up next. The input to amplifier 5 from F_m was set at a voltage of 70 volts through a relay that was disconnected on compute. The spring tension was set through potentiometer F at 30 volts and this problem was run off. It was noticed that integrator 6 saturated with time. The output of amplifier 8 was set so it was limited to -50 volts through D_2, the limiting diode, and potentiometer H. The computer was again run off and it was determined that the integrator saturated after amplifier 8 reached its limited value of -50 volts, therefore it was felt that the saturation of integrator 6 was irrelevant to the armature motion to the stop. The circuit corresponding to function multipliers #2-A, #2-B and #1-B was the final circuit to be checked out. The input i of amplifier 9 was connected and the input 1/u to function multiplier #1-B was also connected. The potentiometer J was set at zero. The output of function multiplier #2-A was then measured and potentiometer K was adjusted for maximum output, which was less than 50 volts. The gain of amplifier 11 was increased to 2 and the process repeated. The output circuit then could be adjusted to the maximum force value of $F_m = 7.02$ newtons $= 70.2$ volts. The top of potentiometer J was then adjusted to -50 volts by applying a voltage to amplifier 8. (This simulated the armature being open at 0.025 centimeters.) The potentiometer was then adjusted so that the output of the function multiplier #2-A corresponded to that shown in

Fig. 12 for an air gap of 0.025 cm. This turned out to be 69.4 volts. It was found that oscillations occurred in function multiplier #1-B. This was corrected by adding a 200 μμf condenser across amplifier 9 to bypass the high frequency gain of the circuit. With this completed the circuits acted stable and no other difficulties of this nature were apparent. With the assembly of the program on the computer, the outline for scaling the amplifiers to the proper voltages described in Chapter IV was followed through.

When the computer was run off with the program fully set up and adjusted, difficulty was encountered in the limiting circuit of diode D_1. Over long time solutions (2-3 seconds) the integrators 6 and 7 appeared to drift before the armature started to move, that is, before $S.B. > F_m$. The (-3) 0 (3) voltmeter on the computer console was used to check the outputs of e_5, e_6, and e_7. It was discovered that the limiter circuit D_1 caused a voltage of -0.7 volts at e_5 when $S.B. < F_m$. During this time the diode is conducting to limit the swing of amplifier 5 to positive directions. The voltage drop across the diode caused this voltage at e_5, since the voltage at the grid of the DC amplifier is essentially zero. In order to overcome this, the limiter circuit shown in Fig. 15 for amplifier 5 was changed to the circuit shown below.



The resistors R_1 and R_2 were used to divide the 105 volt reference on the computer and place about one volt on the leg of the potentiometer X. This gave a vernier for X and made it possible to balance out the -0.07 volts on e_5 easily. The operation of the resistors and potentiometer was to raise the anode end of the diode to 0.7 volts, thereby raising the cathode end to zero volts. The circuit performed much more satisfactorily with this circuit, however a slight amount of drift was still encountered over long time solutions.

Verification of the Computer Program

The behavior of the coil current waveform was encouraging. When the program was run off the observation of the coil current showed the characteristic "cusp" indicated in Fig. 2 at the same time that amplifier 8 shifted from zero (with no residual air gap) to -50 volts. From this evidence it was felt that a comparison of the release times similar to the conditions shown in Fig. 5 would be a good indication of the behavior of the complete program.

Figures 16 through 20 represent the curves recorded for different values of residual air gap and spring tension. Figure 21 is the graph made for the composite values of spring tension and residual air gap for release plus transit times. The time correspondence between the two graphs was considered to be ten to one since the total resistance in the experimental relay was 100 ohms and the total resistance of the computer-simulated relay was 1,000 ohms. The resistance is a constant in determining the time constant of the circuit, so it was felt safe in assuming this. The solid line curves shown in Fig. 21 were compared to Fig. 5. The results were gratifying. The spring tension curves corresponded directly, indicating that the times of the two relays were similar and that the behavior was alike. The residual air gap curves were not quite so similar. At low values of residual air gap, the curve of Fig. 5, the time increased rapidly indicating possibly that the residual magnetism had some effect. Since the curves from the computer were made without employing any residual magnetism in the circuits, it was felt that this was probably the reason for the dissimilarity between the two. It was felt from the results, however, that the relay circuit in Fig. 15 was performing properly except for small values of residual air gap and that accurate predictions on the behavior of the basic parameters could be obtained.

The flux density behavior was analyzed and compared to the behavior of a predicted curve shown in Fig. 23. This curve shows the expected flux variation for the relay during transient conditions. The curve was obtained by drawing the comparative curves for the flux decay for the armature held open, held closed and free to move. The dotted line was drawn to show the predicted variation when the armature was free to move. Since the armature accelerates as it moves, the expected change would be slow at first and then increase rapidly. An investigation has been made in the past[7] which verified this curve by electronically integrating the induced emf in a search coil embedded in the relay coil. From this the flux of the coil was dislayed and found to agree closely with this curve.

Figure 22 represents the flux density of the relay simulated. It can be seen that the coil current decay is much more rapid than the flux decay. The rapid decrease in flux density during armature motion is also shown, agreeing with Fig. 23. It was difficult to obtain a wide variation in the flux density during armature motion for high values of spring tension. This also agrees with the predicted curves, since for short release times the curves in Fig. 23 (open and closed) are close together.

Results

The flux behavior will determine the behavior of the magnetic pull on the armature of the relay. Since the flux density curves shown agreed with the predicted curves, it is believed that the behavior of the magnetic pull on the armature of the relay shown in Fig. 24 is fairly accurate. The air gap was increased to 0.05 cm. so that a more pronounced indication of the armature motion would be shown and the force where the armature was released easily found. The value of spring tension used to obtain this curve was 300 grams, the breaking

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[7] Research Reports, Sandia Corporation, Interim Report on the Dynamic Characteristics of Relays, August, 1959.

point on the "pull" decay where the change in slope occurs, is on the 3 newton line, indicating around 300 grams of magnetic pull, satisfying the condition F_m = S.B. where the armature just starts to move.

The recordings shown in Fig. 25 are for the reluctance of the total magnetic circuit plotted with armature motion and coil current decay. In these curves the reluctance efficiency was assumed to be 100%, in that all reluctances were associated with the air gap of the armature and the core of the relay. With this curve, however, valuable information on the core-gap relationship would be available.

The final set of curves which were made to show the basic behavior of the relay during release is shown in Fig. 26. The acceleration, velocity and displacement of the armature are shown. From these the nonlinearity of the curves is very evident, indicating the difficulties which would be encountered in design relying on the linear set of curves. The acceleration curve is interesting in that it can be said to represent all the forces on the armature instantaneously. As the magnetic pull of the core decreases to zero, the force (using mass times acceleration) on the armature gradually becomes constant, indicating constant spring tension as the armature is resting on the backstop.

The final curve which is shown, is Fig. 27. This is a drawing of a force function curve used in the designing of the relay coil and spring tension requirements. By the amount of shaded areas, each side of the armature motion (travel), the amount of reserve needed for the spring tension, and magnetic pull can be closely determined, thus preventing overdesigning the size of the core and the spring. These types of curves have been obtained through static approximations, taking force measurements over small increments of armature displacement. The curves shown in Figs. 24 and 26 (acceleration) could be quite valuable in design of this type. With special recording instruments (X-Y recorder) the acceleration (total forces on the armature) versus armature displacement, and the magnetic pull on the armature versus the armature displacement could provide important roles in the relay design. Using "ball park" figures on the computer, closer tolerances could be obtained in the magnetic pull and spring tension variables.

The recordings shown are only an indication of the parameters that could be displayed using the analog computer. Using a little discretion the designer could adjust his program so that the parameters of particular interest to him could easily be observed. A list of parameters is presented which are the more common ones used in relay analysis and these could be obtained using the analog computer.

flux versus amp-turns
flux versus gap
force versus gap
force versis amp-turns
B-H curve
flux leakage versus position

Initially it was planned to carry on an investigation into the stability of the armature of the relay during release. Time, however, prevented this from being accomplished. The basis for this investigation was a paper[8] on the analysis of the armature motion of a relay during release. The equations from this paper were programmed and run off on the computer. The results from this agreed with the results obtained experimentally in the paper. The oscillation of the armature when the spring tension was canceled out by the gravitational pull at some equilibrium position occurred as predicted.

It was felt that this work was a duplication of the work covered in the paper and is therefore not presented here. It is recognized, however, that a more exhaustive analysis of the armature motion during release could be performed.

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[8] Cameron, Charles F., and E. F. Allen, Analysis of Armature Motion During Release (Fourth National Conference on Electromagnetic Relays, School of Electrical Engineering, Oklahoma State University), April 17-19, 1956.

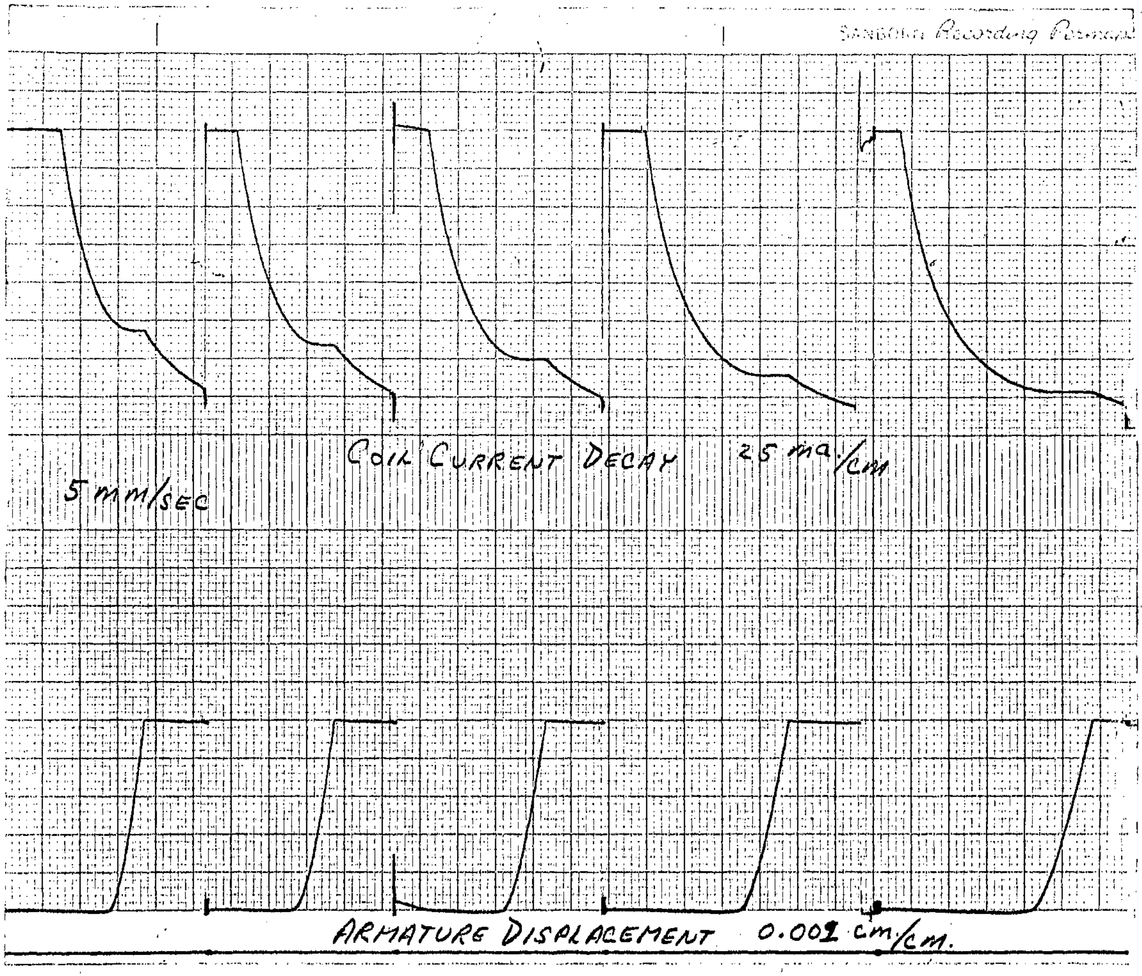


Figure 16 Sanborn Recordings of the Armature Motion and Coil Current Decay for Various Values of Spring Tension and a Residual Air Gap of Approximately Zero.

Spring Tension (left to right) 300, 250, 200, 150, and 100 grams
Number of Coil Turns = 4000
Discharge Resistance = 900 ohms
Coil Resistance = 100 ohms
Air Gap = 0.025 cm.

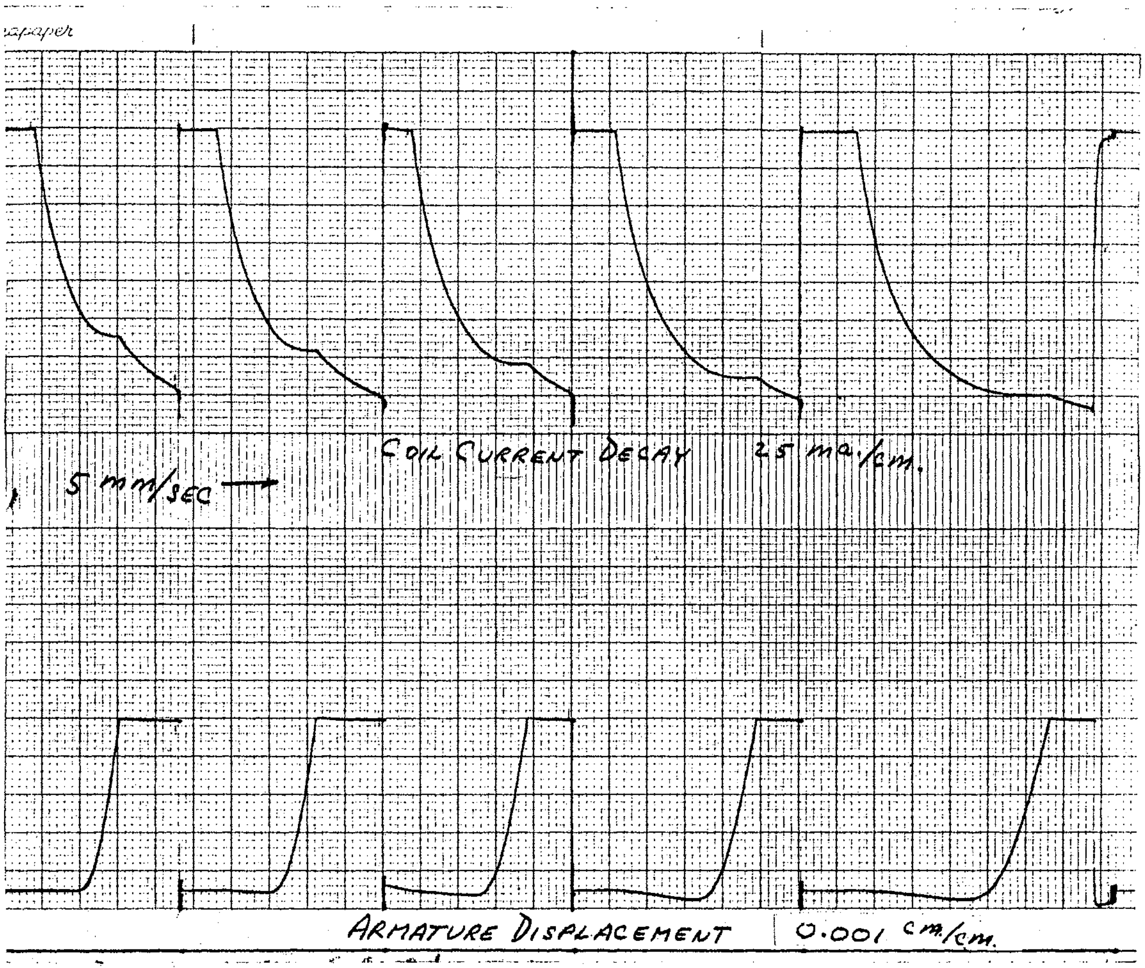


Figure 17 Sanborn Recordings of the Armature Motion and Coil Current Decay for Various Values of Spring Tension and a Residual Air Gap = 0.0025 cm.

Spring Tension (lf to rt) 300, 250, 200, 150 and 100 grams
Number of Coil Turns = 4000
Discharge Resistance = 900 ohms
Coil Resistance = 100 ohms
Air Gap = 0.025 cm.

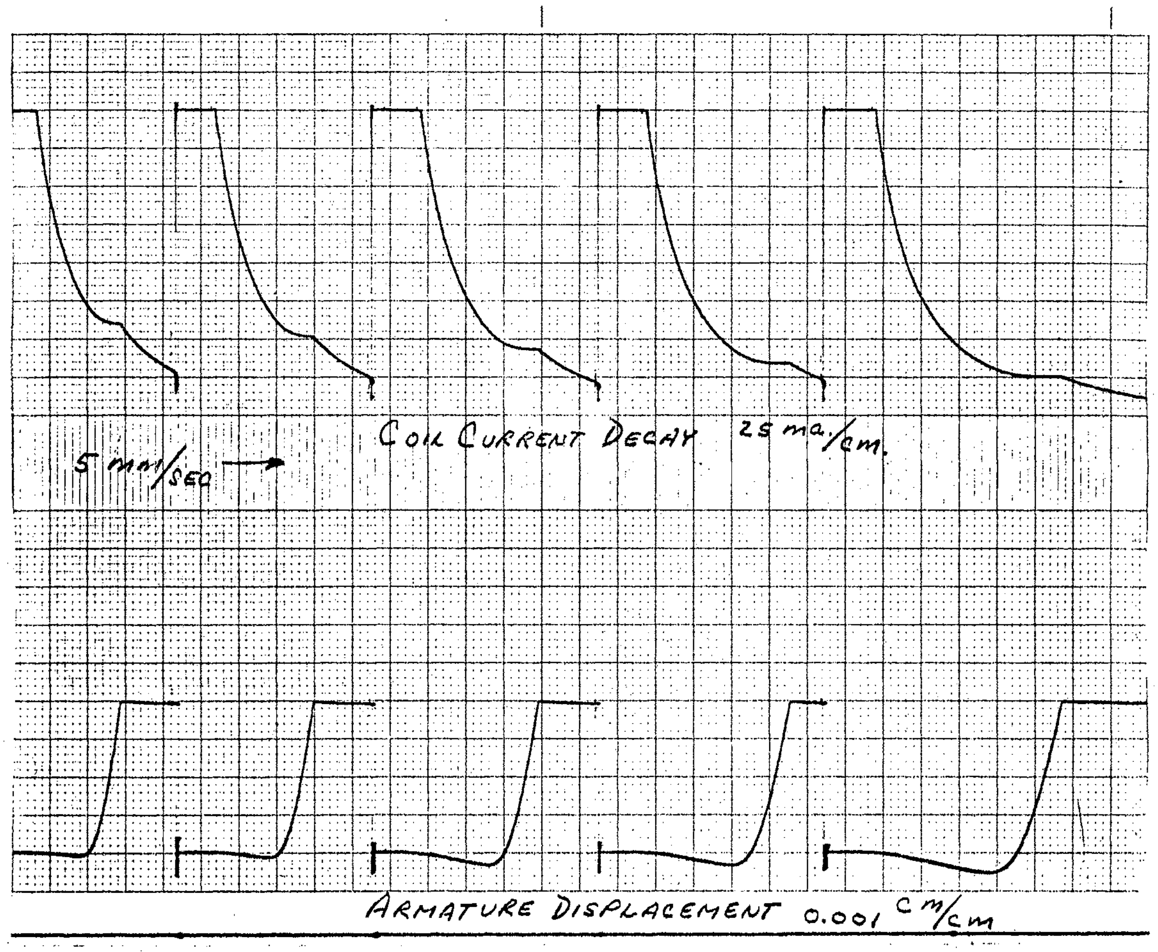


Figure 18 Sanborn Recordings of the Armature Motion and Coil Current Decay for Various Values of Spring Tension and a Residual Air Gap = 0.0050 cm.

Spring Tension (lf to rt) 300, 250, 200, 150, and 100 grams
Number of Coil Turns = 4000
Discharge Resistance = 900 ohms
Coil Resistance = 100 ohms
Air Gap = 0.025 cm.

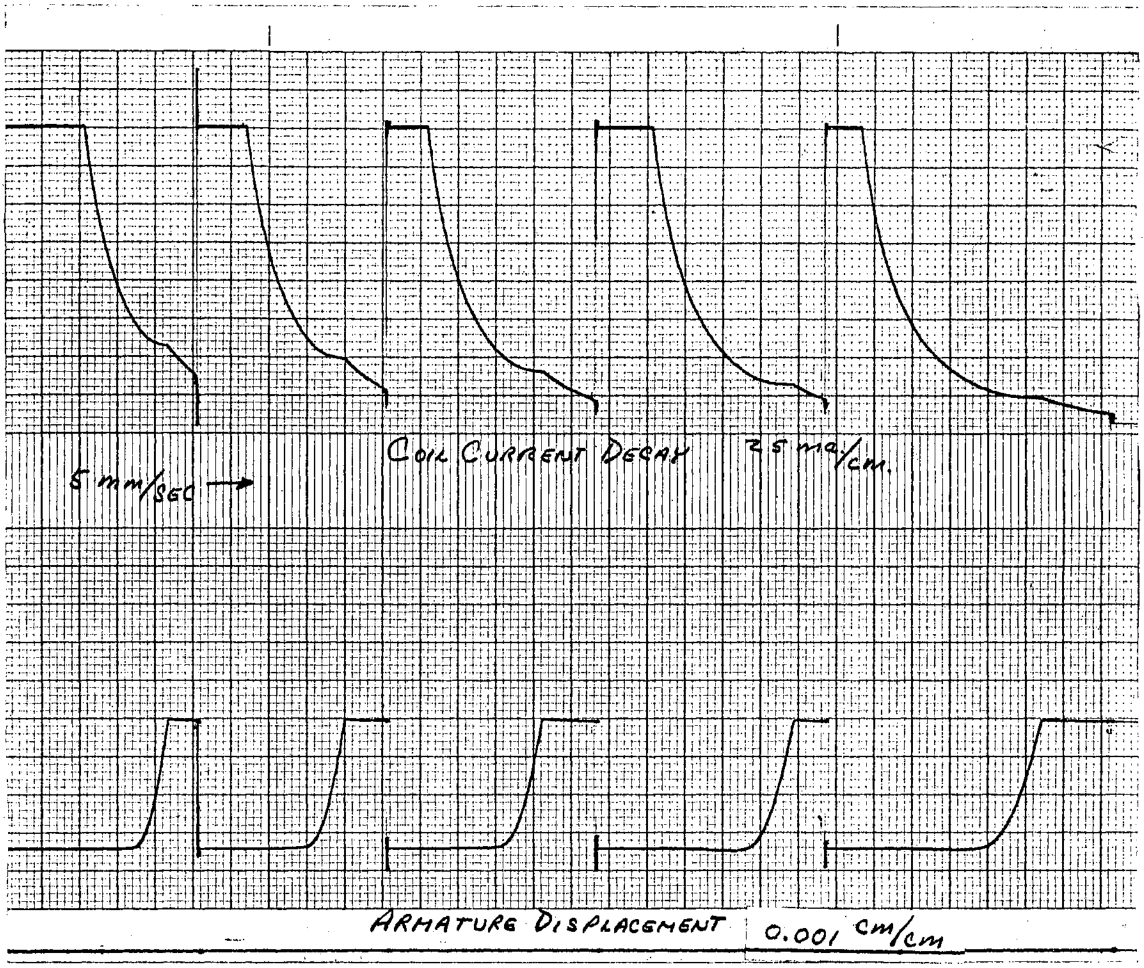


Figure 19 Sanborn Recordings of the Armature Motion and Coil Current Decay for Various Values of Spring Tension and a Residual Air Gap = 0.0075 cm.

Spring Tension (lf to rt) 300, 250, 200, 150 and 100 grams
Number of Coil Turns = 4000
Discharge Resistance = 900 ohms
Coil Resistance = 100 ohms
Air Gap = 0.025 cm.

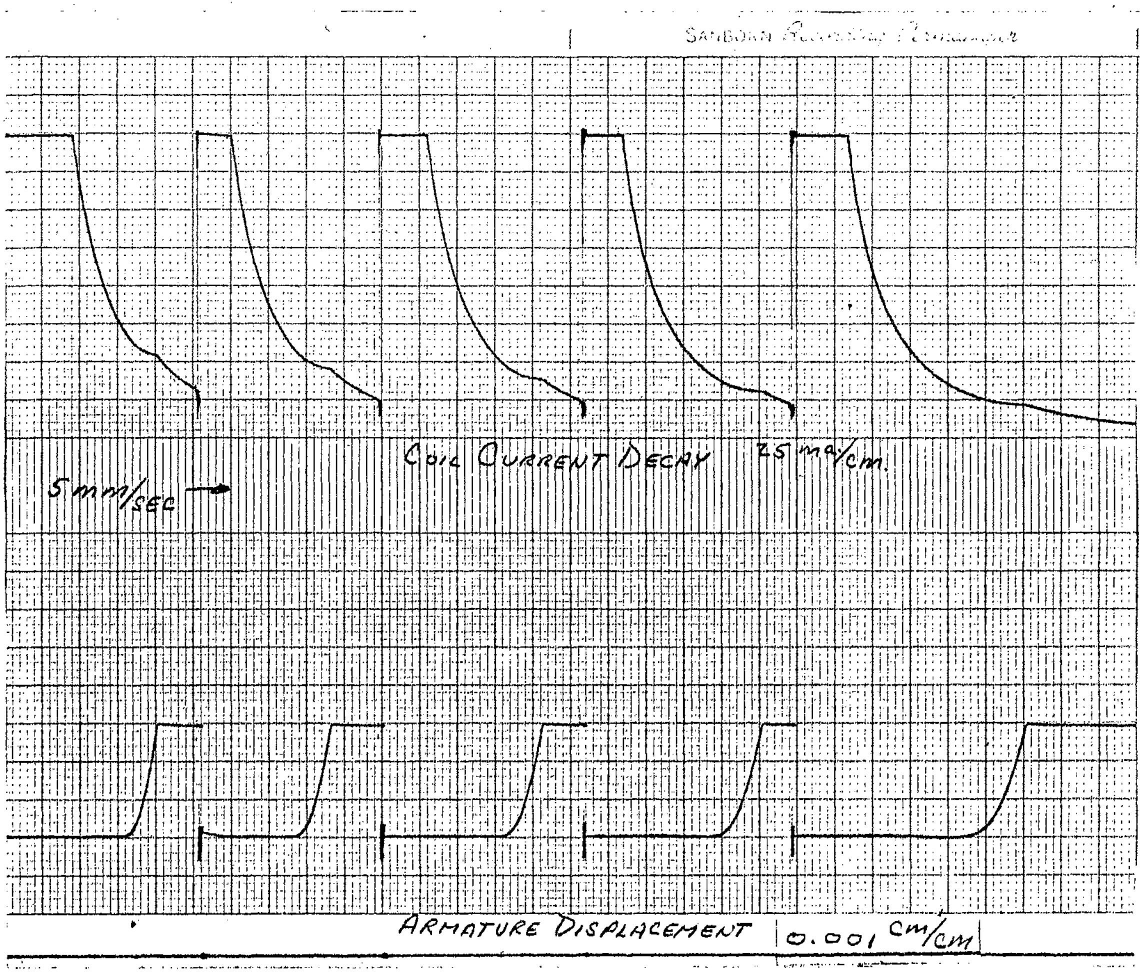


Figure 20 Sanborn Recordings of the Armature Motion and Coil Current Decay for Various Values of Spring Tension and a Residual Air Gap = 0.010 cm.

Spring Tension (lf to rt) 300, 250, 200, 150 and 100 grams
Number of Coil Turns = 4000
Discharge Resistance = 900 ohms
Coil Resistance = 100 ohms
Air Gap = 0.025 cm

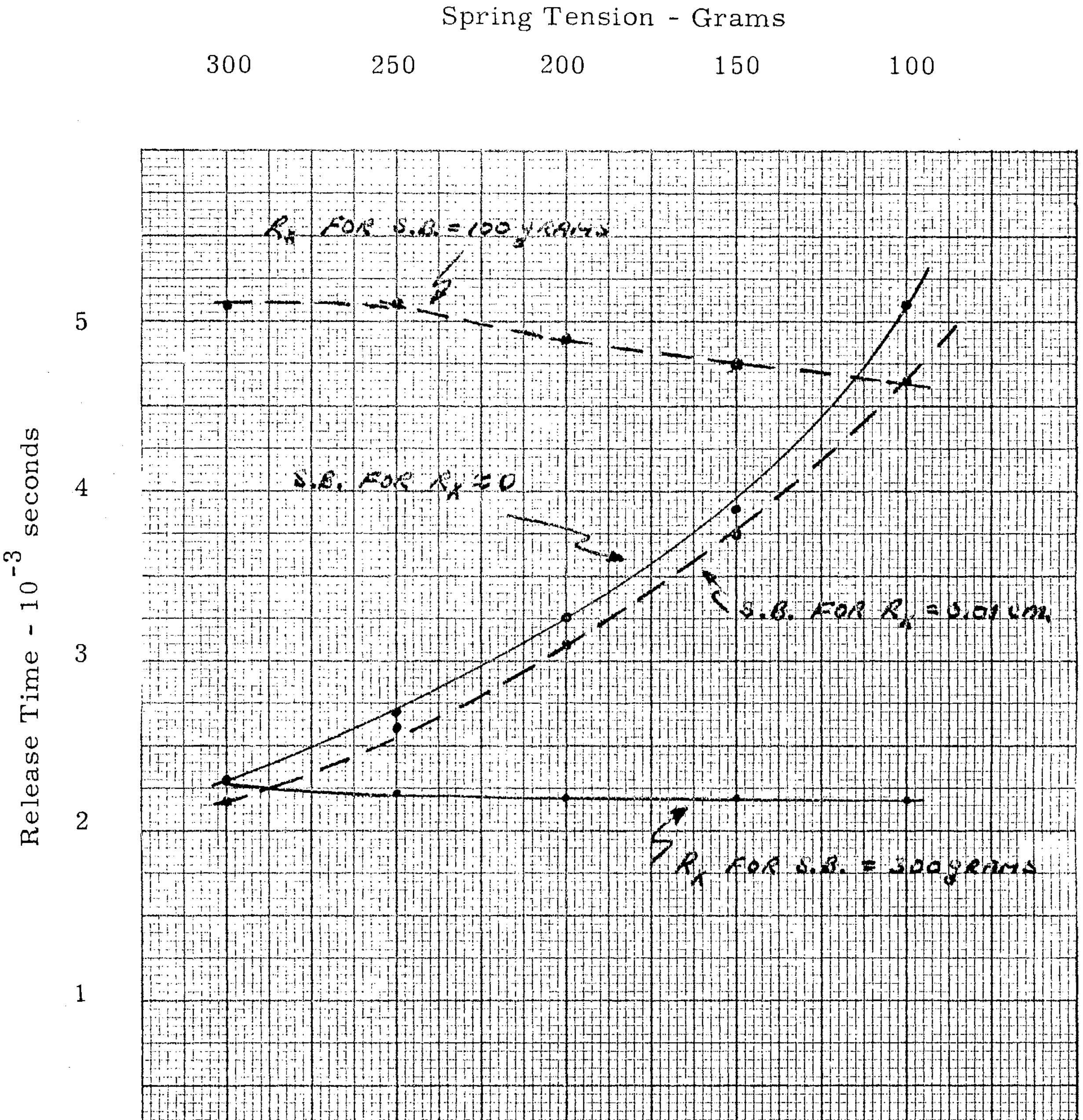


Fig. 21. Graph for the Release Time of the Release Cycle of a Relay for Various Values of Spring Tension and Residual Air Gap

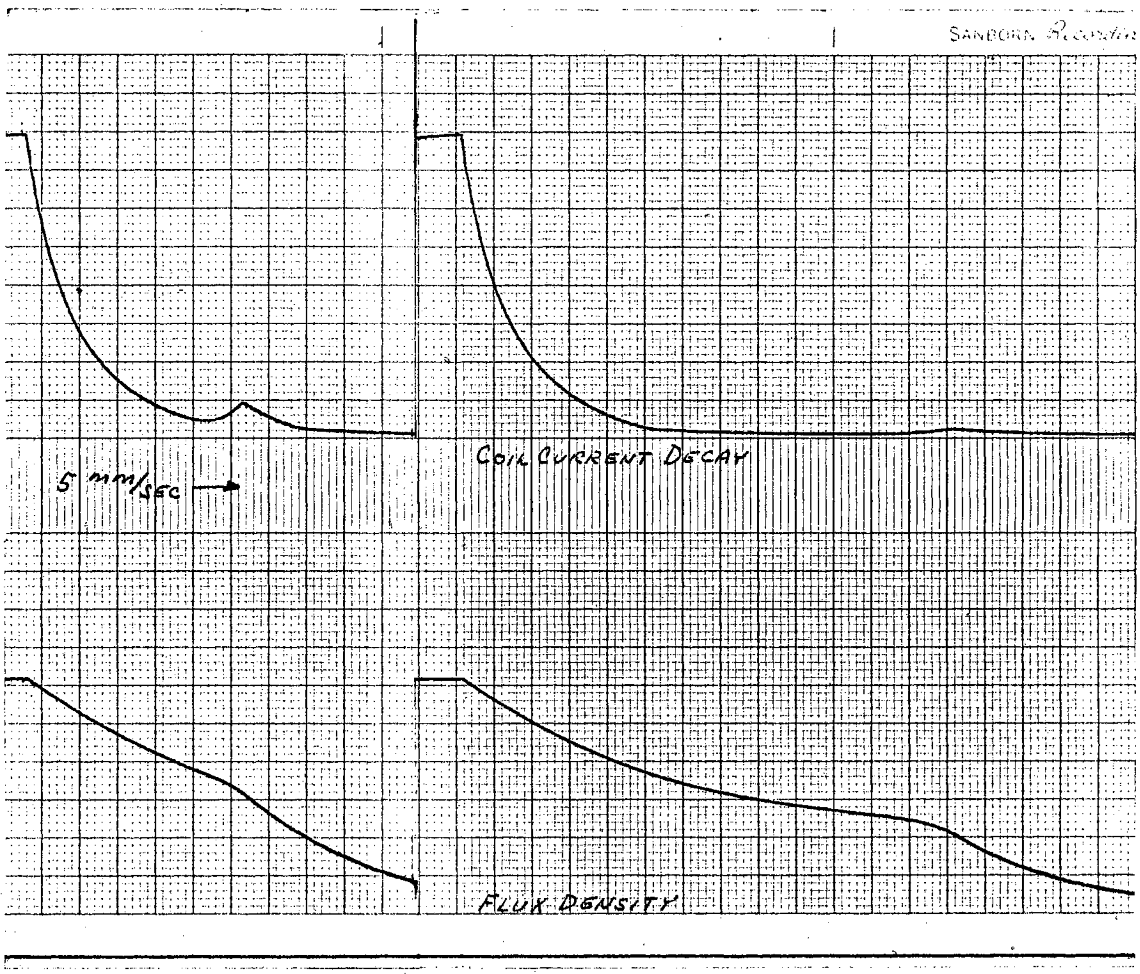
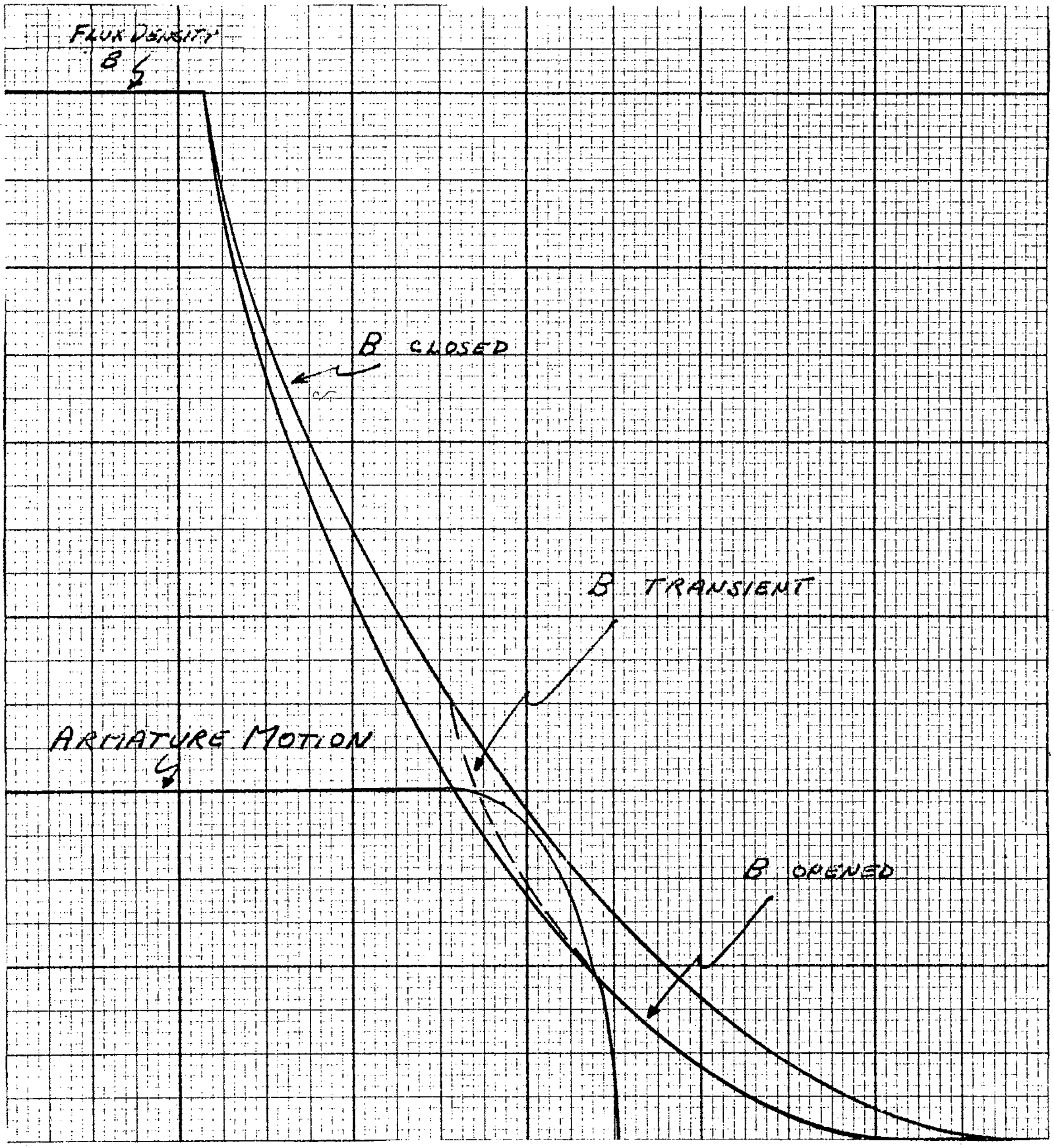


Figure 22 Sanborn Recordings of the Flux Density of the Core of a Relay During the Release Cycle for Two Values of Spring Tension

Spring Tension (lf to rt) 100 grams and 30 grams
Number of Coil Turns = 4000
Discharge Resistance = 900 ohms
Coil Resistance = 100 ohms
Air Gap = 0.025 cm.
Residual Air Gap Approximately Zero



Time

Fig. 23. Composite Curve Representing the Transient Behavior of the Flux Density During Release, for the Armature Held Open, the Armature Held Closed and the Armature Free to Move

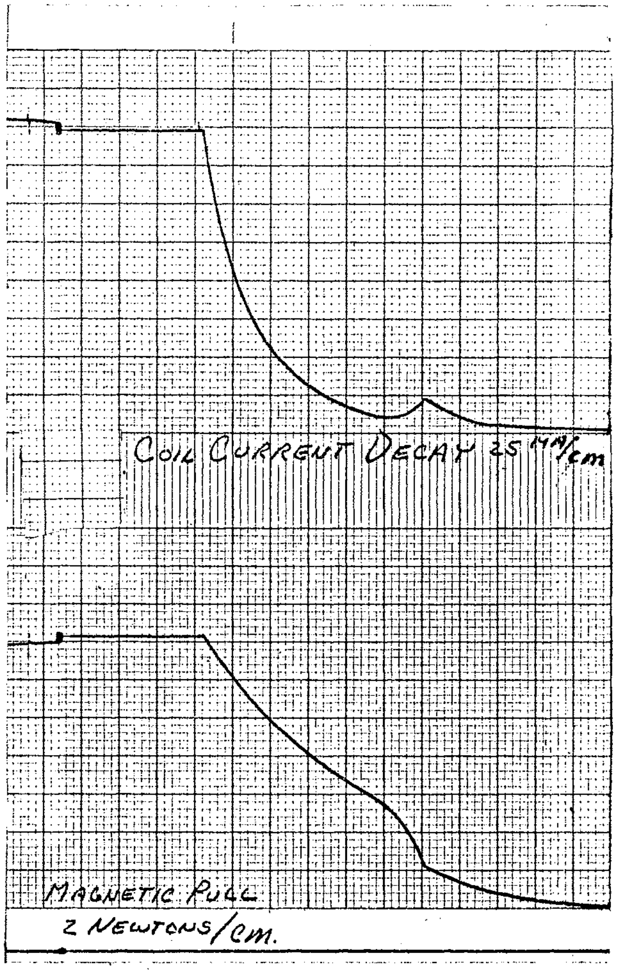


Figure 24 Sanborn Recording of the Magnetic Pull on the Armature of a Relay During Release

Spring Tension = 300 grams
Number of Coil Turns = 4000
Discharge Resistance = 900 ohms
Coil Resistance = 100 ohms
Air Gap = 0.05 cm.
Residual Air Gap Set at Zero

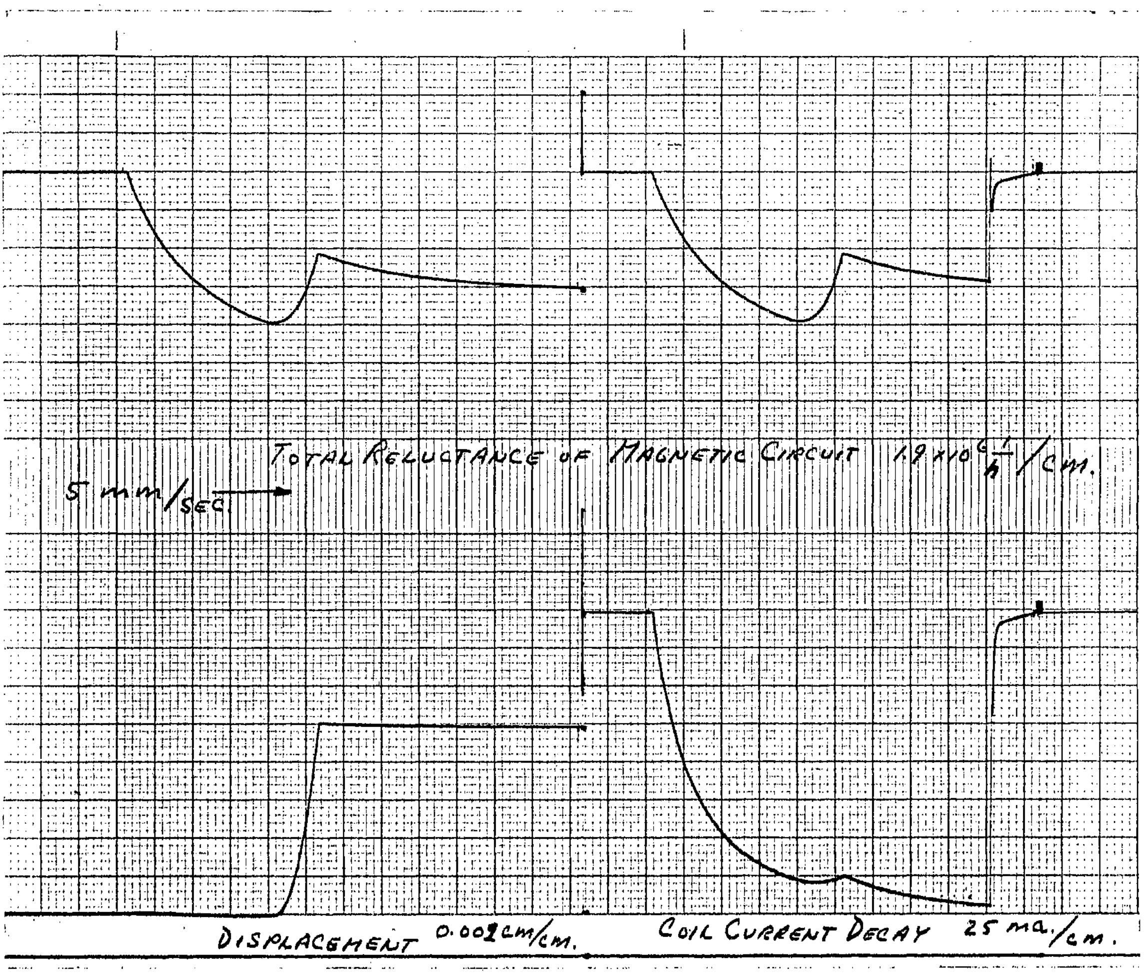


Figure 25 Sanborn Recordings of the Total Reluctance of the Magnetic Circuit Plotted with Coil Current Decay and Armature Motion

Number of Coil Turns = 4000
Discharge Resistance = 900 ohms
Coil Resistance = 100 ohms
Air Gap = 0.025 cm.
Residual Air Gap Approximately Zero
Spring Tension = 100 grams

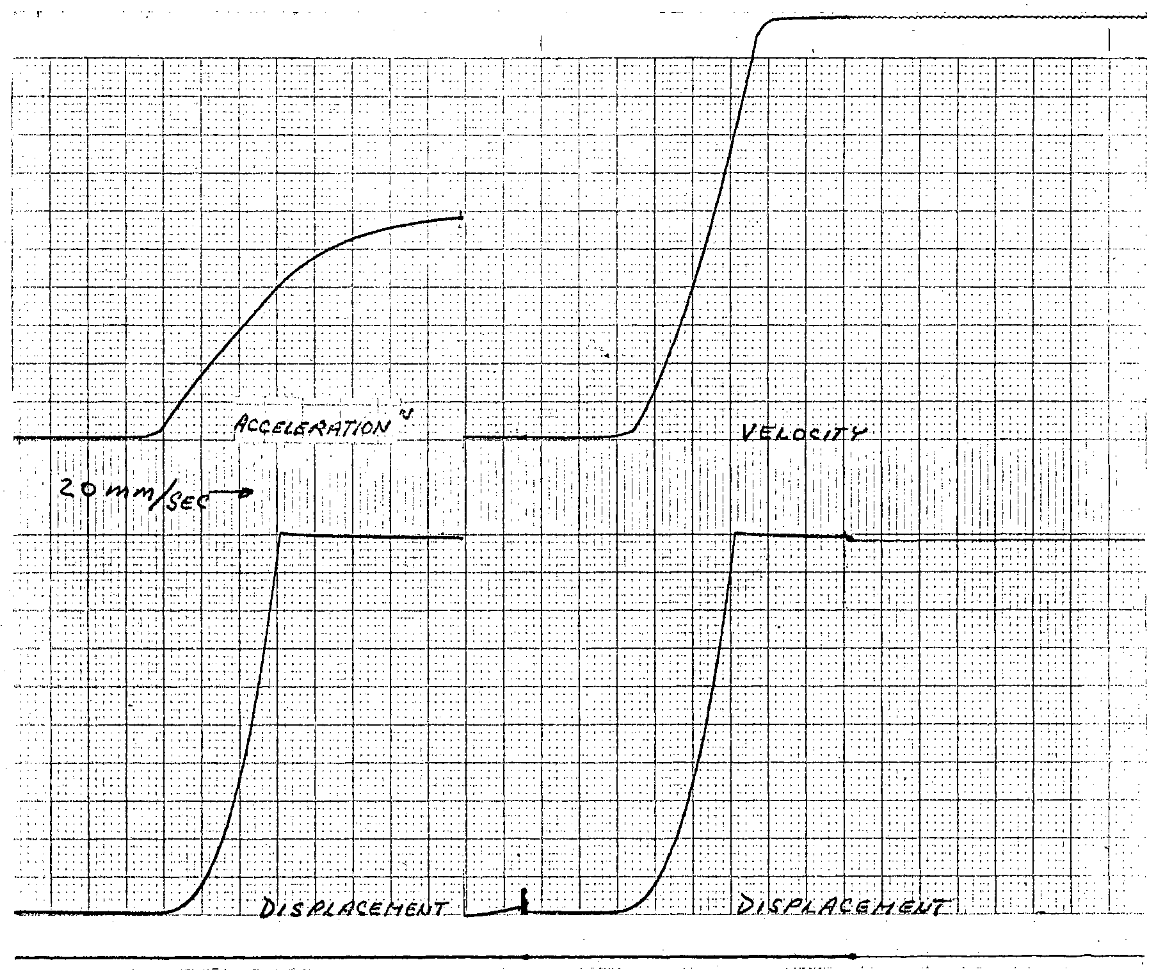


Figure 26 Sanborn Recordings of the Acceleration, Velocity and Displacement of the Armature of a Relay

Scale 10 volts/cm
Air Gap = 50 volts = 0.025 cm.
Residual Air Gap Set at Zero
Spring Tension = 300 grams (approximately 100 grams/cm)
Maximum Magnetic Pull on Armature = 7.02 newtons

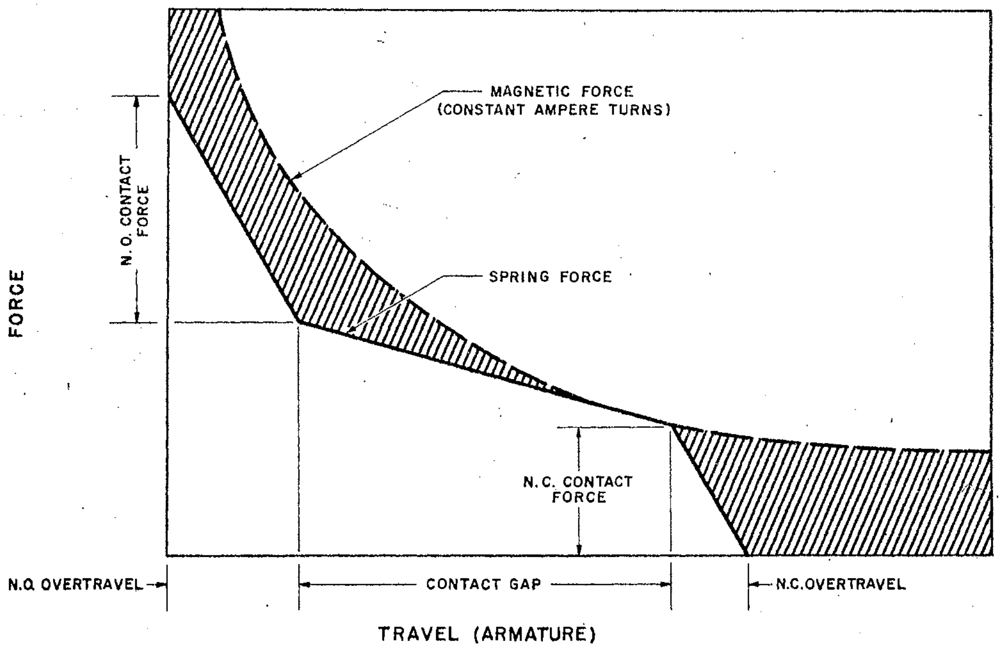


Fig. 27. A Force Function Curve

CHAPTER VI

SUMMARY AND CONCLUSIONS

A method of analysis of the transient behavior of a relay during its release cycle has been shown. A set of equations was derived and a computer program was built. The program was run off on an analog computer and the results were compared to experimental and predicted curves. The comparisons were favorable, and it is felt that the computer program shown in Fig. 15 is valid for the simulation of the release cycle of a relay on an analog computer.

Two basic assumptions were made in regard to deriving the equations for the program: (1) The eddy currents generated in the armature were small, and (2) The residual magnetism of the core was small. There was no evidence revealed that the eddy currents affected the operation; however, more consideration should be given to the redisual magnetism of the core. Evidence of this presents itself in the incompatibility of the release times at low values of residual air gap when comparing the experimental curves with the computer curves. This suggests that the effects of the residual magnetism should be investigated further and perhaps provisions made in the program to incorporate it. The approximation of the permeability of the core with current decay was sufficient for this problem since accurate figures on release time were not desired. For more accurate readings, however, it is felt that a separate function generator should be used for the permeability.

The use of the analog computer for relay analysis could prove to be quite advantageous, especially when working with special relays where unusual RL or RLC circuits are employed, such as relays with slugs or sleeves and the ferromagnetic relay. The design of the miniature relay could be enhanced through the use of the force function and similar techniques to reduce excessive overdesign in the magnetic circuit and in the restoring force.

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APPENDIX A

GLOSSARY OF RELAY TERMS

Air Gap - A term for contact separation or for magnetic air gap.

Ampere-Turns - The product of the number of turns in an electromagnetic coil and the r.m.s. current in amperes passing through the coil.

Armature - The hinged or pivoted moving part of the magnetic circuit of an electromagnetic relay. Sometimes used in a general sense to mean any moving part which actuates contacts in response to a change in coil current.

Armature Chatter - Vibration of armature.

Armature Travel - The total distance traveled during operation by a point on the armature which is nearest the pole-face center when the relay is operated.

Backstop - The part of a relay which limits the movement of the armature away from the pole piece or core.

Coil - One or more windings of wire to which energy is supplied to activate the relay.

Coil Inductance - Primarily a property of the number of turns of wire along with the geometry of the magnetic circuit and its permeability.

Coil Resistance - The DC ohmic resistance of the coil measured at the coil terminals.

Contact Bounce - The uncontrolled making and breaking of contact when relay contacts are moved to the closed position.

Contact Chatter - A sustained rapid opening and closing of contacts caused by variations in the coil current, mechanical vibration and shock, or other causes.

Contacts - Current-carrying parts of a relay which engage or disengage to make or break electrical circuits.

Core - A stationary part of the magnetic circuit of a relay about which the coil is wound.

Discharge Resistance - The ecternal DC ohmic resistance which is effectively in parallel with the relay coil when the coil is de-energized.

Drop-out Current - The maximum value of current for which the contacts of a previously energized relay will always assume their energized positions.

Electromagnetic Relay - A relay whose operation involves the use of a magnetic field, produced by an electromagnet.

Magnetic Air Gap - The nonmagnetic portion of a magnetic circuit.

Operate Time - If a relay has only normally closed contacts, its operate time is the longest time interval given by definition (a) below. If a relay has normally open contacts (regardless of whether or not it has normally closed contacts) its operate time is the longest interval given by definition (b).

(a) Operate Time for Normally Closed Contacts -

The total elapsed time from the instant the coil is energized until the contacts have opened; i.e., the contact current is zero.

(b) Operate Time for Normally Open Contacts -

The total elapsed time from the instant the coil is energized until the contacts are closed and all contact bounce has ceased.

Pole Face - The part of the magnetic structure on the end of the core nearest the armature.

Pull-in Current - The minimum value of current for which the contacts of a previously de-energized relay will always assume their energized position.

Relay - An electromechanical device which is operated by variation in the conditions of one electric circuit to affect the operation of other devices in the same or other electric circuits by either opening contacts or closing contacts or both.

Release Time - If a relay has only normally open contacts, its release time is the longest time interval given by definition (a) below. If a relay has normally closed contacts (regardless of whether or not it has normally open contacts) its operate time is the longest time interval given by definition (b).

(a) Release Time for Normally Open Contacts -

The total elapsed time from the instant the coil current starts to drop from its rated value until the contacts have opened; i.e., the contact current is zero.

(b) Release Time for Normally Closed Contacts -

The total elapsed time from the instant the coil current starts to drop from its rated value until the contacts are closed and all contact bounce has ceased.

Residual Gap - The length of the magnetic air gap between the pole-face center and the nearest point on the armature when the armature is in the energized position.

Tension Spring - A term for "restoring spring".

Transfer Time - The total elapsed time between the breaking of one set of contacts and the making of another set of contacts.

(a) Transfer Time on Operate -

The total elapsed time from the instant the normally closed contacts start to open until the normally open contacts are closed and all contact bounce has ceased.

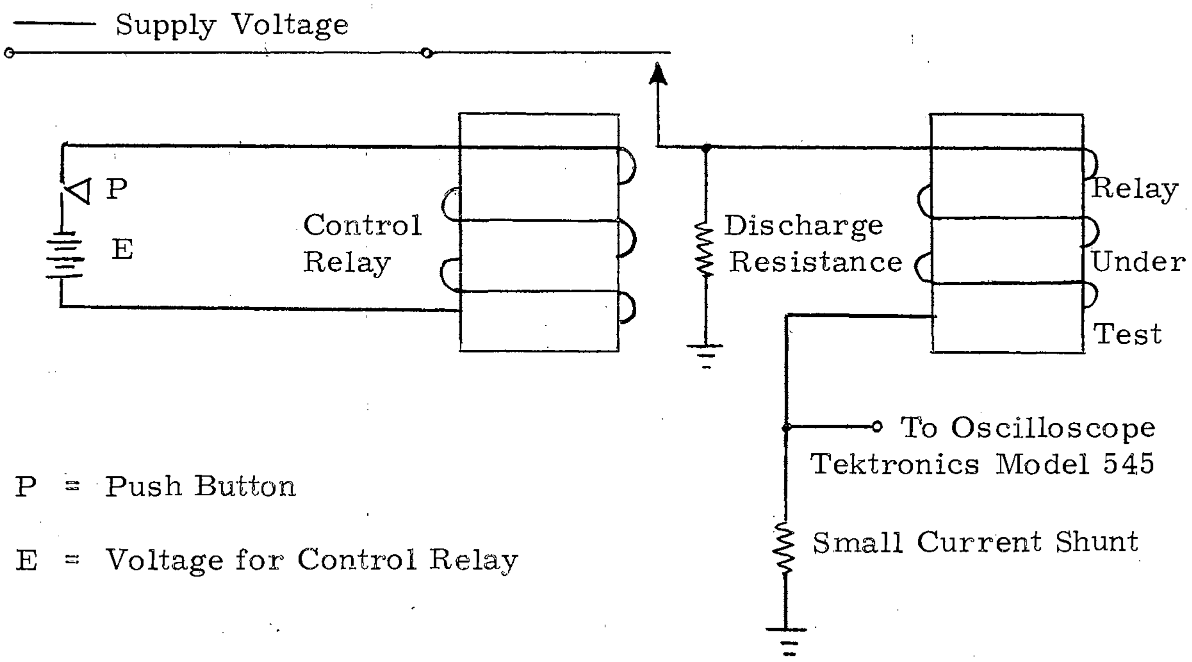
(b) Transfer Time on Release -

The total elapsed time from the instant the normally open contacts start to open until the normally closed contacts are closed and all contact bounce has ceased.

Transit Time - Same as "transfer time".

APPENDIX B

The basic circuit used to obtain the coil current waveforms shown in Figs. 3, 4 and 6, is shown below. The control relay had mercury contacts, thereby eliminating the possibility of contact chatter. A polaroid land camera was used to photograph the current traces.



P = Push Button

E = Voltage for Control Relay

VITA

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Master of Science

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