FLOW CHARACTERISTICS OF MOLTEN LOW-DENSITY POLYETHYLENE FLOWING THROUGH A TUBE

By

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FLOW CHARACTERISTICS OF MOLTEN

LOW-DENSITY POLYETHYLENE

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PREFACE

In the past few years, many new polymers have been developed. Many new plants have been built to produce these new polymers. This increase in the production of polymers has increased the need for determining the behavior of these non-Newtonian polymers.

The flow characteristics of molten low-density polyethylene were determined by measuring the pressure-drop across a circular tube. The tube was heated electrically and the molten plastic was pumped through the tube using a screw extruder. Details of the design of the experimental apparatus are presented in the thesis.

There are many methods available in the literature to analyze and correlate pressure-drop data, but most of them are restricted to a particular non-Newtonian fluid.

In Appendix B, a derivation is presented of a generalized method for correlating data. This method was used to analyze and correlate the experimental data.

I have received guidance and assistance from many during the course of this study. Dr. J. M. Marchello and Dr. R. N. Maddox were helpful in the analyzing of the data and in the formulation of the thesis. Mr. E. E. McCroskey offered many suggestions and gave much aid during the construction and installation of the experimental apparatus. Messrs. R. E. Thompson and P. C. Tully provided a calibration of the pressure gages. I wish to express my gratitude to these individuals for their help and to the many others who have shown an interest in this study.

I am indebted to the School of Chemical Engineering, the National Science Foundation and the Continental Oil Company for personal financial assistance.

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CHAPTER I

INTRODUCTION

During the spring of 1960, the School of Chemical Engineering of Oklahoma State University purchased a one-inch screw extruder. Plans had been made to use the new equipment for the study of non-Newtonian fluids and in the unit operations laboratory. However, before the new extrusion equipment could be used, the extruder needed to be installed and operating instructions needed to be prepared.

As one phase of the thesis project, the screw extruder was to be installed and an operating manual prepared. However, this phase of the thesis project was to be considered as secondary.

The screw extruder could be used as a means to study the flow and heat transfer characteristics of molten thermoplastics in flow through a circular tube. A tube, heated electrically, could be very easily attached to the extruder for this purpose.

Before a complete study of the heat transfer characteristics could be made, the isothermal flow characteristics of the thermoplastic had to be known. As phase two of the thesis project, the flow characteristics of low-density polyethylene were to be determined.

The following goals were set for this thesis:

1. Installation of the screw extruder.
2. Preparation of an operating manual.
3. Determination of the flow characteristics of low-density polyethylene through a circular tube.

CHAPTER II

REVIEW OF NON-NEWTONIAN FLUID FLOW

Fluid Behavior

An important feature of a fluid system is the way it reacts to an applied force. Newton made an attempt to describe the reaction of a fluid to an applied shearing force. Consider a layer of fluid between two parallel plates, one moving and the other stationary (See Figure 1). The plates are separated by a distance, dy. A shearing force, F, is applied to the top plate which causes the top plate to move. Assuming there is no slipping of the fluid at the faces of the plates and the system is at steady-state, the shear force per unit area for a fluid in laminar flow is directly proportional to the velocity gradient. The resulting equation (1) is

$$F/A = \tau = \frac{\mu}{g_c}\frac{du}{dy} \qquad (1)$$

The proportionality constant, μ, is called the dynamic viscosity or the Newtonian viscosity. The term, τ, is the shearing stress, and the velocity gradient, du/dy, is the rate of shear.

The Newtonian viscosity is a property of the fluid and is dependent on the temperature and pressure of the system and independent of the rate of shear (1). At a constant temperature and pressure, the viscosity is the slope of a straight line on a plot of shear stress and shear rate (See Figure 2).

Any fluid that obeys the idealized concept of Newtonian viscosity

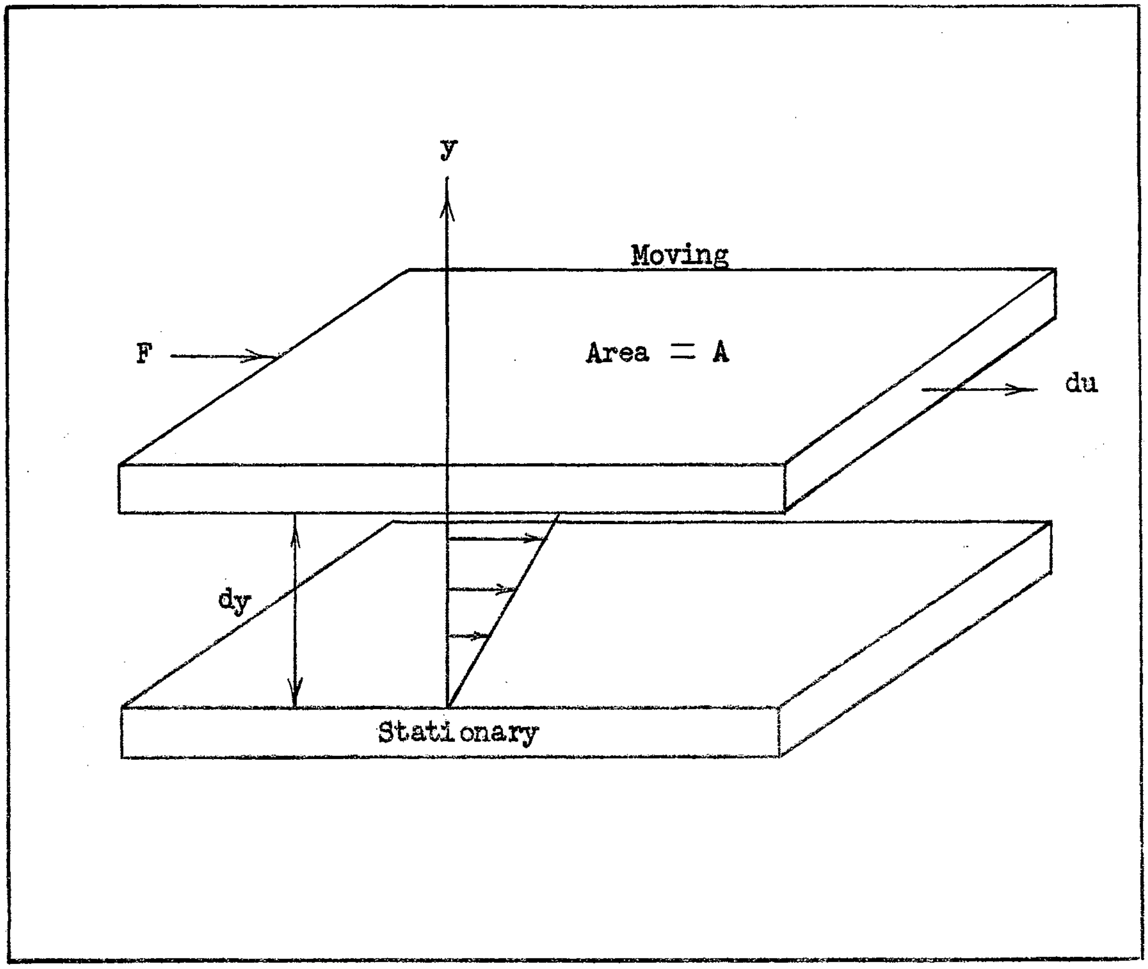


Figure 1. Reaction of A Fluid To An Applied Stress

is called a Newtonian fluid, and any fluid that does not obey the Newtonian equation is called a non-Newtonian fluid.

Non-Newtonian Fluids

The principal characteristic of a non-Newtonian fluid is that the curves on a shear diagram are generally not linear. The viscosity becomes dependent upon the rate of shear and the Newtonian viscosity becomes an apparent viscosity.

Non-Newtonian fluids can be classified according to their reaction to shear. This method of classification is known as the classicial method of classifing non-Newtonian fluids (2). They can be grouped into three categories:

1. Time-independent
2. Time-dependent
3. Viscoelastic

Time-Independent

This group of non-Newtonian fluids is characterized by the independency of fluid properties upon the duration of shear. It can be divided into three types of fluids.

Bingham Plastic. This type of fluid behaves much like a Newtonian fluid. The shear curve is linear (See Figure 2). However, the shear curve does not pass through the origin as for a Newtonian fluid, but intersects the axis at some shear stress, τ_y. Thus, an initial shear stress must be reached in the fluid before the fluid will start to flow. This behavior is represented by the following equation (2):

$$\tau - \tau_y = \frac{\eta}{g_c}\frac{du}{dy} \qquad (2)$$

The term, η, is called the "coefficient of rigidity" or "plastic viscosity".

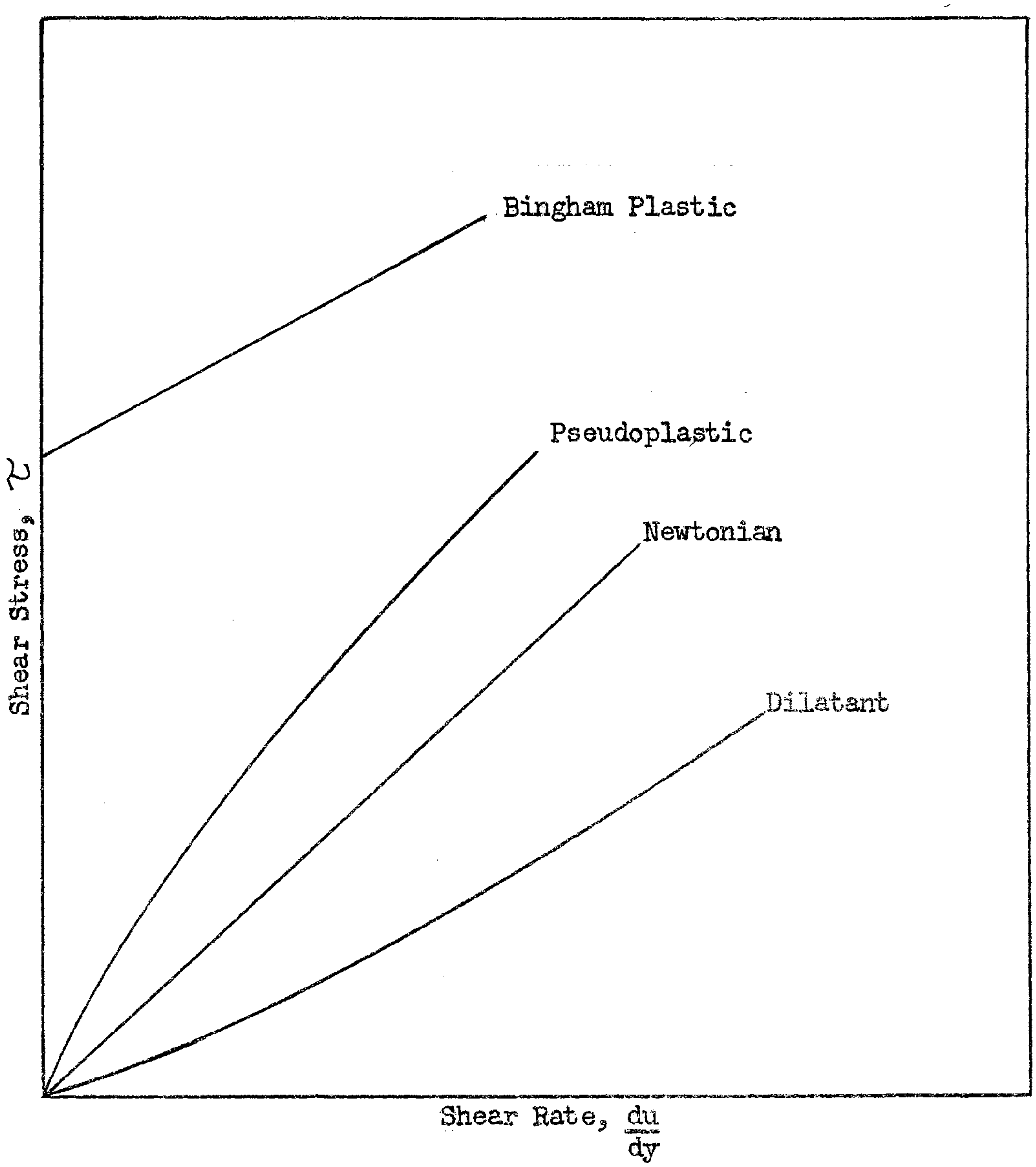


Figure 2. Shear Diagram

Pseudoplastic. This type of fluid is characterized by a shear curve with a decreasing slope on a shear diagram (See Figure 2). The apparent viscosity decreases with increasing shear rate and the shear curve tends to become linear at high rates of shear. The shear curve for this type of plastic can be represented by the power-law equation (2).

$$\tau = K \left(\frac{du}{dy}\right)^n \tag{3}$$

The constant, K, is a measure of the consistency of the fluid; the constant, n, is a measure of the departure from Newtonian behavior. For $n = 1$, the above equation reduces to the Newtonian equation.

Dilatant. This fluid behaves just the opposite of a pseudoplastic fluid. This fluid is represented on a shear diagram by a shear curve that has an increasing slope (See Figure 2). The apparent viscosity increases with increasing shear rate.

Time-Dependent

This group of fluids exhibits a reversible change in the apparent viscosity with duration of shear. The fluids that fall within this group may be subdivided into two categories.

Thixotropic. Thixotropic fluids are those whose apparent viscosity decreases with the duration of shear (3).

Rheopectic. Rheopectic fluids are those whose apparent viscosity increases with the duration of shear (3).

Viscoelastic

The fluids that fall within this group are those which exhibit elastic and viscous properties (3). Energy that is applied to a purely viscous fluid is dissipated as heat as the fluid is worked. However, an elastic material dissipates applied energy by storing it as potential energy. When the applied strain is removed part of the stored energy is recovered. Viscoelastic fluids behave similarly to a combination of a viscous and elastic material.

A simple relationship has been used to describe this behavior. The relationship is a combination of Newton's viscosity for fluids and Hook's law for elastic materials (4).

$$\tau + \lambda \left(\frac{d\tau}{d\theta}\right) = \mu\left(\frac{du}{dy}\right) \qquad (4)$$

The method of classification presented here is restricted. Actually, any particular fluid may not fall within any one of these groups. Probably, the fluid exhibits the properties of part or all of the groups. For example, a fluid could be time-dependent for a short duration and become time-independent over a long duration.

Laminar Flow of Non-Newtonain Fluids

Since the viscosity for a non-Newtonian fluid is a function of the shear rate, pressure-drop predictions for non-Newtonian systems become complex. The ability to predict pressure-drop depends not only on the shear rate but also on the behavior of the fluid (Bingham plastic, pseudoplastic, etc.), and whether or not the rate of shear is dependent on the duration of shear. The problem of predicting pressure-drop for non-Newtonian systems has been studied by many investigators.

Since fluids that behave as Bingham plastics are the simplest of the non-Newtonian fluids, most of the early work was done on this type of fluid. Alves and others (5) presented a review of Bingham's mathematical analysis of a Bingham plastic flowing through a circular duct. The resulting equations are:

$$\frac{8V}{g_c D} = \frac{1}{\eta}\left(\tau_w - 4\tau_y/3\right) \qquad (5)$$

$$\mu_a = \frac{g_c \tau_y D}{6V} + \eta \qquad (6)$$

With viscometer data and the aid of the above equation, the apparent viscosity can be evaluated. Then using the Poiseuille equation, modified by replacing the Newtonian viscosity by the apparent viscosity, the pressure-drop can be predicted (5).

$$\Delta P = \frac{32\mu_a LV}{g_c D^2} \qquad (7)$$

The above equation can be applied to many non-Newtonian fluids over narrow ranges of shear rates.

In 1952 Hedström (6) presented a dimensional analysis based upon Buckingham's equation. The Buckingham equation for a circular tube is (3)

$$Q = \frac{R^4 \Delta P \pi}{8L\mu_p}\left[1 - \frac{4}{3}\left(\frac{2L\tau_y}{R\Delta P}\right) + \frac{1}{3}\left(\frac{2L\tau_y}{R\Delta P}\right)^4\right] \qquad (8)$$

The variables considered by Hedström were ΔP, L, D, τ_y, μ_p, and V. The resulting function was

$$f = \phi(N_{Re}, N_{He}) \qquad (9)$$

A friction factor plot was presented by Hedström (6). The plot gave a family of curves using the Hedström number as a parameter. Hedström's approach is applicable only for fluids that approximate Bingham plastic behavior.

The so called power-law fluids are relatively easy to describe mathematically. Wilkinson (3) has presented a derivation of a relationship to describe the flow of a power-law fluid in a circular tube.

$$Q = \pi R^3 \frac{n}{3n+1}\left(\frac{R\Delta P}{2LK}\right)^{1/n} \qquad (10)$$

Weltman (7) represented the above relationship on a friction factor plot. The following relationships were used to prepare the plot:

$$f = \frac{64}{N_{Re}}\left(\frac{n+3}{4}\right) \qquad (11)$$

$$N_{Re} = \frac{D\rho V}{\text{Viscosity}} \qquad (12)$$

The resulting curves were a family of straight lines, one straight line for each value of n.

Several attemps have been made to develop shear stress-shear rate relationships that are more general. One such relationship was developed by Powell and Eyring (8).

$$\tau = \mu\left(\frac{du}{dy}\right) + \frac{1}{B} \sinh^{-1}\left(\frac{1}{A}\frac{du}{dy}\right) \tag{13}$$

The constants, μ, A, and B, are characteristics of the fluid. The relationship can be applied to many non-Newtonian fluids over a wide range of shear rates. But the application of the relationship to flow problems requires integration which must be done by numerical methods. Christiansen and others (8) made use of the Powell-Eyring equation to derive a relationship to be used for predicting pressure-drop for pipe-line design. The authors used a modified form of the Hagen-Poiseuille equation (8).

$$q = \frac{\pi R^4}{8\mu} \left(-\frac{dp}{dl}\right) \tag{14}$$

Instead of using Newton's equation for the shear stress, the authors used the Powell-Eyring equation. Using the modified Hagen-Poiseuille equation as a model, a dimensional analysis was done. The resulting relationship was (8)

$$\frac{q}{AR^3} = g\left[\left(-\frac{BR}{2}\frac{dp}{dl}\right),\ (AB\mu)\right] \tag{15}$$

In 1933 Schonfield (9) derived an equation, using a geometrical approach, that is applicable to any time-independent fluid flowing through a circular tube. A derivation of the same equation is presented by Oka (10). The equation is

$$\left(-\frac{du}{dr}\right)_w = \frac{3}{4}\left(\frac{8V}{D}\right) + \frac{1}{4}\left(\frac{D\Delta P}{4L}\right)\frac{d(8V/D)}{d(D\Delta P/4L)} \tag{16}$$

Metzner and Reed (11) wrote the equation in the following form:

$$\left(-\frac{du}{dr}\right)_w = \frac{3n' + 1}{4n'} \left(\frac{8V}{D}\right) \tag{17}$$

where

$$n' = \frac{d\ [\ln(D\Delta P/4L)]}{d\ [\ln(8V/D)]} \tag{18}$$

The derivative, Eq. 18, is the slope of the tangent line which is tangent to the curve on a plot of $\ln(D\Delta P/4L)$ versus $\ln(8V/D)$. The equation of the tangent line is (11)

$$\frac{D\Delta P}{4L} = K' \left(\frac{8V}{D}\right)^{n'} \tag{19}$$

Using the definition of the Fanning friction factor and the following relationship,

$$f = 16/N_{Re} \tag{20}$$

Metzner and Reed (11) derived a generalized Reynolds number.

$$N_{Re} = \frac{D^{n'} V^{2-n'} \rho}{\gamma} \tag{21}$$

where

$$\gamma = g_c K' \, 8^{n'-1} \tag{22}$$

Data were presented (11) for various systems to show the utility and applicability of this generalized approach. Metzner (12) has shown specifically that this generalized approach applies to power-law fluids and Bingham plastics. In the first of a series of articles, Bowen (13) shows the adaptability of this approach to pipe-line design.

Screw Extrusion

Extrusion is essentially the process of forcing a molten material through a die which shapes the material into a product of a desired shape (14, 15). There are several methods of extrusion in use. The simplest method, called ram extrusion, is to place a batch of material in a cylinder and force the material through a die with a hydraulic piston. Historically, ram extrusion was probably the first method used for extrusion (16). Another method of extrusion, called screw extrusion, is to feed the material to a screw or worm, which is turning in a heated cylinder and forces the material through the die (15).

Extrusion as a manufacturing process originated about the end of the 19th Century (16). Prior to the late 19th Century the extrusion process was confined to the production of lead pipe and food stuffs such as macaroni and spaghetti. During the mid-19th Century with the rapid growth of transcontinental and transocean telegraph systems, the extrusion process for making and coating wire and cables was established.

The first thermoplastic to be extruded was cellulose nitrate (16). The plastic was first extruded during the late 1870's using a solvent cold-ram extrusion process. Cellulose nitrate was the only plastic material that was extruded in commercial quantities until about World War I, when casein had reached commercial importance. During the 1920's, many new thermoplastics were developed. With the increasing number of new extrudable thermoplastics, the need for a screw extruder designed for thermoplastics arose. In 1931, Heidrich in Germany made the first screw extruder designed for thermoplastics and later in the 1930's similar screw extruders were developed in Great Britain and the United States.

Description of the Screw Extrusion Process

Basically, the screw extruder consists of a screw rotating in a cylinder, called the barrel, with a feed port at one end and a die at the other end (See Figure 3). The plastic material in the form of pellets or powder is fed to the screw through the feed port. As the

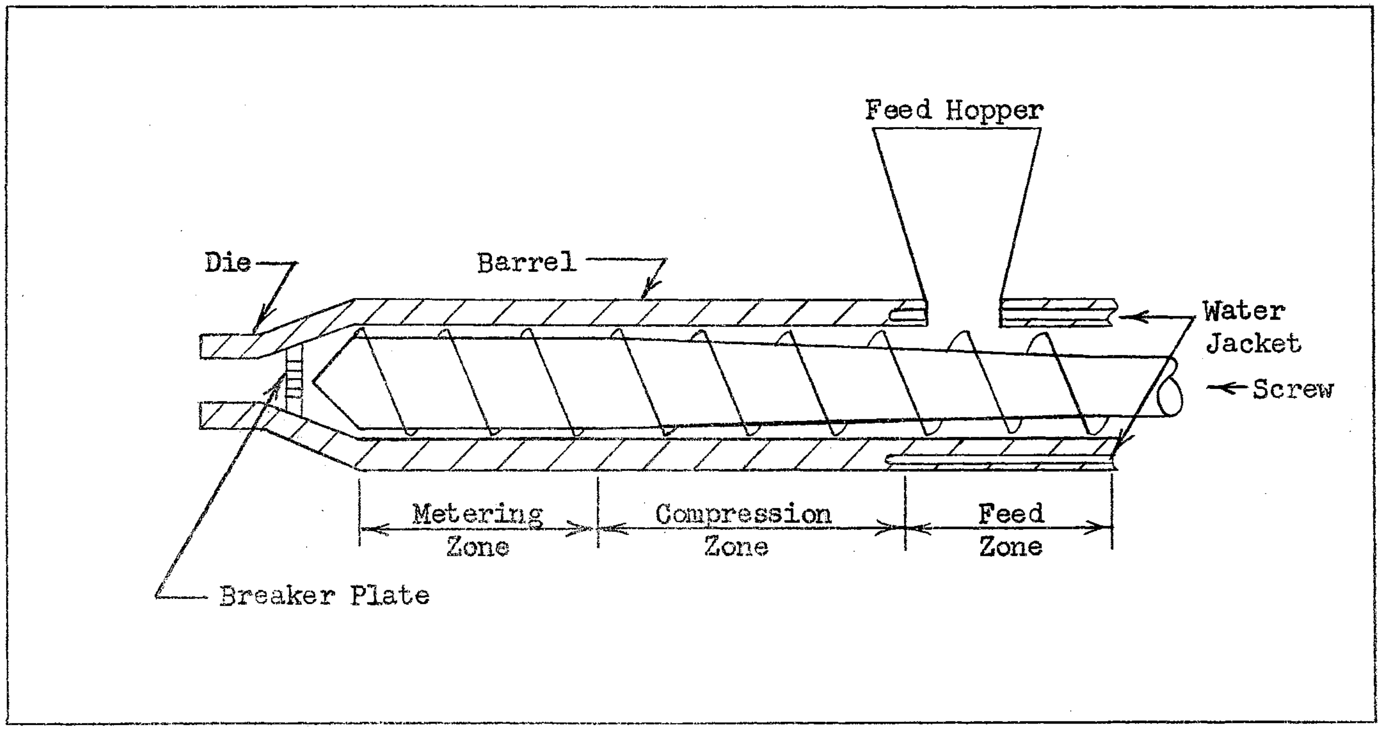


Figure 3. A Typical Extruder

cold plastic pellets are carried forward, they are melted by heat supplied by electrical heaters. The fluid plastic is then forced through the breaker plate and die. To facilitate a more detailed discussion of the screw extrusion process, the extruder barrel can be divided into three zones (16):

1. Feed zone
2. Compression zone
3. Metering zone

Feed Zone. The function of the feed zone is to convey the plastic particles in a continuous stream to the compression zone. The plastic material, usually in the form of cold pellets, enters the feed zone through a feed port. The feed zone is usually cooled with water to keep the temperature of the feed port below the melt temperature of the plastic. If the feed zone temperature rises above the melt temperature, the pellets will melt and stick together, thus clogging the feed port.

The mathematical description of the output of the feed zone is complex. In an article by Darnell and Mol (17), the authors presented a review of previous studies about the feed zone and derived an equation describing the feed zone output. The equation is

$$\frac{Q}{N} = \frac{D_B h(D_B - h) \tan\theta_B \tan\phi}{\tan\theta_B + \tan\phi} \qquad (23)$$

Compression Zone. The plastic pellets are heated in this zone by contact with the hot barrel and by the mechanical action of shearing the plastic. As the name implies, the plastic melt is compressed to force all the air back out the feed zone. The mathematical description of the output from the compression zone involves describing the movement of solid plastic pellets that gradually melt. There has been very little published about studies of this zone.

<u>Metering Zone.</u> The plastic enters the metering zone as a liquid. This zone is designed to act as a metering pump and its function is to deliver plastic to the die at a constant rate and pressure.

More is known about the mechanism of flow in this zone than the other zones. This zone has been studied by many investigators and there is a wealth of information in the literature.

There are three types of flow that contribute to the output from the metering zone:

1. Drag flow
2. Back flow
3. Leakage flow

The drag flow, Q_D, is due to the forward movement imparted by the rotating screw. The back flow, Q_P, is due to the negative pressure gradient in the barrel causing the plastic to flow back towards the feed end of the barrel. The leakage flow, Q_L, is the flow of plastic back over the lands of the screw. The total output can be expressed as:

$$Q = Q_D - Q_P - Q_L \tag{24}$$

The differential equations describing all three types of flow were developed by Navier. For the special case of an isothermal system, the Navier equation for the back flow is (18)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\frac{1}{\mu}\frac{dP}{dz} \tag{25}$$

Integration of this equation gives (18)

$$Q_P = \frac{bh^3 \cos\phi}{\mu F_P}\frac{dP}{dz} \tag{26}$$

where

$$F_P = \frac{1}{12} - \frac{16(h/b)\cos\phi}{\pi^5}\sum\left[\frac{1}{g^5}\tanh\left(\frac{g\pi b\cos\phi}{2h}\right)\right] \tag{27}$$

$$g = 1, 3, 5, \ldots$$

The Navier equation for drag flow is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{28}$$

Integration of this equation gives (18)

$$Q_D = \eta DNb^2 \cos^3 \phi F_D \tag{29}$$

where

$$F_D = \frac{8}{\pi^3} \sum \frac{\cosh\left(\frac{g\pi h}{b\cos\phi} - 1\right)}{g^3 \sinh\left(\frac{g\pi h}{b\cos\phi}\right)} \tag{30}$$

$$g = 1, 3, 5, \ldots$$

Integrating Eq. 25 for the case of leakage flow, the resulting equation is (18)

$$Q_L = \frac{\pi D E \delta^3 \Delta P}{12 \mu e} \tag{31}$$

The above equations describing the output from the metering zone are cumbersome to use. A simplified equation for the output can be derived for an isothermal system considering that velocity changes only in one direction. The Navier equation for these conditions is (19)

$$\frac{\partial^2 u}{\partial x^2} = -\frac{1}{\mu} \frac{dP}{dz} \tag{32}$$

The solution to Eq. 32 is (19)

$$Q = \alpha N - \beta\left(\frac{\Delta P}{\mu}\right) - \gamma\left(\frac{\Delta P}{\mu}\right) \tag{33}$$

where

$$Q_D = \alpha N \tag{34}$$

$$\alpha = \frac{n\pi Dh(t/n - e)\cos^2\phi}{2} \tag{35}$$

$$Q_P = \beta\left(\frac{\Delta P}{\mu}\right) \tag{36}$$

$$\beta = \frac{nh^3(t/n - e)\sin\phi \cos\phi}{2L} \tag{37}$$

$$Q_L = \gamma \frac{\Delta P}{\mu} \tag{38}$$

$$\gamma = \frac{\pi^2 D\, \delta^3 \tan\phi \Delta P}{10 \mu e L} \tag{39}$$

Figure 4. illustrates the definitions of many of the symbols used in the above equations.

Many other authors (16, 20, 21, 22, 23, 24, 25) have presented similar equations for the output of the metering zone. These equations can be used as a basis for experimental studies and design of extruders (26, 27, 28).

Other investigators have based their studies on an adiabatic model rather than an isothermal model. Based on adiabatic considerations, the output from the metering zone is (29, 30)

$$\frac{Q}{N} = \alpha - \frac{\epsilon(\beta+\gamma)}{C} \left(\frac{N\Delta P}{Q\Delta T}\right) \tag{40}$$

where

$$\alpha = \tfrac{1}{2} n\pi Dh(t/n - e)\cos^2\phi \left(\frac{h}{h+\delta}\right) \tag{41}$$

$$\beta = \frac{1}{12} nh^3(t/n - e)\cos\phi \tag{42}$$

$$\gamma = \frac{1}{10}\left(\frac{\pi^2 D^2 \delta^3}{e}\right) \tag{43}$$

$$\epsilon = n\pi^2 D^2 \left[\left(\frac{t - ne}{nh}\right) + \frac{e}{\delta}\right] \tag{44}$$

After leaving the metering section, the liquid plastic is forced through a breaker plate which assists in building up a pressure gradient in the barrel of the extruder. The molten plastic is forced through the die which shapes the plastic into desired form.

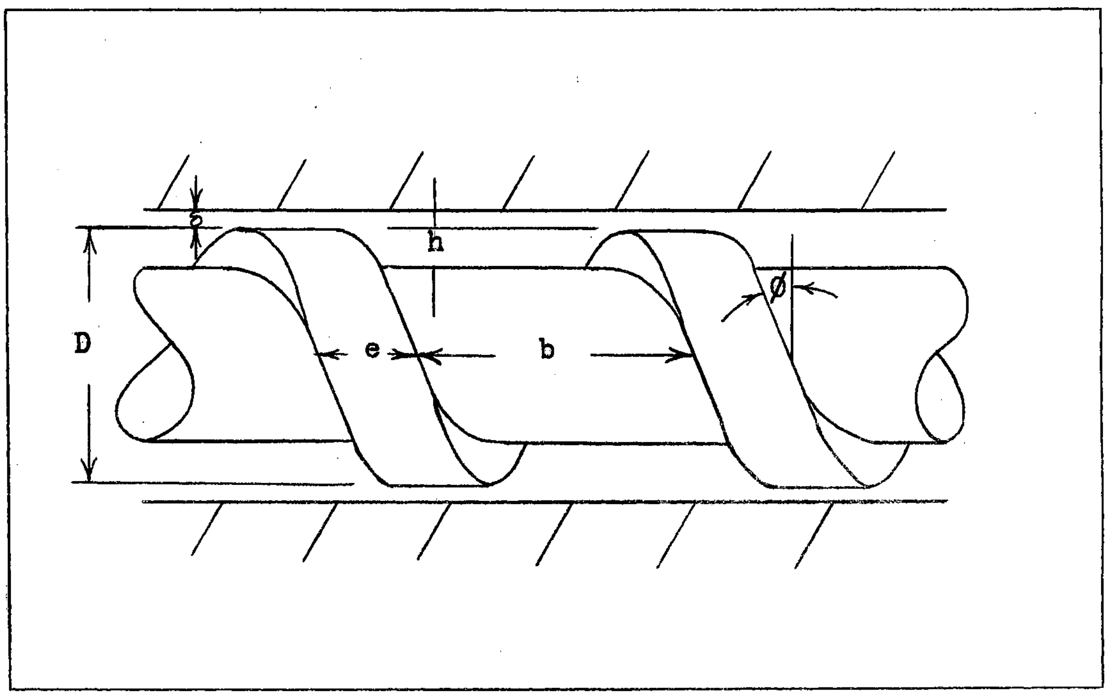


Figure 4. Section of Screw Extruder

CHAPTER III

EXPERIMENTAL APPARATUS

The apparatus consisted of two main parts: the Test Section and the Screw Extruder. The test section was attached to the screw extruder which served as a pump to force the molten thermoplastic through the test section.

Test Section

The test section was composed of a heated tube with attached thermocouples of chromel and iron constantan and two Bourdon pressure gages (See Figure 5).

Heated Tube. A 3/8 in. O.D. by 1/4 in. I.D. stainless steel tube, one foot long, had four thermocouples attached to the tube wall and spaced as shown in Figure 6. The tube was wrapped with an electrical heating tape, $\frac{1}{2}$ in. wide by 48 in. long. The tape was covered with three layers of asbestos tape (about $\frac{1}{4}$ in. thick), with a layer of glass wool (about $\frac{1}{4}$ in. thick), with another layer of asbestos tape, and a layer of aluminium foil. A thermocouple was placed 0.4 in. beneath the surface of the insulation and another on the surface of the insulation.

Front Portion of Test Section. The front end of the heated tube was connected to a $\frac{1}{4}$-inch high pressure tee by high pressure tubing connections (See Figure 7). The tee was wrapped with electrical heating wire and covered with a few layers of asbestos tape. Also two thermocouples were attached to the tee. In front of the tee, a flange coupling was connected to the tee with a $\frac{1}{4}$-inch all-thread nipple. The

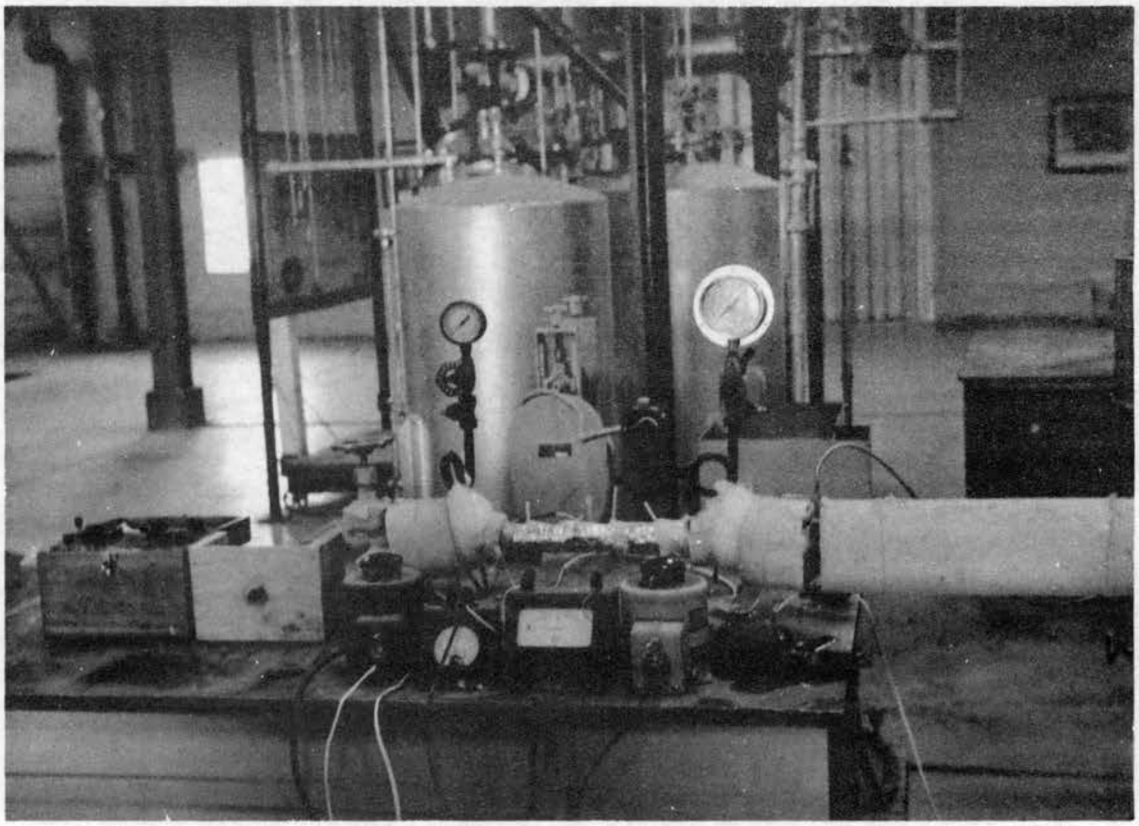


Figure 5. Test Section

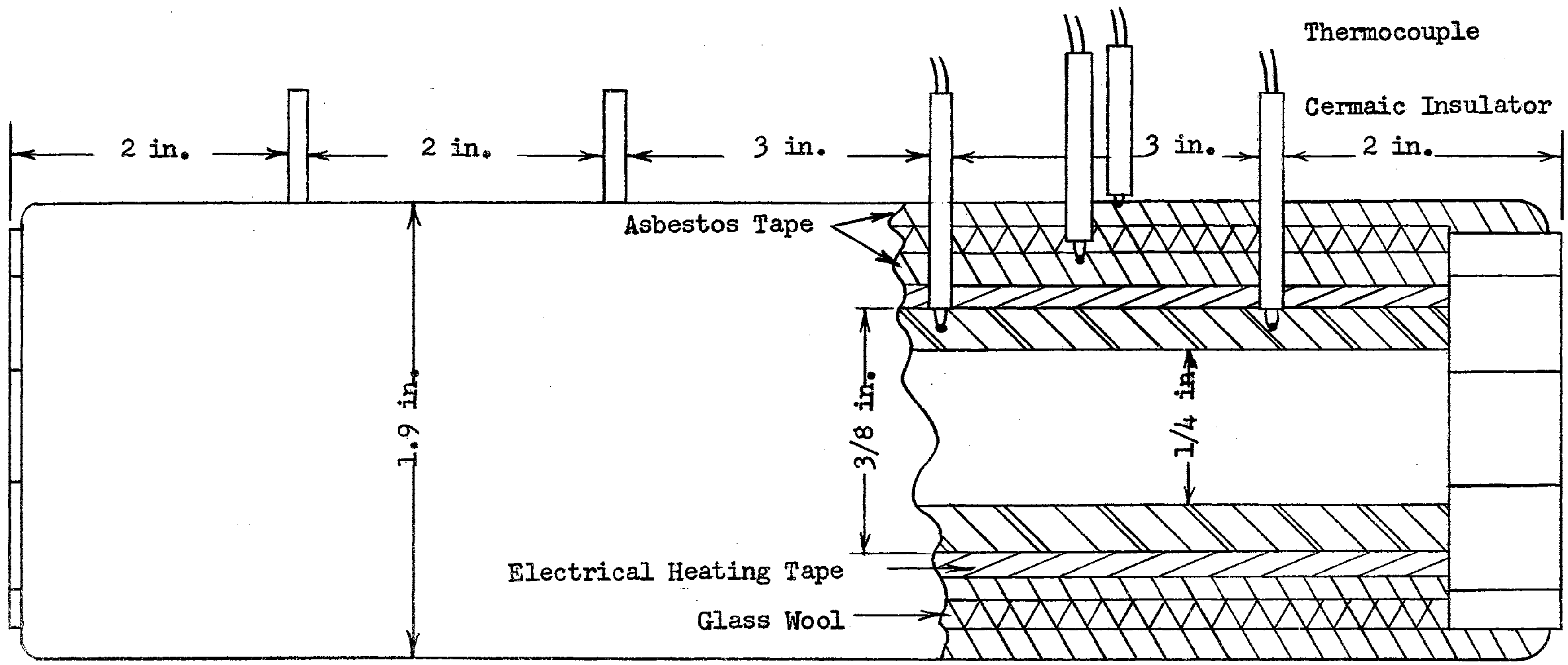


Figure 6. Heated Tube

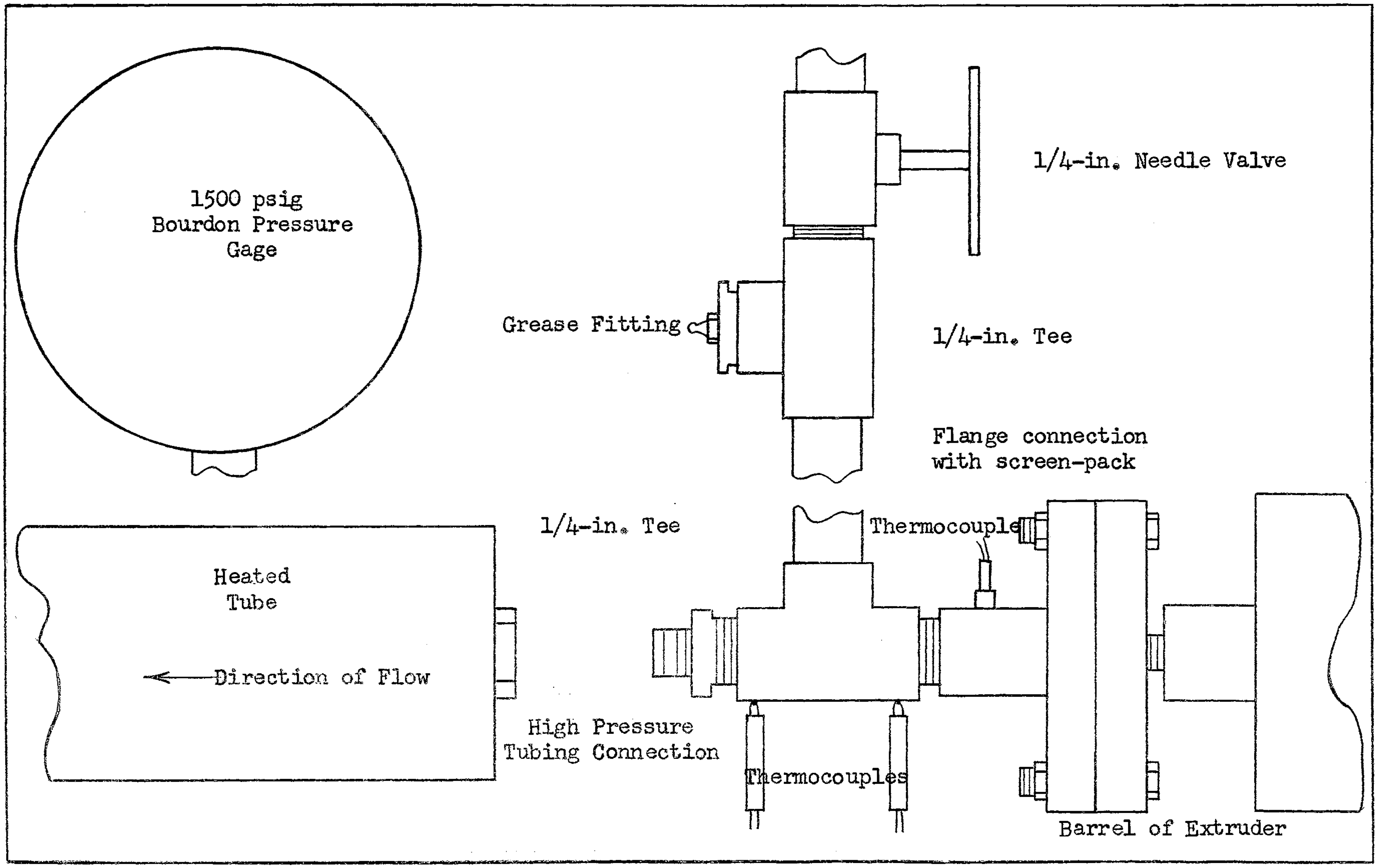


Figure 7. Front Portion of Test Section

flange contained a screen-pack and a thermocouple to measure the inlet temperature of the molten thermoplastic. The flange coupling was connected to the barrel of the extruder. A 1500 psig Bourdon pressure gage was connected into the $\frac{1}{4}$-inch tee through a series of connections: a $\frac{1}{4}$-inch needle valve, a $\frac{1}{4}$-inch tee containing a grease fitting, and a $\frac{1}{4}$-inch coiled siphon tube (See Figure 5 and Figure 7). The coiled siphon tube was filled with Dow-Corning silicone grease. The front portion of the test section was covered with a layer of glass wool and asbestos pipe insulation.

Back Portion of Test Section. The back portion is essentially the same as the front portion. The only difference is that a 1000 psig Bourdon pressure gage was used and the flange coupling was connected to a $\frac{1}{4}$-inch gate valve (See Figure 8). The $\frac{1}{4}$-inch gate valve was wrapped with electrical heating wire and covered with asbestos tape. Also, the flange contained a thermocouple to measure the exit temperature of the thermoplastic leaving the test section.

Auxiliary Equipment. Voltage to the heating tape on the tube was controlled by a Powerstat. An A.C. ammeter and a voltmeter were connected to the tape. Voltage for the heating wire on the $\frac{1}{4}$-inch gate valve was controlled with another Powerstat. The heating wire wrapped around the two $\frac{1}{4}$-inch tees was connected in series to another Powerstat which was used to control the voltage.

A total of 13 chromel-iron constantan thermocouples were used. The thermocouples were connected to a portable potentiometer made by Leeds and Northrup. A selector switch was used to select the proper thermocouple. The positions of the thermocouples and the other auxiliary equipment are shown in Figure 9.

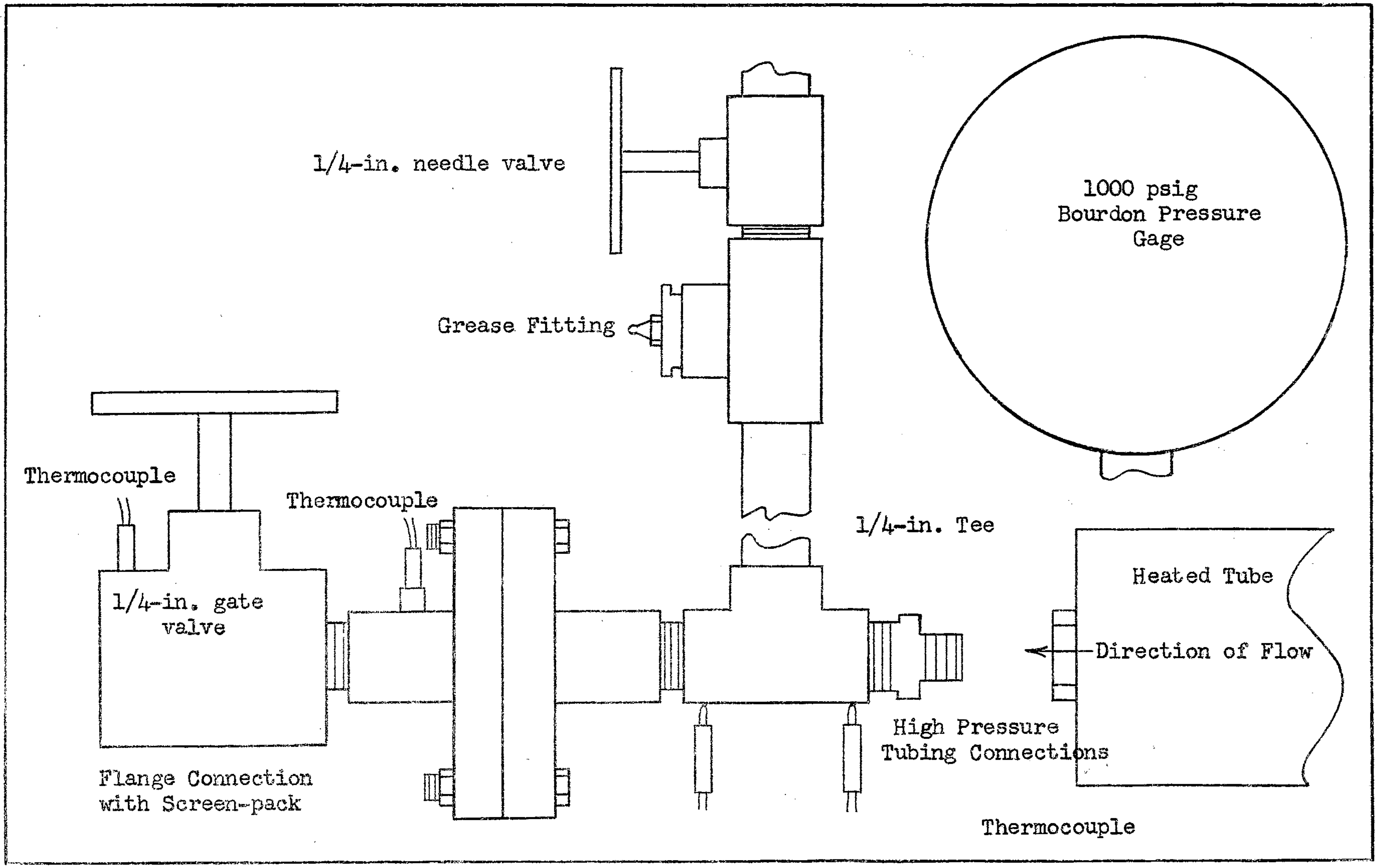
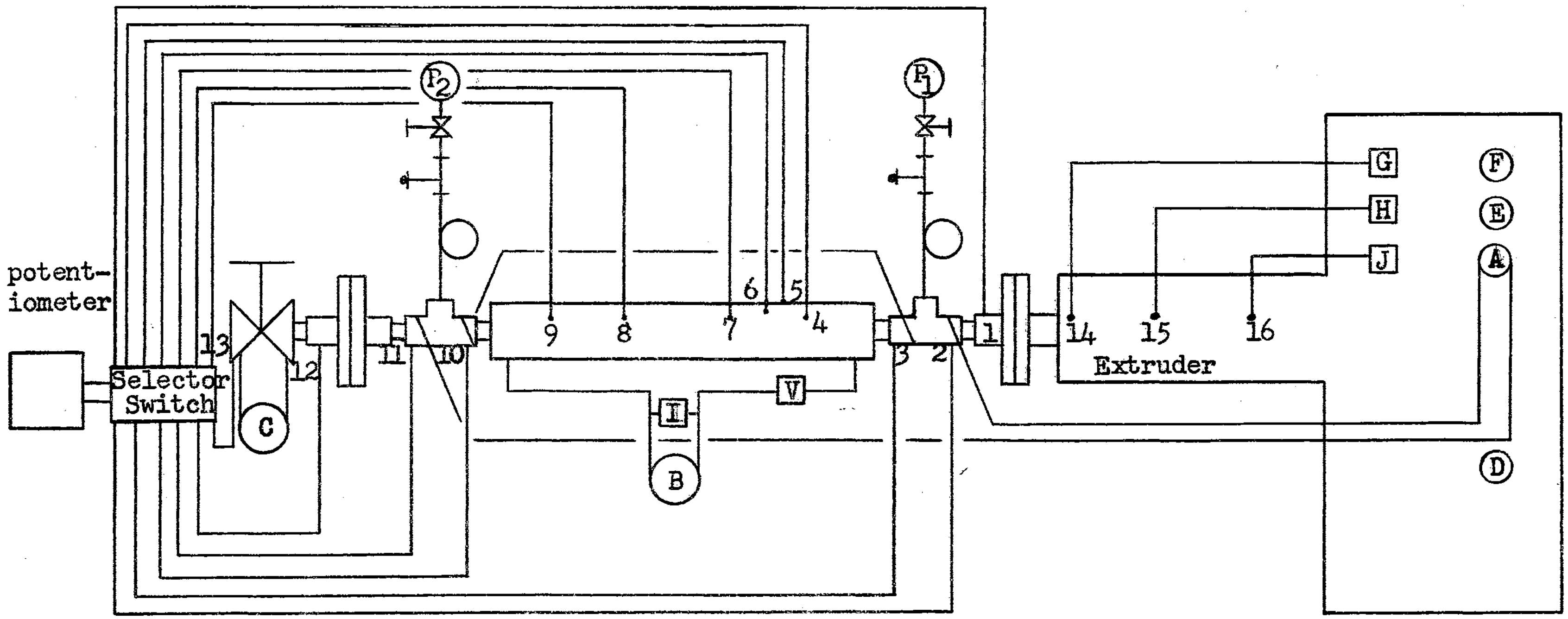


Figure 8. Back Portion of Test Section



LEGEND:

1. Thermocouple-entrance temperature
2-11. Thermocouple
12. Thermocouple-exit temperature
13. Thermocouple
14. Thermocouple-die adapter temperature
15. Thermocouple-meter zone temperature
16. Thermocouple-feed zone temperature

P_1 Entrance Pressure
P_2 Exit Pressure
A. Powerstat-tees heat control
B. Powerstat-tube heat control
C. Powerstat-valve heat control
D. Screw speed control
E. Robert-Shaw temperature controller
F. Robert-Shaw termperature controller
G. Feed zone temperature
H. Meter zone temperature
I. Ammeter
J. Die adapter temperature
V. Voltmeter

Figure 9. Diagram of Experimental Apparatus

Thermoplastic. The following low-density polyethylene plastics were used:

1. Low-density polyethylene manufactured by Phillips Chemical Company.

2. Low-density polyethylene manufactured by Spencer Chemical Company.

3. Low-density polyethylene types 770 and 550E manufactured by Dow Chemical Company.

Screw Extruder

The extrusion equipment described in this section of the thesis is a one-inch floor model screw extruder manufactured by the Killion Tool and Manufacturing Company. The description of the extrusion equipment will be presented in two parts: (1) Extruder, and (2) Auxiliary Equipment.

Extruder

The screw extruder equipment consists of four main parts:

1. Barrel and Screw

2. Control Panel

3. Dies

4. Screw Drive

Barrel and Screw. The barrel, made of stainless steel, is 24 in. long and has an internal bore of 1.0 in. The feed zone is jacketed for water cooling. The front end of the barrel is flanged to hold the die adapter and breaker plate. The feed end of the barrel is attached to the thrust bearing housing with four bolts designed to shear under excessive pressures. A feed port about $1\frac{1}{2}$ in. in diameter is located on top of the barrel next to the thrust bearing housing. Two electric heaters, 600 watts each, are fastened to the barrel (See Figure 10 and Figure 11).

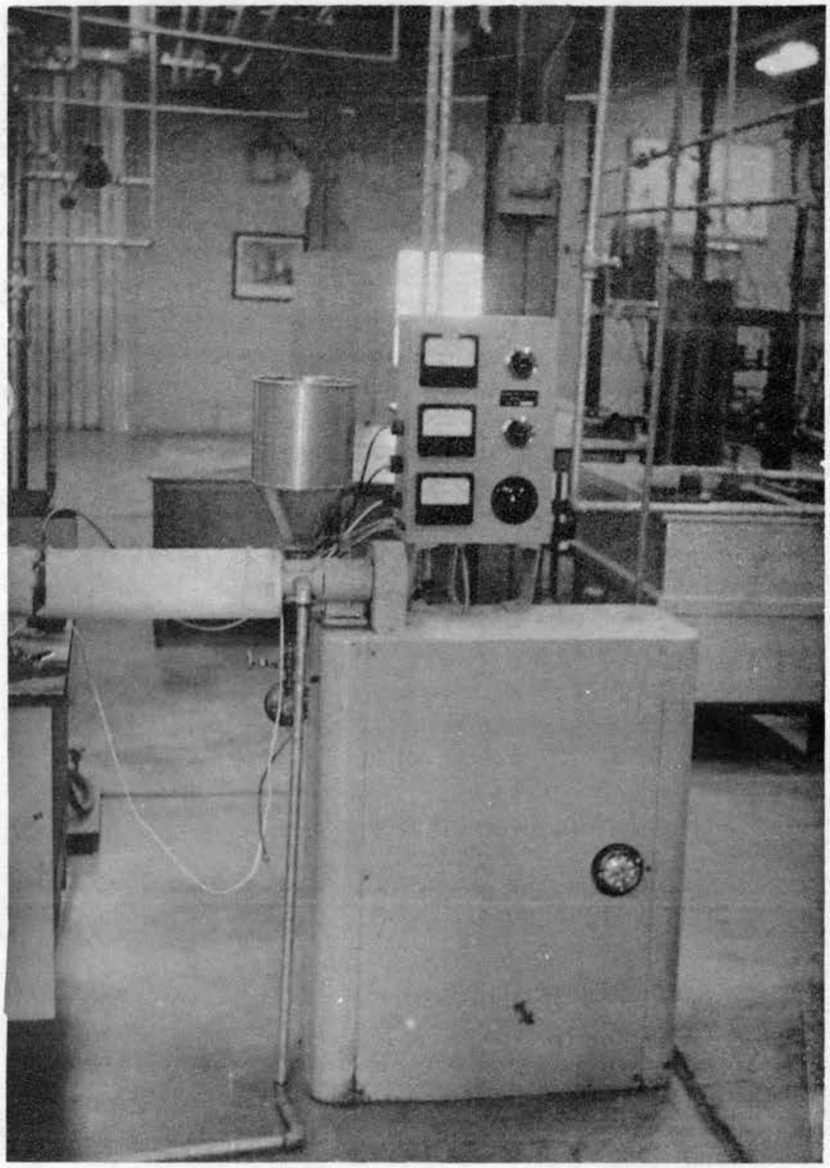


Figure 10. Screw Extruder

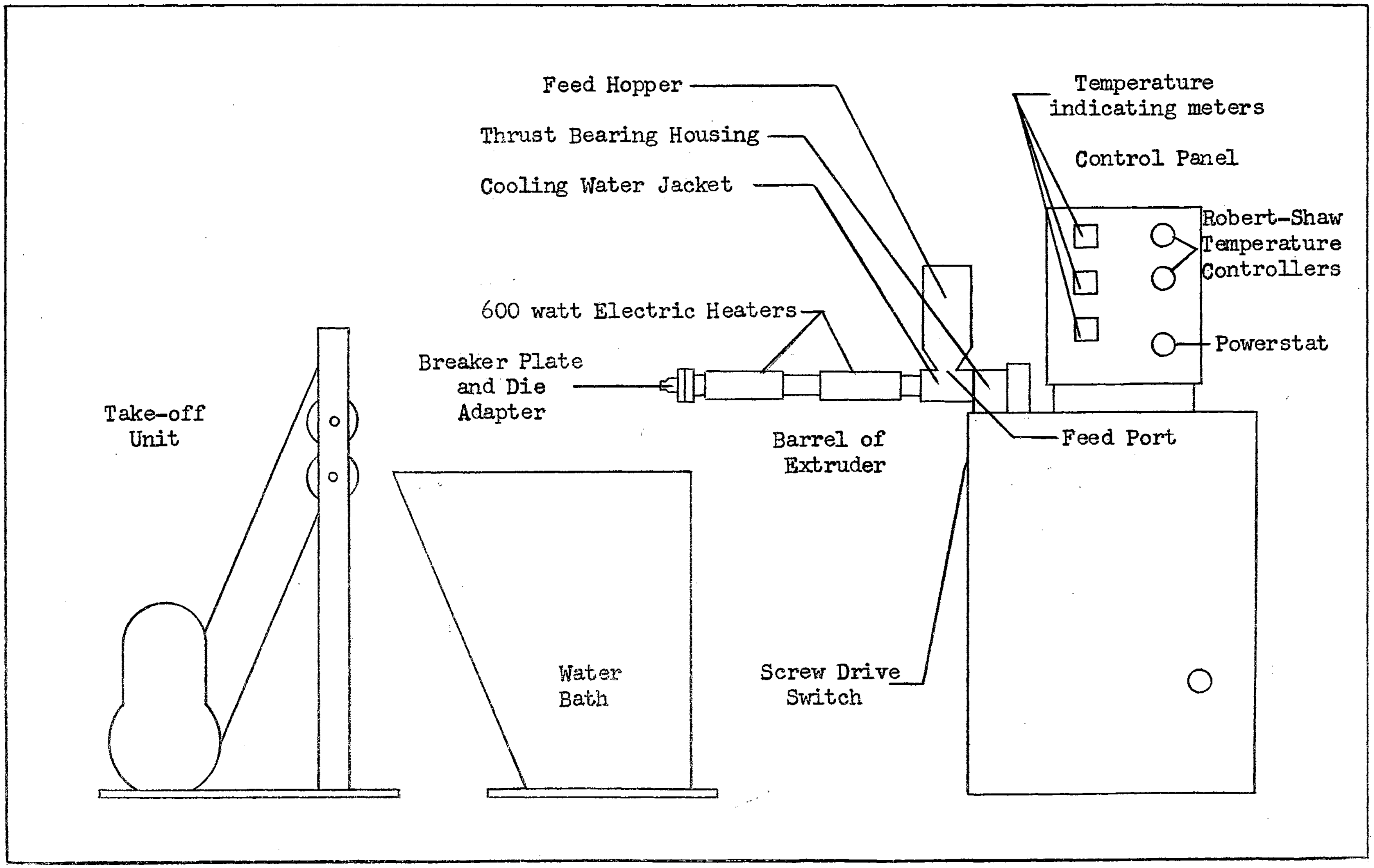


Figure 11. Extrusion Equipment

The screw, made of a heat treated alloy, is about 23 in. long. The screw is a single flight type with a constant pitch and helix angle. The screw is keyed into the thrust bearing. The dimensions of the screw are (See Figure 4):

$D = 0.0996$ in.	$b = 0.753$ in.	$\phi = 18.6$ in.
$e = 0.303$ in.	$L = 7$ in.	$\delta = 0.004$ in.
	$h = 0.104$ in.	

Control Panel. The control panel consists of:

1. Two Robert Shaw on-off temperature controllers for the electric heaters on the barrel.
2. One Powerstat to regulate the voltage to the die heaters for temperature control.
3. Three temperature indicating meters with attached thermocouples.
4. Three electrical outlets for the electric heaters.
5. The screw drive switch.

The control panel is fused for instrument protection.

Dies. There are three dies available for the screw extruder: (1) rod die, (2) tubing die, and (3) film die. The rod die is simply a cylinder with a tapered internal bore. A 50-watt electric heater is attached to the die. The die can produce rods of about 1/8 in. or less. The tubing die is a cylinder containing an annular opening. The annulus is arranged so that air can be blown through a small hole in the center of the annulus (See Figure 12). Two 220-watt electric heaters are attached to the die. The film die is essentially a hollow wedge that shapes the plastic into a film (See Figure 13). Attached to the die are four 110-watt electric heaters.

Screw Drive. The screw is driven by an electric motor (3/4 horsepower) through a variable speed Reeves transmission and a chain drive. The screw drive is housed in the control panel.

Auxiliary Equipment

The auxiliary equipment consists of a take-off unit and a water bath. The take-off unit is essentially two rollers like a ringer on a washing machine powered by an electric motor (3/4 horsepower) through a variable speed Reeves transmission. The take-off unit provides a steady pull on the plastic leaving the extruder to help provide an uniform product.

CHAPTER IV

OPERATING INSTRUCTIONS FOR THE SCREW EXTRUDER

Preparations for Start-Up

The following procedure should be followed in preparation for starting the extruder to insure proper performance and long life of the extrusion equipment. The prestart-up procedure consists of:

1. Lubrication Check
2. Thermocouple Check
3. Barrel and Screw Check

Lubrication Check

Check the oil level in the screw drive transmission and the take-off unit transmission. If the oil is below the proper level, fill the transmission to the proper level with 40 S.A.E. motor oil.

Grease fittings, A and B (See name plate on the electric motors), on the screw drive motor and the take-off unit motor. Use only two or three strokes of the grease gun.

Grease the thrust bearing fitting at feed end of the barrel. Use only three or four stokes of the grease gun.

Grease the six fittings on the carrier bearings on the rollers of the take-off unit. Be sure to grease the fittings behind the drive gears. Use only three or four strokes of the grease gun.

Note. The interval between greasings depends upon the hours of operation of the equipment. The equipment should be greased at least once every two weeks.

Thermocouple Check

The functioning of the thermocouples can be checked by holding the thermocouple tips in a flame. If the thermocouples are functioning, the indicating needle should move rapidly towards the high temperature end of the scale.

The accuracy of the thermocouples can be checked by putting the thermocouple tips in boiling water and comparing the boiling point of water at the barometric conditions prevailing. The indicating needle can be adjusted to read the boiling point of water by turning the adjustment screw in the center of the indicating instrument.

Barrel and Screw Check

Remove the insulating material from the barrel of the extruder. Make sure the electrical connections on the heaters are tight and the heaters are tightly fastened to the barrel.

Remove the four bolts from the flange at the die end of the barrel. If the die adapter appears to be stuck to the barrel, turn on the heater closest to the flange and heat the barrel until the die adapter can be removed. After removing the die adapter, clean the plastic out from between the breaker plate and the head of the screw. Replace the die adapter and tighten the four bolts evenly.

Operating Procedure

In this section, a few general suggestions and precautions will be presented for operating the extruder. The general procedures and precautions should be followed to insure proper operation of the extruder.

After selecting the die to be used, attach the die to the barrel. The die does not have to be tightened firmly to the barrel, but use all the threads possible allowing the die to be in a vertical position pointing downward. Fill the water bath and set the flow rate of water through the bath so that the water is flowing at a constant rate over the weir.

After choosing the plastic to be extruded, set the heater controls to the suggested temperatures (See Appendix G). The barrel heaters are controlled automatically; however, the die heaters are controlled manually. The desired die temperature can be readily attained by setting the Powerstat dial to about 60 and decreasing the Powerstat as the die temperature approaches the desired temperature.

Caution. As the barrel and die are heating, it is imperative that the screws holding the heaters to the barrel and the die are tight. After operating temperatures have been attained, make a final check of the heater screws.

While the barrel and die are heating, turn on the valve that controls the flow of cooling water to the feed zone of the barrel. After operating temperatures have been attained, load plastic in the hopper and start the extruder with the screw operating at its minimum speed. The screw speed may be increased after the plastic has begun to flow out of the die.

Caution. Do not start the extruder until the barrel and die are at their operating temperatures. Do not turn the dials on the transmissions while they are not running.

Shut-Down Procedure

The following shut-down procedures are recommended for ease of restarting the extruder at a later date and for keeping maintenance of the extruder at a minimum.

After the plastic has stopped feeding, allow the extruder to run dry. Turn off the screw drive and electric heaters. Then, remove the die from the barrel. The die should be cleaned while it is hot. The operator should become familiar with the procedure for disassembling the die before it is used. See the Maintenance section for instructions for die disassembly. Next, remove the four bolts holding the die adapter to the barrel. If the bolts are difficult to loosen, allow the barrel to cool until the four bolts can be removed easily. Reheat the barrel by turning on the barrel heater closest to the front of the barrel. Reheating the barrel will soften the plastic making the die adapter easier to remove. Remove the plug of plastic off the breaker plate, scrape any plastic off the flange surface, and reassemble. Shut off the feed zone cooling water and drain the water bath. As an extra precaution, pull the master switch.

Caution. Plastics which tend to decompose should be flushed from the extruder with a heat-stable plastic before the extruder is shut down. When the extruder is restarted, the heat-stable plastic can be flushed from the extruder with the plastic to be extruded.

Suggested Operating Techniques

This section is intended as a guide for operating the screw extruder with the various dies and to suggest some operating techniques for obtaining satisfactory results from the extruder.

The following techniques and procedures apply to each of the three dies available. Each of the dies requires use of the water bath and the take-off unit. When using the dies, always start extrusion at the minimum screw speed and minimum take-off unit speed. After extrusion has started, the screw and take-off unit speeds may be changed.

The quality of the product may be changed by varying screw and take-off unit speeds and operating temperatures. If screw speed is held constant and take-off unit speed is increased, the product will be stretched thus becoming thinner. However, if take-off unit speed is held constant and screw speed is increased, the product becomes thicker. Rate of production of a desired quality can be increased by increasing the screw speed and the take-off unit speed. The operating temperature may be increased, making the product thinner, or the operating temperature may be decreased making the product thicker.

Caution must be exercised to keep the operating temperature above the melt temperature and below the decomposition point of the plastic.

Rod Die

The rod die is the easiest die of the three to use. The temperature of the die is the only operating variable that is critical. If the die temperature is too low, the product will not be uniform and will be lumpy in appearance. The product will become uniform if the die temperature is increased.

Tubing Die

The tubing die is probably the most difficult of the three dies to use. There are four operating variables that are critical:

1. Die temperature
2. Take-off unit speed
3. Screw Speed
4. Air pressure

The correct combination of these four variables for a satisfactory product is difficult to attain. Any fluctuations in the air pressure will cause bulges in the tubing. If the die temperature becomes too hot, the tubing wall will become too thin. Also, an incorrect combination of the take-off unit speed and the screw speed will cause the tubing wall to be thin. A combination of slow take-off unit speed and fast screw speed will increase the wall thickness of the tubing. Holding the take-off unit and screw speed combination constant, adjust the die temperature and air pressure until a satisfactory product results.

Film Die

The film die is only a little more difficult to use than the rod die. The same general considerations as applied to the rod die apply to the film die.

The above suggestions are only general and should be used as a guide. The difference in plastics from run to run will cause the combination of the operating variables to change.

Maintenance

The amount of maintenance will be reduced if the start-up, operating, and shut-down procedures are followed. However, a certain amount of cleaning is necessary. This section presents procedures for cleaning of the extrusion equipment.

As soon as possible after the extruder has been turned off, the die should be detached from the barrel and carefully cleaned. The die is disassembled by removing the appropriate bolts and the plastic is scraped from the die with a soft metal bar (copper or brass). Take extreme care to avoid scratching the die as any nick or grove will appear on the extruded plastic.

Rod Die

The rod die does not have to be disassembled for cleaning. Scrape the plastic off the exterior of the die.

Tubing Die

The tubing die is disassembled in the following manner (See Figure 12):

1. Remove the four bolts from the center of the tubing die face.

2. Remove the sizing ring from the die.

3. Remove the hex-nut from the air intake line at the rear of the die.

4. Push the air intake line forward.

After the die has been reassembled, it may be necessary to recenter the sizing ring by adjusting the centering bolts.

Film Die

The film die is disassembled in the following manner (See Figure 13):

1. Remove the four bolts holding the goose-neck to the die.

2. Remove the four bolts holding each side plate.

3. Remove the electric heaters from the die.

4. Remove the guide bolts from the adjustable plate.

5. Remove the three bolts holding the adjustable plate.

6. Remove the two large remaining bolts.

After the die is reassembled, the gap between the adjustable plates should be checked for uniformity.

Screw

The screw should be removed from the extruder periodically and cleaned. Removing the screw is a simple process. After removing the die, die adapter, breaker plate, and reheating the barrel to a temperature above the melt index of the plastic last extruded, turn on the screw drive briefly to loosen the plastic in the barrel. Then insert

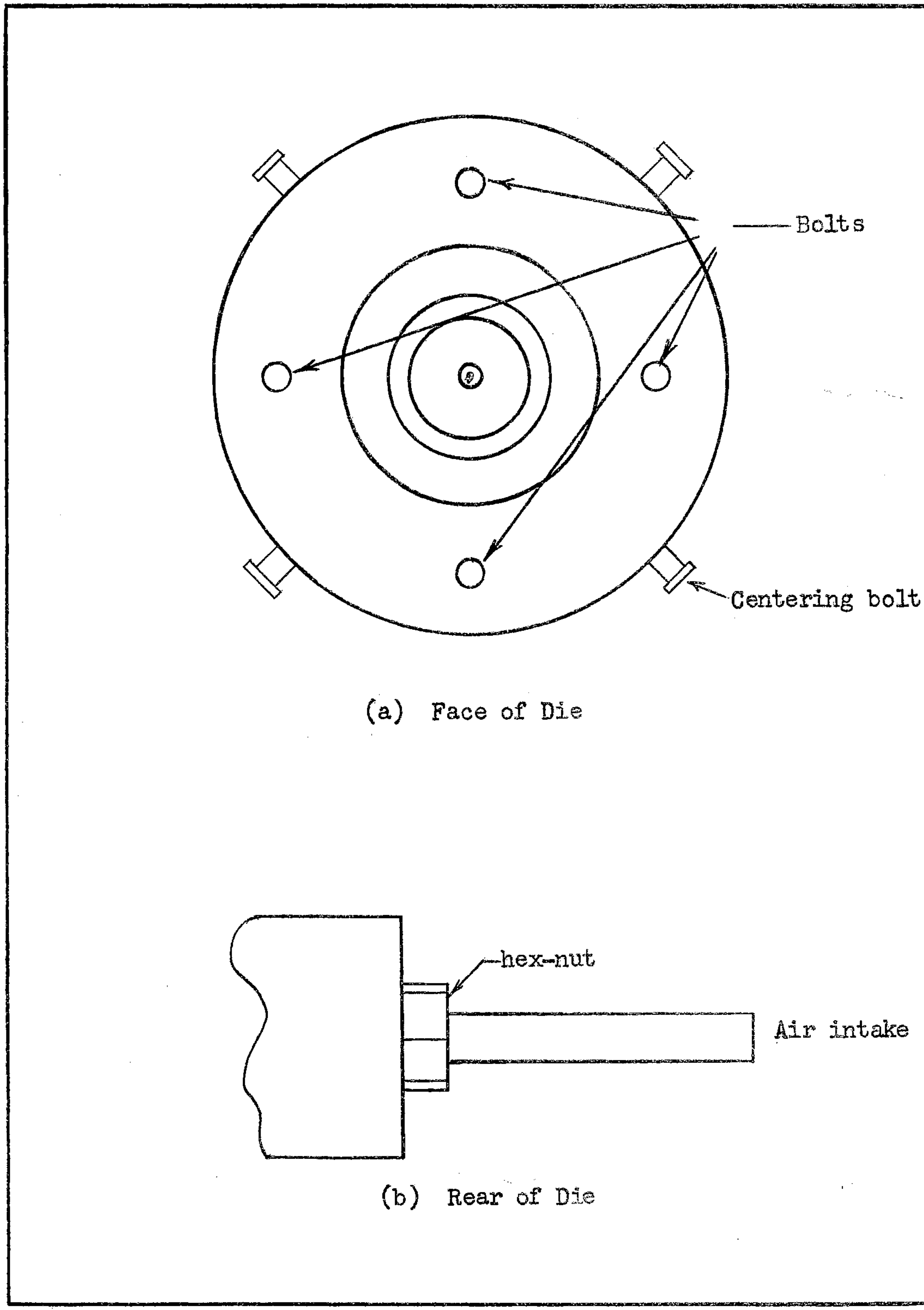


Figure 12. Tubing Die

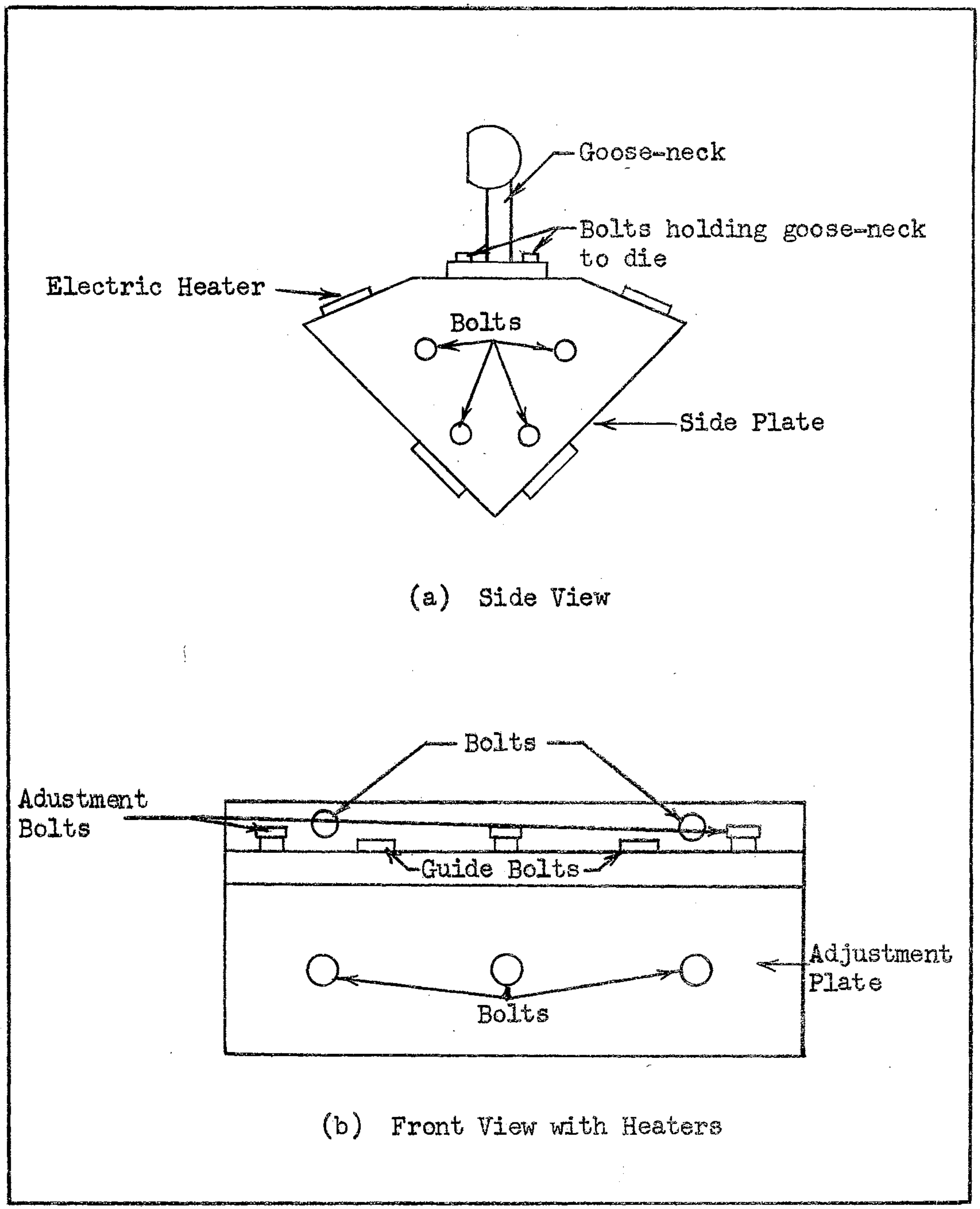


Figure 13. Film Die

a slender rod through the hole in the chain guard at the rear of the barrel. Using this rod, push the screw forward out of the barrel. After the screw has been removed from the barrel, the screw can be cleaned while hot. Scrape the plastic from the screw with a soft metal bar.

If for any reason the barrel of the extruder becomes clogged, the four bolts holding the barrel to the thrust bearing will shear. These four bolts can be easily replaced. Remove the chain guard from the extruder. After disconnecting the master link, remove the chain. Pull the chain sprocket from the shaft. A portion of the four bolts now can be removed. The other portion of the four bolts can be removed by sliding the barrel forward and unscrewing the bolts with a pair of pliers. The four new bolts can be inserted and the extruder parts reassembled.

CHAPTER V

EXPERIMENTAL PROCEDURE

Start-Up Procedure

(After selecting operating temperature for the test section, the feed zone heater control was set at 450° F. and the metering zone heater at 500° F. Powerstat A was set at 30, Powerstat B was set so that the ammeter would read one amp., Powerstat C was set at 50 (See Figure 9). These approximate settings were used for heating up the apparatus before each run.

After the temperature of the test section had reached the selected temperature, the hopper of the extruder was filled with the desired low-density polyethylene, and the cooling water was turned on. The extruder was started at its slowest screw speed.

Next, the coiled tubes were refilled with silicone grease using five or six strokes of the grease gun. Powerstat A was turned off after the plastic had started to flow from the test section. The needle valves were opened so the pressure gages would register. The screw speed was set to the desired speed.)

Operating Procedure

Thermocouple readings and pressure readings were recorded every 15 minutes. Minor adjustments of the Powerstats and heater controls were made as needed for thermocouples, numbers 1 and 12, to read the selected temperature. Thermocouples numbers 1 and 12 were used to measure the entrance and exit temperatures of the low-density polyethylene as it flowed through the heated tube.

After the thermocouple readings and pressure readings had become constant and the temperature readings of thermocouples, numbers 1 and 12, were within 5° F. of each other, the test section was considered as being at steady-state and isothermal conditions. The 15 minute interval readings were continued for 30 to 45 minutes while collecting samples of the plastic for each time interval. The timed samples were weighed on a beam balance to the nearest gram to determine the mass flow rate of the plastic through the test section.

Shut-Down Procedure

After the desired flow-rate, pressure-drop, and temperature data were collected, the following procedure was followed. The hopper of the extruder was emptied, and the extruder was allowed to run dry. The screw speed was reduced to its slowest speed, and the extruder was turned off. The needle valves were closed, and the coiled tubes were refilled with grease. The above procedure was followed to assure that the start-up procedure for the next run could be carried out with no difficulty.

Calibration of Thermocouples

The chromel-iron constantan thermocouples were calibrated against a mercury thermometer. The thermocouples were placed in a paraffin bath with the thermometer. The thermocouples were calibrated over a temperature range of 300° to 400° F.

Density Determination

The density of the plastics was determined for a temperature range of 270° to 380° F. The following procedure was followed.

Cream testing flasks were weighed. Each flask was calibrated with water to determine its volume and the volume corresponding to the scale on each flask. The flasks were partially filled with plastic and placed in a constant temperature bath. The bath was held at a constant temperature for 4 to 6 hours. Then, the level of the plastic in the flask was noted. After volumetric data had been collected for three or four temperatures, the flask containing plastic was weighed.

CHAPTER VI

PRESENTATION AND DISCUSSION OF RESULTS

Isothermal Flow Data

Isothermal flow data were taken for low-density polyethylene manufactured by the following companies:

1. Phillips Chemical Company
2. Spencer Chemical Company
3. Dow Chemical Company

The low-density polyethylene from the Dow Chemical Company consisted of two types, designated as 550E and 770. The data are summarized in Table I.

The data were analyzed using the generalized approach of Metzner and Reed (11). The data were plotted on log-log paper. The abscissa was $8V/D$ and the ordinate was $D\Delta P/4L$. This plot is shown in Figure 14. The data for the Phillips low-density polyethylene were not analyzed. Some difficulties were encountered with the pressure measurements while making the runs for the Phillips low-density polyethylene.

Density Data

The densities for the low-density polyethylenes mentioned previously were determined over the range of operating temperatures. The resulting data are shown in Figure 15. The low-density polyethylene supplied by the Spencer Chemical Company was the most dense; the low-density polyethylene by the Phillips Chemical Company was the least dense. The differences in the densities of the plastics were probably due to differences in the molecular weights.

TABLE I

Summary of Isothermal Data

Brand	Run	T - °F.	DΔP/4L lbs./in.	8V/D Sec.$^{-1}$
Spencer	25	327	2.10	13.9
	26	329	1.98	14.0
	27	329	4.06	38.7
	28	344	2.03	14.4
	29	380	1.46	16.0
	30	345	2.20	16.6
	31	346	2.71	23.2
	32	346	2.94	26.9
	33	348	3.65	39.7
	34	378	1.96	25.0
	35	382	2.74	42.6
	36	384	4.10	69.5
	37	359	1.85	14.1
	38	365	3.46	40.1
Dow	39	328	2.45	16.4
550E	40	328	3.42	29.2
	41	327	4.34	45.9
	42	345	2.17	17.9
	43	342	2.69	25.7
770	44	341	2.32	51.3
	45	361	0.83	19.7
	46	362	1.36	33.8
	47	364	1.83	50.7
	48	384	0.78	24.4
	49	382	1.14	34.7
	50	385	1.48	53.8
	51	342	1.14	21.1
	52	343	1.73	33.6
550E	53	343	3.58	40.8
	54	359	1.70	14.4
	55	363	3.25	43.2
	56	361	2.40	25.4

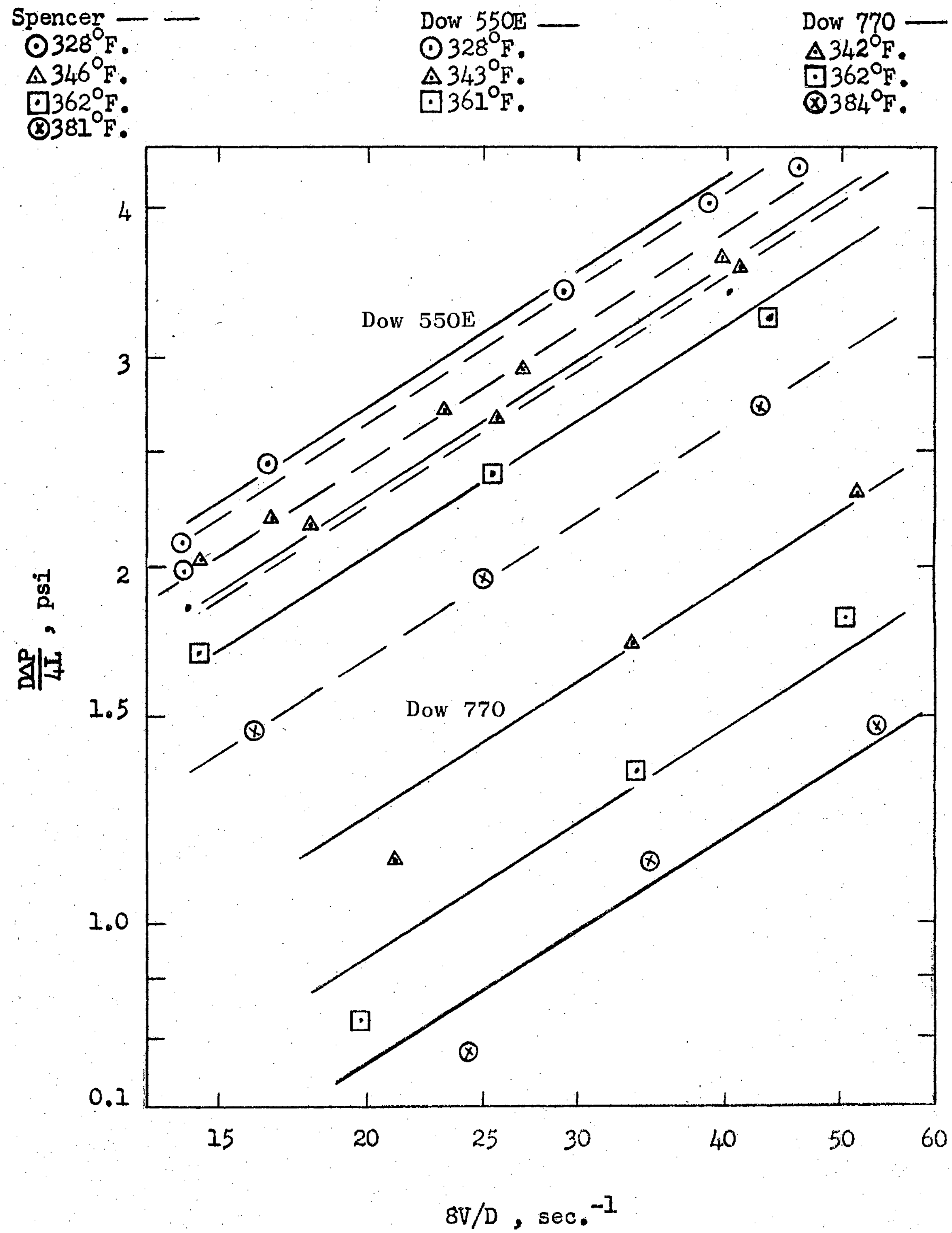


Figure 14. Shear Stress at the Wall

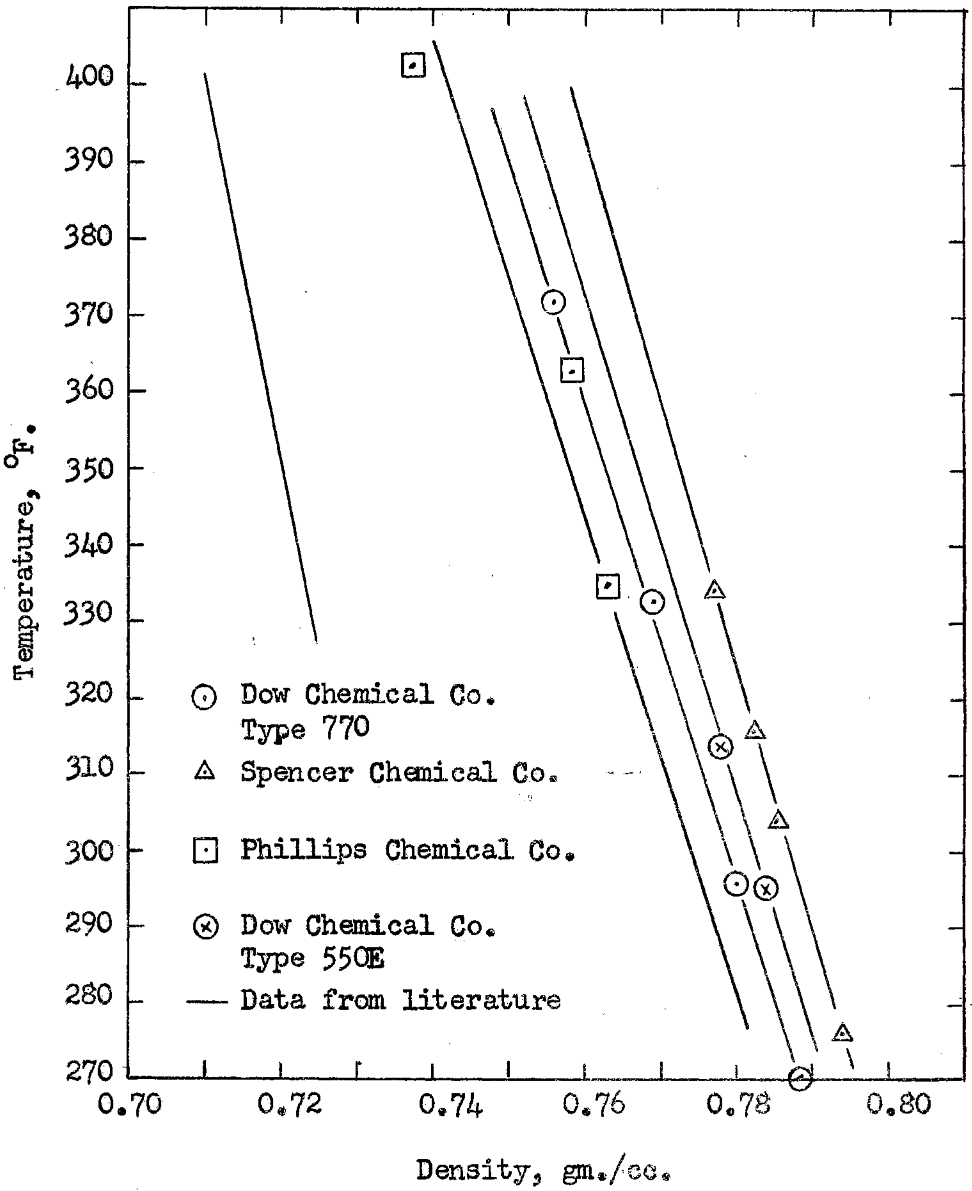


Figure 15. Density of Low-Density Polyethylene

Also shown in Figure 15 are density data for polyethylene from an article by Berhardt (20). The data for the low-density polyethylene differed from the published data for polyethylene by about eight percent.

The Flow Index and the Consistency Index

If the data can be represented by a straight line on a log-log plot of $D\Delta P/4L$ versus $8V/D$, the equation of the line would have the following form (11):

$$\frac{D\Delta P}{4L} = K'\left(\frac{8V}{D}\right)^{n'} \tag{19}$$

As shown in Figure 14, the data could be represented by a straight line.

The data were graphically fitted making the slopes of the straight lines for each low-density polyethylene parallel. The resulting slopes, the flow index $\underline{n'}$, had a value of 0.64 and the intercept, the consistency index, ranged from 0.11 to 0.40 $lb._f/in.\text{-}sec.$ The consistency indices are summarized in Table II. At approximately the same temperature, for example 344°F., the low-density polyethylene by the Spencer Chemical Company had the highest consistency index and the low-density polyethylene type 770 by the Dow Chemical Company had the lowest value for the consistency index. This difference in consistency was probably due to the differences in the molecular weights. This same trend was exhibited by the density data.

Friction Factor

A friction factor was calculated using the following equation (11):

$$f = (D\Delta P/4L)/(V^2\rho/2g_c) \tag{45}$$

For each friction factor, the generalized Reynolds number was calculated (11)

$$N_{Re} = \frac{D^{n'}V^{2-n'}\rho}{g_c K' 8^{n'-1}} \tag{21}$$

TABLE II

Summary of Consistency Indices

Brand	T - F.	K' lb.f/in.-sec.
Spencer	328	0.39
	346	0.36
	362	0.33
	381	0.25
Dow 550E	328	0.40
	343	0.34
	361	0.30
770	342	0.18
	362	0.14
	384	0.11

A friction factor plot calculated from experimental data is shown in Figure 16. Theoretically, the friction factors are given by the following equation (11):

$$f = 16/N_{Re} \qquad (20)$$

The theroretical values of the friction factors are shown in Figure 16.

The friction factors calculated from the experimental data formed a straight line on a friction factor plot, Figure 16.

The calculated friction factors agreed very well with the theoretical values. The calculated friction factors were on the average slightly lower than the theoretical values. The calculated and theroretical values are shown in Table III.

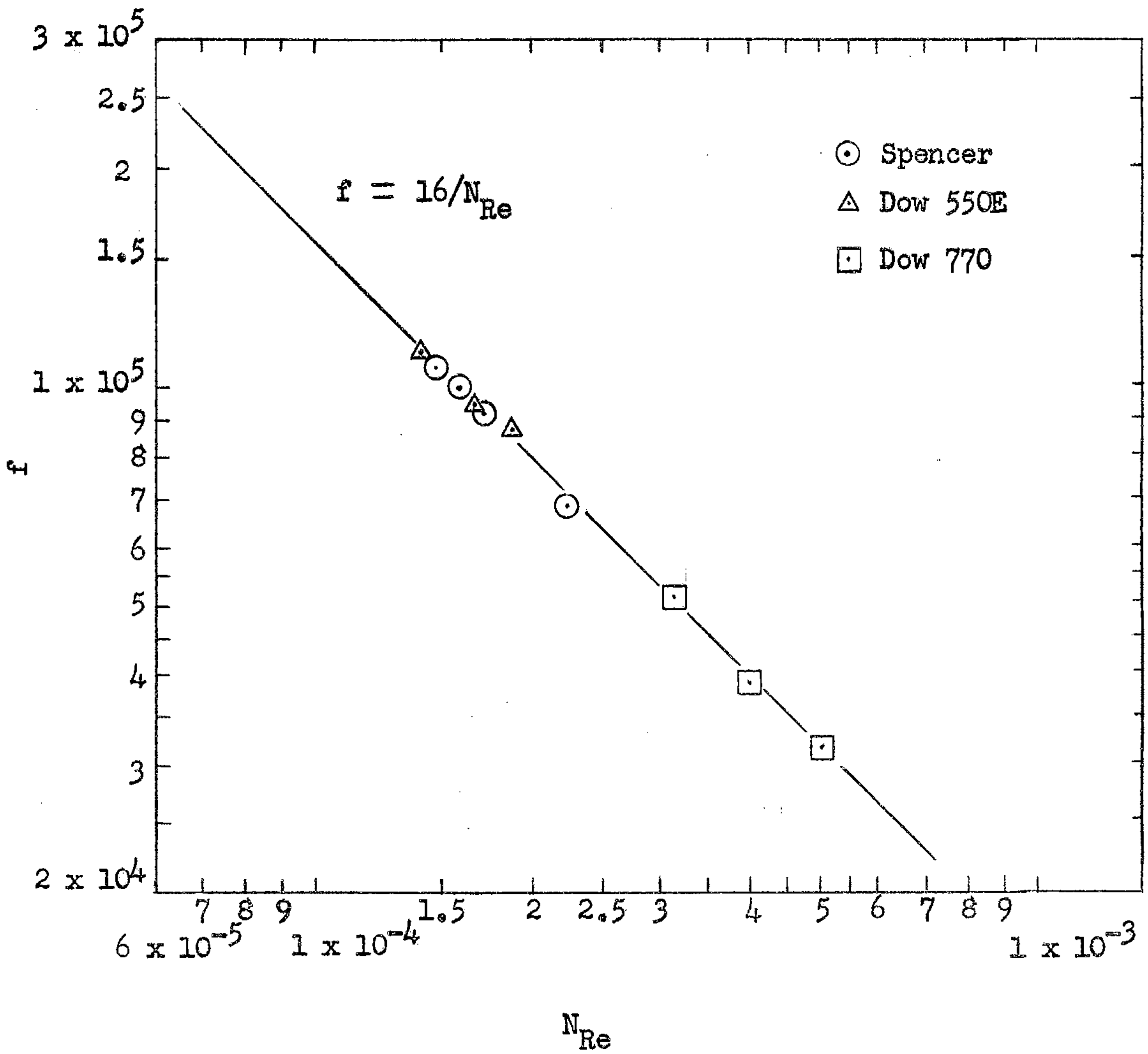


Figure 16. Friction Factor

TABLE III

Comparison of Calculated
Friction Factors to Theoretical Values

$\frac{N_{Re}}{1 \times 10^4}$	$\frac{f}{1 \times 10^{-4}}$	$f = 16/N_{Re}$
1.47	10.60	10.90
1.58	10.00	10.10
1.71	9.20	9.36
2.24	6.94	7.15
1.42	11.20	11.30
1.66	9.46	9.65
1.87	8.78	8.56
3.12	5.08	5.13
3.98	3.90	4.02
5.02	3.16	3.19

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

Thesis Goals

The goals of this thesis were:

1. Installation of the screw extruder.
2. Preparation of operating instructions for the screw extruder.
3. Determination of the flow characteristics of low-density polyethylene through a circular tube.

Conclusions

Installation of the screw extruder has been successfully completed and operating instructions have been presented.

The flow index for the low-density polyethylene was determined to be 0.64. The consistency index had a range of values of 0.11 to 0.40 $lb._f/in.^2$ -sec.

The friction factors agreed very well with the values predicted by theory. The calculated friction factors differed from the theoretical by 0.10 on the average. The generalized approach of Metzner and Reed predicts friction factors for non-Newtonian fluids in laminar flow very well.

Recommendations

It is recommended that future studies be made on systems of thermoplastics flowing through a circular tube. The flow index and consistency index of various thermoplastics would be of value for future heat transfer studies.

In future studies, various lengths and diameters of tubing should be used to check the time-dependency of the thermoplastics and to determine the entrance effect on the flow data.

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APPENDIX A

DEFINITIONS OF SYMBOLS

A - Area or constant in the Powell-Eyring equation.

B - Constant in the Powell-Eyring equation.

b - Width of screw channel.

D - Diameter.

D_B - Diameter of barrel of extruder.

E - Correction term for eccentricity of the screw, generally 1.2.

e - Width of land.

F - Force.

f - Fanning friction factor.

g_c - Conversion factor, 32.2 $(lb._m\text{-}ft.)/(lb._f\text{-}sec.^2)$.

h - Depth of the screw channel.

K - Consistency index for the power-law equation.

K' - Consistency index for the generalized equation.

L - Length of tube or metering zone.

N - Rotational speed of screw.

n - Flow index for the power-law equation.

n' - Flow index for generalized equation.

P - Pressure.

Q - Volumetric flow rate.

R - Radius.

V - Average bulk velocity.

$\frac{du}{dy}$ - Velocity gradient.

$-(\frac{dP}{dl})$ - Pressure gradient.

$(-\frac{du}{dr})_w$ - Velocity gradient at the tube wall.

ΔP - Pressure drop.

τ - Shear stress.

τ_y - Initial shear stress for Bingham plastic.

τ_w - Shear stress at the wall.

μ - Newtonian coefficient of viscosity.

μ_a - Apparent viscosity.

μ_p - Plastic viscosity.

η - Coefficient of rigidity or plastic viscosity.

δ - Clearance between the screw and the barrel of the extruder.

θ - Time.

θ_B - Angle of movement of the outer surface of the solid.

ϕ - Helix angle of the screw.

ρ - Density.

λ - Relaxation time.

N_{Re} - Reynolds number

N_{He} - Hedström number

APPENDIX B

DERIVATION OF THE GENERALIZED EQUATIONS FOR NON-NEWTONIAN FLUID FLOW

The derivation of the generalized equations for non-Newtonian fluid flow, presented here, parallels the derivation by Oka (10).

Consider a circular tube of radius, R, and length, L, with a non-Newtonian fluid flowing through it (See Figure 17). The pressure on the fluid at one end of the tube is $P + \Delta P$ and the pressure on the other end is P.

The following assumptions are made:

1. Incompressible fluid.
2. Fluid is in streamline flow.
3. Fluid is not time-dependent.
4. There is no slip at the tube wall.

Take a cylindrical element of the fluid flowing through the tube (See Figure 18) and make a force balance around it.

$$2\pi r L \tau + \pi r^2 P = (P + \Delta P)\pi r^2 \qquad \text{(B-1)}$$

Solve Eq. B-1 for the shear stress.

$$\tau = \frac{r\Delta P}{2L} \qquad \text{(B-2)}$$

Let the velocity of the fluid flowing through tube be a function of the radius.

$$u = u(r) \qquad \text{(B-3)}$$

Then, the volumetric flow rate through the tube would be

$$Q = 2\pi \int_0^R r\, u(r) dr \qquad \text{(B-4)}$$

Integrating Eq. B-4 by parts, the volumetric flow rate becomes

$$Q = \Big[\pi r^2 u(r)\Big]_0^R - \int_0^R \pi r^2 du$$

or

$$Q = \pi R^2 u(R) - \int_0^R r^2 du \qquad \text{(B-5)}$$

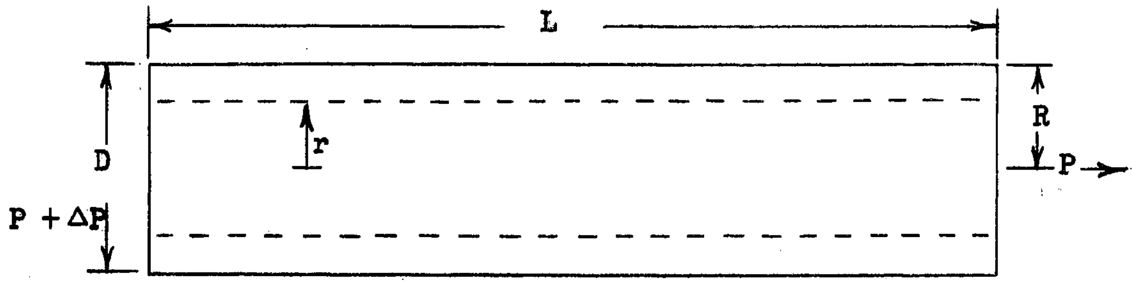

Figure 17. Fluid Flowing Through A Circular Tube

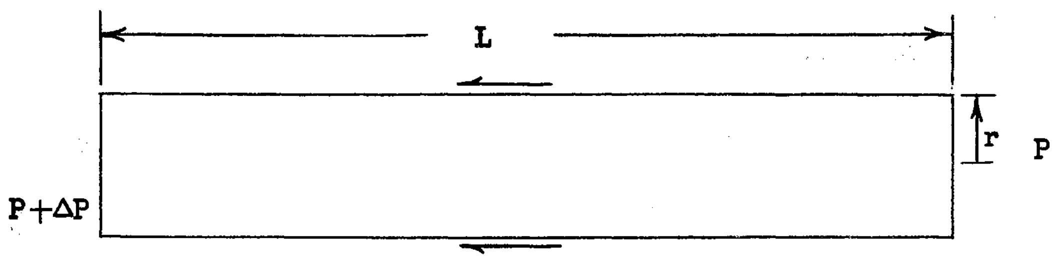

Figure 18. Force Balance for a Cylindrical Section of Fluid Flowing Through a Tube

Since it was assumed that there is no slip at the tube wall, then $u(R) = 0$. Now, the volumetric flow rate can be expressed as

$$Q = -\pi \int_0^R r^2 du \qquad \text{(B-6)}$$

Multiply Eq. B-6 by (dr/dr).

$$Q = \pi \int_0^R r^2 \left(-\frac{du}{dr}\right) dr \qquad \text{(B-7)}$$

The term (-du/dr) in Eq. B-7 is the rate of shear. For a non-Newtonian fluid the rate of shear is a function of the shear stress.

$$\left(-\frac{du}{dr}\right) = f(\tau) \qquad \text{(B-8)}$$

Using Eq. B-8 and Eq. B-2, change the variables in Eq. B-7 to the shear stress, τ. The resulting equation would be

$$Q = \frac{\pi R^3}{\tau_w^3} \int_0^{\tau_w} \tau^2 f(\tau) d\tau \qquad \text{(B-9)}$$

where

$$\tau_w = \frac{R\Delta P}{2L} \qquad \text{(B-10)}$$

For convenience, Eq. B-9 can be put into the following form:

$$\phi(\tau_w) = \frac{4Q}{\pi R^3} = \frac{4}{\tau_w^3} \int_0^{\tau_w} \tau^2 f(\tau) d\tau \qquad \text{(B-11)}$$

By differentiating Eq. B-11, the following equation can be obtained:

$$\frac{d\phi}{d\tau_w} = -\frac{3(4)}{\tau_w^4} \int_0^{\tau_w} \tau^2 f(\tau) d\tau + \frac{4f(\tau_w)}{\tau_w} \qquad \text{(B-12)}$$

Regrouping the terms

$$\frac{d\phi}{d\tau_w} = -\frac{3}{\tau_w} \left[\frac{4}{\tau_w^3} \int_0^{\tau_w} \tau^2 f(\tau) d\tau\right] + \frac{4f(\tau_w)}{\tau_w} \qquad \text{(B-13)}$$

The term in the brackets is equal to $4Q/\pi R^3$, from Eq. B-11.

$$\frac{d\phi}{d\tau_w} = -\frac{3}{\tau_w}\left(\frac{4Q}{\pi R^3}\right) + \frac{4f(\tau_w)}{\tau_w} \tag{B-14}$$

Substituting Eq. B-11 for ϕ and Eq. B-10 for τ_w, Eq. B-14 can be written as

$$\frac{d\left(\frac{4Q}{\pi R^3}\right)}{d\left(\frac{R\Delta P}{2L}\right)} = -\frac{3}{\tau_w}\left(\frac{4Q}{\pi R^3}\right) + \frac{4f(\tau_w)}{\frac{R\Delta P}{2L}} \tag{B-15}$$

Solve Eq. B-15 for $f(\tau_w)$.

$$f(\tau_w) = \frac{3}{4}\left(\frac{4Q}{\pi 3}\right) + \frac{1}{4}\left(\frac{R\Delta P}{2L}\right)\frac{d\left(\frac{4Q}{\pi R^3}\right)}{d\left(\frac{R\Delta P}{2L}\right)} \tag{B-16}$$

Since for a non-Newtonian fluid the shear rate is a function of the shear stress, then

$$f(\tau_w) = \left(-\frac{du}{dr}\right)_w$$

Then, Eq. B-16 can be written as

$$\left(-\frac{du}{dr}\right)_w = \frac{3}{4}\left(\frac{4Q}{\pi R^3}\right) + \frac{1}{4}\left(\frac{R\Delta P}{2L}\right)\frac{d\left(\frac{4Q}{\pi R^3}\right)}{d\left(\frac{R\Delta P}{2L}\right)} \tag{B-17}$$

The volumetric flow rate through the tube can be written in terms of the bulk average flow rate.

$$Q = \frac{\pi D^2 V}{4} \tag{B-18}$$

Substituting Eq. B-18 into B-17 and changing the radius to the diameter, Eq. B-17 can be expressed as

$$\left(-\frac{du}{dr}\right)_w = \frac{3}{4}\left(\frac{8V}{D}\right) + \frac{1}{4}\left(\frac{D\Delta P}{4L}\right)\frac{d(8V/D)}{d(D\Delta P/4L)} \tag{B-19}$$

Substituting

$$d\left[\ln(8V/D)\right] = \frac{d(8V/D)}{8V/D}$$

and

$$d\,[\ln(D\ P/4L)] = \frac{d(D\Delta P/4L)}{D\Delta P/4L}$$

into Eq. B-19

$$\left(-\frac{du}{dr}\right)_w = \frac{3}{4}\left(\frac{8V}{D}\right) + \frac{1}{4}\left(\frac{8V}{D}\right)\frac{d\,[\ln(8V/D)]}{d\,[\ln(D\Delta P/4L)]} \quad \text{(B-20)}$$

Let

$$n' = \frac{d\,[\ln(D\Delta P/4L)]}{d\,[\ln(8V/D)]} \quad \text{(B-21)}$$

Substituting Eq. B-21 into Eq. B-20, one obtains the equation as presented by Metzner and Reed (11).

$$\left(-\frac{du}{dr}\right)_w = \frac{3n'+1}{4n'}\ \frac{8V}{D} \quad \text{(B-22)}$$

The derivative n', Eq. B-21, is the slope of a curve on a plot of ln(DΔP/4L) versus ln(8V/D).

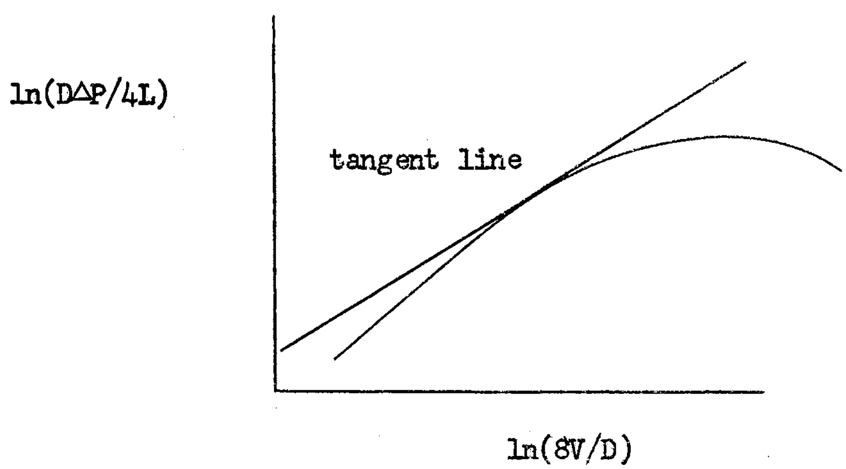

Figure 19. Generalized Shear Stress Diagram

Then, the equation of the tangent line would be

$$\frac{D\Delta P}{4L} = K'\left(\frac{8V}{D}\right)^{n'} \quad \text{(B-23)}$$

The Fanning friction factor is defined as

$$f = (D\Delta P/4L)/(\rho V^2/2g_c) \quad \text{(B-24)}$$

Substituting Eq. B-23 into Eq. B-24, the friction factor can be expressed as

$$f = \frac{2g_c K' 8^{n'}}{D^{n'} V^{2-n'} \rho} \qquad \text{(B-25)}$$

By letting $f = 16/N_{Re}$, one can obtain the expression for a generalized Reynolds number.

$$N_{Re} = \frac{D^{n'} V^{2-n'} \rho}{g_c K' 8^{n'-1}} \qquad \text{(B-26)}$$

Then, for laminar flow of non-Newtonian fluids, the flow equation can be expressed simply as

$$f = 16/N_{Re}$$

where N_{Re} is the generalized Reynolds number, Eq. B-26, instead of the usual definition of the Reynolds number.

APPENDIX C

EXPERIMENTAL AND CALCULATED DATA

TABLE IV

CONTROL SETTING FOR EACH RUN

Brand	Run	Screw Speed Setting D*	Powerstat Setting B*	Powerstat Setting C*	Ammeter Reading I* Amp.	Robert-Shaw Heater Control Setting F* °F.	Robert-Shaw Heater Control Setting E* °F.
Phillips	2	7.6	67	30	1.0	450	475
	8	7.6	68	60	1.0	450	500
	10	7.6	68	45	1.0	475	525
	12	9.0	68	45	1.0	450	450
	15	9.0	68	47	1.0	450	500
	16	9.0	67	52	1.0	450	525
	18	1.0	68	50	1.0	450	450
	19	1.0	69	50	1.0	450	475
	20	2.0	69	50	1.0	475	450
	21	2.0	68	50	1.0	475	475
	22	2.0	67	50	1.0	475	500
	24	1.0	68	40	1.0	450	440
Spencer	25	7.6	68	40	1.0	450	440
	26	7.6	68	40	1.0	450	470
	27	2.0	68	40	1.0	450	425
	28	7.6	64	40	1.0	450	475
	29	7.6	65	50	1.0	450	525
	30	9.0	65	40	1.0	450	475
	31	9.0	65	42	1.0	450	465
	32	1.0	65	40	1.0	450	450
	33	2.0	65	40	1.0	475	450
	34	9.0	67	50	1.0	475	500
	35	2.0	66	50	1.0	475	500
	36	4.0	64	45	1.0	475	475

* Notation refers to that shown in Figure 9.

TABLE IV (Continued)

CONTROL SETTING FOR EACH RUN

Brand	Run	Screw Speed Setting D*	Powerstat Setting B*	Powerstat Setting C*	Ammeter Reading I* Amp.	Robert-Shaw Heater Control Setting F* °F.	Robert-Shaw Heater Control Setting E* °F.
Spencer	37	7.6	65	45	1.0	475	480
	38	2.0	65	45	1.0	475	450
Dow 550E	39	7.6	68	40	1.0	450	460
	40	9.0	63	40	1.0	450	425
	41	2.0	62	30	1.0	450	400
	42	7.6	63	40	1.0	450	470
	43	9.0	63	45	1.0	450	450
Dow 770	44	2.0	65	45	1.0	450	425
	45	7.6	64	45	1.0	450	490
	46	9.0	64	45	1.0	475	480
	47	2.0	64	45	1.0	500	470
	48	7.6	65	45	1.0	475	525
	49	9.0	64	45	1.0	475	500
	50	2.0	64	45	1.0	475	500
	51	7.6	64	38	1.0	475	475
	52	9.0	62	40	1.0	475	440
Dow 550E	53	2.0	64	35	1.0	475	425
	54	7.6	64	45	1.0	475	490
	55	2.0	64	38	1.0	475	450
	56	9.0	64	42	1.0	475	480

TABLE V

EXPERIMENTAL DATA AT STEADY-STATE

Brand	Run	W lb./hr.	P_1 psig	P_2 psig	T_1* °F.	T_{12}* °F.	T_G* °F.	T_H* °F.	T_J* °F.
Phillips	2	2.25	520	100	328	323	280	395	350
	8	2.42	220	45	355	357	285	430	385
	10	2.47	272	42	364	358	305	455	405
	12	3.52	140	35	321	324	280	390	355
	15	4.03	150	55	365	361	265	420	400
	16	4.07	424	50	382	373	265	440	415
	18	4.83	351	85	339	339	265	380	360
	19	4.98	193	65	357	354	267	405	390
	20	6.29	633	200	346	349	215	385	355
	21	6.57	435	100	366	365	275	405	395
	22	6.83	623	45	383	383	270	415	415
	24	4.71	275	63	328	332	265	365	355
Spencer	25	2.21	572	110	324	330	285	385	350
	26	2.23	640	105	327	331	282	395	365
	27	6.18	1290	190	326	332	265	355	355
	28	2.28	650	100	343	345	305	420	380
	29	2.50	465	70	381	378	300	465	420
	30	2.65	712	115	346	345	300	420	385
	31	3.70	865	135	346	346	285	405	375
	32	4.94	1008	150	346	346	280	395	375
	33	6.30	1160	170	346	349	285	385	360
	34	3.93	623	95	379	377	300	440	410
	35	6.67	877	135	383	382	290	420	405
	36	10.88	1290	180	385	384	300	415	415
	37	2.23	590	90	358	360	330	445	395
	38	6.32	1102	62	364	366	300	415	385

* Notation refers to that shown in Figure 9.

TABLE V (Continued)

EXPERIMENTAL DATA AT STEADY-STATE

Brand	Run	W lb./hr.	P_1 psig	P_2 psig	T_1* °F.	T_{12}* °F.	T_G* °F.	T_H* °F.	T_J* °F.
Dow 550E	39	2.58	788	125	327	329	310	395	360
	40	4.62	1095	170	326	329	305	385	355
	41	7.29	1403	225	324	328	300	360	345
	42	2.83	698	110	344	347	310	375	470
	43	406	869	140	342	344	300	400	370
Dow 770	44	8.05	740	110	341	342	300	380	365
	45	3.09	280	60	363	360	300	435	400
	46	5.24	435	70	363	361	305	420	395
	47	7.88	585	90	365	363	315	420	395
	48	3.77	260	55	387	380	315	450	425
	49	5.36	370	65	384	379	310	435	420
	50	8.31	478	80	388	382	300	420	420
	51	3.31	375	70	342	342	315	415	370
	52	5.21	555	90	343	342	315	405	370
Dow 550E	53	6.44	1150	190	343	343	310	390	365
	54	2.25	555	95	360	358	325	455	400
	55	6.76	1052	170	363	362	310	410	390
	56	3.97	780	130	363	359	320	435	395

Sample Calculations

Flow Index and Consistency Index

The experimental data were put into the form as shown in Table I by the following procedure.

The heated tube had an internal diameter of $\frac{1}{4}$ in., a cross-sectional area of 3.51×10^{-4} ft.^{2}, and it was one foot long. The distance between the pressure gages was 1.42 ft.

$$\frac{D\Delta P}{4L} = \frac{1}{4}\text{in.} \times \frac{1\text{ ft.}}{12\text{ in.}} \times \frac{1}{1.42\text{ ft.}} \times \frac{\Delta P\text{ lb.}}{\text{in.}^2} \times \frac{1}{4}$$

$$\frac{D\Delta P}{4L} = 3.68 \times 10^{-3}\,\Delta P \qquad \text{(C-1)}$$

$$\frac{8V}{D} = 8 \times \frac{V\text{ ft.}}{\text{sec.}} \times \frac{1}{\frac{1}{4}\text{ in.}} \times \frac{12\text{ in.}}{\text{ft.}}$$

$$\frac{8V}{D} = 384\,V \qquad \text{(C-2)}$$

$$V = \frac{W}{\rho A}$$

$$V = W\frac{\text{lb.}}{\text{hr.}} \times \frac{1}{\rho}\frac{\text{cc.}}{\text{gm.}} \times \frac{454\text{ gm.}}{\text{lb.}} \times \frac{1\text{ hr.}}{3600\text{ sec.}} \times \frac{1\text{ in.}^3}{(2.54)^3\text{ cc.}} \times \frac{1\text{ ft.}^3}{1728\text{ in.}^3} \times \frac{1}{3.51 \times 10^{-4}\text{ ft.}^2}$$

$$V = 0.0127\,\frac{W}{\rho} \qquad \text{(C-3)}$$

Substituting Eq. C-3 into Eq. C-2,

$$\frac{8V}{D} = 4.88\,\frac{W}{\rho} \qquad \text{(C-4)}$$

From Table IV Run 31

$W = 3.70$ lb./hr. $\qquad T_1 = 346$ °F.
$P_1 = 865$ psig $\qquad T_2 = 346$ °F.
$P_2 = 135$ psig

The average system temperature would be

$$T = \frac{T_1 + T_2}{2}$$

$$T = 346\ ^{o}F.$$

Then, the density of Spencer's low-density polyethylene from Figure 15 at 346^{o} F. is 0.774 gm./cc. From Appendix F, the pressure correction to P_1 is + 18 psig and the pressure correction to P_2 is + 23 psig. The pressure-drop for Run 31 would be

$$\Delta P = (865 - 18)\ \text{psig} + 15\ \text{psi} - \left[(135 - 23)\ \text{psig} + 15\ \text{psi}\right]$$

$$\Delta P = 740\ \text{psi}$$

Substituting into Eq. C-1

$$\frac{D\Delta P}{4L} = 3.68 \times 10^{-3} \times 740\ \text{psi}$$

$$\frac{D\Delta P}{4L} = 2.71\ \text{psi}$$

Substituting the values of W and ρ into Eq. C-4.

$$\frac{8V}{D} = \frac{4.88\ (3.70)}{0.774}$$

$$\frac{8V}{D} = 23.2\ \text{sec.}^{-1}$$

The calculated data are summarized in Table I.

The calculated data were graphically fitted to the following equation:

$$\ln\left(\frac{D\Delta P}{4L}\right) = n' \ln\left(\frac{8V}{D}\right) + \ln K' \qquad \text{(C-5)}$$

where

$$n' = \frac{\ln\left(\frac{D\Delta P}{4L}\right)_2 - \ln\left(\frac{D\Delta P}{4L}\right)_1}{\ln\left(\frac{8V}{D}\right)_2 - \ln\left(\frac{8V}{D}\right)_1} \qquad \text{(C-6)}$$

From Figure 14 for the Spencer low-density polyethylene at 328° F., chose

$$\left(\frac{D\Delta P}{4L}\right)_2 = 4.11 \ , \ \left(\frac{D\Delta P}{4L}\right)_1 = 2.20$$

$$\left(\frac{8V}{D}\right)_2 = 40 \ , \ \left(\frac{8V}{D}\right)_1 = 15$$

$$n' = \frac{\ln 4.11 - \ln 2.20}{\ln 40 - \ln 15} = \frac{1.41 - 0.79}{3.69 - 2.71}$$

$$n' = 0.64$$

By solving Eq. C-5 for ln K' the consistency index was calculated.

$$\ln K' = \ln\left(\frac{D\Delta P}{4L}\right) - n' \ln\left(\frac{8V}{D}\right) \qquad \text{(C-7)}$$

From Figure 14 using the same plastic and temperature, chose

$$\frac{8V}{D} = 20 \ , \ \frac{D\Delta P}{4L} = 2.65$$

Then

$$\ln K' = \ln 2.65 - 0.64 \ln 20$$

$$\ln K' = -0.945$$

$$K' = 0.39 \ \frac{lb._f}{in.^2} - sec.$$

The consistency indices are shown in Table II.

Friction Factor

The friction factor was calculated using the following equation:

$$f = \tau_w / \rho V^2/2gc \qquad \text{(C-8)}$$

For all of the friction factor calculations, a constant value of 8V/D was used. Using a value of 30, the following procedure was used to make the calculation.

$$\frac{8V}{D} = 30 \ sec.^{-1}$$

Solving for V

$$V = \frac{30D}{8}$$

$$V = \frac{30(\frac{1}{4})\ \text{in.}}{8 \times \frac{12\ \text{in.}}{\text{ft.}}}$$

$$V = 0.0782\ \text{ft./sec.}$$

Substituting the value of V into Eq. C-8

$$f = \tau_w \frac{\text{lb.}_f}{\text{in.}^2} \times \frac{1}{\rho}\frac{\text{cc.}}{\text{gm.}} \times \frac{\text{sec.}^2}{(0.0782)^2\text{ft.}^2} \times$$

$$2 \times 32.2 \frac{\text{lb.}_m\text{-ft.}}{\text{lb.}_f\text{-sec.}^2} \times \frac{1\ \text{in.}^3}{(2.54)^3\ \text{cc.}}$$

$$\times \frac{1\ \text{ft.}}{12\ \text{in.}} \times \frac{\text{lb.}_m}{454\ \text{gm.}}$$

$$f = 2.43 \times 10^4\ \tau_w / \rho \qquad \text{(C-9)}$$

The generalized Reynolds number was

$$N_{Re} = \frac{D^{n'} V^{2-n'} \rho}{g_c K'\ 8^{n-1}}$$

$$N_{Re} = \frac{(1/4 \times 1/2)^{0.64} \times (0.0782)^{2-0.64} \rho}{2 \times 32.2 \times K'\ 8^{0.64-1} \times 144}$$

$$N_{Re} = 7.35 \times 10^{-5} \rho / K' \qquad \text{(C-10)}$$

From Figure 14 at $8V/D = 30$, $\tau_w = 3.42$ psi for Spencer's low-density polyethylene at 328° F.; from Figure 14, the density is 0.779 gm./cc., and from Table II, $K' = 0.39\ \text{lb.}_f/\text{in.}^2\text{-sec.}$

Then

$$f = 2.43 \times 10^4 \times 3.42/0.779$$

$$f = 1.09 \times 10^5$$

and

$$N_{Re} = 7.35 \times 10^{-5} \times \frac{0.779}{0.39}$$

$$N_{Re} = 1.47 \times 10^{-4}$$

The friction factor calculations are summarized in Table VI.

TABLE VI

FRICTION FACTOR CALCULATION

Brand	T °F.	W lb.f/in.	8V/D sec.$^{-1}$	gm./cm^3	K' lb.f/in.^{2}-sec.	N_{Re} 1x10^{-4}	f 1x10^{-4}
Spencer	328	3.42	30	0.779	9.39	1.47	10.60
	346	3.18	30	0.774	0.36	1.58	10.00
	362	2.92	30	0.769	0.33	1.71	9.20
	381	2.18	30	0.764	0.25	2.24	6.94
Dow 550E	328	3.55	30	0.774	0.40	1.42	11.20
	343	3.00	30	0.769	0.34	1.66	9.46
	361	2.76	30	0.763	0.30	1.87	8.78
Dow 770	342	1.60	30	0.765	0.18	3.12	5.08
	362	1.22	30	0.759	0.14	3.98	3.90
	384	0.98	30	0.752	0.11	5.02	3.16

APPENDIX D

DENSITY DETERMINATION

TABLE VII

CALIBRATION OF FLASKS

Flask No.	Weight Flask gm.	Temp. °F.	Weight of Water Plus Flask Filled to Scale Reading 0 gm.	50 gm.	100 gm.	Volume of Flask Scale Reading 0 cc.	50 cc.	100 cc.
I	37.52	90	86.76	91.73	96.65	49.49	54.43	59.37
II	36.49	77	84.46	89.44	94.39	48.12	53.07	58.03
III	35.68	77	85.15	90.12	95.11	49.61	54.57	59.56
IV	37.84	77	85.86	90.85	95.79	48.17	53.20	58.15

TABLE VIII

DENSITY DETERMINATION

PHILLIPS LOW-DENSITY POLYETHYLENE

Flask No. I

Temp. °F.	Scale Reading	Volume of Plastic cc.	Weight of Flask and Plastic gm.	Weight of Plastic gm.	Density gm./cc.
336	8	50.3	76.06	38.54	0.763
636	10	50.5	76.06	38.54	0.758
404	26	52.1	76.06	38.54	0.737

TABLE IX

DENSITY DETERMINATION

DOW 770 LOW-DENSITY POLYETHYLENE

Flask No. II

Temp. °F.	Scale Reading	Volume of Plastic cc.	Weight of Flask and Plastic gm.	Weight of Plastic gm.	Density gm./cc.
270	5	48.62	74.80	38.41	0.788
296	10	49.12	74.80	38.41	0.780
333	17	49.81	74.80	38.41	0.769
372	25	50.60	74.80	38.41	0.756

TABLE X

DENSITY DETERMINATION

SPENCER LOW-DENSITY POLYETHYLENE

Flask No. III

Temp. °F.	Scale Reading	Volume of Plastic cc.	Weight of Flask and Plastic gm.	Weight of Plastic gm.	Density gm./cc.
276	7	50.33	75.64	39.96	0.794
304	13	50.92	75.64	39.96	0.785
316	15	51.12	75.64	39.96	0.782
334	18	51.42	75.64	39.96	0.777

TABLE XI

DENSITY DETERMINATION

DOW 550E LOW-DENSITY POLYETHYLENE

Flask No. IV

Temp. °F.	Scale Reading	Volume of Plastic cc.	Weight of Flask and Plastic gm.	Weight of Plastic gm.	Density gm./cc.
295	8	49.00	76.02	38.18	0.784
314	11	49.30	76.02	38.18	0.774

Sample Calculation

From Table IX at 270 °F., the scale reading for Flask No. II was 5. From Figure 20, the corresponding volume was 48.62 cc. The weight of the plastic in Flask No. II was 38.31 gm.

$$\rho = \frac{38.31}{48.62} \frac{\text{gm.}}{\text{cc.}}$$

$$\rho = 0.788 \frac{\text{gm.}}{\text{cc.}}$$

The density calculations are summarized in Tables VIII, IX, X, and XI.

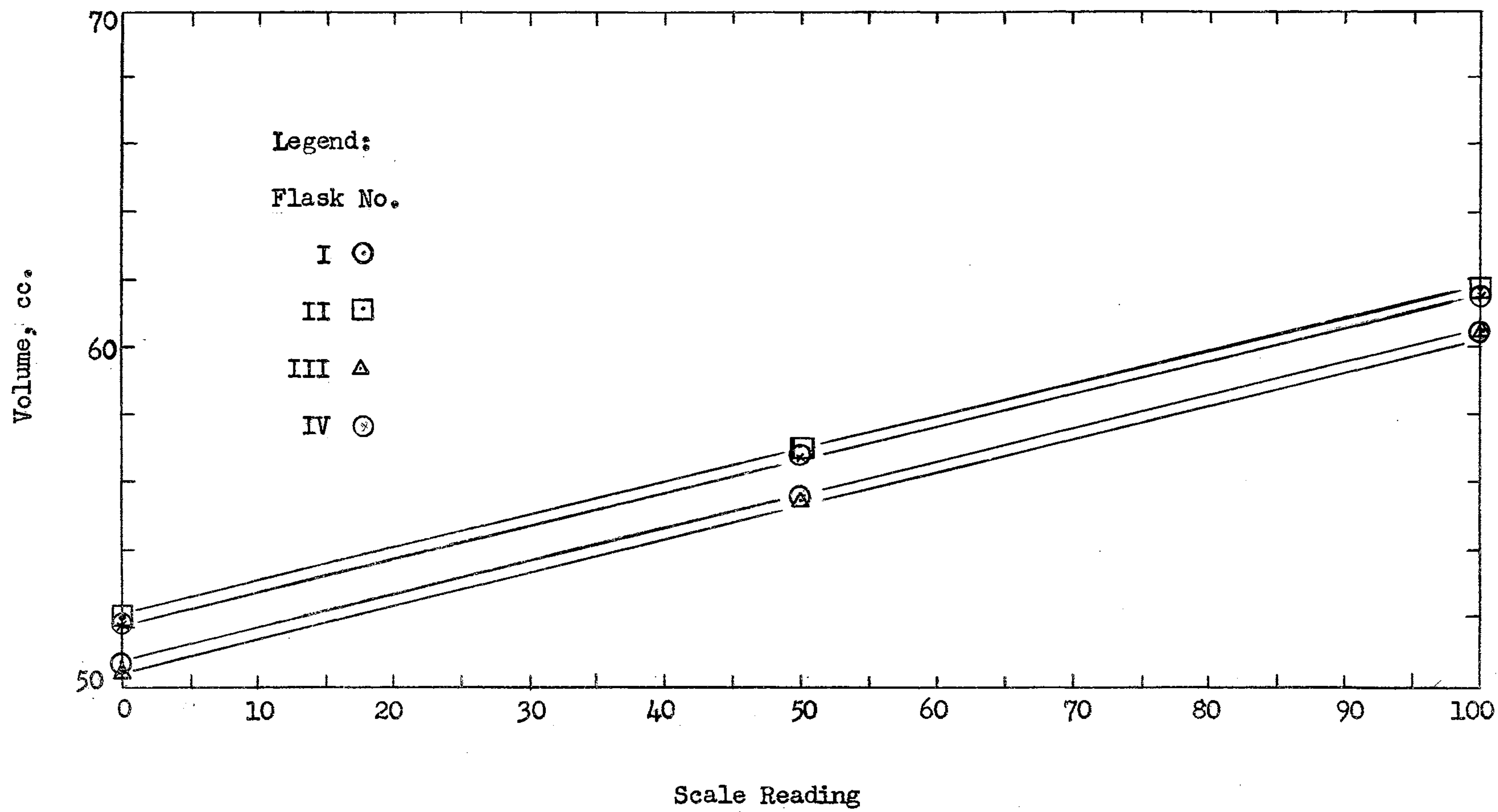

Figure 20. Flask Calibration

APPENDIX E

THERMOCOUPLE CALIBRATION

TABLE XII

THERMOCOUPLE CALIBRATION

T °F.	Emf. mv.	T °F.	Emf. mv.	T °F.	Emf. mv.
458	16.65	388	13.62	330	11.18
428	15.52	316	10.55	464	17.03
408	14.63	458	16.8	386	13.12
293	10.08	320	10.95	458	16.5
275	8.89	458	17.0	408	14.82
246	8.08	404	14.50	360	12.65
508	22.0	358	12.45	320	10.58
458	17.2	318	10.88	408	14.78
424	15.02	364	12.83	402	14.00
438	15.82	350	11.78	393	14.04
392	12.80	343	11.92	322	10.32
308	10.42	290	8.70	456	16.7
458	16.7	403	14.42	406	14.28
328	11.18	350	16.88	288	9.50
406	14.10	462	17.0	362	12.18
402	14.28	324	10.55	346	11.92
459	16.9	298	9.83	422	15.25
440	16.2	345	12.00	385	13.54
460	17.0	350	12.28	344	11.92
465	17.1	304	10.12	395	14.0
456	16.8	345	11.92	390	13.68
302	10.48	335	11.33	468	17.2
456	16.8	408	14.68	395	14.12
314	10.60	336	11.52	458	16.8
465	17.1	394	14.02	402	14.21
352	12.23	342	11.86	460	16.9
464	17.0	346	12.00	395	13.83
288	9.32	338	11.65	455	16.7
463	17.0	394	13.52	386	13.62
350	12.25	298	10.35	260	8.0
449	16.3	463	17.1	398	14.08

Sample Calculation

The thermocouple data in Table XII were fitted to a straight line using the method of the least-mean-squares. The equation of the line in terms of the statistics of the data was

$$T = r_{TE} \frac{\sigma_T}{\sigma_E} (E - \bar{E}) + \bar{T}$$

where

$$r_{TE} = \frac{\sum \frac{TE}{N} - \bar{T}\,\bar{E}}{\sigma_T \sigma_E}$$

$$\sigma_T = \sqrt{\overline{T^2} - (\bar{T})^2}$$

$$\sigma_E = \sqrt{\overline{E^2} - (\bar{E})^2}$$

$$\bar{T} = \frac{\sum T}{N} , \quad E = \frac{\sum E}{N}$$

$$\overline{T^2} = \frac{\sum T^2}{N} , \quad \overline{E^2} = \frac{\sum E^2}{N}$$

The following statistics were calculated from the data:

$$\sum T^2 = 14,483,407$$

$$\sum E^2 = 18,357.17$$

$$\sum TE = 515,038.5$$

$$\sum T = 38,843$$

$$\sum E = 1,302.3$$

$$\bar{T} = 384\ ^{\circ}F.$$

$$\bar{E} = 13.57 \text{ mv.}$$

$$N = 96$$

Then

$$\sigma_T = \sqrt{\frac{14,483,407}{96} - (384)^2}$$

$$\sigma_T = 58.41$$

$$\sigma_E = \sqrt{\frac{18,357.17}{96} - (13.57)^2}$$

$$\sigma_E = 2.66$$

$$r_{TE} = \frac{\frac{515,038.5}{96} - 384(13.57)}{58.41(2.66)}$$

$$r_{TE} = 0.99$$

$$T = 0.99 \left(\frac{58.41}{2.66}\right)(E - 13.57) + 384$$

$$T = 21.74\,E + 89$$

The above equation was used to change the emf-readings to the corresponding temperatures.

APPENDIX F

PRESSURE GAGE CALIBRATION

TABLE XIII

CALIBRATION OF 1500 PSI GAGE

Gage Reading psig	Actual Pressure psig	ΔP psig
235	216.5	18.5
415	394.3	20.7
590	572.2	17.8
770	750.0	20.0
945	927.9	17.1
1120	1105.7	14.3
1300	1283.6	16.4
1475	1461.5	13.5

TABLE XIV

CALIBRATION OF 1000 PSI GAGE

Gage Reading psig	Actual Pressure psig	ΔP psig
60	39.4	20.6
90	67.9	22.1
150	124.7	25.3
200	181.6	18.4
260	238.5	21.5
320	295.4	24.6

The pressure gage calibration was done by Richard Thompson and Phil Tully on the high pressure balance at the School of Chemical Engineering's high-pressure laboratory.

The average pressure correction for the 1500 psi gage was determined to be 18 psig. The average pressure correction for the 1000 psig gage was determined to be 23 psig.

APPENDIX G

DIE TEMPERATURES

TABLE XV

DIE TEMPERATURES (°F.) FOR

VARIOUS PLASTICS

Plastic	Rod Die	Tubing Die	Film Die
Low-density polyethylene	410-450	300-375	300-375
High-density polyethylene	430-480	325-425	325-425
Polyproplyene	430-510	315-450	375-450

VITA

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