PASSIVE-ADAPTIVE COMPENSATION USING A MODEL REFERENCE

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PREFACE

The theory of linear feedback control systems is well documented in the literature. This is the result of a tremendous emphasis on such systems during the postwar years of 1945 through 1955. By the latter date, many techniques of analysis and synthesis had been developed and were in general use in industry. All of these techniques are dependent upon the assumption that the coefficients of the differental equations which described the operation of the systems are constant and well known. This assumption is not generally the case with physical systems, especially if the environmental conditions in which the system is required to operate are variable. For that reason, in the late 1950's the control engineer began searching for methods of control that would enable him to control systems and processes that had variable parameters. The result of this search was a new class of control system that was to eventually be labeled "adaptive".

The primary objective of this thesis is to study in some detail a particular type of adaptive control system. The type to be studied is characterized by the use of a dynamic model of the desired transfer function as the system reference. The resulting system is linear and continuous, and for that reason is classed as passive-adaptive compensation.

A secondary objective of this thesis is to gather into one document some of the ideas of adaptation and to discuss these ideas briefly. This is accomplished in Chapter II. Chapters III and IV are devoted strictly to the passive-adaptive model reference compensation technique mentioned above. Chapter I presents the introductory remarks. Chapter V presents a summary and draws some conclusions from the rest of the thesis.

The author is particularly indebted to Mr. Paul A. McCollum for his assistance, guidance and continuous encouragement during the preparation of this thesis. The author is also indebted to Mr. John Fike for his work with the digital computer computations that were so valuable in the final verification of the theories involved.

An extra special word of thanks goes to my wife Louise for her patience, encouragement, and for typing the original draft.

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INTRODUCTION

Generally, the synthesis problem as applied to automatic control systems can be stated in the following manner. Given a dynamic system, plant, or process that does not yield the desired performance characteristics when operating alone, design a controller(s) and/or compensating device(s) which will cause the resulting system (original system plus controller or compensating device) to yield the desired performance characteristics.

If the original system is linear and its parameters are well known or can be accurately determined then any one of several system synthesis procedures are applicable. All of these synthesis procedures assume one of two prominent forms of compensation. These are cascade and feedback compensation. Both require feedback. The former type places the compensating device in cascade or, as it is sometimes referred to, in series with the plant in the forward transmission path of the system. Feedback compensation places the compensating device in the feedback transmission path. It is sometimes referred to as parallel compensation.

These compensation techniques have been investigated extensively and are well documented in the literature [1, 2, 3]. However, generally the problem is not nearly as straight forward as these defined compensation procedures would imply. For example, there exists, in most physical plants or systems several problems which the cascade and feedback compensation procedures generally ignore. Some of these problems are discussed in the following paragraphs.

First, the system, plant, or process to be controlled is at least partially specified and is not available for adjustment or redesign. This may be the result of technical or economical reasons. This is not a particularly severe problem if the characteristics of the plant are such that one can realize the desired response using the aforementioned compensation techniques.

Secondly, the parameters which describe the transfer characteristics of the plant or system to be controlled are not accurately known. This problem places a definite limitation on the linear compensating techniques mentioned above in that most of the procedures for realizing the desired compensating devices are based on the assumption that the characteristics of the plant are accurately known.

The fact that the properties of the input signal or signals are not always well known also presents some problems in so far as compensating procedures are concerned.

However, the problem area that is of most importance here is the fact that the parameters which describe the characteristics of the system, plant, or process to be controlled are subject to variations during the normal course of system operations. This is true of many physical systems. Generally these variations are due to the changing environment in which the system is required to operate.

A particularly good example of a physical device which exhibits all these problem conditions is the high performance aircraft or missile. In either case the control engineer has little or no control over the physical configuration of the airframe as this is dictated by the aeronautical specifications. Thus, the parameters of the device to be controlled are fixed and not available for adjustment. Generally, the coefficients of the differential equations which describe the motion of the airframe are not well known, at least during the earlier stages of development. Furthermore, these coefficients are functions of the environmental variables such as Mach number, altitude, location of the center-of-gravity, mass, and any number of other factors that normally vary in some manner during flight. It is not uncommon for a particular coefficient to change by a factor of 10 or even 100 during the course of a normal flight. Obviously, a compensating device designed to give a satisfactory performance at the nominal value of such a factor cannot be expected to provide the same performance at some other condition.

As a result of the many problems encountered in the control of systems and processes similiar to the airplane in complexity, the control engineer has found it necessary to resort to techniques of compensation other than those found in conventional texts on automatic feedback control systems. The result of this has been the development of a new class of systems that have been labeled "adaptive" in the literature. This terminology appears to be due to Drenick and Shahbender [4]. It is the intention of this thesis to examine in some detail a particular class of these adaptive control systems and to present a particular configuration which will be shown to provide excellent transient and steady-state performance characteristics for a second order plant whose parameters are subject to large and rapid variations. The unique feature of this configuration is the use of a dynamic model of the desired system transfer characteristics as the system reference.

The next chapter shall be devoted to a brief survey of the available literature on the subject of adaptive control systems. Some of the terminology of such systems is presented and a classification technique that has been proposed in the literature is used as a framework for discussing the ideas of adaptation.

CHAPTER II

GENERAL BACKGROUND - ADAPTIVE CONTROL SYSTEMS

At this writing, a precise definition of adaptive control does not exist. The term "adaptation" was borrowed from the biological sciences where it is defined as a "modification of an animal or plant (or of its parts or organs) fitting it more perfectly for existence under the conditions of its environment."[1] Thus, if one applies the literal biological definition of the term adaptation to the problem of giving a general definition to the phrase "adaptive control system" one automatically implies some sort of modification or adjustment to the system. These modifications or adjustments are designed to fit the system for operations in a changing environment.

Several definitions have been suggested in the literature. To illustrate the wide variation in concepts that have resulted since the introduction of the term "adaptive control" the following representative definitions are quoted from the literature:

> Adaptive control is a method of control aimed at obtaining optimum system performance even when there exists incomplete or inexact analytical or analog model of the process that is being controlled.[5]

> The term adaptive will be applied to any control system which measures continuously or intermittently, the impulse response or some other function which characterizes the system and which makes

[1]"Adaptation", Webster's New Collegiate Dictionary, G. & C. Merriam Co., Publishers, (Springfield, Massachusetts, 1956), p. 10.

use of this system characteristic function to determine and to generate the necessary forcing function to cause the system to behave in a desired manner.[6]

A feedback control system is adaptive if the sensitivity with respect to a variable x is zero over an interval in x of non-zero magnitude.[7]

An adaptive system is any physical system which has been designed with an adaptive viewpoint.[8]

Of these listed definitions, only the last seems to qualify as a general definition of the word adaptation. Consequently, a classical linear feedback control system such as is illustrated in Figure 1 qualifies as an adaptive control system. To see that this is true consider the effects on the plant driving signal, e(t), if suddenly the output signal, c(t), is not equal to the input signal, r(t), as the result of parameter variation in the plant. The error signal, e(t), is modified or adjusted accordingly, so as to fit the overall system more

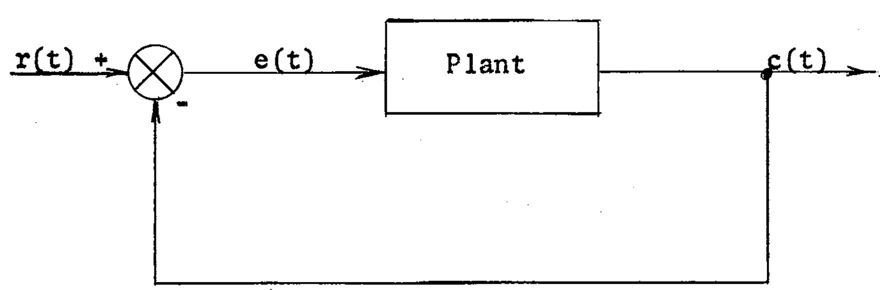


Figure 1. A Feedback Control System

perfectly for its operating environment. Although this is a somewhat simplified example, it does serve as an illustration of the basic definition of adaptation. It can be concluded that any control system that exhibits an ability to adjust to a changing environment is adaptive, whether that control system is a simple feedback control system as illustrated above or one of the more complicated structures discussed later in this chapter.

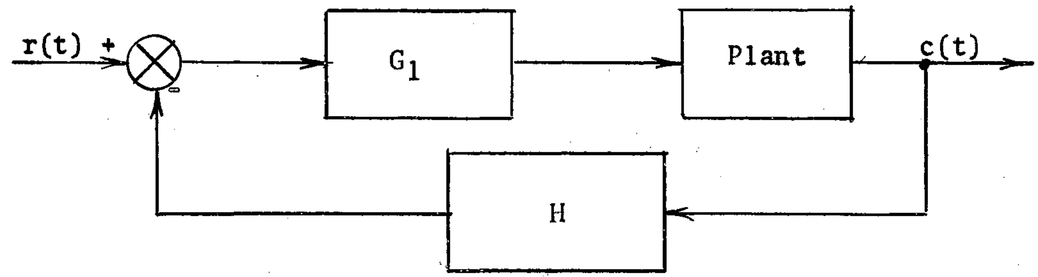
Classification of Adaptive Control Systems

In a paper that has become the byword in so far as terminology is concerned, Aseltine, Mancini, and Sarture [9] attempted to classify the approaches that have been taken in the design of adaptive control systems. Although this classification technique does not always yield a unique classification for any particular system, it does provide a framework for discussion. These classifications and a brief discussion of each are presented in the next several paragraphs.

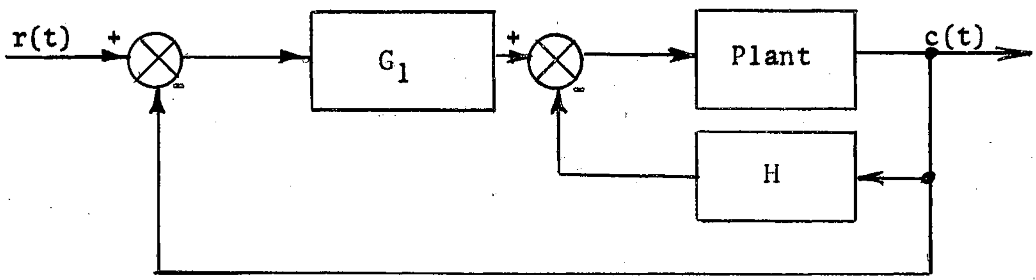
Passive Adaptation. A passive-adaptive system is one that achieves adaptation without system parameter changes. This class of system is adaptive in that it is designed to operate satisfactorily over wide variations in the environmental conditions.

A good example of a passive-adaptive control system is the simple feedback control system illustrated in Figure 1. The original justification for employing feedback in any control system has to do with rendering the system less sensitive to variation in environmental conditions and to improving the system's ability to reject corrupting signals [10]. Consequently, any system which utilizes conventional feedback is adaptive and furthermore can be classified as a passive-adaptive control system. In a sense, passive-adaptive control is more of a compensating technique than a philosophy of control.

There are certain special feedback configurations which display inherent adaptive behavior of a higher degree than the conventional feedback control system illustrated in Figure 1. A pair of such feedback configurations are shown in Figure 2. These special configurations generally achieve their higher degrees of adaptation as a result of the fact



(a)



(b)

Figure 2. Two Feedback Control Systems With Two Degrees of Freedom

that they exhibit two or more degrees of freedom. By the term, two or more degrees of freedom, it is meant that the system designer has the freedom to select two or more independent transfer functions. In the illustrated cases, the designer is free to select G_1 and H and thus has two degrees of freedom. Horowitz [10] has shown that by utilizing a configuration such as those illustrated in Figure 2 and a high loop gain one can control both the system's overall transfer characteristics as well as the system's sensitivity to plant parameter variations.

This class of adaptive control scheme appears to have its greatest potential in systems which involve plants whose parameters vary rapidly compared to the nominal time constants of the system. For that reason a particular subclass of this classification was selected for special study in this thesis. That subclass is characterized by the use of a model of the desired transfer function as the system reference. A more detailed discussion of this type of passive-adaptive control system is presented in Chapters III and IV of this thesis.

Input Signal Adaptation. This class of adaptive control system includes those systems which adjust their parameters in accordance with the input signal characteristics. The essential components of such a system are shown in Figure 3. It is seen that two elements, in addition to the plant, are required. These elements are (1) the input measuring device and (2) the controller.

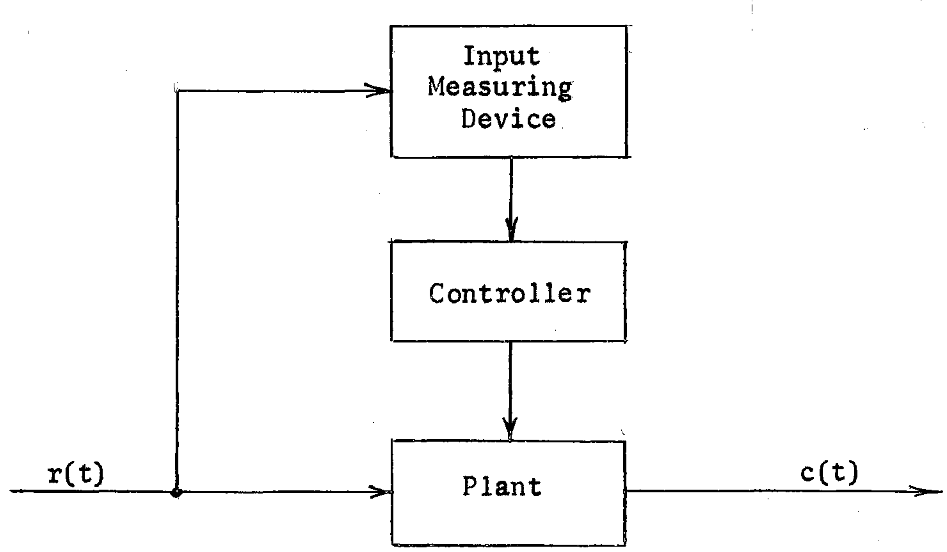


Figure 3. A Generalized Input Sensing Adaptive Control System

The essential nature of such systems can be described in the following manner. The input measuring device monitors the input signal and based on prior knowledge of the plant transfer characteristics causes the controller to adjust the parameters of the plant according to some predetermined function. Several criteria for adjustment have been suggested. Two of the more prominent ones are (1) minimization of the mean-square error between the input signal and the system output in the presence of noise of known statistical characteristics [11] and (2) minimization of the mean-square noise plus squared dynamic error signal [4].

This class of adaptive control system does not consider cases where the plant parameters are subject to variation, and at the same time assumes some prior knowledge of the plant transfer characteristics. As pointed out previously, these are two of the basic problems which caused the control engineer to resort to adaptive control techniques. In that sense, the applicability of input signal adaptation techniques appears to be limited, particularly where plant parameter variations are concerned.

Extremum Adaptation. This classification includes those systems which are designed to operate at or near an extremum of some system variable. The extremum may be either a maximum or a minimum. The variable to be automatically maximized or minimized may be of any variety such as output torque, fuel consumption, or even some artificially derived figure of merit. As pointed out by Cosgriff and Emerling [12] this class of control system differs considerably from the more conventional types where the optimum time response to a specific class of input signals is the ultimate matter of interest. In an extremum adaptive control system the ultimate item of concern is whether or not the variables of interest are maximum or minimum regardless of the character of the input signal.

Draper and Li [9, 13, 14] have proposed four varieties of extremum adaptive systems. The proposed systems of these authors are all based on the concept of dynamically adjusting the controlled system parameters in such a manner as to achieve an extreme condition for a particular system variable. These proposed methods are generally classified according to the manner in which the parameter adjustment signal is derived. The proposed methods are (1) generation of a constant adjustment rate, the sign of which is determined by the sign of the time rate of change of the variable to be optimized, (2) an adjustment technique that is proportional to the response of the plant to a sinusoidal test disturbance signal, (3) an adjustment signal that is proportional to the rate of change of the variable to be optimized with respect to adjustment, and (4) an adjustment signal that is proportional to the difference between the present output and the extremum condition that the variable has held in the past. All of these cases have a common deficiency in that a type of hunting operation is achieved. Furthermore, since the optimum condition is an extremum there are many control applications wherein this feature is not desirable. One example of such an application is the one in which this thesis is most concerned. That application is the problem of insuring satisfactory transient response in the face of gross plant parameter variations.

System Variable Adaptation. This class of adaptive control system includes those systems which base self-adjustment on measurements of system variables such as the output or error signals. These measurements of system variables may consist of magnitude sensing, derivative sensing, algebraic sign sensing, or any other function of a system variable that is useful in achieving the desired system performance. At any rate, certain system control parameters are adjusted either continuously or intermittently according to the characteristics of the system variable being used to actuate the adaptive processes.

A particularly good example of this class of adaptive control that uses intermittent adjustment has been presented by Flugg-Lotz and Taylor [15, 16]. This control system is illustrated in block diagram form in Figure 4.

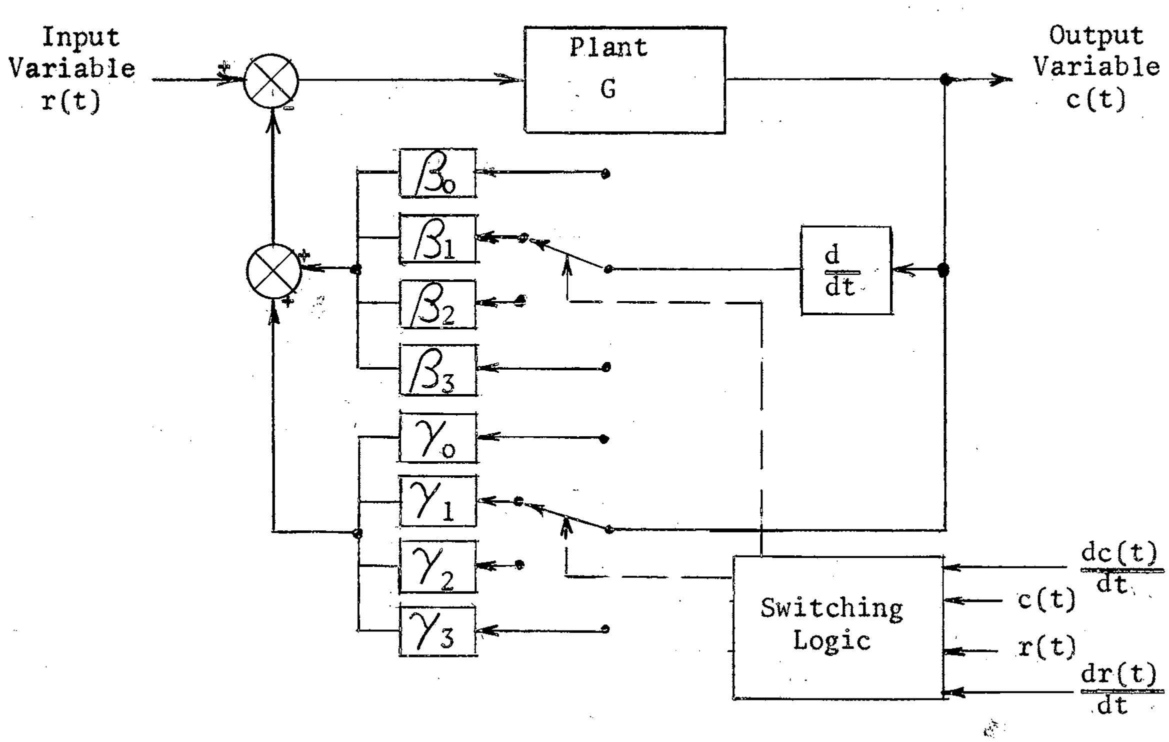


Figure 4. A System Variable Adaptive Control System That Employs Intermittent Adjustment

The essential features of this system are as follows: The plant or controlled system, G, has two feedback loops closed about it. One of these control loops provides a feedback signal that is proportional to the output signal. The other control loop provides a feedback signal that is proportional to the derivative of the output signal. The gains of these two feedback loops are discretely selected by means of logic circuitry that monitors the output and input signals as well as the derivatives of these signals. The feedback gains at any instant are determined by the relative signs of the variables mentioned above.

A good example of a system variable adaptive technique that utilizes semicontinuous system adjustment has been proposed by Whitaker, et al., [17, 18]. This control system is shown in block diagram form in Figure 5. The essential component in the system is the dynamic model that is used to establish the system reference. This model is constructed of conventional analog components and is designed so as to provide the desired dynamic response when subjected to the same input signal as the basic control system. The actual output of the system is compared to the model output thus generating an error signal that is some measure of the difference between the model transfer characteristics and the transfer characteristics of the basic control system. This error is used to analyze the performance of the system and to determine appropriate corrective adjustments for the controller. In the original work Whitaker, et al., adjusted the controller parameters so as to minimize a particular error function over a particular sampling interval such as the rise time or settling time of the model. It was necessary to exercise care in the selection of the proper error function to minimize in controlling a particular system variable. However, it has been demonstrated that such a

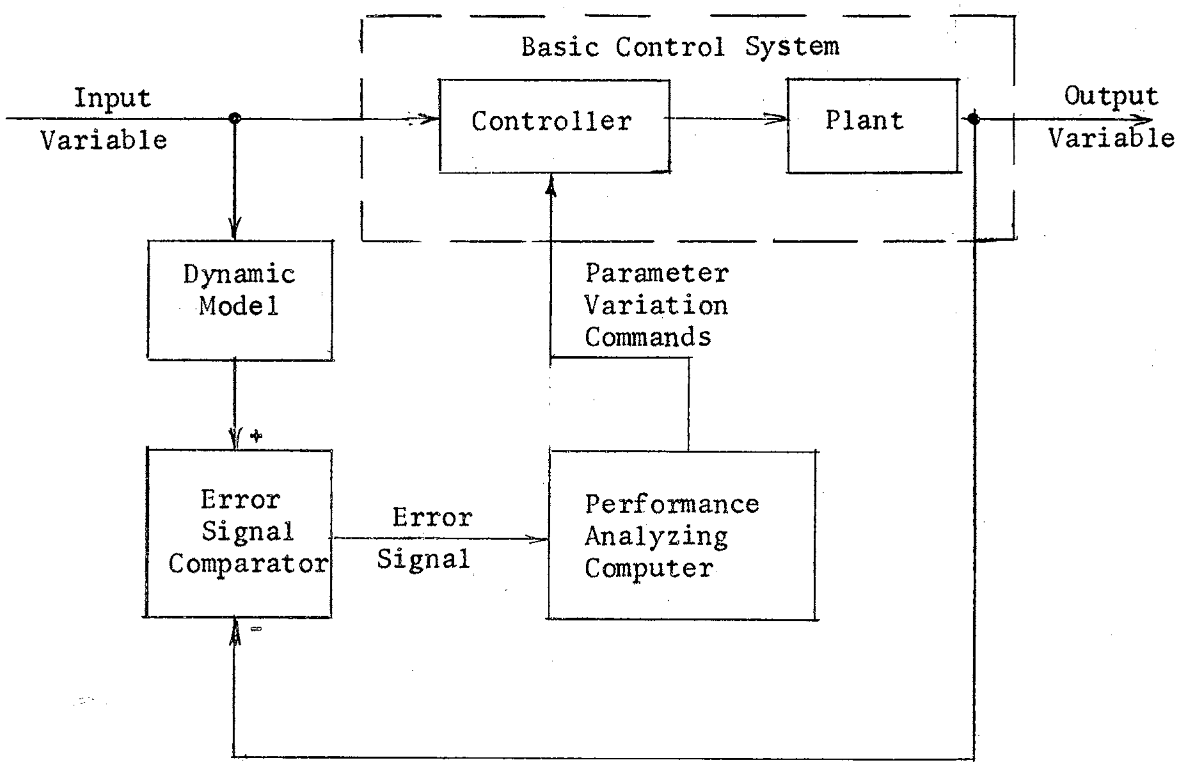


Figure 5. A Model Reference System Variable Adaptive Control System

system is feasible. It has a basic disadvantage that in general a time delay equal to the sampling interval must take place before any corrective action can take place. Even then the corrective action can only start to occur since more sampling intervals must lapse before the error function can be minimized. A type of hunting operation would occur if the basic control system were subjected to rapidly changing parameters.

With these example systems in mind, certain observations can be made concerning system variable adaptive techniques. This class of adaptive control appears to have some promise in so far as finding a solution to the problem of achieving satisfactory transient response with a plant that exhibits grossly varying parameters. The major disadvantages appear to be in the relative complexity that is involved and in determining the variables and criteria that can be used to achieve a particular transient response. Still another disadvantage appears to be the need for dynamic adjustment of controller parameters. This presents an implementation problem.

System Characteristic Adaptation. This class of adaptive control system derives self-adjustment signals from measurements of its own characteristics. By characteristics it is meant the coefficients of the describing differential equations or some other quantity such as second order damping ratio and natural frequency. Quantities such as these completely determine the system response to a particular class of signals. Several ingenious techniques for determining the characteristics of a basic second order system are included in the literature. Some of these techniques are described briefly in the following paragraphs.

Anderson, et al., [19], have described a system in which the impulse response of the plant is measured and used to generate a figure of merit that is a function of the damping ratio of a second order system. This is illustrated in Figure 6. The output of the figure of merit computer is used to adjust the controller parameters so as to achieve the desired value for the figure of merit and in turn the damping ratio. The impulse response of the system is determined by cross-correlation of the white noise input and the resulting plant output as described in [20]. This cross-correlation involves time delay and integration of the product of two functions and is exceedingly difficult to implement in a physical system. Furthermore, the integration time can be excessive as compared to the rate at which the plant parameters might vary. For this reason

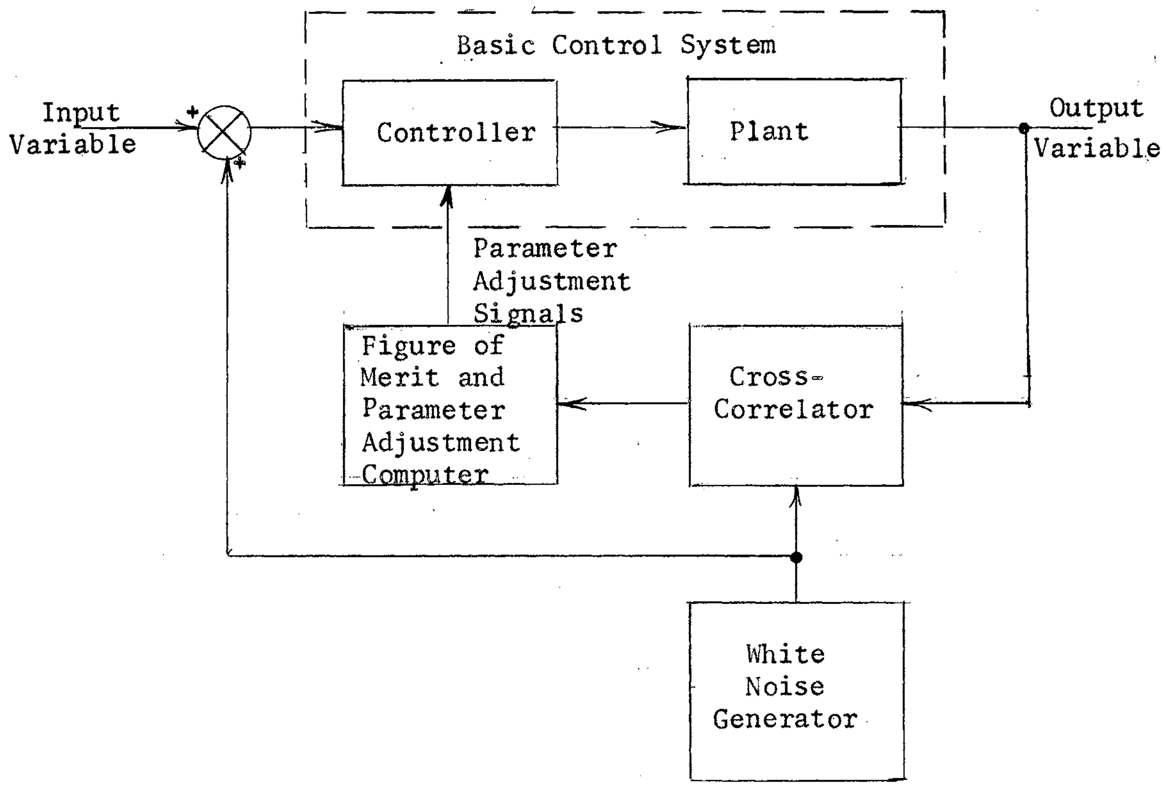


Figure 6. An Impulse Response Adaptive Control System

and the fact that an auxiliary test signal consisting of a low level white noise input is necessary this technique of system measurement is certainly limited.

Several authors [6, 7, 21], have suggested the use of computers in system characteristic adaptation processes. In each case the computer is utilized to determine the system characteristics from certain observations of the input and output signals. These are basically sophisticated techniques for determining the impulse response of the system and for that reason react very much like the example given above.

Advantages and Disadvantages of Adaptive Control

Obviously, the major advantage of the various techniques by which a type of adaptive control can be achieved is the fact that relatively tight specifications on a systems performance are feasible in spite of changing environmental conditions. Whether or not this advantage offsets the major disadvantage of increased complexity and reduced reliability is of course dependent upon the performance and operational requirements. However, one observation does seem appropriate here. That is, until less complicated techniques of system measurement and parameter adjustment becomes available, it does not seem feasible to utilize any of the last four classifications in a control system in which the plant parameters are subject to large and rapid variations. It seems far more reasonable to use one of the passive-adaptive techniques of the first classification in that sort of application. There are no delays for system measurements or identification and no requirement for adjustment of controller parameters. The reduction in complexity seems to far offset the disadvantage of high loop gains. With this in mind a particularly interesting subclass of the passive-adaptive type control system was selected for a more detailed study. The ultimate objective being to find a solution to the problem of providing a satisfactory transient performance when confronted with a plant displaying large and rapid variations in its parameters. The most unique feature of this system to be studied is the fact that the system employs a model of the desired system as a reference.

CHAPTER III

PASSIVE-ADAPTIVE COMPENSATION USING A MODEL REFERENCE

It was stated in Chapter II that the control engineer's problems are generally compounded by certain factors that are related to the parameters of the system or plant to be controlled. One of these factors is the variation of these parameters with environmental conditions such as temperature, atmospheric pressure, humidity, velocity, etc. It was because of problems such as this one that the control engineer resorted to the application of adaptive control techniques such as those described in Chapter II of this thesis. Most of these techniques employ elaborate and complicated methods of measuring the variations in the parameters of the plant. Once the variation in the parameters has been determined the problem of compensation must still be solved. Again, most of the truly adaptive control methods use complicated methods of changing the parameters in the controller or computing new input signals which cause the system to react in a specified manner. If the variation in system parameters is rapid, such complicated techniques of system measurement and implementation could not hope to be adequate. For this reason it is sometimes necessary to resort to techniques of control which mask or hide the effects of the parameter variation in so far as the output of the controlled system is concerned. Such a technique of control is termed passive-adaptive compensation. It is the purpose of this chapter to present a technique of passive-adaptive compensation which utilizes a mathematical model of the desired transfer function as the system reference. The concept of using such a model in a control system was first introduced by Ham and Lang [22]. These authors referred to this method of control as "conditional feedback" and used it to compensate for the effects of a pure time delay (transport lag) in the plant or controlled system.

Before proceeding with the development of the passive-adaptive model reference control system, a discussion of the effects of varying parameters in a simple unity feedback control system will be presented.

Effects of Varying Parameters

It has been stated that one of the reasons for employing feedback in a control system is to reduce the sensitivity of the system to variations in the parameters of the plant or controlled system. Although feedback will reduce the effects of varying parameters it will be shown that the conventional unity feedback that is so extensively discussed in the literature is inadequate in the face of large plant parameter variations. This is particularly true if stringent transient specifications are placed on the system's performance. To illustrate the effects of large parameter variations on the transient performance of a simple unity feedback control system consider the system shown in Figure 7.

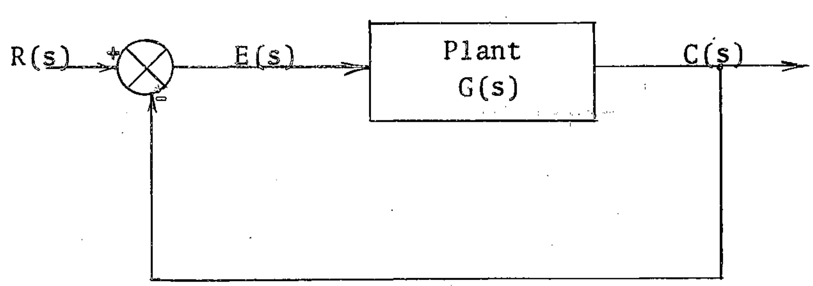


Figure 7. A Unity Feedback Control System

The symbols of Figure 7 are defined as follows:

C(s) = the Laplace transform of the controlled variable, c(t).

R(s) = the Laplace transform of the reference input, r(t).

E(s) = the Laplace transform of the actuating or error signal, e(t).

G(s) = the transfer function of the plant expressed as a function of the complex frequency variable $s = \sigma + j\omega$.

The closed loop transfer function for the system in Figure 7 can be written

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} \text{ .} \qquad \text{III-1}$$

For this example let the plant transfer function be written

$$G(s) = \frac{\beta^2}{S(S + 2\alpha)} \qquad \text{III-2}$$

where α and β are the parameters of the plant. It is the effects of varying these quantities that is to be investigated here. Substituting Equation III-2 into Equation III-1 yields

$$T(s) = \frac{\beta^2}{S^2 + 2\alpha S + \beta^2} \text{ .} \qquad \text{III-3}$$

The justification for using such a simple plant transfer function in this example is now apparent. The closed loop transfer function of Equation III-3 has a second order polynomial in s for a denominator. Many physical systems exhibit transfer functions which are either second order or have dominant second order factors. Thus the results of this illustrative example are representative of many actual systems.

The ultimate item of interest is, of course, the resulting time response of the system controlled variable, c(t), since a system's performance is generally judged according to a set of time domain specifications. However, Laplace transform theory shows that this time domain response, c(t), is completely characterized by the singularities of C(s), where C(s) represents the Laplace transform of the time function c(t). Consequently, it is possible to determine the time response of the system of Figure 7 from a knowledge of the singularities of Equation III-1 and the characteristics of the system input function. For this reason the control engineer has become dependent upon information which relates the location of the singularities of the system transfer function to the parameters in question. With that in mind the effects of varying plant parameters α and β shall be investigated in terms of the resultant effects on the location of the singularities of Equation III-3.

In this case, the complex frequency domain singularities of interest are the zeros of the equation

$$S^2 + 2\alpha S + \beta^2 = 0. \qquad \text{III-4}$$

This equation has zeros located at

$$S_1 = -\alpha + \sqrt{\alpha^2 - \beta^2} \qquad \text{III-5}$$

and

$$S_2 = -\alpha - \sqrt{\alpha^2 - \beta^2} . \qquad \text{III-6}$$

In order to illustrate the effects of the varying parameters, consider the case where α is held constant and β is allowed to vary. This case is shown graphically in Figure 8. It will be noted that this is the familiar root locus of Evans [23] in which the variable parameter is the system open loop gain. Here the locus of constant α lies along the axis of reals between s = 0 and s = -2α and along a vertical line that is defined by the equation

$$R_e(s) = -\alpha \quad . \qquad \text{III-7}$$

The break away from the axis of reals occurs when

$$\beta^2 = \alpha^2 \qquad \text{III-8}$$

The case where β is held constant and α is allowed to vary is generally not as familiar as the preceding case. This case is illustrated in Figure 9 where it is seen that the locus of constant β lies along the negative axis of reals for sufficiently large α and along a circle defined by the equation

$$|s| = \beta \qquad \text{III-9}$$

for

$$\alpha^2 \leq \beta^2 \quad . \qquad \text{III-10}$$

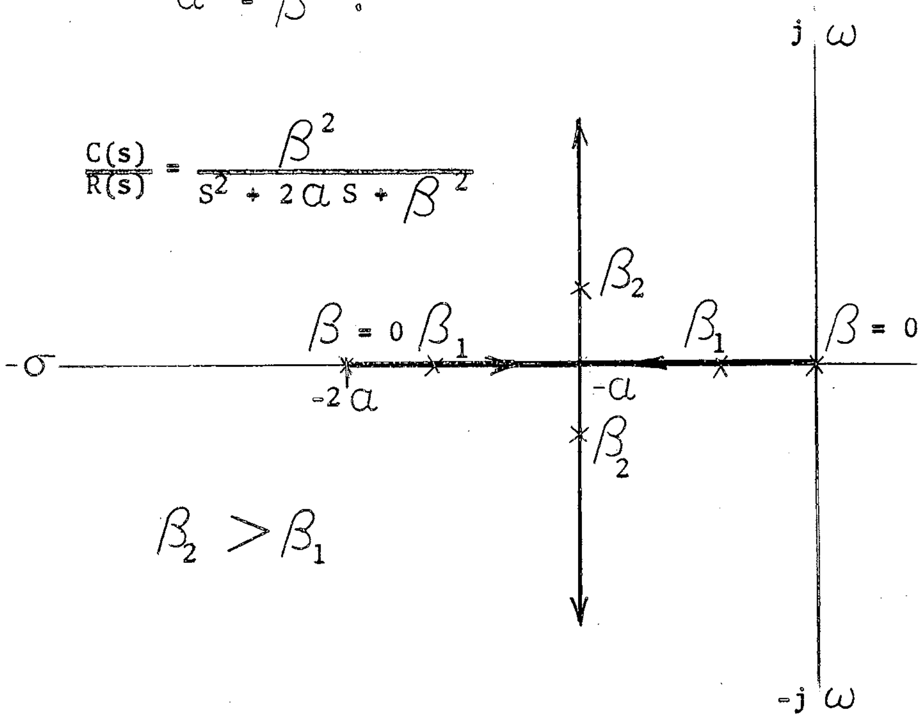


Figure 8. Locus of Singularities of a Second Order System as β Varies

If one ignores the parts of the two loci that lie along the negative real axis in Figures 8 and 9, a composite sketch may be derived. This case is shown in Figure 10 where the vertical lines correspond to constant α and the semi-circles correspond to constant β. Only stable cases are shown (no positive real parts). Thus, when one specifies the ranges of values that the parameters of the second order system may take

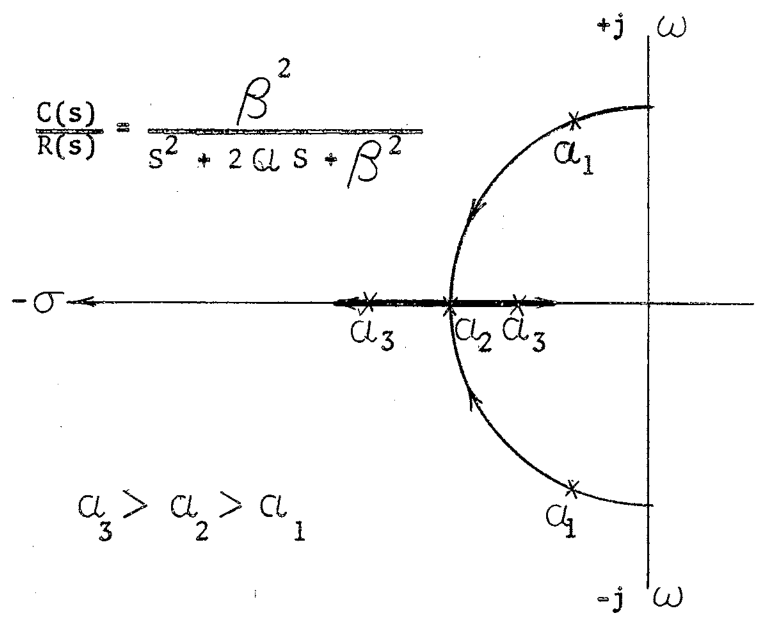


Figure 9. Locus of Singularities of a Second Order System with Variable α

on, one is actually specifying a region in the complex frequency domain in which the singularities of the closed loop system can exist as the result of the allowable variation in the plant parameters. The shaded area of Figure 10 is such a region. Before one can realize the full meaning of this, a review of the time response in terms of the location of the complex conjugate poles of a linear second order system is in order. Figure 11 illustrates a typical case where the system transfer function is

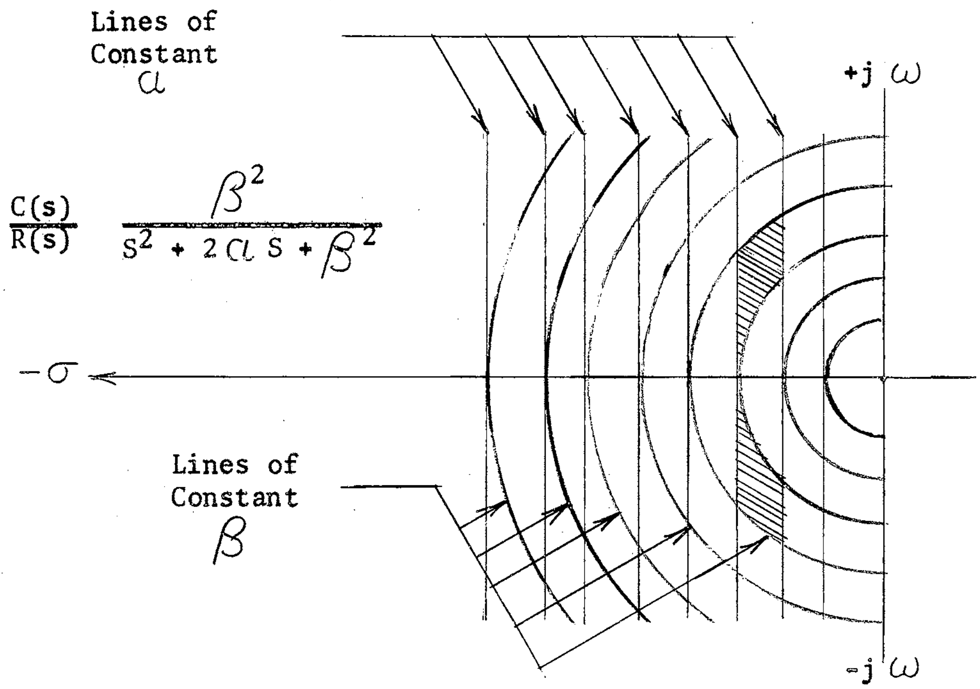


Figure 10. Composite Locus of the Singularities of a Second Order System Having Variable Parameters α and β.

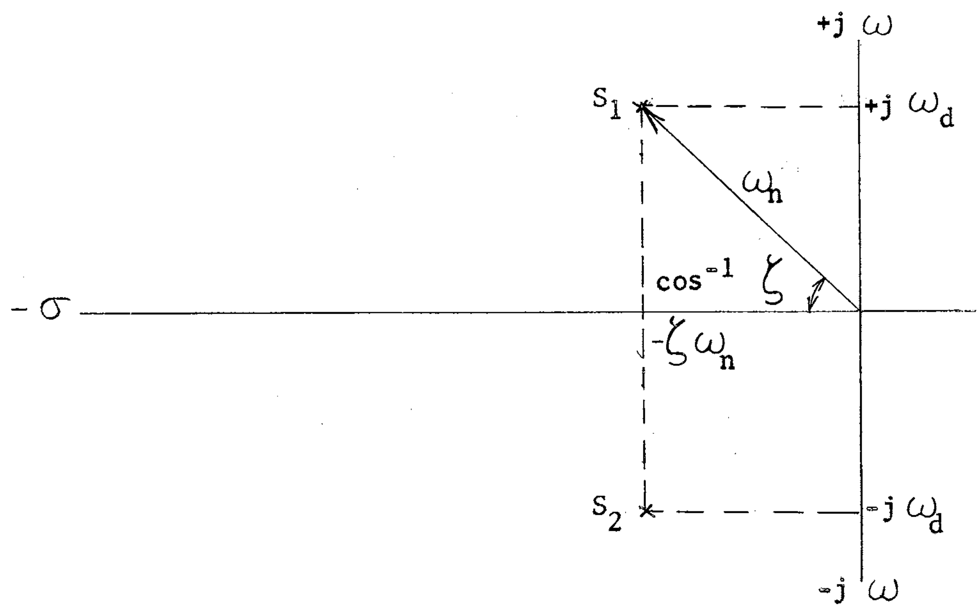


Figure 11. Significance of Pole Location Second Order System

given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{S^2 + 2\zeta\,\omega_n S + \omega_n^2} \quad . \qquad \text{III-11}$$

The corresponding time domain response is given by

$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}}\, \epsilon^{-\zeta\omega_n t} \operatorname{Sin}\left(\omega_d t + \operatorname{Cos}^{-1}\zeta\right) \qquad \text{III-12}$$

where

ζ = damping ratio

ω_n = undamped natural frequency

$\omega_d = \omega_n\sqrt{1-\zeta^2}$ = damped natural frequency.

On comparing Equations III-11 and III-3 it is seen that

$$\omega_n = \beta \qquad \text{III-13}$$

$$\zeta = \alpha/\beta \qquad \text{III-14}$$

$$\omega_d = \sqrt{\beta^2 - \alpha^2} \qquad \text{III-15}$$

To further illustrate the significance of varying parameters in so far as a second order system's transient performance is concerned, consider the example case where the shaded area of Figure 10 is defined by the equations

$$0.707 \leq \alpha \leq 7.07 \qquad \text{III-16}$$

$$\beta^2 = 50 = \text{Constant.} \qquad \text{III-17}$$

On evaluating Equation III-14 at both extremities of Equation III-16, the range of values of the damping ratio is seen to be

$$0.1 \leq \zeta \leq 1.0 \quad . \qquad \text{III-18}$$

Figure 12 shows the resulting time response for a unit step function input signal for several values of damping ratio in the range indicated by Equation III-18. It is obvious that in any application where the overshoot, rise time, and solution time specifications are stringent, then the system defined by Equations III-3, III-16, and III-17, and illustrated in Figure 7 is certainly unsatisfactory.

The next section of this chapter deals with a linear model reference control system that shows some promise in solving the problem of varying parameters.

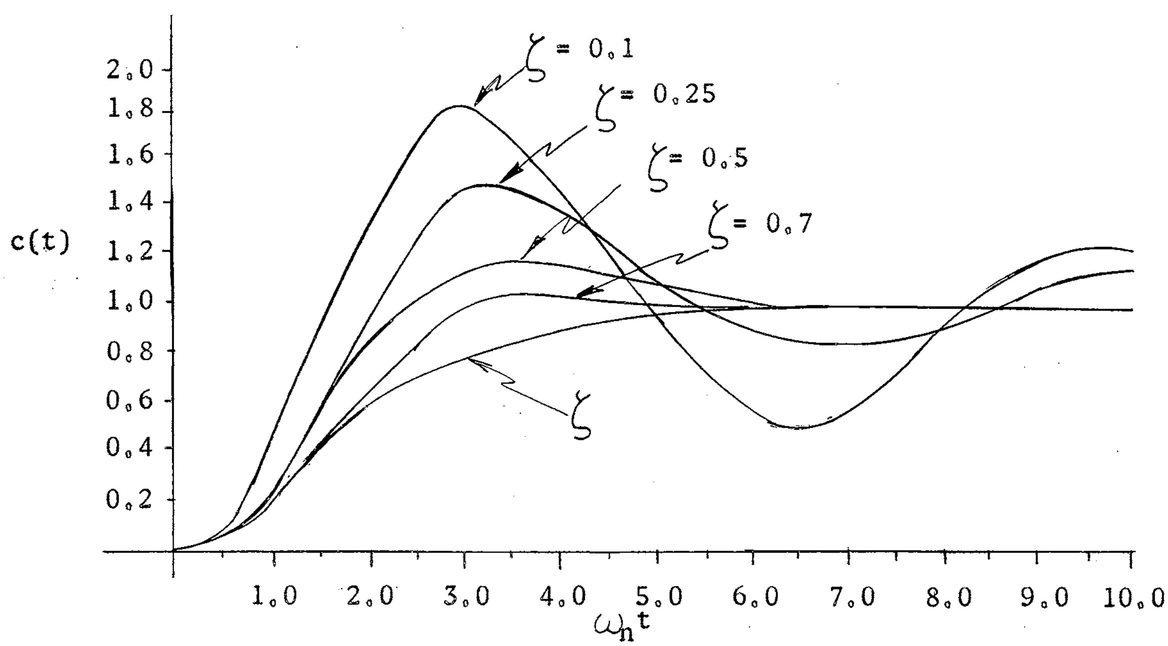


Figure 12. Second Order Transient Response to Step Function

A Passive-Adaptive Model Reference Compensation Technique

In this section, a proposed solution to one of the basic control system problems is presented. This basic problem of control can be stated as follows:

> Given a fixed portion of a system which has parameters that are functions of external variable environmental conditions, devise a compensating technique which will yield a system whose transient performance is essentially insentitive to these parameter variations.

The proposed solution to this basic problem requires that certain assumptions and limitations be placed on the characteristics of the desired system as well as on the form of the transfer characteristics of the plant. These assumptions and limitations are as follows:

1. It is assumed that the overall desired system transfer function, $T_d(s)$, is known or at least can be determined from the system specifications.

2. It is assumed that $T_d(s)$ is of the form

$$T_d(s) = \frac{C(s)}{R(s)} = \frac{K_d}{S^n + a_{n-1} S^{n-1} + \cdots + a_o} \qquad \text{III-19}$$

which can be written

$$T_d(s) = \frac{C(s)}{R(s)} = \frac{K_d}{S^n + \sum_{=0}^{n-1} a_i S^i} \qquad \text{III-20}$$

where

K_d = desired system gain constant

a_i = coefficient of the i^{th} term of the desired characteristic equation.

The symbols C(s) and R(s) are the same as those previously defined. It will be noted that Equation III-20 allows the system characteristic equation to be of any order theoretically. However, there are some practical limitations as will be pointed out later in this chapter.

3. It is assumed that the plant or fixed components of the system to be controlled has a transfer characteristic of the form

$$G(s) = \frac{K_a}{S^m + \gamma_{m-1} S^{m-1} + \dots + \gamma_o} \qquad \text{III-21}$$

which can be written

$$G(s) = \frac{K_a}{S^m + \sum_{i=o}^{m-1} \gamma_i S^i} \qquad \text{III-22}$$

where

$G(s)$ = plant transfer function expressed as a function of the complex frequency variable S

K_a = plant gain constant

γ_i = coefficient of the i^{th} term of the plant's characteristic equation.

The parameters of the plant are therefore K_a and the γ_i of the plant transfer characteristics. For the purpose of these works, K_a shall be assumed to be invariant with the external environmetal conditions while the γ_i terms shall be considered as variables.

4. It will be assumed that the range over which each γ_i varies during normal operating conditions is known or can be estimated with reasonable accuracy.

5. It is also assumed that the coefficient of the i^{th} term of the desired transfer function of Equation III-20 lies somewhere in the range of values that the i^{th} coefficient of the plant transfer function can take on during the normal cycle of operations. That is

$$\gamma_{i_1} \leq a_i \leq \gamma_{i_2} \qquad \text{III-23}$$

where γ_{i_1} and γ_{i_2} represent the minimum and maximum values, respectively, that γ_i can assume during the normal range of environmental changes.

6. It will be assumed that the order of the plant transfer characteristic is equal to the order of the desired system transfer characteristic. This requires

$$m = n \qquad \text{III-24}$$

from Equations III-20 and III-22.

7. It is also assumed that the gain constants of the plant and the desired transfer functions are identical.

In an ordinary feedback control system the basic reference quantity is the input signal and the ideal output is a reproduction of the input signal. Obviously, practical limitations prevent the realization of this ideal output. Consequently, one must design a feedback control system from the standpoint that something less than an ideal output signal will result. The acceptable tolerances on this less than ideal output signal are generally specified by a set of time domain specifications. However, ordinary linear feedback control system synthesis techniques do not provide a means within the system whereby the system may continuously determine whether or not it is meeting these time domain specifications. As far as the system is concerned the ideal output is always a reproduction of the input signal. A compensation technique that was first proposed by Ham and Lang [22] does not suffer from this deficiency in that a dynamic model of the desired transfer function is an integral part of the system. This dynamic model is used to generate a synthetic output signal that is used as a part of the system signal reference scheme. As far as the system is concerned, the ideal output is not a reproduction of the input signal, but is a realizable reproduction of the dynamic model output signal. Figure 13 shows a block diagram of the model reference scheme, as it will be employed to compensate for grossly varying parameters in the plant.

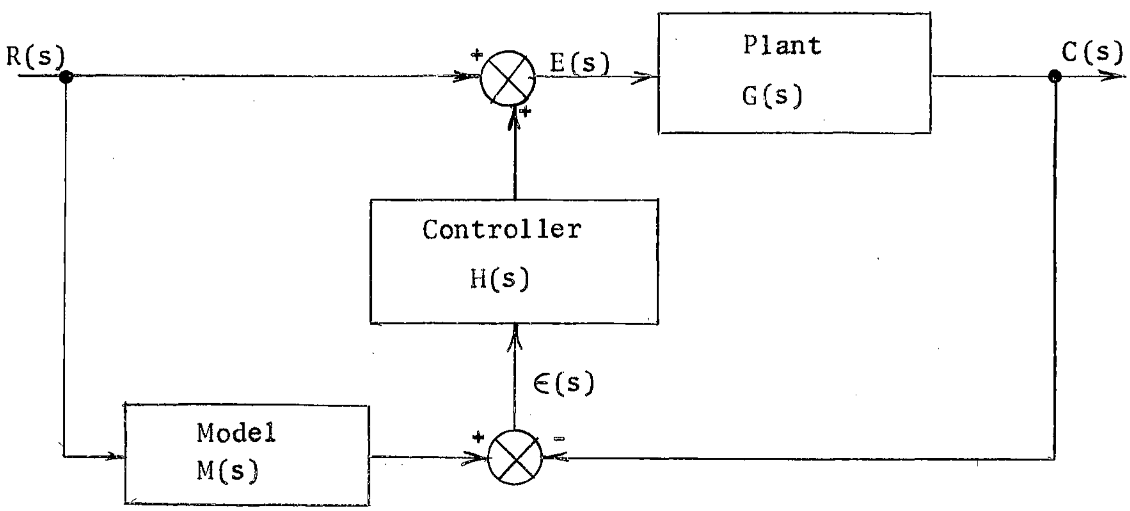


Figure 13. A Model Reference
Passive-Adaptive Control System

The symbols of the block diagram are defined as follows:

M(s) = the desired plant or system transfer function expressed as a function of the complex frequency variable s.

H(s) = the controller transfer function expressed as a function of s also.

$\epsilon(s)$ = the Laplace transform of the difference between the plant output signal and the model output signal. It is assumed here that the device that is used to measure the output of the plant has a unity transfer function, at least in the range of frequencies of interest.

The symbols C(s), R(s), E(s), and G(s) were defined previously.

Basically, the model reference system operates in this manner. If the plant and model transfer functions are identical or if the outputs of the plant and model are identical, then $\epsilon(s)$ is zero and there is no feedback about the plant. However, if these conditions are not satisfied and if H(s) has a relatively high gain then the feedback about G(s) will be appreciable. Also, if the model and the plant outputs differ, the controller can be thought of as a simple computer which operates on the error signal and computes the necessary input signal to cause the output of the plant to be identical to the output of the model. These considerations are readily apparent if one considers the overall system transfer function.

The input signal to the plant can be written as

$$E(s) = R(s) + H(s)\epsilon(s) \qquad \text{III-25}$$

However,

$$E(s) = C(s)/G(s) \qquad \text{III-26}$$

and

$$\epsilon(s) = M(s)\,R(s) - C(s). \qquad \text{III-27}$$

Substituting Equations III-26, and III-27 into Equation III-25 yields

$$C(s)/G(s) = R(s) + H(s)\,[M(s)\,R(s) - C(s)]. \qquad \text{III-28}$$

Solving this equation for the transfer function of the system yields

$$\frac{C(s)}{R(s)} = \left\{\frac{1/M(s) + H(s)}{1/G(s) + H(s)}\right\} M(s). \qquad \text{III-29}$$

This is a most significant equation in that it shows that if

$$\frac{1/M(s) + H(s)}{1/G(s) + H(s)} = 1 \qquad \text{III-30}$$

in the frequency range of interest then

$$\frac{C(s)}{R(s)} = M(s).$$

Under those conditions, the transfer function of the entire control system is simply the transfer function of the dynamic model regardless of the values of the parameters of the plant transfer function G(s).

Of course one can approximate Equation III-30 by requiring that H(s) be very large compared to $[M(s)]^{-1}$ and $[G(s)]^{-1}$ throughout the frequency range in which the system is designed to operate. However, this does not specify what is the most efficient technique for obtaining this high gain from the standpoint of gain conservation. In other words, what configuration must H(s) have in order to approximate Equation III-29 to any desired degree of accuracy with the minimum required gain level?

One technique of approximating Equation III-30 that appears to have some merit is considered here. This technique achieves the approximation by generating a corresponding term in H(s) for every term in $[G(s)]^{-1}$ that differs from the term of the same order in $[M(s)]^{-1}$. Each of these generated terms in H(s) must have a coefficient that is large compared to the coefficients of the corresponding term in $[M(s)]^{-1}$ and $[G(s)]^{-1}$. Consider the case where $[M(s)]$ is taken to be equal to $T_d(s)$. If $T_d(s)$ is defined by Equation III-20 and if G(s) is defined by Equation III-22, it can be written that

$$\frac{1}{M(s)} = [M(s)]^{-1} = \frac{1}{K_d} S^n + \sum_{i=0}^{n-1} \frac{a_i}{K_d} S^i \qquad \text{III-32}$$

and

$$\frac{1}{G(s)} = [G(s)]^{-1} = \frac{1}{K_a} S^m + \sum_{i=0}^{m-1} \frac{\gamma_i}{K_a} S_i. \qquad \text{III-33}$$

If

$$m = n \qquad \text{III-34}$$

and

$$K_a = K_d \qquad \text{III-35}$$

then Equation III-33 becomes

$$\frac{1}{G(s)} = \frac{1}{K_d} S^n + \sum_{i=0}^{n-1} \frac{\gamma_i}{K_d} S^i. \qquad \text{III-36}$$

Now, let H(s) be defined as follows

$$H(s) = K_{n-1} S^{n-1} + \cdots + K_1 S + K_0 \qquad \text{III-37}$$

or

$$H(s) = \sum_{i=0}^{n-1} K_i S^i. \qquad \text{III-38}$$

Now the left-hand side of Equation III-30 can be written as

$$\frac{1/M(s) + H(s)}{1/G(s) + H(s)} = \frac{\frac{1}{K_d} s^n + \sum_{c=0}^{n-1} \left[\frac{a_i}{K_d} + K_i\right] s^i}{\frac{1}{K_d} s^n + \sum_{i=0}^{n-1} \left[\frac{\gamma_i}{K_d} + K_i\right] s^i} . \quad \text{III-39}$$

This factor will be equal to unity only if

$$\frac{a_i}{K_d} + K_i = \frac{\gamma_i}{K_d} + K_i \quad \text{III-40}$$

for all i. But

$$\gamma_i \neq a_i \quad \text{III-41}$$

due to the variations of γ_i caused by the changing environment in which the system is assumed to operate. However, if

$$K_i \gg \frac{a_i}{K_d} \quad \text{III-42}$$

and

$$K_i \gg \frac{\gamma_i}{K_d} \quad \text{III-43}$$

for all conditions of γ_i, then Equation III-40 and hence Equation III-30 can be approximately satisfied. This result leads to the observation that regardless of the value of the parameters of the plant the system can be made to exhibit a transfer function that is very nearly equal to the transfer function of the model. Of course there are some practical considerations pertaining to the physical realization of the higher ordered zeros in H(s) as well as the high gain constants that are required.

Practical Considerations

The preceding paragraphs did not place a restriction on the order of the characteristic equations of the plant and the desired transfer functions other than to specify that both must be of the same order. In reality there are some practical limitations concerning the measurement of higher ordered derivatives of physical signals in conjunction with high gains.

It was noted in Equation III-39 that the controller transfer function H(s) must contain a term corresponding to every term of $[G(s)]^{-1}$ that does not have the same coefficient as the identically ordered term of $[M(s)]^{-1}$. Thus, if $[M(s)]^{-1}$ and $[G(s)]^{-1}$ have terms of order k that do not have equal coefficients then the error between the model and plant must be differentiated k times. The actual differentiation of a physical signal more than twice is extremely difficult particularly if the system happens to be noisy. This difficulty in generating the higher order derivatives of the error signal in conjunction with the required high system gains places practical limitations on the order of the system that may be compensated using the techniques that were presented in the preceding paragraphs.

However, only the first derivative is required to compensate a second order system. Since many practical systems exhibit dominant second order characteristics and therefore can be approximated by a second order transfer function the fact that only the first derivative is required is significant. It allows the model reference technique of compensation for varying plant parameters to be applicable in a practical sense to a large class of control systems. With this in mind, the application of this technique of compensation to a second order system will be studied in detail in the next chapter.

Before proceeding to the detailed examination of the application of the model reference compensation technique, a few words concerning the apparent advantages of the technique are appropriate. The most obvious advantage over other adaptive control systems that the technique enjoys is in the relative simplicity. The technique does not require dynamic adjustment of the system's parameters and gains whereas most of the so called adaptive systems do involve such an adjustment. Along the same lines of thought it is not necessary to provide expensive, complicated, and possibly bulky equipment to measure the variation in plant parameters when the model reference compensation technique is used. The model that is used to determine the desired response can be made of relatively inexpensive, reliable, and time invariant analog components. The fact that the resulting system is continuous is a major advantage in that the overall system will react to faster parameter variations than many of the adaptive systems discussed in Chapter II.

CHAPTER IV

APPLICATION OF A SYSTEM WITH A SECOND ORDER PLANT

In the preceding chapter a model reference passive-adaptive technique for compensating for varying parameters in a control system was introduced. This technique was shown to be dependent upon the various derivatives of an error signal that was derived using a model of the desired system as the reference. The highest order derivative necessary was shown to be determined by the order of the plant transfer function. Because of the physical difficulties encountered in obtaining higher order derivatives of physical signals it was concluded that the usefulness of the technique is limited to systems which require only first or at most the second derivative of the error signal. A second order system is one in which only the first derivative is required. With this in mind a second order system was selected for a detailed investigation.

Development of the Second Order System Transfer Function

The seven basic assumptions listed in Chapter III apply to the following development. For a second order plant transfer function, Equation III-22 reduces to

$$G(s) = \frac{K_d}{S^2 + \gamma_1 S + \gamma_0} \qquad \text{IV-1}$$

where it is assumed that γ_1 and γ_0 are functions of the environment and are therefore subject to variation during the normal operating cycle of the system. In order for the model reference technique to be applicable the desired transfer function must have the same order as G(s) and take the form of Equation III-20. Then

$$T_d(s) = M(s) = \frac{K_d}{S^2 + a_1 S + a_0} \qquad \text{IV-2}$$

is the equation which represents the desired system transfer function as well as the transfer function of the dynamic model. It is convenient to rewrite Equation IV-2 as

$$M(s) = \frac{K_d}{S^2 + 2\zeta_d \omega_d S + \omega_d{}^2} \qquad \text{IV-3}$$

where

$$a_1 = 2\zeta_d \omega_d \qquad \text{IV-4}$$

and

$$a_0 = \omega_d{}^2 \qquad \text{IV-5}$$

It should be noted that ζ_d is the desired system damping ratio and ω_d is the desired system undamped natural frequency. It is also convenient to rewrite Equation IV-1 as

$$G(s) = \frac{K_d}{S^2 + 2\alpha S + \beta^2} \qquad \text{IV-6}$$

where

$$\gamma_1 = 2\alpha \qquad \text{IV-7}$$

and

$$\gamma_0 = \beta^2 \qquad \text{IV-8}$$

The transfer function of the controller for the second order system becomes

$$H(s) = K_1 S + K_0 \qquad \text{IV-9}$$

from Equation III-37.

A block diagram of the resulting system is shown in Figure 14.

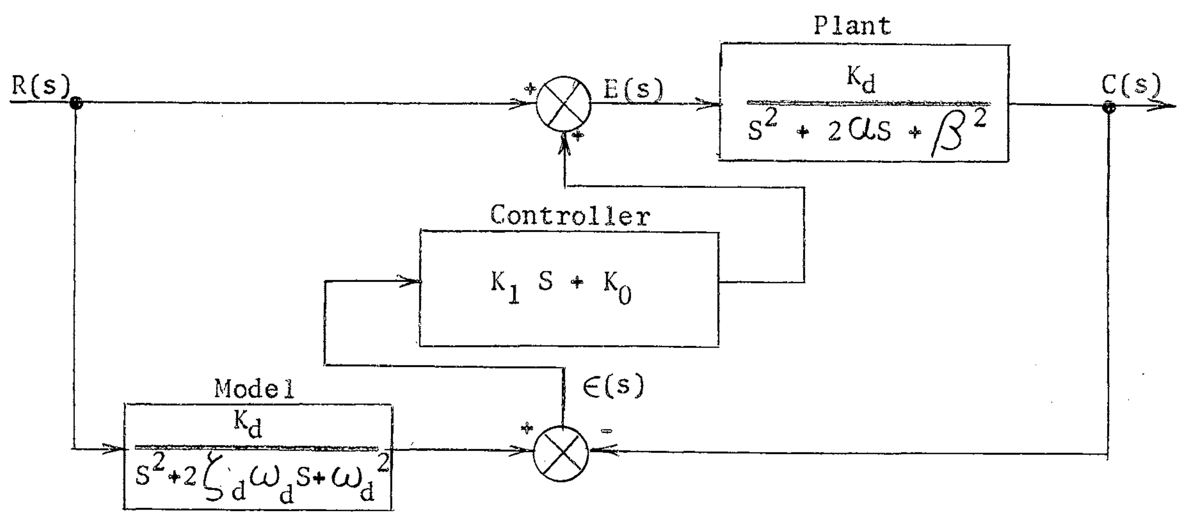


Figure 14. A Second Order Model Reference Passive-Adaptive Control System

The resultant system transfer function can be written by substituting Equations IV-3, IV-6, and IV-9 into Equation III-29. This substitution yields

$$\frac{C(s)}{R(s)} = \frac{K_d}{S^2 + 2\zeta_d \omega_d S + \omega_d^2} \left\{ \frac{S^2 + (2\zeta_d\omega_d + K_1K_d)S + (\omega_d^2 + K_0K_d)}{S^2 + (2\alpha + K_1K_d)S + (\beta^2 + K_0K_d)} \right\} \qquad \text{IV-10}$$

Equation IV-10 is the exact transfer function of the system shown in block diagram form in Figure 14. However, if the desired system transfer function of Equation IV-3 is to be realized the factor in brackets on the right-hand side of Equation IV-10 must approximate unity. The degree to which this approximation is achieved is determined by the magnitude of K_1 and K_0. The larger K_1 and K_0 can be made the more nearly the factor approximates one. Consequently, it is necessary that some technique be devised whereby a set of gains K_1 and K_0 can be determined that will yield the desired degree of approximation. Unfortunately, it does not appear as though a generalized technique exists, however, a procedure that has been successful in the design of second order systems will be presented here. It should be emphasized that the procedure is somewhat arbitrary in the sense that beyond some minimum value, the values of K_1 and K_0 are not critical. It should also be emphasized that any procedure which yields gains that achieve the desired degree of approximation is just as applicable as the one to be presented here.

Controller Design Equations

There are basically two factors to be considered when attempting to select the gains K_1 and K_0 in Equation IV-10. These two factors are (1) the steady-state error between the model output and the actual system output and (2) the transient error between the model output and the actual system output. The selection of K_0 shall be based on minimizing the first of these factors and the selection of K_1 shall be based on minimization of the second factor.

According to basic control system theory the steady-state positional error coefficient, K_p, is defined in the following manner [24].

$$K_p = \frac{\text{Steady-state value of output}}{\text{Steady-state value of input}} \qquad \text{IV-11}$$

It is not difficult to show, for the model unit considered separately, that the model positional error coefficient, K_{pm}, is

$$K_{pm} = \frac{K_d}{\omega_d^2} \quad . \qquad \text{IV-12}$$

It can also be shown that the actual system positional error coefficient, K_{pa}, can be written

$$K_{pa} = \frac{K_d}{\omega_d^2} \left\{ \frac{\omega_d^2 + K_0 K_d}{\beta^2 + K_0 K_d} \right\} . \qquad \text{IV-13}$$

Now, ideally Equations IV-12 and IV-13 should be identical, however, since K_0 cannot be infinite and β is subject to variation while ω_d is constant the ideal cannot be achieved. Hence K_0 is selected such that K_{pa} differs from K_{pm} by no more than some fixed percentage throughout the normal operating range of the system. Therefore, K_0 will be selected such that

$$(1 - d)\ K_{pm} \leq K_{pa} \leq K_{pm}\ (1 + d) \qquad \text{IV-14}$$

for all values of β in the range

$$\beta_1 \leq \beta \leq \beta_2 \qquad \text{IV-15}$$

where d is the arbitrary per unit value by which K_{pa} is allowed to differ from K_{pm}. β_1 and β_2 are the minimum and maximum values, respectively, of β in the normal operating range of the system. Substituting Equations IV-12 and IV-13 into Equation IV-14 yields

$$(1 - d) \leq \frac{\omega_d^2 + K_0 K_d}{\beta^2 + K_0 K_d} \leq (1 + d). \qquad \text{IV-16}$$

Obviously, Equation IV-16 will give two values of K_0 if the two extreme values of β are inserted in the denominator and the appropriate inequality is evaluated. If β equals β_1 and the right side of Equation IV-16 is used then a value

$$K_{0_1} = \frac{\omega_d^2 - \beta_1^2 (1 + d)}{dK_d} \qquad \text{IV-17}$$

would insure that

$$K_{pa} \leq (1 + d) K_{pm} \qquad \text{IV-18}$$

Also if β equals β_2 and the left side of Equation IV-16 is used then a value

$$K_{0_2} = \frac{\beta_2^2 (1 - d) - \omega_d^2}{dK_d} \qquad \text{IV-19}$$

will insure that

$$K_{pa} \geq (1 - d) K_{pm}. \qquad \text{IV-20}$$

As a result of the inequality signs in Equation IV-17 and IV-20 both the right and the left side inequalities of Equation IV-16 will be satisfied if the gain K_0 is made equal to the largest of the two values K_{0_1} and K_{0_2}. The relative magnitude of these two values is dependent upon the magnitudes of ω_d, β_1, and β_2. It can be shown that if

$$\omega_d^2 \geq \frac{\beta_2^2 + \beta_1^2}{2} + \frac{d}{2} (\beta_1^2 - \beta_2^2) \qquad \text{IV-21}$$

then

$$K_{0_1} \geq K_{0_2} \qquad \text{IV-22}$$

and conversely if

$$\omega_d^2 \leq \frac{\beta_2^2 + \beta_1^2}{2} + \frac{d}{2}(\beta_1^2 - \beta_2^2) \qquad \text{IV-23}$$

then

$$K_{0_2} \geq K_{0_1} . \qquad \text{IV-24}$$

It should be noted that the factor

$$\frac{d}{2}(\beta_1^2 - \beta_2^2) << \frac{\beta_2^2 + \beta_1^2}{2}$$

for most practical cases and therefore may be neglected in determining which gain equation is applicable.

As pointed out previously the first derivative gain of the controller unit, K_1, shall be selected so as to control the transient error between the model output and the actual system output. Control over this transient error may be exercised through the use of the gain K_1 to insure that the poles and zeros of the term in the brackets of Equation IV-10 are far removed to the left with respect to the desired poles. That is K_1 shall be selected such that the real part of the poles and zeros of the term in the brackets of Equation IV-10 are n times farther to the left than the real parts of the poles of the model. In this case n is a positive number sufficiently large that an acceptable transient performance is realized. Truxal [25] has indicated that an n of six generally results in a satisfactory transient response in a similar situation. Horowitz [10] uses a figure of n equal to four under similar circumstances.

In the section of Chapter III devoted to evaluating the effects of varying parameters on a second order system, it was shown that if

$$\alpha_1 \leq \alpha \leq \alpha_2 \qquad \text{IV-26}$$

and

$$\beta_1 \leq \beta \leq \beta_2 \qquad \text{IV-27}$$

then the poles of a system having a transfer function

$$G(s) = \frac{K_d}{S^2 + 2\,\alpha\, S + \beta^2} \qquad \text{IV-28}$$

map into an area defined by the inequalities of Equations IV-26 and IV-27. If one is going to control the transient response of the model reference system depicted in Figure 14 by controlling the poles and zeros of Equation IV-10 then one must insure that the entire area over which the poles and zeros of the plant may vary during normal operations is mapped to the left of a line defined by

$$S = -n\,\zeta_d\,\omega_d\,. \qquad \text{IV-29}$$

In Equation IV-29 n is the arbitrary positive number mentioned above and $\zeta_d\,\omega_d$ is the real part of the poles of the model or desired transfer function.

The actual system transfer function of Equation IV-10 has poles located at

$$S_1 = -\zeta_d\,\omega_d + j\,\omega_d\sqrt{1 - \zeta_d^2} \qquad \text{IV-30a}$$

and

$$S_2 = -\zeta_d\,\omega_d - j\,\omega_d\sqrt{1 - \zeta_d^2}\,. \qquad \text{IV-30b}$$

These are the poles of the model and therefore are the desired poles. The actual system also has poles at

$$S_3 = -\left(\alpha + \frac{K_1K_d}{2}\right) + j\sqrt{(\beta^2 + K_0K_d) - \left(\alpha + \frac{K_1K_d}{2}\right)^2} \qquad \text{IV-31a}$$

and

$$S_4 = -\left(\alpha + \frac{K_1K_d}{2}\right) - j\sqrt{(\beta^2 + K_0K_d) - \left(\alpha + \frac{K_1K_d}{2}\right)^2} \qquad \text{IV-31b}$$

It is these latter poles that must be controlled and forced to have real parts to the left of the line defined by Equation IV-29. Therefore K_1 shall be selected such that

$$\alpha + \frac{K_1K_d}{2} \geqq n\zeta_d\,\omega_d \qquad \text{IV-32}$$

for all α in the range of Equation IV-26. Hence when solving for K_1 the minimum value of α shall be employed. Thus solving Equation IV-32 yields

$$K_1 \geqq \left[\frac{n\,\zeta_d\omega_d - \alpha_1}{K_d}\right] 2 \qquad \text{IV-33}$$

where α_1 is the minimum value of α in the normal operating range of the system.

It can be seen that if

$$\alpha_1 \leq \zeta_d\,\omega_d \leq \alpha_2 \qquad \text{IV-34}$$

from the initial basic assumptions of Chapter III then the zeros of the system transfer function are also to the left of the line defined by Equation IV-29 if K_1 is selected using Equation IV-33.

The equations developed in this section have been shown to yield satisfactory time domain response by means of an analog computer investigation. Samples of the results of this computer investigation are presented in the next section of this chapter.

Sample Results of the Computer Investigations

The time domain characteristics of a simple system using the design equations developed in the preceding section were investigated using an analog computer. For the purposes of the computer investigations the plant or system to be compensated was assumed to have a transfer function that could be written

$$G(s) = \frac{50}{S^2 + 2\alpha S + \beta^2} \qquad \text{IV-35}$$

where α and β were both subject to variations in the ranges

$$0.707 \leq \alpha \leq 7.07 \qquad \text{IV-36}$$

and

$$6 \leq \beta \leq 9 \qquad \text{IV-37}$$

The resulting poles of the plant transfer function are located in the shaded region of Figure 15. It can be seen that the assumed conditions represent unusually large variations in the poles of the plant. However, it was felt that presentation of such an example would certainly indicate the feasibility of the model reference compensating technique with more authority than an example with smaller and probably more practical parameter variations.

The system specifications were assumed to be such that a time domain response approximating a second order system with a damping ratio of 0.7 and an undamped natural frequency of 7.07 radians per second would be satisfactory. Consequently, the dynamic model transfer function that was used in this computer investigation could be written

$$M(s) = \frac{50}{S^2 + 9.9S + 50} \qquad \text{IV-38}$$

The poles of Equation IV-38 also lie in the shaded region of Figure 15. This was a requirement if the controller design equations previously presented were to be employed.

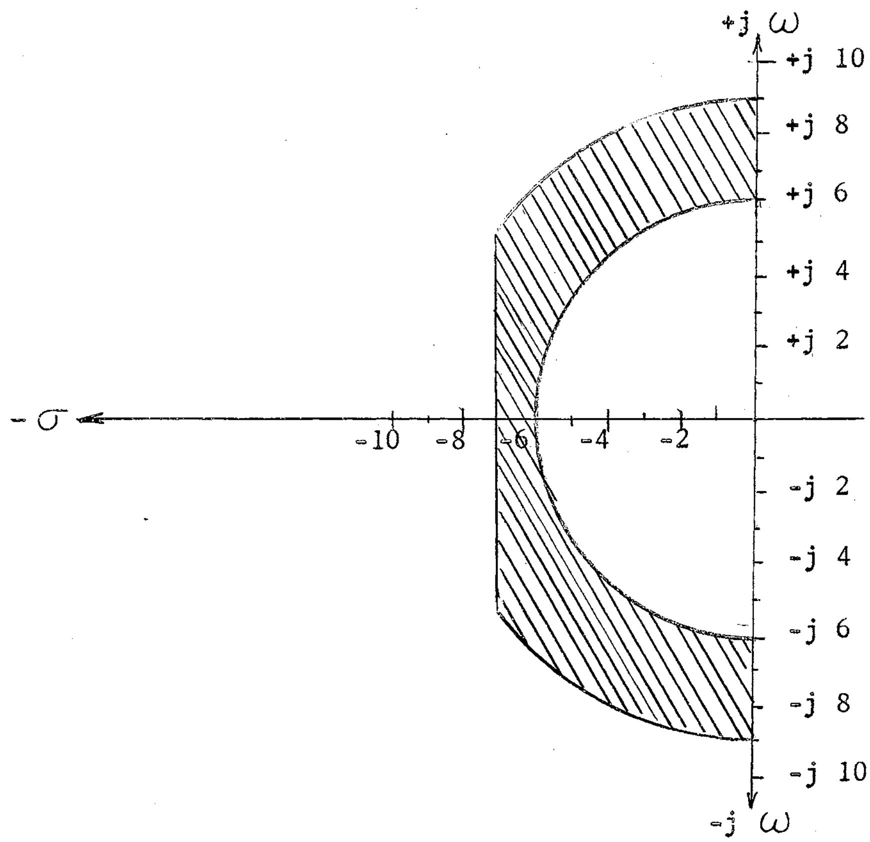


Figure 15. Location of the Poles of the Plant Transfer Function

Application of the controller design equations yields a controller transfer function of

$$H(s) = 1.95\ S + 60.4 \qquad \text{IV-40}$$

when the factors d and n were selected to be 0.01 and 10 respectively. Both of these arbitrarily selected factors were chosen so as to present a rather pessimistic case in this example. That is, a plant with extremely large pole variations was selected for use, while at the same time rather tight transient and steady-state response requirements were assumed to be specified. For example the factor d was chosen to be 0.01 which indicates that the allowable difference between the actual system steady-state positional error coefficient and the corresponding coefficient for the model is one percent based on the model coefficient. This is a rather stringent requirement in the face of the assumed large variations in the plant natural frequency. The fact that the factor n was chosen to be 10 indicates rather close tolerances on the transient response when one considers that eminent authors in the field [10, 25] have suggested the use of factors between four and six in similar circumstances.

Using the plant, model, and controller transfer functions presented above in conjunction with either Equation IV-10 or Equation III-29 the system transfer function could be written as

$$\frac{C(s)}{R(s)} = \frac{50}{S^2 + 9.9S + 50} \left\{ \frac{S^2 + 107.4S + 3070}{S^2 + (2\alpha + 97.5)\,S + (\beta^2 + 3020)} \right\}. \qquad \text{IV-41}$$

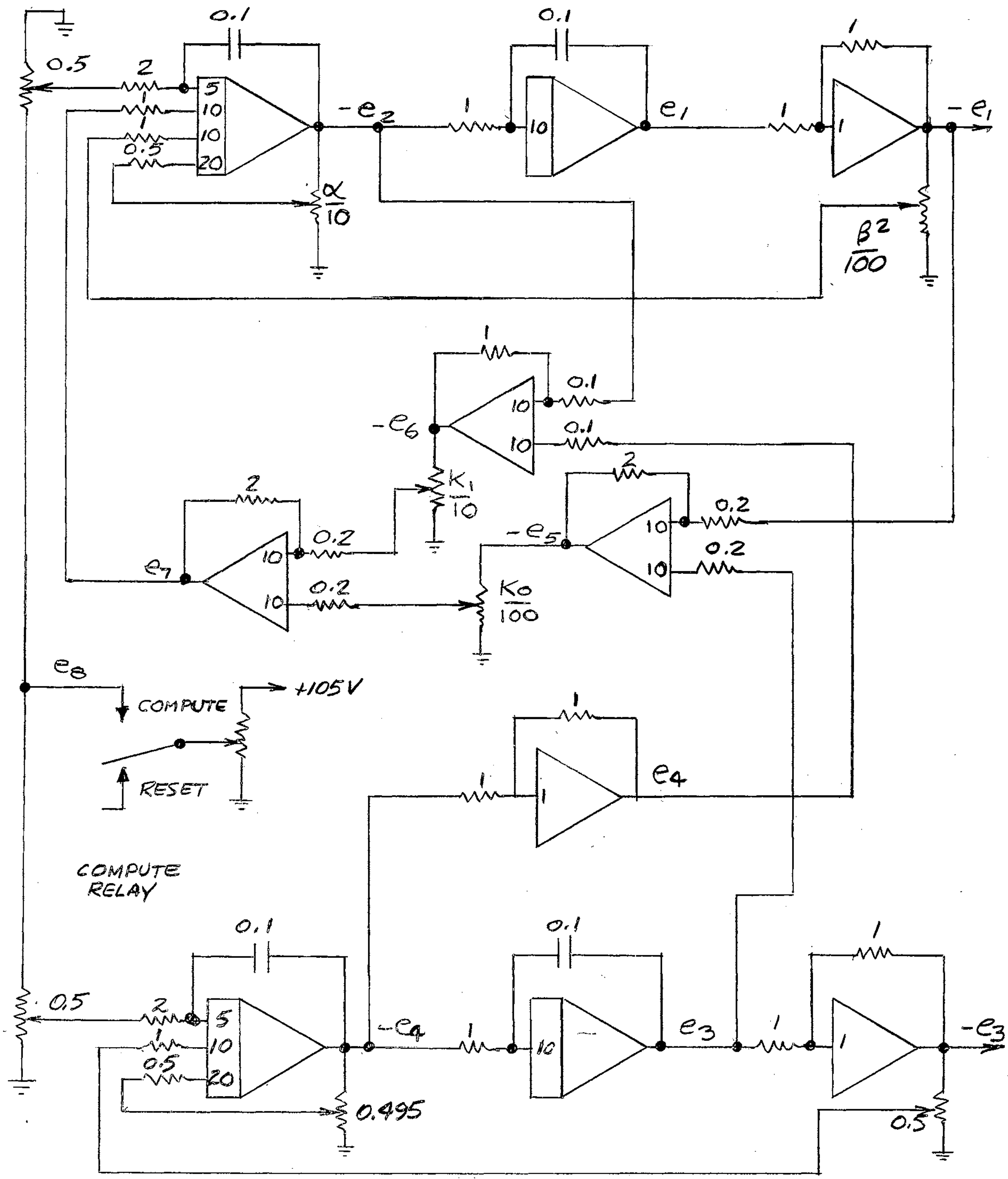
This equation represents the output-input characteristics of the system shown in block diagram form by Figure 14 with the appropriate substitution of values for the various parameters as specified above. It is this equation that was studied rather extensively by means of the analog computer in order to determine the corresponding response in the time domain.

Before proceeding with the presentation of the time domain data it is of interest to note that if the system were ideally compensated then the factor in brackets in Equation IV-41 would be identically equal to unity for all α and β and throughout the frequency range of interest. However, as pointed out previously ideal compensation is possible only if K_1 and K_0 are infinite. Since infinite gains are not possible it is of interest to determine how well the bracketed factor approximates unity. This was accomplished for the example case by evaluating the bracketed term of Equation IV-41 for several combinations of α and β along the periphery of the shaded region of Figure 15. For this evaluation the substitution

$$S = j\omega \qquad \text{IV-42}$$

was used. The factor ω was allowed to vary from a value less than the undamped natural frequency of the model to a value considerably greater than the undamped natural frequency of the model. The actual computation was performed by an IBM-650 general purpose digital computer. The results of this computation indicated that the approximation of unity was quite good. In fact the greatest deviation was found to be approximately an eight percent increase with respect to unity in the magnitude of the bracketed term. The greatest phase shift contributed by the term was found to be on the order of 2.5 degrees. Both of these maximum deviations occurred at frequencies that were considerably greater than the natural frequency of the model and hence were of little concern in so far as the overall response of the system was concerned.

Equation IV-41 was simulated on the analog computer. A schematic diagram of this simulation is shown in Figure 16. For the purposes of the time domain investigation the resulting system was excited by a simulated step function. Table I shows the analogies between the various



Note: All Resistances are in Megohms and all Capacitances are in Microfarads.

Figure 16. Analog Computer Simulation of a Second Order Model Reference Control System

problem variables and the analog computer voltages labeled in Figure 16.

TABLE I

ANALOGIES BETWEEN COMPUTER VOLTAGES
AND PROBLEM VARIABLES

Computer Voltages (See Figure 16)	Problem Variable
e_1	$c(t)$ = plant output expressed as a function of time.
e_2	$\dot{c}(t) = dc(t)/dt$ = time rate of change of the plant output expressed as a function of time.
e_3	$c_d(t)$ = model output expressed as a function of time.
e_4	$\dot{c}_d(t) = dc_d(t)/dt$ = time rate of change of the model output expressed as a function of time.
e_5	$c_d(t) - c(t) = \epsilon(t)$ dynamic error between the model output and the plant output expressed as a function of time.
e_6	$\dot{c}_d(t) - \dot{c}(t) = \dot{\epsilon}(t)$ = time rate of change of the dynamic error between the model output and the plant output.
e_7	$f(\epsilon, \dot{\epsilon})$ = system corrective signal expressed as a function of time.
e_8	$r(t)$ = system input signal, a step function scaled so as to represent 0.01 radians referenced to the output of the control system.[1]

[1]The dimensions of this input signal were arbitrarily assigned for the purposes of the analog computer investigation. Any other dimensions are certainly just as appropriate in that no restrictions have been placed on the type of quantity to be controlled, i.e., angular position, linear velocity, temperature, etc.

During the course of the computer investigations several combinations of α and β were studied so as to determine the effects of the varying plant parameters on the system described by Equation IV-41. Time response data from a sampling of those combinations will be presented and discussed. Each of the cases selected for presentation here result in the poles of the plant being on the periphery of the shaded area of Figure 15. The combinations of α and β selected for presentation are tabulated in Table II.

TABLE II

PLANT PARAMETER COMBINATIONS

Case No.	α	β
I	0.707	6
II	0.707	9
III	7.07	9
IV	6.0	6
V	7.07	7.07

For each of these cases four variables will be shown as a function of time. These variables are (1) the uncompensated plant output, (2) the compensated plant output, (3) the model output, and (4) the error between the compensated plant output and the model output. This time domain data is presented in Figures 17 through 21 in the order that the various cases appear in Table II.

Two items are of particular interest in evaluating the results of the analog computer investigations. These are the transient response

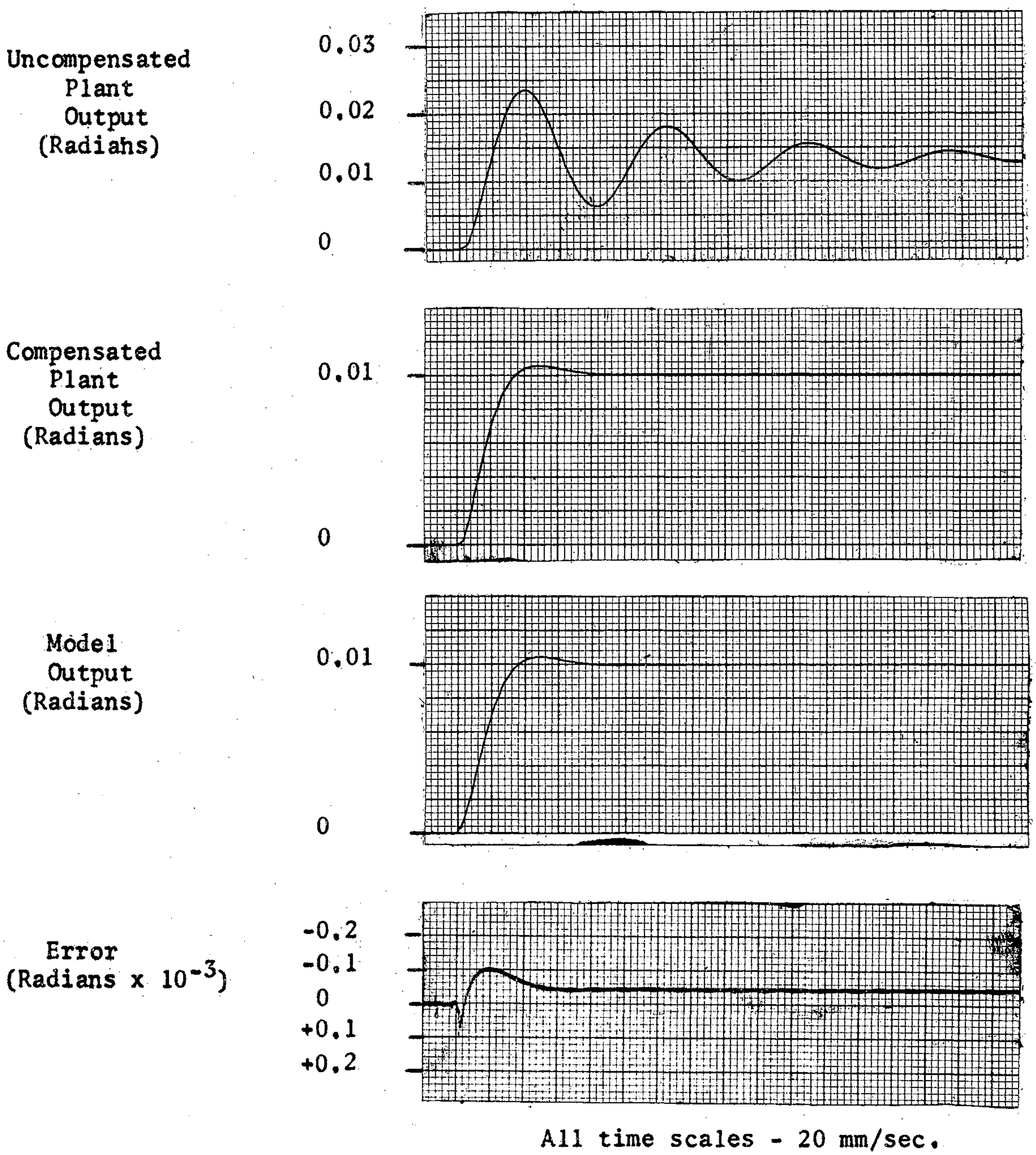


Figure 17

Time Response Data for Case I
$\alpha = 0.707; \beta = 6.0$

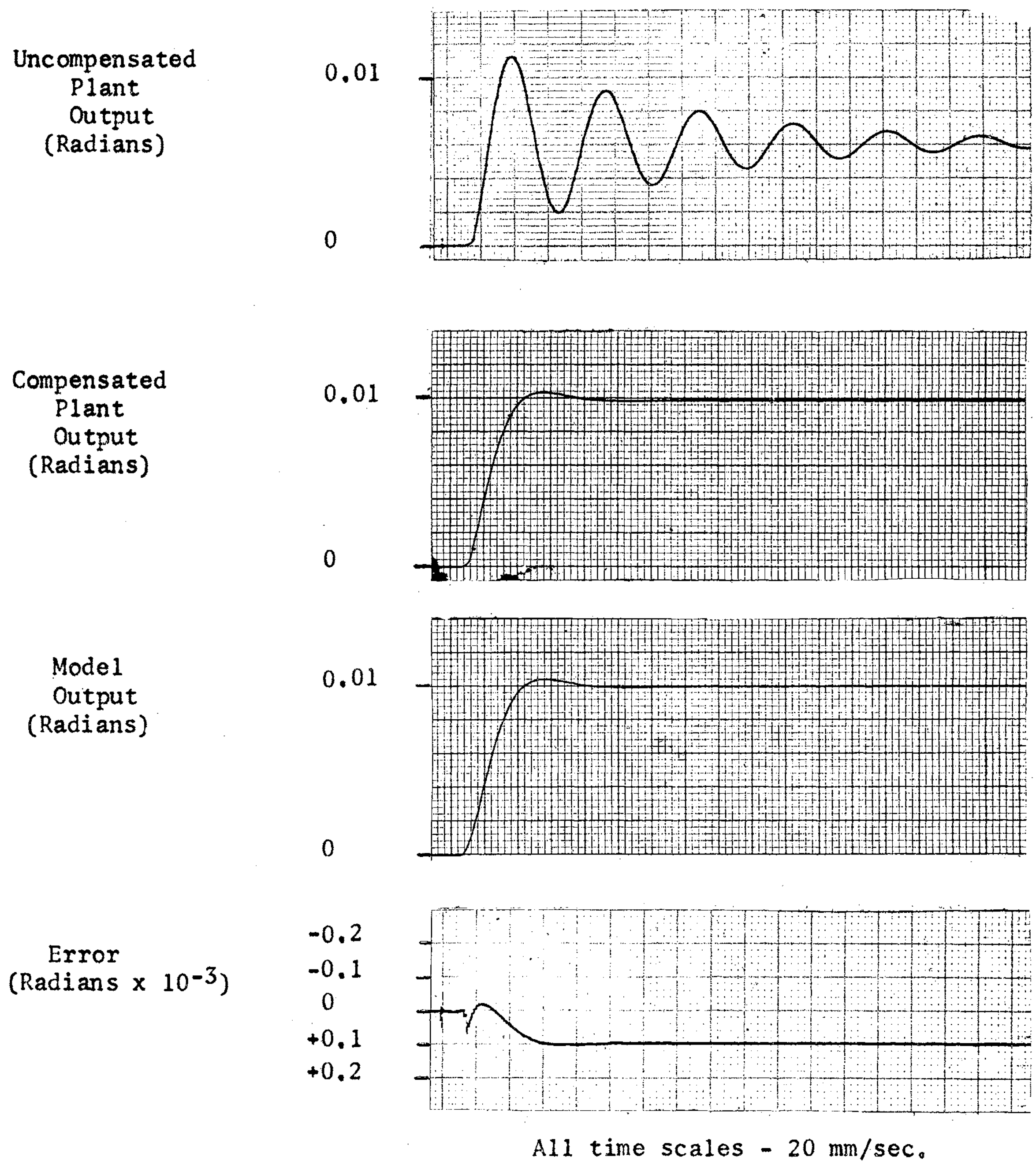


Figure 18

Time Response Data for Case II

$\alpha = 0.707; \beta = 9.0$

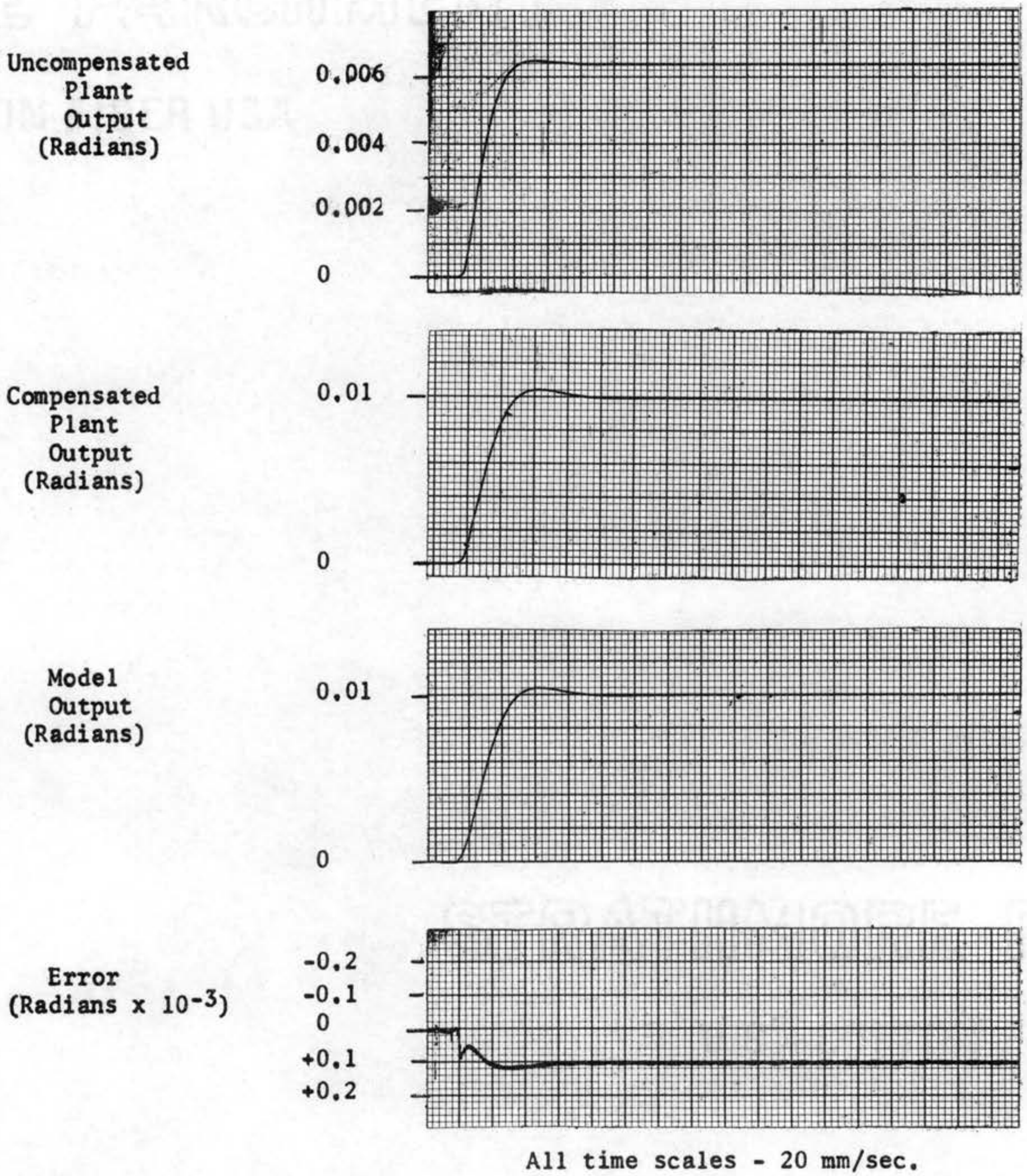


Figure 19

Time Response Data for Case III
$\alpha = 7.07; \beta = 9.0$

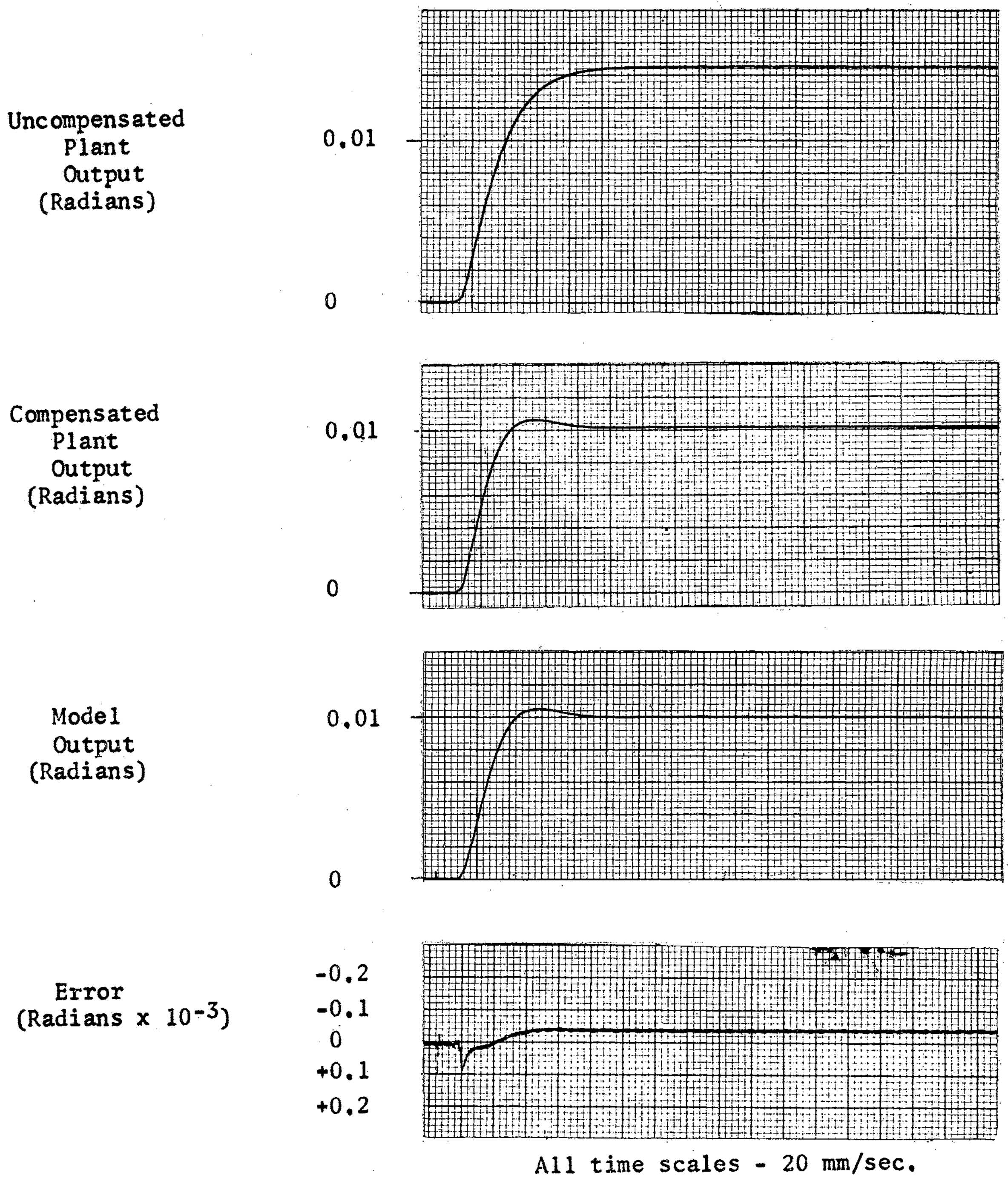


Figure 20

Time Response Data for Case IV
$\alpha = 6.0; \beta = 6.0$

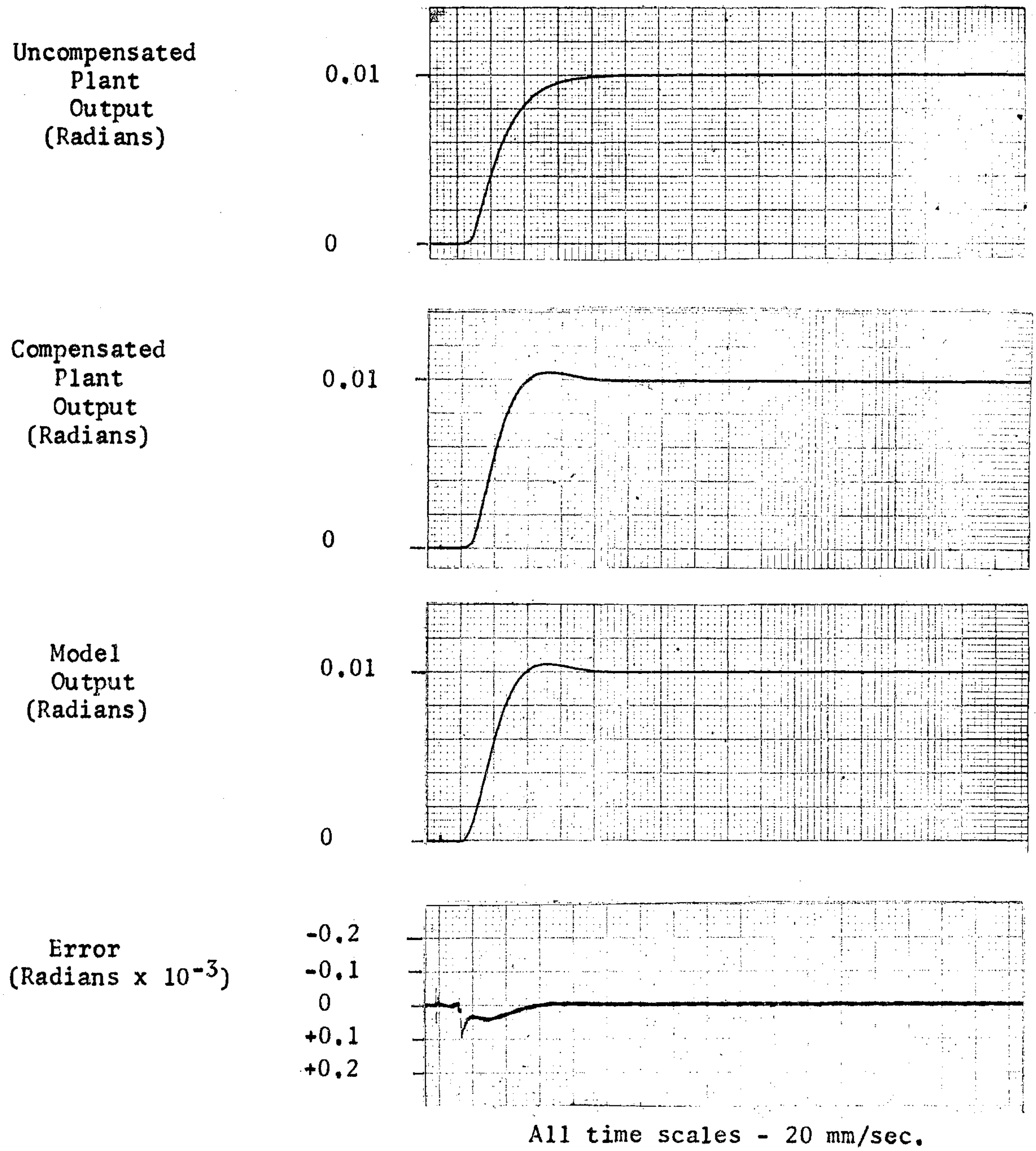


Figure 21

Time Response Data for Case V
$\alpha = 7.07; \beta = 7.07$

and the steady-state or final value error between the model output and the actual system output. The transient performance will be discussed first.

A comparison of Cases I and IV (Figures 17 and 20) best illustrates the transient performance of the model reference passive-adaptive compensation technique. Both of these cases represent second order plants having an undamped natural frequency of six radians per second. However, Case I represents a plant having a damping ratio of approximately 0.12 while Case IV is critically damped. As a basis of comparison, the rise time[1] and the percent overshoot[2] were measured from the transient data presented for these cases. The results of these measurements are tabulated in Table III. It should be pointed out that the values in Table III are at best only good approximations because of the difficulty involved in accurately extracting data from time responses such as those in Figures 17 through 21. However, this data does show that the large variations in transient performance noted in the uncompensated cases, do not appear in the compensated cases. The effects of variations in the plant parameters have been effectively masked from the output signal transient characteristics by means of the model reference passive-adaptive compensation method.

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[1]The generally accepted definition of rise time was employed, i.e., the rise time is the time required for the system output response to go from 10 to 90 percent of its final value. A more detailed discussion of rise time is contained in [26].

[2]The definition of percent overshoot that was employed here and that is generally accepted can be stated as follows: Percent overshoot is the ratio of system output overshoot to the system output final value expressed as a percentage. Additional information concerning this quantity can be found in [26].

TABLE III

A COMPARISON OF TRANSIENT DATA

Case No.	Uncompensated		Compensated		Model or Desired	
	Rise Time Seconds	Percent Over-Shoot	Rise Time Seconds	Percent Over-Shoot	Rise Time Seconds	Percent Over-Shoot
I	0.17	75%	0.275	5.2%	0.3	4.8%
IV	0.6	0%	0.275	5.5%	0.3	4.8%

The remaining item of interest, the steady-state error between the model output and the actual system output, is best illustrated by a comparison of Cases III, IV, and V. It is noted that these cases present data for plants having three different undamped natural frequencies. The steady-state errors were extracted from the time responses for the above named cases and tabulated in Table IV. The quantity tabulated is defined as the difference between the dynamic model output and the plant output. Note that the uncompensated error function is not shown in Figures 17 through 21. That quantity was obtained for Table IV by taking the difference between the model output and the uncompensated plant output traces. Again, it should be emphasized that the values in Table IV are at best good approximations because of the difficulties in obtaining accurate data from the time traces. However, as before the data is certainly indicative of the steady-state performance that is possible using the model reference technique for compensating for varying plant parameters. It is seen that the steady-state errors encountered at the periphery of allowable values of undamped natural frequency are reduced by a factor of 10 at least.

TABLE IV

A COMPARISON OF STEADY-STATE ERROR DATA

Case No.	Uncompensated	Compensated
III	- 0.0014 radian	- 0.0001 radian
IV	0.0046 radian	0.00004 radian
V	0	0

The analog computer data shows that the model reference passive-adaptive technique provides excellent transient and steady-state performance. The configuration is shown to be particularly applicable to a case of very large variations in the parameters of the plant. It can also be concluded that since the system is not required to continuously adjust its parameters, then the technique is certainly applicable to systems which display rapidly varying parameters.

CHAPTER V

SUMMARY AND CONCLUSIONS

A technique whereby a system having variable parameters may be compensated in such a manner as to provide a relatively unchanging transient and steady-state performance throughout the normal range of operating environment is presented in this thesis. The unique feature of this compensating technique is the fact that a dynamic model of the desired system transfer characteristics is employed as the reference quantity rather than the input signal as is the case with conventional feedback control systems.

It is shown that, <u>theoretically</u> at least, the technique can be applied to any ordered plant, system, or servomechanism, provided certain conditions concerning the relative forms of the desired system transfer function and the transfer function of the plant, system, or servomechanism are satisfied. These limiting conditions are (1) the plant and desired transfer characteristics must have the same order, (2) neither the plant nor the desired transfer characteristics may have zeros in the finite complex frequency domain, (3) the plant and the desired transfer characteristics must have identical gain constants, and (4) the plant must, somewhere in its normal operating range of environmental conditions, exhibit poles that are equal to the poles of the desired transfer characteristics.

It is shown that there are some <u>practical</u> limitations concerning the maximum order of the plant to be compensated using the model reference technique. It is seen that the technique is dependent upon the generation of higher ordered derivatives of physical signals. This is not always practical, particularly if the signal happens to be noisy. It is also shown that a second order plant requires only the first derivative of the output signal. Therefore, practical application of the model reference compensation technique to such a system is certainly feasible.

This conclusion concerning feasibility was verified by means of an analog computer investigation based on some second order design equations which are presented in this paper. The results of this computer investigation were excellent. A sampling of the transient data acquired during the analog study is presented to substantiate the theory.

The model reference passive-adaptive compensation technique exhibits certain advantages over the other adaptive control systems that are discussed in this thesis. These advantages are (1) there is no requirement for active adjustment of the system's parameters, (2) the resulting system is strictly linear and therefore ordinary analysis procedures apply, (3) the system designer can totally specify the time response characteristics of the resulting systems within the previously mentioned limits, and (4) the resulting system is more simple than most other adaptive control techniques and is therefore more reliable.

The major disadvantages of the model reference technique of control are (1) the requirement for generating the higher derivatives of a physical signal and (2) the necessity of high loop gains. These two factors are particularly significant if the system is noisy.

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