HYDROCLONE THEORY OF OPERATION

By

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

C	Constant
C_1	Constant
C_2	Constant
C_3	Constant
C_4	Constant
C_o	Constant
C_{D_i}	Coefficient of discharge, inlet
C_{D_o}	Coefficient of discharge, overflow
d_c	Diameter of hydroclone, inches
d_i	Diameter of inlet, inches
d_o	Diameter of overflow, inches
D	Particle diameter, microns
$D_{50\%}$	Diameter separated with 50% efficiency, microns
$D_{90\%}$	Diameter separated with 90% efficiency, microns
F_c	Centrifugal force on particle
F_D	Drag force on particle
F_r	Body force in radial direction
$f(r)$	Function of radius, r,
$f(r_o)$	Function of r_o
K	Constant
N	Number of inlets
Δp	Pressure drop across hydroclone, psi
Δp_c	Pressure drop across cyclone section, psi

Δp_i	Pressure drop across inlet, psi
Δp_o	Pressure drop across overflow, psi
ΔP	Dimensionless hydroclone pressure drop
ΔP_c	Dimensionless cyclone section pressure drop
ΔP_i	Dimensionless inlet pressure drop
ΔP_o	Dimensionless overflow pressure drop
Q	Flow rate, GPM
r	Radial position in hydroclone, inches
r_c	Radius of hydroclone
r_i	Radius of inlet
r_o	Radius of overflow
R	Dimensionless radial position, r/r_c
u	Radial velocity
u_o	Radial velocity at $r = r_o$
v	Tangential velocity
v_c	Tangential velocity at $r = r_c$
v_i	Inlet tangential velocity
v_o	Tangential velocity at $r = r_o$
V	Dimensionless tangential velocity, v/v_c
V_o	Dimensionless velocity, v_o/v_c
w	Axial velocity
z	Axial position
β	Dimensionless inlet size, r_i/r_c
β_e	Effective dimensionless inlet size
δ	Dimensionless overflow size, r_o/r_c
μ	Absolute viscosity

ν	Kinematic viscosity
ρ_L	Density of liquid
ρ_s	Density of solid particles
ϕ	Hydroclone cone apex angle

CHAPTER I

INTRODUCTION AND STATE OF THE ART

Of late, much interest has been generated concerning the possible use of miniature hydroclones to help eliminate some of the filtration problems existing in the missile and space field. Hydroclones offer several attractive advantages over ordinary filters providing that the correct hydroclone configuration is used for the particular job at hand. Until the present time, no one has been able to give a satisfactory theory of hydroclone operation which would allow accurate theoretical prediction of optimum design and performance. Such a theory was probably not needed because the needs were easily fulfilled by existing designs. Because of this large gap in the art, coupled with the need for a hydroclone capable of giving a low micron separation with hydraulic fluid, a theory of operation was devised.

In the following pages, this theory will be presented; and some new design ideas will be introduced which have been shown to greatly increase the hydroclone performance.

There are basically two types of hydroclones:

a) Hydroclones with open underflow,

b) Hydroclones with closed underflow.

See Figure 1-1.

The first type may be considered as being a classifier, i.e., a system which separates the liquid-solid inflow into two parts with one

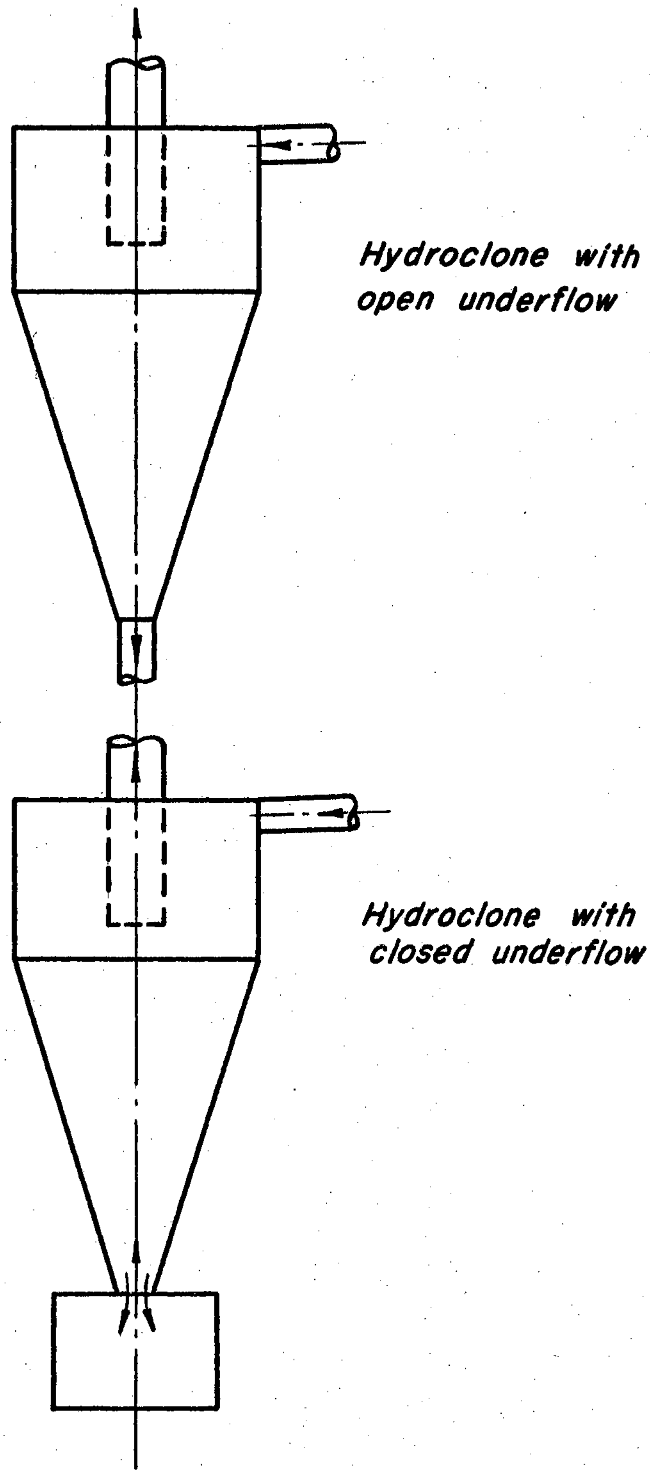

Figure 1-1. Hydroclone Types.

part containing a greater percentage of the larger and more dense particles and the other, containing a greater percentage of smaller and lighter particles. The second type of hydroclone could loosely be termed a filter. That is, a device which removes the larger and more dense particles from the liquid-solid inflow and retains them. The main difference between a hydroclone and an ordinary filter is that the latter removes the particles from the fluid by means of a porous media; that is, it blocks further downstream movement of these particles. The hydroclone, on the other hand, removes these particles from the fluid stream by means of a strong centrifugal field. Thus, there is no blocking of the particles and no possibility of a plugged filter.

A hydroclone may be thought of as a combination of two systems:

a) the cyclone system,

b) the underflow system.

The cyclone system consists of the inlet, the cyclone section, and the overflow. See Figure 1.2. The cyclone section performs the task of separating the solid particles from the liquid. The underflow system may be either of the open or closed type and may consist of any imaginable configuration. The underflow system disposes of the particles which have been separated from the liquid by the cyclone system.

There are two terms of interest in any discussion of hydroclones. These are hydroclone separation efficiency and hydroclone pressure drop. There are several means of describing the separation efficiency. These are:

a) Specification of the largest particle remaining in the overflow.

b) Specification of particle which is separated with some given efficiency, i.e., $D_{50\%}$, $D_{90\%}$, etc.

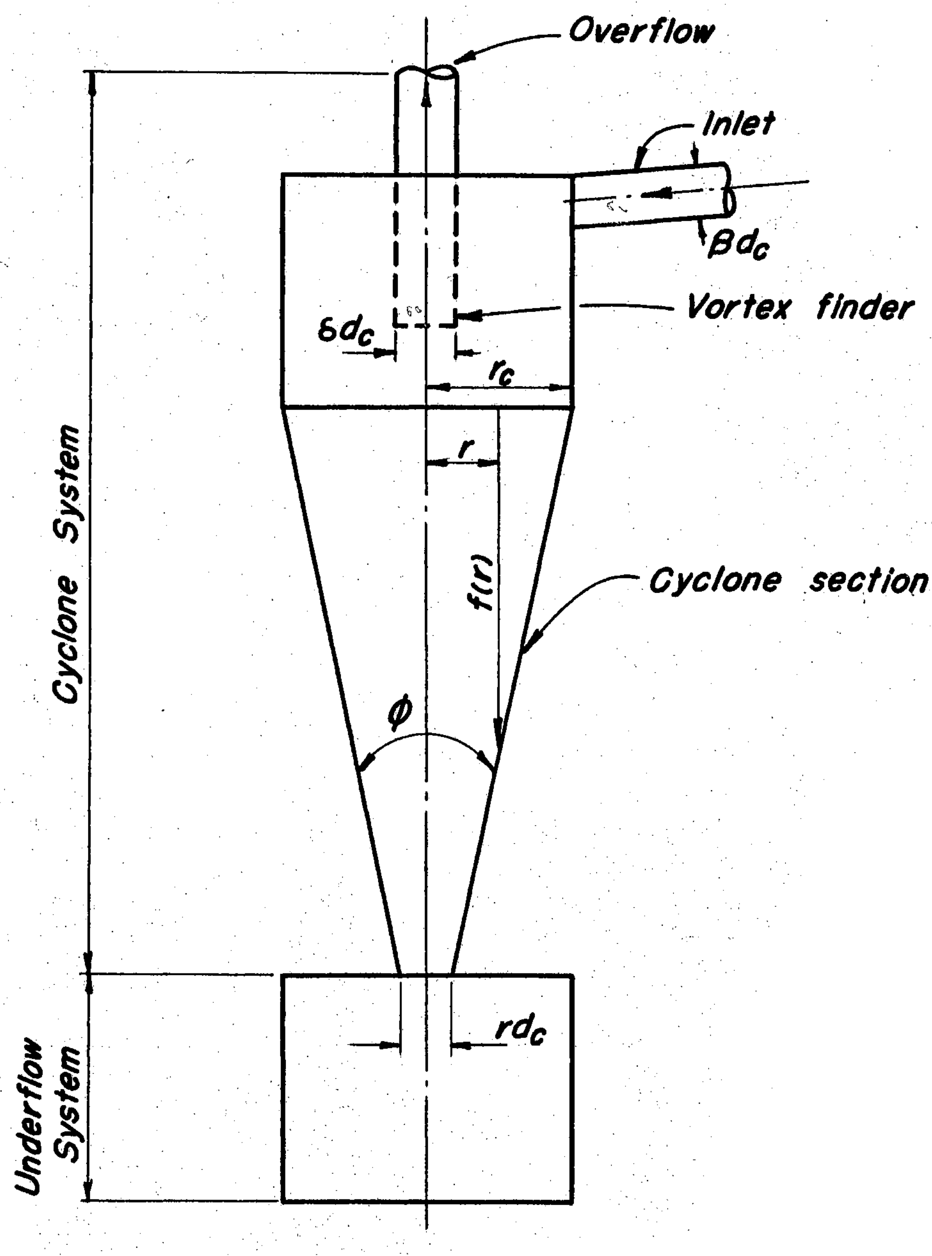

Figure 1-2. Nomenclature.

c) Specification of the relative weight per cent of particles in the inflow and overflow.

An equation for the $D_{50\%}$ particle is derived in Chapter V.

The term "hydroclone pressure drop" refers to the drop in pressure which occurs across the device. This is discussed more thoroughly in Chapter VI.

One should recognize that in order to analytically design a hydroclone for a particular job, it is necessary to know the relationships which exist between the separation efficiency, the pressure drop, the fluid flow rate, and all of the hydroclone design variables.

CHAPTER II

PREVIOUS INVESTIGATION

During the past half-century, much has been written about the cyclone separator for use with both gases and liquids. The majority of investigators have relied heavily on empirical equations and experimental data to attempt to design units for new applications. An extensive review of this literature has been compiled by R. E. Bose (1)[1].

The most useful expression for hydroclone separation efficiency to appear is that of Matschke and Dahlstrom (2) who have shown empirically that

$$D_{50\%} = \frac{C(d_o d_i)^{0.65}}{Q^{0.60}} \left[\frac{1}{\rho_s - \rho_L}\right]^{0.50} \qquad (2\text{-}1)$$

where Q is the flow rate; $(\rho_s - \rho_L)$, the relative density of the liquid and solid; C is a constant; d_o, the hydroclone overflow diameter; and d_i, the hydroclone inlet diameter. The most general empirical equation relating pressure drop and hydroclone parameters is that of Yoshioka and Hatta (3),

$$\Delta P = \frac{K\,Q^2}{d_c^{0.9}\,d_i^{1.2}\,d_o^{1.9}} \qquad (2\text{-}2)$$

where K is a function of the cone angle, surface finish, and type of underflow; and d_c is the hydroclone diameter.

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[1]( )Refers to Selected Bibliography.

There are many other empirical equations available from the literature in addition to those listed on page six. They all have one factor in common; their use is limited. In general, they may be used only to establish trends and to make rough estimates.

Over the past several years, considerable work--both theoretical and experimental--has been done at OSU on hydroclones.

The first investigator at OSU was R. L. Lowery (4). The object of his investigation was to determine if the cyclone separator could be used to separate two immiscible fluids combined in the form of an unstable emulsion. No analytical work was done due to the complexity of the flow fields. Although considerable difficulty was encountered in obtaining consistent results, the experimental work indicated that the cyclone separator may be used to separate two immiscible liquids combined in an unstable emulsion.

The second investigation conducted at OSU was that of J. S. Gilbert (5). This author conducted an analytical and experimental investigation to determine the feasibility of using a hydroclone to remove contaminants from hydraulic fluid.

In Gilbert's research, an equation of motion of a particle in a hydroclone was developed and solved on an analog computer. The only useful data obtained from solutions of this equation of motion was that large particles reach a greater radius faster than small particles. There was no way to determine what size particles would be removed and what size would not. Experimental data showed that large particles were removed better than small particles.

The next investigator at OSU, J. F. Beattie (6), modified the equation of motion derived by Gilbert and obtained more analog solutions.

This change consisted of replacing the molecular viscosity with a value for turbulent viscosity.

Each of these equations of motion, however, have a common fault which yields them useless in determining accurate data. This fault is that they neglect the radial velocity component of the fluid in the computation of the relative particle velocity.

The fourth investigator at OSU, R. E. Bose (1), corrected this basic fault in the equation of motion and obtained some useful data from which to predict hydroclone separation ability. By determining the flow rate at which a particular particle size was in equilibrium at the same radius as the vortex finder radius, a graph was constructed from which one could predict hydroclone performance. Experimental data compared well with these theoretical predictions except at very low flow rates where the vortex patterns break down.

The main drawback of these previous investigations is that they have not yielded any workable equations for determining hydroclone separation efficiency in terms of the hydroclone parameters.

CHAPTER III

STATEMENT OF PROBLEM

The objective of this study was to theoretically develop a set of equations for separation efficiency and pressure drop from which hydroclones could be designed. In addition to this, there was a desire to solve the Navier-Stokes' equations to obtain the expression for the tangential velocity existing in the hydroclone. It was also desired to obtain the cyclone section shape which would yield optimum separation.

A limited number of experimental tests were conducted to attempt to verify, in part, the theoretical equations.

CHAPTER IV

HYDROCLONE VELOCITY FIELD

The flow pattern which exists within a hydroclone is of a very complex nature. Fig. 4-1 is a drawing depicting this pattern. The tangential velocity, v, is the dominant component which exists in the hydroclone. In the region, $r < \delta r_c$, the tangential velocity approaches that of solid body rotation. An exact solution of the Navier-Stokes' equations for the velocity field would be nearly impossible; but fortunately, experimental evidence shows that all three of the velocity components may be assumed to be independent of axial position, thus a two-dimensional solution for the tangential velocity is possible if values are assumed for the radial velocity. In this chapter, an approximate solution will be obtained to the Navier-Stokes' equations for two different hydroclone configurations. Comparisons will be made with experimental data.

First, we must list the assumptions of conditions prevailing within the hydroclone; namely:

1) Steady flow conditions prevail, i.e., derivatives of all variables with respect to time vanish.
2) Tangential velocity is a function of r only and is independent of angular position, ϕ, and axial position, z.
3) Radial velocity is a function of r only and is independent of ϕ and z.
4) The diameter of the inner forced vortex is assumed to be equal to the diameter of the vortex finder.

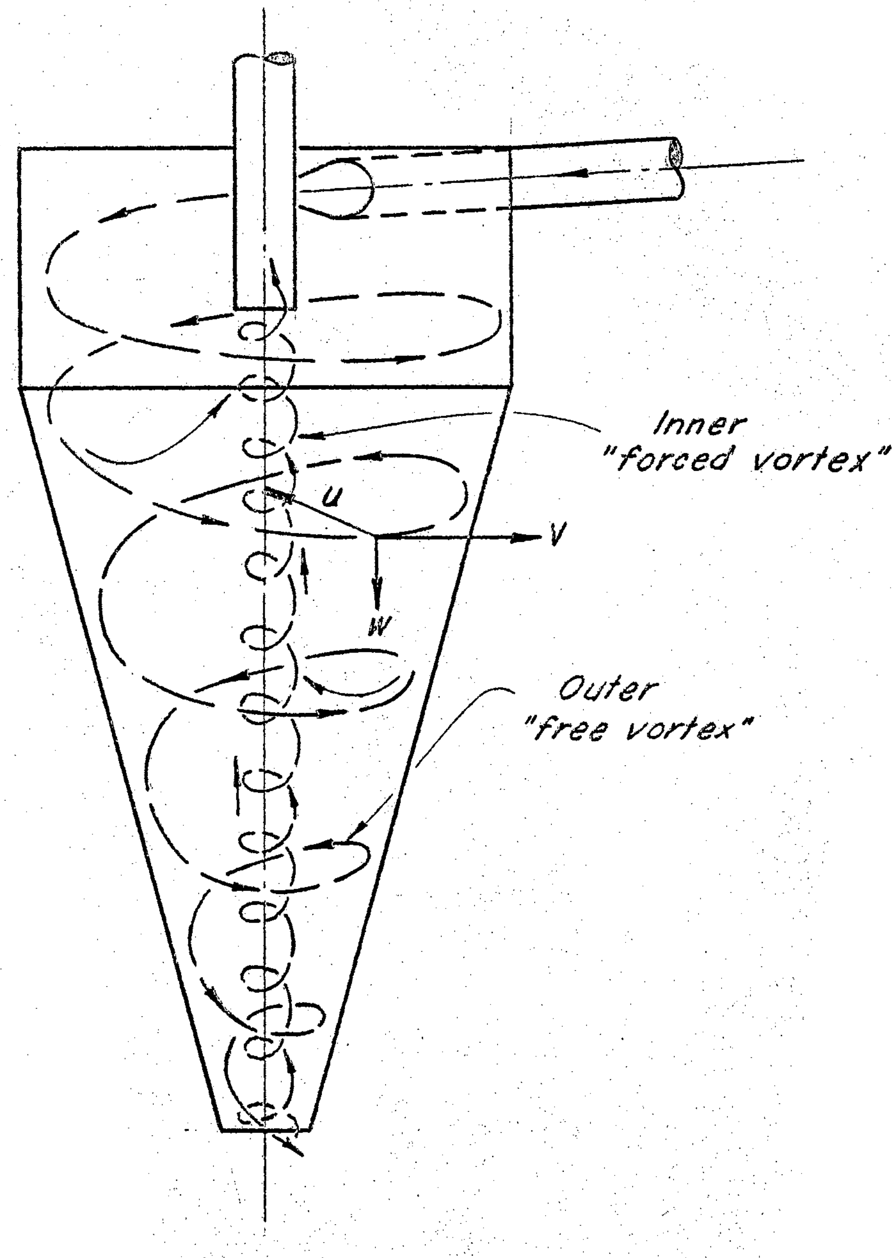

Figure 4-1. Flow Patterns.

From conditions 2) and 3), it is evident that a two-dimensional analysis of the tangential velocity is possible.

Consider the Navier-Stoke's equations for the case of steady flow and polar symmetry. We then have in two dimensions:

$$\frac{u}{r}\frac{\partial(vr)}{\partial r} = \nu \frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial(vr)}{\partial r}\right\} . \qquad (4\text{-}1)$$

The solutions obtained below will be for hydroclones with conic cyclone sections. The only difference between the two configurations under consideration is in their vortex finders. Both types of vortex finders are shown in Fig. 4-2.

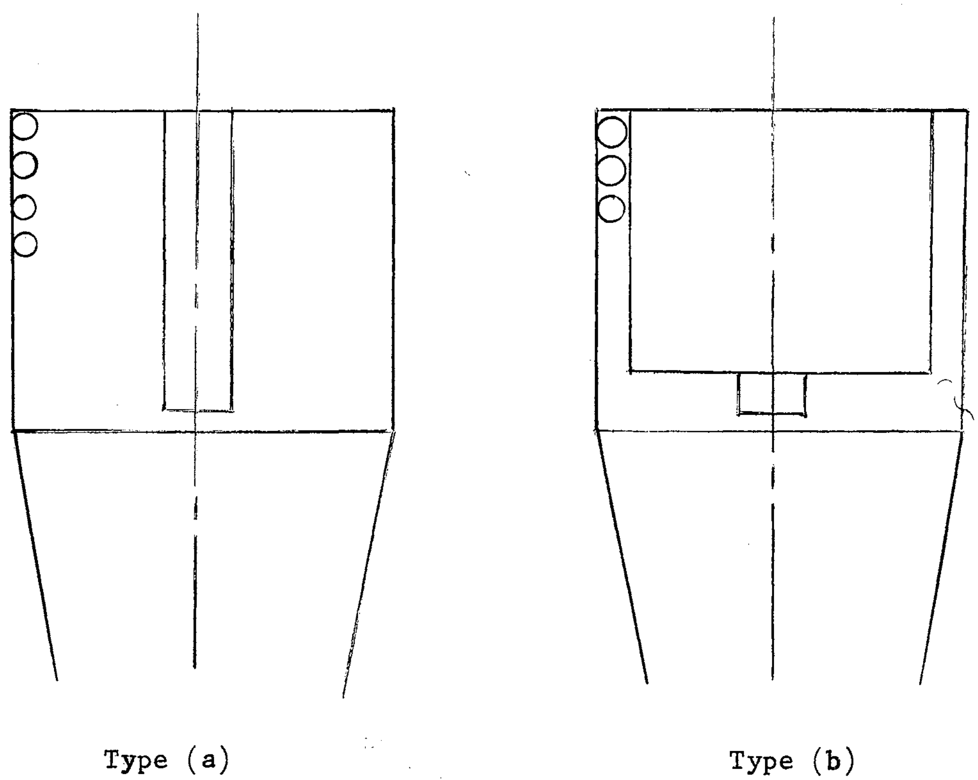

Figure 4-2. Types of Vortex Finders.

Solution for Type A

If we make the assumption that w, the axial velocity, is constant in the region $r > \delta r_c$, then by continuity we would have for the radial velocity

$$u = \frac{-Q(r)}{2\pi r f(r)} \quad , \qquad Q(r) = \frac{(r_c - r)}{(1-\delta)} \frac{Q}{r_c} \tag{4-2}$$

where f(r) is defined in Fig. 1-2. Eq. 4-2 is limited to the region $r > \delta r_c$. One might wonder about the validity of the above assumption, but it will be shown later that the experimental tangential velocity corresponds well enough with the theoretical solution to conclude that the assumption is indeed good. Motivation for the assumption is based on experimental observation.

If we now substitute Eq. 4-1 into Eq. 4-1 and introduce the dimensionless variables

$$R = \frac{r}{r_c} \quad , \quad V = \frac{v}{v_c} \tag{4-3}$$

we have

$$\frac{-A}{R^2} \frac{d(VR)}{dR} = \frac{d}{dR} \left\{ \frac{1}{R} \frac{d(VR)}{dR} \right\} \tag{4-4}$$

where

$$A = \frac{Q \tan \varphi/2}{2\pi(1-\delta) r_c \nu} .$$

Introducing S = d(VR)/dR, Eq. 4-4 becomes

$$\frac{-A}{R} \left\{ \frac{S}{R} \right\} = \frac{d}{dR} \left\{ \frac{S}{R} \right\} . \tag{4-5}$$

In order to completely define the problem we now need to specify the boundary conditions imposed on Eq. 4-5. The first condition is

$$V = 1 \quad \text{at} \quad R = 1. \tag{4-6}$$

In assumption Eq. 4-4 we said the inner forced vorter was the same diameter as the vortex finder. Since the inner forced vortex is defined by V = KR, then the second boundary condition becomes

$$\frac{d\left(\frac{V}{R}\right)}{dR} = 0 \quad \text{at} \quad R = \delta. \tag{4-7}$$

Integrating Eq. 4-5 once, we have

$$S = \frac{d(VR)}{dR} = C_1 R^{-A+1}. \tag{4-8}$$

Integrating Eq. 4-8 yields

$$VR = \frac{C_1 R^{-A+2}}{(2-A)} + C_2 \tag{4-9}$$

or

$$V = \frac{C_2}{R} - \frac{C_1 R^{-A+1}}{(A-2)}. \tag{4-10}$$

Imposing the first boundary condition on Eq. 4-10 yields

$$1 = C_2 - \frac{C_1}{(A-2)}. \tag{4-11}$$

From Eq. 4-10 we have

$$\frac{d\left(\frac{V}{R}\right)}{dR} = \frac{-2C_2}{R^3} + \frac{C_1 A R^{-A-1}}{(A-2)} \quad (4\text{-}12)$$

Now from the second boundary condition, we have

$$0 = -\frac{2C_2}{\delta^3} + \frac{C_1 A \delta^{-A-1}}{(A-2)} \quad (4\text{-}13)$$

Solving for C_1 and C_2 from Eq. 4-11 and Eq. 4-13 yields

$$C_1 = \frac{(A-2)}{\frac{A\delta^{-A+2}}{2} - 1} \quad (4\text{-}14)$$

$$C_2 = 1 + \frac{1}{\frac{A\delta^{-A+2}}{2} - 1} .$$

The solution which has been obtained, defined by Eqs. 4-10 and 4-14, is valid only in the region $\delta \leq R \leq 1$. In the region $0 \leq R \leq \delta$, we have $V = KR$ where K is a constant.

In Fig. 4-3 is a plot of Eq. 4-10 for the case where $\delta = 0.25$. Shown in Fig. 4-4 is a plot of an experimental curve for V obtained by probing the velocity field of a one-inch diameter hydroclone with a ten-degree cone. The fluid used was MIL 5606. Also shown in Fig. 4-4 is a plot of Eq. 4-10 for the case where $A = 3.5$.

It is interesting to note that by solving for ν for this particular case, a value is obtained which is about one hundred times as large as the molecular viscosity of the fluid used. This apparent increase in

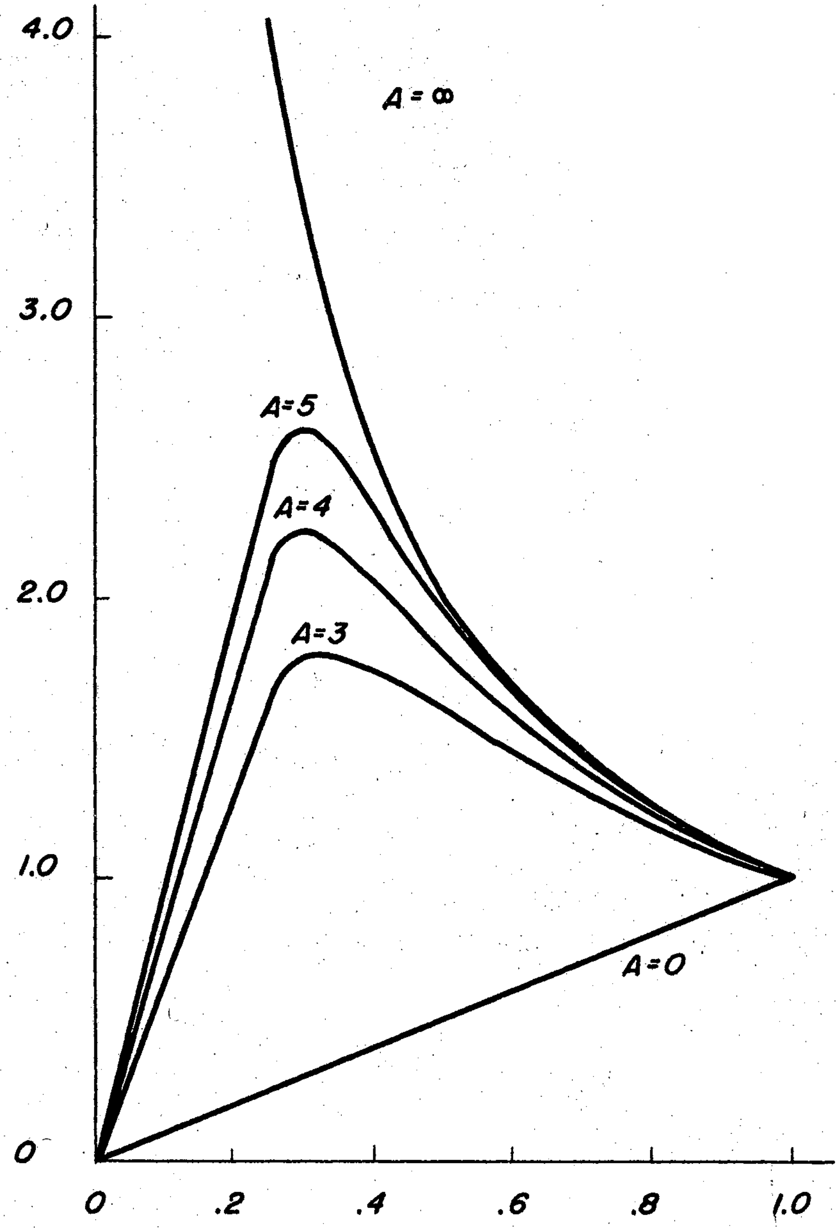

Figure 4-3. Tangential Velocity for Type (a) Vortex Finder.

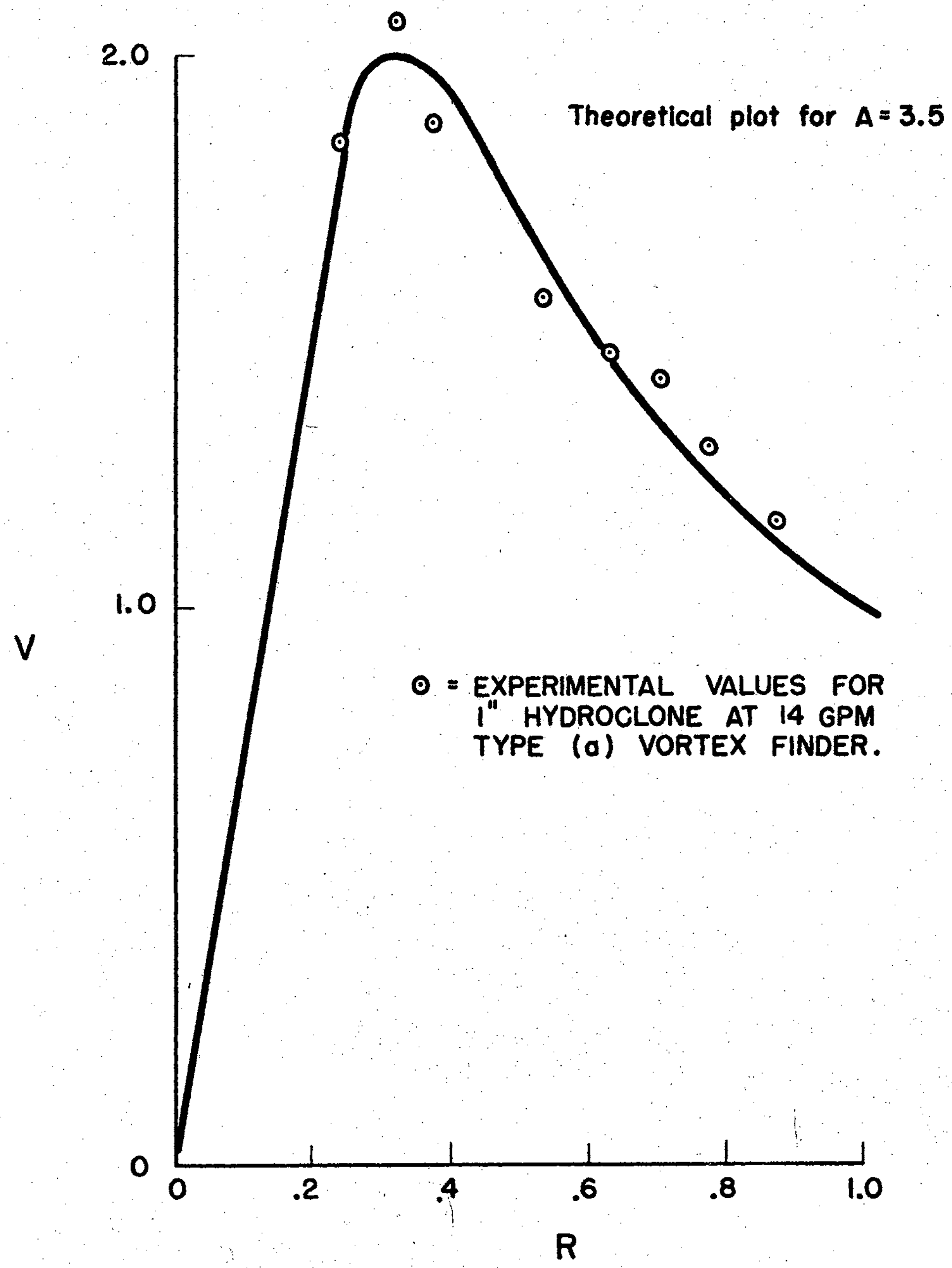

Figure 4-4. Experimental Velocity for Type (a) Vortex Finder.

the viscosity is due to the violent eddying or turbulence which occurs in the hydroclone.

Solution for Type B

For this type, the radial velocity is given approximately by:

$$u = \frac{-Q}{2\pi r f(r)} \quad (4\text{-}15)$$

The radial velocity given in Eq. 4-15 may be approximated by $U = U_o$ = const. without losing much accuracy as may be seen in Fig. 4.5. We now have

$$\frac{-u_o}{\nu}\frac{S}{r} = \frac{d}{dr}\left(\frac{S}{r}\right) \quad (4\text{-}16)$$

where

$$S = \frac{d(vr)}{dr}.$$

Introducing the dimensionless variable $R = r/r_c$, we have

$$\frac{-u_o r_c}{\nu}\frac{S}{R} = \frac{d}{dR}\left(\frac{S}{R}\right), \quad S = \frac{d(VR)}{dR}. \quad (4\text{-}17)$$

Integrating once, yields

$$\ln\left(\frac{S}{R}\right) = -BR + C_3 \quad (4\text{-}18)$$

where $B = \frac{u_o r_c}{\nu}$ or

$$\left(\frac{S}{R}\right) = e^{-BR+C_3}. \quad (4\text{-}19)$$

Substituting for S gives

$$\frac{d(VR)}{dR} = R e^{-BR+C_3}. \quad (4\text{-}20)$$

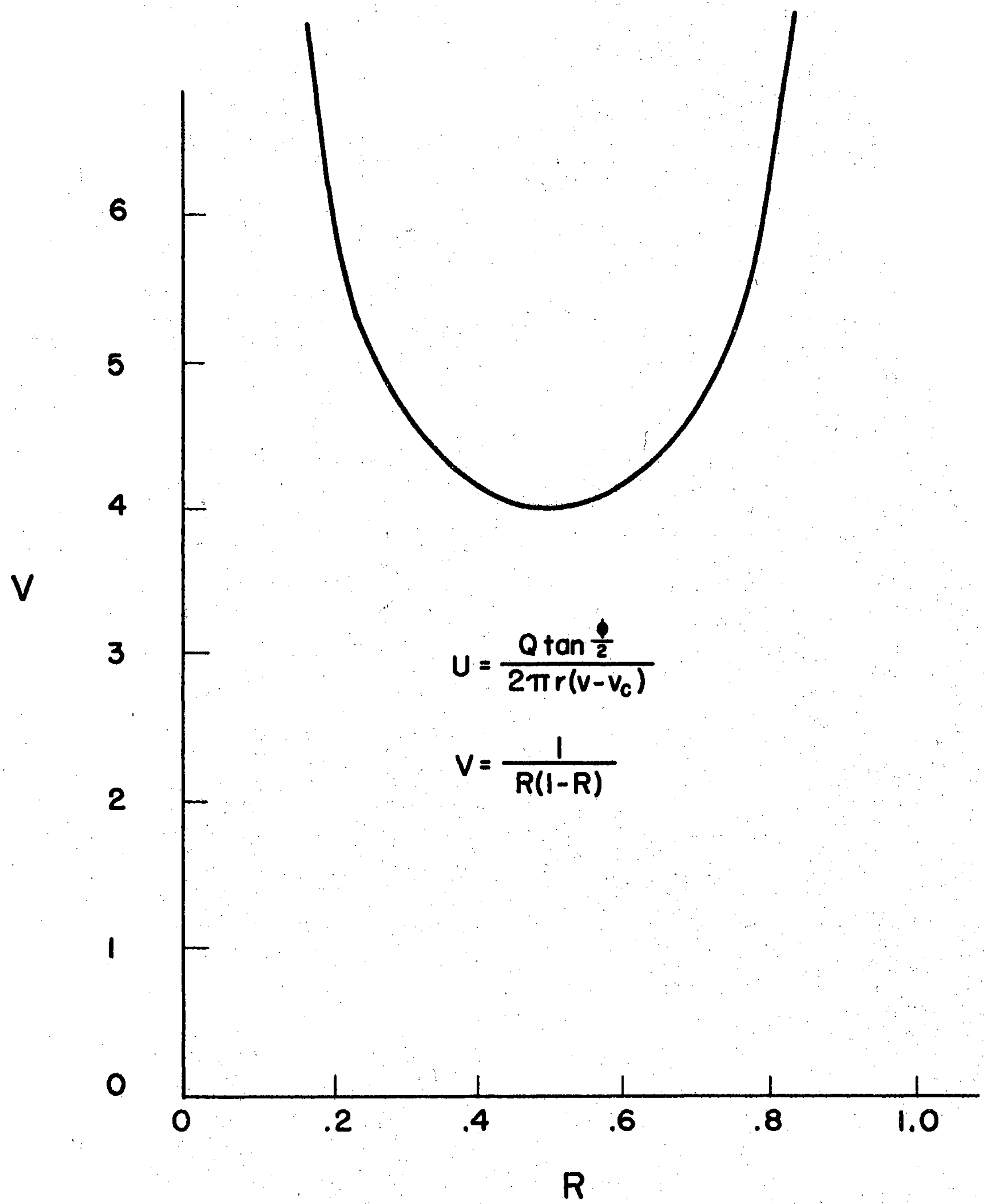

Figure 4-5. Radial Velocity

Integrating again, we have

$$(VR) = e^{C_3}\left\{e^{-BR}\left[-\frac{R}{B} - \frac{1}{B^2}\right]\right\} + C_4 \qquad (4\text{-}21)$$

or

$$V = \frac{C_4}{R} - e^{-BR+C_3}\left[\frac{1}{B} + \frac{1}{B^2R}\right]. \qquad (4\text{-}22)$$

The boundary conditions for this case are the same as in Eq. 4-14. Dividing Eq. 4-22 by R and differentiating yields

$$\frac{d\left(\frac{V}{R}\right)}{dR} = \frac{-2C_4}{R^3} - e^{-BR+C_3}\left[-\frac{1}{BR^2} - \frac{2}{B^2R^3}\right] \qquad (4\text{-}23)$$

$$+\, e^{-BR+C_3}\left[\frac{1}{R} + \frac{1}{BR^2}\right].$$

From the second boundary condition, we have

$$0 = \frac{-2C_4}{\delta^3} + e^{-B\delta+C_3}\left[\frac{1}{\delta} + \frac{2}{B\delta^2} + \frac{2}{B^2\delta^3}\right]. \qquad (4\text{-}24)$$

From the first boundary condition, we have

$$1 = -e^{-B+C_3}\left[\frac{1}{B} + \frac{1}{B^2}\right] + C_4 . \qquad (4\text{-}25)$$

Solving for e^{C_3} from Eq. 4-24 and Eq. 4-25 and equating, we have

$$\frac{2 C_4 e^{B\delta}/\delta^3}{\left[\frac{1}{\delta} + \frac{2}{B\delta^2} + \frac{2}{B^2\delta^3}\right]} = \frac{(C_4 - 1) e^B}{\left[\frac{1}{B} + \frac{1}{B^2}\right]} \quad (4\text{-}26)$$

Solving for C_4 yields

$$C_4 = \frac{1}{(B+1)\left[\frac{1}{(B+1)} - \frac{2e^{B(\delta-1)}}{(B^2\delta^2 + 2B\delta + 2)}\right]} \quad (4\text{-}27)$$

Solving for e^{C_3} from Eq. 4-25, we have

$$e^{C_3} = \frac{(C_4 - 1) e^B}{\left(\frac{1}{B} + \frac{1}{B^2}\right)} \quad (4\text{-}28)$$

In Fig. 4-6 a plot of Eq. 4-22 has been made for the case where $\delta = 0.25$ and B is a variable parameter. In Fig. 4-7 is the plot of an experimental curve for V obtained by probing the velocity field of a one-inch diameter hydroclone with a ten-degree cone. The fluid used was MIL 5606.

The probing of the tangential velocity fields was accomplished by using a small pitot pickup probe inserted into the side of the hydroclone at a point just below the vortex finder. By measuring the velocity pressure with a differential transducer, the velocities could then be computed.

It has been found that on the average about a sixty per cent loss in inlet velocity is suffered by the fluid coming into the hydroclone due to spreading. In terms of the separation efficiency, this represents about the same percentage loss. This is one point of hydroclone design that deserves careful consideration in future study.

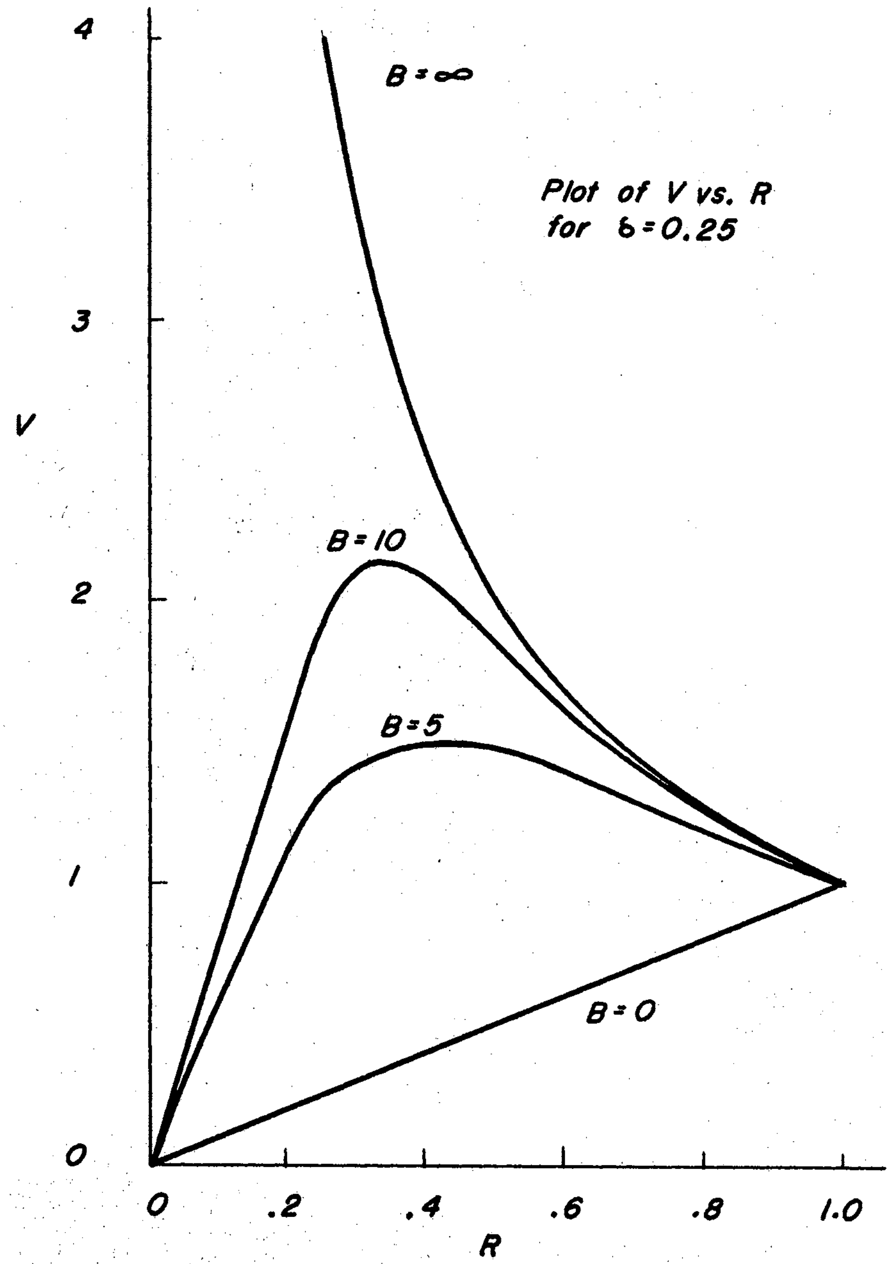

Figure 4-6. Tangential Velocity for Type (b) Vortex Finder

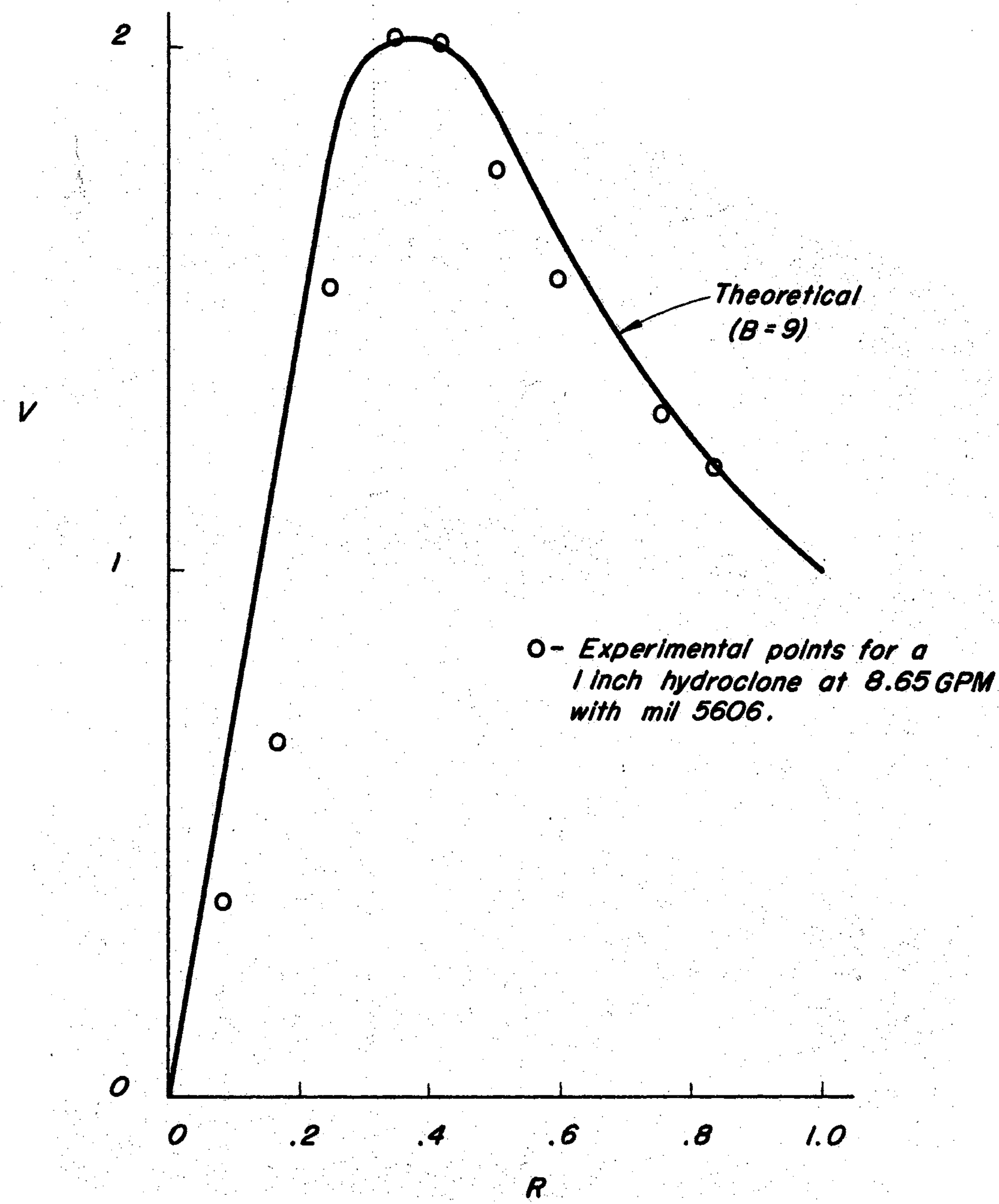

Figure 4-7. Experimental Velocity for Type (b) Vortex Finder.

It is important to remember that this treatment of the tangential velocity is a very simplified account of the actual situation existing within the hydroclone. Secondary flows have been neglected, the boundary layer was not considered, and simplified forms of the axial and radial velocities were assumed. Regardless, however, of the simplification which has been introduced, the good correlation between the analytical and experimental data indicates some usefulness of the solutions presented.

CHAPTER V

DEVELOPMENT OF EXPRESSION FOR THE SEPARATION EFFICIENCY

Let us assume that the hydroclone $D_{50\%}$ particle size is given by the size particle which is in equilibrium at the position $r = r_o$. This appears to be a good assumption since $d_o = 2r_o$ defines the diameter of the inner vortex and particles which are in equilibrium at a radius equal to r_o should report about 50% to the overflow and 50% to the underflow. For a particle in equilibrium, the centrifugal force equals the viscous drag force, or

$$F_{cent.} = F_{drag} \tag{5-1}$$

Assuming all particles to be spherical in shape, each of these forces may be easily calculated. The centrifugal force is given by the product of the particle relative mass and the radial acceleration, or

$$F_c = \frac{\pi}{6} D^3 (\rho_s - \rho_L) \frac{v^2}{r} \; . \tag{5-2}$$

Assuming Stokes' Law to be a valid approximation of the viscous drag, the drag force becomes

$$F_D = 3 \mu \pi D u \; . \tag{5-3}$$

Substituting Eqs. 5-2 and 5-3 into Eq. 5-1 yields

$$\frac{\pi}{6} D^3(\rho_s-\rho_L)\frac{v^2}{r} = 3\pi\mu\, Du\,. \qquad (5\text{-}4)$$

For a particle in equilibrium at $r = r_o$, Eq. 5-4 becomes

$$D^2(\rho_s-\rho_L)\, v_c^2 \frac{V_o^2}{r_o} = 18\, u_o \qquad (5\text{-}5)$$

where $V_o = \dfrac{v_o}{v_c}$.

It has been shown experimentally that in general $v_c \leq v_i$ where

$v_i = \dfrac{Q}{N\pi\beta^2 r_c^2}$. If we now define $v_c\beta_e^2 = v_i\beta^2$ where β_e is the effective dimensionless inlet radius, we have

$$v_c = \frac{Q}{N\pi\,\beta_e^2 r_c^2}\,, \quad u_o = \frac{Q}{2\pi\delta r_c f(r_o)}\,. \qquad (5\text{-}6)$$

Eq. 5-5 now becomes

$$D^2(\rho_s-\rho_L)\frac{Q^2 V_o^2}{N^2\pi^2\beta_e^4 r_c^4 r_o} = \frac{9Q\mu}{\pi\delta r_c f(r_o)}\,. \qquad (5\text{-}7)$$

Solving for D we have

$$D = \frac{3N\beta_e^2}{V_o}\left[\frac{\pi\mu\, r_c^4}{Q f(r_o)(\rho_s-\rho_L)}\right]^{1/2}. \qquad (5\text{-}8)$$

For a conic cyclone section $f(r_o) = \dfrac{(r_c-\delta r_c)}{\tan \varphi/2}$

so that Eq. 5-8 becomes

$$D = \frac{3N\beta e^2}{V_0}\left[\frac{\pi \mu r_c^3 \tan \varphi/2}{Q(\rho_s - \rho_L)(1-\delta)}\right]^{1/2} . \qquad (5\text{-}9)$$

Eq. 5-9 now tells us the size of particle that will be in equilibrium at a radius r_o. Let us now determine the conditions necessary for there to be an absolute separation efficiency. In order for such an absolute separation efficiency to exist, it is necessary for the particle size in equilibrium at $r = r_o$ to increase as R increases;

or

$$\left.\frac{dD}{dR}\right|_{r=r_o} > 0 . \qquad (5\text{-}10)$$

If this were not true, then large particles would have smaller equilibrium radii than small particles. Writing D as a function of R, we would have from Eq. 5-9

$$D(R) = \frac{3N\beta e^2}{V(R)}\left[\frac{\pi \mu r_c^3 \tan \varphi/2}{Q(\rho_s - \rho_L)(1-R)}\right]^{1/2}, \quad \delta \le R \le 1. \qquad (5\text{-}11)$$

Rewriting Eq. 5-11, we have

$$D(R) = \frac{k}{\sqrt{V^2(1-R)}} , \quad k = 3N\beta e^2\left[\frac{\pi \mu r_c^3 \tan \varphi/2}{Q(\rho_s - \rho_L)}\right]^{1/2} . \qquad (5\text{-}12)$$

Differentiating Eq. 5-12 with respect to R, we have

$$\frac{dD}{dR} = \left(-\frac{1}{2}\right)\frac{k}{\left[V^2(1-R)\right]^{3/2}}\left[2V(1-R)\frac{dV}{dR} - V^2\right] . \qquad (5\text{-}13)$$

In order for Eq. 5-10 to be satisfied, we must have

$$\left[2V(1-R)\frac{dV}{dR} - V^2\right] = 0 \quad (5\text{-}14)$$

Eq. 5-14 may be satisfied in two ways:

$$\text{(a)} \quad \frac{dV}{dR} < 0 \quad (5\text{-}15)$$

$$\text{(b)} \quad \frac{dV}{dR} > 0\,, \quad V > 2(1-R)\frac{dV}{dR}\,.$$

If now, Eqs. 5-15 are satisfied at the position R = δ, then Eq. 5-9 is valid for use in computing the $D_{50\%}$ particle size. In Fig. 5-1 a plot has been made of D versus the right hand side of Eq. 5-9. This yields a straight line of slope 1 passing through the origin. Experimental data points have been plotted in Fig. 5-1 for separation efficiencies of 50% and 100%. This experimental data was obtained by injecting AC test dust into the fluid stream above the hydroclone and comparing the particle count data of upstream and downstream samples. Note that in Fig. 5-1 the 50% points seem to fall, in general, around the theoretical 50% line. It may be concluded from this that for the flow ranges used with this particular hydroclone (1 inch diameter with 10° cone) and for MIL 5606 fluid the equation does predict well the $D_{50\%}$ particle size for AC dust. The equation needs to be verified for wide ranges of the variables and for different kinds of contamination to determine exactly the useful limits. Experimental tests of this magnitude were beyond the scope of this study. It should be remembered that certain limitations were imposed on the equation during its development, i.e., the contamination

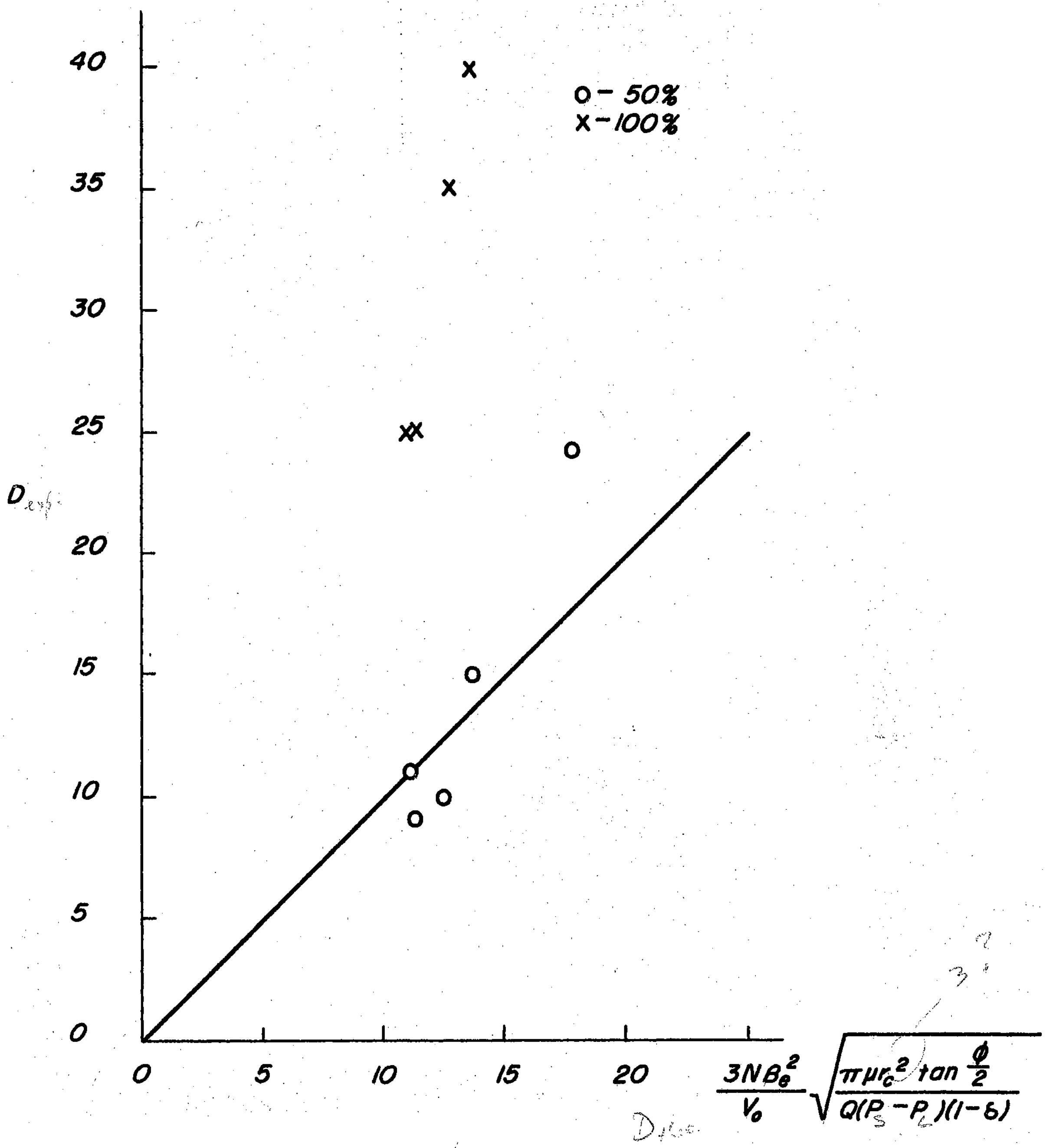

Figure 5-1. Separation Efficiency.

consisted of spherical particles and that Stokes' Law was a valid expression for the viscous drag. The equation would need a correction factor to account for particle shape if other than spherical particles were used. The assumption that Stokes' Law may be used to compute the drag appears to be valid since the relative velocity of the particle in the radial direction yields a Reynolds' number for the particle which is as small as required. One might think that there is inconsistency here since the fluid is turbulent in the hydroclone; however, the relative particle velocity does yield laminar flow of the particle relative to the fluid.

In conclusion, we can say that Eq. 5-9 appears to be a powerful tool in hydroclone design because of the great number of hydroclone parameters which it contains.

CHAPTER VI

HYDROCLONE PRESSURE DROP

The pressure drop across a hydroclone is given by the sum of three terms

$$\Delta p = \Delta p_i + \Delta p_c + \Delta p_o \tag{6-1}$$

in which

Δp_i = pressure drop across inlet

Δp_c = pressure drop across the cyclone section

Δp_o = pressure drop across the overflow.

Considering the inlet(s) and overflow to be nozzles, we can write

$$\Delta p_i = \frac{\rho_L}{2}\left[\frac{Q}{N C_{Di} \pi \beta^2 r_c^2}\right]^2 \tag{6-2}$$

and

$$\Delta p_o = \frac{\rho_L}{2}\left[\frac{Q}{C_{Do} \pi \delta^2 r_c^2}\right]^2 . \tag{6-3}$$

We now need to compute the pressure drop due to the centrifugal field in the cyclone section.

Consider the Euler equation in the r direction, or

$$\rho_L \frac{Du}{Dt} = -\frac{\partial p}{\partial r} + \rho_L F_r \tag{6-4}$$

where F_r is the body force per unit mass in the r direction. Since the radial velocity is negligible compared to the tangential velocity, we may neglect the radial acceleration so that Eq. 6-4 now becomes

$$\frac{dp}{dr} = \rho_L F_r, \qquad (6\text{-}5)$$

or since $F_r = \frac{v^2}{r}$, we now have

$$\Delta p_c = \rho_L \int_{r_0}^{r_c} \frac{v^2}{r} dr \qquad (6\text{-}6)$$

Rewriting Eq. 6-6 in terms of dimensionless variables yields

$$\Delta p_c = \rho_L v_c^2 \int_{\delta}^{1} \frac{V}{R} dR \qquad (6\text{-}7)$$

From Eq. 4-10, Eq. 6-7 becomes for a type (a) vortex finder

$$\Delta p_c = \rho_L v_c^2 \int_{\delta}^{1} \left[\frac{C_2}{R} - \frac{C_1 R^{-A+1}}{(A-2)} \right]^2 \frac{dR}{R} . \qquad (6\text{-}8)$$

Expanding Eq. 6-8 yields

$$\Delta p_c = \rho_L v_c^2 \int_{\delta}^{1} \left\{ \frac{C_2^2}{R^2} - \frac{2 C_1 C_2 R^{-A}}{(A-2)} + \frac{C_1^2 R^{2(1-A)}}{(A-2)^2} \right\} \frac{dR}{R} . \qquad (6\text{-}9)$$

The third term of the above integral is negligible compared with the other two and so will be neglected. Integration now yields

$$\Delta p_c = \rho_L v_c^2 \left\{ \frac{C_2^2}{2} \left[\frac{1}{\delta^2} - 1 \right] + \frac{2 C_1 C_2}{A(A-2)} \left[1 - \frac{1}{\delta^A} \right] \right\} \qquad (6\text{-}10)$$

where C_1 and C_2 are defined in Eq. 4-14. From Eq. 4-6, $v_c = \frac{Q}{N\pi\beta_e^2 r_c^2}$ thus,

$$\Delta p_c = \frac{\rho_L Q^2}{N^2\pi^2\beta_e^4 r_c^4}\left\{\frac{C_2^2}{2}\left[\frac{1}{\delta^2}-1\right]+\frac{2C_1C_2}{A(A-2)}\left[1-\frac{1}{\delta^A}\right]\right\}. \qquad (6\text{-}11)$$

We may now add Eqs. 6-2, 6-3, and 6-11 to obtain Δp, thus

$$\Delta p = \frac{\rho_L Q^2}{2\pi^2 r_c^4}\left\{\frac{1}{(N C_{D_i}\beta^2)^2}+\frac{1}{(C_{D_o}\delta^2)^2}+\frac{1}{(N\beta_e^2)^2}\left[C_2^2\left(\frac{1}{\delta^2}-1\right)-\frac{2C_1C_2}{A(A-2)}\left(\frac{1}{\delta^A}-1\right)\right]\right\}. \qquad (6\text{-}12)$$

Let us now define a dimensionless ΔP given by

$$\Delta P = \Delta P_i + \Delta P_c + \Delta P_o \qquad (6\text{-}13)$$

where

$$\Delta P = \frac{\Delta p}{\rho_L Q/2\pi^2 r_c^4}$$

$$\Delta P_i = \frac{1}{(N C_{D_i}\beta^2)^2}$$

$$\Delta P_o = \frac{1}{(C_{D_o}\delta^2)^2}$$

$$\Delta P_c = \frac{1}{(N\beta_e^2)^2}\left[C_2^2\left(\frac{1}{\delta^2}-1\right)-\frac{2C_1C_2}{A(A-2)}\left(\frac{1}{\delta^A}-1\right)\right].$$

If now values are known for the quantities C_{D_o}, C_{D_i}, β_e, and A in addition to the hydroclone physical parameters r_c, N, and β, then the pressure drop across a given hydroclone may be computed for a given flow rate with a specified fluid. It should be noted that the pressure drop equation which has been presented here was derived for the type (a) vortex finder. Because of the similarity of the tangential velocity profiles for the two types of vortex finders, it would seem that either solution could be used satisfactorily for both vortex finders. Because of this, the simpler of the two solutions was used, i.e., that solution for the type (a) vortex finder.

At the time of this study, not enough experiment equipment was available to check each term of the pressure drop equation individually. It was decided that since for one particular hydroclone configuration, ΔP is a constant, it might be possible to show that ΔP varies as Q^2. If this were true, then the following relation must be satisfied:

$$\ln \Delta P = 2 \ln Q + b \tag{6-14}$$

where b is a constant. In Fig. 6-1 is a plot of ΔP versus Q. The straight line has a slope of 2 and so represents the theoretical curve. Experimental data points have been plotted on the curve also. It should be noted that the theoretical curve is well correlated by the experimental data points. It may be concluded that the pressure drop does vary with the square of the flow rate as predicted by Eq. 6-12. The power of Eq. 6-12 lies in its ability to show the effect of changes in the hydroclone dimensions on the pressure drop. Eq. 6-12 might also be used in conjunction with the separation efficiency equation to theoretically optimize the hydroclone parameters involved. In this way, much experimental work might be eliminated.

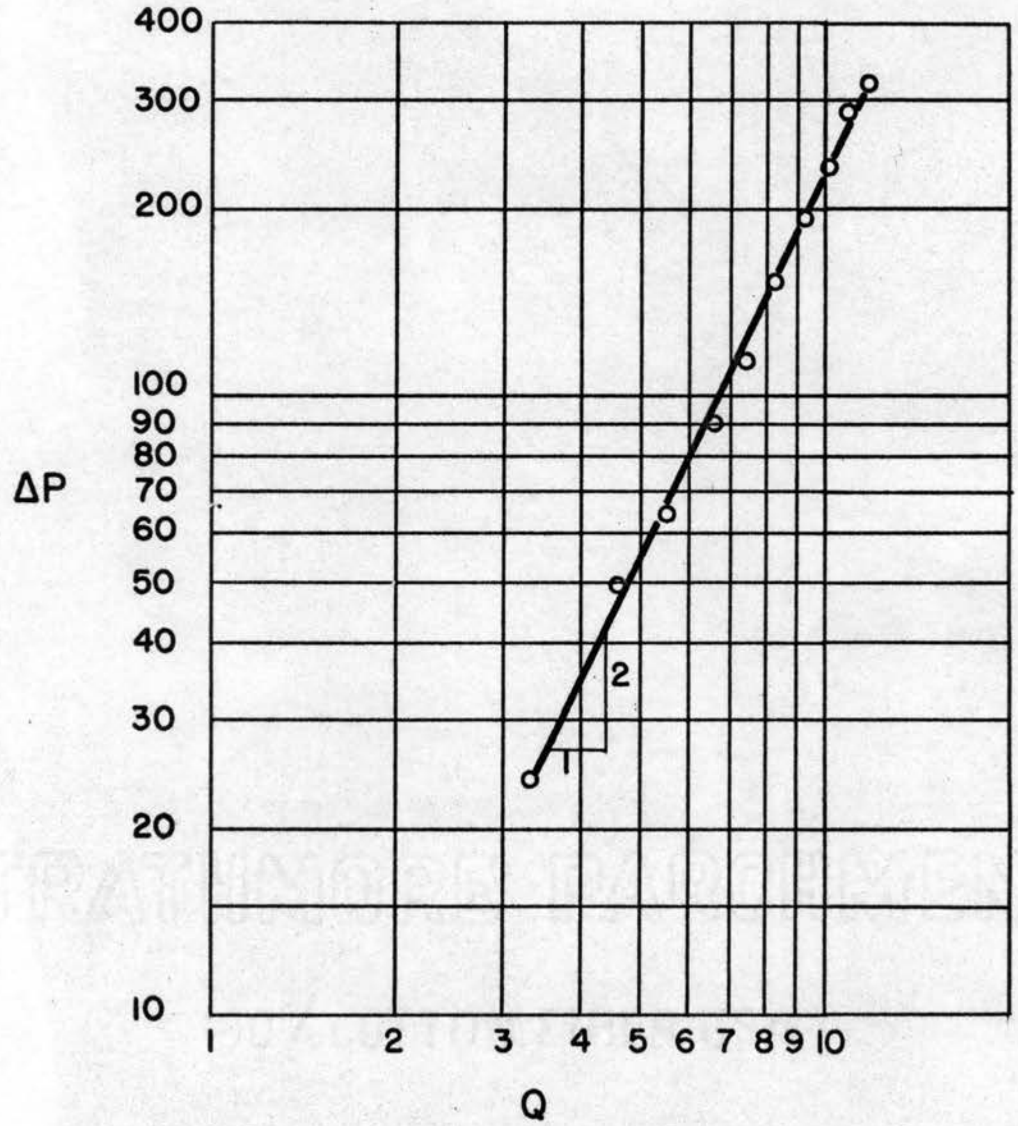

Figure 6-1. Pressure Drop.

CHAPTER VII

DEVELOPMENT OF EXPRESSION FOR THE OPTIMUM CYCLONE SECTION SHAPE

From Eq. 5-4, we have seen that for a particle in equilibrium at radius r we have

$$D = \left[\frac{18\mu\, u r^3}{(\rho_s-\rho_L)(vr)^2}\right] . \tag{7-1}$$

From Eq. 4-5, we have for type (b) vortex finder

$$u = \frac{Q}{2\pi r f(r)} \; , \quad \delta r_c \le r \le r_c . \tag{7-2}$$

Substituting Eq. 7-2 into Eq. 7-1, we have now

$$D = 3\left[\frac{Q u r^2}{\pi(\rho_s-\rho_L) f(r) (vr)^2}\right]^{1/2} = k\left[\frac{r^2}{f(r)(vr)^2}\right]^{1/2} . \tag{7-3}$$

A necessary condition for D to be a minimum with r constant is

$$\frac{\partial D}{\partial f(r)} = 0 . \tag{7-4}$$

Taking the partial derivative of Eq. 7-3 with respect to f(r), we have

$$\frac{\partial D}{\partial f(r)} = \left(-\frac{1}{2}\right) K \sqrt{r^2}\, \frac{\left[(vr)^2 + 2 f(r)(vr)\frac{\partial (vr)}{\partial f(r)}\right]}{\left[f(r)\,(vr)^2\right]} . \tag{7-5}$$

In order that the condition of Eq. 7-4 be satisfied, we must have

$$\left[(vr)^2 + 2f(r)\,(vr)\frac{\partial(vr)}{\partial f(r)}\right] = 0 \tag{7-6}$$

or

$$\frac{\partial(vr)}{\partial f(r)} = -\frac{(vr)}{2f(r)}\,. \tag{7-7}$$

The governing differential which describes (vr) in terms of r and f(r) for $\delta r_c \leq r \leq r_c$ is given by Eq. 4-1, or

$$\frac{-B}{r^2 f(r)}\frac{d(vr)}{dr} = \frac{1}{r}\frac{d^2(vr)}{dr^2} - \frac{1}{r^2}\frac{d(vr)}{dr} \tag{7-8}$$

where $B = \frac{Q}{2\pi\nu}$.

But

$$\frac{d(vr)}{dr} = \frac{\partial(vr)}{\partial f(r)}\frac{df(r)}{dr} = -\frac{(vr)}{2f(r)}\frac{df(r)}{dr} \tag{7-9}$$

and

$$\frac{d^2(vr)}{dr^2} = -\frac{1}{2}\left[\frac{(vr)}{f(r)}\frac{d^2f(r)}{dr^2} + \frac{df(r)}{dr}\,\frac{\left(f(r)\frac{d(vr)}{dr} - (vr)\frac{df(r)}{dr}\right)}{f^2(r)}\right]$$

$$\frac{d^2(vr)}{dr^2} = -\frac{1}{2}\left[\frac{(vr)}{f(r)}\frac{d^2f(r)}{dr^2} - \frac{1}{f^2(r)}\frac{df(r)}{dr}\left(\frac{f(r)}{2}\frac{(vr)}{f(r)}\frac{df(r)}{dr} + (vr)\frac{df(r)}{dr}\right)\right]. \tag{7-10}$$

$$\frac{d^2(vr)}{dr^2} = \frac{(vr)}{2}\left[\frac{1}{f(r)} \cdot \frac{d^2 f(r)}{dr^2} - \frac{3}{2}\frac{1}{f^2(r)}\left(\frac{df(r)}{dr}\right)^2\right].$$

Substituting Eqs. 7-9 and 7-10 into Eq. 7-8, we have

$$\frac{B}{rf(r)}\frac{(vr)}{2f(r)}\frac{df(r)}{dr} = \frac{-(vr)}{2}\left[\frac{1}{f(r)} \cdot \frac{d^2 f(r)}{dr^2} - \frac{3}{2}\frac{1}{f^2(r)}\left(\frac{df(r)}{dr}\right)^2\right] + \frac{1}{r}\frac{(vr)}{2f(r)}\frac{df(r)}{dr}. \tag{7-11}$$

Simplifying Eq. 7-11, we get

$$\frac{d^2 f(r)}{dr^2} = \frac{3}{2}\frac{1}{f(r)}\left(\frac{df(r)}{dr}\right)^2 + \frac{1}{r}\frac{df(r)}{dr}\left(1 - \frac{B}{f(r)}\right). \tag{7-12}$$

Let us assume a solution to Eq. 7-12 of the form

$$f(r) = C_0 + C_1 r^n . \tag{7-13}$$

Substitution of Eq. 7-13 into Eq. 7-12 shows that Eq. 7-12 will be satisfied if

$$C_0 = -B/6 \quad , \quad n = -4 . \tag{7-14}$$

Thus, Eq. 7-13 now becomes

$$f(r) = -B/6 + C_1 r^{-4}$$ where C_1 is arbitrary. (7-15)

The arbitrary constant C_1 may be evaluated from the boundary condition

$$f(r_c) = \alpha r_c . \tag{7-16}$$

We now have

$$f(r) = \left(\alpha r_c + \frac{B}{6}\right)\left(\frac{r_c}{r}\right)^4 - \frac{B}{6} \quad (7\text{-}17)$$

It should be noted that this expression is valid only in the region $\delta r_c \leq r \leq r_c$ which was the limiting condition placed on Eq. 7-2. Eq. 7-17 now is an expression for the cyclone section shape which will yield a minimum particle size in the hydroclone overflow for constant values of Q, v, and r_c. In Fig. 7-1 is shown a plot of Eq. 7-17 for the case where $\alpha = 0.05$ and $B/6r_c = 0.01$.

It would be of interest now to compute the value of (vr) for this optimum cyclone section shape. From Eq. 7-9 we have the relation between f(r) and (vr)

$$\frac{d(vr)}{dr} = -\frac{(vr)}{2f(r)}\frac{df(r)}{dr} \quad . \quad (7\text{-}18)$$

Substituting Eq. 7-17 into Eq. 7-18 yields

$$\frac{d(vr)}{dr} = \frac{2(vr)\left(\alpha r_c + B/6\right)\left(\frac{r_c}{r}\right)^4 \cdot \frac{1}{r}}{\left(\alpha r_c + B/6\right)\left(\frac{r_c}{r}\right)^4 - B/6} \quad . \quad (7\text{-}19)$$

Integration of Eq. 7-19 yields

$$(vr) = C\frac{R^2}{\left|E - R^4\right|^{1/2}}, \quad E = \frac{6\alpha r_c}{B} + 1 \quad (7\text{-}20)$$

where C_1 is an arbitrary constant. From the boundary condition $(vr)\ r = r_c = v_t r_c$ we have

$$(v_c r_c) = \frac{C}{\left|E - 1\right|^{1/2}} \quad (7\text{-}21)$$

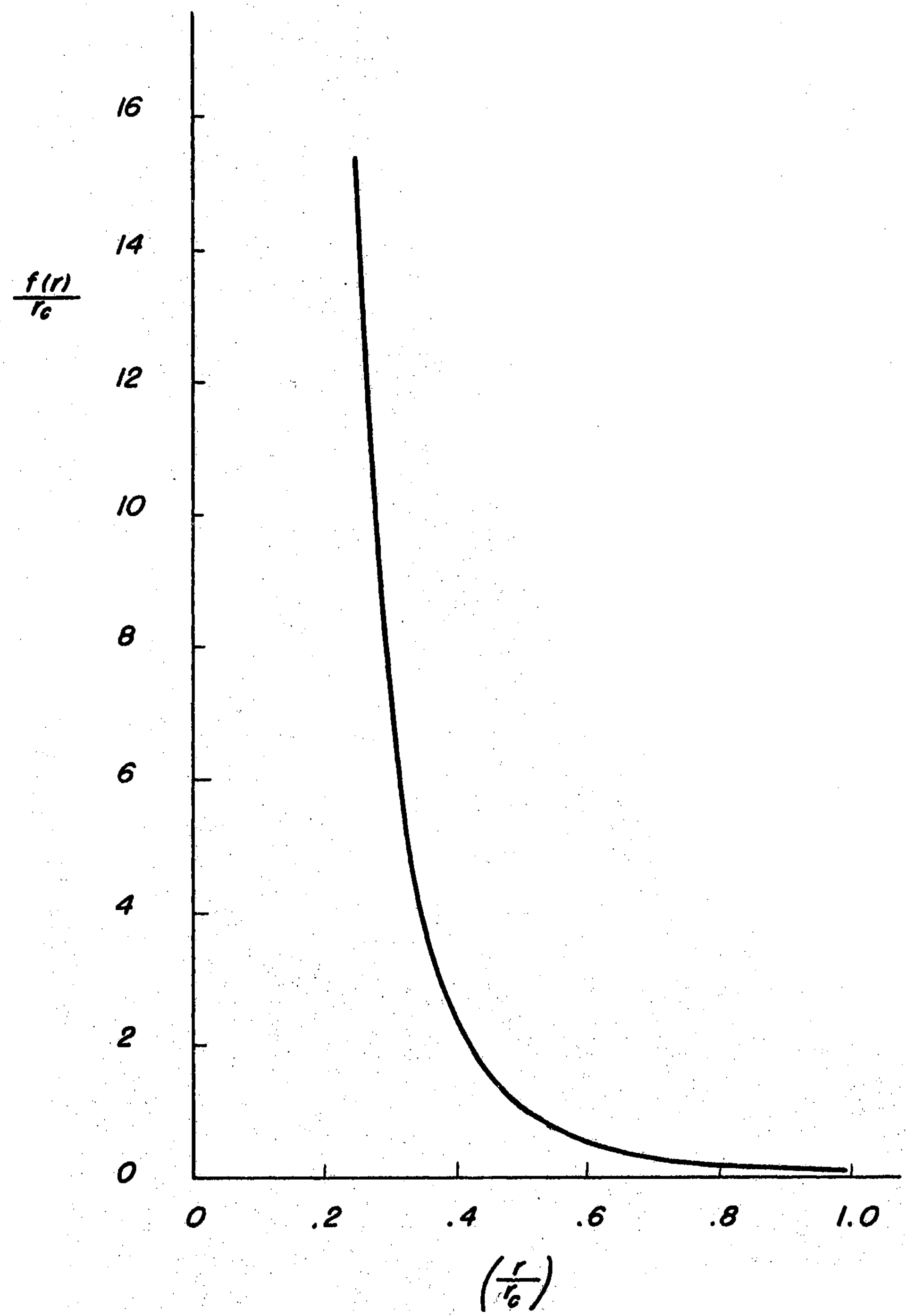

Figure 7-1. Optimum Shape.

thus,

$$(vr) = (v_c r_c) \frac{|E-1|^{1/2}}{|E-R^4|^{1/2}} \cdot R^2 \qquad (7\text{-}22)$$

or finally,

$$V = \left[\left| \frac{E-1}{E-R^4} \right| R \right]^{1/2}, \quad \delta \leq R \leq 1 . \qquad (7\text{-}23)$$

In Fig. 7-2 is shown a plot of Eq. 7-23 for the case where $E = 0.03$. Also plotted in Fig. 7-2 is the tangential velocity field for a potential vortex. The theoretical tangential field for the optimum cyclone section shape is thus predicted to be stronger than for that of a potential vortex which is the limiting case for the tangential field of a cone-shaped cyclone section.

Comparing Fig. 7-2 with Fig. 4-7, it may be seen that for R = 0.3 the value of V for the optimum shape hydroclone is about 1.35 times as large as V for a conventional hydroclone. In terms of separation efficiency, this means that theoretically the optimum shape hydroclone will yield about 25% better efficiency than a conventional hydroclone. In actual practice, this will probably never be achieved as boundary layer effects and other friction losses have been neglected. As of this time, no experimental data has been obtained to indicate the actual performance of a hydroclone with this theoretically optimum shape.

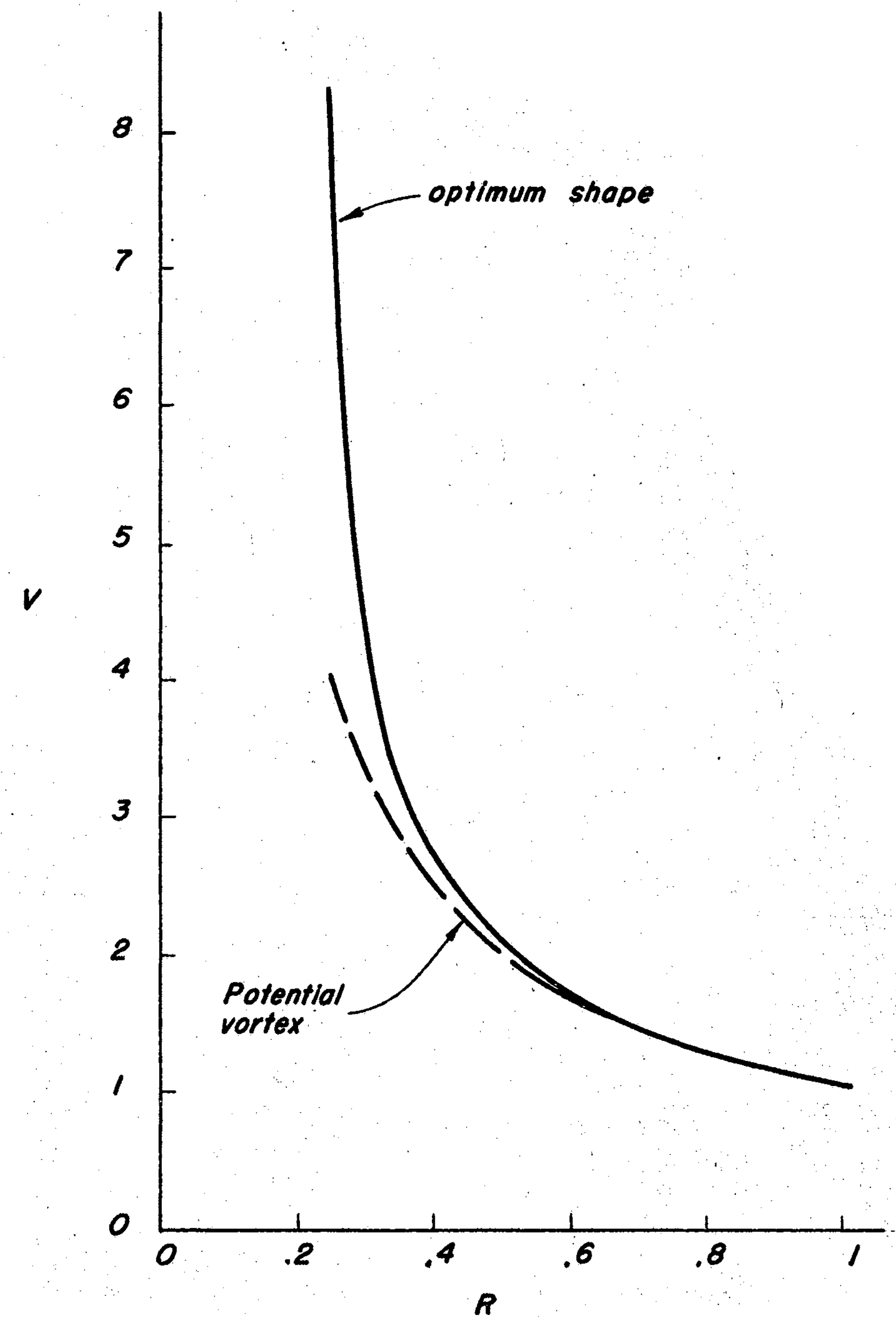

Figure 7-2. Tangential Velocity for Optimum Shape.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

As a result of the experimental studies which were conducted, it may be concluded that the analytical expressions which have been developed for the separation efficiency and pressure drop are accurate for the range of variables used in the tests. For other ranges of these variables (flow rate, hydroclone radius, viscosity, etc.), it is not known at this time if the equations are accurate. It should be noted that there was also excellent agreement between the theoretical and experimental tangential velocity profiles. Note also that because of the similarity between the solutions given for the two types of vortex finders (See Figures 4.3 and 4.6), one might use the simpler of the two solutions, i.e., Eq. 4-10, for both types of vortex finders and not introduce any great errors. As of this time, no conclusive experiments have been conducted to determine the effectiveness of the theoretically derived optimum cyclone section shape; however, calculations do indicate that the maximum possible gain in efficiency over a conventional hydroclone is of the order of 25%.

The feeling of the author is that the main utility of the equations derived in this thesis are to indicate the effect of the various operating variables upon hydroclone operation. If sufficient experimental data is obtained to completely verify these relations, then tables or curves of data could be prepared from which hydroclones could be completely designed for any new application which should arise.

Another utility of these equations would be in the optimization of such parameters as the overflow diameter, the inlet diameter, the cone angle, and the hydroclone size. This would be very beneficial in the elimination of laborious experimental tests.

It is the feeling of the author that there are several great needs for future investigations. These are listed below:

I) Extensive experimental tests to prove or disprove the theoretical equations.

II) Theoretical and experimental optimization of the hydroclone design parameters.

III) Extensive studies to develop more efficient inlet configurations.

IV) Determination of the effects of secondary flows and boundary layer effects upon hydroclone operation.

V) Better underflow design configurations.

The field of the hydroclone separator is not yet touched as far as applications and developments are concerned. The future is bound to yield new and exciting possibilities, but not without many hours of study and experimentation.

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