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FORECASTING THE VOLUME OF STORM RUNOFF

USING METEOROLOGICAL PARAMETERS

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ABSTRACT

FORECASTING THE VOLUME OF STORM RUNOFF

USING METEOROLOGICAL PARAMETERS

by Robert Hasbrouck Cartmill

Major Professor: Jimmie F. Harp, Ph.D.

The volume of runoff from a storm is that small residual volume which remains after the amount of water which infiltrates the soil and is stored in small depressions is subtracted from the total volume of rainfall. An accurate forecast of runoff volume requires an accurate measurement of rainfall and good estimates of the infiltrated volume and to a lesser degree the amount of depression storage.

This work develops a method of forecasting runoff volume without reliance on any empirically derived relationships between meteorological parameters and recorded runoff measurements. The method is therefore applicable to areas where there are no existing meteorological or hydrological records.

This method determines the infiltration rate by use of an approximate solution to the soil moisture diffusion equation. This solution requires knowledge of the existing soil moisture content, the saturated soil mositure content, and the saturated hydraulic conductivity at all depths of the root zone. The variable amount of soil moisture in nine distinct layers of soil is determined daily by maintaining a water budget of the 51 inch layer of soil which is assumed to constitute the root zone. The water budget contains the factors of rainfall, interception losses, runoff, the redistribution of infiltrated rain soon after infiltration, drainage from each layer, and evapotranspiration.

The evapotranspiration factor is determined by first computing from meteorological measurements the evapotranspiration rate provided that moisture were freely available (potential evapotranspiration). This potential rate is then reduced by coefficients which reflect the maturity of the plants and their response to soil moisture deficiencies. The water removed by transpiration is apportioned to each layer by considering the root density of the layer.

After the parameters required to determine the infiltration rate are available, the infiltration rate is compared with the rainfall rate every minute. The excess of rainfall over infiltration is then consigned first to depression storage and then to runoff. This method was applied to two major and thirteen smaller storms over a 208 square mile watershed in South Central Oklahoma. The runoff forecast by the method was compared with the measured runoff from the watershed. Conclusions are reached concerning the area of applicability of the method, the instrumentation required, and the limits of accuracy of the method. Additional comment is made on the advances required in the basic contributing sciences in order to improve the method. Suggestions are made for uses of the ancillary data produced by the method--daily values of soil moisture, potential evapotranspiration, and drainage from the root zone.

PART I - THEORY

CHAPTER 1

INTRODUCTION

The problem of accurately forecasting the runoff hydrograph of storm rainfall has been one of the major problems concerning hydrologists since the development of hydrology as a scientific discipline in the early years of this century. The problem is essentially in two parts. First, the volume of water which will runoff must be estimated and secondly, the time distribution of this volume passing various places must be forecast. The second part of this problem has had considerably more study and effort devoted to it than the first. Particularly the problem of routing hydrographs downstream has been solved by various methods with great accuracy. This problem has been able to draw on the science of hydraulics which is amenable to mathematical study and has had a long history of development. The first problem of estimating what volume to route downstream, however, involves many complex interrelationships between the sciences of soil physics, meteorology, and plant physiology. All of these sciences are much less developed than hydraulics and have had a much shorter period of develcpment. The result has been that hydrologists are now accurately routing downstream very poor estimates of runoff volume. By far the largest errors in hydrograph forecasting arise from poor estimates of runoff volume.

Some of the consequences of poor volume estimates are mitigated by time. In a large river system, hydrograph forecasts near the mouth of the river can be made by waiting until the flow at the upstream stations has been accurately measured. The problem then degenerates into routing these upstream hydrographs downstream to the mouth. Historically, this has provided adequate time to make the necessary preparations for the rise in the river at the major economic areas located downstream. Recently however, urban centers have spread into smaller watersheds and many flood control structures have been built on small watersheds to afford protection. On these smaller watersheds there is often no suitable site to locate an upstream gaging station that will provide adequate information. In addition, the time interval between the storm and the resulting runoff hydrograph is very short. On well-developed large watersheds which have multiple purpose reservoirs constructed on them, the economic value of early forecasts becomes important. For example, if the power pools of hydroelectric facilities are near their upper limit, early decisions on the commencement of flood releases or secondary power releases are of great economic importance. The detailed inflow hydrograph into such a reservoir is not nearly as important as the total volume of flow to be expected in the next week or ten days. The earlier this is known, the greater is the economic benefit that can be realized from the runoff.

This paper will treat the subject of runoff volume forecasting. Recent advances in the sciences affecting this problem will be synthesized to produce a logical method of volume forecasting. The purpose of this paper will be to derive a scheme which is based on available soil, hydrological, and meteorological data which would reasonably be expected to be available to any engineer concerned with the problem. While extensive special instrumentation will be used to develop the scheme, it will be used only for development and evaluation of the method. The method will be designed to be practical and to be developed from data which are now routinely available.

The basic approach to be used will be that of maintaining a soil moisture budget for the drainage basin under consideration. This is a most logical method when it is considered that the disposition of rainfall must either be surface retention, subsurface drainage, or runoff. Obviously the quantity of runoff is dependent on the quantities of water consumed by the other two processes. Reasonable estimates of subsurface drainage in turn depend on the available capacity of soil to hold water. Measurement of soil moisture (and hence storage) over an entire drainage basin is not practicable. Individual measurements are not only subject to instrumentation errors but are really only representative of the conditions at the point of measurement. The approach in this work will be to deduce what the soil moisture of the basin must be from the basic physics governing the sciences of soils, botany, and meteorology based on relatively accurate measurements of the meteorological parameters.

This approach is obviously interdisciplinary. The degree of expertise that one person can bring to the problem is limited. Since the major problem is the rate of moisture flux through the soil, plant, air system and it is at the ground level where there is complete interaction, it is at the surface of ground and air or air and plant where most of the effort will be concentrated. Careful assumptions about the soil and plants must be made. But basically it will be assumed that these factors are constant or vary cyclically with a one year period and that the meteorological conditions vary widely. The total moisture holding capability of the soil once carefully ascertained will remain constant. Likewise the distribution of the various types of vegetation growing in the basin will be assumed constant. This must be done to hold the problem to manageable proportions. Undoubtedly there will be a great temptation to investigate many details of great interest in the fields of soil physics and plant physiology, but the task of this effort must be to determine what is essential in these fields to contribute to the answer. However, careful and mature judgement will be needed if the assumptions made are not to vitiate the results.

A brief description of current forecasting procedures in use is given below.

Antecedent Precipitation Index (API). This scheme is used extensively by the U.S. Weather Bureau and the U.S. Army Corps of Engineers. This procedure is described in detail in references (58) and (50). A variation of this scheme is used by the Soil Conservation Service of the U.S. Department of Agriculture (89). This method uses the parameters of storm duration, total rainfall, antecedent precipitation, and time of year. It is a graphical correlation of these variables with the measured runoff from a specific area. A function of Antecedent Rainfall and time of year is used as a soil moisture index.

Index Stations. Occasionally a stream gaging station is used as a measure of soil moisture conditions upstream. A simple rainfall-runoff relationship can be developed using the pre-storm stream flow as an index.

Infiltration. This approach attempts to construct a curve of infiltration rate vs. time which is compared with a plot of rainfall intensity vs. time. Rainfall intensities in excess of infiltration rates are presumed to contribute to surface storage and runoff. This approach has had wide study because it is the rational way to solve the problem. However, because of many complications and the large amount of calculation required it has not been widely adopted. References (58) and especially (9) present a good summary of the method. This method will be described in detail in Chapter 3.

Clark Method. C. O. Clark (15) has developed an ingenious procedure that requires only the use of some reporting streamflow gages in a reach of a river. The volume of runoff in the channel is determined by current gage readings and the volume of runoff yet to enter the channel is estimated by the rate of rise of the streamflow. Where the necessary gages are available, this procedure gives good results.

Other Methods. A good summary of several different methods is given in reference (64).

All of the above procedures have the following points in common:

a. They are empirical to the extent that a past runoff record is required and the scheme developed can be applied only to the basin studied or hopefully to similar basins in the same geographical area.

b. No water budget system is employed. The sole objective is to forecast runoff without a complete hydrologic description of the basin.

Recently, attempts have been made to provide a more complete hydrologic model of a basin and to introduce more pertinent factors into these models to obtain better forecasts. Two recent models are by Kohler and Richards (52) and Knisel, Baird, and Hartman (49). Both of these models use soil moisture accounting procedures in a two layer soil model. They differ primarily in their computation of evapotranspiration losses. The former uses calculations from meteorological data and the latter uses measured pan evaporation. Both of these schemes retain empirical features since they are not based on general physical considerations.

Recent development of computers and their widespread availability to even small organizational units raises the possibility that a broadly based detailed model of a basin could be constructed. Because the calculations involved no longer preclude practical use of such a model, it could find general acceptance. This is the objective of this paper. This work develops a model which is applicable to medium to coarse textured soils of the prairie grasslands of the southern Great Plains.

CHAPTER 2

THEORY OF THE WATER BUDGET AND THE METEOROLOGICAL APPROACH

The water budget method of solving the runoff forecast problem is based on a very simple basic consideration. The total volume of water falling on a watershed in a given period of time must be consumed in three ways. The water will either be stored temporarily on the surface, infiltrate the surface or runoff. This gives the relation:

$$\text{Runoff} = \text{Rainfall} - \text{Infiltration} - \text{Temporary Storage} \tag{2-1}$$

At the time of the rainfall event, the rainfall can be measured with some degree of accuracy. The only other term in the equation that can be accurately measured over a wide area is runoff after the storm. Since it is runoff that is desired to be forecast, it is necessary to estimate its volume by estimating the volume of water infiltrated and temporarily stored and considering runoff as a residual quantity. In this paper runoff will be considered a dependent variable and a function of the other three independent variables which are themselves functions of many other things and functionally related to each other.

Serious errors in a runoff forecast can arise in two basic ways: there can be an error in the measurement of the volume of rainfall; or there can be an error in the estimate of the other two independent variables. The seriousness of these errors can be simply illustrated. If the true rainfall on a watershed were 1.00 inches and .90 inches infiltrated or was stored temporarily, then the runoff would be .10 inches. If the rainfall were erroneously measured as 1.10 inches (an error of 10%), then even if the infiltration and temporary storage were correctly estimated at .90 inches the estimate of runoff would be .20 inches (an error of 100%). This is an inherent difficulty with a water budget approach and can lead to extremely wild estimates of runoff. The same magnitude of error would apply to a correct measurement of rainfall and a small error in the estimate of the other two independent variables.

In order to test a rational water budget model, it is essential to have accurate measurements of rainfall and runoff from past records so that an accurate model of the infiltration and temporary storage phenomena can be devised. In this study it will be presumed that rainfall measurements are made with a high degree of accuracy. This is in fact the case (see Chapter 6). Likewise, it will be presumed that the measured downstream runoff is accurate. However, the definition of runoff is somewhat imprecise. Linsley, Kohler, and Paulhus (58) divide hydrographs (water-flow vs. time graphs) into the following various components:

> Surface runoff. That water which reaches the streams by traveling over the soil surface is called surface runoff. It is important to emphasize in this connection that the term "stream" includes, not only the larger permanent streams, but also the tiny rills and riverlets which carry water only during and immediately after rains or periods of snowmelt. Surface runoff involves, therefore, not long distances of overland flow, but only the relatively short distances to the nearest minor channel.
>
> Interflow. A portion of the water which infiltrates the soil surface moves laterally through the upper soil horizons until its course is interrupted by a stream channel or until it returns to the surface at some point downslope from its point of infiltration.
>
> Groundwater flow. Whenever the soil in the zone of aeration contains sufficient moisture to permit the passage of gravity water downward, a portion of the rainfall reaches the

groundwater table....

Groundwater flow follows a more devious route to the stream than any of the other components. Its movement is restricted by the percolation rates ordinarily experienced. As a result, the water volume represented by groundwater accretion for a particular storm is discharged into the stream over a long period of time.

Channel precipitation. A fourth source of stream flow is that precipitation which falls directly on the water surfaces of lakes and streams.

This breakdown of the hydrograph components is further complicated by the fact that water which originally entered the stream as surface runoff may fill a channel which is quite permeable and unsaturated. Water will then flow into the banks of the river and add to the groundwater of the alluvial plain. This water will then be discharged days or weeks later as a slow release from groundwater.

These complications make the direct application of equation (2-1) difficult and point out the need for a complete water budget of the whole river basin. No measurement of the components of flow can be made at their source. They must be inferred from hydrograph analysis and other indirect measurements. No description of hydrograph analysis will be presented here. References (58) and (14) are but of few which treat this subject. The necessary specific details will be given in Chapter 7.

The essence of the water budget method is to continuously satisfy the basic hydrologic equation.

$$\text{Inflow} = \text{Outflow} + \text{Change in Storage} \tag{2-2}$$

Inflow is only in the form of precipitation, outflow is in the form of streamflow past the downstream extremity of the watershed and evaporation from the surface and transpiration from plants. The outflow is a function of the distribution of the storages in different zones (i.e., root zone, groundwater, etc.). Consequently, the water budget method requires an accounting of the water volume in each zone.

In the area studied, the volume of outflow in the form of streamflow is but a tiny fraction of the total inflow (approximately 1 part in 12). Since the storage term in equation (2-2) over a period of say one year is usually quite small, it is obvious that the outflow in the form of evaporation and transpiration is very large. These forms of outflow are so intimately mixed with the heat balance of the earth that it is impossible to discuss one without discussing them both. See references (8, 62, and 86). This of necessity leads to a meteorological approach to the water budget method of forecasting. In addition meteorology is the sole element involved in the rainfall term of equation (2-1).

The introduction of the heat energy balance to the water budget concept by Thornthwaite (98) and Penman (70) in 1948 revolutionized the thinking of the plant physiologists. Prior to that time the function of transpiration by plants was not known. A standard botany text (41) of an earlier day, could only state "Bringing the mineral salts (absorbed from the soil and present in great dilution in the sap) quickly to the leaves and concentrating them there, where they are probably principally used in the manufacture of proteins, chlorophyll, and perhaps other substances, may be the single definitely useful role performed by transpiration.... It is highly questionable whether the cooling effect of transpiration is of much importance to the plant." However, once the diurnal description of surface radiant heat flux was quantitatively known and meteorological theory had advanced to the point where the daytime downward radiation surplus could be quantitatively partitioned between sensible and latent heat fluxes away from the surface, the answer became obvious. Plant leaves contain very little water. Hence their ability to store excess heat energy is limited. Plant leaves must transpire if they are to keep their temperatures at viable levels.

Thus by 1948 the theory in general was known and geographers and climatologists were calculating water budgets on a monthly basis. Much more detailed knowledge of water flow through porous unsaturated media and the physiological response of plants to water deficits were required for streamflow forecasts on a daily basis. Only recently has this research been compiled in book form. See references (13, 54, 80, and 87).

Evaporation and transpiration is equal to the net latent heat flux divided by the heat of vaporization of water (neglecting the sublimation of snow). This flux is determined by meteorological parameters. Thus equation (2-2) can be written as

$$\text{Inflow} = (\text{Evaporation} + \text{Streamflow}) + \text{Change in Storage} \qquad (2\text{-}3)$$

The distribution of the storages is also a function of evaporation among other things. The distribution of water storage, particularly in the first few feet of the soil, greatly affects the rate of infiltration. Thus the infiltration term of equation (2-1) is highly dependent upon evaporation and transpiration.

CHAPTER 3

SOIL MOISTURE AND INFILTRATION THEORY

In 1897 Briggs (7) proposed that soil water should be classified into three kinds. These are summarized by Baver (2) as:

1. Hygroscopic water, which is absorbed from an atmosphere of water vapor as a result of attractive forces in the surface of the particles.

2. Capillary water, which is held by surface tension forces as a continuous film around the particles and in the capillary spaces.

3. Gravitational water, which is not held by the soil but drains under the influence of gravity.

The dynamics of soil water is greatly aided by the concept of potential. This has been defined by Taylor (97) as "The amount of work required to remove a unit mass of water from the system in the form of pure free water at the same location and temperature.... Since work must be done on the system to remove water, the potential is negative." Because the specific volume of water is very near 1 cm^3/gm this energy is also the potential of a unit volume. These potentials also may be expressed in terms of hydraulic head by dividing by the acceleration of gravity, g. Water may exist in an air-plant-soil system in various chemical forms or in different phases. For this reason it is often convenient to express potentials in the form of a Gibbs free energy function which is quite general. References (1), (22) and (113), give an adequate description of this function. While this function may be required for development of a model which contains electrical, chemical, or osmotic potentials, the discussion below will express potential in terms of hydraulic head. The utility of this energy potential concept is well explained in reference (85).

Gardner (30) lists soil water potentials in the following classifications:

Matric potential - $\Psi_m = \Psi_a + \Psi_c + \Psi_w$, where
Ψ_a is the adsorption potential operative mainly in the first few molecular layers adjacent to the particle surfaces. Ψ_c is the capillary potential derived from the curved meniscus at the air-water interface and often expressed $\Psi_c = -2\sigma/r$. Ψ_w is due to the attraction between water molecules and ions in the electrical double layer at the charged surfaces of the clay particles.

Osmotic potential - Ψ_π due to the dissolved salts.

Hydrostatic pressure - Ψ_p due to the weight of water above the point of measurement. (positive potential).

Gravitational potential - Ψ_g - which is the potential energy of a mass by virtue of its position above some horizontal datum plane. (positive potential).

Figure 1 illustrates these potentials where osmotic potential has been neglected and the potential is constant throughout the medium (static case).

The potential at P is $\Psi_g + \Psi_m + \Psi_p$, and since Ψ_m is negative, the numerical value of $d = \Psi_m + \Psi_p$ is negative and the potential at point P is identical to that at A. Similarly since Ψ_p at B is zero, the potential at B is identical to that at A.

In the event the potentials are not the same throughout the medium then a flow of water will take place. The rate of flow passing a unit area per unit time (flux density) is given by

$$Q' = -k\,\nabla\Psi, \tag{3-1}$$

where k = hydraulic conductivity, Ψ = total potential = $\Psi_m + \Psi_p + \Psi_g$, and Q' = flux density.

Note that the flow can be in any direction depending on the gradient of the total potential and not solely on the gradient of the free water head. This, of course, is merely a generalization of Darcy's Law to unsaturated media. This generalization seems reasonable and has been verified by Childs and Collis-George (13) among others.

The hydraulic conductivity is not a constant at any location but is a function of Ψ_m. Water flows easily through the larger pores but slowly through small ones. The hydraulic conductivity in unsaturated soils, varies roughly as the fourth power of the radius of the pore size. Ψ_m furthermore is a function of water content, $\theta(cm^3_{water} / cm^3_{total})$, and temperature. See references (2) and (57). And in addition it is not a unique function of θ, as soils exhibit considerable hysteresis effect depending on whether they are wetting or drying. See Figure 2. Notwithstanding, the difficulty presented above, k can often be plotted as a single valued function of soil water content although temperature dependent. See Figure 3. Childs (12) has stated, "Hence it would be expected that some hysteresis should be observed in the relationship between conductivity and moisture content, but it has yet to be recorded, presumably because it is on too small a scale. By contrast, the hysteresis in the relationship between conductivity and suction is naturally as marked as that in the moisture characteristic."

The general equation for water movement can be obtained by combining the flux equation with the equation of continuity or equivalently by use of the divergence theorem. For any volume V bounded by a closed surface S, the rate at which water flows <u>outward</u> from V through an element ds with a unit outward normal $\bar{n}$ is given by

$$-dQ'' = -k(\nabla \Psi) \cdot \bar{n}\, dS. \tag{3-2}$$

The net rate of water flow <u>into</u> V is given by

$$Q'' = \int k(\nabla \Psi) \cdot \bar{n}\, dS = \int \frac{\partial \theta}{\partial t}\, dV. \tag{3-3}$$

Applying Gauss's divergence theorem

$$\int \frac{\partial \theta}{\partial t}\, dV = \int \nabla \cdot k(\nabla \Psi)\, dV, \tag{3-4}$$

and equating the integrands

$$\frac{\partial \theta}{\partial t} = \nabla \cdot k(\nabla \Psi). \tag{3-5}$$

This equation is identical in form to the heat flow equation and there exist a large number of solutions. In order to account for water uptake by plants (a sink), the equation should be put into the form

$$\frac{\partial \theta}{\partial t} = \nabla \cdot k(\nabla \Psi) - Q, \tag{3-6}$$

where Q is the rate of water uptake per unit volume of soil. The method of calculating Q will be discussed in Chapter 5.

One further mathematical transformation will be helpful. Defining $\frac{d\theta}{d\Psi}$ as the specific water capacity (i.e., the rate of change of moisture content with increasing pressure (decreasing suction), the division of k by $\frac{d\theta}{d\Psi}$ (analogous to specific heat) will give a quantity analogous to thermal diffusivity. Thus a new function called soil water diffusivity is defined by the relation

$$D = k\frac{\partial\Psi}{\partial\theta}. \tag{3-7}$$

With this transformation the equation becomes, following Gardner (29) and neglecting Q:

$$\frac{\partial\theta}{\partial t} = \nabla \cdot D \nabla \theta. \tag{3-8}$$

Equivalently when considering only the matric and gravitational potentials

$$\frac{\partial\theta}{\partial t} = \nabla \cdot D \nabla \theta + \frac{\partial k}{\partial z}, \tag{3-9}$$

where the transformation has been applied to Ψ_m only; this expression is mathematically identical with a Fickian molecular equation with a diffusion coefficient that is a function of concentration except for the last term introduced by the gravitational potential.

The advantages of the use of this transformation is that the variable Ψ has been eliminated from the equation and D does not vary so strongly with water content as does k. Gardner (30) has given several methods of measuring conductivities and diffusities. However, there is no existing catalogue of these quantities for a large number of soil types. Crank's book (17) is a standard reference on diffusion, but does not include this special case.

For the one-dimensional case assuming horizontal homogeneity, (the only case considered in this paper), equation (3-9) becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial [D(\partial \theta / \partial Z)]}{\partial z} + \frac{\partial k}{\partial z}, \tag{3-10}$$

where k and D are functions of θ, and θ is a function of z and t only. Using the relationship $\frac{\partial \theta}{\partial t} = - (\frac{\partial \theta}{\partial z})(\frac{\partial z}{\partial t})$ equation (3-10) can be trans-formed into

$$\frac{\partial z}{\partial t} = - \frac{\partial (D \frac{\partial \theta}{\partial z})}{\partial \theta} - \frac{dk}{d\theta}. \tag{3-11}$$

Equation (3-9) is applicable to most cases of flow in porous media. This paper, however, will make use of this equation only for the special case of vertical movement of moisture downward through the surface (infiltration). This equation was numerically solved by Klute (48) and more recently by Philip, presented in a series of brilliant articles on the theory of infiltration. See references (73 through 79).

The concept of soil moisture profiles is essential to a physical understanding of equations (3-10) and (3-11). A typical soil moisture profile during infiltration is shown in Figure 4. It is merely a plot of soil moisture vs. the vertical coordinate at a point. As shown in the figure the profile can be divided into four zones from the surface downward, after Philip (80)

> (1) The saturated zone: a surface zone of presumed saturation extending (in experiments on infiltration from shallow ponded water) to a depth of the order of 1 cm.
> (2) The transmission zone: an upper region in which θ changes quite slowly with both z and t, which lengthens as infiltration proceeds.
> (3) The wetting zone: a region of fairly rapid change of θ with respect to both z and t.
> (4) The wet front: a region of very steep moisture gradient which represents the visible limit of moisture penetration.

Equation (3-10) then is a quantitative statement of the local time rate of change of soil moisture content. And equation (3-11) can be viewed as the velocity of advance downward of the moisture profile for any given θ. Since this rate of advance changes depending on the particular value of θ chosen, the shape of the profile changes with time because some portions of it are advancing more rapidly than others. The depth of water infiltrated between any two successive positions of the profile is the area between the two profiles or mathematically, depth = $\int_o^{surface} \theta_{t2} dz - \int_o^{surface} \theta_{t1} dz$. Philip's solution to equation (3-11) is given in reference (12).

The solutions developed by both Klute and Philip are tedious and difficult to obtain with the required degree of precision. A more practical infiltration equation is needed. Many years ago Green and Ampt (34) suggested a simple formula which still has great merit. A formula of this form has been suggested for practical use by Philip himself (72). The fundamental assumption made by this formula is that the soil moisture profile is a horizontal line. That is, the saturation zone includes the transition zone and the wet front is non-existent and the slope of the wetting zone $(\frac{dz}{d\theta})$ is zero. This approximation of the soil moisture profile is shown in Figure 4. When the water has penetrated to a depth Z_L from a surface which has been kept flooded to a depth P, there exists a saturated column of soil with a uniform saturated conductivity, K, which conducts water with a uniform velocity throughout since there can be no water storage in this zone. Ψ_m in the saturated zone is zero and Ψ_m in the dry zone has a negative value. Water is thus pushed downward by the weight of saturated water column above and pulled downward by the suction of Ψ_m in the dry zone. Direct application of Darcy's law to this situation yields the infiltration rate, i (cm. or in/hr), considered as a positive quantity.

$$i = -Q' = K\ (Z_L + P - \Psi_m)/Z_L. \tag{3-12}$$

This equation is in qualitative agreement with commonly observed facts. If the denominator is small (the beginning of infiltration) then $-\Psi_m$ is the controlling factor and the infiltration rate is large. After infiltration has proceeded a long time and Z_L becomes large compared with P and $-\Psi_m$, the infiltration rate approaches the saturated conductivity. The rate of infiltration, i, is also given by

$$i = (\theta_{sat} - \theta_o)\ \frac{dZ_L}{dt}. \tag{3-13}$$

Equating (3-12) and (3-13) yields

$$\frac{dZ_L}{dt} = \frac{K}{(\theta_{sat} - \theta_o)}\ (1 + (P - \Psi_m)\ /\ Z_L). \tag{3-14}$$

This equation can be integrated to give

$$t = \frac{(\theta_{sat} - \theta_o)}{K}\ (Z_L - (P - \Psi_m)\ \ln[\ 1 + Z_L/(P - \Psi_m)]). \tag{3-15}$$

This explicitly expresses t in terms of Z_L, the depth of wetting, instead of vice-versa. This is not desirable, but it is of little practical significance for a computer problem. A tabulation of Z_L vs. t can always be made. The infiltration rate, i, is obtained from equations (3-12) or (3-13) and (3-14). If i exceeds R, the rainfall rate, then it will be assumed i = R and $\frac{dZ_L}{dt} = \frac{R}{(\theta_{sat} - \theta_o)}$. A typical infiltration curve is shown in Figure 5.

These infiltration equations (3-12) - (3-14), have a versatility far beyond what the development above might indicate. They are, except for K, independent of either the depth or time of the infiltration. Thus at any specific depth, Z_L specified, equation (3-12) is valid if the values of θ_{sat}, θ_o, $-\Psi_m$, P and K are known. Thus i can be determined. A vertical column can be divided into many fine zones and the time t required to reach a given depth, Z_L, can be determined by numerically integrating equation (3-13). Each of these zones could have any value of θ_{sat}, θ_o, $-\Psi_m$, and K and a time to reach a specified depth could be calculated. From these calculations a plot similar to Figure 5 can be made.

The value of K at a given depth does depend on the values of K in the zones above the specified depth. The mean value of K which should be used can easily be calculated from the following expression:

$$K = \frac{Z_L}{\frac{Z_1}{K_1} + \frac{Z_2}{K_2} + \ldots + \frac{Z_n}{K_n}}, \qquad (3\text{-}16)$$

where K is the effective hydraulic conductivity of the entire saturated zone, Z_L is the total depth, and Z_n and K_n are the thicknesses and hydraulic conductivities of the sub-zones, respectively. This expression is derived in Appendix B.

Thus, if the values of saturated conductivities are known for each layer the overall conductivity can be computed. This ability to account for non-homogeneous horizontally layered soil is one of the greatest attractions for the use of infiltration equations in the form of (3-12) - (3-15).

<u>These equations will be used in this paper to determine an infiltration rate curve on which the recorded rainfall rate will be overlayed</u>

to obtain the runoff. The values which are required to use these equations are as follows:

(a) K, the saturated hydraulic conductivity for all layers.

(b) θ_{sat}, the saturated volumetric water content for all layers.

(c) $\Psi_m(\theta)$, the matric potential for all layers and all values of θ.

(d) θ_o, the volumetric water content of the soils for all layers at the initiation of the infiltration.

(e) P, ponded depth over the soil (assumed a very small constant depth).

For any area all of the quantities are constant with respect to time except θ_o. θ as a function of time and depth will be continuously computed by a water budget method as described in Chapter 2. Data on items (a) and (b) above are fairly extensive but not complete. Values for which data are not available can be estimated from other widely available data such as grain size distribution or porosity. Values for item (c) in terms of soil type is fairly common. See Figure 2. Gardner (30) quotes Visser (104) as giving a formula for Ψ_m in the form of

$$\Psi_m = A(\theta_{sat} - \theta)^r/\theta^m \qquad (3\text{-}17)$$

where A, m, and r are constants.

In stark contrast the necessary data required for development of infiltration curves using the numerical solution to the diffusion equation; $k(\theta)$, the unsaturated hydraulic conductivity, and $D(\theta)$, the soil water diffusivity, exists only for those soils which have been subject to specific investigations by a few researchers. This paucity of catalogued data and the extremely difficult process of estimating these quantities from such parameters as grain size distribution, total porosity, etc., renders use of this method impractical at the present time.

The use of infiltration formulae in the form of equations (3-12) - (3-15) was seriously challenged by Horton in 1940 (40). He proposed an empirical equation of the form $i = i_\infty + (i_o - i_\infty)e^{-bt}$ where i_o is the initial infiltration capacity, i_∞ is the final infiltration capacity, and b is a constant depending primarily on soils and vegetation. This function gives a plot of i vs. t similar to Figure 5, except that at $t = 0$, i is finite. i_o is a parameter whose value varies with the soil moisture content at the time of the beginning of the storm event. This formula has been widely used by hydrologists since its introduction.

Horton's objections were based on his interpretations of experimental data and the physical reasoning that "...the soil is not fully saturated during infiltration under natural rainfall conditions, it is difficult to see how capillary pull at the moist front can be transmitted effectively to the soil surface so as in any way to affect or increase the infiltration-capacity in the presence of capillary surfaces exposed to air within the soil" this would in effect be much like trying to use a straw with a hole in it. In view of Figure 5 which shows that the soil is not completely saturated down to the wetting front, this is a valid objection. Horton's rejection of this theory led him to propose that surface conditions were controlling and stress the importance of raindrop impact, colloidal swelling, earthworm activity and so on.

Philip (72) in his proposal of the use of the Green and Ampt formulation, equations (3-12) - (3-15), states that the assumption"... certainly evades the question of the exact status of the 'wetting front' but appears to give useful results nevertheless." He provided a measure of agreement between these equations and the series solutions obtained in reference (12). A Dirac delta function solution of the diffusion equation will yield an expression for t identical in form with equation (3-15), references (76) and (80). The agreement between the series solution (reference (12)) and the Green and Ampt formulation as set forth above is not very good for extremely light clay soils. The Green and Ampt formulation gives values of infiltration that are about four times too large. The values for sandy and initially dry soils will be in very close agreement however. The coarser the medium, the more nearly the actual soil moisture profile approaches the approximation shown on Figure 5, reference (110). Serious errors of estimates of infiltration have been reported by experimental workers using Horton's equation while reasonably good results have been reported using equations (3-12) - (3-15). Lack of good agreement on extremely tight clay soils is not too significant because the actual rate of infiltration is so small that most of the rainfall from an intense storm will runoff and an error of a factor of four in the estimate of infiltration will not significantly change the estimate of runoff. On such soils a simpler formula such as i = c where c is a small constant would suffice for practical purposes. The soils in the study area of this paper are not clay and the agreement between the series solution of reference (12) and equations (3-12) - (3-15) should be good.

Once the soil moisture has infiltrated the soil and the rainfall (hence infiltration) ceases, then the soil moisture is redistributed. Of course, the diffusion concept applies to this case also, but as Youngs (111 and 112) pointed out "...the hysteresis of the moisture characteristics of porous materials makes it impossible to define a unique concentration dependent diffusion coefficient for the draining part of the profile." Recently, however, Rubin (84) has derived a numerical solution to this problem. Qualitatively this redistribution consists of a counter-clockwise rotation of the wetting front line (see Figure 4) about its intersection with the ordinate $(\theta_{sat} + \theta_0)/2$. This results in a redistributed profile which has a nearly vertical shape at the abscissa of the point of rotation extending downward to the point defined where the water volume is identical with the original volume when infiltration ceased. This redistribution occurs in a matter of hours. Black, Gardner, and Thurtell (3) have used a period of two days for redistribution and final smoothing by drainage. This redistribution is also confirmed by numerous field soil moisture measurements taken routinely in the study area by the Agricultural Research Service. At no time did these measurements indicate a large moisture "bulge" in the upper few inches of the soil. <u>This rapid redistribution of soil moisture will be assumed in this paper.</u> Data presented by Youngs (112) and by Hallaire and Henin (35) indicate that in some media this moisture redistribution does not occur at all especially under conditions of an evaporation draft. Jensen and Klute (46) have explained this anamolous situation as being caused by moisture movement in the vapor phase. This behavior appears to be limited to cases where the saturation and transmission zones are very shallow (about 3 cm.). And it does not seem to be of much practical significance as the small volume of water infiltrated is rapidly evaporated away.

Soils also drain under the influence of gravity. Again the diffusion equation does not readily lend itself to a rigorous mathematical description of this process. Gardner (30) has suggested an equation in which the downward flux of water is proportional to water content,

$$- \frac{dW_n}{dt} = AW_n, \tag{3-18}$$

where W_n is the total water content in cm. or in. in a layer of soil and A is a constant. Black, Gardner, and Thurtell (3) suggested a formula derived from Darcy's equation in the form

$$- \frac{dW_n}{dt} = B\ e^{a(W_n - b)}, \tag{3-19}$$

where B, a, and b are empirical constants determined experimentally for a specific soil. The scatter of their data points was large, however. Ogata and Richards (68) and Richards, Gardner, and Ogata (82) determined by empirical curve fitting that the drainage was accurately described by an equation of the form

$$- \frac{dW_n}{dt} = b\ p_{on} D_n\ t^{-(b+1)}, \tag{3-20}$$

where D_n is the depth of the layer and p_{on} and b are constants. An equation of this form was also found by Hewlett and Hibbert (39) for lateral drainage of sloping soil masses which gives rise to interflow. This drainage equation was also found in agreement with measurements made by Nixon and Lawless (67). Although the relationship between equation (3-20) and the diffusion equation is obscure; the agreement with the measurements appears to be uncommonly good. <u>This description of downward and lateral flux will be used hereafter.</u>

Since the diffusion equation (3-9) is general, the flow of water need not always be downward as equation (3-20) would indicate. Evaporation from the top of a layer of soil lying over a water table can cause the flux of water upward. Gardner (26) has examined the mathematics of such a condition. His analysis reveals that there is a limiting rate at which water can be supplied to a soil surface from the water table (29). This rate is given by an expression of the form

$$Q'_{lim} = A_m \; a/d^m, \tag{3-21}$$

where A_m is a constant whose value depends upon m, and a and m are parameters depending on soil type and d is the depth to the water table. Values of the parameters a and m given by Gardner (26) were applied to the average depth, d, to the water table of the study area (about 60 ft). The resulting value of E_{lim} was so small that this process did not have any appreciable significance. Those special areas within the study area which have a shallow water table will be treated as shown in Chapter 7.

Even with a remote table, water can flow upward into the bottom of a root zone where the water has been depleted by drainage and transpiration to a greater extent than the zone immediately below. Because of the obvious difficulties little experimental data are available. Such flow must be very small however, because of the following considerations.

(a) The moisture content of both the root zone and the zone below it will be small because drainage and the tendency of plants to use the water near the surface first (see Chapter 5). Drainage of the lower root zone and the layer beneath it will have gone on a long time before the required differential in matric potential is established by virtue of transpiration from the lower portion of the root zone.

(b) The corresponding hydraulic conductivities will be extremely small. See Figure 3.

(c) Flow must be against the force of gravity.

Although Gardner (30) presents some equations of a quantitative nature which neglect gravity and in which the rate of flow is proportional to $t^{-\frac{1}{2}}$ (total water extraction proportional to $t^{\frac{1}{2}}$), it will be assumed in this paper that once the water had drained through the bottom of the root zone it is unavailable for transpiration. This assumption seems consistent with the drainage data presented by Nixon and Lawless (67).

Finally some accounting must be made for evaporation losses from bare soils or winter time evaporation from vegetatively covered soils when transpiration is near zero. Recent excellent studies on this subject have been provided by Heller (38) and Hanks, Gardner and Fairbourn (36). The latter study also included the effects of soil temperature on evaporation. Some effect was noted but for purposes of this study it did not seem to be significant. The results show that initial evaporation proceeds rapidly at a nearly constant rate as long as soil moisture content is sufficiently high to sustain liquid flow in the upper few centimeters. Soon, however, the water content of the upper few centimeters becomes so low that hydraulic conductivity is essentially zero and water can only flow in the vapor phase. This gives rise to soil moisture profile which has a very small negative slope, $\frac{d\theta}{dz}$, a centimeter or so below the surface, the "bone dry front," to use Heller's terminology, this front descends very slowly and some liquid flow is provided from below. A practical formula based on experiment and theory has been given by Black, Gardner, and Thurtell (3) in the form of

$$E = b\ t^{-\frac{1}{2}}, \tag{3-22}$$

where E is the evaporation rate and b is an empirical constant. This equation is independent of any meteorological factors. Since the quantities of water will be small, it should be adequate provided a check is made that E does not exceed the meteorological requirement for the time period considered. Baver (2) indicates that this extremely low flux of water out of soils in the vapor phase has been the consensus of soil scientists for many years.

One should always bear in mind that there are two quite different regimes of soil water flow at the extremes of soil moisture content. These flow regimes are nothing more than a reflection of the wide variation of unsaturated hydraulic conductivity with moisture content. At high moisture contents, flow quantities are significant. At low moisture content, they are very nearly zero. Thus equations which have a bearing on infiltration and drainage may be almost meaningless for practical use in a dry soil environment. Hydraulic conductivity can vary over several orders of magnitude in a homogeneous soil depending on the soil moisture content. Likewise, it can differ by several orders of magnitude in two different soils with the same moisture content. It is this fact which makes quantitative description of soil moisture flow so extremely difficult.

CHAPTER 4

METEOROLOGICAL FACTORS AFFECTING SOIL MOISTURE DEPLETION

As seen in the preceding chapter the soil moisture profile is required to solve the infiltration equation. After a rainfall the soil moisture profile is affected by the factors described in that chapter. In addition, the profile is in part determined by the effects of evapotranspiration. This term is defined in the Glossary of Meteorology (44) as "the combined processes by which water is transferred from the earth's surface to the atmosphere; evaporation of liquid or solid water plus transpiration from plants." Thus evapotranspiration includes evaporation of dew, intercepted rain, ponded surface water, lakes and flowing streams. So not all evapotranspiration will deplete the soil moisture. Another basic term is potential evapotranspiration defined by the Glossary as "Generally, the amount of moisture which, if available, would be removed from a given land area by evapotranspiration; expressed in units of water depth." As Thornthwaite (98) pointed out, it is maximum evapotranspiration that can occur and depends only on climate. The actual evapotranspiration depends on soil and plant factors described in other chapters. The purpose of this chapter is to present the theory of how potential evapotranspiration can be estimated from routine weather observations.

There are five common methods of determining the vertical flux of water vapor from the surface of the land. These are as follows:

(a) Lysimeter measurements. A lysimeter is a device which consists of a container mounted on a scale which is buried in an excavation. A soil sample with or without plants is placed in the container so that the surface of the sample is flush with the ground. A drainage system is provided and the drained water is collected and weighed. The amount of water evaporated in a given time is determined by subtracting the weight of drained water from the total weight loss of the sample during the time period. A small correction might be made for the increase in weight of the growing plants. These instruments are expensive and difficult to operate correctly, but are very valuable for experimental purposes.

b. Eddy correlation method. The total vertical water vapor movement in a given time period can be determined from the formula

$$E = \overline{\rho w q} \qquad (4\text{-}1)$$

where E is the mass of water vapor transported across a horizontal boundary in a given time period, ρ is the density of the air, w is the vertical wind velocity at the boundary and q is the specific humidity. The bar indicates averaging with respect to time. This method requires fast response instruments to measure w and q. Considerable computing effort is required to perform the time averaging. This method is theoretically straight forward, but the difficulties of instrumentation and data processing limit its accuracy.

c. Gradient method. By this method the flux of water vapor is assumed proportional to the vertical gradient of water vapor. This is expressed as

$$E = - \rho K_E \frac{\Delta q}{\Delta z} \qquad (4\text{-}2)$$

where K_E is the eddy diffusion coefficient and z is the vertical coordinate. This method requires the measurement of humidity at two different levels. In addition K_E varies by about three orders of magnitude during the day and is difficult to determine. It also varies nearly linearly with height. This latter difficulty can be removed if the corresponding equation for momentum transfer is used with the assumption that the eddy viscosity, K_m, is equal to the eddy diffusion coefficient, K_E, and that the logarithmic wind profile holds. With these assumptions equation (4-2) becomes

$$E = - \rho k_v^{\;2} \frac{\Delta u \, \Delta q}{\ln(z_2/z_1)} , \qquad (4\text{-}3)$$

where u is the horizontal wind velocity and k_v is Von Karman's constant.

d. The energy balance method. This method is based on the consideration that net radiant energy flux incident upon a surface must be disposed of in three ways -- a conductive heat flux into the lower layers and a flux of latent and sensible heat between the surface and the atmosphere, so that:

$$R_o + G_o + LE_o + H_o = 0, \qquad (4\text{-}4)$$

where R_o is the net radiation flux at the surface, G_o is the heat flux between the surface and lower layers of soil, H_o is the sensible heat flux between the surface and the atmosphere, and LE_o is the latent heat flux between the surface and the atmosphere (L being the latent heat of vaporization). The sign of each element is positive if the flux is directed toward the surface and negative if directed away from the surface. By employing the gradient equation for sensible heat flux

$$H = -\rho\ c_p\ K_H \frac{\Delta T}{\Delta z}, \tag{4-5}$$

where K_H is the eddy thermal diffusion coefficient, c_p the specific heat at constant pressure, and T the temperature; with the assumption that $K_H = K_E$ the ratio of H_o to E_o is

$$\frac{H_o}{E_o} = c_p \frac{\Delta T}{\Delta q}. \tag{4-6}$$

Using equation (4-4) the expression for E_o becomes

$$E_o = \frac{-R_o - G_o}{L + c_p \frac{\Delta T}{\Delta q}}. \tag{4-7}$$

The formula still requires measurements at two vertical locations as well as measurements of R_o and G_o.

e. Evaporation pan method. This procedure consists of the assumption that potential evapotranspiration is proportional to the measured evaporation from a shallow pan either raised or sunken to be flush with the surface. Difficulties arise in making accurate measurements of pan evaporation and in determining the coefficient of proportionality.

All of these methods except the last require special instrumentation. However, a combination of c and d can be used to avoid this requirement. This procedure has been suggested independently by Penman (70) and Budyko (8). As a first step it is assumed that the total flux does not vary with height in the first meter or two. This is a reasonable assumption based on common experience that there is not a large accumulation or depletion of water vapor in the lowest two meters of the atmosphere during a period of several hours. With this assumption equations (4-2) and (4-5) can be integrated between the surface and a height h to give the expressions

$$H_o = -\rho \; c_p \; D_H \; (T_o - T_h), \tag{4-8}$$

$$E_o = -\rho \; D_E \; (q_o - q_h) = -.622 \frac{D_E}{p} \; (e_o - e_h), \tag{4-9}$$

where q_o is the specific humidity at the surface, q_h is the vapor pressure of the air at screen height, p is the atmospheric pressure, e_o and e_h are the vapor pressures at the surface and screen height, and D_E is the integrated eddy diffusion coefficient given by

$$D_E = \frac{1}{\int_o^h \frac{1}{K_E} \, dz} \; . \tag{4-10}$$

D_H is given by similar expression.

The result of this integration yields an eddy diffusion coefficient D_E that does not vary significantly with height as does K_E. D_E can be determined by equating equation (4-9) with either equation (4-7) or (4-3). Budyko (8) lists the following properties of D_E over land surfaces:

1. It varies only slightly with height above one meter.

2. Mean daily values for the warm season are about .6 - .7 cm /seq and has a mean value during daylight hours of 1.0 - 1.5 cm./sec.

3. It decreases with inversions.

4. It is slightly dependent upon wind velocity. (He indicates that this dependence can be ignored.)

Sellers (86) has suggested a formula for use with average daily data:

$$D_E = a + b\,u, \tag{4-11}$$

where a and b are constants and u is the horizontal wind speed. Significantly a study by Wang and Wang (106) showed that there was nearly zero correlation between windspeed and computed evaporation.

Equation (4-9) could be used directly provided that, q_o, the specific humidity of the surface, or e_o, the vapor pressure of the air at the surface, were known. However, standard meteorological measurements are taken at a height of about two meters and the required surface measurements are not made. What is needed is a good way to estimate the value of e_o. ρ, of course, can be determined from the equation of state.

If equations (4-8) and (4-9) are substituted for the H_o and E_o terms of the heat balance equation, (4-4), an expression is obtained in the form

$$R_o + G_o = A\,(e_o - e_h) + B\,(T_o - T_h), \tag{4-12}$$

where $A = .622 \dfrac{\rho L D_E}{p}$ and $B = \rho\, c_p\, D_H$.

At this point it is assumed that the atmosphere at the surface is saturated with water vapor. This assumption is not as far fetched as it may at first seem. The leaves are surrounded by a very thin laminar flow layer in which only outward diffusion by molecular processes can occur. Since molecular diffusion is quite slow the air adjacent to the leaves will be saturated or nearly so. All of the variables can be determined from the meteorological measurements except e_o and T_o. Remembering that e_o is now the saturation vapor pressure at the surface temperature T_o, these two variables can be related by the Clausius - Claperon equation. Penman (70) uses the finite difference form of this equation to obtain

$$T_o - T_h = \frac{R^* T^2}{.622 \; L \; e_{sh}} (e_{so} - e_{sh}) = \frac{1}{\Delta} (e_{so} - e_{sh}), \qquad (4\text{-}13)$$

where R^* is the gas constant for dry air and e_{sh} or e_{so} is the saturation vapor pressure corresponding to the air temperature, T_h or T_o. Δ is obviously the slope of the saturation vapor pressure versus temperature curve. This is a good approximation as long as the temperature difference is small.

Using equation (4-13), equation (4-12) becomes

$$R_o + G_o = A(e_{so} - e_h) + \frac{B}{\Delta} (e_{so} - e_{sh}). \qquad (4\text{-}14)$$

Noting that $LE_o = A(e_{so} - e_h) = A(e_{so} - e_{sh}) + A(e_{sh} - e_h)$ and defining $LE_h = A(e_{sh} - e_h)$; $(e_{so} - e_{sh})$ becomes

$$(e_{so} - e_{sh}) = \frac{LE_o - LE_h}{A}. \qquad (4\text{-}15)$$

Using these definitions, equation (4-14) can be written as

$$LE_o = R_o + G_o - \frac{B}{\Delta} \; \frac{LE_o - LE_h}{A} \qquad (4\text{-}16)$$

or

$$LE_o \left(1 + \frac{B}{\Delta A}\right) = R_o + G_o + \frac{B}{\Delta A} LE_h. \qquad (4\text{-}17)$$

Defining γ as $\frac{B}{A}$ and solving for LE_o, the following expression is finally obtained

$$LE_o = \frac{(R_o + G_o)\,\Delta + \gamma\, LE_h}{\Delta + \gamma}. \tag{4-18}$$

The potential evapotranspiration rate, E_o, can be obtained from (4-18) by dividing by L.

In using the energy balance equation (4-4) the following forms of energy listed by Sellers (86) will be neglected:

1. Heat of fusion of water (snowmelt).
2. Dissipation of mechanical energy of wind.
3. Heat transfer by precipitation.
4. Expenditure of heat for photosynthesis. (Except as noted in Chapter 7.)
5. Gain of heat by oxidation of biological substances.
6. Combustion, volcanic erruptions, street lighting, flux of heat from the earth's interior, lightning strokes, meteorites, cosmic rays, radiation from stars, radiation from zodiacal light.

All of these sources are several orders of magnitude below the items which are considered by equation (4-4).

The potential evapotranspiration can be calculated from equation (4-18) provided that R_o and G_o can be measured or estimated sufficiently well. R_o is the net radiation flux <u>at the surface</u>. It is the algebraic sum of four components:

a. the incoming short-wave solar radiation flux, S_{od}
b. the outgoing short-wave solar radiation flux, S_{ou}
c. the incoming long-wave infrared radiation flux, I_{od}
d. the outgoing long-wave infrared radiation flux, I_{ou}

The relative magnitudes of these different components is set forth in a table prepared by Miller (62) for his excellent summary of the heat and water budget of the earth's surface. The data are based on the painstaking observations of Fleisher (23) and (24)

AVERAGE DAILY VALUES OF RADIATION FLUXES

AT HAMBERG-FUHLSBITTEL, 1954 (LANGLEYS PER DAY)

	Short-wave radiation	Long-wave radiation	Whole-spectrum radiation
Downward	+213	+659	+872
Upward	- 39	-737	-776
Difference	+174	- 78	+ 96

There is, of course, a large seasonal variation in the value of the short-wave radiation. For example, similar data for May 1954 gives a short-wave radiation difference of +315 and a whole-spectrum difference of +215. This surplus was partitioned as follows: -53, sensible heat; -150, latent heat; -12, soil heat.

Measurements of this net total radiation flux are made at very few locations throughout the country. What is more commonly measured is the incoming short-wave solar radiation, S_{od}. The other three components of R_o must be estimated from other data. Component b, the outgoing short-wave solar radiation, is merely the product of S_{od} and α_s, the short-wave albedo of the surface, since the surface does not emit short-wave radiation. There are several albedo values for different kinds of surfaces listed in references (8, 33, and 86).

As seen from the table above, the magnitude of the infrared radiation is several times that of the short-wave radiation. While these long wave fluxes are relatively constant throughout the year, the simplifying assumption that they are nearly constant and their difference is constant should not be made for calculations based on daily values. Small percentage fluctuations of these components may be significant on an absolute scale compared with the total magnitude of the short-wave radiation.

Sellers (86) lists several methods of computing components c and d above or their difference. The familiar Elsasser Chart appears to be too complicated for daily use when observations of ground level on an average daily basis forms a problem constraint as in this paper. The number of formulae proposed by various authors going back as far as 1916 are legion. All of these formulae give average daily values of the difference of components c and d to an accuracy of 10 percent or better as long as the surface vapor pressure falls between 9 and 27 mb. For larger or smaller values, the errors may be great. This is an error which must be accepted under the constraints established in this study.

The latest of these empirical formulae and one for which the author claims a very high correlation (.99) with observations is the one set forth by Swinbank (94). His forumla is expressed as

$$I_{odc} = a + b\,\sigma\,T_h^{\,4}, \qquad (4\text{-}19)$$

or

$$I_{odc} = B\,\sigma\,T_h^{\,6}, \qquad (4\text{-}20)$$

where I_{odc} is the downward clear sky radiation, σ is the Stefan-Boltzmann constant, T_h is the absolute temperature at screen height and a, b and B are constants. This formula varies from older ones primarily by being independent of e. Whether this formulae represents the radiation at screen height or actually at the surface, of course, depends on the location of the radiation measuring device with which the screen height temperature data was correlated. It can be noted in reference (86) that this difference is negligibly small. The downward sky radiation is increased somewhat with cloudy skies. Geiger (33) and Kondrat'yev (53) both quote the expression given by Boltz (4),

$$I_{od} = I_{odc}\,(1 + k'n'^2), \tag{4-21}$$

where k' is a coefficient which varies with cloud type, and n' is the cloud cover in tenths. The value of k' varies from .04 for cirrus to .24 for stratus. Since cloud type and height is not part of the input data for the computations of this paper this value will be estimated by the dewpoint-temperature difference. The amount of this energy absorbed at the ground is the value of I_{od} times the infrared absorptivity of the surface. Since absorptivity and emissivity are equal and no radiant energy is transmitted through the surface, an expression for I_{od} can be written as

$$I_{od} = \varepsilon\, I_{od} + \alpha_I\, I_{od}, \tag{4-22}$$

where ε is the infrared emissivity, α_I is the long-wave albedo of the surface.

The last component of the net radiation flux remaining to be discussed is upward long-wave radiation flux. This is given simply by the expression :

$$I_{ou} = \epsilon\,\sigma\,T_o^{\,4} + \alpha_I\,I_{od}. \qquad (4\text{-}23)$$

Equations (4-20) thru (4-23) can be combined to give the net infrared flux at the surface as

$$I_{od} - I_{ou} = \epsilon\,B\,\sigma\,T_h^{\,6}\,(1 + k'n'^2) - \epsilon\,\sigma\,T_o^{\,4}. \qquad (4\text{-}24)$$

Here again T_o cannot be measured. It is shown in reference (86), however, that the net infrared flux can be expressed as

$$I_{od} - I_{ou} = \epsilon\,B\,\sigma\,T_h^{\,6}\,(1 + k'n'^2) - \epsilon\,\sigma\,T_h^{\,4} - 4\epsilon\,\sigma\,T_h^{\,3}\,(T_o - T_h). \qquad (4\text{-}25)$$

Now defining R_h as measured and computed net radiation flux based on screen height temperature measurements, (i.e., $I_{hd} = I_{od}$, $I_{hu} = \epsilon\,\sigma\,T_h^{\,4}$)

$$R_h = S_{od}(1 - \alpha_s) + \epsilon B\sigma T_h^{\,6}\,(1 + k'n'^2) - \epsilon\,\sigma\,T_h^{\,4}, \qquad (4\text{-}26)$$

R_o can be defined as

$$R_o = R_h - 4\epsilon\,\sigma\,T_h^{\,3}\,(T_o - T_h). \qquad (4\text{-}27)$$

Substituting this expression into equation (4-12) and transposing the last term of equation (4-27) the heat balance equation can be written as

$$R_h + G_o = A\,(e_o - e_h) + B\,(T_o - T_h), \qquad (4\text{-}28)$$

where B is now defined as $B = \rho\,c_p\,D_H + 4\,\epsilon\,\sigma\,T_h^{\,3}$. This additional term is significant. Equation (4-18) then becomes

$$LE_o = \frac{(R_h + G_o)\,\Delta + \gamma\,L\,E_h}{\Delta + \gamma}, \qquad (4\text{-}29)$$

where γ includes the additional term for B. The only remaining term of equation (4-29) which has not been discussed is G_o, the flux of heat into the lower layers of the soil. This flux is in theory the easiest to measure, but no such data are routinely collected at meteorological observation stations where radiation, temperature, humidity and sky cover measurements are made. Consequently, it must be estimated from these measured data or from general climatological relationships. In general this flux is about one tenth of the value of the latent heat or sensible heat fluxes. It is of course larger in bare ground areas where there is no insulating layer of litter through which the heat must flow.

Both a practical and theoretically clear method of determining the soil flux is the temperature integration method. The flux of ground heat through the surface is given by

$$G_o = - k_g(\frac{\partial T}{\partial z}), \qquad (4\text{-}30)$$

where k_g represents the thermal conductivity of the earth at the surface. For a specific period of time the principle of conservation of energy can be applied. Heat conduction during this time must be equal to the internal energy change within the layer, neglecting, of course, latent heat of any vaporized water which is accounted for by the LE_o term of the energy balance equation. Thus

$$\int_o^t (k_g \frac{\partial T}{\partial z})_o dt = \int_o^{-\infty} C_g \Delta T \, dz, \qquad (4\text{-}31)$$

where ΔT represents the change of temperature in the time interval and

C_g is the specific heat on a volumetric basis of the soil. The practical advantage of using the right hand side of equation (4-31) is that the specific heats of soils can be accurately estimated provided that the soil moisture content, θ, is known; and they vary within a narrow range for all types of soils. The thermal conductivity, k_g, on the other hand increases very rapidly with an increase of moisture and differs greatly from soil to soil.

Studies by many workers agree that the specific heats on a weight basis of the major mineral constituents of soils varies within a very narrow range about .175 cal. $gm.^{-1}C.^{o-1}$. The density of these constituents also varies within a narrow range about 2.65 $gm.\ cm.^{-3}$. The average specific heat and density of soil organic matter is .46 and 1.3 respectively. Thus on a volumetric basis the specific heat of a soil made up of minerals, organic matter, air and water is given by

$$C_g = .46\ X_{min.} + .60\ X_{org.} + 1.00\ \theta, \qquad (4\text{-}32)$$

where $X_{min.}$ and $X_{org.}$ are the volume fractions of the minerals and organic matter respectively. This equation was first given by DeVries (105) and agrees well with the measurements made by Carson and Moses (11) in Illinois. Using equation (4-31) Carson and Moses produced graphs of diurnal heat fluxes for six different months of the year. They also published a graph of the annual cycle of heat flux averaged on a three year basis. The year to year variation in the average daily flux was small (on the order of 10 per cent). Since equation (4-31) is not dependent on the absolute scale of temperature to any great degree, these charts should have wide geographical applicability.

In a manner similar to the development of the diffusion equation, (3-9) of Chapter 3, the relationship of equation (4-31) can be expressed as a diffusion equation, assuming that the soil is homogeneous and k_g does not vary with depth.

$$\frac{\partial T}{\partial t} = \frac{k_g}{C_g}\frac{\partial^2 T}{\partial z^2} = \eta\frac{\partial^2 T}{\partial z^2} \tag{4-33}$$

where $\frac{k_g}{C_g} = \eta$ is the thermal diffusivity. This assumption of homogeneity of k_g is not as constraining as may at first appear. The range of values for different soil types is narrow and as k_g increases so does C_g. For a wide range of moisture values $\frac{k_g}{C_g}$ is nearly constant. If it is assumed that the surface boundary condition is defined as

$$T(0,t) = \bar{T} + \Delta t_o \sin \omega t, \tag{4-34}$$

where $\bar{T}$ is the mean (daily or annual) soil temperature (assumed to be equal at all depths) and ΔT_o is the amplitude of the surface temperature wave. Then the solution (86) to equation (4-33) has the form

$$T(z,t) = \bar{T} + \Delta T_o e^{-z(\frac{\omega}{2\eta})^{\frac{1}{2}}} \sin(\omega t - (\frac{\omega}{2\eta})^{\frac{1}{2}} z). \tag{4-35}$$

Based on this equation the amplitude of the surface wave is reduced to $.01\Delta T_o$ at a depth equal to $4.61\ (\omega/2\eta)^{\frac{1}{2}}$. For most values of η this depth ranges between 48 and 84 cm. for a daily cycle and 9 to 16 meters for an annual cycle. This equation has good correlation with the data gathered by Carson and Moses. More sophisticated forms of a diffusion equation solution have been proposed by Van Wijk and DeVries (108) among others. Sellers (86) states that these equations have had some value in describing the annual cycle but have had difficulty with the diurnal cycle.

An expression for the sensible heat flux into the surface can be obtained by differentiating equation (4-34) with respect to z and multiplying by $-k_g$ and setting z equal to zero. This is equation (4-30). Thus

$$G_o\ (0,t) = \Delta T_o(\omega\ C_g\ k_g)^{\frac{1}{2}}\ \sin(\omega t + \frac{\pi}{4}). \qquad (4\text{-}36)$$

Since ΔT_o is not known or measured, a relationship between ΔT_o and ΔT_h must be used. DeVries (108) has given the relationship $\Delta T_o = 1.25\ \Delta T_h$ for the annual cycle over short grass. No similar relationships can be given to the daily cycle because of the great variability involved.

Thus as a practical method of determining G_o on a daily basis, one has the option of using experimental data like that of Carson and Moses or as was gathered for the "Project Great Plains" at O'Neill, Nebraska or of using equation (4-36). (See reference (57)). Both of these methods will yield only a seasonal average daily value. However, since the magnitude of this flux is small the use of a more refined formulation on a daily basis does not seem justified.

Experimental verification of the Budyko-Penman combinational energy balance-gradient method was conducted by Van Bavel (101). He reports that by comparing an equation similar to equation (4-29) with careful measurements over various surfaces that he obtained "excellent agreement for 24 hour totals and acceptable agreement on an hourly basis." This approach gives good results over open water, wet bare soil, or a vegetated surface (alfalfa). Of course, the pertinent physical constants such as albedo and specific heat must be in accord with the type of surface.

The only other possible approach to estimating potential evapotranspiration without special instrumentation is by use of evaporation pans. The difficulties in establishing suitable pan adjustment coefficients to give the correct value of open water evaporation has been mentioned. There well may be an identifiable relationship between pan evaporation and lake evaporation. However, to extend the argument one step further and say that computed lake evaporation so determined is equal to evaporation from a vegetated surface seems unwarranted. The basic physics are radically different (the deep penetration of short-wave radiation into water, for example). If such a relationship exists, it would be fortuitous and not based on a sound physical basis. Sufficient experimental data is lacking which would allow such a definition of a relationship between pan evaporation and land surface evaporation.

CHAPTER 5

BIOLOGICAL FACTORS AFFECTING SOIL MOISTURE LOSS

Moisture is largely removed from the soil through a biological medium -- the plants. Plants remove water from the soil by three methods. These are in ascending order of importance: guttation, cuticular transpiration and stomatal transpiration. Guttation is the exudation of liquid by non-transpiring plants which is caused by a development of a positive pressure in the xylem of the plant. The liquid is exuded through special pores called hydathodes. The quantity of water lost by this process is extremely small in most species and will be neglected. Cuticular transpiration is the loss of water by diffusion through the outer walls of the epidermal cells of the leaves which are covered by a waxy substance called the cuticle. Crafts (16) states that this form of transpiration can range between 10 and 70 percent of the total transpiration. Most botany texts, however, state that the figure is near the lower value for most plants, references (25 and 41). Thus most of the water loss occurs through the stomata.

The stomata are special pores through the epidermis located usually on the underside of the leaves. Their density ranges from 50,000 to nearly 800,000 per square inch. These pores not only provide the means of diffusion of water vapor, but also provide an opening where the exchange of the intake of products (CO_2) and output of products (O_2) of photosynthesis take place.. Veihmeyer (102) gives this description of the nature of stomata

> The stomata are tiny long openings through epidermis. Each stoma lies between two guard cells, which in turn are bordered by subsidiary cells. As water moves into the guard cells, there is a tendency to stretch them, causing the opening of the stoma. Thus the opening and closing of the stoma is produced by changes in turgor of the guard cells and results from unequal thickening of the guard-cell walls. The important factors affecting such an operation are light, intensity, moisture supply of leaves, temperature of air, humidity, and chemical changes. Stomata usually open in the light and close in the dark. They close with reduced moisture, which causes the guard cells to lose turgor. Temperature affects speed of opening. The guard cells contain chloroplasts, which possess chlorophyll. Within the chloroplasts, carbon dioxide and water react to form sugar. Starch is usually found in the protoplasm of guard cells. The change in turgor of these cells is associated with changes in their starch content. High turgor and open stomata are associated with little or no starch, and low turgor and closed stomata with abundant starch.
>
> When the sugar content of the guard cells increases, as it usually does in daylight hours, the osmotic pressure of the cell sap increases, water is drawn in from adjacent cells, the turgor pressure of the guard cells increases, and the stomata open. When the sugar content decreases, changing to starch, as usually occurs in the dark, the osmotic, and consequently the turgor pressure of the guard cells decreases, and the stomata close.
>
> Contrary to general belief, the stomata exercise a very limited control on the transpiration rate. They close after the wilting or darkness begins, and not in anticipation of it. When the stomata are fully opened, the transpiration rate is determined by the same factors that control evaporation alone. The stomata exert a slight regulatory influence only when they are almost closed.

Thus it can been seen that evapotranspiration is essentially a daytime phenomenon, in contrast to free water evaporation which can proceed at any time there is a suitable vapor pressure gradient. A typical daily evapotranspiration curve is shown in Figure 6. The shape of this curve has been checked by careful lysimeter measurements verifying the form of the curve in Figure 6 which was computed by use of equation (4-29). See reference (101). One must not draw the conclusion that evapotranspiration must necessarily be less than free water surface evaporation because it can occur only during the daylight. There are many physical factors which affect the rate of evaporation-roughness factor, heat conductivities, specific heats, etc. So the actual rate of evapotranspiration depends on all of the physical factors involved which does not preclude the possibility that it may exceed free water evaporation.

Several authors (references (31) and (96) for example) view evapotranspiration in terms of three independent influences. These are the supply of heat at the surface, the vapor pressure gradient between the surface and the bulk air, and the resistance to water vapor diffusion. This resistance may not only be the reciprocal of the natural diffusivity of the air but also the resistance offered by the stomatal passageway. Stomatal resistance has been the subject of intensive study. If this factor materially affects the total water loss from an area, then a provision must be made for it in the theoretical development of the problem. However, as Slayter (87) points out "the area of actual evaporating surface may be much greater than the equivalent land area. Since internal leaf area generally exceeds external leaf surface areas by about an order of magnitude and leaf area frequently exceeds land area." Thus even though there may be significant stomatal resistance the large increase in evaporating area below a surface parallel to the ground enveloping the leaves will soon cause the space below the surface to become saturated. Thus the assumption that there is a surface of area equal to the surface area of the ground which is saturated and at the same temperature as the foliage still appears reasonable.

Several authors, but principally Gates (31, 32, and 45), have stated that plants have evolved in their environment to become efficient devices to grow and reproduce. The absorptance of radiant energy by plant leaves is dependent on both the frequency and angle of incidence. The leaves require a high absorptance -- about .95 -- in the wave length bands .40 to .51 microns and .61 and .70 microns for the photosynthesis process. The absorptance decreases -- to about .75 -- (reflectance increases) somewhat about midway between these wave lengths (.55 microns). This accounts for the green color of growing vegetation. Beginning with the infrared part of the spectrum at .7 microns the absorptance of the leaves falls rapidly to about .05 but then rises again starting at about 1.1 microns and reaches a steady value near .97 for all wave lengths in excess of 2.0 microns. Thus the leaves are good absorbers in the frequency range required for photosynthesis, and good reflectors of sunlight in those regions of the solar spectrum which are not necessary for this process. They are good absorbers, hence good emitters, in the frequency range where terrestrial radiation is large. Thus by evolution plant leaves have become very efficient in disposing of the radiant energy to which they are exposed.

These frequency dependent absorptances vary somewhat among species and depend on the maturity of the plant. Generally young leaves reflect more than old ones, the principal increase in absorption by older leaves occurs in the near infrared portion of the spectrum. This is not in qualitative agreement with the albedo values given by Gieger in Chapter 4. However, the plant physiologists obtain overall solar spectrum albedos for grass and small grain in the range of .09 to .25. Strictly speaking, of course, the proper way to determine the amount of energy absorbed by plant leaves is to apply the spectral absorptance function to the spectrum of the incident radiation. The incident spectrum varies with cloud cover, dust conditions, and altitude of the sun. Leaves are also exposed to various reflected sources which have spectrums differing from that of sunlight. In addition, as mentioned above, the absorptance of leaves is also specular (that is, dependent upon the angle of incidence of the radiation). These various considerations are discussed in reference (45). The authors conclude, however, that for purposes of calculating a heat budget over an extensive area near the constant value of albedo can be adopted.

Despite the fact that leaves are capable of disposing of large radiant energy loads, plants are in constant danger of death unless other means are available to dispose of the radiant energy which the leaves have absorbed. Gates has calculated that a horizontal leaf exposed to direct overhead sunlight would suffer a temperature rise of $100^{o}C$ in less than one minute if there were no means of disposing of the excess radiant energy.

The reason for this is that leaves have very little water volume compared to their surface area. Consequently, their ability to absorb energy without a large temperature rise is very limited. Neglecting the heat conduction within the plant, there are three ways in which the absorbed radiant energy can be eliminated -- infrared radiation, convection and conduction to the atmosphere, and transpiration. The largest amount is disposed of by infrared radiation. The excess radiation is given off as sensible heat and latent heat as described in Chapter 4. While the detailed description of these processes is much more complicated than developed in this chapter, it is worthy of note that the authors of reference (45) conclude that sensible heat transfer is only a slightly varying function of wind speed. They based their calculations on the formula from fluid mechanics for forced convection with laminar or streamline flow across the leaf surface. For broad-leaved plants this function varies as $u^{\frac{1}{2}}$. It is interesting that the calculations of plant physiologists are in qualitative agreement with the climatologist Budyko in that the rate of sensible heat conduction does not vary appreciably with wind speed.

Plants affect evapotranspiration in a way other than their determination of the quantity of radiation absorbed. The flow of water through the soil-plant-atmosphere system is described by exactly the same equation as for soil moisture flow. That is, $q = -A \nabla \Psi$. In other words, for water to flow from the soil into the plant and through the plant and finally into the atmosphere there must be a gradient of potential. Thus the actual rate of evapotranspiration is determined by the point where the flow equation is minimum in the whole system. Analysis of the potentials of the system shows that the potential drop from plant to atmosphere is many, many times the potential drop from soil to plant. See reference (88) for example.

Based on the fact that the free energy available for transpiration existing in the atmosphere so greatly exceeds the free energy of soil and plant water, Viehmeyer and Hendrickson (103) concluded that the transpiration process continues at the maximum rate possible until the soil dries to the point where the hydraulic conductivity is nearly zero (that is, the wilting point of the soil moisture). They supported their thesis with a large amount of experimental data. This view was immediately challenged by Kramer (55) and Mather (61). And indeed in the same year Thornthwaite and Mather (100) published data from the Project Great Plains data at O'Neill, Nebraska, which depict the percent of available heat used in evaporation as a linear function of soil moisture content from field capacity to the permanent wilting point. Pierce (81) in a lysimeter study in Ohio used a function which is intermediate between these two extreme views. He offered no theoretical explanation. Gardner (27) published data indicating that the leaf potential was a function of soil type and soil moisture. These data tended more toward the Veihmeyer view. Denmead and Shaw (19) published data which indicate that the ratio of actual evapotranspiration to potential evaporation is a function of both soil moisture and potential evapotranspiration. Several other authors have described this ratio as a function of plant type as well.

Unfortunately this matter is not of negligible importance. Some authors show the reduction as quite large, which would seriously affect the water budget and the whole concept of the theory of runoff so far developed.

Before adopting a scheme to determine the reduction in evapotranspiration below its potential maximum due to the lack of soil moisture, the concept of potential evapotranspiration should be considered thoroughly. Reference (99) gives a good discussion of this concept. These authors list the conclusions to be drawn from the definition as follows:

(i) water losses from moist surfaces are determined primarily by meteorological processes.
(ii) variations in natural vegetation or crops have little influence on water losses <u>provided</u> <u>that</u> <u>there</u> <u>is</u> <u>abundant</u> <u>water</u>.
(iii) similar agruments apply to soil type.

The idea of potential evapotranspiration is, in fact, simply an expression of the fundamental energy balance concept. Its strength lies in the unchallengable character of the first law of thermodynamics. Its weakness lies in the infinite complexity of natural surfaces, which make generalizations hard to apply in specific cases.

However, if soil moisture is a limiting factor to the actual evaporation taking place, the necessary condition for fulfillment of the definition of potential evapotranspiration cannot be met. As a result the requirements of the energy balance can be met only by increasing the disposition of excess radiant energy by sensible heat convection. The change of the proportion of energy dissipated by sensible and latent heat is shown in Thornthwaite and Mather's (100) data for a late summer drying period as shown below.

HEAT USED FOR CONVECTION, EVAPORATION, AND STORAGE IN SOIL,

AND SOIL-MOISTURE CONTENT ON

DIFFERENT DAYS AT O'NEILL, NEBR., 1953

Date	Heat used for convection (C) (cal/cm^2)	Heat stored in soil (S) (cal/cm^2)	Heat used for evaporation (E) (cal/cm^2)	Total C+S+E (cal/cm^2)	$\frac{E}{C+S+E}$ (%)	Soil moisture in 0 - 18" profile (inches)
Aug. 13,14..	56.3	29.7	377.2	463.2	81	1.65
18,19..	59.1	-4.8	287.8	342.1	84	1.40
22.....	98.4	19.0	216.2	333.6	65	1.20
25.....	181.9	41.5	131.8	355.2	37	1.05
31.....	242.3	28.3	44.5	315.1	14	.75

The result of this increased sensible heat flux is to raise the temperature measured at screen height, and correspondingly the saturation vapor pressure at that temperature. Thus the calculation of potential evaporation by use of equation (4-29) under these conditions will lead to a higher value of potential latent heat loss than would actually occur had the moisture been available. In short, a feedback takes place.

Once the plant fails to respond fully to the potential evaporation demand, the potential is increased by virtue of high temperatures and hence the ratio of actual evaporation to potential evaporation decreases markedly. In fact, Deacon, Priestly, and Swinbank (18) have gone so far as to state "In such circumstances $e_{sh} - e_h$ might be the more realistically regarded as an inverse index of evaporation."

This feedback effect is reflected in the data of Denmead and Shaw (19). Their data showing the values of the ratio of E (the actual evaporation) to E_p (The calculated potential evapotranspiration) as functions of soil moisture and E_p will be used in this paper. See Figure 7.

Since the infiltration equation (3-14) requires an estimate of the matric potential Ψ_m at all depths, it is necessary to estimate the quantities of water removed by evaporation from each zone of soil considered. Unfortunately Denmead and Shaw's data do not contain this information. Common observation indicates that water is removed most readily from those zones having the highest root density.

Gardner (28) gives a very simple method of computing the uptake in each zone. For the nth zone it is given by

$$Q_n = B(\delta - \Psi_{mn} - Z_L)k_n L_n, \qquad (5\text{-}1)$$

where B is a constant, δ is the suction in the roots, assumed constant, and L is the length of roots in the zone. This equation has been verified by experiment with good results. The total water extracted from all the zones is merely the summation of the Q_n, and the surface flux density is

$$Q' = \sum_{n=1}^{N} Z_n Q_n = E. \qquad (5\text{-}2)$$

E will be determined by first computing E_p from equation (4-29) and then applying the E/E_p ratio determined from Denmead and Shaw. The proportion of E to be extracted from each zone will then be determined by using equation (5-1). Since only the proportions are required, determination of B will not be necessary. The value of δ is readily available, Ψ_m as a function of soil moisture (θ) is already required for equation (3-14). Additional data are required for the conductivities, k_n. These vary widely with soil type and are not readily available. However, some reasonable values and formulae are available. See reference (30). Also required is the density of roots for the plants of the study area. The data given in (107) seem adequate to give a good estimation of root density vs. depth. A logarithmic function has often been assumed (29).

Maturation is another way in which plants can affect the quantity of water extracted from the soil. Crops planted as seeds naturally have very little root depth as they start growing. Some crops such as corn in the southern plains will mature and die by August, thus becoming ineffective in removing soil moisture. Natural vegetation, because it is highly varied, has a minimal maturation effect. Moreover, perennials predominate in such covers, and perennials typically show these effects less than annuals and biennials. Reference (81) gives experimental data on the reduction of transpiration due to the influence of varying stages of maturity for meadow plants. Naturally, if a crop of hay is harvested the transpiration from a given area is greatly reduced. This reference also gives quantitative data on the magnitude of this effect.

Finally, plants can influence the water budget by intercepting rainfall before it reaches the ground. Often this effect is not as serious as may at first seem. The intercepted water is immediately available for evaporation and will effectively cool the plant in a manner similar to transpired water. While the quantity of water intercepted may be a suprisingly large fraction of the total annual rainfall, most of this is intercepted during frequent light rains which would not have penetrated the soil to a great depth anyway, and would have been subsequently evaporated quickly from the soil. However, this process must be accounted. The total interception loss (i.e., that which is retained by the aerial portion of the vegetation) is usually taken as a parameter which changes from species to species and during the season, or as a percentage of the storm rainfall which decreases as the total rainfall increases. References (60) and (71) have considerable data on the magnitude of this factor.

PART II - APPLICATIONS

CHAPTER 6

DESCRIPTION OF THE STUDY AREA AND DATA AVAILABLE

The area selected for study is the Little Washita River basin above the stream gaging station at Ninnekah, Oklahoma. This river basin is located about 10 miles southwest of Chickasha, Oklahoma, and about 40 miles southwest of the U.S. Weather Bureau station at the Oklahoma City airport. The drainage area is 207.7 square miles. See Figure 8, Topography of Watershed 522. The watershed is extremely fan shaped (nearly circular), and the river rises in very flat farm and range country and flows southeastward and soon reaches more rugged terrain with many hills ranging in height from 100 to 200 feet. The river flows eastward and then northeastward through this hilly area. The total length of the main watercourse is 24.7 miles. The upper flat country comprises about thirty percent of the total area; the hilly country about sixty-three percent; and an alluvial plain about seven percent. The rate of fall of the stream is not great even in the upper reaches of the stream. There is a marked difference in the character of the stream between the flat and hilly country. In the flat country the banks of the river are steep, deep (about forty feet) and stable. There is some flow in the river even after an extended summer drought. In the hilly country the stream has formed an alluvial valley. The banks are unstable and only about 10 to

15 feet deep. The sandy alluvium is quite porous and several periods of zero flow have been recorded at the stream gage during summer months.

The watershed is largely devoted to agriculture. Only about one percent of the total area could be classified as urban. However a four lane paved turnpike traverses the area diagonally from northeast to southwest. There are about thirty miles of paved two lane highway and an additional eight miles of paved four lane highway in the drainage basin. The western flat country is tilled more than the eastern hill country. About thirty percent of the flat country is plowed and planted, while only about ten percent of the hilly country is devoted to cropping. Almost all of the alluvial plain is cropped, principally as alfalfa. The flat region has few trees and the pasture land is in good condition consisting of eastern prairie grasses. The hilly upland has many wooded areas which are largely scrub oak. Much of this land is abandoned terraced plowland which sustains poor stands of eastern prairie grasses. There is no significant irrigation in the area.

The geologic formations and their exposed surface areas in percent of the total (207.7 square miles) are: Alluvium, 6.9 percent; Cloud Chief, 13.8 percent; Rush Springs, 63.2 percent; Dog-Creek-Blaine and Marlow, 16.1 percent. The Marlow which underlies the Rush Springs is nearly impenetrable as is the Cloud Chief which overlies it. The Rush Springs formation is a good aquifer. Since the dip is to the west the Cloud Chief outcrops in the western portion of the basin and the Marlow along the eastern border. The Rush Springs occupies most of the hilly country. The soils overlying these formations reflect the qualities of their parent materials. Most of the northwestern area is of the Darnell-Noble series or Norge series. There are a great many varieties of soils in the eastern section, the

Kingfisher and Stephanville being common.

The climate of the watershed is that of the southern Great Plains. It is basically characterized by long, hot summers and relatively mild winters. Snow fall is rare and snowmelt is not a significant hydrologic consideration. Precipitation varies widely from year to year but on the average is heaviest in the spring and fall months. The following brief climatological data reflect this description

		Reference
Average Rainfall (1901 - 64 at Chickasha)	31.24 in.	USWB
Maximum Annual Rainfall (1963 - 68)-1968	34.02 in.	(93)
Minimum Annual Rainfall (1963 - 68)-1963	17.72 in.	(93)
Average July Temperature	82°F	USWB
Average January Temperature	40°F	USWB
Average Annual Streamflow (1952 - 67)	2.5 in.	USGS & (93)
Average Annual Deep Percolation (1953 - 56)	2.5 in.	(95)
Maximum Annual Runoff (1964 - 1968)- 1968	1.39 in.	(93)
Minimum Annual Runoff (1964 - 1968) - 1964	.658 in.	(93)

The Little Washita River basin is a portion of a larger area of the Washita River basin currently studied by the Agricultural Research Service. Their study area embraces a total of 1130 square miles. Their study is being conducted in great detail and covers nearly every phase of the hydrology of the area. As a necessary part of their study they have established an elaborate data gathering network. Data pertinent to this paper which are currently being collected are as follows:

ITEM	TYPE OF GAGE	NUMBER AND LOCATION
Rainfall	Recorder	36 gages (Little Washita basin)
Evaporation	Sunken Pan (Young)	2 locations near Chickasha
Soil Moisture	Neutron Probe	16 locations (none in Little Washita basin)
Ground Water Table	Observation wells (recording and non-recording)	4 non-recording 2 recording Little Washita basin

ITEM	TYPE OF GAGE	NUMBER AND LOCATION
Runoff	Recording gage	Ninnekah, Oklahoma and at each soil moisture site.

These data are tabulated in reference (93). Additional unpublished file data and maps have been made available by the Agricultural Research Service. The evaporation pan data are taken approximately once a week. Soil moisture determinations are made about every two weeks at each location. The non-recording well observations are made every month or so.

Other data pertinent to this paper is collected by the U.S. Weather Bureau at Oklahoma City. These data are as follows:

Temperature	Cloud cover (day and night)
Humidity (Dewpoint)	Wind speed
Short Wave Radiation	

These data are taken by standard U.S. Weather Bureau instruments and the observations are published in references (20) and (21).

Groundwater studies have been made of the Rush Springs aquifer by the Oklahoma Geological Survey. The results of these studies are published in references (69) and (95). These reports contain much detail on this aquifer as well as estimates of deep percolation. It is the considered opinion of the authors that no substantial subsurface export of water occurs in the region.

Additional valuable detailed data on specific soil types of the study area are contained in reference (37). These data contain general descriptions of soils, depth of each horizon and engineering data on each specific soil type.

The geographic locations of each soil type were taken from references (90 through 92).

The period of record for which most of these data are concurrent is April 18, 1963 to the present. The period of record studied was from April 18, 1963 to December 31, 1968. During this period there was very little change in the hydrologic factors affecting runoff. The farming practices remained the same and there was no construction of significant water retention structures.

Approximately nineteen percent of the watershed is controlled by some form of water retention structures. The only major structure is Lake Burtschi Dam built in the 1950's by the Oklahoma Fish and Wildlife Commission. This dam controls runoff from about eight square miles (4 percent) of the northernmost portion of the basin. The remainder of the controlled portion of the watershed is controlled by numerous farm ponds scattered throughout the area.

There are several aspects of the study area and period of study which are bad. As shown in the climatological data listed in a preceding table, the percentage of runoff is a very small proportion of the total average rainfall. Furthermore the period of record studied was one in which the rainfall and runoff were considerably less than average. There were no large floods during the study period. This very small runoff will make the study more difficult as the residual between rainfall and runoff is very large. A location of two hundred miles further east would have been much better but there are other considerations which make the selection of this area more desirable.

The desirable feature of the study area is, of course, the instrumentation. The most important data are the high resolution definition of rainfall in both space and time. As noted in Chapter 2 this study is based on the assumption that the rainfall is known to a high degree of precision and there are no errors due to rainfall measurement. A statistical study of this rainfall network has been made, references (65) and (93). The general results of this study indicate that while there are some distinct regional differences, the basic precipitation regime is similar to that of the Middle West. And that the degree of accuracy is similar to that determined by the Illinois State Water Survey. Reference (42) gives a description of their various networks. Analysis of these networks indicates that a density of rain gage spacing such as available in the study area (one gage per 9 square miles) has a standard deviation of 2.4 percent for a 2 inch storm. See reference (43).

Another set of instrumented data which is essential to this study is the soil moisture data. The accuracy of the absolute magnitudes of these measurements is somewhat in doubt, however.

Thus the overriding considerations in selecting a study area were first, detailed instrumentation; second, hydrologic stability; and third, significant areal extent. The Little Washita basin is the only area in the Southern Plains which meets these criteria. The disadvantages of low runoff, short record, and persistent drought during the study period must be overcome if this effort is to be successful.

CHAPTER 7

DESCRIPTION OF THE MODEL

Using the engineering data provided by reference (37) and the geographical descriptions of soil types given by references (90 - 92) the various soils of the Little Washita Basin were grouped into three basic groups. These groups are as follows:

Group No.	Soil Series	Texture of upper horizon	Contributing Drainage Area - sq. mi.	Percent of total
I	Darnell, Yohola, Zavala Noble, Eufaula	Fine Sandy Loam	28	17
II	Kingfisher, Norge, Chickasha, Lawton	Silt Loam	46	28
III	Stephanville, Doughtery, Cobb, Grant, Lucian, Reinach, Konowa	Fine Sandy Loam	91	55

The soils of any group are reasonably consistent with each other as to depths of horizons, texture, parent material, etc. They are not consistent with each other according to the hydrologic classifications of the Soil Conservation Service, reference (89).

The actual area of each group contributing to storm runoff was determined by first subtracting from the total drainage area, the drainage area of Lake Burtschi and then proportioning the drainage area of the controlling farm ponds. A reconnaissance of Lake Burtschi indicated that neither the emergency spillway (a sodded notch in the earth dam) nor the flood pool spillway (a two foot diameter pipe) had ever been in operation. There is some seepage from the dam but there is no controlled outlet. This reconnaissance verified the reports by local residents that the lake has never spilled. A reconnaissance of the drainage area in 1969 after a two inch rainfall revealed that the farm ponds were not contributing to any great extent to the stream runoff. Thus it was assumed that for the period of record studied that areas controlled by farm ponds did not contribute to the storm runoff. For general use however, this matter should be more carefully considered and an appropriate method of estimating the contribution of controlled areas after their retention pools have filled should be adopted. Approximately two square miles were considered as wetted streambed which would produce 100 percent direct runoff. One square mile of urban area was assumed to have 50 percent runoff and .4 square miles of paved roads were assumed to have 90 percent runoff.

The model constructed was designed to be used on a small or medium sized computer. The model developed for this paper was constructed on an IBM 1130 computer using a disk package of scientific library subroutines. Basically this computer worked very well in solving the problem. There was one difficulty involved in constructing the model with a computer of this size. In order to build and test the model it was necessary to process daily values of the pertinent meteorological parameters for the six year period of record studied. Since this volume of data exceeded the storage capacity of the computer, the input data were read in and stored by one year intervals. This inconvenience could have been overcome using a data disk. As it turned out the inconvenience was minor and did not justify the work of using another disk. Even with yearly quantities of data being used, the core storage of the computer was insufficient to hold all parts of the model. Fortunately one of the three major parts of the model was independent of the other two and could be computed independently. The dependency of the other two was in practical terms very limited and they too were developed independently.

However, a distinction should be made between model development and model execution. The development required storage of at least a year of meteorological data. The execution of the developed model would require only the storage of a few days of data in order to update the output information. The IBM 1130 has sufficient core storage to hold the entire model provided that only a few days of input data are required.

The three major components of the model are shown in the following table. All output is available on a daily basis. Inputs are required as shown. The first component applies to all three soil groups. The last two require separate inputs for each soil group.

Model Components

Name	Inputs	Outputs
Potential Evapotranspiration	Monthly soil heat flux - initial Monthly daytime soil heat flux - initial Monthly nighttime soil heat flux - initial Monthly albedos - initial Monthly daylength - initial Solar radiation - daily Average air temp. - daily Minimum air temp. - daily Average dewpoint temp. - daily	Potential evapotranspiration

Name	Inputs	Outputs
	Cloud cover, day - daily Cloud cover, night, - daily	
Soil Moisture Profile	Initial soil moisture content - initial Soil physics parameters - initial Plant physiological curves-initial Dates of killing frosts - initial Rainfall - daily Potential evapotranspiration - daily Storm runoff - daily	Soil moisture profiles Evapotranspiration Deep percolation Interception Depth of water in root zone by layers
Runoff	Soil physics parameters - initial Soil moisture profile - daily Rainfall data - 15 minute intervals	Runoff - 15 minute intervals

The potential evapotranspiration component was a computerized computation of equation (4-29) which is rewritten below for convenience.

$$E_o = E_p = \frac{1}{L} \frac{(R_h + G_o)\Delta + \gamma \, LE_h}{\Delta + \gamma} \tag{7-1}$$

One of the constraints adopted in the development of this paper was that the inputs should not be excessive, and be available on a routine basis. While hourly data are available for most inputs to this component, solar radiation is not. While its hourly value may be inferred from the total daily value and other hourly data, the increase in data input by a factor of 24 in order to obtain radiation values which are still only estimated did not seem justified. However, in order to make maximum use of what daily data were available, the 24 hour day was broken up into two periods -- daytime and nighttime.

Referring back to Chapter 4 it will be noted that the LE_h term and the γ term both contain the integrated eddy diffusion coefficient term D_E. Following Budyko's suggestion that there is a marked difference in the value of this coefficient between daytime and nighttime hours, the division of the 24 hour day into these two periods seemed a logical step to derive the maximum value from daily observations. Budyko's values for daytime ranged between 1.0 and 1.5 cm./sec. with an average daily value of .6 to .7 cm./sec. This implies that the nighttime values must be quite small.

Considerable effort was expended on determining the values of D_E to adopt for the daytime hours. The first step was to calculate using hourly data a single day's potential evapotranspiration. The day selected was a warm June day in 1964 which occurred soon after a day of moderate rainfall so that no feedback effect was to be expected. The daily radiation was apportioned by hours in the same proportions it was measured by Van Bavel (101) on a similar day. The value of D_E for each daylight hour was then varied sinusoidally from 1. at dawn to a maximum of 1.4 and back again to 1. at dusk. The value of D_E was set to .1 during the nighttime hours. The resultant calculations agreed well with Van Bavel's calculations and measurements on a similar day. The results of these calculations are shown in Figure 6.

Next an effort was made to split the day into two distinct periods - daylight and nighttime, and to achieve approximately the same values that would be obtained with hourly measurements. The resultant output also should be consistent with the estimates made by other methods of measurement or calculations. A value of 1.25 cm./sec. for D_E for daylight hours was tried using a mean temperature for daylight hours as the mean of the daily maximum and the daily average temperature. A value of .1 cm./sec. was assigned for nighttime using a temperature as the mean of the daily minimum and daily average temperature. The mean value of potential evapotranspiration for six years using daily data was 59.7 inches per year. This compares with a mean value of 63.2 inches of lake evaporation computed by the U.S. Weather Bureau formula (56) and an average lake evaporation of 64 inches from the Weather Bureau evaporation maps (51). This compares with an average measured sunken pan evaporation for the same six years in the area of 52.0 inches. The correlation coefficient between the daily values computed by the Weather Bureau formula (which requires a value for wind travel) and equation (7-1) using the values of D_E as described above was .95. The value of evapotranspiration computed using hourly values of temperature for June 24, 1964 was .3524 inches and by use of the approximating scheme adopted was .3118 inches. In the absence of more data with which to compare results, it was felt that the procedure used was satisfactory to give daily estimates of potential evapotranspiration.

To estimate the daytime and nighttime values of soil heat flux, G_o, the data of Carson and Moses (11) were used. They give values of soil heat flux which penetrates to a depth of 29 feet for each month, and graphs of diurnal flux penetrating to a depth of 4 feet for every other month. The 24 hour integral of the diurnal values is near zero. The daytime soil heat flux was determined by adding the daytime soil flux as shown on the appropriate graph to the mean value shown on the annual graph.

For nighttime, the flux depicted in diurnal graph for nighttime hours was added to the daily mean for the month as shown by the annual graph.

No cloud height or type data was an input to evapotranspiration formula. Since equation (4-21) for sky backradiation requires this data, it was estimated from the average temperature-dewpoint temperature spread. The linear equation $k' = (.36 - (T - T_d))/150$ was used to give the necessary range of values of k' from .04 to .24. While this equation is quite crude, it should be noted that the exponent of n in equation (4-21) is quite large and the contribution of the $k'n'^2$ term is very small for values of n' of .5 or below. This occurs about 55 percent of the time in the study area. The linear formula above will at least estimate the value of this term on days when it is significant, the removal of this requirement for hourly cloud height or type seemed justified.

The radiation term in equation (7-19) is routinely available on a daily basis. However, the standard Eppley pyranometer used by the Weather Bureau has a glass cover which transmits short wave radiation only between .35 and 2.8 microns; part of the incoming solar radiation is not measured. The amount which is not measured is approximately one percent. Since an amount of energy approximately equivalent to this is required by plants for photosynthesis, it was assumed that the two effects offset each other and the radiation quantities were used directly as reported.

Other terms required for the execution of equation (7-19) are vapor pressure, saturation vapor pressure, atmospheric pressure, and the rate of change of the saturation vapor pressure with temperature, (the Δ term). The atmospheric pressure was assumed constant at the mean value of the Oklahoma City airport which by coincidence was at the same elevation as the study area. The vapor pressures were determined by using the mean and dewpoint temperatures in a formula for saturation vapor pressure set forth by Bosen (6). This formula yields values which agree with the values published by List (59) to less than .05 percent for temperatures below 100^{o}F. The Δ term is simply the temperature derivative of this formula.

The soil moisture content of the overall program required the construction of a logical model of soil moisture flux which could be verified by comparison with measured and estimated moisture fluxes in the study area. Each soil group required the basic inputs of porosity, soil moisture content after one day's drainage from a saturated condition (roughly field capacity), the saturated hydraulic conductivity, a drainage exponent, root length distribution with depth, and the value of soil moisture content at a suction of 15 bars (roughly the wilting point). Since the model was to be compared with soil moisture measurements taken at six inch intervals from 3 to 51 inches in depth, values of these parameters had to be entered for the eight measured levels and for the 0 to 3 inch depth layer which was not measured.

First considering this nine layered model as subject to no evaporation or lateral moisture flux, the only process active is a drainage to equilibrium potentials. After establishing an initial water content in each layer, drainage was conducted according to equation (3-20). This equation is the time derivative of

$$W_n = P_{on} Z_n t^{-b}. \qquad (7\text{-}2)$$

Here the meaning of the constant P_{on} is clear, for setting t = 1, the water contained in the nth layer is equal to the soil moisture content after one day times the depth of the layer. The authors of this equation (68) give a value for P_o of .256 and an empirically derived value for b of .128. The assumption was made that water would drain from layer to layer and the time step adopted for this process would be a one day interval. Differentiating (7-2) with respect to time,

$$-\frac{dW_n}{dt} = b\ P_{on}\ Z_n\ t^{-(b+1)}. \qquad (7\text{-}3)$$

Noting the values for P_o and b listed above it is obvious that after a few time steps the order of drainage whether taken from top to bottom or bottom to top is immaterial as the quantities incremented in or out of each layer are small compared with the water content already existing in the layer. For convenience the computed quantity of water drained from each layer was based on the water contained in that layer at the time of computation. The water content of each layer was then incremented by water coming from the layer above and decremented by the quantity of water drained from itself. For a value of t less than one day the computation is unstable and the maximum drainage permitted for any time step was that given by setting t = 1 in equation (7-3). In general for the range of applicable values of b and t, the larger the value of b the more rapid the drainage. The appropriate value for P_{on} for each soil group could be very well estimated from a series of soil moisture measurements made during the year for each layer. P_{on} would be the largest value of water-filled pore space measured.

When rainfall was allowed to interact with the model, the daily quantity of rainfall less the runoff computed by the runoff component of the model was allowed to fill the unfilled pore space of each layer successively from the top down until the total depth of rain was absorbed. This resulted in soil moisture profiles similar to that shown in Figure 4 as the approximated soil moisture with the exception that the layer immediately below the last saturated layer was used to accommodate the last bit of infiltrated volume. This moisture profile was immediately redistributed by calculating the soil moisture content which was the mean between the θ of the uppermost unaffected layer and the θ_{sat} of the lowermost saturated layer. All water to the right of this calculated mean value of θ was moved downward into successive layers up to that value of θ until all of the water in the saturated layers to the right had been moved downward. This resulted in a soil moisture profile shown in Figure 4 as the soil moisture profile after redistribution. The drainage portion of the program was then executed on this profile. To introduce the effects of evapotranspiration, two regimes were used. A winter regime was used between the average date of the first killing frost in the fall and the average date of the last killing frost in the spring. During this period water was removed from the top 3 inch layer in accordance with equation $-\frac{dw}{dt} = .5\ E_p t^{-\frac{1}{2}}$. This is similar to equation (3-22). The value of t was set to one each day of a rain and incremented by one each day of no rain. The amount of moisture evaporated from the top layer was limited to the moisture content at the 15 bar suction soil moisture content after which water was removed from the lower zones successively.

For the summer routine the calculated values of evapotranspiration from the first component of the overall program were used multiplied by two reducing factors. The first was the ratio of actual evapotranspiration to calculated potential evapotranspiration as developed by

Denmead and Shaw (19). See Figure 7. To use these data the soil moisture content of the entire 51 inch layer was calculated. The abscissae of the plotted points for each soil group was varied to accommodate the differing field capacity and 15 bar percentage points of that group. Any value of E_p which exceeded the maximum value of the lines shown was presumed to fall on the maximum line. The actual value of E/E_p was determined by a two-way linear interpolation of the plotted data points. The value of E_p derived from the processed meteorological data was then multiplied by this ratio. It was also multiplied by a grass maturity factor as suggested by Pierce (81) for the periods of 30 days after the spring frost and 60 days prior to the fall frost.

In both the summer and winter evaporation routines interception losses of up to .04 inches were accumulated from rainfall. The calculated evaporation was first applied to the intercepted water before allowing any water to be evaporated from the soil.

The process described above yields the total amount of moisture to be extracted from the soil. The question remains unanswered as to the layers from which it is extracted. This total quantity was removed from each layer of soil in accordance with equation (5-1), which is rewritten below.

$$\frac{dW_n}{dt} = B(\delta - \Psi_{mn} - Z_L)k_n L_n. \qquad (7\text{-}4)$$

The value of δ was assigned as 15 bars and the values of Ψ_m were determined from equation (3-17) using suggested values of the parameters for each soil group and comparing the results with measured values of Ψ_m for various soil series of similar texture. The values used for L_n were given by a logarithmic distribution with depth varying from 100 at the surface to 1 at a depth of the lowest layer. The value of hydraulic conductivity for each layer was determined from a formula given by Gardner (30)

$$k = K/[(\Psi_m/b)^r + 1] \tag{7-5}$$

where b and r are soil parameters. b is the potential at which $k = K/2$ and s ranges between 1.5 for fine textured soils to as high as 10 or more for sandy soils. This was the only occasion in which the unsaturated hydraulic conductivity, k, was required. Since the purpose of this routine was solely to distribute proportionally an already determined quantity, E, the precise determination of k was not critical. The total evapotranspiration water to be removed from the soil, E, was then proportioned to each layer after each $\frac{dW_n}{dt}$ was determined from equation (7-4). If the matric potential, Ψ_m, exceeded the prescribed 15 bar root suction at any level then no water was removed from that level. This process resulted in using water in the upper zones first and then progressively using water from lower zones as the upper zones dried to the wilting point. This is in agreement with the observed phenomena.

Through the entire evapotranspiration process the drainage continued. It was assumed that equation (7-2) applied to cases where the soil moisture was reduced by evapotranspiration or replaced by downward moving infiltrated water. Thus t was not a real time value, but was continuously computed from soil moisture conditions. Solving for t from equation (7-2),

$$t = \left(\frac{P_{on} Z_n}{W_n}\right)^{1/b} . \tag{7-6}$$

To ensure that all processes were accounted, a water balance was computed daily. This balance was initially zero. And it was maintained by adding all rainfall and subtracting the computed evapotranspiration applied, the intercepted rainfall, the runoff, the water that drained from the lowest layer, and changes in water stored in the 51 inch depth total thickness. This balance should have and did remain exactly zero through the six years of computations.

The last component of the overall model is the runoff computation. This component merely compares the infiltration rate, i, with the storm rainfall rate for successive small equal increments of time. The rainfall rate was determined by use of one of two recording rain gages located near the center of the basin. The rain gage used was the one whose total accumulated rainfall was closer to the average basin rainfall. The accumulated rainfall for the storm was noted every 15 minutes. for the duration of the storm at the index gage. All of the values were then multiplied by the ratio of the basin average rainfall to the total index gage rainfall. This procedure ensured that the storm rainfall for the basin was exactly as computed from the 36 measuring gages. The procedure implies that the total basin rainfall was distributed in time as indicated by the index gage. These ratios varied from .47 to 2.95 but in most of the cases studied the ratio was near 1.

This procedure was dictated by operational considerations. At the present time or in the foreseeable future, determination of accumulated rainfall in intervals of less than 15 minutes does not seem practicable. Present operational rain gage networks have a reporting rain gage spacing which results in a square mile per gage ratio approximately the same as the area studied (207.7 square miles). It is expected that additional radar data will soon become available which will enable a forecaster to increase or decrease the area estimate of rainfall as recorded by the reporting gage. This is discussed further in Chapter 9.

The infiltration rate was determined by equation (3-12) which is written below

$$i = K(Z_L + P - \Psi_m)/Z_L. \tag{7-7}$$

The ponded depth, P, was arbitrarily set equal to .01 inch. The rainfall was determined every fifteen minutes as described above. In order to accommodate the detailed structure of the soil model and to reduce the errors in the numerical integration, a time increment of one minute was adopted. For each one minute increment the infiltration rate in inches per minute was computed using equation (7-7). This was compared with the rainfall rate determined by the index gage. For any time increment, if $i > R$, the depth of saturation was incremented by the amount $R/(\theta_{sat_n} - \theta_{on})$. If $R > i$, the depth was incremented by the amount $i/(\theta_{sat_n} - \theta_n)$ and runoff was set equal to $R - i$. Every 15 minutes a new rainfall rate was computed from the index gage data and the accumulated incremental runoff for the 15 minute period was printed out. If during this process a new layer was reached by the penetrating wet front, a new value of θ_o was used to compute Ψ_m and a new value for effective hydraulic conductivity set by using equation (3-16). Since the rainfall totals were fairly small, less than 3 inches, and the initial air filled pore space reasonably large (θ_o small), only the first three layers (15 inches) of the soil profile were needed to calculate the total runoff. Finally the runoff from channel precipitation, urban areas, and paved roads was calculated by multiplying the total storm rainfall by the appropriate area factor.

Equation (7-7) when using values of Ψ_m such as shown in Figure 2 is extremely sensitive to soil moisture. Also in the absence of a large catalog of typical Ψ_m curves, the exact shape of this curve is somewhat in doubt. Naturally for runoff computations the wetting curve should be used. The curve for Soil Group II is shown on the Figure. Also considering the fact that the values for saturated hydraulic conductivity, K, vary over several orders of magnitude for different soil types, the errors that can occur in determining the value of i can be quite large. For effective use of this equation a large catalog of Ψ_m curves and values of K would be most beneficial. They are not now widely available. The equation does allow considerable latitude for empirical adjustment, however. This is particularly true for the value assigned to K. Reasonable small changes in this number would be expected to radically change the value of runoff from any storm. An attempt was made to do this during the development of the model.

This model artificially introduces an anomaly in the shape of the infiltration curve. In passing from zone to zone the value of θ_o changes which greatly changes the value of the soil suction, Ψ_m, in the numerator of equation (7-7). Thus the i curve has a discontinuity whenever the wet front passes into a deeper zone. If the deeper zone is drier than the zone immediately above, the increase in suction may be greater than the increase in the denominator, Z_L, and an increase in the infiltration rate may be computed. This is never observed. No attempt was made to achieve smoothing of the profiles before execution of the runoff routine.

Finally some comment should be made about the overall model. It was basically sound in that it was stable and self-compensating to a large degree. For example if potential evapotranspiration were over-estimated then the application of the curves shown in Figure 7 would partially correct the error and the amount of water actually transpired would be near the correct value. Likewise, the use of equation (7-3) for the drainage calculation tended to ensure that the water content of each layer was held to a reasonable limit. These features greatly aided in the development of the model. However, the model is overall very complex. The number of interactions is so large that it is virtually impossible to predict what the particular effect of changing any input will be. Any change of input or parameters which is designed to change any particular output may have adverse effects on other outputs. The key to improvement of all complex physical models is not empirical tinkering but more careful refinement of the basic physical inputs and relationships.

CHAPTER 8

RESULTS AND COMPARISONS WITH MEASUREMENTS AND ESTIMATES

The results from the potential evapotranspiration component of the model are shown in Table 1. Although only the monthly totals are shown, a value was computed and used daily. These computations were compared with other formulae and evaporation pan measurements as described in the preceding chapter. There were no daily measurements or other estimates of this factor made during the study period in the study area.

The outputs from the soil moisture profile component of the model were interesting and consistent with measurements and other estimates of the outputs. The Agricultural Research Service takes measurements of soil moisture profiles on eight small rangeland drainage areas. These measurements are taken about every two weeks. At the time of the soil moisture measurement the accumulated runoff since the preceding measurement is calculated from a recording runoff gage for each area (usually about 20 acres). These measured areas are located near recording rain gages. Thus for any area the values of rainfall, runoff, and soil moisture are measured on a frequent basis. Two of these watersheds had soil series similar to Group I and Group II soil series as given in the preceding chapter. None of the measured plots are located in the Little Washita basin, but they are located within 15 miles of it. There are no ground water level observation wells at these plots.

As previously described, these measurements were used to develop the parameters of the soil moisture profile component of the model. The results of the model output as compared with the measured values of the two soil groups are shown in Table 2 and Table 3. These two tables dramatically illustrate the difference in soil moisture content of these two different soils. (Primarily the Darnell series for Group I and R-4 and Kingfisher series for Group II and R-8.) The soil for the Group II model contained nearly twice as much water on the average as the soil for the Group I model. The Group I soil drained rapidly and indicated that the 15 bar suction soil moisture content was quite low. The Group II soil drained slowly and indicated a 15 bar suction soil moisture content of about .10. The measured runoff for the year 1968 from plots R-4 and R-8 was .02 and 3.97 inches respectively.

In comparing the models with the measurements, there were considerable differences in absolute values and rather large percentage differences. The reliability of the absolute values of the measurements is unknown since no measurements of soil water content by weighing wet and oven-dried samples were made. ARS personnel state that there may be considerable error in the absolute values but the incremented changes are reasonably accurate. There is some instrument error in the measurements, of course. The neutron probe is calibrated twice with a known source and then a measurement at each depth interval is made. The conversion of instrument counts to soil moisture content is then made by using the mean of the two calibrations. The error between the two calibration counts is usually less than ten percent. Based on the above information, perfection of the parameters of the model was directed toward improving the differences between incremental changes of the successive model computations and successive measured values -- improving the correlation coefficient.

There were no soil moisture measurements available for soils of Group III. Descriptive literature of these soils indicate that they were more akin to Group I in runoff properties. The literature indicated that they were less well drained and had slightly higher runoff percentages than Group I soils. Consequently, guided by experience gained from perfecting the parameters for the Group I and Group II models, a Group III model was developed which reflected the described properties of these soils. Unfortunately Group III soils comprise 55 percent of the watershed and there was no way to compare the model with measurements.

After the three soil moisture models were developed using 1968 data the entire six year period of rainfall was imposed on each group along with the calculated potential evapotranspiration and runoff calculated from the runoff model. The annual summary of the results is shown in Table 4. All of the data shown in Table 4 are consistent with the soil moisture measurements and with other observed and estimated hydrologic data. Of particular interest is the average value of deep drainage which compares favorably with an estimate of 2.5 inches of groundwater recharge annually made from groundwater studies (95). Referring to the climatological data shown in Chapter 6, it will be noted that the average streamflow is equal to the estimated groundwater recharge. Since some portion of this streamflow is derived from surface runoff, it is obvious that some of the recharge is disposed of in a manner other than groundwater flow. This disposal of excess deep drainage can be logically accounted by considering that there are approximately 16 square miles of streambed and alluvial plain with shallow water tables. This area is generally bordered or covered with deep rooted plants which can reach the water table. During the warm season these plants do not experience a reduction in evapotranspiration but draw on the shallow water table for water. Observation wells in the area show a marked diurnal fluctuation in the water table during the warmest months. During the wintertime these plants are not active and the groundwater table in the alluvial plain rises due to recharge flowing downward from the surrounding hills and the base flow of the stream rises.

While the purpose of this paper was not to develop a complete flow regime for the stream, a rough quantitative check was made of this deep-rooted plant effect. The difference between calculated warm season potential evapotranspiration and the actual evapotranspiration computed by the soil moisture component will be the depth of water in inches removed by the plants growing in the 16 square mile area. By subtracting this volume from the recharge volume a reasonable accounting was made of the total streamflow between surface runoff and groundwater flow.

A model of the streamflow regime would be complicated because of uncertainties in the saturated flow regime from the surrounding aquifers and difficulties in modeling the groundwater flow from the alluvial plain water table. Such a model could be built, however, and the deep drainage values from the soil moisture component would be an essential input. Such a model is beyond the scope of this paper.

The large values of deep drainage shown in Table 4 for the year 1968 arise from the fact that the six month cool season rainfall amounted to 13.92 inches (nearly half the annual total). This rainfall was not subject to large evaporation losses and consequently penetrated to lower levels.

In order to compare the results of the output of the third and final component of the model -- the runoff component, the hydrographs of all rises having a peak flow of 150 cubic feet per second (c.f.s.) were plotted. They numbered 53 for the period of record studied. However, the volumes of water contained in most of them was so minute that only the 15 floods having peak flows of 1000 c.f.s. or greater were studied in detail. The volume of each rise was computed by subtracting an estimated base flow from the hydrograph. Included in the flood volume was a considerable volume of the recession curve. Normally this would be classed as upstream groundwater base flow, but due to the character of the stream banks and alluvial plain which can absorb considerable volume, it was considered as upstream surface runoff which was appearing as return groundwater flow from the adjacent alluvial plain. Data for these floods are shown in Table 5.

Since the purpose of this paper is to develop a generalized model based on scientific principles without empirical cutting and trying, the flood volumes calculated from the hydrographs were compared with the output of the runoff component of the model. The output from the model using the best estimates of parameters is shown in Table 6. A small factor was applied to the total storm rainfall to account for runoff from channel precipitation, highways, and urban areas. This runoff is tabulated separately as channel precipitation runoff. The computation of surface runoff from land areas using equation (7-5) was performed as described in the previous chapter. A deduction of .04 inches was made from the rainfall to account for interception losses. And a value of .05 inches suggested by Musgrave and Holtan (64) was subtracted from the first occuring runoff to account for surface depression storage. The values used for the saturated hydraulic conductivity (in./hr.) of each soil group were as follows: Group I, .30; Group II, .15; Group III, .25. These values were selected from the final infiltration rates given in reference (64). The value for Group I being the upper limit of the above average classification of various soil series listed and the value for Group II being the lower limit of this classification.

The runoff computations did not give any runoff from the smaller storms. This was anticipated and a discussion of the results is given below. In order to provide for some reasonable estimate of runoff for these smaller storms an empirical formula given by Kohler and Richards (52) was used.

$$\text{Runoff} = \left(P_T{}^m + \left(\sum_{n=1}^{3} (\theta_{sat_n} - \theta_{o_n}) D_n\right)^m\right)^{\frac{1}{m}} - \sum_{n=1}^{3} (\theta_{sat} - \theta_o) D_n \qquad (8\text{-}1)$$

where P_T is the storm total precipitation, the last term is the soil depletion from the saturated condition, and m is an empirically derived value given by

$$m = A + \left(\sum_{n=1}^{3} (\theta_{sat_n} - \theta_{o_n}) D_n\right) B \qquad (8\text{-}2)$$

where A and B are empirically derived constants. The values adopted for A and B were derived by making several runs on the program until reasonable agreement was reached between the computed and measured values. The values of A and B adopted were close to those given by the authors. In addition the hydraulic conductivities of each group were adjusted slightly to improve the match of volumes from those larger floods which were computed by the model. The results of these computations are shown in Table 7.

In reviewing these results the most obvious conclusion is that the scheme developed does not account for runoff from the smaller floods. Referring to Table 5, the surface runoff was predicted by all three groups only from the largest floods, nos. 3 and 14 and from no. 4 which occurred the day after no. 3. The Group II routine also forecast some runoff from flood no. 10. This failure to forecast runoff is caused primarily by the basin averaging procedure used. The spacial variability of rainfall especially during summer months is enormous. The runoff occurs only from those areas that receive substantially more than the basin average. This problem has been recognized by hydrologists since the beginning of the science. The only remedy is to decrease the area being averaged. In the instant case a separate soil moisture history could be kept for each of the 36 reporting stations. And each storm could be processed using the rainfall reported by that gage. This would increase the number of computer runs by a factor of at least 12. This is a basic limitation to all rational models of the runoff process. As a result, rational schemes using this averaging procedure are limited to use for large area storms of high rainfall which can reasonably be expected to have a good degree of spacial uniformity and a rainfall depth that exceeds infiltration over all of the area. Fortunately, these are the types of storms which produce the most significant floods.

The most extreme case in point was the storm which produced flood no. 9. The basin average rainfall was but .66 inches, which had it been uniform, would have produced very little runoff. However, the rainfall reported by the 36 gages for this storm ranged from a low of .02 inches to a high of 2.46 inches. As a check on the ability of the scheme to forecast runoff volume from a more detailed definition of rainfall, the runoff from the storm which produced flood no. 10 was computed for the station reporting the heaviest amount of rainfall. The largest rainfall reported in the basin for this storm was 5.59 inches. It was located in an area composed of approximately one half Group II and one half Group III soil types. When processed through eight months of individual rainfall and soil moisture history, .89inches of runoff was forecast by the model from this storm from the area surrounding this gage. Had the whole basin received this amount of rainfall, .69 inches of runoff would have occurred.

Taking into account the inaccuracies introduced by averaging the basin rainfall, the results of the model were good. It did properly forecast that significant runoff processes were taking place from the storms that produced the two largest floods. It also predicted that the flood expected from the largest rainfall (no. 10) was not as serious as either of the largest floods (no. 3 and no. 14). Particularly rewarding were the results of using the best estimates of soil parameters based on available engineering data (Table 5). The error in forecasting the largest floods was reasonable. This indicates that good models can be developed using basic physical theory. These models could be applied to areas where there was no previous rainfall and stream gaging history which would preclude the use of empirically developed runoff-rainfall relationships.

The model also showed a hydrologic phenomenon which is well known to hydrologists but not generally appreciated. Runoff from soils is a phenomenon which occurs only rarely and briefly. The model indicated that significant surface runoff occurred during only five fifteen minute intervals during the entire six year period of rainfall history.

Finally, the model did not account for one observed phenomenon. The volumes of runoff recorded from the small watersheds on which soil moisture measurements are taken are considerably larger for soils of Group II than is given by the model. However, this large runoff is not detected downstream. This indicates that considerable channel losses are present. Since no channel loss or gain model was developed, this phenomenon was not accounted for by the model. Probably the runoff from this soil group is somewhat larger than that indicated by the model. The logical next step would be the development of a flow regime model retaining the important constraint of a water balance. This flow regime model could also account for streamflow derived from interflow and drainage into channels of water which has passed the root zone but not reached the water table.

CHAPTER 9

CONCLUSIONS

The fundamental conclusion to be drawn from the results is that the forecast scheme developed is capable of forecasting the volume of storm runoff for major large area storms. In synthesizing the runoff model from the applicable parts of the three major sciences involved - soil physics, meteorology, and plant physiology - lack of basic knowledge of the infiltration and evapotranspiration process was revealed.

In the field of soil physics the use of such a simplified equation as (7-7) limits the applicability of the model to fairly sandy soils. While Philip has been able to cast his solution of the soil moisture flow equation into the same form as (7-7), the physical meaning of his constant that replaces the saturated hydraulic conductivity, K, is difficult to grasp. The proper value to use is in question unless the complicated solution to the diffusion equation (3-10) is obtained. What is needed for practical use is a relationship between soil texture and the value of this constant. The inability of solutions of the diffusion equation to describe a non-homogeneous medium, makes finding an alternate to equation (7-7) difficult. Also, the simplified method used to redistribute the soil moisture immediately after infiltration was not well founded in theory. Here again the method used was too simple and the complete solutions of numerous diverse cases too complicated for practical use. What is needed is a more sophisticated procedure based on the exact solution of the equations which is still practicable. Equation (7-7) indicates that it is possible for the infiltration curve to be a curve which does not decrease monotonically with time. This is at variance with field tests. Laboratory experiments testing this possibility would be desirable. There also does not exist any clear theoretical relation between the diffusion equation and the empirical drainage equation (7-3). The use of this equation was the principal use of empirical data which was used in the model. A good theoretical development of a drainage equation which would either confirm or deny that equation (7-3) is the proper form for an equation to describe drainage is highly desirable.

The principal difficulties in the meteorological theory used to develop the model are bound up with the current difficulties in formulating an adequate description of atmospheric turbulence in the lowest level of the atmosphere. Until such a formulation can be made, certain empirically derived quantities will be inherent in any calculation of evapotranspiration. More study is needed to determine properties of the integrated eddy diffusion coefficient. The determination of the dependency of this coefficient to wind speed is needed both on theoretical and practical grounds. The assumption of a constant value probably oversimplifies the case, but only further comparison with evaporation rates determined by other methods can reveal if it does have a clearly defined functional relationship to other easily measurable atmospheric variables. In the cases studied by this paper, the value of calculated potential evapotranspiration was not critical. This potential was so large and the interval between floods so great that the soils were reduced to low moisture contents before nearly every flood. As a check on the sensitivity of the soil moisture component of the model to the input, the potential evaporation computed by the U.S. Weather Bureau was used. Although on the average these values were some eight percent larger than those given by the formula developed for this paper, the differences in soil moisture profiles before the day of the floods studied was insignificant. However, in a more humid climate or with a separation of several days between floods, this would not be the case.

The great debate among plant physiologists as to whether transpiration is significantly reduced before the wilting point is reached seems to be slowly resolving in favor of those who hold that it does. However, the data presented in Figure 7 are one of the very few pieces of quantitative data available. What is particularly needed is experimental data similar to those shown which also include measurements of soil moisture losses from several vertical zones. The zonal depletion scheme used, equation (7-4), appears very sound, but experimental evidence is scanty. Also similar type curves are needed for a wide range of soil types. Here again there is good theory extant which can describe the probable shape of the curves for different soils, but experimental data is needed.

Further refinement of the model also depends on integrating the three components with a complete flow regime model which incorporates such important streamflow factors as groundwater flow, water table levels, channel losses and alluvial plain and groundwater recharge from the stream. If such a model were built keeping a strict water budget, the interaction of the components of the runoff with the total hydrologic regime could be studied. This would allow adjustment of parts of the larger model to fit the known physical measurements of the output values.

The application of the evapotranspiration and soil moisture components of the model is not limited to the purpose of forecasting storm runoff. The potential evapotranspiration component would be very valuable in determining water losses from plants which are in permanent contact with a source of water. Phreatrophytes exist in an environment whose evaporation regime is determined by overland, not overwater, turbulence regimes. Consequently, land evaporation calculations should be applied to them. These plants are the cause of serious water losses in arid region streambeds and along the edges of lakes and rivers. The soil moisture component of the model has many practical uses. Obviously groundwater recharge is a function of the deep drainage output of this component. Also soil moisture conditions as a function of depth is the basis of wheat production forecasts. Currently the U.S. Statistical Reporting Service makes hand samplings of soil moisture conditions in the fall and spring in the major wheat growing regions of the United States. Since the correlation between fall soil moisture and the quality of the stand is good, forecasts of the quality of the stand can be made. Likewise, there is a good correlation between soil moisture existing in the early spring and the per acre yield of a crop with given quality of stand. This is particularly true in the more arid wheat growing zones where little help can be expected from precipitation during the rapid growing phase of the crop. Obviously reliable soil moisture models in wheat growing regions which are kept up to date by supplying the necessary meteorological inputs, would eliminate the expensive and often inaccurate hand sampling procedures and supply much broader area coverage which would be continuous in time. The output from the runoff model also gives the time at which significant runoff commences. This information is very important for making complete flood hydrograph forecasts, since the degree of synchronization of runoff from different subunits of an area is highly significant.

Before any conclusions can be reached concerning the area of applicability of the model and the conditions under which it could be used, the factors not treated by the model should be enumerated. These factors were: snowmelt, slope of the land, crop maturity factors for annual crops, crop type distribution and forested areas. These exclusions along with the hydrologic factors neglected and the inadequate description of infiltration into clay soils given by the infiltration equation would limit that area of application to areas similar to the grasslands of the southern Great Plains of the United States. However, a crop maturity factor for simple cropping systems such as winter wheat could be added very easily and the model would then be applicable to winter wheat regions. To add more complicated crop maturity factors and to account for the percentages of each type of crop would add complexity to the model and increase the labor required to apply it to a region of variegated agriculture, but it is not impossible to do. Likewise, snowmelt calculations are difficult, but not directly related to the applicability of the model. The cool season evaporation regime would have to be replaced by a snow water budget, but during the warm season the model would be directly applicable. Finally, if the rapid progress in the related basic sciences continues, the problems first related in this chapter will be reduced to such a magnitude that a model as presented in this paper could be applied to all nonmountainous regions of the world which support agricultural activity.

Before any conclusion can be reached as to the practical application of the model at the present time, the adequacy of the present reporting instrumentation should be considered. The most widely separated instruments are the radiation measuring devices. Present spacing is approximately 200 miles. In this study the calculation of potential evapotranspiration was made using meteorological measurements taken 45 miles from the area to which it was applied. Of course, a few scattered errors did occur, but the errors if scattered did not seem to seriously affect the performance of the model. This experience coupled with the observed movement of major meteorological features would indicate that this density of observations is adequate. For areas located nearly midway between stations, an averaging procedure may be advantageous.

However, the spacing of the reporting raingage stations is not adequate. In addition most of them are not recording gages and they report but three times a day. It should be remembered that this study was based on nearly exact knowledge of the average basin rainfall. Even though only one gage was used to time-distribute the rainfall, the total depth of rainfall was based on observations from a very dense raingage network. Without this knowledge results as good as those reported in Table 7 could not be expected. This implies that radar observations should be considered as a necessary part of the required instrumentation. Radar data can certainly give a good estimate of the time distribution of the rainfall. Digital displays of radar reflectivities integrated for 15 minute intervals are now possible. See reference (47). The grid size of these displays is 4 by 4 nautical miles. The conversion of these radar reflectivities to rainfall rates is best accomplished by calibration with a raingage in the vicinity. This process of determining detailed rainfall rates has succeeded in reducing the errors of estimate to reasonable limits and appear to make this method of determining the rainfall over an area with wide gage spacing practicable. See reference (109). A test of the operational feasibility of this procedure is currently being conducted by the U.S. Weather Bureau. Of course, adoption of this technique would reduce the area of applicability of the model to areas within radar range. In addition, it would require the installation of recording transponder raingages to provide the necessary calibration data. In several areas of high population density and high flood risk, there are already in existence special flash flood warning raingage networks. For these networks the system developed in this paper is directly applicable without any further instrumentation.

The fundamental conclusion of this paper is that the basic sciences contributing to the determination of the volume of storm runoff have advanced to the point where it is not only possible but practicable to construct a computerized model which can be executed on a medium sized computer. While the quality of output of such a model is improved by comparison with measured flood volume, reasonable estimates can be obtained where a significant period of record of streamflow data is not available. The accuracy of the model output can also be improved to almost any degree desired if the required instrumentation is provided and the expenditure of manpower and machine time to increase the detail of the model is made. The practicability of constructing such models depends on the willingness of the society involved to make the necessary expenditures.

TABLE 1

MONTHLY TOTAL POTENTIAL EVAPOTRANSPIRATION - INCHES

Month	1963	1964	1965	1966	1967	1968
Jan.	2.13	3.07	1.99	1.79	2.88	1.43
Feb.	3.27	2.83	2.91	2.25	3.08	2.68
Mar.	5.04	4.48	3.38	4.85	5.01	2.86
Apr.	5.89	6.32	5.45	4.69	5.24	5.92
May	6.57	6.49	5.67	6.95	6.95	5.42
June	7.97	6.76	7.15	7.80	7.21	6.96
July	8.66	9.36	8.80	8.43	7.67	8.22
Aug.	8.22	7.59	7.61	6.35	8.38	7.68
Sept.	6.16	4.58	5.77	4.43	5.41	6.23
Oct.	6.13	4.47	4.29	4.62	4.84	4.74
Nov.	3.35	2.24	2.76	3.21	3.07	2.37
Dec.	1.89	1.95	2.34	1.96	2.06	2.27
TOTAL	65.29	60.13	58.12	57.35	61.80	56.79

TABLE 2

SOIL MOISTURE COMPARISONS -- 1968

GROUP I and ARS PLOT R-4

Date of Measurement	Depth of water (in.) in 3 - 9 inch layer		Depth of water (in.) in 3 - 51 inch layer	
	Measured Plot R-4	Model Group I	Measured Plot R-4	Model Group I
1 - 24 - 68	1.54	1.34	7.84	8.17
2 - 6	1.38	1.14	7.75	8.23
2 - 27	1.47	1.06	8.21	7.63
3 - 25	1.43	1.08	8.29	7.24
4 - 4	1.33	1.03	8.23	7.12
4 - 29	1.01	0.98	7.10	7.21
5 - 7	0.65	0.79	6.44	6.56
5 - 16	1.39	1.37	7.56	7.86
5 - 24	1.03	1.01	6.75	7.32
6 - 10	0.97	0.90	6.67	7.22
6 - 27	0.45	0.68	5.10	6.00
7 - 9	0.46	0.62	4.75	5.53
7 - 23	0.56	0.87	4.76	5.62
8 - 5	0.51	0.82	4.49	5.63
8 - 20	0.80	1.03	4.73	6.03
9 - 20	0.44	0.59	4.15	5.00
10 - 3	0.66	0.64	4.35	4.90
10 - 24	0.86	0.88	4.85	5.54
11 - 22	1.28	1.24	5.66	7.23
12 - 3	1.43	1.30	6.75	9.04
Corr. Coef	.893		.835	

TABLE 3

SOIL MOISTURE COMPARISONS -- 1968

GROUP II and ARS PLOT R-8

Date of Measurement	Depth of water (in.) in 3 - 9 inch layer		Depth of water (in.) in 3 - 51 inch layer	
	Measured Plot R-8	Model Group II	Measured Plot R-8	Model Group II
1 - 25 - 68	1.83	1.83	12.13	12.45
2 - 5	1.82	1.78	12.49	12.95
2 - 19	1.78	1.62	12.27	12.79
3 - 1	1.82	1.76	12.50	12.68
3 - 26	1.87	1.78	13.17	12.93
4 - 4	1.81	1.64	12.83	12.76
4 - 11	1.68	1.58	12.48	12.58
4 - 26	1.77	1.63	12.49	12.22
5 - 2	1.62	1.42	12.26	11.62
5 - 8	1.68	1.46	12.01	11.40
5 - 17	1.77	1.58	12.46	11.69
5 - 23	1.69	1.46	12.29	11.29
5 - 29	1.70	1.51	12.37	11.33
6 - 4	1.81	1.60	13.01	11.53
6 - 11	1.72	1.47	12.92	11.26
6 - 25	1.32	1.26	11.45	10.56
7 - 5	1.66	1.57	11.89	11.08
7 - 12	1.40	1.30	11.13	10.47
7 - 17	1.72	1.59	11.56	10.83
7 - 26	1.41	1.38	11.02	10.56
8 - 8	1.02	1.15	9.42	9.82
8 - 22	0.99	1.21	8.57	9.50
9 - 19	1.31	1.29	8.94	9.51
10 - 2	1.13	1.10	8.36	9.06
10 - 23	1.60	1.44	9.85	9.72
11 - 21	1.77	1.76	10.71	10.76
12 - 2	1.81	1.87	11.77	12.29
12 - 7	1.73	1.61	11.76	12.29
Corr. Coef.	.902		.865	

TABLE 4

ANNUAL WATER BALANCE

(All values in inches)

GROUP I

Year	Rainfall	Evapo-Transpiration	Interception Losses	Surface Runoff	Deep Drainage	51" Layer Δ Storage
1963	17.72	17.20	2.38	0.06	0.82	-2.74
1964	30.79	20.80	2.56	0.93	3.12	+3.38
1965	25.70	21.44	2.66	0.23	2.63	-1.26
1966	19.60	18.65	2.55	0.09	1.58	-3.28
1967	26.33	19.46	2.75	0.17	0.08	+3.87
1968	34.02	22.01	3.41	0.30	7.05	+1.24
Average	25.69	19.93	2.72	0.30	2.55	+0.20

GROUP II

Year	Rainfall	Evapo-Transpiration	Interception Losses	Surface Runoff	Deep Drainage	51" Layer Δ Storage
1963	17.72	17.21	2.38	0.28	0.84	-3.00
1964	30.79	20.16	2.56	2.38	1.68	+4.00
1965	25.70	21.41	2.66	0.60	2.70	-1.67
1966	19.60	18.83	2.55	0.52	1.16	-3.46
1967	26.33	18.95	2.75	0.67	0.11	+3.85
1968	34.02	22.09	3.41	1.56	5.05	+1.90
Average	25.69	19.78	2.72	1.00	1.92	+0.27

GROUP III

Year	Rainfall	Evapo-Transpiration	Interception Losses	Surface Runoff	Deep Drainage	51" Layer Δ Storage
1963	17.72	17.21	2.38	0.06	1.90	-3.83
1964	30.79	20.92	2.56	1.06	2.64	+3.62
1965	25.70	21.67	2.66	0.29	2.35	-1.28
1966	19.60	18.88	2.55	0.10	1.54	-3.48
1967	26.33	19.46	2.75	0.20	0.02	+3.91
1968	34.02	22.38	3.41	0.47	6.33	+1.42
Average	25.69	20.09	2.72	0.36	2.45	+0.06
Six Year Basin Average Using Area Weighing Factors	25.69	19.97	2.72	0.53	2.31	+0.14

TABLE 5

FLOOD DATA

Flood No.	Date	Rainfall in.	Vol. from Hydrographs (c.f.s. - days)	Vol. in inches (165 sq. miles)
1	6 - 23 - 63	0.95	143	0 .03
2	7 - 13 - 63	1.18	186	0 .04
3	5 - 9 - 64	2.11	2392	0 .53
4	5 - 10 - 64	0.75	593	0 .13
5	9 - 20 - 64	1.13	255	0 .06
6	11 - 3 - 64	1.83	404	0 .09
7	11 - 16 - 64	1.42	741	0 .16
8	5 - 26 - 65	1.50	249	0 .06
9	6 - 15 - 65	0.66	438	0 .10
10	8 - 28 - 65	2.42	804	0 .18
11	8 - 31 - 65	1.04	294	0 .07
12	4 - 9 - 67	1.90	609	0 .14
13	4 - 12 - 67	1.35	373	0 .08
14	5 - 31 - 68	1.91	1450	0 .32
15	10 - 9 - 68	1.40	362	0 .08

TABLE 6

RUNOFF ESTIMATES

Flood No.	Group I Surface Runoff (in.)	Group II Surface Runoff (in.)	Group III Surface Runoff (in.)	All Groups Channel Precip. and Urban Runoff (in.)	Total Runoff (165 sq. mi.) c.f.s. - days	Total Runoff (165 sq. mi.) inches
1	0	0	0	0.016	73	0.016
2	0	0	0	0.020	90	0.020
3	0.523	0.706	0.601	0.036	2897	0.653
4	0.046	0.107	0.149	0.012	593	0.133
5	0	0	0	0.019	86	0.019
6	0	0	0	0.031	140	0.031
7	0	0	0	0.024	109	0.024
8	0	0	0	0.025	115	0.025
9	0	0	0	0.011	50	0.011
10	0	0.015	0	0.041	204	0.046
11	0	0	0	0.017	80	0.017
12	0	0	0	0.032	145	0.032
13	0	0	0	0.023	103	0.023
14	0.154	0.400	0.370	0.032	1662	0.375
15	0	0	0	0.024	107	0.024
				Total vol.	6454	
				Corr. Coef. with Table 5	.952	

TABLE 7

RUNOFF ESTIMATES USING MODEL AND EMPIRICAL ESTIMATES

Flood No.	Group I Surface Runoff (in.)	Group II Surface Runoff (in.)	Group III Surface Runoff (in.)	All Groups Channel Precip. and Urban Runoff (in.)	Total Runoff (165 sq. mi.) c.f.s. - days	Total Runoff (165 sq. mi.) inches
1	0.000	0.010	0.001	0.019	102	0.023
2	0.005	0.045	0.008	0.023	187	0.042
3	0.506	0.705	0.518	0.042	2711	0.611
4	0.032	0.106	0.102	0.015	473	0.106
5	0.004	0.033	0.006	0.022	163	0.036
6	0.024	0.138	0.028	0.028	424	0.095
7	0.028	0.176	0.028	0.028	438	0.098
8	0.005	0.041	0.007	0.030	210	0.047
9	0.001	0.016	0.001	0.013	85	0.019
10	0.086	0.032	0.111	0.049	593	0.134
11	0.013	0.071	0.016	0.021	233	0.052
12	0.016	0.089	0.024	0.038	353	0.079
13	0.044	0.162	0.050	0.027	479	0.108
14	0.100	0.379	0.244	0.038	1314	0.296
15	0.019	0.110	0.022	0.024	334	0.075

Total vol. 8099

Corr. Coef. with Table 5 .972

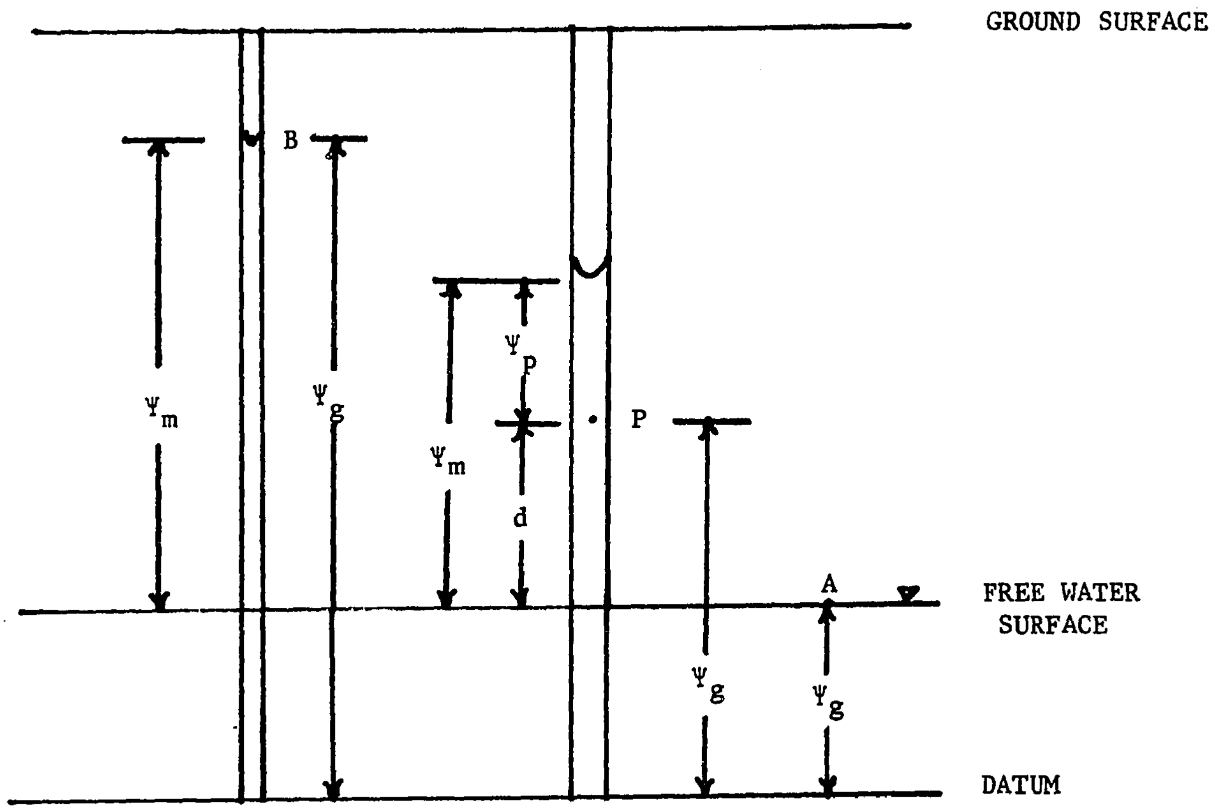

Fig. 1. Illustration of potentials.

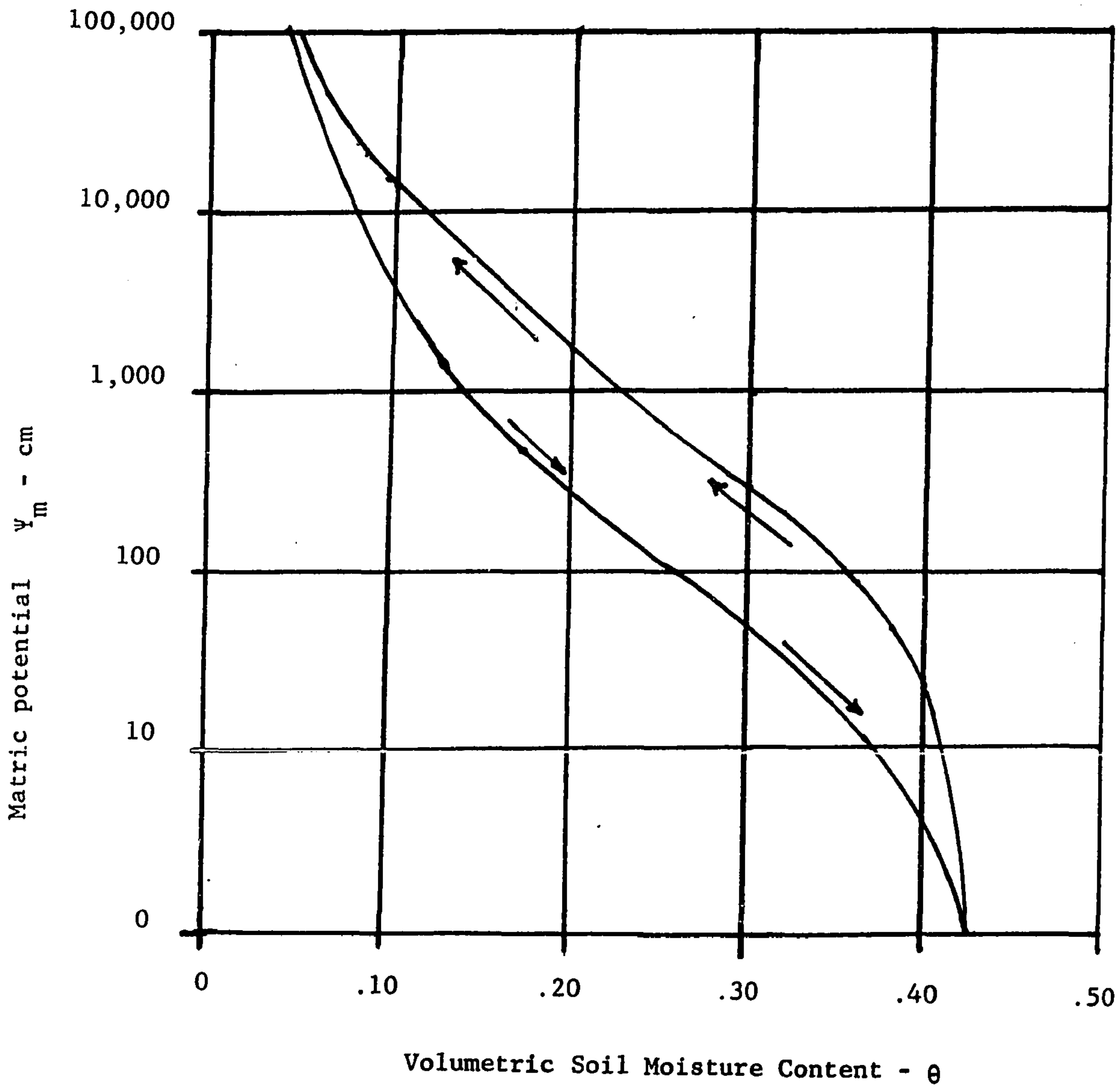

Fig. 2. Typical matric potential curve.

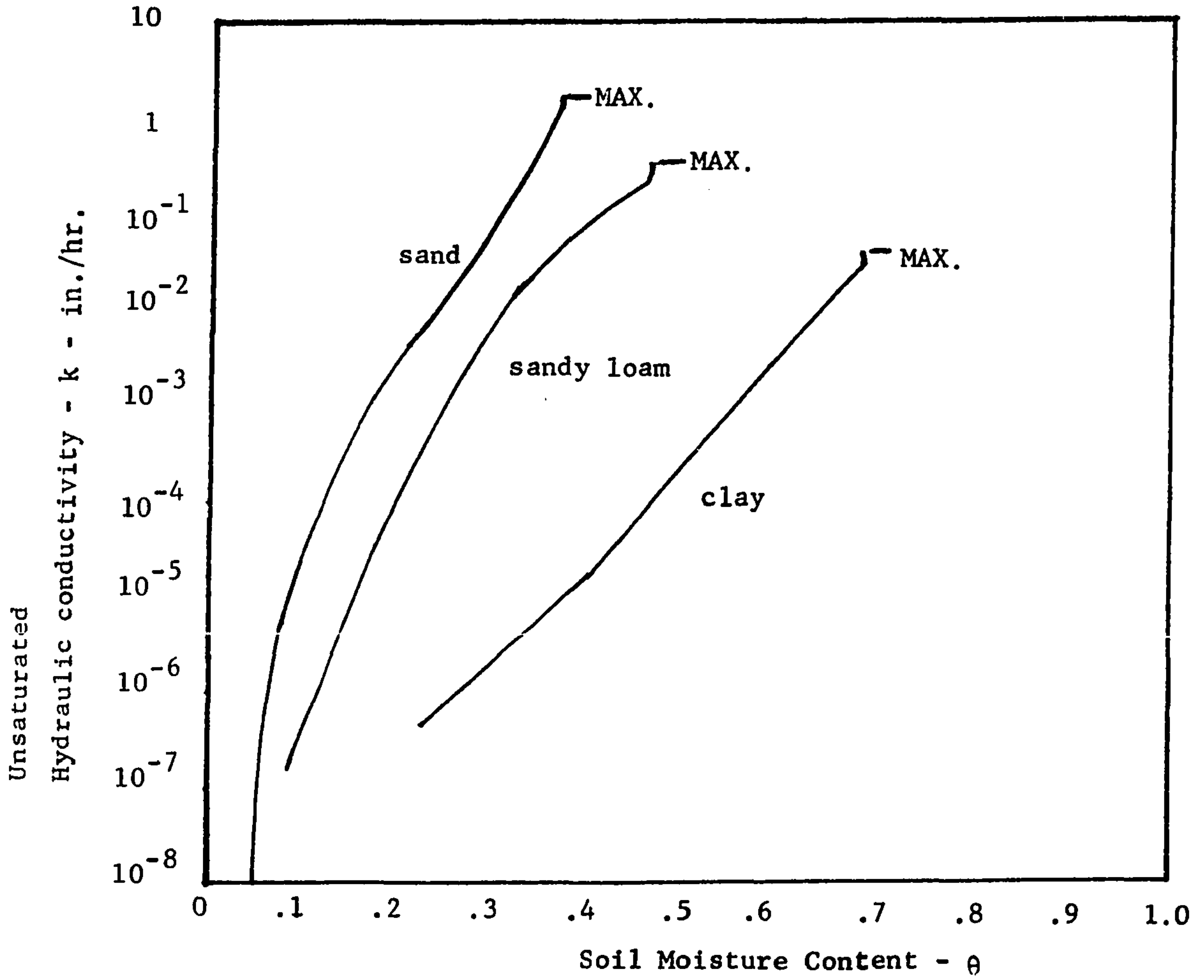

Fig. 3. Hydraulic conductivities as a function of soil moisture content for three different soil types.

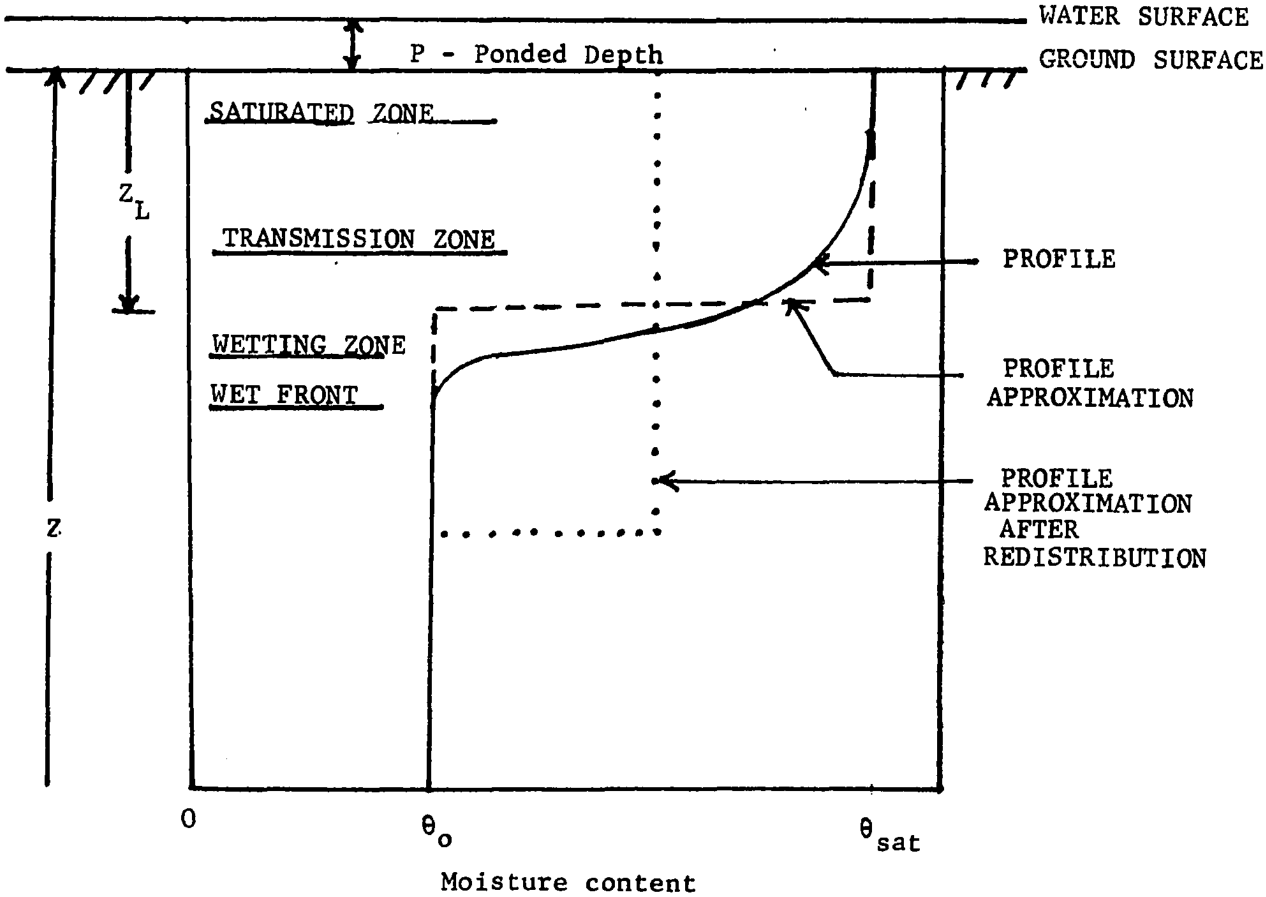

Fig. 4. Typical soil moisture profile during infiltration.

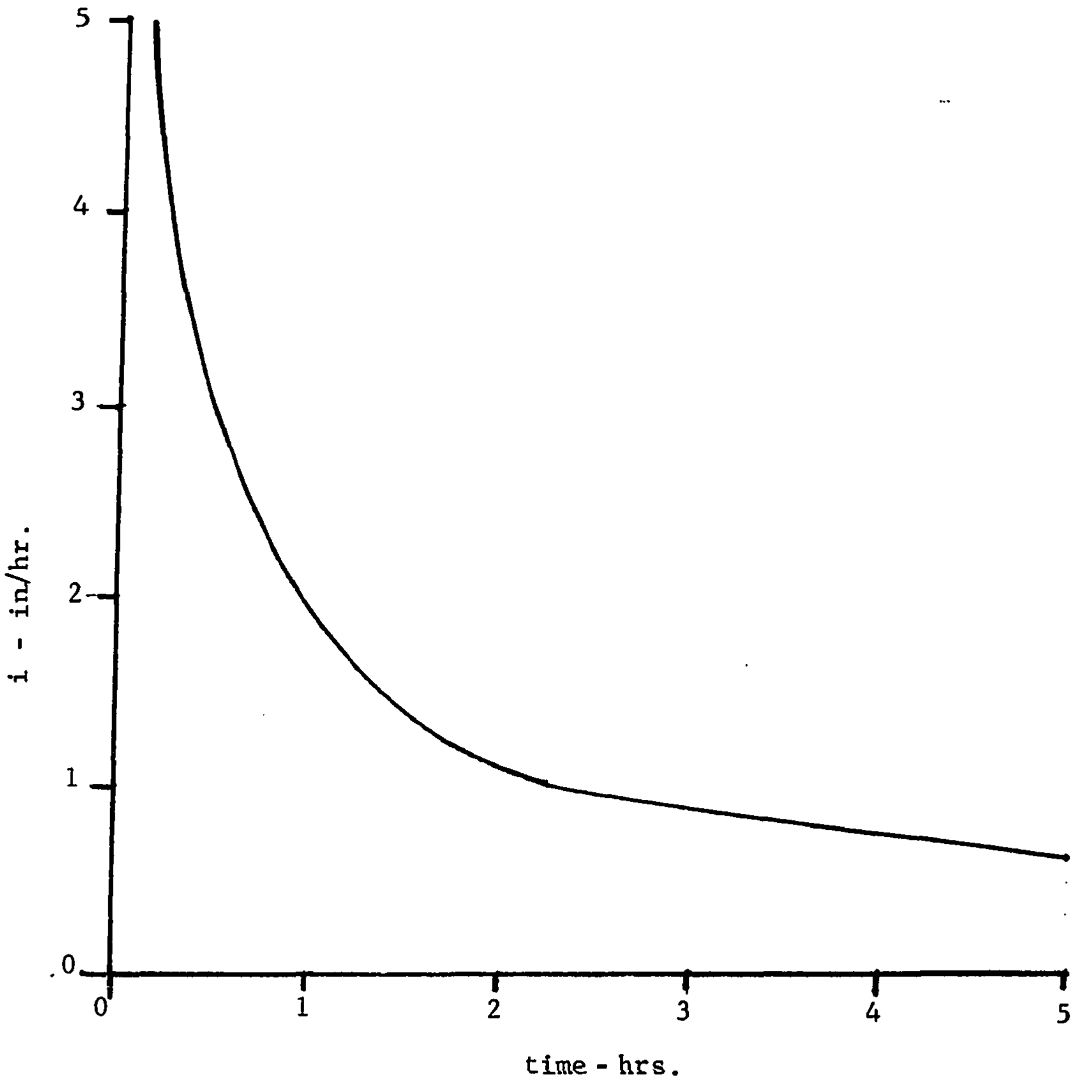

Fig. 5. Typical infiltration curve.

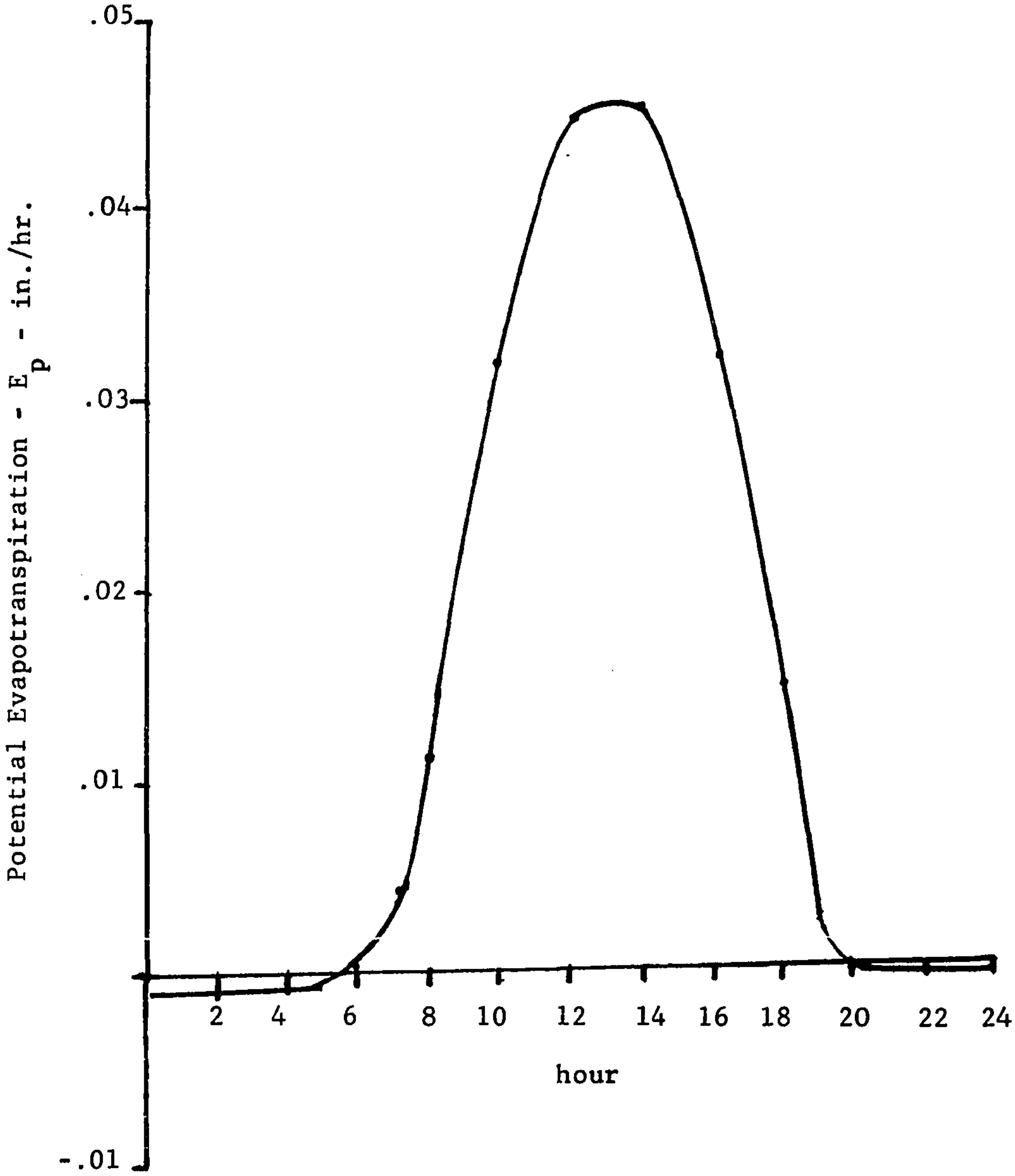

Fig. 6. Computed potential evapotranspiration - June 24, 1964

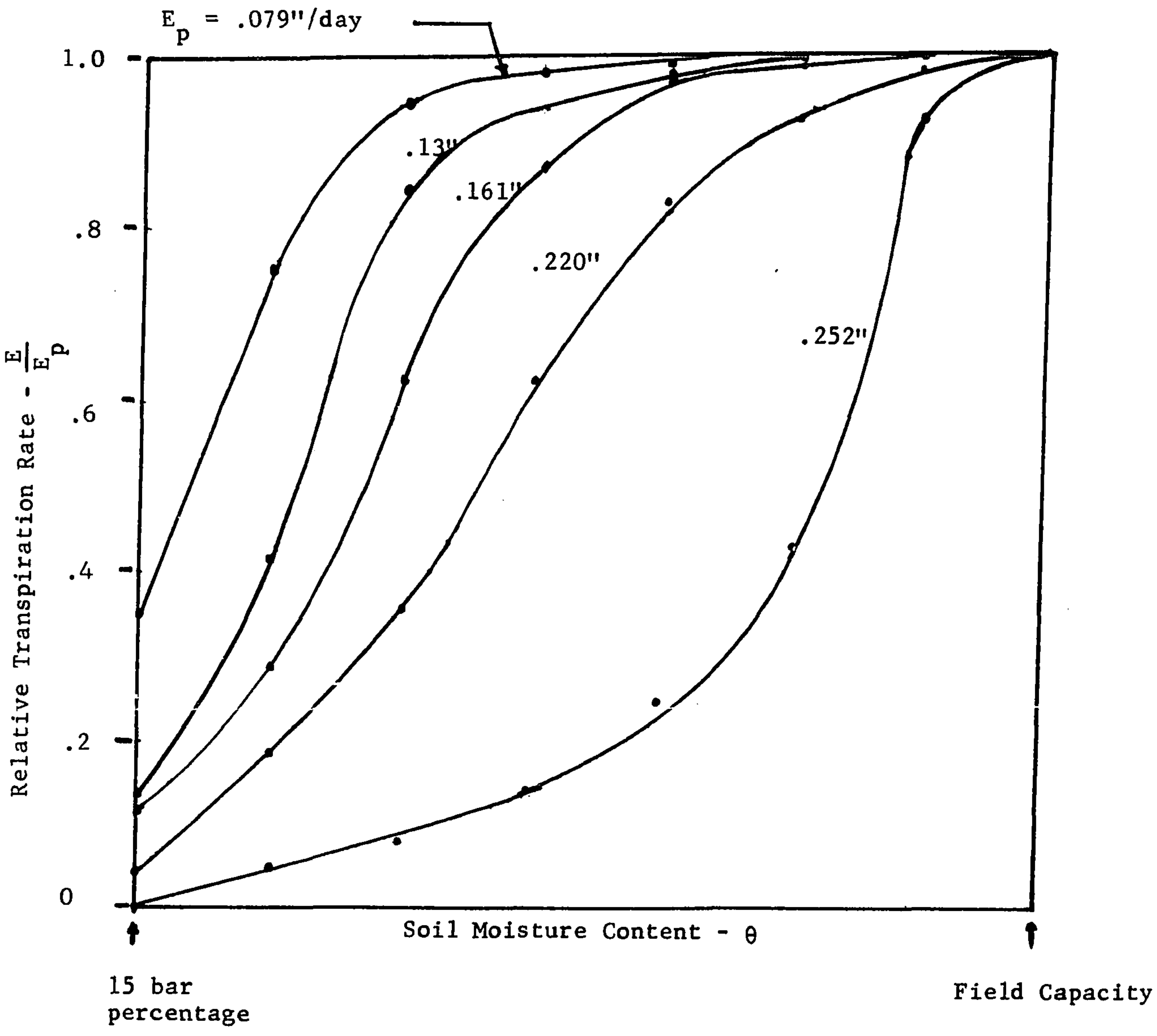

Figure 7. Evapotranspiration reduction curves.

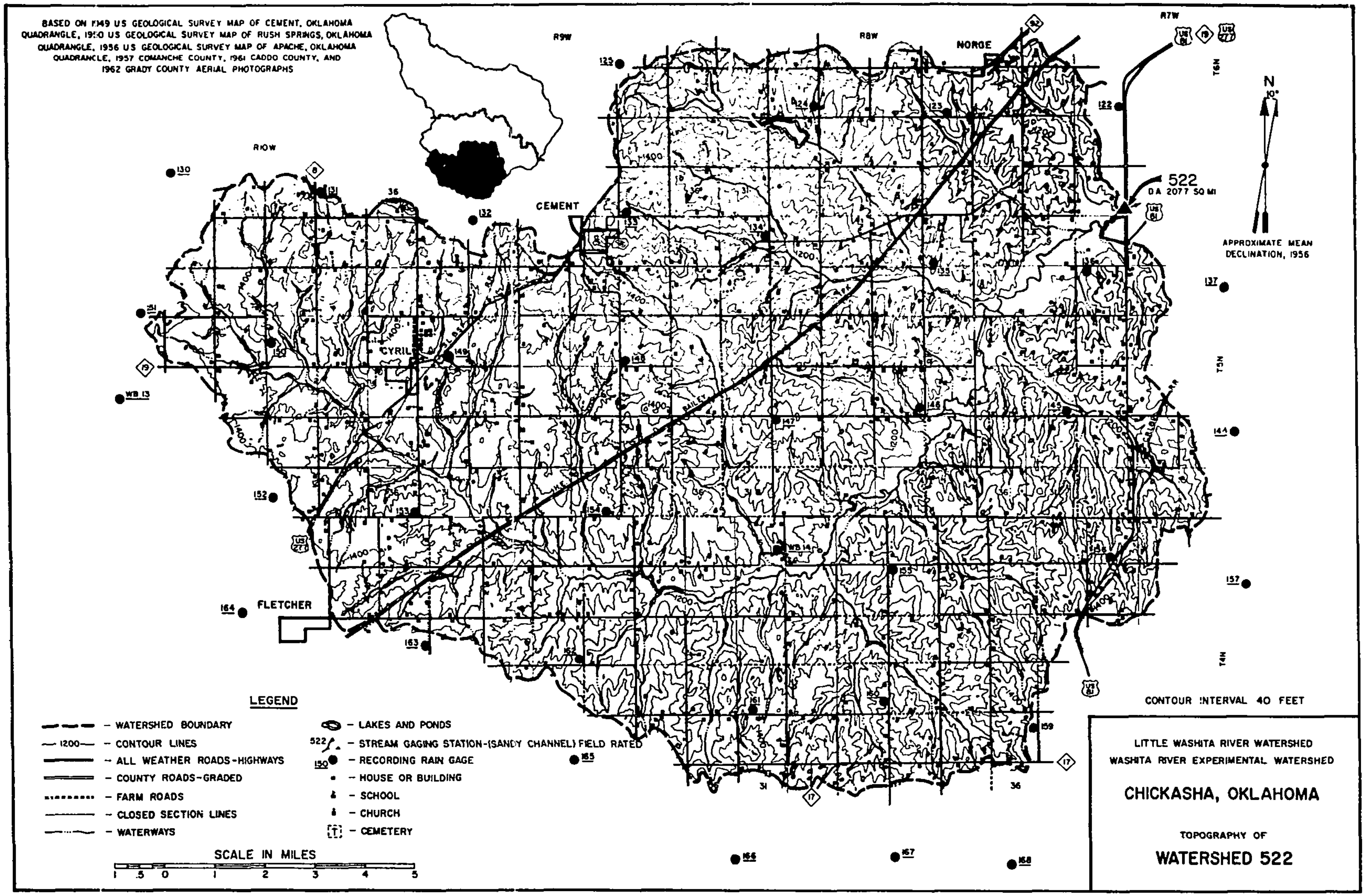

Figure 8. Topography of Watershed 522.

BIBLIOGRAPHY

1. Babcock, K.L. and Overstreet, R., Thermodynamics of soil moisture: A new application, Soil Sci., 80, pp. 257-263, 1965.

2. Baver, L.D., Soil Physics, 2nd Ed., John Wiley and Sons, Inc., New York, 1948.

3. Black, T.A., Gardner, W.R., and Thurtell, G.W., The Prediction of Evaporation, Drainage, and Soil Water Storage Forces on Bare Soil, Dept. of Soil and Water Services, University of Wisconsin, Madison, 1969.

4. Boltz, H.M., Die abhängigkeit der infraroten gegenstrahlung von der bewölkung, Z. f. Met., 3, pp. 201-203, 1949.

5. Boltz, H.M. and Fritz, H., Tabellen und diagram zur berechnung der gengenstrahlung und ausstrahlung, Z. f. Met., 4, pp. 314-317, 1950.

6. Bosen, J.F., Formula for approximation of saturation vapor pressure over water, Monthly Weather Rev., 88, pp. 275-276, 1960.

7. Briggs, L.J., The Mechanics of Soil Moisture, U.S. Dept. of Agriculture Bul. 10, U.S. Dept. of Agriculture, Washington, 1897.

8. Budyko, M.I., Teplovoi Balans Zemnoi Poverkhnosti, Gidrometeorologichesoe Izdatel'stuo, Leningrad, 1956. (English translation: Stepanova, N. A., The Heat Balance of the Earth's Surface. Office of Technical Services, U.S. Dept. of Commerce, Washington, 1958.

9. Butler, S.S., Engineering Hydrology, Prentice Hall, Inc., Englewood Cliffs, N.J., 1957.

10. Carslaw, H.S. and Jaeger, J.C., Conduction of Heat in Solids, Clarendon Press, Oxford, 1959.

11. Carson, J.E., and Moses, H., The annual diurnal heat-exchange cycles in the upper layers of soil, Journ. Appl. Met., 2, pp. 397-406, 1963.

12. Childs, E.C., Soil moisture theory, Advances in Hydroscience, Vol. 4, V.T. Chow, ed., Academic Press, New York, 1967.

13. Childs, E.C., and Collis-George, N., The permeability of porous materials, Proc. Roy. Soc., A201, pp. 392-405, 1950.

14. Chow, V.T., Runoff, Handbook of Applied Hydrology, V.T. Chow, ed., McGraw-Hill, New York, 1964.

15. Clark, C.O., Flood - Storage Accounting, Trans. Am. Geophys. Union, 25, pp. 1013-1023, 1944.

16. Crafts, H.S., Water deficits and physiological processes, Water Deficits and Plant Growth, T.T. Kozlowski, ed., Academic Press, New York, 1968.

17. Crank, J., The Mathematics of Diffusion, Oxford Univ. Press, London and New York, 1956.

18. Deacon, E.L., Priestley, C.H.B., and Swinbank, W.C., Evaporation and the water balance, Arid Zone Research, 10, UNESCO, pp. 9-34, 1958.

19. Denmead, O.T. and Shaw, R.H., Availability of soil water to plants as affected by soil moisture content and meteorological conditions, Agron. Jour. 54, pp. 385-390, 1962.

20. Environmental Data Services, ESSA, U.S. Dept. of Commerce, Climatological Data, National Summary, U.S. Govt. Print. Off., Washington, Jan. 1963 - Dec. 1968.

21. ____________________, Local Climatological Data, Oklahoma City, Oklahoma, Will Rogers Airport, U.S. Govt. Print. Off., Washington, Jan. 1963 - Dec. 1968.

22. Fermi, E., Thermodynamics, Dover Publications, Inc., New York, 1936.

23. Fleischer, R., Registrierung doe Infrastrahlungsstrome der Atmosphrae und des Redbodens, Ann. Meterorol., 8, pp. 115-123, 1957-1958.

24. Fleischer, R., Der Jahredgang der Strahlungsbilanz sowie lher lang- und kurzerlligen Komponenten. Das System Strahlungsbilanz-Globalstrahlung, Ber. Deut. Werrerdienst, 4, No. 22, pp. 32-40, 1956.

25. Fuller, H.J. and Tippo, O., College Botany, Henry Holt and Co., New York, 1949.

26. Gardner, W.R., Some steady state solutions of the unsaturated flow equation with application to evaporation from a water table, Soil Sci., 85, pp. 228-232, 1958.

27. __________, Dynamic aspects of water availability to plants, Soil Sci., 89, pp. 63-73, 1960.

28. __________, Relation of root distribution to water uptake and availability, Agric. Jour., 56, pp. 35-39, 1964.

29. __________, Rainfall, runoff, and return, Agricultural Meteorology, Monograph Vol. 6, No. 28, Am. Met. Soc., Boston, 1965.

30. __________, Availability and measurement of soil water, Water Deficits and Plant Growth, T.T. Kozlowski, ed., Academic Press, New York, 1968.

31. Gates, D.M., Energy Exchange in the Biosphere, Harper and Row, New York, 1962.

32. __________, Radiant energy, its receipt and disposal, Agricultural Meteorology, Monograph Vol. 6, No. 28, Am. Met. Soc., Boston, 1965.

33. Geiger, R., The Climate Near the Ground, Harvard Univ. Press, Cambridge, Mass., 1966.

34. Green, W.H., and Ampt, G.A., Studies on soil physics I. The flow of air and water through soils, Jour. Agr. Sci., 4, pp. 1-24, 1911.

35. Hallaire, M., and Hénin, S., Sur La non-validité de l'équation de conductivité pour exprimer le movement de l'eau non saturante dans le sol, Compt. Rend., Paris, 246, pp. 1720-1722, 1958.

36. Hanks, R.J., Gardner, H.R., and Fairbourn, M.L., Evaporation of water poor soils as influenced by drying with wind and radiation, Soil Sci. Soc. Am. Proc., 31, pp. 593-598, 1967.

37. Hartronft, B.C., Smith, M.D., Hayes, C.J., and McCasland, W., Engineering Classification of Geologic Materials and Related Soils, Oklahoma Highway Department Maintenance Division Seven, Res. and Develop. Div., Okla. Hwy. Dept., Okla. City, 1969.

38. Heller, J.P., The drying through the top surface of a vertical porous column, Soil Sci. Soc. Am. Proc., 32, pp. 778-786, 1968.

39. Hewlett, J.D. and Hibbert, F.R., Moisture and energy conditions within a sloping soil mass during drainage, Jour. Geophys. Res., 68, pp. 1081-1087, 1963.

40. Horton, R.E., An approach toward a physical interpretation of infiltration capacity, Soil Sci. Soc. Am. Proc., 5, pp. 399-417, 1940.

41. Holman, R.M. and Robbins, W.W., A Textbook of General Botany, John Wiley & Sons, New York, 1939.

42. Huff, F.A. and Changnon, S.A., Jr., Development and Utilization of Illinois Precipitation Networks, Ill. State Water Survey, Urbana, Ill., 1966.

43. Huff, F.A., and Niel, J.C., Rainfall Relations on Small Areas in Illinois, Bul. 44, Ill. State Water Survey, Urbana, Ill., 1957.

44. Huschke, R.E., ed., Glossary of Meteorology, Am. Met. Soc., Boston, 1959.

45. Idso, S.B., Baker, D.G., and Gates, D.M., The energy environment of plants, Advances in Agronomy, 18, A.G. Norman, ed., Academic Press, New York, 1966.

46. Jensen, R.D., and Klute, A., Water flow in an unsaturated soil with a step-type initial water content distribution, Soil Sci. Soc. Am. Proc., 31, pp. 289-296, 1967.

47. Kessler, E., and Wilk, K.E., Radar Measurement of Precipitation for Hydrological Purposes, Reports on WMO/IHD Projects, No. 5, Secretariat of the World Meteorological Organization, Geneva, Switzerland, 1968.

48. Klute, A., A numerical method for solving the flow equation in unsaturated materials, Soil Sci., 73, pp. 105-116, 1952.

49. Knisel, W.G., Jr., Baird, R.W., and Hartman, M.A., Runoff volume prediction from daily climatic data, Water Resources Res., 5, pp. 84-94, 1969.

50. Kohler, M.A., and Linsley, R.K., Jr., Predicting the Runoff from Storm Rainfall, U.S. Weather Bureau Research Paper 34, U.S. Weather Bur., U.S. Dept. of Commerce, Washington, 1951.

51. Kohler, M.A., Nordenson, T.J., and Baker, D.R., Evaporation Maps for the United States, U.S. Weather Bureau Technical Paper 37, U.S. Weather Bur., U.S. Dept. of Commerce, Washington, 1959.

52. Kohler, M.A., and Richards, M.M., Multicapacity basin accounting for predicting runoff from storm precipitation, Jour. Geophys. Res., 67, pp. 5187-5197, 1962.

53. Kondrat'yev, K.Ya., Radiative Heat Exchange in the Atmosphere, Pergamon Press, Oxford, 1965.

54. Kozlowski, T.T., ed., Water Deficits and Plant Growth, Vols. 1 & 2, Academic Press, New York, 1968.

55. Kramer, P.J., Discussion of F.J. Veihmeyer and A.H. Hendrickson, Trans. Am. Geophys. Union, 36, pp. 438-439, 1955.

56. Lamoreux, W.W., Modern evaporation formulae adapted to computer use, Monthly Weather Rev., 90, pp. 26-28, 1962.

57. Lettau, H.H., and Davidson B., eds., Exploring the Atmosphere's First Mile, Vol. 2, Pergamon Press, New York, 1957.

58. Linsley, R.K., Jr., Kohler, M.A., and Paulhus, J.L.H., Applied Hydrology, McGraw-Hill Book Co., Inc., New York, 1949.

59. List, R.J., Smithsonian Meteorological Tables, 6th Revised Ed., Smithsonian Miscellaneous Collections, Vol. 114, Smithsonian Inst., Washington, 1966.

60. Lull, H.W., Ecological and silvicultural aspects, Handbook of Applied Hydrology, V.T. Chow, ed., McGraw-Hill Book Co., New York, 1964.

61. Mather, J.R., Discussion of F.J. Veihmeyer and A.H. Hendrickson, Trans. Am. Geophys. Union, 36, pp. 434-435, 1955.

62. Miller, D.H., The heat and water budget of the earth's surface, Advances in Geophysics, 11, H.E. Landsbery and J. Van Mieghen, eds., Academic Press, New York, 1965.

63. Miller, E.E., and Klute, A., The dynamics of soil water, 1, Am. Soc. of Agron. Monograph Vol. 11, pp. 209-219, 1967.

64. Musgrave, G.W., and Holtan, H.N., Infiltration, Handbook of Applied Hydrology, V.T. Chow, ed., McGraw-Hill Book Co., New York, 1964.

65. Nicks, A.D., and Hartman, M.A., Variation of rainfall over a large gaged area, Trans. of A.S.A.E., 9, pp. 437-439, 1966.

66. Nielson, D.R., Biggar, J.W., and Davidson, J.M., Experimental consideration of diffusion analysis in unsaturated flow problems, Soil Sci. Soc. Am. Proc., 26, pp. 107-111, 1962.

67. Nixon, R.R., and Lawless, G.P., Translocation with time in unsaturated profiles, Jour. Geophys. Res., 65, pp. 655-661, 1960.

68. Ogata, G., and Richards, L.A., Water content changes following irrigation of bare-field soil that is protected from evaporation, Soil Sci. Soc. Am. Proc., 21, pp. 355-356, 1957.

69. Oklahoma Geological Survey, Geology and Groundwater Resources of Grady and Northern Stephens Counties, Oklahoma, Bul. 73, Okla. Geo. Sur., Norman, Oklahoma, 1955.

70. Penman, H.L., Natural evaporation from open water, bare soil, and grass, Proc. Roy. Soc., A193, pp. 120-145, 1948.

71. ____________, Vegetation and Hydrology, Commonwealth Bur. of Soils Tech. Comm. 53, Commonwealth Agricultural Bureaux, Farnham Royal, Slough, Bucks, England, 1963.

72. Philip, J.R., An infiltration equation with physical significance, Soil. Sci., 77, pp. 153-157, 1954.

73. ____________, The theory of infiltration: 1. The infiltration equation and its solution, Soil Sci., 83, pp. 345-357, 1957.

74. ____________, The theory of infiltration: 2. The profile at infinity, Soil Sci., 83, pp. 435-448, 1957.

75. ____________, The theory of infiltration: 3. Moisture profiles and relation to experiment, Soil Sci., 84, pp. 163-178, 1957.

76. ____________, The theory of infiltration: 4. Sorptivity and algebraic infiltration, Soil Sci., 84, pp. 257-264, 1957.

77. Philip, J.R., The theory of infiltration: 5. The influence of the initial moisture content, Soil Sci., 84 pp. 329-339, 1957.

78. __________, The theory of infiltration: 6. Effect of water depth over soil, Soil Sci., 85, pp. 278-286, 1958.

79. __________, The theory of infiltration: 7. Soil Sci., 85, pp. 333-337, 1958.

80. __________, Theory of infiltration, Advances in Hydroscience, Vol. 5, V.T. Chow, ed., Academic Press, New York, 1969.

81. Pierce, L.T., Estimating seasonal and short-term fluctuations in evapotranspiration from meadow crops, Bul. Am. Met. Soc., 39, pp. 73-78, 1958.

82. Richards, L.A., Gardner, W.R., and Ogata, G., Physical processes determining water loss from soils, Soil Sci. Soc. Am. Proc., 20, pp. 310-314, 1956.

83. Rider, N.E., and Robinson, G.D., A study of the transfer of heat and water vapor above a surface of short grass, Quart, Jour. Roy. Met. Soc., 77, pp. 375-401, 1951.

84. Rubin, J., Numerical method for analyzing hysteresis-affected, post infiltration redistribution of soil moisture, Soil Sci. Soc. Am. Proc., 31, pp. 13-20, 1967.

85. Russell, M.B., The utility of the energy concept of soil moisture, Soil Sci. Soc. Am. Proc., 7, pp. 90-94, 1942.

86. Sellers, W.D., Physical Climatology, Univ. Chicago Press, Chicago, 1965.

87. Slayter, R.O., Plant-Water Relationships, Academic Press, New York, 1967.

88. Slavík, B., Supply of water to plants, Agricultural Meteorology, Monograph Vol. 6, No. 28, Am. Met. Soc., Boston, 1965.

89. Soil Conservation Service, U.S. Dept. of Agriculture, Hydrology, Sect. 4, Engineering Handbook, Soil Conservation Ser., U.S. Dept. of Agriculture, Washington, 1956.

90. ____________________, Soil Survey of Commanche County, Oklahoma, Soil Con. Ser., Stillwater, Okla., 1967.

91. ____________________, Unpublished soil maps of Caddo County, Oklahoma, Soil Con. Ser., Anadarko, Okla., 1969.

92. ____________________, U.S. Dept. of Agriculture, Unpublished soil maps of Grady County, Oklahoma, Soil Con. Ser., Chickasha, Okla., 1969.

93. Southern Plains Watershed Research Center, U.S. Dept. of Agriculture, Annual Research Report, Southern Plains Branch, Soil and Water Conservation Division, Agricultural Res. Ser., U.S. Dept. of Agriculture, Chickasha, Oklahoma, 1963-1968.

94. Swinbank, W.C., Long-wave radiation from clear skies, Quart. Jour. Roy Met. Soc., 89, pp. 339-348, 1963.

95. Tanaka, H.H., and Davis, L.V., Groundwater Resources of the Rush Springs Sandstone in the Caddo County Area, Cir. 61, Okla. Geo. Sur., Norman, Okla., 1963.

96. Tanner, C.B., Evaporation of water from plants and soils, Water Deficits and Plant Growth, T.T. Kozlowski, ed., Academic Press, New York, 1968.

97. Taylor, S.A., Terminology in plant and soil water relations, Water Deficits and Plant Growth, T.T. Kozlowski, ed., Academic Press, New York, 1968.

98. Thornthwaite, C.W., An approach toward a rational classification of climate, Geog. Rev., 38, pp. 55-94, 1948.

99. Thornthwaite, C.W., and Hare, F.K., The loss of water to the air, Agricultural Meteorology, Monograph Vol. 6, No. 28, Am. Met. Soc., Boston, 1965.

100. Thornthwaite, C.W., and Mather, J.R., The water budget and its use in irrigation, Water, The Yearbook of Agriculture, 1955, U.S. Dept. of Agriculture, Washington, 1955.

101. Van Bavel, C.H.M., Potential evaporation: the combination concept and its experimental verification, Water Resources Res., 2, pp. 455-467, 1966.

102. Veihmeyer, F.J., Evapotranspiration, Handbook of Applied Hydrology, V.T. Chow, ed., McGraw-Hill Book Co., New York, 1964.

103. Veihmeyer, F.J., and Hendrickson, A.H., Does transpiration decrease as the soil moisture decreases?, Trans. Am. Geophys. Union, 36, pp. 425-428, 1955.

104. Visser, W.C., Progress in the Knowledge about the Effects of Soil Moisture Content on Plant Production, Inst. Land-Water Management, Tech. Bul. 45, Wageninger, Netherlands, 1966.

105. Vries, D.A. de, The thermal behavior of soils, Climatology and Micrometeorology, UNESCO, Paris, pp. 109-113, 1958.

106. Wang, J.S., and Wang, S.C., A simple graphical approach to Penman's method for evaporation estimates, Jour. Appl. Met., 1, pp. 582-588, 1962.

107. Weaver, J.E., North American Prairie, Johnsen Pub. Co., Lincoln, Neb., 1954.

108. Wijk, W.R., van, and Vries, D.A. de, Periodic temperature variations in a boundary soil, Physics of Plant Environment, W.R. Van Wijk, ed., North Holland Pub. Co., Amsterdam, 1963.

109. Wilson, J.C., Radar measurements of rainfall for operational purposes, Travelers Research Corporation Final Report 7488360, Travelers Res. Corp., Hartford, Conn., 1969.

110. Youngs, E.G., Moisture profiles during vertical infiltration, Soil Sci., 84, pp. 283-290, 1957.

111. ___________, Redistribution of moisture in porous materials after infiltration: 1., Soil Sci., 86, pp. 117-125, 1958.

112. ___________, Redistribution of moisture in porous materials after infiltration: 2., Soil Sci., 86, pp. 202-207, 1958.

113. Zermansky, M.W., Heat and Thermodynamics, 3rd ed., McGraw-Hill Book Co., New York, 1951.

APPENDIX A

LIST OF SYMBOLS AND UNITS ADOPTED

Symbols and units present some difficulties in this work. Each contributing science has more or less accepted symbols of its own. Unfortunately some of these symbols mean different things in the different disciplines. To avoid confusion, different symbols were created in cases of duplication. The basic sciences all use metric units; however, most of the meteorological measurements and all hydrologic data are in English units. In the list presented below the units listed are those which seem most natural and consistent with their use in this paper. Conversion constants have been omitted from the equations with the understanding that they must be applied where necessary.

General Constants, Coefficients, and Exponents: A, a, B, b, γ, m, r

Subscript Designators -- used with symbols listed below to indicate location or direction, etc.

Subscript		Meaning
c		Clear day conditions
d		Downward
h		Instrument Screen Height
n		Integer designator of layer
o		Ground or vegetative surface
	Also	Groundwater reference datum
	Also	Initial time
sat		Saturated
u		Upward
∞		Ultimate or infinite time

Symbol	Definition	Units
C_g	Specific heat of soil - volume basis	cal cm^{-3} $^oC^{-1}$
c_p	Specific heat of dry air	cal gm^{-1} $^oC^{-1}$
D	Soil moisture diffusivity	cm^2 sec^{-1}
D_E	Integrated eddy diffusion coefficient	cm sec^{-1}
D_H	Integrated thermal eddy diffusion coefficient	cm sec^{-1}
d	Depth to water table	m or ft
E	Flux density of water vapor	gm sec^{-1} cm^{-2}
	Also Actual evapotranspiration rate	in day^{-1}
E_p	Potential evapotranspiration rate	in day^{-1}
e	Base of natural logarithm	
	Also Vapor pressure	mb
e_s	Saturation vapor pressure	mb
G_o	Heat flux through the surface from the ground	ly day^{-1}
g	Acceleration of gravity	cm sec^{-2}
H_o	Sensible heat flux through the surface from the atmosphere	ly day^{-1}
I	Infrared radiation flux	ly day^{-1}
i	Infiltration rate	cm sec^{-1} or in hr^{-1}
K	Saturated hydraulic conductivity,	in hr^{-1}
K_E	Eddy diffusion coefficient	cm^2 sec^{-1}
K_H	Eddy thermal diffusion coefficient	cm^2 sec^{-1}
k	Unsaturated hydraulic conductivity	gm sec^{-1} cm^{-2} or cm sec^{-1}
k'	Cloud cover infrared radiation coefficient	

Symbol	Definition	Units
k_g	Thermal conductivity of soil	ly day^{-1} cm $^oC^{-1}$
k_v	Von Karman's constant	
L	Latent heat of vaporization	cal gm^{-1}
L_n	Length of roots in nth layer	cm cm^{-3}
n'	Cloud cover in tenths	
P	Ponded depth of water above surface	in
P_T	Total storm precipitation	in
p	Atmospheric pressure	mb
P_o	Ratio of water volume to total volume at field capacity	
Q	Rate of water uptake by plants per unit volume	cm^3 day^{-1} cm^{-3}
Q'	Flux density of liquid water	gm sec^{-1} cm^{-2} or cm sec-
Q"	Flux of liquid water	gm sec^{-1} or cm^3 sec^{-1}
q	Specific humidity	
R	Rainfall rate	in hr^{-1}
R_o	Net radiation flux at surface	ly day^{-1}
R^*	Gas constant for dry air	erg gm^{-1} $^oK^{-1}$
S	Short-wave solar radiation flux	ly day^{-1}
T	Temperature	^oC or ^oK
T_d	Dewpoint temperature	^oC
t	Time	sec
u	Horizontal wind velocity	cm sec^{-1}
W_n	Depth of water in the nth layer	in
w	Vertical wind velocity	cm sec^{-1}
X	Volume fraction of soil constituents	
x	Horizontal coordinate	cm
Z_L	Depth of saturated zone	in
Z_n	Thickness of the nth saturated zone	in
z	Vertical coordinate	cm

Symbol	Definition	Units
α_s	Short-wave albedo	
α_I	Infrared albedo	
Δ	Slope of saturation vapor pressure curve with respect to temperature	mb $^oC^{-1}$
δ	Root suction	ergs gm^{-1} expressed as cm of water
ϵ	Infrared emissivity	
η	Thermal diffusivity of soil	cm^2 day^{-1}
θ	Soil moisture content - ratio of water volume to total volume	
ρ	Density of air	kg m^{-3}
σ	Steffan-Boltzman constant	ly day^{-1} $^oK^{-4}$
	Also Surface tension of water	dynes cm^{-1}
Ψ	Soil water potential	ergs gm^{-1} expressed as cm of water
Ψ_m	Matric potential	"
Ψ_a	Adsorption potential	"
Ψ_c	Capillary potential	"
Ψ_w	Electrical bond potential	"
Ψ_π	Osmotic potential	"
Ψ_p	Hydrostatic pressure potential	"
Ψ_g	Gravitational potential	"
ω	Angular velocity	radians day^{-1}

APPENDIX B

DERIVATION OF THE EFFECTIVE HYDRAULIC CONDUCTIVITY

Let the zone of saturation be designated to be a depth Z_L having a total hydraulic level loss H through it. Then from Darcy's law $Q'_L = - K \frac{H}{Z_L}$ where K is the mean hydraulic conductivity. If the zone L is divided into four subzones with depths z_1, z_2, z_3, and z_4 each with a different hydraulic conductivity K_1, K_2, K_3, and K_4 respectively. Then the flow through layer z_1 is given by

$$Q'_1 = -K_1 \frac{H - Z_2 - Z_3 - Z_4}{z_1} = -K_1 \frac{H_1}{z_1}, \tag{B-1}$$

and similarly

$$Q'_2 = -K_2 \frac{H_2}{Z_2}, \quad Q'_3 = -K_3 \frac{H_3}{Z_3}, \quad Q'_4 = -K_4 \frac{H_4}{Z_4}, \tag{B-2}$$

but by continuity

$$Q'_L = Q'_1 = Q'_2 = Q'_3 = Q'_4; \tag{B-3}$$

and

$$H_1 = \frac{Q'_1 Z_1}{K_1}, \quad H_2 = \frac{Q'_2 Z_2}{K_2}, \text{ etc.,} \tag{B-4}$$

and

$$H = H_1 + H_2 + H_3 + H_4 \tag{B-5}$$

So

$$-K = \frac{Q'_L L}{H} = \frac{Q'_L Z_L}{H_1+H_2+H_3+H_4} = -\frac{Q'_L Z_L}{\frac{Q'_L Z_1}{K_1} + \frac{Q'_L Z_2}{K_2} + \frac{Q'_L Z_3}{K_3} + \frac{Q'_L Z_4}{K_4}}, \tag{B-6}$$

or

$$K = \frac{Z_L}{\frac{Z_1}{K_1} + \frac{Z_2}{K_2} + \frac{Z_3}{K_3} + \frac{Z_4}{K_4}}. \tag{B-7}$$