A DETERMINATIVE ANALYSIS OF THE INTRODUCTORY COLLEGE MATHEMATICS COURSE WITH REGARD TO APPROACH EFFECTIVENESS

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PREFACE

A pressing problem in mathematics education today concerns itself with curricular research. More and more mathematical discoveries are being made annually and the student of mathematics should be instructed in such a way that he can benefit from these discoveries.

This study has been conducted in the hope that the evidences presented will be beneficial to the improvement of instruction in mathematics, both at the secondary and college level.

The study was experimental in nature. It was conducted at Central State College, Edmond, Oklahoma, during the fall semester of 1963.

The purpose of the study was to determine which one of three approaches would be most effective in teaching the Introductory College Mathematics Course: the conventional approach, the vector approach, or the set theory approach.

Indebtedness is acknowledged to Dr. W. Ware Marsden, who served as chairman of my advisory committee, for encouraging my interest in the problem and for his guidance throughout the study; to Dr. Richard Rankin and Dr. J. Paschal Twyman for their suggestions and guidance relating to the statistical treatment of the data; and to Dr. Richard Jungers and Mrs. Helen Jones for their assistance in the completion of this study.

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To those members of my family who have aided me in attaining this goal, I can only say thank you. To my wife, Sondra Jo, who by her unselfish sacrifices and encouragement contributed beyond words to the completion of this study, I again humbly express my thanks.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

THE PROBLEM

Introduction

Mathematics is a branch of learning in which all of the major theories of two thousand years ago are still valid, yet never before has there been such a flood of fresh ideas.[1] Mathematics is also one of the fastest growing and most radically changing of all the sciences.

It should be noted that mathematics is man-made. The mathematical concepts, the broad ideas, the logical standards and methods of reasoning which have been steadfastly pursued during these past two thousand years were fashioned by human beings. Through the power of mind, man has surveyed spaces too vast for his imagination to encompass; he has predicted and shown how to control radio waves which none of our senses can perceive; and he has discovered minute particles which cannot be seen even with the most powerful microscope. Using cold symbols and formulae, man has begun to secure a portentous grip on the universe.[2]

As the human race looks to the future, there will be an even greater need for new and more extensive mathematical ideas. Space travel

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[1]George A. W. Boehm, The New World of Mathematics (New York, 1959), p. 17.

[2]Morris Kline, Mathematics and the Physical World (New York, 1959), p. 9.

will require mathematics to compute trajectories and the time of rendezvous with the particular planet to be visited. The thrust of the space vehicle and its capacities for fuel and supplies will all be arrived at by the use of mathematical formulas.

In the field of computer technology, great strides have been made. Computers have been built which will do 5,000 additions a second for hours or days. At present, computers can solve problems much faster and more accurately than human beings can. These computers can solve problems that a man's life is far too short to permit him to do.[3]

Addition is not the only operation that can be done. These "brains" can also multiply, subtract, divide, and retain information which might be useful in solving future problems. Their applicability is quite extensive. They have been used in solving problems dealing with traffic flow, communications, satellite tracking, and other related problems in such fields as physics, economics, education, and industry.

The Historical Development of Mathematics. Mathematics has been a series of changes, adaptations, and alterations. What we now take for granted mathematically would have astounded the mathematicians of the past. This is not because the mathematicians of today are so outstanding, but rather that the bases for present day studies have been laid by such men as Euclid, Gauss, Newton, Leibnitz, Descartes, Hamilton, Cantor, and others.

One of the first concepts to be of great significance was the one-to-one correspondence.[4] This idea was used by the sheepherders of

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[3]Edmund Callis Berkeley, "Giant Brains", A Treasury of Science, (New York, 1959), p. 216.

[4]Francis J. Mueller, Arithmetic, Its Structure and Concepts (Englewood Cliffs, New Jersey, 1956), p.2.

yore in keeping track of their sheep. For every sheep let out of the pens, a rock was placed on a pile. At night the rock was removed as each sheep returned. If there was a rock left over, a sheep was missing. A simple, yet ingenious way of counting. There were limitations on this method, however. The sheepherder still did not know how many sheep he had, or if one were missing, which one it was.

The Egyptians used mathematics in a practical sense. They were more concerned with the construction of their pyramids and statues than they were in understanding the fine points of the mathematical tools they were utilizing. It is true that the Egyptians did compute the distance of the earth from the sun, but the information was used for surveying purposes only. The Egyptians also recorded the rising and falling of the water level of the Nile River. The data collected enabled them to use the river bottom for agricultural purposes.

The Greeks, on the other hand, used mathematics as an intellectual activity. Geometry allowed the use of logic and the Greeks revelled in the philosophical implications involved.

The greatest of all geometers was Euclid. It is said that, "Euclid is the only man to whom there ever came or ever can come the glory of having successfully incorporated in his own writings all the essential parts of the accumulated [mathematical] knowledge of his time."[5] In his Elements, Euclid compiled most of the work in mathematics which had been done up to his time, approximately 300 B.C.

For nearly two thousand years, Euclid reigned supreme in mathematical circles. During the seventeenth century, Friedrich Gauss, Johann

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[5]D.E. Smith, History of Mathematics (New York, 1956), p. 102.

Bolyai, and Nicolai Lobachevsky simultaneously developed a new geometry which was properly titled, Non-Euclidean Geometry.

Of this discovery Bolyai said: "I have made such wonderful discoveries that I am myself lost in astonishment; Out of nothing I have created a new and another world."[6]

Then, in 1854, George Riemann reworded the fifth postulate of Euclid to state: Through a given point outside a given line, no lines can be drawn parallel to the given line. This again revolutionized the mathematical community and gave birth to elliptic geometry. The basic properties of this type of thinking led to space curves and eventually Einstein's theory of relativity.

Calculus has been one of the major accomplishments of man. The invention of the calculus in the seventeenth century initiated a long and exceedingly productive period of research in both pure and applied mathematics, finally growing into the theory of functions of a real and of a complex variable, the study of differential equations, elliptic functions, differential geometry, and other concepts. These subjects proved to be of the greatest interest, not only for the further development of pure mathematics, but also for their applications to the physical world. Newton and Leibnitz are credited with the discovery of calculus.

Between 1871 and 1874, Georg Cantor created a completely new and very special mathematical discipline, the theory of sets. A set is actually nothing more than a collection of elements, but the usefulness is far reaching. The concepts of set theory have been very beneficial in the study of geometry, i.e., a line is a set of points.

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[6]Kline, Mathematics in Western Culture (New York, 1953), p. 410.

Space can be thought of as being the set of all points. Two triangles can be shown to be congruent if there is a one-to-one correspondence between the points of the two triangles and the distance between any two corresponding points is the same for both triangles.

Using sets, a person can describe both finite and infinite sets.

The relationship between sets can be stated in the binary operations union and intersection. For example, the union of two sets A and B is the set of elements which are in A or B or in both A and B. The intersection of the two sets A and B would be the elements which are common to both.

These operations are associative and commutative, two properties which are highly desirable in any mathematical system.

The study of topology and modern algebra are steeped in set theory. Probability and statistics also make a great deal of it in their formalities. A sample space, for example, in probability is a set of outcomes in which an experiment might result. In tossing two coins, the elements of the sample space would include two heads, a head and a tail, a tail and a head, and two tails. Since there are four elements in the sample space, the probability of any one of them occurring is one-fourth.

A population in an experimental study is an example of a set of elements. The sample which is drawn from this population would be a subset of the population since, by definition of a subset, every element in the sample is an element of the population.

Hamilton is given credit for discovering vectors. As reported: It came like a flash, to relieve an intellectual need that had haunted him for fifteen years.[7] Hamilton first called them triplets, because forces

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[7]James R. Newman, The World of Mathematics (New York, 1956), p. 162.

act in three dimensions.

Vectors have many applications. They can be used extensively in physics, in geometry and trigonometry, and in related areas. The path of a guided missile, for example, can be described in terms of vectors as can its velocity and acceleration. Vectors can be incorporated into the solution of systems of equations, and they can be used readily in the simplification of mathematical proofs.

One of the most important factors concerning vectors is that they can be defined as an ordered pair, (X_1, X_2), which is used in Euclidean two-space, or they can be defined as an ordered n-tuple, $(X_1, X_2, \ldots X_n)$. This representation is in Euclidean n-space.

The world of the mathematician is not limited to one world of three dimensions, but rather to a world of many dimensions. Einstein extended our concept of depth by adding a fourth dimension, time.

In solving a system of equations, three equations in three unknowns, each equation is written in linear form, $aX_1 + bX_2 + cX_3 = d$ where a, b, c, and d are constants. The unknowns X_1, X_2, and X_3 are actually elements of a vector, (X_1, X_2, X_3), and vector methods can be used to solve the system of equations.

The solution of this particular system can be illustrated on a Cartesian coordinate system in three dimensions.

Mathematics is currently playing an important role in our everyday living. It is used in industry, business, the social sciences, as well as in the physical sciences.

To keep pace with the demands of industry, technology, and the sciences, mathematicians have had to invent new branches of mathematics and expand the old ones. A superstructure of fresh ideas has been built which people trained in the classical branches of the subject would hardly recognize as mathematics at all.

For some people mathematics is an art to be pursued for art's sake. Whether or not this mathematics has any use is of little consequence.

The very abstractness of mathematics makes it useful. By applying its concepts to worldly problems, the mathematician can oftentimes brush away the obscuring details and reveal simple patterns.

Using mathematics, astronomers are able to calculate the positions of the planets at any time in the past or future and predict the coming and going of comets.

In industry, statistical methods have been developed for controlling quality in high-speed industrial mass production. Mathematics is used to evaluate precisely telephone, radio, and television circuits.

Mathematics is important to our society. It is important in industry where new technical designs require more sophisticated mathematics in solving related problems. In computer technology, new and improved mathematical interpretations are needed in order to develop faster and more accurate computers. Space travel is based on mathematical formulas.

In business, mathematics and computers are used in doing inventories and ordering goods and supplies which have become depleted in the warehouse.

Housewives use mathematics in their everyday work, the extent depending upon their own personal training.

Mathematics is important to each one of us. Its importance will continue to grow as new methods and ideas are introduced.

According to Nicholas Bourbaki, "For twenty-five centuries mathematicians have been in the habit of correcting their errors--and seeing their science enriched rather than impoverished thereby. This gives them the right to contemplate the future with serenity."[8]

Trends in the Development of Mathematics Education. Mathematics education has also undergone vast changes. From the rigors of the Trivium and Quadrivium of Latin times, to the present, the role of mathematics has increased in importance. Mathematics is no longer an activity for the intellectually inclined alone, but a tool to be used in our daily living, in our work, in our play, and in our endeavors to enrich our own personal lives.

Despite the contributions of mathematics to our culture, "Educated people almost universally reject mathematics as an intellectual interest ...School courses and books have presented 'mathematics' as a series of apparently meaningless technical procedures. Such material is as representative of the subject as an account of the name, position, and function of every human bone in the human skeleton is representative of the living, thinking, and emotional being called man."[9]

During the past fifteen years there have been several groups which have attempted to upgrade the level of mathematical knowledge in our elementary and secondary schools. Perhaps the most notable of these is the School Mathematics Study Group.

This group was organized in 1958 with the financial support of the National Science Foundation. The work of the School Mathematics Study Group represents the largest united effort for improvement in the history of mathematics education. The project is national in scope.

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[8] Lucienne Felix, The Modern Aspect of Mathematics (New York, 1960), p. 33.

[9] Kline, ~~Mathematics in Western Culture~~, p. 9.

Dr. E. G. Begle, formerly of Yale University and now of Stanford University, is the director of the group.

The main objective of the School Mathematics Study Group program was to structure and write textbooks for grades four through twelve.

In reference to the junior high school program in mathematics, "Careful attention is paid to the appreciation of abstract concepts, the role of definition, development of the precise vocabulary and thought, experimentation, and proof. Materials are chosen with the intent to capture the fascinating features of mathematics, creation and discovery, rather than utility alone."[10]

With regard to their program in intermediate mathematics, "Careful attention has been taken to give the student some insight into the nature of mathematical thought as well as to prepare him to perform certain manipulations with facility."[11]

The development of the School Mathematics Study Group material is unique in that it represents the combined thinking of many people---psychologists, testmakers, mathematicians from colleges and industry, biologists, and high school teachers.

In their first newsletter, they said:

> "The world of today demands more mathematical knowledge on the part of more people than the world of yesterday, and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased and understanding of the role of mathematics in our society is now a pre-requisite for intelligent citizenship. Since no one can predict with certainty his future profession, it is important that mathematics be taught

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[10]School Mathematics Study Group, Newsletter Number 4, (New Haven, Connecticut, March, 1960), p.1.

[11]Ibid.

> so that students will be able in later life to learn the new mathematical skills which the future will surely demand of them."[12]

Writing on behalf of the School Mathematics Study Group, Dr. Begle stated; "[It]... further agrees that the present mathematics curriculum is out of phase with the actual need of our students as well as with the developments within the field of mathematics itself."[13]

Mathematics has changed. The professional mathematician has a deep understanding of the modern innovations in his subject matter area. But the student also needs to be aware of these changes. He needs to know about sets, vectors, and the other concepts he will encounter in later life. The school is the best place for him to learn about these things.

According to one mathematician, "Widespread unrest exists among professional mathematicians and educators regarding the present mathematics curriculum. This unrest is primarily due to the changed and changing nature of mathematics and its content, and to criticisms regarding the effectiveness, efficiency, coherence, and applicability of the traditional courses."[14]

There are three other groups which have undertaken projects similar to that accomplished by the School Mathematics Study Group. The University of Illinois, Ball State Teachers College, and the University of Maryland were each financed by the National Science Foundation and wrote mathematics programs for the elementary and secondary school.

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[12]School Mathematics Study Group, Newsletter Number 1, (New Haven, Connecticut, March, 1959), p.4.

[13]E. G. Begle, "The School Mathematics Study Group," The Mathematics Teacher, LI, (1958), p.4.

[14]Merle Milligan, "An Inquiry into the Selection of Subject Matter Content for College Freshman Mathematics," (unpub. Ed.D. dissertation, Oklahoma State University, 1961), p.3.

While each of these programs has been extensively utilized, they are not national in scope.

Their basic structure is quite similar to that proposed by the School Mathematics Study Group. At the present time, other programs such as the Cleveland Plan are being introduced. This plan includes kindergarten through grade three.

At the present there is some concern about the mathematics curriculum in our colleges and universities. In a statement by the Commission on Mathematics: "It is futile to expect the secondary schools to move in the direction of a more modern program in mathematics if the colleges do not make extensive changes in the spirit and content of their courses."[15]

A group which is very much interested in the mathematics curriculum of our colleges and universities is the Mathematics Association of America. This group is comprised of both amateur and professional mathematicians. Under the auspices of this association and with the financial support of the National Science Foundation, a committee has been formed to study and make recommendations regarding the undergraduate curriculum in mathematics. This committee is titled, The Committee on the Undergraduate Program in Mathematics (CUPM). Some of its members are John G. Kemeny, Dartmouth University, E.G. Begle, Stanford University, J.L. Kelly, University of California, and Henry Van Engen, University of Wisconsin.

In a recent bulletin, the committee stated:

"Our colleges are being called upon to fill an endless need for professional mathematicians, for mathematically trained scientists, and for a variety of mathematically skilled personnel in hundreds

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[15]Commission on Mathematics, Program for College Preparatory Mathematics, College Entrance Examination Board (New York, 1958), p. 58.

> of activities. Our business schools often demand the very newest techniques developed by the mathematician. Medical research may soon require mathematical training comparable to that required by the nuclear physicist. Our engineers must be prepared to meet the needs of the rapidly changing American technology."[16]

In reference to the education program in our colleges and universities they said:

> "It is fair to say that mathematics will play a central role in the American culture of tomorrow. We must train our young men and women to be able to attack and solve problems that did not exist when they [the present teachers] attended school: problems which require the ability to think mathematically. This requires an educational system that teaches not only fundamental mathematical techniques, but stresses understanding and originality in its mathematics courses."[17]

Mathematics education must be geared to the future needs of its students. The material to be used in mathematics courses, the method to be used in instruction, and the approach to be used in presenting the material will be extremely important. Research can be used to determine a more feasible approach to the study of mathematics.

STATEMENT OF THE PROBLEM

In lieu of the changes in mathematics and in mathematics education, and the important role mathematics plays in our lives, this study has been conducted in mathematics education to determine the following: Which one of three approaches would be most effective in teaching the Introductory College Mathematics Course at Central State College: the conventional approach, the vector approach, or the set theory approach?

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[16]Committee on the Undergraduate Program in Mathematics, Recommendations for the Training of Teachers of Mathematics, The Mathematical Association of America, (1961), p.3.

[17]Ibid., pp. 3-4.

DEFINITION OF TERMS

For the purpose of this study, the following basic terms will be defined. Other terms will be defined as needed.

The Introductory College Mathematics Course will be defined as that combined course in plane trigonometry and advanced algebra which stresses the basic concepts of function, logarithmic and exponential functions, trigonometric functions, complex numbers, solutions of linear equations using simple matrix manipulations, determinants, binomial theorem, permutations and combinations, mathematical induction, and inverse trigonometric functions.

Algebra is one of the three main divisions of pure mathematics. The other two divisions are geometry and trigonometry. The algebra that mathematicians call elementary deals with sets of numbers and with operations performed on these numbers. In this sense algebra resembles ordinary arithmetic. However, algebra provides a symbolism which permits a study of the general properties of sets of numbers and of operations.

Trigonometry is a division of mathematics which deals with the interrelationships of the sides and angles of the triangle and closely related magnitudes. Trigonometry supplies the means for finding distances that cannot be measured directly, such as the distance to a particular star or planet. It has extensive applications in science, engineering, navigation, and related fields, and is fundamental to many branches of higher mathematics.

The Introductory College Mathematics Course is a five hour, one semester course at Central State College and is listed in the Central State College Bulletin as Mathematics 165.

The main objective of this course is to strengthen the student's background in algebra and trigonometry and to prepare him for additional work in mathematics starting with calculus.

The subject matter is structured around algebra and trigonometry with the emphasis being placed on function theory, a study of the relationship between variables. For example, the area of a square is equal to the length of a side squared. In terms of a function, one would say that the area of the square is a function of the length of one side. Using functional notation, this would be stated: $A = f(s)$, or $f(s) = s^2$.

The distance traveled by a person in an automobile is dependent upon two variables, rate and time. Therefore, the distance traveled is a function of both rate and time. Again using functional notation: $d = f(r,t)$, where d represents the distance traveled, r represents the rate or speed, and t represents the time element.

Function theory is extremely important in higher mathematics. In studying calculus, the student uses functions when he computes the area under plane curves, when he calculates the work done in moving a given quantity of liquid a certain distance, or when he finds the center of gravity of any particular geometric solid.

The subject matter in the Introductory College Mathematics Course is more advanced in nature than any algebra the student has encountered in his secondary work. The same thing can be said about trigonometry.

The <u>conventional approach</u> will be defined as that approach to the study of algebra and trigonometry which has been utilized in our secondary schools and colleges during the past twenty-five years. The emphasis is on manipulation and problem solving. Neither vectors nor sets are used in the development of the algebraic and trigonometric principles.

The approach, in general, places the emphasis on learning a particular manipulation, and then utilizing it in a problematic situation. For example, a student learns to factor a quadratic polynomial, then he uses this information in finding the zeros of a quadratic equation or in solving a problem which involves quadratic equations.

The conventional approach can perhaps be said to be a more utilitarian approach to the study of mathematics.

A vector will be defined as a directed line segment.

The vector approach will be defined as that approach to the study of algebra and trigonometry whereby the mathematical ideas are expressed in terms of vectors.

A set will be considered to be a collection of elements (usually numbers), otherwise undefined.

The set theory approach will be defined as that approach to the study of algebra and trigonometry whereby the mathematical ideas are expressed in terms of sets.

SOLUTIONS TO THE PROBLEM

The freshman student in college who is preparing himself to teach mathematics, or to become an industrial mathematician, or to become an engineer, will need adequate mathematical training. He will need to have an understanding of the mathematical tools he is utilizing if he is to be able to perform with any degree of competency. This same student should be aware of the uses and limitations of mathematical concepts, and he should be able to apply mathematical tools with facility.

With regard to this study, there are three possible solutions to the problem. They are the conventional approach, the vector approach, and the set theory approach.

The first approach, the conventional approach, treats the subject matter in the Introductory College Mathematics Course in much the same way that the student has studied it in the secondary school. The emphasis is placed on the basic algebraic manipulations with direct application made to problem solving. The student works with monomials, binomials, and polynomials and the basic arithmetic operations of addition and multiplication, with variations. Trigonometry is developed in a similar way. The emphasis is placed on the trigonometric functions, then later shifted to the solution of triangles.

One of the advocates of the conventional approach to the study of mathematics is Morris Kline. A noted mathematician, Kline feels that mathematics ought to be taught from the concrete standpoint before trying to teach the abstract.[18] A student can know all about the algebraic properties of a mathematical field, and yet not be able to find the sum of two and two. It is also possible that a person with an excellent knowledge of fields might not even be able to make change in a grocery store.

Experiences with the concrete would allow the student to later gain knowledge of the abstract. But knowledge of the abstract does not guarantee facility in the use of the concrete principles of mathematics.

The vector approach to the study of the Introductory College Mathematics Course might be considered by some people to be a novel way of studying algebra and trigonometry. From the standpoint of both applied and pure mathematics, however, vectors have an important role to play. They can be used to describe physical relationships such as velocity and acceleration.

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[18]Kline, "The Ancient vs. the Moderns, A New Battle of the Books," The Mathematics Teacher, LI (1958), p. 241.

Vectors can be introduced as a purely mathematical system, built on the fundamentals of algebra. A vector is defined to be a directed line segment. Using this definition, the basic operations of vector addition and multiplication can be stated. Vectors can be used to describe effectively n-dimensional spaces. Therefore, a vector field is a region of space such that with each point there is associated for complete characterization both a magnitude and a direction, that is, a vector quantity. Common vector fields are the electric, the magnetic, and the gravitational fields. In meteorology, the velocity of the wind at each point of the atmosphere is an example of a vector field.

Using this approach, the student studied algebra in terms of vector notation. For example, the addition and multiplication of vectors, and the geometric interpretation of these operations were discussed.

The student was taught the importance of the algebraic properties of commutativity, associativity and the nature of the distributive law as it is related to vectors.

For example, for vectors a, b, and c, $a+b = b+a$, $a+(b+c) = (a+b) + c$, and using the dot product, $a\cdot(b+c) = a\cdot b + a\cdot c$.

In addition to this, the student was made aware that every vector has an inverse. That is to say, for every vector a there is a vector b such that $a+b = 0$. In this case, b is equal to $-a$.

Trigonometry is the study of triangles, and triangles can be constructed using vectors. In relationship to this, any angle can be described as an angle between two vectors. The area of a triangle can also be computed using vector notation.

Vectors can be used in theoretical mathematics to prove theorems and corollaries. By using vectors, a complex mathematical concept can often times be greatly simplified. As demonstrated, vectors can be used to describe physical relationships, i.e., velocity and acceleration, and in simpler terms, they can be related to a rectangular coordinate system as designed by Descartes.

One of the advocates for the use of vectors in studying algebra and trigonometry is Arthur H. Copeland. He says,"The impression is held that the employment of these modern techniques [vectors] presupposes a high degree of sophistication on the part of the student. This is not the case. The [student] can be prepared for the modern techniques and they should not cause any more difficulty than the older methods."[19]

With regard to the applicability of the modern methods, namely vectors, he continues, "Often a modern method has a much broader range of applicability than the older methods, although the two methods can be applied with comparable difficulty to the development of a given subject. For example, a vector algebra is used to simplify the development of analytic geometry and algebra and to unify the two subjects. The time saved by this simplification is sufficient for the development of the vector algebra. Moreover, vectors are sufficiently similar to numbers so that by studying vectors a student can review his ordinary algebra without being subjected to a course in pure drill."[20]

Set theory can also be used in developing the Introductory College Mathematics Course. A set is nothing more than a collection of elements, but its uses are far reaching.

The set theory approach is more closely oriented to the conventional approach than is the vector approach. However, there is still a great

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[19]Arthur H. Copeland, <u>Geometry, Algebra, and Trigonometry by Vector Methods</u> (New York, 1963), p.3.

[20]Ibid., p.5.

deal of difference between the approaches.

Algebra is a study of the real number system, but the way in which this study is developed is very important.

Using the set theory approach, the student is introduced to sets and their properties. With a short background in logic, the real number system is developed using an axiomatic procedure. For example, the student might be given the set of natural numbers. From this the student can define the set of positive integers: i.e., 1,2,3,... and the set of all integers I, where any integer is the difference between two natural numbers. For example, $3 = 5-2$, $-5 = 8-13$, and so on. While this might appear to be trivial, it does give the student some basis on which to begin his study. The properties of commutativity and associativity can be substantiated, as well as those properties related to identities, inverses, and the distributive law.

In continuing this line of thought, the rational numbers can be defined as being the set of all numbers of the form a/b where a and b are integers and b is not equal to zero.

Again the student is informed of the importance of the mathematical properties involved, namely associativity, commutativity, identities, inverses, and the distributive law.

In solving equations, the student discusses solution sets. For the quadratic equation $x^2-5x+6 = 0$, the solution set would include 2 and 3, since these two values of x satisfy the equation.

Trigonometry is developed using sets and function notations and this information related to the study of complex numbers.

Sets enter mathematics not only in its foundations and language but also in its mathematical structure. For example, the theory of probability is concerned with numerical measures of subsets of some space. Statistics is concerned with 'populations' (which are sets), with measures of subsets of the population which are called 'statistical parameters', and with the test of hypotheses about subsets of the population.

Topology is the study of sets in which certain subsets are distinguished. Modern algebra is developed around the theory of sets. The basic superstructure of ordinary algebra and trigonometry can be developed in a similar way.

In the words of one mathematician:

> "...the language and ideas of sets are gaining importance due to the technical nature of our civilization with its mass production, mass distribution, and increasing socialization in government and industry. Set ideas come into play through statistics, classification, experimental inference, and population studies. They appear in many branches of science such as chemistry, thermodynamics, genetics, and physiology...In fact, the modern change over in the philosophy of science from a mechanistic determinism to a kind of probabilistic indeterminism implicitly involves the idea of set. Moreover, the teachers of statistics tell us that the present lack of familiarity of students with set thinking imposes a serious limitation upon their becoming skillful in statistics. These and many other fields of application reinforce the conclusion arising from modern mathematics itself. Sets are here to stay."[21]

It must be stated that there is a certain degree of overlapping between the three approaches. The one apparent reason is that all approaches are related to the same subject matter areas, algebra and trigonometry. An x^2 using the conventional approach will be interpreted in the same way using the set theory approach, and by the student using the vector approach.

In talking about trigonometry, the basic identity, $\cos^2 t + \sin^2 t = 1$, will have the same significance for all three groups.

This similarity is not critical. Mathematics is a unified subject, and the language of the subject must be conformable. The important implication is that even if mathematical concepts are taught using various approaches,

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[21] W.L. Duren Jr., "The Maneuvers in Set Thinking," The Mathematics Teacher, LI (May, 1958), p. 323.

the basic ideas will still be the same.

CRITERIA FOR CHOOSING BETWEEN SOLUTIONS

In order to choose which one of three approaches, the conventional approach, the vector approach, or the set theory approach, would be most effective in teaching the Introductory College Mathematics Course, the author chose three criteria.

The criteria are: 1) algebraic manipulation; 2) trigonometric achievement; and 3) general problem solving ability.

The student who completes the Introductory College Mathematics Course at Central State College should be able to perform adequately the basic mathematical skills, he should have an understanding of algebra and trigonometry and their inter-relationship, and he should be able to solve basic problems which use algebra and trigonometry in their solution.

The criteria to be used in this study in choosing the appropriate solution to this problem regarding approach effectiveness; i.e., conventional, vectors, or sets, are as follows.

1) The student should be able to perform the basic algebraic manipulations. This includes the addition, subtraction, multiplication, and division of algebraic expressions, factoring, logarithmic operation, and the solution of equations in one unknown.

2) The student should be proficient in trigonometry. He should know the basic trigonometric functions, their usefulness in proving identities, and their role in the solution of triangles.

3) The student should be able to solve problems which involve algebraic and trigonometric principles. These problems will be similar to ones he would encounter in everyday situations or if he were employed in industry.

BACKGROUND FOR THIS STUDY

There are several studies being conducted at the present time dealing with the mathematics curriculum at the college freshman level. One of these is at the University of Kansas. It is the intent of the Department of Mathematics at the University of Kansas to design a two semester sequence of freshman courses in mathematics to be taken by the typical student in the college of liberal arts and sciences. It is to be designed to provide mathematical training for those students with special interests in the social, biological, and management sciences, and for those who would like to take a terminal course in mathematics as a part of a liberal education.

The goals of this program can be stated as follows:

1) To provide mathematical training for those students in fields other than mathematics. This training will allow these students to utilize the mathematical tools in their work.

2) To provide mathematical education which will be an integral part of their liberal education.

3) To provide the type of mathematical training which will enable the student to further educate himself in preparation for the type of mathematics he will need in his work, i.e., statistics, computers, and so on.

During the first semester, the student will study the following topics: a) solutions of systems of linear equations; b) matrices; c) sets, relations, and functions; d) counting problems; e) probability theory for finite sample spaces; and f) flow charts. The plan for the second semester includes an introduction to differential and integral calculus, with applications to the study of continuous probability distributions.

While no quantitative data is available as to the success of this program, indications are that it is being readily received by the students and the mathematics staff members at the University of Kansas.

An experimental study was conducted at Oberlin College during the 1957-58 school year.[22] Its purpose was to gain knowledge which would allow

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[22]John D. Baum, "Mathematics, Self-Taught," The American Mathematical Monthly, 65 (November, 1958), pp. 701-705.

the college to enlarge its student body, utilize its physical facilities, and yet not increase the size of its faculty proportionally to the sizes of increase in the student populace.

In this study, students in the experimental group spent two-thirds of their normal class time in the classroom, the other one-third in outside study. The outside study consisted of independent study on the part of the student. The control group spent the normal time in the classroom.

Two hypotheses were tested in this study:

a) That students who study freshman mathematics by the usual classroom method will not learn significantly more than students who study on their own for approximately one-third of the school year.

b) That students who study freshman mathematics by the usual classroom method will not gain significantly over students who study indepently for approximately one-third of the year in their ability to attack new mathematical concepts.

Based on the t-test of significance, no significant differences were attained and the conclusions reached stated that the students who had done the independent study were probably ahead of the control group. This advantage was attributed to the students' experiences in digging out the material on their own initiative.

There might be questions raised concerning the two-thirds to one-third ratio used in this experimental study. No justification can be given for this choice, but further research in this type of thinking might be beneficial, especially since our colleges and universities are being faced with a student "population explosion."

A new program is currently underway at Tulane University. Its intent is to develop a 'pure' program for able students involving acceleration, but, more basically, laying a foundation for advanced work------"penetration in depth." The eventual goal of the program is to upgrade all freshman mathematics at Tulane University.

There are three factors involved in this program which make this study unique. They are:

1) Small classes (Not more than 25 students)

2) Research mathematicians as teachers

3) High upward mobility of students and the development of a spirit of inquiry.

There are no results available at this time regarding this study.[23]

SUMMARY

Mathematics and mathematics education have undergone and are expected to continue to undergo vast changes. In keeping pace with these changes, it should be quite evident that "we need frequent, repetitive prodding to keep on the best path. We tend to claim that mathematics stimulates clear thinking, then teach it as meaningless memorization and routine having little to do with thought. Being a drillmaster, stressing routines, etc., makes the teacher's life easier but harder to justify."[24]

One of the leading mathematicians in the School Mathematics Study Group has this to say regarding mathematics and education:

> "The world of today demands more mathematical knowledge on the part of more people than the world of yesterday, and the world of tomorrow will demand even more. It is therefore important that mathematics be taught in a vibrant and imaginative way which will make students aware that it is a living, growing subject which plays an important part in the contemporary world."[25]

Mathematics education which makes the student aware that mathematics is a living, growing subject and an important part of our contemporary

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[23] A.D. Wallace, "New Undergraduate Courses in Mathematics", Science Course Improvement Projects, National Science Foundation, (1962), p. 54.

[24] M. Richardson, Fundamentals of Mathematics (New York, 1941), p.4.

[25] E.G. Begle, "The School Mathematics Study Group," op. cit., p. 616.

world can be accomplished. If the language of the subject, the vehicle of communication were improved, the task of teaching mathematics could be greatly implemented.

With an eye to the future, this study has been conducted to determine the following: Which one of three approaches; the conventional, the vector, or the set theory approach, would be most effective in teaching the Introductory College Mathematics Course at Central State College?

CHAPTER II

THE EXPERIMENTAL DESIGN

This study was an experimental study, the chief purpose being to analyze and determine which one of three approaches would be most effective in teaching the Introductory College Mathematics Course at Central State College. In doing this, experimentation was defined as: "The trial of a planned procedure accompanied by control of conditions and/or controlled variation of conditions together with the observation of results for the purpose of discovering relationships and evaluating the reasonableness of a given hypothesis."[26]

In this experiment, the Introductory College Mathematics Course was taught by three different approaches: 1) the conventional approach; 2) the vector approach; and 3) the set theory approach. In each case, the mathematical concepts were basically the same. The important implication here is that perhaps the two groups studying the material from the vector approach and the set theory approach would be more productive than the group using the conventional approach. That is to say, they would be able to perform mathematically the basic skills, yet they would have a knowledge of vectors or sets.

<u>The Experimental Design</u>. During the fall semester of 1963, there were approximately 180 students enrolled in the Introductory College Mathematics Course at Central State College. These students were freshmen

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[26] Carter V. Good, <u>Dictionary of Education</u> (New York, 1945), p. 95.

who indicated an interest in pursuing a major in mathematics, physics, chemistry, or in a professional field such as engineering, law, medicine, and so forth.

By standards previously set, these students were advised to enroll in this course on the basis of scores made on the mathematics section of the test administered by the American College Testing Program prior to their enrollment in college. If a student ranked between the fiftieth and eighty-fifth percentile, he would take the Introductory College Mathematics Course. If his score was above the eighty-fifth percentile, he was advised to take the first course in the calculus. A ranking below the fiftieth percentile indicated a need for remedial work, and the student was advised to take a mathematics course which would prepare him for the advanced sequence of mathematics courses. These advanced courses would include the Introductory College Mathematics Course, calculus, and so on.

The population for this study was defined as the set of mathematics majors and pre-professional students enrolled in the Introductory College Mathematics Course at Central State College during the fall semester of 1963.

The sample for this experiment consisted of three mutually exclusive subsets of the population just defined. Each of these subsets consisted of 24 students, a total of 72 students being involved in the study.

The students who took part in this study were chosen on the following basis:

1) A graduate of a high school in the United States during the spring of 1963.

2) The Introductory College Mathematics Course was the student's initial course in college mathematics.

3) The student was regularly enrolled at Central State College as a full-time student.

4) The student completed this course with a grade of A, B, C, D, or F and took all of the tests administered during the time of the experiment.

The distribution of the sample among the groups was determined by a random selection. The population met prior to enrollment for the fall semester and drew lots which specified the course section in which they would be enrolled. The procedure used five cards numbered 1,2,3,4, and 5 and the students drew these cards successively. The student's name was recorded immediately after the drawing and his name assigned to a particular section. The names were checked after enrollment to make sure that there were no deviations from the class assignment.

A description of the individual groups is as follows:

Group A - This group was the control group. They studied the subject matter from the conventional approach. The textbook used by this group was College Algebra and Trigonometry by Fischer and Ziebuhr. This book is being used by several of the State colleges of Oklahoma and has been in circulation about four years.

Group B - This group was one of two experimental groups. They studied the subject matter from the vector approach. The textbook used was Geometry, Algebra, and Trigonometry by Vector Methods, written by Arthur H. Copeland. This book was published late in the spring of 1963.

Group C - This group was the second experimental group. They studied the subject matter from the set theory approach. A new edition of a textbook by Allendoerfer and Oakley was used. It is titled, Principles of Mathematics. This book is currently being used at the University of Mississippi, the University of California, and the the University of Kansas.

The subject matter was basically the same in each of these groups. The primary difference was in the approach used in instruction. For example, while the conventional approach utilized standard procedures in developing the algebraic structure, the vector approach utilized vectors. The group using the set theory approach was doing basically the same thing utilizing sets and an axiomatic approach.

The duration of this experiment was one semester. It began in September, 1963, and was terminated in January, 1964.

The course was divided into three major areas. The first of these areas dealt with the basic algebraic structures. This included the basic algebraic manipulations, factoring, exponents, fractional expressions, logarithms, and so forth. The second area concerned itself with the study of trigonometry. The student studied trigonometry in its theoretical aspect as well as for its practical or utilitarian values. The third area covered the study of theory of equations, sequences, the binomial theorem, and some probability.

By dividing the course into three subject matter areas, it was a great deal easier to correlate the activities of the three groups and thereby teach about the same material at the same time.

Tests were administered at the beginning of the experiment, during the third and tenth weeks, and again at the end of the experiment. This was done in order to measure the growth of the students in this experiment with regard to mathematical skills.

Selection of the Testing Instruments. One of the major problems in educational research is the proper selection and utilization of testing instruments. In this experiment, five different measuring devices were used. They are as follows:

1) The American College Test is a test which is now being administered throughout the United States. In Oklahoma it is being required for entrance into the state-supported institutions of higher learning. This is a test sequence which covers four areas: English, Social Studies, Natural Science, and Mathematics. There is also a composite score based on the overall test results. The mathematics score is being used by most of the colleges and universities in Oklahoma, including Central State College and Oklahoma State University, for placement purposes.

2) The Cooperative Algebra Test is a test which measures ability to do basic algebraic manipulation. This test is used by Oklahoma State University in its freshman orientation program. The

reliability coefficient for this test is .90 with a standard error of .02. In this study, the test was used to measure the student's ability to do basic algebraic manipulations. This test appears to be well-designed to perform this task. The problems are basic in nature and not unduly complex.

3) The Rassmusson Trigonometry Test measures ability in trigonometry, placing the emphasis on the theory underlying trigonometry, rather than on computation. The reliability coefficient is .93. This test was used to measure trigonometric achievement. The test includes true-false items, multiple choice questions, and a matching section, all of which make it highly comprehensive in nature. The true-false section might be a weak point since the student might be encouraged to guess at the answer. The assumption was made that this possibility was prevalent in all three groups and might not, therefore, be unfavorable to any one of the three groups.

4) The Sequential Test of Educational Progress in Mathematics is a test of general ability in problem solving utilizing both algebra and trigonometry. In the words of one critic, "Mastery of the [mathematical] concepts is tested by problems which require understanding and application in the context of situations of practical significance to the student."[27] A second critic said, "It is outstanding in terms of measuring understanding as opposed to rote memory, application of principles and skills, abilities involved in interpreting and understanding as opposed to rote memory."[28] The problems are quite similar to the type of problem that a student might encounter in a working situation. One of the criticisms of this test is that it is highly verbal in nature. This is a distinct possibility, but on the mathematics section, it did not seem to be so.

5) The Otis Self-Administering Intelligence Test is a device for measuring intelligence. The results of this test were used for descriptive purposes only.

The purposes of the testing program were to measure growth on the part of the student with regard to mathematical achievement. The student was administered a pre-test and a post-test in each of these areas; algebraic manipulation, trigonometric understanding, and general problem solving ability. The Cooperative Algebra Test, the Rassmussen Trigonometry Test, and the Sequential Test of Educational Progress in Mathematics were

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[27]Oscar K. Buros, The Fifth Mental Measurements Yearbook, (Highland Park, New Jersey, 1959), p. 570.

[28]Ibid, p. 571.

for this purpose. The results of the ACT Program in Mathematics were considered to be a pre-test of the students ability in mathematics.

The Testing Schedule. By the nature of the course content and the criteria on which this study is based, the following testing schedule was maintained.

The Sequential Test of Educational Progress in Mathematics (Form 1A) was administered during the first two days of the semester. The test required seventy minutes, therefore two class periods were needed for the proper administration of the examination.

On the third day, the students in the three groups were given the Cooperative Algebra Test, Form Z.

During the third week of instruction, the Rassmussen Trigonometry Test was administered to the three groups. Form A was used in this instance. The three groups had not yet entered into the study of trigonometry in their respective sections and the time lapse gave them a break in the testing sequence.

The Otis Self-Administering Intelligence Test was given to the students during the tenth week of the semester. Since this test was to be used for descriptive purposes only, the time of administration was of little consequence.

The final testing sequence, the post-tests, were taken care of during the final week of the semester. During the last two regular class periods, the Cooperative Algebra Test (Form Y) and the Rassmussen Trigonometry Test (Form B) were administered. During the two-hour final exam period provided for each class, the Sequential Test of Educational Progress in Mathematics (Form 1B) were administered in a straight seventy minute sequence. The choice of final testing was a matter of convenience.

The Statistical Treatment of the Data. In this experiment, the treatment of the data was accomplished by the use of the analysis of variance and the analysis of covariance. These procedures segregate from comparable groups of data the arithmetic means of the products of the paired deviations of two or more variables, measured from the respective means of the variables. The independent variable was the approach; i.e., the conventional approach, the vector approach, or the set theory approach. The dependent variable was the scores obtained on the various tests which were administered to the students participating in the study.

ASSUMPTIONS UNDERLYING THIS STUDY

In order to maintain a logical foundation for this study, the investigator made the following assumptions. From these assumptions, postulates were derived, postulates which lead directly into the hypotheses to be tested in this experiment.

Assumption I - Students who complete the Introductory College Mathematics Course should be well grounded in their ability to do basic algebraic manipulations.

Postulate 1 - Students who complete this course using the conventional approach will be adequately prepared to handle basic algebraic manipulations.

Postulate 2 - Students who complete this course using the vector approach will be adequately prepared to handle the basic algebraic manipulations.

Postulate 3 - Students who complete this course using the set theory approach will be adequately prepared to handle the basic algebraic manipulations.

Assumption II - Students who complete the Introductory College Mathematics Course should be well grounded in their ability to deal with trigonometric problems.

Postulate 1 - Students who complete this course using the conventional approach will be adequately prepared to handle problems in trigonometry.

Postulate 2 - Students who complete this course using vector methods will be adequately prepared to handle problems in trigonometry.

Postulate 3 - Students who complete this course using the set theory approach will be adequately prepared to handle problems in trigonometry.

Assumption III - Students who complete the Introductory College Mathematics Course should be adequately prepared to handle basic mathematical problems involving algebraic and trigonometric principles.

Postulate 1 - Students who complete this course using the conventional approach will be adequately prepared to handle basic mathematical problems involving algebraic and trigonometric principles.

Postulate 2 - Students who complete this course using the vector approach will be adequately prepared to solve basic mathematical problems involving algebraic and trigonometric principles.

Postulate 3 - Students who complete this course using the set theory approach will be adequately prepared to solve basic mathematical problems involving algebraic and trigonometric principles.

Assumption IV - The testing instruments used in this experiment are valid measuring instruments with regard to ascertaining the student's achievement in algebra, trigonometry, and general problem solving ability.

HYPOTHESES TO BE TESTED

On the basis of the assumptions just stated, the following hypotheses were tested in this study:

Hypothesis H_1 - There will be no significant difference in the mean scores of the three groups at the end of the experiment with regard to ability in algebraic manipulation at the 5 per cent level of significance.

Hypothesis H_2 - There will be no significant difference in the mean scores of the three groups at the end of the experiment with regard to achievement in trigonometry at the 5 per cent level of significance.

Hypothesis H_3 - There will be no significant difference in the mean scores of the three groups at the end of the experiment with regard to ability in problem solving at the 5 per cent level of significance.

LIMITATIONS ON THIS STUDY

In any experimental study of this type, there are certain limitations which seem to be prevalent. Some of these include size of the sample, duration of the experiment, outside interests of the student, and so on.

One limitation which could be an important factor was the instructor for each group. It was necessary, in this study, to use three different instructors. However, to use one instructor for all three sections might also be folly. To substantiate the validity of using three instructors, the following criteria were adhered to:

1) Each instructor indicated an interest in taking part in the study.

2) Each instructor indicated an interest and preference for teaching the particular approach to be used in his section of the course.

3) Each instructor held a master's degree in the field of mathematics and had a least 45 semester hours of graduate work in mathematics.

SUMMARY

This was an experimental study in mathematics education. The sample for this experiment was a set of freshman students from Central State College. They were randomly assigned to each of three groups.

During the fall semester of 1963, the students were instructed in algebra and trigonometry with applications. At the beginning of the course, they were tested to measure ability in algebra, trigonometry, and general problem solving ability. At the end of the experiment, they were again tested along these same lines.

The analysis of variance and the analysis of covariance were utilized to determine whether or not there were any significant differences between the three groups with regard to mean scores at the five per cent level of significance.

Three hypotheses were tested. The first hypothesis related itself to algebraic achievement, the second to trigonometric achievement, and the third to general problem solving ability.

CHAPTER III

PRESENTATION OF THE DATA

Introduction

In order to gain a more complete description of the sample, a questionnaire (Appendix A) and a testing program were utilized in this study.

The questionnaire was designed to provide information regarding the background of the student: i.e., the location of his hometown, the size of his high school graduating class, his preparation in mathematics, his professional ambitions and intentions.

The primary purpose of the testing program was to measure the changes which occurred in mathematical achievement with regard to the different approaches. Information was also gathered which could be used to compare the members of the sample as well as to describe them.

The Questionnaire. During the first week of the experimental study, a questionnaire was distributed among the students in the three experimental groups. The information gathered from this questionnaire was used in part to determine whether or not the students had met the requirements for participation in the study as stated in Chapter II.

There were several factors which were considered to be of importance by the investigator with regard to the description of the sample.

For example, the student's hometown might be a key factor. If all of the students in one group came from a large city, and the students in a second group came from a rural area, a bias might be introduced. This could be due to a difference in the educational pattern of the schools, the quality of the instructional staff, or the learning facilities which were available.

Central State College is the fastest-growing of the six state colleges in Oklahoma. Being located just ten miles north of Oklahoma City, a metropolitan area, would make it seem that most of the students would come from the Oklahoma City area. This is true. But there are many students who come from other parts of the state as well.

Table I is a representation of the distance that the students travel to become students at Central State College. To facilitate this operation, the distance 0-25 miles will include the greater Oklahoma City area.

TABLE I

DISTRIBUTION TABLE: DISTANCE OF HOME TOWN FROM CSC

Distance	Conventional	Vectors	Sets
0 - 25 miles	11	12	13
26 - 100 miles	7	9	5
101 - miles	6	3	6

The size of the school system which a student attended might be a factor which is important in a study of this type. As seen in Table I, many of the students did come from the greater Oklahoma City area. However, there appears to be a large number of high schools in this area which are not very large by ordinary standards. Table II offers information regarding the size of the student's high school graduating class.

TABLE II

DISTRIBUTION TABLE: SIZE OF HIGH SCHOOL GRADUATING CLASS

Size	Conventional	Vectors	Sets
0 - 100	6	14	10
101 - 250	10	6	7
251 - 500	8	4	7

A very marked similarity was also noted in the mathematical backgrounds of the students in the three groups.

It is usually recommended that a student have at least two years of high school preparation in algebra and a year of plane geometry before enrolling in the Introductory College Mathematics Course. Approximately ninety per cent of the students in the sample met this requirement. Several students had a much stronger background in mathematics than was required. Trigonometry was taken by fourteen of the students in each group. Matrix algebra, a relatively new course in the secondary curriculum, was taken by only a few students. Mathematical analysis, a course which is considered to be preparatory for calculus, was taken by only three students.

Table III shows the high school preparation in mathematics of the students in the sample.

TABLE III

DISTRIBUTION TABLE: MATHEMATICS COURSES TAKEN IN HIGH SCHOOL

Course	Conventional	Vectors	Sets
Algebra I	24	24	24
Plane Geometry	24	24	24
Algebra II	22	21	21
Trigonometry	14	14	14
Solid Geometry	7	10	8
Matrix Algebra	1	2	2
Mathematical Analysis	2	0	1

Many of the students who attend Central State College do so knowing that they will finish their degree program somewhere else.

This is especially true with those students who wish to become engineers, doctors, lawyers, and so on. Table IV is a representation of this distribution as it affects the three experimental groups.

TABLE IV

DISTRIBUTION TABLE: THE STUDENT'S MAJOR FIELD OF STUDY

Major	Conventional	Vectors	Sets
Mathematics	11	11	10
Engineering	10	11	9
Pre-Professional	3	2	5

During the tenth week of the semester, the Otis Self-Administering Intelligence Test was administered to the students in the three experimental groups. The information gathered from this test is offered here in a descriptive role. Table V shows the distribution of the IQ scores as they are related to the three groups.

TABLE V

DISTRIBUTION TABLE: SCORES FROM THE OTIS INTELLIGENCE TEST

Scores	Conventional	Vectors	Sets
128-132	1	0	1
123-127	4	1	2
118-122	1	3	5
113-117	7	10	7
108-112	6	5	7
103-107	5	3	2
98-102	0	2	0

Grades are a factor which can be used to show achievement in mathematics.

The grades given in this course were evaluations of the teacher independent of the testing program conducted with regard to this study. They are offered here for descriptive purposes and will not be used in the statistical analysis of the data. The distribution of grades according to groups is given in Table VI.

It should be noted here that students who did not successfully complete the course were excluded from the study. Only meaningful grades such as A, B, C, D, or F were acceptable.

TABLE VI

DISTRIBUTION TABLE: GRADES ATTAINED IN MATHEMATICS 165

Grade / Group	F	D	C	B	A
Conventional	3	4	9	5	3
Vectors	4	3	8	5	4
Sets	3	2	10	4	5

<u>The Results of the Testing Program</u>. During the course of this experimental study, several mathematics tests were administered to the members of the sampling body.

These tests included algebraic manipulation, trigonometric achievement, and general problem solving ability in mathematics.

In addition to these tests, the Otis Self-Administering Intelligence Test was also given.

Prior to their enrollment at Central State College, each member of the sample was administered the American College Testing Program. This program yields five test scores. They are: 1) Mathematics; 2) English; 3) Social Studies; 4) Natural Science; and 5) a Composite score.

A complete listing of the raw scores made on each of the tests is given in Appendix B. For the sake of brevity, Table VII is given on the next page to show the mean scores achieved on each test by the three groups as well as the mean score for all three groups. This data is in reference

TABLE VII

TEST SCORES: DEPENDENT VARIABLE STATISTICS

Test	MEANS Conventional M	°/ile	Vectors M	°/ile	Sets M	°/ile	Totals M	°/ile	t_{cv}	t_{cs}	t_{vs}
Cooperative Algebra											
Pre-test	32.8	90	31.8	89	30.9	89	31.8	89	.26	.78	.41
Post-test	34.3	98	32.5	97	33.5	97	33.4	98			
Rassmussen Trigonometry											
Pre-test[a]	53.2	10	55.3	12	55.8	12	54.8	11	-.62	-.84	-.18
Post-test	58.9	20	62.3	28	57.7	18	59.6	23			
Post-test[a]	67.6	40	68.3	40	61.0	28	65.6	35			
STEP Mathematics											
Pre-test	23.9	69	24.0	69	24.4	69	24.1	69	-.07	-.32	-.27
Post-test	26.7	78	28.3	81	27.6	80	27.5	80			
ACT Mathematics	21.6	72	21.0	72	21.0	72	21.2	72	.24	.38	0.00
English	18.0	55	17.2	54	19.1	56	18.1	55	.60	-.94	-1.43
Social Studies	18.7	55	17.4	54	18.1	55	18.1	55	.81	.44	-.52
Natural Sciences	20.9	72	21.1	72	21.4	72	21.1	72	-.13	-.35	-.18
Composite	19.7	69	19.7	69	19.9	69	19.8	69	0.00	-.21	-.22
Otis Intelligence	114.3		112.8		115.3		114.1		.74	-.45	-1.39
n =	24		24		24		72				

a) Only those students with a background in trigonometry were included.

to the pre-test and post-test sequence. In addition, percentile scores are included for each of the tests.

It should be noted that only forty-two scores were recorded for the pre-test in trigonometry. This was done because only these forty-two students had had previous work in trigonometry. For the rest of the sampling body, valid testing results on this particular test were not possible.

The t-test was used to compare the mean scores of the three groups. The results of this test of significance are also listed in Table VII. A t-value of 1.68 was needed at the five per cent level of significance. This did not occur in any case.

In addition to the means and the t-test, the reliability correlation coefficients were computed for the various tests. In Table VIII, Table IX, and Table X, scatter diagrams are used to show the correlation between the pre-test and post-test scores on the Cooperative Algebra Test, the Rassmussen Trigonometry Test, and the Sequential Test of Educational Progress in Mathematics. The highest correlation was recorded on the Cooperative Algebra Test. It was .75. The STEP test had a correlation coefficient of .63, and the Rassmussen Trigonometry Test had a correlation of .58.

Other correlation coefficients were computed and scatter diagrams for these tests are included in Appendix C.

SUMMARY

On the basis of the information gathered using a questionnaire and an extensive testing program, the groups were fairly well matched.

The backgrounds of the students in each of the three groups appeared to be similar. The distance traveled by the students to get to the Central State College campus, the size of the student's high school graduating class, and the preparation of the students in mathematics, were

TABLE VIII

SCATTER DIAGRAM: STEP PRE-TEST SCORES VS. STEP POST-TEST SCORES

Post-test / Pre-test	15-18	19-22	23-26	27-30	31-34	35-38	39-42	43-46
36-39					/		/	/
32-35				/	/	/		
28-31		/	/	////	///	/////	/	
24-27	/	//	/////	///////	///	/		
20-23	/	///	/////	/////		/		
16-19	///	///	/////	/	/			
12-15		//	/	/				

N = 72

r = .63

TABLE IX

SCATTER DIAGRAM: TRIGONOMETRY PRE-TEST SCORES VS. TRIGONOMETRY POST-TEST SCORES

Post-test / Pre-test	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84
71-75							/	
66-70							/	/
61-65			/		//	///	/	/
56-60	/	/		/	/////	/		
51-55	/	//	///	/				/
46-50		/	//	////	/			
41-45			///	/	/			
36-40		/						

N = 42

r = .59

TABLE X

SCATTER DIAGRAM: ALGEBRA PRE-TEST SCORES VS. ALGEBRA POST-TEST SCORES

Post-test / Pre-test	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59
50-54								/	/
45-49				//		/	//	/	
40-44				/	//	//	///		
35-39			/	///	//	////	/		
30-34	/	///		///	///////	/			
25-29		//	//	////////		/			
20-24	//	//	///	//	/				
14-19		///	////						

N = 72

r = .75

all noted to be quite similar.

One factor which might have been a significant one, is the choice of major field of study. However, according to information gathered in the questionnaire, approximately the same number of students in each group indicated an interest in pursuing a mathematics major, pre-engineering, or a professional field of study.

Grades, which might be an indicator of student interest and potential, were also evenly distributed between the three groups. Only students with meaningful grades of A, B, C, D, or F were included in the study.

On the basis of the t-test, no significant differences were noted between the three groups with regard to the pre-test sequence at the five per cent level of significance.

The correlation coefficients indicate a fairly high correlation between the pre-tests and post-tests in algebra, trigonometry, and general problem solving ability as measured by the Cooperative Algebra Test, the Rassmussen Trigonometry Test, and the Sequential Test of Educational Progress in Mathematics.

CHAPTER IV

STATISTICAL TREATMENT OF THE DATA

This study was conducted to answer the following question: Which one of three approaches would be most effective in teaching the Introductory College Mathematics Course at Central State College; 1) the conventional approach, 2) the vector approach, or 3) the set theory approach?

Each of these approaches was defined and discussed in Chapter I. The criteria which were used in making the decision relative to the approach used were algebraic manipulation, trigonometric achievement, and general problem solving ability.

The Cooperative Algebra Test (Form Y) was used to measure the student's performance with regard to algebraic manipulation. The Rassmussen Trigonometry Test (Form B) was used to measure achievement in trigonometry. The Sequential Test of Educational Progress in Mathematics (Form 1B) was used to determine the student's capabilities with regard to general problem solving.

A pre-test sequence had been completed by the first week of the experiment. The students were administered the Cooperative Algebra Test (Form Z), the Rassmussen Trigonometry Test (Form A), and the STEP Test in Mathematics (Form 1A). During their senior year of high school, the students had been given the ACT Program which consisted of sub-tests in mathematics, English, social studies, and natural science. A composite scores is also computed on the basis of the raw scores made on the ACT Program.

In addition to these tests, the Otis Self-Administering Intelligence Test was also given. The results of this test were used only for descriptive purposes.

Using the raw scores attained by the students on the various tests, the following hypotheses were tested:

H_1 - There will be no significant difference in the mean scores of the three groups at the end of the experiment with regard to algebraic manipulation at the five per cent level of significance.

H_2 - There will be no significant difference in the mean scores of the three groups at the end of the experiment with regard to achievement in trigonometry at the five per cent level of significance.

H_3 - There will be no significant difference in the mean scores of the three groups at the end of the experiment with regard to ability in general problem solving at the five per cent level of significance.

In order to determine which type of analysis would be most beneficial in this situation, t-tests were run on each of the tests in the pre-test sequence. No significant differences were found between the mean scores of the set theory group, at the five per cent level of significance.

On the basis of this evidence, it was possible to assume that the sample is, in fact, a randomly drawn sample from a given population, namely the set of all freshman students enrolled in the Introductory College Mathematics Course at Central State College during the fall semester of 1963.

Although there were no significant differences in the achievement of the three groups as measured by the American College Testing Program, and the pre-test sequence in algebra, trigonometry, and general problem solving, the analysis of covariance appeared to be the most appropriate way of analyzing the data obtained from the testing program.

In using the analysis of covariance, the difference between pre-test and post-test scores regarding the particular trait, algebra, trigonometry, or problem solving ability, was noted. This difference was then incorporated into the analysis.

The way in which this was done was to combine both test scores in a linear operation. If Y_2 were the post-test score and Y_1 the pre-test score, then the adjusted score was equal to $Y = (Y_2 - Y_1) + k$, where k is a positive constant.

The adjusted Y score then became the post-test result and the ACT score in mathematics was used as the pre-test score.

The value of k for all three traits, algebraic manipulation, trigonometric achievement, and general problem solving ability, was thirty.

<u>Testing Hypothesis H_1</u>. The hypothesis, H_1, stated that there would be no significant differences between the mean scores of the three groups with respect to the scores attained on a test of algebraic manipulation.

Using a random table of numbers, one of the scores from each group was discarded. This was done to stabilize the analysis.

The computation for the analysis of covariance was conducted. Table XI shows this analysis.

The F-test yielded: $F = \frac{3.6}{29.8} = .121 < 3.14 = F_{2,65,.05}$

An F-value of .121 is not sufficient evidence to reject the hypothesis that there are no significant differences between the mean scores of the three groups with regard to algebraic manipulation.

<u>Testing Hypothesis H_2</u>. The hypothesis, H_2, stated that there would be no significant differences between the mean scores of the three groups with respect to the scores attained on a test in trigonometry.

In this particular analysis, only fourteen scores were used from each group since this is the number of students who had had previous training in trigonometry. For the rest of the sample, the pre-test in trigonometry was not a valid measuring device because these students had not had any experience with trigonometry.

TABLE XI

ANALYSIS OF COVARIANCE: ALGEBRAIC MANIPULATION

Source	df	Sum of Squares	Mean Squares	SD
Among Means	2	7.2	3.6	
Within Groups	65	1935.1	29.8	5.5

Again a score was randomly omitted from each of the three groups in order to obtain a higher degree of stability in the statistical treatment of the data.

The computation for the analysis of covariance was conducted. The information gathered from analysis is shown in Table XII.

The F-test yielded: $F = \frac{133.4}{67.7} = 1.98 < 3.25 = F_{2,35,.05}$

An F-value of 1.98 is not sufficient evidence to reject the hypothesis that there are no significant differences between the mean scores of the three groups with respect to trigonometric achievement.

Testing Hypothesis H_3. The hypothesis, H_3, stated that there would not be a significant difference between the mean scores of the three groups with respect to the scores attained on a test of general problem solving ability.

A single score was omitted from each group in order to lend stability to the analysis.

Table XIII shows the data as related to the analysis of covariance.

TABLE XII

ANALYSIS OF COVARIANCE: TRIGONOMETRIC ACHIEVEMENT

Source	df	Sum of Squares	Mean Squares	SD
Among Means	2	266.9	133.4	
Within Groups	35	2,369.5	67.7	8.2

TABLE XIII

ANALYSIS OF COVARIANCE: PROBLEM SOLVING ABILITY

Source	df	Sum of Squares	Mean Squares	SD
Among Means	2	22.1	11.1	
Within Groups	65	1,427.0	22.0	4.7

The F-test yielded: $F = \frac{11.1}{22.0} = .505 < 3.14 = F_{2,65,.05}$

An F-value of .505 is not sufficient evidence to reject the hypothesis that there are no significant differences between the mean scores of the three groups with regard to achievement in general problem solving ability.

SUMMARY

The purpose of this study was to investigate which of three different approaches to the study of the Introductory College Mathematics Course would be most effective. The three approaches were: 1) the conventional approach; 2) the vector approach; or 3) the set theory approach.

The criteria used in making this decision were: 1) algebraic manipulation; 2) trigonometric achievement; and 3) general problem solving ability.

Tests were given prior to the experiment in order to measure these traits. The t-test was used to determine whether or not there were any significant differences between the three groups in any respect. No differences were found.

On the basis of this information, the analysis of covariance was used to test three hypotheses. These hypotheses stated that there would be no significant differences between the mean scores of the three groups with respect to algebraic manipulation, trigonometric achievement, and general problem solving ability.

In the analysis of covariance, the F-test failed to indicate a significant difference between the mean scores of the three groups at the five per cent level of significance. The hypotheses of no significant differences between the mean scores of the three groups with regard to scores attained were not rejected.

CHAPTER V

INTERPRETATION OF RESULTS

This study was conducted to answer the following question: Which one of three approaches would be most effective in teaching the Introductory College Mathematics Course at Central State College: 1) the conventional approach, 2) the vector approach, or 3) the set theory approach?

The Introductory College Mathematics Course was defined as that five-hour course in college mathematics which includes the study of the basic elements of algebra and trigonometry.

The conventional approach to the study of mathematics was defined as that approach which has been used during the part twenty-five years in many of our secondary schools, colleges and universities.

The subject matter which is taught in the Introductory College Mathematics Course could also be developed using vectors or set theory. The approach which is used could be an important factor in the design of the mathematics curriculum.

Summary

This study was an experimental study. The sample was chosen from a group of freshman students who met the following standards.

1) They were freshmen who were enrolled in their first college mathematics course.

2) They were graduates of a high school in the United States during the spring of 1963.

3) These students had chosen mathematics, pre-engineering, or pre-professional work as their major field of study.

4) They had taken the American College Testing Program prior to their enrollment at Central State College.

The population was defined to be the set of all freshman students at Central State College who were enrolled in the Introductory College Mathematics Course during the fall semester of 1963.

The sample consisted of 72 students. This sample was divided into three sub-groups of 24 students each. One of the groups studied the course material using the conventional approach. The second group used the vector approach, and the third group studied the material utilizing the theory of sets.

The duration of the experiment was one semester.

The students were tested before and after the period of instruction in order to note any changes which could be attributed to the particular approach used.

The pre-test sequence included the American Testing Program, the Cooperative Algebra Test (Form Z), the Rassmussen Trigonometry Test (Form A), and the Sequential Test of Educational Progress in Mathematics (Form 1A).

The ACT Program was administered to the students during their senior year in high school. The results of this testing program are used for placement purposes in the colleges and universities of Oklahoma.

The post-test sequence consisted of the Cooperative Algebra Test (Form Y), the Rassmussen Trigonometry Test (Form B), and the Sequential Test of Educational Progress in Mathematics (Form 1B).

The criteria which were used in making the choice regarding the most effective approach are as follows: 1) algebraic manipulation; 2) trigonometric achievement; and 3) general problem solving ability.

The Cooperative Algebra Test measured the student's ability to handle basic algebraic manipulations. The Rassmussen Trigonometry Test was used to measure the student's achievement in trigonometry. The Sequential Test of Educational Progress in Mathematics was used to measure the student's ability in general problem solving.

The instructors for the three groups were chosen on the basis of interest in the study, background with regard to mathematical preparation and teaching experience, and a declared interest in teaching the course using one of the three approaches.

In treating the data statistically, the t-test was utilized to determine whether or not there was a significant difference between the three groups with respect to the scores achieved on the pre-test sequence. No significant differences were found at the five per cent level of significance.

On the basis of evidence obtained, it was assumed that the sample was actually a random sample from the defined population and that any deviations would in fact be due only to chance factors.

Operating on this assumption, the data was treated using the analysis of covariance.

Conclusions

Three hypotheses were tested in this experimental study. They were:

H_1 - There will be no significant difference in the mean scores of the three groups at the end of the experiment with regard to algebraic manipulations at the five per cent level of significance.

H_2 - There will be no significant difference in the mean scores of the three groups at the end of the experiment with regard to achievement in trigonometry at the five per cent level of significance.

H_3 - There will be no significant difference in the mean scores of the three groups at the end of the experiment with regard to ability in general problem solving at the five per cent level of significance.

In each case, a value of $F = 3.14$ was needed in order to reject the hypotheses of no significant difference between the mean scores of the three groups at the five per cent level of significance.

With regard to algebraic manipulation, the F-test indicated an F value of .121.

The hypothesis of no significant differences between the three groups with regard to algebraic manipulation was not rejected.

With regard to achievement in trigonometry, the F-test yielded an F value of 1.98.

The hypothesis of no significant differences between the three groups with regard to trigonometric achievement was not rejected.

With regard to general problem solving ability, the F-test yielded an F value of .505.

The hypothesis of no significant differences between the three groups with regard to general problem solving ability was not rejected.

One of the limitations on this study was the nature of the testing instruments. Each test favored the conventional group. It was not possible to use a test which could measure the student's ability to manipulate vectors, or sets, as this test would be meaningless for the students using the conventional approach.

Keeping this in mind, and realizing that there were no significant differences between the three groups with regard to algebraic manipulation, trigonometric achievement, and general problem solving ability, the following conclusions were reached:

1) It appears that the students who study the Introductory College Mathematics Course at Central State College using the vector

approach or the set theory approach will have gained more subject-matter wise, than those students who are instructed using the conventional approach.

2) It appears that the students studying algebra and trigonometry using vectors or sets score comparably with the group using the conventional approach with regard to basic algebraic manipulation.

3) It appears that the study of algebra and trigonometry using vectors or sets does not limit the student in achieving in trigonometry.

4) With respect to general problem solving ability, no significant differences were noted. It appears that the approach to be used in this case would be arbitrary.

5) The fact that there were no significant differences between the two groups using the vector and set theory approach could lead to some question as to which one is more effective. At this time, it appears that the choice again is an arbitrary one. The choice would depend to a certain extent upon the curricular design into which the approach would be incorporated.

Implications

While the sample for this particular experiment was a small one, and the population was chosen from a single institution of higher learning, some benefits can undoubtedly be derived from it.

A great deal of research is needed in order to find better ways and means to teach mathematics at the secondary school level and in our colleges and universities.

This study could serve as a pilot study for further studies which could culminate in an ultimate solution to this and similar problems.

More and more advanced mathematical ideas are being introduced into the secondary schools and into the first two years of the college program. How much mathematics can be taught, and to whom, will be questions which will need to be answered.

Research studies such as this one could be beneficial in this respect.

Recommendations

Several aspects of this study suggest areas for further research. Some of these are:

1) There is a definite need in the field of mathematics for standardized tests to measure the various aspects of student achievement in mathematics. These tests need to be structured so as not to be biased toward any one approach in the study of mathematics.

2) There is a definite need for this experiment to be replicated in a number of colleges and universities with a larger sample. A more rigorous testing program should be adhered to in order to evaluate finer differences in student growth.

3) A study of this type should be followed by a study analyzing the student's achievement in a subsequent course. Many of these students will take a course in calculus. How they achieve in this course after having experienced the vector or set theory approach could be significant.

4) More inter-departmental cooperation is definitely needed in order to design courses such as this one, using conventional means, vectors and/or sets, which can be used by scientists, social scientists, psychologists, business students, as well as by a mathematics major.

5) A closer look at the mathematics curriculum is needed. It is possible that the entire four year program should be rewritten to overcome duplication, eliminate unnecessary areas of instruction, and improve instructional techniques.

BIBLIOGRAPHY

BIBLIOGRAPHY

Allendoerfer, Carl B., and Cletus O. Oakley. Principles of Mathematics. New York: McGraw-Hill, 1963.

American College Testing Program. Using the ACT Scores on Your Campus. Chicago; Science Research Associates, 1960.

Baker, I.C. "Not What Can be Taught, but What Should be Taught." The American Mathematical Monthly, LXIX (May, 1962), 426-28.

Begle, E. G. "The School Mathematics Study Group." Mathematics Teacher, LI (December, 1958), 616-18.

________. "A Study of Mathematical Abilities." American Mathematical Monthly, LXIX (December, 1962), 1000-02.

Bernstein, Allen L. "Recent Research in Mathematics Teaching." Michigan Education Journal, XL (November, 1962), 262-73.

Birnbaum, Z. W. Introduction to Probability and Mathematical Statistics. New York: Harper and Brothers, 1962.

Buck, R. C. "The CUPM Catalog Survey." The American Mathematical Monthly, LXIX (April, 1962), 304-06.

Butler, Charles H., and Lynwood F. Wren. The Teaching of Secondary Mathematics. New York: McGraw-Hill, 1960.

Copeland, Arthur H., Sr. Geometry, Algebra, and Trigonometry by Vector Methods. New York: MacMillan, 1962.

Christilles, W. E. "The Need for an Emphasis Upon Creative Development in the Teaching of Mathematics." The American Mathematical Monthly, LXIX (April, 1962), 308-09.

College Entrance Examination Board. A Description of the College Board Achievement Tests. Princeton: College Entrance Examination Board, 1962.

Cook, Desmond L. The Relation of Possible Hawthorne Effect Components to Stages of Experimental Investigation. Columbus: Ohio State University, 1962.

Cunninghame-Greene, Raymond. "Industrial Mathematics." The Mathematical Gazette, XLVI (May, 1962), 106-10.

Dixon, Wilfrid J., and Frank J. Massey, Jr. Introduction to Statistical Analysis. New York: McGraw-Hill, 1957.

Drew, J. William. "The Mathematics Curriculum in the Small College." The American Mathematical Monthly, LXIX (August-September, 1962), 664.

Eves, Howard. *An Introduction to the History of Mathematics.* New York: Rinehart and Company, Inc., 1960.

Fallon, Carlos. "The Teaching of Mathematics for Management Careers." *The American Mathematical Monthly,* LXIX (February, 1962), 148-9.

Fisher, Robert C., and Allen D. Ziebur. *Integrated Algebra and Trigonometry.* Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1960.

Freund, John E. *Modern Elementary Statistics.* Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1960.

Garrett, Henry E. *Statistics in Psychology and Education.* New York: Longmans, Green and Company, 1958.

Good, Carter V. *Dictionary of Education.* New York: McGraw-Hill, 1945.

Grossmarc, George. "Advanced Placement Mathematics - For Whom?" *Mathematics Teacher,* LV (November, 1962), 560-66.

Hoel, Paul G. *Introduction to Mathematical Statistics.* New York: Wiley, 1957.

Huntsberger, David V. *Elements of Statistical Inference.* Boston: Allyn and Bacon, Inc., 1961.

May, Kenneth O. "Small Versus Large Classes." *The American Mathematical Monthly,* LXIX (May, 1962), 433-34.

Milligan, Merle. "An Inquiry into the Selection of Subject Matter Content for College Freshman Mathematics." (Unpub. Ed.D. dissertation, Oklahoma State University, 1961).

Murray, F. J. "Education for Applied Mathematics," *The American Mathematical Monthly,* LXIX (May, 1962), 347-57.

Neeley, J. H. "What to do About a New Kind of Freshman." *American Mathematical Monthly,* LXVI (September, 1959), 584-86.

__________. "A Generation of High School Calculus." *American Mathematical Monthly,* LXVIII (December, 1961), 1004-05.

"Recommendations on the Undergraduate Program for Engineers and Physicists." *The American Mathematical Monthly,* LXIX (May, 1962), 339-47.

Remmers, Herman Henry, N.L. Gage, and J. F. Rummel. *A Practical Introduction to Measurement and Evaluation.* New York: Harper and Brothers, 1960.

Research Design and Analysis. Second Annual Phi Delta Kappa Symposium on Educational Research. Bloomington, Indiana: Phi Delta Kappa, Inc., 1961.

Rosenbloom, Paul C. "Implications for the Colleges of the New School Programs." The American Mathematical Monthly, LXIX (April, 1962), 255-59.

_________. "Large Scale Experimentation with Mathematics Curriculum." Research Design and Analysis. Eds. R. O. Collier and S. M. Elam. Second Annual Phi Delta Kappa Symposium on Educational Research, ch. 1 B, 1961, pp. 5-10.

Sanford, Nevitt. The American College. New York: John Wiley and Sons, Inc., 1962.

_________. "The Successful College." NEA Journal, (November, 1962), 11.

Smith, Robert E. "A Desk, Some Chairs, and a Blackboard." The American Mathematical Monthly, LXIX (August-September, 1962), 658-63.

Snedecor, George W. Statistical Methods. Ames: Iowa State College Press, 1959.

Steel, Robert G. D., and James H. Torrie. Principles and Procedures of Statistics. New York: McGraw-Hill, 1960.

Steller, E. T. "Should There Be A Choice of Questions in an Examination Paper?" The Mathematical Gazette, XLVI (February, 1962), 13-14.

Twenty-Sixth Yearbook. Evaluation in Mathematics. Washington: National Council of Teachers of Mathematics, 1961.

Ulmer, Gilbert, and Donald Eillotsom. "The Teaching of Mathematics at the College and University Level." Review of Educational Research, XXXI (June, 1961), 314-22.

Van Dalen, Deobold B. Understanding Educational Research. New York: McGraw-Hill, 1962.

Van Engen, Henry. "Plans for Reorganization of College Preparatory Mathematics." School Science and Mathematics, LVIII (April, 1958), 279.

Watson, Goodwin. "What Do We Know About Learning?" NEA Journal, LII (March, 1963), 20-22.

APPENDIX

APPENDIX

Appendix A

A Questionnaire

MATHEMATICS QUESTIONNAIRE

Name____ Age________________
(Last) (First) (MI)

High School where Graduated________________ Yr. of Graduation_________

Home Town__________________________________ Size of High School________
(City) (State) Graduating Class

List the mathematics courses you took in high school:

Course	No. of Semesters
1.________________________________	________________
2.________________________________	________________
3.________________________________	________________
4.________________________________	________________
5.________________________________	________________

List the mathematics courses offered by your high school but not taken by you:

1.________________________________

2.________________________________

What is your class standing at Central State College?________________
(i.e., First semester freshman, second semester freshman, etc.)

At the present time, what is your intended major(s)?
(i.e., Mathematics, Physics, Pre-engineering, etc.)

1.________________________________

2.________________________________

Intended Minor(s)?

1.________________________________

2.________________________________

List any mathematics courses you have taken at Central State College prior to this course:

1.________________________________

2.________________________________

APPENDIX

Appendix B

Raw Scores

TABLE XIV

RAW SCORES: STUDENT SCORES ON COOPERATIVE ALGEBRA TEST

Student No.	Conventional		Vectors		Sets	
	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
1	51	58	46	45	33	17
2	46	32	31	37	24	38
3	43	47	38	31	26	34
4	36	27	41	45	48	48
5	42	42	22	18	26	32
6	47	53	39	42	24	31
7	33	34	28	32	31	34
8	26	26	15	25	34	35
9	45	42	17	21	38	40
10	38	46	34	37	36	37
11	23	20	35	44	52	50
12	24	31	31	22	22	19
13	28	32	33	24	26	40
14	28	34	36	32	28	26
15	25	24	35	39	32	24
16	40	40	15	23	29	34
17	22	26	36	41	32	35
18	36	33	42	33	17	27
19	29	31	24	25	31	38
20	16	28	14	20	42	37
21	34	39	42	48	32	40
22	33	30	46	34	22	20
23	15	27	22	27	30	35
24	26	22	40	36	27	32
N =	24	24	24	24	24	24

TABLE XV

RAW SCORES: STUDENT SCORES ON RASSMUSSEN TRIGONOMETRY TEST

Student No.	Conventional		Vectors		Sets	
	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
1	63	83	54	82	56	70
2	--	40	61	72	--	60
3	61	71	--	63	54	58
4	45	58	65	79	--	57
5	--	39	--	54	62	58
6	74	79	64	69	--	62
7	--	65	--	62	59	68
8	--	48	--	51	--	48
9	52	55	51	57	49	52
10	56	67	53	61	55	54
11	--	57	69	75	68	82
12	--	48	42	65	54	50
13	--	59	--	45	--	46
14	--	63	59	68	45	57
15	57	53	--	63	48	57
16	54	46	--	50	--	59
17	50	63	46	60	--	49
18	--	49	57	69	--	59
19	39	54	46	61	65	72
20	56	68	--	50	59	63
21	49	55	--	57	58	46
22	43	59	45	64	--	49
23	46	64	--	53	49	67
24	--	71	62	65	--	41
N =	14	24	14	24	14	24

TABLE XVI

RAW SCORES: STUDENT SCORES ON STEP TEST IN MATHEMATICS

Student	Conventional		Vectors		Sets	
No.	Pre-test	Post-test	Pre-test	Post-test	Pre-test	Post-test
1	38	45	31	27	30	34
2	29	36	32	29	28	32
3	32	31	17	27	19	21
4	19	17	30	38	29	37
5	26	30	17	25	26	31
6	30	38	30	29	24	25
7	23	21	25	35	24	22
8	26	17	19	23	26	33
9	23	35	21	29	25	28
10	38	41	28	26	22	26
11	18	25	33	36	28	39
12	27	24	20	30	25	34
13	21	19	19	18	25	26
14	19	24	23	25	26	27
15	15	20	25	27	21	25
16	29	31	14	28	19	25
17	20	17	22	28	23	28
18	20	25	36	34	17	21
19	21	22	16	17	24	27
20	12	23	16	22	28	22
21	30	29	29	36	24	27
22	23	27	30	30	24	24
23	13	19	26	26	24	28
24	22	24	18	33	25	20
N =	24	24	24	24	24	24

TABLE XVII

RAW SCORES: STUDENT SCORES ON ACT PROGRAM

Student	Mathematics			English			Social Studies			Natural Science			Composite		
No.	Conv.	Vectors	Sets	Conv.	Vectors	Sets	Conv.	Vectors	Sets	Conv.	Vectors	Sets	Conv.	Vectors	Sets
1	25	24	23	22	23	18	28	16	22	31	21	20	29	21	21
2	26	19	13	18	16	20	15	15	22	23	17	28	21	17	21
3	19	25	20	9	23	16	14	20	11	19	26	17	15	24	15
4	17	21	24	16	15	23	25	15	16	20	18	21	20	17	21
5	24	18	14	24	14	18	21	11	16	28	13	23	24	14	18
6	22	28	19	26	18	19	15	15	18	20	20	16	21	21	18
7	23	23	14	17	19	18	15	28	16	17	22	23	18	23	18
8	28	17	23	27	9	18	29	15	20	25	18	18	27	15	20
9	18	18	14	14	20	18	11	15	16	13	22	23	14	19	18
10	24	23	23	24	25	22	25	28	21	23	29	26	22	26	23
11	14	20	28	11	20	17	17	22	16	18	27	24	15	22	22
12	19	24	20	9	20	16	14	15	11	19	23	17	15	21	16
13	20	16	22	18	13	23	15	4	22	18	7	23	18	12	23
14	22	17	23	17	17	25	17	20	22	21	24	20	19	20	23
15	23	20	26	17	9	18	15	11	16	17	12	17	18	13	19
16	21	16	27	18	21	23	29	20	21	30	20	26	25	19	24
17	20	25	18	14	22	20	10	22	15	24	28	18	17	24	18
18	24	25	21	21	19	15	15	25	15	13	28	18	18	24	17
19	19	22	23	17	7	20	14	11	20	19	15	25	17	14	22
20	17	16	18	16	16	20	25	17	15	20	15	20	20	16	18
21	26	18	24	22	16	24	20	15	24	28	22	25	24	18	24
22	22	28	24	19	18	10	20	20	17	19	30	25	20	24	19
23	14	21	21	18	18	20	16	19	24	15	21	18	16	20	21
24	23	22	21	17	18	18	23	20	18	21	28	20	21	20	19
N =	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24

APPENDIX

Appendix C

Scatter Diagrams

TABLE XVIII

SCATTER DIAGRAM: ACT MATHEMATICS SCORES VS. ALGEBRA POST-TEST SCORES

Scores	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60
28-30			/	/		/	/		
25-27		/		////	/	/			/
22-24	/	////	//	/////	//////	//	//	/	
19-21	/		///	///////	///	//	/		
16-18	//	////	//	//	/	/	/		
13-15	/		/	//	//				

N = 72

r = .33

TABLE XIX

SCATTER DIAGRAM: ACT MATHEMATICS SCORES VS. TRIGONOMETRY POST-TEST SCORES

Scores	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84
28-30			/			/	/			/
25-27		/			//	//	//			/
22-24	/		////	//	///	/////	////	///	/	/
19-21		/	///	//	////	//	/	//	//	
16-18			//	/////	////	/	/			
13-15				/	//	//	/			

N = 72

r = .27

TABLE XX

SCATTER DIAGRAM: ACT MATHEMATICS SCORES VS. STEP POST-TEST SCORES

<u>Scores</u>	15-18	19-22	23-26	27-30	31-34	35-38	39-42	43-46
28-30	/			/			/	
25-27			//	///	/	/		/
22-24	/	//	///////	////////	///	///	/	
19-21	/	/////	///	///	///	///		
16-18	//	//	////	///		//		
13-15		//	/	/	//			

N = 72

r = .26

TABLE XXI

SCATTER DIAGRAM: ACT COMPOSITE SCORES VS. ALGEBRA POST-TEST SCORES

Scores	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59
29-32									/
25-28			/		/	/			
21-24	/	//	//	//////	/////	//////	///	//	
17-20		/////	/////	///////////	/////	/	//		
14-16	//	//	///	//		/	/		
10-13					/				

N = 72

r = .38

TABLE XXII

SCATTER DIAGRAM: ACT COMPOSITE SCORES VS. STEP POST-TEST SCORES

Scores	15-18	19-22	23-26	27-30	31-34	35-38	39-42	43-46
29-32								/
25-28	/		//		/			
21-24	/		////	////////////	///	/////	//	
17-20	//	////////	///////	/////	///	//		
13-16	/	//	////	/	///	/		
9-12				/				

N = 72

r = .22

VITA

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Doctor of Education

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